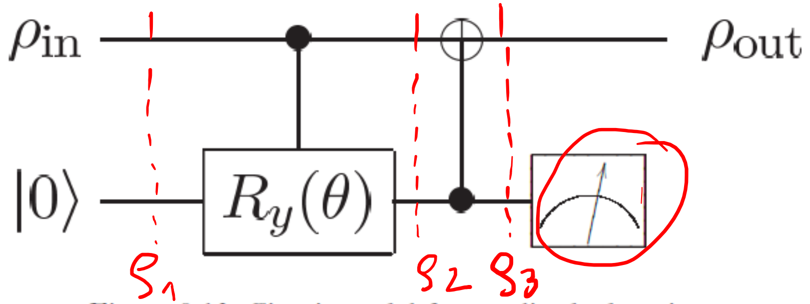


Figure 8.13. Circuit model for amplitude damping

$$cR_y(\theta) = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes R_y(\theta)$$

$$\varrho_2 = cR_y(\theta)\, \varrho_1\, cR_y(\theta)^\dagger$$

Exercise 8.20: (Circuit model for amplitude damping) Show that the circuit in Figure 8.13 models the amplitude damping quantum operation, with $\sin^2(\theta/2) = \gamma$.

$$cX = I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$$

$$\varrho_{in} \longmapsto \varrho_{in} \otimes |0\rangle\langle 0| = \varrho_1$$

$$\longmapsto \left(|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes R_y(\theta)\right)\left(\varrho_{in} \otimes |0\rangle\langle 0|\right)\left(|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes R_y(\theta)^\dagger\right)$$

$$= |0\rangle\langle 0| \varrho_{in} |0\rangle\langle 0| \otimes |0\rangle\langle 0| + |0\rangle\langle 0| \varrho_{in} |1\rangle\langle 1| \otimes |0\rangle\langle 0| R_y(\theta)^\dagger$$

$$+ |1\rangle\langle 1| \varrho_{in} |0\rangle\langle 0| \otimes R_y(\theta)|0\rangle\langle 0| + |1\rangle\langle 1| \varrho_{in} |1\rangle\langle 1| \otimes R_y(\theta)|0\rangle\langle 0| R_y(\theta)^\dagger$$

$$\operatorname{tr}_2\left(|a\rangle\langle b| \otimes |c\rangle\langle d|\right) = |a\rangle\langle b| \langle d|c\rangle \qquad \operatorname{tr}(|i\rangle\langle j|) = \langle j|i\rangle$$

Exercise 8.23: (Amplitude damping of dual-rail qubits) Suppose that a single qubit state is represented by using two qubits, as

$$|\psi\rangle = a\,|01\rangle + b\,|10\rangle\,. \tag{8.113}$$

Show that $\mathcal{E}_{\mathrm{AD}} \otimes \mathcal{E}_{\mathrm{AD}}$ applied to this state gives a process which can be described by the operation elements

$$E_0^{\mathrm{dr}} = \sqrt{1-\gamma}\, I \tag{8.114}$$

$$E_1^{\mathrm{dr}} = \sqrt{\gamma}\left[|00\rangle\langle 01| + |00\rangle\langle 10|\right], \tag{8.115}$$

that is, either nothing (E_0^{dr}) happens to the qubit, or the qubit is transformed (E_1^{dr}) into the state $|00\rangle$, which is orthogonal to $|\psi\rangle$. This is a simple error-detection code, and is also the basis for the robustness of the 'dual-rail' qubit discussed in Section 7.4.

Exercise 8.30: ($T_2 \leq T_1/2$) The T_2 phase coherence relaxation rate is just the exponential decay rate of the off-diagonal elements in the qubit density matrix, while T_1 is the decay rate of the diagonal elements (see Equation (7.144)). Amplitude damping has *both* nonzero T_1 and T_2 rates; show that for amplitude damping $T_2 = T_1/2$. Also show that if amplitude and phase damping are *both* applied then $T_2 \leq T_1/2$.

$$\begin{bmatrix} a & b \\ b^* & 1-a \end{bmatrix} \to \begin{bmatrix} (a-a_0)e^{-t/T_1} + a_0 & be^{-t/T_2} \\ b^* e^{-t/T_2} & (a_0 - a)e^{-t/T_1} + 1 - a_0 \end{bmatrix}, \qquad (7.144)$$

ibm_washington

Exploratory

[IBM Quantum Washington machine specs]

Details

127
Qubits

64
QV

850
CLOPS

Status: Online

Total pending jobs: 140 jobs

Processor type ⓘ: Eagle r1

Version: 1.1.0

Basis gates: CX, ID, RZ, SX, X

Your usage: --

Avg. CNOT Error: 1.000e+0

Avg. Readout Error: 2.826e-2

Avg. T1: 101.21 us

Avg. T2: 96.28 us

Providers with access: --

Supports Qiskit Runtime: Yes

Calibration data

Map view | Graph view | Table view

Qubit:

Frequency (GHz)

Avg 5.064

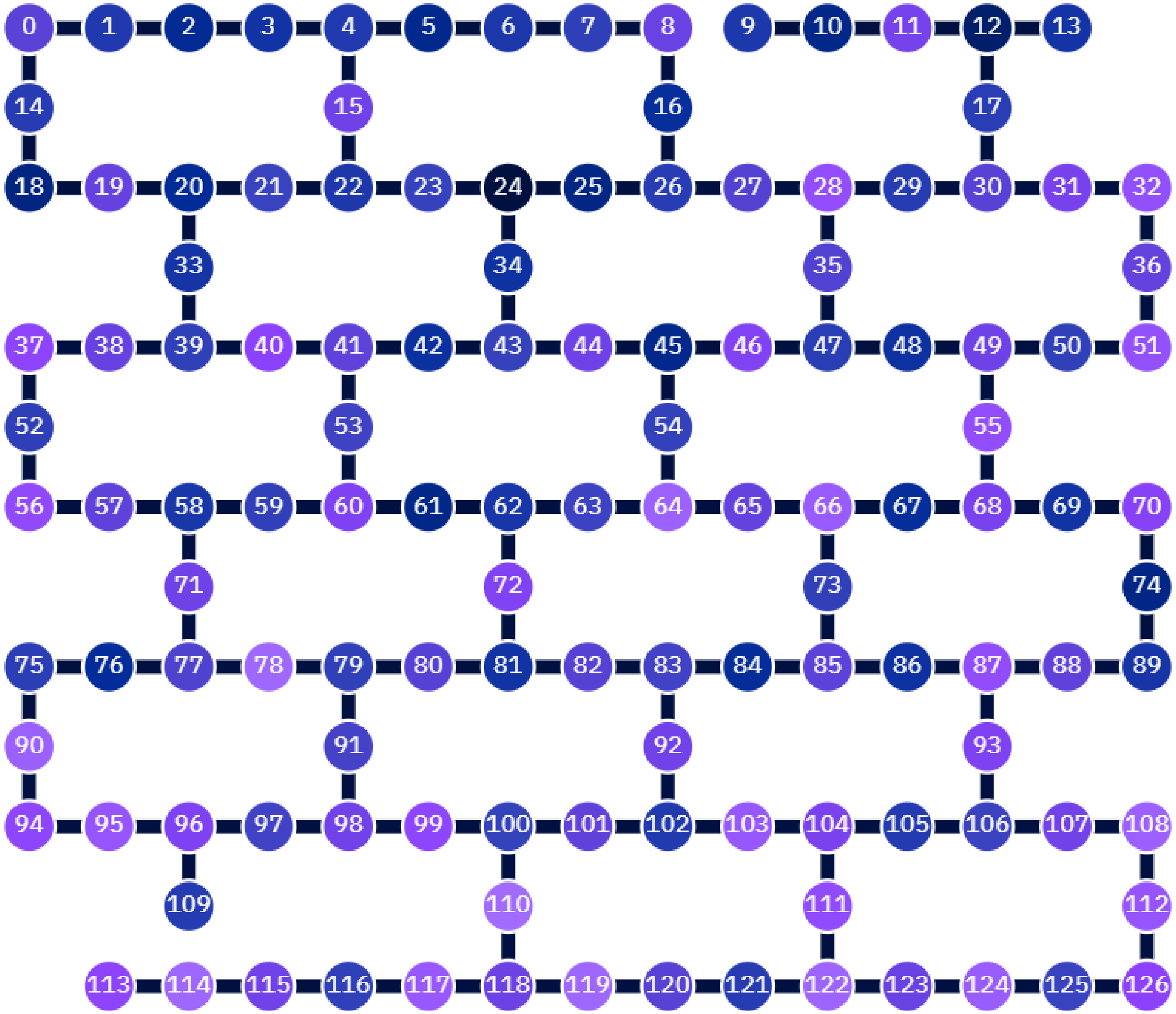
min 4.767 max 5.292

Connection:

CNOT error

Avg 1.000e+0

min 1.000e+0 max 1.000e+0



[1907.09528] Topological and subsystem codes on low-degree graphs with flag qubits (arxiv.org)

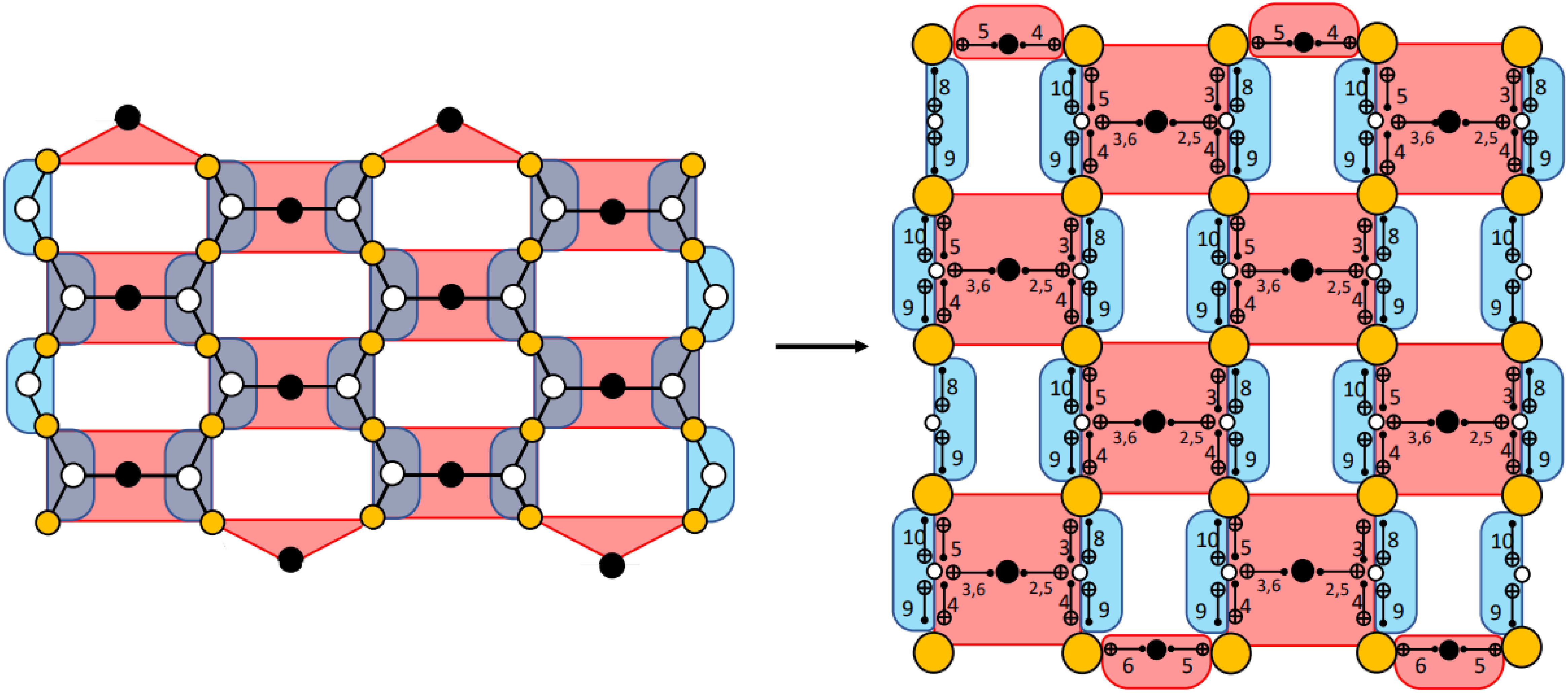


FIG. 2. The left of the figure corresponds to the actual layout of the $d = 5$ heavy hexagon code which encodes one logical qubit. The data qubits are represented by yellow vertices, white vertices are the flag qubits and dark vertices represent the ancilla to measure the X-type gauge generators (red areas) and the Z-type gauge generators (blue areas). In the bulk, products of the two Z-type gauge generators at each white face forms a Z-type stabilizer. The right of the figure provides a circuit illustration of the heavy hexagon code with the scheduling of the CNOT gates used the measure the X-type and Z-type gauge generators.

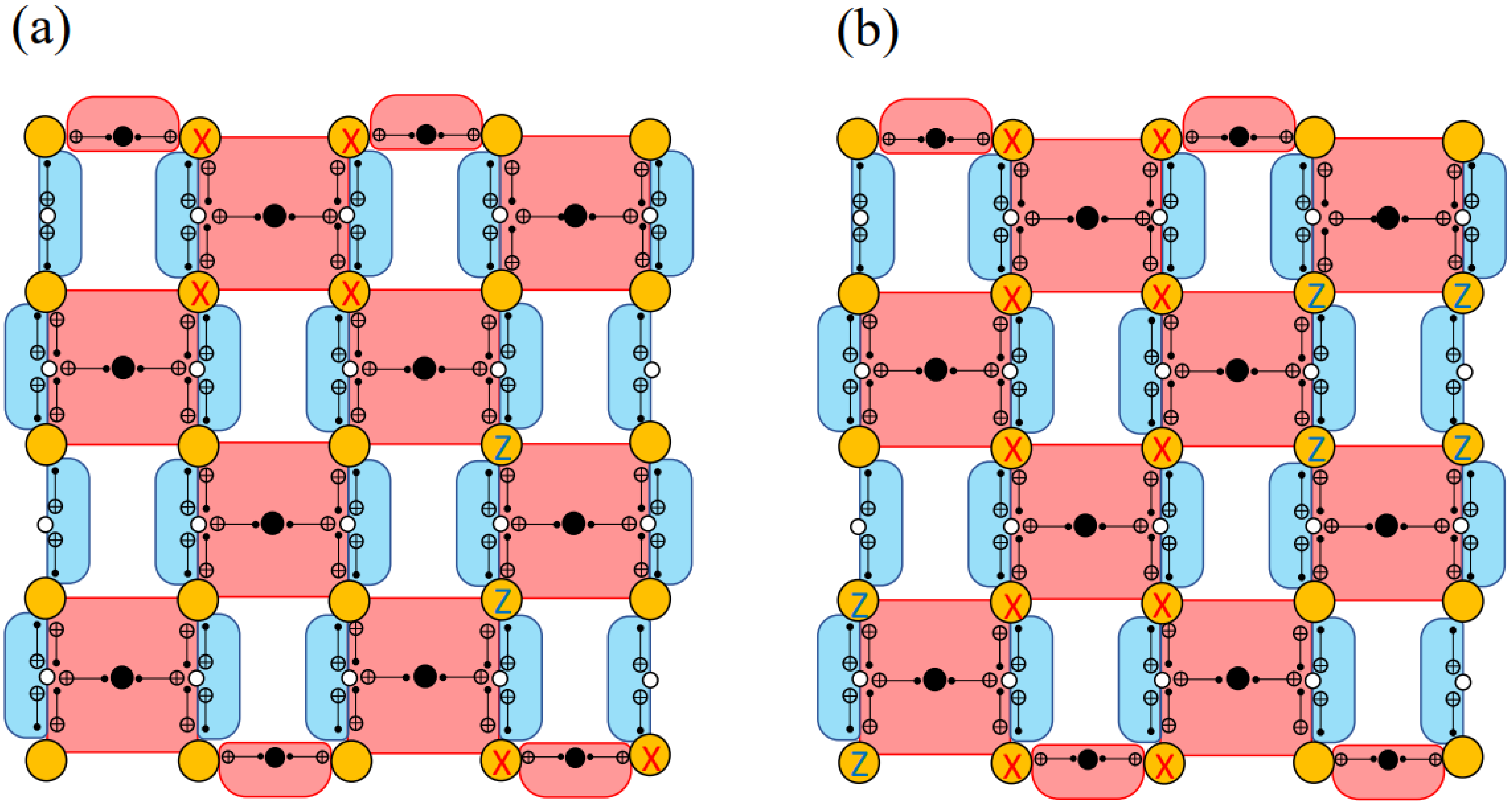


FIG. 3. (a) Gauge generators: weight-four X-type in the bulk, weight-two X-type on the upper and lower boundaries, and weight-two Z-type. (b) Stabilizer operators: a two-column vertical strip with X-type, weight-four Z-type in the bulk, and weight-two Z-type on the left and right boundaries.

The gauge group of the heavy hexagon code is

$$\begin{aligned}\mathcal{G} =& \langle Z_{i,j}Z_{i+1,j},\ X_{i,j}X_{i,j+1}X_{i+1,j}X_{i+1,j+1},\\ & X_{1,2m-1}X_{1,2m},\ X_{d,2m}X_{d,2m+1}\rangle\end{aligned}$$

The stabilizer group which specifies the logical subspace $\mathcal{H}_\mathcal{L}$ is the center of the gauge group or, explicitly,

$$\begin{aligned}\mathcal{S} =& \langle Z_{i,j}Z_{i,j+1}Z_{i+1,j}Z_{i+1,j+1},\ Z_{2m,d}Z_{2m+1,d},\\ & Z_{2m-1,1}Z_{2m,1},\ \prod_i X_{i,j}X_{i,j+1}\rangle \qquad (2)\end{aligned}$$

In general, a distance d version of the code will have d data qubits along each row and each column of the hexagonal lattice so that the code parameters are given by $[\![d^2, 1, d]\!]$. In addition, a distance d implementation of the code requires a total of $\frac{d+1}{2}(d-1)$ syndrome measurement qubits and $d(d-1)$ flag qubits. Hence the total number of qubits in the implementation of the code is $\frac{5d^2-2d-1}{2}$.

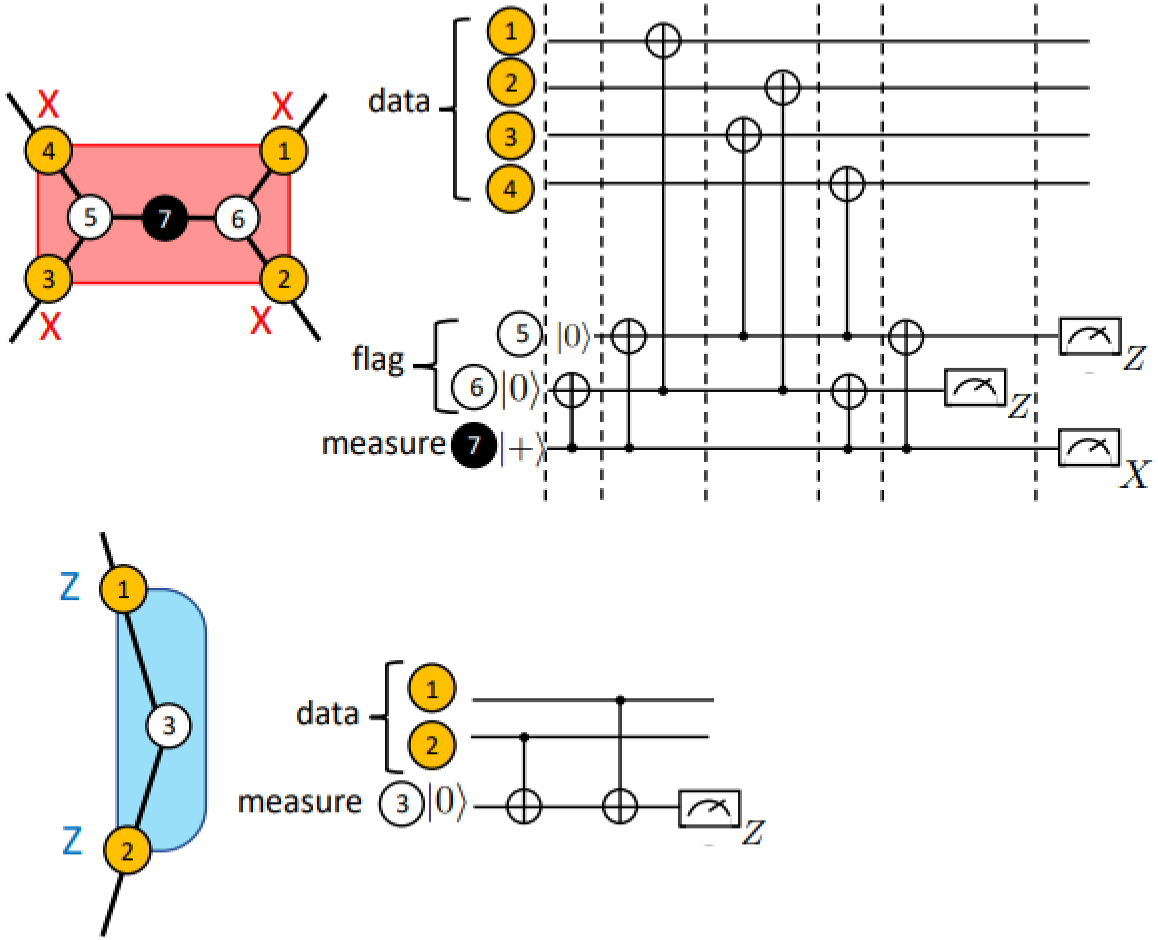
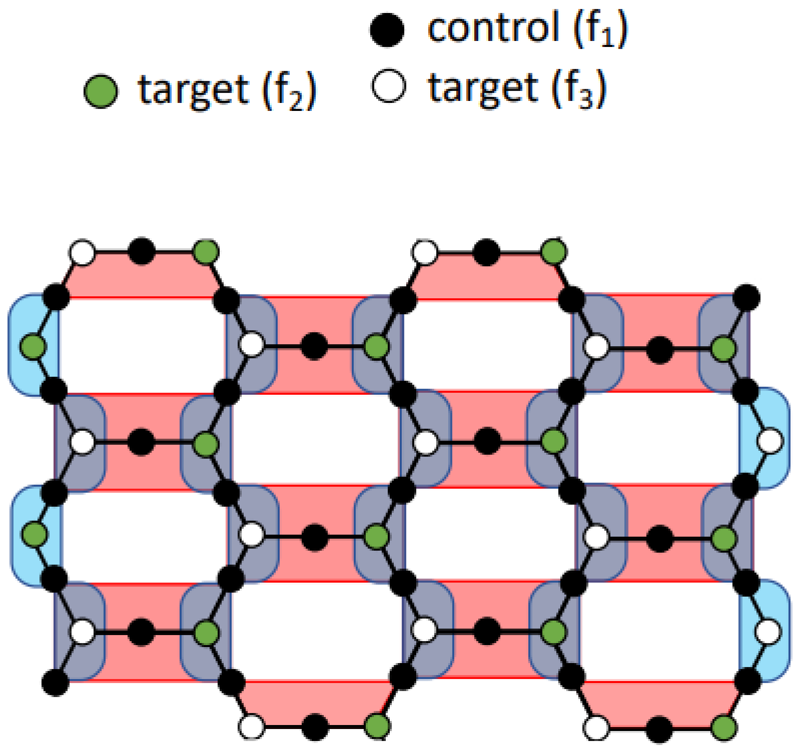
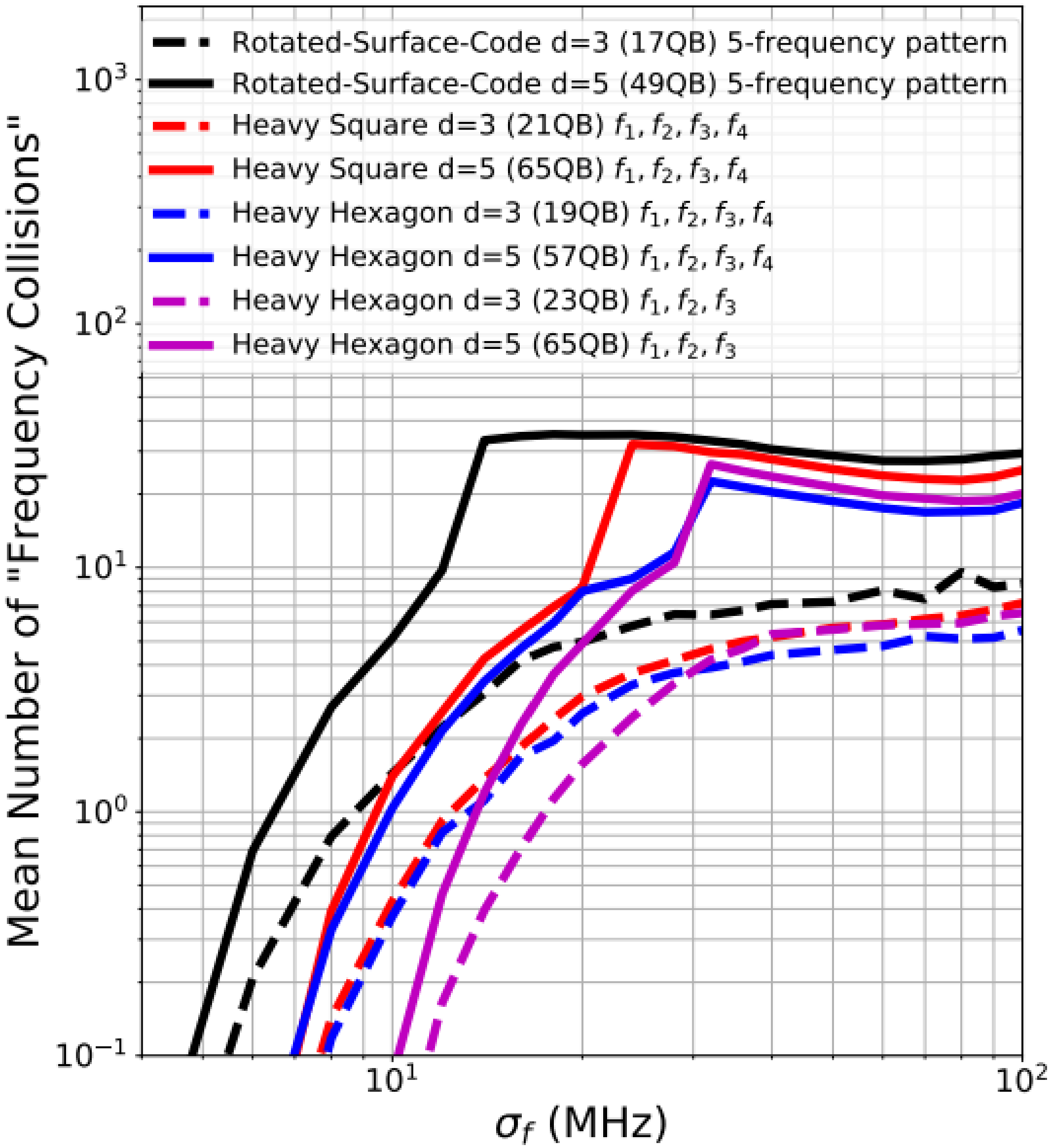


FIG. 4. Circuit to perform the X and Z-type parity measurements of the heavy hexagon code. Two flag qubits (white circles) are used to measure the weight-four X-type gauge generators.



control (f_1)
target (f_2)
target (f_3)



Rotated-Surface-Code d=3 (17QB) 5-frequency pattern
Rotated-Surface-Code d=5 (49QB) 5-frequency pattern
Heavy Square d=3 (21QB) f_1, f_2, f_3, f_4
Heavy Square d=5 (65QB) f_1, f_2, f_3, f_4
Heavy Hexagon d=3 (19QB) f_1, f_2, f_3, f_4
Heavy Hexagon d=5 (57QB) f_1, f_2, f_3, f_4
Heavy Hexagon d=3 (23QB) f_1, f_2, f_3
Heavy Hexagon d=5 (65QB) f_1, f_2, f_3
Mean Number of "Frequency Collisions"
σ_f (MHz)

References

Quantum error correction intro: arXiv:1907.11157
Subsystem codes: arXiv:quant-ph/0508131, arXiv:quant-ph/0506023
Surface codes: arXiv:1208.0928
Flag qubits: arXiv:1705.02329
Heavy hexagon code: arXiv:1907.09528