Partially Observable Markov Decision Processes in Robotics: A Survey

Mikko Lauri, David Hsu and Joni Pajarinen

Abstract—**Noisy sensing, imperfect control, and environment changes are defining characteristics of many real-world robot tasks. The** *partially observable Markov decision process* **(POMDP) provides a principled mathematical framework for modeling and solving robot decision and control tasks under uncertainty. Over the last decade, it has seen many successful applications, spanning localization and navigation, search and tracking, autonomous driving, multi-robot systems, manipulation, and human-robot interaction. This survey aims to bridge the gap between the development of POMDP models and algorithms at one end and application to diverse robot decision tasks at the other. It analyzes the characteristics of these tasks and connects them with the mathematical and algorithmic properties of the POMDP framework for effective modeling and solution. For practitioners, the survey provides some of the key task characteristics in deciding when and how to apply POMDPs to robot tasks successfully. For POMDP algorithm designers, the survey provides new insights into the unique challenges of applying POMDPs to robot systems and points to promising new directions for further research.**

Index Terms—**Autonomous Agents, Planning under Uncertainty, Scheduling and Coordination, AI-Based Methods, partially observable Markov decision process**

I. INTRODUCTION

UNCERTAINTIES are ubiquitous in robot systems due to noisy sensing, imperfect robot control, fast-changing environments, and inaccurate models. A robot must reason about the possible outcomes of its actions based on limited sensor information; it takes actions that not only yield short-term reward but also gather information for long-term success. For example, an autonomous robot vehicle tries to pass through an unsignalized traffic intersection as fast as possible. Instead of accelerating, the vehicle may have to slow down in the short term, in order to gather information on the intentions of pedestrians and other vehicles. This information helps the vehicle to coordinate its actions with others and achieve the overall goal faster in the long term [10], [24]. Similarly, a robot manipulator tries to push an irregular object to a designated pose, with the minimum number of actions. Instead of pushing the objects directly towards the final pose, it may use the first few pushes to gather information on the object's center of mass so that the later pushes become much more effective [81]. The *partially observable Markov decision process* (POMDP) [6] is

a principled general framework for such robot decision-making tasks under uncertainty.

A POMDP models a decision-maker, also called the *agent*, for a system with incomplete state information. At each time step, the agent executes an action that yields some reward depending on the current state and the action, and results in a stochastic transition to a new system state. The agent acquires information about the new state via noisy observations. A solution to a POMDP is a *policy* that prescribes the agent's action conditioned on its past history of actions and observations. The performance of a policy is measured by an objective function, which measures the expected total discounted reward over time. An optimal policy maximizes this objective function. To compute an optimal policy, *planning algorithms* reason about the possible long-term effects of actions on future states, observations, and reward using state transition, observation, and reward models. A prerequisite for planning is to acquire these models. For some robotic tasks, the models are specified by a domain expert, or they are learned from data prior to planning. The model-based POMDP planning approach therefore differs from other control and learning approaches, such as reinforcement learning (e.g., [77]), which do not use system models or sometimes learn such models concurrently with the policy by interacting with the system.

POMDP planning offers key conceptual, algorithmic, and practical advantages for robot systems. First, The POMDP models uncertainties inherent to robot systems. A robust robot system must reason about the effects of noisy sensor observations, imperfect robot actions, and environment changes. This is unavoidable. The POMDP provides a principled conceptual framework to model these uncertainties probabilistically and to identify the underlying conditions required for an optimal solution. The POMDP framework is general and applicable to most robot systems commonly encountered. Second, while POMDP planning is computationally intractable in the worst case, there have been dramatic successes in developing efficient *approximation algorithms* through Monte Carlo sampling and simulation (Section III). Given the POMDP model of a robot system, these algorithms solve for a near-optimal policy automatically, reducing the work required to manually engineer such policies and improving the practical performance of robot systems under uncertainty (Section IV). Finally, some POMDP planning algorithms provide performance guarantees [137], [116], [138], [43], a key requirement for safety-critical applications.

There are several surveys on planning algorithms for POMDPs in the operations research and artificial intelligence research communities [58], [100], [166], [95], [132]. Likewise,

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there are many surveys covering robotics applications such as grasping [21], human robot interaction [85], and autonomous driving [55]. What is lacking, however, is a synthesised view of how to apply the POMDP effectively across different robot systems.

This survey covers the application of POMDP planning to robotics in six broad categories:

- localization and navigation
- autonomous driving
- target search, tracking and avoidance
- manipulation and grasping
- human-robot interaction
- multi-robot coordination.

We identify general challenges of applying POMDPs to robotic tasks, including dealing with high-dimensional, continuous state, action, and observations spaces, and designing accurate and useful models for planning. For practitioners interested in the POMDP as a tool for robot decision making under uncertainty, the survey provides insights in a range of applications and their key characteristics that enable effective POMDP solutions. For researchers specializing in POMDP planning algorithms, this survey serves as an overview of the unique challenges in applying POMDPs to robotic systems and suggests potential directions for future research.

The remainder of this survey is organized as follows. In Section II, we summarize the theoretical foundations of the POMDP model. In Section III, we provide a short review of solution algorithms for computing (near-)optimal policies for POMDPs. Section IV presents the POMDP model and solution algorithms for the six robotic tasks listed above. In Section V, we discuss future challenges most important for the application of POMDPs to robot tasks. Section VI ends with some concluding remarks.

II. POMDP MODELS

In this section, we review the POMDP model for the finite horizon and infinite horizon cases. We defer discussion of practical solution algorithms to the next section. We assume the reader is familiar with the concept of recursive state estimation or Bayesian filtering [39]. For simplicity, we only consider the case where the sets of states, actions, and observations are finite, and the objective function is the expected total discounted reward. We refer the reader interested in further theoretical background to [121], [69], [58].

A. Finite horizon

Fig. 1 shows a single time step of interaction in a POMDP. All signals inside the dashed box are fully observed by the agent, while information regarding the state is only obtained via observations. The dashed line does not necessarily represent boundaries of physical embodiment: the state may refer to, e.g., the acting robot's joint angles. In a finite-horizon POMDP, the task ends after a specified number of time steps.

Formally, a finite-horizon POMDP is a tuple $\langle h, S, A, \Omega, T, O, R, \gamma, b_0 \rangle$ where $h \in \mathbb{N}$ is the time horizon of the problem, S, A, and Ω are the state, action, and observation sets, respectively, T is the stochastic state
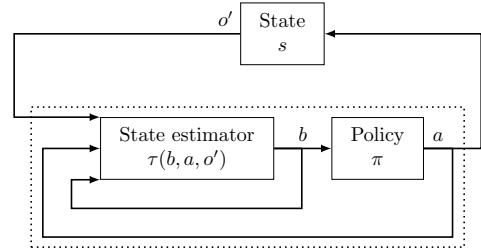

Fig. 1. A single interaction step of an agent (dashed box) in a robot system with a partially observed state s. Note that the state may include both the external environmental state and the internal state of the robot system. An action a is selected according to a policy π as a function of the current belief b. The action causes a transition from s to s', perceived by the agent via an observation o'. Finally, b, a, and o' are used to update the belief using a Bayes filter τ.

transition model, such that $T(s', a, s) := P(s' \mid s, a)$ is the probability that the next state[1] is s' given that the current state is s and action a is executed, O is the probabilistic observation model, such that $O(o', s', a) := P(o' \mid s', a)$ is the probability that observation o' is perceived if state s' was reached after executing action a on the previous time step, R is a bounded reward function, such that $R(s, a)$ is the real-valued reward obtained executing action a in state s, $0 \leq \gamma \leq 1$ is a discount factor that determines the relative value of immediate and future reward[2], and b_0 is an initial probability mass function (pmf) over states such that $b_0(s)$ is equal to the probability that the initial state of the system is s.

The state may include parts relating to an external environmental state and the robot system's internal state. Partial observability can exist in both of these parts. To manage the partial observability, the agent maintains an estimate of the system state b by applying Bayesian filtering. The estimate is a pmf over the system state, and is a sufficient statistic of the history of past actions and observations. As opposed to the history which grows over time, the estimate has a fixed representation size making it a desirable alternative. In the POMDP literature these estimates are referred to as *belief states*. The belief state at any time step is defined as the conditional probability distribution over the state given the history of past actions and observations. As defined above, the initial belief state before taking any actions or perceiving any observations is b_0. Given the current belief state b and action a and a resulting observation o', the updated belief state b' is obtained by Bayes' rule:

$$b'(s') = \frac{P(o' \mid s', a) \sum_s P(s' \mid a, s) b(s)}{\eta(o' \mid b, a)}, \qquad (1)$$

where the denominator is the prior probability of observing o', i.e., $\eta(o' \mid b, a) = \sum_{s'} P(o' \mid s', a) \sum_s P(s' \mid a, s) b(s)$. We shall use a shorthand τ for the Bayes filter defined via Eq. (1) as $b' = \tau(b, a, o')$.

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[1] We indicate quantities on the next time step by symbols such as s', and on the current time step by plain symbols such as s.

[2] The discount factor is not strictly necessary in the finite-horizon case, but we include it for generality and consistency with the infinite-horizon case.

A solution for a finite horizon POMDP is often represented as a sequence $\pi = (\pi_1, \pi_2, \ldots, \pi_h)$ of policies, where the policy π_t is a mapping from belief states to actions when t decisions remain until the end of the horizon. Executing a policy from belief state b when t decisions remain means to first take the action $a = \pi_t(b)$, perceive the observation o', and set $b' = \tau(b, a, o')$. Then, execute the policy from b' when $(t-1)$ decisions remain, now choosing the action according to π_{t-1}, repeating until no decisions remain. The expected total discounted reward collected when executing a policy quantifies its performance. An optimal policy maximizes the performance, as we will see next.

The optimal value function $V_t^*(b)$ is defined to be equal to the expected total discounted reward when the agent executes an optimal policy from belief state b when t decisions remain[3]. Clearly, $V_1^*(b) = \max_a R(b, a)$, where we use the shorthand $R(b, a) := \sum_s R(s, a)b(s)$ for the expected immediate reward of taking action a in belief state b. We can now recursively define V_t^* applying Bellman's principle of optimality [16]: an optimal action when t decisions remain must maximize the sum of the expected immediate reward and the expected future total discounted reward for the remaining $(t-1)$ decisions. Formally,

$$V_t^*(b) = \max_a \left[R(b, a) + \gamma \sum_{o'} \eta(o' \mid b, a) V_{t-1}^*(b') \right], \quad (2)$$

where $b' = \tau(b, a, o')$. As it is not known which next observation o' will be perceived after taking the current action, we take the expectation over o'. The value function V_t^π of an arbitrary policy π is characterized by a similar recursion $V_t^\pi(b) = R(b, \pi_t(b)) + \gamma \sum_{o'} \eta(o' \mid b, \pi_t(b)) V_{t-1}^\pi(\tau(b, \pi_t(b), o'))$, starting from $V_1^\pi(b) = R(b, \pi_1(b))$. Two policies can be compared in terms of their value functions. The optimal value function satisfies $V_t^*(b) \geq V_t^\pi(b)$ for all b, t, and π.

To characterize optimal policies, consider the action-value function $Q_t^*(b, a)$ defined as the expected total discounted reward when action a is executed in belief state b when t decisions remain, and an optimal policy is executed for the remaining $(t-1)$ decisions thereafter. We have $Q_1^*(b, a) = R(b, a)$, and for $t \geq 2$,

$$Q_t^*(b, a) = R(b, a) + \gamma \sum_{o'} \eta(o' \mid b, a) V_{t-1}^*(b'), \quad (3)$$

where $b' = \tau(b, a, o')$. The action-value function Q_t^π of an arbitrary policy π is obtained by replacing the value function on the right hand side of Eq. (3) by V_{t-1}^π. An optimal action is determined by $\pi_t^*(b) \in \arg\max_a Q_t^*(b, a)$, which maximizes the right hand side of Eq. (2).

B. Infinite horizon

Unending tasks can be modeled using infinite horizon POMDPs. We now briefly review their key concepts. Technical details and proofs are found, e.g., in [58], [121].

As the horizon h tends towards infinity, the discount factor is constrained to $0 \leq \gamma < 1$ to ensure the total discounted

[3]Many optimal policies may exist, but the optimal value function is unique.

reward is well-defined. In the infinite horizon case there is always an infinite number of decisions remaining until the end of the horizon. Informally, this is the reason why a solution of an infinite horizon POMDP is a so-called stationary policy, a single policy π applied on every time step.

The infinite-horizon optimal value function V^* is the unique fixed point of Eq. (2) when $t \to \infty$. Formally, $\lim_{t \to \infty} \sup_b |V_t^*(b) - V^*(b)| = 0$. Note the lack of subscripts in V^*, indicating its infinite horizon nature. The infinite horizon optimal action-value function $Q^*(b, a)$ is defined as in Eq. (3), replacing the value function on the right hand side by V^*. An optimal policy is defined via $\pi^*(b) \in \arg\max_a Q^*(b, a)$.

The finite horizon optimal value function V_t^* approximates V^* with a bounded error. In particular, let $\epsilon = \sup_b |V_t^*(b) - V_{t-1}^*(b)|$ be the least upper bound of the change between consecutive iterations of Eq. (2). Then, $\sup_b |V_t^*(b) - V^*(b)| \leq \gamma \epsilon / (1 - \gamma)$. That is, if we wish to approximate V^* with precision δ, we can take any V_t^* for which ϵ is less than $\delta(1 - \gamma)/\gamma$. Solution algorithms for infinite horizon POMDPs can exploit this fact. By solving V_t^* for a sufficiently large t and returning an optimal policy π_t^* we obtain an approximation with an error at most δ.

III. POMDP ALGORITHMS

POMDP solution algorithms compute optimal or approximately optimal value functions and policies. We review the two main categories of solution algorithms: offline and online algorithms. We close the section by discussing heuristics that are widely used for computing POMDP policies in robotic applications. We use the following classical POMDP problem as a running example.

Example (Tiger [34])**.** *An agent stands in front of two doors, one on the right and one on the left. A tiger is behind one door and a treasure behind another, but the agent does not know a priori behind which door the tiger is. The agent may open either door, gaining* 10 *reward for finding the treasure or* −100 *for finding the tiger. Alternatively the agent may listen, gaining* −1 *reward. When listening, the agent makes the correct observation about the tiger's location with probability* 0.85. *The problem resets to a uniformly random tiger location after opening either door.*

We have chosen the Tiger problem as a didactic example for its simplicity. While it lacks meaningful dynamics, which is common in robot tasks, it clearly illustrates the principle used in the POMDP framework for decision making under uncertainty. Another example with more realistic robot dynamics is the *Tag* problem, in which an agent's goal is to search for and tag a moving opponent [115]. Specifically, in contrast to the Tiger problem, the *Tag* problem includes the robot and opponent location as parts of the state that may change as a result of an action while also affecting the reward and the observation.

A. Offline algorithms

The objective of offline algorithms is to compute a policy for all possible belief states before starting policy execution.
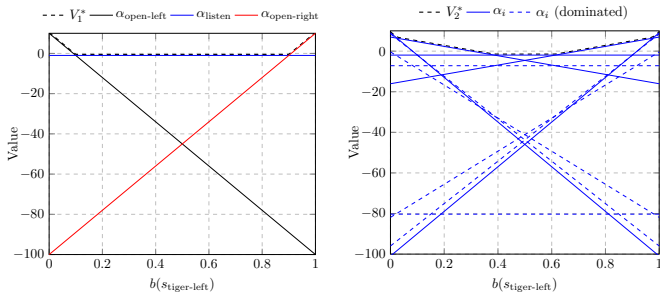
Fig. 2. Optimal value functions (dashed black lines) and their α-vectors (solid lines) in the Tiger problem. **Left:** the one-step optimal value function V_1^*. **Right:** the two-step optimal value function V_2^*. Only α-vectors drawn with solid lines are needed to represent V_2^*. Dominated α-vectors indicated by dashed blue lines do not affect V_2^* and can be pruned.

At execution time, an action is simply looked up from the policy computed offline. Offline algorithms are usually based on iteratively applying the Bellman optimality principle [6], [16]. Solving a POMDP for horizon h is recursively broken down into solving subproblems for horizons $h-1, h-2, \ldots, 1$, similar to the derivation of Eq. (2). We review basic principles of exact and approximate offline algorithms, and refer to the surveys [100], [166], [95], [58], [132] for further details.

The key property employed by offline POMDP algorithms is that the horizon-t optimal value function V_t^* is a piecewise linear and convex (PWLC) function of the belief state [139], [135]. Formally, V_t^* is the convex hull of a finite set of $|S|$-dimensional hyperplanes, also known as α-vectors:

$$V_t^*(b) = \max_{\alpha \in \Gamma_t} \sum_s \alpha(s)b(s), \qquad (4)$$

where Γ_t is the set of α-vectors that represents the horizon-t optimal value function. It is easy to see that $\Gamma_1 = \{\alpha^a | a \in A : \alpha^a(s) = r(s,a)\}$ since only the expected immediate reward matters. We may prove by induction that V_t^* for every finite t is also PWLC. Fig. 2 (left) shows the optimal one-step value function for the Tiger problem. In the tiger problem $b(s_{\text{tiger-left}}) = 1 - b(s_{\text{tiger-right}})$, so the belief is represented by a single real number corresponding to the probability that the tiger is behind the left door. As seen from the figure, the α-vectors corresponding to each action dominate in different regions of the belief space (the horizontal axis). For instance, if the agent is certain that the tiger is on the left, opening the right door is optimal.

Given Γ_{t-1} as input, exact offline algorithms iteratively calculate the set Γ_t of α-vectors representing V_t^* for any number of time steps t remaining. The solution for horizon $(t-1)$ is used as a building block of the solution for horizon t by applying the principle of dynamic programming. We show an example of a single iteration in Fig. 2, where we use the set Γ_1 illustrated on the left together with the POMDP model to compute the set Γ_2 illustrated on the right. By keeping track of the immediate actions associated with each α-vector, we can recover optimal policies π_t^* from Γ_t.

Exact offline algorithms are a suitable choice for small problems with up to a few dozen states. The complexity of the value function representation depends on the number

of actions and observations. In the worst case, the number of α-vectors required is $|\Gamma_t| = |A||\Gamma_{t-1}|^{|\Omega|}$, which grows rapidly as a function of t. However, not all of the α-vectors may be necessary to represent the value function exactly. An α-vector is *dominated* if it is not optimal for any belief state. A dominated α-vector can be removed from Γ_t without affecting the value function representation. Fig. 2 (right) shows examples of dominated α-vectors by the dashed lines. The step of identifying and removing dominated α-vectors is known as *pruning*, see, e.g., [100], [166], [95], [58]. Even with pruning, the growth in the number of α-vectors makes exact offline methods infeasible for problems with large action and observation spaces.

Point-based value iteration algorithms [132], [116], [136], [143], [82] are a class of approximate offline algorithms that represent the value function by maintaining one α-vector per each belief state in a finite set B of belief states. The total number of α-vectors required to represent a value function is bounded by $|B|$, which leads to computational savings compared to exact algorithms. Point-based methods are a useful choice in robotic tasks such as grasping [63] where the set of belief states that are *reachable* under a reasonable policy is only a small subset of the full belief space.

Point-based algorithms may select belief states to be included in B, e.g., randomly [143], by minimizing error bounds [136], [116], or to better approximate the set of reachable belief states [82]. Due to the PWLC property of the value function, we may still easily recover the value function at an arbitrary belief state from a point based representation [116]. We update the value function representation by an iterative procedure similar to the exact case, but only consider the belief states in B. While iterating, we may add new beliefs to B, and bound the approximation quality dependent on how densely B covers the entire space of possible beliefs [116]. Point-based algorithms can address significantly larger problems than exact offline algorithms by sacrificing optimality [116]. However, for very large problems, even point-based algorithms are not computationally tractable. For such problems, we can apply online algorithms.

B. Online algorithms

Online algorithms interleave policy computation and execution. The algorithms compute an optimal or approximately optimal action for the current belief state b_0, execute the action, make an observation, and continue by computing an action for the resulting new belief state. Since a solution is only computed for the current belief state b_0, online algorithms can address larger problems than offline algorithms. Online algorithms commonly work by searching over reachable belief states or equivalent action-observation sequences. We review forward search in online algorithms, and discuss modern sampling-based implementations that have led to major improvements in the scalability of POMDP algorithms to large problem domains. For a general survey of online algorithms, we refer the reader to [128].

Given a fixed initial belief state b_0, let us consider all possible action-observation sequences $(a_0, o_1, \ldots, a_{t-1}, o_t)$ of
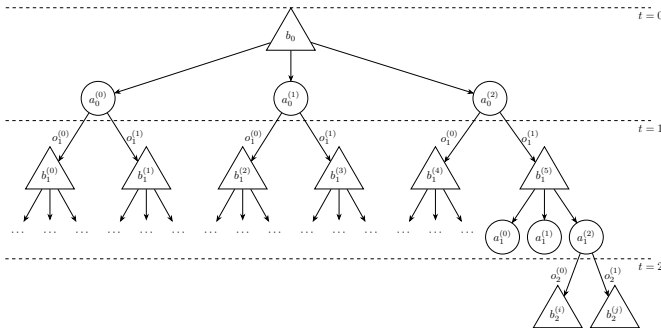
Fig. 3. A tree over action-observation sequences for the Tiger problem. At triangular decision nodes on time step t, the agent selects an action $a_t^{(0)}$ (open-left), $a_t^{(1)}$ (open-right), or $a_t^{(2)}$ (listen). At circular chance nodes, the agent receives an observation $o_t^{(0)}$ (hear-left) or $o_t^{(1)}$ (hear-right). Time steps are indicated by the section labels on the right hand side. Note that the time steps here denote steps since start of the task, *not* steps remaining. The tree is shown fully expanded until the observation at $t = 1$, and partially thereafter. In node labels, subscripts denote time step and superscripts in parentheses an index.
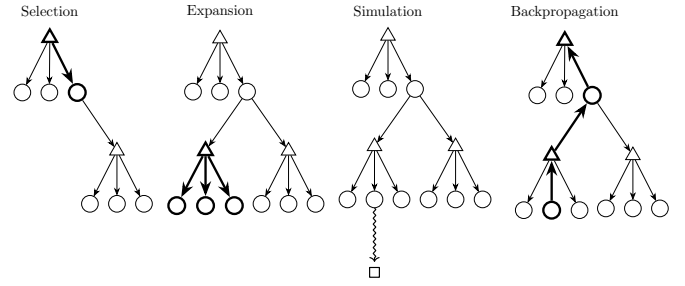

Fig. 4. POMCP repeats the four phases (from left to right) of selection, tree expansion, simulation/rollout, and backpropagation to build a sparse search tree over action-observation histories. Triangle/circle node shapes are as in Fig. 3. Annotations are omitted for clarity.

length $0 \le t \le h$. Such sequences are commonly referred to as *histories*. As shown in Fig. 3, we can build a tree over histories with the root node corresponding to b_0. Each triangular decision node corresponds to the history obtained by concatenating actions and observations along the path from the root to the node. The related belief states may be computed by repeatedly applying the Bayes filter τ.

Online algorithms build a search tree over histories, and return an optimal action to take at the initial belief state [128]. In practice, we use the tree structure to compute the optimal action-value function $Q_h^*(b_0, a)$. For example, consider calculating the optimal action-value of listening on the first time step in the Tiger problem. Contrasting Eq. (3) with Fig. 3 we see that the optimal action-value $Q_h^*(b_0, a_0^{(2)})$ at the root node depends on the expected immediate reward $R(b_0, a_0^{(2)})$ and $V_{h-1}^*(b')$, where b' is either one of the successor beliefs $b_1^{(4)}$ or $b_1^{(5)}$ reached after taking the listening action $a_0^{(2)}$ and perceiving $o_1^{(0)}$ (hear-left) or $o_1^{(1)}$ (hear-right), respectively. We have $Q_h^*(b_0, a_0^{(2)}) = R(b_0, a_0^{(2)}) + \gamma \sum_{o'} \eta(o' \mid b_0, a_0^{(2)}) V_{h-1}^*(b')$ with b' as above depending on o'. To proceed, we recall $V_{h-1}^*(b') = \max_a Q_{h-1}(b', a)$, which is computed by expanding the next layer of chance nodes under b'. This recursion terminates at the leaves of the tree. A naive online algorithm builds a complete search tree as explained above, computing belief states, expected reward, and observation probabilities to return an optimal action $\arg\max_a Q_h^*(b_0, a)$. Infinite horizon problems are tackled by searching over a finite horizon h that reaches the desired approximation quality as described in Sect. II-B.

The number of nodes in the $(t+1)$th level of the search tree is equal to the number of nodes in the tth level multiplied by the branching factor $|A||\Omega|$. Therefore, the tth level of the tree has $|A|^t|\Omega|^t$ decision nodes, corresponding to the number of histories of length t. This exponential increase of tree size as a function of its depth is a major challenge for online algorithms based on tree search. Several techniques have been proposed to address this challenge [128]. For example,

branch-and-bound pruning traverses actions in a descending order according to an upper bound and avoids expanding tree branches that are known to be suboptimal by using lower and upper bounds for the optimal action-value function. The expansion order of branches is sometimes prioritized based on heuristics. Expanding one branch of the tree may reveal that another branch is suboptimal and does not need to be expanded.

The need to explicitly compute the belief states associated with each node is also prohibitively expensive in problems with large state spaces. Instead of explicit belief state computation, state-of-the-art online algorithms for POMDPs draw state samples from the initial belief state and propagate them through the nodes of the search tree by drawing samples from the dynamics model and observation model to determine successor nodes. A Monte Carlo procedure using these samples is used to compute estimates of action-values.

In sampling-based online algorithms for POMDPs, e.g., [133], [138], [147], belief states are represented as collections of particles at decision nodes, and a simulator that allows sampling of the next state, reward, and observation is used for constructing the tree and estimating the action-value functions. It is sufficient to be able draw samples from the state transition and observation models, as no explicit belief state tracking by Bayes filtering is necessary during planning. As a simulator is sufficient for planning, these methods are very flexible and can also handle robotic tasks where an explicit model may be unavailable. The algorithms provide asymptotic optimality guarantees under appropriate assumptions: as the number of iterations tends towards infinity, the solution will approach an optimal solution.

Sampling-based online algorithms differ in their strategies for constructing the search tree. POMCP [133] relies on Monte Carlo tree search (MCTS; e.g., [25]) to iteratively build the search tree as illustrated in Fig. 4. In the selection phase, we traverse the current search tree by selecting actions using the upper confidence bound (UCB) multi-armed bandit rule that balances between exploration and exploitation. Once we reach a leaf node, we add new nodes to the tree in the expansion phase. In the simulation phase, we perform a simulation *rollout* to estimate the long-term value of the leaf node that was reached. In the backpropagation phase, we use the information from the rollout to update the action-value

estimates of nodes along the path from the root node to the leaf node. POMCPOW [147] also uses MCTS, with additional techniques applied for dealing with continuous action and observation spaces. In contrast, determinized sparse partially observable trees (DESPOT) [138] constrains the search to a finite set of randomly sampled *scenarios*, and builds a search tree that covers $|A|^t K$ nodes at depth t, where K is the number of scenarios. The random scenarios determine which branches of the search tree will be expanded.

Online algorithms may also search for a solution in a limited set, such as finite state controllers [13], [12], [110] or other parametric representations [97], [28]. These so-called policy search algorithms gain scalability by limiting the search space, but often lose the ability to bound solution quality.

We conclude by contrasting the reviewed online forward search algorithms to offline dynamic programming based solutions from the previous subsection. Early online algorithms [128] that build a complete search tree or use admissible heuristics to prune it with explicit computation of belief states will return an optimal solution to finite-horizon problems, similarly to exact offline algorithms. MCTS-based algorithms [133], [147] use a greedy heuristic in the selection phase to choose how to expand the search tree, and aggressively exploit the most promising branches of the search tree. They can produce search trees that are shallow in some parts and deep in other parts, potentially discovering solutions that require a very long planning horizon. On the other hand, they are also prone to getting "trapped" in deep branches of the search tree preferred by the heuristic but ultimately not useful, see, e.g., [103, Sect. 2.3.]. While the asymptotic optimality property guarantees the search will eventually avoid such traps, the number of samples required for this may be infeasible in practice. Online algorithms using scenarios [138] can avoid getting stuck due to poor choice of heuristics, but do not exploit promising branches of the search tree as aggressively. The scenario-based methods are therefore in the middle of the spectrum ranging from greedy MCTS solutions to conservative online algorithms that are more similar to offline dynamic programming methods.

C. Heuristics

Heuristics are computationally inexpensive ways to obtain a solution to a POMDP based on simple principles such as taking an action that yields the greatest expected immediate reward. Using heuristics comes at the cost of losing the approximation guarantees provided by offline and online algorithms reviewed above. We aim to provide the reader with a basic understanding of the trade-offs and assumptions underlying the most widely applied heuristics in the literature to find solutions to robotic POMDP problems. For a more comprehensive overview of heuristics, we refer the reader to [58], [18].

The greedy policy heuristic takes an action that maximizes the expected immediate reward: $\operatorname{argmax}_a R(b, a)$. It is quick to compute from the current belief state and reward function. However, it ignores any reasoning about long-term effects of actions. There is also no guarantee on how much worse the greedy policy is compared to an optimal solution.

The QMDP heuristic [93] first computes the optimal action-value function $Q^*_{\mathrm{MDP}} \colon S \times A \to \mathbb{R}$ of the underlying *fully observable* Markov decision process (MDP) [121] where the agent always perceives the true state. QMDP then computes an upper bound

$$\hat{V}(b) := \max_a \sum_s b(s) Q^*_{\mathrm{MDP}}(s, a) \tag{5}$$

for the optimal value function V^* of the POMDP. The QMDP control policy is then obtained by taking an action that maximizes the above equation. Eq. (5) defines $\hat{V}(b)$ as the best expected value in the MDP, when uncertainty about the state will disappear after the current action. Therefore, QMDP is not well suited for problems where explicit information gathering is required for success, but is computationally much cheaper than a full POMDP solution.

Open loop feedback control (OLFC) [18], also known as receding horizon control (RHC) or model predictive control (MPC), is a well-known method from stochastic control that can also be applied to finite-horizon POMDPs. OLFC computes an optimal *open loop policy*, i.e., a sequence of actions with the greatest expected return for the current belief state. Then, the first action in the sequence is executed, and an observation is perceived. A posterior belief state is computed using the Bayes filter, and the process is repeated. This interleaving of computation of open loop policies and observing feedback in the form of observations gives the method its name. There are many fewer possible action sequences than closed loop policies, making the problem computationally cheaper. The performance of OLFC is at least as good as that of simply executing the entire optimal open loop action sequence [18], but no error bound compared to an optimal solution is known.

IV. POMDPs in Robotics

This section discusses robotic application domains where POMDPs have been heavily used: localization and navigation, autonomous driving, search and tracking, manipulation, human-robot interaction, and multi-robot coordination. Each application domain has its specific challenges related to partial observability. How the POMDP model is specified depends not only on the application domain but often also on the specific application requirements. In addition, a large variety of different POMDP algorithms have been used.

Table I summarizes the most commonly used categories of algorithms in robotic applications: point-based, tree search, policy search, and heuristic. For each algorithm, we indicate application areas where it has been used. We also highlight key sources of uncertainty (state, dynamics, or perception) in each application area that POMDP planning techniques address. The last row of the table lists other less frequently used algorithms per application. For each entry on the last row, the first reference introduces the algorithm and/or application, while remaining references indicate applications. By necessity, the table only provides a rough overview and omits aspects such as hierarchical control, details on challenges related to a specific application, or techniques for dealing with high-dimensional observations. We provide further details in the following subsections, adding nuance to the general overview.

TABLE I
KEY SOURCES OF UNCERTAINTY AND COMMON POMDP SOLUTION ALGORITHMS BY APPLICATION AREA IN THE LITERATURE. THE BOTTOM ROW COLLECTS OTHER LESS FREQUENTLY APPLIED ALGORITHMS BY APPLICATION. ACRONYMS USED INCLUDE CROSS ENTROPY (CE), MONTE CARLO (MC), VARIATIONAL BAYES (VB), VALUE ITERATION (VI), POLICY GRADIENT (PG), POLICY ITERATION (PI), POLICY SEARCH (PS).

	Localization & Navigation	Autonomous Driving	Search & Tracking	Manipulation	Human-Robot Interaction	Multi-Robot Coordination
Key Sources of Uncertainty	*State* robot location, environment map	*State* locations of traffic agents *Dynamics* control effects, others' behaviors *Perception* poor weather, occlusion	*State* target location *Dynamics* target behavior	*State* object pose *Dynamics* grasp success/failure *Perception* visual ambiguity, occlusion	*State* human intention	*State* information private to each robot
Point-based						
SARSOP [82]			[56], [7]	[101], [62], [81]	[130], [38], [113]	
PBVI [116]				[63]	[8], [9]	
HSVI [137]			[144]	[26]		
Perseus [143]					[125]	[141]
S-Perseus [119]			[32], [142]		[60], [150]	[32], [142]
Tree search						
POMCP [133]	[89],[75]	[146]	[160], [170], [162], [52], [175]	[170]		[52]
DESPOT [138]	[91]	[10], [147], [120], [96], [30], [65]		[92], [49]	[154], [155]	
ABT [83], [76]		[66]	[158]			
Policy search						
MCVI [13]	[54]		[11]			
PPGI [110]				[109], [110], [111]	[59]	
Random shooting	[42], [97]					[164]
Heuristic						
QMDP [93]	[78],[75]	[165]				[141]
Greedy			[46], [156]			[3]
Others	*Pre-image back-chaining* [70], [71] *FIRM* [1], [2] *MEDUSA* [68] *PLGRIM* [75] *rAMDL* [79] *BPS* [163] *GBS* [67]	*Continuous MCVI* [23], [24] *POMCPOW* [147] *MC on predefined policies* [140] *Tree search* [152] *MPDM* [41]	*PG* [28], [174] *MAA** [149], [86] *Greedy PBVI* [129] *Bayesian Opt.* [97] *Nominal belief* [99] *Search tree VI* [172] *VB-PBVI* [29] *Unknown* [48]	*Pre-image back-chaining* [70], [71] *Most likely state* [64] *POMDP as set of MDPs* [37] *Offline tree policy* [177]	*Oracular POMDP* [5], [126] *LSPI* [84], [164] *Exact* [117]	*Nominal belief* [99], [122] *Tabular VI* [16], [98] *Dec-MCTS* [19], [145] *PG* [28], [174] *MAA** [149], [86] *Masked MC, Graph CE* [107] *Search tree VI* [172] *Heuristic PS* [4] *Approx. PI* [20] *DSGA* [40]

A. Localization and Navigation

Self-localization and navigation are basic functionalities of mobile robots. A warehouse robot needs to be aware of its current location and be able to navigate to the correct place to pick up the next batch of goods. In mining, machines need to navigate in dark underground tunnels and cannot rely on a GPS signal. Outdoor mobile machines need to cope with changing weather and lighting conditions in addition to environmental occlusions. Localization and navigation in cluttered, non-stationary environments with partially occluded visibility and noisy sensors can be challenging. POMDP modeling is thus a natural choice in localization and navigation.

a) Key sources of uncertainty and challenges: In localization and navigation, a key source of uncertainty is the location of the robot which cannot be directly observed due to noisy sensors and sensors that observe only parts of the environment. Another source of uncertainty is the composition of the environment which may not be known a priori.

Localization and navigation is often split into localization, path planning, and path following [78]. When localizing the robot with respect to its surroundings, the environment and a map of it can be either known beforehand or constructed from observations by simultaneous localization and mapping (SLAM, e.g. [45]). Path planning creates a sequence of robot configurations in order to gain more information about the environment or to reach a given goal location while avoiding obstacles and dangerous areas. Path following executes the planned sequence of configurations, taking into account robot dynamics and changes in the environment or task that may occur while traversing the path.

In simple localization and navigation POMDPs, a 2-D point

mass representation can be used. In more complex tasks, we also need to consider the position, orientation, and velocity of the robot. Mobile ground robots often plan paths in 2-D in contrast to path planning in 3-D space in robot manipulation (see Section IV-D). Perceptual aliasing [47] is common due to occluding objects or variable terrain, and leads to multi-modal belief distributions. For POMDP modeling, localization and navigation can be challenging due to continuous states, actions, and observations of which the observation space can be large. Moving obstacles, a dynamic environment, and changing task goals may increase the difficulty of the task. Due to the high uncertainty in observations, localization and navigation is a challenge that has inspired many classical POMDP benchmarks [93].

b) Solution methods: POMDPs are used for both high and low level planning in localization [78], path planning [78], and path following [2]. The environment is often modeled as a set of states including the position and the heading of the robot, the location of obstacles as well as the start and goal locations. It is common to both discretize large continuous state, observation, and action spaces, and to use hierarchical models for the long planning horizons.

Among the first to propose to use a POMDP for indoor navigation, Koenig and Simmons [78] automatically generate the POMDP transition and observation probability distributions from a topological map of the environment and prior observation and dynamic models. To discretize the problem, they map the robot pose to a non-uniform grid, use a discrete set of actions, and map observations to semantic information such as "door open", "wall", or "door closed". They update the POMDP dynamics and observation models using expectation-maximization (EM) [44], [102] to account for uncertainty in the prior models. EM updates the models of the map, sensors, and actuators using observed sequences of actions and observations during task execution. For choosing actions the POMDP model is used at each time step to estimate the belief over possible pose states using previous sensor readings and executed actions. A Gaussian Process [124] based POMDP model for blimp height control with continuous states, actions, and observations is learned by [42] during task execution. In [68], the robot queries a human operator to learn the dynamics and observation models. The queries and when to initiate them are decided by the POMDP policy. The complexity of the POMDP model is reduced by limiting the available set of actions in parts of the belief space in [53], [54]. This helps to simplify the planning task, for example by reducing the car-like dynamics of a robot to rigid body dynamics.

Hierarchical POMDP models (e.g., Fig. 5) are used for long horizon planning and complex dynamics found in navigation tasks. The POMDP is split into high level and low level planning, which makes the planning horizon for both levels shorter compared to planning entirely on the low level. For example, a high level policy may direct the agent from one waypoint to another, and the low level policy decides how to travel between the waypoints. Near real-time replanning is possible, since the problem is split into computationally manageable parts. In [2] a UAV delivers packages while optimizing mission level
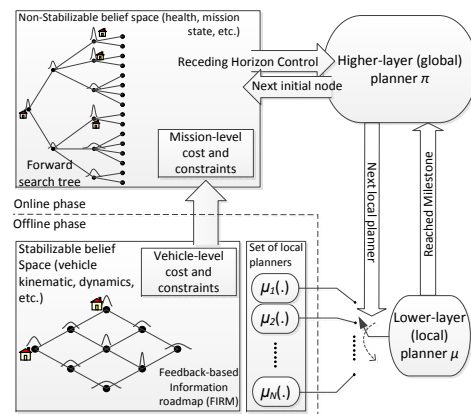

Fig. 5. In localization and navigation, hierarchical control is typical (Figure from [2]). Hierarchical control can make planning more efficient, for example, a high level policy directs the robot from one waypoint to another and the low level policy decides how to travel between the waypoints.

statistics such as battery status, or sensor and actuator health. The UAV uses a high-level graph similar to probabilistic roadmaps [73] for navigation, and a low-level controller to traverse between the nodes of the graph. A two-layer planner is used in [67]. An inner layer predicts posterior beliefs for control actions, and an outer layer uses the predictions to infer a control policy. No discretization is applied, and solutions are found by MPC. Temporally extended open loop sequences of actions, called macro-actions, are used in [91]. They train a generator to output macro-actions, and show that the temporal abstraction helps to address long planning horizons.

Robot navigation can also be framed as a satisfiability problem. The goal is to find a policy that satisfies a set of constraints which guarantee that only belief states considered safe are reached. A satisfiability based safe navigation approach is demonstrated by [163] in simulation with a PR2 robot and uncertain obstacles.

Task and motion planning (TAMP) typically consists of localization, navigation, and manipulation (for manipulation, see Section IV-D). A symbolic pre-image back-chaining [70] based POMDP approach for TAMP is proposed in [71]. Logical expressions describe sets of belief states, and are used as goals and subgoals for POMDP planning. Interleaving high-level planning for information gathering with execution helps the robot to effectively use any newly gained information. Other recent approaches to TAMP which also address partial observability include [114], [50]. These methods may explicitly plan for intelligent information-gathering, which is necessary for many real world tasks.

In SLAM, the belief state captures the uncertainty in the robot pose and environment. *Active SLAM* can employ a belief-dependent reward function that encourages information gathering about the state. For example, [97] uses a Gaussian Process approximation to model how the choice of policy parameters influences the uncertainty of the future belief state. In [89], a mobile ground robot explores an environment and estimates an occupancy grid map by choosing short primitive trajectories as actions. The reward is based on the expected future information gain about the map. To handle the large
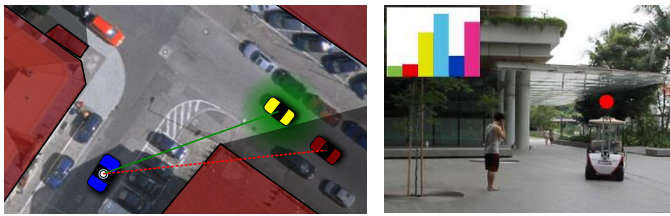
Fig. 6. **Left:** In autonomous driving the agent has to typically reason about occluded vehicles and pedestrians (Figure from [24]). **Right:** The histogram in the top left part shows the belief about intentions of human drivers (Figure from [10]).

discrete observation space, a variant of POMCP combined with MPC is applied. A factor graph belief representation is used in [79], with a Gaussian approximation in the planning stage. The objective is to select a trajectory that leads to a posterior belief with the lowest uncertainty. New landmarks discovered during exploration are added as new state factors. The belief-dependent reward is a natural way to encourage the robot to take actions that reduce future uncertainty. However, additional care is needed to avoid getting stuck in locally optimal solutions due to the limited planning horizon. For example in [89], if no sequence of actions yields a high enough information gain, a frontier-based method selects the next exploration target instead of POMDP planning.

The DARPA Subterranean Exploration Challenge is an interesting example of large-scale real-world application of autonomous localization, mapping and navigation in an unknown environment [31], where POMDP modeling is a natural choice. The PLGRIM [75] framework follows the hierarchical approach to make planning over long time horizons feasible in this context. Locally, PLGRIM uses POMCP to cover the environment search space efficiently. An entropy based objective is used to reduce the uncertainty over possible environments. Globally, PLGRIM employs QMDP allowing for the robot to plan how to reach new areas. QMDP makes the strong assumption that after the first time step the environment is fully observable. This allows for efficient long horizon planning on the global level, but is appropriate only if belief uncertainty in the future can be ignored. Follow-up work [112] avoids using QMDP at the global scale.

B. Autonomous driving

One of the earliest success stories of autonomous driving was the Autonomous Land Vehicle In a Neural Network (ALVINN) project at Carnegie Mellon University [118]. ALVINN trained a neural network to map input images to discrete actions to follow a road. Autonomous driving relies on state estimation, predicting dynamic objects and controlling the car to prevent accidents. In autonomous driving, a POMDP may cope with partial observability due to occlusions caused by the environment and vehicles and uncertainty of human driver and pedestrian intentions. The ability in POMDP planning to reason how current actions affect future uncertainty is important, since a purely reactive control approach may not be able to anticipate a dangerous situation sufficiently early.

a) Key sources of uncertainty and challenges: In autonomous driving, the key sources of uncertainty include other traffic participants such as other vehicles or pedestrians and the environment. The intentions of other traffic participants are commonly partially observable. Parts of the static environment such as buildings and dynamic objects such as other traffic participants block parts of the view of the autonomous vehicle causing partial observability. In autonomous driving, coping with the uncertainty is crucial to prevent accidents.

Similarly to other tasks with navigation, hierarchical control is common in autonomous driving. POMDPs are used in autonomous driving for selecting high-level actions such as lane changing, distance keeping, or overtaking other cars that facilitate safe and efficient driving. Low-level controllers for acceleration and steering are responsible for the execution of these high-level actions. The autonomous driving domain is closely related to the navigation domain (see Section IV-A) with continuous states, action, and observations. Autonomous driving is also related to target avoidance (see Section IV-C) where other vehicles must be incorporated into the dynamics model to predict and avoid collisions. Due to this the state space must be augmented with information about other traffic participants, often including their position and velocity [165] or their internal state including intention and aggressiveness [146]. Modeling of other traffic participants connects autonomous driving with human-robot interaction (see Section IV-E). The reward function in autonomous driving often consists of multiple possibly contradictory objectives. The reward function often assigns negative reward for collisions and positive reward for reaching a target position. Goals such as fuel efficiency can also be taken into account.

b) Solution methods: In the autonomous driving domain, continuous state, action, and observation spaces are often discretized [151] to make computation tractable when applying discrete POMDP methods. Measures to allow for tractable solutions for complex autonomous driving problems include short planning horizons, sparse action alternatives, ruling out illegal or impossible actions, mixed observability, and variable time granularity planning [152].

Many POMDP approaches in autonomous driving focus on a specific subtask. A velocity controller for a car merging into traffic at a T-crossing is developed in [24]. The car follows a pre-defined path, while its view of the environment may be blocked by objects such as buildings or other cars and trucks, see Fig. 6 (left). The position and orientation of dynamic vehicles are modeled as continuous state variables. Observations specify the continuous state of perceived dynamic objects and denote occluded objects as hidden. Velocity control is discretized. The belief state and value function are represented by using a set of samples, resulting in a dynamic discretization of the state and observation spaces. Safe lane changing is addressed in [41], where the agent needs to take into account the behavior of other cars. A state and observation variable are used for each car resulting in large state and observation spaces. Instead of applying an approximate POMDP algorithm, two simplifying assumptions are used to tackle the resulting complexity: 1) a finite set of known policies is sufficient for all vehicles, 2) vehicle dynamics and

observations are deterministic and evaluation using forward simulation is sufficient. These assumptions reduce the decision making problem into choosing from a set of high-level behaviors. However, the scalability of this approach is limited as the number of possible behaviours grows exponentially in the number of traffic participants n. With two possible policies per participant, there are 2^n possible combinations of policies to evaluate.

The MODIA framework [168], [167] proposes a solution for an autonomous vehicle to navigate an intersection. Each external traffic participant is represented by a dynamically instantiated POMDP. Actions for each POMDP are inputs to an executor module responsible for returning the action for the autonomous vehicle. MODIA navigates intersections faster than a naive and cautious policy, with fewer unsafe situations than a baseline that does not model other traffic participants.

The intersection scenario where multiple human drivers arrive at an intersection and the agent needs to decide when to proceed is considered by [140]. The agent predicts the intention of each human driver to safely proceed across the intersection. In [140], the underlying model is a POMDP although a POMDP solver is not used. Instead, a hidden Markov model infers the most likely intention given continuous observations of the relative poses of each human driver. The agent selects the policy yielding the highest cumulative reward by simulating a predefined set of possible policies. This approach cannot take information gathering actions into account, and so does not perform well when there is a need to actively reduce uncertainty about the intention of the human driver prior to acting. A collision avoidance scenario at an intersection is also considered in [66]. The agent controls car acceleration using a POMDP while driving on a predefined path. The state space contains the continuous states of all vehicles. The observation space contains a discrete path choice for each other car. Aggregating small variations in trajectories to a set of path choices reduces the number of possible observations, allowing scaling to more complex intersections.

Driven by the need to reduce the computational cost, a two-level hierarchical planner is used in [96] and [10] for intention-aware autonomous driving among pedestrians, see also Fig. 6 (right). The planner takes the global intention of pedestrians and their local interactions into account. For efficiency reasons states with mixed observability are used where the position and velocity of the pedestrians are considered fully observable but their intentions are only partially observable. To avoid collisions with pedestrians, [120] combines deep reinforcement learning (DRL) with a POMDP solution. [120] uses DESPOT as the underlying POMDP algorithm which then uses state/action value functions, that is, Q-functions from DRL as leaf node approximations. The authors in [65] model the decision making of a car which is uncertain whether a pedestrian intends to cross a street as a POMDP. The pedestrian being either "reckless" or "cautious" and their hidden intent influence the transition probabilities. The demonstrated strength of the POMDP approach is that the computed policy gathers information about the pedestrian's intent when it is considered valuable. The policy could also leverage the autonomous vehicle's own actions to influence
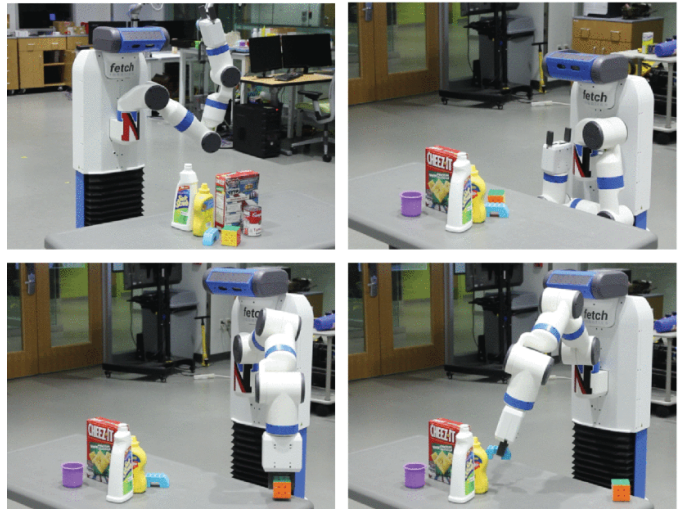

Fig. 7. POMDPs are applied to plan sequential strategies for tasks such as object search which require balancing between information gathering and exploiting current knowledge. **Top left**: the robot is searching for the target light blue toy brick, but it is not visible. **Top right**: moving to a different vantage point, the robot finds the target object blocked by the multicolored cube. **Bottom left**: the robot has pushed the cube out of the way. **Bottom right**: The target object is grasped. (Figure from [170])

pedestrian behavior.

C. Target search, tracking, and avoidance

Robotic applications involving target search, tracking and avoidance include aerial photography, search and rescue, and autonomous flight. While tracking or searching for a target, the robot also needs to navigate and localize, familiar tasks from Section IV-A. Furthermore, the robot needs to model target behavior and sensor uncertainty to reason about how to gather more information about the targets (search, tracking; see Fig. 7) or how to avoid the targets (avoidance).

a) Key sources of uncertainty and challenges: Target behavior is highly domain specific and difficult to model and predict, making it an important source of uncertainty in tracking, search, and avoidance problems. Furthermore, targets may be tracked using sensors that produce high-dimensional observations, for example cameras [97], [142], [144], [174], [7], [156], [160], [170], [162]. It is challenging to determine probabilistic models for target behaviour and high-dimensional observations that are useful for planning while being at the same time computationally feasible.

The state of a single target is often discretized to a finite-dimensional grid, see, e.g., [48], [99], [32], [129]. The number of grid elements equals the number of possible states the target may be in. If multiple targets are tracked, the size of the state space is equal to the product of the individual grid sizes. This leads to an exponential growth of the state space size in the number of targets, leading to computational challenges in solving the related POMDP.

b) Solution methods: Hierarchical approaches including multiple levels of decision-making and control have been widely applied in target search, tracking and avoidance [48], [144], [32], [174], [158], [52], [172], see also Section IV-A

for hierarchical approaches in localization and navigation. POMDP models are applied to plan a strategy for tracking a target using an unmanned aerial vehicle in [158], while flight control is delegated to a low-level controller. A POMDP planner produces waypoints for a robot in [32], [52], deferring the traversal task to a lower level controller. Hierarchical POMDPs for visual problem solving are suggested in [144], [174], and a tree with multiple levels of progressively finer-scale POMDPs is considered in [48]. A hybrid approach that uses an MDP model when a target is visible and a POMDP model when not is proposed in [172]. Hiearchical methods can address long planning horizons by planning on a higher level of abstraction.

Discretization of the target state allows the application of discrete POMDP solvers to tracking and search problems. Grid-based discretization is used in, e.g., [48], [99], [32], [129]. State space factorization is used for object oriented POMDPs in [160], [175], and mixed observability – the fact that some factors of the state may be fully observable – is exploited in [108], [56].

Others directly solve continuous-state tracking and search problems [27], [97], [11], [29]. Multi-modality in dynamics that differ depending on the state is considered in [27]. In [97] Bayesian optimization is applied to directly search for policies while simulation rollouts are used to evaluate policies. For unmanned aerial vehicle collision avoidance with a continuous-state formulation, [11] proposes to use Monte Carlo value iteration. Up to ten continuous state space dimensions in a search problem are handled by the recent variational method that parametrizes the POMDP using Gaussian mixtures [29].

Using image data with POMDP methods has been proposed for object search [174], [160], [170], [162], [175], tracking [142], exploration [97], [156], query answering [144], and object pose estimation [46], [7]. Instead of using images directly as observations in the POMDP which would be computationally prohibitively expensive, the images are abstracted into a low-dimensional representation. For example, in [7] a set of nominal object poses are applied as a basis for estimating arbitrary poses, in [170] observations are formed by detecting objects by point cloud segmentation, and in [97] the landmark detections extracted from the image are used as observations. In [175], a spatial abstraction is obtained via a hierarchical belief representation, where deeper levels represent the environment at a progressively finer resolution. However, similar to [160], dependencies between objects are not considered, and a POMDP problem must be solved for each level of the hierarchy in parallel.

Observations allow estimating the current belief which is then used for decision making. A belief-dependent reward function can be useful for search tasks where the objective is to reduce uncertainty. For example, the entropy of the belief state quantifies the robot's uncertainty about the state. Negative entropy as reward has been applied in the discrete state [61], [174], [86], [129] and continuous state [46], [156] settings. Visual object search is addressed by the top level of the hierarchical method in [174] using the policy gradient planner of [28]. The greedy policy heuristic with simulation-based estimation of expected reward is used in [156]. Cooperative target
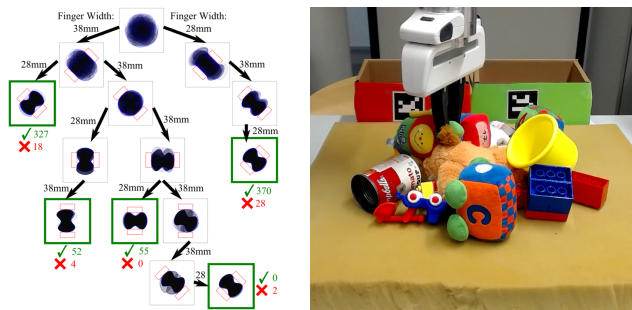

Fig. 8. **Left**: A policy for grasping an object in various configurations to reduce object shape and pose uncertainty (Figure from [177]). **Right:** In multi-object manipulation, occlusions and uncertainty on object identities may result in high partial observability [111].

tracking is treated in [86], computing belief state entropy on a discretized representation projected from a continuous-state tracking filter. Reward submodularity has been exploited in single-robot [129] and multi-robot [61], [40] tracking and exploration to develop approximation algorithms. The advantage of methods using submodularity is that they have performance guarantees for computationally cheap greedy policies.

D. Manipulation and grasping

Robotic arms have been used in the industry since the early 1960s for manipulation and grasping in tasks such as assembly, polishing, and painting. In fact, robotic manipulation is one of the cornerstones of the modern manufacturing industry. In the classical industrial setting uncertainty and partial observability is typically eliminated by exact modeling of the task from start to finish. However, in unknown, unstructured and cluttered environments, using noisy sensors, in collaboration with humans, or in tasks with hard to model objects such as textiles [101] taking uncertainty and partial observability into account is crucial and a POMDP is the de facto model.

a) Key sources of uncertainty and challenges: Robotic manipulation typically occurs in limited space that contains multiple objects including one or more robot arms. Cameras, static or mounted, and tactile sensors are often used for sensing. A key source of uncertainty are occlusions due to objects and robot arms partially blocking the view of objects. Another source of uncertainty are the a priori unknown physical properties and identities of objects. The main challenge is how to manipulate unknown objects with uncertain dynamics based on partially occluded noisy sensor readings.

In more detail, a robot often manipulates objects on a flat surface such as a table. The robot controls the joints of a robot arm or the fingers of a robotic gripper. POMDPs are used for task planning where high level actions indicate which object to manipulate and what kind of manipulation to perform, such as picking up or moving an object. Each object may have different attributes such as color and position that define the state of a single object. However, the number of different combinations of object attributes, and thus the whole state space, grows exponentially with the number of objects.

Observations are typically recorded by imaging sensors observing the robot's workspace. Due to the high dimensionality of image input, the images are usually preprocessed

and mapped to individual objects. In manipulation, partial observability is a result of noisy visual or tactile sensors and occlusions in the environment.

b) Solution methods: In robotic **grasping**, especially when based on tactile sensors [63], [81], [177], the poses of the grasped object and the gripper are partially observed. To handle the naturally continuous action and state spaces, in early work using POMDPs for robotic grasping with tactile feedback, e.g., [63], a model of the environment is used where movements of the robot hand are constrained to a discrete set and the state space is divided into discrete cells. The agent is able to compute a policy using a binary observation signal from its tactile sensor. Scaling to larger problems was later demonstrated by selecting trajectories from a candidate set [64]. In [177], an offline planner solves for a tree policy used online to infer the shape of an object to be grasped based on tactile data, see Fig. 8 (left).

Aligning the fingers of a multi-fingered robot hand and non-prehensile manipulation is used in [62]. The robot pushes a poorly observable object such as a small nut on a table to localize it, after which it can be grasped. To compute a policy, SARSOP is applied on a specifically designed state space where the continuous 2-D location of the object is discretized finely around the estimated object location. Further away from the estimated object location the location is only coarsely discretized to represent the relative displacement from the current object location estimate.

Taking the full grasping motion into account allows for moving the robot hand close to an object and then locating and grasping the object using tactile sensors. In [81], the grasping motion is split into a pre-grasp open-loop movement which is optimized using the A*-algorithm, and into a post-grasp closed-loop POMDP based push-grasping approach which uses partially observed tactile sensors. It is shown that the full POMDP solution outperforms an approach based on the QMDP heuristic, suggesting a closed loop policy is needed to be able to gather necessary information during the task.

A POMDP model for tasks where parts of the system dynamics are fully observable but where state variables are either constant or change deterministically is presented in [37]. The model corresponds to choosing a fully-observable MDP according to a hidden discrete state variable. The model is evaluated in a simulated robot manipulation task where the robot needs to grasp a cup with an uncertain initial position on a planar surface using binary tactile sensors. The position of the robot arm is fully observable and deterministic. The approach discretizes actions, observations, and states.

In another line of grasping research, POMDP solvers and deep learning are combined [49]. First, data are generated by planning grasps on a set of objects using the DESPOT POMDP algorithm. Second, an imitation policy that mimics the behavior of the POMDP policy is learned in a supervised manner. It is empirically shown that the imitation policy generalizes to novel objects not present in the training data.

Multi-object manipulation. Multi-object manipulation requires reasoning about multiple objects and their interactions concurrently [101], [110], [109], [92], [170], [111]. In multi-object manipulation, a POMDP model takes into account partial observability due to sensor noise and objects occluding other objects. Similarly to target search, tracking, and avoidance in Section IV-C a main challenge in multi-object manipulation is the exponential growth of the state space with the number of objects. Techniques such as image rendering or physics simulations are available to model observations and scene dynamics, but it is challenging to find models that are amenable to a POMDP formulation while remaining computationally tractable in practice.

One approach is to approximate states, actions, and observation with discrete values by taking advantage of the inherent properties of the actual task such as cloth separation [101]. A cloth may be removed when it is not entangled with other clothing items. A state space model that counts the number of clothes in two different areas on a table is used, resulting in a POMDP with few states, observations and actions. There are two actions for removing clothes from the table and 20 actions for moving objects from one pile to another chosen using different robot hand finger configurations and visual features.

Other works such as [110], [109], [92], [170], [111] model each object as a separate state variable. A robot observing several objects resting on a table with an RGB-D sensor is considered in [110]. Each state variable specifies the properties of an object such as color or location. An action lifts or moves an object. Observations include information about properties of the objects behind the moved or lifted object. The RGB-D scene is segmented into objects, and the amount of mutual occlusion is estimated. The occlusion information is applied to estimate grasp success probabilities and observation probabilities for the POMDP model to be solved.

In [92], the goal is to find objects from a set of possible known objects resting on a shelf. The location on the planar surface and the orientation of the objects is discretized. Discretized actions allow moving an object onto the shelf, off of the shelf, and tagging the target object. Observations provide the properties of each object in the scene. Similarly to [110], observation probabilities depend on the amount of occlusion which is estimated by projecting objects on top of each other.

Object search using a POMDP model that also considers completely occluded objects is tackled in [170]. The correct segmentation of known object models is assumed to be known, with some uncertainty in object locations. Actions and observations are discretized. Due to the large state space a particle representation is used for the belief, similar to [110], [92]. To generate each belief particle [170] selects object locations from the segmented scene and adds sampled noise. Fully occluded objects are sampled in the occluded volume behind visible objects. A variant of POMCP is used to compute a policy. Fig. 7 shows an example execution of the search policy.

With cluttered unknown objects the RGB-D segmentation of a scene and the number of objects is uncertain, see Fig. 8 (right). Scene uncertainty is incorporated into the POMDP belief in [109], [111], allowing planning of manipulation actions over object hypotheses. Each particle in the belief representation corresponds to a specific set of hypothetical objects. The robot may attempt to grasp and move a hypothetical object. The outcome of the grasp attempt provides information about the actual objects to update the belief.
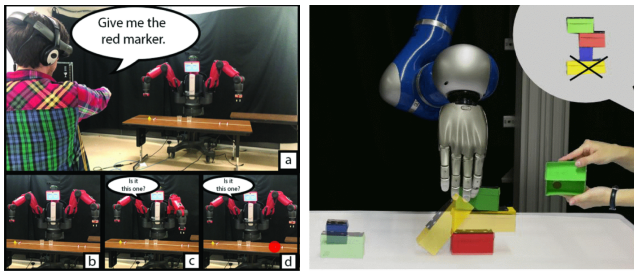
Fig. 9. **Left:** Human-robot interaction uses POMDPs to infer human intent from multimodal interaction data such as gaze, speech, and gestures (Figure from [125]). **Right:** Planning bi-directional communication with a human (Figure from [59]).

E. Human-robot interaction

Human-robot interaction (HRI) is necessary in domains ranging from industry to healthcare. In HRI, a robot typically needs to reason about possible human behavior in order to perform the assigned task efficiently. No sensor exists to directly measure human intentions or mental state, which must therefore be inferred from other data. Furthermore, humans may make decisions based on different sensory input than the robot. POMDPs are used in HRI to make decisions with these kinds of uncertainties in a principled way, see Fig. 9 for examples. The ability to model information gathering allows POMDPs to be used in tasks requiring communication and other kinds of interaction which can be crucial in HRI.

a) Key sources of uncertainty and challenges: A key source of uncertainty specific to HRI is the partially observable human state or intention. A model of human behaviour can help ensure safety and human comfort while the robot performs its tasks. Such models often consider quantities related to the mental state of a human, characteristics such as competence level, and the physical pose and location of a human. The mental state of a human is often defined in terms of discrete states and connected to human models in psychology [60], [150], [38]. In physical human-robot interaction, the pose and location of a human is inherently continuous valued [164] and may require further approximation techniques. While presence of humans increases modeling complexity, humans can also provide information to the robot through communication. By considering communication as a means to gather information, POMDPs are applied to reason about and plan communication [5], [8], [9], [59], [155].

b) Solution methods: In early work on using POMDPs for decision making in HRI, the robot assistant Pearl was deployed in a nursing home [117]. Pearl navigated the nursing home, guiding residents to appointments while answering queries, e.g., concerning the current weather. Due to the intractable complexity of exact POMDP solutions [33], Pearl factors its belief into probability distributions over the location and status state variables. For computational efficiency, the hierarchical POMDP chooses a combination of high and low level actions at each time step. Pre-defined high-level discrete actions such as "Inform" or "Move" are each associated with a set of lower level actions such as "SayTime", "SayWeather", or "VerifyRequest". Pearl demonstrated the feasibility of

POMDPs for high-level decision making in an autonomous robot assistant. Taking into account the needs and capabilities of human partners, e.g., adapting to different walking speeds, was noted to be crucial for successful interaction.

Several POMDP models incorporate latent state variables representing the mental state of the user [60], [150], [38]. Latent state variables can capture both long and short term mental states. In [60], POMDPs are used for monitoring the mental state of elderly people with disabilities and providing assistance in tasks such as hand washing. The system tracks the hand washing of people with dementia and displays messages to the user when needed. The POMDP model consists of discretized observations, a discrete action set of possible messages, a special action for calling a caregiver, and latent state variables that reflect both mental state and hand washing phase. Mental states can be also used in shared-control. The POMDP HRI framework in [150] uses the independence between certain mental states for an efficient algebraic decision diagram [119] based POMDP implementation enabling shared-control in a wheelchair task. While modeling the internal state of a human [60], [150] can yield a comprehensive model, a simpler approach is taken in [38] to model human trust level in a single latent state variable. The dynamics for the state variable are learned from user interaction data which allows predicting when the human would intervene with the robot's table clearing task.

In contrast to mental state models, in [164] the robot models the user as part of a table tennis environment and predicts the current target of the ball for the user's hitting motion. The dynamics and observations are modeled using a Gaussian Process. The robot executes a binary action at each time step to either initiate a table tennis movement or to wait. In [164], the methods used for POMDP computation include Least-Squares Policy Iteration (LSPI) and a model-based random shooting Monte-Carlo method.

Human-robot communication can be crucial in HRI. Some works plan when to ask humans for further information [5], [126], [59] while others focus on bi-directional communication [125], [155]. In the Oracular POMDP (OPOMDP) model [5] the robot may ask a human for full state information at a cost, but does not otherwise receive information. Computational speed-ups are achieved by splitting the task into information gathering oracular actions and domain-level actions. The OPOMDP model is extended by [126] to Human Observation Provider POMDPs (HOP-POMDPs) where human answers are stochastic. The POMDP of [125] models human-robot interaction via speech, gestures, and eye gaze. Instead of asking for full state information, in [59] the robot asks task-specific questions, such as information about an object. The task goal is specified as a set of logic sentences by a human using a graphical user interface. In [59], the PPGI planner [110] is extended to maximize the probability of satisfying the logic sentences. In a shared workspace task with a latent state model for the human teammate's behavior, [155] uses a POMDP to decide which type of communication the robot should engage in. The approach relies on a predefined communication cost model, a human response model, and a human action-selection model with mental states. For policy

optimization, [154], [155] modify the Regularized-DESPOT algorithm [138]: planning is performed during action execution for all possible observations, and the actual perceived observation is then used to choose the next action.

In HRI and in POMDP based robotics in general, the POMDP model is typically specified by an expert either fully or partly. In [8], [9], the robot learns parts of the POMDP model from a user. Cases with an unknown reward function [8] or unknown observation model [9] have been considered. Both works [8], [9] represent the distribution over POMDP parameters drawn from a prior distribution as a finite set of hypothetical POMDPs. At each time step the user provides an action which is executed. The robot updates its set of POMDP model hypotheses to better match the provided discrete-valued action [8] or observation [9]. The approaches are evaluated on a robotic wheelchair control system. Instead of assuming predefined state and observation spaces, [176] learns from human-robot collaboration data the POMDP state and observation spaces using non-parametric probability distributions, and, maximum likelihood transition and observation probabilities. Model confidence intervals are used to estimate a lower bound for the amount of data required to reach a specific POMDP control performance level.

Other tasks where POMDP based HRI has been studied include human-robot social interaction [130], [26], medical diagnosis [113], and assistive tutoring of students [123]. In human-robot social interaction, [26] uses a POMDP to optimize robot social interaction in the task of yielding or going first in a driving simulator. The state space consists of discrete information about human intention, the two cars and the environment. The dynamics model is learned from simulations. In other work, [130] investigates human-robot social interaction by observing human-human-interaction. Contrary to typical POMDP based HRI, in [113] the POMDP decides on the next task in autism spectrum disorder (ASD) diagnosis instead of on high or low level actions. In [113], each task is an interaction sequence between a robot and a child. [123] defines a discrete valued POMDP to tutor 4th grade students in mathematics. The robot chooses a tutoring action before the student attempts a math problem and observes the speed and accuracy of the student. Positive reward is accrued for increased student knowledge, engagement level, and for fast progress.

F. Multi-robot coordination

Multi-robot coordination is useful in applications such as target tracking or logistics (see, e.g., Fig. 10 (top)). In the following, we outline challenges specific to the multi-robot setting typically not present in single-robot cases, and review recent advances in POMDPs for the multi-robot setting. We consider *cooperative* settings with a shared goal, and do not address competitive or adversarial settings. We do not consider multi-robot task allocation, see, e.g., [80] for a survey.

a) Key sources of uncertainty and challenges: The key source of uncertainty in multi-robot coordination is the asymmetry of information available to each robot in the team. If communication is limited, the action-observation history of
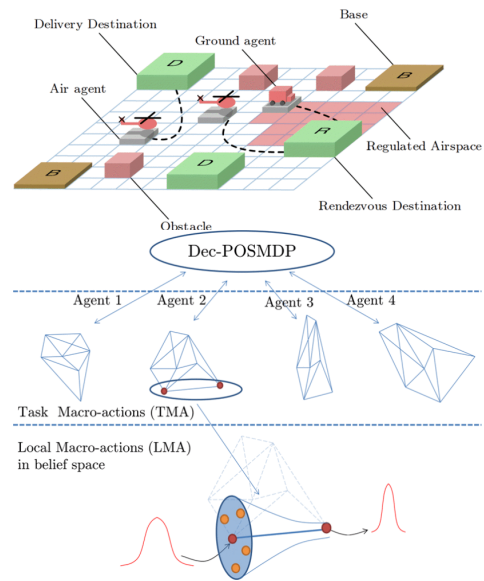

Fig. 10. **Top:** Multi-robot logistics domain with complex interactions under uncertainty. Air and ground agents cooperate to deliver packages to their destinations. **Bottom:** The hierarchical multi-robot planning method of [106], [107] consists of top-level task macro actions (TMAs) that correspond to specific tasks (e.g., picking up a package). Each TMA is a set of local feedback controllers (LMAs) that are executed to transform the initial beliefs of the TMA (shaded region at the bottom) to a goal belief (red circle). Figures from [106].

robot j is in general not known to another robot i. This means that it is not possible for i to predict which action j will take, as this depends on both j's policy and j's private action-observation history. This makes coordination challenging.

One way to avoid the complexity of a decentralized solution is to assume that robots can freely communicate so-as to share a single shared belief state [105, Sec. 2.4.3.]. In the resulting multi-agent POMDP the action space consists of joint action tuples $(a^1, a^2, \ldots, a^n)$ of each of the n robots in the team, and the observation space consists of joint observation tuples $(z^1, \ldots, z^n)$. This multi-agent POMDP is simply a POMDP where the actions and observations have a tuple structure, and standard POMDP solvers may be applied to compute a policy. Such a policy is often realized by deploying a centralized coordinator that uses the current belief state to determine the next individual action a^i of each robot i, and then receives the individual observations z^i to update the belief state via Bayes' rule. This approach has been successfully applied in many multi-robot coordination problems, see, e.g., [141], [142], [3], [40], [159], [172], [20]. The communication requirement is a major practical challenge of applying multi-agent POMDPs.

A *decentralized POMDP* (Dec-POMDP) takes the approach of "centralized planning with decentralized execution" [105]. Like the multi-agent POMDP, the Dec-POMDP also replaces the action and observation spaces by the joint action and joint observation space. However, the solution of a Dec-POMDP is a *decentralized* policy that each robot in the team can execute individually, without having to know the action-observation histories of other robots, or sharing a belief state. The policy is computed centrally, and then distributed to each robot for decentralized execution. No communication at

execution time is assumed, but any available communication can be explicitly modeled [105, Sec. 8.3]. This approach is used for multi-robot coordination in, e.g., [106], [107], [4], [86]. While the Dec-POMDP approach is extremely general, it is also computationally more challenging than solving a POMDP [17]. We refer the reader to [105] for technical details.

b) Solution methods: As mentioned above, if the robots in the team can always communicate with each other, coordination can be reached by solving a multi-agent POMDP, see, e.g., [141], [142], [3], [40], [159], [172], [20]. The solution methods do not differ from the single-robot case. Therefore, we focus here on two distinct approaches to planning in multi-robot coordination problems with limited communication. First, a Dec-POMDP can be solved offline to agree on a joint strategy before plan execution. Then, no communication at the time of policy execution is needed. The Dec-POMDP is either solved directly, or it is decomposed into simpler single-agent planning problems solved locally. Secondly, methods that use local communication during plan execution can reach a consensus, e.g., on a shared belief state, and use this to coordinate actions online.

A Dec-POMDP modeling a multi-robot task can be solved approximately [122] or exactly [86]. In [86], a multi-robot target tracking problem with a reward dependent on the robot team's joint belief is considered. The theoretical foundations of such multi-robot information gathering were investigated in [88], [87]. Due to the computational complexity of Dec-POMDPs, these approaches are limited to small problems with a handful of agents, actions, and observations. Alternatively, the *semi*-Markov decision process setting allows temporal abstraction by modeling varying durations of actions [4], [106], [107]. This abstraction improves scalability by introducing additional levels of hierarchy through macro-actions composed of simpler closed-loop controllers, see Fig. 10 (bottom). Each high-level macro-action for a task executes lower-level closed loop controllers that transform an initial belief to a goal belief where the task is completed. In [107], Gaussian beliefs are used at the lowest level, allowing application of macro-actions in continuous problems. The problem is solved by Monte Carlo and cross-entropy based algorithms. The method is demonstrated in a package delivery domain with four quadrotor robots with no online communication. The completion times of macro-actions such as "go to delivery location j" are probabilistically modeled in the semi-Markov framework. The MacDec-POMDP framework, see, e.g., [4], further removes the restriction that the completion time step of a macro action is determined before execution.

Some works map the Dec-POMDP problem to a set of local single-agent problems that are easier to solve [32], [173], [98], [94], at the cost of losing optimality. In [32], each agent plays one of several possible roles corresponding to a specific local reward function. At run-time, each agent solves a multi-agent POMDP to compute the value of each potential role. An auction algorithm allocates tasks to best-performing agents. The auction requires communication between robots, but overall the method avoids the computational burden of solving a Dec-POMDP. The method of [32] is demonstrated in robotic environmental monitoring and cooperative tracking.

Role-based POMDP abstractions are also proposed in [173], where agents leave clues that are observable to the other agents, allowing inference of the roles and future actions of others without direct communication. In [98], a homogeneous team of robots explores an environment. The discrete Dec-POMDP problem is approximated as a set of local MDPs. If communication is not possible, a robot acts independently by solving its local MDP. Robots interact via communication with nearby other robots, affecting each others' local MDP value functions. In [94], multi-robot navigation and collision avoidance is cast as a set of POMDPs, one for each homogeneous robot in the team. A single shared policy is learned, which still induces individual behaviour as each agent conditions its actions on its individual observations.

When robots communicate during policy execution they can reach a consensus, e.g., by communicating and updating local beliefs [174], [32] or observations [122], [52]. In case of communication breaks, missing data from other team members can be predicted based on the model parameters, see, e.g., [98], [32]. Alternatively, coordination is achieved in [19] by periodically communicating policies with other robots. Notably, no centralized planning phase is assumed, which makes the method fully distributed. Each robot locally plans a best response policy using MCTS, simulating other robots' actions according to the received policies. The method of [19] was applied to agricultural multi-robot fruit detection in [145]. A limitation of this approach is that it searches for open loop policies, potentially with a lower performance compared to an optimal solution. The approaches above allow a robot to continue executing its policy even if communication is interrupted.

V. CHALLENGES AND FUTURE DIRECTIONS

POMDPs have succeeded in many robot tasks that require reasoning about uncertain future outcomes using incomplete or noisy data. This section outlines some of the outstanding challenges that we believe crucial for further progress in the field. Accurate models are required for planning, but difficult to acquire. Robotics tasks are often inherently continuous, while the classic POMDP framework has mainly focused on the discrete case. Finally, in safety-critical robotic tasks, more robust probabilistic guarantees are needed to guarantee safety.

Model uncertainty. In many realistic robotic tasks that could be modeled as a POMDP, there is considerable uncertainty about the model parameters. Most approaches reviewed in this survey assume that dynamics, observation, and reward models are given. However, obtaining such models either by learning or expert design can be challenging in robotics. Kinematic modeling of rigid robots is well understood but efficient modeling of heavy hydraulic machines [171], soft robots [74], or, complex physics in man-made and natural environments [22] are unsolved active research topics. POMDP approaches often require model simplifications for tractability, and handling problems with intricacies such as non-stationary or switching-mode dynamics may also be challenging.

Recent works have proposed several learning methods for robotic tasks that can help tackle model uncertainty, for example combination of planning methods with imitation learning

for grasping [49], representation learning for beliefs [104], and multi-robot reinforcement learning for navigation [161] or temporal abstraction [169]. Techniques such as Bayesian reinforcement learning (BRL) [51] could be applied when the model has been identified up to a handful of unknown parameters. BRL treats the unknown parameters as additional state variables. After defining a prior on the unknown parameters, a policy that optimally gathers information on the parameters is solved for [14], [127], [134]. When considerably less prior information about the model parameters is available, e.g., when identifying a model for image observations, models could be learned directly from data and then used for planning. A promising recent trend is task-driven end-to-end learning, which integrates planning with model learning. For example, we could learn latent space models for planning with image data [57] and for long-horizon tasks [131]. We could also train a neural network to produce plans, thus making the planning algorithm differentiable. The Differentiable Algorithm Network (DAN) leverages the latest advances in deep learning to learn a model most useful for planning, even though it may deviate from the ground truth [72]. Translating these advances to robotic tasks is an important future direction.

Continuous state, action, observation, and time. The classic POMDP model consists of discrete states, actions, observations, and time, while robots operate in the physical world and often require continuous models for control. Discretizing continuous values can result in inaccurate solutions especially in high dimensional tasks. One approach is to assume specific probability distributions, policies, or, state, action, observation representations. For example, linear quadratic Gaussian control assumes Gaussian distributions and a linear feedback control policy [157]. Modern online POMDP algorithms, specifically, POMCP [133] and DESPOT [138], relax these restrictive assumptions. They directly handle continuous states and observations through Monte Carlo sampling. However, the actions remain discrete. One future direction is to transfer recent advances in continuous POMDP methods [147] to novel robotic tasks. Practical real world control often assumes discrete time steps, even under full observability. However, some systems can benefit from continuous time control, in robotics especially non-linear systems that require fast responses. Continuous time MDPs have been investigated as semi-MDPs [148] which consist of a sequence of continuous time fragments. While existing work on continuous time POMDPs is scarce [36], novel continuous time contributions could enable solutions to new robotic tasks.

Safe POMDPs in robotics. Safety is an inherent requirement in several robotic applications such as human-robot interaction or autonomous driving where partial observability plays an important role. However, typically safety requires constrained POMDP solutions [43], [90] that can guarantee how often unsafe events may happen on average [35], [153], [43], [90]. There is an opportunity for scaling and extending constrained POMDP methods such that they become a standard tool in robotics. At the moment these methods are however limited to only a few robotic applications. On the one hand, these limitations are due to computational complexity, such as the "curse of dimensionality" due to state space discretization [163], that limit the scale of problems that can be addressed. On the other hand, safety-constrained methods are safe with respect to the POMDP *model* of the task, whereas there may still be a gap between the model and the real task.

Robots are systems where knowing which sensors and actuators are fully operational is crucial for safety and robustness. Such questions are tackled by system health monitoring, which is becoming increasingly unified with decision-making, see [15] and references therein. This implies further possibilities for leveraging POMDP planning for objectives such as greater fault resiliency in robotics.

VI. CONCLUSION

Uncertainties in action effects, sensor data, and environment states are inherent to robot systems operating in the physical world and pose significant challenges to their robust performance. In this survey, we have reviewed the POMDP framework for robot decision-making under uncertainty, including both efficient algorithms for POMDP planning and their applications in robotics. The success of POMDP planning on a wide range of different robot tasks clearly demonstrate its effectiveness and generality for robot decision making under uncertainty. Currently, sampling-based online POMDP planning algorithms stand out as a favored choice because of their scalability in high dimensional spaces and ability to handle dynamic environments naturally. Dealing with continuous state and action spaces, planning with high-dimensional sensory data, and acquiring accurate dynamics and sensor models are among the key challenges shared among many application areas of POMDPs in robotics.

In future work, we expect to see tighter integration of POMDP model learning and planning. Further, by establishing a strong link to the theoretical guarantees provided by state-of-the-art POMDP solution algorithms, these new results will contribute to reliable and safe decision making under uncertainty in many robotic systems.

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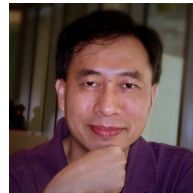
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