# Exercise class 3

**Learning Objectives:**

- Algebraic form of LP problems
- The Simplex algorithm

## Demo 1: The simplex algorithm

Transform the linear problem into the standard form and solve it using the tabular Simplex algorithm.

$$\begin{aligned} \max.\ \ & 5x_1 + 4x_2 \\ \text{s.t.}\ \ & x_1 + x_2 \leq 4, \\ & 6x_1 + 3x_2 \leq 18, \\ & x_1 \geq 0,\, x_2 \geq 0. \end{aligned}$$

## Demo 2: Variants in the standard form

Transform the linear problem into minimisation standard form (without changing the problem to maximisation) and solve it using the tabular Simplex algorithm.

$$\begin{aligned} \min. \quad & -2x_1 + 3x_2 \\ \text{s.t.} \quad & 8x_1 + 3x_2 \leq 6, \\ & x_1 + 2x_2 \leq 2, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

## Problem 1: The Simplex algorithm

Transform the linear problem into standard form and solve it using the tabular Simplex algorithm

$$\begin{aligned} \max. \quad & 5x_1 + 6x_2 \\ \text{s.t.} \quad & x_2 \leq 4, \\ & 2x_1 + x_2 \leq 6, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

## Problem 2: Variants in the standard form

Transform the linear problem into the standard form and solve it using the tabular Simplex algorithm.

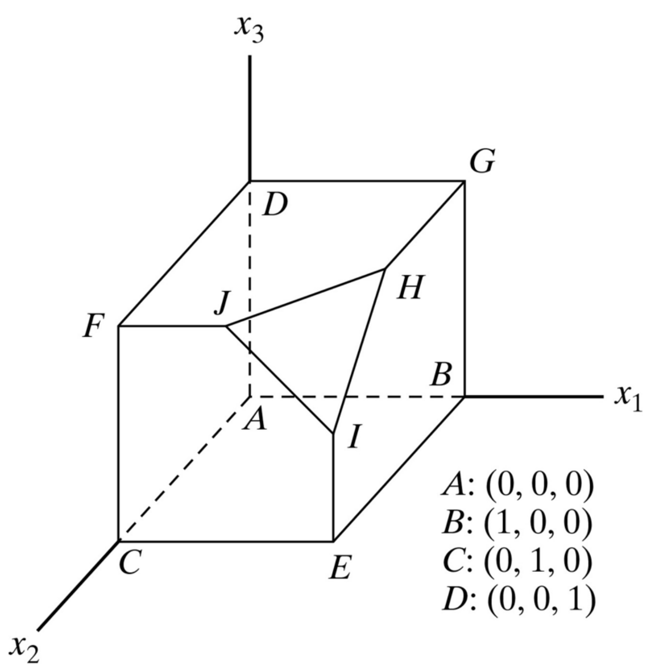
$$\begin{aligned} \min. \quad & 5x_1 - 6x_2 \\ \text{s.t.} \quad & -x_1 + 3x_2 \leq 5, \\ & -x_1 \leq 4, \\ & x_1 \leq 0,\ x_2 \geq 0. \end{aligned}$$

## Problem 3: Variants in the standard form

Transform the linear problem into standard form and solve it using the tabular Simplex algorithm:

$$\begin{aligned} \max. \quad & 4x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 \leq 3, \\ & 3x_1 + x_2 \leq 8, \\ & x_1 \geq 0,\ x_2 \in \mathbf{R}. \end{aligned}$$

## Problem 4: The Simplex algorithm




Consider the three-dimensional LP solution space in the figure , whose feasible extreme points are A,B, ..., and J.

(a) Which of the following pairs of corner points cannot represent successive simplex iterations: $(A,\ B)$, $(H,\ I\ )$, $(E,H\ )$, and $(A,\ I\ )$? Explain why.

(b) Suppose that the simplex iterations start at $A$ and that the optimum occurs at $H$. Indicate whether any of the following paths are not legitimate for the simplex algorithm, and state the reason.

(i) $A \to B \to G \to H$.

(ii) $A \to D \to F \to C \to A \to B \to G \to H$.

(iii) $A \to C \to I \to H$.

### Problem 5: The Simplex algorithm

Consider the following system of equations:

$$
\begin{cases}
x_1 + 2x_2 - 3x_3 + 5x_4 + x_5 & = 8 \\
5x_1 - 2x_2 \qquad\quad + 6x_4 \qquad + x_6 & = 16 \\
2x_1 + 3x_2 - 2x_3 + 3x_4 \qquad\qquad + x_7 & = 6 \\
-x_1 \qquad\ + \ x_3 - 2x_4 \qquad\qquad\qquad + x_8 & = 0
\end{cases}
$$

$$x_1, x_2, ..., x_8 \geq 0$$

Let $x_5, x_6, ..., x_8$ be a given initial basic feasible solution. Suppose that $x_1$ becomes basic. Which of the given basic variables in the initial tableau must become nonbasic to guarantee that all the variables remain non-negative, and what is the value of $x_1$ in the new solution? Repeat this procedure for $x_2$, $x_3$, and $x_4$.

### Home Exercise 3: The Simplex algorithm

Transform the linear problem into the standard form and solve using the tableau Simplex algorithm.

$$
\begin{aligned}
\min. \quad & 2x_1 + x_2 \\
\text{s.t.} \quad & 7x_1 - 3x_2 \leq 4, \\
& x_1 + 2x_2 \leq 7, \\
& x_1 \geq 0,\ x_2 \in \mathbf{R}.
\end{aligned}
$$