Quantum Information Spring 2023 Problem Set 3

Solutions are due on Sunday May 14, 23:59.

1. **Quantum teleportation (programming)**
   In this problem you will implement the quantum teleportation protocol using Qiskit. You can use the template notebook "teleportation.ipynb" which you can find on the MyCourses page. Follow the notebook and fill in the missing parts. I recommend using `jupyter.cs.aalto.fi`. Return your answer as a .ipynb-notebook (you can download the notebook from JupyterHub if you used the department cloud installation.)

2. **Grover search on two qubits (programming)**
   This time you will implement the Grover search algorithm. Use the template "grover.ipynb" and fill in the missing parts which are:

   - Implementation of the oracle
   - Implementation of the phase flip
   - The correct number of Grover iterations

3. **Grover iteration**
   Recall the definitions

$$|\alpha\rangle \equiv \frac{1}{\sqrt{N-M}} \sum_x{}'' |x\rangle \tag{1}$$

$$|\beta\rangle \equiv \frac{1}{\sqrt{M}} \sum_x{}' |x\rangle \ , \tag{2}$$

   where $\sum_x{}'$ is a sum over all solutions to the search problem and $\sum_x{}''$ is a sum over x which are not solutions. Show that in the $|\alpha\rangle$, $|\beta\rangle$ basis, we may write the Grover iteration as

$$G = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \ , \tag{3}$$

   where θ is a real number in the range 0 to $\pi/2$ chosen so that

$$\sin\theta = \frac{2\sqrt{M(N-M)}}{N} \ . \tag{4}$$

   You can assume for simplicity that $M \leq N/2$.

4. **Circuit identities**
   Show that the following circuit identities hold. Subscripts denote on which qubit a gate acts and C denotes the CNOT-gate with the first qubit as control and the second qubit as target.

(a) $CX_1C = X_1X_2$

(b) $CZ_1C = Z_1$

(c) $CY_1C = Y_1X_2$

5. **Measuring an operator**
Suppose we have a single qubit operator U with eigenvalues ± 1, so that U is both Hermitian and unitary, so it can be regarded both as an observable and a quantum gate. Suppose we wish to measure the observable U. That is, we desire to obtain a measurement result indicating one of the two eigenvalues, and leaving a post-measurement state which is the corresponding eigenvector. Show that the following circuit implements a measurement of U: