$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$X|1\rangle = |0\rangle$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$X|\psi\rangle = \alpha \underbrace{X|0\rangle}_{=|1\rangle} + \beta \underbrace{X|1\rangle}_{=|0\rangle} = \beta|0\rangle + \alpha|1\rangle$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = |-\rangle$$

cX, CNOT

$$cX_{01}|00\rangle = |00\rangle , \quad cX_{01}|01\rangle = |11\rangle , \quad cX_{01}|10\rangle = |10\rangle$$

$$cX_{01}|11\rangle = |01\rangle$$

$$cX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} , \quad |00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad |10\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vdots$$

$$\underbrace{H^{\otimes n}}_{=H\otimes H\otimes\cdots\otimes H} \underbrace{|0\rangle^{\otimes n}}_{=|0\rangle\otimes|0\rangle\otimes\cdots\otimes|0\rangle} = \left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\right)^{\otimes n}$$

$$= \frac{1}{2^{n/2}} \sum_{x=0}^{2^n-1} |x\rangle$$

$$(|0\rangle + |1\rangle)\otimes(|0\rangle + |1\rangle)$$

$$= |0\rangle\otimes|0\rangle + |0\rangle\otimes|1\rangle$$

$$+ |1\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle$$

$|00\ldots0\rangle + |0\ldots1\ldots0\rangle \cdots$

A, B are 2×2 matrices

$$(A, B) = \frac{1}{2}\mathrm{tr}(A^\dagger B)$$

$$(\sigma_i, \sigma_j) = \frac{1}{2}\mathrm{tr}(\sigma_i \sigma_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

$0 \oplus 0 = 0$

$1 \oplus 0 = 1$

$0 \oplus 1 = 1$

$1 \oplus 1 = 0$

mod 2

$$|00\rangle \xmapsto{H\otimes I} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

$$\xmapsto{cX_{12}} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = |\Phi\rangle$$

$$|\langle 00|\Phi\rangle|^2 = \frac{1}{2}$$

$$|\langle 11|\Phi\rangle|^2 = \frac{1}{2}$$

$$(\langle\psi_1|\langle\phi_1|)(|\psi_2\rangle|\phi_2\rangle) = \langle\psi_1|\psi_2\rangle\langle\phi_1|\phi_2\rangle$$

$$\underline{\underline{\langle\psi|\phi\rangle}} = \underbrace{(\langle\psi|\langle 0|)\overset{I = U^\dagger U}{}(|\phi\rangle|0\rangle)}_{= \langle\psi|\phi\rangle\underbrace{\langle 0|0\rangle}_{=1}} = \underbrace{\langle\psi|\langle 0|U^\dagger}\ \underbrace{U|\phi\rangle|0\rangle}_{|\phi\rangle|\phi\rangle}$$

$$(U|\psi\rangle|0\rangle)^\dagger = (|\psi\rangle|\psi\rangle)^\dagger = \langle\psi|\langle\psi|$$

$$= (\langle\psi|\langle\psi|)(|\phi\rangle|\phi\rangle)$$

$$= \langle\psi|\phi\rangle\langle\psi|\phi\rangle = \underline{\underline{\langle\psi|\phi\rangle^2}}$$

$a = a^2$ only if

$\underline{a=0 \text{ or } a=1}$

cX

$$|00\rangle \longmapsto |00\rangle$$

$$|10\rangle \overset{CX}{\longmapsto} |11\rangle$$

$$(A \otimes B)(C \otimes D) = AC \otimes BD$$

$$\langle \psi | \phi \rangle = \begin{pmatrix} \psi_1 & \cdots & \psi_n \end{pmatrix} \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$$