Exercise class 4

Learning Objectives:

- Simplex method: special cases

Demo 1: The simplex M-method

Solve the linear problem using the M-method.

$$\begin{aligned} \min. \quad & 4x_1 + 6x_2 \\ \text{s.t.} \quad & x_1 + x_2 \geq 5, \\ & 3x_1 + 8x_2 \geq 24, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

Demo 2: The simplex 2-Phase method

Solve the linear problem using the 2-Phase method.

$$\begin{aligned} \min. \quad & 4x_1 + 6x_2 \\ \text{s.t.} \quad & x_1 + x_2 \geq 5, \\ & 3x_1 + 8x_2 \geq 24, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

Problem 1: M-method

Solve the linear problem using the M-method.

$$\begin{aligned} \max. \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_1 + x_2 = 7, \\ & x_1 + 4x_2 = 16, \\ & 3x_1 + 2x_2 = 18, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

Problem 2: 2-Phase method

Solve the linear problem using 2 phase method.

$$\begin{aligned} \max. \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_1 + x_2 = 7, \\ & x_1 + 4x_2 = 16, \\ & 3x_1 + 2x_2 = 18, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

Problem 3: M-method formulation

Consider the following set of constraints:

$$-2x_1 + 3x_2 = 3 \tag{1}$$
$$4x_1 + 5x_2 \geq 10 \tag{2}$$
$$x_1 + 2x_2 \leq 5 \tag{3}$$
$$6x_1 + 7x_2 \leq 3 \tag{4}$$
$$4x_1 + 8x_2 \geq 5 \tag{5}$$
$$x_1,\ x_2 \geq 0 \tag{6}$$

For each of the following problems, develop the z-row after substituting out the artificial variables:

1. Maximise $z = 5x_1 + 6x_2$ subject to (1), (3), and (4).
2. Maximise $z = 2x_1 - 7x_2$ subject to (1), (2), (4), and (5).
3. Minimise $z = 3x_1 + 6x_2$ subject to (3), (4), and (5).
4. Minimise $z = 4x_1 + 6x_2$ subject to (1), (2), and (5).
5. Minimise $z = 3x_1 + 2x_2$ subject to (1) and (5).

Problem 4: 2-Phase method formulation

For each subproblem in Problem 3, write the corresponding Phase 1 objective function.

Problem 5: Unbounded solution

Solve the linear problem using the M-method.

$$\begin{aligned} \max. \quad & 3x_1 + 5x_2 \\ \text{s.t.} \quad & x_1 - 2x_2 \leq 6, \\ & x_1 \leq 10, \\ & x_2 \geq 1, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

Home Exercise 4:

Solve the following linear programming (LP) problems by the graphical method, and answer which problems have/are:

1. a unique optimal solution
2. multiple solutions
3. infeasible
4. unbounded

Problem 1:

$$\begin{aligned} \max. \ & z = x_1 + x_2 \\ \text{s.t.} \ & x_1 + x_2 \leq 4 \\ & x_1 - x_2 \geq 5 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

Problem 2:

$$\begin{aligned} \max. \ & z = 4x_1 + x_2 \\ \text{s.t.} \ & 8x_1 + 2x_2 \leq 16 \\ & 5x_1 + 2x_2 \leq 12 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

Problem 3:

$$\begin{aligned} \max. \ & z = -x_1 + 3x_2 \\ \text{s.t.} \ & x_1 - x_2 \leq 4 \\ & x_1 + 2x_2 \geq 4 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

Problem 4:

$$\begin{aligned} \max. \ & z = 3x_1 + x_2 \\ \text{s.t.} \ & 2x_1 + x_2 \leq 6 \\ & x_1 + 3x_2 \leq 9 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$