# Exercise class 11

**Learning Objectives:**

- Newton's method for constrained problems
- Barrier method and interior point method

## Demo 1: Newton-Raphson for a system of nonlinear equations

Consider the system of equations:

$$\begin{aligned}
&x_1^2 + x_2^2 - 4x_3 = 0 \\
&x_1^2 + x_3^2 = \frac{1}{4} \\
&x_1^2 + x_2^2 + x_3^2 = 1
\end{aligned}$$

Compute the first two iterations of the Newton-Raphson method by hand, and then implement in Julia to find a solution for the system. Consider $x^0 = [1, 1, 1]^\top$ and $\varepsilon = 0.001$.

## Demo 2: Newton for a system of linear equations

$$\begin{aligned} \min.\ & x_1 + x_2 \\ \text{s.t. } & 3x_1 + x_2 \leq 18 \\ & x_2 \leq 6 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

a) Write the Newton system for this problem, with intial point $x^0 = (2, 4)$, $\rho^0 = 10$ and $\beta = 0.5$ .

b) Perform one iteration of interior point method.

   **Hint.** Use Julia with the \ (backslash) operator.

## Problem 1: Nonlinear optimisation

$$\begin{aligned} \min.\ & x_1^4 + 2x_2^2 \\ \text{s.t. } & x_1 + 2x_2 = 10 \end{aligned}$$

Solve the optimisation problem using Newton method with equality constraint by hand for one iteration and fully solve in Julia. With starting point $x_0 = (2, 4)$ and tolerance 0.00001.

## Problem 2: Barrier method

$$\begin{aligned} \min.\ & (x + 4)^2 \\ \text{s.t. } & x \geq 0 \end{aligned}$$

a) Write the optimality conditions for $\rho = 2,\ 1,\ 0.5$ and find the optimal $x$.

b) Show that as $\rho \to 0$, $x(\rho) \to x^*$.

## Problem 3: Barrier method for constrained problems (Interior point method)

$$\begin{aligned} \min.\ & x_1^2 + 2x_2^2 \\ \text{s.t. } & 2x_1 + x_2 \leq 9 \\ & x_1 + 2x_2 = 10 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

a) Formulate the barrier problem.

b) Form the newton system for $\rho^0 = 5$, $\beta = 0.5$ and initial solution (2,4).