Home Exercise 1: Formulate an LP problem

Bev's Beverage Products is considering producing a wine cooler that would be a blend of a white wine, a rosé wine, and fruit juice. To meet taste specifications, the wine cooler must consist of at least 50% white wine, at least 20% and no more than 30% rosé, and exactly 20% fruit juice. Bev purchases the wine from local wineries and the fruit juice from a processing plant in California. For the current production period, 10,000 litres of white wine and 8,000 litres of rosé wine can be purchased; and unlimited amount of fruit juice can be ordered. The costs for the wine are $1.00 per litre for the white and $1.50 per litre for the rosé; the fruit juice can be purchased for $0.50 per litre. Bev's Beverages can sell all of the wine cooler they can produce for $2.50 per litre.

Formulate a linear program to determine the blend of the three ingredients that will maximise total profit contribution. Solve the linear program to determine the number of litres of each ingredient Bev should purchase and the total profit contribution they will realise from this blend.

Let's carefully check each conditions

- a wine cooler that would be a blend of a white wine, a rose wine, and fruit juice.

=> Let's call white wine percentage as w, rose wine percentage as r and fruit juice percentage as f**.** Wine cooler must consist of at least 50% white wine, at least 20% and no more than 30% rose, and exactly 20% fruit juice

=> The constraints are:

- **w >= 0.5,**
- **r >= 0.2, r <= 0.3**
- **f = 0.2**

- For the current production period, 10,000 litres of white wine and 8,000 litres of rose wine can be purchased; and unlimited amount of fruit juice can be ordered.

=> Let's call white wine liters as W, rose wine liters as R and fruit juice liters as F. **The constraints are:**

- **W <= 10000**
- **R <= 8000**

- The costs for the wine are $1.00 per litre for the white and $1.50 per litre for the rose; the fruit juice can be purchased for $0.50 per liter. Bev's Beverages can sell all of the wine cooler they can produce for $2.50 per liter.\

=> Let's call the total liter of production as T liters, so **W + R + F = T**

The total gross profits will be **G = 2.5T $ or G = 2.5(W + R + F) $**
The cost of buying the ingredients are **C = 1W + 1.5R + 0.5F $**
=> Total profit contribution: **P = G - C = 1.5W + 1R + 2F $**

Question 1: Determine the blend of the three ingredients that will maximize total profit contribution?
=> Objective function: **maximize P = 1.5W + 1R + 2F $**
From the blending constraints, we can analyze as follows:

- w >= 0.5 => W/T >= 0.5 => W/(W+R+F) >= 0.5 => W >= 0.5(W+R+F)
  => Constraint: 0.5W - 0.5R - 0.5F >= 0
- r >= 0.2
  => Constraint: -0.2W + 0.8R - 0.2F >= 0
- r <= 0.3
  => Constraint: -0.3W + 0.7R - 0.3F <= 0
- f = 0.2
  => Constraint: -0.2W - 0.2R + 0.8F = 0

Solve the optimization problem

maximize 1.5W + 1R + 2F
So that W <= 10000
R <= 8000
0.5W - 0.5R - 0.5F >= 0
-0.2W + 0.8R - 0.2F >= 0
-0.3W + 0.7R - 0.3F <= 0
-0.2W - 0.2R + 0.8F = 0

I used Julia to solve the linear program:

```
using JuMP, Cbc # modelling language and solver
d1 = Model(with_optimizer(Cbc.Optimizer,logLevel = 0)) # create the model, select the solver
@variable(d1, x[1:3] >= 0) #creates the non-negative variables x1, x2 and x3
@constraint(d1, x[1] <= 10000)
@constraint(d1, x[2] <= 8000)
@constraint(d1, 0.5 * x[1] - 0.5 * x[2] - 0.5 * x[3] >= 0)
@constraint(d1, -0.2 * x[1] + 0.8 * x[2] - 0.2 * x[3] >= 0)
@constraint(d1, -0.3 * x[1] + 0.7 * x[2] - 0.3 * x[3] <= 0)
@constraint(d1, -0.2 * x[1] - 0.2 * x[2] + 0.8 * x[3] == 0)

@objective(d1, Max, 1.5 * x[1] + 1 * x[2] + 2 * x[3]) # declare the objective function
optimize!(d1) # solve the optimization problem

# printing out the solution
x_value = value.(x)
print("Optimal values: $(x_value), \nOptimal objective: $(objective_value(d1))\n")
```

```
Optimal values: [10000.0, 6000.0, 4000.0],
Optimal objective: 29000.0
```

The blend of the three ingredients that will maximize total profit contribution:
T = 10000 + 6000 + 4000 = 20000 liters
Composition of the blend

- White wine: 10000/20000 = 50%
- Rose wine: 6000/20000 = 30%
- Fruit juice: 4000/20000 = 20%

Question 2: Determine the number of liters of each ingredient Bev should purchase?
From the result above, amount of liters of each ingredient Bev should purchase is:

- White wine: 10000 liters
- Rose wine: 6000 liters
- Fruit juice: 4000 liters

Question 3: Determine the total profit contribution they will realize from this blend:
From the result provided by Julia in optimal objective, the total profit contribution would be 29000$
Profit contribution of each ingredient:

- White wine: 10000 * 1.5 / 29000 = 51.724 %
- Rose wine: 6000 * 1 / 29000 = 20.689%
- Fruit juice: 4000 * 2 / 29000 = 27.586%

The Julia code annex:

```
using JuMP, Cbc # modelling language and solver
d1 = Model(with_optimizer(Cbc.Optimizer,logLevel = 0)) # create the model

@variable(d1, x[1:3] >= 0) #creates the non-negative variables x1, x2 and x3
@constraint(d1, x[1] <= 10000)
@constraint(d1, x[2] <= 8000)
@constraint(d1, 0.5 * x[1] - 0.5 * x[2] - 0.5 * x[3] >= 0)
@constraint(d1, -0.2 * x[1] + 0.8 * x[2] - 0.2 * x[3] >= 0)
@constraint(d1, -0.3 * x[1] + 0.7 * x[2] - 0.3 * x[3] <= 0)
@constraint(d1, -0.2 * x[1] - 0.2 * x[2] + 0.8 * x[3] == 0)
@objective(d1, Max, 1.5 * x[1] + 1 * x[2] + 2 * x[3]) # declare the objective function

optimize!(d1) # solve the optimization problem
# printing out the solution
x_value = value.(x)
print("Optimal values: $(x_value), \nOptimal objective: $(objective_value(d1))\n")
```