

# Task 2: Multiple constraints

## Task 2\_1: multi-constraint material selection for passenger car and commercial airplane frame structures

In this report the frame structures are simplified to be consisting of triangulated beams of rectangular cross-section. The beams thickness is kept as constant ( $h$ ) to keep the outer and inner dimensions of the vehicles constant while leaving the width as variable depending on the chosen material and required load constraints. It is assumed that the beams would primarily be loaded in tension and compression in normal operation. Worst case scenario would nevertheless be outside object hitting the centre of the beam and it deflecting more than constant ( $f$ ) and hitting the passenger of the vehicle. In addition, both vehicles are supposed to be mass produced so cost of the material is equally important factor in the material selection process. It should be noted that the lightest material is possibly not the cheapest so trade off analysis is presented in the end. These design requirements and objectives can be seen in table form in table 1 below.

Table 1, design requirements for the task

Design requirements for vehicle frame structure	
<b>Function</b>	Frame triangulated beam, light, cheap, stiff beam with worst case scenario in simply supported bending with point force at beam centre
<b>Constraints</b>	Length, $L$ Max bending displacement, $f$ , under load $F$ ->Max bending stiffness, $S^*$ Cross-section height, $h$ Cross-section shape ( $h \times w$ rectangle)
<b>Objective</b>	Minimize the mass, $m$ Minimize the cost, $C$
<b>Free variables</b>	Choice of material (young's modulus, density, cost per mass) Cross-section area, $A$

Following Ashby method the two objectives to minimize can be formatted as following ( $p$  is the materials price per kg):

$$m = L * h * w * \rho$$

$$C = p * L * h * w * \rho$$

The bending stiffness ( $S$ ) of the beam is defined through requirement of maximum deflection ( $f$ ) in the beam under the determined load ( $F$ ) written as following:

$$S = \frac{F}{f}$$

Maximum deflection of simply supported beam loaded with point load  $F$  at the centre can be written as:

$$f = \frac{F * L^3}{48 * E * I}$$

Then substituting bending stiffness (S) to the maximum deflection formula above one can get the formula below. This formula is then compared to the wanted minimum value of S\*.

$$S = \frac{48 * E * I}{L^3} \geq S *$$

Where I is the second moment of area for the beam:

$$I = \frac{w * h^3}{12}$$

Then by substituting the formula of second moment of area into the formula for bending stiffness it can be written as:

$$S = \frac{4 * E * w * h^3}{L^3}$$

Then solving for variable width (w) the above formula can be written as and substituted to the mass formula:

$$m = L * h * \rho * \frac{L^3 * S}{4 * E * h^3}$$

Taking all the constant values out from the equation, we can define the first metric representing the mass of the solution (M<sub>1</sub>) to minimize as:

$$M_1 = \frac{\rho}{E}$$

Then, the second index (M<sub>2</sub>) taking the cost into account can be defined as:

$$M_2 = \frac{p * \rho}{E}$$

These metrics can then be plotted in with Granta level 2 dataset materials as shown in figure 1. Additionally figure 1 displays arbitrarily chosen selection line removing some of the dominated solutions from further analysis.

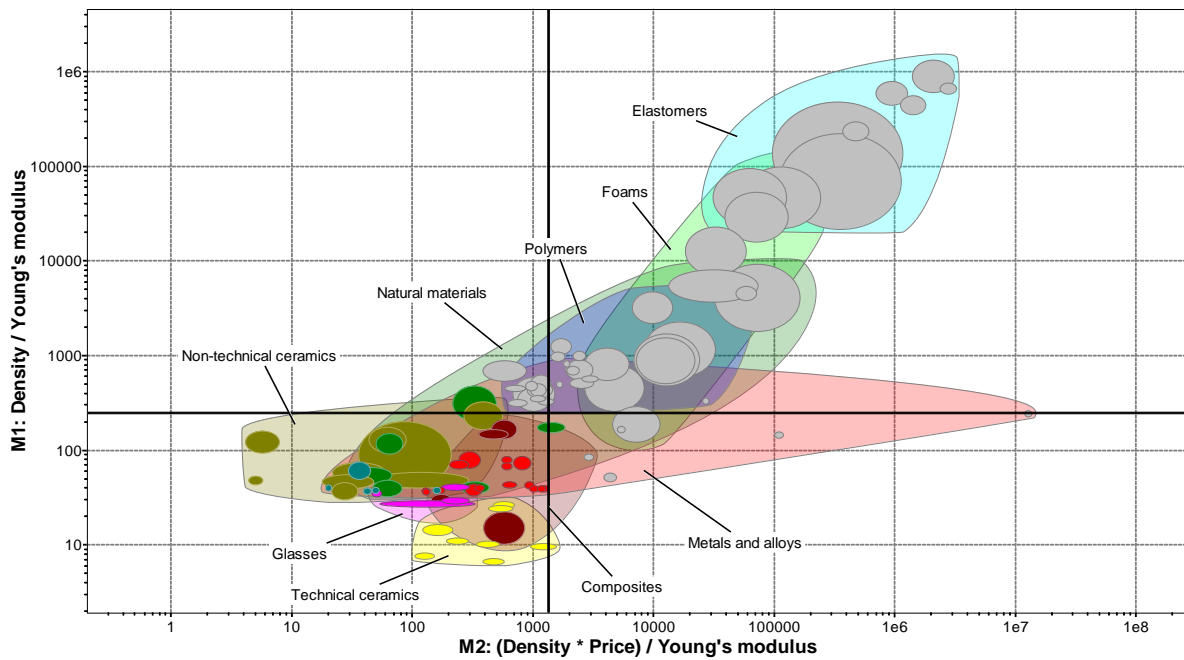


Figure 1, Granta Edupack level 2 dataset plotted with used metrics as axis

As further analysis a penalty function  $Z$  can be formatted with known factors  $\alpha$  to approximate mass and costs weights. For passenger car  $\alpha = 5 \frac{\text{€}}{\text{kg}}$  and for commercial airplane  $\alpha = 250 \frac{\text{€}}{\text{kg}}$ . Meaning that essentially a single additional unit of mass costs 50 times more in operation of the commercial airplane compared to a passenger car. A generalized penalty function to minimize could then be written as:

$$Z = M_2 + M_1 * \alpha$$

It can be seen that the metric 2 is already showing generalized cost and thus does not need the factor  $\alpha$ . The minimizable penalty can then be drawn on plot with materials presented in figure 1 with linear line of slope  $\frac{-1}{\alpha}$  for both vehicle types as seen in figure 2.

Analysing the optimal solutions as per presented metrics, it can be seen that ceramics and glasses are well presented in the results. They need to be discarded from further examination due to their inherent brittleness. This would cause problems in the highly dynamic load conditions present in vehicle in motion. For the commercial airplane this would leave CFRP as seemingly clearly best option. Although it should be noted that Granta Edupack level 2 dataset only presents CFRP as isotropic material while in reality it is highly anisotropic. Taking this into account with constraints for multiple loading directions could make different metals or even bamboo more promising solutions. As for the passenger vehicle with higher emphasis on the component cost, CFRP is clearly outshined by more affordable metals (steels, cast iron, aluminum) and some wood materials. Low alloy steel would seemingly be good option for affordable, yet stiff car frame.

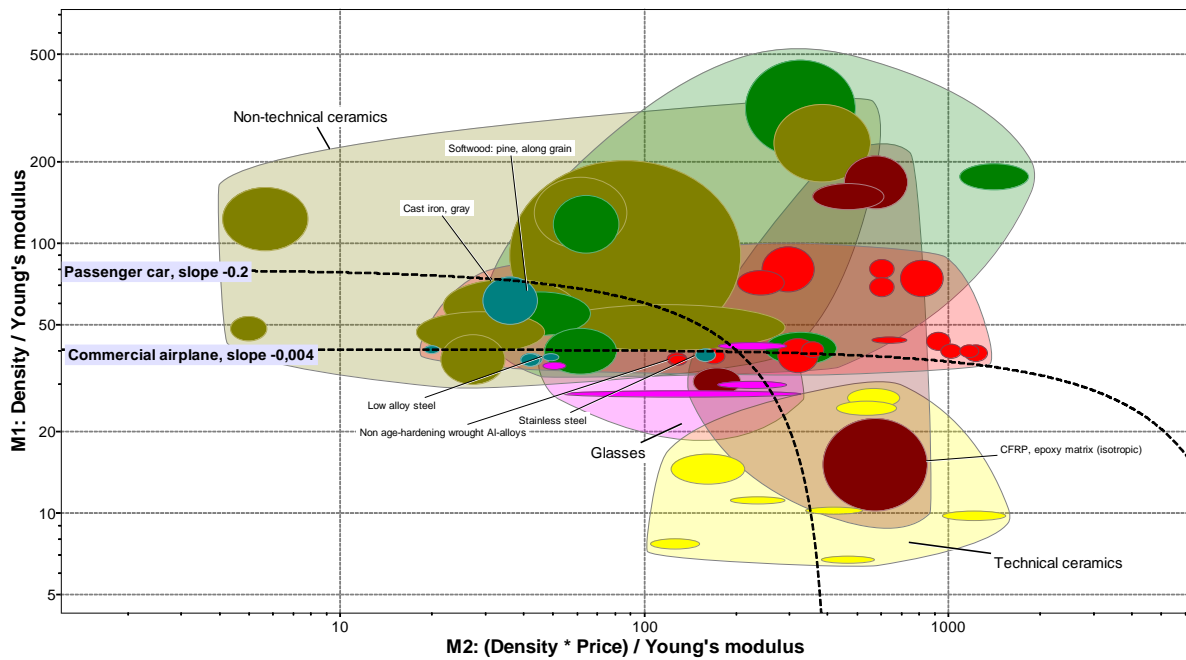


Figure 2, chosen materials from figure 1 with penalty function

#### Task 2\_2: portable electronic device casing material comparison to baseline ABS

Following example set in chapter 8.6 of Materials Selection in Mechanical Design textbooks 4<sup>th</sup> edition (Ashby). In this report material values for ABS plastic are as taken from Granta Edupack level 2 dataset as average and rounded to one decimal accuracy:

$$E_0 = 2.4 \text{ GPa}$$

$$\rho_0 = 1.05e3 \frac{\text{kg}}{\text{m}^3}$$

The constructed comparison chart can be seen in figure 3. As it can be seen that all the examined material examples used in modern electronics can be produced to be thinner while keeping bending stiffness as constant. Almost all of them even 50 % thinner compared to the baseline. Glasses, most composites, aluminum and magnesium based alloys are additionally more lightweight than the baseline ABS and thus dominating it as material option, at least with the used metrics. The heavier-than-baseline materials could be used as they could create more premium feeling to the product. Especially titanium and stainless steel can be used to create impression of sturdy and high quality product while making sacrifices on the mass.

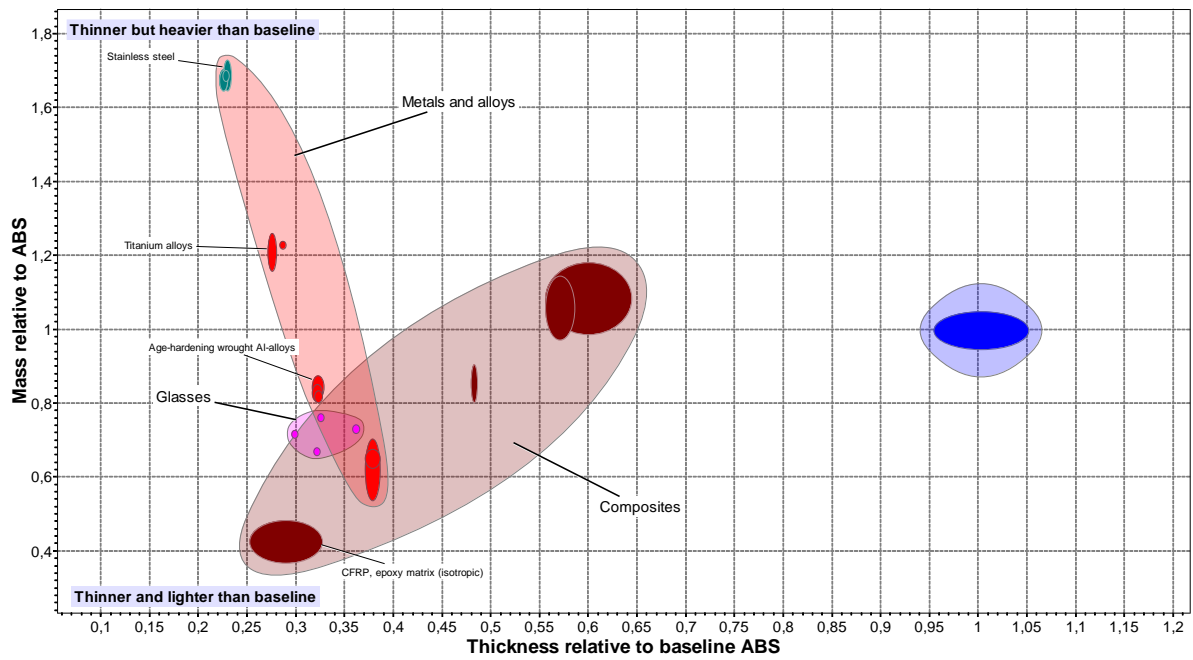


Figure 3, comparison chart showcasing baseline ABS plate thickness and mass and comparing modern used materials