Aalto University
School of Engineering

MEC-E1070 Selection of Engineering Materials

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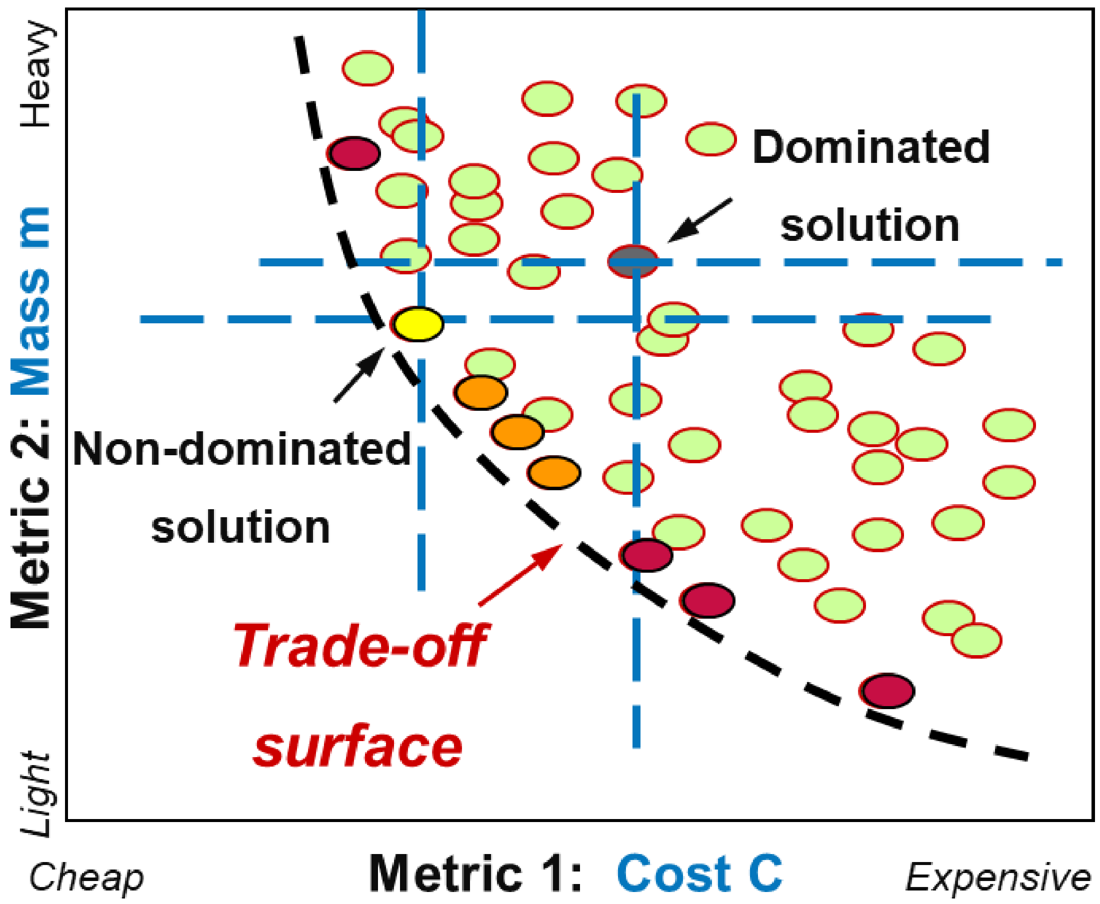
Objectives in conflict



- **Multiple objectives**: may conflict
- **Trade-off** methods
- **Penalty functions** and exchange constants
- **Multi-objective optimization**

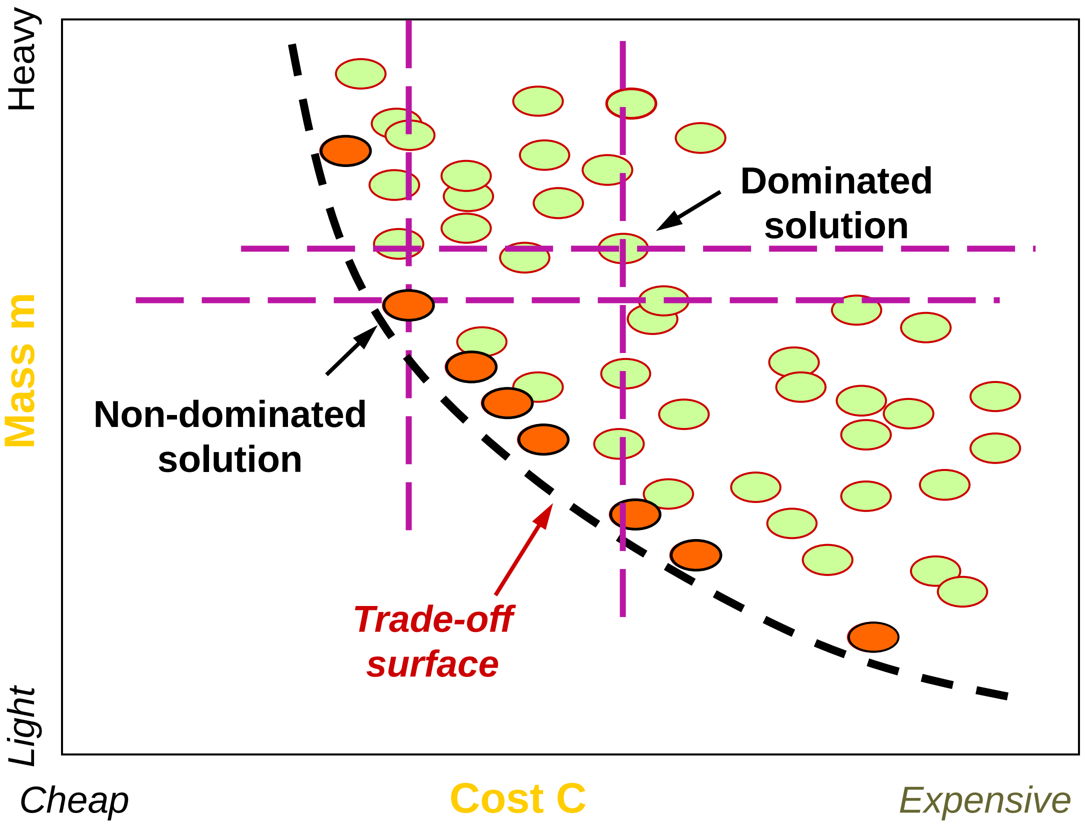
Multi-objective optimization

- ***"Solution":*** one candidate that meets the constraints, but not necessarily optimum by either objective

- Plot solutions.
(***Convention***: express objectives to be ***minimized***)

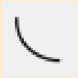
- ***"Dominated solution":*** one that is definitely non-optimal

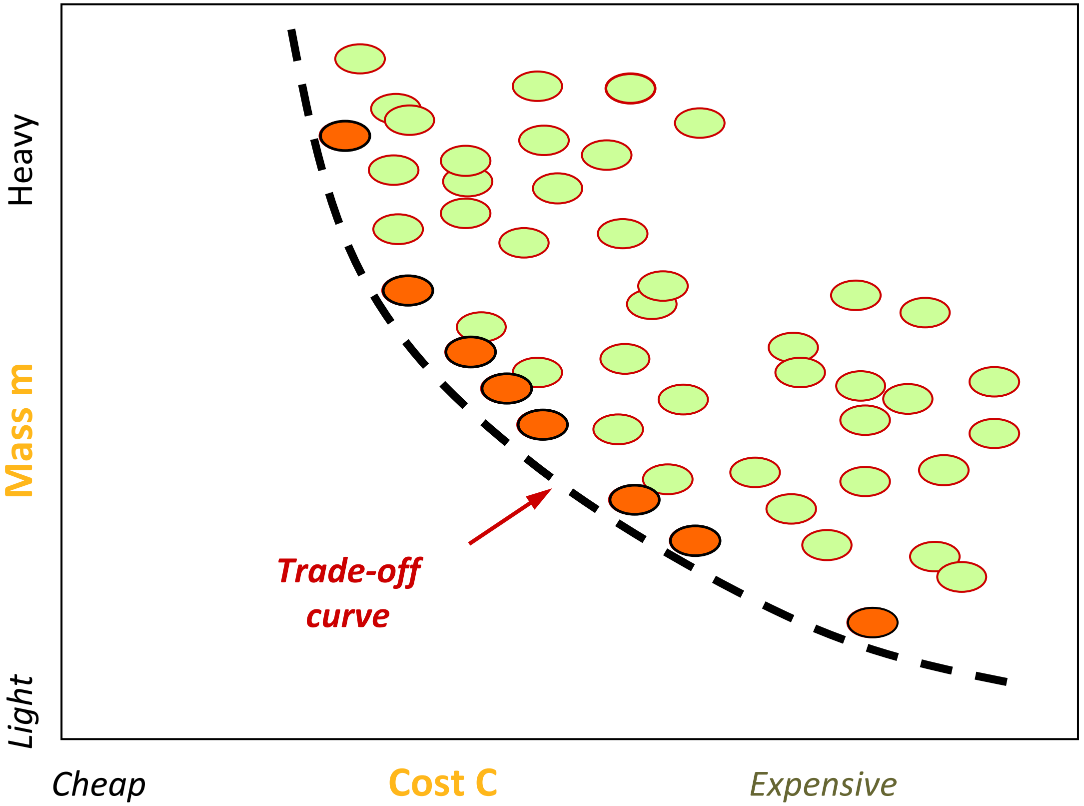
- ***"Non-dominated solution":*** one that is optimal by one metric (but not usually by both)



- ***"Trade-off surface":*** the surface on which the non-dominated solutions lie (Pareto Front). In 2-dimensional case: trade-off curve

Multi-objective optimization: documentation

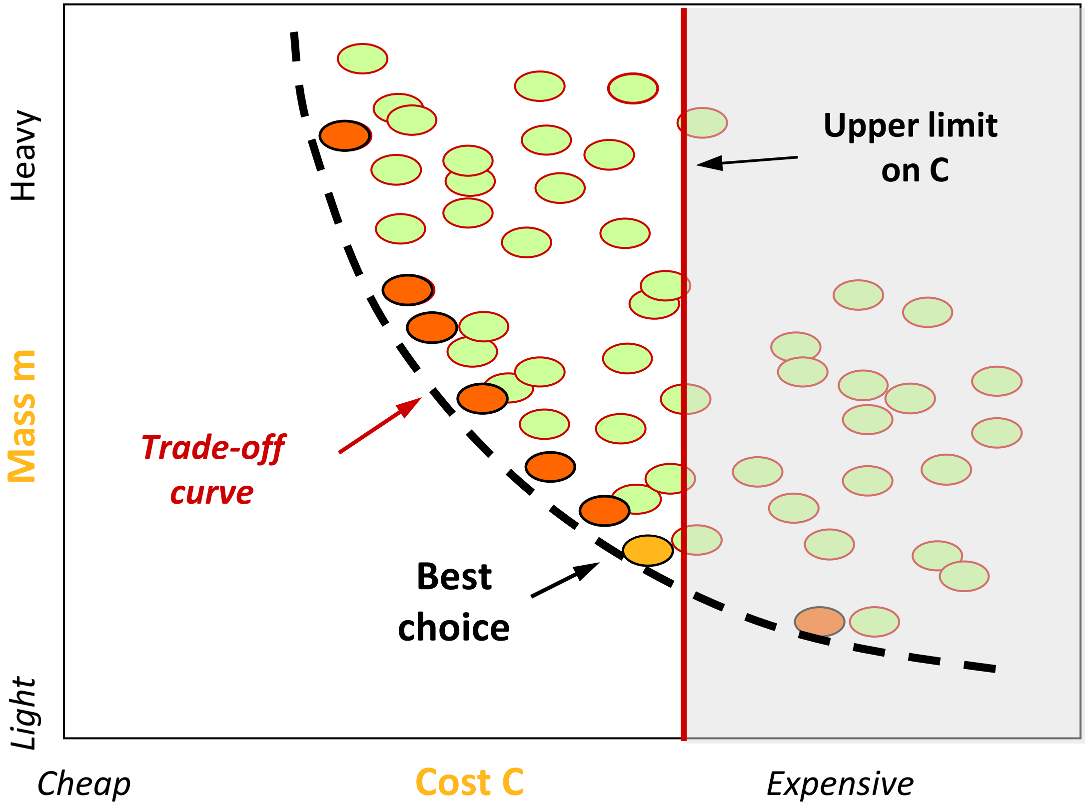
- **Make a trade-off plot** 
- **Sketch a trade-off curve** 
- **Use intuition** to select a solution on the trade-off curve



Multi-objective optimization: constraint

- ***Reformulate*** all but one of the objectives as constraints, setting an upper limit for it

e.g. budget limit



Multi-objective optimization: penalty function

Define locally-linear
Penalty function Z

$$Z = C + \alpha\, m$$

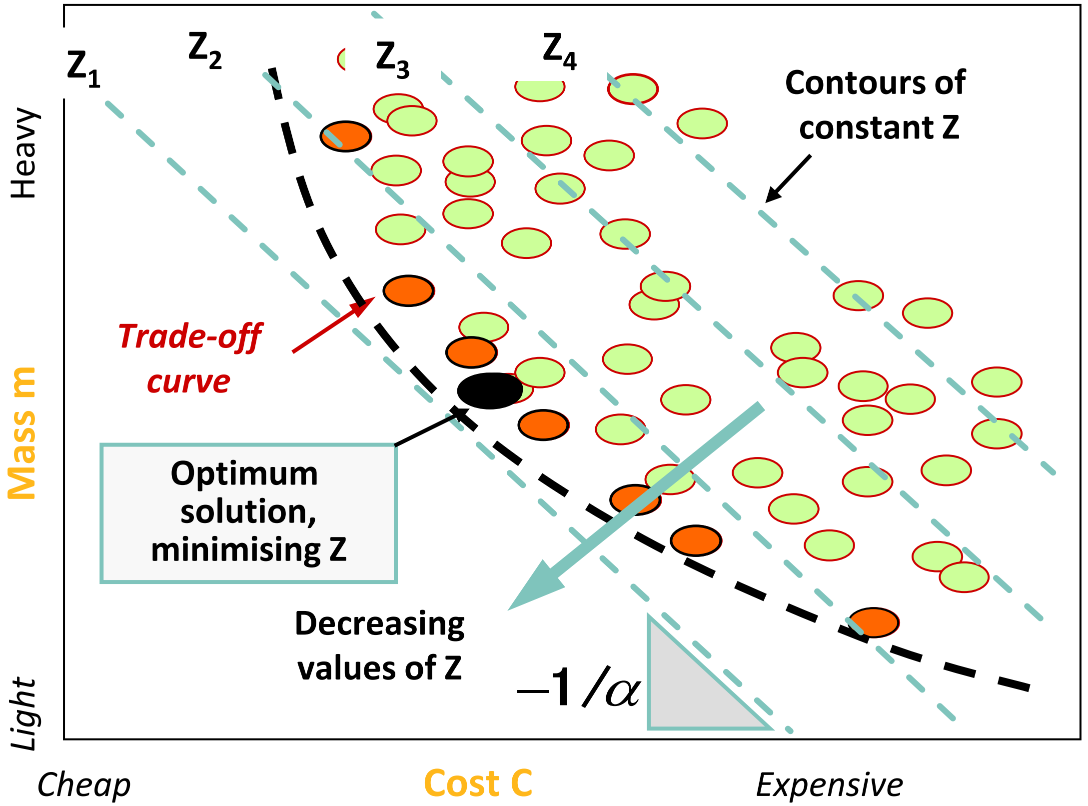
Seek solution with smallest Z

- Make **trade-off plot**

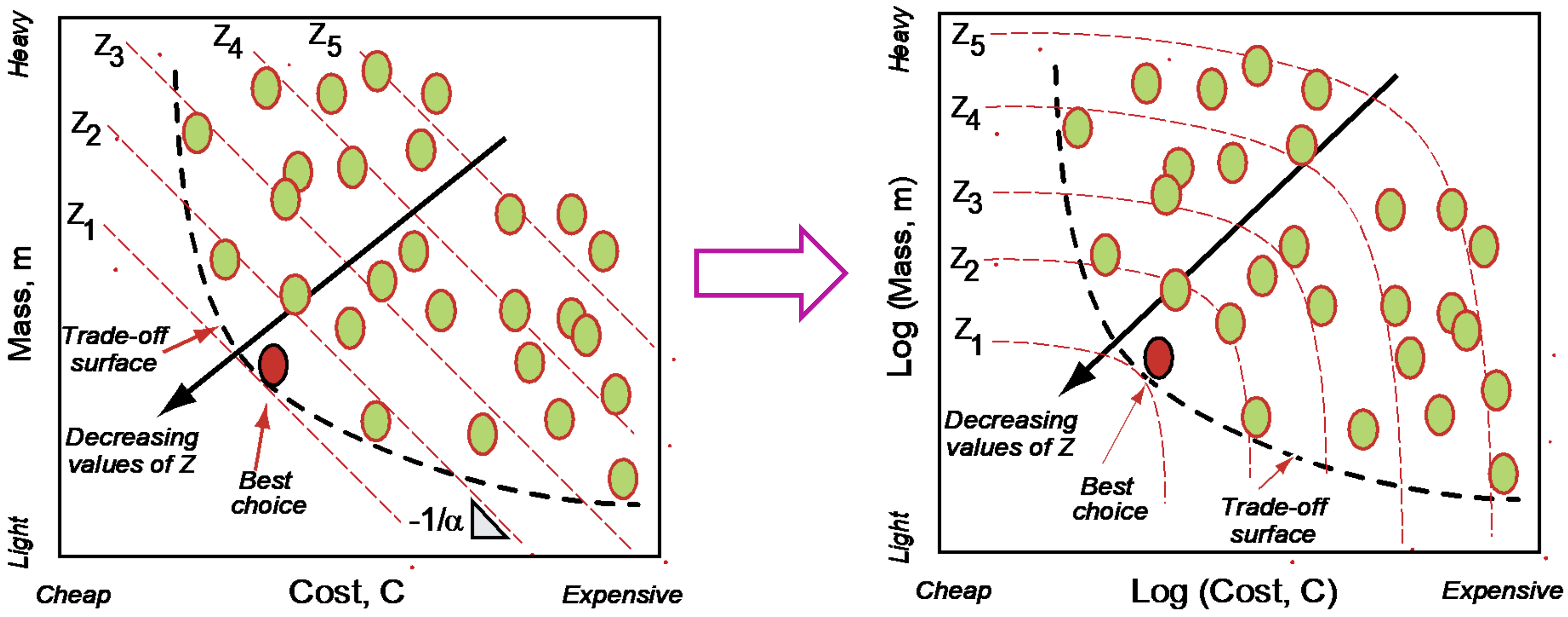
Rearranging the equation for Z

$$m = \boxed{-\frac{1}{\alpha}} C + \frac{1}{\alpha} Z$$

Lines of Z have slope -1/α



Linear penalty functions - linear axes



- Set axes to linear before plotting property charts for linear penalty functions
- Logarithmic scales give the same best choice with Z as curves

Two-objective Log chart

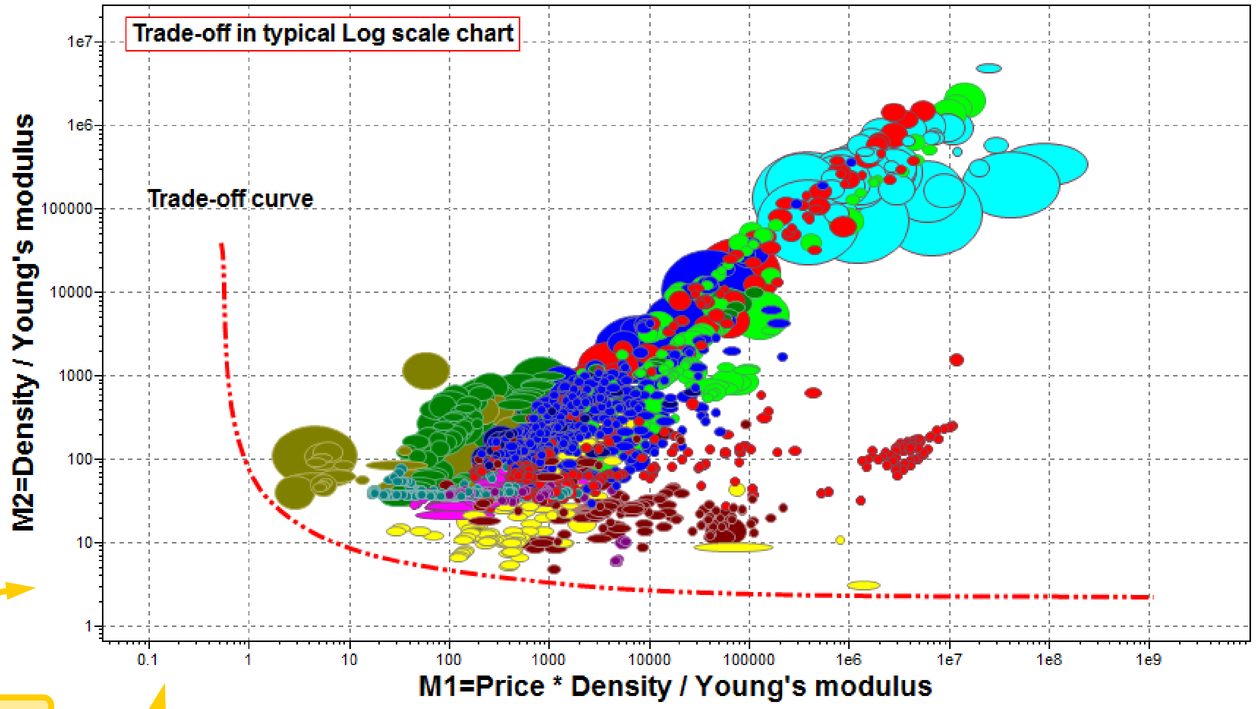
Minimum mass and cost for member in tensile or compressive load and stiffness-limited design

Maximize

$$\frac{E}{\rho} = M$$

Minimize

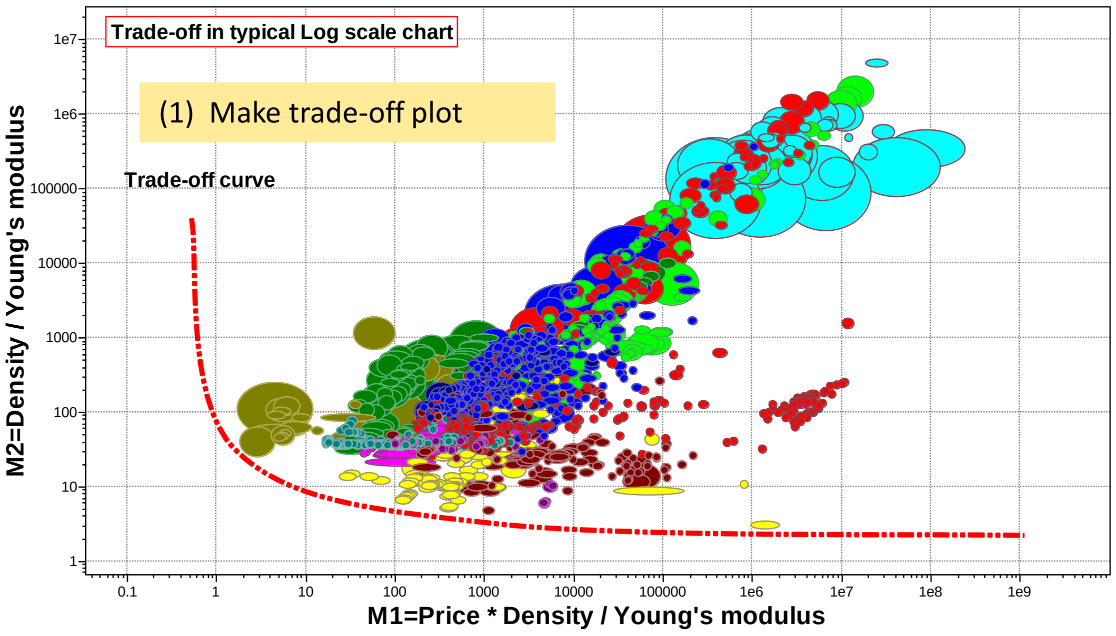
$M2 = \rho / E$

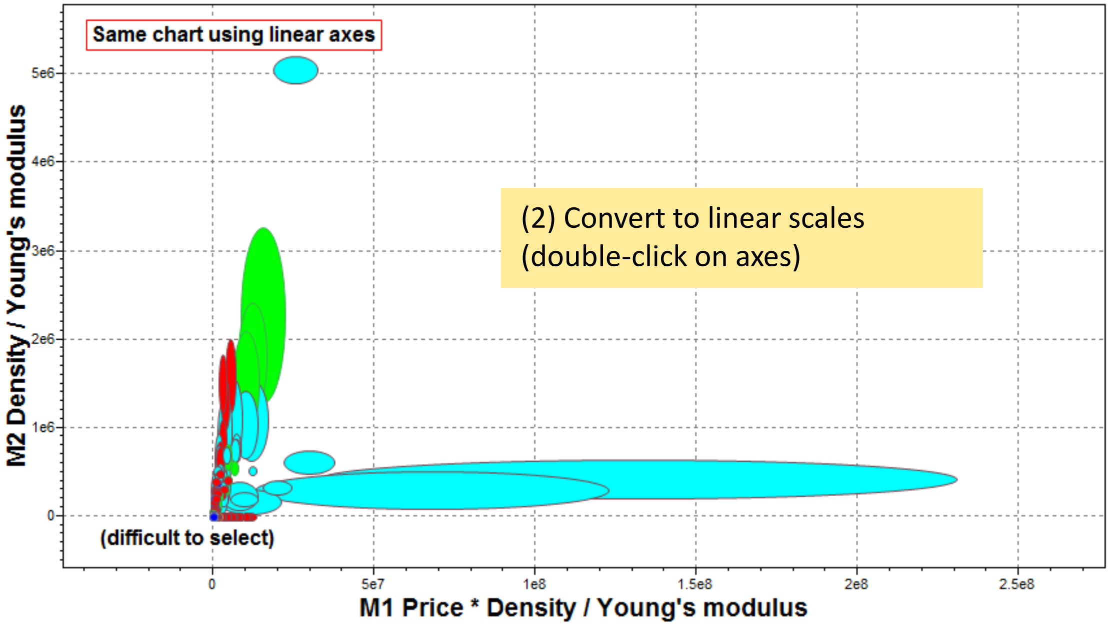


Log scales

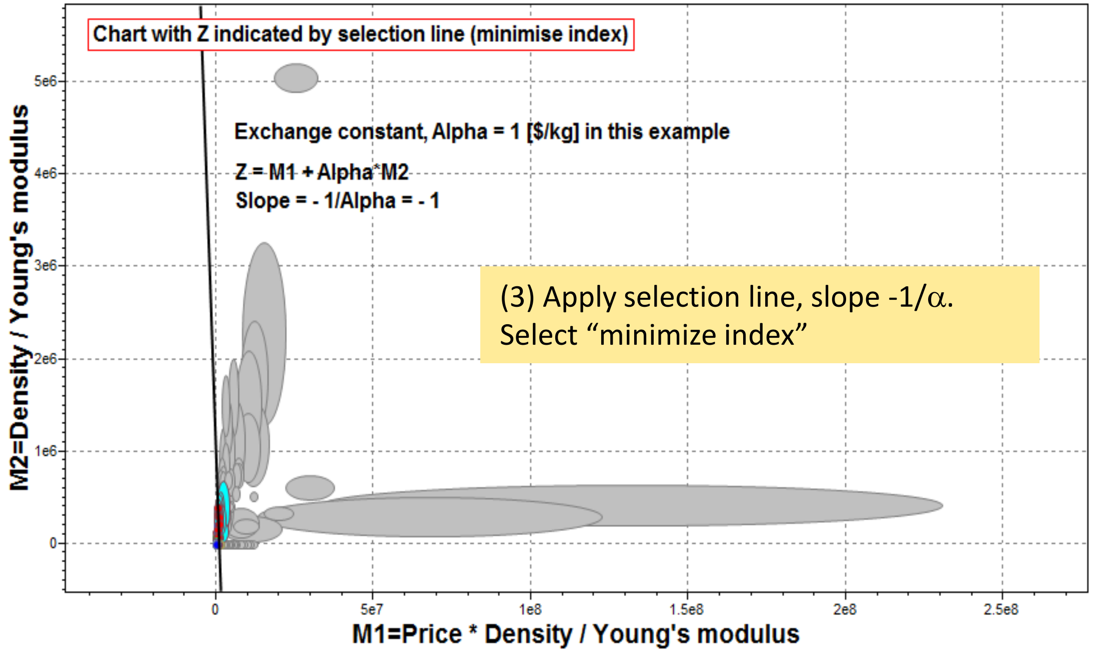
Minimize $M1 = C_m * \rho / E$

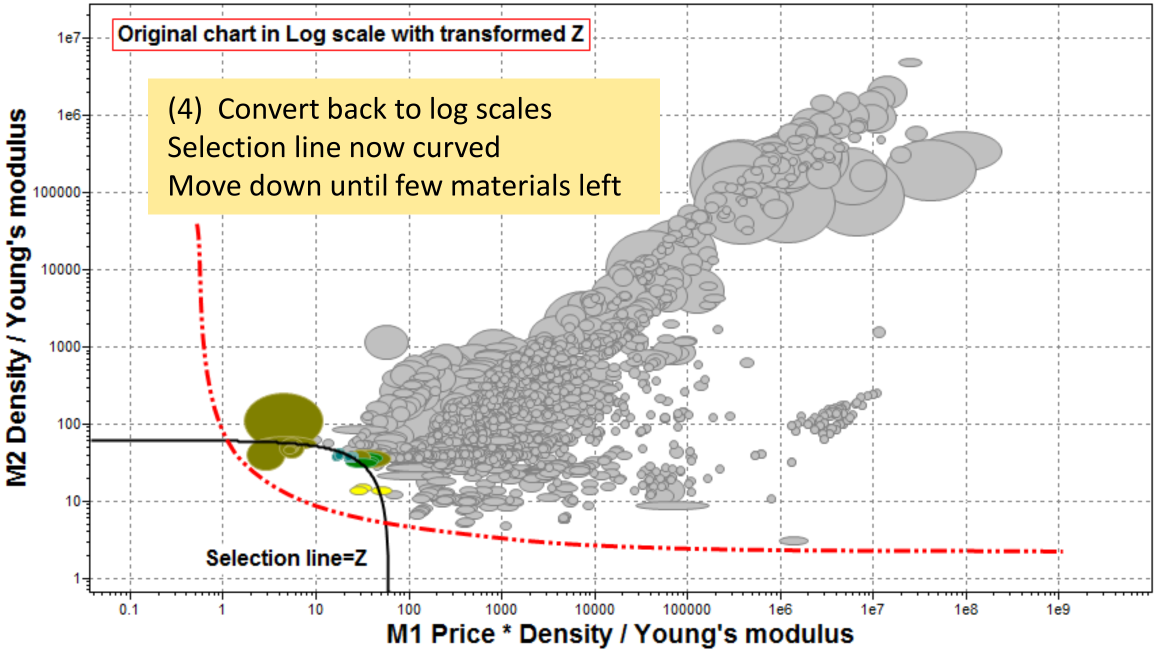
Penalty function with log axes





Penalty function with log axes





Bar chart selection using the penalty function

$$Z = \frac{\rho}{\sigma_y^{2/3}}\left(C_m + \alpha\right)$$

*Use the "**Advanced**" facility to make the penalty function*

(Density / (Yield strength^0.66))

***(Price + 10)**

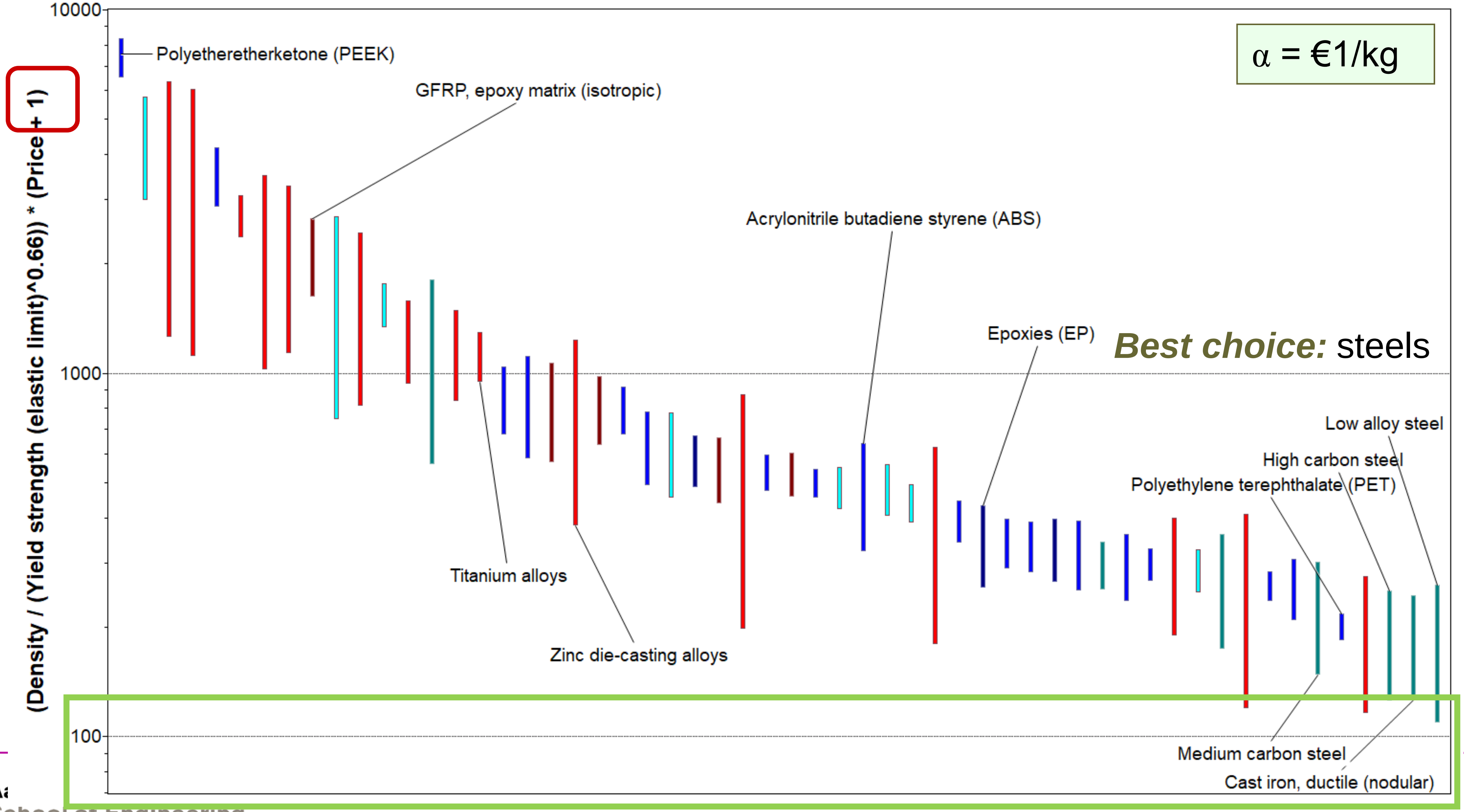
The value of the exchange constant

+	-	/	*	^	(	)

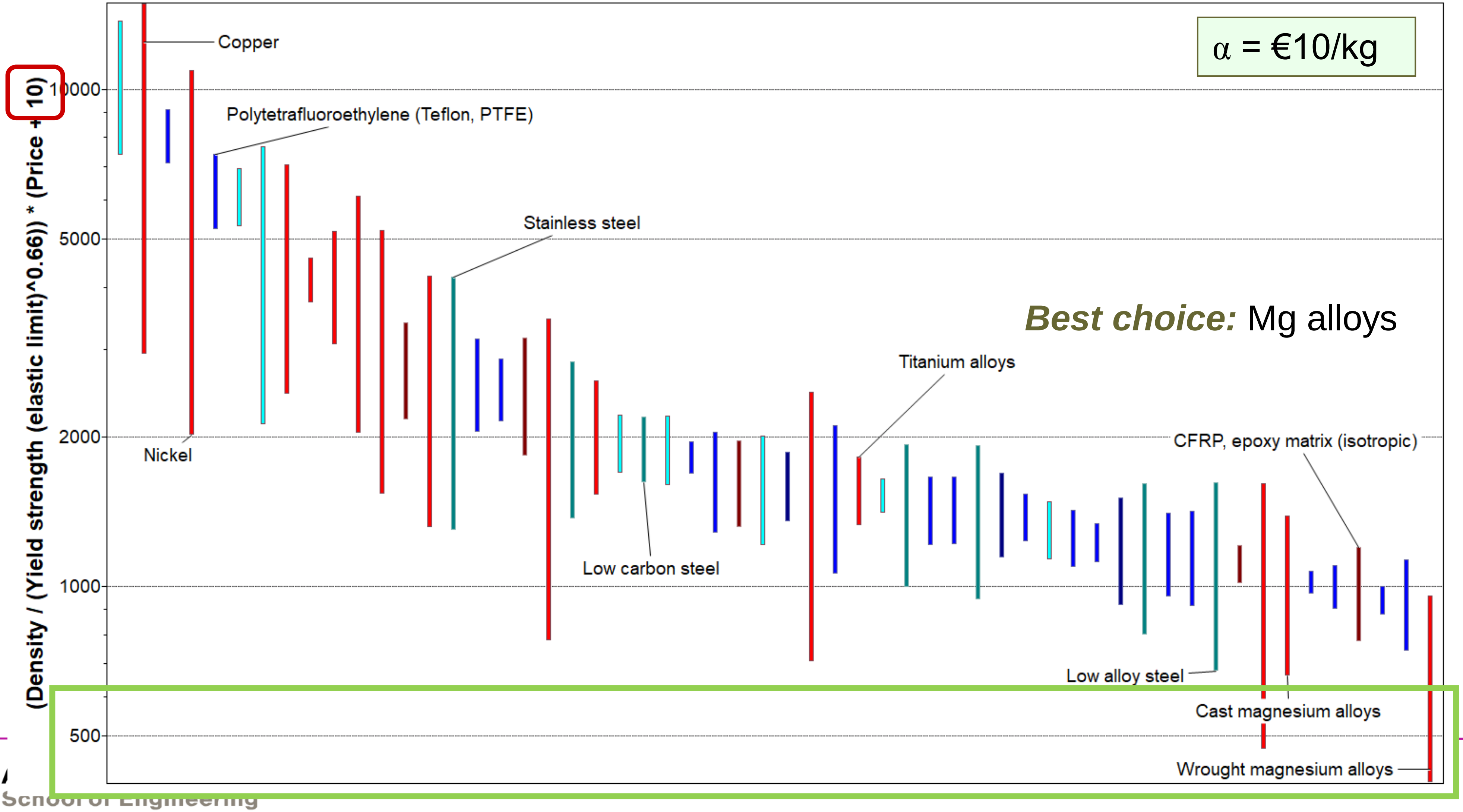
List of properties

- Density
- Price
- Tensile strength
- etc

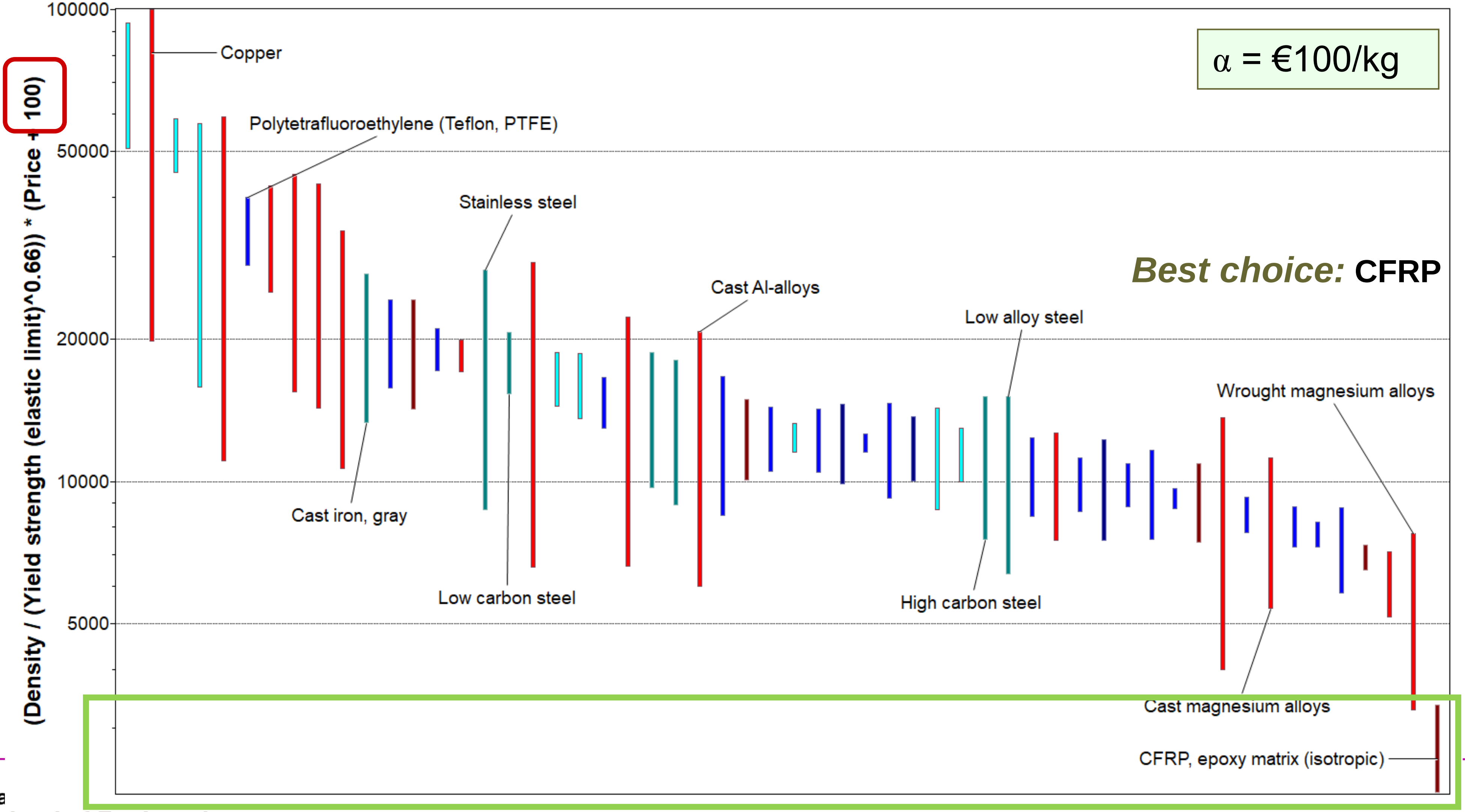
Penalty function with α = 1



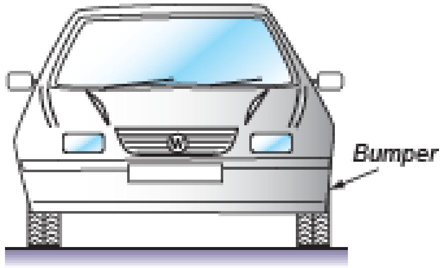
Penalty function with $\alpha = 10$



Penalty function with α = 100



Bubble chart selection using penalty function



Minimize weight ⇨ $M_2 = \dfrac{\rho}{\sigma_y^{2/3}}$

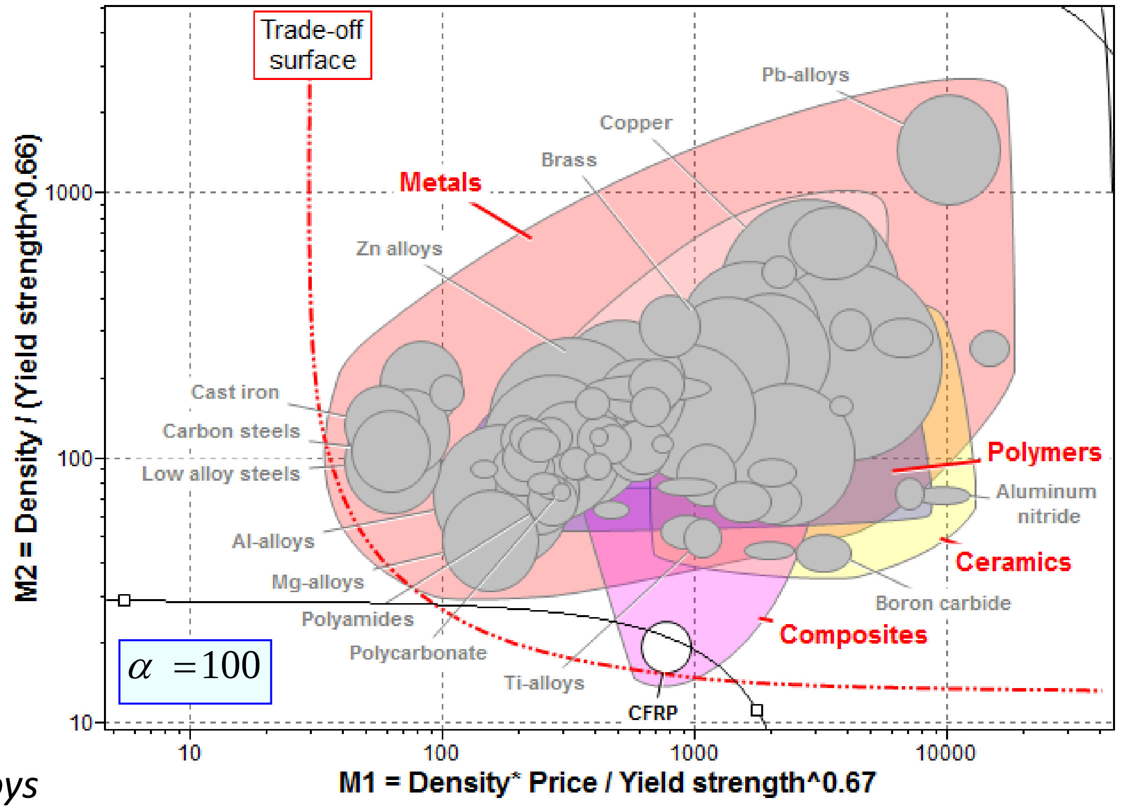
Minimize cost ⇨ $M_1 = \dfrac{C_m \rho}{\sigma_y^{2/3}}$

Penalty function $Z = M_1 + \alpha M_2$

α = 1 \$/kg ⇨ *Low alloy steels, Carbon steels,*

α = 10 \$/kg ⇨ *Aluminum alloys, Magnesium alloys*

α = 100 \$/kg ⇨ *Carbon-fiber reinforced composites*



Performance Index finder

Identify function: Panel in bending

Identify free variables:

Panel thickness

Identify variables and constraints:

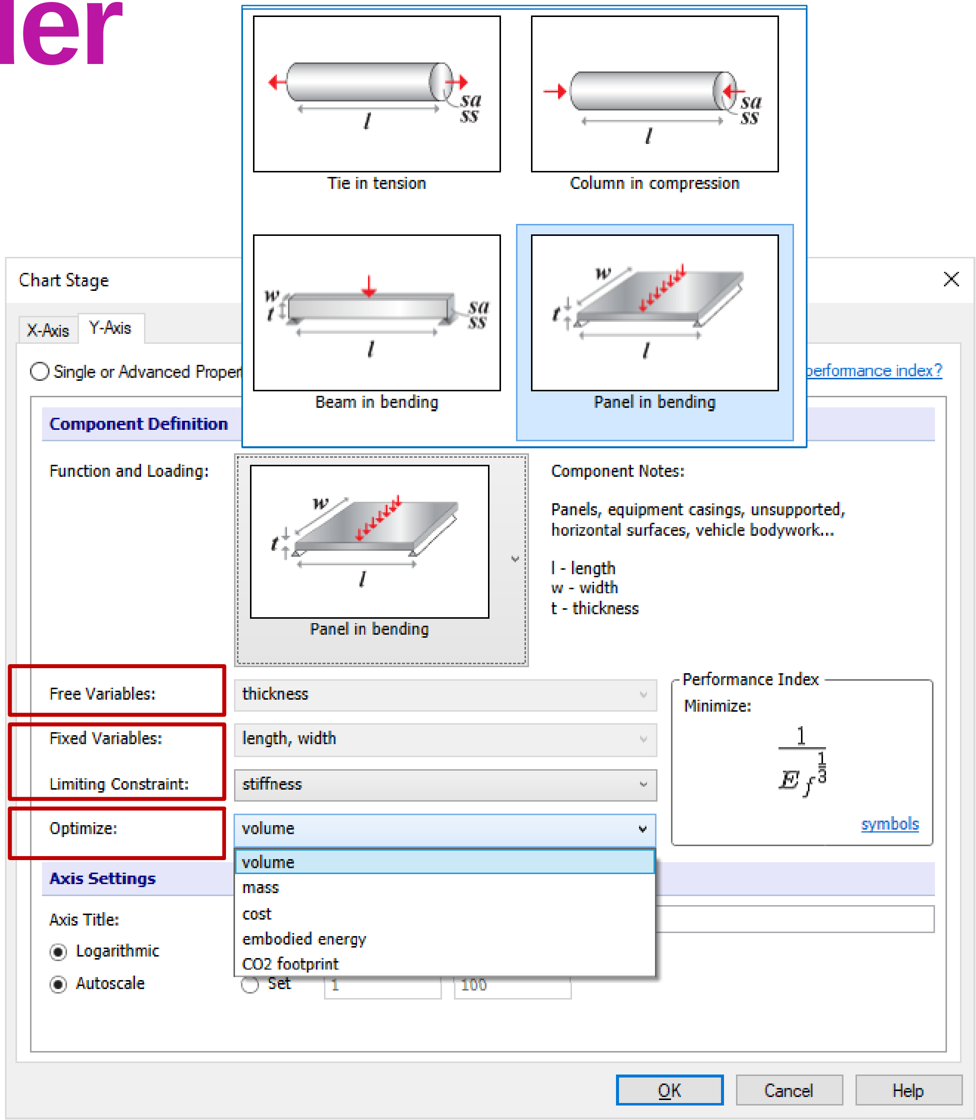
Panel length

Panel width

Stiffness limited design

Identify objective

Default is to minimize



Summary

- Real design involves conflicting objectives –

often ***technical performance*** vs. ***economic performance*** (cost).

- **Trade-off plots** reveal options

- If the **exchange constant** is known –

penalty function allows unambiguous choice