Sebastian Pouttu

MEC-E1070

Prof. Junhe Lian

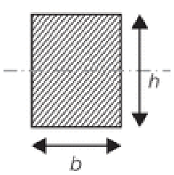
Prof. Sven Bossuyt

DL: Friday, 29 September 2023

Task 3

This report will be dedicated to identifying shape factors and how to identify them. Shape factors are necessary to taking into account the effect that shape has on selecting materials. Shape factors are measures of the efficiency of material usage. The shape factors are calculated using the cross-sections of certain shapes. Below are examples of shape factors being calculated for solid square, solid rectangle, hollow square, hollow round pipe, and I-beam profiles:

a) solid square profile, width, and height 100 mm.

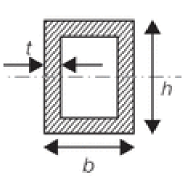


$$\phi_B^e = \frac{h}{b} = \frac{100}{100} = 1 \tag{1}$$

b) solid rectangle profile, width 500 mm, height 100 mm.

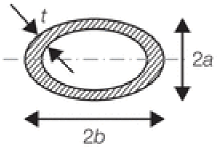
$$\phi_B^e = \frac{h}{b} = \frac{100}{500} = 0.2 \tag{2}$$

c) hollow square profile, outer width and height 100 mm, wall thickness 10 mm.



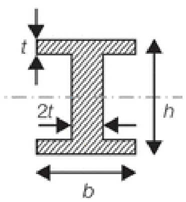
$$\phi_B^e = \frac{1}{2}\frac{h}{t}\frac{\left(1+\frac{3b}{h}\right)}{\left(1+\frac{b}{h}\right)^2} = \frac{1}{2}\frac{100}{10}\frac{\left(1+\frac{3\cdot 100}{100}\right)}{\left(1+\frac{100}{100}\right)^2} = 5 \tag{3}$$

d) hollow round pipe, outer diameter 500 mm, wall thickness 10 mm.



$$\phi_B^e = \frac{3}{\pi}\frac{a}{t}\frac{\left(1+\frac{3b}{a}\right)}{\left(1+\frac{b}{a}\right)^2} = \frac{3}{\pi}\frac{250}{10}\frac{\left(1+\frac{3\cdot 250}{250}\right)}{\left(1+\frac{250}{250}\right)^2} = 23.87 \tag{4}$$

e) a spar of the wind turbine blade, whose cross-section is either an I-beam with a width of 500 mm, height of 100 mm, flange thickness of 10 mm, and web thickness of 20 mm, or a box girder with a width of 500 mm, height 100 mm, wall thickness 10 mm



$$\phi_B^e = \frac{1}{2}\frac{h}{t}\frac{\left(1+\frac{3b}{h}\right)}{\left(1+\frac{b}{h}\right)^2} = \frac{1}{2}\frac{100}{10}\frac{\left(1+\frac{3\cdot 500}{100}\right)}{\left(1+\frac{500}{100}\right)^2} = 2.22 \tag{5}$$

Task 3.1.2

Using these calculated shape factors they can be integrated to modulus vs density plots which allow a visual comparison of materials with different surface profiles. This is done by dividing the moduli and densities by their respective shape factor. Figure 5 depicts nine profiles involving three different materials; GFRP, high-carbon steel, and hardwood.

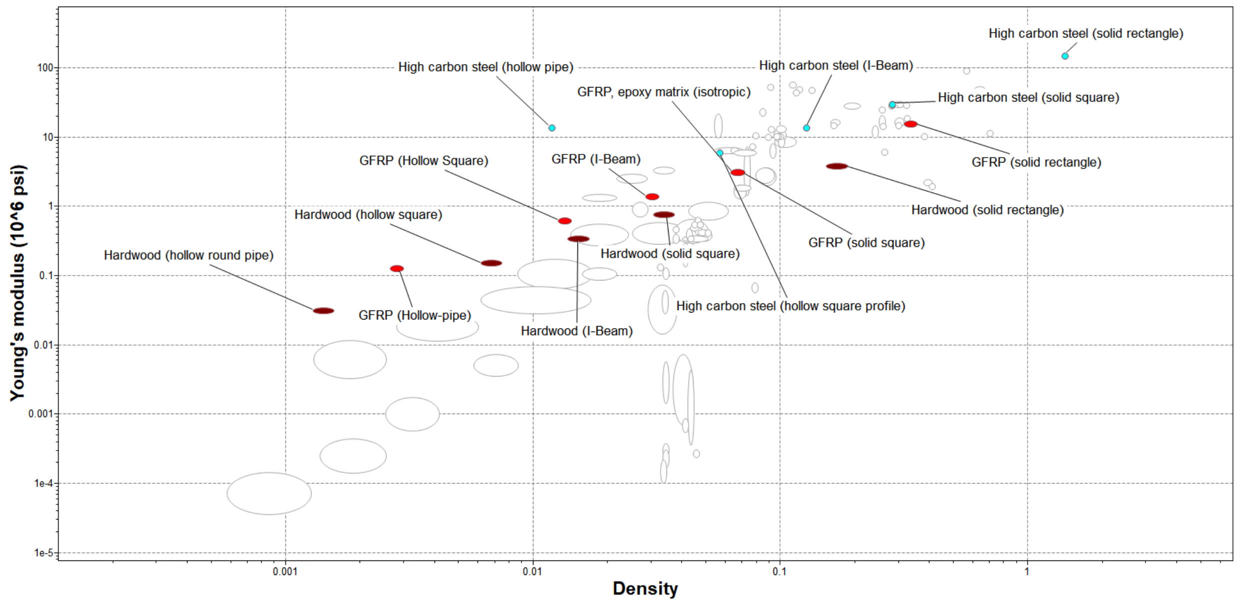


Figure 5: Modulus vs Density plot including nine different combinations of surface profiles and materials

It can be seen that in all materials, the solid rectangle has the highest modulus and density, so having a bar of such shape for either material would not be ideal. Hollow round pipes seem to have the highest modulus with a relatively low density. More specifically hollow round pipes made of high carbon steel. If someone were wanting to have the lightest round pipe with a relatively good modulus, then a carbon fiber round pipe would prove to be sufficient. Wood low density material, relatively speaking, but it does not offer as good of a modulus. Where density is not as much of an issue it is understandable why so many structures are made of high-carbon steel pipes. Below is a numerical depiction of Figure 5.

Material property values	Reference shape (Profile a) $\phi_e = 1$	Profile b $\phi_e = 0.2$	Profile c $\phi_e = 5$	Profile d $\phi_e = 23.87$	Profile e $\phi_e = 2.22$
GFRP	E = 3.10 ρ = 0.0672	E = 15.5 ρ = 0.336	E = 0.621 ρ = 0.0134	E = 0.130 ρ = 0.002815	E = 1.395 ρ = 0.0303
High carbon steel	E = 30.5 ρ = 0.282	E = 150 ρ = 1.41	E = 6.1 ρ = 0.0564	E = 13.8 ρ = 0.0118	E = 0.691 ρ = 0.0153
Hardwood	E = 0.767 ρ = 0.03395	E = 3.84 ρ = 0.17	E = 0.1535 ρ = 0.00679	E = 0.03215 ρ = 0.001425	E = 0.3455 ρ = 0.0153

Table 1: Numerical values of points in modulus vs density plot

Task 3.2.1

While it would be logical to assume that the higher the safety factor the better, there are limits which prevent placing material structures into scenarios which result in failure. Equation 6 describes the relationship between the elastic modulus and the yield strength and the role they play in discovering the safety limits.

$$(\phi_B^e)_{max} \approx 2.3\left(\frac{E}{\sigma_f}\right)^{0.5} \tag{6}$$

Material	Low Alloy Steel	Aluminum (cast)	CFRP	Wood
Young's Modulus (max) [x10^6 psi]	30.5	11	21.8	3.65
Yield Strength (max) [x10^3 psi]	232	38.1	152	7.66

Table 2: Values of maximum modulus and yield strength for Low-Alloy Steel, Aluminum, CFRP, and Wood

Plugging these values into equation 6 provides the following theoretical results for the maximum shape factor.

Material	Low Alloy Steel	Aluminum (cast)	CFRP	Wood (Hardwood)
$(\phi_B^e)_{max}$	26.37	39	27.5	50.2

Table 3: Values of calculated maximum safety factor for Low-Alloy Steel, Aluminum, CFRP, and Wood

Comparing these to the following empirical values:

Material	Steel	Aluminum (cast)	CFRP	Wood
$(\phi_B^e)_{max}$	65	44	39	5

Table 4: Values of empirical maximum safety factor for Low-Alloy Steel, Aluminum, CFRP, and Wood

There are certainly some differences to discuss. It can be inferred that the values being used for moduli or yield strengths for the materials are different from the difference in maximum factor values in all materials; most of all Steel and Wood. These were very off, so there is either an error in calculation or a misunderstood general value for each material. The values for Aluminum and CFRP were fairly to each other.

Task 3.2.2

Modulus vs Density material property charts display properties of materials. A combination of a modulus vs density plot, modulus vs second moment of area plot, section area vs density plot, and section area vs second moment of area plot get split into four quadrants. Connecting these quadrants which consider the material, stiffness constraint, section shape, and performance allow the user to easily detect a material's placement in all four quadrants. See Figure 6 to the material selection of a stiffness-limit design.

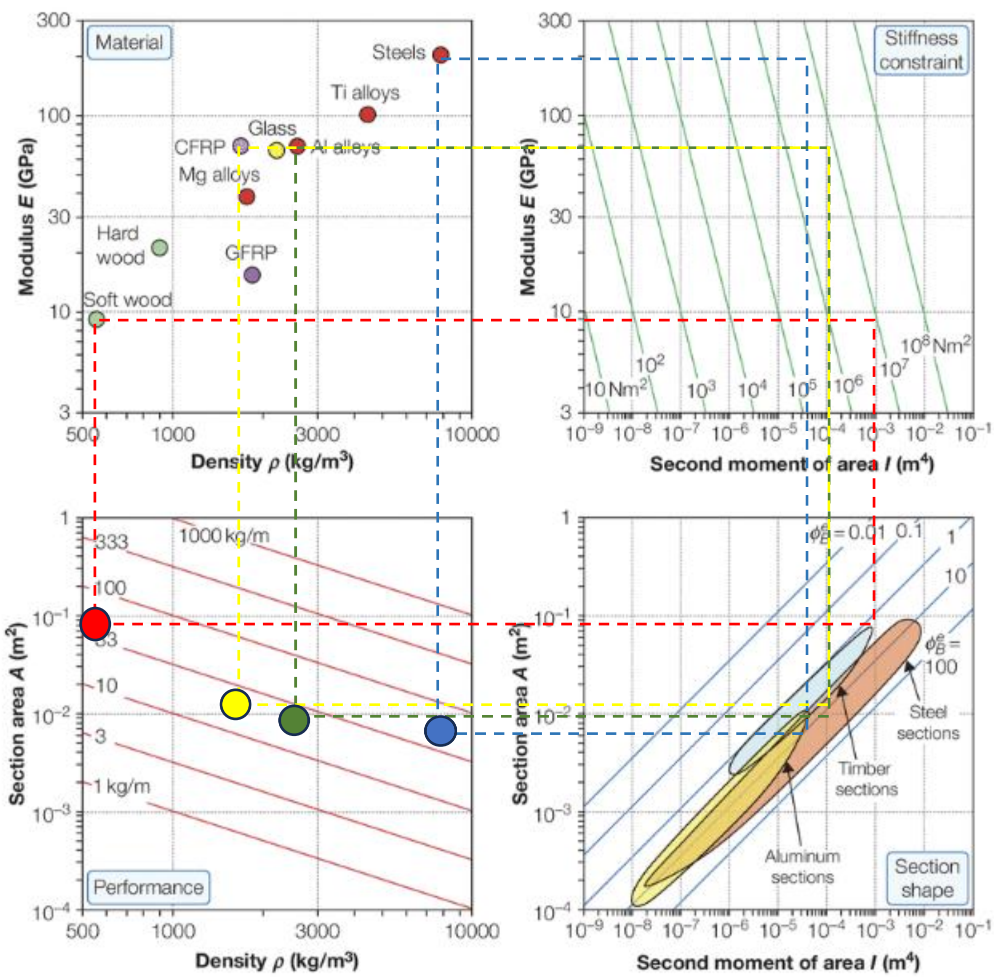


Figure 6; A comparison of steels, aluminum, CFRP, and wood sections for a stiffness-limited design with EI = 10^6 Nm^2

Seeing that on all four quadrants CFRP seems to be in an optimal balanced position in the plots, it could be inferred that CFRP would be the best choice for a light bending beam with good stiffness.