

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\langle 0|0\rangle = 1 = \langle 1|1\rangle$$

$$\langle 0|1\rangle = 0$$

$$(A \otimes B)(C \otimes D) = AC \otimes BD$$

$$A - \lambda I = \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} - \lambda & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} - \lambda \end{pmatrix}$$

$$U A U^\dagger$$

$$\xrightarrow{\det} \underbrace{\left(\frac{3}{2} - \lambda\right)^2 - \left(-\frac{1}{2}\right)^2}_{=0} = 0$$

$$M_m = |m\rangle\langle m|, \quad M_m^\dagger = M_m$$

$$M_m^2 = (|m\rangle\langle m|)^2 = |m\rangle \underbrace{\langle m|m\rangle}_{=1} \langle m| = |m\rangle\langle m| = M_m$$

$$\langle \psi | \underbrace{M_m^\dagger M_m}_{=M_m^2=M_m} | \psi \rangle = \underbrace{\langle \psi | m \rangle \langle m | \psi \rangle}_{\langle m | \psi \rangle^*} = \underline{\underline{|\langle m | \psi \rangle|^2}} = p(m)$$

$$|\psi\rangle = |m'\rangle$$

$$\langle m' | A | m' \rangle = \langle m' | \sum_m m |m\rangle\langle m| | m' \rangle$$

$$\begin{aligned}
 \langle m' | A | m' \rangle &= \langle m' | \sum_m m | m \rangle \langle m | m' \rangle \\
 &= \sum_m m \underbrace{\langle m' | m \rangle \langle m | m' \rangle}_{= \begin{cases} 1 & \text{if } m' = m \\ 0 & \text{otherwise} \end{cases}} = m'
 \end{aligned}$$

$$\sigma_A^2 = 0$$