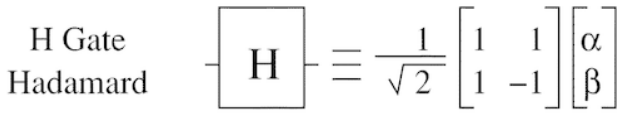
1. Mixed States

- slides 4-5 from the lecture slides on 1.5.

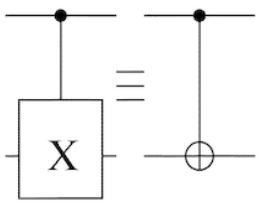
2. Entangling gates

H Gate
Hadamard



$$\equiv \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \frac{\alpha+\beta|0\rangle + \alpha-\beta|1\rangle}{\sqrt{2}}$$

Controlled Not
Controlled X
CNot



$$\equiv \equiv \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix} = \begin{matrix}a|00\rangle + b|01\rangle +\\ d|10\rangle + c|11\rangle\end{matrix}$$

Z Gate
Phase−flip

$$\equiv \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \alpha|0\rangle - \beta|1\rangle$$

$$\rho = |\psi\rangle\langle\psi|$$

3. Entanglement Entropy

Definition (Entanglement entropy)
The entanglement entropy $S(\rho)$ of a quantum state ρ is

$$S(\rho) = -\operatorname{tr}(\rho \log \rho) = -\sum_i p_i \log p_i \ ,$$

where $\{p_i\}$ are the eigenvalues of ρ.

(slide 27)

4. Trace Distance

$$D(\rho, \sigma) = \tfrac{1}{2}\operatorname{tr}|\rho - \sigma|$$

$$|A| \equiv \sqrt{A^\dagger A}$$

5. Fidelity

$$F(\rho, \sigma) = \operatorname{tr}\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}$$