Quantum Information Spring 2023 Problem Set 6

Solutions are due on Sunday June 04, 23:59.

1. **QKD–1: One-time pad**
   Show how to get the message coded using a random sequence of 0's and 1's. Assume that the message is only a letter 'T'. Its ascii is 84, with the command bin(84) in python you get the corresponding binary sequence. Create some random secret key (8 bits) and make the message that Alice sends to Bob using the one-time pad-method.

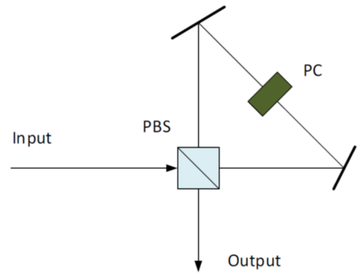
   Show how Bob regenerates the original message.

2. **QKD–2: Attenuated laser**
   Assume 1 GHz pulse repetition rate for pulses that are used for creating secret keys with a highly attenuated semiconductor laser. Assume that the used protocol can stand 4 % pulses containing more than one photon (semiclassical treatment).

   Calculate how many photons you can at maximum use for key production per second. For a laser telecommunication wavelength of $\lambda = 1.55$ μm, what is the corresponding optical power? (photon energy is $\hbar\omega \equiv \hbar(2\pi c/\lambda)$, where
   $\hbar = 1.064 \cdot 10^{-34}$ J·s is the Planck constant.)

3. **Cavity-based quantum memory**
   Cavity-based quantum memory device with a cavity length of 100 cm and a polarization control unit (Pockels cell) with a switching time of 50 ps is used to store light pulses of 200 fs $= 2 \cdot 10^{-13}$ s duration.
   - How many distinct pulses N_{stored} can be stored in a single memory unit?
   - What is the lowest overall transmittance of the cavity (per one pass) T_{cav}, optical elements included, that allows storing a pulse with 80% efficiency during $K = 200$ passes of the cavity?

   Assume non-dispersive medium and negligibly small optical length (= time delay) in both the beam-splitter and Pockels cell.
   Reference values of reflectance (power ratio between the reflected and the incident beams) are: $\approx 96\%$ for metal mirrors; $\approx 99.5\%$ for dielectric multilayer mirrors; up to $(1 - 3 \cdot 10^{-7})$ for main mirrors in LIGO gravitational wave interferometer.

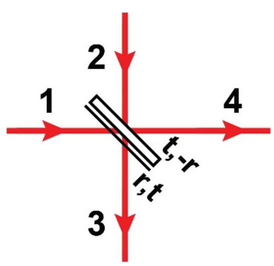


4. **Hong-Ou-Mandel interference – 1**

Let the light at each of the two input ports of a beam-splitter to be in the Fock state $|n\rangle_{\mathrm{F}}$ – that is, the number of photons (n) is fixed. Let the numbers of photons be $n_1 = M$ and $n_2 = 0$.
(1) Use the following properties of creation ($\hat{a}$) and annihilation ($\hat{a}^\dagger$) operators

$$\begin{aligned} \hat{a}\,|n\rangle &= \sqrt{n}\,|n-1\rangle \ , \\ \hat{a}^\dagger\,|n-1\rangle &= \sqrt{n}\,|n\rangle \ , \\ \hat{a}\,|0\rangle &= 0 \cdot |0\rangle = 0 \ , \end{aligned}$$

to express the input state $|M\rangle_{1\,\mathrm{F}}\,|0\rangle_{2\,\mathrm{F}}$ via the operators $\hat{a}_1^\dagger$, $\hat{a}_2^\dagger$.
(2) Using the input-output relations between $\hat{a}_1^\dagger$, $\hat{a}_2^\dagger$ and $\hat{a}_3^\dagger$, $\hat{a}_4^\dagger$ (Figure below and lecture materials), substitute the equations for $\hat{a}_1^\dagger$ and $\hat{a}_2^\dagger$ to the solution of (a). Write down the light state at the output of the beam-splitter. Use the photon-number basis.



$$\begin{pmatrix} \hat{a}_3^\dagger \\ \hat{a}_4^\dagger \end{pmatrix} = \begin{pmatrix} r & t \\ t & -r \end{pmatrix} \begin{pmatrix} \hat{a}_1^\dagger \\ \hat{a}_2^\dagger \end{pmatrix}$$

5. **Hong-Ou-Mandel interference – 2**

Using the setting and the results of the previous exercise, evaluate the output state if the input one is

$$|1\rangle_{1\,\mathrm{F}}\,|0\rangle_{2\,\mathrm{F}} \ .$$

If the photons at ports 1 and 2 are used as a path-entangled qubit ($|1\rangle_{1\,\mathrm{F}}\,|0\rangle_{2\,\mathrm{F}} \equiv |0\rangle_{logical}$, $|0\rangle_{1\,\mathrm{F}}\,|1\rangle_{2\,\mathrm{F}} \equiv |1\rangle_{logical}$), what is the equivalent quantum circuit that performs the same transformation on the input state?