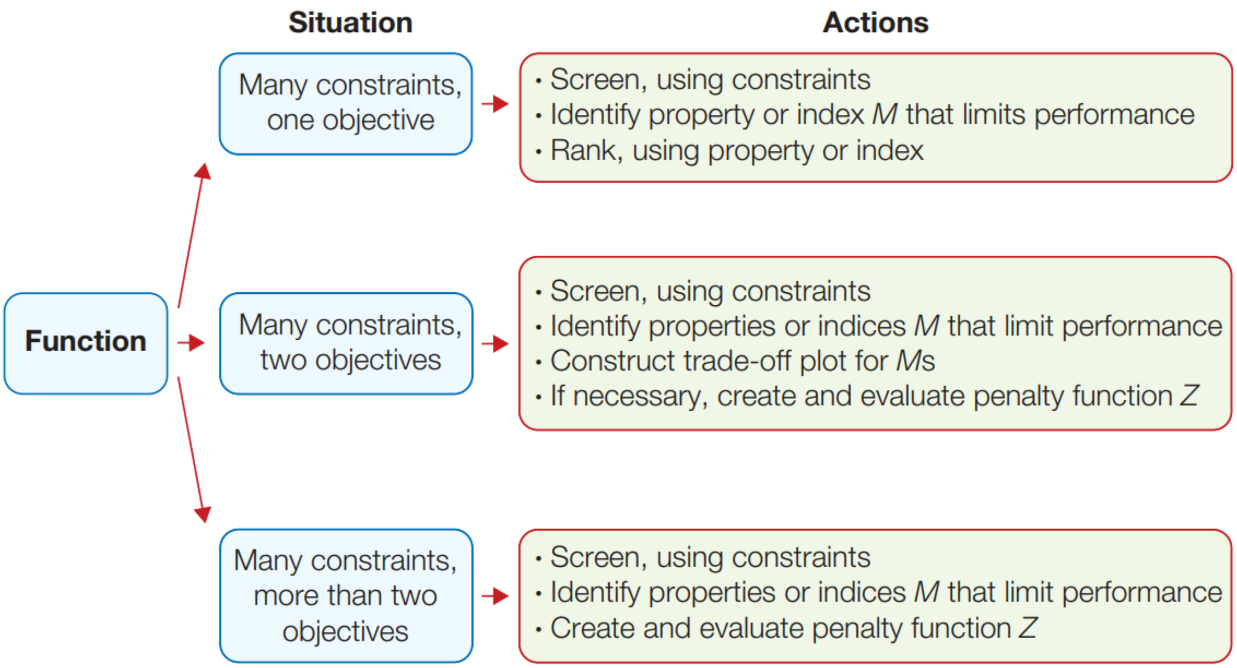
Task 2: Multiple constraints

Instructions: Check the following questions and exercises. Read chapters 7 and 8 of the course textbook (4th edition).



Criterion: A good report on this task demonstrates a good understanding of:

- the concept of solutions that dominate others when there are multiple objectives, and trade-offs when there are multiple un-dominated solutions
- how to combine multiple material performance indices into a penalty function, using a trade-off parameter
- how to graphically represent this on a chart with multiple performance indices as axes, and
- how to express this as relative performance compared to a reference solution

Task 2.1: Sufficient stiffness is required from the frame structures used in various transportation vehicles. In addition, those frame structures should be as light as possible to minimize fuel consumption. On the other hand, the frame should also be as cost-efficient as possible.

The vehicle frame can be simplified as panels, or beams, or a combination of both. Different loading conditions can be possible, therefore you may have different answers from your peers. Hence, you need to **specify the situation for your own structure at the beginning**.

Find out the materials that meet your requirements. Choose **two** vehicles from the listed application table below. Explore suitable materials for each of your chosen vehicles with the corresponding exchange constants (α) provided in the table. In this case, each exchange constant represents the value of weight savings in that vehicle.

Note: Suitable indices for frame structures can be found in the textbook. **A step-by-step derivation must be included in your report**.

Hint: It is recommended to perform the material comparison by calculating the values of a *penalty function (Z)*. A graph of the results needed for this task can be drawn from the GRANTA EduPack by choosing the Z-function values as the Y-axis in *Advanced*-tools, and various material groups for the X-axis by leaving the attribute of X-axis empty or by using the *Trees*-tool. Draw these maps on **level 2**.

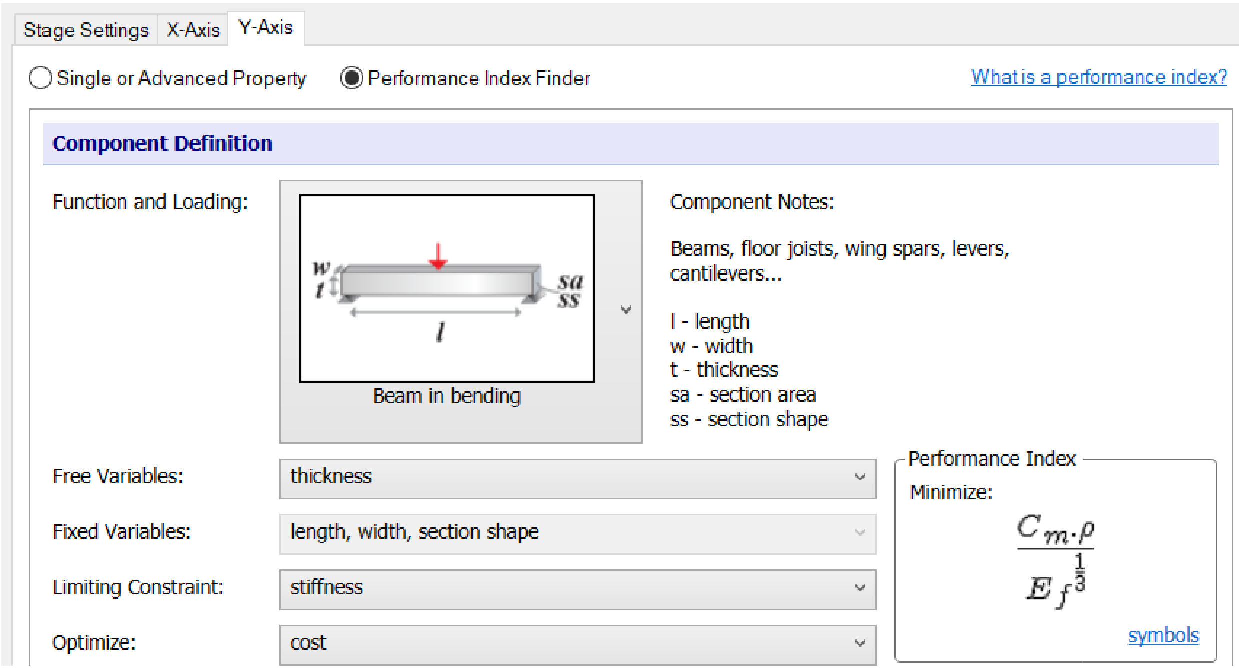
Application	Alpha [€/kg]
Passenger car	5
Truck	15
Commercial airplane	250
Fighter jet	750
Space shuttle	5000

In this exercise, I choose the Passenger car and the Space shuttle to observe the extreme opposite effect of exchange constant α in material selection. In this case, I choose the beam structure that is subject to normal bending force. Thus, this panel should have a good stiffness.

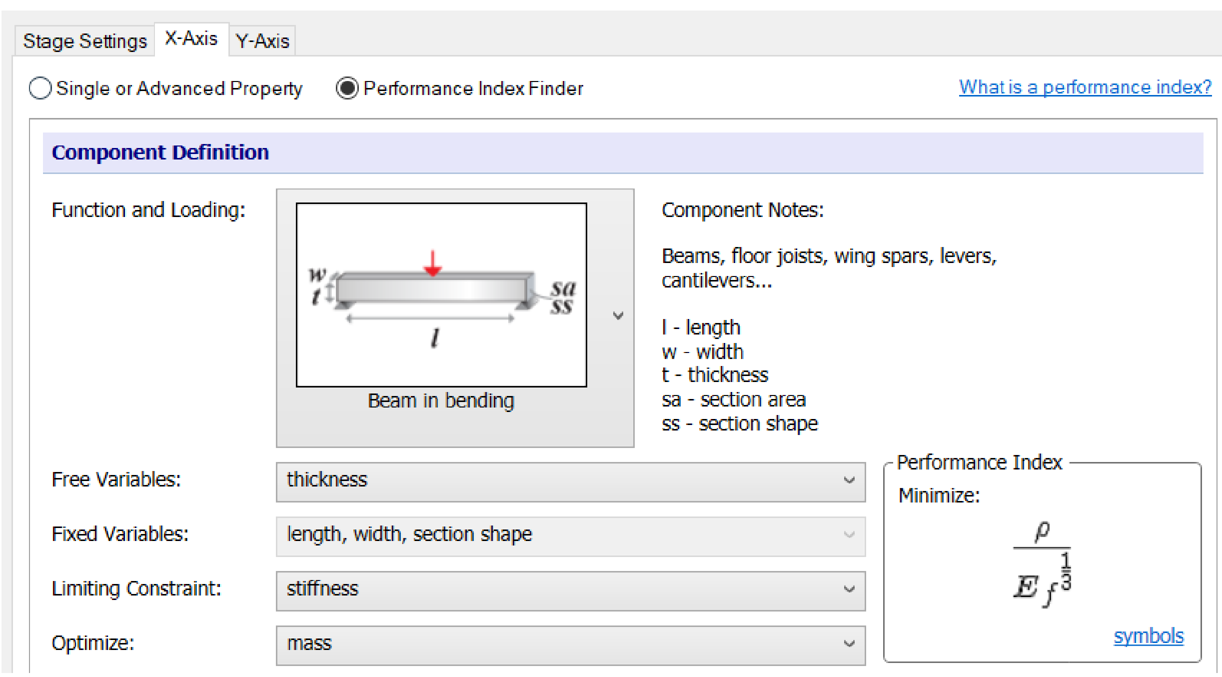
The problem can be defined as follows:

Function	❖ Vehicle frame under different loading conditions ❖ Shape is a beam frame under bending force
Constraints	❖ Fixed beam length, width and cross section shape ❖ Given bending stiffness S*
Objective	❖ Minimizing mass ❖ Minimizing cost
Free variables	❖ Choice of material ❖ Thickness t ❖ Any other material properties

The material performance index for minimizing cost is $M_1 = (C_m\rho)/E_f^{(1/3)}$



The material performance index for minimizing mass is $M_2 = \rho/E_f^{(1/3)}$



How to derive the material index:

For $M_1 = (C_m\rho)/E_f^{(1/3)}$

Given that the shape is a beam frame under bending force, the bending stiffness S* is given by:

$$\mathbf{S^* = E_f \times L}$$

where E_f is the flexural modulus and I is the second moment of area. For a rectangular cross-section beam, inertia I can be expressed as:

$$\mathbf{I = (w * t^3)/12}$$

where w is the width and t is the thickness of the beam.
The volume V of the beam is given by:

$$\mathbf{V = l * w * t}$$

where l is the length of the beam.
The mass m of the beam can be expressed as:

$$\mathbf{m = \rho * V = \rho * l * w * t}$$

where ρ is the density of the material.
The cost C of the beam can be expressed as:

$$\mathbf{C = C_m * m = C_m * \rho * l * w * t}$$

where C_m is the material cost per unit kg.
We need to minimize cost while maintaining a given bending stiffness S*. Therefore, we can express cost C in terms of S*:

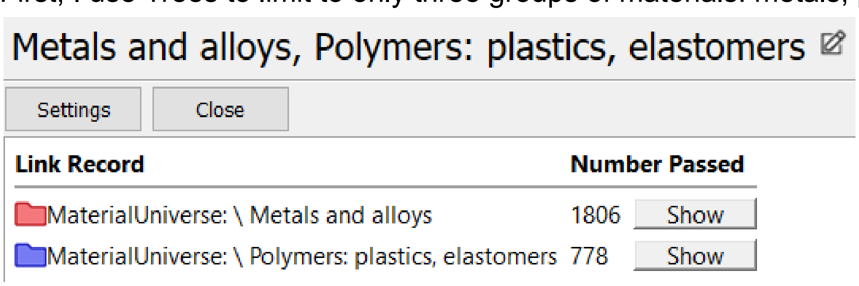
$$\mathbf{C = C_m * \rho * l * w * t = C_m * \rho * l * (12S^*/E_f)^{(1/3)}}$$

From this, we can see that to minimize cost for a given stiffness, we need to minimize the terms dependent on the material properties:

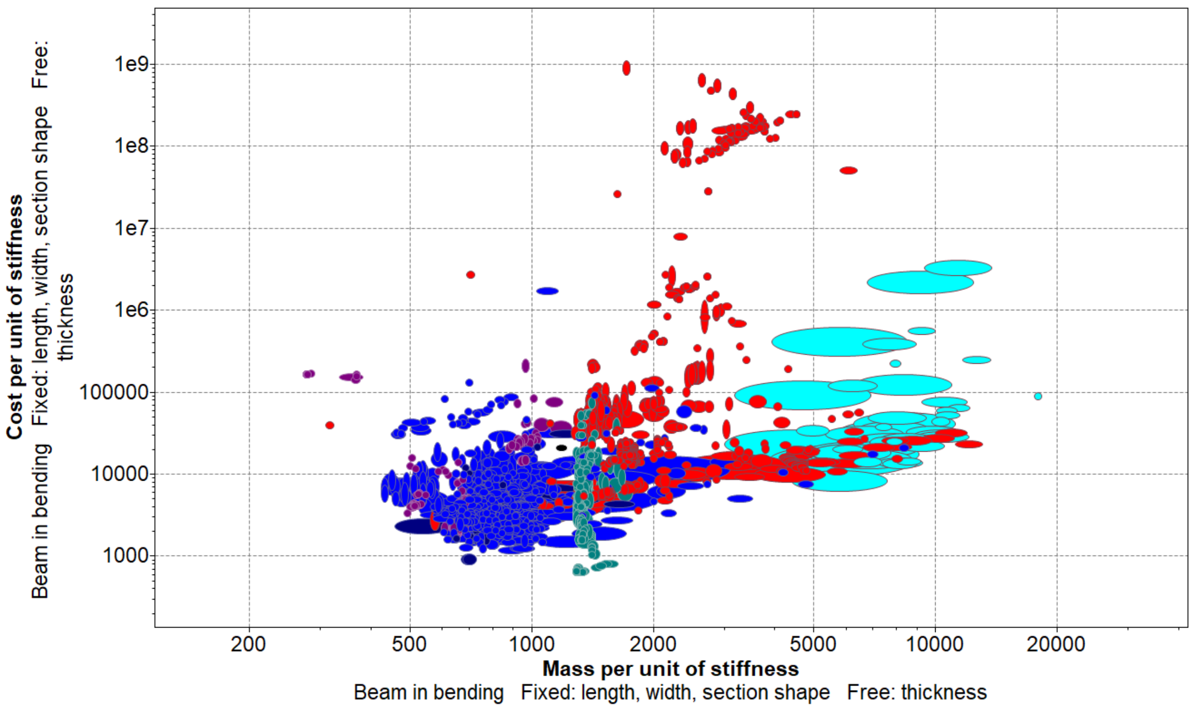
$$\mathbf{M_1 = (C_m\rho)/E_f^{(1/3)}}$$

We can derive the index M_2 minimizing the mass similarly like above

Next task is to plot the graphs.
First, I use Trees to limit to only three groups of materials: metals, polymers and fibers.



Then, I proceed to plot, which looks like this

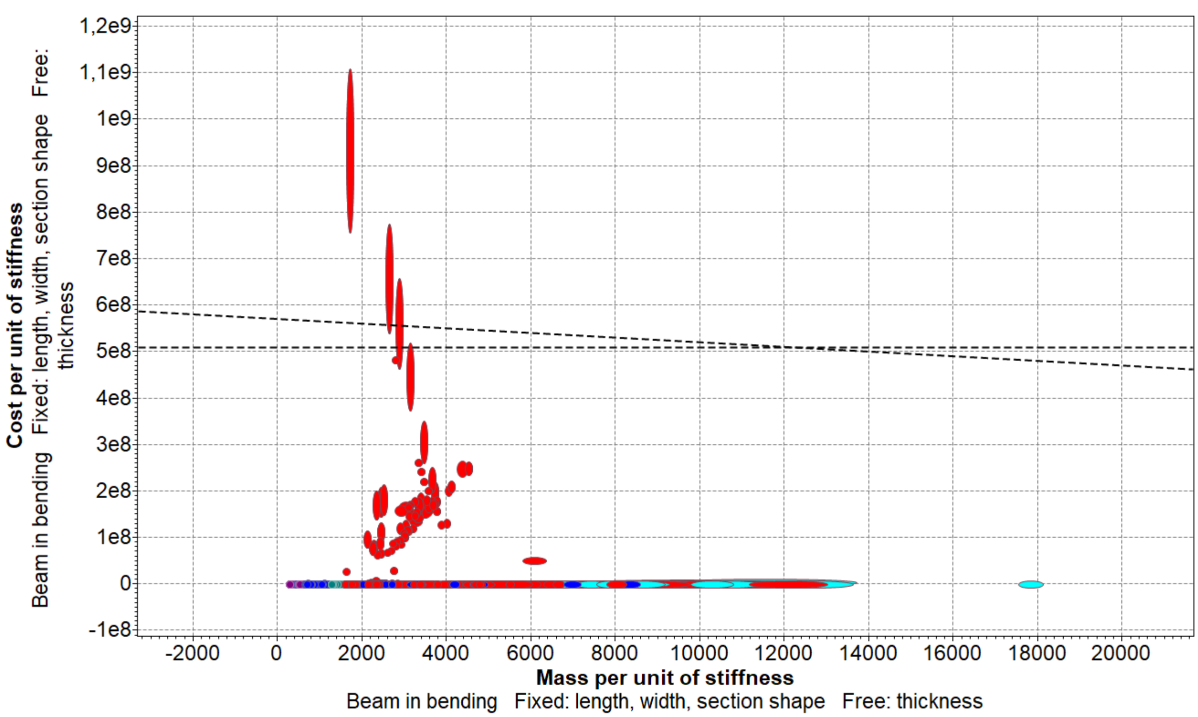


My penalty function is:

Z = $M_1 + \alpha M_2$, which basically says that the total cost should be minimized based on the material cost per unit and exchange cost reduction. In this case, α = 5 for cars and α = 5000 for the space shuttle. By rearranging, we have this equation:

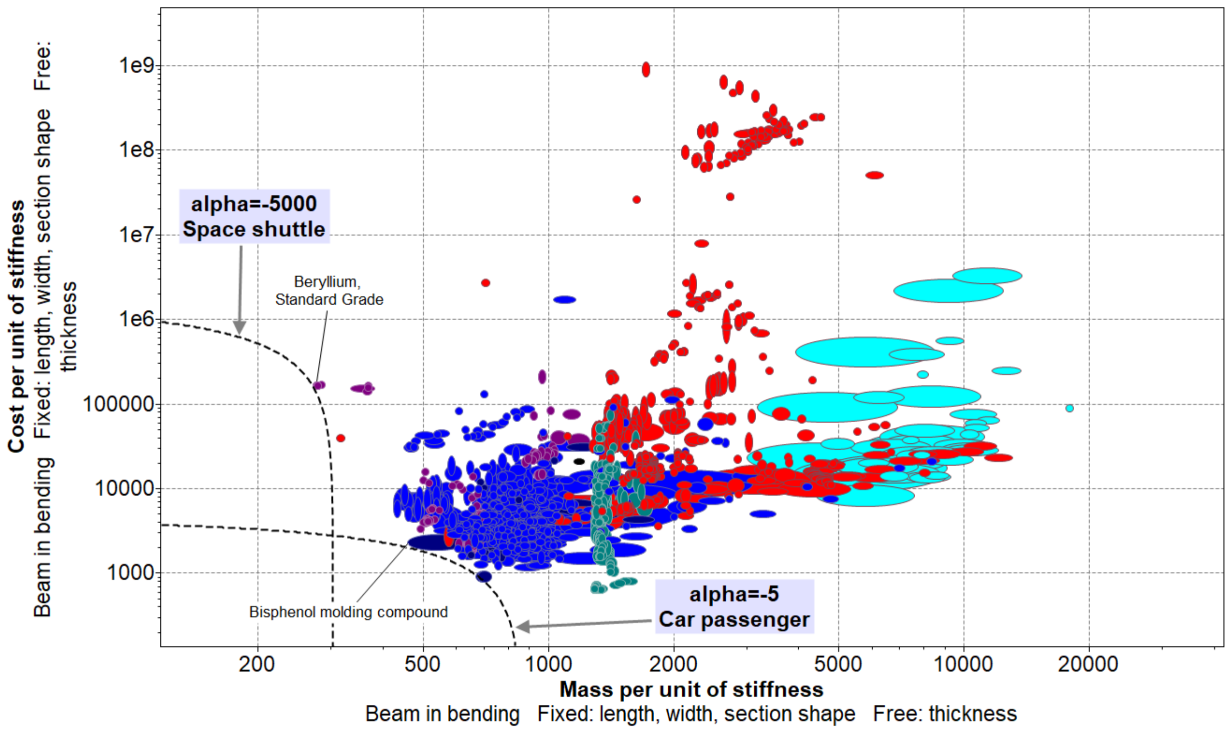
$$M_1 = -\alpha M_2 + Z$$

So the slope of the limiting constraint line is the negative of α. Set both axes to Linear scale and add the two limiting index lines as below:



Finally, we can switch back the log scale, where the dotted lines are the penalty line with different α values. We can now determine that:

- Optimal material for passenger car is **Bisphenol molding**
- Optimal material for the space shuttle is **Standard grade Beryllium**



Note, the chosen materials are quite novel and harder to produce. In reality, kenaf fiber is used quite commonly in the automotive industry [1][2], while carbon fibers are used in the spacecraft and aerospace industry [3][4], such as RRC. Result could have been different if it is conducted on Level 2 Database

References:

[1] Mohd Radzuan NA, Ismail NF, Fadzly Md Radzi MK, Razak ZB, Tharizi IB, Sulong AB, Che Haron CH, Muhamad N. Kenaf Composites for Automotive Components: Enhancement in Machinability and Moldability. Polymers (Basel). 2019 Oct 17;11(10):1707. doi: 10.3390/polym11101707. PMID: 31627431; PMCID: PMC6836254.

[2] https://www.plasticstoday.com/sustainability/kenaf-composite-toyota-concept-car

[3]https://www.smicomposites.com/the-role-of-carbon-fiber-in-aerospace-materials/#:~:text=Spacecraft,shields%20and%20rocket%20motor%20nozzles.

[4] https://www.nature.com/articles/d41586-023-00834-3

Task 2.2: Not so long ago, the most prevalent casing material for mobile phones and portable music players was ABS-plastic.

The challenge: to identify materials for casings that are at least as stiff as a 2-mm ABS case but thinner and lighter. We must recognize that the thinnest may not be the lightest, and vice versa. A trade-off will be needed.

From the Textbook:

Table 8.10 The Design Requirements: Casing for Portable Electronics

Function	Light, thin (cheap) casing
Constraints	Bending stiffness S^* specified Dimensions L and W specified
Objectives	Minimize thickness of casing Minimize mass of casing (Minimize material cost)
Free variables	Thickness t of casing wall Choice of material

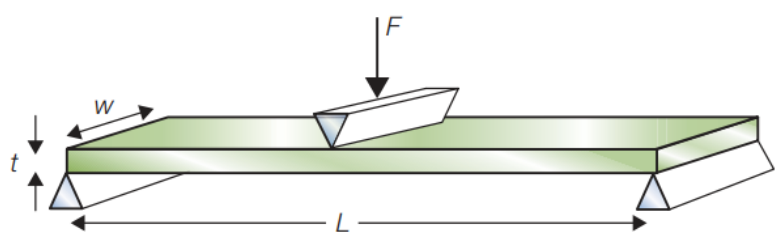


FIGURE 8.10
The casing can be idealized as a panel of dimensions $L \times W$ and thickness t, loaded in bending.

The thinnest panel is that made from the material with the smallest value of the index:

$$M_1 = \frac{1}{E^{1/3}}$$

The mass of the panel per unit area, m_a, is just ρt, where ρ is its density—the lightest panel is that made from the material with the smallest value of

$$M_2 = \frac{\rho}{E^{1/3}} \qquad (8.25)$$

We use the existing ABS panel, stiffness S^*, as the standard for comparison. If ABS has a modulus E_o and a density ρ_o, then a panel made from any other material (modulus E, density ρ) will, according to Equation (8.24), have a thickness t relative to that of the ABS panel t_o given by

$$\frac{t}{t_o} = \left(\frac{E_o}{E}\right)^{1/3} \tag{8.26}$$

and a relative mass per unit area of

$$\frac{m_a}{m_{a,o}} = \left(\frac{\rho}{E^{1/3}}\right)\left(\frac{E_o^{1/3}}{\rho_o}\right) \tag{8.27}$$

We wish to explore the trade-off between t/t_o and $m_a/m_{a,o}$ for possible solutions.

Where E_0 and ρ_0 are the Young's modulus and density of ABS

Hint*:* In this task the casing material is approximated as a bent sheet structure, for which the limiting factor is stiffness. Alternative parameters that should be optimized are mass and wall thickness. Construct a relative material property map where the alternative materials are located in a WIN-LOSE – four-quadrant map. Place the reference material choice (ABS) so that its coordinates are (1,1). The relative material properties can be expressed using the following formula: $\frac{P(M_n)}{P(M_{nABS})}$. In your answer, use the material properties for ABS, which can be found from GRANTA EduPack, level 2 materials, or from the textbook appendices.

Draw a relative material property map following the example from the textbook: 8.6 "*Wafer-thin casings for must-have electronics*".

Step 1: Search for ABS material and find its Young's modulus and density

Acrylonitrile butadiene styrene (ABS)

Datasheet view: All properties | Show/Hide | Find Similar

Polymers and elastomers > Polymers > Thermoplastics >

Description

Image

Density	ⓘ	1,03e3	-	1,06e3	kg/m^3
Young's modulus	ⓘ	2,07	-	2,76	GPa

Step 2: Plot with advanced option the formula for x-axis. Take the average of ABS E_0

$$\frac{t}{t_o} = \left(\frac{E_o}{E}\right)^{1/3} \tag{8.26}$$

Axis Settings

Axis Title: (2,41 / Young's modulus)^(1 / 3)

Absolute values | Relative values

Logarithmic | Linear

Autoscale | Set 0,0121289 min - 77,9213 max

and plot similarly for y-axis this formula. Take the average of ABS's density

$$\frac{m_a}{m_{a,o}} = \left(\frac{\rho}{E^{1/3}}\right)\left(\frac{E_o^{1/3}}{\rho_o}\right) \tag{8.27}$$

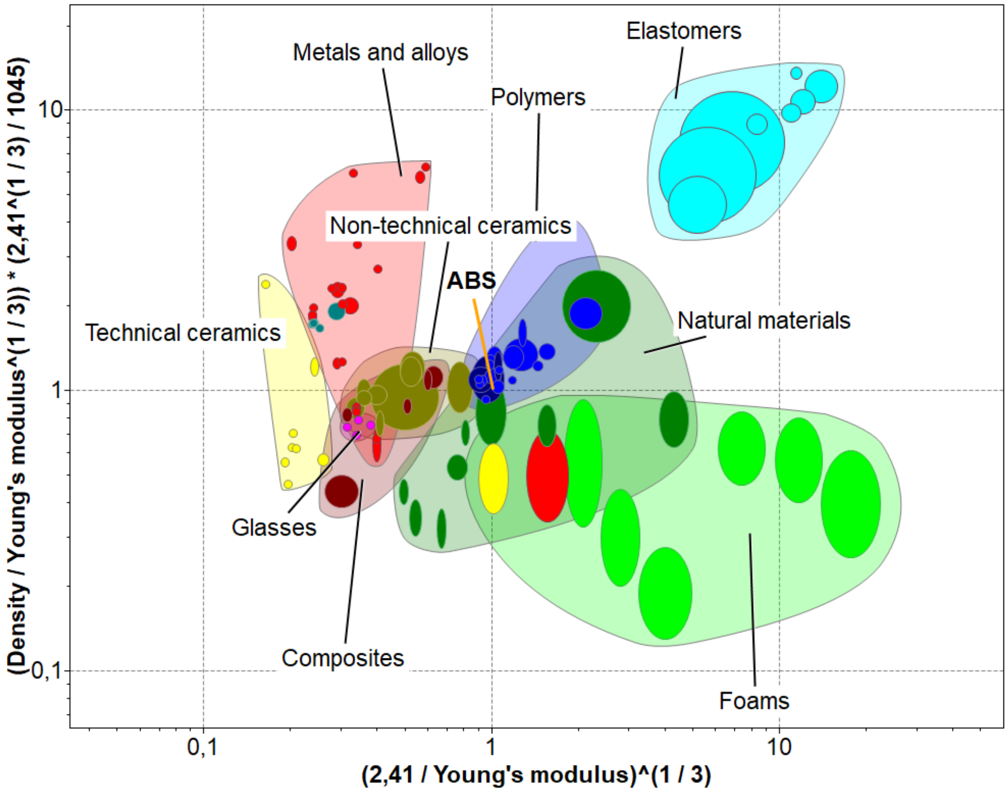
Axis Settings

Axis Title: (Density / Young's modulus^(1 / 3)) * (2,41^(1 / 3) / 1045)

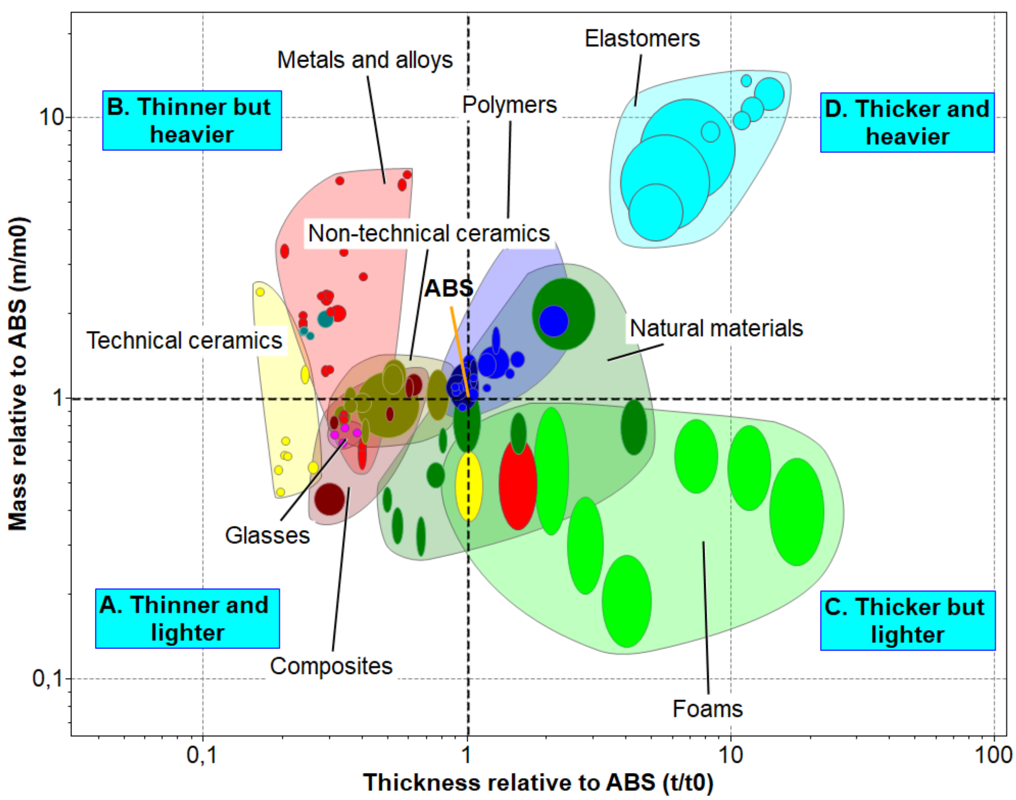
Absolute values | Relative values

Logarithmic | Linear

We arrive at this graph:



Step 3: Renaming the x-axis and y-axis and add the WIN - LOSE quadrant textbox



Step 4: Draw the Pareto Front line and the penalty function line as the final answer. According to the textbook, the penalty function is:

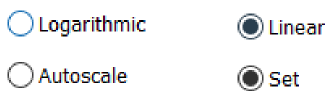
This is already enough to suggest choices that offer savings in thickness and in weight. If we want to go further we must formulate a *relative penalty function*. Define Z^*, measured in units of currency, as

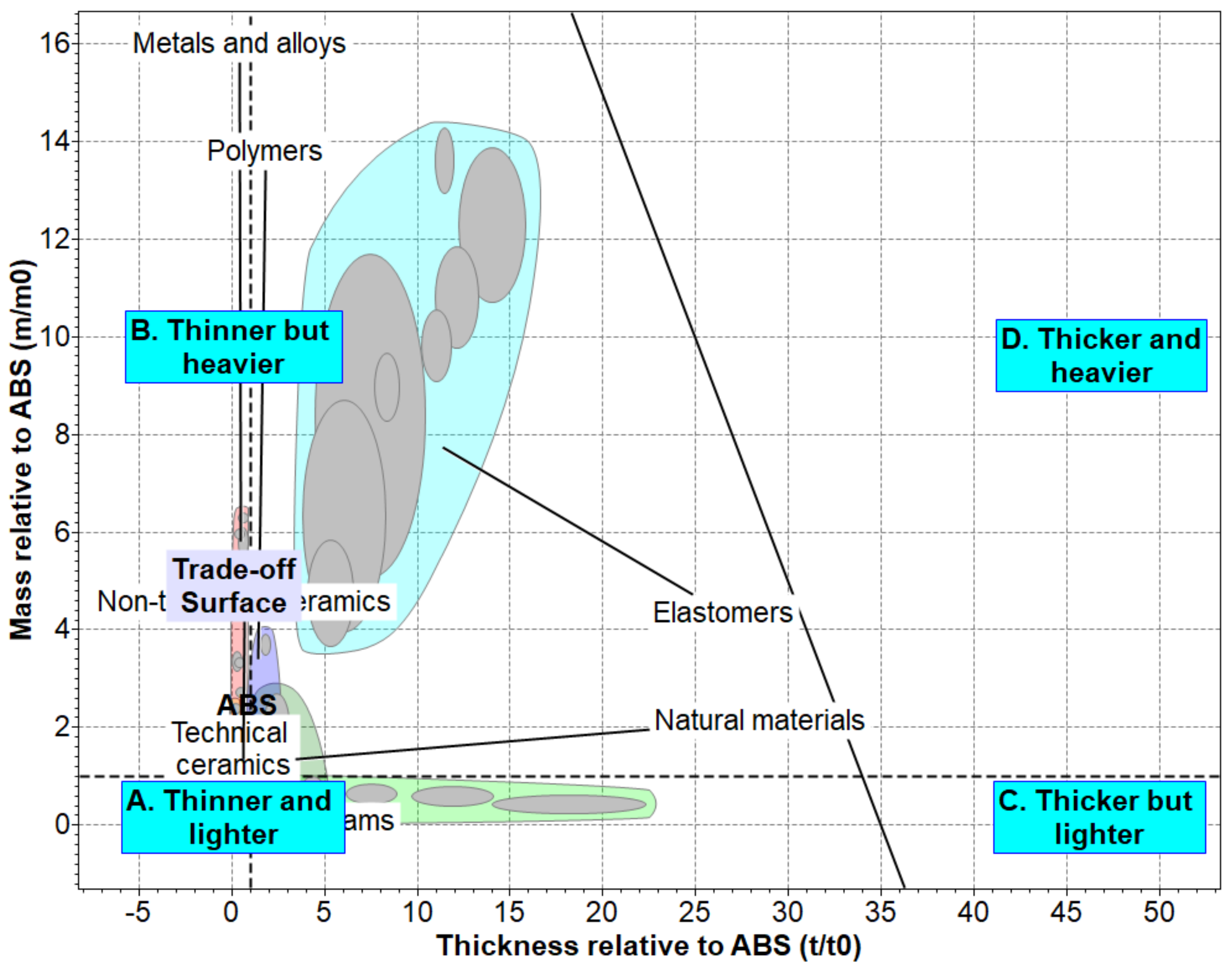
$$Z^* = \alpha_t^* \frac{t}{t_o} + \alpha_m^* \frac{m_a}{m_{a,o}} \qquad (8.28)$$

As an example, set $\alpha_t^* = \alpha_m^*$, meaning that we value both equally. Then solutions with equal penalty Z^* are those on the contour

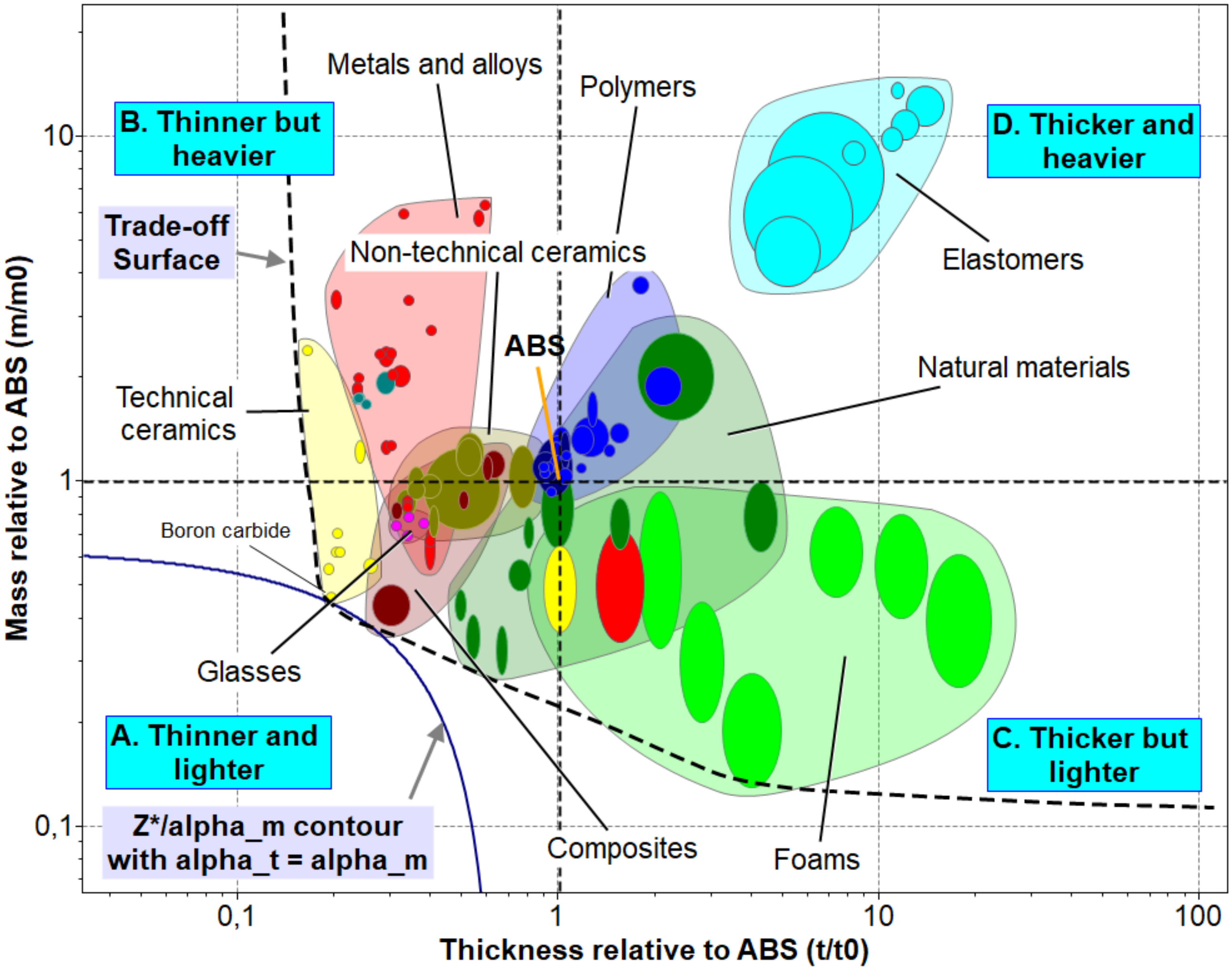
$$\frac{Z^*}{\alpha_m^*} = \frac{t}{t_o} + \frac{m_a}{m_{a,o}} \qquad (8.29)$$

Change both axes to linear to plot the line Z = X + Y, or Y = -X + Z. The slope will be -1





Then, switch both axes to Log again. The index line will be a log curve



The best material according to the Pareto Front and the penalty function is **Boron Carbide**

Compare the casing materials from nowadays (e.g., aluminum, carbon fiber and glass) to ABS. **Describe** and **explain** your observations.

- Aluminum is more rigid and durable than ABS. While ABS can be prone to warping under certain conditions, aluminum remains stable. However, aluminum is more thermally and electrically conductive than ABS, which is not preferable for casing.
- Carbon fiber is significantly stronger and stiffer than ABS. While ABS can be injection molded, carbon fiber parts are typically made using a layup process, which can be more labor-intensive. Carbon fiber is also more expensive than ABS.
- Glass is more brittle than ABS. However, glass offers better optical clarity and UV resistance. ABS can turn yellow when exposed to sunlight, while glass remains clear.
- In this case, I choose Boron Carbide as the best material for the casing. It minimizes the penalty function while also lying on the Pareto Front