

Problem 1

Consider the two qubit state described by

$$\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \in \mathcal{H}_A \otimes \mathcal{H}_B .$$

a)

Is the state pure?

b)

Are the two qubits entangled?

Problem 2

Show that the n -qubit quantum Fourier transform

$$U_{QFT} = \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} \sum_{k=0}^{2^n-1} e^{-2\pi i j k / 2^n} |k\rangle\langle j|$$

is unitary. That is, show that

$$U_{QFT}^\dagger U_{QFT} = \mathbb{I}_{2^n} = \sum_{k=0}^{2^n-1} |k\rangle\langle k| .$$

Problem 3

Describe the error correction protocol for the Shor 9-qubit code including (i) encoding, (ii) syndrome measurements and (iii) recovery operations.

Problem 4

Provide a quantum circuit for simulating one timestep of the dynamics of a chain of three interacting spin- $\frac{1}{2}$ particles (modelled by single qubits) with the Hamiltonian

$$H = \mu \sum_{i=1}^3 \sigma_x^{(i)} + \lambda \sigma_x^{(1)} \sigma_y^{(2)} \sigma_z^{(3)} ,$$

where $\sigma_k^{(i)}$ is the k 'th Pauli matrix acting on the i 'th particle.

Problem 5

Suppose you want to measure the expectation value of some observable M in a quantum state you prepare on a quantum computer. That is, you are interested in $\langle M \rangle$ computed in some state $|\psi\rangle$.

a)

What is quantum error mitigation and why would one want to use it instead of quantum error correction for this computation?

b)

Assume your quantum circuit which prepares $|\psi\rangle$ consists of only single qubit gates and CNOT gates. Further assume that the error rate of single qubit gates is zero and the all CNOT gates have an error rate ϵ .

You have measured $\langle M \rangle$ at two different error rates:

$$\langle M \rangle (\epsilon) = x$$

$$\langle M \rangle (3\epsilon) = y .$$

Calculate the estimate for $\langle M \rangle (0)$ using Richardson extrapolation.

c)

Give one method for increasing the error rate ϵ in the measurement of $\langle M \rangle$.