Task 3

Task 3.1.1

The shape factor for bending stiffness of a Glass-reinforced (GRP) bar.

a) Solid square profile,

$b = h$,

$$\phi_B^e = \frac{12I}{A^2} = \frac{12bh^3}{12A^2} = \frac{12A^2}{12A^2} = 1 \quad (1)$$

b) Solid rectangle profile,

$b = 500mm$, $h = 100mm$,

$$\phi_B^e = \frac{12I}{A^2} = \frac{12bh^3}{b^2h^2} = \frac{h}{b} = \frac{100mm}{500mm} = 0.2 \quad (2)$$

c) Hollow square profile,

$b = h = 100mm$, $t = 10mm$,

$$\phi_B^e = \frac{12I}{A^2} = \frac{12h^3t(1+3b/h)}{4t^2(b+h)^2} \quad (3)$$

due to $b = h$, we get,

$$\phi_B^e = 3\frac{h^3(1+3)}{t(2h)^2} = 3\frac{4h^3}{4th^2} = \frac{3h}{t} = \frac{300mm}{10mm} = 30 \quad (4)$$

d) Hollow round pipe,

$r = 500mm$, $t = 10mm$,

$$\phi_B^e = \frac{12I}{A^2} = \frac{12\pi r^3 t}{(2\pi rt)^2} = \frac{12r}{4\pi t} = \frac{3r}{\pi t} = \frac{1500mm}{\pi 10mm} = \frac{150}{\pi} \approx 48 \quad (5)$$

e) A spar of the wind turbine blade,

Cross-section is an I-beam, with $b = 500mm$, $h = 100mm$, $t_{flange} = 10mm$, $t_{web} = 20mm$. Because $t_{web} = 2t_{flange}$, following second moment of inertia I and surface area A can be used for the calculations,

$$I = \frac{h^3 t(1+3\frac{b}{h})}{6}, A = 2t(h+b) \tag{6}$$

$$\phi_B^e = \frac{12I}{A^2} = \frac{12h^3 t(1+3\frac{b}{h})}{6(2t(h+b))^2} = \frac{12}{24}\frac{h^3 t(1+3\frac{b}{h})}{t^2(h+b)^2} \tag{7}$$

$$(h+b)^2 = h^2(1+\frac{b}{h})^2 \tag{8}$$

$$\phi_B^e = \frac{h^3(1+3\frac{b}{h})}{2th^2(1+\frac{b}{h})^2} = \frac{h}{2t}\frac{1+3\frac{b}{h}}{(1+\frac{b}{h})^2} = \frac{100mm}{20mm}\frac{1+3\frac{500mm}{100mm}}{(1+\frac{500mm}{100mm})^2} = 5\frac{16}{6^2} \approx 2.2 \tag{9}$$

Task 3.1.2

After calculating the bending factors ϕ_B^e, to use them for material selection, own material records must be created. In *Granta Edupack*, Young's modulus and density values are given as if the materials had a square section. Bending factor ϕ_B^e for basic beam with square section is 1. To take into account the shape factors, material records were created for each shape factor calculated in task 3.1.1 . Table 1 shows the density and Young's modulus for each ϕ_B^e using the material values from *Granta Edupack* level 2 database. Equations that were used to calculate the influence of bending factor to the material properties were,

$$E^* = \frac{E}{\phi_B^e} \tag{10}$$

$$\rho^* = \frac{\rho}{\phi_B^e} \tag{11}$$

Figure 1 shows the materials in Young's modulus vs. density chart. Materials that were chosen to be compared against classical material that is used for building the spars of a wind turbine blade, glass fibre reinforced polymer (GFRP), are age-hardening wrought aluminum and polyetheretherketone (PEEK). All density values and Young's modulus values plotted in the chart are given in tables 1 and 2.

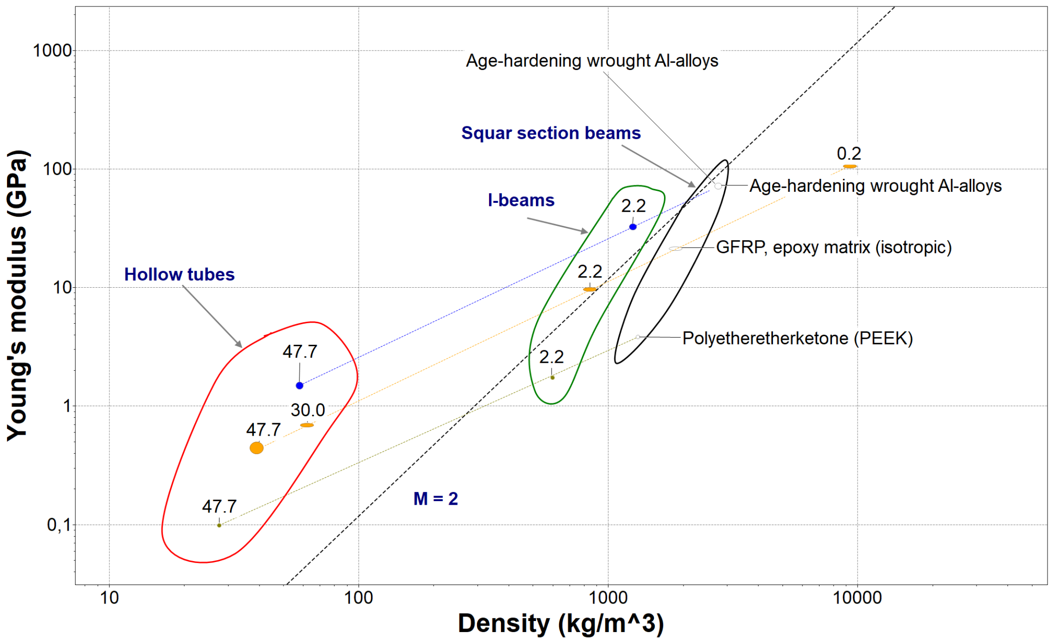

Figure 1: All the materials compared in Young's modulus vs. Density chart. Material index line $M = E^{1/2}/\rho$ has the slope of 2, and is drawn as a dotted line on top of the chart. Granta Edupack, level 2 database.

The material index $M = \frac{\sqrt{E}}{\rho}$ was derived in previous tasks, and it can be used define which of the materials is the lightest and stiffest. In figure 1 material index is displayed as a dotted line with slope of 2. When the material records from tables 1 and 2 were ranked by the material index, the aluminum with $\phi_B^e = 47.7$ seems to be the best option. However, aluminum cannot be shaped so easily, so bending factor as high as 47.7 can't realistically be reached. PEEK on the other hand can be shaped easily. The drawback for PEEK is the poor Young's modulus. If reinforced with carbon fibre however, the stiffness of PEEK could be enhanced. This appears to be an interesting material option. However, very efficient shapes can be achieved for GFRP. This is why GFRP is nowadays commonly used as a material for building a spar for a wind turbine blade.

Table 1: Table showing the Young's modulus and density values for glass fibre reinforced polymer (GFRP), with corresponding shape factors calculated in task 3.1.1.

Material	ϕ_B^e	Density (kg/m^3)	Young's modulus (GPa)
GFRP	1	1750,0 - 1970,0	21,0 - 21,8
	0,2	8750,0 - 9850,0	105,0 - 109,0
	30	58,3 - 65,7	0,7 - 0,7
	47,7	36,7 - 41,3	0,4 - 0,5
	2,2	795,5 - 895,5	9,5 - 9,9

Table 2: Table showing the Young's modulus and density values for PEEK and age-hardened aluminum, with corresponding shape factors calculated in task 3.1.1.

Material	ϕ_B^e	Density (kg/m^3)	Young's modulus (GPa)
PEEK	1	1300,0 - 1320,0	3,8 - 4,0
	2,2	590,9 - 600,0	1,7 - 1,8
	47,7	27,3 - 27,7	0,1 - 0,1
Aluminum	1	2670,0 - 2840,0	68,0 - 76,0
(age-hardening	2,2	1213,6 - 1290,9	30,9 - 34,5
wrought)	47,7	56,0 - 59,5	1,4 - 1,6

3.2.1

Theoretical max ϕ_B^e-values for steel, aluminum, CFRP and hardwood were calculated and listed in table 3. These values were then compared to experimental values in table 4.

When theoritical and experimental values are compared, there is a lot of variety how these values correspond to each other. For steel, much bigger experimental value is measured than the theoretical value suggests. On the other hand for hardwood, theoretical max ϕ_B^e-value is much bigger than experimental. This is important to note when selecting material for engineering purpose. We can only rely on experimental values, and even those might give us the wrong idea. For some materials, achieving certain shape might be too expensive, thus the material with best experimental ϕ_B^e-value might not be the correct choice due to budgeting reasons.

Table 3: Calculated theoretical max ϕ_B^e-values, Young's modulus and Yield strength values from Granta edupack, level 2 database.

Material	Young's modulus (Gpa)	Yield strength (MPa)	Max ϕ_B^e
Low alloy steel	200,0 - 210,0	469,0 - 1600,0	25,7 - 48,7
Aluminum age-hardening wrought	68,0 - 76,0	241,0 - 520,0	26,3 - 40,8
CFRP	69,0 - 150,0	550,0 - 1050,0	18,6 - 38,0
Hardwood	5,0 - 5,6	4,3 - 5,2	71,2 - 83,2

Table 4: Experimental max ϕ_B^e-values.

Material	$(\varphi_B^e)_{max}$
Steel	65
6061 aluminum	44
CFRP	39
Polymer	12
Wood	5
Elastomers	<6

3.2.2

Each material was plotted to the 4 chart with shape factor approximately matching the values in table 5. Straight lines were drawn to vertical and horizontal directions from upper left chart and lower right chart in figure 2. This way stiffness constraint and performance values were received. Based on these values, we can roughly say, that only steel and CFRP from the table 5 could be selected as a material for a bending beam with minimum stiffness requirement of $10^7 Nm^2$, since aluminum and wood do not fulfill this criteria. From steel and CFRP, CFRP is clearly the best candidate, because it's lighter.

Table 5: Materials and their bending factors.

Material	ϕ_B^e
Steel(I-beam)	15
Aluminum(I-beam)	10
CFRP(tube)	10
Wood(beam)	2

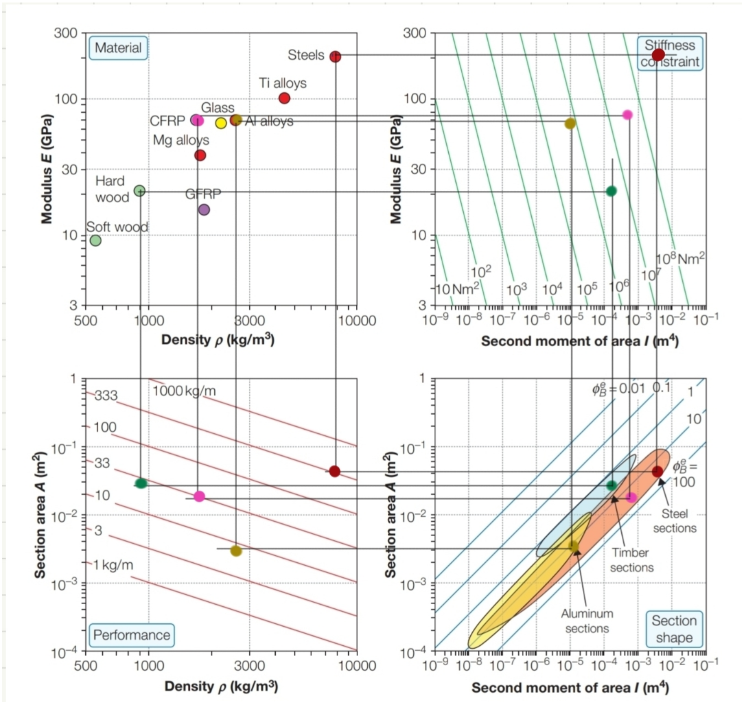

Figure 2: 4-field method for choosing the best material for beam out of steel, aluminum, CFRP and wood.