Aalto University
School of Engineering

MEC-E1070 Selection of Engineering Materials

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Learning objectives for this Lecture

Knowledge and Understanding

Understanding of the concept of **shape efficiency**

Skills and Abilities

Ability to select efficient **material-shape combinations**

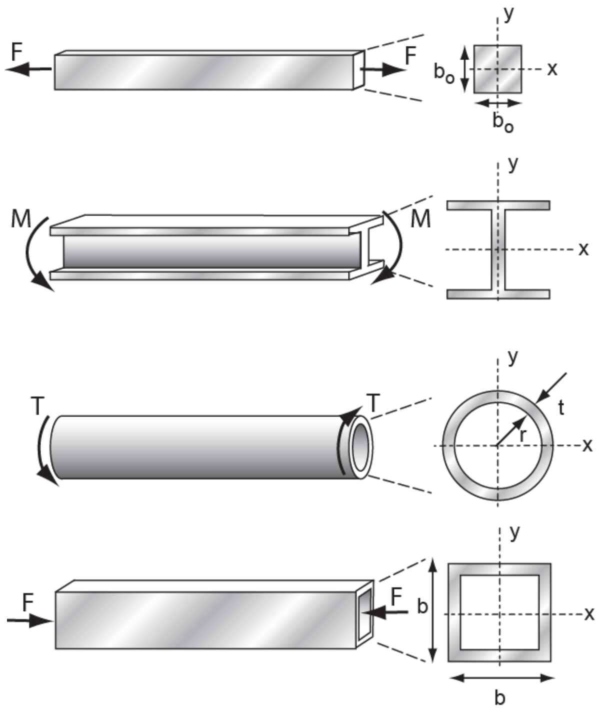
Values and Attitudes

Awareness of how materials and shape interact

Resources

- Text: *"**Materials: engineering, science, processing and design**"* 4th edition by M.F. Ashby, H.R. Shercliff and D. Cebon, Butterworth Heinemann, Oxford, 2011, Chapters 9-10.

Outline of Lecture



- Efficient shapes: tubes, I-beams etc
- The shape factor and shape limits
- Material indices that include shape
- Graphical ways of dealing with shape

Shape efficiency

- When materials are loaded in bending, in torsion, or are used as slender columns, section shape becomes important

- ”Shape” = cross section formed to a

  tubes
  I-sections
  hollow box-section
  sandwich panels
  ribbed panels

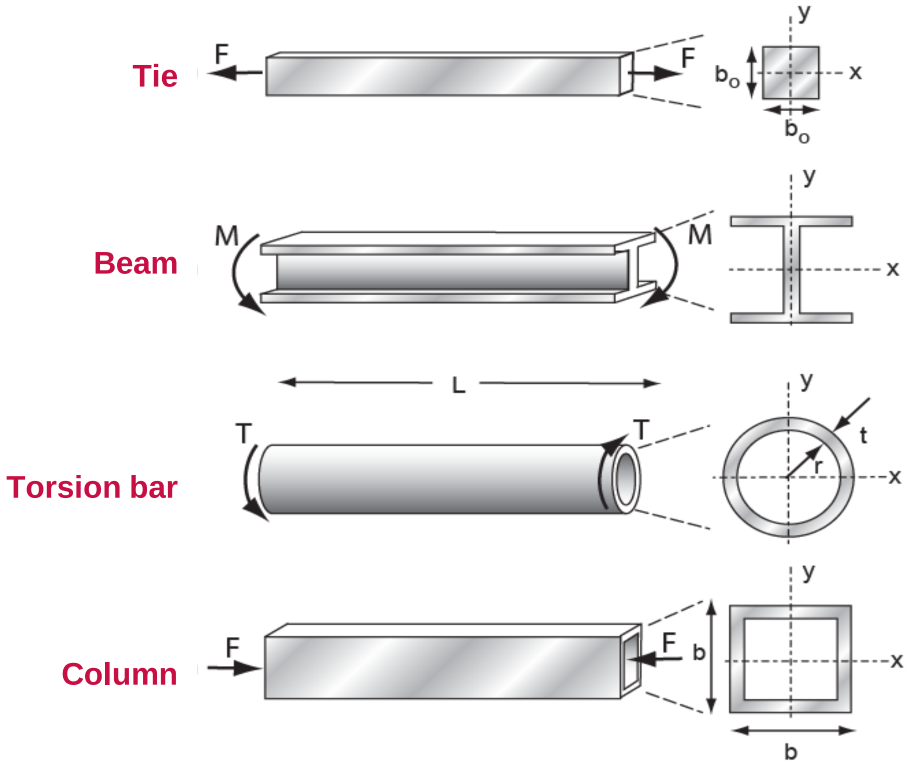
- “Efficient” = use least material for given stiffness or strength

- Shapes to which a material can be formed are limited by the material itself

- Goals: understand the limits to shape develop methods for co-selecting material and shape

Shape and mode of loading

Standard structural members



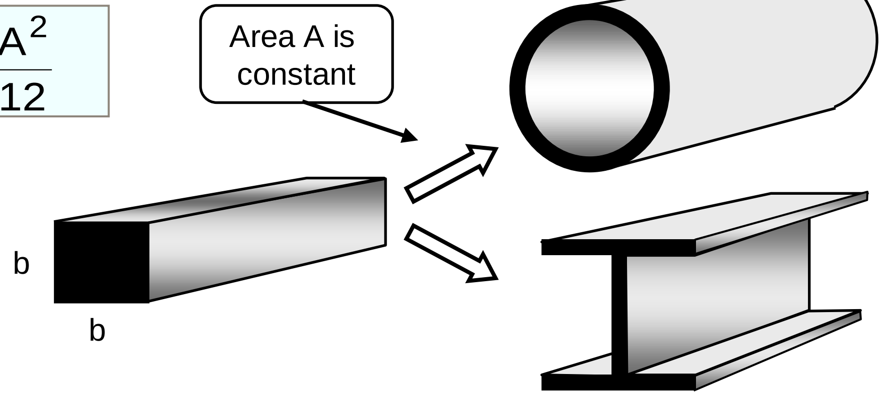
Shape efficiency: bending stiffness

- Take ratio of bending stiffness S of shaped section to that (S_o) of a neutral reference section of the same cross-section area
- Define a standard reference section: a solid square with area $A = b^2$

- Second moment of area is I; stiffness scales as EI.

$$I_o = \frac{b^4}{12} = \frac{A^2}{12}$$

Area $A = b^2$



$$I = \int y^2 b(y)\,dy$$

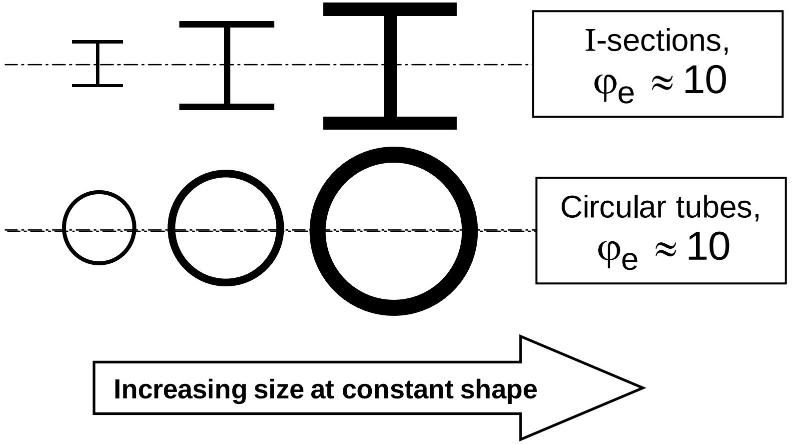
Area A and modulus E unchanged

- Define **shape factor for elastic bending,** measuring efficiency, as

$$\varphi_e = \frac{S}{S_o} = \frac{EI}{EI_o} = 12\frac{I}{A^2}$$

Properties of the shape factor

- The shape factor is dimensionless – a pure number.
- It characterizes shape.



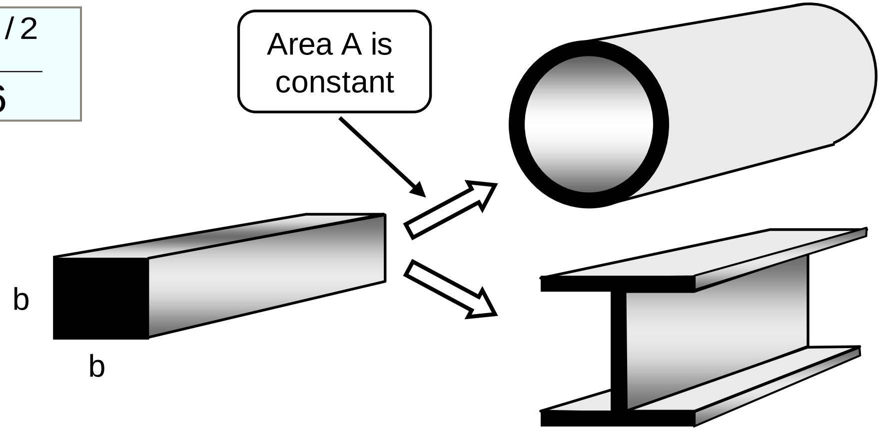
- Each of these is roughly 10 times stiffer in bending than a solid square section of the same cross-sectional area

Shape efficiency: bending strength

- Take ratio of bending strength F_f of shaped section to that ($F_{f,o}$) of a neutral reference section of the same cross-section area
- Section modulus of area is Z; strength scales as $\sigma_y Z$

$$Z_o = \frac{b^3}{6} = \frac{A^{3/2}}{6}$$

Area $A = b^2$



$$Z = \frac{I}{y_{max}}$$

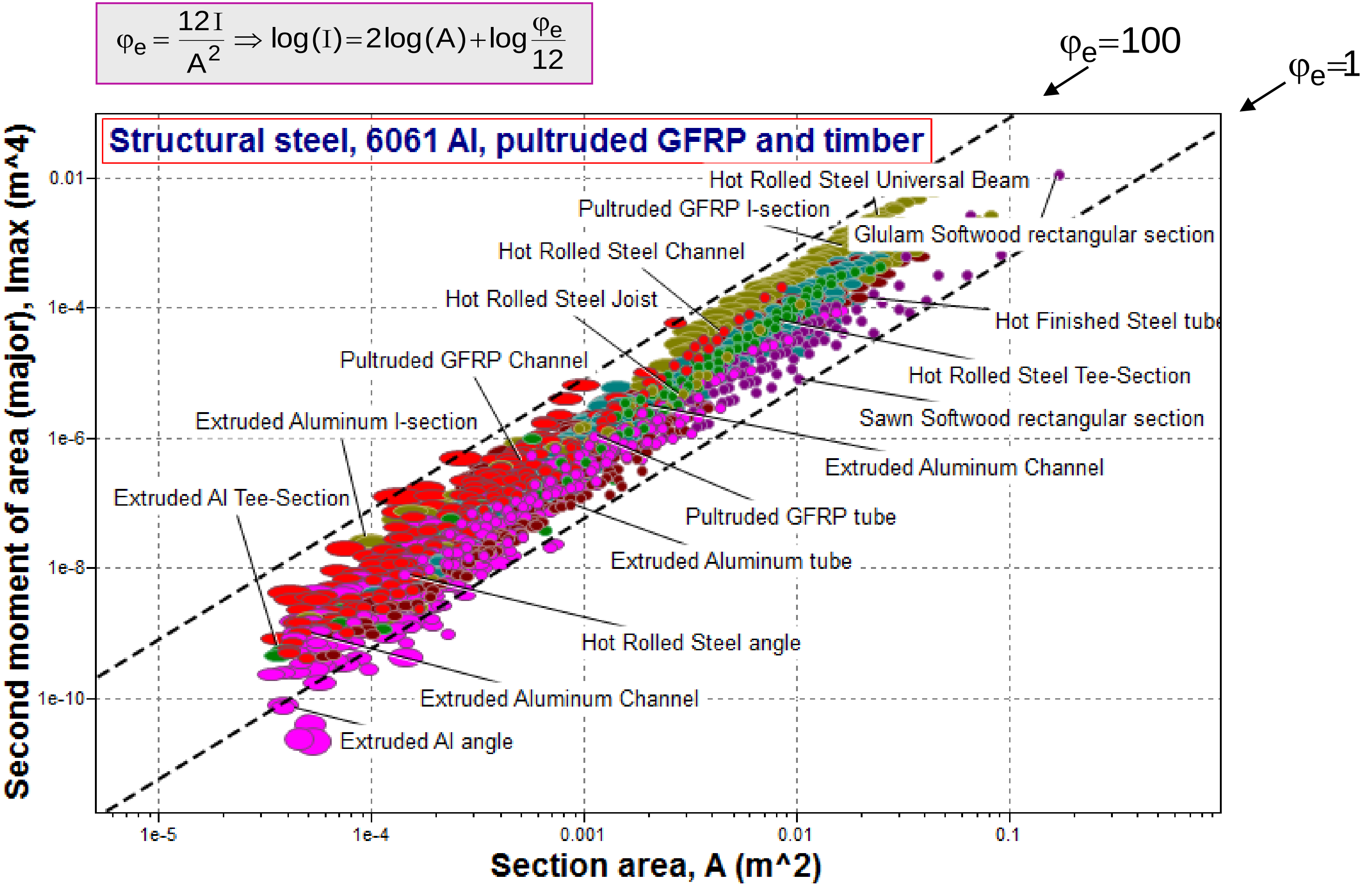
Area A and yield strength σ_y unchanged

- Define **shape factor for onset of plasticity (failure),** measuring efficiency, as

$$\varphi_f = \frac{F_f}{F_{fo}} = \frac{\sigma_y Z}{\sigma_y Z_o} = 6\frac{Z}{A^{3/2}}$$

What values of φ_e exist in reality?

$$\varphi_e = \frac{12I}{A^2} \Rightarrow \log(I) = 2\log(A) + \log\frac{\varphi_e}{12}$$



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Limits for Shape Factors φ_e and φ_f

- There is an upper limit to shape factor for each material

Material	Max φ_e	Max φ_f
Steels	65	13
Aluminum alloys	44	10
GFRP and CFRP	39	9
Unreinforced polymers	12	5
Woods	8	3
Elastomers	<6	-
Other materials	...can calculate	

- Limit set by: (a) manufacturing constraints
  (b) local buckling

- Theoretical limit: $\varphi_e \approx 2\sqrt{\dfrac{E}{\sigma_y}}$

Modulus

Yield strength

Indices that include shape

Function: *Beam* (shaped section).

Constraint: *Bending stiffness* = S:

$$\boxed{S = \frac{CEI}{L^3}}$$

I is the second moment of area:

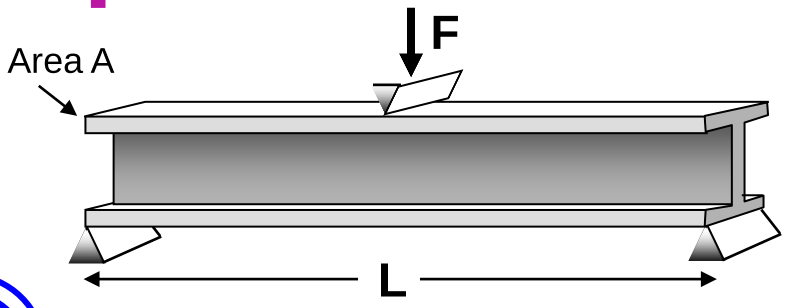
$$\boxed{\varphi_e = 12\frac{I}{A^2} \qquad A = \left(\frac{12I}{\varphi_e}\right)^{1/2}}$$

Objective: *Minimise mass*, m, where:

$$\boxed{m = AL\rho}$$

$$m = \left(\frac{12\, S\, L^5}{C}\right)^{1/2}\left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$$

Chose materials with smallest $\left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$



- m = mass
- A = area
- L = length
- ρ = density
- b = edge length
- S = stiffness
- I = second moment of area
- E = Youngs Modulus

Selecting material-shape combinations

Materials for stiff, *shaped* beams of minimum weight

- Fixed shape (φ_e fixed): choose materials with low $\dfrac{\rho}{E^{1/2}}$
- Shape φ_e a variable: choose materials with low $\dfrac{\rho}{(\varphi_e E)^{1/2}}$

Material	ρ, Mg/m³	E, GPa	$\varphi_{e,max}$	$\rho / E^{1/2}$	$\rho / (\varphi_{e,max} E)^{1/2}$
1020 Steel	7.85	205	65	0.55	0.068
6061 T4 Al	2.70	70	44	0.32	0.049
GFRP	1.75	28	39	0.35	0.053
Wood (oak)	0.9	13	8	0.25	0.088

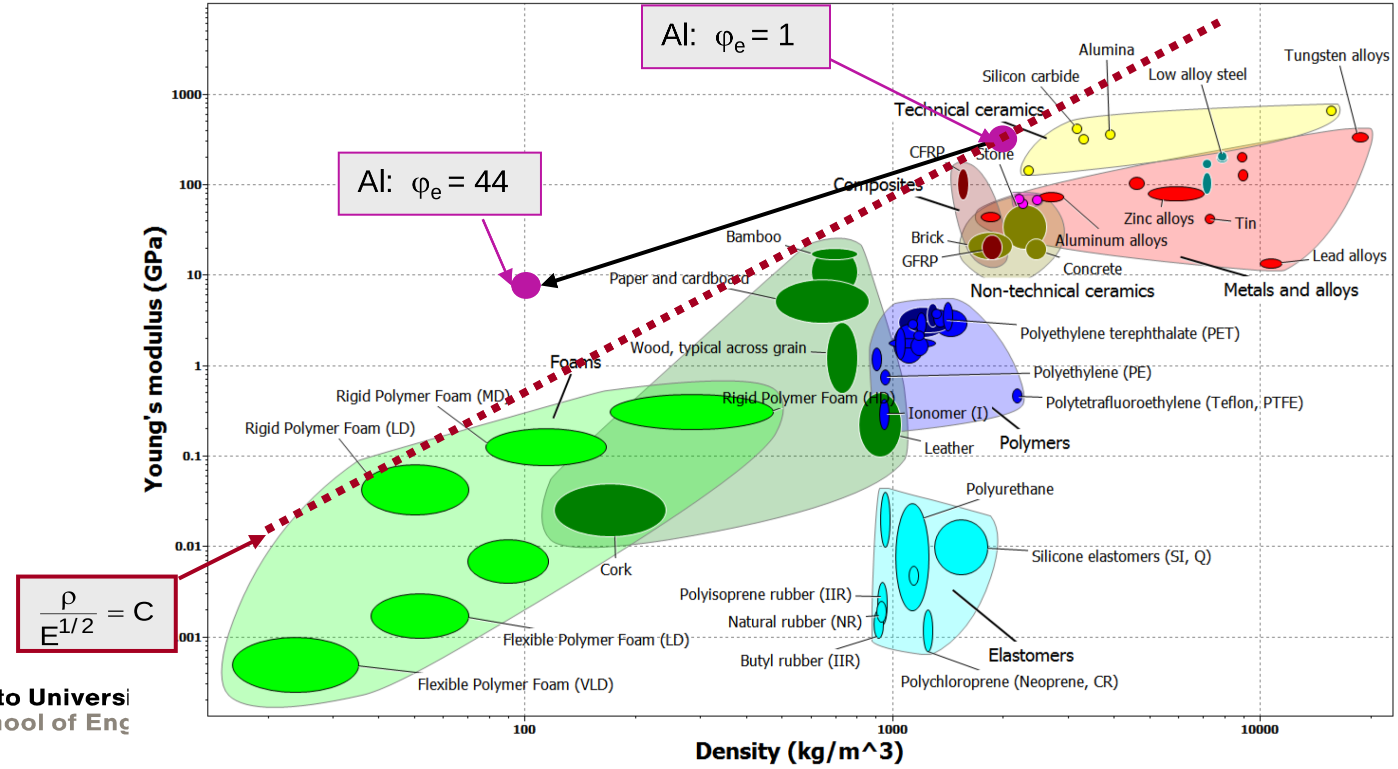
- Commentary: Fixed shape (up to $\varphi_e = 8$): wood is best

  Maximum shape ($\varphi_e = \varphi_{e,max}$): Al-alloy is best

  Steel recovers some performance through high $\varphi_{e,max}$

Shape on selection charts

- Note that $\dfrac{\rho}{(\varphi_e E)^{1/2}} = \dfrac{\rho/\varphi_e}{(E/\varphi_e)^{1/2}}$ New material with $\begin{cases} \rho^* = \rho/\varphi_e \\ E^* = E/\varphi_e \end{cases}$



Remarks

- When materials carry bending, torsion or axial compression, the section shape becomes important.

- The “shape efficiency” is the amount of material needed to carry the load. It is measured by the shape factor, φ.

- If two materials have the *same* shape, the standard indices for bending (e.g. $\rho / E^{1/2}$) guide the choice.

- If materials can be made -- or are available -- in different shapes, then indices which include the shape (e.g. $\rho / (\varphi E)^{1/2}$) guide the choice.