Exercise class 2

Learning Objectives:

- Formulation of linear programming problems
- Solving linear optimisation problems graphically and with Julia

Demo 1: Solving LP problems graphically

Solve the following linear programming problems graphically:

a)

$$
\begin{array}{llcll}
\min. & x_1 & + & x_2 & \\
\\
\text{s.t.} & 2x_1 & + & 2x_2 & \geq 4 \\
& x_1 & & & \leq 3 \\
& & & x_2 & \geq 1 \\
& x_1 \geq 0, & & x_2 \geq 0 &
\end{array}
$$

b) Example 2.2-1 Reddy Mikks model from Hamdy A. Taha's book Operations Research: An Introduction.

$$
\begin{array}{llcll}
\max. & 5x_1 & + & 4x_2 & \\
\\
\text{s.t.} & 6x_1 & + & 4x_2 & \leq 24 \\
& x_1 & + & 2x_2 & \leq 6 \\
& -x_1 & + & x_2 & \leq 1 \\
& & & x_2 & \leq 2 \\
& x_1 \geq 0, & & x_2 \geq 0 &
\end{array}
$$

Demo 2: Algebraic form of LPs

MG Auto has three source plants i and two major distribution centers j. The quarterly capacities of the three plants are a_i, and the demands at the two distribution centers for the same period are b_j. The trucking company in charge of transporting the cars charges c_{ij} per unit. The amount shipped is x_{ij} is the amount of cars shipped from source i to destination j.

The objective of the model is to minimise the total transportation cost while satisfying all the supply and demand restrictions.

Formulate the algebraic form of the LP.

Problem 1: Solving LP problems graphically

Solve the following LP graphically and using JuMP.

a)

$$
\begin{array}{llcll}
\max. & 8x_1 & + & 3x_2 & \\
\\
\text{s.t.} & 3x_1 & + & 8x_2 & \leq 48 \\
& 4x_1 & + & 3x_2 & \leq 21 \\
& x_1 & - & 2x_2 & \leq 1 \\
& x_1 \geq 0, & & x_2 \geq 0 &
\end{array}
$$

b)

$$\begin{array}{llcll} \min. & 6x_1 & + & 5x_2 & \\ \text{s.t.} & 3x_1 & + & 5x_2 & \geq 15 \\ & 7x_1 & + & 2x_2 & \leq 14 \\ & -x_1 & + & 3x_2 & \leq 9 \\ & x_1 \geq 0, & & x_2 \geq 0 & \end{array}$$

Problem 2: Solving LPs graphically

Recall Demo 2 from exercise class 1: Matti is a farmer and wants to decide how many acres of rye and wheat to plant for the coming year. Each acre of wheat produces 25 loads of wheat and requires 10 hours of labour per week. An acre of rye produces 10 loads of rye and requires 4 hours of labour per week. The wheat sells at €4 per load and the rye €3 per load. Matti has 7 acres of farmland available and 40 hours of labour per week. Government regulations require that at least 30 loads of rye is produced during a given year. Let x_1 be the number of acres of wheat planted, and x_2 be the number of acres of rye planted. Then we have the following LP:

$$\max.\ 100x_1 + 30x_2 \tag{1}$$
$$x_1 + x_2 \leq 7 \tag{2}$$
$$10x_1 + 4x_2 \leq 40 \tag{3}$$
$$10x_2 \geq 30 \tag{4}$$
$$x_1, x_2 \geq 0 \tag{5}$$

Solve this graphically and answer the following questions:

(a) Is $x_1 = 2,\ x_2 = 3$ in the feasible region?

(b) Is $x_1 = 4,\ x_2 = 3$ in the feasible region?

(c) Is $x_1 = 3,\ x_2 = 2$ in the feasible region?

(d) Which of the constraints (2)-(4) are binding?

Problem 3: Solving LPs graphically

Aalto Chemical manufactures three chemicals: A, B, and C. These chemicals are produced via two production processes: 1 and 2. Running process 1 for an hour costs €4 and yields 3 units of A, 1 of B and 1 of C. Running process 2 for an hour costs €1 and produces 1 unit of A and one unit of B. To meet customer demands, at least 10 units of A, 5 of B and 3 units of C must be produced daily. Graphically determine a daily production plan that minimizes the cost of meeting Aalto Chemical's daily demands.

Hint. The decision variables should be the number of hours of process 1 and number of hours of process 2.

Problem 4: Formulate LP and solve graphically

Happy Farms uses at least 800 grams of special feed daily. The special feed is a mixture of corn and soybean meal with the following compositions:

Feedstuff	Gram per gram of feedstuff		Cost (€/g)
	Protein	Fiber	
Corn	0.09	0.02	0.30
Soybean meal	0.60	0.06	0.90

The dietary requirements of the special feed are at least 30% protein and at most 5% fiber. The aim is to formulate the LP and solve graphically the daily minimum cost-feed mix.

Problem 5: Formulate & solve transportation problem

A textile company makes blouses in two factories, one in San Diego and another in Seattle. The company sells their products through three vendors located in New York, Chicago, and Miami. The table below summarises the values for supply and demand by location and how much it costs to transport one unit from the factories to the vendors.

	Vendors			Supply
	3-New York	**4-Chicago**	**5-Miami**	
1-Seattle	2.5	1.7	1.8	350
2-San Diego	2.5	1.8	1.4	600
Demand	325	300	275	

a) Formulate the LP model that minimises the transportation costs of the company

b) Consider the following limitations on capacities of the arcs between the origins and destinations in the table below. Formulate the LP with the additional restrictions.

Capacities	3-New York	4-Chicago	5-Miami
1-Seattle	150	200	150
2-San Diego	200	250	200

c) Now consider that the network has a distribution center in Pittsburgh for the products going to the vendors in New York and Chicago. The table below shows the unit transportation cost between Pittsburgh and the factories and vendors. Alter the formulation of the problem to include the distribution center.

	1-Seattle	2-San Diego	3-New York	4-Chicago
Pittsburgh	2.0	1.8	0.5	0.7

d) The company now sells two products: blouses and trousers. The table below shows the supply and demand of the two products. Modify the LP to consider the two products in addition to the previous (b)-(c) restrictions.

Supply	1-Seattle	2-San Diego
Blouses	250	150
Trousers	200	300

Demand	3-New York	4-Chicago	5-Miami
Blouses	125	100	175
Trousers	200	200	100

e) Formulate the complete LP model using algebraic language (summations etc.) to consider all parts (a)-(d) simultaneously.

Home Exercise 2: Solving LPs graphically

For each of the following, determine the direction in which the objective function increases:

(1) $z = 4x_1 - x_2$

(2) $z = -x_1 + 2x_2$

(3) $z = -x_1 - 3x_2$

Hint. Think of the gradient of the objective function.