Home Exercise 2: Solving LPs graphically

For each of the following, determine the direction in which the objective function increases:

(1) $z = 4x_1 - x_2$

(2) $z = -x_1 + 2x_2$

(3) $z = -x_1 - 3x_2$

Hint. Think of the gradient of the objective function.

Home Exercise 2:

The functions's increase is in direction of their gradients

$$\nabla z = \left(\frac{\partial z}{\partial x_1}, \frac{\partial z}{\partial x_2}\right)$$

(1) $z = 4x_1 - x_2$

$$\Rightarrow \nabla z = \left(\frac{\partial}{\partial x_1}(4x_1 - x_2), \frac{\partial}{\partial x_2}(4x_1 - x_2)\right) = (4, -1)$$

The vector $(4, -1)$ is direction in which the objective function increases

(2) $z = -x_1 + 2x_2$

$$\Rightarrow \nabla z = \left(\frac{\partial}{\partial x_1}(-x_1 + 2x_2), \frac{\partial}{\partial x_2}(-x_1 + 2x_2)\right) = (-1, 2)$$

The vector $(-1, 2)$ is direction in which the objective function increases

(3) $z = -x_1 - 3x_2$

$$\Rightarrow \nabla z = \left(\frac{\partial}{\partial x_1}(-x_1 - 3x_2), \frac{\partial}{\partial x_2}(-x_1 - 3x_2)\right) = (-1, -3)$$

The vector $(-1, -3)$ is direction in which the objective function increases