Home Exercise 3: The Simplex algorithm

Transform the linear problem into the standard form and solve using the tableau Simplex algorithm.

$$\begin{aligned} \min. \quad & 2x_1 + x_2 \\ \text{s.t.} \quad & 7x_1 - 3x_2 \leq 4, \\ & x_1 + 2x_2 \leq 7, \\ & x_1 \geq 0,\ x_2 \in \mathbf{R}. \end{aligned}$$

Exercise 3: Simplex Algorithm

min. $2x_1 + x_2$ — x_2 is unconstrained variable

s.t. $7x_1 - 3x_2 \leq 4$ ⇒ Let $x_2 = t^+ - t^-$, $t^+, t^- \geq 0$

$x_1 + 2x_2 \leq 7$ Also, standard form will maximize the objective function

$x_1 \geq 0, x_2 \in R$ ⇒ max $-z = -2x_1 + x_2$

⇒ Standard form of the linear problem

max $-z = 2x_1 + t^+ - t^-$

s.t $7x_1 - 3t^+ + 3t^- + s_1 = 4$

$x_1 + 2t^+ - 2t^- + s_2 = 7$

$x_1, t^+, t^-, s_1, s_2 \geq 0$

- The Simplex Algorithm

⇒

basic	x_1	t^+	t^-	s_1	s_2	Sol.	Ratio
$-z$	2	1	-1	0	0	0	
s_1	7	-3	(3)	1	0	4	4/3 ← Pivot row
s_2	1	2	-2	0	1	7	

↑ Pivot column

⇒

basic	x_1	t^+	t^-	s_1	s_2	Sol.	Row operation
$-z$	13/3	0	0	1/3	0	4/3	$(+R_2)$
t^-	7/3	-1	1	1/3	0	4/3	
s_2	17/3	0	0	2/3	1	29/3	$(+2R_2)$

First row doesn't have any negative element anymore ⇒ optimal solution is reached

⇒ Basic variables: $t^- = 4/3$, $s_2 = 29/3$ and $z = -4/3$

Non-basic variables: $x_1 = t^+ = 0$

We have $x_2 = t^+ - t^- = 0 - 4/3 = -4/3$

⇒ Optimal solution: min. $z = -\frac{4}{3}$ at $(x_1, x_2) = \left(0, -\frac{4}{3}\right)$ and $s_2 = \frac{29}{3}$