Exercise class 1

Learning Objectives:

- Use of Excel Solver and Julia
- Formulation of optimisation problems

Demo 1: Excel Solver and Julia

Solve the optimisation problem

$$\begin{array}{lrcrl} \max. & 3x_1 & + & x_2 & \\ \\ \text{s.t.} & 2x_1 & + & 5x_2 & \leq 8 \\ & -4x_1 & + & 2x_2 & \geq -5 \\ & x_1, & & x_2 & \geq 0 \end{array}$$

by using Excel Solver and Julia.

Demo 2: Formulation of linear optimisation problem

Matti is a farmer and wants to decide how many acres of rye and wheat to plant for the coming year. Each acre of wheat produces 25 loads of wheat and requires 10 hours of labour per week. An acre of rye produces 10 loads of rye and requires 4 hours of labour per week. The wheat sells at €4 per load and the rye €3 per load. Matti has 7 acres of farmland available and 40 hours of labour per week. Government regulations require that at least 30 loads of rye is produced during a given year. Let x_1 be the number of acres of wheat planted, and x_2 be the number of acres of rye planted.

Formulate an LP that will maximise Matti's total profit from wheat and rye, and solve with Excel Solver and Julia.

Problem 1: Solving simple linear programming problems

Solve with Excel Solver or Julia:

a)

$$\begin{array}{lrcrl} \min. & x & + & y & \\ \text{s.t.} & 3x & + & y & \geq 2 \\ & x & - & y & \geq 0 \\ & x, & & y & \geq 0 \end{array}$$

b)

$$\begin{array}{lrcrl} \max. & 3x_1 & + & 2x_2 & \\ \text{s.t.} & 2x_1 & + & x_2 & \leq 100 \\ & x_1 & + & x_2 & \leq 80 \\ & x_1 & & & \leq 40 \\ & x_1, & & x_2 & \geq 0 \end{array}$$

Problem 2: Formulation of a linear programming (LP) problem

A cargo plane has three compartments for storing cargo: front, centre and rear. These compartments have the following limits on both weight and space:

Compartment	**Weight Capacity (tonnes)**	**Space Capacity (m^3)**
Front	10	6800
Center	16	8700
Rear	8	5300

Table 1: Weight & space capacities

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the plane, e.g. if the front compartment has 90% of its weight capacity utilised (90% full) then the centre and the rear have to have 90% of their respective weight capacities used also.

The following four cargoes are available for shipment on the next flight:

Cargo	**Weight (tonnes)**	**Volume (m^3/tonne)**	**Profit (€/tonne)**
Cargo 1	18	480	310
Cargo 2	15	650	380
Cargo 3	23	580	350
Cargo 4	12	390	285

Table 2: Cargoes

Any proportion of these cargos can be distributed between any or all of the compartments.

Formulate an LP to determine how much (if any) of each cargo (C1, C2, C3 and C4) should be accepted and how to distribute each among the compartments so that the total profit for the flight is maximised. Solve in Excel Solver or Julia.

Problem 3: Formulation of an LP problem

Putte the Pig is arranging his name day party, again. He is expecting 100 guests to his party and he has to make delicacies for all of them. Putte has decided to make only three kind of delicacies: cake, cookies and buns.

In one cake there is enough for 10 people, one cookie for one person, and one batch of buns for 20 people. Unfortunately, there is only 10 h until the party and everything has to be ready by then. Baking a cake takes 30 min, one cookie takes 1 min on average, and one batch of buns takes 40 min.

The yearly party is expensive so Putte tries to minimise the costs. The ingredients for a cake cost €5, for a cookie €1, and for a batch of buns €10.

How many of each delicacies should Putte make? Also notice that Putte thinks that there should be enough for at least 20 persons of every delicacy and there should be cake for at least twice as many persons as there are cookies for.

Formulate as LP problem and solve with Excel Solver or Julia.

Problem 4: Formulation of an LP problem

A construction company owns 800 hectares (ha) of land and is about to build one-family, two-family and three-family houses. The company estimates that a one-family house will profit the company €200,000, it requires an area of 1 ha, building costs are €145,000, and the water consumption is 2000 l per day. The key figures for a two-family house are €240,000, 1.5 ha, €165,000 and 2700 l per day, and for a three-family house are €300,000, 2 ha, €215,000 and 3200 l per day.

- At least half of the houses have to be one-family houses.
- The water consumption for the area cannot surpass 850,000 l per day.
- For every 200 families there has to be at least one recreational area. It requires 0.5 ha of area. Building costs are €125,000, and water consumption is 2500 l per day.
- Streets etc. require 15 % of the total area.

Formulate LP problem and find out how many of each house types should the company build to maximise its profits?

Problem 5: Formulation of an LP problem*

Stockmann requires different numbers of full-time employees on different days of the week. The number of full-time staff needed is given in Table 3. Union rules state that each full-time employee must work 5 consecutive days then receive 2 days off, for example if they work Monday - Friday then they must have Saturday and Sunday off. Stockmann wants to meet its daily requirements using only full-time staff.

Day	Mon	Tues	Weds	Thurs	Fri	Sat	Sun
# full-time staff needed	17	13	15	19	14	16	11

Table 3: Full-time staff requirements

a) Formulate an LP to minimise the number of full-time employees and solve in Excel Solver or Julia

b) Now suppose full-time employees work 8 hours per day, and Stockmann may meet daily labour requirements by using full- and part-time staff. Full-time staff work 8h/day for 5 straight days at 15€/h. Part-time staff work 4h/day for 5 straight days at 10€/h. Union requirements limit part-time staff labour to 25% of weekly labour requirements.
Formulate an LP to minimise cost of labour and solve in Excel Solver or Julia.

Home Exercise 1: Formulate an LP problem

Bev's Beverage Products is considering producing a wine cooler that would be a blend of a white wine, a rosé wine, and fruit juice. To meet taste specifications, the wine cooler must consist of at least 50% white wine, at least 20% and no more than 30% rosé, and exactly 20% fruit juice. Bev purchases the wine from local wineries and the fruit juice from a processing plant in California. For the current production period, 10,000 litres of white wine and 8,000 litres of rosé wine can be purchased; and unlimited amount of fruit juice can be ordered. The costs for the wine are \$1.00 per litre for the white and \$1.50 per litre for the rosé; the fruit juice can be purchased for \$0.50 per litre. Bev's Beverages can sell all of the wine cooler they can produce for \$2.50 per litre.

Formulate a linear program to determine the blend of the three ingredients that will maximise total profit contribution. Solve the linear program to determine the number of litres of each ingredient Bev should purchase and the total profit contribution they will realise from this blend.