Aalto University
School of Engineering

MEC-E1070
Selection of Engineering Materials

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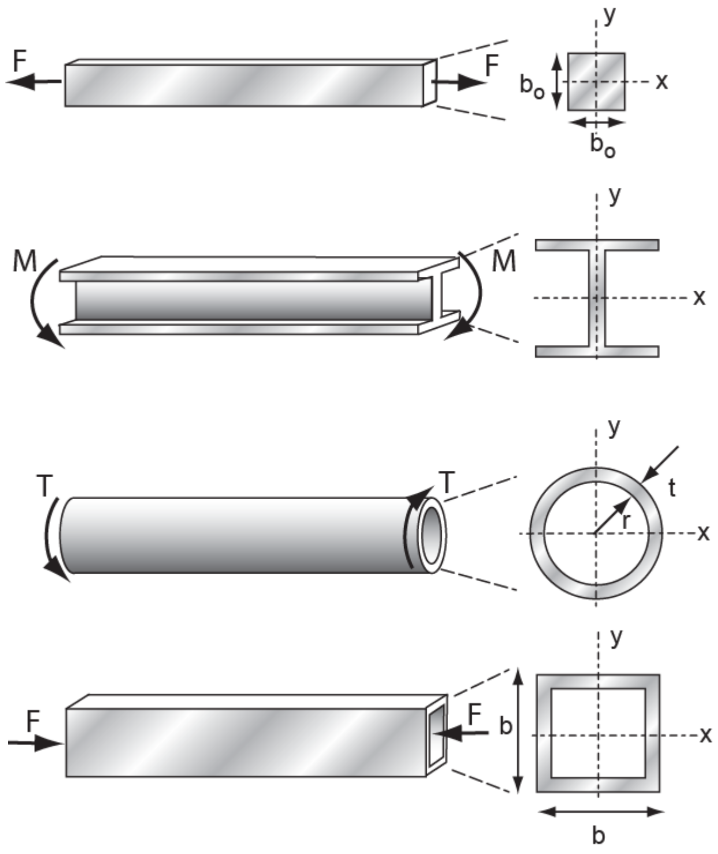
Shape factor



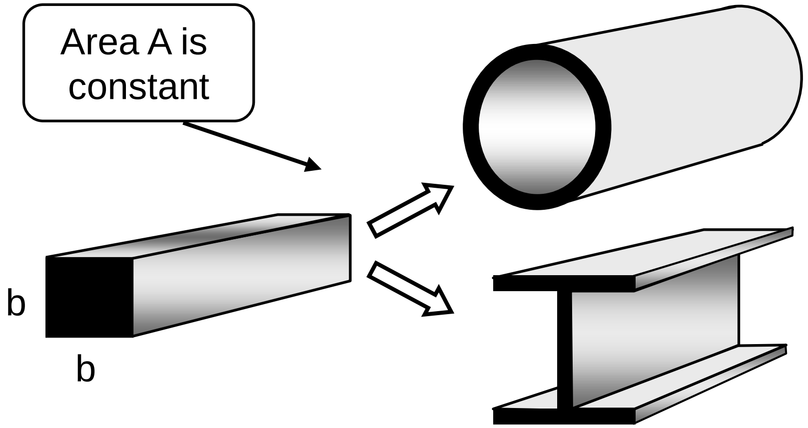
- Efficient shapes: I-beams, tubes, etc.
- The shape factor and shape limits
- Material indices that include shape
- Graphical ways of dealing with shape

Shape factor: bending stiffness

- Take ratio of bending stiffness S of shaped section to that (S_o) of a neutral reference section of the same cross-section area
- Define a standard reference section: a solid square with area $A = b^2$
- Second moment of inertia is I; **stiffness constraint** as EI.

$$I_o = \frac{b^4}{12} = \frac{A^2}{12}$$

Area $A = b^2$



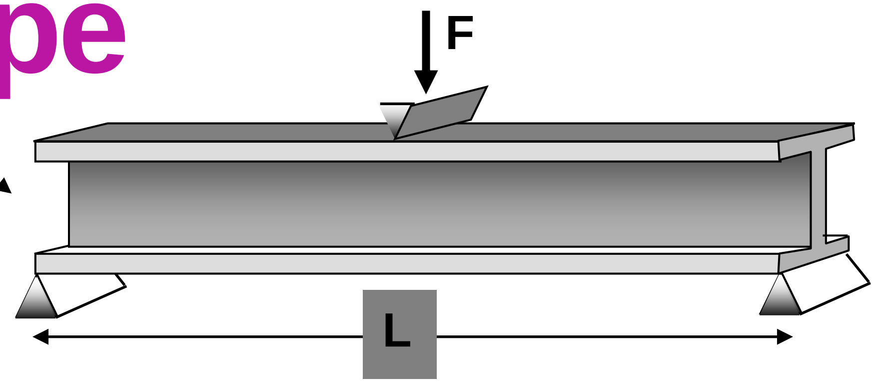
$$I = \int y^2 \, dA$$

Area A and modulus E unchanged

- Define **shape factor for elastic bending,** measuring efficiency, as

$$\varphi_e = \frac{S}{S_o} = \frac{EI}{EI_o} = 12\frac{I}{A^2}$$

Indices that include shape



Function *Beam* (shaped section).

Constraint *Bending stiffness* = S: $\boxed{S = \frac{CEI}{L^3}}$

$$\boxed{\varphi_e = 12\frac{I}{A^2} \qquad A = \left(\frac{12I}{\varphi_e}\right)^{1/2}}$$

Objective *Minimise mass*, m, where:

$$\boxed{m = AL\rho}$$

$$m = \left(\frac{12\,S\,L^5}{C}\right)^{1/2}\left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$$

Chose materials with smallest $\left(\frac{\rho}{(\varphi_e E)^{1/2}}\right)$

- m = mass
- A = area
- L = length
- ρ = density
- b = edge length
- S = stiffness
- I = second moment of inertia
- E = Youngs Modulus

Selection chart – with shaped-material index

Fixed shape ($\varphi_e = 1$): $\frac{\rho}{E^{1/2}}$

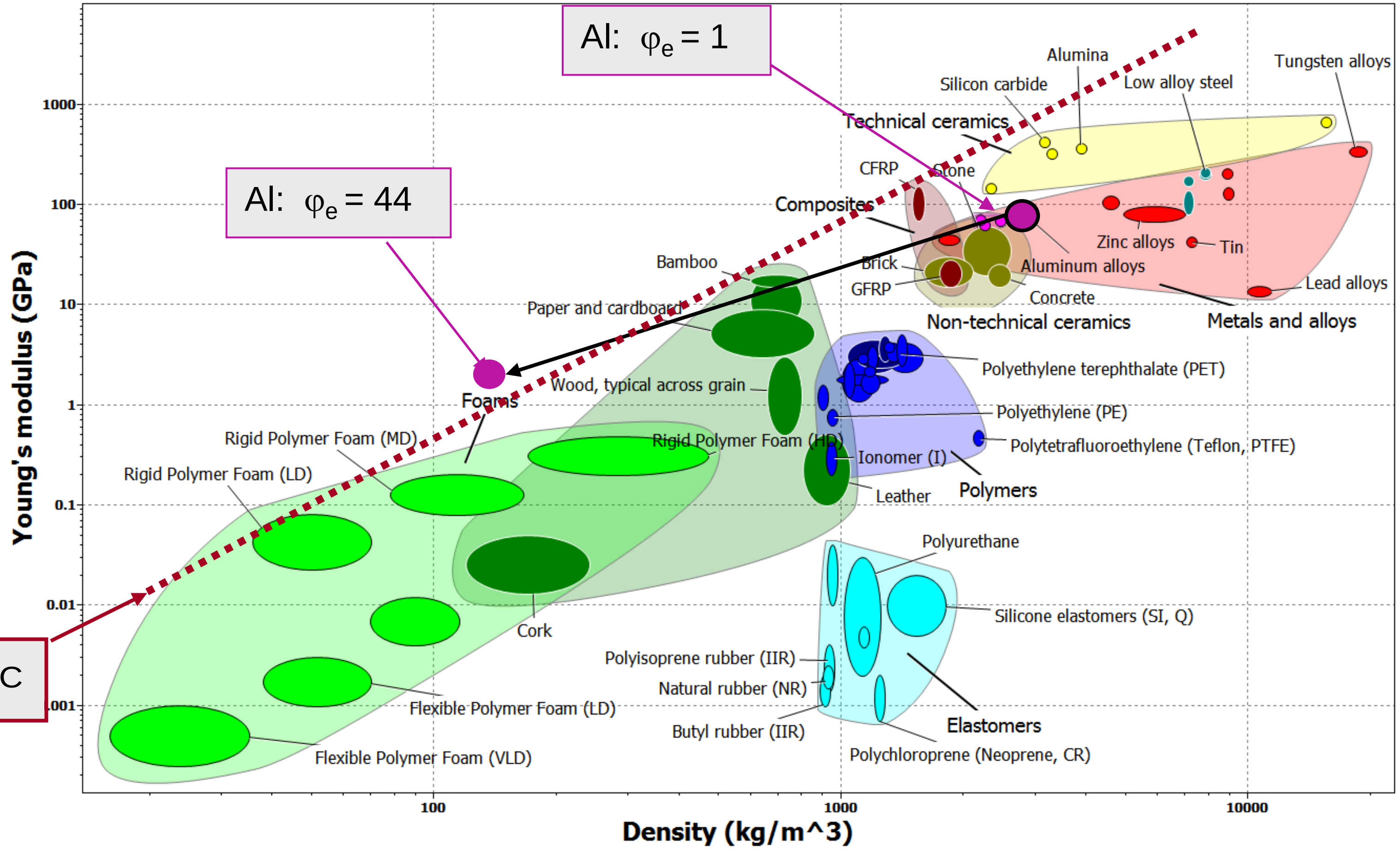
Shaped-material index

$$\frac{\rho}{(\varphi_e E)^{1/2}} = \frac{\boxed{\rho / \varphi_e}}{(\boxed{E / \varphi_e})^{1/2}}$$

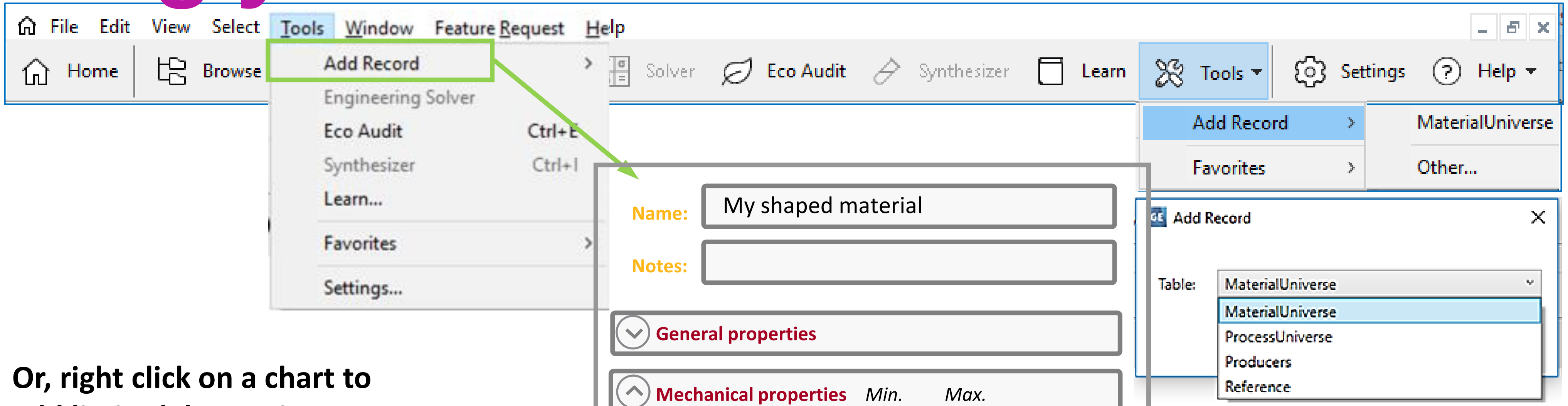
New material with

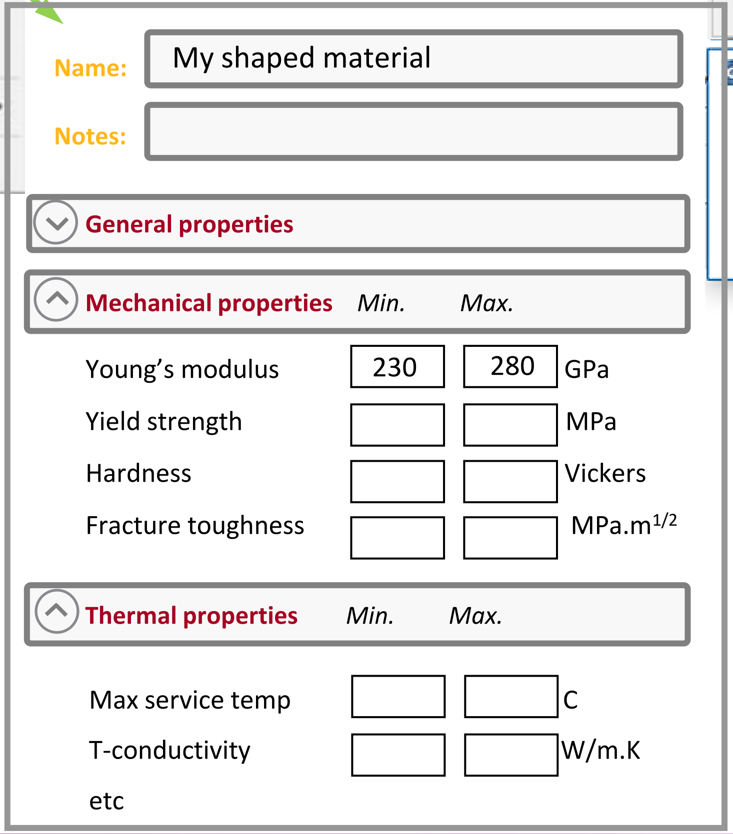
$$\begin{cases} \rho^* = \rho / \varphi_e \\ E^* = E / \varphi_e \end{cases}$$

$$\frac{\rho}{E^{1/2}} = C$$



Making your own records

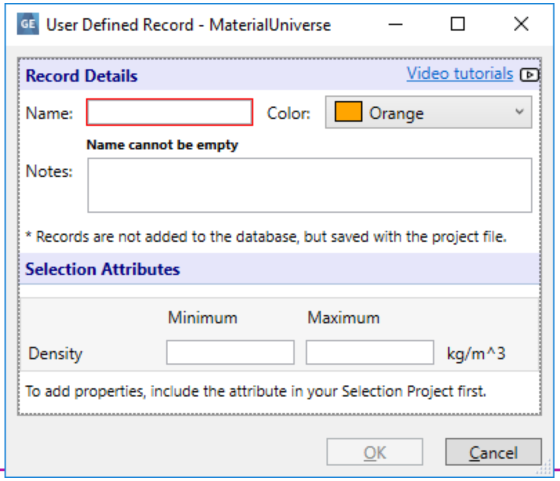




$$E^* = E / \varphi_e$$

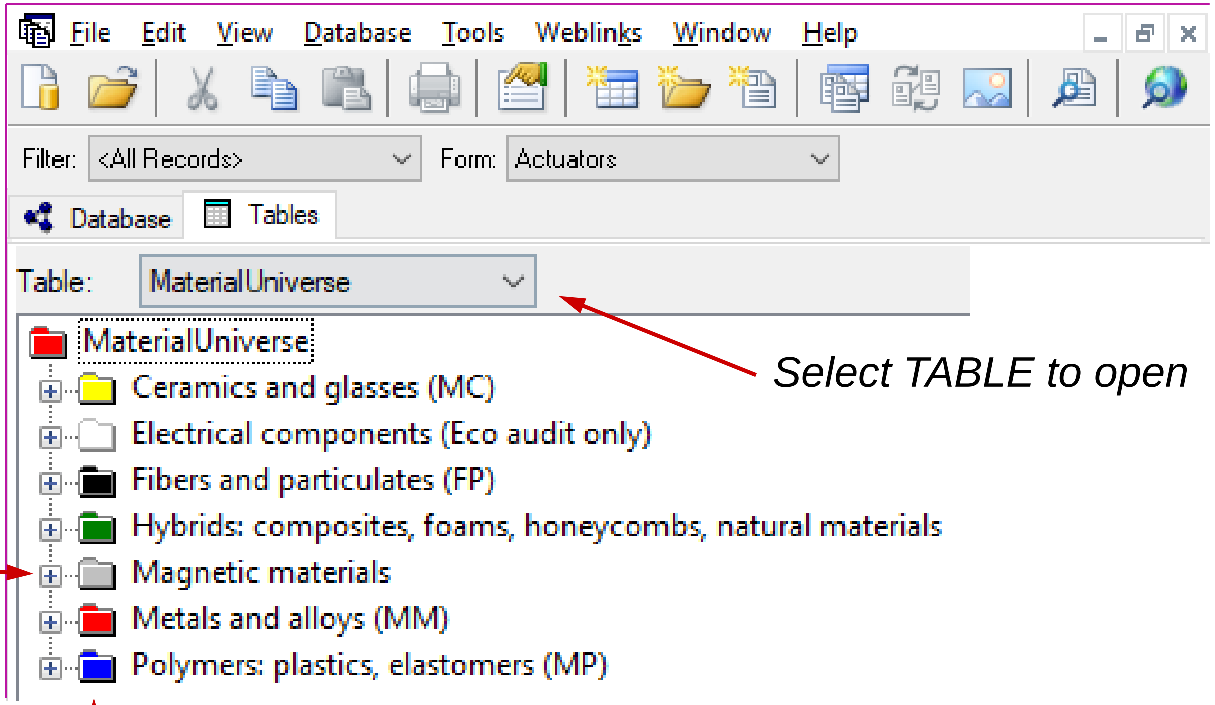
$$\rho^* = \rho / \varphi_e$$

Or, right click on a chart to add limited data to it…



Opening a data-table for editing

Toolbar



Select TABLE to open

Expand and locate record for editing

*The **FILTER** determines which folders and records appear*

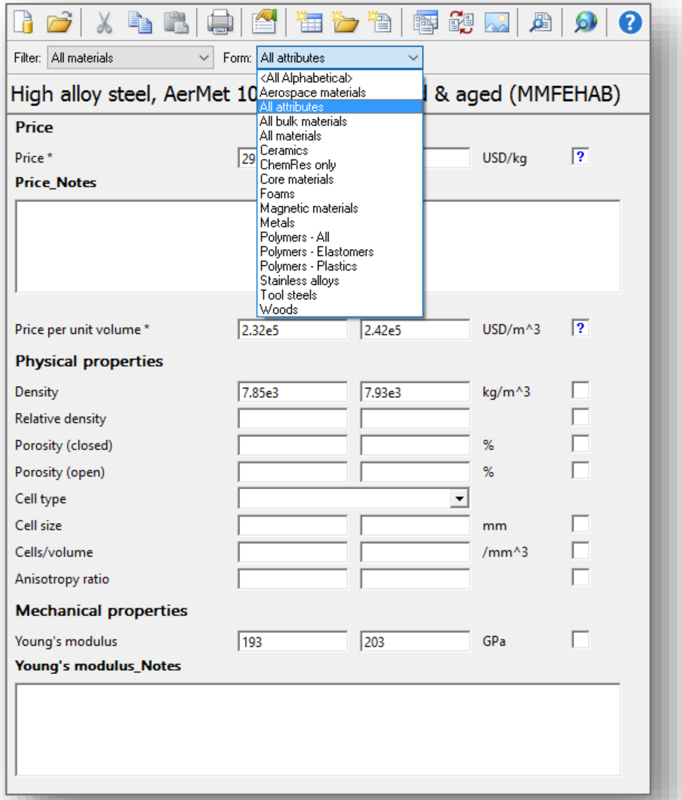
Editing a Record

- Select which properties to display with the ‘Form’ drop-down

- Closing the form automatically saves the data
  don’t need to click ‘Save’

- Ensure enter values in the unit displayed
  Or change the unit system:
  Tools > Options > General
  > Preferred Unit System

- Question mark ? On the right
  indicates the value is estimated



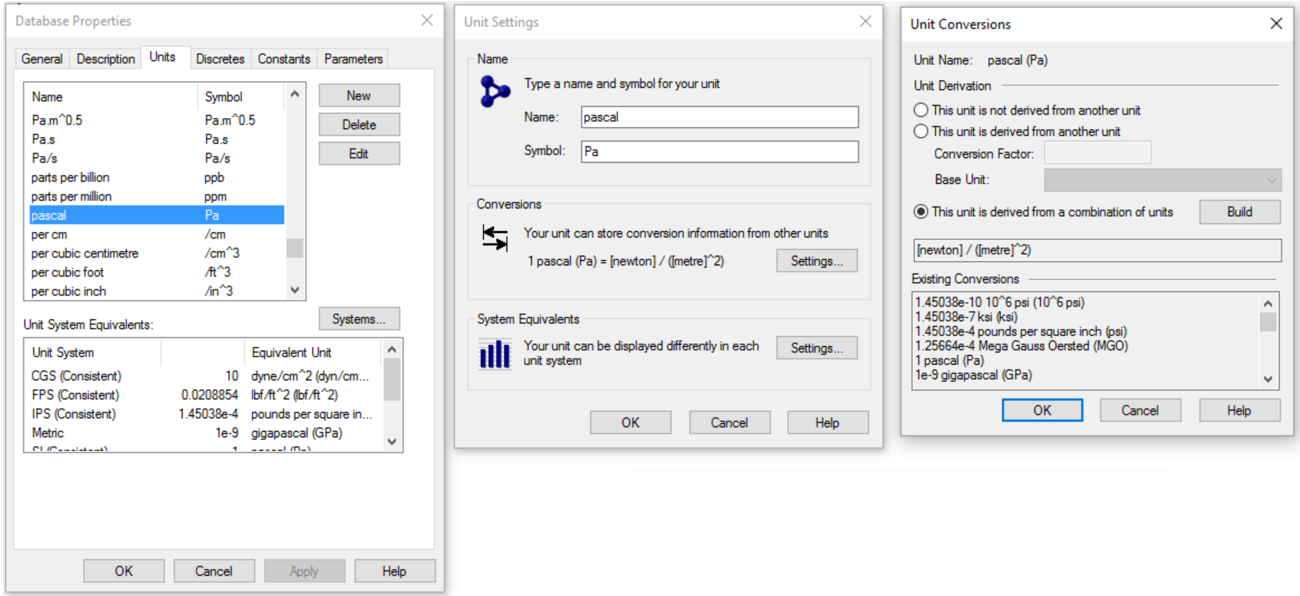
Customize units

Adding a new unit

- Database menu > Database settings > Units
- Conversions can be defined by their derivation from other base units, eg:

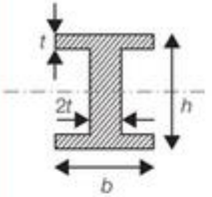
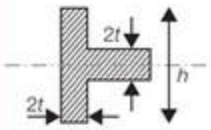
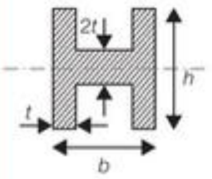
    [Pa] = [newton] / [metre]^2

    [inch] = 0.0254 * [metre]



Task 3.1.1 Second moment of area of I-beam – flange and web thickness

Table 9.3 *continued*

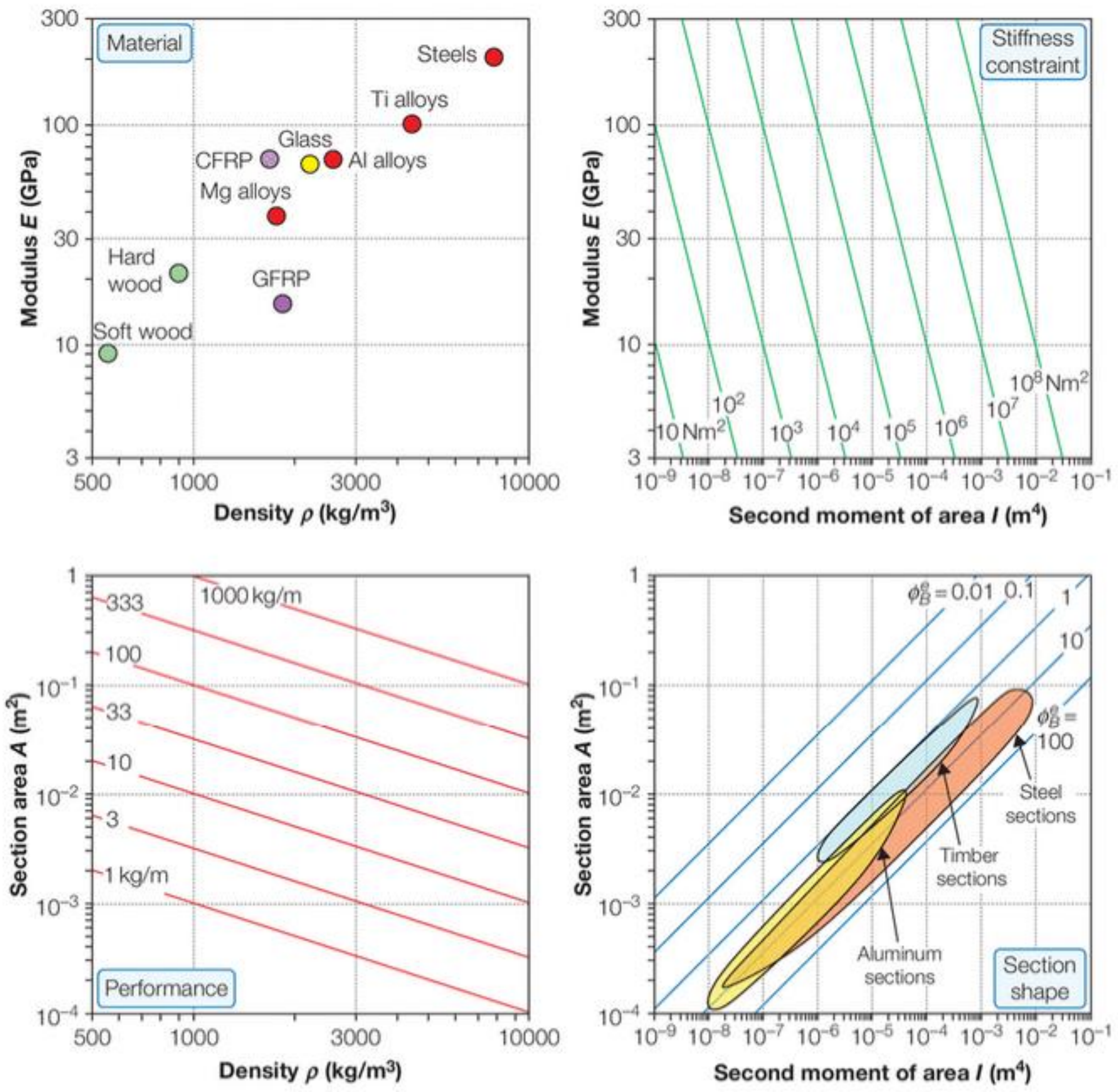
Section Shape	Bending Factor ϕ_B^e	Torsional Factor ϕ_T^e	Bending Factor ϕ_B^f	Torsional Factor ϕ_T^f
	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ $(h,b>>t)$	$1.19\left(\frac{t}{b}\right)\frac{\left(1+\frac{4h}{b}\right)}{\left(1+\frac{h}{b}\right)^2}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{3b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$1.13\sqrt{\frac{t}{b}}\frac{\left(1+\frac{4h}{b}\right)}{\left(1+\frac{h}{b}\right)^{3/2}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+4bt^2/h^3)}{(1+b/h)^2}$ $(h,b\ t)$	$0.595\left(\frac{t}{h}\right)\frac{\left(1+\frac{8b}{h}\right)}{\left(1+\frac{b}{h}\right)^2}$	$\frac{3}{4}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{4bt^2}{h^3}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$0.565\sqrt{\frac{t}{h}}\frac{\left(1+\frac{8b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+4bt^2/h^3)}{(1+b/h)^2}$ $(h,b>>t)$	$1.19\left(\frac{t}{h}\right)\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^2}$	$\frac{3}{4}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{4bt^2}{h^3}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$1.13\sqrt{\frac{t}{h}}\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$

Task 3.1.2 calculation to include shape

Material property values	Reference shape (Profile a) $\varphi_e = 1$	Profile b $\varphi_e = ?$	Profile c $\varphi_e = ?$	Profile d $\varphi_e = ?$	Profile e $\varphi_e = ?$
Material X	Ex (GPa), ρx($kg.m{-3}$)	Exb(GPa), ρxb($kg.m{-3}$)	Exc(GPa), ρxc ($kg.m{-3}$)	…	…
Material Y	Ey(GPa), ρy ($kg.m{-3}$)	Eyb(GPa), ρyb ($kg.m{-3}$)	Eyc(GPa), ρyc ($kg.m{-3}$)		
Material Z	…			…	…

Task 3.2.1 theoretical / empirical shape factor

Task 3.2.2 four-field chart (Figure 9.9)



Summary

- When materials carry bending, torsion or axial compression, the section shape becomes important.
- The "shape efficiency" is the amount of material needed to carry the load. It is measured by the shape factor, φ.
- If two materials have the *same* shape, the standard indices for bending (e.g. $\rho / E^{1/2}$) guide the choice.
- If materials can be made -- or are available -- in different shapes, then indices which include the shape (e.g. $\rho / (\varphi E)^{1/2}$) guide the choice.

Demo - bending stiffness map w. shape

$\varphi_e = 1$	ρ $(kg.m{-}3)$	E (GPa)
Plywood	700-800	5-8
Mg Alloys	1500-1950	42-47
CFRP	1500-1600	69-150

$\varphi_e = 5$	ρ $(kg.m{-}3)$	E (GPa)
Plywood	140-160	1-1.6
Mg Alloys	300-390	8.4-9.4
CFRP	300-320	13.8-30

$\varphi_e = 0.2$	ρ $(kg.m{-}3)$	E (GPa)
Plywood	3500-4000	25-40
Mg Alloys	…	…
CFRP	…	…

$$\rho^* = \rho / \varphi_e$$

$$E^* = E / \varphi_e$$