Home Exercise 4:

Solve the following linear programming (LP) problems by the graphical method, and answer which problems have/are:

1. a unique optimal solution
2. multiple solutions
3. infeasible
4. unbounded

Problem 1:

$$\begin{aligned} \max.\ z &= x_1 + x_2 \\ \text{s.t. } x_1 + x_2 &\leq 4 \\ x_1 - x_2 &\geq 5 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

Problem 2:

$$\begin{aligned} \max.\ z &= 4x_1 + x_2 \\ \text{s.t. } 8x_1 + 2x_2 &\leq 16 \\ 5x_1 + 2x_2 &\leq 12 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

Problem 3:

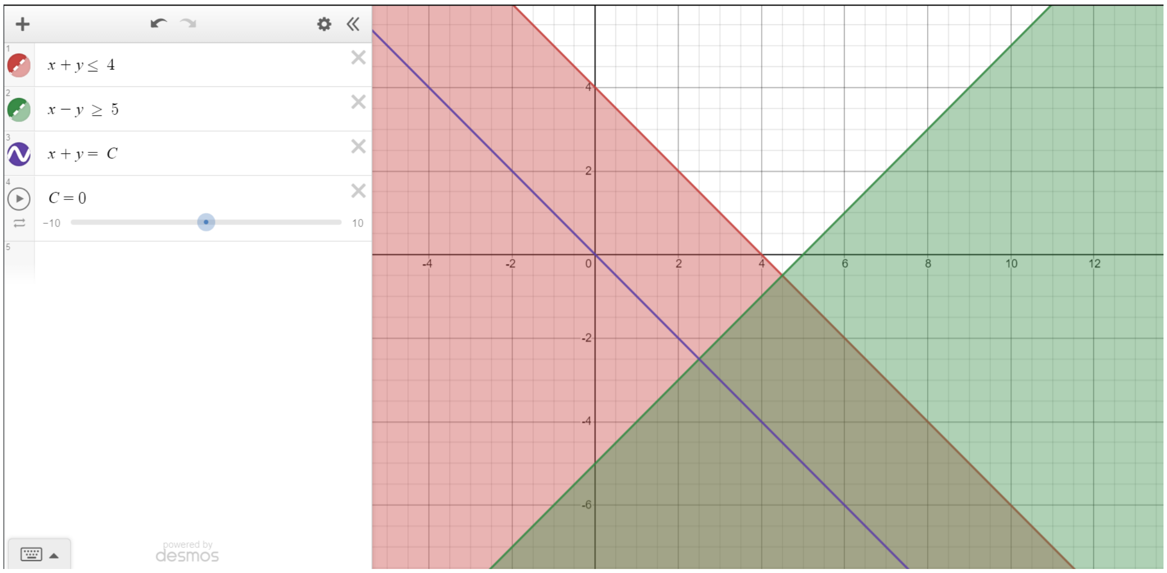
$$\begin{aligned} \max.\ z &= -x_1 + 3x_2 \\ \text{s.t. } x_1 - x_2 &\leq 4 \\ x_1 + 2x_2 &\geq 4 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

Problem 4:

$$\begin{aligned} \max.\ z &= 3x_1 + x_2 \\ \text{s.t. } 2x_1 + x_2 &\leq 6 \\ x_1 + 3x_2 &\leq 9 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

I solve this exercise using the online graphing tool Desmos.
Address at https://www.desmos.com/calculator
Problem 1:

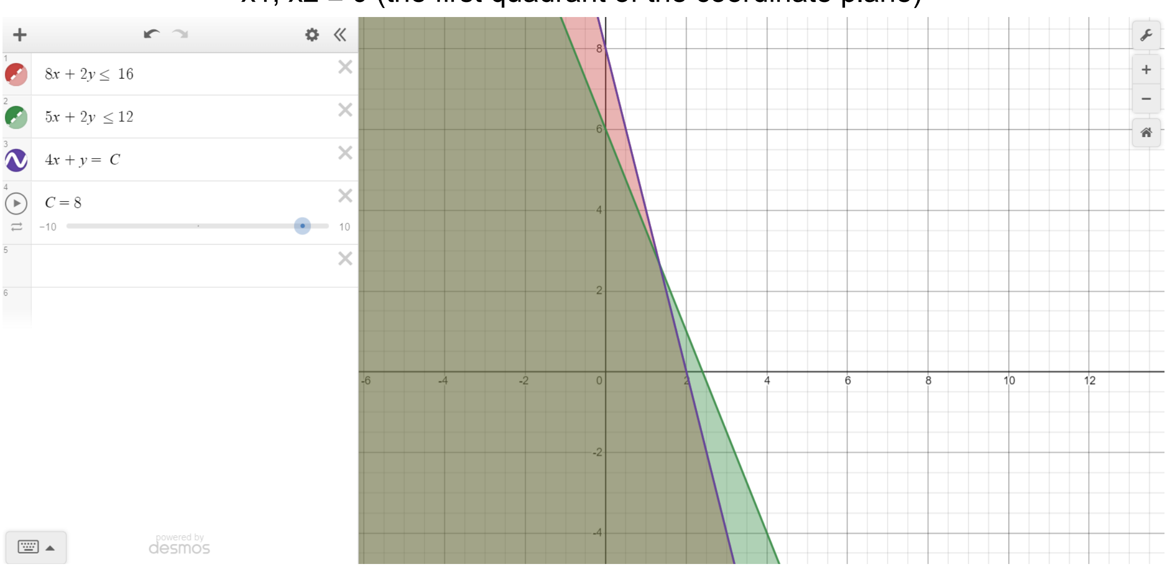
max . z = x1 + x2 (purple line)
s.t. x1 + x2 ≤ 4 (red area)
x1 − x2 ≥ 5 (green area)
x1, x2 ≥ 0 (the first quadrant of the coordinate plane)



Since the common area of red and green area does not lie in the first quadrant, this linear programming problem is infeasible (answer)

Problem 2:

max . z = 4x1 + x2 (purple line)
s.t. 8x1 + 2x2 ≤ 16 (red)
5x1 + 2x2 ≤ 12 (green)
x1, x2 ≥ 0 (the first quadrant of the coordinate plane)



In the graph above, there is a common area shared by the limited regions, which means that there is a possible solution to the maximum objective function. The maximum value of the objective function is obtained at the far right edge of the convex polygon. Since the solution is an edge, there are multiple optimal solutions to this linear programming problem (answer). The maximum value is 8 and the solution of x1 and x2 lies on the line 4x1 + x2 = 8

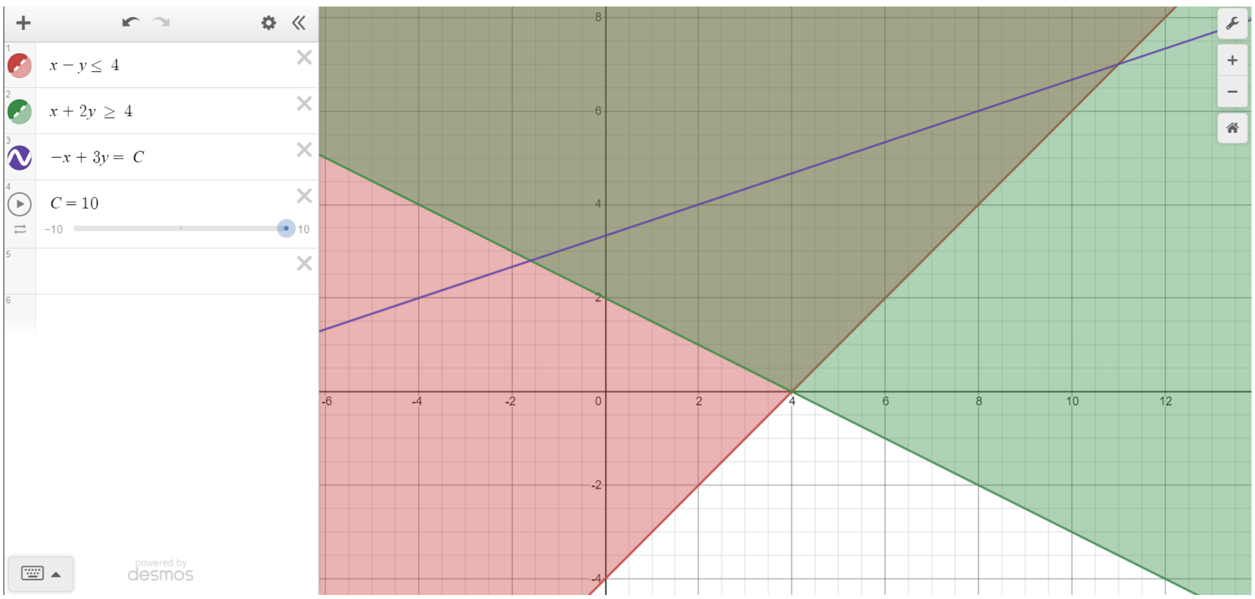
Problem 3:

max . z = -x1 + 3x2 (purple line)

s.t. x1 - x2 ≤ 4 (red)

x1 + 2x2 ≥ 4 (green)

x1, x2 ≥ 0 (the first quadrant of the coordinate plane)



In the graph above, there is a common area shared by the limited regions. The objective function is to maximize the purple line. However, the common area is unbounded towards infinity in the north direction and the objective function will increase when it advances towards the north => This linear programming problem is unbounded (to the north) (answer)

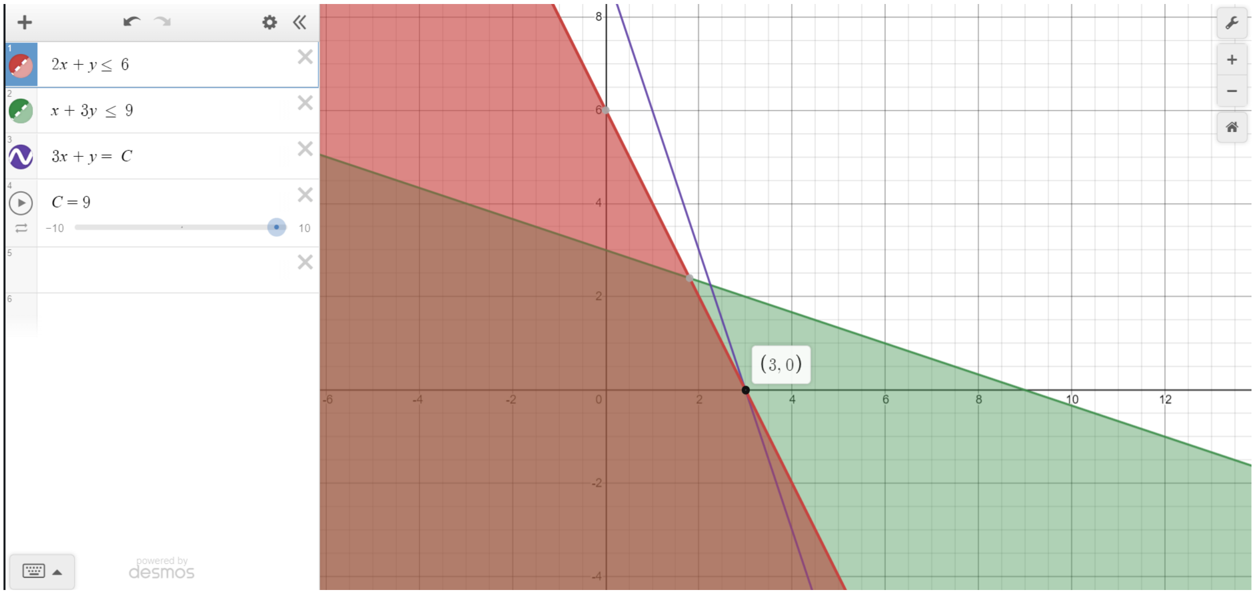
Problem 4:

max . $z = 3x_1 + x_2$ (purple line)

s.t. $2x_1 + x_2 \leq 6$ (red)

$x_1 + 3x_2 \leq 9$ (green)

$x_1, x_2 \geq 0$ (the first quadrant of the coordinate plane)



In the graph above, there is a common area shared by the limited regions. The purple line can be maximized by hitting at exactly one point in the far right of the common area limited by the boundaries. This point coordinate is (3,0) and C = 9 => This linear programming problem has a unique optimal solution, where max z = 9 at x1 = 3 and x2 = 0 (answer)