MEC-E1070 - Selection of Engineering Materials 2023

Task 3.1.1:

Calculate the shape factor for bending stiffness for Glass-reinforced plastic (GRP) bar, when the profile of the bar is:

a. **solid square profile, width, and height 100 mm.**
b. **solid rectangle profile, width 500 mm, height 100 mm.**
c. **hollow square profile, outer width and height 100 mm, wall thickness 10 mm.**
d. **hollow round pipe, outer diameter 500 mm, wall thickness 10 mm.**
e. **a spar of the wind turbine blade, whose cross-section is either an I-beam with a width of 500 mm, height of 100 mm, flange thickness of 10 mm, and web thickness of 20 mm, or a box girder with a width of 500 mm, height 100 mm, wall thickness 10 mm**

Note: Including the calculation procedure in your report is beneficial. In case you make mathematical mistakes at the end, there are still points for the correct solution steps.

The shape factor is given by

$$\phi_B^e = \frac{S}{S_0} = \frac{EI}{EI_0} = \frac{12I}{A^2}$$

The second moment of inertias mentioned in the below are taken from reference book:

Section Shape	Area (A)	Moment (I)
Square profile	bh (Here, b=h)	$\frac{bh^3}{12}$
Rectangular profile	bh	$\frac{bh^3}{12}$
Hollow square shape	$hb - [(h-2t)*(b-2t)]$ (Here, $b = h$)	$\frac{1}{6}h^3t\left(1+3\frac{b}{h}\right)$
Hollow round pipe	$\pi\left(r_o^2 - r_i^2\right)$	$\frac{\pi}{4}\left(r_o^4 - r_i^4\right)$
I-beam profile	$2t(h+b) - 4t^2$	$\frac{1}{6}h^3t\left(1+3\frac{b}{h}\right)$

Using the above table, the shape factor is calculated as follows:

a. Solid square profile, width and height 100 mm:
$$\phi_B^e = \frac{12I}{A^2} = 1$$
b. Solid rectangle profile, width 500mm, height 100mm:
$$\phi_B^e = \frac{12I}{A^2} = 0.2$$
c. Hollow square profile, outer width and height 100mm, wall thickness 10 mm:
$$\phi_B^e = \frac{12I}{A^2} = 5$$
d. Hollow round pipe, outer diameter 500 mm, wall thickness 10 mm:
$$\phi_B^e = \frac{12I}{A^2} = 23.875$$

e. A spar of the wind turbine blade, which cross-section is either an I-beam with width of 500 mm, height 100 mm, flange thickness 10 mm and web thickness 20 mm:

$$\phi_B^e = \frac{12I}{A^2} = 2.378$$

From the above shape factor calculations, it can be seen that the hollow round shape has the greatest shape factor, while the rectangular profile has the lowest. This indicates that, for the same area, the beam with a hollow round profile is 23 times stiffer than the reference beam with a square cross section and 5 times less stiff than the reference.

Task 3.1.2

First, apply all 5 profiles to the "level 2" bending stiffness map for the GRP bar. Then, select 2 profiles with 2 different materials (e.g., Carbon-fiber-reinforced polymers (CFRP) and 6061 T6 aluminum), and add these 4 records to the GRP bending stiffness map. Compare the overall 9 records among these 3 materials, and discuss your observations regarding the locus of these points on the selection chart, preferably with figures and tables.

Hint: Find the properties of each material, apply the calculated factors, and use the "Add records" tool (Tools – Add records) to construct the map.

As I was going through the levels in Granta EduPack materials for selection I could not find the GRP material hence I have selected GFRP material for deriving my observations, whose material properties are available.

The table below shows the new materials that correlate to the various shape factors parameters that are taken into account for this assignment. Here, E* and ρ* are assessed for the highlighted new materials below (highlighted in yellow) and defined in accordance with various shape factor values.

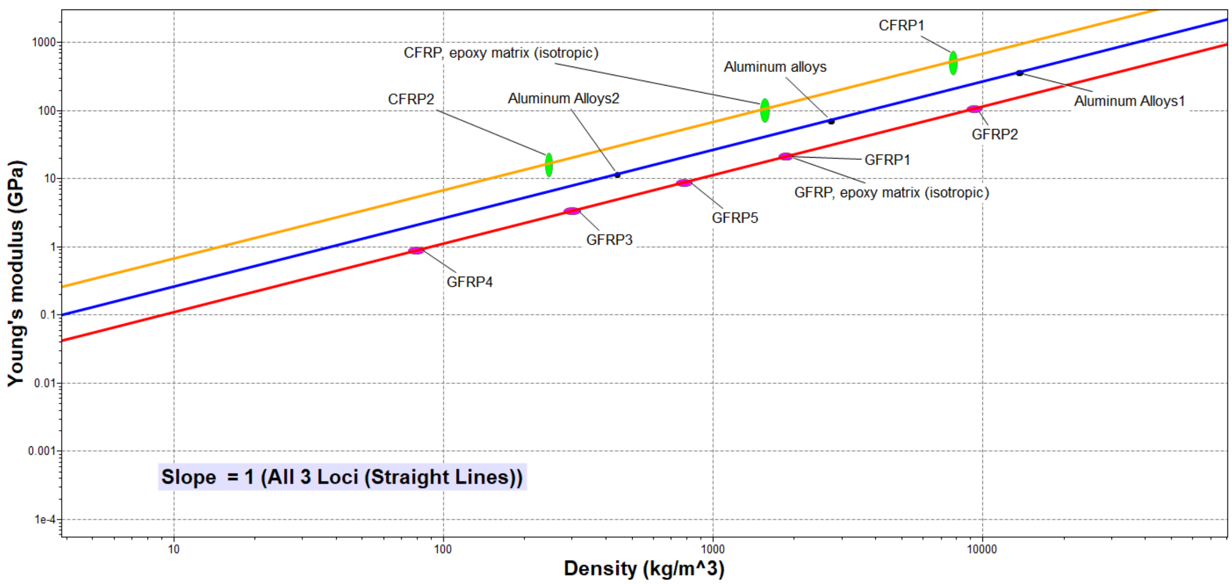
Material	Shape Factor Used (Φ_B^e, from Task 3.1.1)	E (Young's modulus)	ρ (Density)	E* = E/Φ_B^e	ρ* = ρ/Φ_B^e
GFRP	-	21 – 21.8 GPa	1.75e3 – 1.97e3 kg/m^3	-	-
GFRP1	1	21 – 21.8 GPa	1.75e3 – 1.97e3 kg/m^3	21 – 21.8 GPa	1.75e3 – 1.97e3 kg/m^3
GFRP2	0.2	21 – 21.8 GPa	1.75e3 – 1.97e3 kg/m^3	105 - 109 GPa	8.75e3 – 9.85e3 kg/m^3
GFRP3	6.172	21 – 21.8 GPa	1.75e3 – 1.97e3 kg/m^3	3.4 – 3.53 GPa	0.28e3 – 0.32e3 kg/m^3
GFRP4	23.417	21 – 21.8 GPa	1.75e3 – 1.97e3 kg/m^3	0.89 – 0.93 GPa	0.074e3 – 0.084e3 kg/m^3
GFRP5	2.378	21 – 21.8 GPa	1.75e3 – 1.97e3 kg/m^3	8.8 – 9.17 GPa	0.73e3 – 0.83e3 kg/m^3
CFRP	-	69 – 150 GPa	1.5e3 – 1.6e3 kg/m^3	-	-
CFRP1	0.2	69 – 150 GPa	1.5e3 – 1.6e3 kg/m^3	345 – 750 GPa	7.5e3 – 8e3 kg/m^3
CFRP2	6.172	69 – 150 GPa	1.5e3 – 1.6e3 kg/m^3	11.18 – 24.3 GPa	0.24e3 – 0.25e3 kg/m^3
Aluminum Alloys	-	69 – 75 GPa	2.64e3 – 2.81e3 kg/m^3	-	-
Aluminum Alloys1	0.2	69 – 75 GPa	2.64e3 – 2.81e3 kg/m^3	345 – 375 GPa	13.2e3 – 14.05e3 kg/m^3
Aluminum Alloys2	6.172	69 – 75 GPa	2.64e3 – 2.81e3 kg/m^3	11.18 – 12.15 GPa	0.43e3 – 0.45e3 kg/m^3

These new materials in accordance with all of the aforementioned data are plotted on the bending stiffness map. As a result, using the values of E* and ρ* determined from the table above for each of the new customized specified materials, we obtain the bending stiffness map shown below.

Here, we can see that the bending stiffness map's locus for each point for a certain material group type—for instance, GFRP, GFRP1, GFRP2, GFRP3, GFRP4, and GFRP5—falls on a straight line with a slope of 1 (on the logarithmic scale of X and Y). Similar results may be seen for the groups of materials known as CFRP, CFRP1, and CFRP2, as well as for the Aluminum Alloys, Aluminum Alloys1, and Aluminum Alloys2 group.

Here, only the varying values for all these straight-line loci are the values of intersection of the line at the Y-axis, which gives a different value of log C, [where C = constant value] for each of these loci. The general equation of the map can be written as:

$$\log E = (1)\log \rho + \log C$$



In conclusion, we can say that for a particular material, all the geometric shapes and their corresponding shape factors fall on the same curve of logarithmic chart of bending stiffness map.

Moreover, here on reducing the shape factor, the material moves towards increased values on the locus, and on increasing the shape factor value, the material moves towards lower values on the locus. Thus, taking a shape factor greater than 1 reduces the requirement of high mass/weight, and thus for lower usage of mass, we can obtain the same amount of bending stiffness. Also, on comparing all the 9 custom derived materials on the bending stiffness map, it was found that GFRP4 has the lowest E and ρ, and thus this design can be selected to obtain the required bending stiffness with reduced use of material mass.

Task 3.2.1

Calculate the theoretical maximum shape factor for a low alloy steel, aluminum, CFRP, and wood, according to equation 9.15a from the textbook (4th edition): $(\varphi_B^e)_{max} \approx 2.3\left(\frac{E}{\sigma_f}\right)^{0.5}$.

<u>Compare</u> your results to the empirical maximum shape factors found in Table 9.4 of the textbook (4th edition) reproduced below. <u>Discuss</u> the extent to which theoretical and empirical results agree or differ.

Material	$(\varphi_B^e)_{max}$
Steel	65
6061 Aluminum	44
CFRP	39
Polymer	12
Wood	5
Elastomers	<6

The theoretical maximum shape factor is calculated using the equation 3 (as per reference book text)

Material	E (Young's Modulus, Pa)	σ_f (Yield strength, Pa)	Theoretical minimum - maximum shape factor $(\varphi_B^e)_{max} \approx 2.3\left(\frac{E}{\sigma_f}\right)^{0.5}$	Empirical maximum shape factor (from reference table)	% Difference of theoretical maximum shape factor (in reference to empirical maximum shape factor)
Low Alloy Steel	2 e11 – 2.10 e11	4.69 e8 – 1.60 e9	25.715 – 48.669	65	- 26.93 %
Aluminum (values from internet)	6.89 e10 – 7.50 e10	1.09 e8 – 4.39 e9	28.835 – 60.332	44	-17.41 %
CFRP	6.9 e10 – 1.50 e11	5.5 e8 – 1.05 e9	18.645 – 37.983	39	-33.94 %
Wood (along grain)	5.00 e8 – 2.90 e9	1.26 e 6 -3.78 e6	26.453 – 110.342	5	612.12 %

As can be observed from the above table, several of the materials have values that are comparable to one another. For instance, CFRP has empirical values of 39 and a calculated range of 18.645 to 37.983, which is virtually within range. Aluminum has an empirical value of 44, falling within the computed range of 28.835 to 60.332. Also, The empirical value for steel is 65 while the calculated range is 25.715 – 48.669, which is above the range. This may be due to the unknown category of the steel in empirical values, since it can be high alloy steel or stainless steel, which will alter the values.

On the contrast, the value of theoretical maximum shape factor for wood is significantly higher than that of the empirically derived maximum shape factor value. Now, this can be reasoned to the fact that there are practical limits for the slenderness of sections, and these determine the maximum attainable shape factor. These limits may be imposed by manufacturing constraints: The difficulty or expense of making an efficient shape may simply be too great. More often they are imposed by the properties of the material itself because these determine the failure mode of the section.

Therefore, we cannot solely rely on the values of theoretical maximum shape factor derived using the equation 2 presented above, and would have to look for another evaluation criterion for obtaining closer values to the empirically derived shape factor values.

Task 3.2.2:

Which of these materials, structural steel, aluminum, wood, or CFRP, would be the best choice for a bending beam? The beam must have a stiffness $S = EI_{max} > 10^7$ Nm^2 and the beam has to be as light as possible. Use the following shape factors in your solution: steel (I-beam) 15, aluminum (I-beam) 10, CFRP (tube) 10, and wood (beam) 2. Resolve the problem using the 4-field method, and present your answers together with the decision-making process with text and figures.

Hint: You do not need to draw the 4-field map by yourself, you can use the one from the book.

We extract the following figure of 4-quadrant chart assembly for exploring structural sections from the reference textbook for stiffness-limited design for evaluating best material out of structural steel, aluminum, wood and CFRP during bending.

Following shape factors are used for locating the point on the shape factor quadrant:

1. Steel (I-beam) - 15
2. Aluminum (I-beam) - 10
3. CFRP (tube) - 10
4. Wood (beam) - 2

Now, following the steps given in the reference textbook, we draw the lines on the 4-quadrant chart assembly figure so that we obtain the values on the performance chart for mass per unit length corresponding to each of the material (structural steel, aluminum, CFRP, and wood) and the chosen shape factor.

We follow the below mentioned steps for evaluating mass/length values for each material chosen using the 4-field method:

1. We first identify the materials for which we are going to evaluate the mass/length. Same has been done in material quadrant.
2. Now, as $S = EI > 10^7$ Nm^2, therefore we drop a horizontal line for each material on the stiffness constraint quadrant at 10^7 Nm^2 minimum stiffness value which we require.
3. Further, we drop a vertical line for each material in the section shape quadrant corresponding to each shape factor line given in the problem statement for that particular material (shown above).
4. We then draw a horizontal line to performance quadrant corresponding to each material.
5. Similarly, we drop a vertical line from the material quadrant to performance quadrant to identify the points of intersection of the horizontal and vertical lines on the performance quadrant.
6. Finally, we evaluate the mass/length values corresponding to each of the materials and present them in the table shown below for performance evaluation. Thus, we obtain following values from the "Performance" quadrant:

S. No	Material	Section Area (m^2)	Density (kg/m3)	Mass/Length (kg/m)
1	Steels	0.55 e-2	7700	42.35
2	Aluminum alloys	0.15 e -1	2650	39.75
3	CFRP	0.15 e-1	1750	26.25
4	Hard wood	0.6 e -1	900	54

In conclusion, we observe from above table that the least mass/length value (since we have to choose the lightest material here) is given by CFRP material, which is 26.25 kg/meter. Thus, we can conclude that for the mentioned stiffness constraint and the shape factor, the best material to use for designing a bending beam is CFRP.