Problem Solutions

By
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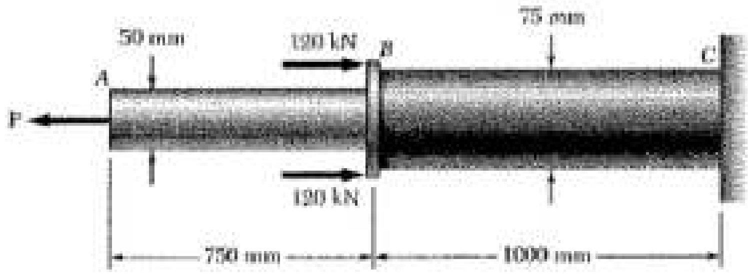
Problem 1.1

1.1 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Determine the magnitude of the force P for which the tensile stress in rod AB is twice the magnitude of the compressive stress in rod BC.



$A_{AB} = \frac{\pi}{4}(50)^2 = 1963.5\ mm^2$

$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{1963.5}$

$= 509.3 \times 10^{-6} P$

$A_{BC} = \frac{\pi}{4}(75)^2 = 4417.9\ mm^2$

$\sigma_{BC} = \frac{(2)(120) - P}{A_{AB}}$

$= \frac{240 - P}{4417.9} = 0.0543 - 226.4 \times 10^{-6} P$

Equating σ_{AB} to $2\sigma_{BC}$

$509.3 \times 10^{-6} P = 2(0.0543 - 226.4 \times 10^{-6} P)$

$P = 112.9\ kN$ ◄

Problem 1.2

1.2 In Prob. 1.1, knowing that $P = 160$ kN, determine the average normal stress at the midsection of (a) rod AB, (b) rod BC.

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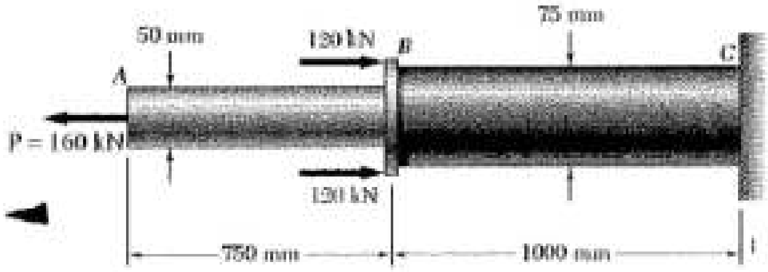


(a) Rod AB.

$P = 160\ kN$ (tension)

$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi (50)^2}{4} = 1963.5\ mm^2$

$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{160 \times 10^3}{1963.5} = 81.5\ MPa$ ◄

(b) Rod BC.

$F = 160 - (2)(120) = -80\ kN$ i.e. 80 kN compression.

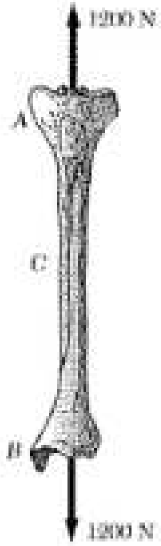
$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi (75)^2}{4} = 4417.9\ mm^2$

$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-80 \times 10^3}{4417.9}$

$\sigma_{BC} = -18.1\ MPa$ ◄

Problem 1.5

1.5 A strain gage located at *C* on the surface of bone *AB* indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at *C* to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at *C*.



$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $\quad A = \frac{\pi}{4}(d_1^2 - d_2^2)$

$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

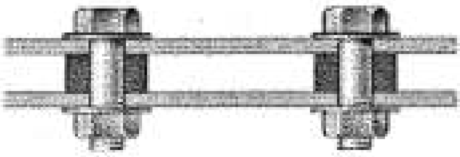
$$d_2^2 = (25\times10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80\times10^6)}$$

$$= 222.9\times10^{-6}\ m^2$$

$$d_2 = 14.93\times10^{-3}\ m \qquad d_2 = 14.93\ mm$$

Problem 1.6

1.6 Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.



At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time the spacer pushes that plate upward with a compressive force P_s. In order to maintain equilibrium

$$P_b = P_s$$

For the bolt, $\quad \sigma_b = \frac{F_b}{A_b} = \frac{4P_b}{\pi d_b^2} \quad$ or $\quad P_b = \frac{\pi}{4}\sigma_b d_b^2$

For the spacer, $\quad \sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)} \quad$ or $\quad P_s = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

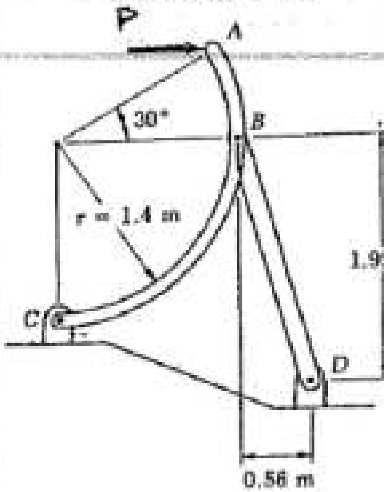
Equating P_b and P_s,

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)}\, d_b = \sqrt{1 + \frac{200}{130}}\,(16) \qquad d_s = 25.2\ mm$$

Problem 1.8

1.8 Knowing that the central portion of the link BD has a uniform cross-sectional area of 800 mm², determine the magnitude of the load **P** for which the normal stress in that portion of BD is 50 MPa.



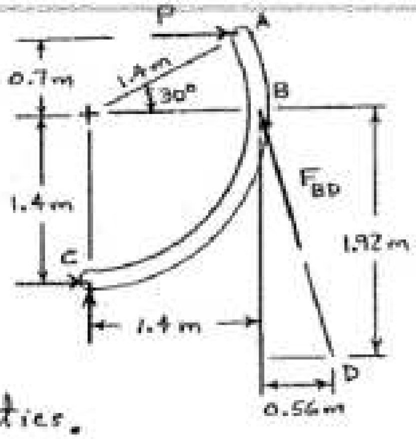
$F_{BD} = \sigma A$

$= (50 \times 10^6)(800 \times 10^{-6})$

$= 40 \times 10^3$ N

$BD = \sqrt{(0.56)^2 + (1.92)^2}$

$= 2.00$ m



Use Free Body AC for statics.

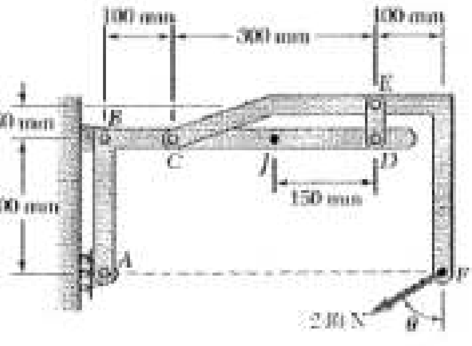
$\circlearrowleft \Sigma M_C = 0:$ $\quad \frac{0.56}{2.00}(40 \times 10^3)(1.4) + \frac{1.92}{2.00}(40 \times 10^3)(1.4)$

$- P(0.7 + 1.4) = 0$

$P = 33.1 \times 10^3$ N $\qquad$ $P = 33.1$ kN

Problem 1.9

1.9 Knowing that link DE is 25 mm wide and 3 mm thick, determine the normal stress in the central portion of that link when (a) $\theta = 0$; (b) $\theta = 90°$.



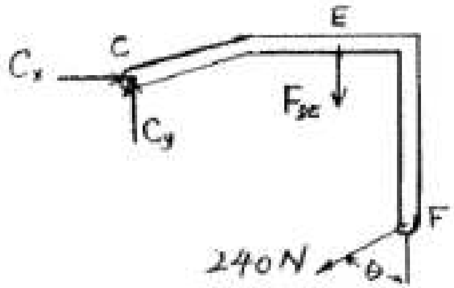
Use member CEF as a free body

$\circlearrowleft \Sigma M_C = 0$

$-0.3\, F_{DE} - (0.2)(240 \sin\theta) - (0.4)(240 \cos\theta) = 0$

$F_{DE} = -160 \sin\theta - 320 \cos\theta$ N

$A_{DE} = (0.025)(0.003) = 75 \times 10^{-6}$ m²



$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$

(a) $\theta = 0$: $\quad F_{DE} = -320$ N

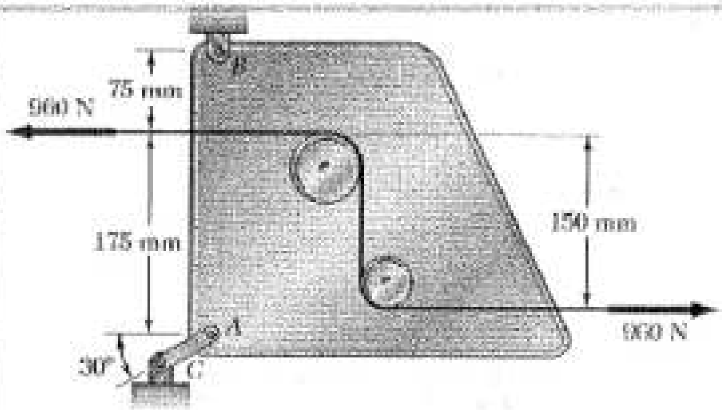
$\sigma_{DE} = \frac{-320}{75 \times 10^{-6}} = -4.27$ MPa

(b) $\theta = 90°$: $\quad F_{DE} = -160$ N

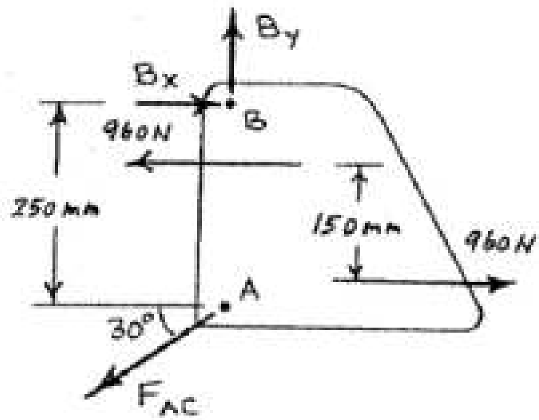
$\sigma_{DE} = \frac{-160}{75 \times 10^{-6}} = -2.13$ MPa

Problem 1.10

1.10 Link AC has a uniform rectangular cross section 3 mm thick and 12 mm wide. Determine the normal stress in the central portion of the link.



Free Body Diagram of Plate



Note that the two 960-N forces form a couple of moment

$(960\ N)(0.15\ m) = 144\ N.m$

$\circlearrowleft \Sigma M_B = 0: \quad 144\ Nm - (F_{AC} \cos 30°)(0.25\ m) = 0$

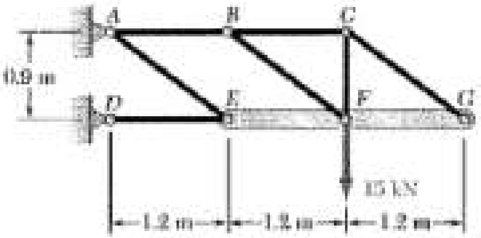
$F_{AC} = 665.1\ N$

Area of link: $A_{AC} = (3\ mm)(12\ mm) = 36\ mm^2$

Stress $\sigma_{AC} = \dfrac{F_{AC}}{A_{AC}} = \dfrac{665.1}{36} = 18.475\ MPa \qquad \sigma_{AC} = 18.5\ MPa$

Problem 1.11

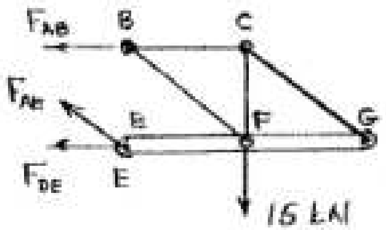
1.11 The rigid bar EFG is supported by the truss system shown. Knowing that the member CG is a solid circular rod of 18 mm diameter, determine the normal stress in CG.



Using portion EFGCB as a free body

$+\uparrow \Sigma F_y = 0: \frac{0.9}{1.5} F_{AE} - 15 = 0$

$F_{AE} = 25 \text{ kN}$



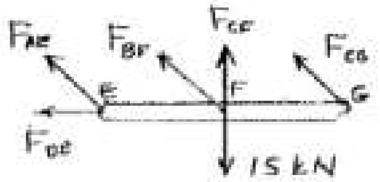
Using beam EFG as a free body

$\circlearrowleft M_F = 0: -(1.2)\frac{0.9}{1.2} F_{AE} + (1.2)\left(\frac{0.9}{1.2} F_{CG}\right) = 0$

$F_{CG} = F_{AE} = 25 \text{ kN}$

Cross sectional area of member CG

$A_{CG} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.018)^2 = 254.4 \times 10^{-6} \text{ m}^2$

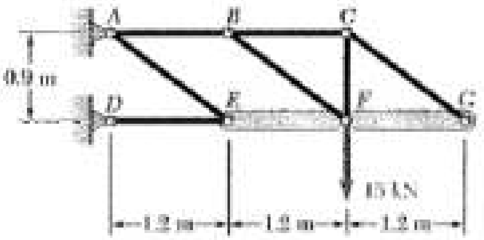


Normal stress in CG.

$\sigma_{CG} = \frac{F_{CG}}{A_{CG}} = \frac{25}{254.4 \times 10^{-6}} = 98.3 \text{ MPa}$ ◄

Problem 1.12

1.12 The rigid bar EFG is supported by the truss system shown. Determine the cross-sectional area of member AE for which the normal stress in the member is 105 MPa.

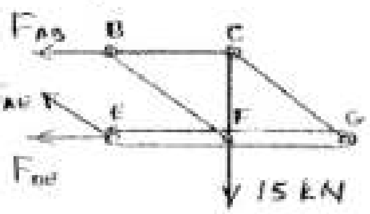


Using portion EFGCB as a free body

$+\uparrow \Sigma F_y = 0: \frac{0.9}{1.5} F_{AE} - 15 = 0$

$F_{AE} = 25 \text{kN}$

Stress in member AE $\qquad \sigma_{AE} = 105 \text{ MPa}$

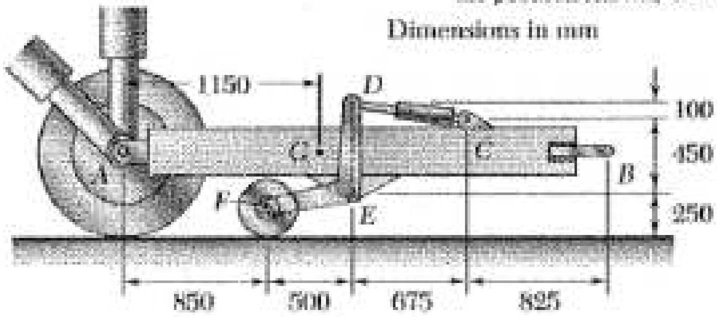


$\sigma_{AE} = \frac{F_{AE}}{A_{AE}}$

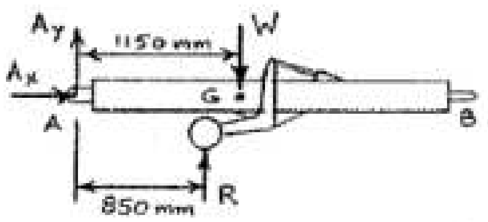
$A_{AE} = \frac{F_{AE}}{\sigma_{AE}} = \frac{25 \times 10^3}{105 \times 10^6} = 238.1 \times 10^{-6} \text{ m}^2$ ◄

Problem 1.14

1.14 An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units DEF. The mass of the entire tow bar is 200 kg, and its center of gravity is located at G. For the position shown, determine the normal stress in the rod.



FREE BODY – ENTIRE TOW BAR:



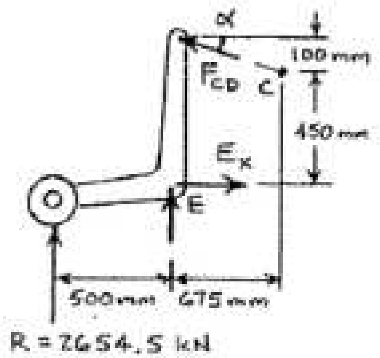
$W = (200\ \text{kg})(9.81\ \text{m/s}^2) = 1962.00\ \text{N}$

$+\circlearrowleft \Sigma M_A = 0:$

$850R - 1150(1962.00\ \text{N}) = 0$

$R = 2654.5\ \text{N}$

FREE BODY – BOTH ARM & WHEEL UNITS:



$\tan\alpha = \dfrac{100}{675} \qquad \alpha = 8.4270°$

$+\circlearrowleft \Sigma M_E = 0:$

$(F_{CD}\cos\alpha)(550) - R(500) = 0$

$F_{CD} = \dfrac{500}{550\cos 8.4270°}(2654.5\ \text{N})$

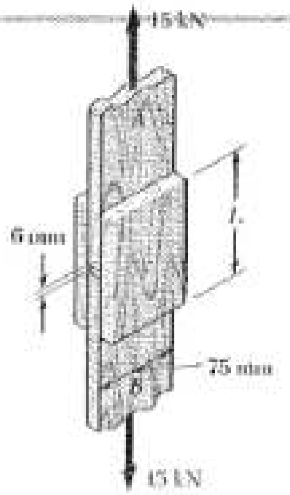
$= 2439.5\ \text{N} \quad \text{(COMP.)}$

$\sigma_{CD} = -\dfrac{F_{CD}}{A_{CD}} = -\dfrac{2439.5\ \text{N}}{\pi(0.0125\ \text{m})^2}$

$= -4.9697 \times 10^6\ \text{Pa}$

$\sigma_{CD} = -4.97\ \text{MPa}$ ◀

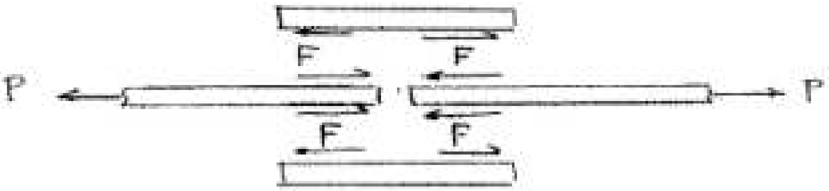
Problem 1.15



1.15 The wooden members *A* and *B* are to be joined by plywood splice plates that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be 6 mm, determine the smallest allowable length *L* if the average shearing stress in the glue is not to exceed 700 kPa.

There are four separate areas that are glued. Each of these areas transmits one half the the 15 kN load. Thus

$$F = \tfrac{1}{2}P = \tfrac{1}{2}(15) = 7.5 \text{ kN} = 7500 \text{ N}$$



Let l = length of one glued area and $w = 75 \text{ mm} = 0.075 \text{ m}$ be its width.

For each glued area, $A = lw$

Average shearing stress: $\tau = \dfrac{F}{A} = \dfrac{F}{lw}$

The allowable shearing stress is $\tau = 700 \times 10^3$ Pa

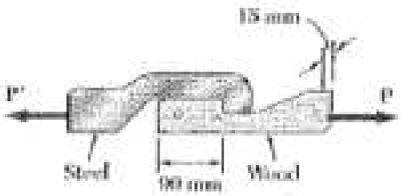
Solving for l, $l = \dfrac{F}{\tau w} = \dfrac{7500}{(700\times10^3)(0.075)} = 0.14286 \text{ m} = 142.85 \text{ mm}$

Total length L: $L = l + (\text{gap}) + l = 142.85 + 6 + 142.85$

$L = 292$ mm ◄

Problem 1.16

1.16 When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.



Area being sheared

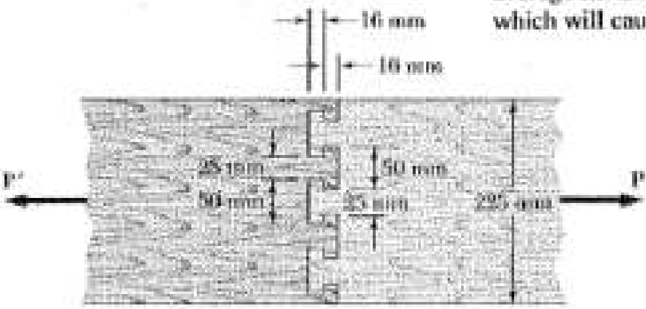
$A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{ m}^2$

Force $P = 8 \times 10^3$ N

Shearing stress $\tau = \dfrac{P}{A} = \dfrac{8\times10^3}{1350\times10^{-6}} = 5.93\times10^6 \text{ Pa} = 5.93 \text{ MPa}$ ◄

Problem 1.17

1.17 Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load which will cause the joint to fail.



Six areas must be sheared off when the joint fails. Each of these areas has dimensions 16 mm × 12 mm, its area being

$$A = (16)(12) = 192 \text{ mm}^2 = 192 \times 10^{-6} \text{ m}^2$$

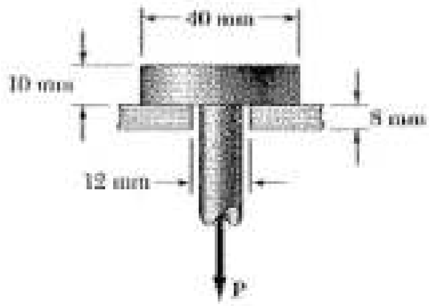
At failure the force F carried by each of areas is

$$F = \tau A = (8 \times 10^6)(192 \times 10^{-6}) = 1536 \text{ N} = 1.536 \text{ kN}$$

Since there are six failure areas $\quad P = 6F = (6)(1.536) = 9.22 \text{ kN}$

Problem 1.18

1.18 A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.



For the steel rod,

$$A_1 = \pi d_1 t_1 = (\pi)(0.012)(0.010)$$
$$= 376.99 \times 10^{-6} \text{ m}^2$$

$$\tau_1 = \frac{P}{A_1} \rightarrow P_1 = \tau_1 A_1$$

$$P_1 = (180 \times 10^6)(376.99 \times 10^{-6}) = 67.86 \times 10^3 \text{ N}$$

For the aluminum plate,

$$A_2 = \pi d_2 t_2 = (\pi)(0.040)(0.008) = 1.00531 \times 10^{-3} \text{ m}^2$$

$$\tau_2 = \frac{P_2}{A_2} \Rightarrow P_2 = \tau_2 A_2$$

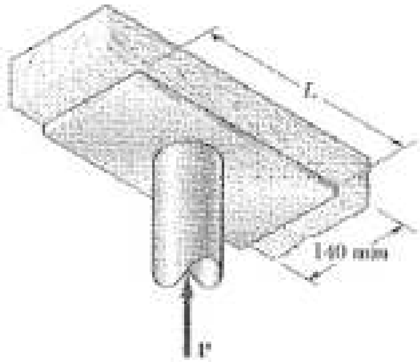
$$P_2 = (70 \times 10^6)(1.0053 \times 10^{-6}) = 70.372 \times 10^3 \text{ N}$$

The limiting value for the load P is the smaller of P_1 and P_2.

$P = 67.86 \times 10^3$ N $\qquad P = 67.9$ kN

Problem 1.19

1.19 The axial force in the column supporting the timber beam shown is $P = 75$ kN. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 3.0 MPa.



SOLUTION

$$\sigma_b = \frac{P}{A} = \frac{P}{Lw}$$

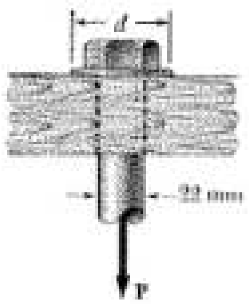
Solving for L: $L = \frac{P}{\sigma_b w} = \frac{75\times10^3}{(3.0\times10^6)(0.140)}$

178.6×10^{-3} m

$L = 178.6$ mm

Problem 1.20

1.20 The load **P** applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.



Steel rod: $A = \frac{\pi}{4}(0.022)^2 = 380.13\times10^{-6}\ m^2$

$\sigma = 35\times10^6$ Pa

$P = \sigma A = (35\times10^6)(380.13\times10^{-6})$

$= 13.305\times10^3$ N

Washer: $\sigma_b = 5\times10^6$ Pa

Required bearing area:

$$A_b = \frac{P}{\sigma_b} = \frac{13.305\times10^3}{5\times10^6} = 2.6609\times10^{-3}\ m^2$$

But, $A_b = \frac{\pi}{4}(d^2 - d_i^2)$

$d^2 = d_i^2 + \frac{4A_b}{\pi}$

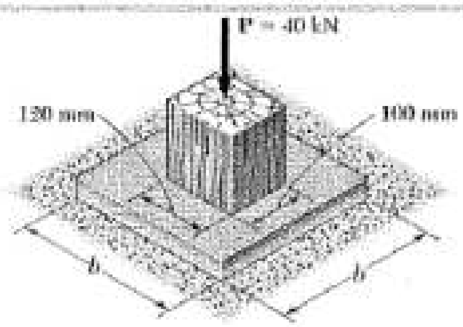
$= (0.025)^2 + \frac{(4)(2.6609\times10^{-3})}{\pi}$

$= 4.013\times10^{-3}\ m^2$

$d = 63.3\times10^{-3}$ m $\qquad d = 63.3$ mm

Problem 1.21

1.21 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.



(a) Bearing stress on concrete footing.

$P = 40 \text{ kN} = 40 \times 10^3 \text{ N}$

$A = (100)(120) = 12 \times 10^3 \text{ mm}^2 = 12 \times 10^{-3} \text{ m}^2$

$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.33 \times 10^6 \text{ Pa}$

3.33 MPa ◄

(b) Footing area. $P = 40 \times 10^3 \text{ N}$ $\sigma = 145 \text{ kPa} = 45 \times 10^3 \text{ Pa}$

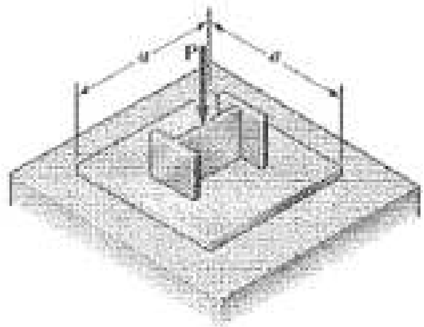
$\sigma = \frac{P}{A}$ $A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586 \text{ m}^2$

Since the area is square, $A = b^2$

$b = \sqrt{A} = \sqrt{0.27586} = 0.525 \text{ m}$ $b = 525 \text{ mm}$ ◄

Problem 1.22

1.22 An axial load **P** is supported by a short W200 × 59 column of cross-sectional area A = 7560 mm² and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 200 MPa and that the bearing stress on the concrete foundation must not exceed 20 MPa, determine the side *a* of the plate that will provide the most economical and safe design.



For the column $\sigma = \frac{P}{A}$

or $P = \sigma A = (200 \times 10^6)(7560 \times 10^{-6}) = 1512 \text{ kN}$

For the a × a plate, $\sigma = 20 \text{ MPa}$

$A = \frac{P}{\sigma} = \frac{1.512}{20} = 0.0756 \text{ m}^2$

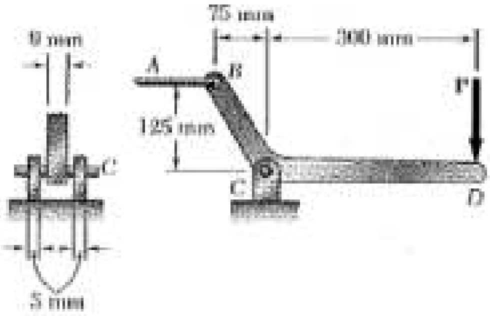
Since the plate is square $A = a^2$

$a = \sqrt{A} = \sqrt{0.0756} = 0.275 \text{ m}$

$= 275 \text{ mm}$

Problem 1.23

1.23 A 6-mm-diameter pin is used at connection C of the pedal shown. Knowing that $P = 500$ N, determine (*a*) the average shearing stress in the pin, (*b*) the nominal bearing stress in the pedal at C, (*c*) the nominal bearing stress in each support bracket at C.



Draw free body diagram of ACD. Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.

From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm.

$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325} C - P = 0 \qquad C = 2.6 P = (2.6)(500) = 1300$ N

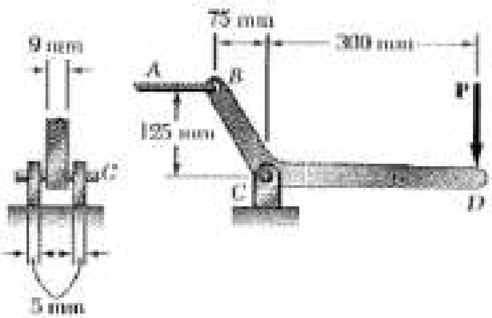
(a) $\tau_{pin} = \frac{\frac{1}{2}C}{A_{pin}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} = \frac{2C}{\pi d^2} = \frac{(2)(1300)}{\pi(6\times10^{-3})^2} = 23.0\times10^6$ Pa $\qquad \tau_{pin} = 23.0$ MPa ◄

(b) $\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1300}{(6\times10^{-3})(9\times10^{-3})} = 24.1\times10^6$ Pa $\qquad \sigma_b = 24.1$ MPa ◄

(c) $\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1300}{(2)(6\times10^{-3})(5\times10^{-3})} = 21.7\times10^6$ Pa $\qquad \sigma_b = 21.7$ MPa ◄

Problem 1.24

1.24 Knowing that a force **P** of magnitude 750 N is applied to the pedal shown, determine (*a*) the diameter of the pin at C for which the average shearing stress in the pin is 40 MPa, (*b*) the corresponding bearing stress in the pedal at C, (*c*) the corresponding bearing stress in the each support bracket at C.



Draw free body diagram of ACD. Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.

From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm

$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325} C - P = 0 \qquad C = 2.6 P = (2.6)(750) = 1950$ N

(a) $\tau_{pin} = \frac{\frac{1}{2}C}{A_{pin}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} \qquad d = \sqrt{\frac{2C}{\pi\tau_{pin}}} = \sqrt{\frac{(2)(1950)}{\pi(40\times10^6)}} = 5.57\times10^{-3}$ m $\qquad d = 5.57$ mm ◄

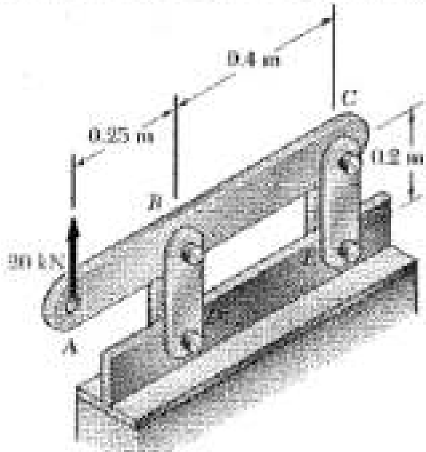
(b) $\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1950}{(5.57\times10^{-3})(9\times10^{-3})} = 38.9\times10^6$ Pa $\qquad \sigma_b = 38.9$ MPa ◄

(c) $\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1950}{(2)(5.57\times10^{-3})(5\times10^{-3})} = 35.0\times10^6$ Pa $\qquad \sigma_b = 35.0$ MPa ◄

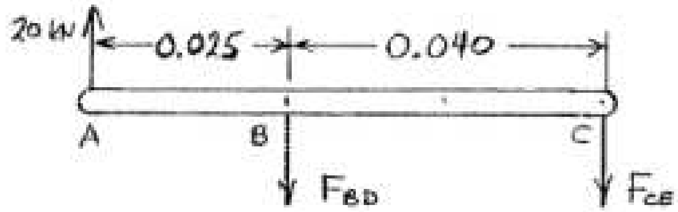
Problem 1.27

1.27 For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.



Use bar ABC as a free body.



$$\circlearrowleft \Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(a) Shear pin at B. $\quad \tau = \dfrac{F_{BD}}{2A}$ for double shear,

where $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \qquad \tau = 80.8 \text{ MPa} \blacktriangleleft$$

(b) Bearing: link BD. $\quad A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \qquad \sigma_b = 127.0 \text{ MPa} \blacktriangleleft$$

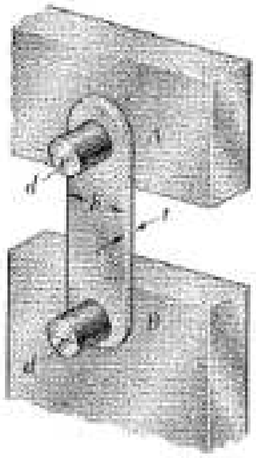
(c) Bearing in ABC at B.

$$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \qquad \sigma_b = 203 \text{ MPa} \blacktriangleleft$$

Problem 1.28

1.28 Link AB, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -140 MPa, and that the average shearing stress in each of the two pins is 80 MPa, determine (*a*) the diameter d of the pins, (*b*) the average bearing stress in the link.



Rod AB is in compression.

$A = bt$ where $b = 50\,\text{mm}$ and $t = 6\,\text{mm}$

$A = (0.050)(0.006) = 300 \times 10^{-6}\ \text{m}^2$

$P = -\sigma A = -(-140 \times 10^{6})(300 \times 10^{-6})$
$= 42 \times 10^{3}\ \text{N}$

For the pin, $A_P = \frac{\pi}{4} d^2$ and $\tau = \frac{P}{A_P}$

$A_P = \frac{P}{\tau} = \frac{42 \times 10^{3}}{80 \times 10^{6}} = 525 \times 10^{-6}\ \text{m}^2$

(a) Diameter d.

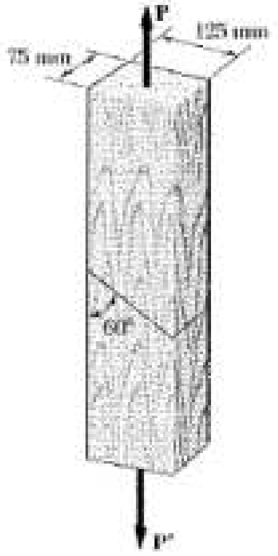
$d = \sqrt{\frac{4A_P}{\pi}} = \sqrt{\frac{(4)(525 \times 10^{-6})}{\pi}} = 2.585 \times 10^{-3}\ \text{m}$

$d = 25.9\ \text{mm}$

(b) $\sigma_b = \frac{P}{dt} = \frac{42 \times 10^{3}}{(25.85 \times 10^{-3})(0.006)} = 271 \times 10^{6}\ \text{Pa}$

$\sigma_b = 271\ \text{MPa}$

Problem 1.29



1.29 The 5.6-kN load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$P = 6227\text{N} \qquad \theta = 90° - 60° = 30°$

$A_o = (0.125)(0.075) = 9.375 \times 10^{-3}\ \text{m}^2$

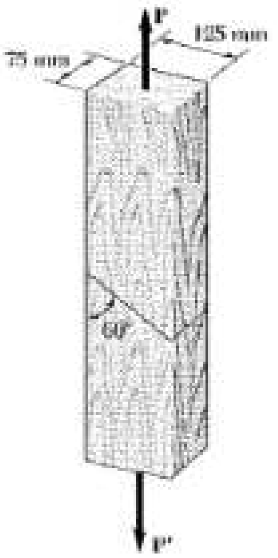
$\sigma = \dfrac{P\cos^2\theta}{A_o} = \dfrac{(6.227 \times 10^3)(\cos 30°)^2}{9.375 \times 10^{-3}}$

$\sigma = 0.498\ \text{MPa}$ ◄

$\tau = \dfrac{P\sin 2\theta}{2A_o} = \dfrac{(6.227 \times 10^3)\sin 60°}{(2)(9.375 \times 10^{-3})}$

$\tau = 0.288\ \text{MPa}$ ◄

Problem 1.30



1.30 Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 525 kPa, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding tensile stress in the splive.

$A_o = (0.125)(0.075) = 9.375 \times 10^{-3}\ \text{m}^2$

$\theta = 90° - 60° = 30°$

$\sigma = \dfrac{P\cos^2\theta}{A_o}$

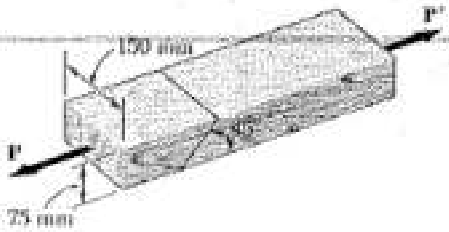
(a) $P = \dfrac{\sigma A_o}{\cos^2\theta} = \dfrac{(525 \times 10^3)(9.375 \times 10^{-3})}{\cos^2 30°} = 6562\text{N}$

$P = 6.562\ \text{kN}$ ◄

(b) $\tau = \dfrac{P\sin 2\theta}{2A_o} = \dfrac{(6562)\sin 60°}{(2)(9.375 \times 10^{-3})}$

$\tau = 0.303\ \text{MPa}$ ◄

Problem 1.31



1.31 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

$\theta = 90° - 45° = 45°$

$P = 11 \text{ kN} = 11 \times 10^3 \text{ N}$

$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$

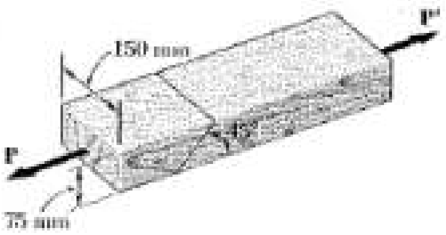
$$\sigma = \frac{P \cos^2\theta}{A_o} = \frac{(11 \times 10^3)\cos^2 45°}{11.25 \times 10^{-3}}$$

$= 489 \times 10^3$ Pa $\qquad \sigma = 489$ kPA

$$\tau = \frac{P \sin 2\theta}{2A_o} = \frac{(11 \times 10^3)(\sin 90°)}{(2)(11.25 \times 10^{-3})}$$

$= 4.89 \times 10^4$ Pa $\qquad \tau = 489$ kPA

Problem 1.32



1.32 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 560 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding shearing stress in the splice.

$\theta = 90° - 45° = 45°$

$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$

$\sigma = 560 \text{ kPa} = 560 \times 10^3 \text{ Pa}$

$$\sigma = \frac{P \cos^2\theta}{A_o}$$

(a) $$P = \frac{\sigma A_o}{\cos^2\theta} = \frac{(560 \times 10^3)(11.25 \times 10^{-3})}{\cos^2 45°}$$

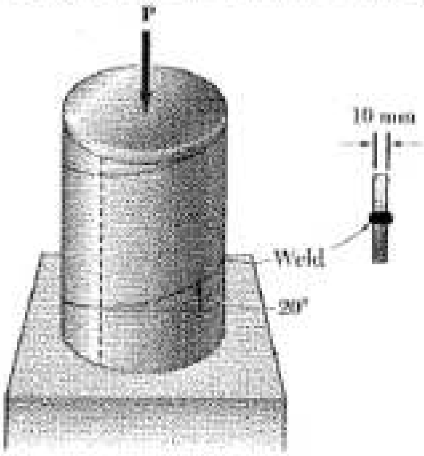
$= 12.60 \times 10^3$ N $\qquad P = 12.60$ kN

(b) $$\tau = \frac{P \sin\theta \cos\theta}{A_o} = \frac{(12.60 \times 10^3)(\sin 45°)(\cos 45°)}{11.25 \times 10^{-3}}$$

$= 560 \times 10^3$ Pa $\qquad \tau = 560$ kPa

Problem 1.35

1.35 A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are σ = 60 MPa and τ = 36 MPa, determine the magnitude P of the largest axial force that can be applied to the pipe.



$d_o = 0.400$ m $\quad r_o = \frac{1}{2} d_o = 0.200$ m

$r_i = r_o - t = 0.200 - 0.010 = 0.190$ m

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2)$

12.25×10^{-3} m^2

$\theta = 20°$

Based on $|\sigma| = 60$ MPa: $\quad \sigma = \frac{P}{A_o}\cos^2\theta$

$$P = \frac{A_o \sigma}{\cos^2\theta} = \frac{(12.25\times 10^{-3})(60\times 10^6)}{\cos^2 20°} = 833 \times 10^3 \text{ N}$$

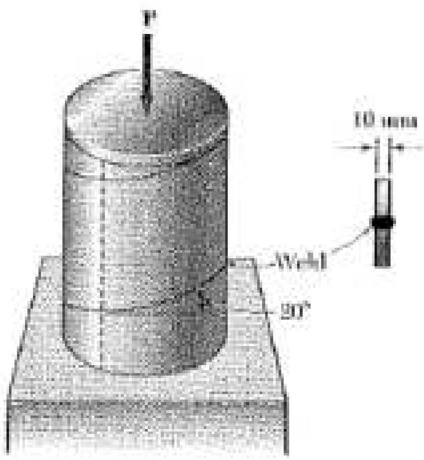
Based on $|\tau| = 30$ MPa $\quad \tau = \frac{P}{2A_o}\sin 2\theta$

$$P = \frac{2A_o\tau}{\sin 2\theta} = \frac{(2)(12.25\times 10^{-3})(36\times 10^6)}{\sin 40°} = 1372\times 10^3 \text{ N}$$

Smaller value is the allowable value of P. $\quad P = 833$ kN

Problem 1.36

1.36 A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 300-kN axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.



$d_o = 0.400$ m $\quad r_o = \frac{1}{2} d_o = 0.200$ m

$r_i = r_o - t = 0.200 - 0.010 = 0.190$ m

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2)$

$= 12.25 \times 10^{-3}$ m^2

$\theta = 20°$

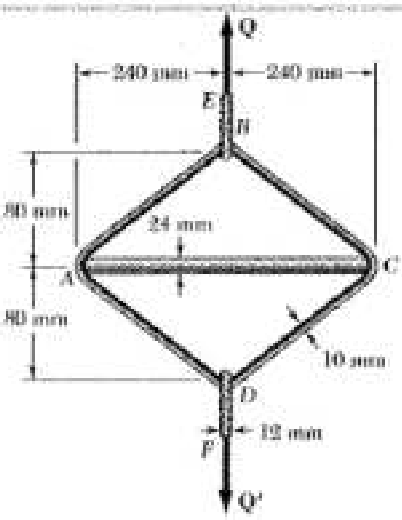
$$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{-300\times 10^3 \cos^2 20°}{12.25\times 10^{-3}}$$

$= -21.6\times 10^6$ Pa $\quad \sigma = -21.6$ MPa

$$\tau = \frac{P}{2A_o}\sin 2\theta = \frac{-300\times 10^3 \sin 40°}{(2)(12.25\times 10^{-3})}$$

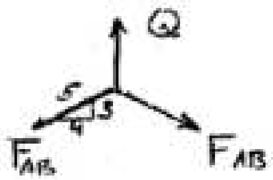
$= -7.87 \times 10^6 \quad \tau = 7.87$ MPa

Problem 1.37



1.37 A steel loop *ABCD* of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of 12-mm diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

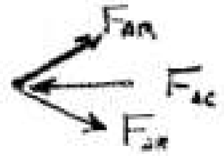
Using joint B as free body and considering symmetry,



$$2 \cdot \frac{3}{5} F_{AB} - Q = 0$$

$$Q = \frac{6}{5} F_{AB}$$

Using joint A as a free body and considering symmetry,



$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$

Based on strength of cable BE.

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop.

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N}$$

Based on strength of rod AC.

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N}$$

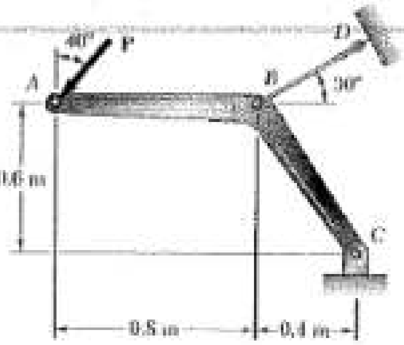
Actual ultimate load Q_U is the smallest. $\therefore$ $Q_U = 45.24 \times 10^3$ N

Allowable load $Q = \frac{Q_U}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3$ N

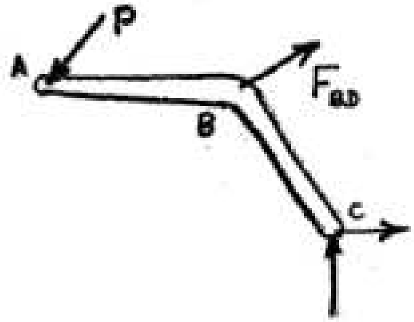
$Q = 15.08$ kN ◄

Problem 1.38

1.38 Member *ABC*, which is supported by a pin and bracket at *C* and a cable *BD*, was designed to support the 16-kN load **P** as shown. Knowing that the ultimate load for cable *BD* is 100 kN, determine the factor of safety with respect to cable failure.



Use member ABC as a free body, and note that member BD is a two-force member.



$$\circlearrowleft \Sigma M_C = 0:$$

$$(P\cos 40°)(1.2) + (P\sin 40°)(0.6) - (F_{BD}\cos 30°)(0.6) - (F_{BD}\sin 30°)(0.4) = 0$$

$$1.30493\ P - 0.71962\ F_{BD} = 0$$

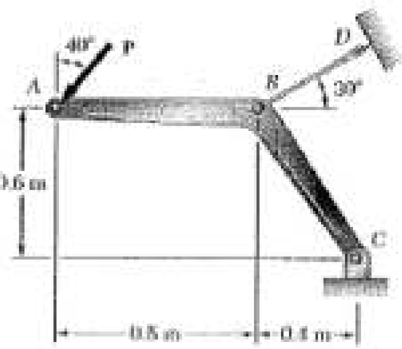
$$F_{BD} = 1.81335\ P = (1.81335)(16\times10^3) = 2.9014\times10^3\ \text{N}$$

$$F_{ult} = 100\times10^3\ \text{N}$$

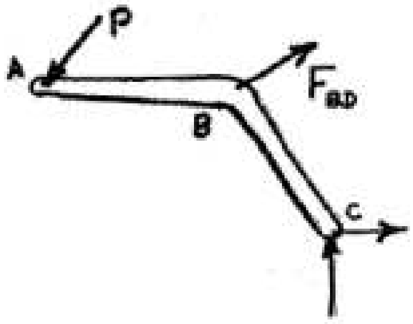
$$F.S. = \frac{F_{ult}}{F_{BD}} = \frac{100\times10^3}{2.9014\times10^3} \qquad F.S. = 3.45 \blacktriangleleft$$

Problem 1.39

1.39 Knowing that the ultimate load for cable *BD* is 100 kN and that a factor of safety of 3.2 with respect to cable failure is required, determine the magnitude of the largest force **P** which can be safely applied as shown to member *ABC*.



Use member ABC as a free body, and note that member BD is a two-force member.



$$\circlearrowleft \Sigma M_C = 0:$$

$$(P\cos 40°)(1.2) + (P\sin 40°)(0.6) - (F_{BD}\cos 30°)(0.6) - (F_{BD}\sin 30°)(0.4) = 0$$

$$1.30493\ P - 0.71962\ F_{BD} = 0$$

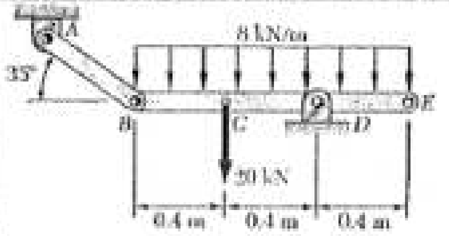
$$P = 0.55404\ F_{BD}$$

Allowable value of F_{BD}. $\quad F_{BD} = \dfrac{F_{ult}}{F.S.} = \dfrac{100\text{ kN}}{3.2} = 3.125\text{ kN}$

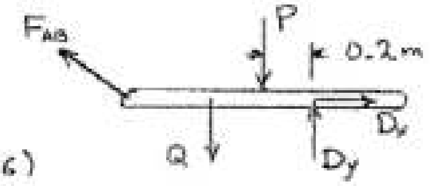
$$P_{all} = (0.55404)(3.125) \qquad P_{all} = 1.732\text{ kN} \blacktriangleleft$$

Problem 1.42

1.42 Link *AB* is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for *AB* for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at *A* and *B*.



$P = (1.2)(8) = 9.6\ \text{kN}$



$+\circlearrowleft \Sigma M_D = 0:$

$-(0.8)(F_{AB}\sin 35°) + (0.2)(9.6) + (0.4)(20) = 0$

$F_{AB} = 21.619\ \text{kN} = 21.619 \times 10^3\ \text{N}$

$\sigma_{AB} = \dfrac{F_{AB}}{A_{AB}} = \dfrac{\sigma_{ult}}{F.S.}$

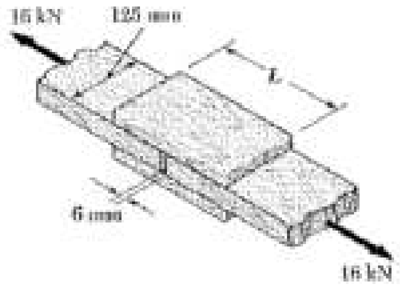
$A_{AB} = \dfrac{(F.S.)F_{AB}}{\sigma_{ult}} = \dfrac{(3.50)(21.619\times 10^3)}{450 \times 10^6}$

$= 168.1 \times 10^{-6}\ \text{m}^2$

$A_{AB} = 168.1\ \text{mm}^2$ ◀

Problem 1.43

1.43 The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.



There are 4 separate areas of glue. Each glue area must transmit 8 kN of shear load.

$$P = 8\text{ kN} = 8\times10^3\text{ N}$$

Required ultimate load.

$$P_U = (F.S.)P = (2.75)(8\times10^3) = 22\times10^3\text{ N}$$

Required length of each glue area.

$$P_U = \tau_U A = \tau_U l w \qquad l = \frac{P_U}{\tau_U w} = \frac{22\times10^3}{(2.5\times10^6)(0.125)} = 70.4\times10^{-3}\text{ m}$$

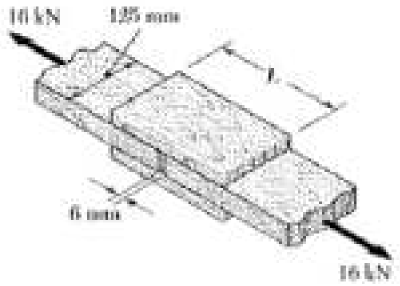
Length of splice: $L = 2l + c = (2)(70.4\times10^{-3}) + 0.006 = 0.1468\times10^{-3}\text{ m}$

$L = 146.8\text{ mm}$ ◄

Problem 1.44

1.44 For the joint and loading of Prob. 1.43, determine the factor of safety, knowing that the length of each splice is $L = 180$ mm.

1.43 The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.



There are 4 separate areas of glue. Each glue area must transmit 8 kN of shear load.

$$P = 8\text{ kN} = 8\times10^3\text{ N}$$

Length of splice. $L = 2l + c$ where l = length of glue and c = clearance. $l = \frac{1}{2}(L-c) = \frac{1}{2}(0.180 - 0.006) = 0.087\text{ m}$

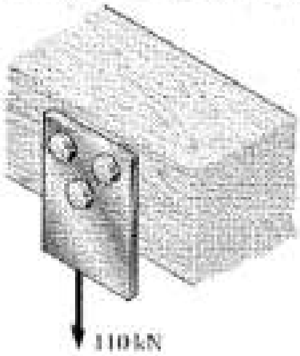
Area of glue. $A = lw = (0.087)(0.125) = 10.875\times10^{-3}\text{ m}$

Ulimate load. $P_U = \tau_U A = (2.5\times10^6)(10.875\times10^{-3}) = 27.1875\times10^3\text{ N}$

Factor of safety. $F.S. = \frac{P_U}{P} = \frac{27.1875\times10^3}{8\times10^3}$ $\qquad F.S. = 3.40$ ◄

Problem 1.45

1.45 Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.



For each bolt, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2$

$= 254.47 \times 10^{-6} \text{ m}^2$

$P_U = A\tau_U = (254.47 \times 10^{-6})(360 \times 10^6)$

$= 91.609 \times 10^3 \text{ N}$

For the three bolts, $P_U = (3)(91.609 \times 10^3)$

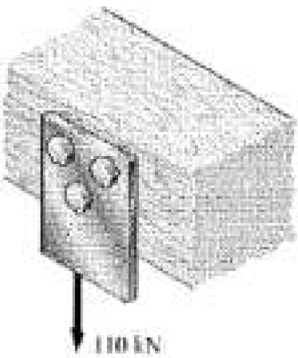
$= 274.83 \times 10^3 \text{ N}$

Factor of safety.

$$F.S. = \frac{P_U}{P} = \frac{274 \times 10^3}{110 \times 10^3} \qquad F.S. = 2.50$$

Problem 1.46

1.46 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110 kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.



For each bolt, $P = \frac{110}{3} = 36.667 \text{ kN}$

Required $P_U = (F.S.)P = (3.35)(36.667) = 122.83 \text{ kN}$

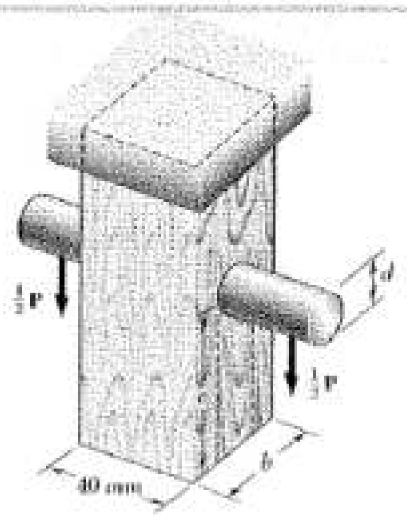
$$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$$

$$d = \sqrt{\frac{4P_U}{\pi \tau_U}} = \sqrt{\frac{(4)(122.83 \times 10^3)}{\pi(360 \times 10^6)}} = 20.8 \times 10^{-3} \text{ m}$$

$d = 20.8 \text{ mm}$

Problem 1.48

1.48 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.



Based on double shear in pin

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$
$$= \frac{\pi}{4}(2)(0.012)^2(145\times10^6) = 32.80\times10^3\ N$$

Based on tension in wood

$$P_U = A\sigma_U = w(b-d)\sigma_U$$
$$= (0.040)(0.040-0.012)(60\times10^6)$$
$$= 67.2\times10^3\ N$$

Based on double shear in the wood

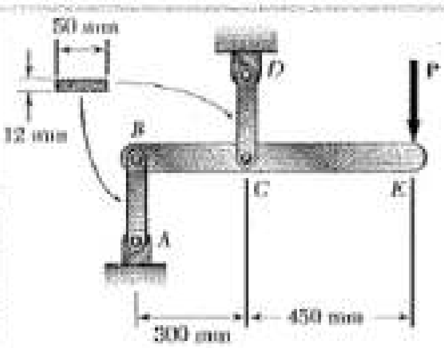
$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$
$$= 33.0\times10^3\ N$$

Use smallest $\quad P_U = 32.8\times10^3\ N$

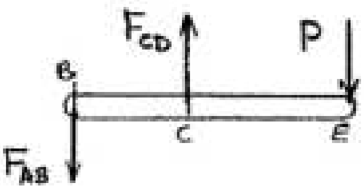
Allowable $\quad P = \frac{P_U}{F.S.} = \frac{32.8\times10^3}{3.2} = 10.25\times10^3\ N$

10.25 kN ◄

Problem 1.51



1.51 Each of the steel links *AB* and *CD* is connected to a support and to member *BCE* by 25-mm-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 210 MPa for the steel used in the pins and that the ultimate normal stress is 490 MPa for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.0 is desired. (Note that the links are not reinforced around the pin holes.)



Use member BCE as free body.

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.3 F_{CD} - 0.75 P = 0$$

$$P = \frac{2}{5} F_{CD}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad 0.3 F_{AB} - 0.45 P = 0$$

$$P = \frac{2}{3} F_{AB}$$

Both links have the same area, pin diameter and material, Therefore, they have the same ultimate load.

Failure by pin in single shear. $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.025)^2 = 490.9 \times 10^{-6}\ m^2$

$$F_U = \tau_U A = (210 \times 10^6)(490.9 \times 10^{-6}) = 103.09\ kN$$

Failure by tension in link. $A = (b - d)t = (0.05 - 0.025)0.012 = 3 \times 10^{-4}\ m^2$

$$F_U = \sigma_U A = (490 \times 10^6)(3 \times 10^{-4}) = 147\ kN$$

Ultimate load for link and pin is the smaller.

$$F_U = 103.09\ kN$$

Allowable values of F_{CD} and F_{AB}.

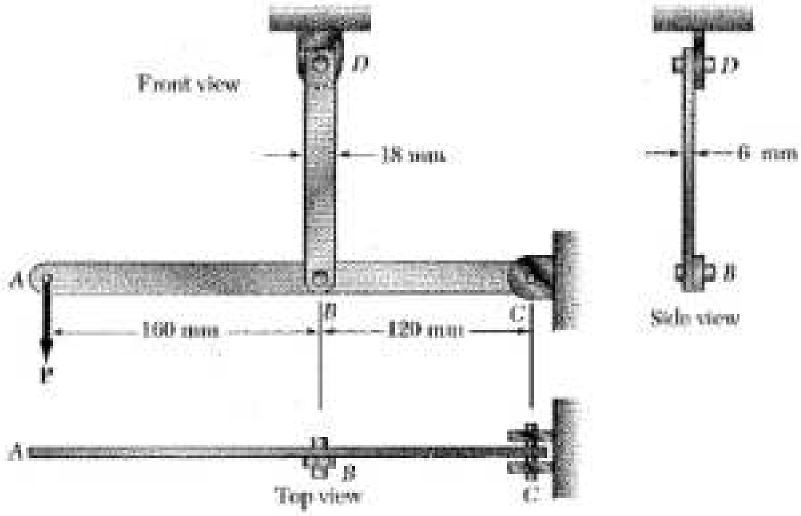
$$F_{all} = \frac{F_U}{F.S.} = \frac{103.09}{3.0} = 34.36\ kN$$

Allowable load for structure is the smaller of $\frac{2}{3} F_{all}$ and $\frac{2}{5} F_{all}$.

$$P = \frac{2}{5}(34.36) \qquad P = 13.7\ kN \blacktriangleleft$$

Problem 1.53

1.53 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at A. Note that link BD is not reinforced around the pin holes.



Use free body ABC.

$+\circlearrowleft \Sigma M_C = 0:$

$0.280\,P - 0.120\,F_{BD} = 0$

$P = \frac{3}{7} F_{BD} \qquad (1)$

$+\circlearrowleft \Sigma M_B = 0:$

$0.160\,P - 0.120\,C = 0$

$P = \frac{3}{4} C \qquad (2)$

Tension on net section of link BD.

$$F_{BD} = \sigma A_{net} = \frac{\sigma_U}{F.S.} A_{net} = \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-10)(10^{-3}) = 6.40\times10^3\ \text{N}$$

Shear in pins at B and D.

$$F_{BD} = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(10\times10^{-3})^2 = 3.9270\times10^3\ \text{N}$$

Smaller value of F_{BD} is 3.9270×10^3 N.

From (1) $\quad P = \left(\frac{3}{7}\right)(3.9270\times10^3) = 1.683\times10^3\ \text{N}$

Shear in pin at C

$$C = 2\tau A_{pin} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3\ \text{N}$$

From (2) $\quad P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3\ \text{N}$

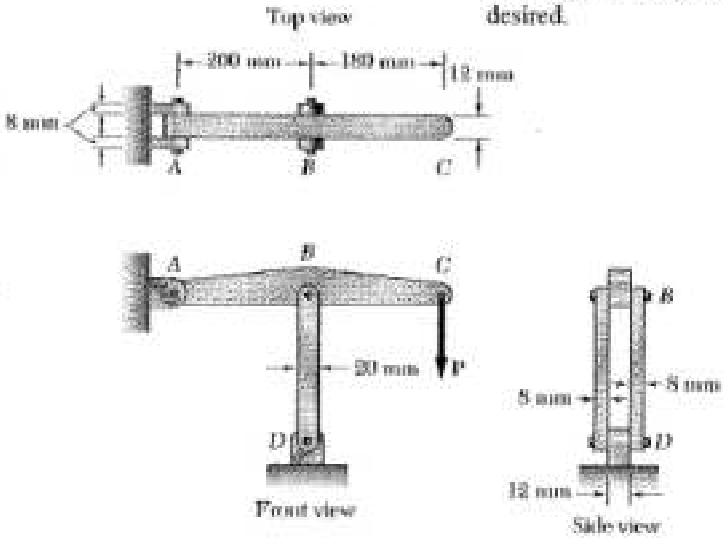
Smaller value of P is allowable value. $\quad P = 1.683\times10^3\ \text{N}$

$P = 1.683\ \text{kN}$

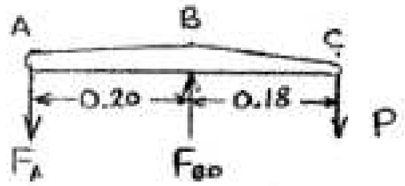
Problem 1.56

1.56 In an alternative design for the structure of Prob. 1.55, a pin of 10-mm-diameter is to be used at *A*. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

1.55 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



Statics: Use ABC as free body.



$\circlearrowleft \Sigma M_B = 0: \quad 0.20\,F_A - 0.18\,P = 0$

$P = \frac{10}{9} F_A$

$\circlearrowleft \Sigma M_A = 0: \quad 0.20\,F_{BD} - 0.38\,P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ m^2$

$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(78.54 \times 10^{-6})}{3.0} = 5.236 \times 10^3\ N$

$P = \frac{10}{9} F_A = 5.82 \times 10^3\ N$

Based on double shear in pins at B and D.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ m^2$

$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\ N$

Based on compression in links BD.

For one link $A = (0.020)(0.008) = 160 \times 10^{-6}\ m^2$

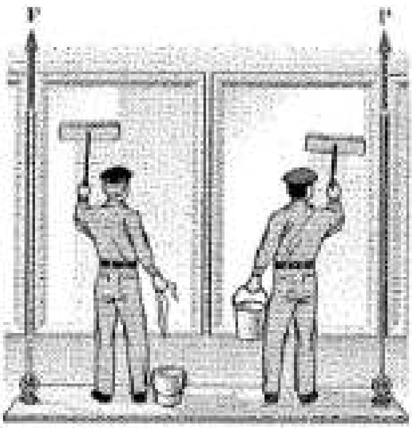
$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\ N$

Allowable value of P is smallest. $\therefore \quad P = 3.97 \times 10^3\ N$

$P = 3.97\ kN$ ◄

Problem 1.58



*1.58 The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 72 kg and each of the window washers is assumed to weigh 88 kg with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\phi}$$

$$= \frac{(1.2)(\frac{1}{2}\times 72) + (1.5)(\frac{3}{4}\times 2\times 88)}{0.85}$$

$$= 283.76 \text{ kg} = 2.78 \text{ kN}$$

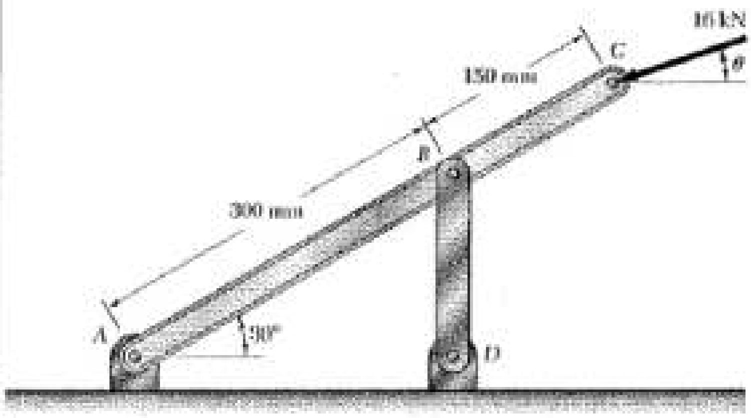
Conventional factor of safety

$$P = P_D + P_L = \tfrac{1}{2}\times 72 + 0.75\times 2\times 88 = 168 \text{ kg} = 1.648 \text{ kN}$$

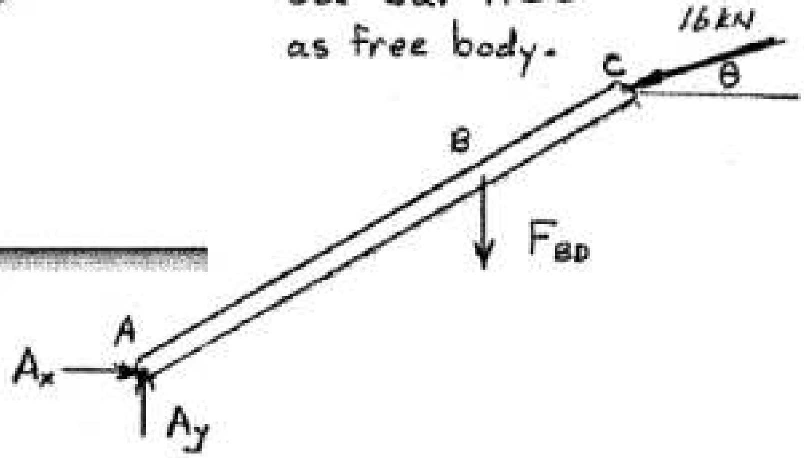
$$F.S. = \frac{P_U}{P} = \frac{2.784}{1.648} = 1.69$$

Problem 1.59

1.59 Link *BD* consists of a single bar 24 mm wide and 12 mm thick. Knowing that each pin has a 9 mm diameter, determine the maximum value of the average normal stress in link *BD* if (*a*) $\theta = 0$, (*b*) $\theta = 90°$.



Use bar ABC as free body.



(a) $\underline{\theta = 0.}$

$+\circlearrowleft \Sigma M_A = 0: \quad (0.45 \sin 30°)(16) - (0.3 \cos 30°) F_{BD} = 0$

$F_{BD} = 13.856 \quad kN \qquad$ (tension)

Area for tension loading: $A = (b - d)t = (24 - 9)(12) = 180 \text{ mm}^2$

Stress: $\sigma = \dfrac{F_{BD}}{A} = \dfrac{13.856 \times 10^3}{180 \times 10^{-6}} \qquad \sigma = 77 \text{ MPa}$ ◄

(b) $\underline{\theta = 90°.}$

$+\circlearrowleft \Sigma M_A = 0: \quad -(0.45 \cos 30°)(16) - (0.3 \cos 30°) F_{BD} = 0$

$F_{BD} = -24 \text{ kN} \qquad$ i.e. compression.

Area for compression loading: $A = bt = (24)(12) = 288 \text{ mm}^2$

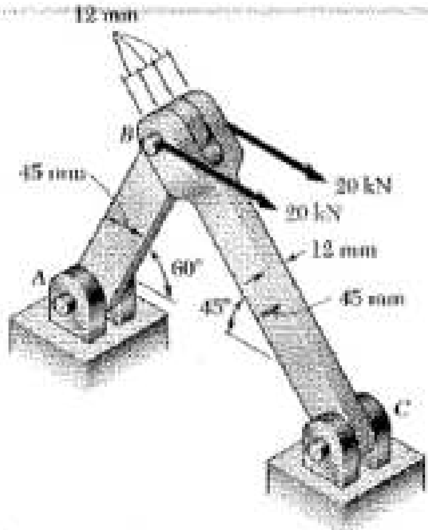
Stress: $\sigma = \dfrac{F_{BD}}{A} = \dfrac{-24000}{288 \times 10^{-6}}$

$\sigma = -83.3 \text{ kN}$ ◄

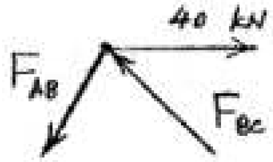
Problem 1.61

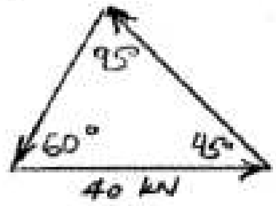
1.61 For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.

1.60 Two horizontal 20-kN forces are applied to pin *B* of the assembly shown. Knowing that a pin of 20-mm diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.



Use joint B as free body.





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{40}{\sin 95^\circ} \qquad F_{BC} = 34.77 \text{ kN}$$

(a) <u>Shearing stress in pin at C.</u> $\tau = \frac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4} d^2 = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2$$

$$\tau = \frac{34770}{(2)(314.16 \times 10^{-6})} = 55.338 \times 10^6 \qquad \tau = 55.3 \text{ MPa}$$

(b) <u>Bearing stress at C in member BC.</u> $\sigma_b = \frac{F_{BC}}{A}$

$$A = t\,d = (12)(20) = 240 \text{ mm}^2$$

$$\sigma_b = \frac{34770}{240 \times 10^{-6}} = 144.875 \times 10^6 \qquad \sigma_b = 144.9 \text{ MPa}$$

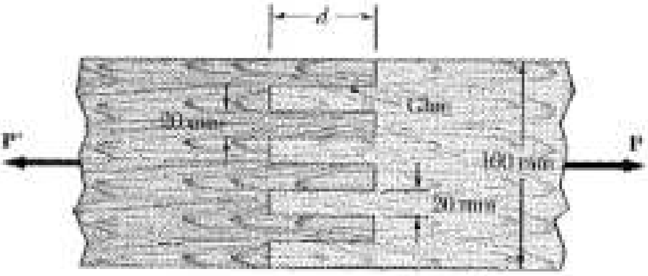
(c) <u>Bearing stress at B in member BC.</u> $\sigma_b = \frac{F_{BC}}{A}$

$$A = 2t\,d = 2(12)(20) = 480 \text{ mm}^2$$

$$\sigma_b = \frac{34770}{480 \times 10^{-6}} = 72.437 \times 10^6 \qquad \sigma_b = 72.4 \text{ MPa}$$

Problem 1.62

1.62 Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.



Seven surfaces carry the total load $P = 7.6$ kN $= 7.6 \times 10^3$.

Let $t = 22$ mm.

Each glue area is $A = dt$

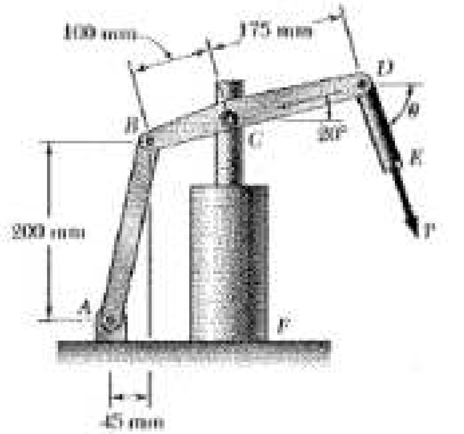
$$\tau = \frac{P}{7A} \qquad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3} \text{ m}^2$$

$$= 1.32404 \times 10^3 \text{ mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2 \qquad d = 60.2 \text{ mm}$$

Problem 1.63

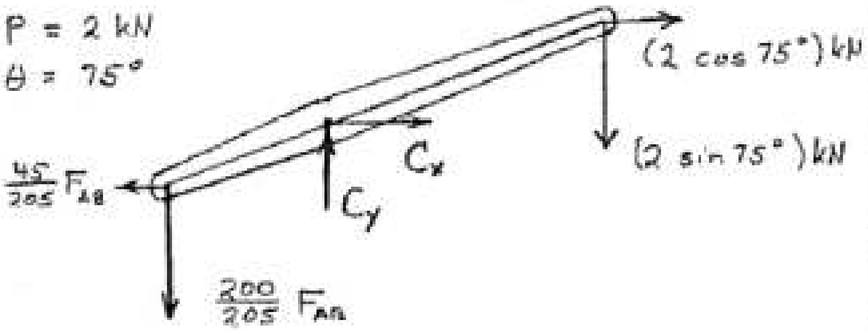
1.63 The hydraulic cylinder CF, which partially controls the position of rod DE, has been locked in the position shown. Member BD is 15 mm thick and is connected to the vertical rod by a 9-mm-diameter bolt. Knowing that $P = 2$ kN and $\theta = 75°$, determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at C in member BD.



Use member BCD as a free body, and note that AB is a two force member.

$P = 2$ kN

$\theta = 75°$



Length of member AB.

$l_{AB} = \sqrt{200^2 + 45^2}$

$= 205$ mm

$\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{200}{205} F_{AB}\right)(100 \cos 20°) - \left(\frac{45}{205} F_{AB}\right)(100 \sin 20°)$

$\qquad - (2 \cos 75°)(175 \sin 20°) - (2 \sin 75°)(175 \cos 20°) = 0$

$84.1696\, F_{AB} - 348.668 = 0 \qquad F_{AB} = 4.1424$ kN

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - \frac{45}{205}(4.1424) + 2\cos 75° = 0 \qquad C_x = 0.3917$ kN

$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{200}{205}(4.1424) - 2\sin 75° = 0 \qquad C_y = 5.9732$ kN

Reaction at C: $\quad C = \sqrt{C_x^2 + C_y^2} \qquad C = 5.9860$ kN

(a) Shearing stress in bolt (single shear).

$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.009)^2 = 63.617 \times 10^{-6}\ \text{m}^2$

$\tau = \frac{C}{A_{bolt}} = \frac{5.9860 \times 10^3}{63.617 \times 10^{-6}} = 94.09 \times 10^6$ Pa $\qquad \tau = 94.1$ MPa ◄

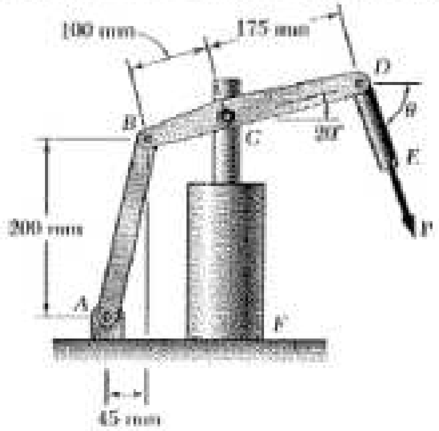
(b) Bearing stress at C in member BD.

$A_b = dt = (0.009)(0.015) = 135 \times 10^{-6}\ \text{m}^2$

$\sigma_b = \frac{C}{A_b} = \frac{5.9860 \times 10^3}{135 \times 10^{-6}} = 44.34 \times 10^6$ Pa $\qquad \sigma_b = 44.3$ MPa ◄

Problem 1.64

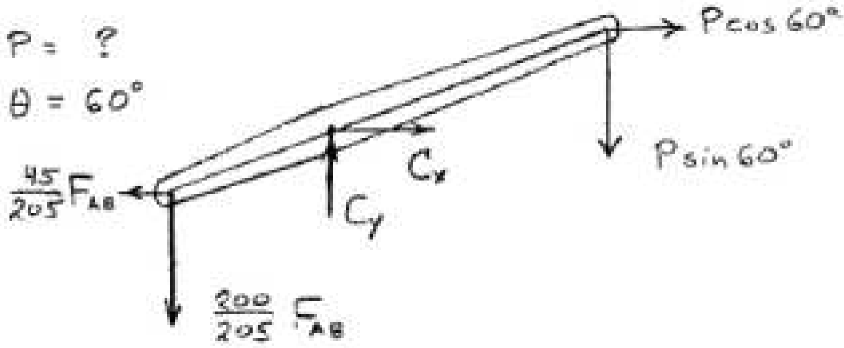
1.64 The hydraulic cylinder CF, which partially controls the position of rod DE, has been locked in the position shown. Link AB has a uniform rectangular cross section of 12 × 25 mm and is connected at B to member BD by an 8-mm-diameter pin. Knowing that the maximum allowable average shearing stress in the pin is 140 MPa, determine (a) the largest force **P** which may be applied at E when $\theta = 60°$, (b) the corresponding bearing stress at B in link AB, (c) the corresponding maximum value of the normal stress in link AB.



Use member BCD as a free body, and note that AB is a two force member.

$P = ?$

$\theta = 60°$



Length of member AB.

$l_{AB} = \sqrt{200^2 + 45^2}$

$= 205 \text{ mm}$

$$+\circlearrowleft \Sigma M_C = 0: \left(\frac{200}{205}F_{AB}\right)(100\cos 20°) - \left(\frac{45}{205}F_{AB}\right)(100\sin 20°)$$
$$-(P\cos 60°)(175\sin 20°) - (P\sin 60°)(175\cos 20°) = 0$$

$$84.1696\, F_{AB} - 172.3414\, P = 0 \qquad F_{AB} = 2.0475\, P$$

(a) Allowable load P. Pin at A is in single shear.

$A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.008)^2 = 50.2655 \times 10^{-6} \text{ m}^2$

$\tau = 140 \times 10^6 \text{ Pa}$

$\tau = \frac{F_{AB}}{A_{pin}} \qquad 140 \times 10^6 = \frac{2.0475\, P}{50.2655 \times 10^{-6}}$

$P = 3.4370 \times 10^3 \text{ N} \qquad P = 3.44 \text{ kN}$

(b) Bearing stress at B in link AB. $d = 8 \text{ mm}$, $t = 12 \text{ mm}$

$A_b = dt = (0.008)(0.012) = 96 \times 10^{-6} \text{ m}^2$

$F_{AB} = (2.0475)(3.4375 \times 10^3) = 7.0383 \times 10^3 \text{ N}$

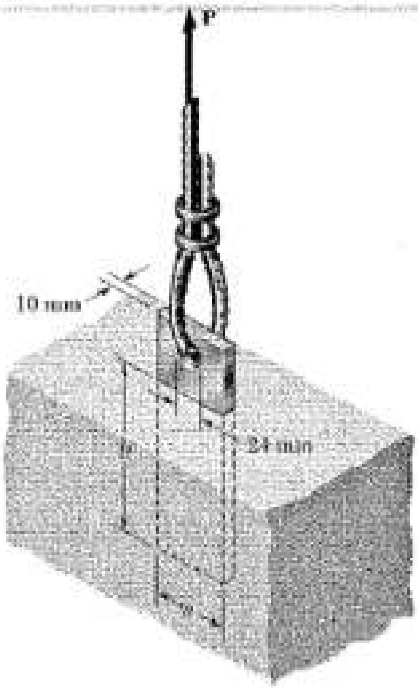
$\sigma_b = \frac{F_{AB}}{A_b} = \frac{7.0383 \times 10^3}{96 \times 10^{-6}} = 73.3 \times 10^6 \text{ Pa} \qquad \sigma_b = 73.3 \text{ MPa}$

(c) Maximum normal stress in link AB. $b = 25 \text{ mm}$, $t = 0.012 \text{ mm}$

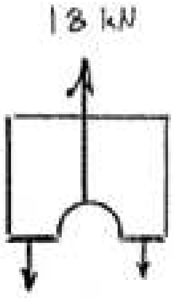
$A_{net} = (b - d)(t) = (0.025 - 0.008)(0.012) = 204 \times 10^{-6} \text{ m}^2$

$\sigma_{AB} = \frac{F_{AB}}{A_{net}} = \frac{7.0383 \times 10^3}{204 \times 10^{-6}} = 34.5 \times 10^6 \text{ Pa} \qquad \sigma_{AB} = 34.5 \text{ MPa}$

Problem 1.67



1.67 A steel plate 10 mm thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is 24 mm, the ultimate strength of the steel used is 250 MPa, and the ultimate bonding stress between plate and concrete is 2.1 MPa. Knowing that a factor of safety of 3.60 is desired when $P = 18$ kN, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)



(a) Based on tension in the plate.

$A = (a-d)t$

$P_U = \sigma_U A$

$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a-d)t}{P}$

Solving for a,

$a = d + \frac{(F.S.)P}{\sigma_U t} = 0.024 + \frac{(3.60)(18\times10^3)}{(250\times10^6)(0.010)}$

$a = 0.04992$ m $\qquad a = 49.9$ mm

(b) Based on shear between plate and concrete slab.

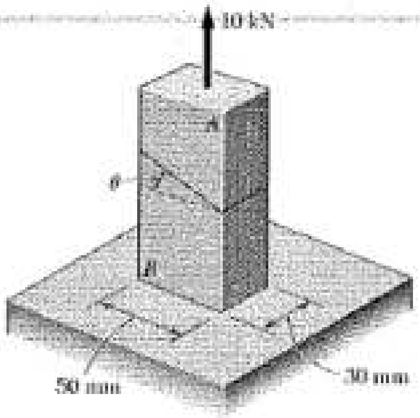
$A = \text{perimeter} \times \text{depth} = (2a+2t)\,b \qquad \tau_U = 2.1\times10^6$ Pa

$P_U = \tau_U A = 2\tau_U (a+t)\,b \qquad F.S. = \frac{P_U}{P}$

Solving for b, $\quad b = \frac{(F.S.)P}{2(a+t)\tau_U} = \frac{(3.60)(18\times10^3)}{(2)(0.04992+0.010)(2.1\times10^6)}$

$b = 0.25748$ m $\qquad b = 257$ mm

Problem 1.68



1.68 The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

$A_o = (0.05)(0.03) = 0.0015\ m^2$

$P = 10\ kN \qquad P_U = (F.S)P = 30\ kN$

Based on tensile stress

$\sigma_U = \frac{P_U}{A_o}\cos^2\theta$

$\cos^2\theta = \frac{\sigma_U A_o}{P_U} = \frac{(17\times10^6)(0.0015)}{30\times10^3} = 0.85$

$\cos\theta = 0.922 \qquad \theta = 22.8° \qquad \theta \geq 22.8°$

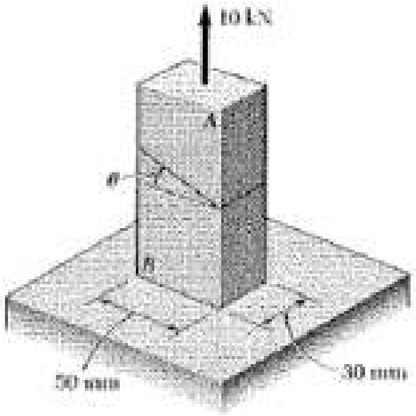
Based on shearing stress $\qquad \tau_U = \frac{P_U}{A_o}\sin\theta\cos\theta = \frac{P_U}{2A_o}\sin 2\theta$

$\sin 2\theta = \frac{2A_o\tau_U}{P_U} = \frac{(2)(0.0015)(9\times10^6)}{30\times10^3} = 0.9$

$2\theta = 64.16° \qquad \theta = 32.1° \qquad \theta \leq 32.1°$

Hence $\qquad 22.8° \leq \theta \leq 32.1°$

Problem 1.69



1.69 The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint:* Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

$A_o = (0.05)(0.03) = 0.0015\ m^2$

At the optimum angle $\quad (F.S.)_\sigma = (F.S.)_\tau$

Normal stress : $\sigma = \frac{P}{A_o}\cos^2\theta \quad \therefore \quad P_{U,\sigma} = \frac{\sigma_U A_o}{\cos^2\theta}$

$(F.S.)_\sigma = \frac{P_{U\sigma}}{P} = \frac{\sigma_U A_o}{P\cos^2\theta}$

Shearing stress : $\tau = \frac{P}{A_o}\sin\theta\cos\theta \quad \therefore \quad P_{U,\tau} = \frac{\tau_U A_o}{\sin\theta\cos\theta}$

$(F.S.)_\tau = \frac{P_{U\tau}}{P} = \frac{\tau_U A_o}{P\sin\theta\cos\theta}$

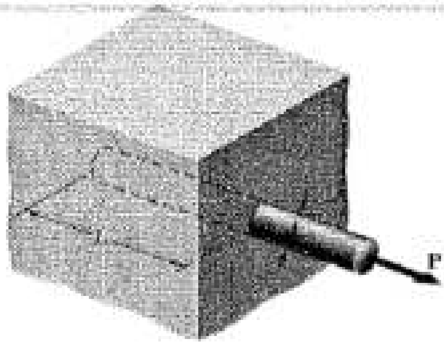
Equating: $\frac{\sigma_U A_o}{P\cos^2\theta} = \frac{\tau_U A_o}{P\sin\theta\cos\theta}$;

Solving: $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\tau_U}{\sigma_U} = \frac{9}{17} = 0.529 \qquad$ (a) $\theta_{opt} = 27.9°$

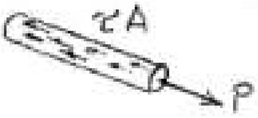
(b) $\quad P_U = \frac{\sigma_U A_o}{\cos^2\theta} = \frac{(17\times10^6)(0.0015)}{\cos^2 27.9°} = 32.65\ kN$

$F.S. = \frac{P_U}{P} = \frac{32.64}{10} = 3.26$

Problem 1.70



1.70 A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)



For shear, $A = \pi d L$

$P = \tau_{all} A = \tau_{all} \pi d L$

For tension, $A = \frac{\pi}{4} d^2$

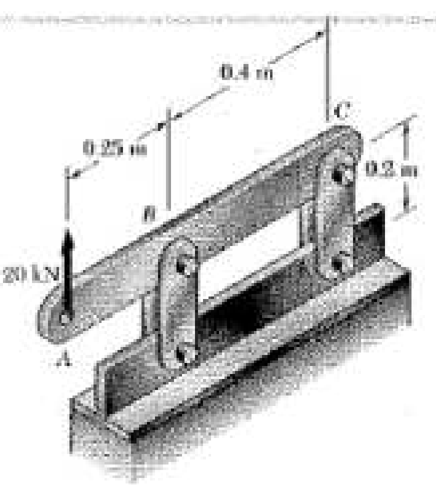
$P = \sigma_{all} A = \sigma_{all} \left(\frac{\pi}{4} d^2 \right)$

Equating, $\tau_{all} \pi d L = \sigma_{all} \frac{\pi}{4} d^2$

Solving for L,

$L_{min} = \sigma_{all} d / 4 \tau_{all}$

PROBLEM 1.C2



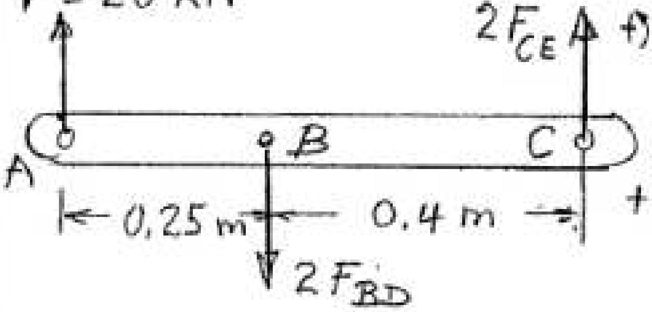
1.C2 A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (2) the average normal stress in the links connecting pins *C* and *E*, (3) the average shearing stress in pin *B*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *B* in member *ABC*, (6) the average bearing stress at *C* in member *ABC*. (*b*) Check your program by comparing the values obtained for *d* = 16 mm with the answers given for Probs. 1.7 and 1.27. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve part *c*, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

FORCES IN LINKS

$P = 20\text{ kN}$

F.B. DIAGRAM OF ABC:



$+\circlearrowleft \Sigma M_C = 0:\ 2F_{BD}(BC) - P(AC) = 0$

$F_{BD} = P(AC)/2(BC)$ (TENSION)

$+\circlearrowleft \Sigma M_B = 0:\ 2F_{CE}(BC) - P(AB) = 0$

$F_{CE} = P(AB)/2(BC)$ (COMP.)

(1) LINK BD

Thickness $= t_L$

$A_{BD} = t_L(w_L - d)$

$\sigma_{BD} = +F_{BD}/A_{BD}$

(2) LINK CE

Thickness $= t_L$

$A_{CE} = t_L w_L$

$\sigma_{CE} = -F_{CE}/A_{CE}$

(3) PIN B

$\tau_B = F_{BD}/(\pi d^2/4)$

(4) PIN C

$\tau_C = F_{CE}/(\pi d^2/4)$

(5) BEARING STRESS AT B

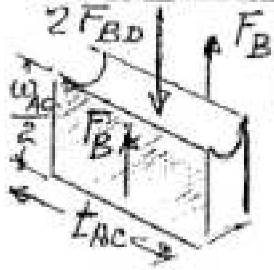
Thickness of member $AC = t_{AC}$

Sig Bear $B = F_{BD}/(d\, t_{AC})$

(6) BEARING STRESS AT C

Sig Bear $C = F_{CE}/(d\, t_{AC})$

SHEARING STRESS IN ABC UNDER PIN B



$F_B = \tau_{AC}\, t_{AC}\,(w_{AC}/2)$

$\Sigma F_y = 0:\ 2F_B = 2F_{BD}$

$\tau_{AC} = \dfrac{2F_{BD}}{t_{AC}\, w_{AC}}$

(CONTINUED)

INPUT DATA FOR PARTS (a), (b), (c): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 10 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C	
10.00	78.13	-21.70	206.90	79.58	325.00	125.00	
11.00	81.25	-21.70	170.99	65.77	295.45	113.64	
12.00	84.64	-21.70	143.68	55.26	270.83	104.17	
13.00	88.32	-21.70	122.43	47.09	250.00	96.15	
14.00	92.33	-21.70	105.56	40.60	232.14	89.29	
15.00	96.73	-21.70	91.96	35.37	216.67	83.33	
16.00	101.56	-21.70	80.82	31.08	203.12	78.13	← (b)
17.00	106.91	-21.70	71.59	27.54	191.18	73.53	
18.00	112.85	-21.70	63.86	24.56	180.56	69.44	
19.00	119.49	-21.70	57.31	22.04	171.05	65.79	
20.00	126.95	-21.70	51.73	19.89	162.50	62.50	
21.00	135.42	-21.70	46.92	18.04	154.76	59.52	
22.00	145.09	-21.70	42.75	16.44	147.73	56.82	
23.00	156.25	-21.70	39.11	15.04	141.30	54.35	
24.00	169.27	-21.70	35.92	13.82	135.42	52.08	
25.00	184.66	-21.70	33.10	12.73	130.00	50.00	
26.00	203.13	-21.70	30.61	11.77	125.00	48.08	
27.00	225.69	-21.70	28.38	10.92	120.37	46.30	
28.00	253.91	-21.70	26.39	10.15	116.07	44.64	
29.00	290.18	-21.70	24.60	9.46	112.07	43.10	
30.00	338.54	-21.70	22.99	8.84	108.33	41.67	

(c) ANSWER : 16 mm ≤ *d* ≤ 22 mm (c)

CHECK: For d = 22 mm, Tau AC = 65 MPa < 90 MPa O.K.

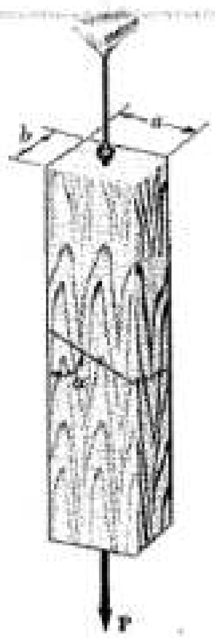
INPUT DATA FOR PART (d): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 8 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	406.25	156.25
11.00	81.25	-21.70	170.99	65.77	369.32	142.05
12.00	84.64	-21.70	143.68	55.26	338.54	130.21
13.00	88.32	-21.70	122.43	47.09	312.50	120.19
14.00	92.33	-21.70	105.56	40.60	290.18	111.61
15.00	96.73	-21.70	91.96	35.37	270.83	104.17
16.00	101.56	-21.70	80.82	31.08	253.91	97.66
17.00	106.91	-21.70	71.59	27.54	238.97	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	156.25	-21.70	39.11	15.04	176.63	67.93
24.00	169.27	-21.70	35.92	13.82	169.27	65.10
25.00	184.66	-21.70	33.10	12.73	162.50	62.50
26.00	203.13	-21.70	30.61	11.77	156.25	60.10
27.00	225.69	-21.70	28.38	10.92	150.46	57.87
28.00	253.91	-21.70	26.39	10.15	145.09	55.80
29.00	290.18	-21.70	24.60	9.46	140.09	53.88
30.00	338.54	-21.70	22.99	8.84	135.42	52.08

(d) ANSWER : 18 mm ≤ *d* ≤ 22 mm (d)

CHECK: For d = 22 mm, Tau AC = 81.25 MPa < 90 MPa O.K.

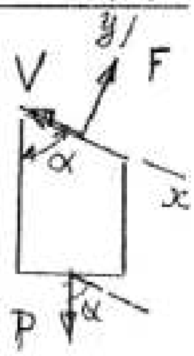
PROBLEM 1.C5



1.C5 A load **P** is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U, respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of a, b, P, σ_U and τ_U, and for values of α from 5 to 85° at 5° intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.31, knowing that $\sigma_U = 1.26$ MPa and $\tau_U = 1.50$ MPa for the glue used in Prob. 1.29, and that $\sigma_U = 1.03$ MPa and $\tau_U = 1.47$ MPa for the glue used in Prob. 1.31. (*c*) Verify in each of these two cases that the shearing stress is maximum for $\alpha = 45°$.

SOLUTION

(1) and (2) Draw the F.B. diagram of lower member:



$+\searrow \Sigma F_x = 0: \quad -V + P\cos\alpha = 0 \qquad V = P\cos\alpha$

$+\nearrow \Sigma F_y = 0: \quad F - P\sin\alpha = 0 \qquad F = P\sin\alpha$

Area $= ab/\sin\alpha$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/ab)\sin^2\alpha$$

Shearing stress: $\tau = \dfrac{V}{\text{Area}} = (P/ab)\sin\alpha\cos\alpha$

(3) F.S. for tension (normal stresses)

$FSN = \sigma_U/\sigma$

(4) F.S. for shear:

$FSS = \tau_U/\tau$

(5) OVERALL F.S.:

FS = The <u>smaller</u> of FSN and FSS.

(CONTINUED)

PROGRAM OUTPUTS

Problem 1.31
a = 150 mm
b = 75 mm
P = 11 kN
SIGU = 1.26 MPa
TAUU = 1.50 MPa

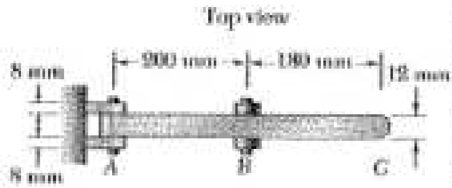
ALPHA	SIG (MPa)	TAU (MPa)	FSN	FSS	FS	
5	.007	.085	169.644	17.669	17.669	
10	.029	.167	42.736	8.971	8.971	
15	.065	.244	19.237	6.136	6.136	
20	.114	.314	11.016	4.773	4.773	
25	.175	.375	7.215	4.005	4.005	
30	.244	.423	5.155	3.543	3.543	
35	.322	.459	3.917	3.265	3.265	
40	.404	.481	3.119	3.116	3.116	
45	.489	.489	2.577	3.068	2.577	◄ (b), (c)
50	.574	.481	2.196	3.116	2.196	
55	.656	.459	1.920	3.265	1.920	
60	.733	.423	1.718	3.543	1.718	
65	.803	.375	1.569	4.005	1.569	
70	.863	.314	1.459	4.773	1.459	
75	.912	.244	1.381	6.136	1.381	
80	.948	.167	1.329	8.971	1.329	
85	.970	.085	1.298	17.669	1.298	

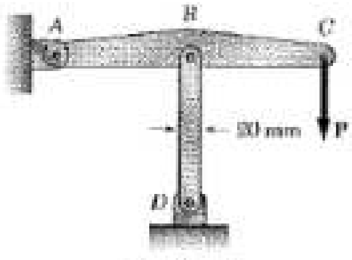
Problem 1.29
a = 125 mm
b = 75 mm
P = 5600 N
SIGU = 1.05 MPa
TAUU = 1.5 MPa

ALPHA	SIG (kPa)	TAU (kPa)	FSN	FSS	FS
5	4.693	56.728	211.574	26.408	26.408
25	116.69	250.243	8.998	5.986	5.986
45	326.669	321.769	3.214	4.586	3.214
60	490.00	282.905	2.143	5.295	2.143
85	648.368	55.877	1.619	26.408	1.619

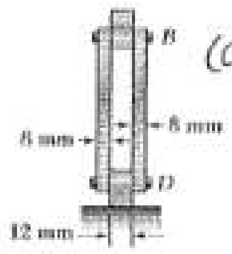
PROBLEM 1.C6

1.C6 Member *ABC* is supported by a pin and bracket at *A* and by two links, which are pin-connected to the member at *B* and to a fixed support at *D*. (*a*) Write a computer program to calculate the allowable load P_{all} for any given values of (1) the diameter d_1 of the pin at *A*, (2) the common diameter d_2 of the pins at *B* and *D*, (3) the ultimate normal stress σ_U in each of the two links, (4) the ultimate shearing stress τ_U in each of the three pins, (5) the desired overall factor of safety *F.S.* Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at *A*, or the shearing stress in the pins at *B* and *D*. (*b* and *c*) Check your program by using the data of Probs. 1.55 and 1.56, respectively, and comparing the answers obtained for P_{all} with those given in the text. (*d*) Use your program to determine the allowable load P_{all}, as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MP for aluminum links, $\tau_U = 100$ MPa for steel pins, and $F.S. = 3.2$.





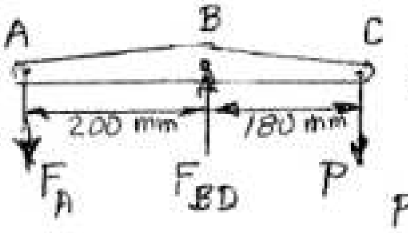
Front view



Side view

SOLUTION

(a) F.B DIAGRAM OF ABC:



$\Sigma M_A = 0$: $P = \frac{200}{380} F_{BD}$

$\Sigma M_B = 0$: $P = \frac{200}{180} F_A$

(1) For given d_1 of pin A: $F_A = 2(\tau_U / FS)(\pi d_1^2/4)$, $P_1 = \frac{200}{180} F_A$

(2) For given d_2 of pins B and D: $F_{BD} = 2(\tau_U / FS)(\pi d_2^2/4)$, $P_2 = \frac{200}{380} F_{BD}$

(3) For ultimate stress in links BD: $F_{BD} = 2(\sigma_U / FS)(0.02)(0.008)$, $P_3 = \frac{200}{380} F_{BD}$

(4) For ult. shearing stress in pins: P_4 is the smaller of P_1 and P_2

(5) For desired overall F.S.: P_5 is the smaller of P_3 and P_4

If $P_3 < P_4$, stress is critical in links

If $P_4 < P_3$ and $P_1 < P_2$, stress is critical in pin A

If $P_4 < P_3$ and $P_2 < P_1$, stress is critical in pins B and D

PROGRAM OUTPUTS

(b) Prob. 1.53. DATA: $d_1 = 8$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, F.S. = 3.0

$P_{all} = 3.72$ kN. Stress in pin A is critical ◀

(c) Prob. 1.54 DATA: $d_1 = 10$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, F.S. = 3.0

$P_{all} = 3.97$ kN. Stress in pins B and D is critical ◀

(d) DATA: $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa, $\tau_U = 100$ MPa, F.S. = 3.2

$P_{all} = 5.79$ kN. Stress in links is critical ◀

Problem 2.8

2.8 An 80-m-long wire of 5-mm diameter is made of a steel with $E = 200$ GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine (*a*) the largest allowable tension in the wire, (*b*) the corresponding elongation of the wire.

(a) $\sigma_U = 400 \times 10^6$ Pa $\qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(5)^2 = 19.635 \text{ mm}^2 = 19.635 \times 10^{-6} \text{ m}^2$

$$P_U = \sigma_U A = (400 \times 10^6)(19.635 \times 10^{-6}) = 7854 \text{ N}$$

$$P_{all} = \frac{P_U}{F.S.} = \frac{7854}{3.2} = 2454 \text{ N} \qquad P_{all} = 2.45 \text{ kN}$$

(b) $$\delta = \frac{PL}{AE} = \frac{(2454)(80)}{(19.635 \times 10^{-6})(200 \times 10^9)} = 50.0 \times 10^{-3} \text{ m}$$

$$\delta = 50.0 \text{ mm}$$

Problem 2.9

2.9 A block of 250-mm length and 50 × 40 mm cross section is to support a centric compressive load **P**. The material to be used is a bronze for which $E = 95$ GPa. Determine the largest load which can be applied, knowing that the normal stress must not exceed 80 MPa and that the decrease in length of the block should be at most 0.12% of its original length.

$A = (50)(40) = 2000 \text{ mm}^2$
$= 2 \times 10^{-3} \text{ m}^2$

$\sigma_a = 80 \text{ MPa} = 80 \times 10^6 \text{ Pa} \qquad E = 95 \times 10^9 \text{ Pa}$

Considering allowable stress:

$\sigma = \frac{P}{A} \qquad P = A\sigma = (2 \times 10^{-3})(80 \times 10^6) = 160 \times 10^3 \text{ N}$

Considering allowable deformation:

$\delta = \frac{PL}{AE} \qquad P = AE\left(\frac{\delta}{L}\right) = (2 \times 10^{-3})(95 \times 10^9)(0.0012) = 228 \times 10^3 \text{ N}$

The smaller value governs. $\quad P = 160 \times 10^3$ N $\qquad P = 160.0$ kN

Problem 2.10

2.10 A 1.5-m-long aluminum rod must not stretch more than 1 mm and the normal stress must not exceed 40 MPa when the rod is subjected to a 3-kN axial load. Knowing that $E = 70$ GPa, determine the required diameter of the rod.

$L = 1.5 \text{ m}$

$\delta = 1 \times 10^{-3} \text{ m}, \quad \sigma = 40 \times 10^6 \text{ Pa}, \quad E = 70 \times 10^9 \text{ Pa}, \quad P = 3 \times 10^3 \text{ N}$

<u>Stress:</u> $\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{3 \times 10^3}{40 \times 10^6} = 75 \times 10^{-6} \text{ m}^2 = 75 \text{ mm}^2$

<u>Deformation:</u> $\delta = \frac{PL}{AE}$

$$A = \frac{PL}{E\delta} = \frac{(3 \times 10^3)(1.5)}{(70 \times 10^9)(1 \times 10^{-3})} = 64.29 \times 10^{-6} \text{ m}^2 = 64.29 \text{ mm}^2$$

Larger value of A governs. $\qquad A = 75 \text{ mm}^2$

$A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(75)}{\pi}} \qquad d = 9.77 \text{ mm}$

Problem 2.11

2.11 An aluminum control rod must stretch 2 mm when a 2-kN tensile load is applied to it. Knowing that $\sigma_{all} = 154$ MPa and $E = 70$ GPa, determine the smallest diameter and shortest length which may be selected for the rod.

$P = 2 \text{ kN}, \quad \delta = 2 \text{ mm} \qquad \sigma_{all} = 154 \text{ MPa}$

$\sigma = \frac{P}{A} \leq \sigma_{all} \qquad A \geq \frac{P}{\sigma_{all}} = \frac{2000}{154} = 12.987 \text{ mm}^2$

$A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(12.987)}{\pi}} \qquad d_{min} = 4.07 \text{ mm}$ ◄

$\sigma = E\varepsilon = \frac{E\delta}{L} \leq \sigma_{all}$

$L \geq \frac{E\delta}{\sigma_{all}} = \frac{(70 \times 10^9)(0.002)}{154 \times 10^6} = 0.909 \text{ m} \qquad L_{min} = 0.91 \text{ m}$ ◄

Problem 2.12

2.12 A square aluminum bar should not stretch more than 1.4 mm when it is subjected to a tensile load. Knowing that $E = 70$ GPa and that the allowable tensile strength is 120 MPa, determine (*a*) the maximum allowable length of the pipe, (*b*) the required dimensions of the cross-section if the tensile load is 28 kN.

$\sigma = 120 \times 10^6 \text{ Pa}$

$E = 70 \times 10^9 \text{ Pa} \qquad \delta = 1.4 \times 10^{-3} \text{ m}$

(a) $\delta = \frac{PL}{AE} = \frac{\sigma L}{E} \qquad L = \frac{E\delta}{\sigma} = \frac{(70 \times 10^9)(1.4 \times 10^{-3})}{120 \times 10^6} = 0.817 \text{ m}$

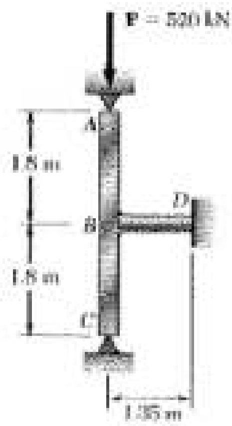
$L = 817 \text{ mm}$ ◄

(b) $\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{28 \times 10^3}{120 \times 10^6} = 233.333 \times 10^{-6} \text{ m}^2 = 233.333 \text{ mm}^2$

$A = a^2 \qquad a = \sqrt{A} = \sqrt{233.333} \qquad a = 15.28 \text{ mm}$ ◄

Problem 2.13

2.13 Rod *BD* is made of steel ($E = 200$ GPa) and is used to brace the axially compressed member *ABC*. The maximum force that can be developed in member *BD* is $0.02P$. If the stress must not exceed 126 MPa and the maximum change in length of *BD* must not exceed 0.001 times the length of *ABC*, determine the smallest-diameter rod that can be used for member *BD*.



$F_{BD} = 0.02P = (0.02)(520) = 10.4 \text{ kN}$

Considering stress: $\sigma = 126 \text{ MPa}$

$\sigma = \frac{F_{BD}}{A} \therefore A = \frac{F_{BD}}{\sigma} = \frac{10400}{126} = 82.54 \text{ mm}^2$

Considering deformation: $\delta = (0.001)(3600) = 3.6 \text{ mm}$

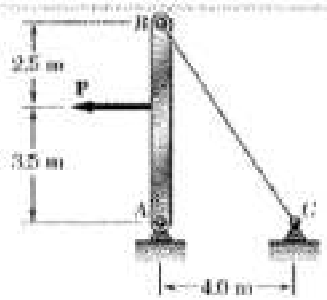
$\delta = \frac{F_{BD}L_{BD}}{AE} \therefore A = \frac{F_{BD}L_{BD}}{E\delta} = \frac{(10.4 \times 10^3)(1350)}{(200 \times 10^3)(3.6)} = 19.5 \text{ mm}^2$

Larger area governs. $\quad A = 19.5 \text{ mm}^2$

$A = \frac{\pi}{4}d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(19.5)}{\pi}} \qquad d = 5 \text{ mm}$ ◄

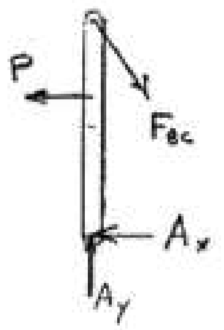
Problem 2.14

2.14 The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.



$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$

Use bar AB as a free body.



$\circlearrowleft \Sigma M_A = 0 \qquad 3.5P - (6)\left(\frac{4}{7.2111} F_{BC}\right) = 0$

$P = 0.9509 \, F_{BC}$

Considering allowable stress: $\sigma = 190 \times 10^6$ Pa

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$

$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$

Considering allowable elongation: $\delta = 6 \times 10^{-3}$ m

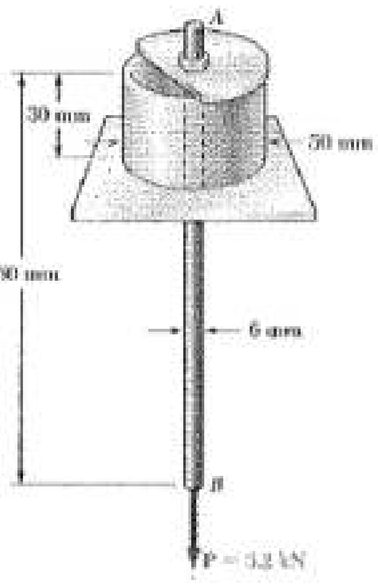
$\delta = \frac{F_{BC} L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$

Smaller value governs. $\qquad F_{BC} = 2.091 \times 10^3 \text{ N}$

$P = 0.9509 \, F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N} \qquad P = 1.988 \text{ kN}$

Problem 2.15

2.15 A 3-mm-thick hollow polystyrene cylinder ($E = 3$ GPa) and a rigid circular plate (only part of which is shown) are used to support a 250-mm-long steel rod AB ($E = 200$ GPa) of 6 mm diameter. If an 3.2 kN load **P** is applied at B, determine (*a*) the elongation of rod AB, (*b*) the deflection of point B, (*c*) the average normal stress in rod AB.



Rod AB: $P_{AB} = 3200$ N $\quad L_{AB} = 0.25$ m $\quad d = 6$ mm

$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.006)^2 = 28.27 \times 10^{-6} \text{ m}^2$

(a) $\delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(3200)(0.25)}{(200 \times 10^9)(28.27 \times 10^{-6})} = 0.1415 \times 10^{-3} \text{ m}$

Hollow cylinder: $d_o = 0.05$ m $\quad d_i = (0.05 - (0.003)(2)) = 0.044$ m

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(0.05^2 - 0.044^2) = 0.000443 \text{ m}^2$

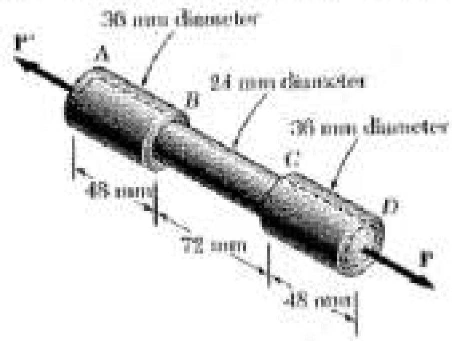
$L = 0.03$ m, $\qquad P = 3200$ N

$\delta_{cyl} = \frac{PL}{EA} = \frac{(3200)(0.03)}{(3 \times 10^9)(0.000443)} = 0.0722 \times 10^{-3} \text{ m}$

(b) Deflection of point B $\qquad \delta_B = \delta_{AB} + \delta_{cyl} = 0.214 \times 10^{-3} \text{ m} \downarrow$

(c) Stress in rod AB $\qquad \sigma = \frac{P}{A_{AB}} = \frac{3200}{28.27 \times 10^{-6}} = 113.2 \text{ MPa}$

Problem 2.16



2.16 The specimen shown is made from a 24-mm-diameter cylindrical steel rod with two 36-mm-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 200$ GPa, determine (*a*) the load **P** so that the total deformation is 0.04 mm, (*b*) the corresponding deformation of the central portion *BC*.

(a) $\delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \sum \frac{L_i}{A_i}$

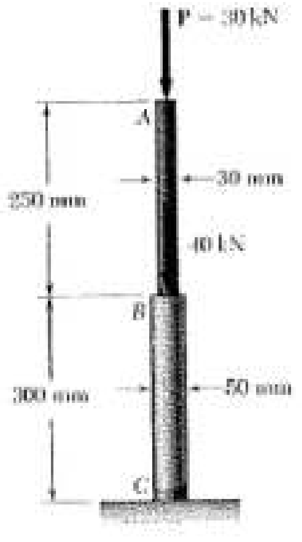
$P = E\delta \left(\sum \frac{L_i}{A_i}\right)^{-1}$ $\qquad A_i = \frac{\pi}{4} d_i^2$

	L, mm	d, mm	A, mm²	L/A, mm⁻¹
AB	48	36	1017.9	0.04716
BC	72	24	452.4	0.15915
CD	48	36	1017.9	0.04716
				0.25347 ← sum

$P = (200 \times 10^3)(0.04)(0.25347)^{-1} = 31.562 \times 10^3$ N $\qquad P = 31.6$ kN ◄

(b) $\delta_{BC} = \frac{P L_{BC}}{A_{BC} E} = \frac{P}{E}\frac{L_{BC}}{A_{BC}} = \frac{31.562 \times 10^3}{200 \times 10^3}(0.15915)$ $\qquad \delta = 0.025$ mm ◄

Problem 2.17



2.17 Two solid cylindrical rods are joined at *B* and loaded as shown. Rod *AB* is made of steel ($E = 200$ GPa) and rod *BC* of brass ($E = 105$ GPa). Determine (*a*) the total deformation of the composite rod *ABC*, (*b*) the deflection of point *B*.

Rod AB: $\quad F_{AB} = -P = -30 \times 10^3$ N

$L_{AB} = 0.250$ m $\qquad E_{AB} = 200 \times 10^9$ GPa

$A_{AB} = \frac{\pi}{4}(30)^2 = 706.85 \text{ mm}^2 = 706.85 \times 10^{-6} \text{ m}^2$

$\delta_{AB} = \frac{F_{AB} L_{AB}}{E_{AB} A_{AB}} = -\frac{(30 \times 10^3)(0.250)}{(200 \times 10^9)(706.85 \times 10^{-6})}$

$= -53.052 \times 10^{-6}$ m

Rod BC: $\quad F_{BC} = 30 + 40 = 70$ kN $= 70 \times 10^3$ N

$L_{BC} = 0.300$ m $\qquad E_{BC} = 105 \times 10^9$ Pa

$A_{BC} = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$

$\delta_{BC} = \frac{F_{BC} L_{BC}}{E_{BC} A_{BC}} = -\frac{(70 \times 10^3)(0.300)}{(105 \times 10^9)(1.9635 \times 10^{-3})}$

$= -101.859 \times 10^{-6}$ m

(a) Total deformation: $\quad \delta_{tot} = \delta_{AB} + \delta_{BC} = -154.9 \times 10^{-6}$ m $= -0.1549$ mm ◄

(b) Deflection of point B. $\quad \delta_B = \delta_{BC} \qquad \delta_B = 0.1019$ mm ↓ ◄

Problem 2.19

2.19 Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

(a) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ \text{m}^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\ \text{m}^2$

Force in member AB is P tension.

Elongation. $\delta_{AB} = \dfrac{P L_{AB}}{E A_{AB}} = \dfrac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})} = 72.756 \times 10^{-6}\ \text{m}$

Force in member BC is Q − P compression.

Shortening. $\delta_{BC} = \dfrac{(Q-P)L_{BC}}{E A_{BC}} = \dfrac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})} = 2.5263\times10^{-9}(Q-P)$

For zero deflection at A, $\delta_{BC} = \delta_{AB}$

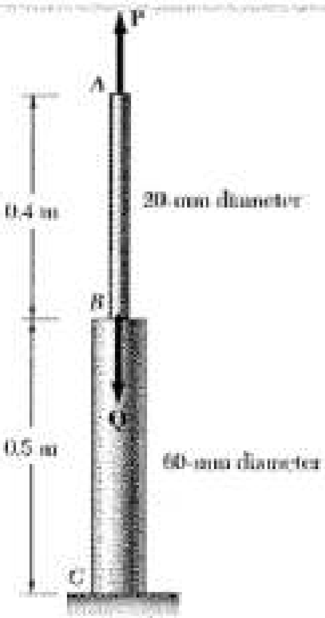
$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6} \quad \therefore \quad Q - P = 28.8\times10^3\ \text{N}$

$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3\ \text{N} = 32.8\ \text{kN}$ ◀

(b) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}\ \text{m} = 0.0728\ \text{mm} \downarrow$ ◀

Problem 2.20

2.20 The rod ABC is made of an aluminum for which $E = 70$ GPa. Knowing that $P = 6$ kN and $Q = 42$ kN, determine the deflection of (*a*) point A, (*b*) point B.



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ m^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\ m^2$$

$$P_{AB} = P = 6 \times 10^3\ N$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3\ N$$

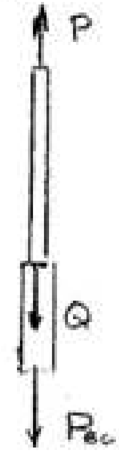
$$L_{AB} = 0.4\ m \qquad L_{BC} = 0.5\ m$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)}$$

$$= 109.135 \times 10^{-6}\ m$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)}$$

$$= -90.947 \times 10^{-6}\ m$$



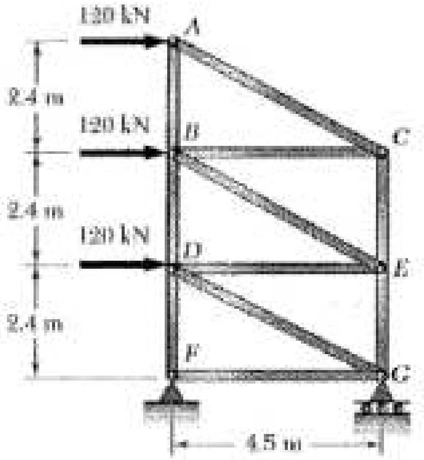
(a) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6}\ m = 18.19 \times 10^{-6}\ m$

$= 0.01819\ mm \uparrow$ ◄

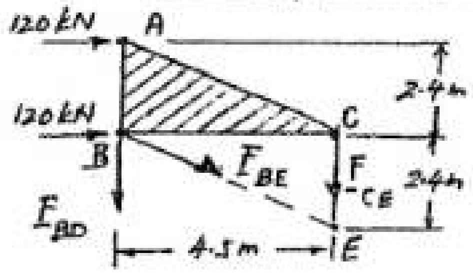
(b) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6}\ m = -0.0909\ mm$ or $0.0919\ mm \downarrow$ ◄

Problem 2.22

2.22 For the steel truss (E = 200 GPa) and loading shown, determine the deformations of the members BD and DE, knowing that their cross-sectional areas are 1250 mm^2 and 1875 mm^2, respectively.



Free Body: Portion ABC of truss

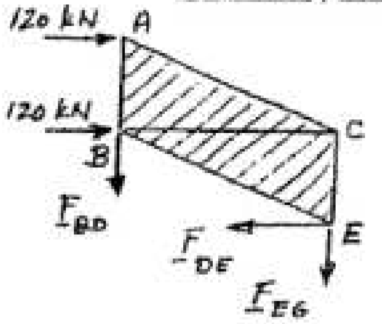


$+\circlearrowleft \Sigma M_E = 0:$

$F_{BD}(4.5 \text{ m}) - (120 \text{ kN})(2.4 \text{ m}) - (120 \text{ kN})(4.8 \text{ m}) = 0$

$F_{BD} = +192 \text{ kN}$

Free Body: Portion ABEC of truss



$\xrightarrow{+} \Sigma F_x = 0:$

$120 \text{ kN} + 120 \text{ kN} - F_{DE} = 0$

$F_{DE} = +240 \text{ kN}$

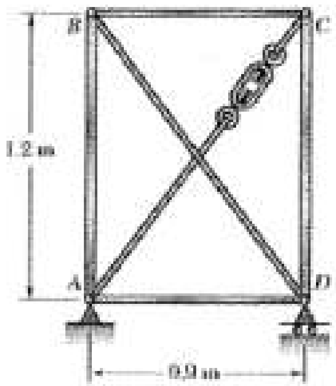
$$\delta_{BD} = \frac{PL}{AE} = \frac{(+192 \times 10^3 \text{ N})(2400 \text{ mm})}{(1250 \text{ mm}^2)(200000 \text{ MPa})}$$

$\delta_{BD} = +1.84 \text{ mm}$ ◀

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+240 \times 10^3 \text{ N})(4500 \text{ mm})}{(1875 \text{ mm}^2)(200000 \text{ MPa})}$$

$\delta_{DE} = +2.88 \text{ mm}$ ◀

Problem 2.23



2.23 Members AB and CD are 30-mm-diameter steel rods, and members BC and AD are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 200$ GPa, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 1.0 mm.

$\delta_{AB} = \delta_{CD} = 1\,mm \qquad h = 1.2\,m \qquad = L_{CD}$

$A_{CD} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.03)^2 = 706.9 \times 10^{-6}\, m^2$

$\delta_{CD} = \frac{F_{CD} L_{CD}}{E A_{CD}}$

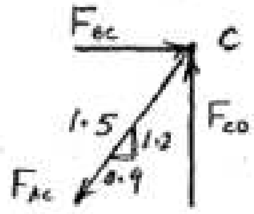
$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(200\times 10^9)(706.9\times 10^{-6})(0.001)}{1.2}$

$= 117.8\ kN$

Use joint C as a free body

$+\uparrow \Sigma F_y = 0: \quad F_{CD} - \frac{1.2}{1.5} F_{AC} = 0 \quad \therefore F_{AC} = \frac{1.5}{1.2} F_{CD}$

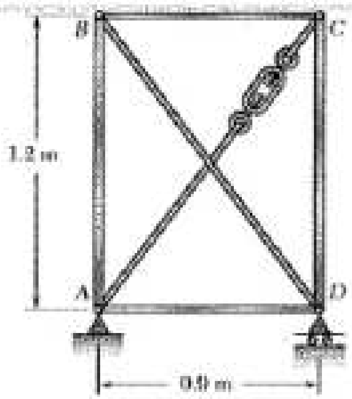
$F_{AC} = \frac{1.5}{1.2}(117.8) = 147.3\,kN.$



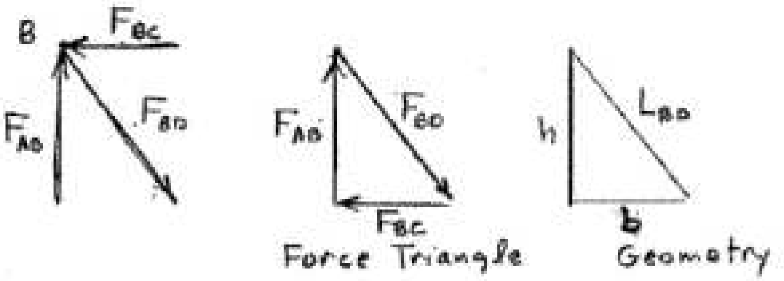
Problem 2.24

2.24 For the structure in Prob. 2.23, determine (*a*) the distance *h* so that the deformations in members *AB*, *BC*, *CD*, and *AD* are equal to 1 mm, (*b*) the corresponding tension in member *AC*.

2.23 Members *AB* and *CD* are 30-mm-diameter steel rods, and members *BC* and *AD* are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member *AC* is put in tension. Knowing that $E = 200$ GPa and $h = 1.2$ m, determine the largest allowable tension in *AC* so that the deformations in members *AB* and *CD* do not exceed 1 mm.



(a) Statics: Use joint B as a free body



From similar triangles

$$\frac{F_{AB}}{h} = \frac{F_{BC}}{b} = \frac{F_{BD}}{L_{BC}}$$

$$F_{AB} = \frac{h}{b} F_{BC}$$

For equal deformations

$$\delta_{AB} = \delta_{BC} \therefore \quad \frac{F_{AB}\, h}{E A_{AB}} = \frac{F_{BC}\, b}{E A_{BC}} \quad \therefore \quad F_{AB} = \frac{b}{h} \cdot \frac{A_{AB}}{A_{BC}} F_{BC}$$

Equating expressions for F_{AB}

$$\frac{h}{b} F_{BC} = \frac{b}{h} \frac{A_{AB}}{A_{BC}} F_{BC} \qquad \frac{h^2}{b^2} = \frac{A_{AB}}{A_{BC}} = \frac{\frac{\pi}{4} d_{AB}^2}{\frac{\pi}{4} d_{BC}^2} = \frac{d_{AB}^2}{d_{BC}^2}$$

$$\frac{h}{b} = \frac{d_{AB}}{d_{BC}} = \frac{30}{22} = \frac{15}{11} \qquad b = 0.9\text{ m}$$

$$h = \frac{15}{11} b = \frac{15}{11}(0.9) = 1.227\text{ m}$$

(b) Setting $\delta_{AB} = \delta_{BC} = 1\text{ mm}$

$$\delta_{BC} = \frac{F_{BC}\, b}{E A_{BC}} \therefore \quad F_{BC} = \frac{E A_{BC} \delta_{BC}}{b} = \frac{(200\times10^9)\frac{\pi}{4}(0.022)^2(0.001)}{0.9}$$

$$= 84.5\text{ kN}$$

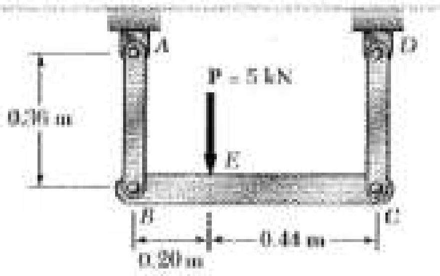
$$F_{AB} = \frac{h}{b} F_{BC} = \frac{15}{11}(84.5) = 115.2\text{ kN}$$

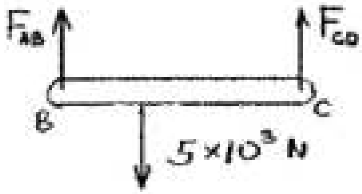
From the force triangle

$$F_{BD} = F_{AC} = \sqrt{F_{BC}^2 + F_{AB}^2} = 142.9\text{ kN}$$

Problem 2.27

2.27 Each of the links *AB* and *CD* is made of aluminum ($E = 75$ GPa) and has a cross-sectional area of 125 mm^2. Knowing that they support the rigid member *BC*, determine the deflection of point *E*.





Use member BC as a free body.

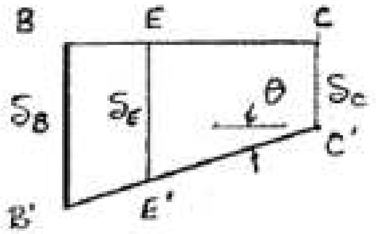
$\circlearrowleft \Sigma M_C = 0: \quad -(0.64)F_{AB} + (0.44)(5\times10^3) = 0 \qquad F_{AB} = 3.4375\times10^3 \text{ N}$

$\circlearrowleft \Sigma M_B = 0: \quad (0.64)F_{CD} - (0.20)(5\times10^3) = 0 \qquad F_{CD} = 1.5625\times10^3 \text{ N}$

For links AB and CD, $\quad A = 125 \text{ mm}^2 = 125\times10^{-6} \text{ m}^2$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA} = \frac{(3.4375\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 132.00\times10^{-6} \text{ m} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA} = \frac{(1.5625\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 60.00\times10^{-6} \text{ m} = \delta_C$$



Deformation diagram.

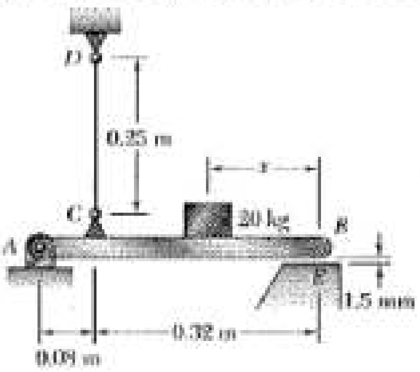
$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{l_{BC}} = \frac{72.00\times10^{-6}}{0.64} = 112.5\times10^{-6} \text{ rad}$$

$$\delta_E = \delta_C + l_{EC}\theta$$

$$= 60.00\times10^{-6} + (0.44)(112.5\times10^{-6})$$

$$= 109.5\times10^{-6} \text{ m} \qquad \delta_E = 0.1095 \text{ mm} \downarrow$$

Problem 2.28



2.28 The length of the 2-mm-diameter steel wire *CD* has been adjusted so that with no load applied, a gap of 1.5 mm exists between the end *B* of the rigid beam *ACB* and a contact point *E*. Knowing that $E = 200$ GPa, determine where a 20-kg block should be placed on the beam in order to cause contact between *B* and *E*.

Rigid rod ACE rotates through angle θ to close gap.

$$\theta = \frac{1.5 \times 10^{-3}}{0.40} = 3.75 \times 10^{-3} \text{ rad}$$

Point C moves downward.

$$\delta_c = 0.08\,\theta = (0.08)(3.75 \times 10^{-3}) = 300 \times 10^{-6} \text{ m}$$

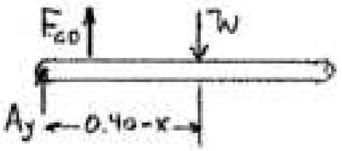
$$\delta_{CD} = \delta_c = 300 \times 10^{-6} \text{ m}$$

$$A_{CD} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2)^2 = 3.1416 \text{ mm}^2 = 3.1416 \times 10^{-6} \text{ m}^2$$

$E = 200 \times 10^9$ Pa $\qquad \delta_{CD} = \dfrac{F_{CD} L_{CD}}{E A_{CD}}$

$$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(200 \times 10^9)(3.1416 \times 10^{-6})(300 \times 10^{-6})}{0.25} = 753.98 \text{ N}$$

Use beam ACB as a free body. $\qquad W = mg = (20)(9.81) = 196.2$ N



$$+\circlearrowleft \Sigma M_A = 0: \quad 0.08\,F_{CD} - (0.40 - x)\,W = 0$$

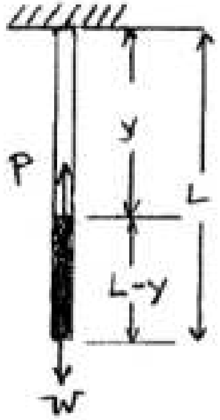
$$0.40 - x = \frac{(0.08)(753.98)}{196.2} = 0.30743 \text{ m}$$

$$x = 0.0926 \text{ m} = 92.6 \text{ mm}$$

For contact, $x < 92.6$ mm ◄

Problem 2.29

2.29 A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.



(a) For element at point identified by coordinate y

P = weight of portion below the point

$= \rho g A(L-y)$

$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A(L-y)\,dy}{EA} = \frac{\rho g(L-y)}{E}\,dy$$

$$\delta = \int_0^L \frac{\rho g(L-y)}{E}\,dy = \frac{\rho g}{E}\left(Ly - \frac{1}{2}y^2\right)\Big|_0^L$$

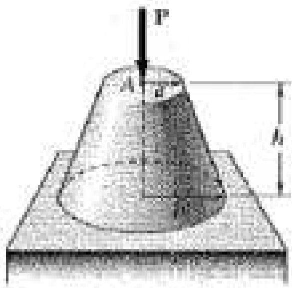
$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right) \qquad \delta = \frac{1}{2}\frac{\rho g L^2}{E}$$

(b) Total weight. $W = \rho g A L$

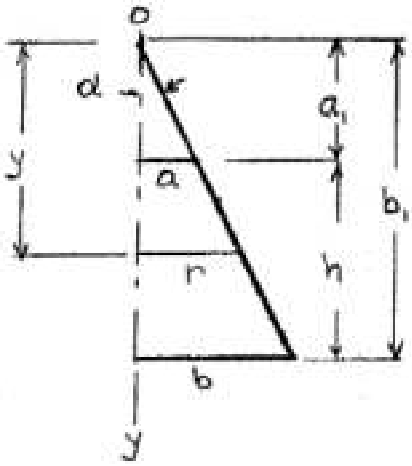
$$F = \frac{EA\delta}{L} = \frac{EA}{L}\cdot\frac{1}{2}\frac{\rho g L^2}{E} = \frac{1}{2}\rho g A L \qquad F = \frac{1}{2}W$$

Problem 2.30

2.30 A vertical load **P** is applied at the center *A* of the upper section of a homogeneous frustum of a circular cone of height *h*, minimum radius *a*, and maximum radius *b*. Denoting by *E* the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point *A*.



Extend the slant sides of the cone to meet at a point O and place the origin of the coordinate system there.



From geometry, $\tan\alpha = \dfrac{b-a}{h}$

$$a_1 = \frac{a}{\tan\alpha}, \quad b_1 = \frac{b}{\tan\alpha}, \quad r = y\tan\alpha$$

At coordinate point y, $A = \pi r^2$

Deformation of element of height dy: $d\delta = \dfrac{P\,dy}{AE}$

$$d\delta = \frac{P}{E\pi}\frac{dy}{r^2} = \frac{P}{\pi E\tan^2\alpha}\frac{dy}{y^2}$$

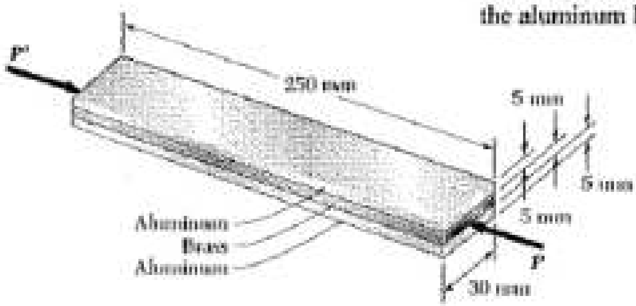
Total deformation.

$$\delta_A = \frac{P}{\pi E\tan^2\alpha}\int_{a_1}^{b_1}\frac{dy}{y^2} = \frac{P}{\pi E\tan^2\alpha}\left(-\frac{1}{y}\right)\Bigg|_{a_1}^{b_1} = \frac{P}{\pi E\tan^2\alpha}\left(\frac{1}{a_1}-\frac{1}{b_1}\right)$$

$$= \frac{P}{\pi E\tan^2\alpha}\frac{b_1-a_1}{a_1 b_1} = \frac{P(b_1-a_1)}{\pi E ab} \qquad \delta_A = \frac{Ph}{\pi Eab}\downarrow$$

Problem 2.33

2.33 A 250-mm bar of 15 × 30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.



For each layer,

$A = (30)(5) = 150 \text{ mm}^2 = 150 \times 10^{-6} \text{ m}^2$

Let P_a = load on each aluminum layer.

P_b = load on brass layer.

Deformation. $\delta = \dfrac{P_a L}{E_a A} = \dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5 P_a$

Total force. $P = 2P_a + P_b = 3.5 P_a$

Solving for P_a and P_b, $\qquad P_a = \frac{2}{7} P \qquad P_b = \frac{3}{7} P$

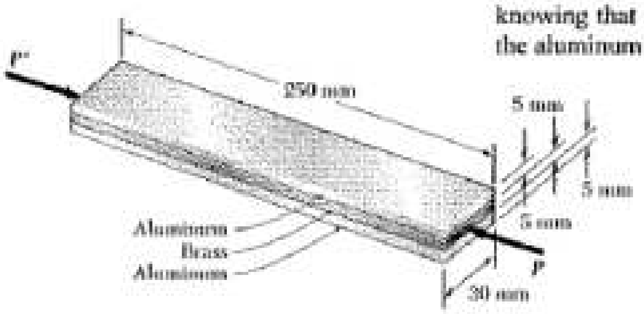
(a) $\sigma_a = -\dfrac{P_a}{A} = -\dfrac{2}{7}\dfrac{P}{A} = -\dfrac{2}{7}\dfrac{30 \times 10^3}{150 \times 10^{-6}} = -57.1 \times 10^6 \text{ Pa}$ $\qquad \sigma_a = -57.1 \text{ MPa}$ ◄

(b) $\sigma_b = -\dfrac{P_b}{A} = -\dfrac{3}{7}\dfrac{P}{A} = -\dfrac{3}{7}\dfrac{30 \times 10^3}{150 \times 10^{-6}} = -85.7 \times 10^6 \text{ Pa}$ $\qquad \sigma_b = -85.7 \text{ MPa}$ ◄

Problem 2.34

2.34 Determine the deformation of the composite bar of Prob. 2.33 if it is subjected to centric forces of magnitude $P = 45$ kN.

2.33 A 250-mm bar of 15 × 30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.



For each layer,

$A = (30)(5) = 150 \text{ mm}^2 = 150 \times 10^{-6} \text{ m}^2$

Let P_a = load on each aluminum layer.

P_b = load on brass layer.

Deformation. $\delta = -\dfrac{P_a L}{E_a A} = -\dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5 P_a$

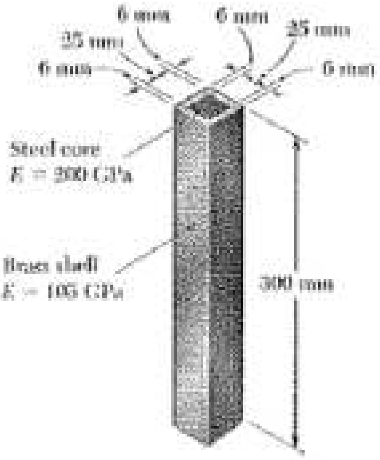
Total force. $P = 2P_a + P_b = 3.5 P_a \qquad P_a = \frac{2}{7} P$

$\delta = -\dfrac{P_a L}{E_a A} = -\dfrac{2}{7}\dfrac{PL}{E_a A} = -\dfrac{2}{7}\dfrac{(45 \times 10^3)(250 \times 10^{-3})}{(70 \times 10^9)(150 \times 10^{-6})} = -306 \times 10^{-6} \text{ m}$

$\delta = -0.306 \text{ mm}$ ◄

Problem 2.36

2.36 The length of the assembly decreases by 0.15 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (*b*) the corresponding stress in the steel core.



Let P_b = portion of axial force carried by brass shell.

P_s = portion of axial force carried by steel core.

$$\delta = \frac{P_b L}{A_b E_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s)\frac{\delta}{L}$$

$$A_s = (0.025)^2 = 625 \times 10^{-6}\ m^2$$

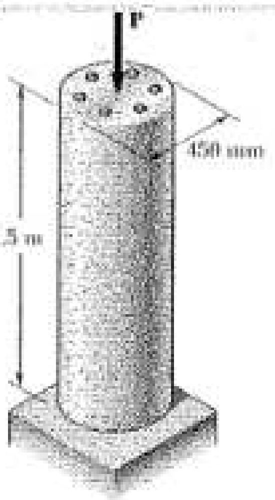
$$A_b = (0.037)^2 - (0.025)^2 = 744 \times 10^{-6}\ m^2$$

(a) $P = [(105 \times 10^9)(744 \times 10^{-6}) + (200 \times 10^9)(625 \times 10^{-6})]\frac{0.00015}{0.3} = 101.6\ kN$

(b) $\sigma_s = E_s \epsilon = E_s \frac{\delta}{L} = (200 \times 10^9)\frac{0.00015}{0.3} = 100\ MPa$

Problem 2.37

2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force **P** is applied to the post.



Let P_c = portion of axial force carried by concrete.
P_s = portion carried by the six steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_c A_c + E_s A_s}$$

$$A_s = 6\frac{\pi}{4}d_s^2 = \frac{6\pi}{4}(28)^2 = 3.6945\times10^3\ \text{mm}^2 = 3.6945\times10^{-3}\ \text{m}^2$$

$$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(450)^2 - 3.6945\times10^3 = 155.349\times10^3\ \text{mm}^2$$
$$= 153.349\times10^{-3}\ \text{m}^2$$

$$L = 1.5\ \text{m}$$

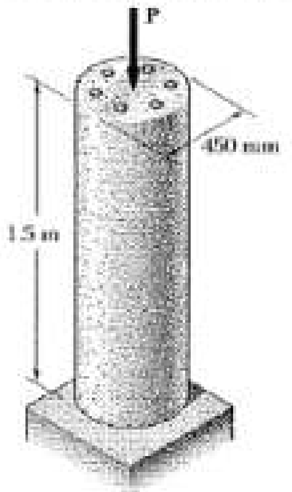
$$\varepsilon = \frac{1550\times10^3}{(25\times10^9)(155.349\times10^{-3}) + (200\times10^9)(3.6945\times10^{-3})} = 335.31\times10^{-6}$$

$$\sigma_s = E_s\varepsilon = (200\times10^9)(-335.31\times10^{-6}) = 67.1\times10^6\ \text{Pa} = 67.1\ \text{MPa} \blacktriangleleft$$

$$\sigma_c = E_c\varepsilon = (25\times10^9)(-335.31\times10^{-6}) = 8.38\times10^6\ \text{Pa} = 8.38\ \text{MPa} \blacktriangleleft$$

Problem 2.38

2.38 For the post of Prob. 2.37, determine the maximum centric force which can be applied if the allowable normal stress is 160 MPa in the steel and 18 MPa in the concrete.



2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force **P** is applied to the post.

Determine allowable strain in each material.

Steel: $\varepsilon_s = \dfrac{\sigma_s}{E_s} = \dfrac{160 \times 10^6}{200 \times 10^9} = 800 \times 10^{-6}$

Concrete: $\varepsilon_c = \dfrac{\sigma_c}{E_c} = \dfrac{18 \times 10^6}{25 \times 10^9} = 720 \times 10^{-6}$

Smaller value governs $\quad \varepsilon = \dfrac{\delta}{L} = 720 \times 10^{-6}$

Let P_c = portion of load carried by concrete.

P_s = portion carried by six steel rods.

$\delta = \dfrac{P_c L}{E_c A_c}, \quad P_c = E_c A_c \dfrac{\delta}{L} = E_c A_c \varepsilon$

$\delta = \dfrac{P_s L}{E_s A_s}, \quad P_s = E_s A_s \dfrac{\delta}{L} = E_s A_s \varepsilon$

$P = P_c + P_s = (E_c A_c + E_s A_s)\varepsilon$

$A_s = 6 \frac{\pi}{4} d_s^2 = \frac{6\pi}{4}(28)^2 = 3.6945 \times 10^3 \text{ mm}^2 = 3.6945 \times 10^{-3} \text{ m}^2$

$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4}(450)^2 - 3.6945 \times 10^3 = 155.349 \times 10^3 \text{ mm}^2$

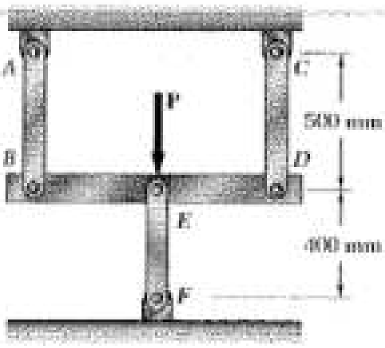
$= 155.349 \times 10^{-3} \text{ m}^2$

$P = [(25 \times 10^9)(155.349 \times 10^{-3}) + (200 \times 10^9)(3.6945 \times 10^{-3})](720 \times 10^{-6})$

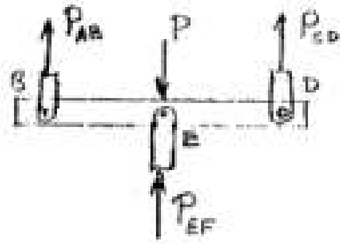
$= 3.33 \times 10^6$ N $\qquad$ 3330 kN ◄

Problem 2.39

2.39 Three steel rods ($E = 200$ GPa) support a 36-kN load **P**. Each of the rods AB and CD has a 200-mm^2 cross-sectional area and rod EF has a 625-mm^2 cross-sectional area. Neglecting the deformation of rod BED, determine (*a*) the change in length of rod EF, (*b*) the stress in each rod.



Use member BED as a free body.



By symmetry, or by $\Sigma M_E = 0$

$P_{CD} = P_{AB}$

$\Sigma F_y = 0:$

$P_{AB} + P_{CD} + P_{EF} - P = 0$

$P = 2P_{AB} + P_{EF}$

$\delta_{AB} = \dfrac{P_{AB}L_{AB}}{EA_{AB}}, \quad \delta_{CD} = \dfrac{P_{CD}L_{CD}}{EA_{CD}}, \quad \delta_{EF} = \dfrac{P_{EF}L_{EF}}{EA_{EF}}$

Since $L_{AB} = L_{CD}$ and $A_{AB} = A_{CD}$, $\delta_{AB} = \delta_{CD}$

Since points A, C, and E are fixed, $\delta_B = \delta_{AB}$, $\delta_D = \delta_{CD}$, $\delta_E = \delta_{EF}$

Since member BED is rigid, $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB}L_{AB}}{EA_{AB}} = \frac{P_{EF}L_{EF}}{EA_{EF}} \quad \therefore \quad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF}$$

$$= 0.256\, P_{EF}$$

$P = 2P_{AB} + P_{EF} = (2)(0.256)P_{EF} + P_{EF} = 1.512\, P_{EF}$

$P_{EF} = \dfrac{P}{1.512} = \dfrac{36 \times 10^3}{1.512} = 23.810 \times 10^3$ N

$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3$ N

(a) $\delta = \delta_{EF} = \dfrac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6}$ m

$= 0.0762$ mm

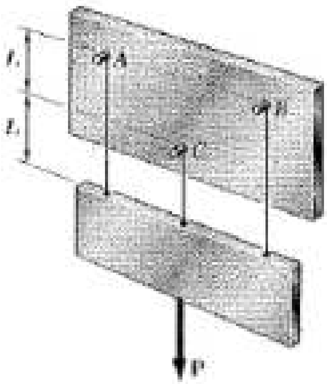
or $\delta = \delta_{AB} = \dfrac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6}$ m

(b) $\sigma_{AB} = \sigma_{CD} = \dfrac{P_{AB}}{A_{AB}} = \dfrac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6$ Pa $= 30.5$ MPa

$\sigma_{EF} = -\dfrac{P_{EF}}{A_{EF}} = -\dfrac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6$ Pa $= 38.1$ MPa

Problem 2.40

2.40 Three wires are used to suspend the plate shown. Aluminum wires are used at A and B with a diameter of 3 mm and a steel wire is used at C with a diameter of 2 mm. Knowing that the allowable stress for aluminum (E = 70 GPa) is 98 MPa and that the allowable stress for steel (E = 200 GPa) is 126 MPa, determine the maximum load **P** that can be applied.



By symmetry $P_A = P_B$, and $\delta_A = \delta_B$

Also, $\delta_C = \delta_A = \delta_B = \delta$

Strain in each wire.

$$\varepsilon_A = \varepsilon_B = \frac{\delta}{2L}, \qquad \varepsilon_C = \frac{\delta}{L} = 2\varepsilon_A$$

Determine allowable strain.

Wires A & B: $\varepsilon_A = \dfrac{\sigma_A}{E_A} = \dfrac{98}{70 \times 10^3} = 1.4 \times 10^{-3}$

$\varepsilon_C = 2\,\varepsilon_A = 4.8 \times 10^{-3}$

Wire C: $\varepsilon_C = \dfrac{\sigma_C}{E_C} = \dfrac{126}{200 \times 10^3} = 0.63 \times 10^{-3}$

$\varepsilon_A = \varepsilon_B = \frac{1}{2}\varepsilon_C = 0.315 \times 10^{-3}$

Allowable strain for wire C governs $\therefore$ $\sigma_C = 126$ MPa

$\sigma_A = E_A \varepsilon_A \qquad P_A = A_A E_A \varepsilon_A = \frac{\pi}{4}(0.003)^2(70 \times 10^9)(0.315 \times 10^{-3})$

$= 155.87$ N

$P_B = 155.87$ N

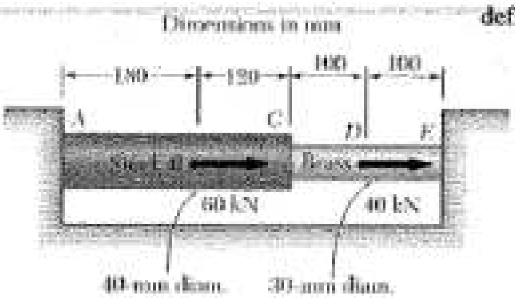
$\sigma_C = E_C \varepsilon_C \qquad P_C = A_C \sigma_C = \frac{\pi}{4}(0.002)^2(126 \times 10^6) = 395.84$ N

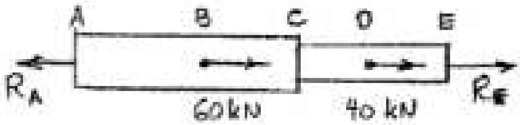
For equilibrium of the plate,

$$P = P_A + P_B + P_C = 707.6 \text{ N}$$

Problem 2.41

2.41 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.





A to C: $E = 200\times10^9$ Pa

$A = \frac{\pi}{4}(40)^2 = 1.25664\times10^3\ mm^2 = 1.25664\times10^{-3}\ m^2$

$EA = 251.327\times10^6$ N

C to E: $E = 105\times10^9$ Pa

$A = \frac{\pi}{4}(30)^2 = 706.86\ mm^2 = 706.86\times10^{-6}\ m^2$

$EA = 74.220\times10^6$ N

A to B: $P = R_A$ $\qquad L = 180\ mm = 0.180\ m$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327\times10^6} = 716.20\times10^{-12} R_A$$

B to C: $P = R_A - 60\times10^3$ $\qquad L = 120\ mm = 0.120\ m$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A-60\times10^3)(0.120)}{251.327\times10^6} = 447.47\times10^{-12} R_A - 26.848\times10^{-6}$$

C to D: $P = R_A - 60\times10^3$ $\qquad L = 100\ mm = 0.100\ m$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A-60\times10^3)(0.100)}{74.220\times10^6} = 1.34735\times10^{-9} R_A - 80.841\times10^{-6}$$

D to E: $P = R_A - 100\times10^3$ $\qquad L = 100\ mm = 0.100\ m$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A-100\times10^3)(0.100)}{74.220\times10^6} = 1.34735\times10^{-9} R_A - 134.735\times10^{-6}$$

A to E: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.85837\times10^{-9} R_A - 242.424\times10^{-6}$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(a) $3.85837\times10^{-9} R_A - 242.424\times10^{-6} = 0$ $\qquad R_A = 62.831\times10^3$ N $\qquad$ 62.8 kN ←

$R_E = R_A - 100\times10^3 = 62.8\times10^3 - 100\times10^3 = -37.2\times10^3$ N $\qquad$ 37.2 kN ←

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367\times10^{-9} R_A - 26.848\times10^{-6}$

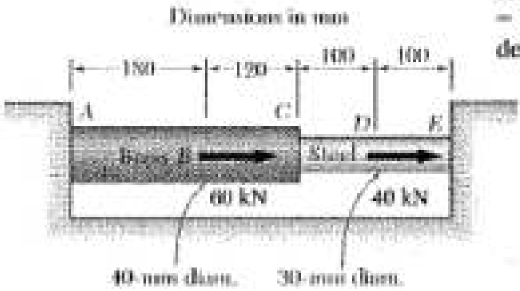
$= (1.16369\times10^{-9})(62.831\times10^3) - 26.848\times10^{-6}$

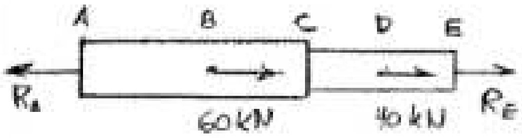
$= 46.3\times10^{-6}$ m $\qquad \delta_C = 46.3\ \mu m$ →

Problem 2.42

2.42 Solve Prob. 2.41, assuming that rod *AC* is made of brass and rod *CE* is made of steel.

2.41 Two cylindrical rods, one of steel and the other of brass, are joined at *C* and restrained by rigid supports at *A* and *E*. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at *A* and *E*, (*b*) the deflection of point *C*.





A to C: $E = 105 \times 10^9$ Pa

$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$

$EA = 131.947 \times 10^6$ N

C to E: $E = 200 \times 10^9$ Pa

$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$

$EA = 141.372 \times 10^6$ N

A to B: $P = R_A$ $\qquad L = 180 \text{ mm} = 0.180 \text{ m}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947 \times 10^6} = 1.36418 \times 10^{-9} R_A$$

B to C: $P = R_A - 60 \times 10^3$ $\qquad L = 120 \text{ mm} = 0.120 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{131.947 \times 10^6} = 909.456 \times 10^{-12} R_A - 54.567 \times 10^{-6}$$

C to D: $P = R_A - 60 \times 10^3$ $\qquad L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 42.441 \times 10^{-6}$$

D to E: $P = R_A - 100 \times 10^3$ $\qquad L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 70.735 \times 10^{-6}$$

A to E: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6}$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(a) $3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6} = 0 \qquad R_A = 45.479 \times 10^3$ N $\qquad$ 45.5 kN ←

$R_E = R_A - 100 \times 10^3 = 45.479 \times 10^3 - 100 \times 10^3 = -54.521 \times 10^3 \qquad$ 54.5 kN ←

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364 \times 10^{-9} R_A - 54.567 \times 10^{-6}$

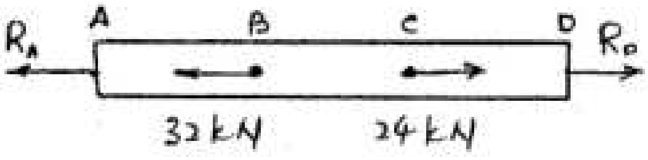
$= (2.27364 \times 10^{-9})(45.479 \times 10^3) - 54.567 \times 10^{-6}$

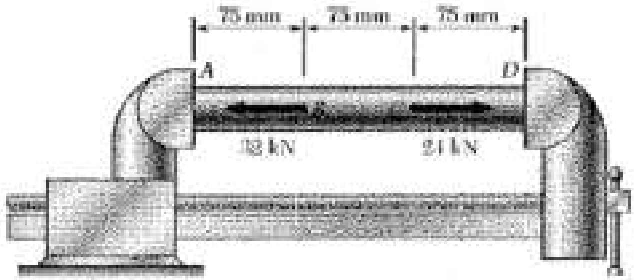
$= 48.8 \times 10^{-6}$ m $\qquad \delta_C = 48.8\ \mu\text{m} \rightarrow$

Problem 2.44

2.44 Solve Prob. 2.43, assuming that after the forces have been applied, the vise is adjusted to increase the distance between its jaws by 0.1 mm.

2.43 A steel tube (E_s = 200 GPa) with a 30-mm outer diameter and a 3-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (*a*) the forces exerted by the vise on the tube at *A* and *D*, (*b*) the change in length of the portion *BC* of the tube.





For the tube $d_o = 30$ mm

$$d_i = d_o - 2t = 0.03 - 2(0.003) = 0.024 \text{ m}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(0.03^2 - 0.024^2) = 254.5 \times 10^{-6} \text{ m}^2$$

A to B: $P = R_A$ $\quad L = 0.075$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.075)}{(200\times10^9)(254.5\times10^{-6})} = 1.473\times10^{-9} R_A \text{ m.}$$

B to C: $P = R_A + 32\text{kN}$, $\quad L = 0.075$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A + 32000)(0.075)}{(200\times10^9)(254.5\times10^{-6})} = 1.473\times10^{-9} R_A + 47.15\times10^{-6} \text{ m}$$

C to D: $P = R_A + 8000$ N $\quad L = 0.075$ m.

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A + 8000)(0.075)}{(200\times10^9)(254.5\times10^{-6})} = 1.473\times10^{-9} R_A + 11.79\times10^{-6} \text{ m}$$

A to D: $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 4.419\times10^{-9} R_A + 58.94\times10^{-6}$

Given jaw movement $\delta_{AD} = -0.0001$ m

(a) $4.419\times10^{-9} R_A + 58.94\times10^{-6} = -0.0001$

$R_A = -35967$ N $\qquad R_A = 35.97$ kN $\rightarrow$

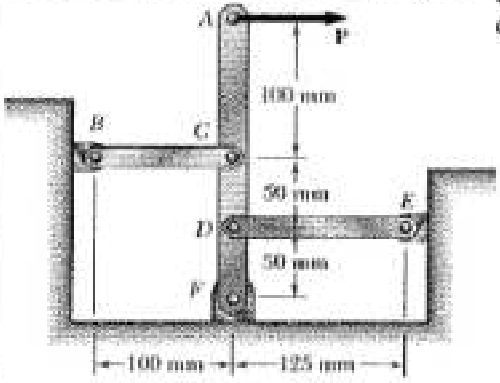
$R_D = R_A + 8000 = -27967$ $\qquad R_D = 27.97$ kN $\leftarrow$

(b) $\delta_{BC} = (1.473\times10^{-9})(-35967) + 47.15\times10^{-6} = -5.83\times10^{-6}$ m

$= -5.83\times10^{-3}$ mm

Problem 2.45

2.45 Links *BC* and *DE* are both made of steel (E = 200 GPa) and are 12 mm wide and 6 mm thick. Determine (*a*) the force in each link when a 2.4-kN force **P** is applied to the rigid member *AF* shown, (*b*) the corresponding deflection of point *A*.



Let the rigid member ACDF rotate through small angle θ clockwise about point F.

Then $\delta_C = \delta_{BC} = 0.1\theta \text{ m} \rightarrow$

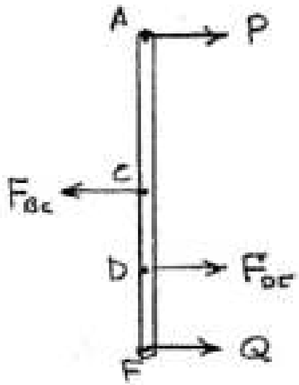
$\delta_D = -\delta_{DE} = 0.05\theta \text{ m} \rightarrow$

$$\delta = \frac{FL}{EA} \quad \text{or} \quad F = \frac{EA\delta}{L}$$

For links: $A = (0.012)(0.006) = 72\times10^{-6}\text{ m}^2$, $L_{BC} = 0.1\text{ m}$, $L_{DE} = 0.125\text{ m}$

$$F_{BC} = \frac{EA\delta_{BC}}{L_{BC}} = \frac{(200\times10^9)(72\times10^{-6})(0.1\theta)}{0.1} = 14.4\times10^6\,\theta$$

$$F_{DE} = \frac{EA\delta_{DE}}{L_{DE}} = \frac{(200\times10^9)(72\times10^{-6})(-0.05\theta)}{0.125} = -5.76\times10^6\,\theta$$



Use member ACDF as a free body.

$$\circlearrowright + \Sigma M_F = 0: \ 0.2P - 0.1F_{BC} + 0.05F_{DE} = 0$$

$$P = \tfrac{1}{2}F_{BC} - \tfrac{1}{4}F_{DE}$$

$$2400 = \tfrac{1}{2}(14.4\times10^6)\theta - \tfrac{1}{4}(-5.76\times10^6)\theta = 8.64\times10^6\,\theta$$

$$\theta = 0.27778\times10^{-3}\text{ rad} \circlearrowright$$

(a) $F_{BC} = (14.4\times10^6)(0.27778\times10^{-3})$ $\qquad F_{BC} = 4\text{ kN}$

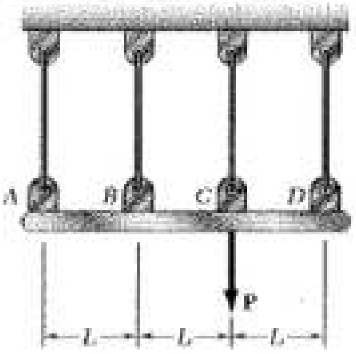
$F_{DE} = -(5.76\times10^6)(0.27778\times10^{-3})$ $\qquad F_{DE} = -1.6\text{ kN}$

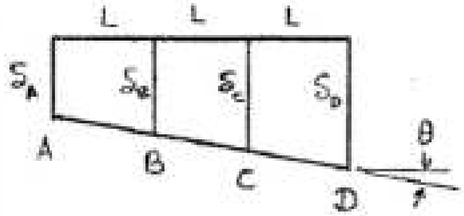
(b) <u>Deflection at point A.</u>

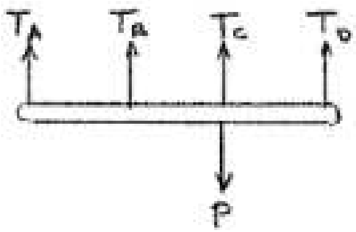
$\delta_A = 0.2\theta = (0.2)(0.27778\times10^{-3})$ $\qquad \delta_A = 0.056\text{ mm} \rightarrow$

Problem 2.46

2.46 The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load **P** shown.







<u>Deformations</u> Let θ be the rotation of bar ABCD and $\delta_A, \delta_B, \delta_C$ and δ_D be the deformations of wires A, B, C, and D.

From geometry, $\theta = \dfrac{\delta_B - \delta_A}{L}$

$$\delta_B = \delta_A + L\theta$$

$$\delta_C = \delta_A + 2L\theta = 2\delta_B - \delta_A \qquad (1)$$

$$\delta_D = \delta_A + 3L\theta = 3\delta_B - 2\delta_A \qquad (2)$$

Since all wires are identical, the forces in the wires are proportional to the deformations.

$$T_C = 2T_B - T_A \qquad (1')$$

$$T_D = 3T_B - 2T_A \qquad (2')$$

Use bar ABCD as a free body.

$$+\circlearrowleft \Sigma M_C = 0: \quad -2LT_A - LT_B + LT_D = 0 \qquad (3)$$

$$+\uparrow \Sigma F_y = 0: \quad T_A + T_B + T_C + T_D - P = 0 \qquad (4)$$

Substituting (2') into (3) and dividing by L,

$$-4T_A + 2T_B = 0 \qquad T_B = 2T_A \qquad (3')$$

Substituting (1'), (2'), and (3') into (4),

$$T_A + 2T_A + 3T_A + 4T_A - P = 0 \qquad 10T_A = P$$

$$T_A = \tfrac{1}{10}P \quad ◄$$

$$T_B = 2T_A = (2)(\tfrac{1}{10})P \qquad T_B = \tfrac{1}{5}P \quad ◄$$

$$T_C = (2)(\tfrac{1}{5}P) - (\tfrac{1}{10}P) \qquad T_C = \tfrac{3}{10}P \quad ◄$$

$$T_D = (3)(\tfrac{1}{5}P) - (2)(\tfrac{1}{10}P) \qquad T_D = \tfrac{2}{5}P \quad ◄$$

Problem 2.48

2.48 Solve Prob. 2.47, assuming that the core is made of steel (E = 200 GPa, α = $11.7 \times 10^{-6}/°C$) instead of brass.

2.47 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15 °C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195 °C.

Aluminum shell
$E = 70$ GPa
$\alpha = 23.6 \times 10^{-6}/°C$

Let L be the length of the assembly.

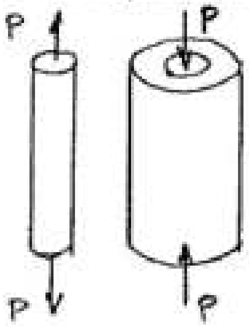
Free thermal expansion.

$\Delta T = 195 - 15 = 180°C$

Steel core: $(\delta_T)_s = L\alpha_s(\Delta T)$

Aluminum shell: $(\delta_T)_a = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core. $\delta = L(\alpha_a - \alpha_s)(\Delta T)$



Let P be the tensile force in the core and the compressive force in the shell.

Steel core: $E_s = 200 \times 10^9$ Pa

$A_s = \frac{\pi}{4}(25)^2 = 490.87\ mm^2 = 490.87 \times 10^{-6}\ m^2$

$(\delta_P)_s = \frac{PL}{E_s A_s}$

Aluminum shell: $E_a = 70 \times 10^9$ Pa $\qquad (\delta_P)_a = \frac{PL}{E_a A_a}$

$A_a = \frac{\pi}{4}(60^2 - 25)^2 = 2.3366 \times 10^3\ mm^2 = 2.3366 \times 10^{-3}\ m^2$

$\delta = (\delta_P)_s + (\delta_P)_a$

$L(\alpha_a - \alpha_s)(\Delta T) = \frac{PL}{E_s A_s} + \frac{PL}{E_a A_a} = KPL$

where $K = \frac{1}{E_s A_s} + \frac{1}{E_a A_a} = \frac{1}{(200 \times 10^9)(490.87 \times 10^{-6})} + \frac{1}{(70 \times 10^9)(2.3366 \times 10^{-3})}$

$= 16.2999 \times 10^{-9}\ N^{-1}$

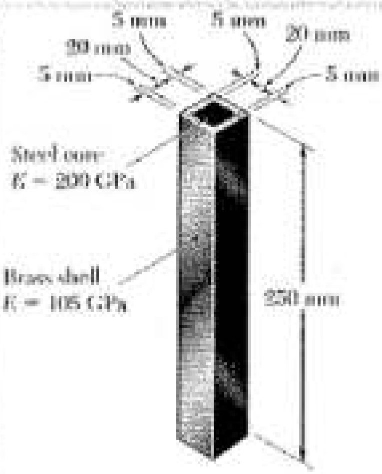
Then $P = \frac{(\alpha_a - \alpha_s)(\Delta T)}{K} = \frac{(23.6 \times 10^{-6} - 11.7 \times 10^{-6})(180)}{16.2999 \times 10^{-9}} = 131.41 \times 10^3$ N

Stress in aluminum. $\sigma_a = -\frac{P}{A_a} = -\frac{131.19 \times 10^3}{2.3366 \times 10^{-3}} = -56.2 \times 10^6$ Pa

$\sigma_a = -56.2$ MPa ◄

Problem 2.50

2.50 The brass shell ($\alpha_b = 20.9 \times 10^{-6}/°C$) is fully bonded to the steel core ($\alpha_s = 11.7 \times 10^{-6}/°F$). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 55 MPa.



Let P_s = axial force developed in the steel core

For equilibrium with zero total force the compressive force in the brass shell is P_s.

Strains $\quad \varepsilon_s = \dfrac{P_s}{E_s A_s} + \alpha_s(\Delta T)$

$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

Matching, $\varepsilon_s = \varepsilon_b$

$$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)(\Delta T)$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6} \text{ m}^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6} \text{ m}^2$$

$$\alpha_b - \alpha_s = 9.2 \times 10^{-6} /°C$$

$$P_s = \sigma_s A_s = (55 \times 10^6)(400 \times 10^{-6}) = 22 \times 10^3 \text{ N}$$

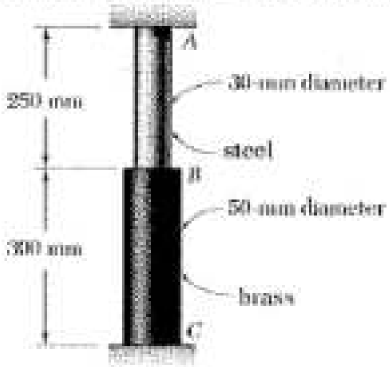
$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(200 \times 10^9)(400 \times 10^{-6})} + \frac{1}{(105 \times 10^9)(500 \times 10^{-6})} = 31.55 \times 10^{-9} \text{ N}^{-1}$$

$$(31.55 \times 10^{-9})(22 \times 10^3) = (9.2 \times 10^{-6})(\Delta T)$$

$$\Delta T = 75.4 \text{ °C} \blacktriangleleft$$

Problem 2.51

2.51 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/^\circ\mathrm{C}$) and portion BC is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/^\circ\mathrm{C}$). Knowing that the rod is initially unstressed, determine the compressive force induced in ABC when there is a temperature rise of 50°C.



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

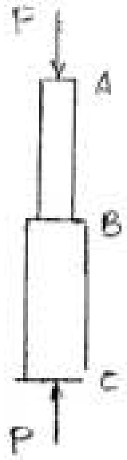
$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

<u>Free thermal expansion.</u>

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$

$$= (0.250)(11.7\times10^{-6})(50) + (0.300)(20.9\times10^{-6})(50)$$

$$= 459.75\times10^{-6} \text{ m}$$



<u>Shortening due to induced compressive force P.</u>

$$\delta_P = \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}}$$

$$= \frac{0.250\,P}{(200\times10^9)(706.86\times10^{-6})} + \frac{0.300\,P}{(105\times10^9)(1.9635\times10^{-3})}$$

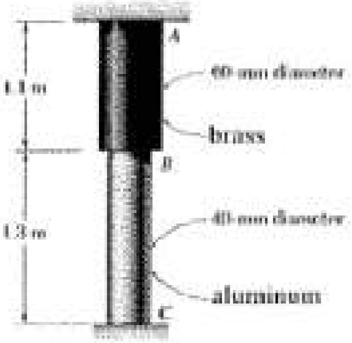
$$= 3.2235\times10^{-9}\,P$$

For zero net deflection, $\delta_P = \delta_T$

$$3.2235\times10^{-9}\,P = 459.75\times10^{-6}$$

$$P = 142.62\times10^3 \text{ N} \qquad P = 142.6 \text{ kN}$$

Problem 2.52



2.52 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of brass (E_b = 105 GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) and portion BC is made of aluminum (E_a = 72 GPa, $\alpha_a = 23.9 \times 10^{-6}/°C$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 42°C, (*b*) the corresponding deflection of point B.

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(60)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

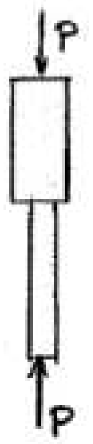
$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(40)^2 = 1.2566 \times 10^3 \text{ mm}^2 = 1.2566 \times 10^{-3} \text{ m}^2$$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_b(\Delta T) + L_{BC}\alpha_a(\Delta T)$$

$$= (1.1)(20.9 \times 10^{-6})(42) + (1.3)(23.9 \times 10^{-6})(42)$$

$$= 2.2705 \times 10^{-3} \text{ m}$$



Shortening due to induced compressive force

$$\delta_P = \frac{PL_{AB}}{E_b A_{AB}} + \frac{PL_{BC}}{E_a A_{BC}}$$

$$= \frac{1.1\ P}{(105 \times 10^9)(2.8274 \times 10^{-3})} + \frac{1.3\ P}{(72 \times 10^9)(1.2566 \times 10^{-3})}$$

$$= 18.074 \times 10^{-9}\ P$$

For zero net deflection $\quad \delta_P = \delta_T$

$$18.074 \times 10^{-9}\ P = 2.2705 \times 10^{-3}$$

$$P = 125.62 \times 10^3 \text{ N}$$

(a) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{125.62 \times 10^3}{2.8274 \times 10^{-3}} = -44.4 \times 10^6 \text{ Pa} = -44.4 \text{ MPa}$$ ◄

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{125.62 \times 10^3}{1.2566 \times 10^{-3}} = -100.0 \times 10^6 \text{ Pa} = -100.0 \text{ MPa}$$ ◄

(b) $$\delta_B = +\frac{PL_{AB}}{E_b A_{AB}} - L_{AB}\alpha_b(\Delta T)$$

$$= \frac{(125.62 \times 10^3)(1.1)}{(105 \times 10^9)(2.8274 \times 10^{-3})} - (1.1)(20.9 \times 10^{-6})(42)$$

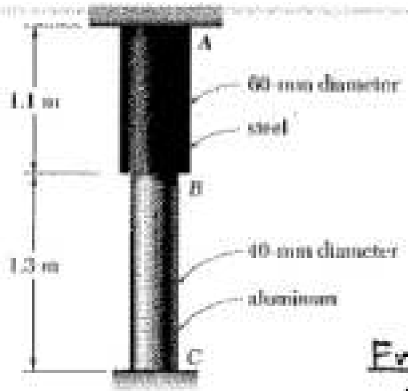
$$= -500 \times 10^{-6} \text{ m} = -0.500 \text{ mm}$$

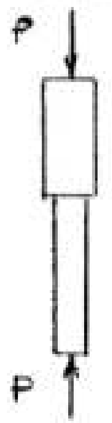
i.e. 0.500 mm ↓ ◄

Problem 2.53

2.53 Solve Prob. 2.52, assuming that portion AB of the composite rod is made of steel and portion BC is made of brass.

2.52 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of brass (E_b = 105 GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) and portion BC is made of aluminum (E_a = 72 GPa, $\alpha_a = 23.9 \times 10^{-6}/°C$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 42°C, (*b*) the corresponding deflection of point B.





$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.06)^2 = 2.8274 \times 10^{-6}\ m^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.04)^2 = 1.2566 \times 10^{-6}\ m^2$

Free thermal expansion. $\Delta T = 42°$

$(\delta_T)_{AB} = L_{AB}\,\alpha_s(\Delta T) = (1.1)(11.7 \times 10^{-6})(42) = 0.541 \times 10^{-3}\ m$

$(\delta_T)_{BC} = L_{BC}\,\alpha_b(\Delta T) = (1.3)(23.9 \times 10^{-6})(42) = 1.305 \times 10^{-3}\ m$

Total $\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 1.846 \times 10^{-3}\ m$

Shortening due to induced compressive force P.

$(\delta_P)_{AB} = \frac{P L_{AB}}{E_s A_{AB}} = \frac{1.1\,P}{(200 \times 10^9)(2.8274 \times 10^{-6})} = 1945.25 \times 10^{-9}\ P$

$(\delta_P)_{BC} = \frac{P L_{BC}}{E_b A_{BC}} = \frac{1.3\,P}{(72 \times 10^9)(1.2566 \times 10^{-6})} = 14368.58 \times 10^{-9}\ P$

Total $\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 16313.8 \times 10^{-9}\ P$

For zero net deflection $\delta_P = \delta_T$

$16313.8 \times 10^{-9} P = 1.846 \times 10^{-3}$ $\qquad P = 113.16\ N$

(a) $\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{113.16}{2.8274 \times 10^{-6}} = -40.02\ MPa$ $\qquad \sigma_{AB} = -40\ MPa$ ◄

$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{113.16}{1.2566 \times 10^{-6}} = -90.05\ MPa$ $\qquad \sigma_{BC} = -90.1\ MPa$ ◄

(b) $(\delta_P)_{AB} = (1945.25 \times 10^{-9})(113.16) = 0.22 \times 10^{-3}\ m$

$\delta_B = (\delta_T)_{AB}\downarrow + (\delta_P)_{AB}\uparrow = 0.541 \times 10^{-3}\downarrow + 0.22 \times 10^{-3}\uparrow$

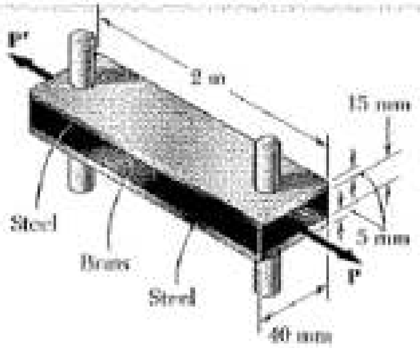
$\delta_B = 0.32\ mm \downarrow$ ◄

or $(\delta_P)_{BC} = (14368.58 \times 10^{-9})(113.16) = 1.626 \times 10^{-3}\ m$

$\delta_B = (\delta_T)_{BC}\uparrow + (\delta_P)_{BC}\downarrow = 1.305 \times 10^{-3}\uparrow + 1.626 \times 10^{-3}\downarrow$

$\delta_B = 0.32\ mm \downarrow$ (checks)

Problem 2.55



2.55 Two steel bars (E_s = 200 GPa and α_s = 11.7 × 10^{-6}/°C) are used to reinforce a brass bar (E_b = 105 GPa, α_b = 20.9 × 10^{-6}/°C) which is subjected to a load P = 25 kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

(a) Required temperature change for fabrication.

$$\delta_T = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

Temperature change required to expand steel bar by this amount

$$\delta_T = L\alpha_s \Delta T, \quad 0.5\times10^{-3} = (2.00)(11.7\times10^{-6})(\Delta T), \quad \Delta T =$$

$$0.5\times10^{-3} = (2)(11.7\times10^{-6})(\Delta T)$$

$$\Delta T = 21.368\ ^\circ\text{C} \qquad 21.4^\circ\text{C}$$

(b) Once assembled, a tensile force P^* develops in the steel and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400\times10^{-6} \text{ m}^2$

$$(\delta_P)_s = \frac{P^*L}{A_sE_s} = \frac{P^*(2.00)}{(400\times10^{-6})(200\times10^{9})} = 25\times10^{-9}\,P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600\times10^{-6} \text{ m}^2$

$$(\delta_P)_b = \frac{P^*L}{A_bE_b} = \frac{P^*(2.00)}{(600\times10^{-6})(105\times10^{9})} = 31.746\times10^{-9}\,P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit

$$(\delta_P)_s + (\delta_P)_b = 0.5\times10^{-3}, \quad 56.746\times10^{-9}\,P^* = 0.5\times10^{-3}$$

$$P^* = 8.811\times10^{3} \text{ N}$$

Stresses due to fabrication.

Steel: $$\sigma_s^* = \frac{P^*}{A_s} = \frac{8.811\times10^{3}}{400\times10^{-6}} = 22.03\times10^{6} \text{ Pa} = 22.03 \text{ MPa}$$

Brass: $$\sigma_b^* = -\frac{P^*}{A_b} = -\frac{8.811\times10^{3}}{600\times10^{-6}} = -14.68\times10^{6} \text{ Pa} = -14.68 \text{ MPa}$$

To these stresses must be added the stresses due to the 25 kN load.

continued

Problem 2.55 continued

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^{9})}{2.00}\delta' = 40\times10^{6}\,\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^{9})}{2.00}\delta' = 31.5\times10^{6}\,\delta'$$

Total $P = P_s + P_b = 25\times10^{3}$ N

$$40\times10^{6}\,\delta' + 31.5\times10^{6}\,\delta' = 25\times10^{3} \qquad \delta' = 349.65\times10^{-6}\text{ m}$$

$$P_s = (40\times10^{6})(349.65\times10^{-6}) = 13.986\times10^{3}\text{ N}$$

$$P_b = (31.5\times10^{6})(349.65\times10^{-6}) = 11.140\times10^{3}\text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986\times10^{3}}{400\times10^{-6}} = 34.97\times10^{6}\text{ Pa}$$

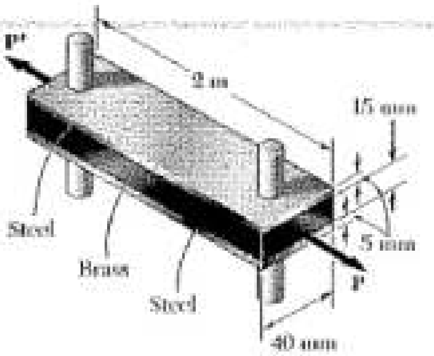
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140\times10^{3}}{600\times10^{-6}} = 18.36\times10^{6}\text{ Pa}$$

Add stress due to fabrication. <u>Total stresses.</u>

$$\sigma_s = 34.97\times10^{6} + 22.03\times10^{6} = 57.0\times10^{6}\text{ Pa} = 57.0\text{ MPa}$$

$$\sigma_b = 18.36\times10^{6} - 14.68\times10^{6} = 3.68\times10^{6}\text{ Pa} = 3.68\text{ MPa}$$

Problem 2.56



2.56 Determine the maximum load P that may be applied to the brass bar of Prob. 2.55 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

2.55 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) which is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

See solution to Problem 2.55 to obtain the fabrication stresses.

$\sigma_s^* = 22.03$ MPa $\qquad \sigma_b^* = -14.68$ MPa

Allowable stresses: $\sigma_{s,all} = 30$ MPa, $\sigma_{b,all} = 25$ MPa

Available stress increase from load.

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$
$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains.

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97\times10^6}{200\times10^9} = 39.85\times10^{-6}$$
$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68\times10^6}{105\times10^9} = 377.9\times10^{-6}$$

Smaller value governs $\therefore \quad \varepsilon = 39.85\times10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400\times10^{-6} \text{ m}^2$
$A_b = (15)(40) = 600 \text{ mm}^2 = 600\times10^{-6} \text{ m}^2$

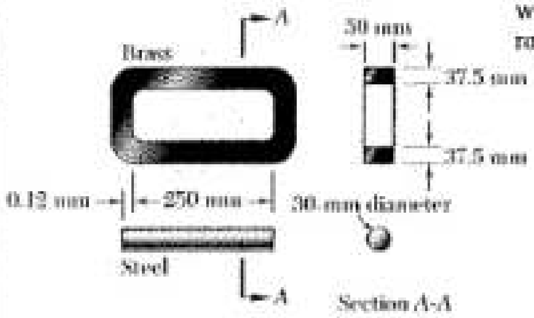
$$P_s = E_s A_s \varepsilon = (200\times10^9)(400\times10^{-6})(39.85\times10^{-6}) = 3.188\times10^3 \text{ N}$$
$$P_b = E_b A_b \varepsilon = (105\times10^9)(600\times10^{-6})(39.85\times10^{-6}) = 2.511\times10^3 \text{ N}$$

<u>Total allowable additional force.</u>

$$P = P_s + P_b = 3.188\times10^3 + 2.511\times10^3 = 5.70\times10^3 \text{ N}$$
$$= 5.70 \text{ kN}$$

Problem 2.57

2.57 An brass link (E_b = 105 GPa, α_b = 20.9 × 10^{-6}/°C) and a steel rod (E_s = 200 GPa, α_s = 11.7 × 10^{-6}/°C) have the dimensions shown at a temperature of 20°C. The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to 45°C. Determine (*a*) the final stress in the steel rod, (*b*) the final length of the steel rod.



Initial dimensions at T = 20°C.

Final dimensions at T = 45°C.

$\Delta T = 45 - 20 = 25°C$

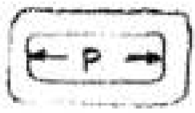
Free thermal expansion of each part.

Brass link: $(\delta_T)_b = \alpha_b(\Delta T)L' = (20.9\times10^{-6})(25)(0.250) = 130.625\times10^{-6}$ m

Steel rod: $(\delta_T)_s = \alpha_s(\Delta T)L = (11.7\times10^{-6})(25)(0.250) = 73.125\times10^{-6}$ m

At the final temperature the difference between the free length of the steel rod and the brass link is

$\delta = 120\times10^{-6} + 73.125\times10^{-6} - 130.625\times10^{-6} = 62.5\times10^{-6}$ m



Add equal but opposite forces P to elongate the brass link and contract the steel rod.

Brass link: $E = 105\times10^9$ Pa

$A_b = (2)(50)(37.5) = 3750\ \text{mm}^2 = 3.750\times10^{-3}\ \text{m}^2$

$(\delta_P) = \dfrac{PL}{EA} = \dfrac{P(0.250)}{(105\times10^9)(3.750\times10^{-3})} = 634.92\times10^{-12}P$

Steel rod: $E = 200\times10^9$ Pa $\qquad A_s = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^3 = 706.86\times10^{-6}\ \text{m}^2$

$(\delta_P)_s = \dfrac{PL}{E_sA_s} = \dfrac{P(0.250)}{(200\times10^9)(706.86\times10^{-6})} = 1.76838\times10^{-9}P$

$(\delta_P)_b + (\delta_P)_s = \delta \qquad 2.4033\times10^{-9}P = 62.5\times10^{-6}$

$P = 26.006\times10^3$ N

(a) Stress in steel rod. $\sigma_s = -\dfrac{P}{A_s} = -\dfrac{(26.006\times10^3)}{706.86\times10^{-6}}$

$= -36.8\times10^6$ Pa $\qquad \sigma_s = -36.8$ MPa ◄

(b) Final length of steel rod. $L_f = L_o + (\delta_T)_s - (\delta_P)_s$

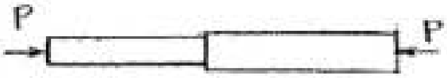
$L_f = 0.250 + 120\times10^{-6} + 73.125\times10^{-6} - (1.76838\times10^{-9})(26.003\times10^3)$

$= 0.250147$ m $\qquad L_f = 250.147$ mm ◄

Problem 2.58

2.58 Knowing that a 0.5-mm gap exists when the temperature is 24°C, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to −75 MPa, (*b*) the corresponding exact length of the aluminum bar.

Bronze	Aluminum
$A = 1500\ mm^2$	$A = 1800\ mm^2$
$E = 105$ GPa	$E = 73$ GPa
$\alpha = 21.6 \times 10^{-6}/°C$	$\alpha = 23.2 \times 10^{-6}/°C$



$\sigma_a = -75$ MPa

$P = -\sigma_a A_a = -(75\times10^{6})(1800\times10^{-6}) = -135$ kN

Shortening due to P

$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$

$$= \frac{(135000)(0.35)}{(105\times10^{9})(1500\times10^{-6})} + \frac{(135000)(0.45)}{(73\times10^{9})(1800\times10^{-6})}$$

$$= 762.3\times10^{-6}\ m.$$

Available elongation for thermal expansion

$$\delta_T = (0.0005 + 762.3\times10^{-6}) = 1.2623\times10^{-3}\ m$$

But $\delta_T = L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)$

$$= (0.35)(21.6\times10^{-6})(\Delta T) + (0.45)(23.2\times10^{-6})(\Delta T) = (18\times10^{-6})\Delta T$$

Equating $(18\times10^{-6})\Delta T = 1.2623\times10^{-3}$ $\quad \Delta T = 70.1°C$

(a) $T_{hot} = T_{cold} + \Delta T = 24 + 70.1 = 94.1°C$

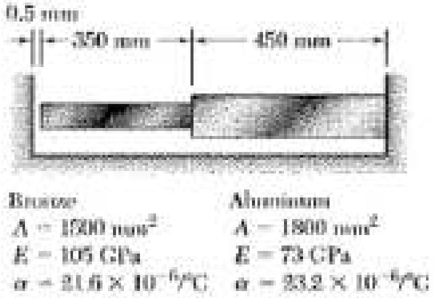
(b) $\delta_a = L_a\alpha_a(\Delta T) - \frac{PL_a}{E_a A_a}$

$$= (0.45)(23.2\times10^{-6})(70.1) - \frac{(135000)(0.45)}{(73\times10^{9})(1800\times10^{-6})} = 0.27\times10^{-3}\ m.$$

$L_{exact} = 0.45 + 0.27\times10^{-3} = 0.45027$ m

Problem 2.59

2.59 Determine (*a*) the compressive force in the bars shown after a temperature rise of 82°C, (*b*) the corresponding change in length of the bronze bar.

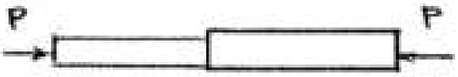


Thermal expansion if free of constraint

$$\delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$$

$$= (0.35)(21.6\times10^{-6})(82) + (0.45)(23.2\times10^{-6})(82)$$

$$= 1.476\times10^{-3} \text{ m}$$



Constrained expansion. $\delta = 0.5$ mm

Shortening due to induced compressive force P

$$\delta_P = 1.476\times10^{-3} - 0.0005 = 0.976\times10^{-3} \text{ m}$$

But $\delta_P = \dfrac{PL_b}{E_b A_b} + \dfrac{PL_a}{E_a A_a} = \left(\dfrac{L_b}{E_b A_b} + \dfrac{L_a}{E_a A_a}\right)P$

$$= \left(\frac{0.35}{(105\times10^9)(1500\times10^{-6})} + \frac{0.45}{(73\times10^9)(1800\times10^{-6})}\right)P = 5.647\times10^{-9}.$$

(a) Equating $5.647\times10^{-9} P = 0.976\times10^{-3}$ $\qquad P = 172835$ N

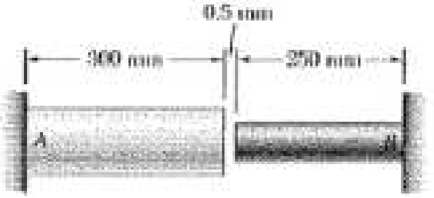
172.8 kN ◄

(b) $\delta_b = L_b \alpha_b (\Delta T) - \dfrac{PL_b}{E_b A_b}$

$$= (0.35)(21.6\times10^{-6})(82) - \frac{(172835)(0.35)}{(105\times10^9)(1500\times10^{-6})} = 0.2358\times10^{-3} \text{ m} = 0.236 \text{ mm}$$ ◄

Problem 2.60

2.60 At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.



Aluminum	Stainless steel
$A = 2000\ \text{mm}^2$	$A = 800\ \text{mm}^2$
$E = 75\ \text{GPa}$	$E = 190\ \text{GPa}$
$\alpha = 23 \times 10^{-6}/°\text{C}$	$\alpha = 17.3 \times 10^{-6}/°\text{C}$

$\Delta T = 140 - 20 = 120\,°\text{C}$

Free thermal expansion.

$$\delta_T = L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)$$
$$= (0.300)(23\times10^{-6})(120) + (0.250)(17.3\times10^{-6})(120)$$
$$= 1.347\times10^{-3}\ \text{m}$$

Shortening due to P to meet constraint.

$$\delta_P = 1.347\times10^{-3} - 0.5\times10^{-3} = 0.847\times10^{-3}\ \text{m}$$

$$\delta_P = \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P$$
$$= \left(\frac{0.300}{(75\times10^9)(2000\times10^{-6})} + \frac{0.250}{(190\times10^9)(800\times10^{-6})}\right)P = 3.6447\times10^{-9}\,P$$

Equating; $3.6447\times10^{-9}\,P = 0.847\times10^{-3}$ $\qquad P = 232.39\times10^3\ \text{N}$

(a) $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{232.39\times10^3}{2000\times10^{-6}} = -116.2\times10^6\ \text{Pa}$ $\qquad -116.2\ \text{MPa}$ ◀

(b) $\delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_aA_a}$

$$= (0.300)(23\times10^{-6})(120) - \frac{(232.39\times10^3)(0.300)}{(75\times10^9)(2000\times10^{-6})} = 363\times10^{-6}\ \text{m}$$

0.363 mm ◀

Problem 2.61

2.61 In a standard tensile test, an aluminum rod of 20-mm diameter is subjected to a tension force of $P = 30$ kN. Knowing that $\nu = 0.35$ and $E = 70$ GPa, determine (*a*) the elongation of the rod in an 150-mm gage length, (*b*) the change in diameter of the rod.

$P = 30 \times 10^3$ N

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(20)^2 = 314.159 \text{ mm}^2 = 314.159 \times 10^{-6} \text{ m}^2$

$\sigma_y = \frac{P}{A} = \frac{30 \times 10^3}{314.159 \times 10^{-6}} = 95.493 \times 10^6$ Pa

$\varepsilon_y = \frac{\sigma_y}{E} = \frac{95.493 \times 10^6}{70 \times 10^9} = 1.36418 \times 10^{-3}$

(a) $\delta_y = L\varepsilon_y = (150 \times 10^{-3})(1.36418 \times 10^{-3}) = 205 \times 10^{-6}$ m

0.205 mm

$\varepsilon_x = -\nu\varepsilon_y = -(0.35)(1.36418 \times 10^{-3}) = -477.46 \times 10^{-6}$

(b) $\delta_x = d\,\varepsilon_x = (20 \times 10^{-3})(-477.46 \times 10^{-6}) = -9.55 \times 10^{-6}$ m

-0.00955 mm

Problem 2.62

2.62 A 20-mm-diameter rod made of an experimental plastic is subjected to a tensile force of magnitude $P = 6$ kN. Knowing that an elongation of 14 mm and a decrease in diameter of 0.85 mm are observed in a 150-mm length, determine the modulus of elasticity, the modulus of rigidity and Poisson's ratio for the material.

Let the y-axis be along the length of the rod and the x-axis be transverse.

$A = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$ $\qquad P = 6 \times 10^3$ N

$\sigma_y = \frac{P}{A} = \frac{6 \times 10^3}{314.16 \times 10^{-6}} = 19.0985 \times 10^6$ Pa

$\varepsilon_y = \frac{\delta_y}{L} = \frac{14 \text{ mm}}{150 \text{ mm}} = 0.093333$

Modulus of elasticity: $E = \frac{\sigma_y}{\varepsilon_y} = \frac{19.0985 \times 10^6}{0.093333} = 204.63 \times 10^6$ Pa

$E = 205$ MPa

$\varepsilon_x = \frac{\delta_x}{d} = -\frac{0.85}{20} = -0.0425$

Poisson's ratio: $\nu = -\frac{\varepsilon_x}{\varepsilon_y} = -\frac{-0.0425}{0.093333}$ $\qquad \nu = 0.455$

Modulus of rigidity: $G = \frac{E}{2(1+\nu)} = \frac{204.63 \times 10^6}{(2)(1.455)} = 70.31 \times 10^6$ Pa

$G = 70.3$ MPa

Probem 2.64

2.64 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column and carries a centric axial load of 640 kN. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

$d_o = 240$ mm $\quad t = 10$ mm $\quad d_i = d_o - 2t = 220$ mm

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^3$ mm^2

$= 7.2257 \times 10^{-3}$ m^2

$P = 640 \times 10^3$ N

(a) $\delta = -\frac{PL}{AE} = -\frac{(640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3}$ m

$= -2.43$ mm

$\varepsilon = \frac{\delta}{L} = -\frac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$

$\varepsilon_{lat} = -\nu\varepsilon = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$

(b) $\Delta d_o = d_o \varepsilon_{lat} = (240)(400.4 \times 10^{-6}) = 0.0961$ mm

(c) $\Delta t = t \varepsilon_{lat} = (10)(400.4 \times 10^{-6}) = 0.00400$ mm

Problem 2.65

2.65 The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine the internal force in the bolt, if the diameter is observed to decrease by 13 μm.

$$\delta_y = -13 \times 10^{-6} \text{ m} \qquad d = 60 \times 10^{-3} \text{ m}$$

$$\varepsilon_y = \frac{\delta_y}{d} = -\frac{13 \times 10^{-6}}{60 \times 10^{-3}} = -216.67 \times 10^{-6}$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} \quad \therefore \quad \varepsilon_x = -\frac{\varepsilon_y}{\nu} = \frac{216.67 \times 10^{-6}}{0.29} = 747.13 \times 10^{-6}$$

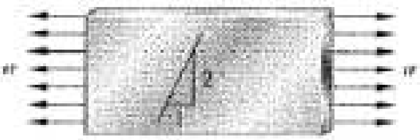
$$\sigma_x = E\varepsilon_x = (200 \times 10^9)(747.13 \times 10^{-6}) = 149.43 \times 10^6 \text{ Pa}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(60)^2 = 2.827 \times 10^3 \text{ mm}^2 = 2.827 \times 10^{-3} \text{ m}^2$$

$$F = \sigma_x A = (149.43 \times 10^6)(2.827 \times 10^{-3}) = 422 \times 10^3 \text{ N}$$

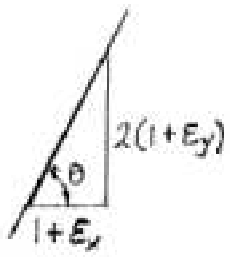
$$= 422 \text{ kN}$$

Problem 2.66



2.66 An aluminum plate ($E = 74$ GPa and $\nu = 0.33$) is subjected to a centric axial load that causes a normal stress σ. Knowing that, before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when $\sigma = 125$ MPa.

The slope after deformation is $\tan\theta = \dfrac{2(1+\varepsilon_y)}{1+\varepsilon_x}$

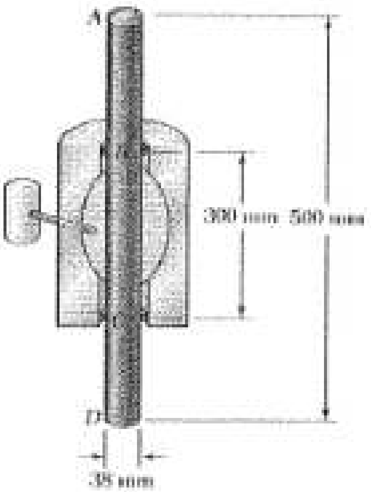


$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{125 \times 10^6}{74 \times 10^9} = 1.6892 \times 10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.33)(1.6892 \times 10^{-3}) = 0.5574 \times 10^{-3}$$

$$\tan\theta = \frac{2(1 - 0.0005574)}{1 + 0.0016892} = 1.99551$$

Problem 2.68



2.68 For the rod of Prob. 2.67, determine the forces that should be applied to the ends *A* and *D* of the rod (*a*) if the axial strain in portion *BC* of the rod is to remain zero as the hydrostatic pressure is applied, (*b*) if the total length *AD* of the rod is to remain unchanged.

2.67 The aluminum rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 42 MPa to the 300-mm portion *BC* of the rod. Knowing that $E = 70$ GPa and $\nu = 0.36$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

Over the pressurized portion BC

$$\sigma_x = \sigma_z = -p \qquad \sigma_y = \sigma_y$$

$$(\varepsilon_y)_{BC} = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}(2\nu p + \sigma_y)$$

(a) $(\varepsilon_y)_{BC} = 0 \qquad 2\nu p + \sigma_y = 0$

$$\sigma_y = 2\nu p = -(2)(0.36)(42\times10^6)$$

$$= -30.24 \text{ MPa}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.038)^2 = 1.134\times10^{-3}\ m^2$$

$$F = A\sigma_y = (1.134\times10^{-3})(-30.24\times10^6) = -34292 \text{ N}$$

i.e. 34.3 kN compression

(b) Over unpressurized portions AB and CD $\qquad \sigma_x = \sigma_z = 0$

$$(\varepsilon_y)_{AB} = (\varepsilon_y)_{CD} = \frac{\sigma_y}{E}$$

For no change in length

$$\delta = L_{AB}(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} + L_{CD}(\varepsilon_y)_{CD} = 0$$

$$(L_{AB} + L_{CD})(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} = 0$$

$$(0.5-0.3)\frac{\sigma_y}{E} + \frac{0.3}{E}(2\nu p + \sigma_y) = 0$$

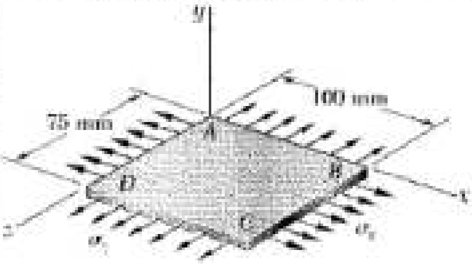
$$\sigma_y = -\frac{0.6\nu p}{0.1} = -\frac{(0.6)(0.36)(42\times10^6)}{0.1} = -90.72 \text{ MPa}$$

$$P = A\sigma_y = (1.134\times10^{-3})(-90.72\times10^6) = -102876 \text{ N}$$

i.e. 102.9 kN compression

Problem 2.69

2.69 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.



$$\sigma_x = 120 \times 10^6 \text{ Pa}, \quad \sigma_y = 0, \quad \sigma_z = 160 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{87 \times 10^9}\left[120 \times 10^6 - (0.34)(160 \times 10^6)\right]$$

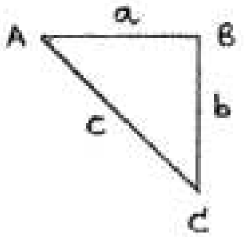
$$= 754.02 \times 10^{-6}$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{87 \times 10^9}\left[-(0.34)(120 \times 10^6) + 160 \times 10^6\right]$$

$$= 1.3701 \times 10^{-3}$$

(a) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100 \text{ mm})(754.02 \times 10^{-6}) = 0.0754 \text{ mm}$

(b) $\delta_{BC} = (\overline{BC})\varepsilon_z = (.75 \text{ mm})(1.3701 \times 10^{-6}) = 0.1028 \text{ mm}$



Label sides of right triangle ABC as a, b, and c.

$$c^2 = a^2 + b^2$$

Obtain differentials by calculus.

$$2c\,dc = 2a\,da + 2b\,db$$

$$dc = \frac{a}{c}\,da + \frac{b}{c}\,db$$

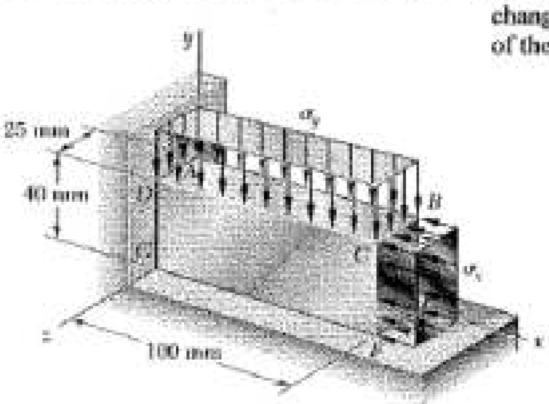
But, $a = 100$ mm, $b = 100$ mm, $c = \sqrt{100^2 + 75^2)} = 125$ mm

$da = \delta_{AB} = 0.0754$ mm $\qquad db = \delta_{BC} = 0.1370$ mm

(c) $\delta_{AC} = dc = \frac{100}{125}(0.0754) + \frac{75}{125}(0.1028) = 0.1220 \text{ mm}$

Problem 2.70

2.70 The block shown is made of a magnesium alloy for which $E = 45$ GPa and $\nu = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.



$\delta_y = 0 \qquad \varepsilon_y = 0 \qquad \sigma_z = 0$

$\varepsilon_y = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$

$\sigma_y = \nu\sigma_x = (0.35)(-180 \times 10^6)$

$= -63 \times 10^6$ Pa $\qquad \sigma_y = -63$ MPa ◄

$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.35)(-243 \times 10^6)}{45 \times 10^9} = -1.89 \times 10^{-3}$

$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x - \nu\sigma_y}{E} = -\frac{157.95 \times 10^6}{45 \times 10^9} = -3.51 \times 10^{-3}$

(b) $A_o = L_x L_z$

$A = L_x(1+\varepsilon_x) L_z(1+\varepsilon_z) = L_x L_z(1 + \varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$

$\Delta A = A - A_o = L_x L_z(\varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z) \approx L_x L_z(\varepsilon_x + \varepsilon_z)$

$\Delta A = (100 \text{ mm})(25 \text{ mm})(-3.51 \times 10^{-3} - 1.89 \times 10^{-3}) \qquad \Delta A = -13.50 \text{ mm}^2$ ◄

(c) $V_o = L_x L_y L_z$

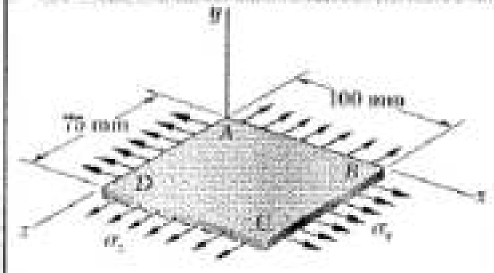
$V = L_x(1+\varepsilon_x) L_y(1+\varepsilon_y) L_z(1+\varepsilon_z)$

$= L_x L_y L_z(1 + \varepsilon_x + \varepsilon_y + \varepsilon_z + \varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x + \varepsilon_x\varepsilon_y\varepsilon_z)$

$\Delta V = V - V_o = L_x L_y L_z(\varepsilon_x + \varepsilon_y + \varepsilon_z + \text{small terms})$

$\Delta V = (100)(40)(25)(-3.51 \times 10^{-3} + 0 - 1.89 \times 10^{-3}) \qquad \Delta V = -540 \text{ mm}^3$ ◄

Problem 2.71



2.71 The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\varepsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0 / ε_z.

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}(\sigma_x - \nu\sigma_0)$$

(a) $\sigma_x = \nu\sigma_0$ ◄

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{E}(-\nu^2\sigma_0 - 0 + \sigma_0) = \frac{1-\nu^2}{E}\sigma_0$

$$\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$$ ◄

Problem 2.72

2.72 For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypothenuses of the triangles shown in Fig. 2.54, which represent respectively an element before and after deformation, (*b*) using the values of the corresponding stresses σ' and σ_x shown in Fig. 1.40, and the generalized Hooke's law.

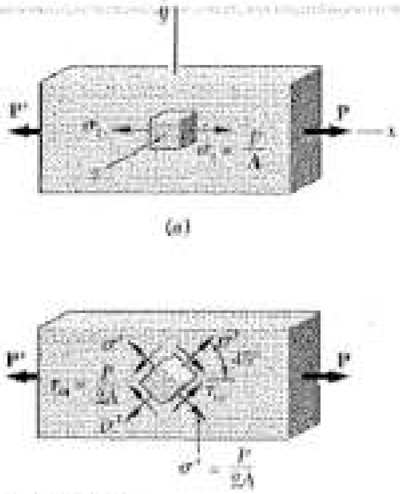


Fig. 1.40 (*b*)

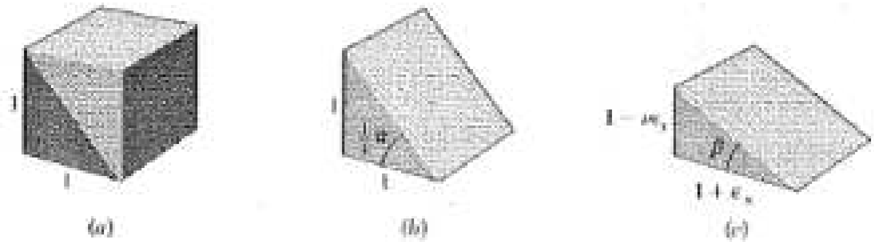
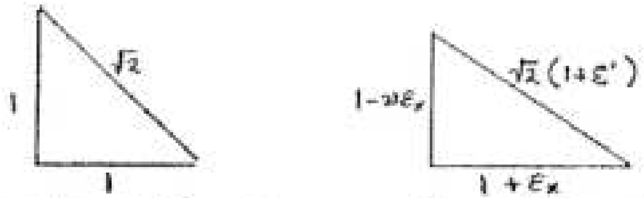


Fig. 2.54



(a) Before deformation After deformation

$$[\sqrt{2}(1+\varepsilon')]^2 = (1+\varepsilon_x)^2 + (1-\nu\varepsilon_x)^2$$

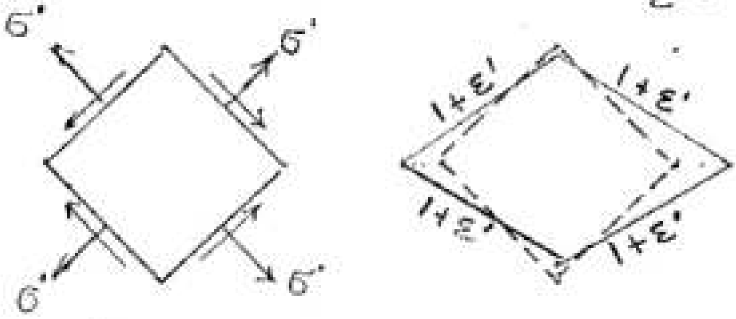
$$2(1 + 2\varepsilon' + \varepsilon'^2) = 1 + 2\varepsilon_x + \varepsilon_x^2 + 1 - 2\nu\varepsilon_x + \nu^2\varepsilon_x^2$$

$$4\varepsilon' + 2\varepsilon'^2 = 2\varepsilon_x + \varepsilon_x^2 - 2\nu\varepsilon_x + \nu^2\varepsilon_x^2$$

Neglect squares as small $\quad 4\varepsilon' = 2\varepsilon_x - 2\nu\varepsilon_x$

$$\varepsilon' = \frac{1-\nu}{2}\varepsilon_x$$

(b)



$$\varepsilon' = \frac{\sigma'}{E} - \frac{\nu\sigma'}{E}$$

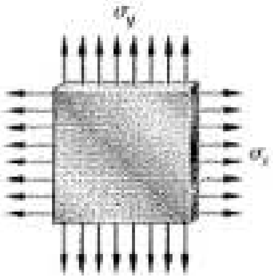
$$= \frac{1-\nu}{E} \cdot \frac{P}{2A}$$

$$= \frac{1-\nu}{2E}\sigma_x$$

$$= \frac{1-\nu}{2}\varepsilon_x$$

Problem 2.73

2.73 In many situations it is known that the normal stress in a given direction is zero, for example $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y and ε_z as follows:



$$\sigma_x = E\frac{\varepsilon_x + \nu\varepsilon_y}{1-\nu^2} \qquad \sigma_y = E\frac{\varepsilon_y + \nu\varepsilon_x}{1-\nu^2} \qquad \varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad (1) \qquad \varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y) \quad (2)$$

Multiplying (2) by ν and adding to (1)

$$\varepsilon_x + \nu\varepsilon_y = \frac{1-\nu^2}{E}\sigma_x \quad \text{or} \quad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

Multiplying (1) by ν and adding to (2)

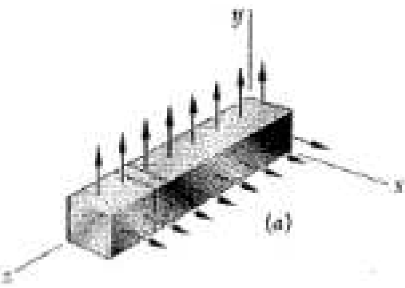
$$\varepsilon_y + \nu\varepsilon_x = \frac{1-\nu^2}{E}\sigma_y \quad \text{or} \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{E}\cdot\frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y + \varepsilon_y + \nu\varepsilon_x)$$

$$= -\frac{\nu(1+\nu)}{1-\nu^2}(\varepsilon_x + \varepsilon_y) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

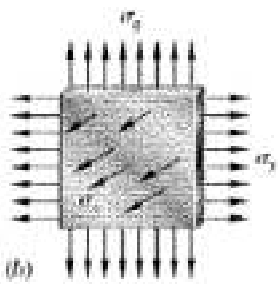
Problem 2.74

2.74 In many situations physical constraints prevent strain from occurring in a given direction, for example $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain*, we can express σ_z, ε_x and ε_y as follows:



$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y] \qquad \varepsilon_y = \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$



$$\varepsilon_z = 0 = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = \nu(\sigma_x + \sigma_y)$$

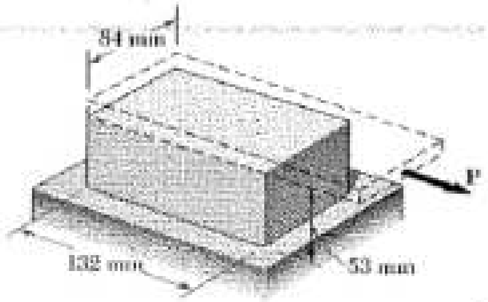
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}[\sigma_x - \nu\sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

$$= \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y]$$

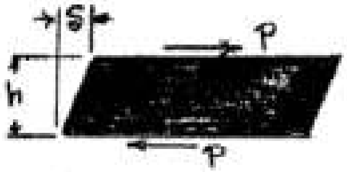
$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1}{E}[-\nu\sigma_x + \sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

$$= \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$

Problem 2.75



2.75 The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used $G = 385$ MPa, determine the deflection of the plate when $P = 36$ kN.



Consider the plastic block. The shearing force carried is $P = 36$ kN

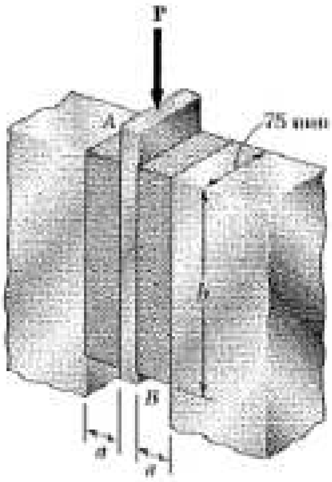
The area is $A = (0.084)(0.132) = 11.088 \times 10^{-3}\ m^2$

<u>Shearing stress.</u> $\tau = \dfrac{P}{A} = \dfrac{36 \times 10^3}{11.088 \times 10^{-3}} = 3.24675$ MPa.

<u>Shearing strain.</u> $\gamma = \dfrac{\tau}{G} = \dfrac{3.24675}{385} = 0.008433$

But $\gamma = \dfrac{\delta}{h}$ $\therefore$ $\delta = h\gamma = (53)(0.008433) = 0.45$ mm

Problem 2.76



2.76 A vibration isolation unit consists of two blocks of hard rubber bonded to plate AB and to rigid supports as shown. For the type and grade of rubber used $\tau_{all} = 1.54$ MPa and $G = 12.6$ MPa. Knowing that a centric vertical force of magnitude $P = 13$ kN must cause a 2.5-mm vertical deflection of the plate AB, determine the smallest allowable dimensions a and b of the block.

Consider the rubber block on the right. It carries a shearing force equal to $\frac{1}{2}P$

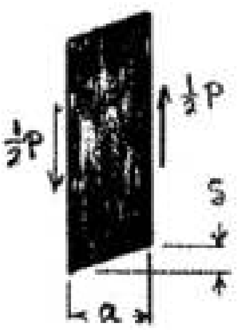
The <u>shearing stress</u> is $\tau = \dfrac{\frac{1}{2}P}{A}$

or required $A = \dfrac{P}{2\tau} = \dfrac{13 \times 10^3}{(2)(1.54 \times 10^6)} = 4.221 \times 10^{-3}\ m^2$

But $A = (3.0)\,b$

Hence $b = \dfrac{A}{0.075} = 0.05628$ m $\qquad b_{min} = 56.3$ mm

Use $b = 56.3$ mm and $\tau = 1.54$ MPa



<u>Shearing strain.</u> $\gamma = \dfrac{\tau}{G} = \dfrac{1.54}{12.6} = 0.12222$

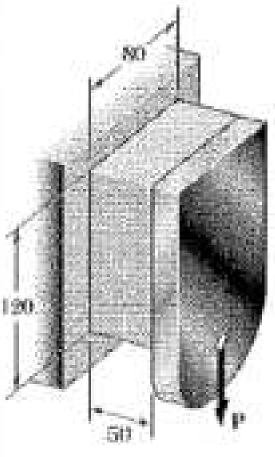
But, $\gamma = \dfrac{\delta}{a}$

Hence, $a = \dfrac{\delta}{\gamma} = \dfrac{2.5}{0.12222} = 20.45$ mm

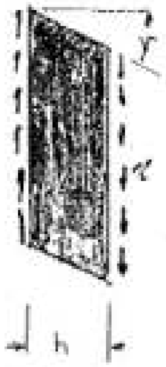
$a_{min} = 20.5$ mm

Problem 2.77

2.77 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load **P** is applied. Knowing that for the plastic used $G = 1050$ MPa, determine the deflection of the plate.



Dimensions in mm



$A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$

$P = 240 \times 10^3 \text{ N}$

$\tau = \frac{P}{A} = \frac{240 \times 10^3}{9.6 \times 10^{-3}} = 25 \times 10^6 \text{ Pa}$

$G = 1050 \times 10^6 \text{ Pa}$

$\gamma = \frac{\tau}{G} = \frac{25 \times 10^6}{1050 \times 10^6} = 23.810 \times 10^{-3}$

$h = 50 \text{ mm} = 0.050 \text{ m}$

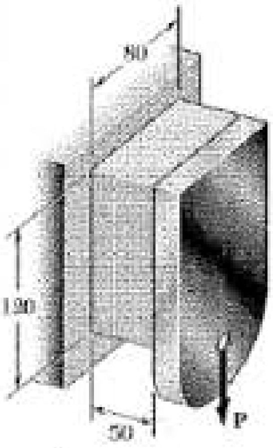
$\delta = h\gamma = (0.050)(23.810 \times 10^{-3})$

$= 1.190 \times 10^{-3} \text{ m}$ 1.091 mm ↓

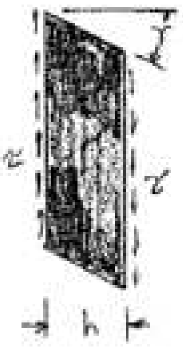
Problem 2.78

2.78 What load **P** should be applied to the plate of Prob. 2.77 to produce a 1.5-mm deflection?

2.77 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load **P** is applied. Knowing that for the plastic used $G = 1050$ MPa, determine the deflection of the plate.



Dimensions in mm



$\delta = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

$h = 50 \text{ mm} = 50 \times 10^{-3} \text{ m}$

$\gamma = \frac{\delta}{h} = \frac{1.5 \times 10^{-3}}{50 \times 10^{-3}} = 30 \times 10^{-3}$

$G = 1050 \times 10^6 \text{ Pa}$

$\tau = G\gamma = (1050 \times 10^6)(30 \times 10^{-3})$

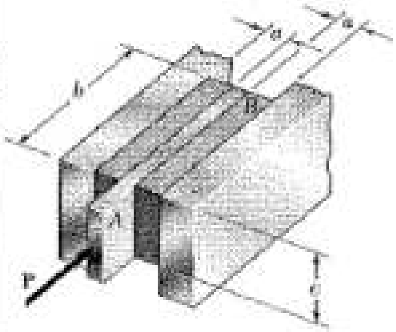
$= 31.5 \times 10^6 \text{ Pa}$

$A = (120)(80) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$

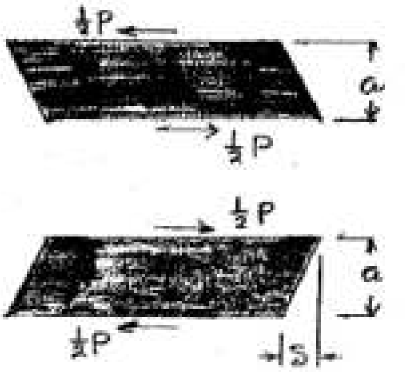
$P = \tau A = (31.5 \times 10^6)(9.6 \times 10^{-3}) = 302 \times 10^3 \text{ N}$ 302 kN

NOTE: In problems 2.75 through 2.82, 2.87, 2.88, and 2.132 it is assumed that γ is the tangent of the angle change. This makes the relative displacement proportional to the force.

Problem 2.79



2.79 Two blocks of rubber with a modulus of rigidity $G = 12$ MPa are bonded to rigid supports and to a plate AB. Knowing that $c = 100$ mm and $P = 40$ kN, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm.



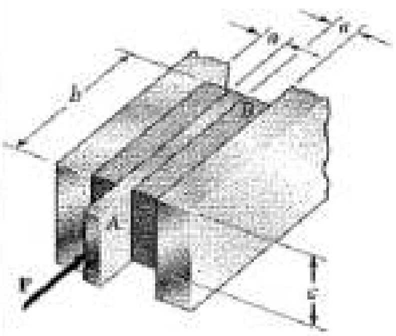
Shearing strain $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(12\times10^6)(0.005)}{1.4\times10^6} = 0.04286 \text{ m} = 43 \text{ mm.}$$

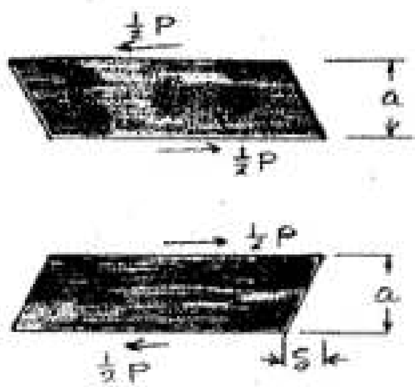
Shearing stress $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{40\times10^3}{(2)(0.1)(1.4\times10^6)} = 0.143 \text{ m} = 143 \text{ mm.}$$

Problem 2.80



2.80 Two blocks of rubber with a modulus of rigidity $G = 10$ MPa are bonded to rigid supports and to a plate AB. Knowing that $b = 200$ mm and $c = 125$ mm, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 1.5 MPa and the deflection of the plate is to be at least 6 mm.



Shearing stress $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$$P = 2bc\tau = (2)(0.2)(0.125)(1.5\times10^6) = 75 \text{ kN}$$

Shearing strain $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(10\times10^6)(0.006)}{1.5\times10^6} = 0.04 \text{ m} = 40 \text{ mm}$$

Problem 2.83

*2.83 A 150-mm diameter solid steel sphere is lowered into the ocean to a point where the pressure is 50 MPa (about 5 km below the surface). Knowing that $E = 200$ GPa and $\nu = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

For a solid sphere $\quad V_0 = \frac{\pi}{6} d_0^3 = \frac{\pi}{6}(0.15)^3 = 1.767 \times 10^{-3}\ m^3.$

$$\sigma_x = \sigma_y = \sigma_z = -p = 50 MPa.$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = -\frac{(1-2\nu)p}{E} = -\frac{(0.4)(50 \times 10^6)}{200 \times 10^9} = -100 \times 10^{-6}$$

Likewise $\quad \varepsilon_y = \varepsilon_z = -100 \times 10^{-6}$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -300 \times 10^{-6}$$

(a) $-\Delta d = -d_0 \varepsilon_x = -(0.15)(100 \times 10^{-6}) = 15 \times 10^{-6} m.$

(b) $-\Delta V = -V_0 e = -(1.767 \times 10^{-3})(-300 \times 10^{-6}) = 530 \times 10^{-9}\ m^3$

(c) Let m = mass of sphere. $\qquad m$ = constant.

$$m = \rho_0 V_0 = \rho V = \rho V_0(1+e)$$

$$\frac{\rho - \rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{V_0(1+e)} \cdot \frac{V_0}{m} - 1 = \frac{1}{1+e} - 1$$

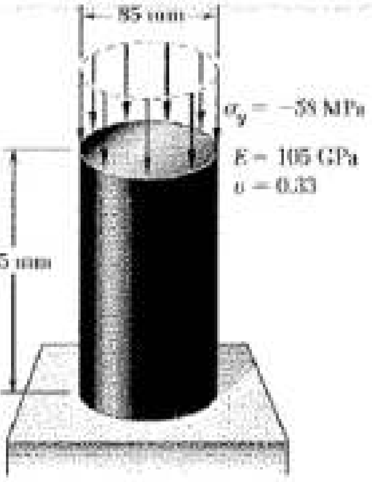
$$= (1 - e + e^2 - e^3 + \dots) - 1 = -e + e^2 - e^3 + \dots$$

$$\approx -e = 300 \times 10^{-6}$$

$$\frac{\rho - \rho_0}{\rho_0} \times 100\% = (300 \times 10^{-6})(100\%) = 0.03\%$$

Problem 2.84

*2.84 (*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a*, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.



$h_o = 135 \text{ mm} = 0.135 \text{ m}$

$A_o = \frac{\pi}{4} d_o^2 = \frac{\pi}{4}(85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$

$V_o = A_o h_o = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$

(a) $\sigma_x = 0$, $\sigma_y = -58 \times 10^6$ Pa, $\sigma_z = 0$

$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{\sigma_y}{E}$

$= -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6}$

$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6}) = -0.0746 \text{ mm}$ ◀

$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9}$

$= -187.81 \times 10^{-6}$

$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) = -143.9 \text{ mm}^3$ ◀

(b) $\sigma_x = \sigma_y = \sigma_z = -70 \times 10^6$ Pa $\qquad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6$ Pa

$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1-2\nu}{E}\sigma_y$

$= \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6}$

$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6}) = -0.0306 \text{ mm}$ ◀

$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$

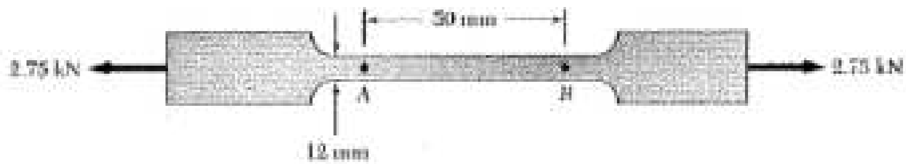
$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) = -521 \text{ mm}^3$ ◀

Problem 2.86

*2.86 Determine the change in volume of the 50-mm gage length segment AB in Prob. 2.63 (*a*) by computing the dilatation of the material, (*b*) by subtracting the original volume of portion AB from its final volume.

2.63 A 2.75 kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200$ GPa, $\nu = 0.30$). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion AB of the test coupon, (*c*) in the thickness of portion AB, (*d*) in the cross-sectional area of portion AB.

All calculations here are in mm



(a) $A_0 = (12)(1.6) = 19.2 \text{ mm}^2 = 19.2 \times 10^{-6} \text{ m}^2$

Volume $\quad V_0 = L_0 A_0 = (50)(19.2) = 960 \text{ mm}^3$

$\sigma_x = \frac{P}{A_0} = \frac{2.75 \times 10^3}{19.2 \times 10^{-6}} = 143.229 \times 10^6 \text{ Pa} \qquad \sigma_y = \sigma_z = 0$

$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$

$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(716.15 \times 10^{-7}) = -214.84 \times 10^{-6}$

$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 286.46 \times 10^{-6}$

$\Delta V = V_0 e = (960)(286.46 \times 10^{-6}) = 0.275 \text{ mm}^3$ ◄

(b) From the solution to Problem 2.62

$\delta_x = 0.03581 \text{ mm} \qquad \delta_y = -0.002578 \text{ mm} \qquad \delta_z = -0.0003437 \text{ mm}$

The dimensions when under the 2.75 kN load are:

length $\quad L = L_0 + \delta_x = 50 + 0.03581 \quad 50.03581 \text{ mm}$

width $\quad w = w_0 + \delta_y = 12 - 0.002578 = 11.997422 \text{ mm}$

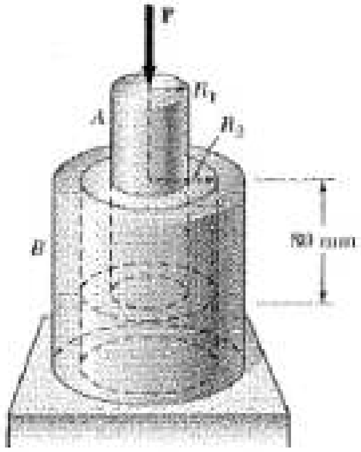
thickness $\quad t = t_0 + \delta_z = 1.6 - 0.0003437 = 1.5996563 \text{ mm}$

volume $\quad V = Lwt = (50.03581)(11.997422)(1.5996563) = 960.275 \text{ mm}^3$

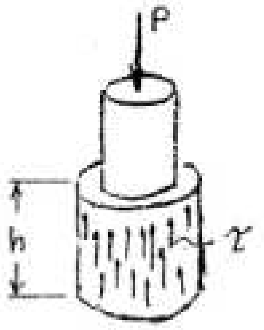
$\Delta V = V - V_0 = 960.275 - 960 = 0.275 \text{ mm}^3$ ◄

Problem 2.88

*2.88 A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_2 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 10.93$ MPa. Determine the required value of the ratio R_2/R_1 if a 10-kN force **P** is to cause a 2-mm deflection of rod A.



Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \le r \le R_2$

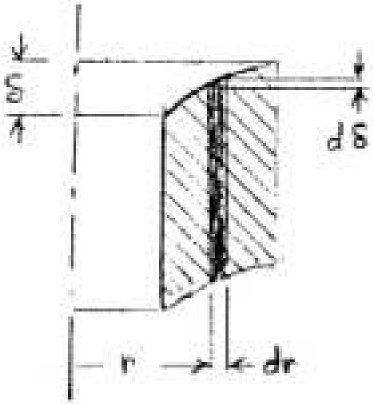


Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length dr

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi G h}\frac{dr}{r}$$

Total deformation

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi G h}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1}$$

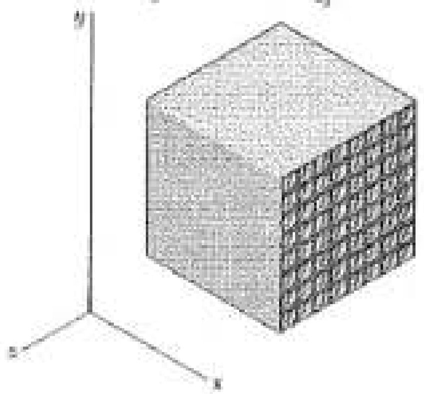
$$\ln\frac{R_2}{R_1} = \frac{2\pi G h \delta}{P} = \frac{(2\pi)(10.93\times10^6)(0.080)(0.002)}{10\times10^3} = 1.0988$$

$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

Problem 2.89

*2.89 A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the x direction. The cube is constrained against deformations in the y and z directions and is subjected to a tensile load of 65 kN in the x direction. Determine (*a*) the change in the length of the cube in the x direction, (*b*) the stresses σ_x, σ_y, and σ_z.

$E_x = 50$ GPa $\quad \nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\quad \nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\quad \nu_{zy} = 0.428$



Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \qquad (1) \qquad\qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \qquad (4)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \qquad (2) \qquad\qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \qquad (5)$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \qquad (3) \qquad\qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \qquad (6)$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z = \frac{\nu_{xy}}{E_x}\sigma_x \qquad (7)$$

$$-\frac{\nu_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{\nu_{xz}}{E_x}\sigma_x \qquad (8)$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\sigma_z = 0.077216\,\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\sigma_y + \sigma_z = 0.077216\,\sigma_x$$

Solving simultaneously, $\sigma_y = \sigma_z = 0.134993\,\sigma_x$

Using (4) and (5) in (1), $\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{xy}}{E_x}\sigma_y - \frac{\nu_{xz}}{E}\sigma_z$

$$\varepsilon_x = \frac{1}{E_x}\left[1 - (0.254)(0.134993) - (0.254)(0.134993)\right]\sigma_x$$

$$= \frac{0.93142\,\sigma_x}{E_x}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600\times10^{-6} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65\times10^3}{1600\times10^{-6}} = 40.625\times10^6 \text{ Pa}$$

continued

Problem 2.89 continued

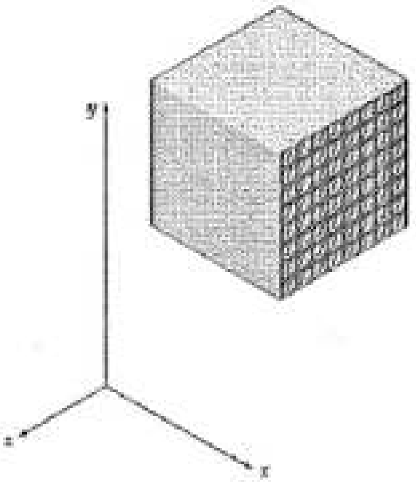
$$\varepsilon_x = \frac{(0.93142)(40.625\times10^3)}{50\times10^9} = 756.78\times10^{-6}$$

(a) $\delta_x = L_x\varepsilon_x = (40\text{ mm})(756.78\times10^{-6}) = 0.0303\text{ mm}$

(b) $\sigma_x = 40.625\times10^6\text{ Pa} = 40.6\text{ MPa}$

$\sigma_y = \sigma_z = (0.134993)(40.625\times10^6) = 5.48\times10^6\text{ Pa}$
$= 5.48\text{ MPa}$

Problem 2.90



*2.90 The composite cube of Prob. 2.89 is constrained against deformation in the z direction and elongated in the x direction by 0.035 mm due to a tensile load in the x direction. Determine (*a*) the stresses σ_x, σ_y, and σ_z, (*b*) the change in the dimension in the y direction.

$E_x = 50$ GPa $\quad \nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\quad \nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\quad \nu_{zy} = 0.428$

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \qquad (1) \qquad\qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \qquad (4)$$

$$\epsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \qquad (2) \qquad\qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \qquad (5)$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \qquad (3) \qquad\qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \qquad (6)$$

Constraint condition. $\quad \varepsilon_z = 0$

Load condition. $\quad \sigma_y = 0$

From equation (3), $\quad 0 = -\dfrac{\nu_{xz}}{E_x}\sigma_x + \dfrac{1}{E_z}\sigma_z$

$$\sigma_z = \frac{\nu_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\,\sigma_x$$

continued

Problem 2.91

*2.91 The material constants E, G, k, and ν are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (*a*) $k = GE/(9G - 3E)$ and (*b*) $\nu = (3k - 2G)/(6k + 2G)$.

$$k = \frac{E}{3(1-2\nu)} \quad \text{and} \quad G = \frac{E}{2(1+\nu)}$$

(a) $1 + \nu = \dfrac{E}{2G}$ or $\nu = \dfrac{E}{2G} - 1$

$$k = \frac{E}{3\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E}$$

$$= \frac{EG}{9G - 6E}$$

(b) $\dfrac{k}{G} = \dfrac{2(1+\nu)}{3(1-2\nu)}$

$$3k - 6k\nu = 2G + 2G\nu$$

$$3k - 2G = 2G + 6k$$

$$\nu = \frac{3k - 2G}{6k + 2G}$$

Problem 2.92

*2.92 Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [*Hint*: Refer to Eq. (2.43) and to Sec. 2.13.]

$$G = \frac{E}{2(1+\nu)} \quad \text{or} \quad \frac{E}{G} = 2(1+\nu)$$

Assume $\nu \geq 0$ for almost all materials and $\nu < \frac{1}{2}$ for a positive bulk modulus.

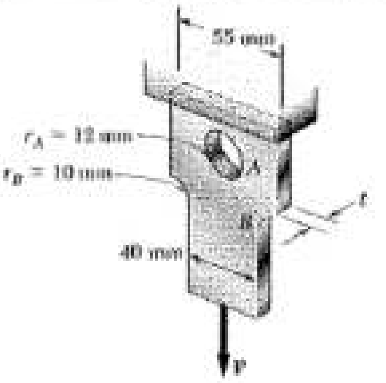
Applying the bounds, $2 \leq \dfrac{E}{G} < 2(1 + \frac{1}{2}) = 3$

Taking the reciprocals, $\frac{1}{2} \geq \dfrac{G}{E} > \frac{1}{3}$

or $\frac{1}{3} \leq \dfrac{G}{E} \leq \frac{1}{2}$

Problem 2.95

2.95 For $P = 35$ kN, determine the minimum plate thickness t required if the allowable stress is 125 MPa.



At the hole: $r_A = 12\,mm \quad d_A = 55 - 24 = 31$

$$\frac{r_A}{d_A} = \frac{12}{31} = 0.39$$

From Fig 2.64 a $\quad K = 2.26$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{max}}$$

$$t = \frac{(2.26)(35000)}{(0.031)(125\times10^6)} = 0.0204\,m = 20.4\,mm$$

At the fillet $\quad D = 55\,mm \quad d_B = 40\,mm \quad \frac{D}{d_B} = \frac{55}{40} = 1.375$

$$r_B = \tfrac{3}{8} = 10\,mm \qquad \frac{r_B}{d_B} = \frac{10}{40} = 0.25$$

From Fig 2.64 b $\quad K = 1.70 \qquad \sigma_{max} = \frac{KP}{A_{min}} = \frac{KP}{d_B t}$

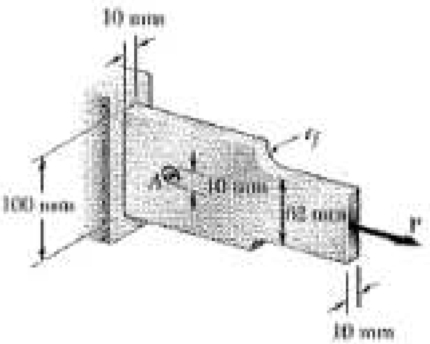
$$t = \frac{KP}{d_B \sigma_{max}} = \frac{(1.70)(35000)}{(0.04)(125\times10^6)} = 0.0119\,m = 11.9\,mm$$

The larger value is the required minimum plate thickness

$$t = 20.4\,mm$$

Problem 2.96

2.96 Knowing that the hole has a diameter of 10 mm, determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 105 MPa.



For the circular hole $\quad r = \frac{10}{2} = 5\,mm$

$$d = 100 - 10 = 90\,mm \qquad \frac{r}{d} = \frac{5}{90} = 0.056$$

$$A_{net} = dt = (0.09)(0.01) = 0.0009\,m^2$$

From Fig 2.64 a $\quad K_{hole} = 2.8$

$$\sigma_{max} = \frac{K_{hole} P}{A_{net}}$$

(b) $$P = \frac{A_{net}\sigma_{max}}{K_{hole}} = \frac{(0.0009)(105\times10^6)}{2.8} = 33.75\,kN$$

(a) For fillet $\quad D = 100\,mm \quad d = 62\,mm \quad \frac{D}{d} = \frac{100}{62} = 1.61$

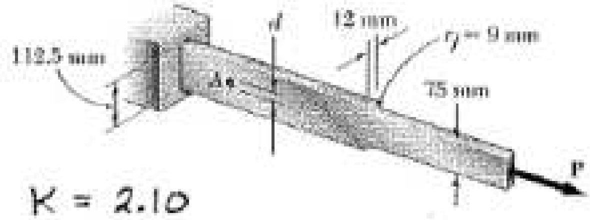
$$A_{min} = dt = (0.062)(0.01) = 0.00062\,m^2$$

$$\sigma_{max} = \frac{K_{fillet} P}{A_{min}} \quad \therefore \quad K_{fillet} = \frac{A_{min}\sigma_{max}}{P} = \frac{(0.00062)(105\times10^6)}{33750} = 1.93$$

From Fig 2.64 b $\quad \frac{r_f}{d} \approx 0.18 \quad \therefore \quad r_f \approx 0.18d = (0.18)(62) = 11.16\,mm$

Problem 2.97

2.97 A hole is to be drilled in the plate at *A*. The diameters of the bits available to drill the hole range from 12 to 24 mm in 3-mm increments. (*a*) Determine the diameter *d* of the largest bit that can be used if the allowable load at the hole is to exceed that at the fillets. (*b*) If the allowable stress in the plate is 145 MPa, what is the corresponding allowable load **P**?



At the fillets, $r = 9$ mm $\quad d = 75$ mm

$D = 112.5$ mm $\quad \frac{D}{d} = \frac{112.5}{75} = 1.5$

$\frac{r}{d} = \frac{9}{75} = 0.12 \quad$ From Fig 2.64 b $\quad K = 2.10$

$A_{min} = (75)(12) = 900\ mm^2 = 900\ mm^2 = 900 \times 10^{-6}\ m^2$

$$\sigma_{max} = K\frac{P_{all}}{A_{min}} = \sigma_{all} \quad \therefore \quad P_{all} = \frac{A_{min}\sigma_{all}}{K} = \frac{(900\times10^{-6})(145\times10^{6})}{2.10}$$

$$= 62.1\times10^{3}\ N = 62.1\ kN$$

At the hole, $\quad A_{net} = (D - 2r)t, \quad \frac{r}{d} = \frac{r}{D-2r}$

where $\quad D = 112.5$ mm $\quad r =$ radius of circle $\quad t = 12$ mm

K is taken from Fig 2.64 a.

$$\sigma_{max} = K\frac{P}{A_{net}} = \sigma_{all} \quad \therefore \quad P_{all} = \frac{A_{net}\sigma_{all}}{K}$$

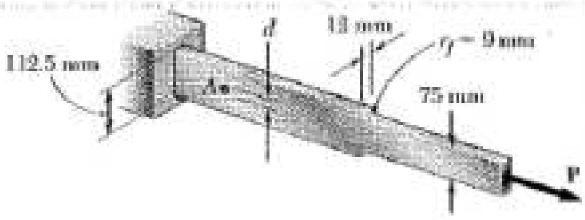
Hole diam	r	d = D − 2r	r/d	K	A_{net} (m^2)	P_{all}, (N)
12 mm	6.0 mm	100.5	0.060	2.80	1206×10^{-6}	62.5×10^{3}
15 mm	7.5 mm	97.5 mm	0.077	2.75	1170×10^{-6}	61.7×10^{3}
18 mm	9.0 mm	94.5 mm	0.095	2.71	1134×10^{-6}	60.7×10^{3}
21 mm	10.5 mm	91.5 mm	0.115	2.67	1098×10^{-6}	59.6×10^{3}
24 mm	12.0 mm	88.5 mm	0.136	2.62	1062×10^{-6}	58.8×10^{3}

(a) Bit diameter, $\quad$ 1 = 12 mm ◄

(b) Allowable load. $\quad$ 62.1 kN ◄

Problem 2.98

2.98 (*a*) For $P = 58$ kN and $d = 12$ mm, determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.



Maximum stress at hole

Use Fig. 2.64 a for values of K

$$\frac{r}{d} = \frac{6}{112.5 - 12} = 0.0597, \qquad K = 2.80$$

$$A_{net} = (12)(112.5 - 12) = 1206 \text{ mm}^2 = 1206 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{net}} = \frac{(2.80)(58 \times 10^3)}{1206 \times 10^{-6}} = 134.7 \times 10^6 \text{ Pa}$$

Maximum stress at fillets

Use Fig. 2.64 b

$$\frac{r}{d} = \frac{9}{75} = 0.12, \qquad \frac{D}{d} = \frac{112.5}{75} = 1.50, \qquad K = 2.10$$

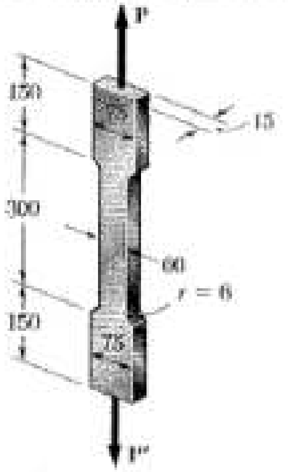
$$A_{min} = (12)(75) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{min}} = \frac{(2.10)(58 \times 10^3)}{900 \times 10^{-6}} = 135.3 \times 10^6 \text{ Pa}$$

(a) With hole and fillets. $\sigma_{max} = 134.7$ MPa ◄

(b) Without hole. $\sigma_{max} = 135.3$ MPa ◄

Problem 2.99



Dimensions in mm

2.99 The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P. (*a*) Knowing that $E = 70$ GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60 × 15-mm rectangular cross section.

$\sigma_{all} = 200 \times 10^6$ Pa $\qquad E = 70 \times 10^9$ Pa

$A_{min} = (60\text{ mm})(15\text{ mm}) = 900\text{ mm}^2 = 900 \times 10^{-6}\text{ m}^2$

(a) Test specimen. $\qquad D = 75$ mm, $d = 60$ mm, $r = 6$ mm

$\frac{D}{d} = \frac{75}{60} = 1.25 \qquad \frac{r}{d} = \frac{6}{60} = 0.10$

From Fig. 2.64(b) $\qquad K = 1.95 \qquad \sigma_{max} = K\frac{P}{A}$

$$P = \frac{A\sigma_{max}}{K} = \frac{(900 \times 10^{-6})(200 \times 10^6)}{1.95} = 92.308 \times 10^3\text{ N}$$

$P = 92.3$ kN

Wide area $\qquad A^* = (75\text{ mm})(15\text{ mm}) = 1125\text{ mm}^2 = 1.125 \times 10^{-3}\text{ m}^2$

$$\delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E}\sum\frac{L_i}{A_i} = \frac{92.308 \times 10^3}{70 \times 10^9}\left[\frac{0.150}{1.125 \times 10^{-3}} + \frac{0.300}{900 \times 10^{-6}} + \frac{0.150}{1.125 \times 10^{-3}}\right]$$

$= 791 \times 10^{-6}$ m $\qquad \delta = 0.791$ mm

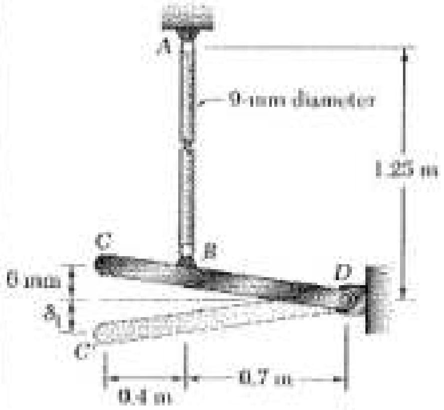
(b) Uniform bar.

$P = A\sigma_{all} = (900 \times 10^{-6})(200 \times 10^6) = 180 \times 10^3$ N $\qquad P = 180.0$ kN

$$\delta = \frac{PL}{AE} = \frac{(180 \times 10^3)(0.600)}{(900 \times 10^{-6})(70 \times 10^9)} = 1.714 \times 10^{-3}\text{ m}$$

$\delta = 1.714$ mm

Problem 2.101



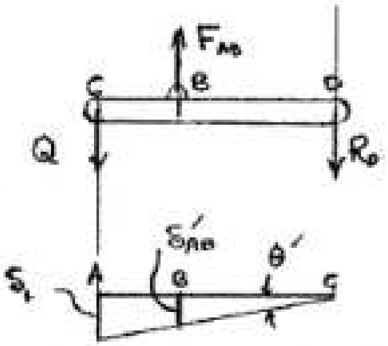
2.101 Rod AB is made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 345 MPa. After the rod has been attached to the rigid lever CD, it is found that end C is 6 mm too high. A vertical force **Q** is then applied at C until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$

Since rod AB is to be stretched permanently,

$(F_{AB})_{max} = A_{AB}\sigma_Y = (63.617 \times 10^{-6})(345 \times 10^6)$

$= 21.948 \times 10^3$ N



$+\circlearrowleft \Sigma M_D = 0: \quad 1.1\,Q - 0.7\,F_{AB} = 0$

$Q_{max} = \frac{0.7}{1.1}(21.948 \times 10^3) = 13.967 \times 10^3$ N

13.97 kN ◄

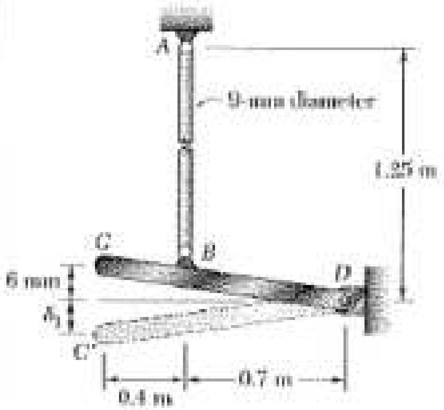
$\delta'_{AB} = \frac{(F_{AB})_{max} L_{AB}}{E A_{AB}} = \frac{(21.948 \times 10^3)(1.25)}{(200 \times 10^9)(63.617 \times 10^{-6})} = 2.15625 \times 10^{-3}$ m

$\theta' = \frac{\delta'_{AB}}{0.7} = 3.0804 \times 10^{-3}$ rad

$\delta_1 = 1.1\,\theta' = 3.39 \times 10^{-3}$ m

3.39 mm ◄

Problem 2.102



2.102 Solve Prob. 2.101, assuming that the yield point of the mild steel is 250 MPa.

2.101 Rod AB is made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 345 MPa. After the rod has been attached to the rigid lever CD, it is found that end C is 6 mm too high. A vertical force **Q** is then applied at C until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$

Since rod AB is to be stretched permanently,

$(F_{AB})_{max} = A_{AB}\sigma_Y = (63.617 \times 10^{-6})(250 \times 10^6)$

$= 15.9043 \times 10^3$ N

$+\circlearrowleft \Sigma M_D = 0 \quad 1.1\,Q - 0.7\,F_{AB} = 0$

$Q_{max} = \frac{0.7}{1.1}(15.9043 \times 10^3) = 10.12 \times 10^3$ N

10.12 kN ◄

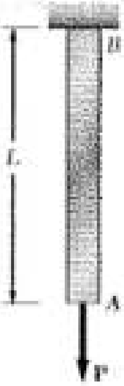
$\delta'_{AB} = \frac{(F_{AB})_{max} L_{AB}}{E A_{AB}} = \frac{(15.9043 \times 10^3)(1.25)}{(200 \times 10^9)(63.617 \times 10^{-6})} = 1.5625 \times 10^{-3}$ m

$\theta' = \frac{\delta'_{AB}}{0.7} = 2.2321 \times 10^{-3}$ rad

$\delta_1 = 1.1\,\theta' = 2.46 \times 10^{-3}$ m

2.46 mm ◄

Problem 2.103



2.103 The 30-mm square bar *AB* has a length $L = 2.2$ m; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar until end *A* has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing that (*a*) $\delta_m = 4.5$ mm, (*b*) $\delta_m = 8$ mm.

$$A = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.2)(345 \times 10^6)}{200 \times 10^9} = 3.795 \times 10^{-3} = 3.795 \text{ mm}$$

IF $\delta_m \geq \delta_Y$ $\quad P_m = A\sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3 \text{ N}$

<u>Unloading</u>: $\delta' = \dfrac{P_m L}{AE} = \dfrac{\sigma_Y L}{E} = \delta_Y = 3.795 \text{ mm}$

$$\delta_P = \delta_m - \delta'$$

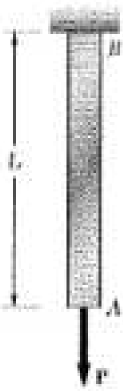
(a) $\delta_m = 4.5 \text{ mm} > \delta_Y$ $\quad P_m = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$ ◄

$\delta_{perm} = 4.5 \text{ mm} - 3.795 \text{ mm} = 0.705 \text{ mm}$ ◄

(b) $\delta_m = 8 \text{ mm} > \delta_Y$ $\quad P_m = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$ ◄

$\delta_P = 8.0 \text{ mm} - 3.795 \text{ mm} = 4.205 \text{ mm}$ ◄

Problem 2.104



2.104 The 30-mm square bar *AB* has a length $L = 2.5$ m; it is made of mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar and then removed to give it a permanent set δ_p. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (*a*) 3.5 mm, (*b*) 6.5 mm.

$$A = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.5)(345 \times 10^6)}{200 \times 10^9} = 4.3125 \times 10^{-3} \text{ m} = 4.3125 \text{ mm}$$

When δ_m exceeds δ_Y, thus producing a permanent stretch of δ_p, the maximum force is

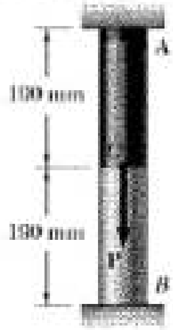
$$P_m = A\sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$$ ◄

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \therefore \quad \delta_m = \delta_p + \delta_Y$$

(a) $\delta_p = 3.5 \text{ mm}$ $\quad \delta_m = 3.5 \text{ mm} + 4.3125 \text{ mm} = 7.81 \text{ mm}$ ◄

(b) $\delta_p = 6.5 \text{ mm}$ $\quad \delta_m = 6.5 \text{ mm} + 4.3125 \text{ mm} = 10.81 \text{ mm}$ ◄

Problem 2.107



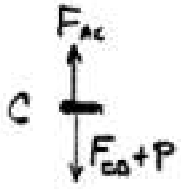
2.107 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm^2. Portion AC is made of a mild steel with E = 200 GPa and σ_Y = 250 MPa, and portion CB is made of a high-strength steel with E = 200 GPa and σ_Y = 345 MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

Displacement at C to cause yielding of AC

$$\delta_{C,Y} = L_{AC}\,\varepsilon_{Y,AC} = \frac{L_{AC}\,\sigma_{Y,AC}}{E} = \frac{(0.190)(250\times10^6)}{200\times10^9} = 0.2375\times10^{-3}\ \text{m}$$

Corresponding force $\quad F_{AC} = A\,\sigma_{Y,AC} = (1750\times10^{-6})(250\times10^6) = 437.5\times10^3\ \text{N}$

$$F_{CB} = -\frac{EA\,\delta_C}{L_{CB}} = -\frac{(200\times10^9)(1750\times10^{-6})(0.2375\times10^{-3})}{0.190} = -437.5\times10^3\ \text{N}$$



For equilibrium of element at C

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 875\times10^3\ \text{N}$$

Since applied load $P = 975\times10^3\ \text{N} > 875\times10^3\ \text{N}$, portion AC yields.

$$F_{CB} = F_{AC} - P = 437.5\times10^3 - 975\times10^3\ \text{N} = -537.5\times10^3\ \text{N}$$

(a) $$\delta_C = -\frac{F_{CB}L_{CB}}{EA} = \frac{(537.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.29179\times10^{-3}\ \text{m} = 0.292\ \text{mm}$$ ◄

(b) Maximum stresses: $\quad \sigma_{AB} = \sigma_{Y,AB} = 250\ \text{MPa}$ ◄

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{537.5\times10^3}{1750\times10^{-6}} = -307.14\times10^6\ \text{Pa} = -307\ \text{MPa}$$ ◄

(c) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{CB}} = -P'_{AC}$$

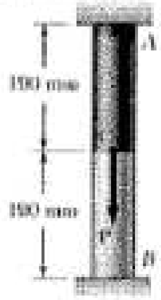
$$P' = 975\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \qquad P'_{AC} = 487.5\times10^3\ \text{N}$$

$$\delta' = \frac{(487.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.26464\times10^{-3}\ \text{m}$$

$$\delta_p = \delta_m - \delta' = 0.29179\times10^{-3} - 0.26464\times10^{-3} = 0.02715\times10^{-3}\ \text{m}$$

$$= 0.027\ \text{mm}$$ ◄

Problem 2.108



2.108 For the composite rod of Prob. 2.107, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.03$ mm and then decreased back to zero, determine, (*a*) the maximum value of P, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C after the load is removed.

2.107 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm². Portion AC is made of a mild steel with E = 200 GPa and σ_Y = 250 MPa, and portion CB is made of a high-strength steel with E = 200 GPa and σ_Y = 345 MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

Displacement at C is $\delta_m = 0.30$ mm. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789 \times 10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789 \times 10^{-3}$$

Strains at initial yielding

$$\varepsilon_{Y,AC} = \frac{\sigma_{Y,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,CB} = -\frac{\sigma_{Y,CB}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \qquad \text{(elastic)}$$

(a) <u>Forces</u>: $F_{AC} = A\sigma_Y = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^{-3}$ N

$$F_{CB} = EA\varepsilon_{CB} = (200 \times 10^9)(1750 \times 10^{-6})(-1.5789 \times 10^{-3}) = -552.6 \times 10^{-3} \text{ N}$$

For equilibrium of element at C, $F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CD} = 437.5 \times 10^3 + 552.6 \times 10^3 = 990.1 \times 10^3 \text{ N} = 990 \text{ kN} \blacktriangleleft$$

(b) <u>Stresses</u>: AC $\sigma_{AC} = \sigma_{Y,AC} = 250$ MPa ◀

CB $\sigma_{CB} = \frac{F_{CB}}{A} = -\frac{552.6 \times 10^3}{1750 \times 10^{-6}} = -316 \times 10^6 \text{ Pa} = -316 \text{ MPa}$ ◀

(c) <u>Deflection and forces for unloading.</u>

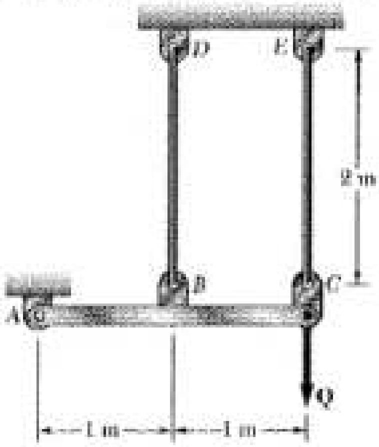
$$\delta' = \frac{P'_{AC} L_{AC}}{EA} = -\frac{P'_{CB} L_{CB}}{EA} \quad \therefore P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P_{AC}$$

$$P' = P'_{AC} - P'_{CB} = 2P'_{AC} = 990.1 \times 10^3 \text{ N} \quad \therefore P'_{AC} = 495.05 \times 10^3 \text{ N}$$

$$\delta' = \frac{(495.05 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.26874 \times 10^{-3} \text{ m} = 0.26874 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 0.30 \text{ mm} - 0.26874 \text{ mm} = 0.031 \text{ mm} \blacktriangleleft$$

Problem 2.109



2.109 Each cable has a cross-sectional area of 100 mm² and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable BD, (*b*) the maximum deflection of point C, (*c*) the final displacement of point C. (*Hint*: In part *c*, cable CE is not taut.)

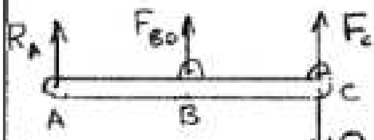
Elongation constraints for taut cables.

Let θ = rotation angle of rigid bar ABC

$$\theta = \frac{\delta_{BD}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \qquad (1)$$

Equilibrium of bar ABC.



$$+\circlearrowleft M_A = 0: \quad L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \qquad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^3$ N

From (2), $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^3 - 34.5\times10^3) = 31.0\times10^3$ N

Since $F_{BD} < A\sigma_Y = 34.5\times10^3$ N, cable BD is elastic when $Q = 50$ kN.

(a) Maximum stresses. $\qquad \sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^3}{100\times10^{-6}} = 310\times10^6 \text{ Pa} \qquad \sigma_{BD} = 310 \text{ MPa}$$

(b) Maximum of deflection of point C.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^3)(2)}{(200\times10^9)(100\times10^{-6})} = 3.1\times10^{-3} \text{ m}$$

From (1) $\quad \delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3}$ m $\qquad$ 6.20 mm ↓

Permanent elongation of cable CE: $(\delta_{CE})_p = (\delta_{CE}) - \frac{\sigma_Y L_{CE}}{E}$

$$(\delta_{CE})_p = (\delta_{CE})_{max} - \frac{F_{CE}L_{CE}}{EA} = (\delta_{CE})_{max} - \frac{\sigma_Y L_{CE}}{E} =$$

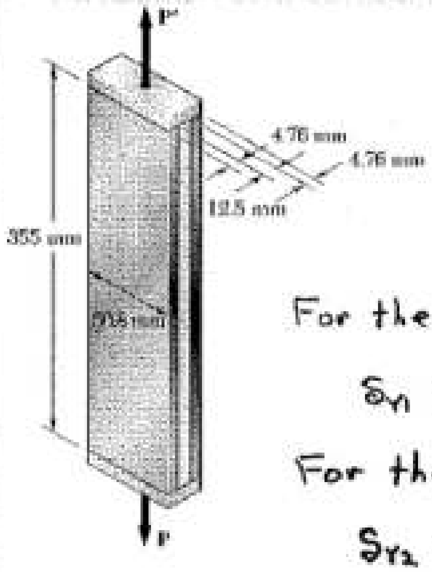
$$= 6.20\times10^{-3} - \frac{(345\times10^6)(2)}{200\times10^9} = 2.75\times10^{-3} \text{ m}$$

(c) Unloading. Cable CE is slack ($F_{CE} = 0$) at $Q = 0$.

From (2), $\quad F_{BD} = 2(Q - F_{CE}) = 2(0-0) = 0$

Since cable BD remained elastic, $\quad \delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = 0$

Problem 2.111



2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

For the mild steel $\quad A_1 = (0.0125)(0.0508) = 635 \times 10^{-6}\ m^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(0.355)(345\times10^6)}{200\times10^9} = 0.612\times10^{-3}\ m$$

For the tempered steel $\quad A_2 = 2(0.00476)(0.0508) = 483.6\times10^{-6}\ m^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(0.355)(690\times10^6)}{200\times10^9} = 1.225\times10^{-3}\ m.$$

Total area: $\quad A = A_1 + A_2 = 1118.6\times10^{-6}\ m^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ The mild steel yields. Tempered steel is elastic.

(a) Forces $\quad P_1 = A_1\sigma_{Y1} = (635\times10^{-6})(345\times10^6) = 219.075$ kN

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(200\times10^9)(483.6\times10^{-6})(0.001016)}{0.355} = 276.81\ kN$$

$$P = P_1 + P_2 = 495.9\ kN$$

(b) Stresses $\quad \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 345$ MPa

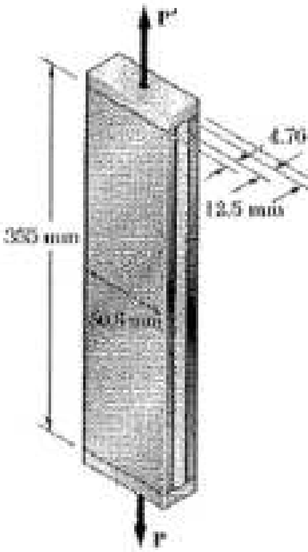
$$\sigma_2 = \frac{P_2}{A_2} = \frac{276810}{483.6\times10^{-6}} = 572.4\ MPa$$

Unloading $\quad \delta' = \frac{PL}{EA} = \frac{(495900)(0.355)}{(200\times10^9)(1118.6\times10^{-6})} = 0.787\times10^{-3}$ m.

(c) Permanent set $\quad \delta_p = \delta_m - \delta' = 1.016 - 0.787 = 0.229$ mm

$= 0.23$ mm

Problem 2.112



2.112 For the composite bar of Prob. 2.111, if P is gradually increased from zero to 436 kN and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

Mild steel $\quad A_1 = (0.0125)(0.0508) = 635\times10^{-6}\ m^2$

Tempered steel $\quad A_2 = 2(0.00476)(0.0508) = 483.6\times10^{-6}\ m^2$

Total: $\quad A = A_1 + A_2 = 1118.6\times10^{-6}\ m^2$

orce to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1118.6\times10^{-6})(345\times10^{6}) = 385.9\ kPa$$

$P > P_Y$, therefore mild steel yields.

Let P_1 = force carried by mild steel
P_2 = force carried by tempered steel

$$P_1 = A_1\sigma_1 = (635\times10^{-6})(345\times10^{6}) = 219075\ N$$

$$P_1 + P_2 = P \qquad P_2 = P - P_1 = 436000 - 219075 = 216925\ N$$

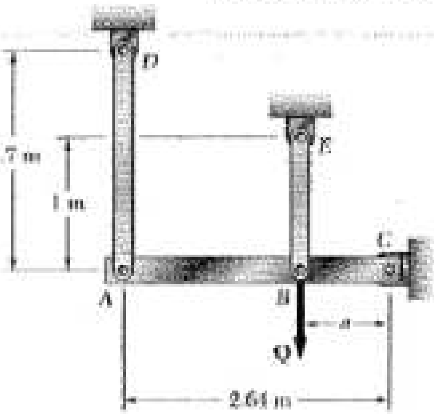
(a) $\quad \delta_m = \dfrac{P_2 L}{E A_2} = \dfrac{(216925)(0.355)}{(200\times10^{9})(483.6\times10^{-6})} = 0.796\times10^{-3}\ m = 0.8\ mm$

(b) $\quad \sigma_2 = \dfrac{P_2}{A_2} = \dfrac{(216925)}{483.6\times10^{-6}} = 448.562\ MPa$

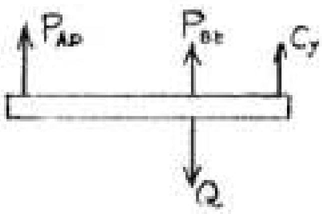
Unloading $\quad \delta' = \dfrac{PL}{EA} = \dfrac{(436000)(0.355)}{(200\times10^{9})(1118.6\times10^{-6})} = 0.692\times10^{-3}\ m = 0.692\ mm$
$= 0.7\ mm$

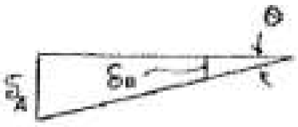
(c) $\quad \delta_p = \delta_m - \delta' = 0.796 - 0.692 = 0.104\ mm$

Problem 2.113



2.113 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5 × 6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 250 MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that a = 0.640 m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point B.





Statics: $\Sigma M_C = 0: \; 0.640(Q - P_{BE}) - 2.64\,P_{AD} = 0$

Deformation: $\delta_A = 2.64\,\theta, \quad \delta_B = a\theta = 0.640\,\theta$

Elastic Analysis:

$$A = (37.5)(6) = 225 \text{ mm}^2 = 225\times10^{-6}\text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\,\delta_A$$

$$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6\,\theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times10^9\,\theta$$

$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B$$

$$= (45\times10^6)(0.640\,\theta) = 28.80\times10^6\,\theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 128\times10^9\,\theta$$

From Statics, $Q = P_{BE} + \frac{2.64}{0.640}P_{AD} = P_{BE} + 4.125\,P_{AD}$

$$= [28.80\times10^6 + (4.125)(69.88\times10^6)]\,\theta = 317.06\times10^6\,\theta$$

θ_Y at yielding of link AD $\qquad \sigma_{AD} = \sigma_Y = 250\times10^6 = 310.6\times10^9\,\theta$

$\theta_Y = 804.89\times10^{-6}$

$Q_Y = (317.06\times10^6)(804.89\times10^{-6}) = 255.2\times10^3$ N

(a) Since $Q = 260\times10^3 > Q_Y$, link AD yields. $\quad \sigma_{AD} = 250$ MPa ◄

$$P_{AD} = A\sigma_Y = (225\times10^{-6})(250\times10^{-6}) = 56.25\times10^3\text{ N}$$

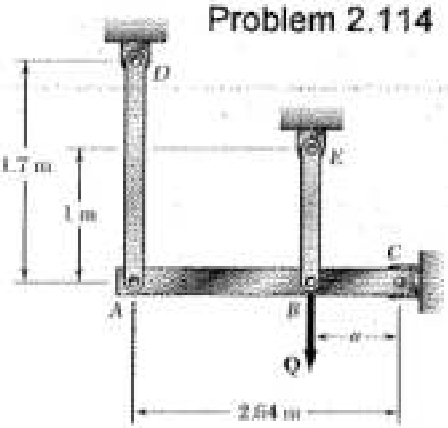
From Statics, $P_{BE} = Q - 4.125\,P_{AD} = 260\times10^3 - (4.125)(56.25\times10^3)$

$P_{BE} = 27.97\times10^3$ N $\qquad \sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97\times10^3}{225\times10^{-6}} = 124.3\times10^6$ Pa

$= 124.3$ MPa ◄

(b) $\delta_B = \frac{P_{BE}L_{BE}}{EA} = \frac{(27.97\times10^3)(1.0)}{(200\times10^9)(225\times10^{-6})} = 621.53\times10^{-6}$ m

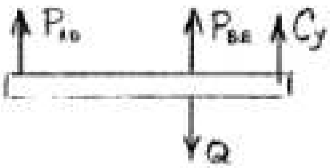
$\delta_B = 0.622$ mm ↓ ◄



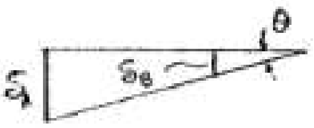
Problem 2.114

2.114 Solve Prob. 2.113, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at B is gradually increased from zero to 135 kN.

2.113 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point B.



Statics: $\Sigma M_C = 0: \; 1.76(Q - P_{BE}) - 2.64 P_{AD} = 0$



Deformation: $\delta_A = 2.64\,\theta, \quad \delta_B = 1.76\,\theta$

Elastic Analysis:

$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$

$P_{AD} = \frac{EA}{L_{AD}} \delta_A = \frac{(200\times10^9)(225\times10^{-6})}{1.7} \delta_A = 26.47\times10^6 \, \delta_A$

$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6 \, \theta$

$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times10^9 \, \theta$

$P_{BE} = \frac{EA}{L_{BE}} \delta_B = \frac{(200\times10^9)(225\times10^{-6})}{1.0} \delta_B = 45\times10^6 \, \delta_B = (45\times10^6)(1.76\,\theta)$

$= 79.2\times10^6 \, \theta \qquad \sigma_{BE} = \frac{P_{BE}}{A} = 352\times10^9 \, \theta$

From Statics, $\quad Q = P_{BE} + \frac{2.64}{1.76} P_{AD} = P_{BE} + 1.500\, P_{AD}$

$= [73.8\times10^6 + (1.500)(69.88\times10^6)]\,\theta = 178.62\times10^6 \, \theta$

θ_Y at yielding of link BE $\qquad \sigma_{BE} = \sigma_Y = 250\times10^6 = 352\times10^9 \, \theta_Y$

$\theta_Y = 710.23\times10^{-6}$

$Q_Y = (178.62\times10^6)(710.23\times10^{-6}) = 126.86\times10^3$ N

Since $Q = 135\times10^3 \text{ N} > Q_Y$ link BE yields $\quad \sigma_{BE} = \sigma_Y = 250$ MPa

$P_{BE} = A\,\sigma_Y = (225\times10^{-6})(250\times10^6) = 56.25\times10^3$ N

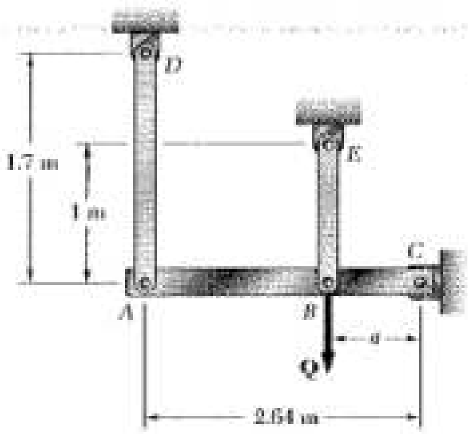
From Statics, $\quad P_{AD} = \frac{1}{1.500}(Q - P_{BE}) = 52.5\times10^3$ N

$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5\times10^3}{225\times10^{-6}} = 233.3\times10^6 = 233$ MPa

From elastic analysis of AD, $\qquad \theta = \frac{P_{AD}}{69.88\times10^6} = 751.29\times10^{-6}$ rad

$\delta_B = 1.76\,\theta = 1.322\times10^{-3}$ m $\qquad \delta_B = 1.322 \text{ mm} \downarrow$

Problem 2.115



*2.115 Solve Prob. 2.113, assuming that the magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (*a*) the residual stress in each link, (*b*) the final deflection of point B. Assume that the links are braced so that they can carry compressive forces without buckling.

2.113 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5 × 6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point B.

See solution to Problem 2.113 for the normal stresses in each link and the deflection of point B after loading

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa} \qquad \sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$

$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to PROBLEM 2.113 applies to the unloading

$$Q' = 317.06 \times 10^6\ \theta'$$

$$\theta' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

$$\sigma'_{AD} = 310.6 \times 10^9\ \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$

$$\sigma'_{BE} = 128 \times 10^9\ \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$

$$\delta'_B = 0.640\ \theta' = 524.82 \times 10^{-6} \text{ m}$$

(a) Residual stresses

$$\sigma_{AD,res} = \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^{-6} = -4.70 \times 10^6 \text{ Pa} = -4.70 \text{ MPa}$$

$$\sigma_{BE,res} = \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} = 19.34 \text{ MPa}$$

(b) $$\delta_{B,P} = \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6} = 96.71 \times 10^{-6} \text{ m} = 0.0967 \text{ mm} \downarrow$$

Problem 2.116

2.116 A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 7°C. The steel is assumed to be elastoplastic with $\sigma_Y = 250$ MPa and $E = 200$GPa. Knowing that $\alpha = 11.7 \times 10^{-6}/°C$, determine the stress in the bar (*a*) when the temperature is raised to 160°C, (*b*) after the temperature has returned to 7°C.

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{250\times10^6}{(200\times10^9)(11.7\times10^{-6})} = -106.8\,°C$$

But $\Delta T = 160 - 7 = 153\,°C > (\Delta T_Y)$

(a) Yielding occurs $\qquad \sigma = -\sigma_Y = 250\,\text{MPa}$

Cooling: $\quad (\Delta T)' = 153°C.$

$$\delta' = \delta_P' + \delta_T' = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)' =$$

$$= -(200\times10^9)(11.7\times10^{-6})(153) = -358\,\text{MPa}.$$

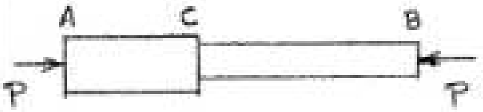
(b) Residual stress

$$\sigma_{res} = -\sigma_Y - \sigma' = -250\times10^6 + 358\times10^6 = 108\,\text{MPa}$$

108 MPa

Problem 2.117

2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150 °C. Knowing that $\alpha = 11.7 \times 10^{-6}/°\text{C}$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.



$A_{AC} = 500\times10^{-6}\ \text{m}^2 \qquad L_{AC} = 0.150\ \text{m}$

$A_{CB} = 300\times10^{-6}\ \text{m}^2 \qquad L_{CB} = 0.250\ \text{m}$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion CB.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\,\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(250\times10^{6}) = 75\times10^{3}\ \text{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

$$= \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right) = 90.812\ °\text{C}$$

Actual ΔT: $150°\text{C} - 25°\text{C} = 125°\text{C} > (\Delta T)_Y$ Yielding occurs.

For $\Delta T > (\Delta T)_Y$ $\quad P = P_Y = 75\times10^{3}\ \text{N}$

(a) $\sigma_{AC} = -\dfrac{P_Y}{A_{AC}} = -\dfrac{75\times10^{3}}{500\times10^{-6}} = -150\times10^{6}\ \text{Pa}$ $\qquad \sigma_{AC} = -150\ \text{MPa}$ ◄

$\sigma_{CB} = -\dfrac{P_Y}{A_{CB}} = -\sigma_Y$ $\qquad \sigma_{CB} = -250\ \text{MPa}$ ◄

(b) For $\Delta T > (\Delta T)_Y$, portion AC remains elastic.

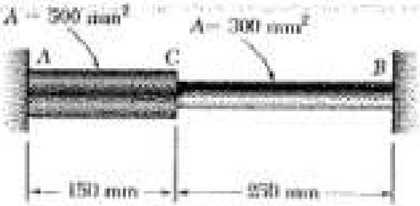
$$\delta_{C/A} = -\frac{P_Y L_{AC}}{EA_{AC}} + L_{AC}\,\alpha(\Delta T)$$

$$= -\frac{(75\times10^{3})(0.150)}{(200\times10^{9})(500\times10^{-6})} + (0.150)(11.7\times10^{-6})(125) = 106.9\times10^{-6}\ \text{m}$$

Since point A is stationary, $\delta_C = \delta_{C/A} = 106.9\times10^{-6}\ \text{m}$

$\delta_C = 0.1069\ \text{mm} \rightarrow$ ◄

Problem 2.118

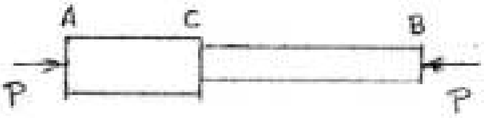


*2.118 Solve Prob. 2.117, assuming that the temperature of the rod is raised to 150 °C and then returned to 25°C.

2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with E = 200 GPa and σ_Y = 250 MPa. The temperature of both portions of the rod is then raised to 150 °C. Knowing that $\alpha = 11.7 \times 10^{-6}/°C$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

$A_{AC} = 500 \times 10^{-6}\ m^2$ $\qquad L_{AC} = 0.150\ m$

$A_{CB} = 300 \times 10^{-6}\ m^2$ $\qquad L_{CB} = 0.250\ m$



Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion CB.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\,\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

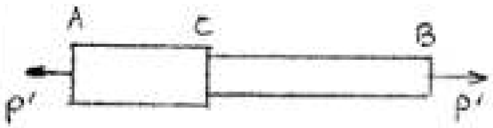
At yieding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(250\times10^{6}) = 75\times10^{3}\ N$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

$$= \frac{75\times10^3}{(0.400)(200\times10^9)(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right) = 90.812\ °C$$

Actual ΔT: $150°C - 25°C = 125°C > (\Delta T)_Y$ $\qquad$ Yielding occurs.

For $\Delta T > (\Delta T)_Y$ $\qquad P = P_Y = 75\times10^3\ N$



Cooling: $(\Delta T)' = 125\ °C$

$$P' = \frac{EL_{AB}\,\alpha(\Delta T)'}{\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)}$$

$$P' = \frac{(200\times10^9)(0.400)(11.7\times10^{-6})(125)}{\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}} = 103.235\times10^3\ N$$

Residual force: $P_{res} = P' - P_Y = 103.235\times10^3 - 75\times10^3 = 28.235\times10^3\ N$ (tension)

(a) Residual stresses.

$\sigma_{AC} = \dfrac{P_{res}}{A_{AC}} = \dfrac{28.235\times10^3}{500\times10^{-6}}$ $\qquad \sigma_{AC} = 56.5$ MPa ◄

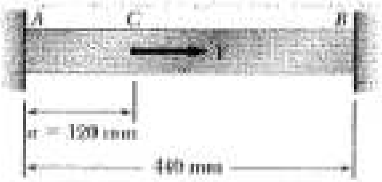
$\sigma_{CB} = \dfrac{P_{res}}{A_{CB}} = \dfrac{28.235\times10^3}{300\times10^{-6}}$ $\qquad \sigma_{CB} = 94.1$ MPa ◄

(b) Permanent deflection of point C. $\qquad \delta_C = \dfrac{P_{res}L_{AC}}{EA_{AC}}$ $\qquad \delta_C = 0.0424\ mm \rightarrow$ ◄

Problem 2.120

*2.120 Solve Prob. 2.119, assuming that $a = 180$ mm.

*2.119 Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force $\mathbf{F}$ increases from 0 to 520 kN and then decreases to zero, determine (a) the permanent deflection of point C, (b) the residual stress in the bar.



$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion AC: $P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$
$= 300 \times 10^3$ N

For equilibrium $\quad F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$
$$= -220 \times 10^3 \text{ N}$$

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.238333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}$$

Unloading

$$\delta_C' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} = \frac{(F - P_{AC}') L_{CB}}{EA} \therefore \qquad P_{AC}'\left(\frac{L_{AC}}{EA} + \frac{L_{CB}}{EA}\right) = \frac{F L_{CB}}{EA}$$

$$P_{AC}' = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.180)}{0.440} = 307.273 \times 10^3 \text{ N}$$

$$P_{CB}' = P_{AC}' - F = 307.273 \times 10^3 - 520 \times 10^3 = -212.727 \times 10^3 \text{ N}$$

$$\delta_C' = \frac{(307.273 \times 10^3)(0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.230455 \times 10^{-3} \text{ m}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{307.273 \times 10^3}{1200 \times 10^{-6}} = 256.061 \times 10^6 \text{ Pa}$$

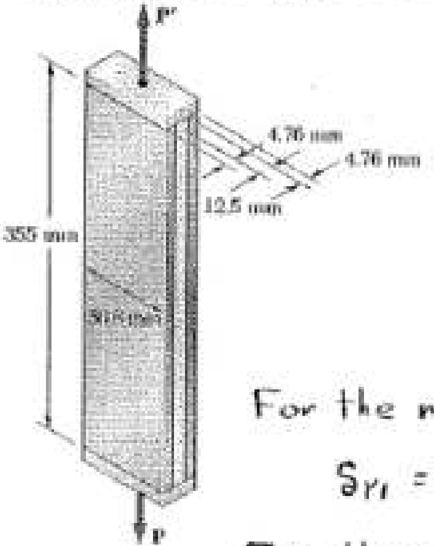
$$\sigma_{CB}' = \frac{P_{CB}'}{A} = \frac{-212.727 \times 10^3}{1200 \times 10^{-6}} = -177.273 \times 10^6 \text{ P}$$

(a) $\quad \delta_{C,p} = \delta_C - \delta_C' = 0.238333 \times 10^{-3} - 0.230455 \times 10^{-3} = 0.00788 \times 10^{-3}$ m
$= 0.00788$ mm ◀

(b) $\quad \sigma_{AC,res} = \sigma_{AC} - \sigma_{AC}' = 250 \times 10^6 - 256.061 \times 10^6 = -6.06 \times 10^6$ Pa
$= -6.06$ MPa ◀

$\sigma_{CB,res} = \sigma_{CB} - \sigma_{CB}' = -183.333 \times 10^6 + 177.273 \times 10^6 = -6.06 \times 10^6$ Pa
$= -6.06$ MPa ◀

Problem 2.122



*2.122 For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 10$ mm and is then decreased back to zero.

2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

For the mild steel $\quad A_1 = (0.0125)(0.0508) = 635 \times 10^{-6}\ m^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(0.355)(345 \times 10^6)}{200 \times 10^9} = 0.612 \times 10^{-3}\ m$$

For the tempered steel $\quad A_2 = 2(0.00476)(0.0508) = 483.6 \times 10^{-6}\ m^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(0.355)(690 \times 10^6)}{200 \times 10^9} = 1.225 \times 10^{-3}\ m$$

Total area: $\quad A = A_1 + A_2 = 1118.6 \times 10^{-6}\ m^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ The mild steel yields. Tempered steel is elastic.

Forces $\quad P_1 = A_1\sigma_{Y1} = (635 \times 10^{-6})(345 \times 10^6) = 219.075$ kN

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(200 \times 10^9)(483.6 \times 10^{-6})(0.001016)}{0.355} = 276.81 \text{ kN}$$

Stresses $\quad \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 345$ MPa

$$\sigma_2 = \frac{P_2}{A_2} = \frac{276.81 \times 10^3}{483.6 \times 10^{-6}} = 572.4 \text{ MPa}$$

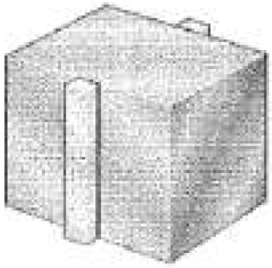
Unloading $\quad \sigma' = \frac{P}{A} = \frac{495900}{1118.6 \times 10^{-6}} = 443.3$ MPa.

Residual stresses

$$\sigma_{1,res} = \sigma_1 - \sigma' = 345 - 443.3 = -98.3 \text{ MPa}$$

$$\sigma_{2,res} = \sigma_2 - \sigma' = 572.4 - 443.3 = 129.1 \text{ MPa}$$ ◄

Problem 2.123



*2.123 A narrow bar of aluminum is bonded to the side of a thick steel plate as shown. Initially, at $T_1 = 21°C$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 400 MPa. Assume $\alpha_a = 23 \times 10^{-6}/°C$ for the aluminum and $\alpha_s = 11.7 \times 10^{-6}/°C$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 75$ GPa and $\sigma_Y = 400$ MPa. (*Hint:* Neglect the small stresses in the plate.)

Determine temperature change to cause yielding

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T_Y) = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{400 \times 10^6}{(75 \times 10^9)(23 - 11.7)(10^{-6})} = 472°C$$

(a) $T_{2Y} = T_1 + (\Delta T)_Y = 21 + 472 = 493°C$ ◄

After yielding

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{res} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set $\sigma_{res} = -\sigma_Y$

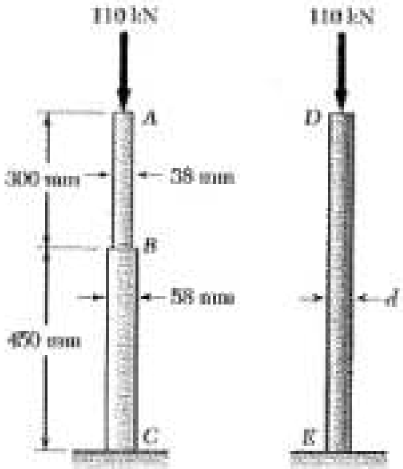
$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{400 \times 10^6}{(75 \times 10^9)(23 - 11.7)(10^{-6})} = 944°C$$

(b) $T_2 = T_1 + \Delta T = 21 + 944 = 965°C$ ◄

If $T_2 > 965°C$ the aluminum bar will most likely yield in compression.

Problem 2.124



2.124 The aluminum rod ABC (E = 70 GPa), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE (E = 200 GPa) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 165 MPa.

Deformation of aluminum rod

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{110\times10^3}{70\times10^9}\left(\frac{0.3}{\frac{\pi}{4}(0.038)^2} + \frac{0.45}{\frac{\pi}{4}(0.058)^2}\right) = 0.683\times10^{-3}\ \text{m.}$$

Steel rod $\delta = 0.683\times10^{-3}\ \text{m.}$

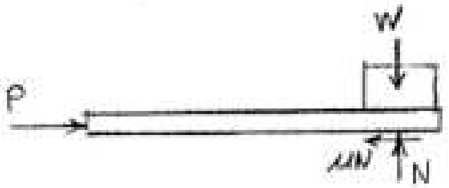
$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(110\times10^3)(0.75)}{(200\times10^9)(0.683\times10^{-3})} = 604\times10^{-6}\ \text{m}^2$$

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{110\times10^3}{165\times10^6} = 0.667\times10^{-3}\ \text{m}^2$$

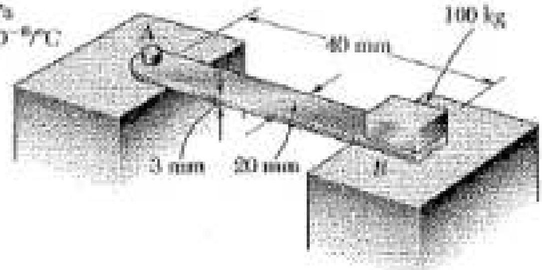
Required area is the larger value $A = 0.667\times10^{-3}\ \text{m}^2.$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.667\times10^{-3})}{\pi}} = 0.029\ \text{m} = 29\ \text{mm}$$

Problem 2.125



2.125 The brass strip *AB* has been attached to a fixed support at *A* and rests on a rough support at *B*. Knowing that the coefficient of friction is 0.60 between the strip and the support at *B*, determine the decrease in temperature for which slipping will impend.



$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \qquad N = W$

$\xrightarrow{+} \Sigma F_x = 0: \quad P - \mu N = 0 \qquad P = \mu W = \mu m g$

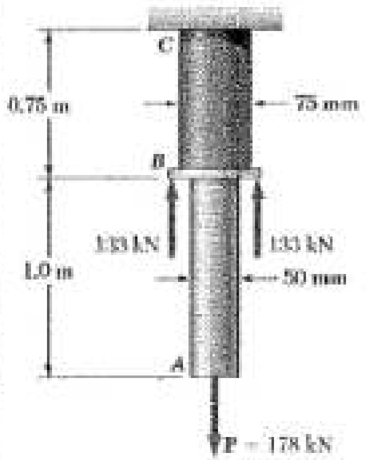
$\delta = -\frac{PL}{EA} + L\alpha(\Delta T) = 0 \qquad \Delta T = \frac{P}{EA\alpha} = \frac{\mu m g}{EA\alpha}$

Data: $\mu = 0.60 \qquad A = (20)(3) = 60\ \text{mm}^2 = 60 \times 10^{-6}\ \text{m}^2$

$m = 100\ \text{kg} \qquad g = 9.81\ \text{m/s}^2 \qquad E = 105 \times 10^9\ \text{Pa}$

$\Delta T = \frac{(0.60)(100)(9.81)}{(105 \times 10^9)(60 \times 10^{-6})(20 \times 10^{-6})} = 4.67°C$

Problem 2.126



2.126 Two solid cylindrical rods are joined at *B* and loaded as shown. Rod *AB* is made of steel (E = 200 GPa), and rod *BC* of brass (E = 105 GPa). Determine (*a*) the total deformation of the composite rod *ABC*, (*b*) the deflection of point *B*.

Portion AB: $P_{AB} = 178\ \text{kN} \qquad L_{AB} = 1\ \text{m} \qquad d = 50\ \text{mm}$

$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.05)^2 = 0.001963\ \text{m}^2 \quad E_{AB} = 200\ \text{GPa}$

$\delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(178 \times 10^3)(1)}{(200 \times 10^9)(0.001963)} = 0.45\ \text{m} \times 10^{-3}\ \text{m}$

Portion BC: $P_{BC} = -88\ \text{kN} \qquad L_{BC} = 0.75\ \text{m}, \quad d = 75\ \text{mm}$

$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.075)^2 = 0.00442\ \text{m}^2 \qquad E_{BC} = 105\ \text{GPa}$

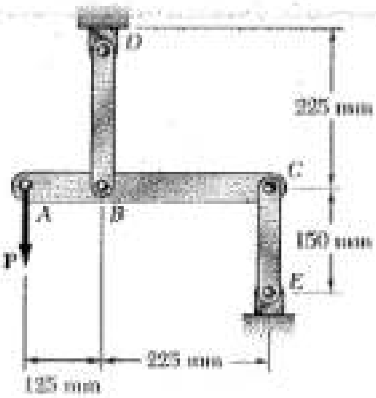
$\delta_{BC} = \frac{P_{BC} L_{BC}}{E_{BC} A_{BC}} = \frac{(-88 \times 10^3)(0.75)}{(105 \times 10^9)(0.00442)} = -0.142 \times 10^{-3}\ \text{m}$

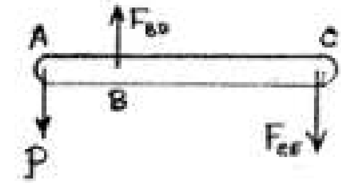
(a) $\delta = \delta_{AB} + \delta_{BC} = 0.45 - 0.142 = 0.308\ \text{mm} \qquad \delta = 0.308\ \text{mm}$

(b) $\delta_B = -\delta_{BC} \qquad \delta_B = 0.142\ \text{mm}$

Problem 2.127

2.127 Link BD is made of brass (E = 105 GPa) and has a cross-sectional area of 250 mm². Link CE is made of aluminum (E = 72 GPa) and has a cross-sectional area of 450 mm². Determine the maximum force **P** that can be applied vertically at point A if the deflection of A is not to exceed 0.35 mm.





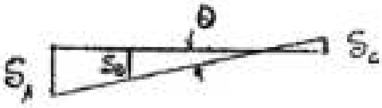
Use member ABC as a free body

$\circlearrowleft \Sigma M_C = 0, \quad 350P - 225F_{BD} = 0, \qquad F_{BD} = 1.5556\ P$

$\circlearrowleft \Sigma M_B = 0 \quad 125P \quad 225F_{CE} = 0, \qquad F_{CE} = 0.5556\ P$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.5556P)(0.225)}{(105\times10^9)(250\times10^{-6})} = 13.3334\times10^{-9}\ P \downarrow$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.5556)(0.15)}{(72\times10^9)(450\times10^{-6})} = 2.5722\times10^{-9}\ P \uparrow$$



Deformation Diagram

From the deformation diagram

$$\text{Slope } \theta = \frac{\delta_B + \delta_C}{l_{BC}} = \frac{15.9056\times10^{-9}\ P}{0.225}$$

$$= 70.6916\times10^{-9}\ P$$

$$\delta_A = \delta_B + l_{AB}\theta$$

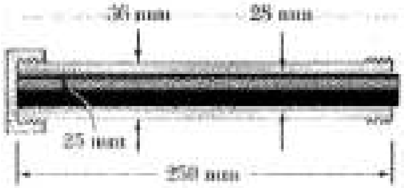
$$= 13.3334\times10^{-9}P + (0.125)(70.69\times10^{-9})P$$

$$= 22.17\times10^{-9}P$$

Apply displacement limit $\delta_A = 0.00035 = 22.17\times10^{-9}P$.

$$P = \frac{0.00035}{22.17\times10^{-9}} = 15.787\ \text{kN} = 15.79\ \text{kN}$$

Problem 2.128



2.128 A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12\times10^{-6} \text{ m}^2$$

$$A_{rod} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87\times10^{-6} \text{ m}^2$$

$$\delta_{tube} = \frac{PL}{E_{tube}A_{tube}} = \frac{P(0.250)}{(70\times10^9)(402.12\times10^{-6})} = 8.8815\times10^{-9}\,P$$

$$\delta_{rod} = -\frac{PL}{E_{rod}A_{rod}} = \frac{P(0.250)}{(105\times10^6)(490.87\times10^{-6})} = -4.8505\times10^{-9}\,P$$

$$\delta^* = (\tfrac{1}{4}\text{ turn})\times 1.5 \text{ mm} = 0.375 \text{ mm} = 375\times10^{-6} \text{ m}$$

$$\delta_{tube} = \delta^* + \delta_{rod} \quad \text{or} \quad \delta_{tube} - \delta_{rod} = \delta^*$$

$$8.8815\times10^{-9}P + 4.8505\times10^{-9}P = 375\times10^{-6}$$

$$P = \frac{0.375\times10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308\times10^3 \text{ N}$$

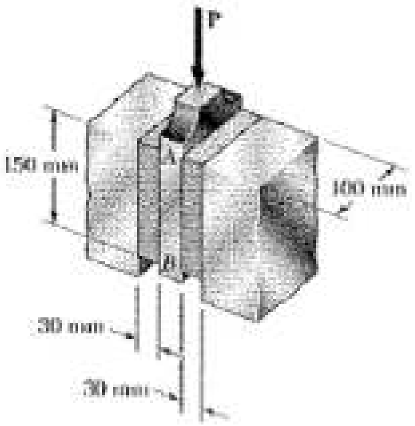
(a) $$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{27.308\cdot10^3}{402.12\times10^{-6}} = 67.9\times10^6 \text{ Pa} = 67.9 \text{ MPa}$$

$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{27.308\times10^3}{490.87\times10^{-6}} = -55.6\times10^6 \text{ Pa} = -55.6 \text{ MPa}$$

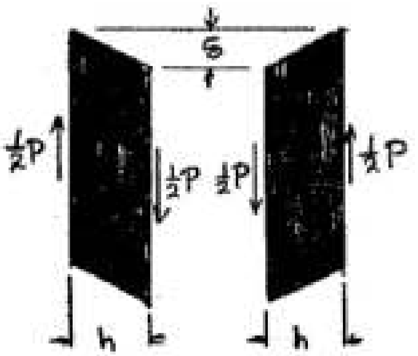
(b) $$\delta_{tube} = (8.8815\times10^{-9})(27.308\times10^3) = 242.5\times10^{-6} \text{ m} = 0.2425 \text{ mm}$$

$$\delta_{rod} = -(4.8505\times10^{-9})(27.308\times10^3) = -132.5\times10^{-6} \text{ m} = -0.1325 \text{ mm}$$

Problem 2.132



2.132 A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 19$ MPa bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.



Shearing strain $\quad \gamma = \dfrac{\delta}{h}$

Shearing stress $\quad \tau = G\gamma = \dfrac{G\delta}{h}$

Force $\quad \frac{1}{2}P = A\tau = \dfrac{GA\delta}{h}$

$$P = \frac{2GA\delta}{h}$$

Effective spring constant

$$k = \frac{P}{\delta} = \frac{2GA}{h}$$

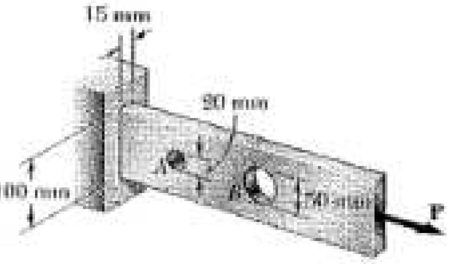
Data: $\quad G = 19$ MPa

$A = (0.15)(0.1) = 0.015 \text{ m}^2$

$h = 30$ mm

$$k = \frac{2(19\times10^6)(0.015)}{0.03} = 19 \text{ N/m}.$$

Problem 2.133



2.133 Knowing that $\sigma_{all} = 120$ MPa, determine the maximum allowable value of the centric axial load **P**.

At hole A: $r = \frac{1}{2}(20) = 10$ mm

$d = D - 2r = 100 - 20 = 80$ mm

$A_{net} = dt = (80)(15) = 1200 \text{ mm}^2 = 1.20 \times 10^{-3} \text{ m}^2$

$\dfrac{r}{d} = \dfrac{10}{80} = 0.125$

From Fig. 2.64a, $K = 2.65$

$\sigma_{max} = \dfrac{KP}{A_{net}} \qquad P = \dfrac{A_{net}\sigma_{max}}{K} = \dfrac{(1.20\times10^{-3})(120\times10^6)}{2.65} = 54.3\times10^3$ N

At hole B: $r = \frac{1}{2}(50) = 25$ mm $\qquad d = 100 - 50 = 50$ mm

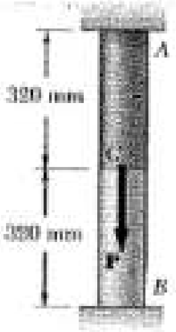
$A_{net} = (50)(15) = 750 \text{ mm}^2 = 750\times10^{-6} \text{ m}^2, \quad \dfrac{r}{d} = \dfrac{25}{50} = 0.50, \quad K = 2.16$

$P = \dfrac{(750\times10^{-6})(120\times10^6)}{2.16} = 41.7\times10^3$ N

Allowable value of P is the smaller. $\qquad P = 41.7\times10^3 \text{ N} = 41.7$ kN

Problem 2.134

2.134 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 2950 mm^2. Portion AC is made of a mild steel with E = 200 GPa and σ_Y = 250 MPa, and portion CB is made of a high-strength steel with E = 200 GPa and σ_Y = 345 MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 1625 kN and then reduced back to zero, (*b*) the permanent deflection of C.

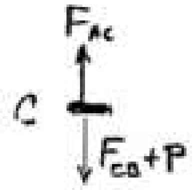


Displacement at C to cause yielding of AC

$$\delta_{C,Y} = L_{AC}\,\varepsilon_{Y,AC} = \frac{L_{AC}\,\sigma_{Y,AC}}{E} = \frac{(0.320)(250\times10^6)}{200\times10^9} = 0.400\times10^{-3}\ \text{m}$$

Corresponding force $\quad F_{AC} = A\,\sigma_{Y,AC} = (2950\times10^{-6})(250\times10^6) = 737.5\times10^3\ \text{N}$

$$F_{CB} = -\frac{EA\,\delta_C}{L_{CB}} = -\frac{(200\times10^9)(2950\times10^{-6})(0.400\times10^{-3})}{0.320} = -737.5\times10^3\ \text{N}$$



For equilibrium of element at C,

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 1475\times10^3\ \text{N}$$

Since applied load $P = 1625\times10^3\ \text{N} > 1475\times10^3\ \text{N}$, portion AC yields.

$$F_{CB} = F_{AC} - P = 737.5\times10^3 - 1625\times10^3\ \text{N} = -887.5\times10^3\ \text{N}$$

(a) $$\delta_C = -\frac{F_{CB}L_{CB}}{EA} = \frac{(887.5\times10^3)(0.320)}{(200\times10^9)(2950\times10^{-6})} = 0.48136\times10^{-3}\ \text{m} = 0.481\ \text{mm}\downarrow$$

Maximum stresses. $\quad \sigma_{AC} = \sigma_{Y,AC} = 250\ \text{MPa}$

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{887.5\times10^3}{2950\times10^{-6}} = -300.81\times10^6\ \text{Pa} = -301\ \text{MPa}$$

(b) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore\ P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{CB}} = -P'_{AC}$$

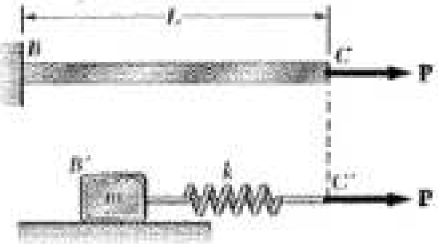
$$P' = 1625\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \qquad P'_{AC} = 812.5\times10^3\ \text{N}$$

$$\delta' = \frac{(812.5\times10^3)(0.320)}{(200\times10^9)(2950\times10^{-6})} = 0.44068\times10^{3}\ \text{m}$$

$$\delta_p = \delta_m - \delta' = 0.48136\times10^{-3} - 0.44068\times10^{-3} = 0.04068\times10^{-3}\ \text{m}$$

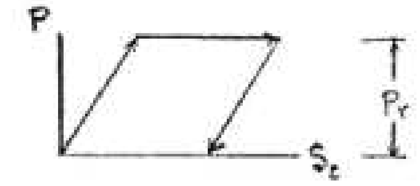
<u>Permanent deflection of C</u> $\qquad \delta_p = 0.0407\ \text{mm}\downarrow$

Problem 2.135



2.135 The uniform rod BC has a cross-sectional area A and is made of a mild steel which can be assumed to be elastoplastic with a modulus of elasticity E and a yield strength σ_Y. Using the block-and-spring system shown, it is desired to simulate the deflection of end C of the rod as the axial force **P** is gradually applied and removed, that is, the deflection of points C and C' should be the same for all values of P. Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (*a*) the required mass m of the block, (*b*) the required constant k of the spring.

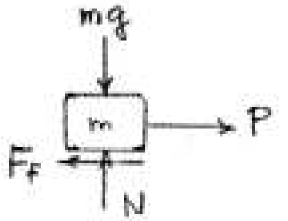
Force-deflection diagram for point C of rod BC.



For $P < P_Y = A\sigma_Y$

$$\delta_C = \frac{PL}{EA} \qquad P = \frac{EA}{L}\delta_C$$

$$P_{max} = P_Y = A\sigma_Y$$



Force-deflection diagram for point C' of block and spring system.

$+\uparrow \Sigma F_y = 0: \quad N - mg = 0 \qquad N = mg$

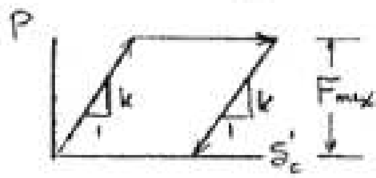
$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_f = 0 \qquad P = F_f$

If block does not move, i.e $F_f < \mu N = \mu mg$ or $P < \mu mg$,

then $\delta_C' = \dfrac{P}{k}$ or $P = k\delta_C'$

If $P = \mu mg$, then slip at $P = F_m = \mu mg$ occurs.

If the force P is the removed, the spring returns to its initial length.



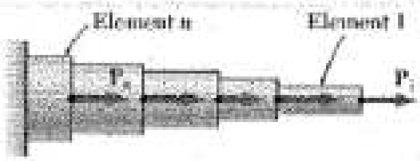
(a) Equating P_Y and F_{max},

$$A\sigma_Y = \mu mg \qquad m = \frac{A\sigma_Y}{\mu g}$$

(b) Equating slopes,

$$k = \frac{EA}{L}$$

PROBLEM 2.C1



2.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (*a*) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (*b*) Use this program to solve Probs. 2.20 and 2.126.

SOLUTION

FOR EACH ELEMENT, ENTER

L_i, A_i, E_i

COMPUTE DEFORMATION

UPDATE AXIAL LOAD $P = P + P_i$

COMPUTE FOR EACH ELEMENT

$\sigma_i = P/A_i$

$\delta_i = PL_i/A_iE_i$

TOTAL DEFORMATION:

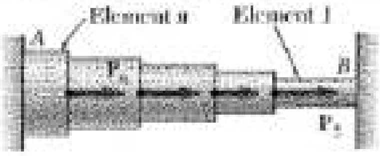
UPDATE THROUGH n ELEMENTS

$\delta = \delta + \delta_i$

PROGRAM OUTPUT

```
Problem  2.20
  Element  Stress (MPa)  Deformation (mm)

     1       19.0986        .1091
     2      -12.7324       -.0909
  Total Deformation =     .0182 mm

Problem  2.126
  Element  Stress (MPa)  Deformation (mm)

     1        90.6775        0.4534
     2       -19.9095        0.1422
  Total Deformation =  0.3112
```

PROBLEM 2.C2



2.C2 Rod AB is horizontal with both ends fixed; it consists of n elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (*a*) Write a computer program which can be used to determine the reactions at A and B, the average normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.41 and 2.42.

SOLUTION

WE CONSIDER THE REACTION AT B REDUNDANT AND RELEASE THE ROD AT B

COMPUTE δ_B WITH $R_B = 0$

FOR EACH ELEMENT, ENTER

L_i, A_i, E_i

UPDATE AXIAL LOAD

$P = P + P_i$

COMPUTE FOR EACH ELEMENT

$\sigma_i = P/A_i$

$\delta_i = PL_i/A_iE_i$

UPDATE TOTAL DEFORMATION

$\delta_B = \delta_B + \delta_i$

COMPUTE δ_B DUE TO UNIT LOAD AT B

UNIT $\sigma_i = 1/A_i$

UNIT $\delta_i = L_i/A_iE_i$

UPDATE TOTAL UNIT DEFORMATION

UNIT δ_B = UNIT δ_B + UNIT δ_i

SUPERPOSITION

FOR TOTAL DISPLACEMENT AT B = ZERO

$\delta_B + R_B$ UNIT $\delta_B = 0$

SOLVING:

$R_B = -\delta_B/$UNIT δ_B

THEN:

$R_A = \Sigma P_i + R_B$

CONTINUED

FOR EACH ELEMENT

$$\sigma = \sigma_i + R_B \text{ UNIT } \sigma_i$$

$$\delta = \delta_i + R_B \text{ UNIT } \delta_i$$

PROGRAM OUTPUT

```
Problem  2.41
     RA =   -62.809 kN
     RB =   -37.191 kN
   Element  Stress (MPa)  Deformation (mm)

     1        -52.615          -.05011
     2          3.974           .00378
     3          2.235           .00134
     4         49.982           .04498

Problem  2.42
     RA =   -45.479 kN
     RB =   -54.521 kN
   Element  Stress (MPa)  Deformation (mm)

     1        -77.131          -.03857
     2        -20.542          -.01027
     3        -11.555          -.01321
     4         36.191           .06204
```

PROBLEM 2.C3

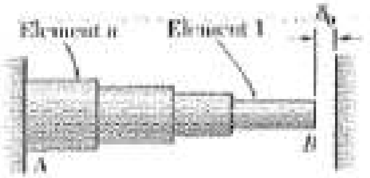


Fig. P2.C3

2.C3 Rod AB consists of n elements, each of which is homogeneous and of uniform cross section. End A is fixed, while initially there is a gap δ_0 between end B and the fixed vertical surface on the right. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and its coefficient of thermal expansion by α_i. After the temperature of the rod has been increased by ΔT, the gap at B is closed and the vertical surfaces exert equal and opposite forces on the rod. (*a*) Write a computer program which can be used to determine the magnitude of the reactions at A and B, the normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.51, 2.59, and 2.60.

SOLUTION

WE COMPUTE THE DISPLACEMENTS AT B ASSUMING THERE IS NO SUPPORT AT B:

ENTER L_i, A_i, E_i, α_i

ENTER TEMPERATURE CHANGE T

COMPUTE FOR EACH ELEMENT

$$\delta_i = \alpha_i L_i T$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE δ_B DUE TO UNIT LOAD AT B

$$\text{UNIT } \delta_i = L_i / A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

COMPUTE REACTIONS

FROM SUPERPOSITION

$$R_B = (\delta_B - \delta_0) / \text{UNIT } \delta_B$$

THEN

$$R_A = - R_B$$

FOR EACH ELEMENT

$$\sigma_i = - R_B / A_i$$

$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

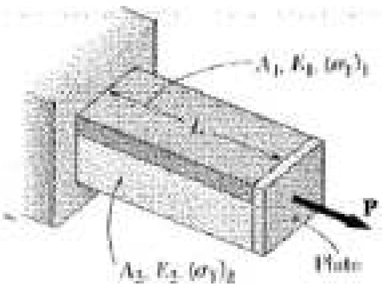
CONTINUED

PROBLEM 2.C3 CONTINUED

PROGRAM OUTPUT

```
Problem  2.51
 R =    125.628 kN

 Element     Stress (MPa)    Deform.(microm)

    1          -44.432           .500
    2          -99.972          -.500

Problem  2.59
 R =     172.835 kN

 Element     Stress (MPa)    Deform.(mm)

    1          -115.223        0.236
    2           -96.019        0.24

Problem  2.60
 R =    232.390 kN

 Element     Stress (MPa)    Deform.(microm)

    1         -116.195       363.220
    2         -290.487       136.780
```

PROBLEM 2.C4



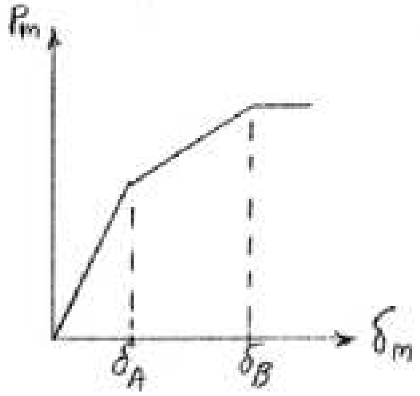
2.C4 Bar *AB* has a length *L* and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load **P** which is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (*a*) Write a computer program which, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (*b*) Use this program to solve Probs. 2.111, and 2.112.

SOLUTION

NOTE: THE FOLLOWING ASSUMES $(\sigma_Y)_1 < (\sigma_Y)_2$

DISPLACEMENT INCREMENT

$$\delta_m = 0.05\,(\sigma_Y)_2 L/E_2$$



DISPLACEMENTS AT YIELDING

$$\delta_A = (\sigma_Y)_1 L/E_1 \qquad \delta_B = (\sigma_Y)_2 L/E_2$$

FOR EACH DISPLACEMENT

IF $\delta_m < \delta_A$:

$$\sigma_1 = \delta_m E_1/L$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = (\delta_m/L)(A_1E_1 + A_2E_1)$$

IF $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = A_1\sigma_1 + (\delta_m/L)A_2E_2$$

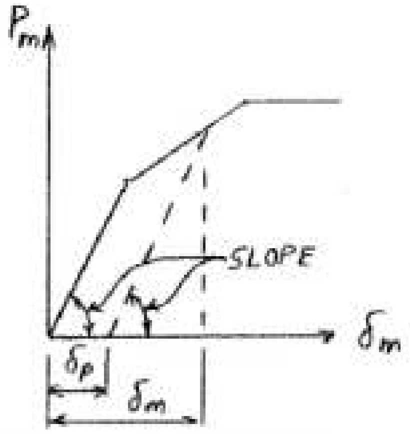
IF $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_Y)_1 \qquad \sigma_2 = (\sigma_Y)_2$$
$$P_m = A_1\sigma_1 + A_2\sigma_2$$

PERMANENT DEFORMATIONS, RESIDUAL STRESSES

SLOPE OF FIRST (ELASTIC) SEGMENT

$$SLOPE = (A_1E_1 + A_2E_2)/L$$
$$\delta_p = \delta_m - (P_m/SLOPE)$$
$$(\sigma_1)_{res} = \sigma_1 - (E_1 P_m/(L\ SLOPE))$$
$$(\sigma_2)_{res} = \sigma_2 - (E_2 P_m/(L\ SLOPE))$$



CONTINUED

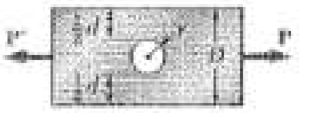
PROBLEM 2.C4 CONTINUED

PROGRAM OUTPUT

Problems 2.109 and 2.110

DM mm	PM kN	SIGM(1) MPa	SIGM(2) MPa	DP mm	SIGR(1) MPa	SIGR(2) MPa	
0	0	0	0	0	0	0	
0.368	233.571	207.000	207.000	0	0	0	
0.796	436.000	345.000	448.562	0.104	-44.892	58.580	◄ P2.110
1.016	495.883	345.000	572.395	0.229	-98.560	128.835	◄ P2.109
1.471	556.000	345.000	700.000	0.593	-150.248	198.305	

PROBLEM 2.C5



2.C5 The plate has a hole centered across the width. The stress concentration factor for a flat bar under axial loading with a centric hole is:

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where r is the radius of the hole and D is the width of the bar. Write a computer program to determine the allowable load **P** for the given values of r, D, the thickness t of the bar, and the allowable stress σ_{all} of the material. Knowing that $r = 6$ mm, D = 75 mm and $\sigma_{all} = 110$ MPa, determine the allowable load **P** for values of t from 3 mm to 18 mm, using 3 mm increments.

SOLUTION

ENTER

$r,\ D,\ t,\ \sigma_{all}$

COMPUTE K

$RD = 2.0\ r/D$

$K = 3.00 - 3.13\ RD + 3.66\ RD^2 - 1.53\ RD^3$

COMPUTE AVERAGE STRESS

$\sigma_{ave} = \sigma_{all}/K$

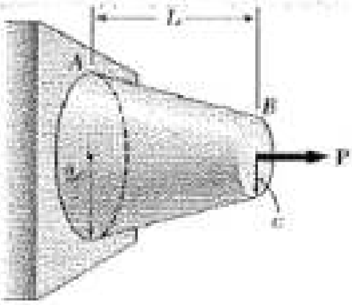
ALLOWABLE LOAD

$P_{all} = \sigma_{ave}\ (D - 2.0\,r)\,t$

PROGRAM OUTPUT

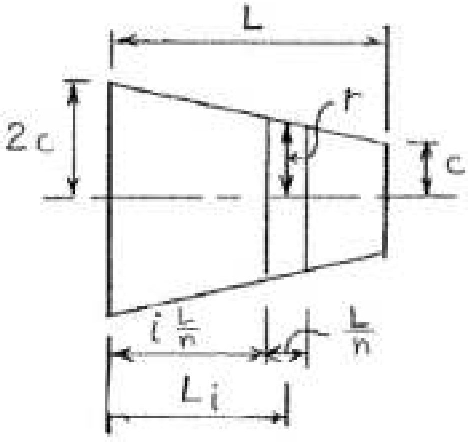
Radius (mm)	Allowable Load (kN)
3	16.405
6	16.023
9	15.368
12	14.391
15	13.165
18	11.721

PROBLEM 2.C6



2.C6 A solid truncated cone is subjected to an axial force **P** as shown. The exact elongation is $(PL)/(2\pi c^2 E)$. By replacing the cone by n circular cylinders of equal thickness, write a computer program that can be used to calculate the elongation of the truncated cone. What is the percentage error in the answer obtained from the program using (a) $n = 6$, (b) $n = 12$, (c) $n = 60$.

SOLUTION



FOR $i = 1$ TO n:

$$L_i = (i + 0.5)(L/n)$$

$$r_i = 2c - c(L_i/L)$$

AREA:

$$A = \pi r_i^2$$

DISPLACEMENT:

$$\delta = \delta + P(L/n)/(AE)$$

EXACT DISPLACEMENT:

$$\delta_{EXACT} = PL/(2.0\pi c^2 E)$$

PERCENTAGE ERROR:

$$PERCENT = 100(\delta - \delta_{EXACT})/\delta_{EXACT}$$

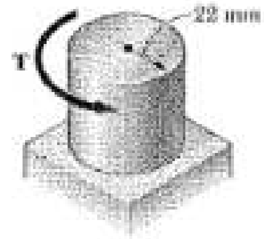
PROGRAM OUTPUT

n	Approximate	Exact	Percent
6	0.15852	0.15915	-.40083
12	0.15899	0.15915	-.10100
60	0.15915	0.15915	-.00405

Chapter 3

Problem 3.1

3.1 For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude $T = 1.5$ kN · m.



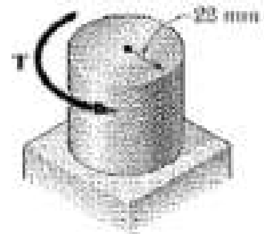
$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4$$

$$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(1500)}{\pi(0.022)^3} = 89.682\times10^6$$

$$\tau_{max} = 89.7 \text{ MPa}$$

Problem 3.2

3.2 Determine the torque **T** that causes a maximum shearing stress of 80 MPa in the steel cylindrical shaft shown.



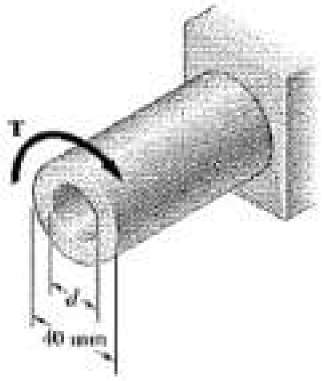
$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4$$

$$T = \frac{\pi}{2}c^3\tau_{max} = \frac{\pi}{2}(0.022)^3(80\times10^6)$$

$$= 1338.1 \text{ N}\cdot\text{m} \qquad T = 133.8 \text{ kN}\cdot\text{m}$$

Problem 3.3

3.3 Knowing that the internal diameter of the hollow shaft shown is $d = 22$ mm, determine the maximum shearing stress caused by a torque of magnitude $T = 900$ N · m.



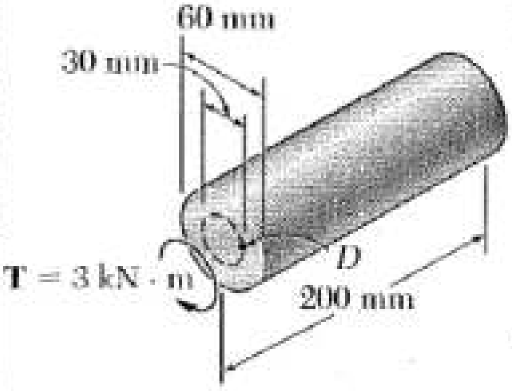
$$c_2 = \frac{1}{2}d_2 = (\tfrac{1}{2})(40) = 20 \text{ mm} \qquad c = 0.02 \text{ m}$$

$$c_1 = \frac{1}{2}d_1 = (\tfrac{1}{2})(22) = 11 \text{ mm}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(20^4 - 11^4) = 228329 \text{ mm}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(900)(0.02)}{228329\times10^{-12}} = 78.8 \text{ MPa}$$

Problem 3.6



3.6 A torque $T = 3$ kN · m is applied to the solid bronze cylinder shown. Determine (*a*) the maximum shearing stress, (*b*) the shearing stress at point *D* which lies on a 15-mm-radius circle drawn on the end of the cylinder, (*c*) the percent of the torque carried by the portion of the cylinder within the 15-mm radius.

(a) $c = \frac{1}{2}d = 30 \text{ mm} = 30 \times 10^{-3} \text{ m}$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30 \times 10^{-3})^4 = 1.27235 \times 10^{-6} \text{ m}^4$

$T = 3 \text{ kN} = 3 \times 10^3 \text{ N}$

$$\tau_m = \frac{Tc}{J} = \frac{(3 \times 10^3)(30 \times 10^{-3})}{1.27235 \times 10^{-6}} = 70.736 \times 10^6 \text{ Pa}$$

$\tau_m = 70.7 \text{ MPa}$ ◀

(b) $\rho_D = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$

$$\tau_D = \frac{\rho_D}{c}\tau = \frac{(15 \times 10^{-3})(70.736 \times 10^{-6})}{(30 \times 10^{-3})} \qquad \tau_D = 35.4 \text{ MPa}$$ ◀

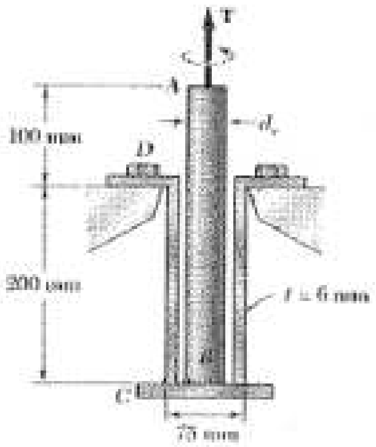
(c) $\tau_D = \frac{T_D \rho_D}{J_D} \qquad T_D = \frac{J_D \tau_D}{\rho_D} = \frac{\pi}{2}\rho_D^3 \tau_D$

$T_D = \frac{\pi}{2}(15 \times 10^{-3})^3(35.368 \times 10^6) = 187.5 \text{ N} \cdot \text{m}$

$$\frac{T_D}{T} \times 100\% = \frac{187.5}{3 \times 10^3}(100\%) = \qquad 6.25\%$$ ◀

Problem 3.7

3.7 The solid spindle AB is made of a steel with an allowable shearing stress of 84 MPa, and sleeve CD is made of a brass with an allowable shearing stress of 50 MPa. Determine (*a*) the largest torque **T** that can be applied at A if the allowable shearing stress is not to be exceeded in sleeve CD, (*b*) the corresponding required value of the diameter d_s of spindle AB.



(a) Sleeve CD: $c_2 = \frac{1}{2} d_2 = \frac{1}{2}(75) = 37.5$ mm

$c_1 = c_2 - t = 37.5 - 6 = 31.5$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0375^4 - 0.0315^4) = 1.56 \times 10^{-6}\ \text{m}^4$

$\tau_{max} = \dfrac{Tc_2}{J}$

$T_{CD} = \dfrac{J\tau_{CD}}{c_2} = \dfrac{(1.56 \times 10^{-6})(50 \times 10^6)}{0.0375} = 2080\ \text{N.m.}$

For equilibrium $\qquad T = 2.08$ kN

(b) Solid spindle AB: $\qquad T = 2080$ N.m.

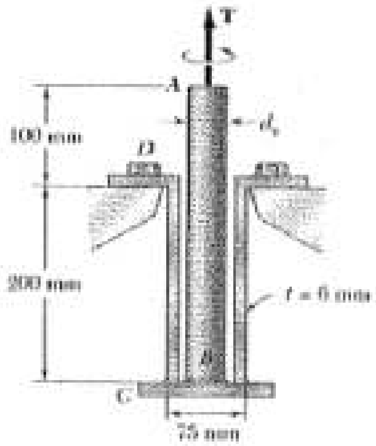
$\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$c = \sqrt[3]{\dfrac{2T}{\pi \tau}} = \sqrt[3]{\dfrac{(2)(2080)}{\pi(84 \times 10^6)}} = 0.025$ m $= 25$ mm

$d_s = 2c = (2)(25)$ $\qquad d_s = 50$ mm

Problem 3.8

3.8 The solid spindle AB has a diameter $d_s = 38$ mm and is made of a steel with an allowable shearing stress of 84 MPa, while sleeve CD is made of a brass with an allowable shearing stress of 50 MPa. Determine the largest torque **T** that can be applied at A.



Solid spindle AB: $\quad c = \frac{1}{2} d_s = \frac{1}{2}(38) = 19$ mm

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.019)^4 = 204.7 \times 10^{-9}$

$\tau_{max} = \dfrac{Tc}{J}$

$T_{AB} = \dfrac{J\tau_{all}}{c} = \dfrac{(204.7 \times 10^{-9})(84 \times 10^6)}{0.019} = 905$ Nm.

Sleeve CD: $\quad c_2 = \frac{1}{2} d_2 = \frac{1}{2}(75) = 37.5$ mm.

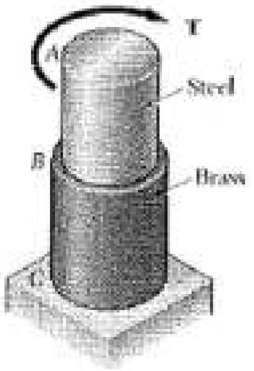
$c_1 = c_2 - t = 37.5 - 6 = 31.5$ mm.

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0375^4 - 0.0315^4) = 1.56 \times 10^{-6}\ \text{m}^4$

$T_{CD} = \dfrac{J\tau_{all}}{c_2} = \dfrac{(1.56 \times 10^{-6})(50 \times 10^6)}{0.0375} = 2080$ Nm

Allowable value of torque T is the smaller. $\qquad T = 2.08$ kN.m

Problem 3.15



3.15 The allowable shearing stress is 100 MPa in the steel rod *AB* and 60 MPa in the brass rod *BC*. Knowing that a torque of magnitude $T = 900$ N · m is applied at *A* and neglecting the effect of stress concentrations, determine the required diameter of (*a*) rod AB, (*b*) rod *BC*.

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}c^4, \quad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

Shaft AB: $\tau_{max} = 100 \text{ MPa} = 100 \times 10^6 \text{ Pa}$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(100 \times 10^6)}} = 17.894 \times 10^{-3} \text{ m} = 17.894 \text{ mm}$$

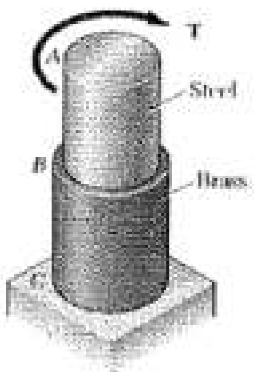
$$d_{AB} = 2c = 35.8 \text{ mm}$$

Shaft BC: $\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(60 \times 10^6)}} = 21.216 \times 10^{-3} \text{ m} = 21.216 \text{ mm}$$

$$d_{BC} = 2c = 42.4 \text{ mm}$$

Problem 3.16



3.16 The allowable shearing stress is 100 MPa in the 36-mm-diameter steel rod *AB* and 60 MPa in the 40-mm-diameter rod *BC*. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at *A*.

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}c^4, \quad T = \frac{\pi}{2}\tau_{max}c^3$$

Shaft AB: $\tau_{max} = 100 \text{ MPa} = 100 \times 10^6 \text{ Pa}$

$$c = \frac{1}{2}d_{AB} = \frac{1}{2}(36) = 18 \text{ mm} = 0.018 \text{ m}$$

$$T_{AB} = \frac{\pi}{2}(100 \times 10^6)(0.018)^3 = 916 \text{ N-m}$$

Shaft BC: $\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$

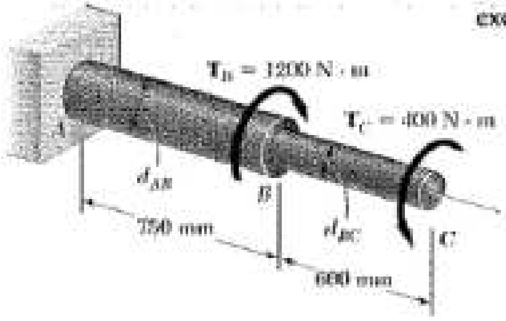
$$c = \frac{1}{2}d_{BC} = \frac{1}{2}(40) = 20 \text{ mm} = 0.020 \text{ m}$$

$$T_{BC} = \frac{\pi}{2}(60 \times 10^6)(0.020)^3 = 754 \text{ N-m}$$

The allowable torque is the smaller of T_{AB} and T_{BC}.

$$T = 754 \text{ N-m}$$

Problem 3.17



3.17 The solid shaft shown is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

$\tau_{max} = 55$ MPa $= 55 \times 10^6$ Pa

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

Shaft AB: $T_{AB} = 1200 - 400 = 800$ N·m

$$c = \sqrt[3]{\frac{(2)(800)}{\pi(55\times10^6)}} = 21.00\times10^{-3}\text{ m} = 21.0\text{ m}$$

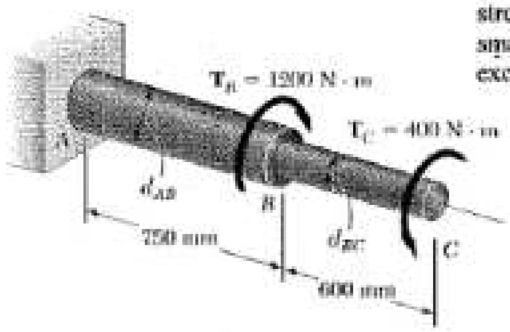
minimum $d_{AB} = 2c = 42.0$ mm ◄

Shaft BC: $T_{BC} = 400$ N·m

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55\times10^6)}} = 16.667\times10^{-3}\text{ m} = 16.67\text{ mm}$$

minimum $d_{BC} = 2c = 33.3$ mm ◄

Problem 3.18



3.18 Solve Prob. 3.17 assuming that the direction of T_C is reversed.

3.17 The solid shaft shown is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

Note that the direction of T_C has been reversed in the figure to the left.

$\tau_{max} = 55$ MPa $= 55\times10^6$ Pa

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

Shaft AB: $T_{AB} = 1200 + 400 = 1600$ N·m

$$c = \sqrt[3]{\frac{(2)(1600)}{\pi(55\times10^6)}} = 26.46\times10^{-3}\text{ m} = 26.46\text{ mm}$$

minimum $d_{AB} = 2c = 52.9$ mm ◄

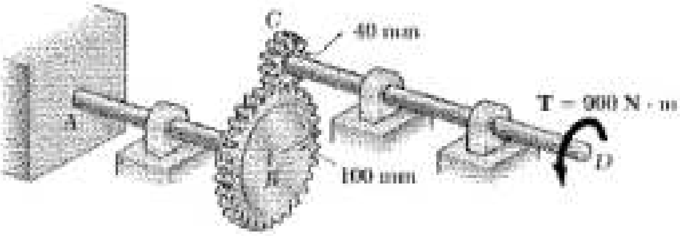
Shaft BC: $T_{BC} = 400$ N·m

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55\times10^6)}} = 16.667\times10^{-3}\text{ m} = 16.67\text{ mm}$$

minimum $d_{BC} = 2c = 33.3$ mm ◄

Problem 3.21

3.21 A torque of magnitude $T = 900$ N · m is applied at D as shown. Knowing that the allowable shearing stress is 50 MPa in each shaft, determine the required diameter of (*a*) shaft AB, (*b*) shaft CD.



$T_{CD} = 900 \text{ Nm.}$

$T_{AB} = \dfrac{r_B}{r_C} T_{AB} = \dfrac{100(900)}{40} = 2250 \text{ Nm.}$

$\tau_{max} = 50 \text{ MPa.}$

$\tau_{max} = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$c = \sqrt[3]{\dfrac{2T}{\pi \tau_{max}}}$

(a) Shaft AB: $c = \sqrt[3]{\dfrac{(2)(2250)}{\pi(50\times10^6)}} = 0.0306 \text{ m} = 30.6 \text{ mm}$

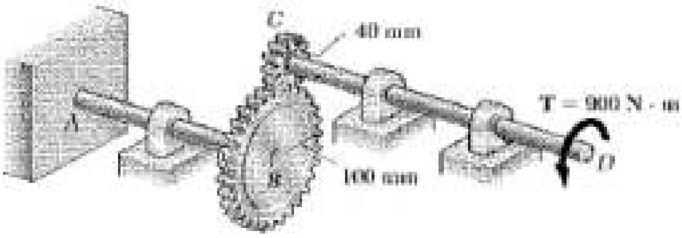
$d_{AB} = 2c = 61.2 \text{ mm}$ ◄

(b) Shaft CD: $c = \sqrt[3]{\dfrac{(2)(900)}{\pi 50\times10^6}} = 0.0225 \text{ m} = 22.5 \text{ mm}$

$d_{CD} = 2c = 45 \text{ mm}$ ◄

Problem 3.22

3.22 A torque of magnitude $T = 900$ N · m is applied at D as shown. Knowing that the diameter of shaft AB is 60 mm and that the diameter of shaft CD is 45 mm, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.



$T_{CD} = 900 \text{ Nm}$

$T_{AB} = \dfrac{r_B}{r_C} T_{CD} = \dfrac{100}{40}(900) = 2250 \text{ Nm.}$

(a) Shaft AB: $c = \frac{1}{2} d_{AB} = 30 \text{ mm}$

$\tau_{max} = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$\tau_{max} = \dfrac{(2)(2250)}{\pi(0.03)^3} = 53.05 \text{ MPa}$

(a) $\tau_{max} = 53.05 \text{ MPa}$ ◄

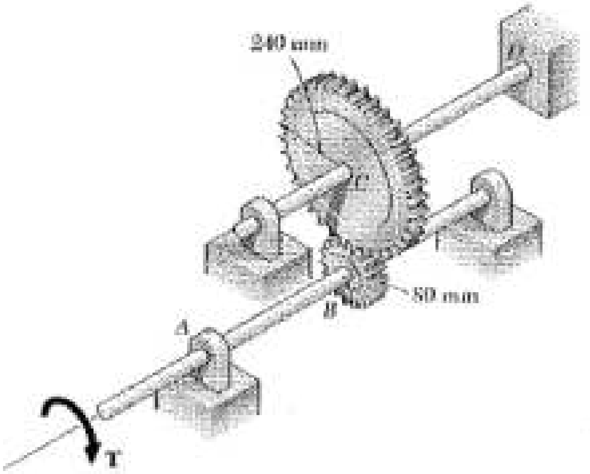
(b) Shaft CD: $c = \frac{1}{2} d_{CD} = 22.5 \text{ mm}$

$\tau_{max} = \dfrac{2T}{\pi c^3} = \dfrac{(2)(900)}{\pi(0.0225)^3} = 50.3 \text{ MPa}$

(b) $\tau_{max} = 50.3 \text{ MPa.}$ ◄

Problem 3.23

3.23 Two solid steel shafts are connected by the gears shown. A torque of magnitude $T = 900$ N · m is applied to shaft AB. Knowing that the allowable shearing stress is 50 MPa and considering only stresses due to twisting, determine the required diameter of (*a*) shaft AB, (*b*) shaft CD.



$$T_{AB} = T = 900 \text{ N·m}$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{240}{80}(900) = 2700 \text{ N·m}$$

(a) Shaft AB: $\tau_{max} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}, \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(50 \times 10^6)}} = 22.55 \times 10^{-3} \text{ m} = 22.55 \text{ mm}$$

$$d_{AB} = 2c = 45.1 \text{ mm}$$

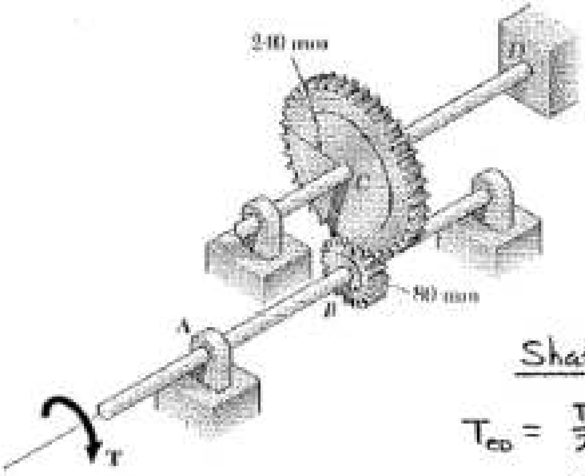
(b) Shaft CD: $\tau_{max} = 50 \text{ MPa}$

$$c = \sqrt[3]{\frac{(2)(2700)}{\pi(50 \times 10^6)}} = 32.52 \times 10^{-3} \text{ m} = 32.52 \text{ mm}$$

$$d_{CD} = 2c = 65.0 \text{ mm}$$

Problem 3.24

3.24 Shaft CD is made from a 66-mm-diameter rod and is connected to the 48-mm-diameter shaft AB as shown. Considering only stresses due to twisting and knowing that the allowable shearing stress is 60 MPa for each shaft, determine the largest torque **T** that can be applied.



$$\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

Shaft AB: $c = \frac{1}{2} d_{AB} = 24 \text{ mm} = 0.024 \text{ m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad T = \frac{\pi}{2} \tau_{max} c^3$$

$$T = \frac{\pi}{2}(60 \times 10^6)(0.024)^3 = 1303 \text{ N·m}$$

Shaft CD: $c = \frac{1}{2} d_{CD} = 33 \text{ mm} = 0.033 \text{ m}$

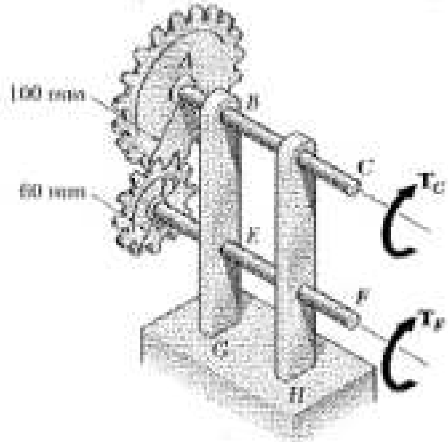
$$T_{CD} = \frac{\pi}{2}(60 \times 10^6)(0.033)^3 = 3387 \text{ N·m}$$

$$T = \frac{r_B}{r_C} T_{CD} = \frac{80}{240}(3387) = 1129 \text{ N·m}$$

The allowable torque is the smaller of the two calculated values.

$$T = 1129 \text{ N·m} = 1.129 \text{ kN·m}$$

Problem 3.25



3.25 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 50 MPa. Knowing the diameters of the two shafts are, respectively, $d_{BC} = 40$ mm and $d_{EF} = 30$ mm determine the largest torque T_C that can be applied at C.

$\tau_{max} = 50\ MPa$

Shaft BC: $d_{BC} = 40\ mm \qquad c = \frac{1}{2}d = 20\ mm.$

$$T_C = \frac{J\tau_{max}}{c} = \frac{\pi}{2}\tau_{max}c^3 = \frac{\pi}{2}(50\times10^6)(0.02)^3$$
$$= 628\ N.m.$$

Shaft EF: $d_{EF} = 30\ mm \qquad c = \frac{1}{2}d = 15\ mm.$

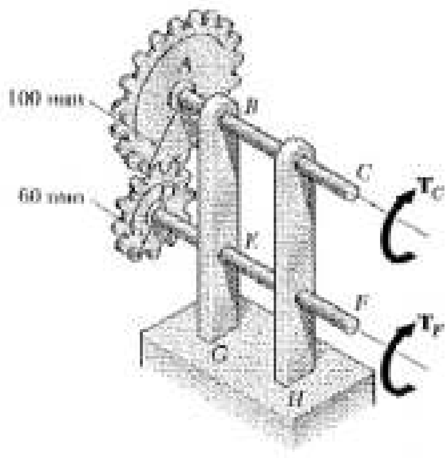
$$T_F = \frac{J\tau_{max}}{c} = \frac{\pi}{2}\tau_{max}c^3$$
$$= \frac{\pi}{2}(50\times10^6)(0.015)^3 = 265\ N.m.$$

By statics $\quad T_C = \frac{r_A}{r_D}T_F = \frac{10}{6}(265) = 441.7\ N.m.$

Allowable value of T_C is the smaller.

$$T_C = 441.7\ Nm$$

Problem 3.26



3.26 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 60 MPa. Knowing that a torque of magnitude $T_C = 600$ N · m is applied at C and that the assembly is in equilibrium, determine the required diameter of (*a*) shaft BC, (*b*) shaft EF.

$\tau_{max} = 60\ MPa$

(a) Shaft BC: $T_C = 600\ Nm$

$$\tau_{max} = \frac{T_C}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi\tau_{max}}}$$

$$c = \sqrt[3]{\frac{(2)(600)}{\pi(60\times10^6)}} = 0.0185\ m$$

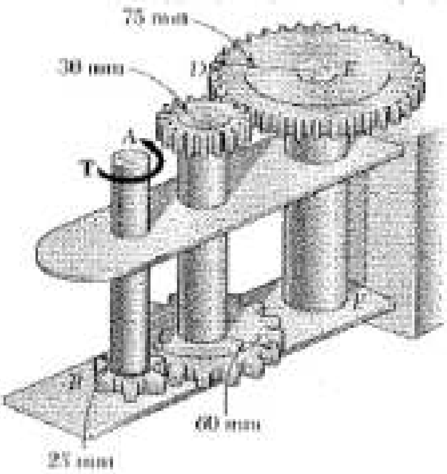
$$d_{BC} = 2c = 37\ mm$$

Shaft EF: $T_F = \frac{r_D}{r_A}T_C = \frac{60}{100}(600) = 360\ N\,m.$

$$c = \sqrt[3]{\frac{2T}{\pi\tau_{max}}} = \sqrt[3]{\frac{(2)(360)}{\pi(60\times10^6)}} = 0.0156\ m$$

$$d_{EF} = 2c = 31.2\ mm$$

Problem 3.27



3.27 A torque of magnitude $T = 120$ N · m is applied to shaft AB of the gear train shown. Knowing that the allowable shearing stress is 75 MPa in each of the three solid shafts, determine the required diameter of (*a*) shaft AB, (*b*) shaft CD, (*c*) shaft EF.

STATICS

Shaft AB. $T_{AB} = T_A = T_B = T$

Gears B and C. $r_B = 25\text{mm}$, $r_C = 60\text{ mm}$

Force on gear circles. $F_{BC} = \frac{T_B}{r_B} = \frac{T_C}{r_C}$

$$T_C = \frac{r_C}{r_B} T_B = \frac{60}{25} T = 2.4 T$$

Shaft CD. $T_{CD} = T_C = T_D = 2.4 T$

Gears D and E. $r_D = 30\text{ mm}$, $r_E = 75\text{ mm}$

Force on gear circles. $F_{DE} = \frac{T_D}{r_D} = \frac{T_E}{r_E}$

$$T_E = \frac{r_E}{r_D} T_D = \frac{75}{30}(2.4T) = 6T$$

Shaft EF. $T_{EF} = T_E = T_F = 6T$

REQUIRED DIAMETERS

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau}} \qquad d = 2c = 2\sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$\tau_{max} = 75 \times 10^6 \text{ Pa}$$

(a) Shaft AB. $T_{AB} = T = 120$ N·m

$$d_{AB} = 2\sqrt[3]{\frac{2(120)}{\pi(75\times10^6)}} = 20.1\times10^{-3}\text{ m} \qquad d_{AB} = 20.1\text{ mm}$$

(b) Shaft CD. $T_{CD} = (2.4)(120) = 288$ N·m

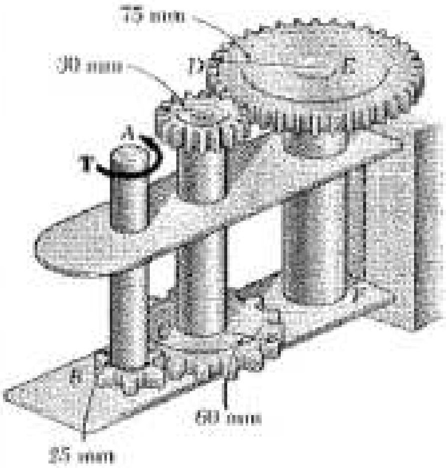
$$d_{CD} = 2\sqrt[3]{\frac{(2)(288)}{\pi(75\times10^6)}} = 26.9\times10^{-3}\text{ m} \qquad d_{CD} = 26.9\text{ mm}$$

(c) Shaft EF. $T_{EF} = (6)(120) = 720$ N·m

$$d_{EF} = 2\sqrt[3]{\frac{(2)(720)}{\pi(75\times10^3)}} = 36.6\times10^{-3}\text{ m} \qquad d_{EF} = 36.6\text{ mm}$$

Problem 3.28

3.28 A torque of magnitude $T = 100$ N · m is applied to shaft AB of the gear train shown. Knowing that the diameters of the three solid shafts are, respectively, $d_{AB} = 21$ mm, $d_{CD} = 30$ mm, and $d_{EF} = 40$ mm, determine the maximum shearing stress in (a) shaft AB, (b) shaft CD, (c) shaft EF.



STATICS

Shaft AB. $T_{AB} = T_A = T_B = T$

Gears B and C. $r_B = 25$ mm, $r_C = 60$ mm

Force on gear circles. $F_{BC} = \frac{T_B}{r_B} = \frac{T_C}{r_C}$

$$T_C = \frac{r_C}{r_B} T_B = \frac{60}{25} T = 2.4\,T$$

Shaft CD. $T_{CD} = T_C = T_D = 2.4\,T$

Gears D and E. $r_D = 30$ mm, $r_E = 75$ mm

Force on gear circles. $F_{DE} = \frac{T_D}{r_D} = \frac{T_E}{r_E}$

$$T_E = \frac{r_E}{r_D} T_D = \frac{75}{30}(2.4\,T) = 6\,T$$

Shaft EF. $T_{EF} = T_E = T_F = 6\,T$

MAXIMUM SHEARING STRESSES

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

(a) Shaft AB. $T = 100$ N·m $\quad c = \frac{1}{2}d = 10.5$ mm $= 10.5 \times 10^{-3}$ m

$$\tau_{max} = \frac{(2)(100)}{\pi(10.5\times10^{-3})^3} = 55.0\times10^6 \text{ Pa} \qquad \tau_{max} = 55.0 \text{ MPa}$$

(b) Shaft CD. $T = (2.4)(100) = 240$ N·m

$c = \frac{1}{2}d = 15$ mm $= 15\times10^{-3}$ m

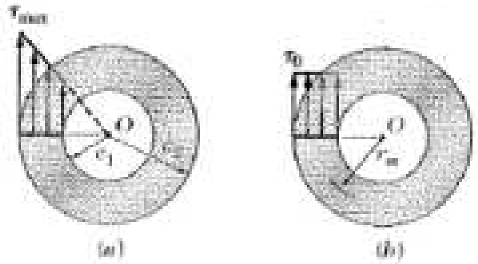
$$\tau_{max} = \frac{(2)(240)}{\pi(15\times10^{-3})^3} = 45.3\times10^6 \text{ Pa} \qquad \tau_{max} = 45.3 \text{ MPa}$$

(c) Shaft EF. $T = (6)(100) = 600$ N·M

$c = \frac{1}{2}d = 20$ mm $= 20\times10^{-3}$ m

$$\tau_{max} = \frac{(2)(600)}{\pi(20\times10^{-3})^3} = 47.7\times10^6 \text{ Pa} \qquad \tau_{max} = 47.7 \text{ MPa}$$

Problem 3.29



3.29 While the exact distribution of the shearing stresses in a hollow-cylindrical shaft is as shown in Fig. P3.29*a*, an approximate value can be obtained for τ_{max} by assuming that the stresses are uniformly distributed over the area A of the cross section, as shown in Fig. P 3.29*b*, and then further assuming that all of the elementary shearing forces act at a distance from O equal to the mean radius $\frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio τ_{max}/τ_0 of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2 respectively equal to 1.00, 0.95, 0.75, 0.50 and 0.

For a hollow shaft: $\tau_{max} = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)} = \frac{2Tc_2}{\pi(c_2^2 - c_1^2)(c_2^2 + c_1^2)}$

$$= \frac{2Tc_2}{A(c_2^2 + c_1^2)}$$

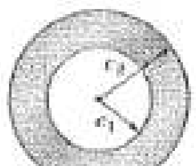
By definition, $\tau_0 = \frac{T}{Ar_m} = \frac{2T}{A(c_2 + c_1)}$

Dividing, $\frac{\tau_{max}}{\tau_0} = \frac{c_2(c_2 + c_1)}{c_2^2 + c_1^2} = \frac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2}$ ◄

c_1/c_2	1.0	0.95	0.75	0.5	0.0
τ_{max}/τ_0	1.0	1.025	1.120	1.200	1.0

◄

Problem 3.30



3.30 (*a*) For a given allowable shearing stress, determine the ratio T/w of the maximum allowable torque T and the weight per unit length w for the hollow shaft shown. (*b*) Denoting by $(T/w)_0$ the value of this ratio for a solid shaft of the same radius c_2, express the ratio T/w for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2.

w = weight per unit length, ρg = specific weight

W = total weight, L = length

$$w = \frac{W}{L} = \frac{\rho g L A}{L} = \rho g A = \rho g \pi (c_2^2 - c_1^2)$$

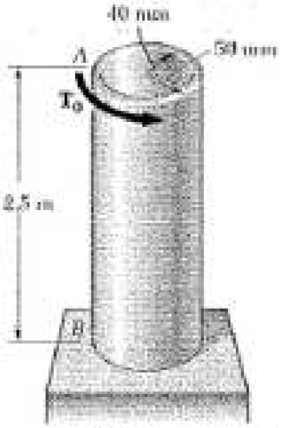
$$T = \frac{J\tau_{al}}{c_2} = \frac{\pi}{2}\frac{c_2^4 - c_1^4}{c_2}\tau_{al} = \frac{\pi}{2}\frac{(c_2^2 + c_1^2)(c_2^2 - c_1^2)}{c_2}\tau_{al}$$

(a) $\frac{T}{W} = (c_1^2 + c_2^2)\tau_{al}$ $\quad \frac{T}{W} = \frac{(c_1^2 + c_2^2)\tau_{al}}{2\rho g c_2}$ (hollow shaft) ◄

$c_1 = 0$ for solid shaft: $\left(\frac{T}{W}\right)_0 = \frac{c_2 \tau_{al}}{2\rho g}$ (solid shaft)

(b) $\frac{(T/w)_h}{(T/w)_0} = 1 + \frac{c_1^2}{c_2^2}$ $\quad \left(\frac{T}{W}\right) = \left(\frac{T}{W}\right)_0\left(1 + \frac{c_1^2}{c_2^2}\right)$ ◄

Problem 3.31



3.31 (*a*) For the aluminum pipe shown (G = 27 GPa), determine the torque T_0 causing an angle of twist of 2°. (*b*) Determine the angle of twist if the same torque T_0 is applied to a solid cylindrical shaft of the same length and cross-sectional area.

(a) $c_o = 50 \text{ mm} = 0.050 \text{ m}, \quad c_i = 40 \text{ mm} = 0.040 \text{ m}$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}(0.050^4 - 0.040^4)$$
$$= 5.7962 \times 10^{-6} \text{ m}^4$$

$\varphi = 2° = 34.907 \times 10^{-3}$ rad $\qquad L = 2.5$ m

$G = 27 \times 10^9$ Pa

$$\varphi = \frac{TL}{GJ} \qquad T_0 = \frac{GJ\varphi}{L} = \frac{(27\times10^9)(5.7962\times10^{-6})(34.907\times10^{-3})}{2.5}$$
$$= 2.1851 \times 10^3 \text{ N}\cdot\text{m} \qquad T_0 = 2.19 \text{ kN}\cdot\text{m}$$

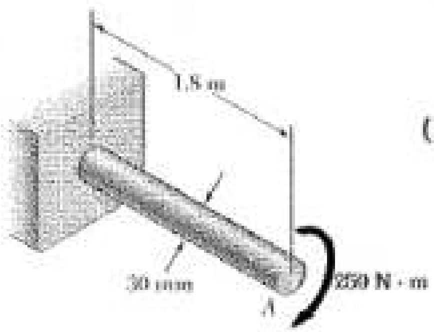
Area of pipe: $A = \pi(c_o^2 - c_i^2) = \pi(0.050^2 - 0.040^2) = 2.8274 \text{ m}^2$

(b) Radius of solid of same area: $c_s = \sqrt{\frac{A}{\pi}} = 0.030$ m

$$J_s = \frac{\pi}{2}c_s^4 = \frac{\pi}{2}(0.030)^4 = 1.27235 \times 10^{-6} \text{ m}^2$$

$$\varphi_s = \frac{T_0 L}{GJ} = \frac{(2.1851\times10^3)(2.5)}{(27\times10^9)(1.27235\times10^{-6})} = 0.15902 \text{ rad} \qquad \varphi_s = 9.11°$$

Problem 3.32



3.32 (*a*) For the solid steel shaft shown (G = 77 GPa), determine the angle of twist at A. (*b*) Solve part *a*, assuming that the steel shaft is hollow with a 30-mm-outer diameter and a 20-mm-inner diameter.

(a) $c = \frac{1}{2}d = 0.015$ m, $\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4$

$J = 79.522 \times 10^{-9} \text{ m}^4$, $\quad L = 1.8$ m, $\quad G = 77 \times 10^9$ Pa

$T = 250$ N·m $\qquad \varphi = \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77\times10^9)(79.522\times10^{-9})} = 73.49 \times 10^{-3} \text{ rad}$$

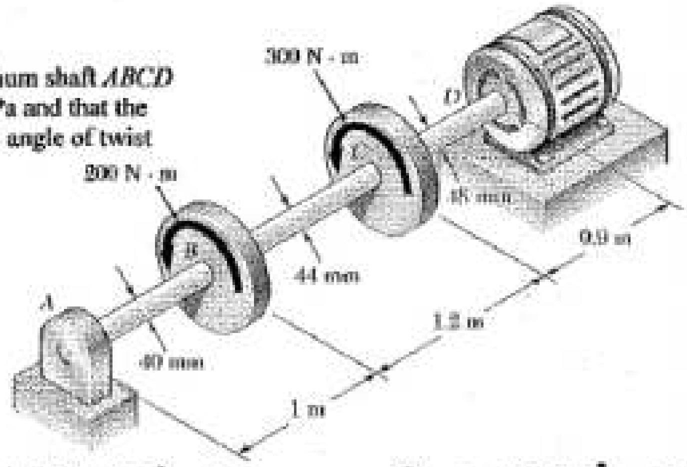
$$\varphi = \frac{(73.49\times10^{-3})180}{\pi} = 4.21°$$

(b) $c_2 = 0.015$ m, $\quad c_1 = \frac{1}{2}d_1 = 0.010$ m, $\quad J = \frac{\pi}{2}(c_2^4 - c_1^4)$

$J = \frac{\pi}{2}(0.015^4 - 0.010^4) = 63.814 \times 10^{-9} \text{ m}^4 \qquad \varphi = \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77\times10^9)(63.814\times10^{-9})} = 91.58\times10^{-3} \text{ rad} = \frac{180}{\pi}(91.58\times10^{-3}) = 5.25°$$

Problem 3.35



3.35 The electric motor exerts a 500 N · m torque on the aluminum shaft *ABCD* when it is rotating at a constant speed. Knowing that $G = 27$ GPa and that the torques exerted on pulleys *B* and *C* are as shown, determine the angle of twist between (*a*) *B* and *C*, (*b*) *B* and *D*.

(a) Angle of twist between B and C.

$T_{BC} = 200\ N\cdot m, \quad L_{BC} = 1.2\ m$

$c = \frac{1}{2}d = 0.022\ m, \quad G = 27\times10^9\ Pa$

$J_{BC} = \frac{\pi}{2}c^4 = 367.97\times10^{-9}\ m$

$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(200)(1.2)}{(27\times10^9)(367.97\times10^9)} = 24.157\times10^{-3}\ rad$

$\varphi_{B/C} = 1.384°$

(b) Angle of twist between B and D.

$T_{CD} = 500\ N\cdot m, \quad L_{CD} = 0.9\ m, \quad c = \frac{1}{2}d = 0.024\ m, \quad G = 27\times10^9\ Pa$

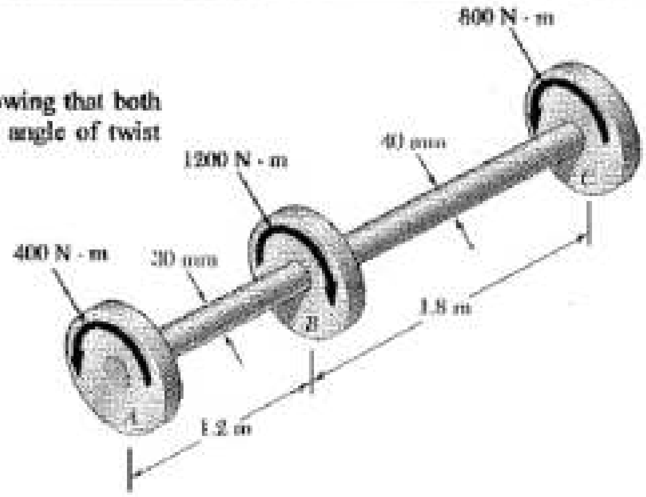
$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.024)^4 = 521.153\times10^{-9}\ m^4$

$\varphi_{C/D} = \frac{(500)(0.9)}{(27\times10^9)(521.153\times10^9)} = 31.980\times10^{-3}\ rad$

$\varphi_{B/D} = \varphi_{B/C} + \varphi_{C/D} = 24.157\times10^{-3} + 31.980\times10^{-3} = 56.137\times10^{-3}\ rad$

$\varphi_{B/D} = 3.22°$

Problem 3.36



3.36 The torques shown are exerted on pulleys *A*, *B*, and *C*. Knowing that both shafts are solid and made of brass ($G = 39$ GPa), determine the angle of twist between (*a*) *A* and *B*, (*b*) *A* and *C*.

(a) Angle of twist between A and B.

$T_{AB} = 400\ N\cdot m, \quad L_{AB} = 1.2\ m$

$c = \frac{1}{2}d = 0.015\ m \quad G = 39\times10^9\ Pa$

$J_{AB} = \frac{\pi}{2}c^4 = 79.52\times10^{-9}\ m^4$

$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(400)(1.2)}{(39\times10^9)(79.52\times10^{-9})}$

$= 0.154772\ rad$ ↺

$\varphi_{A/B} = 8.87°$ ↺

(b) Angle of twist between A and C.

$T_{BC} = 800\ N\cdot m, \quad L_{BC} = 1.8\ m, \quad c = \frac{1}{2}d = 0.020\ m, \quad G = 39\times10^9\ Pa$

$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.327\times10^{-9}\ m^4$

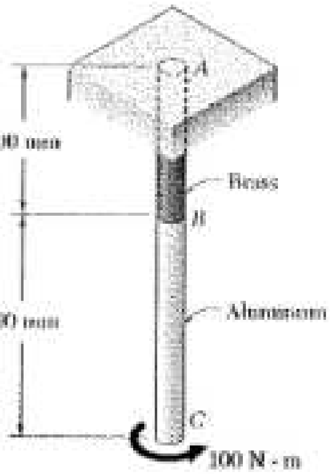
$\varphi_{B/C} = \frac{(800)(1.8)}{(39\times10^9)(251.327\times10^{-9})} = 0.146912\ rad$ ↻

$\varphi_{A/C} = \varphi_{A/B} + \varphi_{B/C} = 0.154772 - 0.146912 = 0.007860\ rad$ ↺

$\varphi_{C/A} = 0.450°$ ↺

Problem 3.37

3.37 The aluminum rod *BC* (G = 26 GPa) is bonded to the brass rod *AB* (G = 39 GPa). Knowing that each rod is solid and has a diameter of 12 mm, determine the angle of twist (*a*) at *B*, (*b*) at *C*.



Both portions $c = \frac{1}{2}d = 6\text{ mm} = 6\times10^{-3}\text{ m}$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(6\times10^{-3})^4 = 2.03575\times10^{-9}\text{ m}^4$

$T = 100\text{ N}\cdot\text{m}$

Rod AB: $G_{AB} = 39\times10^9\text{ Pa}$, $L_{AB} = 0.200\text{ m}$

(a) $\varphi_B = \varphi_{AB} = \frac{TL_{AB}}{G_{AB}J} = \frac{(100)(0.200)}{(39\times10^9)(2.03575\times10^{-9})}$

$= 0.25191\text{ rad}$ $\qquad \varphi_B = 14.43°$

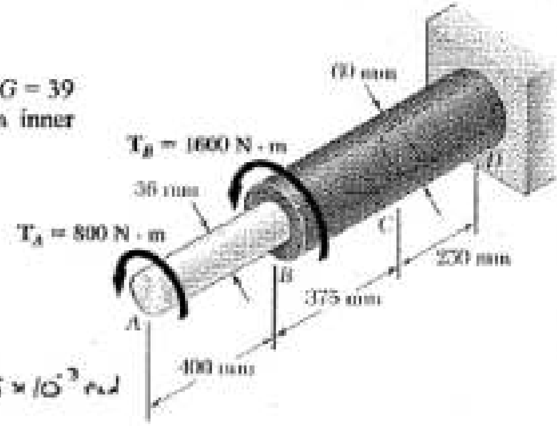
Rod BC: $G_{BC} = 26\times10^9\text{ Pa}$, $L_{BC} = 0.300\text{ m}$

$\varphi_{BC} = \frac{TL_{BC}}{G_{BC}J} = \frac{(100)(0.300)}{(26\times10^9)(2.03575\times10^{-9})} = 0.56679\text{ rad}$

(b) $\varphi_C = \varphi_B + \varphi_{BC} = 0.25191 + 0.56679 = 0.81870\text{ rad}$ $\qquad \varphi_C = 46.9°$

Problem 3.38

3.38 The aluminum rod *AB* (G = 27 GPa) is bonded to the brass rod *BD* (G = 39 GPa). Knowing that portion *CD* of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at *A*.



Rod AB: $G = 27\times10^9\text{ Pa}$, $L = 0.400\text{ m}$

$T = 800\text{ N}\cdot\text{m}$ $\qquad c = \frac{1}{2}d = 0.018\text{ m}$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896\times10^{-9}\text{ m}$

$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27\times10^9)(164.896\times10^{-9})} = 71.875\times10^{-3}\text{ rad}$

Part BC: $G = 39\times10^9\text{ Pa}$ $\quad L = 0.375\text{ m}$, $\quad c = \frac{1}{2}d = 0.030\text{ m}$

$T = 800 + 1600 = 2400\text{ N}\cdot\text{m}$, $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234\times10^{-6}\text{ m}^4$

$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39\times10^9)(1.27234\times10^{-6})} = 18.137\times10^{-3}\text{ rad}$

Part CD: $c_1 = \frac{1}{2}d_1 = 0.020\text{ m}$ $\qquad c_2 = \frac{1}{2}d_2 = 0.030\text{ m}$, $\quad L = 0.250\text{ m}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102\times10^{-6}\text{ m}^4$

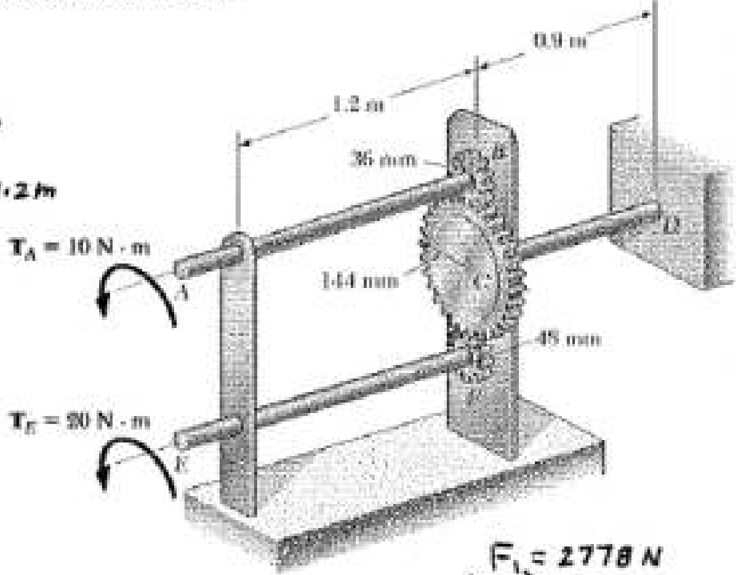
$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39\times10^9)(1.02102\times10^{-6})} = 15.068\times10^{-3}\text{ rad}$

Angle of twist at A. $\quad \varphi_A = \varphi_{A/B} + \varphi_{B/C} + \varphi_{C/D}$

$= 105.080\times10^{-3}\text{ rad}$ $\qquad \varphi_A = 6.02°$

Problem 3.39

3.39 Three solid shafts, each of 18-mm diameter, are connected by the gears shown. Knowing that $G = 77$ GPa, determine (*a*) the angle through which end A of shaft AB rotates, (*b*) the angle through which end E of shaft EF rotates.



Geometry:

$r_B = 0.036\,m$, $r_C = 0.144\,m$, $r_F = 0.048$

$L_{AB} = 1.2\,m$, $L_{CD} = 0.9\,m$, $L_{EF} = 1.2\,m$

Statics: $T_A = 10$ Nm

$T_E = 20$ Nm

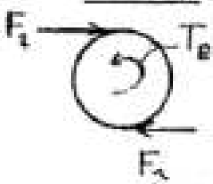
Gear B. $+\circlearrowleft \Sigma M_B = 0$:

$-r_B F_1 + T_A = 0$

$-0.036 F_1 + 10 = 0$

$F_1 = 277.8$ N

Gear F. $+\circlearrowleft \Sigma M_F = 0$:

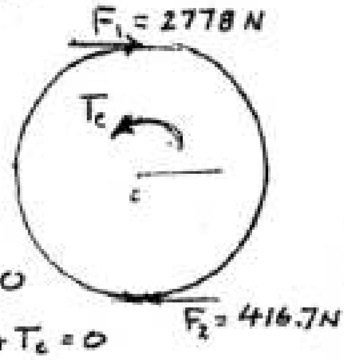


$-r_F F_2 + T_E = 0$

$-0.048 F_2 + 20 = 0$

$F_2 = 416.7$ N

Gear C.



$+\circlearrowleft \Sigma M_C = 0$

$-r_C F_1 + r_C F_2 + T_C = 0$

$-(0.144)(277.8) - (0.144)(10) + T_C = 0$

$T_C = 41.4$ Nm

Deformations:

For all shafts $c = \frac{1}{2} d = 0.009$ m

$J = \frac{\pi}{2} c^4 = 10.306 \times 10^{-9}\ m^4$

$$\varphi_{A/B} = \frac{T_{AB} L_{AB}}{GJ} = \frac{(10)(1.2)}{(77\times10^9)(10.306\times10^{-9})} = 0.01512 \text{ rad}$$

$$\varphi_{E/F} = \frac{T_{EF} L_{EF}}{GJ} = \frac{(20)(1.2)}{(77\times10^9)(10.306\times10^{-9})} = 0.03024 \text{ rad}$$

$$\varphi_{C/D} = \frac{T_{CD} L_{CD}}{GJ} = \frac{(41.4)(0.9)}{(77\times10^9)(10.306\times10^{-9})} = 0.04695 \text{ rad}$$

Kinematics: $\varphi_C = \varphi_{C/D} = 0.04695$ rad

$r_B \varphi_B = r_C \varphi_C$ $\quad \varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{0.144}{0.036}(0.04695) = 0.1878$ rad

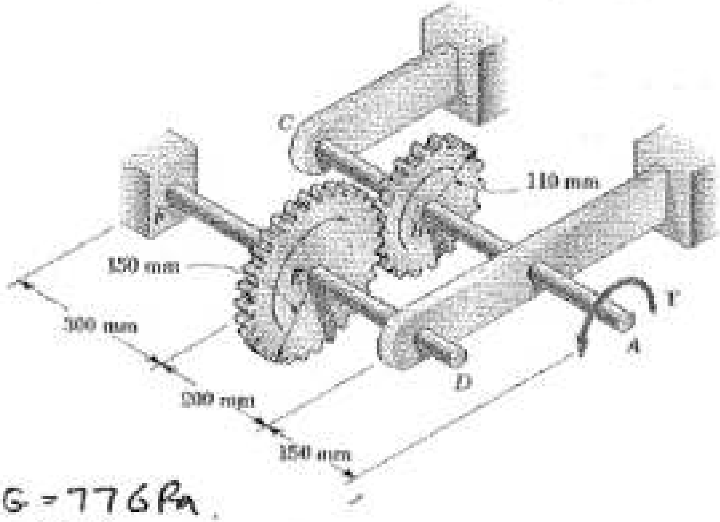
(a) $\varphi_{A/C} = \varphi_B + \varphi_{A/B} = 0.1878 + 0.01512 = 0.20292$ rad $\quad \varphi_B = 11.6°$

$r_F \varphi_F = r_C \varphi_C$ $\quad \varphi_F = \frac{r_C}{r_F}\varphi_C = \frac{0.144}{0.048}(0.04695) = 0.14085$ rad

(b) $\varphi_E = \varphi_F + \varphi_{E/F} = 0.14085 + 0.03024 = 0.17109$ rad $\quad \varphi_F = 9.8°$

Problem 3.40

3.40 Two shafts, each of 22-mm diameter are connected by the gears shown. Knowing that $G = 77$ GPa and that the shaft at F is fixed, determine the angle through which end A rotates when a 130 N · m torque is applied at A.



Calculation of torques.

Circumferential contact force between gears B and E.

$$F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E} \qquad T_{EF} = \frac{r_E}{r_B} T_{AB}$$

$T_{AB} = 130$ N m.

$$T_{EF} = \frac{150}{110}(130) = 177.3 \text{ Nm}$$

Twist in shaft FE.

$L = 300$ mm, $c = \frac{1}{2}d = 11$ mm, $G = 77$ GPa.

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.011)^4 = 23 \times 10^{-9} \text{ m}^4$$

$$\varphi_{E/F} = \frac{TL}{GJ} = \frac{(177.3)(0.3)}{(77\times 10^9)(23\times 10^{-9})} = 0.03 \text{ rad}$$

Rotation at E. $\varphi_E = \varphi_{E/F} = 0.03$ rad

Tangential displacement at gear circle $S = r_E \varphi_E = r_B \varphi_B$

Rotation at B $\varphi_B = \frac{r_E}{r_B}\varphi_E = \frac{15}{11}(0.03) = 0.0409$ rad

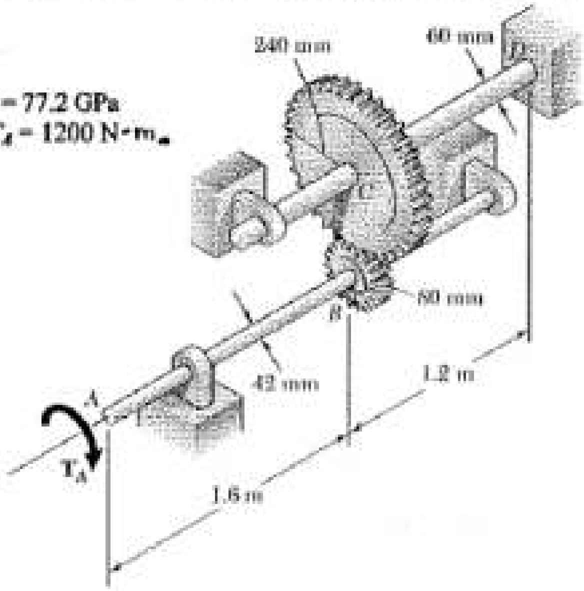
Twist in shaft BA. $L = 0.35$ m $J = 23\times 10^{-9}$ m^4

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(130)(0.35)}{(77\times 10^9)(23\times 10^{-9})} = 0.0257 \text{ rad}$$

Rotation at A. $\varphi_A = \varphi_B + \varphi_{A/B} = 0.0666$ rad $\varphi_A = 3.82^\circ$

Problem 3.41

3.41 Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200$ N·m.



<u>Calculation of torques.</u>

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \qquad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$T_{AB} = 1200$ N·m

$$T_{CD} = \frac{240}{80}(1200) = 3600 \text{ N·m}$$

<u>Twist in shaft CD:</u> $c = \frac{1}{2}d = 0.030$ m, $L = 1.2$ m, $G = 77.2 \times 10^9$ Pa

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(1.27234 \times 10^{-6})} = 43.981 \times 10^{-3} \text{ rad}$$

<u>Rotation angle at C.</u> $\varphi_C = \varphi_{C/D} = 43.981 \times 10^{-3}$ rad

Circumferential displacement at contact points of gears B an C

$$s = r_C \varphi_C = r_B \varphi_B$$

<u>Rotation angle at B.</u> $\varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{240}{80}(43.981 \times 10^{-3}) = 131.942 \times 10^{-3}$ rad

<u>Twist in shaft AB:</u> $c = \frac{1}{2}d = 0.021$ m, $L = 1.6$ m, $G = 77.2 \times 10^9$ Pa

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.021)^4 = 305.49 \times 10^{-9} \text{ m}^4$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(305.49 \times 10^{-9})} = 81.412 \times 10^{-3} \text{ rad}$$

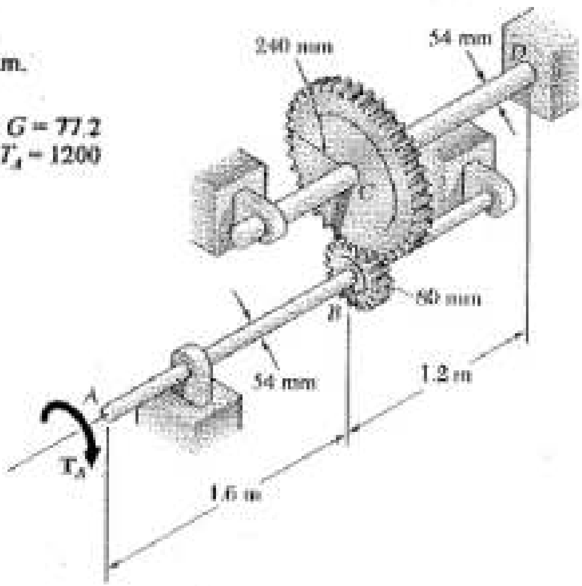
<u>Rotation angle at A.</u> $\varphi_A = \varphi_B + \varphi_{A/B} = 213.354 \times 10^{-3}$ rad

$$\varphi_A = 12.22°$$

Problem 3.42

3.42 Solve Prob. 3.41, assuming that the diameter of each shaft is 54 mm.

3.41 Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200$ N · m.



<u>Calculation of torques</u>

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \qquad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = 1200 \text{ N}\cdot\text{m}$$

$$T_{CD} = \frac{240}{80}(1200) = 3600 \text{ N}\cdot\text{m}$$

<u>Twist in shaft CD:</u> $c = \frac{1}{2}d = 0.027$ m, $L = 1.2$ m, $G = 77.2\times10^9$ Pa

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.027)^4 = 834.79\times10^{-9} \text{ m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2\times10^9)(834.78\times10^{-9})} = 67.033\times10^{-3} \text{ rad}$$

<u>Rotation angle at C.</u> $\varphi_C = \varphi_{C/D} = 67.033\times10^{-3}$ rad

Circumferential displacement at contact points of gears B and C

$$\delta = r_C\varphi_C = r_B\varphi_B$$

<u>Rotation angle at B.</u> $\varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{240}{3}(67.033\times10^{-3}) = 201.10\times10^{-3}$ rad

<u>Twist in shaft AB:</u> $c = \frac{1}{2}d = 0.027$ m, $L = 1.6$ m, $G = 77.2\times10^9$ Pa

$$J = 834.79\times10^{-9} \text{ m}$$

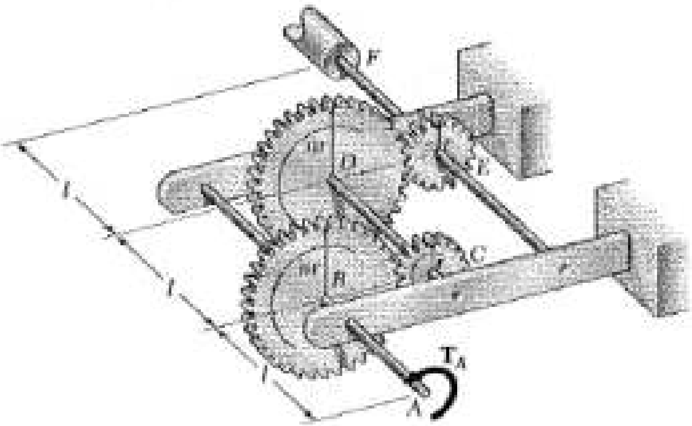
$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2\times10^9)(834.79\times10^{-9})} = 29.792\times10^{-3} \text{ rad}$$

<u>Rotation angle at A.</u> $\varphi_A = \varphi_B + \varphi_{A/B} = 230.89\times10^{-3}$ rad

$$\varphi_A = 13.23^\circ \blacktriangleleft$$

Problem 3.43

3.43 A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.



$T_{AB} = T_A$

$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{T_{AB}}{n} = \frac{T_A}{n}$

$T_{EF} = \frac{r_E}{r_D} T_{CD} = \frac{T_{CD}}{n} = \frac{T_A}{n^2}$

$\varphi_E = \varphi_{EF} = \frac{T_{EF} l_{EF}}{GJ} = \frac{T_A l}{n^2 GJ}$

$\varphi_D = \frac{r_E}{r_D}\varphi_E = \frac{\varphi_E}{n} = \frac{T_A l}{n^3 GJ}$

$\varphi_{CD} = \frac{T_{CD} l_{CD}}{GJ} = \frac{T_A l}{n GJ}$

$\varphi_C = \varphi_D + \varphi_{CD} = \frac{T_A l}{n^3 GJ} + \frac{T_A l}{n GJ} = \frac{T_A l}{GJ}\left(\frac{1}{n^3} + \frac{1}{n}\right)$

$\varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{\varphi_C}{n} = \frac{Tl}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2}\right)$

$\varphi_{AB} = \frac{T_{AB} l_{AB}}{GJ} = \frac{T_A l}{GJ}$

$\varphi_A = \varphi_B + \varphi_{AB} = \frac{T_A l}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2} + 1\right)$

Problem 3.44

3.44 For the gear train described in Prob. 3.43, determine the angle through which end A rotates when $T = 0.6$ N · m, $l = 60$ mm, $d = 2$ mm, $G = 77$ GPa, and $n = 2$.

3.43 A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.

See solution to PROBLEM 3.43 for development of equation for φ_A

$\varphi_A = \frac{Tl}{GJ}\left(1 + \frac{1}{n^2} + \frac{1}{n^4}\right)$

Data: $T = 0.6$ Nm $\quad l = 60$ mm, $\quad c = \frac{1}{2}d = 1$ mm $\quad G = 77\times10^9$ Pa.

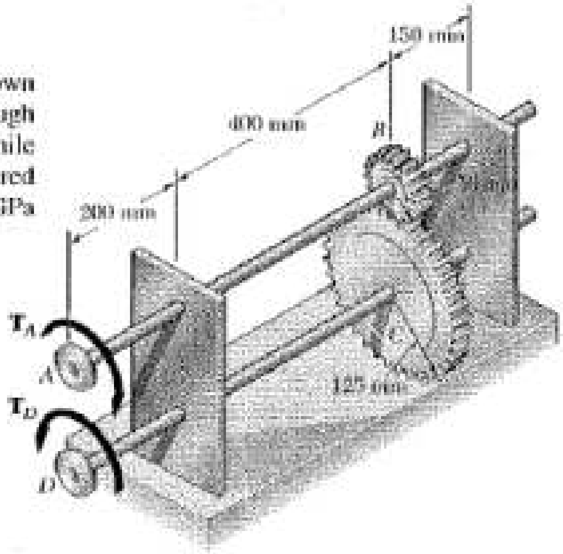
$n = 2$, $\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.001)^4 = 1.57\times10^{-12}$ m^4.

$\varphi_A = \frac{(0.6)(0.06)}{(77\times10^9)(1.57\times10^{-12})}\left(1 + \frac{1}{4} + \frac{1}{16}\right) = 0.391$ rad

$\varphi_A = 22.4°$

Problem 3.47

3.47 The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts and that the angle through which pulley A will rotate when subjected to a 200 N · m torque T_A while pulley D is held fixed will not exceed 7.5°. Determine the required diameter of the shafts if both shafts are made of a steel with G = 77 GPa and τ_{all} = 84 MPa.



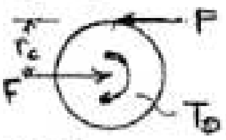
Statics. Gear B.

$+\circlearrowleft \Sigma M_B = 0:$

$r_B F - T_A = 0 \qquad F = T_B / r_B$

Gear C $\quad +\circlearrowleft \Sigma M_C = 0:$

$r_C F - T_D = 0$



$T_D = r_C F = \frac{r_C}{r_B} T_A = n T_B$

$n = \frac{r_C}{r_B} = \frac{125}{50} = 2.5$

Torques in shafts. $\quad T_{AB} = T_A = T_B \qquad T_{CD} = T_C = nT_B = nT_A$

Deformations: $\quad \varphi_{C/D} = \frac{T_{CD} L}{GJ} = \frac{nT_A L}{GJ}$

$\varphi_{A/B} = \frac{T_{AB} L}{GJ} = \frac{T_A L}{GJ}$

Kinematics: $\quad \varphi_D = 0 \qquad \varphi_C = \varphi_D + \varphi_{C/D} = 0 + \frac{nT_A L}{GJ}$

$r_B \varphi_B = -r_C \varphi_B \qquad \varphi_B = -\frac{r_C}{r_B}\varphi_C = -n\varphi_C \qquad \varphi_B = \frac{n^2 T_A L}{GJ}$

$\varphi_A = \varphi_C + \varphi_{B/C} = \frac{n^2 T_A L}{GJ} + \frac{T_A L}{GJ} = \frac{(n^2+1) T_A L}{GJ}$

Diameter based on stress. $\quad$ Largest torque: $T_m = T_{CD} = nT_A$

$\tau_m = \frac{T_m c}{J} = \frac{2nT_A}{\pi c^3} \qquad \tau_m = \tau_{all} = 84 MPa, \qquad T_A = 200$ N m

$c = \sqrt[3]{\frac{2nT_A}{\pi \tau_m}} = \sqrt[3]{\frac{(2)(2.5)(2\times10^2)}{\pi(84000\times10^3)}} = 0.0156$ m $\qquad d = 2c = 0.0312$ m

Diameter based on rotation limit. $\quad \varphi = 7.5° = 0.1309$ rad

$\varphi = \frac{(n^2+1)T_A L}{GJ} = \frac{(2)(7.25)T_A L}{\pi c^4 G} \qquad L = 0.2 + 0.4 = 0.6$ m

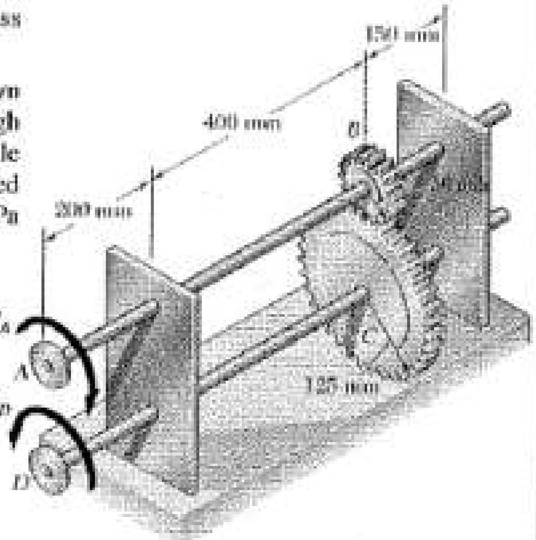
$c = \sqrt[4]{\frac{(2)(7.25)T_A L}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(7.25)(2\times10^2)(0.6)}{\pi(72000\times10^6)(0.1309)}} = 0.01557$ m $\qquad d = 2c = 0.03114$ m

Choose the larger diameter. $\qquad d = 31.2$ mm ◄

Problem 3.48

3.48 Solve Prob. 3.47, assuming that both shafts are made of a brass with $G = 39$ GPa and $\tau_{all} = 56$ MPa.

3.47 The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts and that the angle through which pulley A will rotate when subjected to a 200 N · m torque $\mathbf{T}_A$ while pulley D is held fixed will not exceed 7.5°. Determine the required diameter of the shafts if both shafts are made of a steel with $G = 77$ GPa and $\tau_{all} = 84$ MPa.

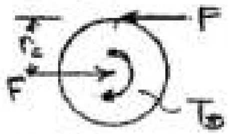


Statics. Gear B

$+\circlearrowleft \Sigma M_B = 0:$

$r_B F - T_A = 0 \qquad F = T_B / r_B$

Gear C $\quad +\circlearrowleft \Sigma M_C = 0:$



$r_C F - T_D = 0$

$T_D = r_C F = \dfrac{r_C}{r_B} T_A = n T_B \qquad n = \dfrac{r_C}{r_B} = \dfrac{125}{50} = 2.5$

Torques in shafts. $\quad T_{AB} = T_A = T_B \qquad T_{CD} = T_C = nT_B = nT_A$

Deformations: $\quad \varphi_{C/D} = \dfrac{T_{CD} L}{GJ} \circlearrowleft = \dfrac{nT_A L}{GJ} \circlearrowleft$

$\varphi_{A/B} = \dfrac{T_{AB} L}{GJ} \circlearrowright = \dfrac{T_A L}{GJ} \circlearrowright$

Kinematics: $\quad \varphi_D = 0 \qquad \varphi_C = \varphi_D + \varphi_{C/D} = 0 + \dfrac{nT_A L}{GJ} \circlearrowright$

$r_B \varphi_B = -r_C \varphi_B \qquad \varphi_B = -\dfrac{r_C}{r_B} \varphi_C = -n\varphi_C \qquad \varphi_B = \dfrac{n^2 T_A L}{GJ} \circlearrowright$

$\varphi_A = \varphi_C + \varphi_{B/C} = \dfrac{n^2 T_A L}{GJ} + \dfrac{T_A L}{GJ} = \dfrac{(n^2+1) T_A L}{GJ} \circlearrowright$

Diameter based on stress. $\quad$ Longest torque $\quad T_m = T_{CD} = nT_A$

$\tau_m = \dfrac{T_m c}{J} = \dfrac{2nT_A}{\pi c^3} \qquad \tau_m = \tau_{all} = 56 \text{ MPa}, \quad T_A = 200 \text{ Nm}$

$c = \sqrt[3]{\dfrac{2nT_A}{\pi \tau_m}} = \sqrt[3]{\dfrac{(2)(2.5)(2\times10^2)}{\pi(56000\times10^3)}} = 0.0178 \text{ m} \qquad d = 2c = 0.0356 \text{ m}$

Diameter based on rotation limit $\quad \varphi = 7.5° = 0.1309$ rad

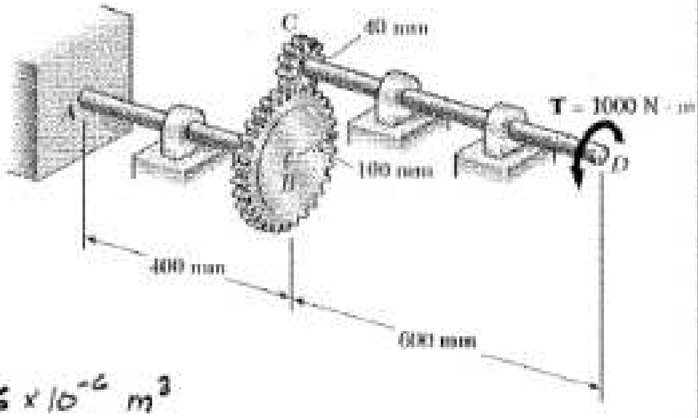
$\varphi = \dfrac{(n^2+1)T_A L}{GJ} = \dfrac{(2)(7.25)T_A L}{\pi c^4 G} \qquad L = 0.2 + 0.4 = 0.6 \text{ m}$

$c = \sqrt[4]{\dfrac{(2)(7.25)T_A L}{\pi G \varphi}} = \sqrt[4]{\dfrac{(2)(7.25)(2\times10^2)(0.6)}{\pi(39000\times10^6)(0.1309)}} = 0.01815 \qquad d = 2c = 0.0363 \text{ m}$

Choose the larger diameter. $\qquad d = 36.3$ mm

Problem 3.49

3.49 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both AB and CD. It is further required that $\tau_{max} \le 60$ MPa and that the angle ϕ_D through which end D of shaft CD rotates not exceed 1.5°. Knowing that $G = 77$ GPa, determine the required diameter of the shafts.



$T_{CD} = T_D = 1000$ N·m

$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500$ N·m

For design based on stress, use larger torque. $T_{AB} = 2500$ N·m.

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2500)}{\pi(60 \times 10^6)} = 26.526 \times 10^{-6}$ m³

$c = 29.82 \times 10^{-3}$ m $= 29.82$ mm, $\quad d = 2c = 59.6$ mm

Design based on rotation angle $\varphi_D = 1.5° = 26.18 \times 10^{-3}$ rad

Shaft AB: $T_{AB} = 2500$ N·m, $L = 0.4$ m

$\varphi_{AB} = \frac{TL}{GJ} = \frac{(2500)(0.4)}{GJ} = \frac{1000}{GJ}$

Gears $\begin{cases} \varphi_B = \varphi_{AB} = \frac{1000}{GJ} \\ \varphi_C = \frac{r_B}{r_C}\varphi_B = \frac{100}{40} \cdot \frac{1000}{GJ} = \frac{2500}{GJ} \end{cases}$

Shaft CD $\quad T_{CD} = 1000$ N·m, $L = 0.6$ m

$\varphi_{CD} = \frac{TL}{GJ} = \frac{(1000)(0.6)}{GJ} = \frac{600}{GJ}$

$\varphi_D = \varphi_C + \varphi_{CD} = \frac{2500}{GJ} + \frac{600}{GJ} = \frac{3100}{GJ} = \frac{3100}{G\frac{\pi}{2}c^4}$

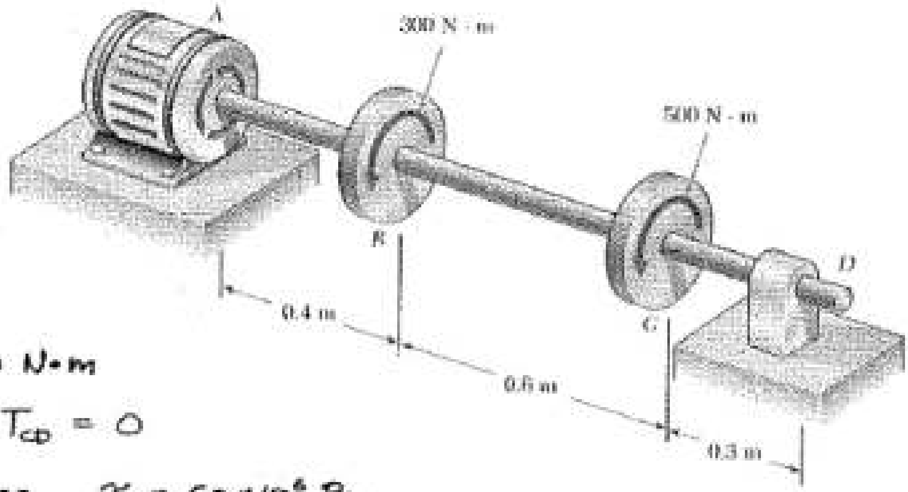
$c^4 = \frac{(2)(3100)}{\pi G \varphi_D} = \frac{(2)(3100)}{\pi(77 \times 10^9)(26.18 \times 10^{-3})} = 979.06 \times 10^{-9}$ m⁴

$c = 31.46 \times 10^{-3}$ m $= 31.46$ mm, $\quad d = 2c = 62.9$ mm

Design must use larger value for d. $\quad d = 62.9$ mm ◄

Problem 3.50

3.50 The electric motor exerts a torque of 800 N·m on the steel shaft $ABCD$ when it is rotating at constant speed. Design specifications require that the diameter of the shaft be uniform from A to D and that the angle of twist between A and D not exceed 1.5°. Knowing that $\tau_{max} \le 60$ MPa and $G = 77$ GPa, determine the minimum diameter shaft that may be used.



Torques

$T_{AB} = 300 + 500 = 800$ N·m

$T_{BC} = 500$ N·m, $\quad T_{CD} = 0$

Design based on stress $\quad \tau = 60 \times 10^6$ Pa

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(800)}{\pi(60\times10^6)} = 8.488\times10^{-6}\ \text{m}^3$

$c = 20.40\times10^{-3}$ m $= 20.40$ mm, $\qquad d = 2c = 40.8$ mm

Design based on deformation $\quad \varphi_{D/A} = 1.5° = 26.18\times10^{-3}$ rad

$\varphi_{D/C} = 0$

$\varphi_{C/B} = \frac{T_{BC}L_{BC}}{GJ} = \frac{(500)(0.6)}{GJ} = \frac{600}{GJ}$

$\varphi_{B/A} = \frac{T_{AB}L_{AB}}{GJ} = \frac{(800)(0.4)}{GJ} = \frac{320}{GJ}$

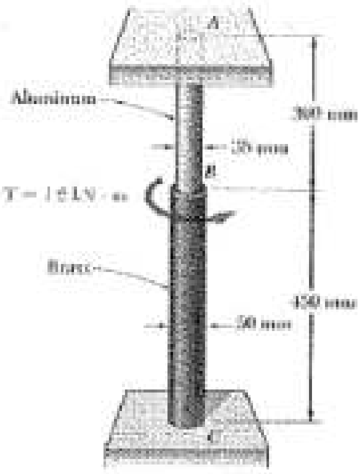
$\varphi_{D/A} = \varphi_{D/C} + \varphi_{C/B} + \varphi_{B/A} = \frac{620}{GJ} = \frac{620}{G\frac{\pi}{2}c^4} = \frac{(2)(620)}{\pi G c^4}$

$c^4 = \frac{(2)(620)}{\pi G \varphi_{D/A}} = \frac{(2)(620)}{\pi(77\times10^9)(26.18\times10^{-3})} = 195.80\times10^{-9}\ \text{m}^4$

$c = 21.04\times10^{-3}$ m $= 21.04$ mm, $\qquad d = 2c = 42.1$ mm

Design must use larger value of d. $\qquad d = 42.1$ mm ◄

Problem 3.52



3.52 Solve Prob. 3.51, assuming that cylinder AB is made of steel, for which G = 77 GPa.

3.51 The solid cylinders AB and BC are bonded together at B and are attached to fixed supports at A and C. Knowing that the modulus of rigidity is 26 GPa for aluminum and 39 GPa for brass, determine the maximum shearing stress (*a*) in cylinder AB, (*b*) in cylinder BC.

The torques in cylinders AB and BC are statically indeterminate. Match the rotation φ_B for each cylinder.

<u>Cylinder AB</u> $c = \frac{1}{2}d = 0.019 m$ $L = 0.3 m$

$$J = \frac{\pi}{2}c^4 = 204.7 \times 10^{-9} m^4$$

$$\varphi_B = \frac{T_{AB}L}{GJ} = \frac{T_{AB}(0.3)}{(77\times10^9)(204.7\times10^{-9})} = 19.03 \times 10^{-6}\, T_{AB}$$

<u>Cylinder BC</u> $c = \frac{1}{2}d = 0.025 m$ $L = 0.45 m$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.025)^4 = 613.6 \times 10^{-9} m^4$$

$$\varphi_B = \frac{T_{BC}L}{GJ} = \frac{T_{BC}(0.45)}{(39\times10^9)(613.6\times10^{-9})} = 18.8 \times 10^{-6}\, T_{BC}$$

Matching expressions for φ_B, $19.03 \times 10^{-6}\, T_{AB} = 18.8 \times 10^{-6}\, T_{BC}$

$$T_{BC} = 1.01223\, T_{AB} \qquad (1)$$

Equilibrium of connection at B: $T_{AB} + T_{BC} - T = 0$ $T = 1200 Nm$

$$T_{AB} + T_{BC} = 1200 \qquad (2)$$

Substituting (1) into (2) $2.01223\, T_{AB} = 1200$

$T_{AB} = 596$ Nm $T_{BC} = 603$ Nm

(a) <u>Maximum stress in cylinder AB.</u>

$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{(596)(0.019)}{204.7\times10^{-9}} = 55.32 \text{ MPa}$$

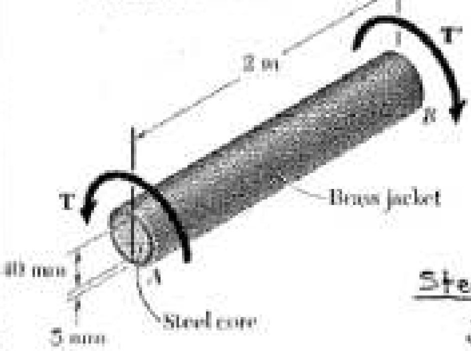
$\tau_{AB} = 55.3$ MPa ◄

(b) <u>Maximum stress in cylinder BC.</u>

$$\tau_{BC} = \frac{T_{BC}c}{J} = \frac{(603)(0.025)}{613.6\times10^{-9}} = 24.57 \text{ MPa}$$

$\tau_{BC} = 24.6$ MPa ◄

Problem 3.53



3.53 The composite shaft shown consists of a 5-mm-thick brass jacket (G_{brass} = 39 GPa) bonded to a 40-mm-diameter steel core (G_{steel} = 77.2 GPa). Knowing that the shaft is subjected to a 600 N · m torque, determine (*a*) the maximum shearing stress in the brass jacket, (*b*) the maximum shearing stress in the steel core, (*c*) the angle of twist of *B* relative to *A*.

$L = 2\text{ m}$

<u>Steel core:</u> $c = \frac{1}{2}d = 20\text{ mm} = 0.020\text{ m}$

$J_1 = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33\times10^{-9}\text{ m}^4$

$G_1J_1 = (77.2\times10^9)(251.33\times10^{-9}) = 19.4025\times10^3\text{ N}\cdot\text{m}^2$

Torque carried by steel core $\quad T_1 = G_1J_1\frac{\varphi}{L}$

<u>Brass jacket:</u> $c_1 = \frac{1}{2}d_1 = 20\text{ mm} = 0.020\text{ m} \qquad c_2 = 20+5 = 25\text{ mm} = 0.025\text{ m}$

$J_2 = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.025^4 - 0.020^4) = 362.265\times10^{-9}\text{ m}^4$

$G_2J_2 = (39\times10^9)(362.265\times10^{-9}) = 14.1283\times10^3\text{ N}\cdot\text{m}^2$

Torque carried by brass jacket $\qquad T_2 = G_2J_2\frac{\varphi}{L}$

<u>Total torque,</u> $\qquad T = T_1 + T_2 = (G_1J_1 + G_2J_2)\frac{\varphi}{L}$

$$\frac{\varphi}{L} = \frac{T}{G_1J_1 + G_2J_2} = \frac{600}{19.4025\times10^3 + 14.1283\times10^3} = 17.894\times10^{-3}\text{ rad/m}$$

(a) <u>Maximum shearing stress in brass jacket.</u>

$\tau_{max} = G\gamma_{max} = G_2c_2\frac{\varphi}{L} = (39\times10^9)(0.025)(17.894\times10^{-3})$

$= 17.45\times10^6\text{ Pa} \qquad\qquad \tau_{brass} = 17.45\text{ MPa}$ ◀

(b) <u>Maximum shearing stress in steel core.</u>

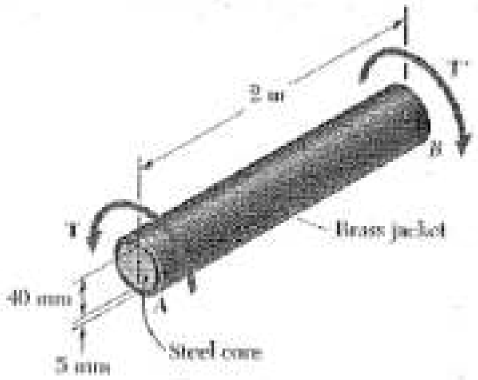
$\tau_{max} = G\gamma_{max} = G_1c\frac{\varphi}{L} = (77.2\times10^9)(0.020)(17.894\times10^{-3})$

$= 27.6\times10^6\text{ Pa} \qquad\qquad \tau_{steel} = 27.6\text{ MPa}$ ◀

(c) <u>Angle of twist.</u>

$\varphi = L\frac{\varphi}{L} = (2)(17.894\times10^{-3}) = 35.788\times10^{-3}\text{ rad} \qquad\qquad \varphi = 2.05°$ ◀

Problem 3.54



3.54 For the composite shaft of Prob. 3.53 the allowable shearing stress in the brass jacket is 20 MPa and 45 MPa in the steel core. Determine (*a*) the largest torque which can be applied to the shaft, (*b*) the corresponding angle of twist of *B* relative to *A*.

3.53 The composite shaft shown consists of a 5-mm-thick brass jacket (G_{brass} = 39 GPa) bonded to a 40-mm-diameter steel core (G_{steel} = 77.2 GPa). Knowing that the shaft is subjected to a 600 N · m torque, determine (*a*) the maximum shearing stress in the brass jacket, (*b*) the maximum shearing stress in the steel core, (*c*) the angle of twist of *B* relative to *A*.

$$\tau_{max} = G\gamma_{max} = G c_{max} \frac{\varphi}{L}$$

$$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G c_{max}} \quad \text{for each material}$$

$$T_i = G_i J_i \frac{\varphi}{L} \quad \text{for each material}$$

Brass jacket: $\tau_{all} = 20 \times 10^6$ Pa, $c_1 = 20$ mm $= 0.020$ m, $c_2 = 20 + 5 = 25$ mm $= 0.025$ m

$$\frac{\varphi_{all}}{L} = \frac{20 \times 10^6}{(39 \times 10^9)(0.025)} = 20.513 \times 10^{-3} \text{ rad/m}$$

$$J_2 = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.025^4 - 0.020^4) = 362.265 \times 10^{-9} \text{ m}^4$$

Steel core: $\tau_{all} = 45 \times 10^6$ Pa, $c = 0.020$ m

$$\frac{\varphi_{all}}{L} = \frac{45 \times 10^6}{(77.2 \times 10^9)(0.020)} = 29.145 \times 10^{-3} \text{ rad/m}$$

$$J_1 = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Smaller value of $\frac{\varphi_{all}}{L}$ governs $\qquad \frac{\varphi_{all}}{L} = 20.513 \times 10^{-3}$ rad/m

Torque carried by brass sleeve

$$T_2 = G_2 J_2 \frac{\varphi_{all}}{L} = (39 \times 10^9)(362.265 \times 10^{-9})(20.513 \times 10^{-3}) = 289.8 \text{ N·m}$$

Torque carried by steel core

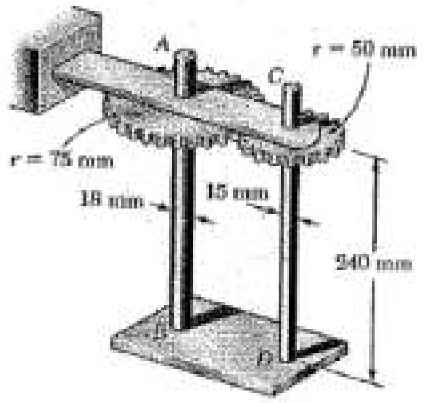
$$T_1 = G_1 J_1 \frac{\varphi_{all}}{L} = (77.2 \times 10^9)(251.33 \times 10^{-9})(20.513 \times 10^{-3}) = 398.0 \text{ N·m}$$

(a) Allowable torque. $\quad T = T_1 + T_2 = 687.8$ N·m $\qquad T_{all} = 688$ N·m ◄

(b) Allowable twist angle.

$$\varphi_{all} = L\frac{\varphi_{all}}{L} = (2)(20.513 \times 10^{-3}) = 41.026 \times 10^{-3} \text{ rad} \qquad \varphi_{all} = 2.35° \;◄$$

Problem 3.56



3.56 Solve Prob. 3.55, assuming that the 80-N · m torque is applied to end *C* of shaft *CD*.

3.55 At a time when rotation is prevented at the lower end of each shaft, a 50-N · m torque is applied to end *A* of shaft *AB*. Knowing that $G = 77.2$ GPa for both shafts, determine (*a*) the maximum shearing stress in shaft *CD*, (*b*) the angle of rotation at *A*.

Let T_C = torque applied at C = 50 N·m

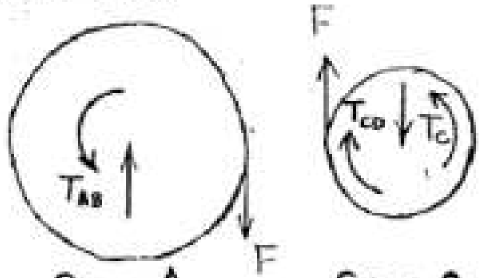
T_{CD} = torque in shaft CD.

T_{AB} = torque in shaft AB.

<u>Statics:</u>

$$T_{AB} - r_A F = 0$$

$$T_C - T_{CD} - r_C F = 0$$



Gear A   Gear C

$$T_{AB} = \frac{r_A}{r_C}(T_C - T_{CD}) = \frac{3}{2}(T_C - T_{CD})$$

<u>Kinematics:</u> $r_A \varphi_A = r_C \varphi_C$ $\varphi_C = \frac{r_A}{r_C}\varphi_A = \frac{3}{2}\varphi_A$

<u>Angles of twist:</u> $\varphi_C = \dfrac{T_{CD} L}{G J_{CD}}$ $\varphi_A = \dfrac{T_{AB} L}{G J_{AB}} = \dfrac{3}{2}\dfrac{(T_C - T_{CD})L}{G J_{AB}}$

$$\frac{T_{CD} L}{G J_{CD}} = \frac{3}{2}\cdot\frac{3}{2}\;\frac{T_C - T_{CD}}{G J_{AB}}$$

$$\left(\frac{J_{AB}}{J_{CD}} + \frac{9}{4}\right)T_{CD} = \left(\left(\frac{18}{15}\right)^4 + \frac{9}{4}\right)T_{CD} = \frac{9}{4}T_C$$

$$T_{CD} = 0.4796\,T_C = (0.52040\,(80) = 41.632 \text{ N·m}$$

$$T_{AB} = \frac{3}{2}(80 - 41.632) = 57.552 \text{ N·m}$$

(a) <u>Maximum shearing stress in CD.</u>

$$\tau_{CD} = \frac{T_{CD}\,c}{J_{CD}} = \frac{2T_{CD}}{\pi c^3} = \frac{(2)(41.632)}{\pi(0.0075)^3} = 62.8\times10^6 \text{ Pa}$$

$\tau_{CD} = 62.8$ MPa ◀

(b) <u>Angle of rotation at A.</u>

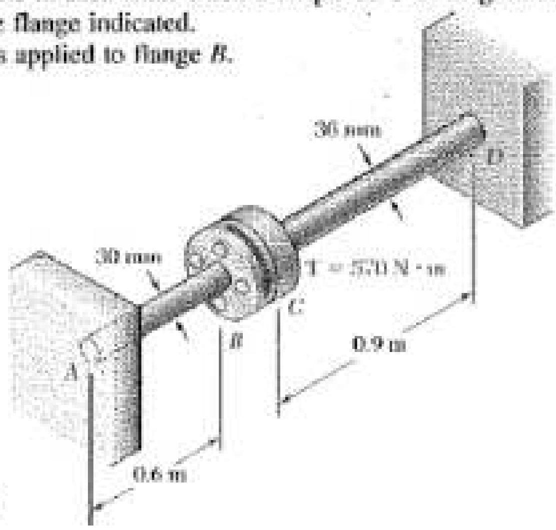
$$\varphi_A = \frac{T_{AB} L}{G J_{AB}} = \frac{2T_{AB}L}{\pi G c^4} = \frac{(2)(57.552)(0.240)}{\pi(77.2\times10^9)(0.009)^4} = 17.36\times10^{-3} \text{ rad}$$

$\varphi_A = 0.995°$ ◀

Problem 3.57

3.57 and 3.58 Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 77$ GPa, determine the maximum shearing stress in each shaft when a torque of **T** of magnitude 570 N · m is applied to the flange indicated.

3.57 The torque **T** is applied to flange *B*.



Shaft AB

$T = T_{AB}$, $L_{AB} = 0.6m$, $c = \frac{1}{2}d = 0.015m$

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.5 \times 10^{-9} m^4$

$\varphi_B \quad \frac{T_{AB} L_{AB}}{G J_{AB}}$

$T_{AB} = \frac{G J_{AB} \varphi_B}{L_{AB}} - \frac{(77 \times 10^9)(79.5 \times 10^{-9})}{0.6} \varphi_B$

$= 10202.5 \; \varphi_B$

Shaft CD

$T = T_{CD}$, $L_{CD} = 0.9m$, $c = \frac{1}{2}d = 0.018m$

$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.9 \times 10^{-9} m^4$

$\varphi_c = \frac{T_{CD} L_{CD}}{G J_{CD}}$

Applied torque
$T = 570 Nm$

$T_{CD} = \frac{G J_{CD} \varphi_c}{L_{CD}} = \frac{(77 \times 10^9)(164.9 \times 10^{-9})}{0.9} \varphi_c = 14108.1 \; \varphi_c$

Clearance rotation for flange B : $\varphi_B' = 1.5° = 26.18 \times 10^{-3}$ rad

Torque to remove clearance: $T_{AB}' = (10202.5)(26.18 \times 10^{-3}) = 267.1$ Nm

Torque T'' to cause additional rotation φ'': $T'' = 570 - 267.1 = 302.9$ Nm

$T'' = T_{AB}'' + T_{CD}''$

$302.9 = (10202.5)\varphi'' + (14108.1)\varphi'' \qquad \therefore \quad \varphi'' = 12.46 \times 10^{-3}$ rad

$T_{AB}'' = (10202.5)(12.46 \times 10^{-3}) = 127.1$ Nm

$T_{CD}'' = (14108.1)(12.46 \times 10^{-3}) = 175.8$ Nm

Maximum shearing stress in AB.

$\tau_{AB} = \frac{T_{AB} c}{J_{AB}} = \frac{(267.1 + 127.1)(0.015)}{79.5 \times 10^{-9}} = 74.38$ MPa $\qquad \tau_{AB} = 74.4$ MPa

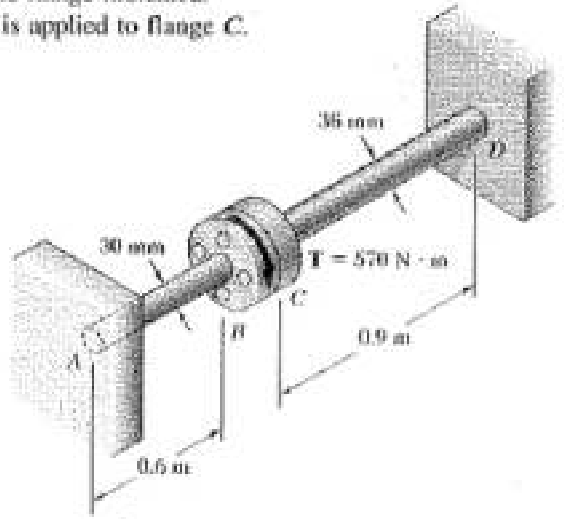
Maximum shearing stress in CD.

$\tau_{CD} = \frac{T_{CD} c}{J_{CD}} = \frac{(175.8)(0.018)}{164.9 \times 10^{-9}} = 19.19$ MPa $\qquad \tau_{CD} = 19.2$ MPa

Problem 3.58

3.57 and 3.58 Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 77$ GPa, determine the maximum shearing stress in each shaft when a torque of **T** of magnitude 570 N · m is applied to the flange indicated.

3.58 The torque **T** is applied to flange C.



Shaft AB

$T = T_{AB}$, $L_{AB} = 0.6\,m$, $c = \frac{1}{2}d = 0.015\,m$

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.5\times10^{-9}\,m^4$

$\varphi_B \quad \frac{T_{AB}L_{AB}}{GJ_{AB}}$

$T_{AB} = \frac{GJ_{AB}\varphi_B}{L_{AB}} = \frac{(77\times10^9)(79.5\times10^{-9})}{0.6}\varphi_B$

$= 10202.5 \quad \varphi_B$

Shaft CD

$T = T_{CD}$, $L_{CD} = 0.9\,m$, $\quad c = \frac{1}{2}d = 0.018\,m$

$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.9\times10^{-9}\,m^4$

$\varphi_C = \frac{T_{CD}L_{CD}}{GJ_{CD}}$

$T_{CD} = \frac{GJ_{CD}\varphi_C}{L_{CD}} = \frac{(77\times10^9)(164.9\times10^{-9})}{0.9}\varphi_C = 14108.1\,\varphi_C$

Applied torque

$T = 570\,Nm$

Clearance rotation for flange C: $\varphi_C' = 1.5° = 26.18\times10^{-3}$ rad

Torque to remove clearance: $T_{CD}' = (14108.1)(26.18\times10^{-3}) = 369.4$ Nm

Torque T″ to cause additional rotation φ'': $T'' = 570 - 369.4 = 200.6$ Nm

$T'' = T_{AB}'' + T_{CD}''$

$200.6 = (10202.5)\varphi'' + (14108.1)\varphi'' \quad \therefore \quad \varphi'' = 8.2515\times10^{-3}$ rad

$T_{AB}'' = (10202.5)(8.2515\times10^{-3}) = 84.2$ Nm

$T_{CD}'' = (14108.1)(8.2515\times10^{-3}) = 116.4$ Nm

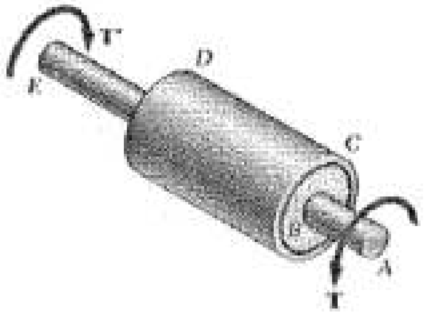
Maximum shearing stress in AB.

$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(84.2)(0.015)}{79.5\times10^{-9}} = 15.89$ MPa $\qquad \tau_{AB} = 15.9$ MPa ◄

Maximum shearing stress in CD.

$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(369.4 + 116.4)(0.018)}{164.9\times10^{-9}} = 53.03$ MPa $\qquad \tau_{CD} = 53$ MPa ◄

Problem 3.59



3.59 The steel jacket CD has been attached to the 40-mm-diameter steel shaft AE by means of *rigid* flanges welded to the jacket and to the rod. The outer diameter of the jacket is 80 mm and its wall thickness is 4 mm. If 500 N · m torques are applied as shown, determine the maximum shearing stress in the jacket.

Solid shaft: $c = \frac{1}{2}d = 0.020$ m

$$J_S = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Jacket: $c_2 = \frac{1}{2}d = 0.040$ m

$$c_1 = c_2 - t = 0.040 - 0.004 = 0.036 \text{ m}$$

$$J_J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.040^4 - 0.036^4) = 1.3829 \times 10^{-6} \text{ m}^4$$

Torque carried by shaft. $T_S = GJ_S\varphi/L$

Torque carried by jacket. $T_J = GJ_J\varphi/L$

Total torque. $T = T_S + T_J = (J_S + J_J)G\varphi/L \quad \therefore \quad \frac{G\varphi}{L} = \frac{T}{J_S + J_J}$

$$T_J = \frac{J_J}{J_S + J_J}T = \frac{(1.3829 \times 10^{-6})(500)}{1.3829 \times 10^{-6} + 251.33 \times 10^{-9}} = 423.1 \text{ N}\cdot\text{m}$$

Maximum shearing stress in jacket.

$$\tau = \frac{T_J c}{J_J} = \frac{(423.1)(0.040)}{1.3829 \times 10^{-6}} = 12.24 \times 10^6 \text{ Pa} \qquad 12.24 \text{ MPa}$$

Problem 3.60

3.60 The mass moment of inertia of a gear is to be determined experimentally by using a torsional pendulum consisting of a 1.8-m steel wire. Knowing that $G = 77$ GPa, determine the diameter of the wire for which the torsional spring constant will be 6 N · m/rad.

Torsion spring constant $K = 6$ N.m/rad

$$K = \frac{T}{\varphi} = \frac{T}{TL/GJ} = \frac{GJ}{L} = \frac{\pi G c^4}{2L}$$

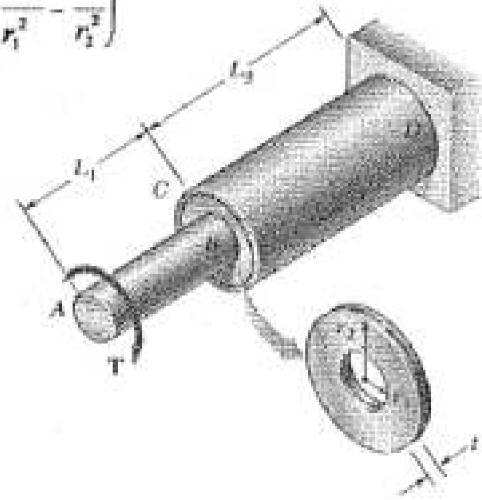
$$c^4 = \frac{2LK}{\pi G} = \frac{(2)(1.8)(6)}{\pi(77 \times 10^9)} = 89.292 \times 10^{-12} \text{ m}^4$$

$c = 0.00307$ m $\qquad d = 2c = 6$ mm

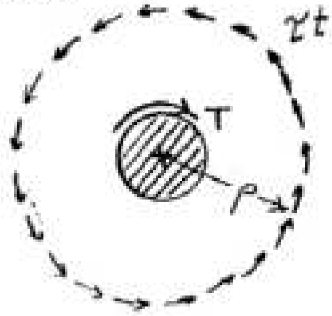
Problem 3.61

3.61 An annular plate of thickness t and modulus of rigidity G is used to connect shaft AB of radius r_1 to tube CD of inner radius r_2. Knowing that a torque $\mathbf{T}$ is applied to end A of shaft AB and that end D of tube CD is fixed, (a) determine the magnitude and location of the maximum shearing stress in the annular plate, (b) show that the angle through which end B of the shaft rotates with respect to end C of the tube is

$$\phi_{B/C} = \frac{T}{4\pi G t}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$



Use a free body consisting of shaft AB and an inner portion of the plate BC, the outer radius of this portion being ρ



The force per unit length of circumference is τt.

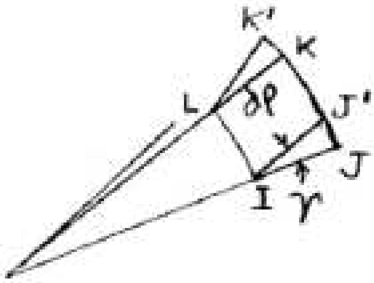
$\Sigma M = 0$

$$\tau t(2\pi\rho)\rho - T = 0$$

$$\tau = \frac{T}{2\pi t \rho^2}$$

(a) Maximum shearing stress occurs at $\rho = r_1$ $\qquad \tau_{max} = \frac{T}{2\pi t r_1^2}$ ◄

Shearing strain $\quad \gamma = \frac{\tau}{G} = \frac{T}{2\pi G t \rho^2}$



The relative circumferential displacement in radial length $d\rho$ is

$$dS = \gamma\, d\rho = \rho\, d\varphi$$

$$d\varphi = \gamma \frac{d\rho}{\rho}$$

$$d\varphi = \frac{T}{2\pi G t \rho^2}\frac{d\rho}{\rho} = \frac{T\, d\rho}{2\pi G t\, \rho^3}$$

$$\text{(b)}\quad \varphi_{B/C} = \int_{r_1}^{r_2} \frac{T}{2\pi G t}\frac{d\rho}{\rho^3} = \frac{T}{2\pi G t}\int_{r_1}^{r_2}\frac{d\rho}{\rho^3} = \frac{T}{2\pi G t}\left\{-\frac{1}{2\rho^2}\right\}\Bigg|_{r_1}^{r_2}$$

$$= \frac{T}{2\pi G t}\left\{-\frac{1}{2r_2^2} + \frac{1}{2r_1^2}\right\} = \frac{T}{4\pi G t}\left\{\frac{1}{r_1^2} - \frac{1}{r_2^2}\right\}$$ ◄

Problem 3.71

3.71 A steel drive shaft is 1.8 m long and its outer and inner diameters are respectively equal to 56 mm and 42 mm. Knowing that the shaft transmits 180 kW while rotating at 1800 rpm, determine (*a*) the maximum shearing stress, (*b*) the angle of twist of the shaft ($G = 77$ GPa).

$L = 1.8\ m \qquad c_2 = \frac{1}{2} d_o = 28\ mm \qquad c_1 = \frac{1}{2} d_i = 21\ mm$

$P = 180\ kW$

$f = \frac{1800}{60} = 30\ Hz \qquad T = \frac{P}{2\pi f} = \frac{180 \times 10^3}{2\pi (30)} = 955.10\ Nm$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.028^4 - 0.021^4) = 660 \times 10^{-9}\ m^4$

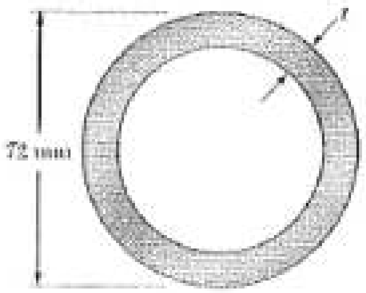
(a) Maximum shearing stress. $\qquad \tau = \frac{Tc_2}{J}$

$\tau = \frac{(955)(0.028)}{660 \times 10^{-9}} = 40.5\ MPa \qquad \tau = 40.5\ MPa$ ◄

(b) Angle of twist. $\qquad \varphi = \frac{TL}{GJ}$

$\varphi = \frac{(955)(1.8)}{(77 \times 10^9)(660 \times 10^{-9})} = 0.0338\ rad \qquad \varphi = 1.94°$ ◄

Problem 3.72



3.72 A steel pipe of 72-mm outer diameter is to be used to transmit a torque of 2500 N · m without exceeding an allowable shearing stress of 55 MPa. A series of 72-mm-outer-diameter pipes is available for use. Knowing that the wall thickness of the available pipes varies from 4 mm to 10 mm in 2-mm increments, choose the lightest pipe that can be used.

$c_2 = \frac{1}{2} d_o = 36\ mm = 0.036\ m$

$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$

$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.036^4 - \frac{(2)(2500)(0.036)}{\pi(55 \times 10^6)} = 637.875 \times 10^{-9}$

$c_1 = 28.26 \times 10^{-3}\ m = 28.26\ mm$

$t = c_2 - c_1 = 36\ mm - 28.26\ mm = 7.74\ mm \qquad$ Use $t = 8\ mm$ ◄

Problem 3.75

3.75 A 2.5-m-long solid steel shaft is to transmit 10 kW at a frequency of 25 Hz. Determine the required diameter of the shaft, knowing that $G = 77.2$ GPa, that the allowable shearing stress is 30 MPa, and that the angle of twist must not exceed 4°.

$P = 10\ kW = 10\times10^3\ W \qquad f = 25\ Hz \qquad \varphi = 4° = 69.813\times10^{-3}\ rad$

$$T = \frac{P}{2\pi f} = \frac{10\times10^3}{2\pi(25)} = 63.662\ N\cdot m$$

Stress requirement. $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(63.662)}{\pi(30\times10^6)}} = 11.055\times10^{-3}\ m = 11.055\ mm$$

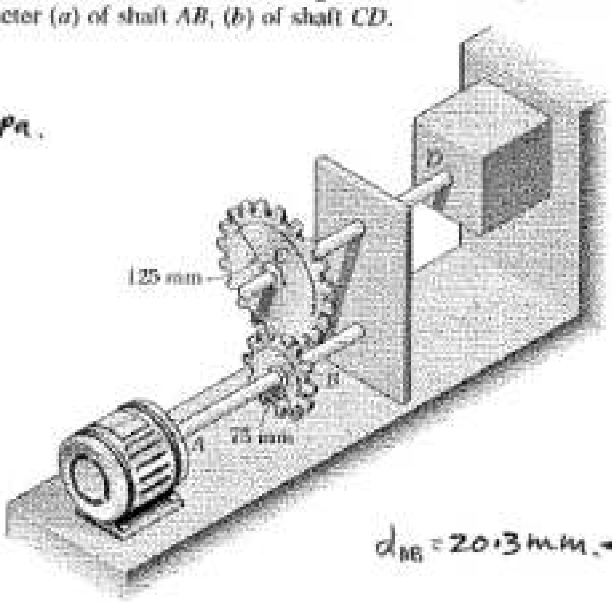
Twist angle requirement. $\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$

$$c = \sqrt[4]{\frac{2TL}{\pi G\varphi}} = \sqrt[4]{\frac{(2)(63.662)(2.5)}{\pi(77.2\times10^9)(69.813\times10^{-3})}} = 11.709\ mm$$

Use the larger value. $c = 11.709\ mm \qquad d = 2c = 23.4\ mm$ ◄

Problem 3.76

3.76 The two solid shafts and gears shown are used to transmit 12 kW from the motor at *A* operating at a speed of 1260 rpm, to a machine tool at *D*. Knowing that the maximum allowable shearing stress is 55 MPa, determine the required diameter (*a*) of shaft *AB*, (*b*) of shaft *CD*.



(a) Shaft AB : $P = 12\ kW$

$f = \frac{1260}{60} = 21\ Hz \qquad \tau = 55\ MPa.$

$$T_{AB} = \frac{P}{2\pi f} = \frac{12\times10^3}{2\pi(21)} = 91\ Nm$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi\tau}}$$

$$c = \sqrt[3]{\frac{(2)(91)}{\pi(55\times10^6)}} = 0.01017\ m$$

$d_{AB} = 2c = 20.3\ mm$

$d_{AB} = 20.3\ mm$ ◄

(b) Shaft CD

$$T_{CD} = \frac{r_C}{r_B}T_{AB} = \frac{5}{3}(91) = 151.7\ Nm$$

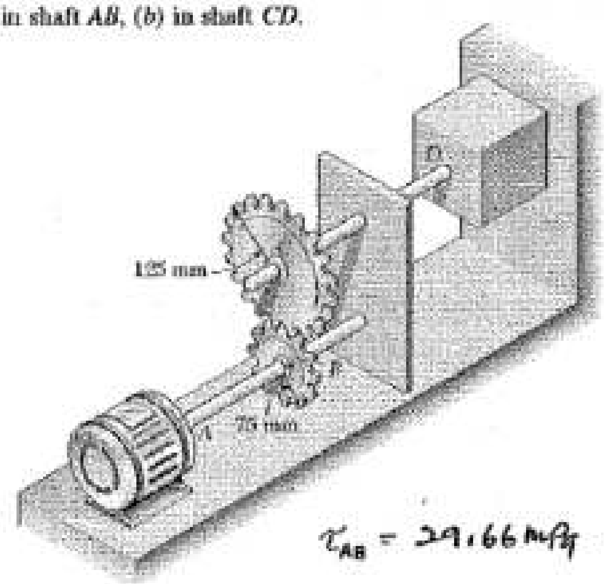
$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(151.7)}{\pi(55\times10^6)}} = 0.01206\ m$$

$d_{CD} = 2c = 0.024\ m$

$d_{CD} = 24\ mm$ ◄

Problem 3.77

3.77 The two solid shafts and gears shown are used to transmit 12 kW from the motor at A operating at a speed of 1260 rpm, to a machine tool at D. Knowing that each shaft has a diameter of 25 mm, determine the maximum shearing stress (*a*) in shaft AB, (*b*) in shaft CD.



(a) Shaft AB: $P = 12\text{ kW}$

$f = \frac{1260}{60} = 21\text{ Hz}$

$T_{AB} = \frac{P}{2\pi f} = \frac{12\times10^3}{2\pi(21)} = 91\text{ Nm}$

$c = \frac{1}{2}d = 12.5\text{ mm}$

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$= \frac{(2)(91)}{\pi(0.0125)^3} = 29.66\text{ MPa}.$

$\tau_{AB} = 29.66\text{ MPa}$

(b) Shaft CD

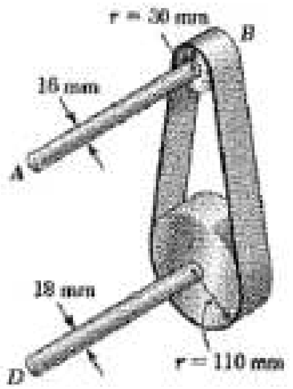
$T_{CD} = \frac{r_C}{r_B}T_{AB} = \frac{5}{3}(91) = 151.7\text{ Nm}.$

$\tau = \frac{2T}{\pi c^3} = \frac{(2)(151.7)}{\pi(0.0125)^3} = 49.44\text{ MPa}.$

$\tau_{CD} = 49.44\text{ MPa}$

Problem 3.78

3.78 The shaft-disk-belt arrangement shown is used to transmit 2 kW from point A to point D. (*a*) Using an allowable shearing stress of 66 MPa, determine the required speed of shaft AB. (*b*) Solve part *a*, assuming that the diameters of shafts AB and CD are, respectively, 18 mm and 15 mm.



SOLUTION

$\tau = 66\,\text{MPa}, \quad P = 2\,\text{kW/s}$

$\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3} \qquad T = \dfrac{\pi}{2} c^3 \tau$

Allowable torques

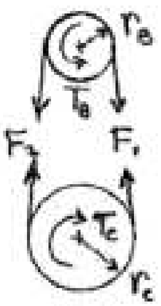
16 mm diameter shaft

$c = 8\,\text{mm}, \quad T_{all} = \dfrac{\pi}{2}(0.008)^3(66\times10^6) = 53\,\text{Nm}.$

18 mm diameter shaft

$c = 9\,\text{mm}, \quad T_{all} = \dfrac{\pi}{2}(0.009)^3(66\times10^6) = 75.6\,\text{Nm}.$

Statics:



$T_B = r_B(F_1 - F_2) \qquad T_C = r_C(F_1 - F_2)$

$T_B = \dfrac{r_B}{r_C} T_C = \dfrac{30}{110} T_C = 0.2727\, T_C$

(a) Allowable torques $\quad T_{B,all} = 53\,\text{Nm}, \quad T_{C,all} = 75.6\,\text{Nm}.$

Assume $T_C = 75.6\,\text{Nm}$ $\quad$ Then $T_B = (0.2727)(75.6) = 20.6\,\text{Nm}$ $< 53\,\text{Nm}$ (okay)

$P = 2\pi f T \qquad f_{AB} = \dfrac{P}{2\pi T_B} = \dfrac{2000}{2\pi(20.6)} \qquad f_{AB} = 15.45\,\text{Hz}$ ◄

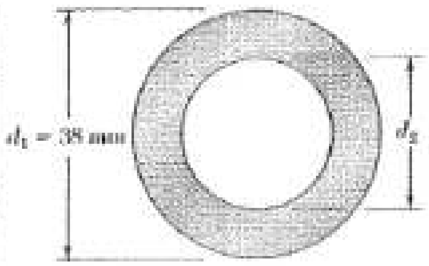
(b) Allowable torques $\quad T_{B,all} = 75.6\,\text{Nm}, \quad T_{C,all} = 53\,\text{Nm}.$

Assume $T_C = 53\,\text{Nm}$ $\quad$ Then $T_B = (0.2727)(53) = 14.5\,\text{Nm}$ $< 75.6\,\text{Nm}$

$P = 2\pi f T \qquad f_{AB} = \dfrac{P}{2\pi T_B} = \dfrac{2000}{2\pi(14.5)} \qquad f_{AB} = 21.95\,\text{Hz}$ ◄

Problem 3.83

3.83 A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 is to be made of a steel for which $\tau_{all} = 65$ MPa and $G = 77.2$ GPa. Knowing that the angle of twist must not exceed 4° when the shaft is subjected to a torque of 600 N · m, determine the largest inner diameter d_2 that can be specified in the design.



$L = 1.5\text{ m} \qquad c_2 = \frac{1}{2} d_o = 19\text{ mm} = 0.019\text{ m}$

$\tau = 65 \times 10^6 \text{ Pa} \qquad \varphi = 4° = 69.813 \times 10^{-3}\text{ rad}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4)$

Stress requirement. $\qquad \tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$

$c_1 = \sqrt[4]{c_2^4 - \frac{2Tc_2}{\pi\tau}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(0.019)}{\pi(65\times10^6)}} = 11.689\times10^{-3}\text{ m} = 11.689\text{ mm}$

Twist angle requirement. $\qquad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G(c_2^4 - c_1^4)}$

$c_1 = \sqrt[4]{c_2^4 - \frac{2TL}{\pi G\varphi}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(1.5)}{\pi(77.2\times10^9)(69.813\times10^{-3})}}$

$c_1 = 12.448\times10^{-3}\text{ m} = 12.448\text{ mm}$

Use smaller value of c_1 $\qquad c_1 = 11.689$ mm $\qquad d_2 = 2c_1 = 23.4$ mm ◄

Problem 3.84

3.84 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 12$ mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 50 MPa.

$d = 125\text{ mm} \quad D = 150\text{ mm} \quad r = 12\text{ mm}$

$\frac{D}{d} = \frac{150}{125} = 1.20 \qquad \frac{r}{d} = \frac{12}{125} = 0.096 \qquad$ From Fig 3.32 $K = 1.35$

For the smaller side $\qquad c = \frac{1}{2}d = 62.5\text{ mm} \qquad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

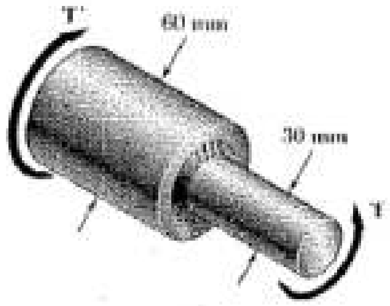
$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.0625)^3(50\times10^6)}{(2)(1.35)} = 14.2\text{ kNm}$

$f = 450\text{ rpm} = 7.5\text{ Hz}$

Power $\quad P = 2\pi f T = 2\pi(7.5)(14.2\times10^3) = 669\text{ kW/s}$

$P = 669\text{ kW}$ ◄

Problem 3.87



3.87 Knowing that the stepped shaft shown must transmit 45 kW at a speed of 2100 rpm, determine the minimum radius *r* of the fillet if an allowable shearing stress of 50 MPa is not to be exceeded.

$$f = \frac{2100}{60} = 35 \text{ Hz}$$

$$P = 45 \times 10^3 \text{ W}$$

$$T = \frac{P}{2\pi f} = \frac{45 \times 10^3}{2\pi(35)} = 204.63 \text{ N}\cdot\text{m}$$

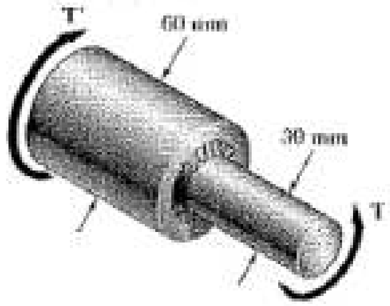
For smaller side $\quad c = \frac{1}{2}d = 15 \text{mm} = 0.015 \text{m} \qquad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

$$K = \frac{\pi \tau c^3}{2T} = \frac{\pi(50\times10^6)(0.015)^3}{(2)(204.63)} = 1.295$$

$\frac{D}{d} = \frac{60}{30} = 2 \qquad$ From Fig. 3.32 $\quad \frac{r}{d} = 0.17$

$r = 0.17 d = (0.17)(30) = 5.1 \text{ mm} \qquad\qquad r = 5.1 \text{ mm}$ ◄

Problem 3.88



3.88 The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is *r* = 6 mm, determine the smallest permissible speed of the shaft.

$\frac{r}{d} = \frac{6}{30} = 0.2 \qquad \frac{D}{d} = \frac{60}{30} = 2$

From Fig. 3.32 $\quad K = 1.26$

For smaller side $\quad c = \frac{1}{2}d = 15 \text{mm} = 0.015 \text{ m}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

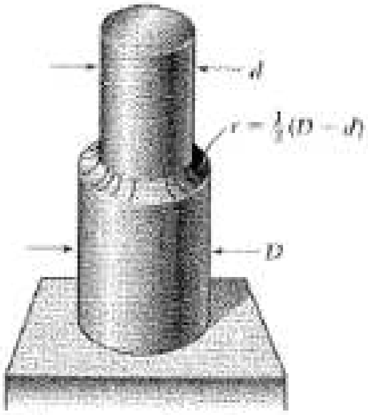
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.015)^3(40\times10^6)}{(2)(1.26)} = 168.30 \times 10^3 \text{ N}\cdot\text{m}$$

$P = 45 \text{ kW} = 45 \times 10^3 \qquad P = 2\pi f T$

$f = \frac{P}{2\pi T} = \frac{45\times10^3}{2\pi(168.30\times10^3)} = 42.6 \text{ Hz} \qquad\qquad f = 42.6 \text{ Hz}$ ◄

Problem 3.89

3.89 In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 80 MPa. Knowing that $D = 30$ mm, determine the largest allowable torque that can be applied to the shaft if (*a*) $d = 26$ mm, (*b*) $d = 24$ mm.



Full quarter-circular fillet extends to edge of larger shaft

$\tau = 80 \times 10^6$ Pa

(a) $\frac{D}{d} = \frac{30}{26} = 1.154 \qquad r = \frac{1}{2}(D-d) = 2\text{ mm}$

$\frac{r}{d} = \frac{2}{26} = 0.0768$

From Fig. 3.32, $K = 1.36$

Smaller side, $c = \frac{1}{2}d = 13\text{ mm} = 0.013\text{ m}$

$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.013)^3(80 \times 10^6)}{(2)(1.36)}$

$= 203$ N·m $\qquad T = 203$ N·m ◄

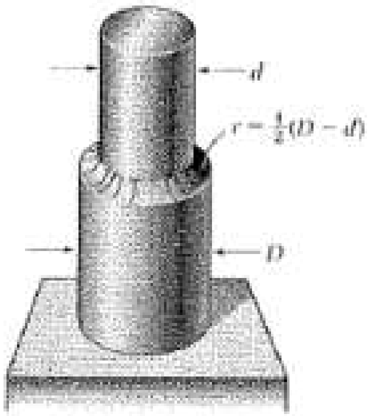
(b) $\frac{D}{d} = \frac{30}{24} = 1.25 \qquad r = \frac{1}{2}(D-d) = 3\text{ mm} \qquad \frac{r}{d} = \frac{3}{24} = 0.125$

From Fig. 3.32, $K = 1.31 \qquad c = \frac{1}{2}d = 12\text{ mm} = 0.012\text{ m}$

$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.012)^3(80 \times 10^6)}{(2)(1.31)} = 165.8$ N·m $\qquad T = 165.8$ N m ◄

Problem 3.90

3.90 In the stepped shaft shown, which has a full quarter-circular fillet, $D = 30$ mm and $d = 25$ mm. Knowing that the speed of the shaft is 2400 rpm and that the allowable shearing stress is 50 MPa, determine the maximum power that can be trasmitted by the shaft.



Full quarter-circular fillet extends to edge of larger shaft

$\frac{D}{d} = \frac{30}{25} = 1.2 \qquad r = \frac{1}{2}(D-d) = 2.5\text{ mm}$

$\frac{r}{d} = \frac{2.5}{25} = 0.1$

From Fig. 3.32 $\qquad K = 1.34$

For smaller side $\qquad c = \frac{1}{2}d = 12.5\text{ mm}$

$\tau = \frac{KTc}{J} \qquad T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K}$

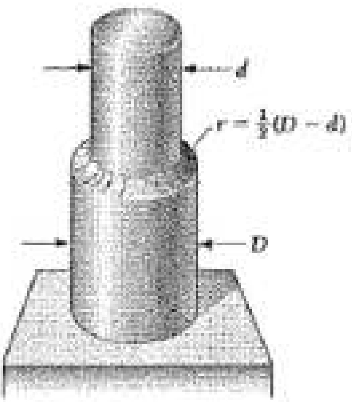
$T = \frac{\pi(0.0125)^3(50 \times 10^6)}{(2)(1.34)} = 114.5$ Nm

$f = 2400\text{ rpm} = 40\text{ Hz}$

$P = 2\pi f T = 2\pi(40)(114.5) = 28.8$ kW

$P = 28.8$ kW ◄

Problem 3.91



Full quarter-circular fillet extends to edge of larger shaft

3.91 A torque of magnitude $T = 22$ N · m is applied to the stepped shaft shown, which has a full quarter-circular fillet. Knowing that $D = 25$ mm, determine the maximum shearing stress in the shaft when (*a*) $d = 20$ mm, (*b*) $d = 23$ mm.

(a) $\frac{D}{d} = \frac{25}{20} = 1.25 \qquad r = \frac{1}{2}(D-d) = 2.5\text{ mm}$

$\frac{r}{d} = \frac{2.5}{20} = 0.125$

From Fig. 3.32 $\quad K = 1.31$

For smaller side $\quad c = \frac{1}{2}d = 10$ mm

$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} = \frac{(2)(1.31)(22)}{\pi(0.01)^3} = 18.3\text{ MPa}$

$\tau = 18.3\text{ MPa}$

(b) $\frac{D}{d} = \frac{25}{23} = 1.09 \qquad r = \frac{1}{2}(D-d) = 1\text{ mm}$

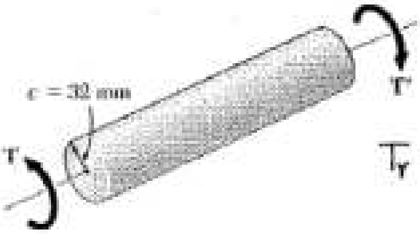
$\frac{r}{d} = \frac{1}{23} = 0.0435 \qquad$ From Fig. 3.32 $\quad K = 1.48$

For smaller side $\quad c = \frac{1}{2}d = 11.5$ mm

$\tau = \frac{2KT}{\pi c^3} = \frac{(2)(1.48)(22)}{\pi(11.5\times10^{-3})^3} = 13.6\text{ MPa}.$

$\tau = 13.6\text{ MPa}$

Problem 3.92



3.92 The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the magnitude T of the applied torque when the plastic zone is (*a*) 16 mm deep, (*b*) 24 mm deep.

$c = 32\text{ mm} = 0.32\text{m} \qquad \tau_Y = 145\times10^6\text{ Pa}$

$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.032)^3(145\times10^6)$

$= 7.4634\times10^3\text{ N·m}$

(a) $t_p = 16\text{ mm} = 0.016\text{ m} \qquad \rho_Y = c - t_p = 0.032 - 0.016 = 0.016\text{ m}$

$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634\times10^3)\left(1 - \frac{1}{4}\frac{0.016^3}{0.032^3}\right)$

$= 9.6402\times10^3\text{ N·m} \qquad T = 9.64\text{ kN·m}$

(b) $t_p = 24\text{mm} = 0.024\text{ m} \qquad \rho_Y = c - t_p = 0.032 - 0.024 = 0.008\text{ m}$

$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634\times10^3)\left(1 - \frac{1}{4}\frac{0.008^3}{0.032^3}\right)$

$= 9.9123\times10^3\text{ N·m} \qquad T = 9.91\text{ kN·m}$

Problem 3.93

3.93 A 30-mm-diameter solid rod is made of an elastoplastic material with $\tau_Y = 35$ MPa. Knowing that the elastic core of the rod is of diameter 25 mm, determine the magnitude of the torque applied to the rod.

$c = \frac{1}{2}d = 0.015\text{ m}$ $\quad \tau_Y = 35\text{ MPa}$ $\quad \rho_Y = \frac{1}{2}d_Y = 0.0125\text{ m}$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.015)^3(35\times10^6) = 185.6\text{ Nm}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(185.6)\left(1 - \frac{1}{4}\frac{0.0125^3}{0.015^3}\right) = 211.7\text{ Nm}$$

$T = 211.7\text{ Nm.}$ ◀

Problem 3.94

3.94 A 50-mm-diameter solid shaft is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 140$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (*a*) 3 kN · m, (*b*) 4 kN · m.

$c = \frac{1}{2}d = 25\text{ mm}$ $\quad \tau_Y = 140\text{ MPa}$

Compute T_Y $\quad T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.025)^3(140\times10^6) = 3436\text{ N}$

(a) $T = 3000 < T_Y$ $\quad$ elastic $\quad \rho = c = 25\text{ mm}$ ◀

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(3000)}{\pi(0.025)^3} = 122.2\text{ MPa}$$

$\tau_{max} = 122.2\text{ MPa}$ ◀

(b) $T = 4000 > T_Y$ $\quad$ plastic region with elastic core

The maximum shearing stress is $\quad \tau_{max} = \tau_Y = 140\text{ MPa}$ ◀

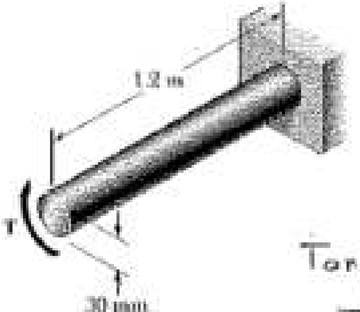
$$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(4000)}{3436} = 0.5076 \qquad \frac{\rho_Y}{c} = 0.8$$

$\rho_Y = 0.8 \quad = (0.8)(25)$ $\qquad \rho_Y = 20\text{ mm}$ ◀

Problem 3.99

3.99 For the solid circular shaft of Prob. 3.95, determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 600$ N · m, (*b*) $T = 1000$ N · m.



3.95 The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (*a*) $T = 600$ N · m, (*b*) $T = 1000$ N · m. $(G = 77.2\ \text{GPa})$.

$c = \frac{1}{2}d = 15\ \text{mm} = 15\times10^{-3}\ \text{m}$

Torque at onset of yielding $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$T_Y = \frac{\pi c^3 \tau_Y}{2} = \frac{\pi(15\times10^{-3})^3(145\times10^6)}{2} = 768.71\ \text{N}\cdot\text{m}$$

(a) $T = 600\ \text{N}\cdot\text{m}$. Since $T \leq T_Y$ the shaft is elastic.

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G} = \frac{(2)(600)(1.2)}{\pi(15\times10^{-3})^4(77.2\times10^9)} = 0.11728\ \text{rad} \qquad \varphi = 6.72^\circ$$

(b) $T = 1000\ \text{N}\cdot\text{m}$. $T > T_Y$ A plastic zone has developed.

$$T = \frac{4}{3}T_Y\left[1 - \left(\frac{\varphi_Y}{\varphi}\right)^3\right] \qquad \frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - 3\left(\frac{T}{T_Y}\right)}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(768.71)(2.1)}{\pi(15\times10^{-3})^4(77.2\times10^9)} = 0.15026\ \text{rad}$$

$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - \frac{(3)(1000)}{768.71}} = 0.46003$$

$$\varphi = \frac{\varphi_Y}{0.46003} = \frac{0.15026}{0.46003} = 0.32663\ \text{rad} \qquad \varphi = 18.71^\circ$$

Problem 3.102

3.102 A 18-mm-diameter solid circular shaft is made of material that is assumed to be elastoplastic with $\tau_Y = 140$ MPa and $G = 77$ GPa. For a 1.2-m length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 200 N · m torque.

$\tau_Y = 140\text{MPa}, \qquad c = \frac{d}{2} = 9\text{mm}, \qquad L = 1.2\text{m}, \qquad T = 200\text{Nm}$

$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.009)^3(140\times10^6) = 160\text{ Nm}$

$T > T_Y$ Plastic region with elastic core $\qquad \tau_{max} = \tau_Y = 140\text{MPa}$ ◄

$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(160)(1.2)}{\pi(0.009)^4(77\times10^9)} = 0.242 \text{ rad}$

$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi^3}{\varphi_Y^3}\right)$

$\left(\frac{\varphi_Y}{\varphi}\right)^3 = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(200)}{160} = 0.25 \qquad \frac{\varphi_Y}{\varphi} = 0.63$

$\varphi = \frac{\varphi_Y}{0.90471} = \frac{0.242}{0.63} = 0.384 \text{ rad} \qquad \varphi_Y = 22^\circ$ ◄

Problem 3.103

3.103 A solid circular rod is made of a material that is assumed to be elastoplastic. Denoting by T_Y and φ_Y, respectively, the torque and the angle of twist at the onset of yield, determine the angle of twist if the torque is increased to (*a*) $T = 1.1\ T_Y$, (*b*) $T = 1.25\ T_Y$, (*c*) $T = 1.3\ T_Y$.

$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y^3}{\varphi^3}\right)$

$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - \frac{3T}{T_Y}} \quad \text{or} \quad \frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_Y}}}$

(a) $\frac{T}{T_Y} = 1.10 \qquad \frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4-(3)(1.10)}} = 1.126 \qquad \varphi = 1.126\,\varphi_Y$ ◄

(b) $\frac{T}{T_Y} = 1.25 \qquad \frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4-(3)(1.25)}} = 1.587 \qquad \varphi = 1.587\,\varphi_Y$ ◄

(c) $\frac{T}{T_Y} = 1.3 \qquad \frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4-(3)(1.3)}} = 2.15 \qquad \varphi = 2.15\,\varphi_Y$ ◄

Problem 3.104

3.104 A 0.9-m-long solid shaft has a diameter of 62 mm and is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 147$ MPa and $G = 77$ GPa. Determine the torque required to twist the shaft through an angle of (*a*) 2.5°, (*b*) 5°.

$L = 0.9\,m$, $\quad c = \frac{1}{2}d = 0.031\,m \quad \tau_Y = 147\,MPa$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.031)^4 = 1.45 \times 10^{-6}\,m^4$$

$$\tau_Y = \frac{T_Y c}{J} \qquad T_Y = \frac{J\tau_Y}{c} = \frac{(1.45\times10^{-6})(147\times10^{6})}{0.031} = 6.876\times10^{3}\,Nm$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(6.876\times10^{3})(0.9)}{(77\times10^{9})(1.45\times10^{-6})} = 55.427\times10^{-3}\,rad = 3.176°$$

(a) $\varphi = 2.5° = 43.633\times10^{-3}$ rad $\qquad \varphi < \varphi_Y$ The shaft remains elastic.

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L} = \frac{(77\times10^{9})(1.45\times10^{-6})(43.633\times10^{-3})}{0.9}$$

$$= 5.413\times10^{3}\,N.m \qquad T = 5.4\,kNm$$

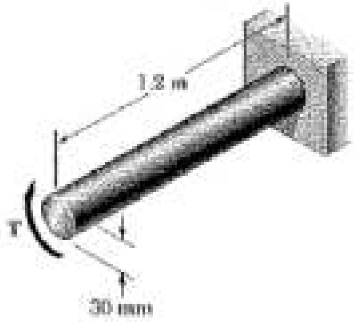
(b) $\varphi = 5° = 87.266\times10^{-3}$ rad $\qquad \varphi > \varphi_Y$ A plastic zone occurs.

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\left(\frac{\varphi_Y}{\varphi}\right)^3\right] = \frac{4}{3}(6.876\times10^{3})\left[1 - \frac{1}{4}\left(\frac{55.427\times10^{-3}}{87.266\times10^{-3}}\right)^3\right]$$

$$= 8.58\times10^{3}\,N.m \qquad T = 8.6\,kNm$$

Problem 3.105

3.105 For the solid shaft of Prob. 3.95, determine (*a*) the magnitude of the torque **T** required to twist the shaft through an angle of 15°, (*b*) the radius of the corresponding elastic core.



3.95 The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (*a*) $T = 600$ N · m, (*b*) $T = 1000$ N · m. $(G = 77.2 \text{ GPa})$.

$$c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$\varphi = 15° = 0.2618 \text{ rad.}$$

$$\varphi_Y = \frac{L\gamma_Y}{c} = \frac{L\tau_Y}{cG} = \frac{(1.2)(145 \times 10^6)}{(15 \times 10^{-3})(77.2 \times 10^9)} = 0.15026 \text{ rad}$$

(a) Since $\varphi > \varphi_Y$, there is a plastic zone.

$$\tau_Y = \frac{T_Y c}{J} = \frac{2T}{\pi c^3}$$

$$T_Y = \frac{\pi c^3 \tau_Y}{2} = \frac{\pi (15 \times 10^{-3})^3 (145 \times 10^6)}{2} = 768.71 \text{ N·m}$$

$$T = \frac{4}{3} T_Y \left[1 - \frac{1}{4}\left(\frac{\varphi_Y}{\varphi}\right)^3\right] = \frac{4}{3}(768.71)\left[1 - \frac{1}{4}\left(\frac{0.15026}{0.2618}\right)^3\right]$$

$$= 976.5 \text{ N·m}$$

$T = 977$ N·m ◄

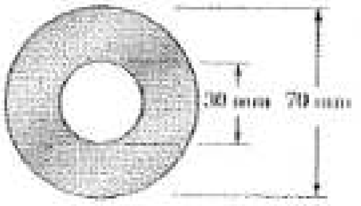
(b) $L\gamma_Y = \rho_Y \varphi = c\varphi_Y$

$$\rho_Y = \frac{c\varphi_Y}{\varphi} = \frac{(15 \times 10^{-3})(0.15026)}{0.2618} = 8.63 \times 10^{-3} \text{ m}$$

$\rho_Y = 8.61$ mm ◄

Problem 3.106

3.106 A hollow shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with $\tau_Y = 180$ MPa and $G = 77.2$ GPa. Determine the applied torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.



(a) At the onset of yield, the stress distribution is the elastic distribution with $\tau_{max} = \tau_Y$

$$c_2 = \tfrac{1}{2} d_2 = 0.035\text{ m}, \quad c_1 = \tfrac{1}{2} d_1 = 0.015\text{ m}.$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.035^4 - 0.015^4) = 2.2777 \times 10^{-6}\text{ m}$$

$$\tau_{max} = \tau_Y = \frac{T_Y c_2}{J} \quad \therefore \quad T_Y = \frac{J\tau_Y}{c_2} = \frac{(2.2777\times10^{-6})(180\times10^6)}{0.035} = 11.714\times10^3\text{ N}\cdot\text{m}$$

$$= 11.71\text{ kN}\cdot\text{m}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(11.714\times10^3)(0.9)}{(77.2\times10^9)(2.2777\times10^{-6})} = 59.96\times10^{-3}\text{ rad} \qquad \varphi_Y = 3.44^\circ$$

(b) $t = 0.010$ m $\qquad \rho_Y = c_2 - t = 0.035 - 0.010 = 0.025$ m

$$\gamma = \frac{\rho\varphi}{L} = \frac{\rho_Y \varphi}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi = \frac{\tau_Y L}{G\rho_Y} = \frac{(180\times10^6)(0.9)}{(77.2\times10^9)(0.025)} = 83.94\times10^{-3}\text{ rad} \qquad \varphi = 4.81^\circ$$

Torque T_1 carried by elastic portion $c_1 \le \rho \le \rho_Y$

$\tau = \tau_Y$ at $\rho = \rho_Y$. $\quad \tau_Y = \dfrac{T_1 \rho_Y}{J_1}$ where $J_1 = \frac{\pi}{2}(\rho_Y^4 - c_1^4)$

$$J_1 = \frac{\pi}{2}(0.025^4 - 0.015^4) = 534.07\times10^{-9}\text{ m}^4$$

$$T_1 = \frac{J_1 \tau_Y}{\rho_Y} = \frac{(534.07\times10^{-9})(180\times10^6)}{0.025} = 3.845\times10^3\text{ N}\cdot\text{m}$$

Torque T_2 carried by plastic portion

$$T_2 = 2\pi\int_{\rho_Y}^{c_2} \tau_Y \rho^2\, d\rho = 2\pi\tau_Y \frac{\rho^3}{3}\Big|_{\rho_Y}^{c_2} = \frac{2\pi}{3}\tau_Y(c_2^3 - \rho_Y^3)$$

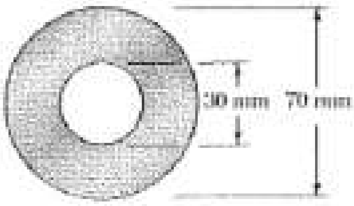
$$= \frac{2\pi}{3}(180\times10^6)(0.035^3 - 0.025^3) = 10.273\times10^3\text{ N}\cdot\text{m}$$

Total torque

$$T = T_1 + T_2 = 3.845\times10^3 + 10.273\times10^3 = 14.12\times10^3\text{ N}\cdot\text{m}$$

$$14.12\text{ kN}\cdot\text{m}$$

Problem 3.107



3.107 A hollow shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with $\tau_Y = 180$ MPa and $G = 77.2$ GPa. Determine the (*a*) angle of twist at which the section first becomes fully plastic, (*b*) the corresponding magnitude of the applied torque.

$$c_1 = \frac{1}{2}d_1 = 0.015\text{ m} \qquad c_2 = \frac{1}{2}d_2 = 0.035\text{ m}$$

(a) For onset of fully plastic yielding, $\rho_Y = c_1$

$$\tau = \tau_Y \quad \therefore \quad \gamma = \frac{\tau_Y}{G} = \frac{\rho_Y \varphi}{L} = \frac{c_1 \varphi}{L}$$

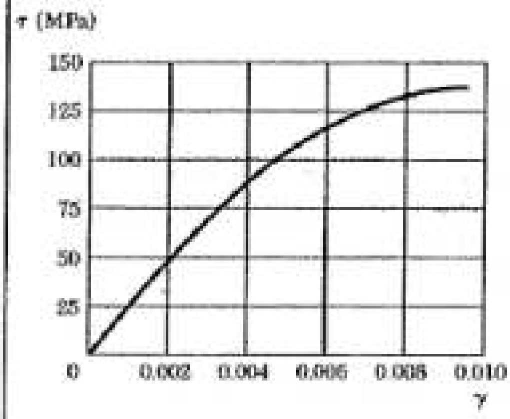
$$\varphi = \frac{L\tau_Y}{c_1 G} = \frac{(0.9)(180\times10^6)}{(0.015)(77.2\times10^9)} = 139.90\times10^{-3}\text{ rad} \qquad \varphi = 8.02^\circ$$

(b) $$T_p = 2\pi\int_{c_1}^{c_2}\tau_Y\rho^2\,d\rho = 2\pi\tau_Y\frac{\rho^3}{3}\Big|_{c_1}^{c_2} = \frac{2\pi}{3}\tau_Y(c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3}(180\times10^6)(0.035^3 - 0.015^3) = 14.89\times10^3\text{ N}\cdot\text{m}$$

$$= 14.89\text{ kN}\cdot\text{m}$$

Problem 3.112



3.112 A solid aluminum rod of 40-mm diameter is subjected to a torque that produces in the rod a maximum shearing strain of 0.008. Using the τ-γ diagram shown for the aluminum alloy used, determine (*a*) the magnitude of the torque applied to the rod, (*b*) the angle of twist in a 750-mm length of the rod.

$\gamma_{max} = 0.008 \qquad c = \frac{1}{2}d = 0.020\text{ m}$

$L = 750\text{ mm} = 0.750\text{ m}$

(a) Let $z = \frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}$

$$T = 2\pi\int_0^c \rho^2 \tau\, d\rho = 2\pi c^3 \int_0^1 z^2 \tau\, dz = 2\pi c^3 I$$

where the integral I is given by $\qquad I = \int_0^1 z^2 \tau\, dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$ we get the values given in the table below.

z	γ	τ, MPa	$z^2\tau$, MPa	w	$wz^2\tau$, MPa
0	0	0	0	1	0
0.25	0.002	48	3.0	4	12.0
0.5	0.004	88	22.0	2	44.0
0.75	0.006	115	64.7	4	258.8
1	0.008	133	133.0	1	133.0
					447.8 ← $\sum wz^2\tau$

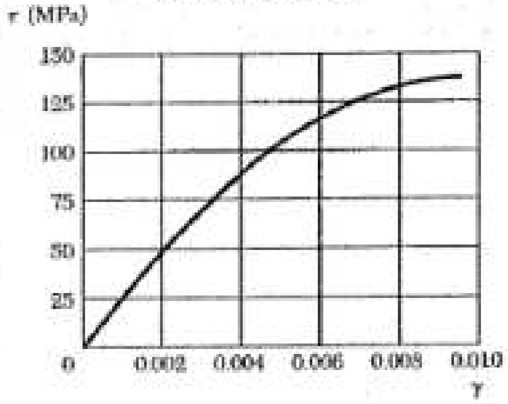
$$I = \frac{(0.25)(447.8)}{3} = 37.3\text{ MPa} = 37.3\times 10^6\text{ Pa}$$

$$T = 2\pi(0.020)^3(37.3\times10^6) = 1.876\times10^3\text{ N·m} \qquad T = 1.876\text{ kN·m} ◄$$

(b) $\gamma_{max} = \frac{c\varphi}{L}$

$$\varphi = \frac{L\gamma_{max}}{c} = \frac{(0.750)(0.008)}{0.020} = 300\times10^{-3}\text{ rad} \qquad \varphi = 17.19° ◄$$

Problem 3.113



3.113 The curve shown in Fig. P3.112 can be approximated by the relation

$$\tau = 27.8\times10^{9}\gamma - 1.390\times10^{12}\gamma^2$$

Using this relation and Eqs. (3.2) and (3.26), solve Prob. 3.112.

3.112 A solid aluminum rod of 40-mm diameter is subjected to a torque which produces in the rod a maximum shearing strain of 0.008. Using the τ-γ diagram shown for the aluminum alloy used, determine (*a*) the magnitude of the torque applied to the rod, (*b*) the angle of twist in a 750-mm length of the rod.

$\gamma_{max} = 0.008 \qquad c = \frac{1}{2}d = 0.020\text{ m}$

$L = 750\text{ mm} = 0.750\text{ m}$

(a) Let $z = \frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}$

$$T = 2\pi\int_0^c \rho^2\tau\,d\rho = 2\pi c^3\int_0^1 z^2\tau\,dz$$

The given stress-strain curve is $\tau = B\gamma + C\gamma^2 = B\gamma_{max}z + C\gamma_{max}^2 z^2$

where $B = 27.8\times10^9$ and $C = -1.390\times10^{12}$

$$T = 2\pi c^3\int_0^1 z^2(B\gamma_{max}z + C\gamma_{max}^2 z^2)\,dz$$

$$= 2\pi c^3\left\{B\gamma_{max}\int_0^1 z^3\,dz + C\gamma_{max}^2\int_0^4 z^4\,dz\right\}$$

$$= 2\pi c^3\left\{\tfrac{1}{4}B\gamma_{max} + \tfrac{1}{5}C\gamma_{max}^2\right\}$$

$$= 2\pi(0.020)^3\left\{\tfrac{1}{4}(27.8\times10^9)(0.008) + \tfrac{1}{5}(-1.390\times10^{12})(0.008)^2\right\}$$

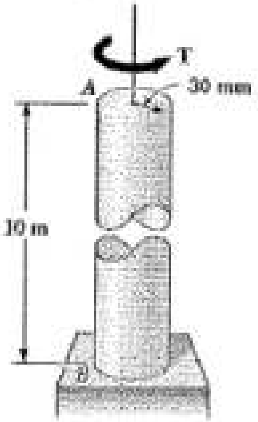
$$= 1.900\times10^3\text{ N}\cdot\text{m} \qquad T = 1.900\text{ kN}\cdot\text{m}$$

(b) $\gamma_{max} = \frac{c\varphi}{L}$

$$\varphi = \frac{L\gamma_{max}}{c} = \frac{(0.750)(0.008)}{0.020} = 300\times10^{-3}\text{ rad} \qquad \varphi = 17.19°$$

Problem 3.114

3.114 The solid circular drill rod AB is made of a steel that is assumed to be elastoplastic with $\tau_Y = 154$ MPa and $G = 77$ GPa. Knowing that a torque $T = 8475$ N · m is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.



SOLUTION

$c = 30mm \qquad L = 10m$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.03)^4 = 1.272 \times 10^{-6} m^4$

$T_Y = \frac{J\tau_Y}{c} = \frac{(1.272\times10^{-6})(154\times10^6)}{0.03} = 6530\ Nm$

Loading: $T = 8475$

$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$

$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(8475)}{6530} = 0.1064$

$\frac{\rho_Y}{c} = 0.4739 \qquad \rho_Y = 0.4739 = 14.2\ mm$

Unloading: $\tau' = \frac{T\rho}{J}$ where $T = 8475\ Nm$

At $\rho = c$ $\quad \tau' = \frac{(8475)(0.03)}{1.272\times10^{-6}} = 200\ MPa.$

At $\rho = \rho_Y$ $\quad \tau' = \frac{(8475)(0.0142)}{1.272\times10^{-6}} = 95\ MPa$

Residual: $\tau_{res} = \tau_{load} - \tau'$

At $\rho = c$ $\quad \tau_{res} = 154 - 200 = -46\ MPa$

At $\rho = \rho_Y$ $\quad \tau_{res} = 154 - 95 = 59\ MPa.$

maximum $\tau_{res} = 59\ MPa$ ◄

Problem 3.115

3.115 In Prob. 3.114, determine the permanent angle of twist of the rod.

3.114 The solid circular drill rod *AB* is made of a steel that is assumed to be elastoplastic with $\tau_Y = 154$ MPa and $G = 77$ GPa. Knowing that a torque $T = 8475$ N · m is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

From the solution to PROBLEM 3.114

$c = 30$ mm $\quad J = 1.272 \times 10^{-6}$ m^4 $\quad \frac{\rho_Y}{c} = 0.4739$, $\rho_Y = 14.2$ mm

After loading $\quad \gamma = \frac{\rho\varphi}{L} \therefore \quad \varphi = \frac{L\gamma}{\rho} = \frac{L\gamma_Y}{\rho_Y} = \frac{L\tau_Y}{\rho_Y G} \qquad L = 10$ m

$$\varphi_{load} = \frac{(10)(154 \times 10^6)}{(0.0142)(77 \times 10^9)} = 1.4085 \text{ rad} = 80.7$$

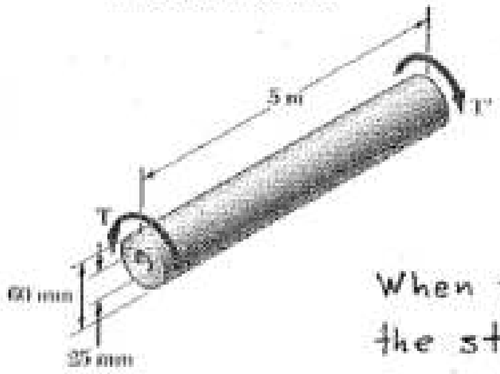
During unloading $\quad \varphi' = \frac{TL}{GJ}$ (elastic) $\qquad T = 8475$ N.

$$\varphi' = \frac{(8.475 \times 10^3)(10)}{(77 \times 10^9)(1.272 \times 10^{-6})} = 0.8653 \text{ rad} = 49.6°$$

Permanent twist angle

$$\varphi_{perm} = \varphi_{load} - \varphi' = 1.4085 - 0.8653 = 0.5432 \qquad \varphi = 31.1°$$

Problem 3.116



3.116 The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.

$$c_1 = \tfrac{1}{2} d_1 = 12.5 \text{ mm} \qquad c_2 = \tfrac{1}{2} d_2 = 30 \text{ mm}$$

When the plastic zone reaches the inner surface, the stress is equal to τ_Y. The corresponding torque is calculated by integration.

$$dT = \rho \tau \, dA = \rho \tau_Y (2\pi \rho \, d\rho) = 2\pi \tau_Y \rho^2 d\rho$$

$$T = 2\pi \tau_Y \int_{c_1}^{c_2} \rho^2 \, d\rho = \frac{2\pi}{3} \tau_Y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3} (145 \times 10^6)\left[(30 \times 10^{-3})^3 - (12.5 \times 10^{-3})^3\right] = 7.6064 \times 10^3 \text{ N}\cdot\text{m}$$

<u>Unloading</u>. $\quad T' = 7.6064 \times 10^3$ N·m

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}\left[(30)^4 - (12.5)^4\right] = 1.234 \times 10^6 \text{ mm}^4 = 1.234 \times 10^{-6} \text{ m}^4$$

$$\tau_1' = \frac{T' c_1}{J} = \frac{(7.6064 \times 10^3)(12.5 \times 10^{-3})}{1.234 \times 10^{-6}} = 77.05 \times 10^6 \text{ Pa} = 77.05 \text{ MPa}$$

$$\tau_2' = \frac{T' c_2}{J} = \frac{(7.6064 \times 10^3)(30 \times 10^{-3})}{1.234 \times 10^{-6}} = 192.63 \times 10^6 \text{ Pa} = 192.63 \text{ MPa}$$

<u>Residual stress</u>.

Inner surface: $\tau_{res} = \tau_Y - \tau_1' = 145 - 77.05 = 67.95$ MPa

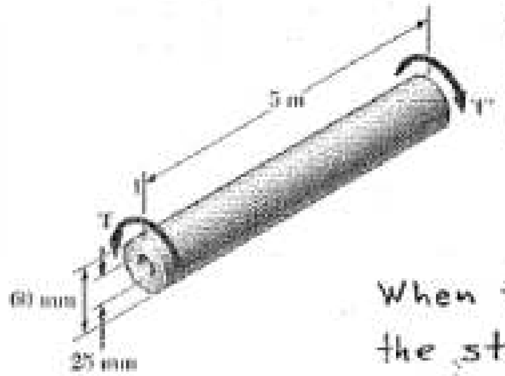
Outer surface: $\tau_{res} = \tau_Y - \tau_2' = 145 - 192.63 = -47.63$ MPa

<u>Maximum residual stress:</u> 68.0 MPa at inner surface. ◀

Problem 3.117

3.117 In Prob. 3.116, determine the permanent angle of twist of the rod.

3.116 The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.



$c_1 = \frac{1}{2} d_1 = 12.5$ mm $\qquad c_2 = \frac{1}{2} d_2 = 30$ mm

When the plastic zone reaches the inner surface, the stress is equal to τ_Y. The corresponding torque is calculated by integration.

$$dT = \rho \tau \, dA = \rho \tau_Y (2\pi \rho \, d\rho) = 2\pi \tau_Y \rho^2 d\rho$$

$$T = 2\pi \tau_Y \int_{c_1}^{c_2} \rho^2 \, d\rho = \frac{2\pi}{3} \tau_Y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3} (145 \times 10^6)[(30 \times 10^{-3})^3 - (12.5 \times 10^{-3})^3] = 7.6064 \times 10^3 \text{ N}\cdot\text{m}$$

Rotation angle at maximum torque.

$$\frac{c_1 \varphi_{max}}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi_{max} = \frac{\tau_Y L}{G c_1} = \frac{(145 \times 10^6)(5)}{(77.2 \times 10^9)(12.5 \times 10^{-3})} = 0.75130 \text{ rad}$$

Unloading $\qquad T' = 7.6064 \times 10^3$ N·m

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(30)^4 - (12.5)^4] = 1.234 \times 10^6 \text{ mm}^4 = 1.234 \times 10^{-6} \text{ m}^4$$

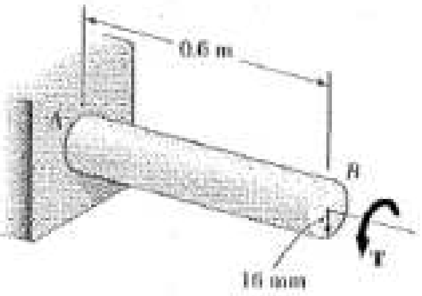
$$\varphi' = \frac{T'L}{GJ} = \frac{(7.6064 \times 10^3)(5)}{(77.2 \times 10^9)(1.234 \times 10^{-6})} = 0.39922 \text{ rad}$$

Permanent angle of twist.

$$\varphi_{perm} = \varphi_{max} - \varphi' = 0.75130 - 0.39922 = 0.35208 \text{ rad}$$

$$\varphi_{perm} = 20.2^\circ \quad \blacktriangleleft$$

Problem 3.118



3.118 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The torque is increased in magnitude until the shaft has been twisted through 6°; the torque is then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist.

$c = 0.016$ m $\quad \varphi = 6° = 104.72 \times 10^{-3}$ rad

$$\gamma_{max} = \frac{c\varphi}{L} = \frac{(0.016)(104.72 \times 10^{-3})}{0.6} = 0.0027925$$

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{145 \times 10^6}{77.2 \times 10^9} = 0.0018782$$

$$\frac{\rho_Y}{c} = \frac{\gamma_Y}{\gamma_{max}} = \frac{0.0018}{0.0027925} = 0.67260$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.016)^4 = 102.944 \times 10^{-9}\ m^4$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.016)^3(145 \times 10^6) = 932.93\ N \cdot m$$

At end of loading. $T_{load} = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(932.98)\left[1 - \frac{1}{4}(0.67433)^3\right]$

$= 1.14855 \times 10^3$ N·m

Unloading: $\quad$ elastic $\quad T' = 1.14855 \times 10^3$ N·m

At $\rho = c \quad \tau' = \frac{T'c}{J} = \frac{(1.14855 \times 10^3)(0.016)}{102.944 \times 10^{-9}} = 178.52 \times 10^6$ Pa

At $\rho = \rho_Y \quad \tau' = \frac{T'c}{J}\frac{\rho_Y}{c} = 178.52 \times 10^6 (0.67433) = 120.38 \times 10^6$ Pa

$$\varphi' = \frac{T'L}{GJ} = \frac{(1.14855 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 86.71 \times 10^{-3}\ \text{rad} = 4.97°$$

Residual: $\quad \tau_{res} = \tau_{load} - \tau' \qquad \varphi_{perm} = \varphi_{load} - \varphi'$

(a) At $\rho = c \quad \tau_{res} = 145 \times 10^6 - 178.52 \times 10^6 = -33.52 \times 10^6$ Pa

-33.5 MPa

At $\rho = \rho_Y \quad \tau_{res} = 145 \times 10^6 - 120.38 \times 10^6 = 24.62 \times 10^6$ Pa

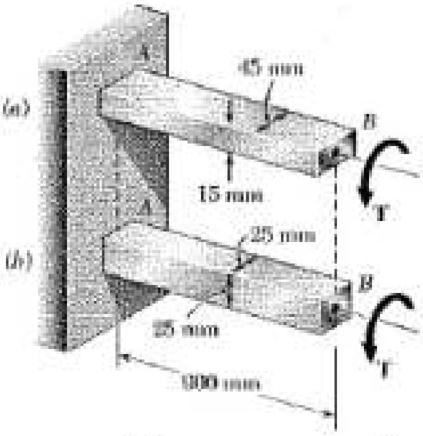
24.6 MPa

Maximum residual stress: 33.5 MPa at $\rho = 16$ mm ◄

(b) $\varphi_{perm} = 104.72 \times 10^{-3} - 86.71 \times 10^{-3} = 17.78 \times 10^{-3}$ rad $\qquad \varphi_{perm} = 1.032°$ ◄

Problem 3.121

3.121 Using $\tau_{all} = 70$ MPa and $G = 27$ GPa, determine for each of the aluminum bars shown the largest torque **T** that can be applied and the corresponding angle of twist at end B.



$\tau_{all} = 70\times10^6$ Pa $\quad G = 27\times10^9$ Pa $\quad L = 0.900$ m

(a) $a = 45$ mm $\quad b = 15$ mm $\quad \frac{a}{b} = 3.0$

From Table 3.1, $\quad c_1 = 0.267, \; c_2 = 0.263$

$\tau_{max} = \dfrac{T}{c_1 a b^2} \qquad T = c_1 a b^2 \tau_{max}$

$T = (0.267)(0.045)(0.015)^2(70\times10^6) = 189.2$ N·m

$T = 189.2$ N·m ◄

$\varphi = \dfrac{TL}{c_2 a b^3 G} = \dfrac{(189.2)(0.900)}{(0.263)(0.045)(0.015)^3(27\times10^9)} = 157.9\times10^{-3}$ rad $\qquad \varphi = 9.05°$ ◄

(b) $a = 25$ mm $\quad b = 25$ mm, $\frac{a}{b} = 1.0$. From Table 3.1, $c_1 = 0.208$, $c_2 = 0.1406$

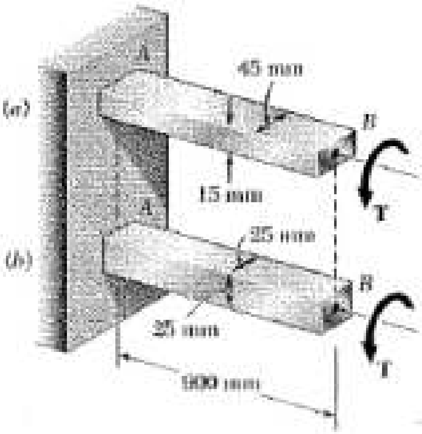
$\tau_{max} = \dfrac{T}{c_1 a b^2} \qquad T = c_1 a b^2 \tau_{max} = (0.208)(0.025)(0.025)^2(70\times10^6)$

$= 227.5$ N·m $\qquad T = 228$ N·m ◄

$\varphi = \dfrac{TL}{c_2 a b^3 G} = \dfrac{(227.5)(0.900)}{(0.1406)(0.025)(0.025)^3(27\times10^9)} = 138.1\times10^{-3}$ rad $\quad \varphi = 7.91°$ ◄

Problem 3.122

3.122 Knowing that the magnitude of the torque **T** is 200 N · m and that $G = 27$ GPa, determine for each of the aluminum bars shown the maximum shearing stress and the angle of twist at end B.



$T = 200$ N·m $\quad L = 0.900$ m $\quad G = 27\times10^9$ Pa

(a) $a = 45$ mm, $b = 15$ mm, $\frac{a}{b} = 3.0$.

From Table 3.1, $\quad c_1 = 0.267 \quad c_2 = 0.263$

$\tau_{max} = \dfrac{T}{c_1 a b^2} = \dfrac{200}{(0.267)(0.045)(0.015)^2}$

$= 74.0\times10^6$ Pa $\qquad \tau_{max} = 74.0$ MPa ◄

$\varphi = \dfrac{TL}{c_2 a b^3 G} = \dfrac{(200)(0.900)}{(0.263)(0.045)(0.015)^3(27\times10^9)}$

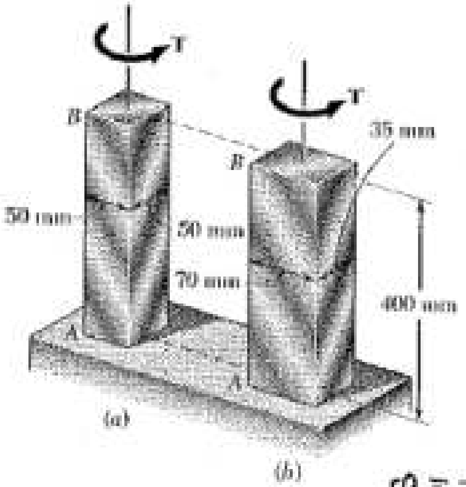
$= 166.9\times10^{-3}$ rad $\qquad \varphi = 9.56°$ ◄

(b) $a = 25$ mm, $b = 25$ mm, $\frac{a}{b} = 1.0$. From Table 3.1, $c_1 = 0.208$, $c_2 = 0.1406$

$\tau_{max} = \dfrac{T}{c_1 a b^2} = \dfrac{200}{(0.208)(0.025)(0.025)^2} = 61.5\times10^6$ Pa $\qquad \tau_{max} = 61.5$ MPa ◄

$\varphi = \dfrac{TL}{c_2 a b^3 G} = \dfrac{(200)(0.900)}{(0.1406)(0.025)(0.025)^3(27\times10^9)} = 121.4\times10^{-3}$ rad $\quad \varphi = 6.95°$ ◄

Problem 3.123



3.123 Using $\tau_{all} = 50$ MPa and knowing that $G = 39$ GPa, determine for each of the cold-rolled yellow brass bars shown the largest torque **T** that can be applied and the corresponding angle of twist at end B.

$\tau_{al} = 50 MPa \qquad L = 0.4 m$

(a) $a = 50 mm \quad b = 50 mm \quad \frac{a}{b} = 1.0$

From Table 3.1 $\quad c_1 = 0.208, \quad c_2 = 0.1406$

$\tau_{max} = \frac{T}{c_1 a b^2} \qquad T = c_1 a b^2 \tau_{max}$

$T = (0.208)(0.05)(0.05)^2(50\times10^6) = 1300 Nm$

$T = 1.3 kNm$ ◄

$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(1.3\times10^3)(0.4)}{(0.1406)(0.05)(0.05)^3(39\times10^9)}$

$= 15.173\times10^{-3}$ rad $\qquad \varphi = 0.87°$ ◄

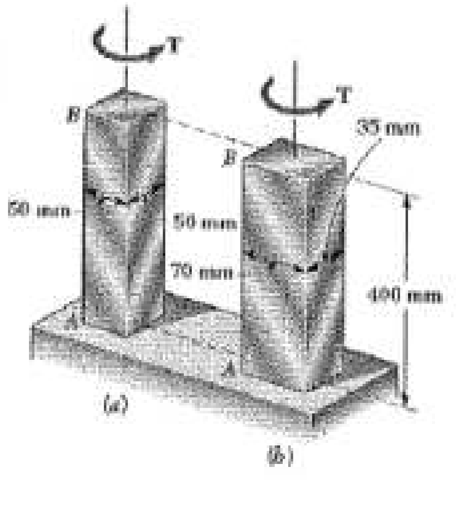
(b) $a = 70mm \quad b = 35mm \quad \frac{a}{b} = 2.0$ From Table 3.1 $c_1 = 0.246, \; c_2 = 0.229$

$\tau_{max} = \frac{T}{c_1 a b^2} \qquad T = c_1 a b^2 \tau_{max} = (0.246)(0.07)(0.035)^2(50\times10^6) = 1055 Nm$

$T = 1.055 kN.m$ ◄

$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(1.055\times10^3)(0.4)}{(0.229)(0.07)(0.035)^3(39\times10^9)} = 15.74\times10^{-3}$ rad $\qquad \varphi = 0.902°$ ◄

Problem 3.124



3.124 Knowing that $T = 800$ N · m and that $G = 39$ GPa, determine for each of the cold-rolled yellow brass bars shown the maximum shearing stress and the angle of twist of end B.

$T = 800 N/m \qquad L = 0.4 m.$

(a) $a = 50 mm \quad b = 50 mm \quad \frac{a}{b} = 1.0$

From Table 3.1, $\quad c_1 = 0.208, \quad c_2 = 0.1406$

$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{800}{(0.208)(0.05)(0.05)^2} = 30.77 MPa$

$\tau_{max} = 30.77 MPa$ ◄

$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(800)(0.4)}{(0.1406)(0.05)(0.05)^3(39000\times10^6)}$

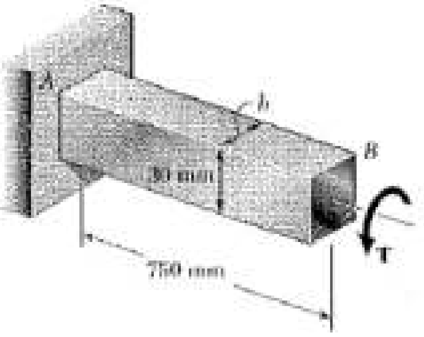
$= 9.34\times10^{-3}$ rad $\qquad \varphi = 0.535°$ ◄

(b) $a = 70mm \quad b = 35mm \quad \frac{a}{b} = 2.0$ From Table 3.1 $c_1 = 0.246, \; c_2 = 0.229$

$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{800}{(0.246)(0.07)(0.035)^2} = 37.9 MPa \qquad \tau_{max} = 37.9 MPa$ ◄

$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(800)(0.4)}{(0.229)(0.07)(0.035)^3(39\times10^9)} = 0.01194$ rad $\qquad \phi = 0.684°$ ◄

Problem 3.125



3.125 The torque **T** causes a rotation of 2° at end *B* of the stainless steel bar shown. Knowing that $b = 20$ mm and $G = 75$ GPa, determine the maximum shearing stress in the bar.

$a = 30 \text{ mm} = 0.030 \text{ m}, \quad b = 20 \text{ mm} = 0.020 \text{ m}$

$\varphi = 2° = 34.907 \times 10^{-3} \text{ rad}$

$\varphi = \dfrac{TL}{c_2 ab^3 G} \quad \therefore \quad T = \dfrac{c_2 a b^3 G \varphi}{L}$

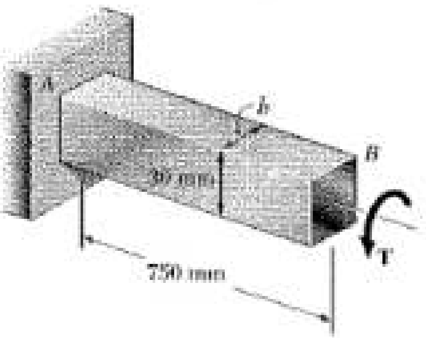
$\tau_{max} = \dfrac{T}{c_1 a b^2} = \dfrac{c_2 a b^3 G \varphi}{c_1 a b^2 L} = \dfrac{c_2 b G \varphi}{c_1 L}$

$\dfrac{a}{b} = \dfrac{30}{20} = 1.5.$ From Table 3.1, $c_1 = 0.231$, $c_2 = 0.1958$

$$\tau_{max} = \frac{(0.1958)(20 \times 10^{-3})(75 \times 10^{9})(34.907 \times 10^{-3})}{(0.231)(750 \times 10^{-3})} = 59.2 \times 10^{6} \text{ Pa}$$

$\tau_{max} = 59.2 \text{ MPa}$ ◄

Problem 3.126



3.126 The torque **T** causes a rotation of 0.6° at end *B* of the aluminum bar shown. Knowing that $b = 15$ mm and $G = 26$ GPa, determine the maximum shearing stress in the bar.

$a = 30 \text{ mm} = 0.030 \text{ m}, \quad b = 15 \text{ mm} = 0.015 \text{ m}$

$\varphi = 0.6° = 10.472 \times 10^{-3} \text{ rad}$

$\varphi = \dfrac{TL}{c_2 ab^3 G} \quad \therefore \quad T = \dfrac{c_2 a b^3 G \varphi}{c_1 L}$

$\tau_{max} = \dfrac{T}{c_1 a b^2} = \dfrac{c_2 a b^3 G \varphi}{c_1 a b^2 L} = \dfrac{c_2 b G \varphi}{c_1 L}$

$\dfrac{a}{b} = \dfrac{30}{15} = 2.0.$ From Table 3.1, $c_1 = 0.246$, $c_2 = 0.229$

$$\tau_{max} = \frac{(0.229)(15 \times 10^{-3})(26 \times 10^{9})(10.472 \times 10^{-3})}{(0.246)(750 \times 10^{-3})} = 5.07 \times 10^{6} \text{ Pa}$$

$\tau_{max} = 5.07 \text{ MPa}$ ◄

Problem 3.127

3.127 Determine the largest allowable square cross section of a steel shaft of length 6 m if the maximum shearing stress is not to exceed 120 MPa when the shaft is twisted through one complete revolution. Use $G = 77.2$ GPa.

$L = 6\text{ m}, \quad \tau = 120\times10^6\text{ Pa}, \quad G = 77.2\times10^9\text{ Pa} \qquad \varphi = 1\text{ rev} = 2\pi\text{ rad}$

$\tau = \dfrac{T}{c_1 a b^2} \qquad \varphi = \dfrac{TL}{c_2 a b^3 G}$

Divide to eliminate T; then solve for b. $\dfrac{\varphi}{\tau} = \dfrac{c_1 a b^2 L}{c_2 a b^3 G} = \dfrac{c_1 L}{c_2 b G}$

$b = \dfrac{c_1 L \tau}{c_2 G \varphi}$ For a square cross section $\dfrac{a}{b} = 1.0$

From Table 3.1 $\quad c_1 = 0.208, \quad c_2 = 0.1406$

$b = \dfrac{(0.208)(6)(120\times10^6)}{(0.1406)(77.2\times10^9)(2\pi)} = 2.20\times10^{-3}\text{ m}$ $\qquad b = 2.20\text{ mm}$ ◄

Problem 3.128

3.128 Determine the largest allowable length of a stainless steel shaft of 9 × 18 mm cross section if the shearing stress is not to exceed 105 MPa when the shaft is twisted through 15°. Use $G = 77$ GPa.

$a = 18\text{ mm}, \qquad b = 9\text{ mm}, \qquad \tau_{max} = 105\text{ MPa}$

$\varphi = 15° = \dfrac{15\pi}{180}\text{ rad} = 0.26180\text{ rad}$

$\tau_{max} = \dfrac{T}{c_1 a b^2} \quad (1) \qquad \varphi = \dfrac{TL}{c_2 a b^3 G} \quad (2)$

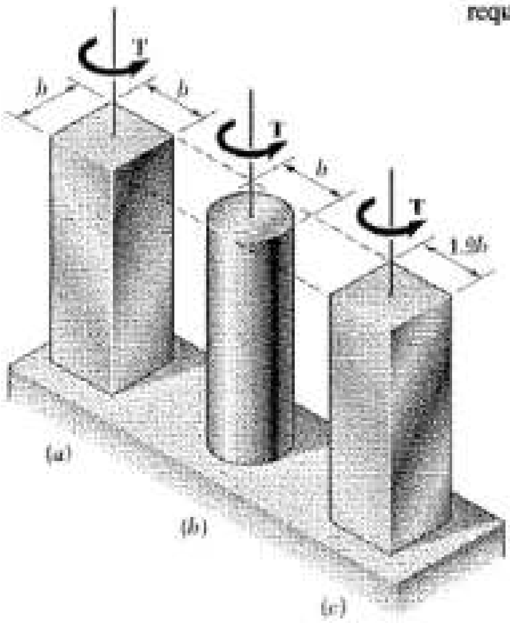
Divide (2) by (1) to eliminate T. $\qquad \dfrac{\varphi}{\tau_{max}} = \dfrac{c_1 a b^2 L}{c_2 a b^3 G} = \dfrac{c_1 L}{c_2 b G}$

Solve for L. $\qquad L = \dfrac{c_2 b G \varphi}{c_1 \tau_{max}}$

$\dfrac{a}{b} = \dfrac{18}{9} = 2 \qquad$ Table 3.1 gives $c_1 = 0.246, \; c_2 = 0.229$

$L = \dfrac{(0.229)(0.009)(77\times10^9)(0.26180)}{(0.246)(105\times10^6)} = 1.609\text{ m} \qquad L = 1.6\text{ m}$ ◄

Problem 3.129



3.129 Each of the three steel bars shown is subjected to a torque of magnitude $T = 275\ \text{N}\cdot\text{m}$. Knowing that the allowable shearing stress is 50 MPa, determine the required dimension b for each bar.

$T = 275\ \text{N}\cdot\text{m}, \quad \tau_{max} = 50\times10^6\ \text{Pa}$

(a) Square: $a = b \quad \frac{a}{b} = 1.0$

From Table 3.1 $\quad c_1 = 0.208$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{c_1 b^3}$$

$$b = \sqrt[3]{\frac{T}{c_1 \tau_{max}}} = \sqrt[3]{\frac{275}{(0.208)(50\times10^6)}}$$

$= 29.8\times10^{-3}\ \text{m} \qquad b = 29.8\ \text{mm}$

(b) Circle: $c = \frac{1}{2}b \qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

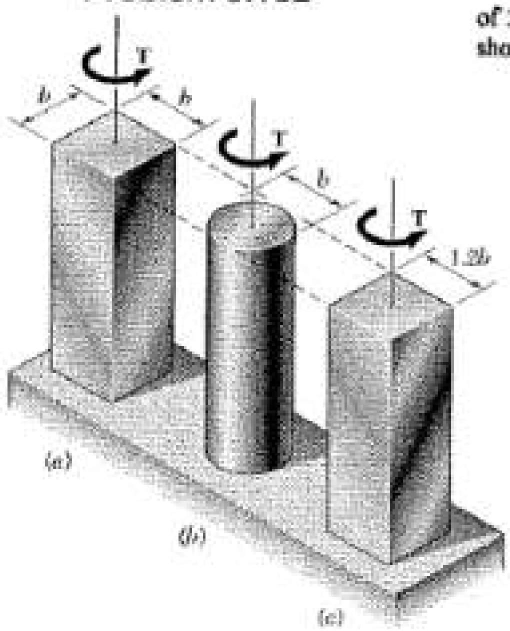
$$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(275)}{\pi(50\times10^6)}} = 15.18\times10^{-3}\ \text{m}$$

$c = 15.18\ \text{m} \qquad b = 2c = 30.4\ \text{mm}$

(c) Rectangle: $a = 1.2b \quad$ From Table 3.1 $\quad c_1 = 0.219 \quad \tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{1.2 c_1 b^3}$

$$b = \sqrt[3]{\frac{T}{1.2 c_1 \tau_{max}}} = \sqrt[3]{\frac{275}{(1.2)(0.219)(50\times10^6)}} = 27.6\times10^{-3}\ \text{m} \qquad b = 27.6\ \text{mm}$$

Problem 3.132



3.132 Each of the three aluminum bars shown is to be twisted through an angle of 2°. Knowing that $b = 30$ mm, $\tau_{all} = 50$ MPa, and $G = 27$ GPa, determine the shortest allowable length of each bar.

$\varphi = 2° = 34.907 \times 10^{-3}$ rad $\qquad \tau = 50 \times 10^{6}$ Pa

$G = 27 \times 10^{9}$ Pa $\qquad b = 30$ mm $= 0.030$ m

For square and rectangle

$$\tau = \frac{T}{c_1 a b^2} \qquad \varphi = \frac{TL}{c_2 a b^3 G}$$

Divide to eliminate T; then solve for L

$$\frac{\varphi}{\tau} = \frac{c_1 a b^2 L}{c_2 a b^3 G} \qquad L = \frac{c_2 b G \varphi}{c_1 \tau}$$

(a) Square: $\frac{a}{b} = 1.0$

From Table 3.1 $\quad c_1 = 0.208, \; c_2 = 0.1406$

$$L = \frac{(0.1406)(0.030)(27 \times 10^{9})(34.907 \times 10^{-3})}{(0.208)(50 \times 10^{6})} = 382 \times 10^{-3} \text{ m} \qquad L = 382 \text{ mm}$$

(b) Circle $\quad c = \frac{1}{2}b = 0.015$ m $\qquad \tau = \frac{Tc}{J} \qquad \varphi = \frac{TL}{GJ}$

Divide to eliminate T; then solve for L $\qquad \frac{\varphi}{\tau} = \frac{JL}{cGJ} = \frac{L}{cG}$

$$L = \frac{cG\varphi}{\tau} = \frac{(0.015)(27 \times 10^{9})(34.907 \times 10^{-3})}{50 \times 10^{6}} = 283 \times 10^{-3} \text{ m} \qquad L = 283 \text{ mm}$$

(c) Rectangle: $a = 1.2b \qquad \frac{a}{b} = 1.2 \qquad$ From Table 3.1 $\quad c_1 = 0.219, \; c_2 = 0.1661$

$$L = \frac{(0.1661)(0.030)(27 \times 10^{9})(34.907 \times 10^{-3})}{(0.219)(50 \times 10^{6})} = 429 \times 10^{-3} \text{ m} \qquad L = 429 \text{ mm}$$

Problem 3.135

3.135 A 340 N · m torque is applied to a 1.8-m-long steel angle with an L102 × 102 × 9.5 cross section. From Appendix C, we find that the thickness of the section is 9.5 mm and that its area is 1850 mm². Knowing that $G = 77$ GPa, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the angle of twist.

$A = 1850\,\text{mm}^2, \quad b = 9.5\ \text{mm}, \qquad a = \frac{A}{b} = \frac{1850}{9.5} = 194.7\ \text{mm}$

$\frac{a}{b} = \frac{194.7}{9.5} = 20.5 \qquad c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3230$

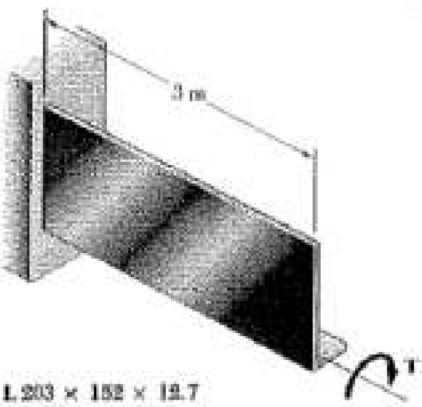
(a) $\tau_{max} = \frac{T}{c_1 a b^2} = \frac{340}{(0.3230)(0.1947)(0.0095)^2} = 59.9\ \text{MPa}$ ◄

(b) $\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(340)(1.8)}{(0.323)(0.1947)(0.0095)^3\, 77\times 10^9} = 147.4 \times 10^{-3}\ \text{rad}$

$\varphi = 8.45°$ ◄

Note: $L = 1.8$ m.

Problem 3.136



L203 × 152 × 12.7

3.136 A 3-m-long steel angle has an L203 × 152 × 12.7 cross section. From Appendix C we find that the thickness of the section is 12.7 mm and that its area is 4350 mm². Knowing that $\tau_{all} = 50$ MPa and that $G = 77.2$ GPa, and ignoring the effect of stress concentration, determine (*a*) the largest torque **T** that can be applied, (*b*) the corresponding angle of twist.

$A = 4350\ \text{mm}^2 \qquad b = 12.7\ \text{mm} \qquad a = ?$

Equivalent rectangle, $\quad a = \frac{A}{b} = \frac{4350}{12.7} = 342.52\ \text{mm}$

$\frac{a}{b} = 26.97$

$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.32555$

(a) $\tau_{max} = \frac{T}{c_1 a b^2} \qquad \tau_{max} = 50\times 10^6\ \text{Pa}$

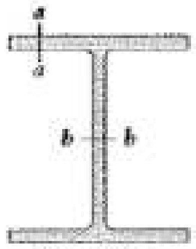
$T = c_1 a b^2 \tau_{max} = (0.32555)(26.97\times 10^{-3})(12.7\times 10^{-3})^2(50\times 10^6)$

$= 70.807\ \text{N}\cdot\text{m}$ $\qquad T = 70.8\ \text{N}\cdot\text{m}$ ◄

(b) $\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(70.807)(3)}{(0.32555)(26.97\times 10^{-3})(12.7\times 10^{-3})(77.2\times 10^9)}$

$= 0.15299\ \text{rad}$ $\qquad \varphi = 8.77°$ ◄

Problem 3.137



W200 × 46.1

3.137 An 2.4-m steel member with a W200 × 46.1 cross section is subjected to a 560 N · m torque. The properties of the rolled-steel section are given in Appendix C. Knowing that $G = 77$ GPa, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint:* consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

SOLUTION

<u>Flange:</u> $a = 203\text{ mm}, \quad b = 11\text{ mm}, \quad \frac{a}{b} = \frac{203}{11} = 18.45$

$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.322 \qquad \varphi_F = \frac{T_F L}{c_2 a b^3 G}$

$T_F = c_2 a b^3 \frac{G\varphi_F}{L} = K_F \frac{G\varphi}{L} \qquad \text{where } K_F = c_2 a b^3$

$K_F = (0.322)(0.203)(0.011)^3 = 87 \times 10^{-9}\text{ m}^3$

<u>Web:</u> $a = 203 - (2)(11) = 181\text{ mm}, \qquad b = 7.2\text{ mm}, \quad \frac{a}{b} = \frac{181}{7.2} = 25.14$

$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3249 \qquad \varphi_W = \frac{T_W L}{c_2 a b^3 G}$

$T_W = c_2 a b^3 \frac{G\varphi_W}{L} = K_W \frac{G\varphi}{L} \qquad \text{where } K_W = c_2 a b^3$

$K_W = (0.3249)(0.181)(0.0072)^3 = 21.95 \times 10^{-9}\text{ m}^3$

For matching twist angles $\quad \varphi_F = \varphi_W = \varphi$

Total torque $\quad T = 2T_F + T_W = (2K_F + K_W)\frac{G\varphi}{L}$

$\frac{G\varphi}{L} = \frac{T}{2K_F + K_W}, \quad T_F = \frac{K_F T}{2K_F + K_W}, \quad T_W = \frac{K_W T}{2K_F + K_W}$

$T_F = \frac{(87\times10^{-9})(560)}{(2)(87\times10^{-9}) + 21.95\times10^{-9}} = 248.6\text{ Nm}, \quad T_W = \frac{(21.95\times10^{-9})(560)}{(2)(87\times10^{-9}) + 21.95\times10^{-9}} = 62.7\text{ Nm}$

(a) $\tau_F = \frac{T_F}{c_1 a b^2} = \frac{248.6}{(0.3219)(0.203)(0.011)^2} = 31.44\text{ MPa}$ ◀

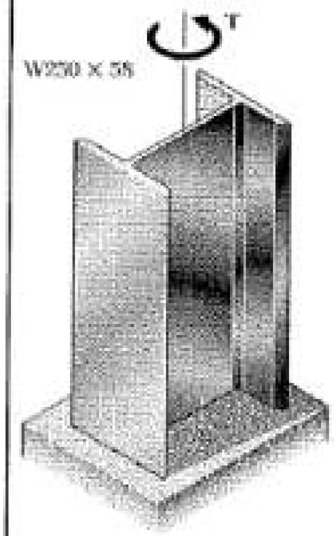
(b) $\tau_W = \frac{T_W}{c_1 a b^2} = \frac{62.7}{(0.3249)(0.181)(0.0072)^2} = 20.57\text{ MPa}$ ◀

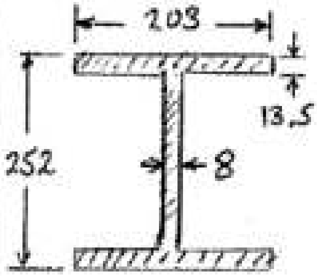
(c) $\frac{G\varphi}{L} = \frac{T}{2K_F + K_W} \quad \therefore \ \varphi = \frac{TL}{G(2K_F + K_W)} \quad \text{where } L = 2.4\text{ m}$

$\varphi = \frac{(560)(2.4)}{(77\times10^{9})[(2)(87\times10^{-9}) + 21.95\times10^{-9}]} = 89.1\times10^{-3}\text{ rad} = 5.1^\circ$ ◀

Problem 3.138

3.138 A 3-m-long steel member has a W250 × 58 cross section. Knowing that $G = 77.2$ GPa and that the allowable shearing stress is 35 MPa, determine (*a*) the largest torque **T** that can be applied, (*b*) the corresponding angle of twist. Refer to Appendix C for the dimensions of the cross section and neglect the effect of stress concentrations. (See hint of Prob. 3.137.)





Flange: $a = 203$ mm, $b = 13.5$ mm, $\frac{a}{b} = 15.04$

$$C_1 = C_2 = \tfrac{1}{3}\left(1 - 0.630\tfrac{b}{a}\right) = 0.3194$$

$$\varphi_F = \frac{T_F L}{C_2 a b^3 G} \quad \therefore \quad T_F = C_2 a b^3 \frac{G\varphi}{L} = K_F \frac{G\varphi}{L}$$

$$K_F = (0.3194)(0.203)(0.0135)^3 = 159.53 \times 10^{-9} \text{ m}^4$$

Web: $a = 252 - (2)(13.5) = 225$ mm, $b = 8$ mm

$$\frac{a}{b} = 28.13, \quad C_1 = C_2 = \tfrac{1}{3}\left(1 - 0.63\tfrac{b}{a}\right) = 0.3259$$

$$\varphi_W = \frac{T_W L}{C_2 a b^3 G} \quad \therefore \quad T_W = C_2 a b^3 \frac{G\varphi}{L} = K_W \frac{G\varphi}{L}$$

$$K_W = (0.3259)(0.225)(0.008)^3 = 37.54 \times 10^{-9} \text{ m}^4$$

For matching twist angles $\varphi_F = \varphi_W = \varphi$

Total torque: $T = 2T_F + T_W = (2K_F + K_W)\frac{G\varphi}{L}$

$$\frac{G\varphi}{L} = \frac{T}{2K_F + K_W}, \quad T_F = \frac{K_F T}{2K_F + K_W} \quad \therefore \quad T = \frac{2K_F + K_W}{K_F} T_F$$

$$T_W = \frac{K_W T}{2K_F + K_W} \quad \therefore \quad T = \frac{2K_F + K_W}{K_W} T_W$$

Allowable value for T based on allowable value for T_F:

$$T_F = c_1 a b^2 \tau = (0.3194)(0.203)(0.0135)^2(35 \times 10^6) = 413.6 \text{ N}\cdot\text{m}$$

$$T = \frac{(2)(159.53) + (37.54)}{159.53}(413.6) = 924.5 \text{ N}\cdot\text{m}$$

Allowable value for T based on allowable value for T_W:

$$T_W = c_1 a b^2 \tau = (0.3259)(0.225)(0.008)^2(35 \times 10^6) = 164.25 \text{ N}\cdot\text{m}$$

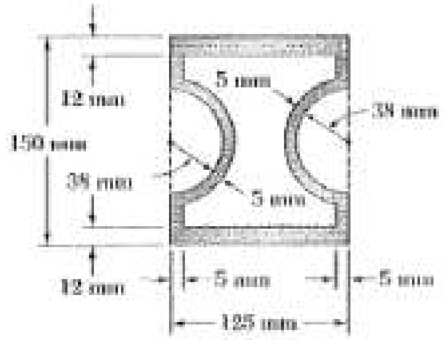
$$T = \frac{(2)(159.53) + 37.54}{37.54}(164.25) = 1560 \text{ N}\cdot\text{m}$$

Choose smaller value $\qquad T = 925$ N·m ◄

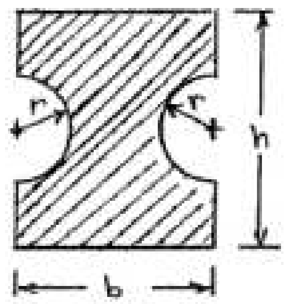
$$\varphi = \frac{TL}{(2K_F + K_W)G} = \frac{(924.5)(3.00)}{(356.6 \times 10^{-9})(77.2 \times 10^9)} = 100.7 \times 10^{-3} \text{ rad} \qquad \varphi = 5.77° \blacktriangleleft$$

Problem 3.143

3.143 A hollow brass shaft has the cross section shown. Knowing that the shearing stress must not exceed 84 MPa and neglecting the effect of stress concentrations, determine the largest torque that can be applied to the shaft.



Calculate the area bounded by the center line of the wall cross section.
The area is a rectangle with two semi-circular cutouts.



$b = 125 - 5 = 120$ mm

$h = 150 - 12 = 138$ mm

$r = 38 - 2.5 = 35.5$ mm.

$$a = bh - 2\left(\frac{\pi}{2}r^2\right) = (120)(138) - \pi(35.5)^2 = 12600 \text{ mm}^2 = 12.6 \times 10^{-3} \text{ m}^2$$

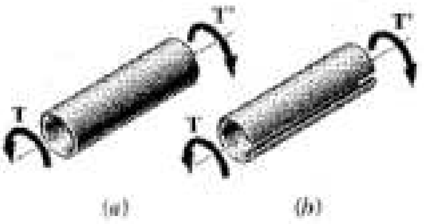
$\tau_{max} = \dfrac{T}{2a t_{min}}$ $\qquad \tau_{max} = 84$ MPa $\qquad t_{min} = 0.005$ m

$$T = 2a\, t_{min}\, \tau_{max} = (2)(12.6 \times 10^{-3})(0.005)(84 \times 10^6) = 10584 \text{ Nm}$$

$T = 10.58$ kNm

Problem 3.149

3.149 Equal torques are applied to thin-walled tubes of the same length L, same thickness t, and same radius c. One of the tubes has been slit lengthwise as shown. Determine (*a*) the ratio τ_b/τ_a of the maximum shearing stresses in the tubes, (*b*) the ratio φ_b/φ_a of the angles of twist of the shafts.



Without slit

Area bounded by centerline: $a = \pi c^2$

$$\tau_a = \frac{T}{2ta} = \frac{T}{2\pi c^2 t}$$

$$J \approx 2\pi c^3 t \qquad \varphi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 t G}$$

With slit: $\quad a = 2\pi c, \quad b = t, \quad \frac{a}{b} = \frac{2\pi c}{t} >> 1$

$$c_1 = c_2 = \frac{1}{3}$$

$$\tau_b = \frac{T}{c_1 a b^2} = \frac{3T}{2\pi c t^2}$$

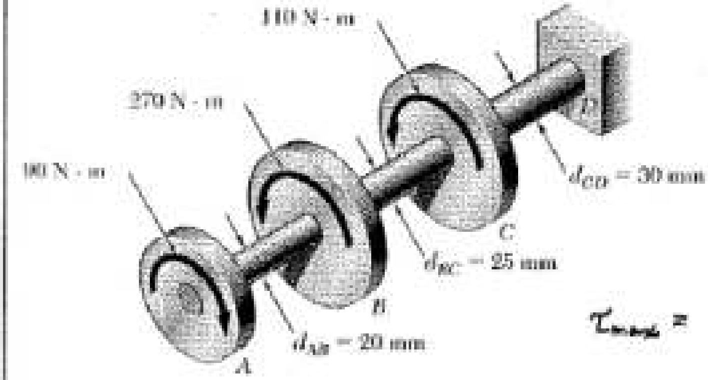
$$\varphi_b = \frac{T}{c_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G}$$

(a) Stress ratio: $\quad \frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \frac{3c}{t} \qquad \frac{\tau_b}{\tau_a} = \frac{3c}{t}$ ◄

(b) Twist ratio: $\quad \frac{\varphi_b}{\varphi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 t G}{TL} = \frac{3c^2}{t^2} \qquad \frac{\varphi_b}{\varphi_a} = \frac{3c^2}{t^2}$ ◄

Problem 3.151

3.151 Knowing that a 10 mm-diameter hole has been drilled through each of the shafts *AB*, *BC*, and *CD*, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.



Hole: $c_1 = \frac{1}{2}d_1 = \frac{1}{2}(10) = 5\text{mm}$

Shaft AB: $T = 90\text{Nm}$

$c_2 = \frac{1}{2}d_2 = 10\text{mm}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.01^4 - 0.005^4)$

$= 14.73 \times 10^{-9}\,\text{m}^4$

$\tau_{max} = \frac{Tc_2}{J} = \frac{(90)(0.01)}{14.73\times10^{-9}} = 61.1\text{ MPa (largest)}$

$c_2 = \frac{1}{2}d_2 = 12.5\text{ mm}$

Shaft BC: $T = -90 + 270 = 180\text{Nm}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0125^4 - 0.005^4) = 37.37\times10^{-9}\,\text{m}^4$

$\tau_{max} = \frac{Tc_2}{J} = \frac{(180)(0.0125)}{37.37\times10^{-9}} = 60.2\text{ MPa}$

Shaft CD: $T = -90 + 270 + 110 = 290\text{ Nm}$ $\qquad c_2 = \frac{1}{2}d_2 = 15\text{ mm}$

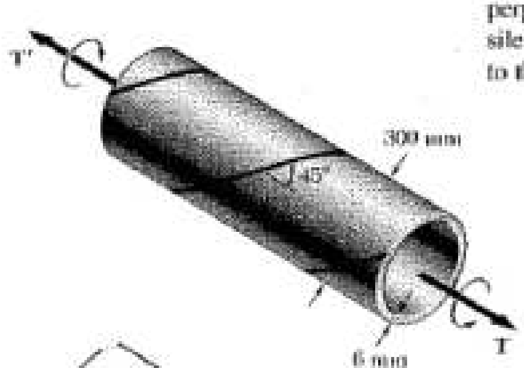
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.015^4 - 0.005^4) = 78.54\times10^{-9}\,\text{m}^4$

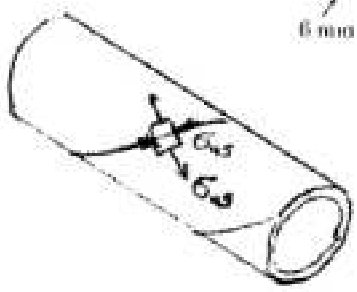
$\tau_{max} = \frac{Tc_2}{J} = \frac{(290)(0.015)}{78.54\times10^{-9}} = 55.39\text{ MPa}$

Answers: (a) Shaft AB $\qquad$ (b) 61.1 MPa

Problem 3.152

3.152 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 45° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable tensile stress in the weld is 84 MPa, determine the largest torque that can be applied to the pipe.





From equation (3.14) of the textbook,

$\sigma_{45} = \tau_{max}$

hence, $\tau_{max} = 84\,\text{MPa}$

$c_2 = \frac{1}{2} d_o = \frac{1}{2}(300) = 150\,\text{mm}$

$c_1 = c_2 - t = 150 - 6 = 144\,\text{mm}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}\left[(0.15)^4 - (0.144)^4\right]$

$= 119.8 \times 10^{-6}\,\text{m}^4$

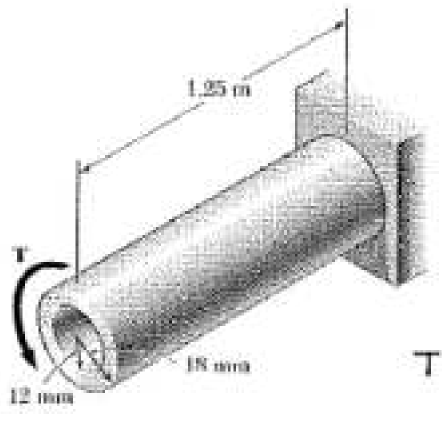
$\tau_{max} = \frac{Tc}{J} \qquad T = \frac{\tau_{max} J}{c}$

$$T = \frac{(84\times10^{6})(119.8\times10^{-6})}{0.15} = 67.088$$

$T = 67\,\text{kN}$

Problem 3.153

3.153 For the aluminum shaft shown ($G = 27$ GPa), determine (*a*) the torque **T** that causes an angle of twist of 4°, (*b*) the angle of twist caused by the same torque **T** in a solid cylindrical shaft of the same length and cross-sectional area.



(a) $\varphi = \frac{TL}{GJ}, \qquad T = \frac{GJ\varphi}{L}$

$\varphi = 4° = 69.813\times10^{-3}\,\text{rad}, \quad L = 1.25\,\text{m}$

$G = 27\,\text{GPa} = 27\times10^{9}\,\text{Pa}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.018^4 - 0.012^4) = 132.324\times10^{-9}\,\text{m}^4$

$$T = \frac{(27\times10^{9})(132.324\times10^{-9})(69.813\times10^{-3})}{1.25}$$

$= 199.539\,\text{N}\cdot\text{m} \qquad T = 199.5\,\text{N}\cdot\text{m}$

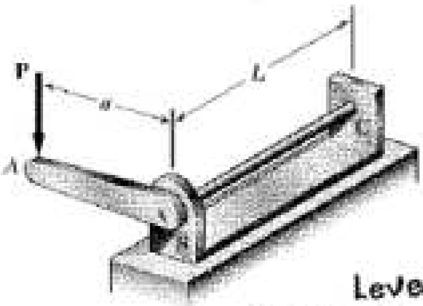
(b) Matching areas $\quad A = \pi c^2 = \pi(c_2^2 - c_1^2)$

$c = \sqrt{c_2^2 - c_1^2} = \sqrt{0.018^2 - 0.012^2} = 0.013416\,\text{m}$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.013416)^4 = 50.894\times10^{-9}\,\text{m}^4$

$$\varphi = \frac{TL}{GJ} = \frac{(195.539)(1.25)}{(27\times10^{9})(50.894\times10^{-9})} = 181.514\times10^{-3}\,\text{rad} \qquad \varphi = 10.40°$$

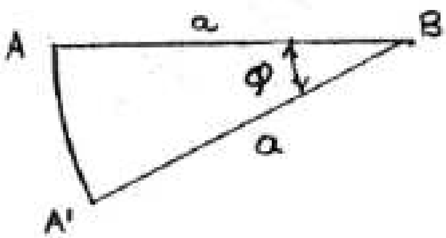
Problem 3.154



3.154 The solid cylindrical rod BC is attached to the rigid lever AB and to the fixed support at C. The vertical force **P** applied at A causes a small displacement Δ at point A. Show that the corresponding maximum shearing stress in the rod is

$$\tau = \frac{Gd}{2La}\Delta$$

where d is the diameter of the rod and G its modulus of rigidity.



Lever AB turns through angle φ to position A'B as shown in the auxiliary figure.

Vertical displacement is $\Delta = a \sin\varphi$

from which $\varphi = \arcsin \frac{\Delta}{a}$

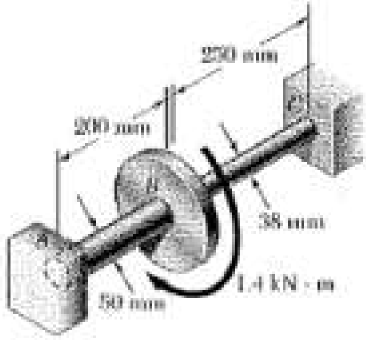
The maximum shearing stress in rod BC is

$$\tau_{max} = G\gamma_{max} = G\frac{c\varphi}{L} = G\frac{d\varphi}{2L} = \frac{Gd}{2L}\arcsin\frac{\Delta}{a}$$

For small $\frac{\Delta}{a}$, $\arcsin\frac{\Delta}{a} \approx \frac{\Delta}{a}$

$$\tau_{max} = \frac{Gd\Delta}{2La}$$

Problem 3.155



3.155 Two solid steel shafts (G = 77.2 GPa) are connected to a coupling disk B and to fixed supports at A and C. For the loading shown, determine (*a*) the reaction at each support, (*b*) the maximum shearing stress in shaft AB, (*c*) the maximum shearing stress in shaft BC.

Shaft AB

$T = T_{AB}$, $L_{AB} = 0.200\text{ m}$, $c = \frac{1}{2}d = 25\text{ mm} = 0.025\text{ m}$

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.025)^4 = 613.59\times10^{-9}\text{ m}^4$

$\varphi_B = \frac{T_{AB}L_{AB}}{GJ_{AB}}$

$$T_{AB} = \frac{GJ_{AB}}{L_{AB}}\varphi_B = \frac{(77.2\times10^9)(613.59\times10^{-9})}{0.200}\varphi_B = 236.847\times10^3\ \varphi_B$$

Shaft BC

$T = T_{BC}$, $L_{BC} = 0.250\text{ m}$, $c = \frac{1}{2}d = 19\text{ mm} = 0.019\text{ m}$

$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.019)^4 = 204.71\times10^{-9}\text{ m}^4$ $\qquad \varphi_B = \frac{T_{BC}L_{BC}}{GJ_{BC}}$

$$T_{BC} = \frac{GJ_{BC}}{L_{BC}}\varphi_B = \frac{(77.2\times10^9)(204.71\times10^{-9})}{0.250} = 63.214\times10^3\ \varphi_B$$

Equilibrium of coupling disk. $\quad T = T_{AB} + T_{BC}$

$$1.4\times10^3 = 236.847\times10^3\ \varphi_B + 63.214\times10^3\ \varphi_B$$

$$\varphi_B = 4.6657\times10^{-3}\text{ rad.}$$

$$T_{AB} = (236.847\times10^3)(4.6657\times10^{-3}) = 1.10506\times10^3\text{ N}\cdot\text{m}$$

$$T_{BC} = (63.214\times10^3)(4.6657\times10^{-3}) = 294.94\text{ N}\cdot\text{m}$$

(a) Reactions at supports.

$T_A = T_{AB} = 1105\text{ N}\cdot\text{m}$ ◄

$T_C = T_{BC} = 295\text{ N}\cdot\text{m}$ ◄

(b) Maximum shearing stress in AB.

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(1.10506\times10^3)(0.025)}{613.59\times10^{-9}} = 45.0\times10^6\text{ Pa}$$

$\tau_{AB} = 45.0\text{ MPa}$ ◄

(c) Maximum shearing stress in BC.

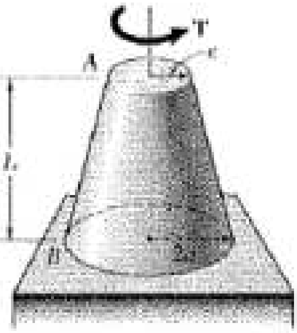
$$\tau_{BC} = \frac{T_{BC}c}{J_{BC}} = \frac{(294.94)(0.019)}{204.71\times10^{-9}} = 27.4\times10^9\text{ Pa}$$

$\tau_{BC} = 27.4\text{ MPa}$ ◄

Problem 3.158

3.158 A torque **T** is applied as shown to a solid tapered shaft *AB*. Show by integration that the angle of twist at *A* is

$$\phi = \frac{7TL}{12\pi G c^4}$$

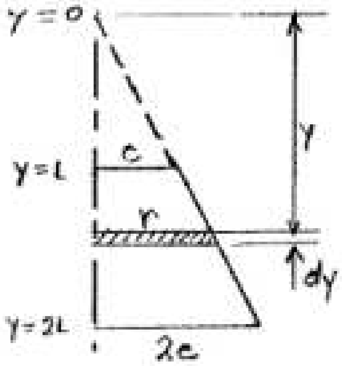


Introduce coordinate y as shown.

$$r = \frac{cy}{L}$$

Twist in length dy

$$d\varphi = \frac{T\,dy}{GJ} = \frac{T\,dy}{G\frac{\pi}{2}r^4} = \frac{2TL^4}{\pi G c^4}\frac{dy}{y^4}$$



$$\varphi = \int_L^{2L} \frac{2TL^4}{\pi G c^4}\frac{dy}{y^4} = \frac{2TL}{\pi G c^4}\int_L^{2L}\frac{dy}{y^4}$$

$$= \frac{2TL^4}{\pi G c^4}\left\{-\frac{1}{3y^3}\right\}_L^{2L} = \frac{2TL^4}{\pi G c^4}\left\{-\frac{1}{24L^3} + \frac{1}{3L^3}\right\}$$

$$= \frac{2TL^4}{\pi G c^4}\left\{\frac{7}{24L^3}\right\} = \frac{7TL}{12\pi G c^4}$$

Problem 3.159

3.159 A 38-mm-diameter steel shaft of length 1.2 m will be used to transmit 45 kW between a motor and a pump. Knowing that $G = 77$ GPa, determine the lowest speed of rotation at which the stress does not exceed 60 MPa and the angle of twist does not exceed 2°.

$c = \frac{1}{2}d = 19\text{mm}$ $\quad L = 1.2\text{ m}$ $\quad P = 45\text{ kW/s}$

Stress requirement $\quad \tau = 60\text{ MPa}$ $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$T = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(60\times10^6)(0.019)^3 = 646\text{ Nm}$$

Twist angle requirement $\quad \varphi = 2° = 34.907\times10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$T = \frac{\pi G c^4 \varphi}{2L} = \frac{\pi(77\times10^9)(0.019)^4(34.907\times10^{-3})}{(2)(1.2)} = 458\text{ Nm}$$

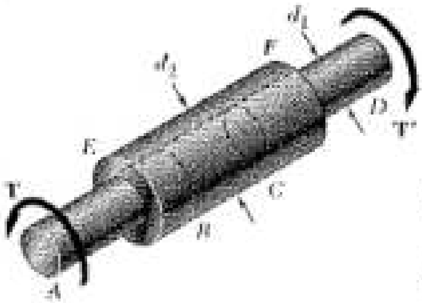
Maximum allowable torque is the smaller value. $\quad T = 458\text{ Nm}$

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{45\times10^3}{2\pi(458)} = 15.6\text{ Hz} \qquad f = 936\text{ rpm}$$

Problem 3.160

3.160 Two solid brass rods AB and CD are brazed to a brass sleeve EF. Determine the ratio d_2/d_1 for which the same maximum shearing stress occurs in the rods and in the sleeve.



Let $c_1 = \frac{1}{2}d_1$ and $c_2 = \frac{1}{2}d_2$

Shaft AB. $\tau_1 = \frac{Tc_1}{J_1} = \frac{2T}{\pi c_1^3}$

Sleeve EF. $\tau_2 = \frac{Tc_2}{J_2} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$

For equal stresses, $\frac{2T}{\pi c_1^3} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$

$c_2^4 - c_1^4 = c_1^3 c_2$

Let $x = \frac{c_2}{c_1}$ $\quad x^4 - 1 = x \quad$ or $\quad x = \sqrt[4]{1+x}$

Solve by successive approximations starting with $x_0 = 1.0$.

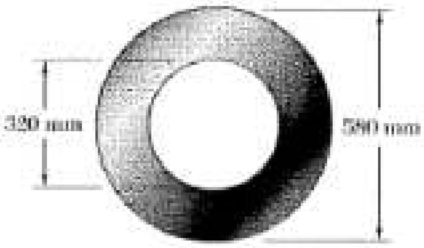
$x_1 = \sqrt[4]{2} = 1.189$, $\quad x_2 = \sqrt[4]{2.189} = 1.216$, $\quad x_3 = \sqrt[4]{2.216} = 1.220$

$x_4 = \sqrt[4]{2.220} = 1.221$, $\quad x_5 = \sqrt[4]{2.221} = 1.221$ (converged).

$x = 1.221 \qquad \frac{c_2}{c_1} = 1.221 \qquad\qquad \frac{d_2}{d_1} = 1.221$ ◄

Problem 3.161

3.161 One of the two hollow steel drive shafts of an ocean liner is 75 m long and has the cross section shown. Knowing that $G = 77.2$ GPa and that the shaft transmits 44 MW to its propeller when rotating at 144 rpm, determine (*a*) the maximum shearing stress in the shaft, (*b*) the angle of twist of the shaft.



$L = 75\ m,\qquad f = 144\ rpm = \frac{144}{60} = 2.4\ Hz$

$P = 44\ MW = 44 \times 10^6\ W$

$P = 2\pi f T \therefore\ T = \frac{P}{2\pi f} = \frac{44 \times 10^6}{2\pi(2.4)} = 2.9178 \times 10^6\ N\cdot m$

$c_1 = \frac{d_1}{2} = \frac{320}{2} = 160\ mm = 0.160\ m$

$c_2 = \frac{d_2}{2} = \frac{580}{2} = 290\ mm = 0.290\ m$

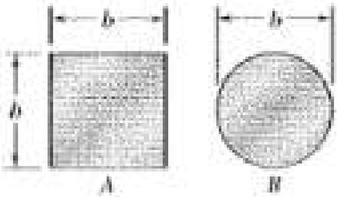
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.290^4 - 0.160^4) = 10.08 \times 10^{-3}\ m$

(a) $\tau = \frac{Tc_2}{J} = \frac{(2.9178 \times 10^6)(0.290)}{10.08 \times 10^{-3}} = 83.9 \times 10^6\ Pa = 83.9\ MPa$ ◄

(b) $\varphi = \frac{TL}{GJ} = \frac{(2.9178 \times 10^6)(75)}{(77 \times 10^9)(10.08 \times 10^{-3})} = 281.9 \times 10^{-3}\ rad = 16.15°$ ◄

Problem 3.162

3.162 Two shafts are made of the same material. The cross section of shaft *A* is a square of side *b* and that of shaft *B* is a circle of diameter *b*. Knowing that the shafts are subjected to the same torque, determine the ratio τ_A/τ_B of the maximum shearing stresses occurring in the shafts.



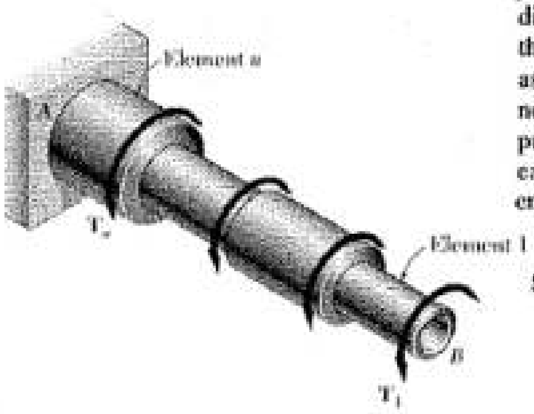
A. Square: $\frac{a}{b} = 1,\quad c_1 = 0.208$ (Table 3.1)

$\tau_A = \frac{T}{c_1 a b^2} = \frac{T}{0.208\, b^3}$

B. Circle: $c = \frac{1}{2}b \qquad \tau_B = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16\,T}{\pi b^3}$

Ratio $\frac{\tau_A}{\tau_B} = \frac{1}{0.208} \cdot \frac{\pi}{16} = 0.3005\,\pi \qquad \frac{\tau_A}{\tau_B} = 0.944$ ◄

PROBLEM 3.C1



3.C1 Shaft *AB* consists of *n* homogeneous cylindrical elements, which can be solid or hollow. Its end *A* is fixed, while its end *B* is free, and it is subjected to the loading shown. The length of element *i* is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, its modulus of rigidity by G_i, and the torque applied to its right end by $\mathbf{T}_i$, the magnitude T_i of this torque being assumed to be positive if $\mathbf{T}_i$ is observed as counterclockwise from end *B* and negative otherwise. (Note that $ID_i = 0$ if the element is solid.) (*a*) Write a computer program that can be used to determine the maximum shearing stress in each element, the angle of twist of each element, and the angle of twist of the entire shaft. (*b*) Use this program to solve Probs. 3.35 and 3.38.

SOLUTION

FOR EACH CYLINDRICAL ELEMENT, ENTER

L_i , OD_i , ID_i , G_i , T_i

AND COMPUTE

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

OUTLINE OF PROGRAM

UPDATE TORQUE $\quad T = T + T_i$

AND COMPUTE

$$TAU_i = T\,(OD_i/2)/J_i$$

$$PHI_i = TL_i/G_iJ_i$$

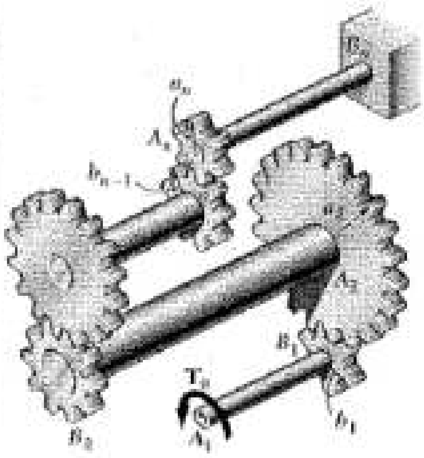
ANGLE OF TWIST OF ENTIRE SHAFT, STARTING WITH $\Theta = 0$, UPDATE THROUGH n^{th} ELEMENT

$$\Theta = \Theta + PHI_i$$

PROGRAM OUTPUT

```
Problem   3.35
Element   Maximum Stress    Angle of Twist
              (MPa)            (degrees)

 1.0000      11.9575           1.3841
 2.0000      23.0259           1.8323
Angle of twist for entire shaft =    3.2164 °

Problem   3.38
Element   Maximum Stress    Angle of Twist
              (MPa)            (degrees)

 1.0000      87.3278           4.1181
 2.0000      56.5884           1.0392
 3.0000      70.5179           0.8633
Angle of twist for entire shaft =    6.0206 °
```

PROBLEM 3.C2



3.C2 The assembly shown consists of n cylindrical shafts, which can be solid or hollow, connected by gears and supported by brackets (not shown). End A_1 of the first shaft is free and is subjected to a torque T_0, while end B_n of the last shaft is fixed. The length of shaft A_iB_i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, and its modulus of rigidity by G_i. (Note that $ID_i = 0$ if the element is solid.) The radius of gear A_i is denoted by a_i, and the radius of gear B_i by b_i. (a) Write a computer program that can be used to determine the maximum shearing stress in each shaft, the angle of twist of each shaft, and the angle through which end A_i rotates. (b) Use this program to solve Prob. 3.40 and 3.44.

SOLUTION

TORQUE IN SHAFTS. ENTER $T_i = T_0$

$$T_{i+1} = T_i\,(A_{i+1}/B_i)$$

FOR EACH SHAFT, ENTER

$L_i \quad OD_i \quad ID_i \quad G_i$

COMPUTE: $J_i = (\pi/32)(OD_i^4 - ID_i^4)$

$$TAU_i = T_i\,(OD_i/2)/J_i$$

$$PHI_i = T_i L_i / G_i J_i$$

ANGLE OF ROTATION AT END A_1

COMPUTE ROTATION AT THE "A" END OF EACH SHAFT

START WITH ANGLE = PHI_n AND UPDATE

FROM n TO 1, AND ADD PHI_i

$$ANGLE = ANGLE\,(A_i)/B_{i-1} + PHI_{i-1}$$

PROGRAM OUTPUT

Shaft No.	Max Stress (MPa)	Angle of Twist (degrees)
1	62.17	1.472
2	84.80	1.719

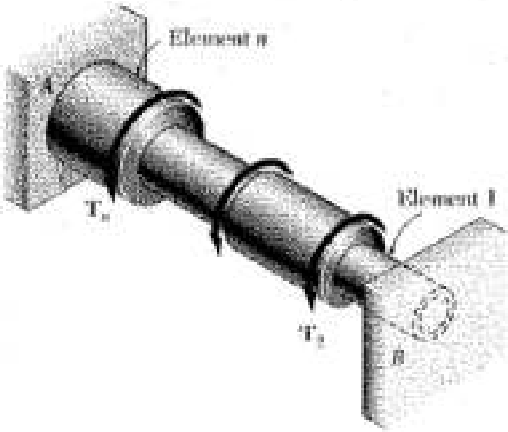
Angle through which A1 rotates = 3.816°

PROGRAM OUTPUT

Shaft No.	Max Stress (MPa)	Angle of Twist (degrees)
1	382.17	22.403
2	191.09	11.202
3	95.55	5.601

Angle through which A1 rotates = 22.403°

PROBLEM 3.C3



3.C3 Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Both of its ends are fixed, and it is subjected to the loading shown. The length of element i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, its modulus of rigidity by G_i, and the torque applied to its right end by $\mathbf{T}_i$, the magnitude T_i of this torque being assumed to be positive if $\mathbf{T}_i$ is observed as counterclockwise from end B and negative otherwise. Note that $ID_i = 0$ if the element is solid and also that $T_1 = 0$. Write a computer program that can be used to determine the reactions at A and B, the maximum shearing stress in each element, and the angle of twist of each element. Use this program (*a*) to solve Prob. 3.155,(*b*) to determine the maximum shearing stress in the shaft of Example 3.05.

SOLUTION WE CONSIDER THE REACTION AT B AS REDUNDANT AND RELEASE THE SHAFT AT B.

COMPUTE Θ_B WITH $T_B = 0$:

FOR EACH ELEMENT, ENTER

L_i , OD_i , ID_i , G_i , T_i (NOTE $T_1 = T_B = 0$)

COMPUTE

$J_i = (\pi/32)(OD_i^4 - ID_i^4)$

UPDATE TORQUE

$T = T + T_i$

AND COMPUTE FOR EACH ELEMENT

$TAU_i = T\,(OD_i/2)/J_i$

$PHI_i = T\,L_i/G_iJ_i$

COMPUTE Θ_B: STARTING WITH $\Theta = 0$ AND UPDATING THROUGH n ELEMENTS

$\Theta_i = \Theta_i + PHI_i$; $\Theta_B = \Theta_n$

COMPUTE Θ_B DUE TO UNIT TORQUE AT B

UNIT $TAU_i = OD_i/2J_i$

UNIT $PHI_i = L_i/G_iJ_i$

FOR n ELEMENTS:

UNIT$\Theta_B(i)$ = UNIT$\Theta_B(i)$ + UNIT PHI_i

SUPERPOSITION

FOR TOTAL ANGLE AT B TO BE ZERO. $\Theta_B + T_B(\text{UNIT}\Theta_B(n)) = 0$

$T_B = -\Theta_B/(\text{UNIT}\Theta_B(n))$

THEN $T_A = \sum T(i) + T_B$

FOR EACH ELEMENT: MAX STRESS: TOTAL $TAU_i = TAU_i + T_B(\text{UNIT } TAU_i)$

ANGLE OF TWIST: TOTAL $PHI_i = PHI_i + T_B(\text{UNIT } PHI_i)$

PROGRAM OUTPUT

Problem 3.155

TA = -0.295 kN*m

TB = -1.105 kN*m

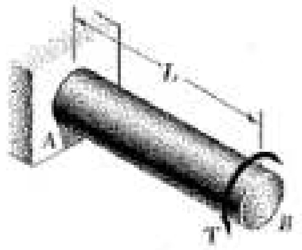
Element	tau max (MPa)	Angle of Twist (degrees)
1	-45.024	-0.267
2	27.375	-.267

Problem 3.05

TA = -69.767 Nm

TB = -50.232 Nm

PROBLEM 3.C4



3.C4 The homogeneous, solid cylindrical shaft AB has a length L, a diameter d, a modulus of rigidity G, and a yield strength τ_Y. It is subjected to a torque **T** that is gradually increased from zero until the angle of twist of the shaft has reached a maximum value ϕ_m and then decreased back to zero. (*a*) Write a computer program that, for each of 16 values of ϕ_m equally spaced over a range extending from 0 to a value 3 times as large as the angle of twist at the onset of yield, can be used to determine the maximum value T_m of the torque, the radius of the elastic core, the maximum shearing stress, the permanent twist, and the residual shearing stress both at the surface of the shaft and at the interface of the elastic core and the plastic region. (*b*) Use this program to obtain approximate answers to Probs. 3.114, 3.115, and 3.118.

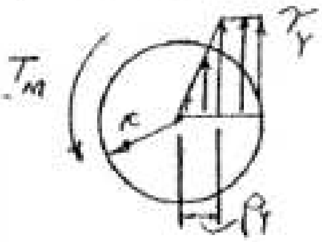
SOLUTION

AT ONSET OF YIELD

$$T_Y = \tau_Y \frac{J}{c} = \frac{\pi}{2}\tau_Y c^3$$

$$\phi_Y = \frac{T_Y L}{GJ} = \left(\frac{\tau_Y J}{c}\right)\frac{L}{GJ} = \frac{\tau_Y L}{c G}$$

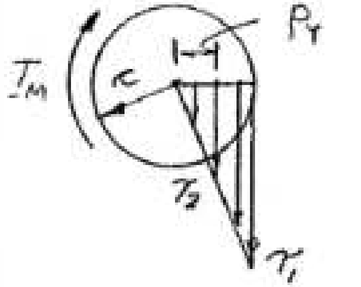
LOADING: $T_M > T_Y$



$$T_M = \frac{4}{3}T_M\left[1 - \frac{1}{4}\left(\frac{\phi_Y}{\phi_M}\right)^3\right] \qquad \text{EQ.(1)}$$

$$\rho_Y = c\frac{\phi_Y}{\phi_M} \qquad \text{EQ.(2)}$$

UNLOADING (ELASTIC)



$\phi_U = \dfrac{T_M L}{GJ}$ ϕ_U = ANGLE OF TWIST FOR UNLOADING

$\tau_1 = T_M \dfrac{c}{J}$ τ_1 = TAU AT $\rho = c$

$\tau_2 = \tau_1 \dfrac{\rho_Y}{c}$ τ_2 = TAU AT $\rho = \rho_Y$

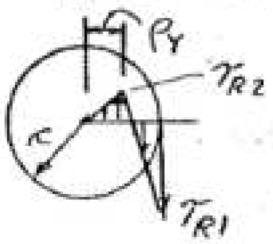
SUPERPOSE LOADING AND UNLOADING

FOR $\phi = 0$ TO $\phi = 3\phi_Y$ USING $0.2\,\phi_Y$ INCREMENTS

WHEN $\phi < \phi_Y$: $T_M = T_Y \dfrac{\phi}{\phi_Y}$ $\rho_Y = \frac{1}{2}d$ $\phi_M = \phi_Y \dfrac{\phi}{\phi_Y}$

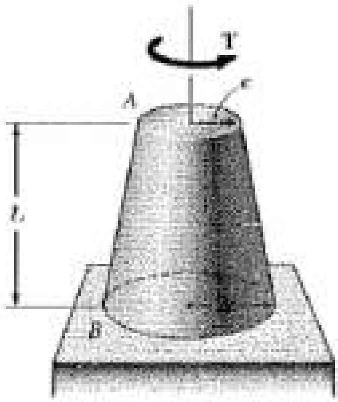
WHEN $\phi > \phi_Y$: T_M USE EQ(1) ρ_Y USE EQ(2)

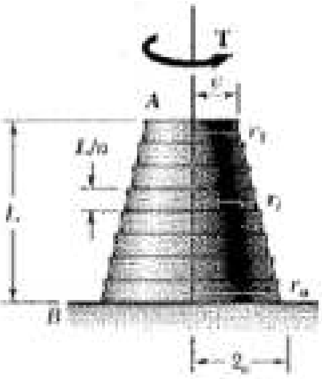
RESIDUAL: $\phi_P = \phi_M - \phi_U$ $\tau_{R1} = \tau_1 - \tau_Y$ $\tau_{R2} = \tau_2 - \tau_Y$



CONTINUED

PROBLEM 3.C5





3.C5 The exact expression is given in Prob. 3.158 for the angle of twist of the solid tapered shaft *AB* when a torque **T** is applied as shown. Derive an approximate expression for the angle of twist by replacing the tapered shaft by *n* cylindrical shafts of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for *T*, *L*, *G*, and *c* values of your choice, determine the percentage error in the approximate expression when (*a*) $n = 4$, (*b*) $n = 8$, (*c*) $n = 20$, (*d*) $n = 100$.

SOLUTION

FROM PROB. 3.158 EXACT EXPRESSION:

$$\phi = \frac{7TL}{12\pi G c^4}$$

$$\text{OR,}\quad \phi = \left(\frac{7}{12\pi}\right)\frac{TL}{Gc^4} = 0.18568\,\frac{TL}{Gc^4}$$

CONSIDER TYPICAL ith SHAFT

$$r_i = (n + i - \tfrac{1}{2})(c/n)$$

L/n

$$J_i = \frac{\pi}{2}(r_i)^4$$

$$\Delta\phi = \frac{T(L/n)}{G J_i}$$

ENTER UNIT VALUES OF T, L, G, AND c.
(NOTE: SPECIFIC VALUES CAN BE ENTERED)

ENTER INITIAL VALUE OF ZERO FOR ϕ
ENTER n = NUMBER CYLINDRICAL SHAFTS

FOR $i = 1$ TO n, UPDATE ϕ

$$\phi = \phi + \Delta\phi$$

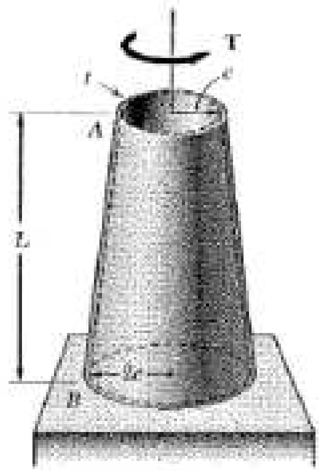
OUTPUT OF PROGRAM

COEFFICIENT of TL/Gc^4

Exact coefficient from Prob. 3.158 is 0.18568
Number of elemental disks = n

n	approximate	exact	percent error
4	0.17959	0.18568	-3.28185
8	0.18410	0.18568	-0.85311
20	0.18542	0.18568	-0.13810
100	0.18567	0.18568	-0.00554

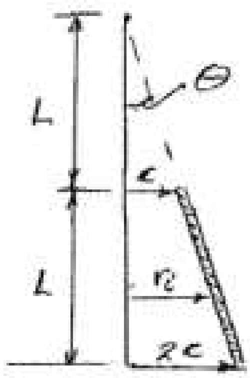
PROBLEM 3.C6



3.C6 A torque **T** is applied as shown to the long, hollow, tapered shaft AB of uniform thickness t. The exact expression for the angle of twist of the shaft can be obtained from the expression given in Prob. 3.153. Derive an approximate expression for the angle of twist by replacing the tapered shaft by n cylindrical rings of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for T, L, G, c and t values of your choice, determine the percentage error in the approximate expression when (a) $n = 4$, (b) $n = 8$, (c) $n = 20$, (d) $n = 100$.

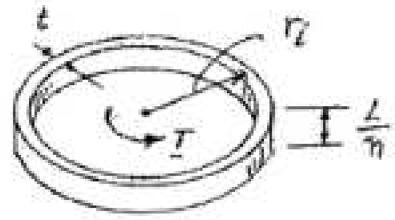
SOLUTION

SINCE THE SHAFT IS LONG $c \ll L$, THE ANGLE θ IS SMALL AND WE CAN USE t AS THE THICKNESS OF THE n CYLINDRICAL RINGS.



FOR $c \ll L$

$$\theta = \tan\theta = \frac{2c - c}{L} = \frac{c}{L}$$



$$r_i = (n + i - \tfrac{1}{2})(\tfrac{c}{n})$$

$$J_i \approx (\text{AREA})\, r_i^2 = (2\pi r_i t) r_i^2 = 2\pi t\, r_i^3$$

$$\Delta\phi = \frac{T(L/n)}{G J_i}$$

ENTER UNIT VALUES FOR T, L, G, t, AND c

(NOTE: SPECIFIC VALUES CAN BE ENTERED IF DESIRED)

ENTER INITIAL VALUE OF ZERO FOR ϕ

ENTER n = NUMBER OF CYLINDRICAL RINGS

FOR $i = 1$ TO n, UPDATE ϕ

$$\phi = \phi + \Delta\phi$$

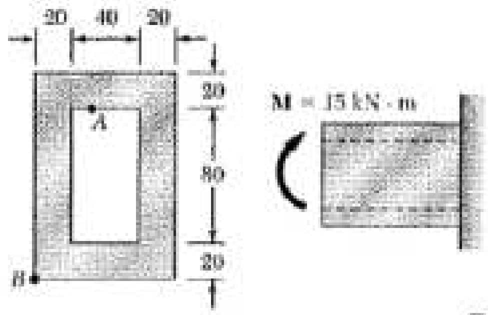
OUTPUT OF PROGRAM

```
COEFFICIENT of TL/Gtc^3

Exact coefficient from Prob. 3.153 is 0.05968
Number of elemental disks = n

  n     approximate     exact       percent error

  4     0.058559       0.059683     -1.883078
  8     0.059394       0.059683     -0.483688
 20     0.059637       0.059683     -0.078022
100     0.059681       0.059683     -0.003127
```

Chapter 4

Problem 4.1

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm

For rectangle $I = \frac{1}{12} bh^3$

Outside rectangle: $I_1 = \frac{1}{12}(80)(120)^3$

$I_1 = 11.52 \times 10^6 \text{ mm}^4 = 11.52 \times 10^{-6} \text{ m}^4$

Cutout: $I_2 = \frac{1}{12}(40)(80)^3$

$I_2 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$

Section: $I = I_1 - I_2 = 9.81333 \times 10^{-6} \text{ m}^4$

(a) $y_A = 40 \text{ mm} = 0.040 \text{ m}$ $\quad \sigma_A = -\frac{My_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$

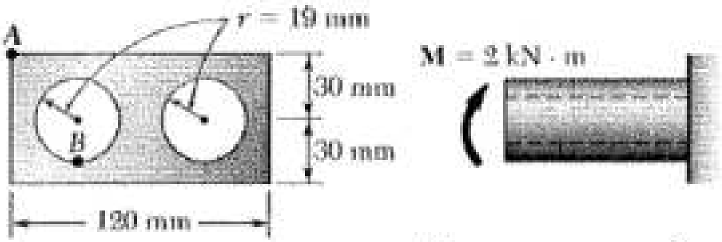
$\sigma_A = -61.6 \text{ MPa}$ ◄

(b) $y_B = -60 \text{ mm} = -0.060 \text{ m}$ $\quad \sigma_B = -\frac{My_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$

$\sigma_B = 91.7 \text{ MPa}$ ◄

Problem 4.2

4.1 and *4.2* Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



For the solid rectangle,

$I_1 = \frac{1}{12} bh^3 = \frac{1}{12}(0.12)(0.06)^3$

$= 2.16 \times 10^{-6} \text{ m}^4$

For one circular cutout, $I_2 = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.019)^4$

$= 102.35 \times 10^{-9} \text{ m}^4$

For the section, $I = I_1 - 2I_2 = 2.16 \times 10^{-6} - (2)(102.35 \times 10^{-9}) = 1.955 \times 10^{-6} \text{ m}^4$

(a) $\underline{y_A = 0.03 \text{ m}}$ $\quad \sigma_A = -\frac{My_A}{I} = -\frac{(2000)(0.03)}{1.955 \times 10^{-6}} = -30.69 \text{ MPa}$

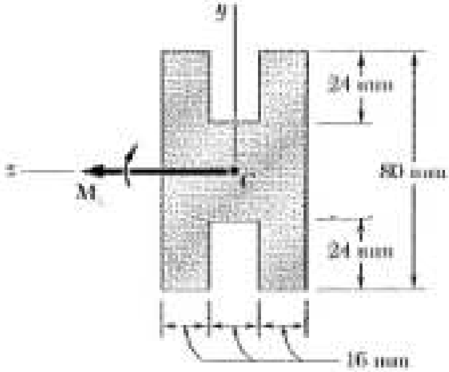
$\sigma_A = -30.7 \text{ MPa}$ ◄

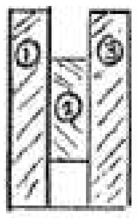
(b) $\underline{y_B = -0.019 \text{ m}}$ $\quad \sigma_B = -\frac{My_B}{I} = -\frac{(2000)(-0.019)}{1.955 \times 10^{-6}} = 19.44 \text{ MPa}$

$\sigma_B = 19.4 \text{ MPa}$ ◄

Problem 4.3

4.3 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.





Allowable stress $= \dfrac{\sigma_U}{F.S.} = \dfrac{450}{3} = 150$ MPa $= 150 \times 10^6$ Pa

Moment of inertia about z axis.

$I_1 = \frac{1}{12}(16)(80)^3 = 682.67 \times 10^3 \text{ mm}^4$

$I_2 = \frac{1}{12}(16)(32)^3 = 43.69 \times 10^3 \text{ mm}^4$

$I_3 = I_1 = 682.67 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 1.40902 \times 10^6 \text{ mm}^4 = 1.40902 \times 10^{-6} \text{ m}^4$

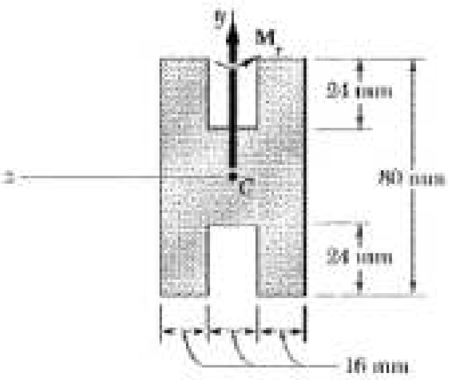
$\sigma = \dfrac{Mc}{I}$ with $c = \frac{1}{2}(80) = 40 \text{ mm} = 0.040 \text{ m}$

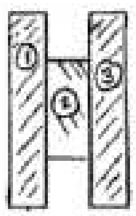
$M = \dfrac{I\sigma}{c} = \dfrac{(1.40902 \times 10^{-6})(150 \times 10^6)}{0.040} = 5.28 \times 10^3 \text{ N}\cdot\text{m}$ $\quad M = 5.28 \text{ kN}\cdot\text{m}$ ◄

Problem 4.4

4.4 Solve Prob. 4.3, assuming that the beam is bent about the y axis.

4.3 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.





Allowable stress $= \dfrac{\sigma_U}{F.S.} = \dfrac{450}{3.00} = 150$ MPa $= 150 \times 10^6$ Pa

Moment of inertia about y-axis.

$I_1 = \frac{1}{12}(80)(16)^3 + (80)(16)(16)^2 = 354.987 \times 10^3 \text{ mm}^4$

$I_2 = \frac{1}{12}(32)(16)^3 = 10.923 \times 10^3 \text{ mm}^4$

$I_3 = I_1 = 354.987 \times 10^3 \text{ mm}^4$

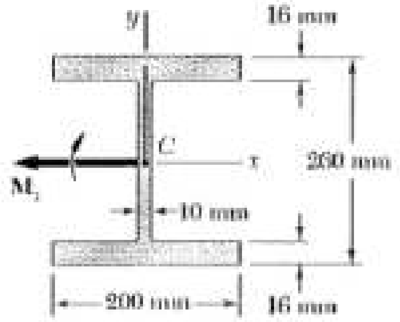
$I = I_1 + I_2 + I_3 = 720.9 \times 10^3 \text{ mm}^4 = 720.9 \times 10^{-9} \text{ m}^4$

$\sigma = \dfrac{Mc}{I}$ with $c = \frac{1}{2}(48) = 24 \text{ mm} = 0.024 \text{ m}$

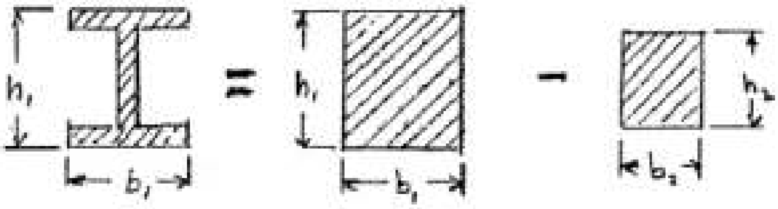
$M = \dfrac{I\sigma}{c} = \dfrac{(720.9 \times 10^{-9})(150 \times 10^6)}{0.024} = 4.51 \times 10^3 \text{ N}\cdot\text{m}$ $\quad M = 4.51 \text{ kN}\cdot\text{m}$ ◄

Problem 4.5

4.5 The steel beam shown is made of a grade of steel for which $\sigma_Y = 250$ MPa and $\sigma_U = 400$ MPa. Using a factor of safety of 2.50, determine the largest couple that can be applied to the beam when it is bent about the x axis.



The moment of inertia I_x is equivalent to that of a rectangle with a cutout



Larger rectangle: $b_1 = 200$ mm $\quad h_1 = 260$ mm $\quad I_1 = \frac{1}{12} b_1 h_1^3$

$$I_1' = \frac{1}{12}(200)(260)^3 = 292.933 \times 10^6 \text{ mm}^4$$

Smaller rectangle: $b_2 = 200 - 10 = 190$ mm $\quad h_2 = 260 - (2)(16) = 228$ mm

$$I_2 = \frac{1}{12}(190)(228)^3 = 187.662 \times 10^6 \text{ mm}^4$$

Section: $I_x = I_1 - I_2 = 105.271 \times 10^6 \text{ mm}^4 = 105.271 \times 10^{-6} \text{ m}^4$

$$c = \frac{260}{2} = 130 \text{ mm} = 0.130 \text{ m}$$

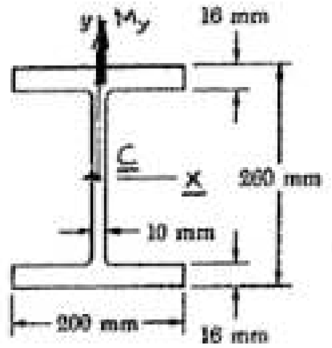
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{400}{2.50} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$\sigma_{all} = \frac{M_x c}{I_x} \qquad M_x = \frac{I_x \sigma_{all}}{c} = \frac{(105.271 \times 10^{-6})(160 \times 10^6)}{0.130}$$

$$= 129.564 \times 10^3 \text{ N}\cdot\text{m}$$

$M_x = 129.6$ kN·m ◄

Problem 4.6



4.6 Solve Prob. 4.5, assuming that the steel beam is bent about the y axis by a couple of moment M_y.

4.5 The steel beam shown is made of a grade of steel for which $\sigma_Y = 250$ MPa and $\sigma_U = 400$ MPa. Using a factor of safety of 2.50, determine the largest couple that can be applied to the beam when it is bent about the x axis.

For one flange, $I_F = \frac{1}{12} b_F h_F^3$

$b_F = 16$ mm $\quad h_F = 200$ mm

$I_F = \frac{1}{12}(16)(200)^3 = 10.6667 \times 10^6$ mm^4

For the web $\quad I_w = \frac{1}{12} b_w h_w^3$

$b_w = 260 - (2)(16) = 228$ mm $\quad h_w = 10$ mm

$I_w = \frac{1}{12}(228)(10)^3 = 19 \times 10^3$ mm^4

Section: $I_y = 2I_F + I_w = 21.352 \times 10^6$ mm^4 $= 21.352 \times 10^{-6}$ m^4

$c = \frac{200}{2} = 100$ mm $= 0.100$ m

$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{400}{2.50} = 160$ MPa $= 160 \times 10^6$ Pa

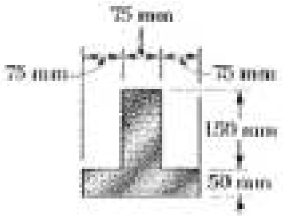
$\sigma_{all} = \frac{M_y c}{I_y} \qquad M_y = \frac{I_y c}{\sigma_{all}} = \frac{(21.352 \times 10^{-6})(160 \times 10^6)}{0.100}$

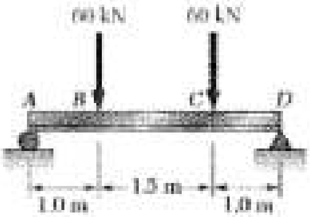
$= 34.163 \times 10^3$ N·m

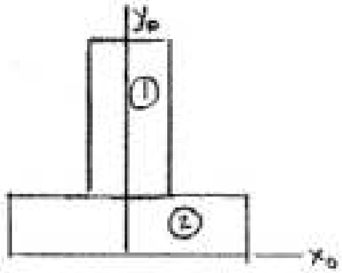
$M_y = 34.2$ kN·m

Problem 4.7

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.



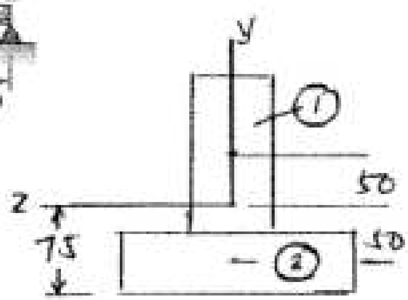




	A	$\bar{y}_0$	$A\bar{y}_0$
1	11250	125	1406250
2	11250	25	281250
Σ	22500		1687500

$$\bar{Y}_0 = \frac{1687500}{22500} = 75 \text{ mm}$$

Neutral axis lies 75 mm above the base.

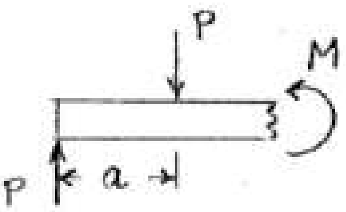


$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(0.075)(0.15)^3 + (0.01125)(0.05)^2 = 49.22 \times 10^{-6} \text{ m}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(0.225)(0.05)^3 + (0.01125)(0.05)^2 = 30.47 \times 10^{-6} \text{ m}^4$$

$$I = I_1 + I_2 = 79.7 \times 10^{-6} \text{ m}^4$$

$y_{top} = 125$ mm $\quad y_{bot} = -75$ mm



$$M - Pa = 0$$

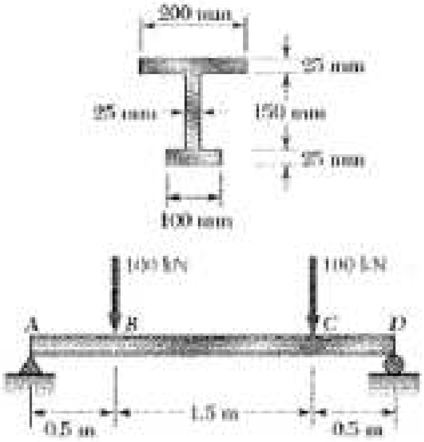
$$M = Pa = (60)(1) = 60 \text{ kN.m}$$

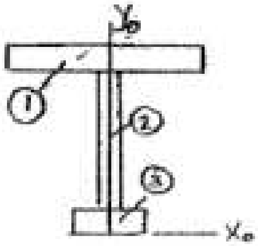
$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(60000)(0.125)}{79.7 \times 10^{-6}} = -94.1 \text{ MPa (compression)}$$

$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(60000)(-0.075)}{79.7 \times 10^{-6}} = 56.5 \text{ MPa (tension)}$$

Problem 4.8

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

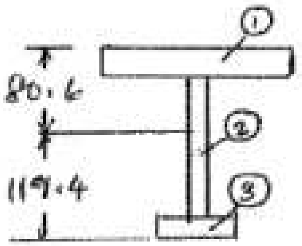




	A $(\times 10^3)$	$\bar{y}_o$	$A\bar{y}_o$ $(\times 10^3)$
①	5	187.5	937
②	3.75	100	375
③	2.5	12.5	31.25
Σ	11.25		1343.25

$$\bar{Y}_o = \frac{1343.25}{11.25} = 119.4 \text{ mm}$$

Neutral axis lies 119.4 above the base.



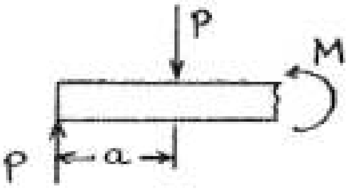
$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(200)(25)^3 + (5000)(68.1)^2$$
$$= 23.448 \times 10^{+6} \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(25)(150)^3 + (3750)(19.4)^2$$
$$= 8.443 \times 10^{6} \text{ mm}^4$$

$$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12}(100)(25)^3 + (2500)(106.9)^2 = 28.699 \times 10^{6} \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 60.59 \times 10^{-6} \text{ m}^4$$

$y_{top} = 80.6$ mm $\qquad y_{bot} = -119.4$ mm



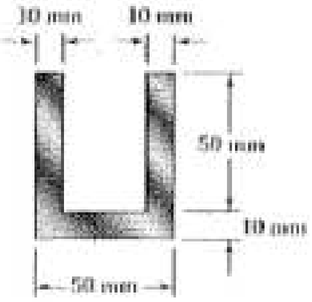
$$M - Pa = 0$$
$$M = Pa = (60)(1) = 60 \text{ kN.m}$$

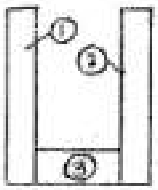
$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(60000)(0.0806)}{60.59 \times 10^{-6}} = -79.8 \text{ MPa} \quad \text{(compression)} \blacktriangleleft$$

$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(60000)(0.1194)}{60.59 \times 10^{-6}} = 118.2 \text{ MPa} \quad \text{(tension)} \blacktriangleleft$$

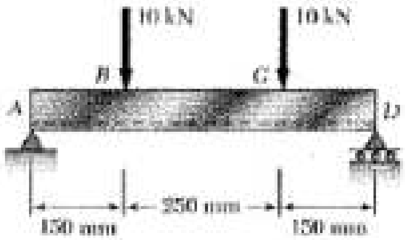
Problem 4.9

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

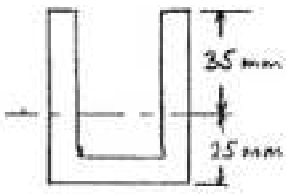




	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3



$$\bar{Y}_o = \frac{37.5\times10^3}{1500} = 25 \text{ mm}$$



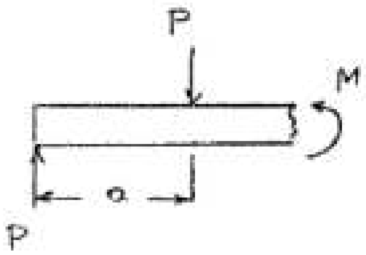
Neutral axis lies 25 mm above the base.

$$I_1 = \tfrac{1}{12}(10)(60)^3 + (600)(5)^2 = 195\times10^3 \text{ mm}^4 \qquad I_2 = I_1 = 195 \text{ mm}^4$$

$$I_3 = \tfrac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 512.5\times10^3 \text{ mm}^4 = 512.5\times10^{-9} \text{ m}^4$$

$$y_{top} = 35 \text{ mm} = 0.035 \text{ m} \qquad y_{bot} = -25 \text{ mm} = -0.025 \text{ m}$$



$$a = 150 \text{ mm} = 0.150 \text{ m} \qquad P = 10\times10^3 \text{ N}$$

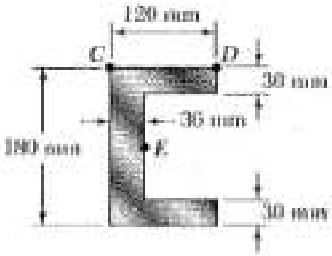
$$M = Pa = (10\times10^3)(0.150) = 1.5\times10^3 \text{ N}\cdot\text{m}$$

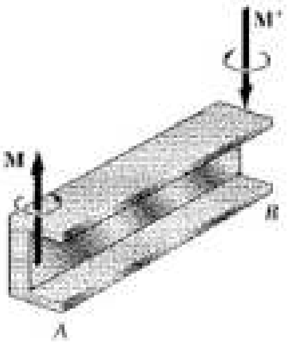
$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1.5\times10^3)(0.035)}{512.5\times10^{-9}} = -102.4\times10^6 \text{ Pa} \qquad \sigma_{top} = -102.4 \text{ MPa}$$
(compression)

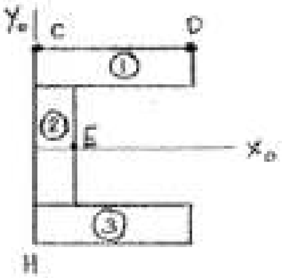
$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1.5\times10^3)(-0.025)}{512.5\times10^{-9}} = 73.2\times10^6 \text{ Pa} \qquad \sigma_{bot} = 73.2 \text{ MPa}$$
(tension)

Problem 4.10

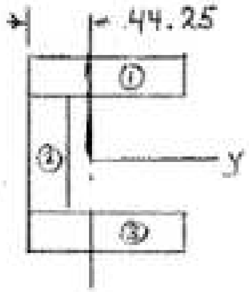
4.10 Two equal and opposite couples of magnitude $M = 25$ kN · m are applied to the channel-shaped beam AB. Observing that the couples cause the beam to bend in a horizontal plane, determine the stress at (*a*) point C, (*b*) point D, (*c*) point E.







	A, mm²	$\bar{x}_0$, mm	$A\bar{x}_0$, mm³
①	3600	60	216×10^3
②	4320	18	77.76×10^3
③	3600	60	216×10^3
Σ	11520		509.76×10^3



$$\bar{X} = \frac{509.76 \times 10^3}{11520} = 44.25 \text{ mm}$$

$$y_C = -44.25 \text{ mm} = -0.04425 \text{ m}$$

$$y_D = 120 - 44.25 = 75.75 \text{ mm} = 0.07575 \text{ m}$$

$$y_E = 36 - 44.25 = -8.25 \text{ mm} = -0.00825 \text{ m}$$

$$d_1 = 60 - 44.25 = 15.75 \text{ mm}$$
$$d_2 = 44.25 - 18 = 26.26 \text{ mm}$$
$$d_3 = d_1$$

$$I_1 = I_3 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(30)(120)^3 + (3600)(15.75)^2 = 5.2130 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(120)(26)^3 + (4320)(26.25)^2 = 3.4433 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 6.5187 \times 10^6 \text{ mm}^4 = 13.8694 \times 10^{-6} \text{ m}^4$$

$$M = 15 \times 10^3 \text{ N-m}$$

(a) Point C: $\sigma_C = -\dfrac{M y_C}{I} = -\dfrac{(25 \times 10^3)(-0.04425)}{13.8694 \times 10^{-6}} = 79.8 \times 10^6$ Pa

$\sigma_C = 79.8$ MPa ◄

(b) Point D: $\sigma_D = -\dfrac{M y_D}{I} = -\dfrac{(25 \times 10^3)(0.07575)}{13.8694 \times 10^{-6}} = -136.5 \times 10^6$ Pa

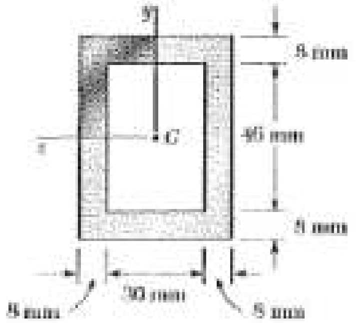
$\sigma_D = -136.5$ MPa ◄

(c) Point E: $\sigma_E = -\dfrac{M y_E}{I} = -\dfrac{(25 \times 10^3)(-0.00825)}{13.8694 \times 10^{-6}} = 14.87 \times 10^6$ Pa

$\sigma_E = 14.87$ MPa ◄

Problem 4.11

4.11 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 900 N · m, determine the total force acting on the shaded portion of the beam.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

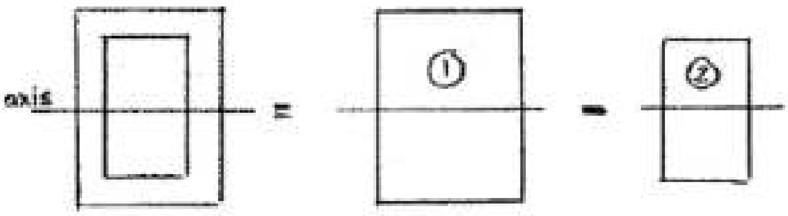
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

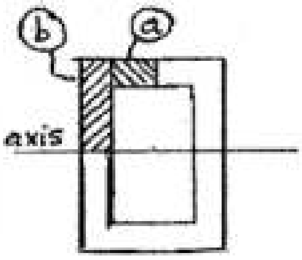
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\,dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



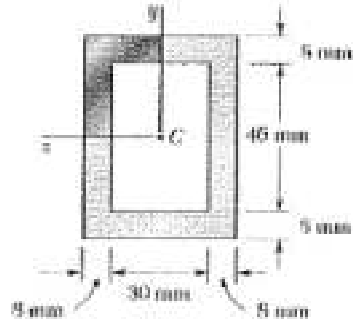
$$I = I_1 - I_2$$
$$= \frac{1}{12}b_1h_1^3 - \frac{1}{12}b_2h_2^3$$
$$= \frac{1}{12}(46)(62)^3 - \frac{1}{12}(30)(46)^3$$
$$= 0.67025 \times 10^6 \text{ mm}^4$$



$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$
$$= (27)(23)(8) + (11.5)(23)(8) = 7084 \text{ mm}^3$$

$$F = \frac{M\bar{y}^*A^*}{I} = \frac{(900)(7084 \times 10^{-9})}{0.67025 \times 10^{-6}} = 9.51 \text{ kN}$$

Problem 4.12



4.12 Solve Prob. 4.11, assuming that the beam is bent about a vertical axis and that the bending moment is 900 N · m.

4.11 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 900 N · m, determine the total force acting on the shaded portion of the beam.

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

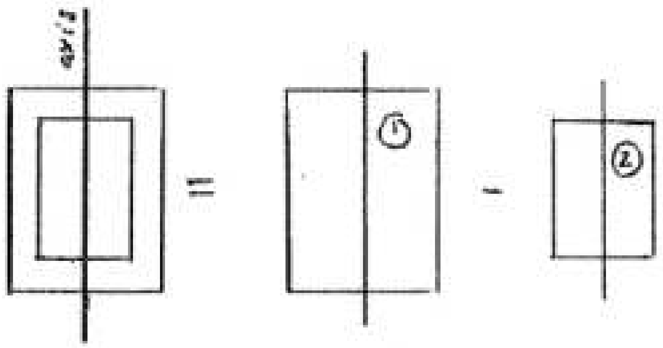
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

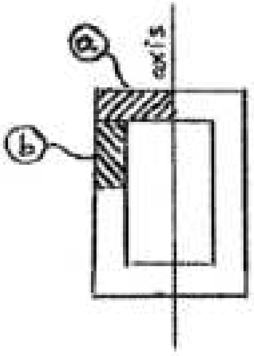
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y \, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



$$I = I_1 - I_2$$
$$= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3$$
$$= \frac{1}{12}(62)(46)^3 - \frac{1}{12}(46)(30)^3$$
$$= 0.399408 \times 10^6 \text{ mm}^4$$

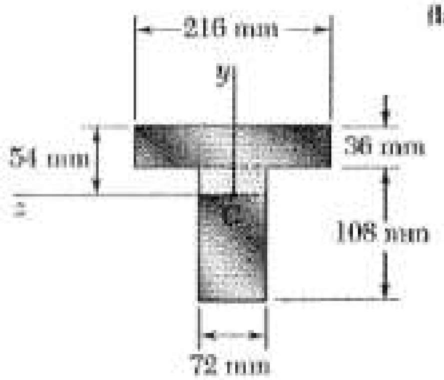


$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$
$$= (11.5)(8)(23) + (18)(23)(8)$$
$$= 5428 \text{ mm}^3.$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(900)(5428 \times 10^{-9})}{0.399403 \times 10^{-6}} = 12.2 \text{ kN.}$$

Problem 4.13

4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN · m, determine the total force acting on the top flange.



The stress distribution over the entire cross-section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

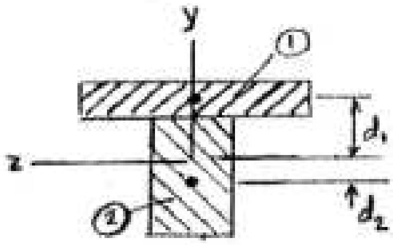
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA_1 = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



$d_1 = 54 - 18 = 36$ mm

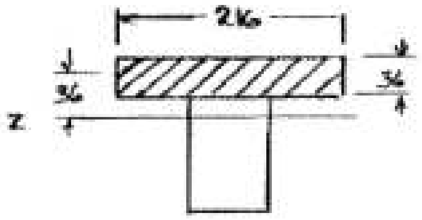
$d_2 = 54 - 36 - 54 = 36$mm

Moment of inertia of entire cross section.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(216)(36)^3 + (216)(36)(36)^2 = 10.9175 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(72)(108)^3 + (72)(108)(36)^2 = 17.6360 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 28.5535 \times 10^6 \text{ mm}^4 = 28.5535 \times 10^{-6} \text{ m}^4$$



For the shade area

$A^* = (216)(36) = 7776 \text{ mm}^2$

$\bar{y}^* = 36$ mm

$A^* \bar{y}^* = 279.936 \times 10^3 \text{ mm}^3 = 279.936 \times 10^{-6} \text{ m}^3$

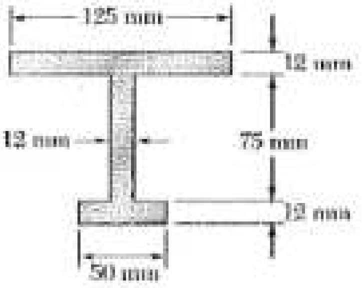
$$F = \left| \frac{M A^* \bar{y}^*}{I} \right| = \frac{(6 \times 10^3)(279.936 \times 10^{-6})}{28.5535 \times 10^{-6}}$$

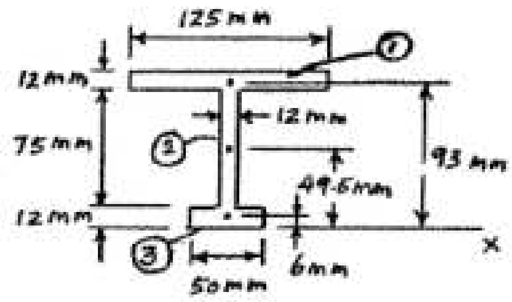
$$= 58.8 \times 10^3 \text{ N}$$

$F = 58.8$ kN

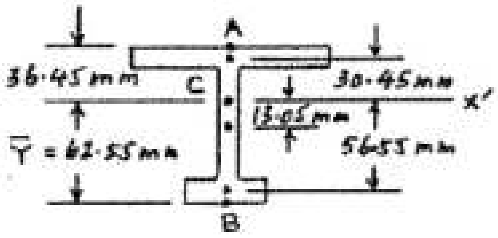
Problem 4.15

4.15 Knowing that for the casting shown the allowable stress is 42 MPa in tension and 105 MPa in compression, determine the largest couple **M** that can be applied.





	A, mm^2	$\bar{y}, mm$	$A\bar{y}, mm^3$
①	1500	93	139500
②	900	49.5	44550
③	600	6	3600
Σ	3000		187650



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{187650}{3000} = 62.55 \text{ mm}$$

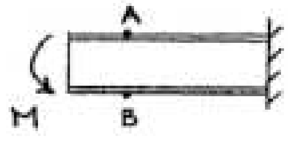
$$I_{x'} = \Sigma\left(\tfrac{1}{12}bh^3 + Ad^2\right)$$

$$= \tfrac{1}{12}(125)(12)^3 + (125)(12)(30.45)^2$$

$$+ \tfrac{1}{12}(12)(75)^3 + (12)(75)(13.05)^2$$

$$+ \tfrac{1}{12}(50)(12)^3 + (50)(12)(56.55)^2$$

$$= 3909900 \text{ mm}^4 = 3.91\times10^{-6} \text{ m}^4$$



ASSUME: $\sigma_A = +\sigma_{ALL} = +42 \text{ MPa}$

$$M = \sigma_A \frac{I_{x'}}{c} = (42\times10^6)\frac{3.91\times10^{-6}}{0.03645} = 4505 \text{ Nm}$$

ASSUME: $\sigma_B = -\sigma_{ALL} = -6$

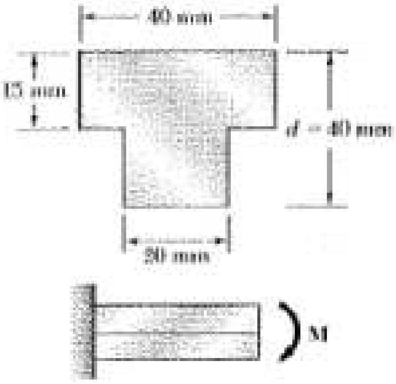
$$M = \sigma_B \frac{I_{x'}}{c} = (105\times10^6)\frac{3.91\times10}{0.06255} = 6563.5 \text{ Nm}$$

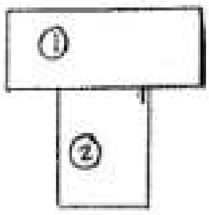
CHOOSE THE SMALLER M: $M_{ALL} = 4.5 \text{ kNm}$ ◀

Problem 4.17

4.17 Solve Prob. 4.16, assuming that $d = 40$ mm.

4.16 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.





	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³
①	600	32.5	19.5×10^3
②	500	12.5	6.25×10^3
Σ	1100		25.75×10^3

$$\bar{Y}_o = \frac{25.75 \times 10^3}{1100} = 23.41 \text{ mm}$$

The neutral axis lies 23.41 mm above the bottom.

$y_{top} = 40 - 23.41 = 16.59 \text{ mm} = 0.01659 \text{ m}$

$y_{bot} = -23.41 \text{ mm} = -0.02341 \text{ m}$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(9.09)^2 = 60.827 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(20)(25)^3 + (500)(10.91)^2 = 85.556 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 146.383 \times 10^3 \text{ mm}^4 = 146.383 \times 10^{-9} \text{ m}^4$$

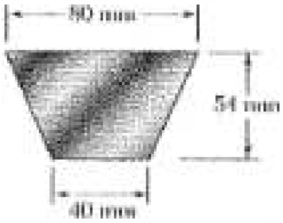
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

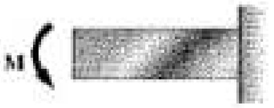
Top: tension side. $M = \dfrac{(24 \times 10^6)(146.383 \times 10^{-9})}{0.01659} = 212 \text{ N}\cdot\text{m}$

Bottom: compression. $M = \dfrac{(30 \times 10^6)(146.383 \times 10^{-9})}{0.02341} = 187.6 \text{ N}\cdot\text{m}$

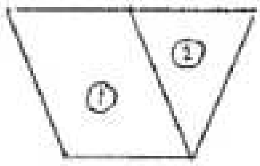
Choose smaller value. $M = 187.6 \text{ N}\cdot\text{m}$ ◄

Problem 4.18





4.18 and 4.19 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.



	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³	d, mm
①	2160	27	58320	3
②	1080	36	38880	3
Σ	3240		97200	

$$\bar{Y} = \frac{97200}{3240} = 30 \text{ mm}$$

The neutral axis lies 30 mm above the bottom.

$$y_{top} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m}$$
$$y_{bot} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \tfrac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \tfrac{1}{12}(40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^3 \text{ mm}^4$$

$$I_2 = \tfrac{1}{36} b_2 h_2^3 + A_2 d_2^2 = \tfrac{1}{36}(40)(54)^3 + \tfrac{1}{2}(40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

top: tension side $\quad M = \dfrac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

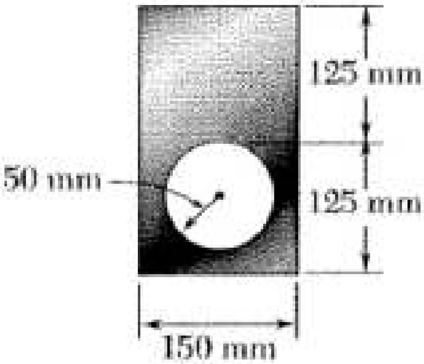
bottom: compression $\quad M = \dfrac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

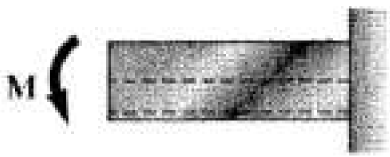
Choose the smaller as M_{all}. $\quad M_{all} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

$$M_{all} = 3.79 \text{ kN}\cdot\text{m}$$

Problem 4.19

4.18 and *4.19* Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.





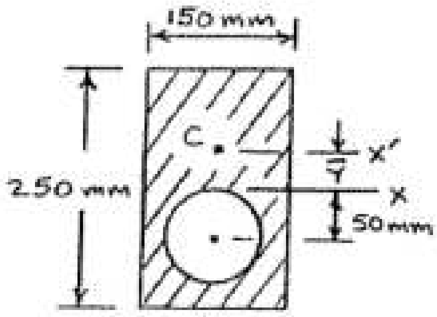
① rectangle ② circular cutout

$$A_1 = (150)(250) = 37.5\times10^3 \ mm^2$$
$$A_2 = -\pi(50)^2 = 7.85398\times10^3 \ mm^2$$
$$A = A_1 - A_2 = 29.64602\times10^3 \ mm^2$$

$\bar{y}_1 = 0 \ mm$

$\bar{y}_2 = -50 \ mm$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$



$$\bar{Y} = \frac{(37.5\times10^3)(0) + (-7.85393\times10^3(-50)}{29.64602\times10^3}$$
$$= 13.2463 \ mm$$

$$I_{x'} = \Sigma(I + Ad^2) = I_1 - I_2$$
$$= \left[\tfrac{1}{12}(150)(250)^3 + (37.5\times10^3)(13.2463)^2\right]$$
$$- \left[\tfrac{\pi}{4}(50)^4 + (7.85398\times10^3)(50 + 13.2463)^2\right]$$
$$= 201.892\times10^6 - 36.3254\times10^6 = 165.567\times10^6 \ mm^4 = 165.567\times10^{-6} \ m^4$$

Top: (Tension side) $c = 125 - 13.2463 = 111.7537 \ mm = 0.11175 \ m$

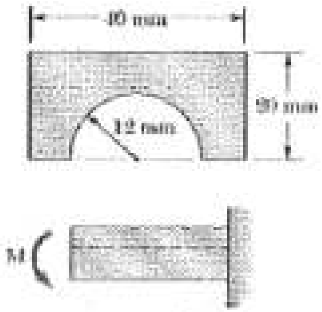
$$\sigma = \frac{Mc}{I} \qquad M = \frac{I\sigma}{c} = \frac{(165.567\times10^{-6})(120\times10^6)}{0.11175}$$
$$= 177.79\times10^3 \ N\cdot m$$

Bottom: (Compression side) $c = 125 + 13.2463 = 138.2463 \ mm$
$= 0.13825 \ m$

$$\sigma = \frac{Mc}{I} \qquad M = \frac{I\sigma}{c} = \frac{(165.567\times10^{-6})(150\times10^6)}{0.13825}$$
$$= 179.64\times10^3 \ N\cdot m$$

Choose the smaller. $M = 177.8\times10^3 \ N\cdot m$ $\quad M = 177.8 \ kN\cdot m$

Problem 4.20



4.20 Knowing that for the beam shown the allowable stress is 84 MPa in tension and 110 MPa in compression, determine the largest couple **M** that can be applied.

① = rectangle ② = semi-circular cutout

$A_1 = (40)(20) = 800 \text{ mm}^2$

$A_2 = \frac{\pi}{2}(12)^2 = 226.2 \text{ mm}^2$

$A = 800 - 226.2 = 573.8 \text{ mm}^2$

$\bar{y}_1 = 10 \text{ mm}$ $\bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(12)}{3\pi} = 5.1 \text{ mm}$

$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(800)(10) - (226.2)(5.1)}{573.8} = 11.9 \text{ mm}$

Neutral axis lies 11.9 mm above the bottom

Moment of inertia about the base

$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(40)(20)^3 - \frac{\pi}{8}(12)^4 = 104075 \text{ mm}^4$

Centroidal moment of inertia

$\bar{I} = I_b - A\bar{Y}^2 = 104075 - (573.8)(11.9)^2$

$= 22819 \text{ mm}^4$

$y_{top} = 20 - 11.9 = 8.1 \text{ mm}$, $y_{bot} = -11.9 \text{ mm}$

$|\sigma| = \left|\frac{My}{I}\right|$ $M = \left|\frac{\sigma I}{y}\right|$

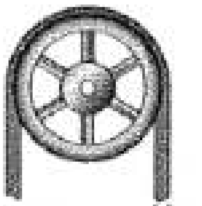
Top: tension side $M = \frac{(84\times10^6)(22819\times10^{-12})}{0.0081} = 236.6 \text{ Nm}$

Bottom: compression $M = \frac{(110\times10^6)(22819\times10^{-12})}{0.0119} = 210.9 \text{ Nm}$

Choose the smaller value $M = 210.9 \text{ Nm}$

Problem 4.21

4.21 A steel band blade, that was originally straight, passes over 200 mm diameter pulleys when mounted on a band saw. Determine the maximum stress in the blade, knowing that it is 0.5 mm thick and 16 mm wide. Use $E = 200$ GPa.



Band blade thickness: $t = 0.5\text{mm}$

Radius of pulley: $r = \frac{1}{2}d = 100\text{mm}$

Radius of curvature of center line of blade:

$$\rho \doteq r + \frac{1}{2}t = 100.25\text{ mm}$$

$$c = \frac{1}{2}t = 0.25\text{ mm}$$

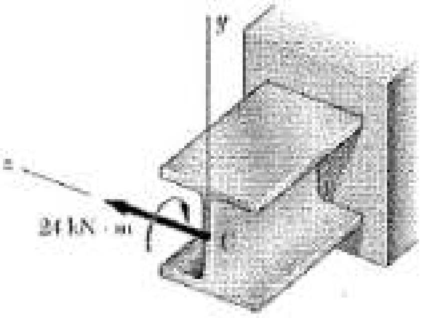
Maximum strain: $\varepsilon_m = \dfrac{c}{\rho} = \dfrac{0.25}{100.25} = 0.002494$

Maximum stress: $\sigma_m = E\varepsilon_m = (200\times10^9)(0.002494)$

$\sigma_m = 498.8$ MPa $\qquad \sigma_m = 498.8$ MPa

Problem 4.24

4.24 A 24 kN · m couple is applied to the W200 × 46.1 rolled-steel beam shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the beam. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.



For W 200 × 46.1 rolled steel section

$I_x = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4$

$S_x = 448 \times 10^3 \text{ mm}^3 = 448 \times 10^{-6} \text{ m}^3$

$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$

$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$

(a) $M_z = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{448 \times 10^{-6}} = 53.6 \times 10^6 \text{ Pa} = 53.6 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(45.5 \times 10^{-6})} = 2.637 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 379 \text{ m}$$

(b) $M_y = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$

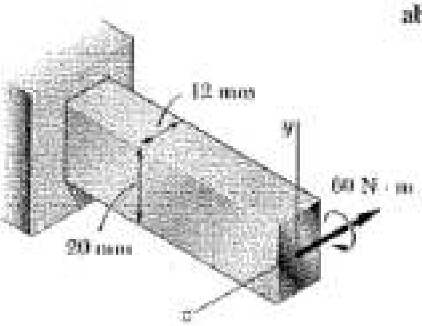
$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{151 \times 10^{-6}} = 158.9 \times 10^6 \text{ Pa} = 158.9 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(15.3 \times 10^{-6})} = 7.84 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 127.5 \text{ m}$$

Problem 4.25

4.25 A 60 N · m couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.



(a) Bending about z-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(20)^3 = 8 \times 10^3 \text{ mm}^4 = 8 \times 10^{-9} \text{ m}^4$$

$$c = \frac{20}{2} = 10 \text{ mm} = 0.010 \text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.010)}{8 \times 10^{-9}} = 75.0 \times 10^6 \text{ Pa} \qquad \sigma = 75.0 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200 \times 10^9)(8 \times 10^{-9})} = 37.5 \times 10^{-3} \text{ m}^{-1} \qquad \rho = 26.7 \text{ m}$$

(b) Bending about y-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(20)(12)^3 = 2.88 \times 10^3 \text{ mm}^4 = 2.88 \times 10^{-9} \text{ m}^4$$

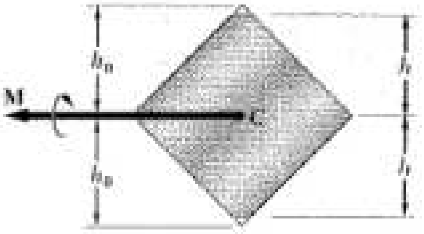
$$c = \frac{12}{2} = 6 \text{ mm} = 0.006 \text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.006)}{2.88 \times 10^{-9}} = 125.0 \times 10^6 \text{ Pa} \qquad \sigma = 125.0 \text{ MPa}$$

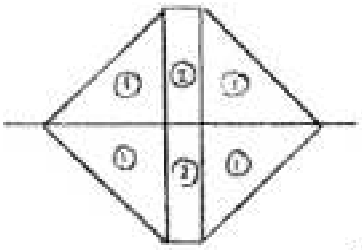
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200 \times 10^9)(2.88 \times 10^{-9})} = 104.17 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 9.60 \text{ m}$$

Problem 4.27



4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.



$$I = 4I_1 + 2I_2$$

$$= (4)(\tfrac{1}{12})h\,h^3 + (2)(\tfrac{1}{3})(2h_0 - 2h)(h^3)$$

$$= \tfrac{1}{3}h^4 + \tfrac{4}{3}h_0h^3 - \tfrac{4}{3}h\,h^3 = \tfrac{4}{3}h_0h^3 - h^4$$

$$c = h$$

$$\sigma_1 = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^2}$$

For the original square, $h = h_0$, $c = h_0$

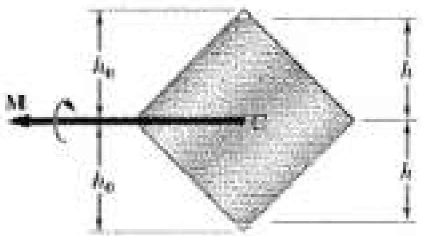
$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9h_0^2)} = 0.950$$

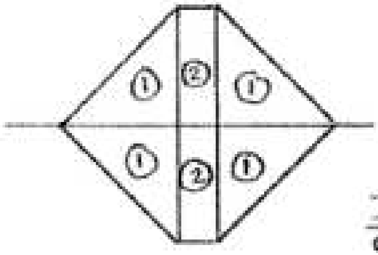
$\sigma = 0.950\,\sigma_0$ $\qquad\qquad k = 0.950$

Problem 4.28

4.28 In Prob. 4.27, determine (*a*) the value of h for which the maximum stress σ_m is as small as possible, (*b*) the corresponding value of k.



4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.



$I = 4I_1 + 2I_2$

$= (4)(\frac{1}{12})hh^3 + (2)(\frac{1}{3})(2h_0 - 2h)h^3$

$= \frac{1}{3}h^4 - \frac{4}{3}h_0h^3 - \frac{4}{3}h^3 \quad = \frac{4}{3}h_0h^3 - h^4$

$c = h \qquad \frac{I}{c} = \frac{4}{3}h_0h^2 - h^3$

$\frac{I}{c}$ is maximum at $\frac{d}{dh}\left[\frac{4}{3}h_0h^2 - h^3\right] = 0$

$\frac{8}{3}h_0h - 3h^2 = 0 \qquad h = \frac{8}{9}h_0$

$\frac{I}{c} = \frac{4}{3}h_0(\frac{8}{9}h_0)^2 - (\frac{8}{9}h_0)^3 = \frac{256}{729}h_0^3 \qquad \sigma = \frac{Mc}{I} = \frac{729}{256}\frac{M}{h_0^3}$

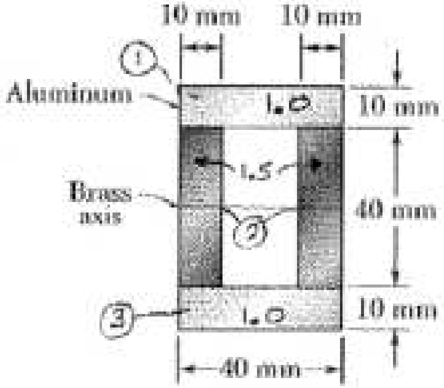
For the original square, $h = h_0 \quad c = h_0 \quad \frac{I_0}{c_0} = \frac{1}{3}h_0^3$

$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^3}$

$\frac{\sigma}{\sigma_0} = \frac{729}{256}\cdot\frac{1}{3} = \frac{729}{768} = 0.949 \qquad k = 0.949$

Problem 4.33

4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.



	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section

$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(40)(10)^3 + (1.0)(40)(10)(25)^2 = 253.333 \times 10^3 \text{ mm}^4$

$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(20)(40)^3 = 160 \times 10^3 \text{ mm}^4$

$I_3 = I_1 = 253.333 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 666.67 \times 10^3 \text{ mm}^4 = 666.67 \times 10^{-9} \text{ m}^4$

$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$

Aluminum: $n = 1.0$, $|y| = 30 \text{ mm} = 0.030 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

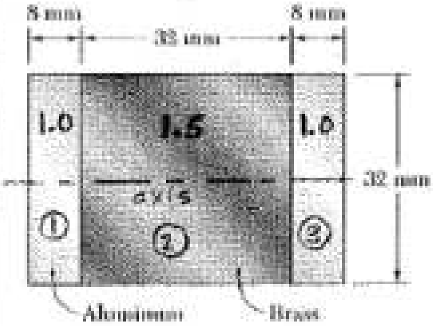
$$M = \frac{(100 \times 10^6)(666.67 \times 10^{-9})}{(1.0)(0.030)} = 2.2222 \times 10^3 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$ $|y| = 20 \text{ mm} = 0.020 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(666.67 \times 10^{-9})}{(1.5)(0.020)} = 3.5556 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 2.22 \times 10^3$ N·m $\qquad M = 2.22$ kN·m ◄

Problem 4.34



4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 174.7626 \times 10^3 \text{ mm}^4 = 174.7626 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

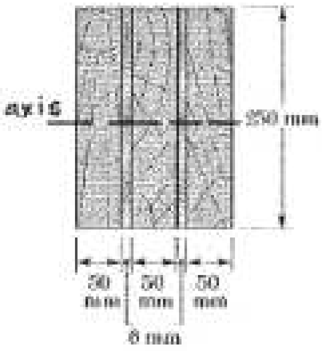
$$M = \frac{(100 \times 10^6)(174.7626 \times 10^{-9})}{(1.0)(0.016)} = 1.0923 \times 10^3 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(174.7626 \times 10^{-9})}{(1.5)(0.016)} = 1.1651 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.092 \times 10^3$ N·m $\qquad M = 1.092$ kN·m

Problem 4.37



4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity	14 GPa	200 GPa
Allowable stress	14 MPa	150 MPa

Use wood as the reference material

For wood $n_w = 1$

For steel $n = E_s/E_w = 200/14 = 14.29$

Properties of the geometric section.

Steel: $I_s = \frac{1}{12}(6+6)(250)^3 = 15.625 \times 10^6 \text{ mm}^4$

Wood: $I_w = \frac{1}{12}(50+50+50)(250^3) = 195.3125 \times 10^6 \text{ mm}^4$

Transformed section

$$I_{trans} = n_s I_s + n_w I_w = (14.29)(15.625\times10^6) + (1)(195.3125\times10^6)$$
$$= 418.6 \times 10^6 \text{ mm}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Wood: $n = 1$ $\quad |y| = 125 \text{ mm}$ $\quad \sigma = 14 \text{ MPa}$

$$M = \frac{(14\times10^6)(418.6\times10^{-6})}{(1)(0.125)} = 46.9\times10^3 \text{ Nm}$$

Steel: $n = 14.29$ $\quad |y| = 125 \text{ mm}$ $\quad \sigma = 150 \text{ MPa}$

$$M = \frac{(150\times10^6)(418.6\times10^{-6})}{(14.29)(0.125)} = 35.2\times10^3 \text{ Nm}$$

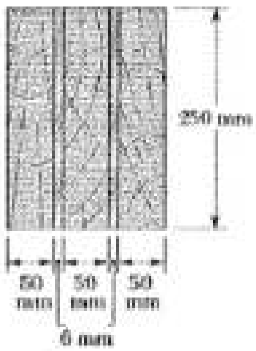
Choose the smaller value $\quad M = 35.2\times10^3 \text{ Nm}$

$M = 35.2 \text{ kN.m}$ ◄

Problem 4.38

4.38 For the composite member of Prob. 4.37, determine the largest permissible bending moment when the member is bent about a vertical axis.

4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.



	Wood	Steel
Modulus of elasticity	14 GPa	200 GPa
Allowable stress	14 MPa	150 MPa

Use wood as the reference material

For wood $n_w = 1$

For steel $n_s = E_s/E_w = 200/14 = 14.29$.

Properties of the geometric section

Total: $I_t = \frac{1}{12} hb^3 = \frac{1}{12}(250)(162)^3 = 88573500 \text{ mm}^4$

Steel: $I_s = \frac{1}{12} h(B^3 - b^3) = \frac{1}{12}(250)(62^3 - 50^3) = 2361000 \text{ mm}^4$

Wood: $I_w = I_t - I_s = 86212500 \text{ mm}^4$

Transformed section

$I_{trans} = n_s I_s + n_w I_w = (14.29)(2361000) + (1)(86212500) = 120 \times 10^6 \text{ mm}^4$

$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$

Wood: $n = 1 \qquad |y| = 81 \text{ mm} \qquad \sigma = 14 \text{ MPa}$

$M = \frac{(14 \times 10^6)(120 \times 10^{-6})}{(1)(0.081)} = 20.74 \times 10^3 \text{ N.m}$

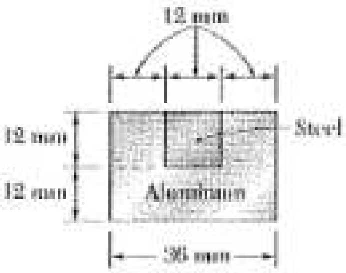
Steel: $n = 14.29 \qquad |y| = 31 \text{ mm} \qquad \sigma = 150 \text{ MPa}$

$M = \frac{(150 \times 10^6)(120 \times 10^{-6})}{(14.29)(0.031)} = 40.6 \times 10^3 \text{ N.m}$

Choose the smaller value. $M = 20.74 \text{ kNm}$

$M = 20.74 \text{ kNm}$ ◄

Problem 4.40



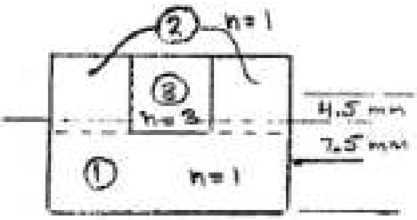
4.39 and 4.40 A steel bar (E_s = 210 GPa) and an aluminum bar (E_a = 70 GPa) are bonded together to form the composite bar shown. Determine the maximum stress in (*a*) the aluminum, (b) the steel, when the bar is bent about a horizontal axis, with M = 200 N · m.

Use aluminum as the reference material.

For aluminum, $n = 1$

For steel, $n = E_s/E_a = 210/70 = 3$

Transformed section.



	A, mm²	nA, mm²	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm³
1	432	432	6	2592
2	288	288	18	5184
3	144	432	18	7776
		1152		15552

$\bar{Y}_o = \frac{15552}{1152} = 13.5$ mm The neutral axis lies 13.5 mm above the bottom.

$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2 = \frac{1}{12}(36)(12)^3 + (432)(7.5)^2 = 29.484 \times 10^3 \text{ mm}^4$

$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{1}{12}(24)(12)^3 + (288)(14.5)^2 = 9.288 \times 10^3 \text{ mm}^4$

$I_3 = \frac{n_3}{12}b_3h_3^3 + n_3A_3d_3^2 = \frac{3}{12}(12)(12)^3 + (432)(4.5)^2 = 13.932 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 52.704 \times 10^3 \text{ mm}^4 = 52.704 \times 10^{-9} \text{ m}^4$

$M = 60 \text{ N}\cdot\text{m}$

$\sigma = -\frac{nMy}{I}$

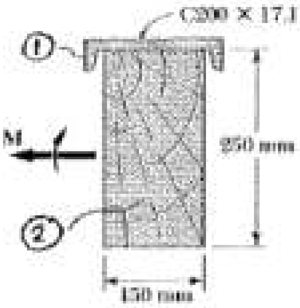
(a) Aluminum: $n = 1$, $y = -13.5 \text{ mm} = -0.0135 \text{ m}$

$\sigma_a = -\frac{(1)(200)(-0.0135)}{52.704 \times 10^{-9}} = 51.2 \times 10^6 \text{ Pa}$ $\sigma_a = 51.2 \text{ MPa}$ ◄

(b) Steel: $n = 3$, $y = 10.5 \text{ mm} = 0.0105 \text{ m}$

$\sigma_s = -\frac{(3)(200)(0.0105)}{52.704 \times 10^{-9}} = -119.5 \times 10^6 \text{ Pa}$ $\sigma_s = -119.5 \text{ MPa}$ ◄

Problem 4.42



4.41 and 4.42 The 150 × 250 mm timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 12 GPa and for steel 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 50$ kN · m, determine the maximum stress in (*a*) the wood, (*b*) the steel.

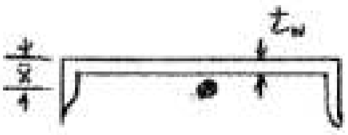
Use wood as the reference material.

For wood $\quad n = 1$

For steel $\quad n = E_s/E_w = 200/12 = 16.67$

For C8×11.5 channel section, $A = 2170 \text{ mm}^2$

$t_w = 5.6$ mm, $\bar{x} = 14.4$ mm, $I_y = 538000$ mm



For the composite section the centroid of the channel lies $250 + 5.6 - 14.4 = 241.2$ mm above the base. $\bar{y}_o = 241.2$ mm for channel.

Transformed section

$\bar{Y}_o = \dfrac{13412669}{73674}$

$= 182.1$ mm

The neutral axis lies 182.1 mm above the bottom.

	A, mm²	nA, mm²	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm³
①	2170	36174	241.2	8725169
②	37500	37500	125	4687500
		73674		13412669

$I_1 = n_1 \bar{I}_x + n_1 A_1 d_1^2 = (16.67)(538000) + (36174)(241.2 - 182.1)^2 = 135.32 \times 10^6 \text{ mm}^4$

$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(150)(250)^3 + (37500)(182.1 - 125)^2 = 317.58 \times 10^6 \text{ mm}^4$

$I = I_1 + I_2 = 452.9 \times 10^6 \text{ mm}^4.$

$M = 50$ kNm $\qquad \sigma = -\dfrac{nMy}{I}$

(a) Wood: $\quad n = 1, \quad y = -8.433$ in.

$$\sigma_w = -\frac{(50\times10^3)(-182.1\times10^{-3})}{452.9\times10^{-6}} = 20.1 \text{ MPa} \qquad \sigma_w = 20.1 \text{ MPa}$$

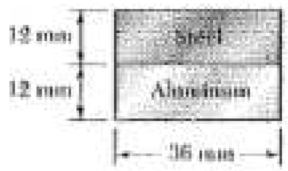
(b) Steel: $\quad n = 16.67 \qquad y = 250 + 5.6 - 182.1 = 73.5$ mm

$$\sigma_s = -\frac{(16.67)(50\times10^3)(73.5\times10^{-3})}{452.9\times10^{-6}} = -135.3 \text{ MPa} \qquad \sigma_s = -135.3 \text{ MPa}$$

Problem 4.43

4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 200 N · m.

4.43 Bar of Prob. 4.39



See solution to Problem 4.39 for calculation of I.

$I = 67.392 \times 10^{-9}\ m^4 \qquad E_a = 70 \times 10^9\ Pa$

$M = 200\ N \cdot m$

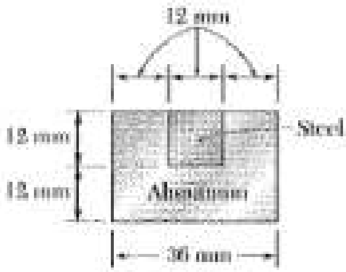
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{200}{(70 \times 10^9)(67.392 \times 10^{-9})} = 42.396 \times 10^{-3}\ m^{-1}$$

$\rho = 23.6\ m$ ◄

Problem 4.44

4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 200 N · m.

4.44 Bar of Prob. 4.40



See solution to Problem 4.40 for calculation of I.

$I = 52.704 \times 10^{-9}\ m^4 \qquad E_a = 70 \times 10^6\ Pa$

$M = 200\ N \cdot m$

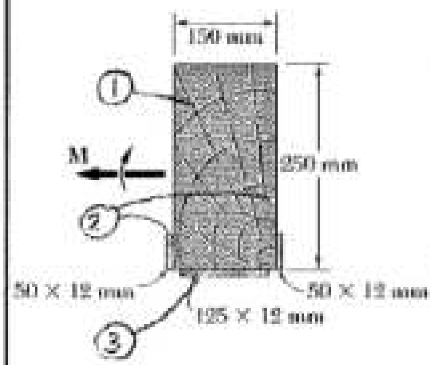
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{200}{(70 \times 10^9)(52.704 \times 10^{-9})} = 54.211\ m^{-1}$$

$\rho = 18.45 m$ ◄

Problem 4.45

4.45 and 4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 50 kN · m.

4.45 Beam of Prob. 4.41



See solution to Problem 4.41 for calculation of I.

$I = 491.53 \times 10^{-6}\ m^4 \qquad E_w = 12 \times 10^9\ Pa$

$M = 50 \times 10^3\ Nm$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{50 \times 10^3}{(12 \times 10^9)(491.53 \times 10^{-6})} = 8.4769 \times 10^{-3}\ m^{-1}$$

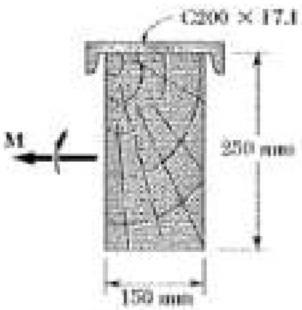
$\rho = 117.96 m = 118\ m$ ◄

Problem 4.46

4.45 and 4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 50 kN · m.

4.45 Bar of Prob. 4.41.

4.46 Bar of Prob. 4.42.



See solution to Problem 4.42 for calculation of I.

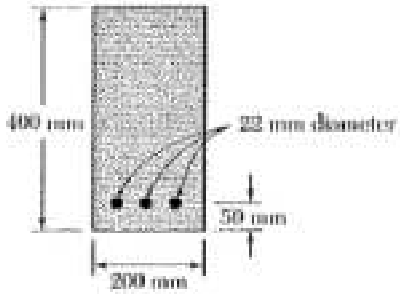
$I = 452.9 \times 10^6\ mm^4 \qquad E_w = 12\ GPa$

$M = 50\ kN/m$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{50 \times 10^3}{(12 \times 10^9)(452.9 \times 10^{-6})} = 0.0092\ m^{-1}$$

$\rho = 108.7 m$ ◄

Problem 4.47

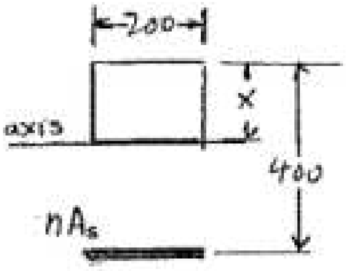


4.47 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress of 9.45 MPa for the concrete and 140 MPa for the steel, determine the largest allowable positive bending moment in the beam.

$$n = \frac{E_s}{E_c} = \frac{200\times10^9}{20\times10^9} = 10$$

$$A_s = 3\,\frac{\pi}{4}d^2 = 3\left(\frac{\pi}{4}\right)(22)^2 = 1140.4\ \text{mm}^2$$

$$nA_s = 11404\ \text{mm}^2$$



Locate neutral axis.

$$200x\,\frac{x}{2} - (11404)(400-x) = 0$$

$$100x^2 + 11404x - 4561600 = 0$$

Solve for x $\quad x = \dfrac{-11404 + \left[11404^2 + (4)(100)(4561600)\right]^{1/2}}{2(100)} = 164\ \text{mm}$

$$400 - x = 236\ \text{mm}$$

$$I = \frac{1}{3}200x^3 + nA_s(400-x)^2 = \frac{1}{3}(200)(164)^3 + (11404)(236)^2$$

$$= 296.75\times10^6\ \text{mm}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $\quad n = 1.0, \quad |y| = 164\text{mm}, \quad |\sigma| = 9.45\text{MPa}$

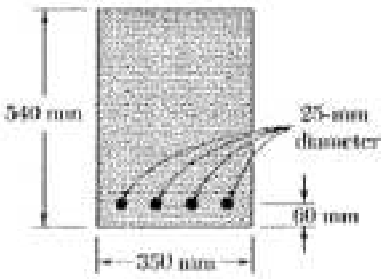
$$M = \frac{(9.45\times10^6)(296.75\times10^{-6})}{(1.0)(0.164)} = 17.1\ \text{kNm}$$

Steel: $\quad n = 10, \quad |y| = 236\text{mm}, \quad \sigma = 140\text{MPa}$

$$M = \frac{(140000\times10^3)(296.75\times10^{-6})}{(10)(0.236)} = 17.6\ \text{kNm}$$

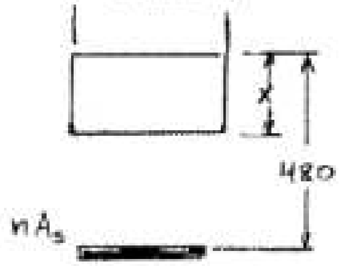
Choose the smaller value $\quad M = 17.1\ \text{kNm}$

Problem 4.49



4.49 Solve Prob. 4.48, assuming that the 300-mm width is increased to 350 mm.

4.48 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.



$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4\,\frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$nA_s = 15.708 \times 10^3 \text{ mm}^2$$

Locate the neutral axis

$$350\, x\, \frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$175 x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x. $\quad x = \dfrac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(175)(7.5398 \times 10^6)}}{(2)(175)}$

$$x = 167.48 \text{ mm}, \qquad 480 - x = 312.52 \text{ mm}$$

$$I = \tfrac{1}{3}(350)x^3 + (15.708 \times 10^3)(480 - x)^2$$

$$= \tfrac{1}{3}(350)(167.48)^3 + (15.708 \times 10^3)(312.52)^2$$

$$= 2.0823 \times 10^9 \text{ mm}^4 = 2.0823 \times 10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

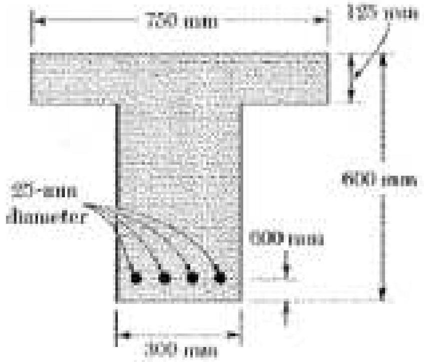
(a) Steel: $\quad y = -312.52 \text{ mm} = -0.31252 \text{ m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.31252)}{2.0823 \times 10^{-3}} = 210 \times 10^6 \text{ Pa} = 210 \text{ MPa} \blacktriangleleft$$

(b) Concrete: $\quad y = 167.48 \text{ mm} = 0.16748 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.16748)}{2.0823 \times 10^{-3}} = -14.08 \times 10^6 \text{ Pa} = -14.08 \text{ MPa} \blacktriangleleft$$

Problem 4.51

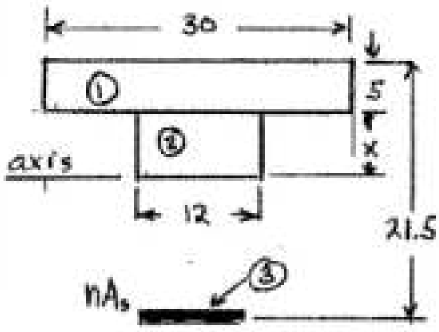


4.51 Knowing that the bending moment in the reinforced concrete beam is +200 kN · m and that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

$$n = \frac{E_s}{E_c} = \frac{200\times10^9}{25\times10^9} = 8$$

$$A_s = 4\frac{\pi}{4}d^2 = 4\left(\frac{\pi}{4}\right)(25)^2 = 1963.5 \text{ mm}^2$$

$$nA_s = 15708 \text{ mm}^2$$



Locate the neutral axis

$$(750)(125)(x+62.5) + 300x\,x/2 - (15708)(415-x) = 0$$

$$93750x + 5859375 + 150x^2 - 6518820 + 15708x = 0$$

$$150x^2 + 109458x - 659445 = 0$$

Solve for x
$$x = \frac{-109458 + \left[(109458)^2 + (4)(150)(659445)\right]^{1/2}}{2(150)} = 6 \text{ mm}$$

$$415 - x = 409 \text{ mm}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(750)(125)^3 + (750)(125)(68.125)^2 = 557.166\times10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{3}b_2x^3 = \frac{1}{3}(12)(6)^3 = 864 \text{ mm}^4$$

$$I_3 = nA_sd_3^2 = (15708)(409)^2 = 2627.65\times10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 3.1848\times10^9 \text{ mm}^4$$

$$\sigma = -\frac{nMy}{I} \quad \text{where} \quad M = 200 \text{ kNm}$$

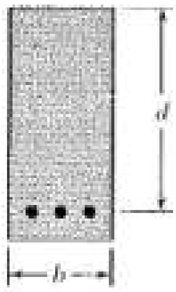
(a) Steel $n = 8.0$, $y = -409$ mm

$$\sigma = -\frac{(8.0)(200\times10^3)(409\times10^{-3})}{3.1848\times10^{-3}} = 205.5 \text{ MPa}$$ ◄

(b) Concrete $n = 1.0$, $y = 131$ mm

$$\sigma = -\frac{(1.0)(200\times10^3)(0.131)}{3.1848\times10^{-3}} = -8.2 \text{ MPa}$$ ◄

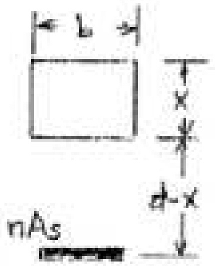
Problem 4.52



4.52 The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.



$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

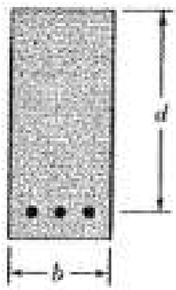
$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

$$x = \frac{d}{1 + \dfrac{E_c \sigma_s}{E_s \sigma_c}}$$

Problem 4.54

4.54 For the concrete beam shown, the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel. Knowing that $b = 200$ mm and $d = 450$ mm and using an allowable stress of 12.5 MPa for the concrete and 140 MPa for the steel, determine (*a*) the required area A_s of the steel reinforcement if the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.52 for definition of a balanced beam.)



$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{25 \times 10^9} = 8.0$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.0}\cdot\frac{140 \times 10^6}{12.5 \times 10^6} = 2.40$$

$$x = 0.41667\,d = (0.41667)(450) = 187.5 \text{ mm}$$

Locate neutral axis

$$bx\frac{x}{2} - nA_s(d-x)$$

(a) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(200)(187.5)^2}{(2)(8.0)(262.5)} = 1674 \text{ mm}^2$$ (a) $A_s = 1674 \text{ mm}^2$ ◄

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(200)(187.5)^3 + (8.0)(1674)(262.5)^2$$

$$= 1.3623 \times 10^9 \text{ mm}^4 = 1.3623 \times 10^{-3} \text{ m}^4$$

$$\sigma = \frac{nMy}{I} \qquad M = \frac{I\sigma}{ny}$$

Concrete: $n = 1.0$ $\quad y = 187.5 \text{ mm} = 0.1875 \text{ m}$ $\quad \sigma = 12.5 \times 10^6$ Pa

$$M = \frac{(1.3623 \times 10^{-3})(12.5 \times 10^6)}{(1.0)(0.1875)} = 90.8 \times 10^3 \text{ N·m}$$

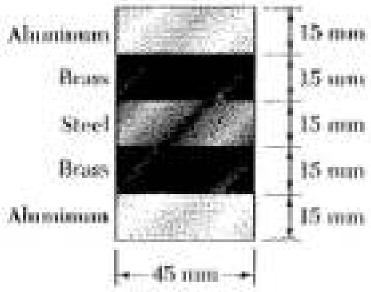
Steel: $n = 8.0$ $\quad y = 262.5 \text{ mm} = 0.2625 \text{ m}$ $\quad \sigma = 140 \times 10^6$ Pa

$$M = \frac{(1.3623 \times 10^{-3})(140 \times 10^6)}{(8.0)(0.2625)} = 90.8 \times 10^3 \text{ N·m}$$

Note that both values are the same for balanced design.

(b) $M = 90.8$ kN·m ◄

Problem 4.55



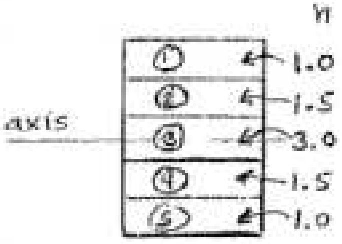
4.55 and 4.56 Five metal strips, each of 15 × 45-mm cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment 1400 N · m, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

Use aluminum as the reference material.

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel.

$n = \frac{E_b}{E_a} = \frac{105}{70} = 1.5$ in brass.

$n = 1.0$ in aluminum.



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.0}{12}(45)(15)^3 + (1.0)(45)(15)(30)^2 = 620.16 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(45)(15)^3 + (1.5)(45)(15)(15)^2 = 246.80 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3}{12}(45)(15)^3 = 37.97 \times 10^3 \text{ mm}^4$$

$I_4 = I_2 = 246.80 \times 10^3 \text{ mm}^4$ $\qquad$ $I_5 = I_1 = 620.16 \times 10^3 \text{ mm}^4$

$$I = \sum_1^5 I_i = 1.77189 \times 10^6 \text{ mm}^4 = 1.77189 \times 10^{-6} \text{ m}^4$$

(a) Aluminum: $y = 37.5 \text{ mm} = 0.0375 \text{ m}$ $\qquad$ $n = 1.0$

$$\sigma = \frac{nMy}{I} = \frac{(1.0)(1400)(0.0375)}{1.77189 \times 10^{-6}} = 29.6 \times 10^6 \text{ Pa} \qquad \sigma_a = 29.6 \text{ MPa}$$

Brass: $y = 22.5 \text{ mm} = 0.0225 \text{ m}$ $\qquad$ $n = 1.5$

$$\sigma = \frac{nMy}{I} = \frac{(1.5)(1400)(0.0225)}{1.77189 \times 10^{-6}} = 26.7 \times 10^6 \text{ Pa} \qquad \sigma_b = 26.7 \text{ MPa}$$

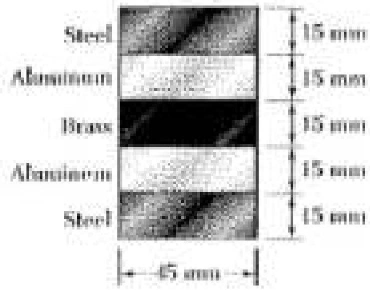
Steel: $y = 7.5 \text{ mm} = 0.0075 \text{ m}$ $\qquad$ $n = 3.0$

$$\sigma = \frac{nMy}{I} = \frac{(3.0)(1400)(0.0075)}{1.77189 \times 10^{-6}} = 17.78 \times 10^6 \text{ Pa} \qquad \sigma_s = 17.78 \text{ MPa}$$

(b) $$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{1400}{(70 \times 10^9)(1.77189 \times 10^{-6})} = 11.287 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 88.6 \text{ m}$$

Problem 4.56



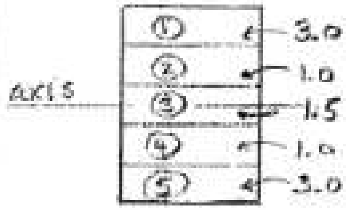
4.55 and 4.56 Five metal strips, each of 15 x 45-mm cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment 1400 N · m, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

Use aluminum as the reference material.

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{210}{105} = 1.5 \text{ in brass.}$$

$$n = 1.0 \text{ in aluminum}$$



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{3.0}{12}(45)(15)^3 + (3.0)(45)(15)(30)^2 = 1.860469 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12}(45)(15)^3 + (1.0)(45)(15)(15)^2 = 164.53 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12}(45)(15)^3 = 18.98 \times 10^3 \text{ mm}^4$$

$$I_4 = I_2 = 164.53 \times 10^3 \text{ mm}^4 \qquad I_5 = I_1 = 1.860469 \times 10^6 \text{ mm}^4$$

$$I = \sum_1^5 I_i = 4.0690 \times 10^6 \text{ mm}^4 = 4.0690 \times 10^{-6} \text{ m}^4$$

(a) Steel: $y = 37.5 \text{ mm} = 0.0375 \text{ m} \qquad n = 3.0$

$$\sigma = \frac{nMy}{I} = \frac{(3.0)(1400)(0.0375)}{4.0690 \times 10^{-6}} = 38.7 \times 10^6 \text{ Pa} \qquad \sigma_s = 38.7 \text{ MPa}$$

Aluminum: $y = 22.5 \text{ mm} = 0.0225 \text{ m} \qquad n = 1.0$

$$\sigma = \frac{nMy}{I} = \frac{(1.0)(1400)(0.0225)}{4.0690 \times 10^{-6}} = 7.74 \times 10^6 \text{ Pa} \qquad \sigma_a = 7.74 \text{ MPa}$$

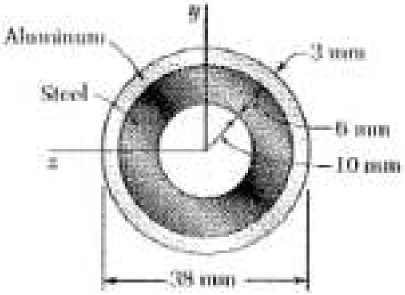
Brass: $y = 7.5 \text{ mm} = 0.0075 \text{ m} \qquad n = 1.5$

$$\sigma = \frac{nMy}{I} = \frac{(1.5)(1400)(0.0075)}{4.0690 \times 10^{-6}} = 3.87 \times 10^6 \text{ Pa} \qquad \sigma_b = 3.87 \text{ MPa}$$

(b) $$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{1400}{(70 \times 10^9)(4.0690 \times 10^{-6})} = 4.9152 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 203 \text{ m}$$

Problem 4.57



4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(16^4 - 10^4) = 130.85 \times 10^3 \text{ mm}^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 181.73 \times 10^3 \text{ mm}^4 = 181.73 \times 10^{-9} \text{ m}^4$

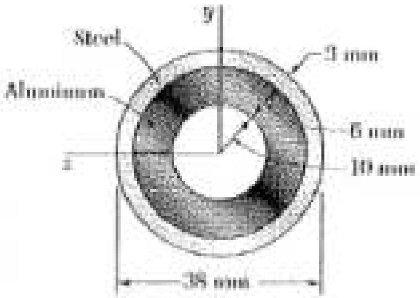
(a) Aluminum: $c = 19 \text{ mm} = 0.019 \text{ m}$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.019)}{181.73 \times 10^{-9}} = 52.3 \times 10^6 \text{ Pa} = 52.3 \text{ MPa}$$ ◄

(b) Steel: $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\frac{nMc}{I} = \frac{(3.0)(500)(0.016)}{181.73 \times 10^{-9}} = 132.1 \times 10^6 \text{ Pa} = 132.1 \text{ MPa}$$ ◄

Problem 4.58



4.58 Solve Prob. 4.57, assuming that the 6-mm-thick inner pipe is made of aluminum and that the 3-mm-thick outer pipe is made of steel.

4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel.

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(19^4 - 16^4) = 152.65 \times 10^3 \text{ mm}^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(16^4 - 10^4) = 43.62 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 196.27 \times 10^3 \text{ mm}^4 = 196.27 \times 10^{-9} \text{ m}^4$

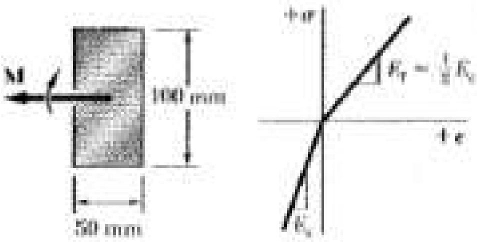
(a) Aluminum: $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.016)}{196.27 \times 10^{-9}} = 40.8 \times 10^6 \text{ Pa} = 40.8 \text{ MPa}$$

(b) Steel: $c = 19 \text{ mm} = 0.019 \text{ m}$

$$\sigma = \frac{nMc}{I} = \frac{(3.0)(500)(0.019)}{196.27 \times 10^{-9}} = 145.2 \times 10^6 \text{ Pa} = 145.2 \text{ MPa}$$

Problem 4.59

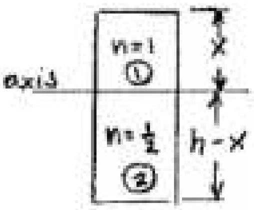


4.59 The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600\ \mathrm{N\cdot m}$, determine the maximum (*a*) tensile stress, (*b*) compressive stress.

$n = \frac{1}{2}$ on the tension side of neutral axis.

$n = 1$ on the compression side.

Locate neutral axis.



$$n_1 b x \frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \qquad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421\,h = 41.421\ \mathrm{mm}$$

$$h - x = 58.579\ \mathrm{mm}$$

$$I_1 = n_1 \tfrac{1}{3} b x^3 = (1)\left(\tfrac{1}{3}\right)(50)(41.421)^3 = 1.1844\times10^6\ \mathrm{mm}^4$$

$$I_2 = n_2 \tfrac{1}{3} b(h-x)^3 = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{3}\right)(50)(58.579)^3 = 1.6751\times10^6\ \mathrm{mm}^4$$

$$I = I_1 + I_2 = 2.8595\times10^6\ \mathrm{mm}^4 = 2.8595\times10^{-6}\ \mathrm{m}^4$$

(a) <u>tensile stress</u>: $n = \frac{1}{2}$, $y = -58.579\ \mathrm{mm} = -0.058579\ \mathrm{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times10^{-6}} = 6.15\times10^6\ \mathrm{Pa}$$

$$\sigma_T = 6.15\ \mathrm{MPa} \quad \blacktriangleleft$$

(b) <u>compressive stress</u>: $n = 1$, $y = 41.421\ \mathrm{mm} = 0.041421\ \mathrm{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times10^{-6}} = -8.69\times10^6\ \mathrm{Pa}$$

$$\sigma_C = -8.69\ \mathrm{MPa} \quad \blacktriangleleft$$

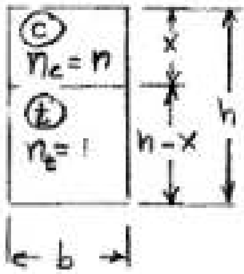
Problem 4.60

*4.60 A rectangular beam is made of material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{\left(\sqrt{E_t} + \sqrt{E_c}\right)^2}$$



Use E_t as the reference modulus.

Then $E_c = nE_t$

Locate neutral axis.

$$nbx\frac{x}{2} - b(h-x)\frac{h-x}{2} = 0$$

$$nx^2 - (h-x)^2 = 0 \qquad \sqrt{n}\,x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \qquad h-x = \frac{\sqrt{n}\,h}{\sqrt{n}+1}$$

$$I_{trans} = \frac{n}{3}bx^3 + \frac{1}{3}b(h-x)^3 = \left[\frac{n}{3}\left(\frac{1}{\sqrt{n}+1}\right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^3\right]bh^3$$

$$= \frac{1}{3}\frac{n + n^{3/2}}{(\sqrt{n}+1)^3}bh^3 = \frac{1}{3}\frac{n(1+\sqrt{n})}{(\sqrt{n}+1)^3}bh^3 = \frac{1}{3}\cdot\frac{n}{(\sqrt{n}+1)^2}bh^3$$

$$\frac{1}{\rho} = \frac{M}{E_t I_{trans}} = \frac{M}{E_r I} \qquad \text{where } I = \frac{1}{12}bh^3$$

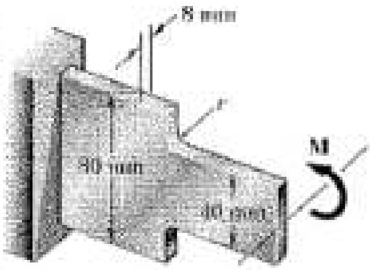
$$E_r I = E_t I_{trans}$$

$$E_r = \frac{E_t I_{trans}}{I} = \frac{12}{bh^3}\cdot E_t \cdot \frac{n}{3(\sqrt{n}+1)^2}bh^3$$

$$= \frac{4E_t E_c/E_t}{\left(\sqrt{E_c/E_t}+1\right)^2} = \frac{4E_t E_c}{\left(\sqrt{E_c}+\sqrt{E_t}\right)^2}$$

Problem 4.61

4.61 Knowing that the allowable stress for the beam shown is 90 MPa, determine (*a*) the allowable bending moment *M* when the radius *r* of the fillets is (*a*) 8 mm, (*b*) 12 mm.



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667\times10^3\ \text{mm}^4 = 42.667\times10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

(a) $\frac{r}{d} = \frac{8\ \text{mm}}{40\ \text{mm}} = 0.2$ From Fig. 4.31 $K = 1.50$

$$\sigma_{max} = K\frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma_{max} I}{Kc} = \frac{(90\times10^6)(42.667\times10^{-9})}{(1.50)(0.020)}$$

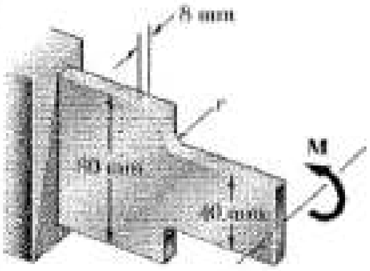
$$= 128\ \text{N}\cdot\text{m}$$ ◄

(b) $\frac{r}{d} = \frac{12\ \text{mm}}{40\ \text{mm}} = 0.3$ From Fig 4.31 $K = 1.35$

$$M = \frac{(90\times10^6)(42.667\times10^{-9})}{(1.35)(0.020)} = 142\ \text{N}\cdot\text{m}$$ ◄

Problem 4.62

4.62 Knowing that $M = 250$ N·m, determine the maximum stress in the beam shown when the radius *r* of the fillets is (*a*) 4 mm, (*b*) 8 mm.



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667\times10^3\ \text{mm}^4 = 42.667\times10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

(a) $\frac{r}{d} = \frac{4\ \text{mm}}{40\ \text{mm}} = 0.10$ From Fig. 4.31 $K = 1.87$

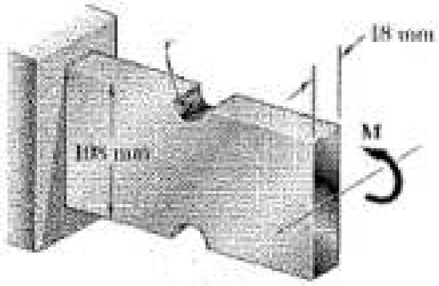
$$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667\times10^{-9}} = 219\times10^6\ \text{Pa} = 219\ \text{MPa}$$ ◄

(b) $\frac{r}{d} = \frac{8\ \text{mm}}{40\ \text{mm}} = 0.20$ From Fig. 4.31 $K = 1.50$

$$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667\times10^{-9}} = 176\times10^6\ \text{Pa} = 176\ \text{MPa}$$ ◄

Problem 4.63

4.63 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 450$ N · m, determine the maximum stress in the member when the radius r of the semicircular grooves is (a) $r = 9$ mm, (b) $r = 18$ mm.



(a) $d = D - 2r = 108 - (2)(9) = 90$ mm

$\frac{D}{d} = \frac{108}{90} = 1.20 \qquad \frac{r}{d} = \frac{9}{90} = 0.1$

From Fig. 4.32, $K = 2.07$

$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(90)^3 = 1.0935 \times 10^6 \text{ mm}^4 = 1.0935 \times 10^{-6} \text{ m}^4$

$c = \frac{1}{2} d = 45 \text{ mm} = 0.045 \text{ m}$

$\sigma_{max} = \frac{KMc}{I} = \frac{(2.07)(450)(0.045)}{1.0935 \times 10^{-6}} = 38.3 \times 10^6 \text{ Pa} \qquad \sigma_{max} = 38.3 \text{ MPa}$ ◄

(b) $d = D - 2r = 108 - (2)(18) = 72$ mm $\qquad \frac{D}{d} = \frac{108}{72} = 1.5 \qquad \frac{r}{d} = \frac{18}{72} = 0.25$

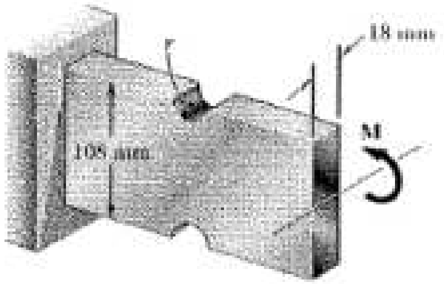
From Fig. 4.32, $K = 1.61 \qquad c = \frac{1}{2} d = 72 \text{ mm} = 0.036 \text{ m}$

$I = \frac{1}{12}(18)(72)^3 = 559.87 \times 10^3 \text{ mm}^4 = 559.87 \times 10^{-9} \text{ m}^4$

$\sigma_{max} = \frac{KMc}{I} = \frac{(1.61)(450)(0.036)}{559.87 \times 10^{-9}} = 46.6 \times 10^6 \text{ Pa} \qquad \sigma_{max} = 46.6 \text{ MPa}$ ◄

Problem 4.64

4.64 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 60 MPa, determine the largest bending moment that can be applied to the member when (a) $r = 9$ mm, (b) $r = 18$ mm.



(a) $d = D - 2r = 108 - (2)(9) = 90$ mm

$\frac{D}{d} = \frac{108}{90} = 1.20 \qquad \frac{r}{d} = \frac{9}{90} = 0.1$

From Fig. 4.32, $K = 2.07$

$I = \frac{1}{12}(18)(90)^3 = 1.0935 \times 10^6 \text{ mm}^4 = 1.0935 \times 10^{-6} \text{ m}^4$

$c = \frac{1}{2} d = 45 \text{ mm} = 0.045 \text{ m}$

$\sigma = \frac{KMc}{I} \qquad M = \frac{\sigma I}{Kc} = \frac{(60 \times 10^6)(1.0935 \times 10^{-6})}{(2.07)(0.045)} = 704 \qquad M = 704 \text{ N·m}$ ◄

(b) $d = 108 - (2)(18) = 72$ mm $\qquad \frac{D}{d} = \frac{108}{72} = 1.5 \qquad \frac{r}{d} = \frac{18}{72} = 0.25$

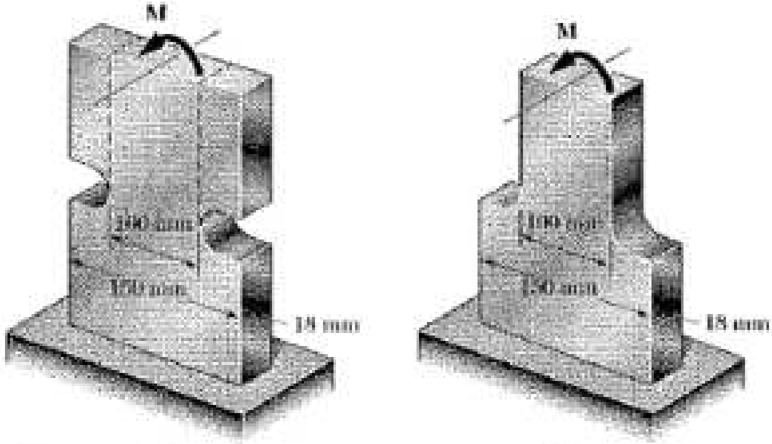
$c = \frac{1}{2} d = 36 \text{ mm} = 0.036 \text{ m} \qquad$ From Fig. 4.32, $K = 1.61$

$I = \frac{1}{12}(18)(72)^3 = 559.87 \times 10^3 \text{ mm}^4 = 559.87 \times 10^{-9} \text{ m}^4$

$M = \frac{\sigma I}{Kc} = \frac{(60 \times 10^6)(559.87 \times 10^{-9})}{(1.61)(0.036)} = 580 \qquad M = 580 \text{ N·m}$ ◄

Problem 4.65

4.65 A couple of moment $M = 2$ kN · m is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 10$ mm, as shown in Fig. 4.65*a*, (*b*) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65*b*.



(*a*) (*b*)

For both configurations

$D = 150$ mm; $d = 100$ mm

$r = 10$ mm

$\frac{D}{d} = \frac{150}{100} = 1.50$

$\frac{r}{d} = \frac{10}{100} = 0.10$

For configuration (a),
Fig 4.32 gives $K_a = 2.21$.

For configuration (b), Fig. 4.31 gives $K_b = 1.79$.

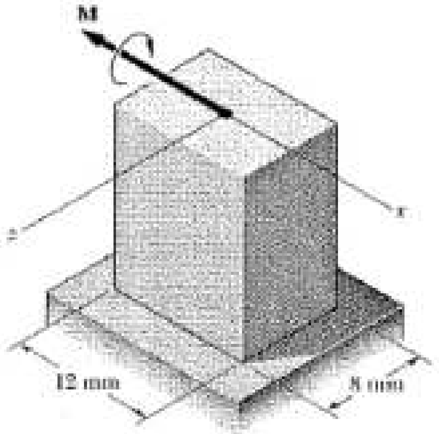
$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$

$c = \frac{1}{2}d = 50 \text{ mm} = 0.05 \text{ m}$

(a) $\sigma = \frac{KMc}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147 \times 10^6 \text{ Pa} = 147 \text{ MPa}$ ◄

(b) $\sigma = \frac{KMc}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119 \times 10^6 \text{ Pa} = 119 \text{ MPa}$ ◄

Problem 4.67



4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the x axis. Determine the moment M of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(8)^3 = 512 \text{ mm}^4$
$= 512 \times 10^{-12} \text{ m}^4$

$c = \frac{1}{2}h = 4 \text{ mm} = 0.004 \text{ m}$

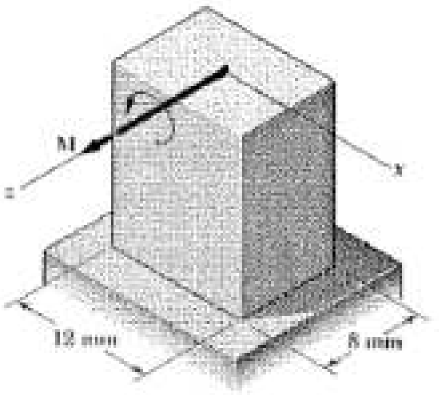
$M_Y = \dfrac{\sigma_Y I}{c} = \dfrac{(300\times10^6)(512\times10^{-12})}{0.004}$
$= 38.4 \text{ N}\cdot\text{m}$ $\qquad M_Y = 38.4 \text{ N}\cdot\text{m}$ ◄

(b) $y_Y = \frac{1}{2}(4) = 2 \text{ mm}$ $\qquad \dfrac{y_Y}{c} = \dfrac{2}{4} = 0.5$

$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(38.4)\left[1 - \frac{1}{3}(0.5)^2\right] = 52.8 \text{ N}\cdot\text{m}$

$M = 52.8 \text{ N}\cdot\text{m}$ ◄

Problem 4.68



4.68 Solve Prob. 4.67, assuming that the couple **M** is parallel to the z axis.

4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the x axis. Determine the moment M of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(12)^3 = 1.152\times10^3 \text{ mm}^4$
$= 1.152\times10^{-9} \text{ m}^4$

$c = \frac{1}{2}h = \quad \text{mm} = 0.006 \text{ m}$

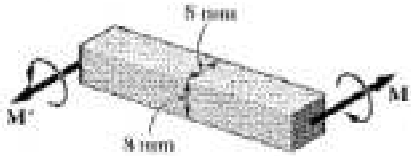
$M_Y = \dfrac{\sigma_Y I}{c} = \dfrac{(300\times10^6)(1.152\times10^{-9})}{0.006}$
$= 57.6 \text{ N}\cdot\text{m}$ $\qquad M_Y = 57.6 \text{ N}\cdot\text{m}$ ◄

(b) $y_Y = \frac{1}{2}(4) = 2 \text{ mm}$ $\qquad \dfrac{y_Y}{c} = \dfrac{2}{6} = \dfrac{1}{3}$

$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(57.6)\left[1 - \frac{1}{3}\left(\frac{1}{3}\right)^2\right] = 83.2 \text{ N}\cdot\text{m}$

$M = 83.2 \text{ N}\cdot\text{m}$ ◄

Problem 4.69



4.69 For the steel bar of Prob. 4.70, determine the thickness of the plastic zones at the top and bottom of the bar when (*a*) $M = 30$ N.m, (*b*) $M = 35$ N.m.

4.70 A bar having the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 330$ MPa. Determine the bending moment M at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 2 mm thick.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(8)^3 = 341.3 \text{ mm}^4$$

$$c = \frac{1}{2}h = 4 \text{ mm}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(330\times10^6)(341.3\times10^{-12})}{0.004} = 28.16 \text{ Nm}$$

$$M_y = 28.16 \text{ Nm}$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

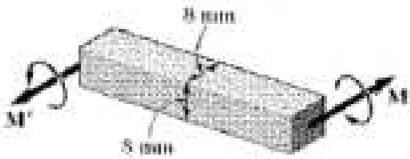
(a) $M = 30$ Nm $\qquad \frac{y_Y}{c} = \sqrt{3 - 2\left(\frac{30}{28.16}\right)} = 0.9324$

$y_Y = (0.9324)(4) = 3.73$ mm $\qquad t_p = 4 - 3.73 = 0.27$ mm

(b) $M = 35$ Nm $\qquad \frac{y_Y}{c} = \sqrt{3 - 2\left(\frac{35}{28.16}\right)} = 0.7171$

$y_Y = (0.7171)(4) = 2.87$ mm $\qquad t_p = c - y_Y = 1.13$ mm

Problem 4.70



4.70 A bar having the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 330$ MPa. Determine the bending moment M at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 2 mm thick.

(a) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(8)^3 = 341.3 \text{ mm}^4$$

$$c = \frac{1}{2}h = 4 \text{ mm}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(330\times10^6)(341.3\times10^{-12})}{0.004} = 28.16 \text{ Nm}$$

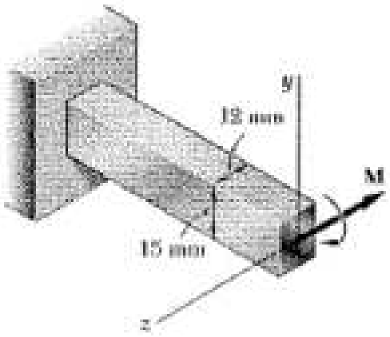
$M_Y = 28.16$ Nm

(b) $y_Y = c - t_p = 4 - 2 = 2$ mm

$$M_p = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(28.16)\left[1 - \frac{1}{3}\left(\frac{2}{4}\right)^2\right]$$

$M_p = 38.72$ Nm

Problem 4.71



4.71 The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, is subjected to a couple of 162 N · m parallel to the z axis. Determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

(a) $I = \frac{1}{12}(0.012)(0.015)^3 = 3.375 \times 10^{-9}\ m^4$

$c = \frac{1}{2}(0.015) = 0.0075\ m$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(3.375 \times 10^{-9})}{0.0075}$

$= 135\ Nm$

$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^3\right]$

$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(162)}{135}} = 0.7746$

$y_Y = (0.7746)(0.0075) = 5.81 \times 10^{-3}\ m$

Thickness of elastic core: $2y_Y = 11.6\ mm$

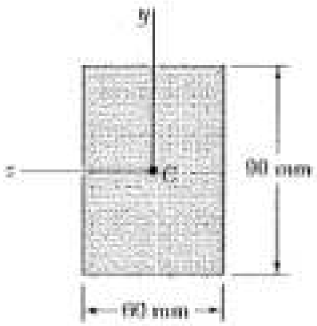
(b) $y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho$

$\rho = \frac{y_Y E}{\sigma_Y} = \frac{(5.81 \times 10^{-3})(200 \times 10^9)}{300 \times 10^6} = 3.873\ m$

$\rho = 3.9\ m$

Problem 4.73

4.73 and 4.74 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30-mm thick.

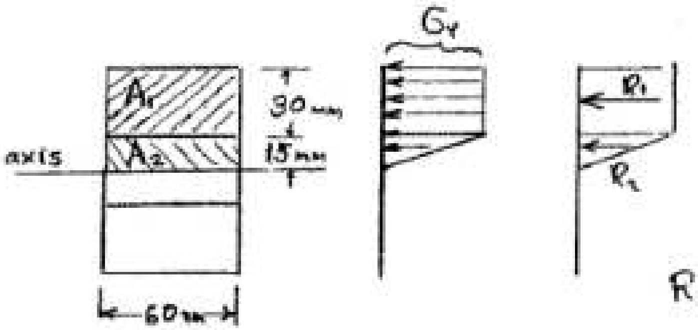


(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \ \text{mm}^4 = 3.645 \times 10^{-6} \ \text{m}^4$

$c = \frac{1}{2} h = 45 \ \text{mm} = 0.045 \ \text{m}$

$M_Y = \dfrac{\sigma_Y I}{c} = \dfrac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10 \ \text{N} \cdot \text{m}$

$M_Y = 19.44 \ \text{kN} \cdot \text{m}$



$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.060)(0.030)$
$= 432 \times 10^3 \ \text{N}$

$y_1 = 15 \ \text{mm} + 15 \ \text{mm} = 0.030 \ \text{m}$

$R_2 = \frac{1}{2} \sigma_Y A_2 = (\frac{1}{2})(240 \times 10^6)(0.060)(0.015)$
$= 108 \times 10^3 \ \text{N}$

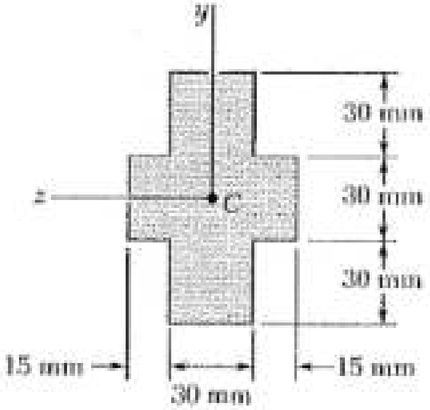
$y_2 = \frac{2}{3}(15 \ \text{mm}) = 10 \ \text{mm} = 0.010 \ \text{m}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(432 \times 10^3)(0.030) + (108 \times 10^3)(0.010)]$
$= 28.08 \times 10^3 \ \text{N} \cdot \text{m}$

$M = 28.1 \ \text{kN} \cdot \text{m}$

Problem 4.74

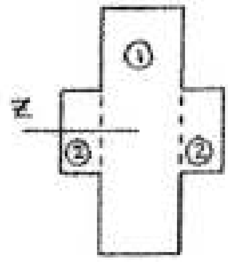
4.73 and 4.74 A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30 mm thick.



(a) $I_{①} = \frac{1}{12} b_1 h_1^3$

$= \frac{1}{12}(30)(90)^3$

$= 1.8225 \times 10^6 \text{ mm}^4$



$I_{②} = \frac{1}{12} b_2 h_2^3$

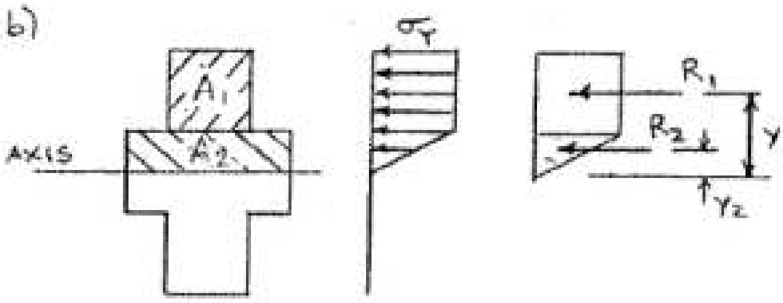
$= \frac{1}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$

$I = I_{①} + I_{②} = 1.89 \times 10^6 \text{ mm}^4 = 1.89 \times 10^{-6} \text{ m}^4$ $\qquad c = 45 \text{ mm} = 0.045 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(1.89 \times 10^{-6})}{0.045} = 10.08 \times 10^3 \text{ N·m}$

$M_Y = 10.08 \text{ kN·m}$ ◄

(b)



$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.030)(0.030) = 216 \times 10^3 \text{ N}$

$y_1 = 15 + 15 = 30 \text{ mm} = 0.030 \text{ m}$

$R_2 = \frac{1}{2} \sigma_Y A_2 = \frac{1}{2}(240 \times 10^6)(0.060)(0.015) = 108 \times 10^3 \text{ N}$

$y_2 = \frac{2}{3}(15) = 10 \text{ mm} = 0.010 \text{ m}$

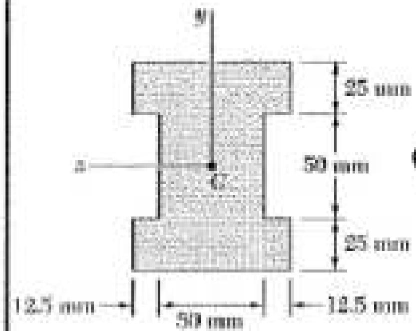
$M = 2(R_1 y_1 + R_2 y_2)$

$= 2[(216 \times 10^3)(0.030) + (108 \times 10^3)(0.015)]$

$= 16.12 \times 10^3 \text{ N·m}$ $\qquad M = 16.12 \text{ kN·m}$ ◄

Problem 4.75

4.75 and 4.76 A beam of the cross section shown is made of a steel that is assumed to be elastoplastic $E = 200$ GPa and $\sigma_Y = 300$ MPa. For bending about the z axis, determine the bending moment and radius of curvature at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 25 mm thick.



(a) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(25)^3 + (75)(25)(37.5)^2 = 2734375 \text{ mm}^4$

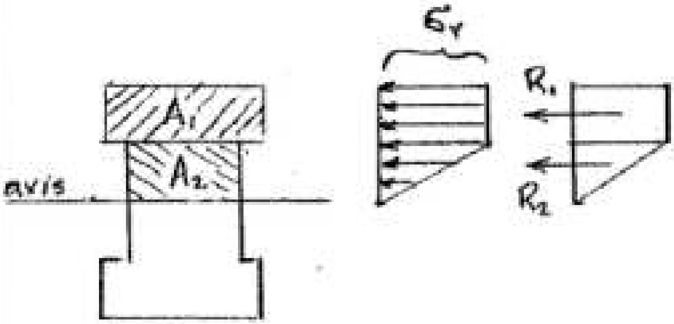
$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(50)(50)^3 = 520833 \text{ mm}^4$

$I_3 = I_1 = 2734375 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 5989583 \text{ mm}^4$

$c = 50 \text{ mm}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(5989583 \times 10^{-12})}{.050} = 35.9 \text{ kNm}$



$R_1 = \sigma_Y A_1 = (300 \times 10^6)(0.075)(0.025) = 562.5 \text{ kN}$

$y_1 = 25 + 12.5 = 37.5 \text{ mm}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300 \times 10^6)(0.05)(0.025)$

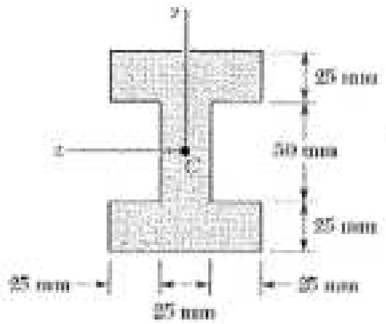
$= 187.5 \text{ kN}$

$y_2 = \frac{2}{3}(25) = 16.67 \text{ mm}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(562.5)(0.0375) + (187.5)(0.01667)] = 48.5 \text{ kN.m}$

Problem 4.76

4.75 and 4.76 A beam of the cross section shown is made of a steel that is assumed to be elastoplastic $E = 200$ GPa and $\sigma_Y = 300$ MPa. For bending about the z axis, determine the bending moment and radius of curvature at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 25 mm thick.



(a) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(25)^3 + (75)(25)(37.5)^2$

$= 2734375 \text{ mm}^4$

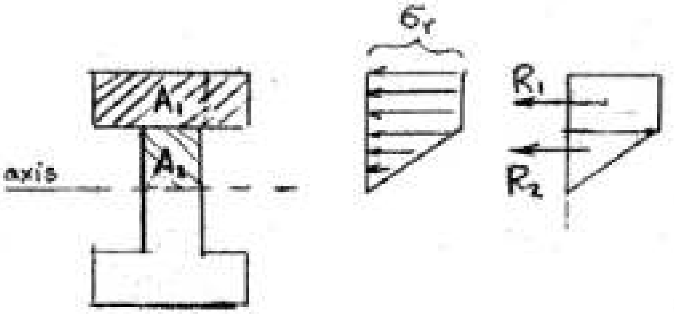
$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(25)(50)^3 = 260417 \text{ mm}^4$

$I_3 = I_1 = 2734375 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 5729167 \text{ mm}^4$

$c = 50 \text{ mm}$

$M_y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(5729167\times10^{-12})}{0.05} = 34.4 \text{ kNm}$



$R_1 = \sigma_Y A_1 = (300\times10^6)(0.075)(0.025)$

$= 562.5 \text{ kN}$

$y_1 = 25 + 12.5 = 37.5 \text{ mm}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times10^6)(25\times10^{-3})^2$

$= 93.75 \text{ kN}$

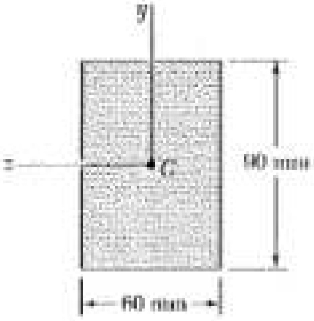
$y_2 = \frac{2}{3}(25) = 16.67 \text{ mm.}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2\left[(562.5)(0.0375) + (93.75)(0.01667)\right] = 45.3 \text{ kNm.}$

Problem 4.77

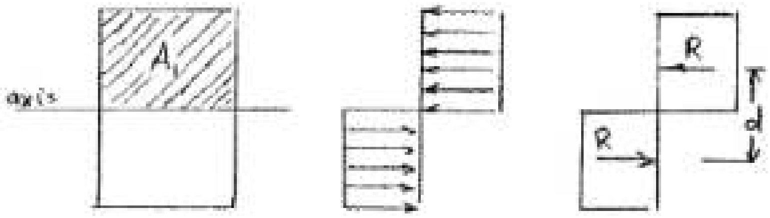
4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.77 Beam of Prob. 4.73.



From PROBLEM 4.73 $E = 200$ GPa and $\sigma_Y = 240$ MPa.

$$A_1 = (60)(45) = 2700 \text{ mm}^2 = 2700 \times 10^{-6} \text{ m}^2$$



$$R = \sigma_Y A_1 = (240 \times 10^6)(2700 \times 10^{-6}) = 648 \times 10^3 \text{ N}$$

$$d = 45 \text{ mm} = 0.045 \text{ m}$$

(a) $M_p = Rd = (648 \times 10^3)(0.045) = 29.16 \times 10^3 \text{ N}\cdot\text{m}$ $\quad M_p = 29.2 \text{ kN}\cdot\text{m}$ ◄

(b) $I = \frac{1}{12} bh^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$

$$c = 45 \text{ mm} = 0.045 \text{ m}$$

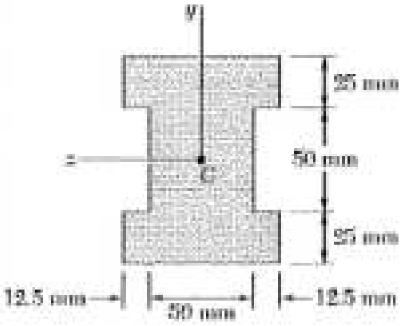
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10^3 \text{ N}\cdot\text{m}$$

$$k = \frac{M_p}{M_Y} = \frac{29.16}{19.44}$$ $\quad k = 1.500$ ◄

Problem 4.79

4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

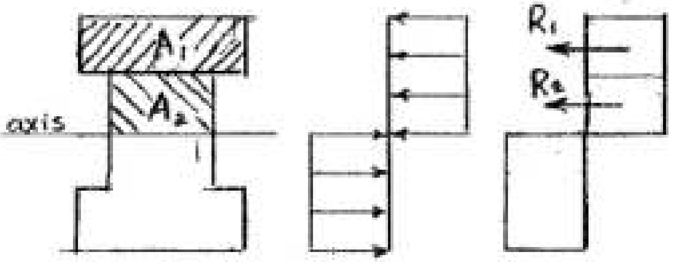
4.79 Beam of Prob. 4.75.



From PROBLEM 4.76 $E = 29\times10^6$ and $\sigma_Y = 300$ MPa.

$R_1 = \sigma_Y A_1 = (300\times10^6)(0.075)(0.025) = 562.5$ kN

$y_1 = 25 + 12.5 = 37.5$ mm



$R_2 = \sigma_Y A_2 = (300\times10^6)(0.05)(0.025) = 375$ kN

$y_2 = \frac{1}{2}(25) = 12.5$ mm

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(562.5)(0.0375) + (375)(0.0125)] = 51.6 \text{ kN}$$

(b) $I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(75)(25)^3 + (75)(25)(37.5)^2 = 2734375 \text{ mm}^4$

$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(50)(50) = 520833 \text{ mm}^4$

$I_3 = I_1 = 2734375 \text{ mm}^4$

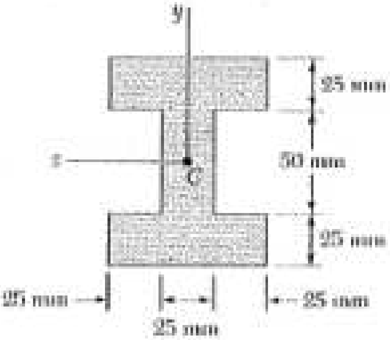
$I = I_1 + I_2 + I_3 = 5989583$

$c = 50$ mm

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(5989583\times10^{-12})}{0.05} = 35.9 \text{ kNm}$$

$$k = \frac{M_p}{M_Y} = \frac{51.6}{35.9} = 1.437$$

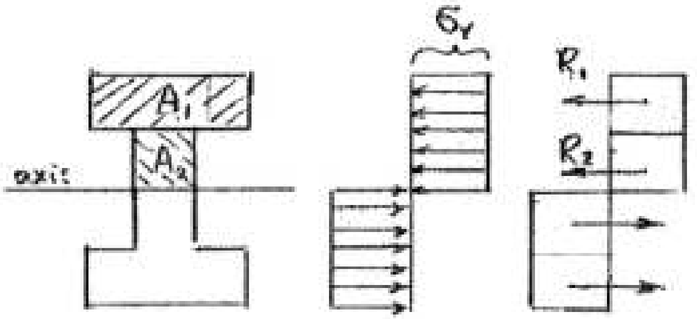
Problem 4.80



4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.77 Beam of Prob. 4.73.
4.78 Beam of Prob. 4.74.
4.79 Beam of Prob. 4.75.
4.80 Beam of Prob. 4.76.

From PROBLEM 4.75 $E = 200\times10^{9}$ GPa and $\sigma_Y = 300$ MPa



$R_1 = \sigma_Y A_1 = (300\times10^{6})(0.075)(0.025) = 562.5$ kN

$\bar{y}_1 = 25 + 12.5 = 37.5$ mm

$R_2 = \sigma_Y A_2 = (300\times10^{6})(0.025)(0.025)$
$= 187.5$ kN

$\bar{y}_2 = \frac{1}{2}(25) = 12.5$ mm

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2\left[(562.5)(0.0375) + (187.5)(0.0125)\right] = 46.9 \text{ kN m}$$

(b) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(25)^3 + (75)(25)(37.5)^2 = 2734375$ mm^4.

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(25)(50)^3 = 260417$ mm^4

$I_3 = I_1 = 2734375$ mm^4

$I = I_1 + I_2 + I_3 = 5729167$ mm^4

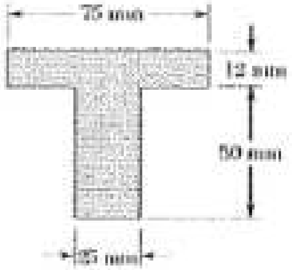
$c = 50$ mm

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^{6})(5729167\times10^{-12})}{0.05} = 34.4 \text{ kNm}$$

$$k = \frac{M_p}{M_Y} = \frac{46.9}{34.4} = 1.363$$

Problem 4.85

4.85 Determine the plastic moment M_p of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 330 MPa.



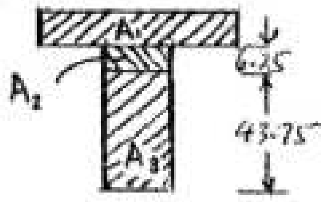
Total area: $A = (75)(12) + (50)(25) = 2150 \text{ mm}^2$

$\frac{1}{2}A = 1075 \text{ mm}^2 \qquad y_n = \frac{\frac{1}{2}A}{b} = \frac{1075}{25} = 43 \text{ mm}$

$A_1 = (75)(12) = 900 \text{ mm}^2, \quad \bar{y}_1 = 12.25 \text{ mm}, \quad A_1\bar{y}_1 = 11025 \text{ mm}^3$

$A_2 = (6.25)(25) = 156.25 \text{ mm}^2, \quad \bar{y}_2 = 3.125 \text{ mm} \quad A_2\bar{y}_2 = 488.3 \text{ mm}^3$

$A_3 = (43.75)(25) = 1093.75 \text{ mm}^2 \quad \bar{y}_3 = 21.875 \text{ mm} \quad A_3\bar{y}_3 = 23925.8$

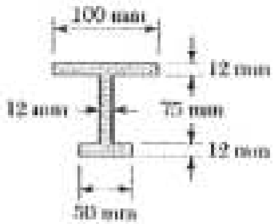


$M_p = \sigma_Y (A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3)$

$= 330\times10^6 (11025 + 488.3 + 23925.8)(1\times10^{-9}) = 11.7 \text{ kN m}.$ ◄

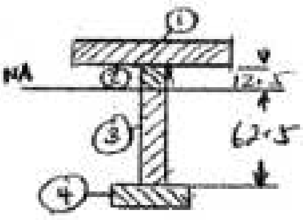
Problem 4.86

4.86 Determine the plastic moment M_p of the cross section shown assuming the steel to be elastoplastic with a yield strength of 250 MPa.



Total area: $A = (100\times12) + (75)(12) + (50)(12) = 2700 \text{ mm}^2.$

$\frac{1}{2}A = 1350 \text{ mm}^2.$



$A_1 = 1200 \text{ mm}^2, \quad \bar{y}_1 = 18.5, \quad A_1 y_1 = 22200 \text{ mm}^3$

$A_2 = 150 \text{ mm}^2, \quad \bar{y}_2 = 6.25, \quad A_2\bar{y}_2 = 937.5 \text{ mm}^3$

$A_3 = 750 \text{ mm}^2, \quad \bar{y}_3 = 31.25, \quad A_3\bar{y}_3 = 23437.5 \text{ mm}^3$

$A_4 = 600 \text{ mm}^2, \quad \bar{y}_4 = 68.5, \quad A_4\bar{y}_4 = 41100 \text{ mm}^3$

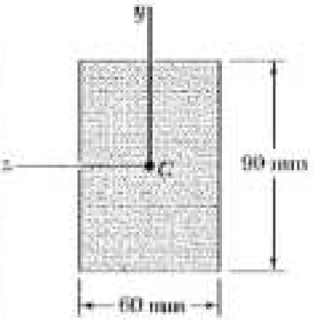
$M_p = \sigma_Y (A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4)$

$= (250\times10^6)(22200 + 937.5 + 23437.5 + 41100)(1\times10^{-9}) = 21.9 \text{ kN m}.$ ◄

Problem 4.87

4.87 and 4.88 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45$ mm.

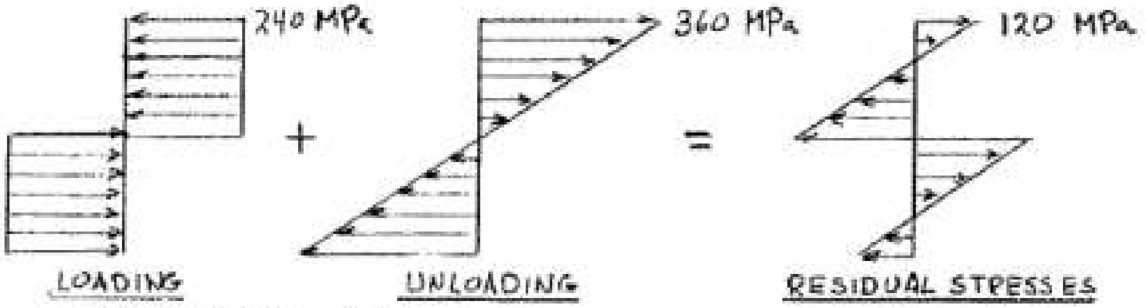
4.87 Beam of Prob. 4.73.



$M_p = 29.16 \times 10^3$ N·m (See solutions to Problems 4.73 and 4.77)

$I = 3.645 \times 10^{-6}$ m^4, $c = 0.045$ m

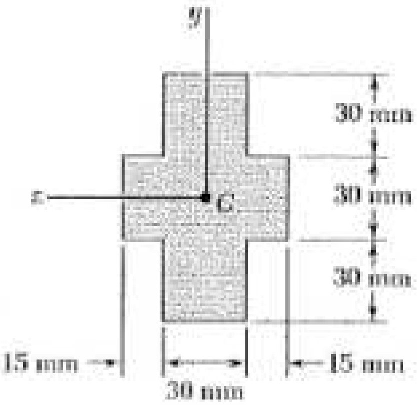
$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I}$ at $y = c = 45$ mm.



$$\sigma' = \frac{(29.16 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 360 \times 10^6 \text{ Pa}$$

$$\sigma_{res} = \sigma' - \sigma_Y = 360 \times 10^6 - 240 \times 10^6 = 120 \times 10^6 \text{ Pa} \qquad \sigma_{res} = 120.0 \text{ MPa}$$

Problem 4.88



4.87 and 4.88 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45$ mm.

4.88 Beam of Prob. 4.74.

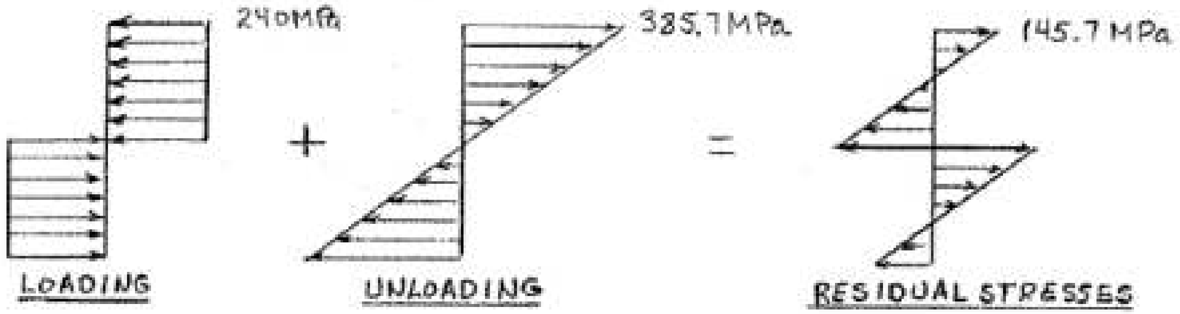
$M_p = 16.20 \times 10^3$ N·m from the solution to PROBLEM 4.78

$I = 1.86 \times 10^{-6}\ m^4$

$c = 0.045$ m

$$\sigma' = \frac{M_{max} c}{I} = \frac{M_p c}{I} = \frac{(16.20 \times 10^3)(0.045)}{1.89 \times 10^{-6}}$$

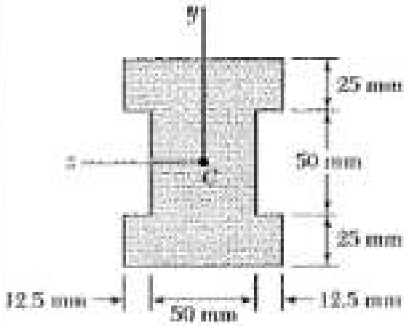
$$= 385.7 \times 10^6 \text{ Pa} = 385.7 \text{ MPa}$$



At $y = 45$ $\quad \sigma = 240$ MPa $\quad \sigma' = 385.7$ MPa

Residual stress $\quad \sigma_{res} = \sigma' - \sigma_Y \quad \sigma_{res} = 145.7$ MPa

Problem 4.89



4.89 and 4.90 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 290 MPa, determine the residual stress at (*a*) $y = 25$ mm, (*b*) $y = 50$ mm.

4.89 Beam of Prob. 4.75.

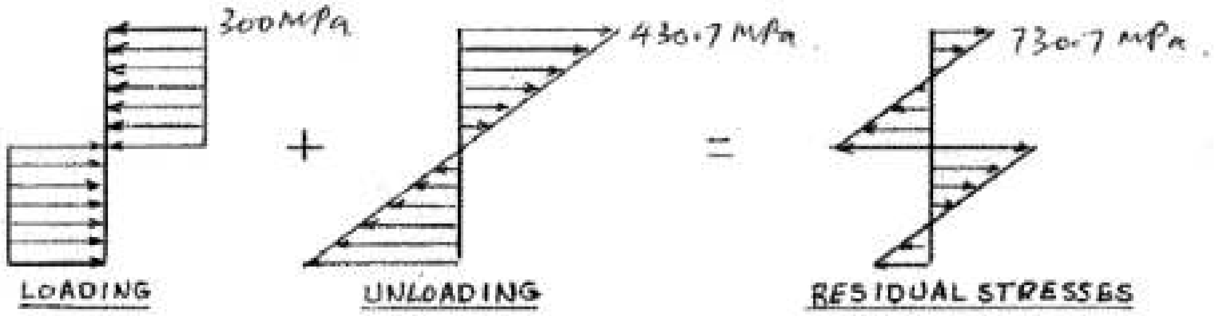
4.90 Beam of Prob. 4.76.

$M_p = 51.6$ kN (See solutions to Problems 4.76 and 4.80)

$I = 5989583 \text{ mm}^4, \quad c = 50 \text{ mm}.$

$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{for } y = c$$

$$\sigma' = \frac{(51.6\times10^3)(0.05)}{5989583\times10^{-12}} = 430.7 \text{ MPa}.$$



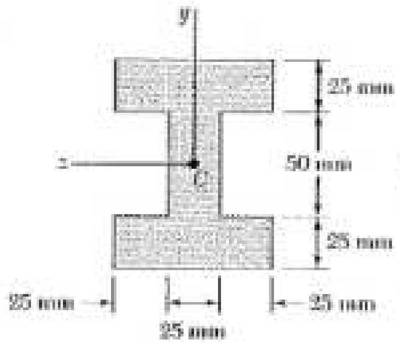
(a) At $y = 25 \text{ mm} = \frac{1}{2}c \quad \sigma' = \frac{1}{2}(430.7) = 215.35$ MPa.

$\sigma_{res} = -300 + 215.35 = -84.65$ MPa.

(b) At $y = 50 \text{ mm} = c \quad \sigma' = 430.7$ MPa

$\sigma_{res} = -300 + 430.7 = 730.7$ MPa.

Problem 4.90



4.89 and 4.90 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 290 MPa, determine the residual stress at (*a*) $y = 25$ mm, (*b*) $y = 50$ mm.

4.89 Beam of Prob. 4.75.

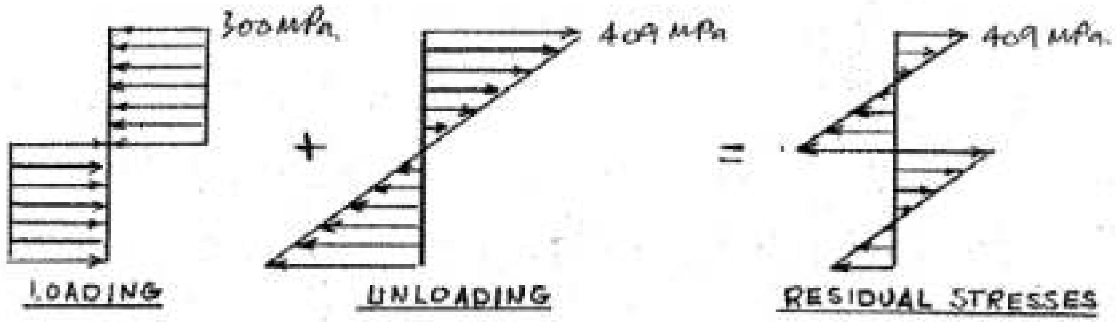
4.90 Beam of Prob. 4.76.

$M_p = 46.9$ kNm (See solutions to Problems 4.75 and 4.79)

$I = 5729167 \text{ mm}^4 \quad c = 50 \text{ mm}.$

$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{at } y = c.$$

$$\sigma' = \frac{(46.9 \times 10^3)(0.05)}{5729167 \times 10^{-12}} = 409 \text{ MPa}.$$



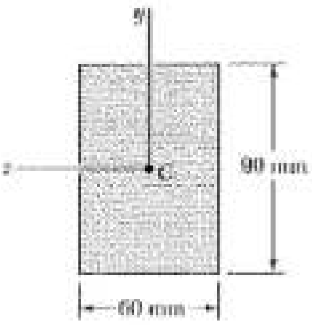
(a) At $y = 25 \text{ mm} = \frac{1}{2}c \qquad \sigma' = \frac{1}{2}(409) = 204.5$ MPa.

$\sigma_{res} = -300 + 204.5 = -95.5$ MPa.

(b) At $y = 50 \text{ mm} = c \qquad \sigma' = 409$ MPa.

$\sigma_{res} = -300 + 409 = 109$ MPa.

Problem 4.91



4.91 A bending couple is applied to the beam of Prob. 4.73, causing plastic zones 30 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 45$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

See SOLUTION to PROBLEM 4.73 for bending couple and stress distribution during loading.

$M = 28.08 \times 10^3$ N-m $\qquad y_Y = 15$ mm $= 0.015$ m

$E = 200$ GPa $\qquad \sigma_Y = 240$ MPa

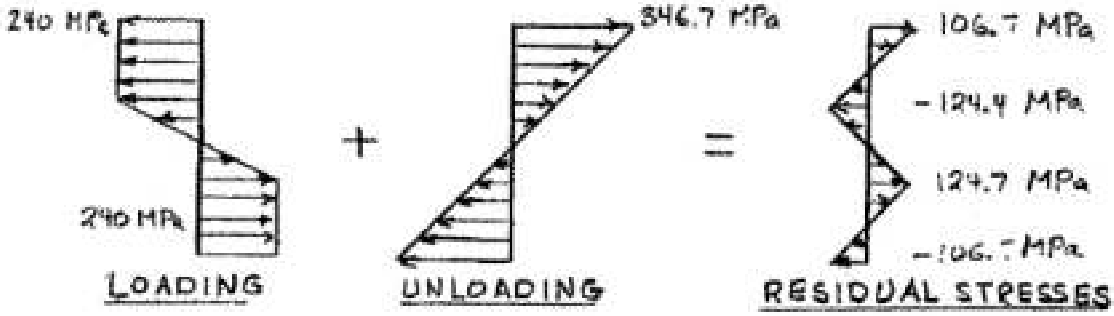
$I = 3.645 \times 10^{-6}$ m^4 $\qquad c = 0.045$ m

(a) $\sigma' = \frac{Mc}{I} = \frac{(28.08\times10^3)(0.045)}{3.645\times10^{-6}} = 346.7\times10^6$ Pa $= 346.7$ MPa

$\sigma'' = \frac{My_Y}{I} = \frac{(28.08\times10^3)(0.015)}{3.645\times10^{-6}} = 115.6\times10^6$ Pa $= 115.6$ MPa

At $y = c$, $\quad \sigma_{res} = \sigma' - \sigma_Y = 346.7 - 240 \qquad \sigma_{res} = 106.7$ MPa ◄

At $y = y_Y$, $\quad \sigma_{res} = \sigma'' - \sigma_Y = 115.6 - 240 \qquad \sigma_{res} = -124.4$ MPa



(b) $\sigma_{res} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_Y = 0$

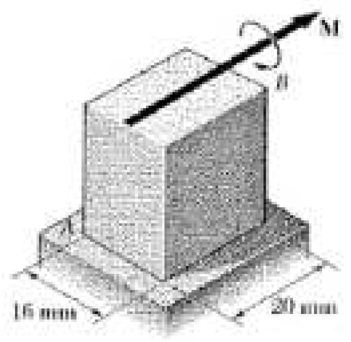
$y_0 = \frac{I\sigma_Y}{M} = \frac{(3.645\times10^{-6})(240\times10^6)}{28.08\times10^3} = 31.15\times10^{-3}$ m $= 31.15$ mm

ans. $y_0 = -31.15$ mm, 0, 31.15 mm ◄

(c) At $y = y_Y$, $\quad \sigma_{res} = -124.4\times10^6$ Pa

$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(200\times10^9)(0.015)}{-124.4\times10^6} \qquad \rho = 24.1$ m ◄

Problem 4.95



4.95 The prismatic bar AB is made of a steel that is assumed to be elastoplastic and for which $E = 200$ GPa. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 350$ N · m is applied as shown, determine (*a*) the yield strength of the steel, (*b*) the thickness of the elastic core of the bar.

$$M = \frac{3}{2} M_Y \left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)$$
$$= \frac{3}{2}\frac{\sigma_Y I}{c}\left(1 - \frac{1}{3}\frac{\rho^2 \sigma_Y^2}{E^2 c^2}\right)$$
$$= \frac{3}{2}\frac{\sigma_Y b (2c)^3}{12 c}\left(1 - \frac{1}{3}\frac{\rho^2 \sigma_Y^2}{E^2 c^2}\right)$$
$$= \sigma_Y b c^2 \left(1 - \frac{1}{3}\frac{\rho^2 \sigma_Y^2}{E^2 c^2}\right)$$

(a) $bc^2 \sigma_Y \left(1 - \frac{\rho^2 \sigma_Y^2}{3E^2 c^2}\right) = M$ Cubic equation for σ_Y

Data: $E = 200 \times 10^9$ Pa, $M = 420$ N·m $\rho = 2.4$ m

$b = 20$ mm $= 0.020$ m $c = \frac{1}{2}h = 8$ mm $= 0.008$ m

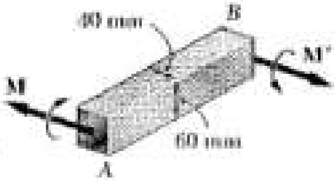
$$(1.28 \times 10^{-6})\,\sigma_Y \left[1 - 750 \times 10^{-21}\,\sigma_Y^2\right] = 350$$
$$\sigma_Y \left[1 - 750 \times 10^{-21}\,\sigma_Y^2\right] = 273.44 \times 10^6$$

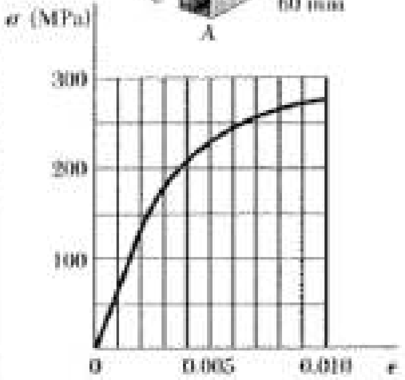
Solving by trial $\sigma_Y = 292 \times 10^6$ Pa $\sigma_Y = 292$ MPa ◄

(b) $y_Y = \frac{\sigma_Y \rho}{E} = \frac{(292 \times 10^6)(2.4)}{200 \times 10^9} = 3.504 \times 10^{-3}$ m $= 3.504$ mm

thickness of elastic core $= 2y_Y = 7.01$ mm ◄

Problem 4.96





4.96 The prismatic bar *AB* is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine (*a*) the radius of curvature of the bar when the maximum stress is 250 MPa, (*b*) the corresponding value of the bending moment. (*Hint*: For part *b*, plot σ versus *y* and use an approximate method of integration.)

(a) $\sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$

$\varepsilon_m = 0.0064$ from curve

$c = \frac{1}{2}h = 30 \text{ mm} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1}$$

$\rho = 4.69 \text{ m}$ ◄

(b) Strain distribution. $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple.

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

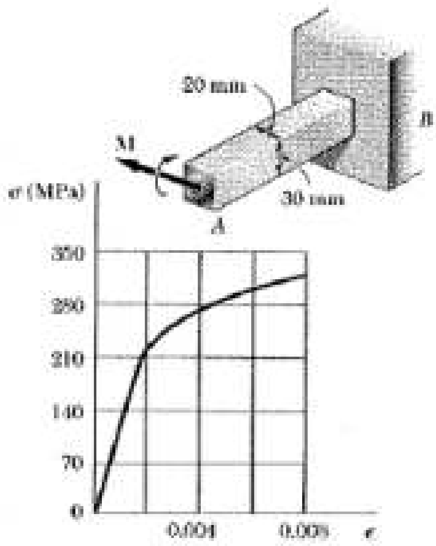
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, (MPa)	$u\lvert\sigma\rvert$, (MPa)	w	$wu\lvert\sigma\rvert$, (MPa)
0	0	0	0	1	0
0.25	0.0016	110	27.5	4	110
0.5	0.0032	180	90	2	180
0.75	0.0048	225	168.75	4	675
1.00	0.0064	250	250	1	250
					1215 ← $\sum wu\lvert\sigma\rvert$

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$$

$M = (2)(0.040)(0.030)^2(101.25 \times 10^6) = 7.29 \times 10^3 \text{ N}\cdot\text{m}$ $\qquad M = 7.29 \text{ kN}\cdot\text{m}$ ◄

Problem 4.97



4.97 The prismatic bar AB is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ϵ diagram is the same in compression as in tension, determine (*a*) the maximum stress in the bar when the radius of curvature of the bar is 2.5 m, (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.96.)

(a) $\rho = 2500$ mm, $b = 20$ mm, $c = 15$ mm.

$$\varepsilon_m = \frac{c}{\rho} = \frac{15}{2500} = 0.006$$

From the curve $\sigma_m = 300$ MPa ◄

(b) Strain distribution $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y \sigma b\, dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 u |\sigma| du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u |\sigma| du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3} \Sigma\, w u |\sigma|$$

where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

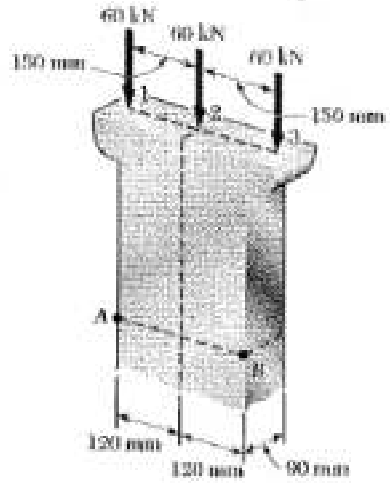
u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, MPa	$u\lvert\sigma\rvert$, MPa	w	$wu\lvert\sigma\rvert$, MPa
0	0	0	0	1	0
0.25	0.0015	175	43.8	4	175
0.5	0.003	252	126	2	252
0.75	0.0045	280	210	4	840
1.00	0.006	300	300	1	300
					1567 ← $\Sigma\, wu|\sigma|$

$$J = \frac{(0.25)(1567)}{3} = 130.6 \text{ MPa}$$

$$M = (2)(0.02)(0.015)^2(130.6 \times 10^6) = 1.175 \text{ kN/m.}$$ ◄

Problem 4.99

4.99 Determine the stress at points A and B, (*a*) for the loading shown, (*b*) if the 60-kN loads are applied at points 1 and 2 only.



(a) Loading is centric.

$P = 180\ \text{kN} = 180\times10^3\ \text{N}$

$A = (90)(240) = 21.6\times10^3\ \text{mm}^2 = 21.6\times10^{-3}\ \text{m}^2$

At A and B $\sigma = -\frac{P}{A} = -\frac{180\times10^3}{21.6\times10^{-3}} = -8.33\times10^6\ \text{Pa}$

$= -8.33\ \text{MPa}$ ◄

(b) Eccentric loading

$P = 120\ \text{kN} = 120\times10^3$

$M = (60\times10^3)(150\times10^{-3}) = 9.0\times10^3\ \text{N}\cdot\text{m}$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(90)(240)^3 = 103.68\times10^6\ \text{mm}^4 = 103.68\times10^{-6}\ \text{m}^4$

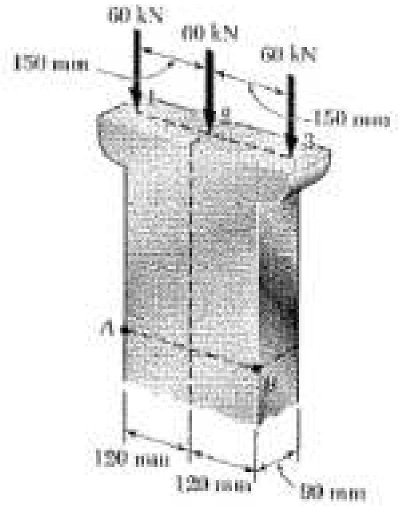
$c = 120\ \text{mm} = 0.120\ \text{m}$

At A $\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120\times10^3}{21.6\times10^{-3}} - \frac{(9.0\times10^3)(0.120)}{103.68\times10^{-6}} = -15.97\times10^6\ \text{Pa} = -15.97\ \text{MPa}$ ◄

At B $\sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{120\times10^3}{21.6\times10^{-3}} + \frac{(9.0\times10^3)(0.120)}{103.68\times10^{-6}} = 4.86\times10^6\ \text{Pa} = 4.86\ \text{MPa}$ ◄

Problem 4.100

4.100 Determine the stress at points A and B, (*a*) for the loading shown, (*b*) if the 60-kN loads applied at points 2 and 3 are removed.



(a) Loading is centric. $P = 180\ \text{kN}$

$A = (0.09)(0.24) = 0.0216\ \text{m}^2$

At A and B $\sigma = -\frac{P}{A}$

$\sigma = -\frac{180\times10^3}{0.0216}$ $\sigma = -8.3\ \text{MPa}$ ◄

(b) Eccentric loading. $P = 60\ \text{kN}$

$M = (60)(0.15) = 9\ \text{kNm}$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.09)(0.24)^3 = 103.68\times10^{-6}\ \text{m}^4$

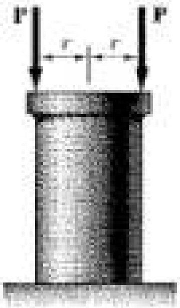
$c = 0.12\ \text{m}$

At A $\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{60\times10^3}{0.0216} - \frac{(9\times10^3)(0.12)}{103.68\times10^{-6}} = -13.19\ \text{MPa}$ $\sigma_A = -13.2\ \text{MPa}$ ◄

At B $\sigma = -\frac{P}{A} + \frac{Mc}{I} = -\frac{60\times10^3}{0.0216} + \frac{(9\times10^3)(0.12)}{103.68\times10^{-6}} = 7.64\ \text{MPa}$ $\sigma_B = 7.6\ \text{MPa}$ ◄

Problem 4.101

4.101 Two forces **P** can be applied separately or at the same time to a plate that is welded to a solid circular bar of radius r. Determine the largest compressive stress in the circular bar, (*a*) when both forces are applied, (*b*) when only one of the forces is applied.



For a solid circular section $A = \pi r^2$, $I = \frac{\pi}{4} r^4$, $c = r$

Compressive stress $\sigma = -\frac{F}{A} - \frac{Mc}{I}$

$= -\frac{F}{\pi r^2} - \frac{4M}{\pi r^3}$

(a) Both forces applied. $F = 2P$, $M = 0$

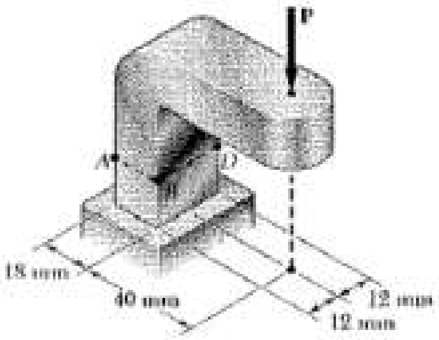
$\sigma = -2P/\pi r^2$ ◄

(b) One force applied. $F = P$, $M = Pr$

$\sigma = -\frac{F}{\pi r^2} - \frac{4Pr}{\pi r^3}$ $\qquad \sigma = -5P/\pi r^2$ ◄

Problem 4.102

4.102 Knowing that the magnitude of the vertical force **P** is 2 kN, determine the stress at (*a*) point *A*, (*b*) point *B*.



$A = (24)(18) = 432 \text{ mm}^2 = 432 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(24)(18)^3 = 11.664 \times 10^3 \text{ mm}^4$
$= 11.664 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2}(18) = 9 \text{ mm} = 0.009 \text{ m}$

$e = \frac{1}{2}(18) + 40 = 49 \text{ mm} = 0.049 \text{ m}$

$M = Pe = (2 \times 10^3)(0.049) = 98 \text{ N}\cdot\text{m}$

(a) $\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = -\frac{2 \times 10^3}{432 \times 10^{-6}} + \frac{(98)(0.009)}{11.664 \times 10^{-9}} = 71.0 \times 10^6 \text{ Pa}$

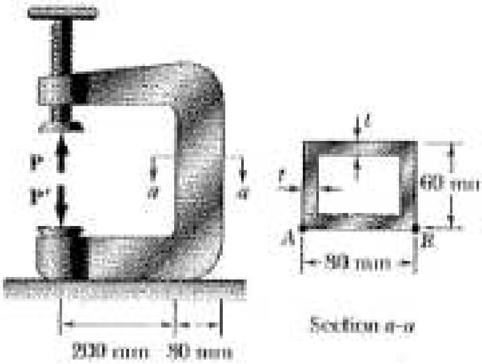
$\sigma_A = 71.0 \text{ MPa}$ ◄

(b) $\sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2 \times 10^3}{432 \times 10^{-6}} - \frac{(98)(0.009)}{11.664 \times 10^{-9}} = -80.2 \times 10^6 \text{ Pa}$

$\sigma_B = -80.2 \text{ MPa}$ ◄

Problem 4.103

4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (a) point A, (b) point B.



Rectangular cutout is 60 mm × 40 mm.

$A = (80)(60) - (60)(40) = 2.4 \times 10^3 \text{ mm}^2 = 2.4 \times 10^{-3} \text{ m}^2$

$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84 \times 10^6 \text{ mm}^4$

$= 1.84 \times 10^{-6} \text{ m}^4$

$c = 40 \text{ mm} = 0.040 \text{ m}$ $\quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$

$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N}\cdot\text{m}$

(a) $\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^3}{2.4 \times 10^{-3}} + \frac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = 112.7 \times 10^6 \text{ Pa}$

$\sigma_A = 112.8 \text{ MPa}$

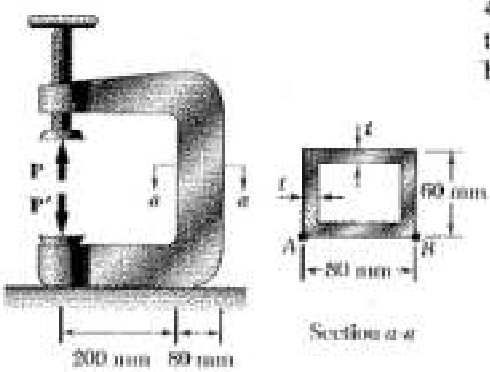
(b) $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{2.4 \times 10^{-3}} - \frac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = 96.0 \ 10^6 \text{ Pa}$

$\sigma_B = -96.0 \text{ MPa}$

Problem 4.104

4.104 Solve Prob. 4.103, assuming that $t = 8$ mm.

4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (a) point A, (b) point B.



Rectangular cutout is 64 mm × 44 mm

$A = (80)(60) - (64)(44) = 1.984 \times 10^3 \text{ mm}^2$

$= 1.984 \times 10^{-3} \text{ m}^2$

$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(44)(64)^3 = 1.59881 \times 10^6 \text{ mm}^4$

$= 1.59881 \times 10^{-6} \text{ m}^4$

$c = 40 \text{ mm} = 0.004 \text{ m}$ $\quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$

$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N-m}$

(a) $\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} + \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = 130.2 \times 10^6 \text{ Pa}$

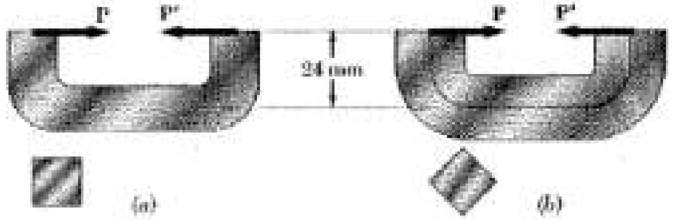
$\sigma_A = 130.2 \text{ MPa}$

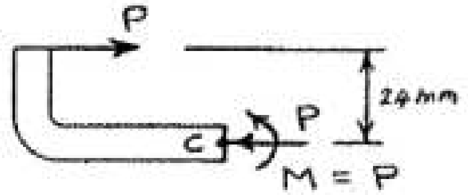
(b) $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} - \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = -110.0 \times 10^6 \text{ Pa}$

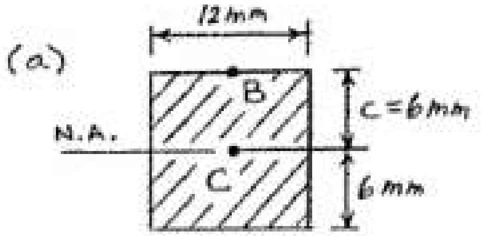
$\sigma_B = -110.0 \text{ MPa}$

Problem 4.105

4.105 Portions of a 12 × 12 mm square bar have been bent to form the two machine components shown. Knowing that the allowable stress is 105 MPa, determine the maximum load that can be applied to each component.







The maximum stress occurs at point B

$$\sigma_B = -105 \text{ MPa}$$

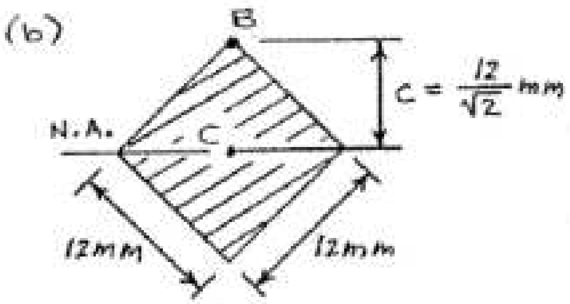
$$\sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP$$

where $K = \frac{1}{A} + \frac{ec}{I}$

$$A = (0.012)(0.012) = 144 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(0.012)(0.012)^3 = 1.728 \times 10^{-9} \text{ m}^4$$

$$e = 0.024 \text{ m}$$



(a) $c = 0.006$ m

$$K = \frac{1}{144 \times 10^{-6}} + \frac{(0.024)(0.006)}{1.728 \times 10^{-9}} = 90277.8 \text{ m}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-105 \times 10^6)}{90277.8}$$

$$P = 1163 \text{ N}$$

(b) $c = \frac{0.012}{\sqrt{2}} = 0.008485$ m

For a square I is the same for all centroidal axes.

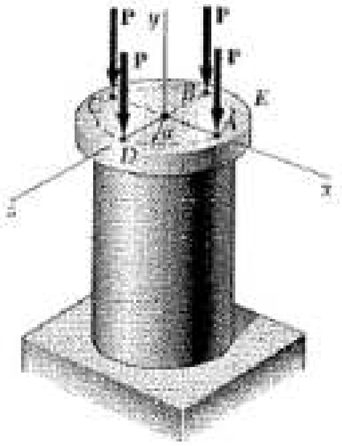
$$I = 1.728 \times 10^{-9} \text{ m}^4$$

$$K = \frac{1}{144 \times 10^{-6}} + \frac{(0.024)(0.008485)}{1.728 \times 10^{-9}} = 124722.2 \text{ m}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-105 \times 10^6)}{124722.2}$$

$$P = 842 \text{ N}$$

Problem 4.106



4.106 The four forces shown are applied to a rigid plate supported by a solid steel post of radius a. Knowing that $P = 100$ kN and $a = 40$ mm, determine the maximum stress in the post when (a) the force at D is removed, (b) the forces at C and D are removed.

For a solid circular section of radius a

$$A = \pi a^2 \qquad I = \frac{\pi}{4} a^4$$

(a) Centric force. $F = 4P$, $M_x = M_z = 0$

$$\sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2}$$

(b) Force at D is removed.

$F = 3P \qquad M_x = -Pa, \qquad M_z = 0$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4}a^2} = -\frac{7P}{\pi a^2}$$

(c) Forces at C and D are removed.

$F = 2P \qquad M_x = -Pa \qquad M_z = -Pa$

Resultant bending couple $M = \sqrt{M_x^2 + M_z^2} = \sqrt{2}\, Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2}\,Pa\,a}{\frac{\pi}{4}a^4} = -\frac{2+4\sqrt{2}}{\pi}\frac{P}{a^2} = -2.437\,P/a^2$$

Numerical data: $P = 100 \times 10^3$ N $\qquad a = 0.040$ m

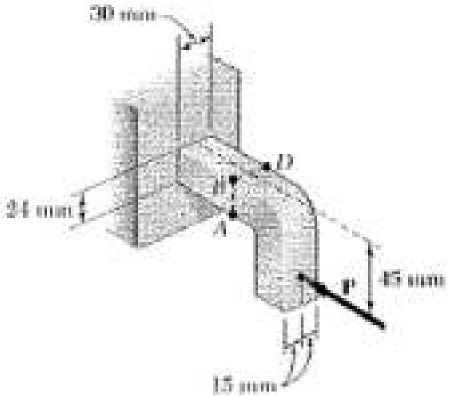
Answers: (a) $\sigma = -\frac{(4)(100\times10^3)}{\pi(0.040)^2} = -79.6\times10^6$ Pa $\qquad -79.6$ MPa ◄

(b) $\sigma = -\frac{(7)(100\times10^3)}{\pi(0.040)^2} = -139.3\times10^6$ Pa $\qquad -139.3$ MPa ◄

(c) $\sigma = -\frac{(2.437)(100\times10^3)}{(0.040)^2} = -152.3\times10^6$ Pa $\qquad -152.3$ MPa ◄

Problem 4.111

4.111 Knowing that the allowable stress in section *ABD* is 70 MPa, determine the largest force **P** that can be applied to the bracket shown.



$$A = (0.03)(0.024) = 72 \times 10^{-5}\ m^2$$

$$I = \frac{1}{12}(0.03)(0.024)^3 = 34.56 \times 10^{-9}\ m^4$$

$$c = \frac{1}{2}(0.024) = 0.012\ m$$

$$e = 0.045 - \frac{0.024}{2} = 0.021\ m$$

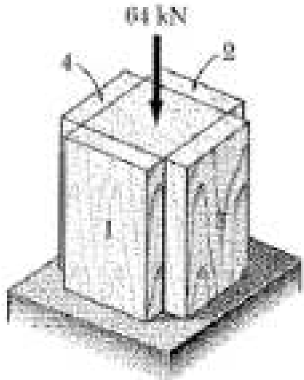
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = PK$$

where $K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{72 \times 10^{-5}} + \frac{(0.021)(0.012)}{34.56 \times 10^{-9}} = 8680.6\ m^{-2}$

$$P = \frac{\sigma}{K} = \frac{70 \times 10^6}{8680.6}$$

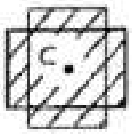
$P = 8.06$ kN

Problem 4.112



4.112 A short column is made by nailing four 25 × 100 mm planks to a 100 × 100 mm timber. Determine the largest compressive stress created in the column by a 64-kN load applied as shown in the center of the top section of the timber if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) planks 1 and 2 are removed, (*d*) planks 1, 2, and 3 are removed, (*e*) all planks are removed.

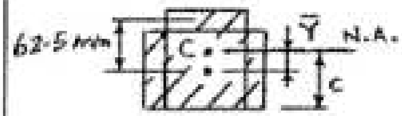
(a) Centric loading. $M = 0$ $\quad \sigma = -\frac{P}{A}$



$A = (100)(100) + (4)(25)(100) = 2\times10^4\ mm^2$

$\sigma = -\frac{64\times10^3}{0.02} = \qquad \sigma = -3.2\ MPa$

(b) Eccentric loading. $M = Pe$ $\quad \sigma = -\frac{P}{A} - \frac{Pec}{I}$

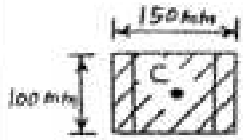


$A = (100)(100) + (3)(25)(100) = 17500\ mm^2 \qquad e = \bar{y}$

$\bar{y} = \frac{\Sigma A\bar{y}}{A} = \frac{(25)(100)(62.5)}{17500} = 8.93\ mm$

$I = \Sigma(\bar{I} + Ad^2) = \frac{1}{12}(150)(100)^3 + (150)(100)(8.93)^2 + \frac{1}{12}(100)(25)^3 + (100)(25)(53.75)^2$
$= 21.05\times10^6\ mm^4$

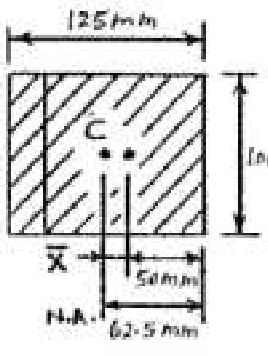
$\sigma = -\frac{64\times10^3}{17500\times10^{-6}} - \frac{(64\times10^3)(8.93\times10^{-3})(58.93\times10^{-3})}{21.05\times10^{-6}} \qquad \sigma = -5.26\ MPa$



(c) Centric loading. $M = 0$ $\quad \sigma = -\frac{P}{A}$

$A = (150)(100) = 15000\ mm^2$

$\sigma = -\frac{64\times10^3}{0.015} \qquad \sigma = -4.3\ MPa$



(d) Eccentric loading. $M = Pe$ $\quad \sigma = -\frac{P}{A} - \frac{Pec}{I}$

$A = (100)(100) + (1)(100)(25) = 12.5\times10^3\ mm^2 \qquad e = \bar{x}$

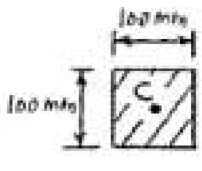
$\bar{x} = 62.5 - 50 = 12.5\ mm$

$I = \frac{1}{12}(100)(125)^3 = 16.276\times10^6\ mm^4$

$\sigma = -\frac{64\times10^3}{12.5\times10^3} - \frac{(64\times10^3)(0.0125)(0.0625)}{16.276\times10^{-6}}$

$\sigma = -8.2\ MPa$

(e) Centric loading. $M = 0$ $\quad \sigma = -\frac{P}{A}$

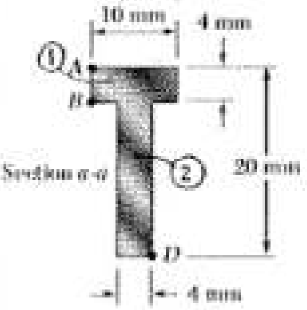


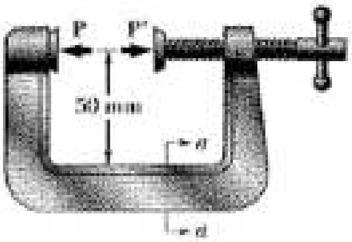
$A = (100)(100) = 1\times10^4\ mm^2$

$\sigma = -\frac{64\times10^3}{0.01} \qquad \sigma = -6.4\ MPa$

Problem 4.113

4.113 Knowing that the clamp shown has been tightened on wooden planks being glued together until $P = 400$ N, determine in section a-a (*a*) the stress at point A, (*b*) the stress at point D, (*c*) the location of the neutral axis.





Locate centroid.

Part	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³
①	40	18	720
②	64	8	512
	104		1232

$$\bar{Y}_o = \frac{1232}{104} = 11.846 \text{ mm}$$

The centroid lies 11.846 mm above point D.

Eccentricity: $e = -(50 + 20 - 11.846) = -58.154$ mm

Bending couple: $M = Pe = (400)(-58.154\times10^{-3}) = -23.262$ N·m

$$A = 104 \text{ mm}^2 = 104\times10^{-6} \text{ m}^2$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(10)(4)^3 + (40)(6.154)^2 = 1.5682\times10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(4)(16)^3 + (64)(3.846)^2 = 2.3120\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 3.8802\times10^3 \text{ mm}^4 = 3.8802\times10^{-9} \text{ m}^4$$

(a) Stress at point A. $y = 20 - 11.846 = 8.154$ mm $= 8.154\times10^{-3}$ m

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{400}{104\times10^{-6}} - \frac{(-23.262)(8.154\times10^{-3})}{3.8802\times10^{-9}} = 52.7\times10^6 \text{ Pa}$$

$$\sigma_A = 52.7 \text{ MPa}$$

(b) Stress at point D. $y = -11.846$ mm $= -11.846\times10^{-3}$ m

$$\sigma_D = \frac{P}{A} + \frac{My}{I} = \frac{400}{104\times10^{-6}} - \frac{(-23.262)(-11.846\times10^{-3})}{3.8802\times10^{-9}} = -67.2\times10^6 \text{ Pa}$$

$$\sigma_D = -67.2 \text{ MPa}$$

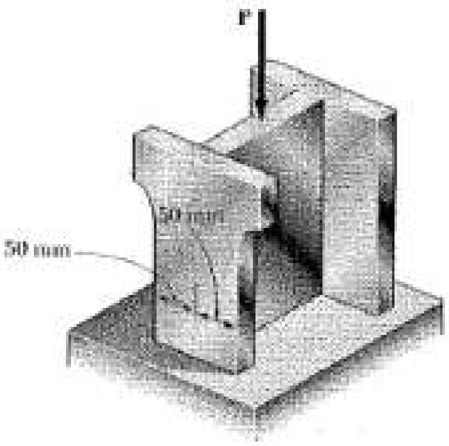
(c) Location of neutral axis. $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0$$

$$y = \frac{I}{Ae} = \frac{3.8802\times10^{-9}}{(104\times10^{-6})(-58.154\times10^{-3})} = -0.642\times10^{-6} \text{ m} = -0.642 \text{ mm}$$

Neutral axis lies $11.846 - 0.642 = 11.205$ mm 11.20 mm above D

Problem 4.114



4.114 Three steel plates, each of 25 × 150-mm cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 25-mm strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 100 MPa, determine the largest force **P** (*a*) that could be applied to the original column, (*b*) that can be applied to the modified column.

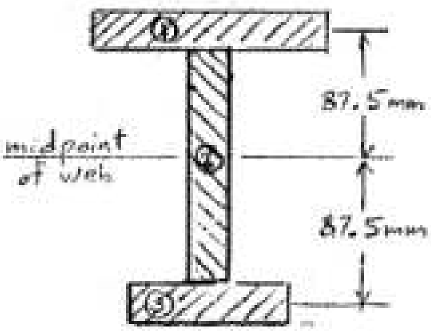
(a) Centric loading $\qquad \sigma = -\frac{P}{A}$

$A = (3)(150)(25) = 11.25 \times 10^3\ mm^2 = 11.25 \times 10^{-3}\ m^2$

$P = -\sigma A = -(-100 \times 10^6)(11.25 \times 10^{-3})$

$= 1.125 \times 10^6\ N \qquad P = 1125\ kN$

(b) Eccentric loading (reduced cross section)



	$A, 10^3\ mm^2$	$\bar{y}$, mm	$A\bar{y}\ (10^3\ mm^3)$	d, mm
1	3.75	87.5	328.125	76.5625
2	3.75	0	0	10.9375
3	2.50	-87.5	-218.75	98.4375
Σ	10.00		109.375	

$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{109.375 \times 10^3}{10.00 \times 10^3} = 10.9375\ mm$

The centroid lies 10.9375 mm from the midpoint of the web.

$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(150)(25)^3 + (3.75 \times 10^3)(76.5625)^2 = 22.177 \times 10^6\ mm^4$

$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(25)(150)^3 + (3.75 \times 10^3)(10.9375)^2 = 7.480 \times 10^6\ mm^4$

$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12}(100)(25)^3 + (2.50 \times 10^3)(98.4375)^2 = 24.355 \times 10^6\ mm^4$

$I = I_1 + I_2 + I_3 = 54.012 \times 10^6\ mm^4 = 54.012 \times 10^{-6}\ m^4$

$c = 10.9375 + 75 + 25 = 110.9375\ mm = 0.1109375\ m$

$M = Pe \quad$ where $e = 10.4375\ mm = 10.4375 \times 10^{-3}\ m$

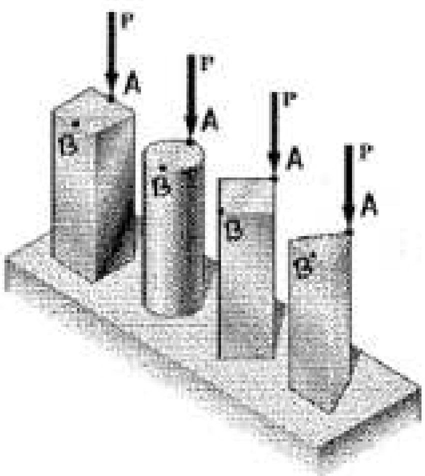
$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP \qquad A = 10.00 \times 10^{-3}\ m^2$

$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{10.00 \times 10^{-3}} + \frac{(10.9375 \times 10^{-3})(0.1109375)}{54.012 \times 10^{-6}} = 122.465\ m^{-2}$

$P = -\frac{\sigma}{K} = -\frac{(-100 \times 10^6)}{122.465} = 817 \times 10^3\ N \qquad P = 817\ kN$

Problem 4.117

4.117 The four bars shown have the same cross-sectional area. For the given loadings, show that (*a*) the maximum compressive stresses are in the ratio 4:5:7:9, (*b*) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note*: the cross section of the triangular bar is an equilateral triangle.)



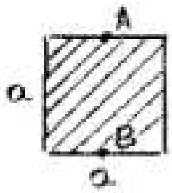
Stresses

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Pec_A}{I}$

$$= -\frac{P}{A}\left(1 + \frac{Aec_A}{I}\right)$$

At B $\quad \sigma_B = -\frac{P}{A} + \frac{Pec_B}{I}$

$$= \frac{P}{A}\left(\frac{Aec_B}{I} - 1\right)$$



$$A_1 = a^2, \quad I_1 = \tfrac{1}{12}a^4, \quad c_A = c_B = \tfrac{1}{2}a, \quad e = \tfrac{1}{2}a$$

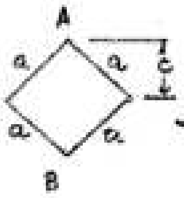
$$\sigma_A = -\frac{P}{A}\left(1 + \frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^2}\right) = -4\frac{P}{A_1}$$

$$\sigma_B = \frac{P}{A}\left(\frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^2} - 1\right) = 2\frac{P}{A_1}$$

$$A_2 = \pi c^2 = a^2 \quad \therefore c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4}c^4$$

$$\sigma_A = -\frac{P}{A_2}\left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4}\right) = -5\frac{P}{A_2}$$

$$\sigma_B = \frac{P}{A_2}\left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4} - 1\right) = 3\frac{P}{A_2}$$



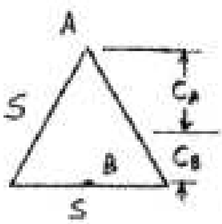
$$A_3 = a^2 \qquad c = \frac{\sqrt{2}}{2}a \qquad I_3 = \tfrac{1}{12}a^4 \qquad e = c$$

$$\sigma_A = -\frac{P}{A_3}\left(1 + \frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4}\right) = -7\frac{P}{A_3}$$

$$\sigma_B = \frac{P}{A_3}\left(\frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4} - 1\right) = 5\frac{P}{A_3}$$

$$A_4 = \tfrac{1}{2}(s)(\tfrac{\sqrt{3}}{2}s) = \tfrac{\sqrt{3}}{4}s^2 \qquad I_4 = \tfrac{1}{36}s(\tfrac{\sqrt{3}}{2}s)^3 = \tfrac{\sqrt{3}}{96}s^4$$

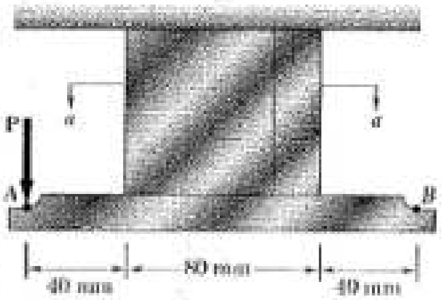
$$c_A = \tfrac{2}{3}\tfrac{\sqrt{3}}{2}s = \tfrac{s}{\sqrt{3}} = e \qquad c_B = \tfrac{s}{2\sqrt{3}}$$

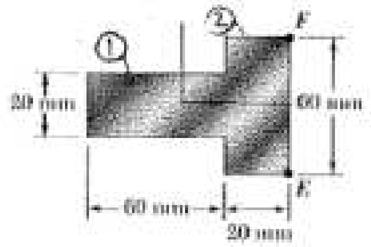


$$\sigma_A = -\frac{P}{A_4}\left(1 + \frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4}\right) = -9\frac{P}{A_4}$$

$$\sigma_B = \frac{P}{A_4}\left(\frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{2\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4} - 1\right) = 3\frac{P}{A_4}$$

Problem 4.118





Section a-a

4.118 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (*a*) the largest vertical force **P** that can be applied at point *A*, (*b*) the corresponding location of the neutral axis of section *a-a*.

Locate centroid.

	A, mm^2	$\bar{y}_0, mm$	$A\bar{y}_0, mm^3$
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{Y}_0 = \frac{\Sigma A\bar{y}_0}{\Sigma A} = \frac{120\times10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple: $M = Pe$

$$e = 40 + 50 = 90 \text{ mm} = 0.090 \text{ m}$$

$$I_1 = \frac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840\times10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360\times10^6 \text{ mm}^4 = 1.360\times10^{-6} \text{ m}^4, \qquad A = 2400\times10^{-6} \text{ m}^2$$

(a) Based on tensile stress at left edge: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = KP$$

$$K = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400\times10^{-6}} - \frac{(0.090)(-0.050)}{1.360\times10^{-6}} = 3.7255\times10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150\times10^6}{3.7255\times10^3} = 40.3\times10^3 \text{ N} = 40.3 \text{ kN}$$ ◄

(b) Location of neutral axis: $\sigma = 0$

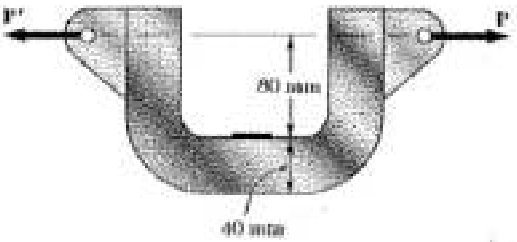
$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360\times10^{-6}}{(2400\times10^{-6})(0.090)} = 6.30\times10^{-3} \text{ m} = 6.30 \text{ mm}$$

The neutral axis lies 6.30 mm to the right of the centroid or 56.30 mm from the left face. ◄

Problem 4.120

4.120 The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and the strain on the inner edge was measured and found to be 450 μ, determine the magnitude P of the forces. Use $E = 200$ GPa.



At the strain gage location

$\sigma = E\varepsilon = (200\times10^9)(450\times10^{-6})$

$= 90\,\text{MPa}$

$A = (40)(40) = 1600\ \text{mm}^2$

$I = \frac{1}{12}(40)(40)^3 = 213333.3\ \text{mm}^4$

$e = 80 + 20 = 100\ \text{mm}.$

$c = 20\ \text{mm}.$

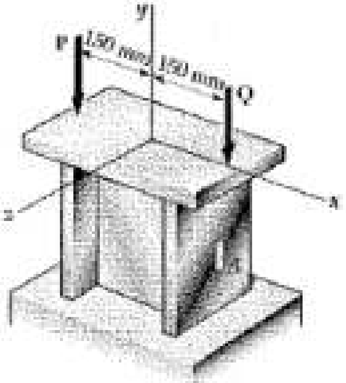
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

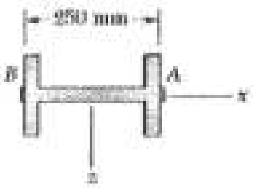
$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600\times10^{-6}} + \frac{(0.1)(0.02)}{213333.3\times10^{-12}} = 10000\ \text{m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{90000\times10^3}{10000} = 9000\,\text{N}$$

$P = 9\,\text{kN}$ ◄

Problem 4.121





$A = 6450 \text{ mm}^2$
$I_z = 114 \times 10^6 \text{ mm}^4$

4.121 A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points *A* and *B* on the center line of the outer faces of the flanges have been measured and found to be

$$\epsilon_A = -400 \times 10^{-6} \text{ mm/mm} \qquad \epsilon_B = -300 \times 10^{-6} \text{ mm/mm}$$

Knowing that $E = 200$ GPa, determine the magnitude of each load.

Stresses at A and B from strain gages

$$\sigma_A = E\varepsilon_A = (200\times10^9)(-400\times10^{-6}) = -80 \text{ MPa}$$

$$\sigma_B = E\varepsilon_B = (200\times10^9)(-300\times10^{-6}) = -60 \text{ MPa}$$

Centric force $F = P + Q$

Bending couple $M = 6P - 6Q$

$c = 125$ mm.

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{6450\times10^{-6}} + \frac{(0.15P - 0.15Q)(0.125)}{114\times10^{-6}}$$

$$-80\times10^6 = 9.5P - 319.5Q. \qquad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{6450\times10^{-6}} - \frac{(0.15P - 0.15Q)(0.125)}{114\times10^{-6}}$$

$$-60\times10^6 = -319.5P + 9.5Q \qquad (2)$$

Solving (1) and (2) simultaneously

$P = 195.4$ kN ◄

$Q = 256.2$ kN ◄

Problem 4.123

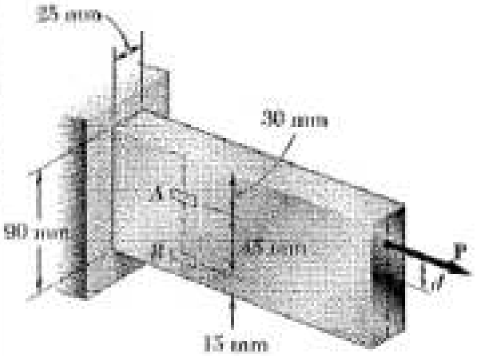
4.123 Solve prob. 4.122, assuming that the measured strains are

$$\varepsilon_A = +600\,\mu \qquad \varepsilon_B = +420\,\mu$$

4.122 An eccentric force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\,\mu \qquad \varepsilon_B = -70\,\mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.



$h = 15 + 45 + 30 = 90$ mm

$b = 25$ mm $\qquad c = \frac{1}{2}h = 45$ mm $= 0.045$ m

$A = bh = (25)(90) = 2.25\times10^3$ mm^2 $= 2.25\times10^{-3}$ m^2

$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times10^6$ mm^4 $= 1.51875\times10^{-6}$ m^4

$y_A = 60 - 45 = 15$ mm $= 0.015$ m, $\quad y_B = 15 - 45 = -30$ mm $= -0.030$ m

Stresses from strain gages at A and B:

$\sigma_A = E\varepsilon_A = (200\times10^9)(600\times10^{-6}) = 120\times10^6$ Pa

$\sigma_B = E\varepsilon_B = (200\times10^9)(420\times10^{-6}) = 84\times10^6$ Pa

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting, $\quad \sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(36\times10^6)}{0.045} = -1215 \text{ N·m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting,

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(84\times10^6) - (-0.030)(120\times10^6)]}{0.045}$$

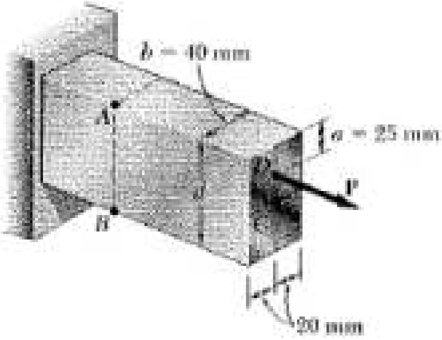
$$= 243\times10^3 \text{ N}$$

$M = -Pd$

(a) $\quad \therefore\ d = -\dfrac{M}{P} = -\dfrac{-1215}{243\times10^3} = 5\times10^{-3}$ m $\qquad d = 5.00$ mm

(b) $\qquad P = 243$ kN

Problem 4.124



4.124 The eccentric axial force **P** acts at point *D*, which must be located 25 mm below the top surface of the steel bar shown. For *P* = 60 kN, determine (*a*) the depth *d* of the bar for which the tensile stress at point *A* is maximum, (*b*) the corresponding stress at point *A*.

$$A = bd \qquad I = \frac{1}{12} b d^3$$

$$c = \frac{1}{2} d \qquad e = \frac{1}{2} d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

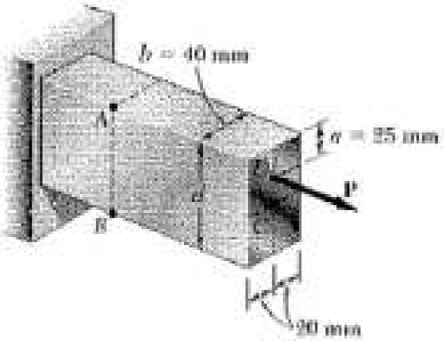
$$\sigma_A = \frac{P}{b}\left\{\frac{1}{d} + \frac{12(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^3}\right\} = \frac{P}{b}\left\{\frac{4}{d} - \frac{6a}{d^2}\right\}$$

(a) Depth d for maximum σ_A. Differentiate with respect to d.

$$\frac{d\sigma_A}{dd} = \frac{P}{b}\left\{-\frac{4}{d^2} + \frac{12a}{d^3}\right\} = 0 \qquad d = 3a = 75 \text{ mm}$$

(b) $$\sigma_A = \frac{60\times10^3}{40\times10^{-3}}\left\{\frac{4}{75\times10^{-3}} - \frac{(6)(25\times10^{-3})}{(75\times10^{-3})^2}\right\} = 40\times10^6 \text{ Pa} = 40 \text{ MPa}$$

Problem 4.125



4.125 For the bar and loading of Prob. 4.124, determine (*a*) the depth *d* of the bar for which the compressive stress at point *B* is maximum, (*b*) the corresponding stress at point *B*.

4.124 The eccentric axial force **P** acts at point *D*, which must be located 25 mm below the top surface of the steel bar shown. For *P* = 60 kN, determine (*a*) the depth *d* of the bar for which the tensile stress at point *A* is maximum, (*b*) the corresponding stress at point *A*.

$$A = bd \qquad I = \frac{1}{12} b d^3$$

$$c = \frac{1}{2} d \qquad e = \frac{1}{2} d - a$$

$$\sigma_B = \frac{P}{A} - \frac{Pec}{I}$$

$$\sigma_B = \frac{P}{b}\left\{\frac{1}{d} - \frac{(12)(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^3}\right\} = \frac{P}{b}\left\{-\frac{2}{d} + \frac{6a}{d^2}\right\}$$

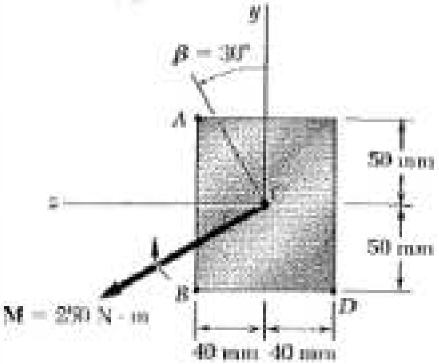
(a) Depth d for maximum σ_B: Differentiate with respect to d.

$$\frac{d\sigma_B}{dd} = \frac{P}{b}\left\{\frac{2}{d^2} - \frac{12a}{d^3}\right\} = 0 \qquad d = 6a = 150 \text{ mm}$$

(b) $$\sigma_B = \frac{60\times10^3}{40\times10^{-3}}\left\{-\frac{2}{150\times10^{-3}} + \frac{(6)(25\times10^{-3})}{(150\times10^{-3})^2}\right\} = -10\times10^6 \text{ Pa} = -10 \text{ MPa}$$

Problem 4.126

4.126 through *4.128* The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$I_z = \frac{1}{12}(80)(100)^3 = 6.6667 \times 10^6 \text{ mm}^4 = 6.6667 \times 10^{-6} \text{ m}^4$

$I_y = \frac{1}{12}(100)(80)^3 = 4.2667 \times 10^6 \text{ mm}^4 = 4.2667 \times 10^{-6} \text{ m}^4$

$y_A = -y_B = -y_D = 50 \text{ mm}$

$z_A = z_B = -z_D = 40 \text{ mm}$

$M_y = -250 \sin 30° = -125 \text{ N}\cdot\text{m} \qquad M_z = 250 \cos 30° = 216.51 \text{ N}\cdot\text{m}$

(a) $\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(0.040)}{4.2667 \times 10^{-6}}$

$= -2.80 \times 10^6 \text{ Pa}$ $\qquad -2.80 \text{ MPa}$ ◄

(b) $\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(-0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(0.040)}{4.2667 \times 10^{-6}}$

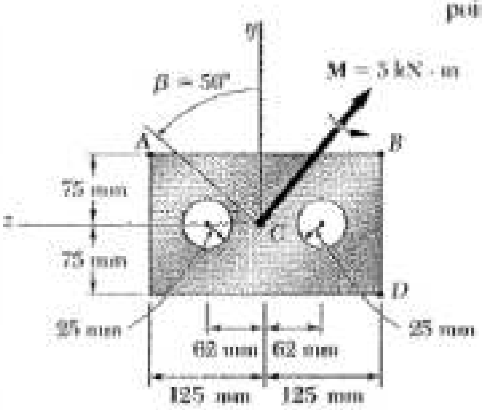
$= 0.452 \times 10^3 \text{ Pa}$ $\qquad 0.452 \text{ MPa}$ ◄

(c) $\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(-0.040)}{4.2667 \times 10^{-6}}$

$= 2.80 \times 10^6 \text{ Pa}$ $\qquad 2.80 \text{ MPa}$ ◄

Problem 4.128

4.126 through *4.128* The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$$M_z = -5 \sin 40° = -3.214 \text{ kNm}$$

$$M_y = 5 \cos 40° = 3.83 \text{ kNm}$$

$$y_A = y_B = -y_D = 75 \text{ mm}$$

$$z_A = -z_B = -z_D = 125 \text{ mm}$$

$$I_z = \frac{1}{12}(250)(150)^3 - 2\left[\frac{\pi}{4}(25)^4\right] = 69.7\times10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12}(150)(250)^3 - 2\left[\frac{\pi}{4}(25)^4 + \pi(25)^2(62)^2\right] = 187.2\times10^6 \text{ mm}^4$$

(a) $\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-3214)(0.075)}{69.7\times10^{-6}} + \frac{(3830)(0.125)}{187.2\times10^{-6}}$

$= 6.016 \text{ MPa}$ $\qquad \sigma_A = 6.02 \text{ MPa}$ ◄

(b) $\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-3214)(0.075)}{69.7\times10^{-6}} + \frac{(3830)(-0.125)}{187.2\times10^{-6}}$

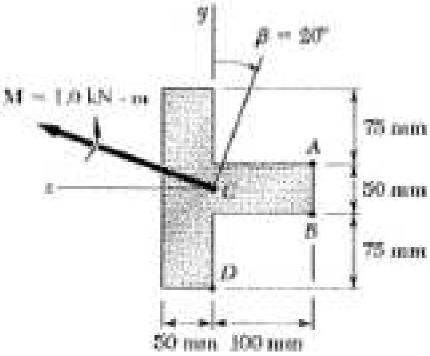
$= 0.9 \text{ MPa}$ $\qquad \sigma_B = 0.9 \text{ MPa}$ ◄

(c) $\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-3214)(-0.075)}{69.7\times10^{-6}} + \frac{(3830)(-0.125)}{187.2\times10^{-6}}$

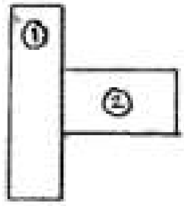
$= -6.016 \text{ MPa}$ $\qquad \sigma_D = -6.02 \text{ MPa}$ ◄

Problem 4.129

4.129 through 4.131 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



Locate centroid



	A, mm^2	$\bar{z}, mm$	$A\bar{z}, mm^3$
①	10000	-25	-250 000
②	5000	50	250 000
Σ	15000		0

The centroid lies at point C

$$I_z = \frac{1}{12}(50)(200)^3 + \frac{1}{12}(100)(50)^3 = 34.375 \times 10^6 \, mm^4.$$

$$I_y = \frac{1}{3}(200)(50)^3 + \frac{1}{3}(50)(100)^3 = 25 \times 10^6 \, mm^4.$$

$$y_A = -y_B = 25 mm, \qquad y_D = -100 mm.$$

$$z_A = z_B = -100 mm, \qquad z_D = 0$$

$$M_z = 1 \cos 20° = 0.94 \, kNm.$$

$$M_y = 1 \sin 20° = 0.342 \, kNm$$

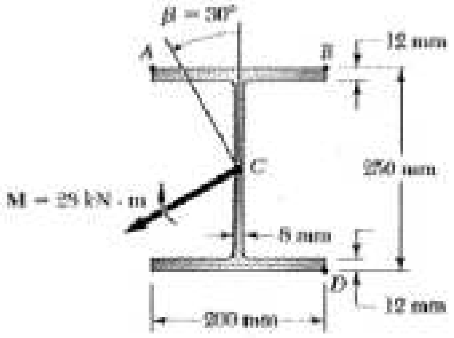
(a) $\sigma_A = -\dfrac{M_z y_A}{I_z} + \dfrac{M_y z_A}{I_y} = -\dfrac{(940)(0.025)}{34.375 \times 10^{-6}} + \dfrac{(342)(-0.1)}{25 \times 10^{-6}} = 2.05 \, MPa.$ ◄

(b) $\sigma_B = -\dfrac{M_z y_B}{I_z} + \dfrac{M_y z_B}{I_y} = -\dfrac{(940)(-0.025)}{34.375 \times 10^{-6}} + \dfrac{(342)(-0.1)}{25 \times 10^{-6}} = -0.684 \, MPa.$ ◄

(c) $\sigma_D = -\dfrac{M_z y_D}{I_z} + \dfrac{M_y z_D}{I_y} = -\dfrac{(940)(-0.1)}{34.375 \times 10^{-6}} + \dfrac{(342)(0)}{25 \times 10^{-6}} = 2.73 \, MPa.$ ◄

Problem 4.130

4.129 through 4.131 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



Flange: $I_z = \frac{1}{12}(200)(12)^3 + (200)(12)(119)^2$

$= 34015200 \text{ mm}^4.$

$I_y = \frac{1}{12}(12)(200)^3 = 8\times10^6 \text{ mm}^4.$

Web: $I_z = \frac{1}{12}(8)(226)^3 = 7695451 \text{ mm}^4$

$I_y = \frac{1}{12}(226)(8)^3 = 9643 \text{ mm}^4.$

Total: $I_z = (2)(34015200) + 7695451 = 75725851 \text{ mm}^4$

$I_y = (2)(8\times10^6) + 9643 = 16009643 \text{ mm}^4$

$y_A = y_B = -y_D = 125 \text{ mm}; \quad z_A = -z_B = -z_D = 100 \text{ mm}.$

$M_z = 28 \cos 30° = 24.25 \text{ kNm}.$

$M_y = -28 \sin 30° = -14 \text{ kNm}$

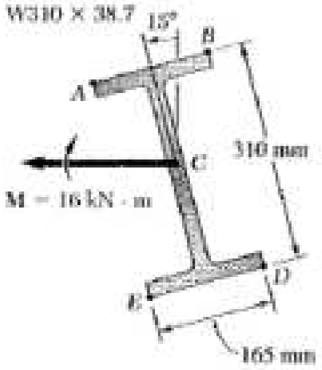
(a) $\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(24.25\times10^3)(0.125)}{75725851\times10^{-12}} + \frac{(-14\times10^3)(0.1)}{16009643\times10^{-12}} = -127.5 \text{ MPa}$ ◄

(b) $\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(24250)(0.125)}{75725851\times10^{-12}} + \frac{(-14000)(-0.1)}{16009643\times10^{-12}} = 47.4 \text{ MPa}$ ◄

(c) $\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(24250)(-0.125)}{75725851\times10^{-12}} + \frac{(-14000)(-0.1)}{16009643\times10^{-12}} = 127.5 \text{ MPa}$ ◄

Problem 4.132

4.132 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



For W 310 × 38.7 rolled steel shape,

$$I_z = 85.1 \times 10^6 \text{ mm}^4 = 85.1 \times 10^{-6} \text{ m}^4$$

$$I_y = 7.27 \times 10^6 \text{ mm}^4 = 7.27 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = -y_E = (\tfrac{1}{2})(310) = 155 \text{ mm}$$

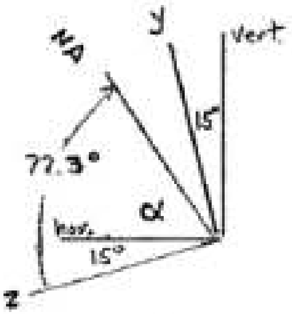
$$z_A = z_E = -z_B = -z_D = (\tfrac{1}{2})(165) = 82.5 \text{ mm}$$

$$M_z = (16 \times 10^3) \cos 15^\circ = 15.455 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_y = (16 \times 10^3) \sin 15^\circ = 4.1411 \times 10^3 \text{ N} \cdot \text{m}$$

(a) $$\tan \varphi = \frac{I_z}{I} \tan \theta = \frac{85.1 \times 10^{-6}}{7.27 \times 10^{-6}} \tan 15^\circ = 3.1365$$

$$\varphi = 72.3^\circ$$



$$\alpha = 72.3 - 15 = 57.3^\circ$$ ◄

(b) Maximum tensile stress occurs at point E.

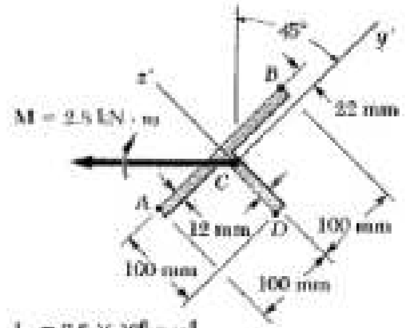
$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455 \times 10^3)(-155 \times 10^{-3})}{85.1 \times 10^{-6}} + \frac{(4.1411 \times 10^3)(82.5 \times 10^{-3})}{7.27 \times 10^{-6}}$$

$$= 75.1 \times 10^6 \text{ Pa}$$

$\sigma_E = 75.1$ MPa ◄

Problem 4.135

4.135 and 4.136 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



$I_{y'} = 2.8 \times 10^6\ \text{mm}^4$
$I_{z'} = 8.9 \times 10^6\ \text{mm}^4$

$I_{z'} = 8.9 \times 10^6\ \text{mm}^4, \quad I_{y'} = 2.8 \times 10^6\ \text{mm}^6$

$z_A' = z_B' = 22\ \text{mm}.$

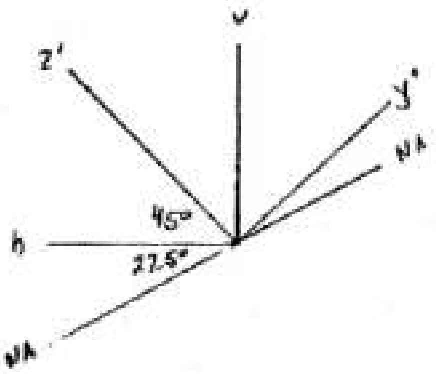
$z_D = -100 + 22 = -78$

$y_A = -100\ \text{m}, \quad y_B = 100\ \text{m}, \quad y_D = -12\ \text{mm}$

$M_{y'} = -2.8 \sin 45° = -1.98\ \text{kNm}.$

$M_{z'} = 2.8 \cos 45° = 1.98\ \text{kNm}.$

(a) $\tan \varphi = \dfrac{I_{z'}}{I_{y'}} \tan \theta = \dfrac{8.9}{2.8} \tan(-45°) = -3.1786.$



$\varphi = -72.5°$

$\alpha = 72.5° - 45° = 27.5°$

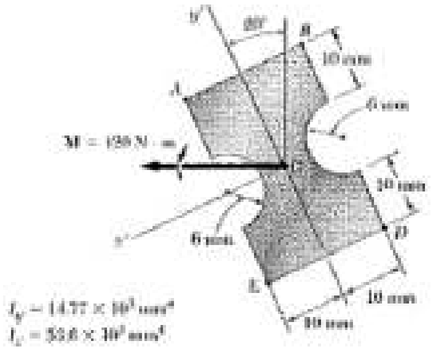
(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(1980)(-0.012)}{8.9 \times 10^{-6}} + \frac{(-1980)(-0.078)}{2.8 \times 10^{-6}}$$

$$= 57.8\ \text{MPa}$$

Problem 4.136

4.135 and 4.136 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

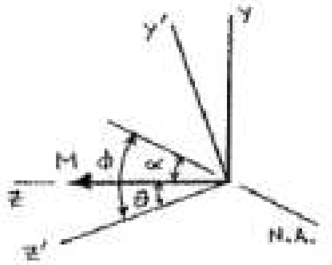


$I_{z'} = 53.6 \times 10^3\ mm^4 = 53.6 \times 10^{-9}\ m^4$

$I_{y'} = 14.77 \times 10^3\ mm^4 = 14.77 \times 10^{-9}\ m^4$

$M_{z'} = 120 \sin 70° = 112.763\ N \cdot m$

$M_{y'} = 120 \cos 70° = 41.042\ N \cdot m$



(a) $\theta = 20°$.

$\tan \varphi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{53.6 \times 10^{-9}}{14.77 \times 10^{-9}} \tan 20°$

$= 1.32054$

$\varphi = 52.871°$

$\alpha = 52.871° - 20°$ $\qquad \alpha = 32.9°$ ◀

(b) The maximum tensile stress occurs at point E.

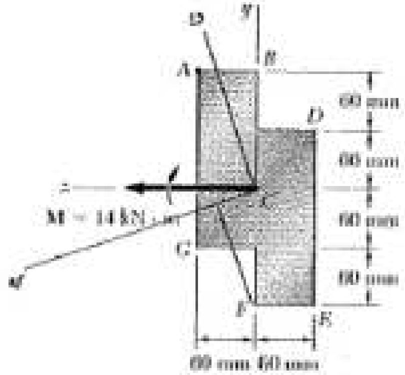
$y'_E = -16\ mm = -0.016\ m, \qquad z'_E = 10\ mm = 0.010\ m$

$\sigma_E = -\frac{M_{z'} y'_E}{I_{z'}} + \frac{M_{y'} z'_E}{I_{y'}} = -\frac{(112.763)(-0.016)}{53.6 \times 10^{-9}} + \frac{(41.042)(0.010)}{14.77 \times 10^{-9}}$

$= 61.448 \times 10^6\ Pa$ $\qquad \sigma_E = 61.4\ MPa$ ◀

Problem 4.137

*4.137 through *4.139 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.



$$I_y = 2\left\{\tfrac{1}{3}(180)(60)^3\right\} = 25.92\times10^6\ mm^4$$

$$I_z = 2\left\{\tfrac{1}{12}(60)(180)^3 + (60)(180)(30)^2\right\} = 77.76\times10^6\ mm^4$$

$$I_{yz} = 2\{(60)(180)(30)(30)\} = 19.44\times10^6\ mm^4$$

Using Mohr's circle determine the principal axes and principal moments of inertia.

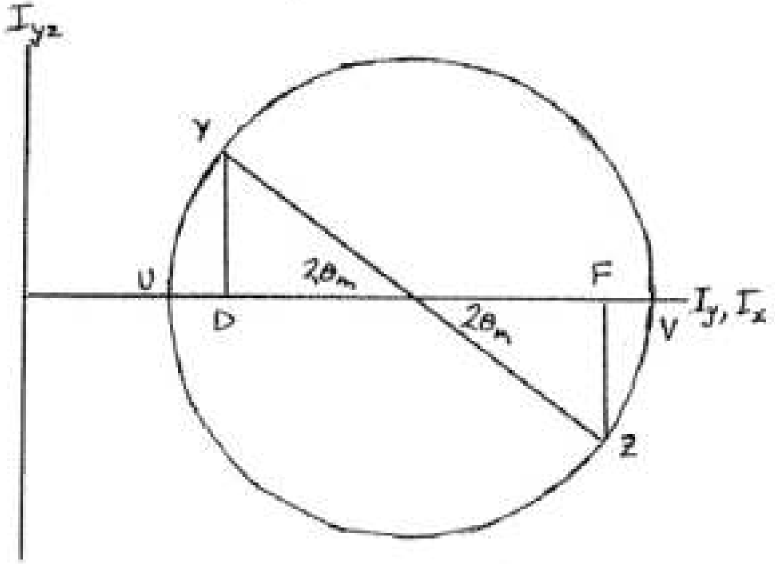
$Y = (25.92\times10^6\ mm^4,\ 19.44\times10^6\ mm^4)$

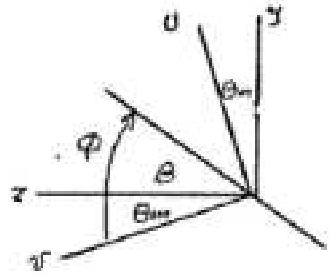
$Z = (77.76\times10^6\ mm^4,\ -19.44\times10^6\ mm^4)$

$E = (51.84\times10^6\ mm^4,\ 0)$

$$\tan 2\theta_m = \frac{DY}{DE} = \frac{19.44}{25.92}$$

$2\theta_m = 36.87^\circ \qquad \theta_m = 18.435^\circ$





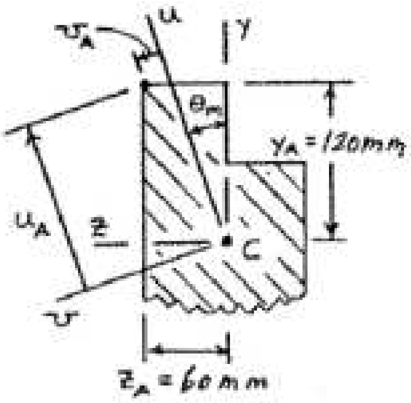
$R = \sqrt{\overline{DE}^2 + \overline{DY}^2} = 32.4\times10^6\ mm^4$

$I_u = (51.84 - 32.4)10^6 = 19.44\times10^6\ mm^4$

$I_v = (51.84 + 32.4)10^6$
$= 84.24\times10^6\ mm^4$

$M_u = 14\sin 18.435^\circ = 4.43\ kNm$

$M_v = 14\cos 18.435^\circ = 13.3\ kNm$



$u_A = 0.12\cos 18.435^\circ + 0.06\sin 18.435^\circ = 0.1328\ m$

$v_A = -0.12\sin 18.435^\circ + 0.06\cos 18.435^\circ = 0.01897\ m$

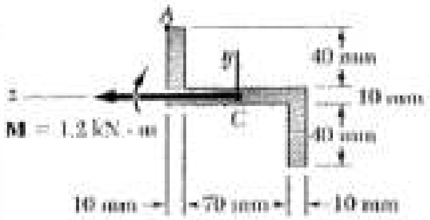
$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u}$$

$$= -\frac{(13300)(0.1328)}{84.24\times10^{-6}} + \frac{(4430)(0.01897)}{19.44\times10^{-6}}$$

$= -16.64\ MPa \qquad \sigma_A = -16.6\ MPa$

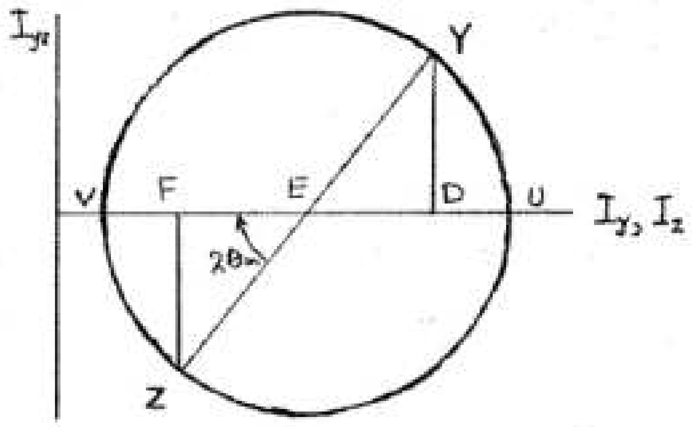
Problem 4.138

*4.137 through *4.139 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.



$I_y = 1.894 \times 10^6\ \text{mm}^4$
$I_z = 0.614 \times 10^6\ \text{mm}^4$
$I_{yz} = +0.800 \times 10^6\ \text{mm}^4$

Using Mohr's circle, determine the principal axes and the principal moments of inertia.



$Y\ (1.894, 0.800) \times 10^6\ \text{mm}^4$

$Z\ (0.614, 0.800) \times 10^6\ \text{mm}^4$

$E\ (1.254, 0\ ) \times 10^6\ \text{mm}^4$

$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-6} = 1.0245 \times 10^6\ \text{mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6\ \text{mm}^4 = 0.2295 \times 10^6\ \text{mm}^4 = 0.2295 \times 10^{-6}\ \text{m}^4$$

$$I_u = (1.254 + 1.0245) \times 10^6\ \text{mm}^4 = 2.2785 \times 10^6\ \text{mm}^4 = 2.2785 \times 10^{-6}\ \text{m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \qquad \theta_m = 25.67°$$

$$M_v = M \cos\theta_m = (1.2 \times 10^3)\cos 25.67° = 1.0816 \times 10^3\ \text{N}\cdot\text{m}$$

$$M_u = -M \sin\theta_m = -(1.2 \times 10^3)\sin 25.67° = -0.5198 \times 10^3\ \text{N}\cdot\text{m}$$

$$u_A = y_A \cos\theta_m - z_A \sin\theta_m = 45 \cos 25.67° - 45 \sin 25.67° = 21.07\ \text{mm}$$

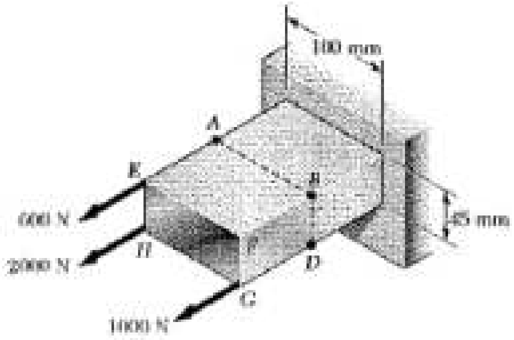
$$v_A = z_A \cos\theta_m + y_A \sin\theta_m = 45 \cos 25.67° + 45 \sin 25.67° = 60.05\ \text{mm}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6\ \text{Pa} \qquad \sigma_A = 113.0\ \text{MPa}$$

Problem 4.140

4.140 For the loading shown, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

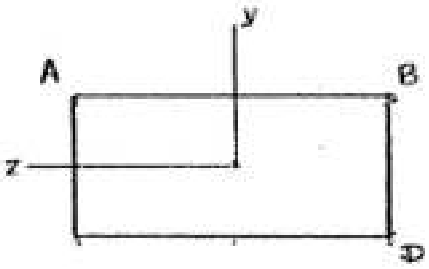


Add y and z axes as shown

$A = (100)(45) = 4500 \text{ mm}^2.$

$I_z = \frac{1}{12}(100)(45)^3 = 759375 \text{ mm}^4$

$I_y = \frac{1}{12}(45)(100)^3 = 3.75 \times 10^6 \text{ mm}^4$



Resultant force and bending couples

$P = 600 + 2000 + 1000 = 3600 \text{ N}$

$M_z = [-(600)(0.225) + (2000)(0.225) + (1000)(0.225)](1 \times 10^{-1})$
$= 54 \text{ N mm}$

$M_y = (600)(0.05) + (2000)(0.05) - (1000)(0.05)$
$= 80 \text{ Nm}$

(a) $\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{(54)(0.0225)}{759375 \times 10^{-12}} + \frac{(80)(0.05)}{3.75 \times 10^{-6}} = 266.7 \text{ kPa}$ ◄

$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{(54)(0.0225)}{1.944} + \frac{(800)(-2)}{9.6} = -1.87 \text{ MPa}$ ◄

(b) Intersection of neutral axis with line AB or its extension.

$\sigma = 0, \quad y = 0.0225 \text{ m}, \; z = ?$

$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y y}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{(54)(0.0225)}{759375 \times 10^{-12}} + \frac{80z}{3.75 \times 10^{-6}}$

$-0.8 \times 10^6 + 21.33z = 0 \qquad z = 0.0375 \text{ m}$

Intersects AB at 12.5 mm from A ◄

Intersection of neutral axis with line BD or its extension.

$\sigma = 0 \qquad z = -50 \text{ mm} \quad y = ?$

$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{54y}{759375 \times 10^{-12}} + \frac{(80)(-0.05)}{3.75 \times 10^{-6}}$

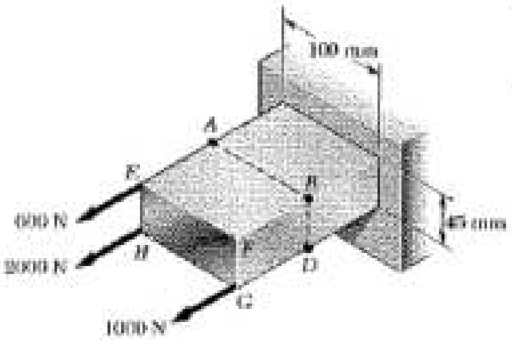
$-0.2667 \times 10^6 - 71.1111y = 0 \qquad y = -0.00375 \text{ m}$

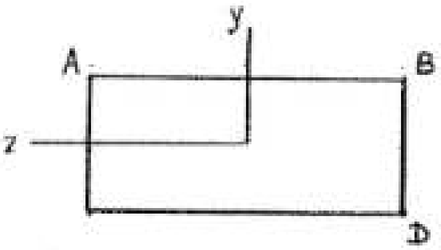
Intersects BD at 18.75 mm from D ◄

Problem 4.141

4.141 Solve Prob. 4.140, assuming that the magnitude of the force applied at G is increased from 1.0 kN to 1.6 kN.

4.140 For the loading shown, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.





Add y and z axes as shown.

$A = (100 \times 45) = 4500 \text{ mm}^2$

$I_z = \frac{1}{12}(100)(45)^3 = 759375 \text{ mm}^4$

$I_y = \frac{1}{12}(45)(100)^3 = 3.75 \times 10^6 \text{ mm}^4$

Resultant force and bending couples

$P = 600 + 2000 + 1600 = 4200 \text{ N}.$

$M_z = -(600)(0.0225) + (2000)(0.0225) + (1600)(0.0225)$
$= 67.5 \text{ Nm}$

$M_y = (600)(0.05) + (2000)(0.05) - (1600)(0.05)$
$= 50 \text{ Nm}.$

(a) $\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{4200}{0.0045} - \frac{(67.5)(0.0225)}{759375 \times 10^{-12}} + \frac{(50)(0.05)}{3.75 \times 10^{-6}} = -400 \text{ kPa}$ ◄

$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{4200}{0.0045} - \frac{(67.5)(0.0225)}{759375 \times 10^{-12}} + \frac{(50)(-0.05)}{3.75 \times 10^{-6}} = -1.73 \text{ MPa}$ ◄

(b) Intersection of neutral axis with line AB or its extension.

$\sigma = 0, \quad y = 0.9 \text{ in.} \quad z = ?$

$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{4200}{0.0045} - \frac{(67.5)(0.0225)}{759375 \times 10^{-12}} + \frac{50z}{3.75 \times 10^{-6}}$

$-1.0667 \times 10^6 + 13.3333 z = 0 \quad z = 0.08 \text{ m} = 80 \text{ mm}$

Does not intersect AB ◄

Intersection of neutral axis with line BD or its extension.

$\sigma = 0 \quad z = -2 \text{ in.} \quad y = ?$

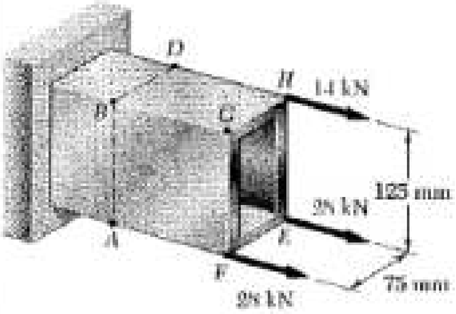
$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{4200}{0.0045} - \frac{67.5y}{759375 \times 10^{-12}} + \frac{(50)(-0.05)}{3.75 \times 10^{-6}}$

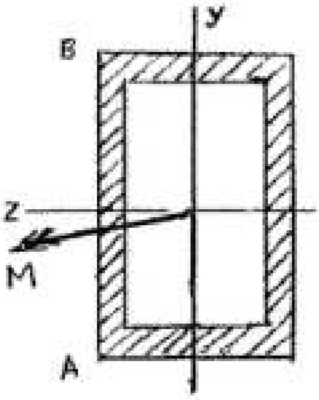
$10^6 \times 0.26667 - 88.8889 y = 0 \quad y = 0.003 \text{ m} = 3 \text{ mm}.$

Intersects BD at 19.5 mm from B ◄

Problem 4.142

4.142 The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.





Add y- and z-axes as shown. Cross section is a 75 mm × 125 mm rectangle with a 51 mm × 101 mm rectangular cutout.

$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283\times10^6\ \text{mm}^4 = 7.8283\times10^{-6}\ \text{m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781\times10^6\ \text{mm}^4 = 3.2781\times10^{-6}\ \text{m}^4$$

$$A = (75)(125)-(51)(101) = 4.224\times10^3\ \text{mm}^2 = 4.224\times10^{-3}\ \text{m}^2$$

Resultant force and bending couples:

$$P = 14+28+28 = 70\ \text{kN} = 70\times10^3\ \text{N}$$

$$M_z = -(62.5\ \text{mm})(14\ \text{kN}) + (62.5\ \text{mm})(28\ \text{kN}) + (62.5\ \text{mm})(28\ \text{kN}) = 2625\ \text{N·m}$$

$$M_y = -(37.5\ \text{mm})(14\ \text{kN}) + (37.5\ \text{mm})(28\ \text{kN}) + (37.5\ \text{mm})(28\ \text{kN}) = -525\ \text{N·m}$$

(a) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{70\times10^3}{4.224\times10^{-3}} - \frac{(2625)(-0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= 31.524\times10^6\ \text{Pa} \qquad \sigma_A = 31.5\ \text{MPa}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{70\times10^3}{4.224\times10^{-3}} - \frac{(2625)(0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= -10.39\times10^6\ \text{Pa} \qquad \sigma_B = -10.39\ \text{MPa}$$

(b) Let point H be the point where the neutral axis intersects AB.

$$z_H = 0.0375\ \text{m}, \quad y_H = \ ?, \quad \sigma_H = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

$$y_H = \frac{I_z}{M_z}\left(\frac{P}{A} + \frac{M_y z_H}{I_y}\right) = \frac{7.8283\times10^{-6}}{2625}\left[\frac{70\times10^3}{4.224\times10^{-3}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}\right]$$

$$= 0.03151\ \text{m} = 31.51\ \text{mm}$$

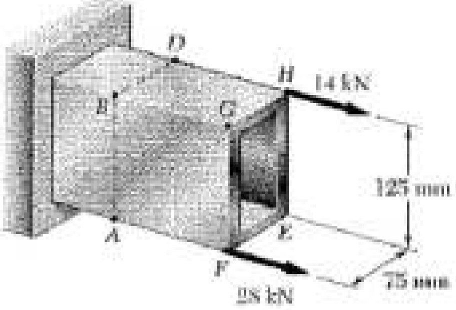
$$31.51 + 62.5 = 94.0\ \text{mm}$$

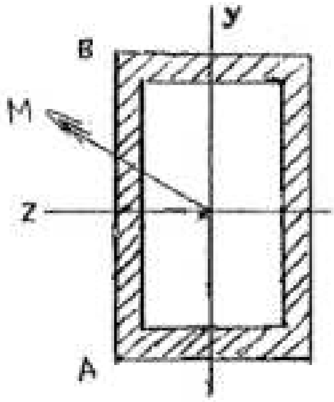
Answer: 94.0 mm above point A.

Problem 4.143

4.143 Solve Prob. 4.142, assuming that the 28-kN force at point E is removed.

4.142 The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points A and B, (*b*) the point where the neutral axis intersects line ABD.





Add y- and z-axes as shown. Cross section is a 75 mm × 125 mm rectangle with a 51 mm × 101 mm rectangular cutout.

$$I_z = \tfrac{1}{12}(75)(125)^3 - \tfrac{1}{12}(51)(101)^3 = 7.8283\times10^6\ \text{mm}^4$$
$$= 7.8283\times10^{-6}\ \text{m}^4$$
$$I_y = \tfrac{1}{12}(125)(75)^3 - \tfrac{1}{12}(101)(51)^3 = 3.2781\times10^6\ \text{mm}^4$$
$$= 3.2781\times10^{-6}\ \text{m}^4$$
$$A = (75)(125) - (51)(101) = 4.224\times10^3\ \text{mm}^2$$
$$= 4.224\times10^{-3}\ \text{m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 = 42\ \text{kN} = 42\times10^3\ \text{N}$$
$$M_z = -(62.5\ \text{mm})(14\ \text{kN}) + (62.5\ \text{mm})(28\ \text{kN})$$
$$= 875\ \text{N·m}$$
$$M_y = -(37.5\ \text{mm})(14\ \text{kN}) - (37.5\ \text{mm})(28\ \text{kN})$$
$$= 525\ \text{N·m}$$

(a) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{42\times10^3}{4.224\times10^{-3}} - \frac{(875)(-0.0625)}{7.8283\times10^{-6}} + \frac{(525)(0.0375)}{3.2781\times10^{-6}}$$
$$= 22.935\times10^6\ \text{Pa} \qquad \sigma_A = 22.9\ \text{MPa}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{42\times10^3}{4.224\times10^{-3}} - \frac{(875)(0.0625)}{7.8283\times10^{-6}} + \frac{(525)(0.0375)}{3.2781\times10^{-6}}$$
$$= 8.9831\times10^6\ \text{Pa} \qquad \sigma_B = 8.96\ \text{MPa}$$

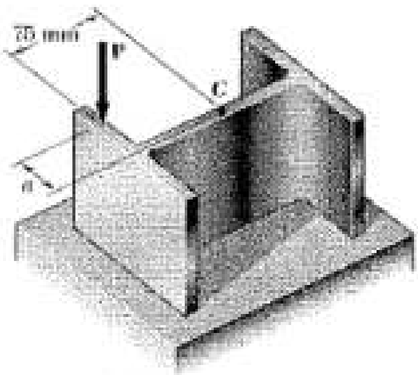
(b) Let point K be the point where the neutral axis intersects BD.

$$z_K = ?,\quad y_K = 0.0625\ \text{m},\quad \sigma_K = 0$$
$$0 = \frac{P}{A} - \frac{M_z y_K}{I_z} + \frac{M_y z_K}{I_y}$$
$$z_K = \frac{I_y}{M_y}\left(\frac{M_z y_K}{I_z} - \frac{P}{A}\right) = \frac{3.2781\times10^{-6}}{525}\left[\frac{(875)(0.0625)}{7.8283\times10^{-6}} - \frac{42\times10^3}{4.224\times10^{-3}}\right]$$
$$= -0.018465\ \text{m} = -18.465\ \text{mm}$$
$$37.5 + 18.465 = 56.0\ \text{mm}$$

Answer: 56.0 mm to the right of point B.

Problem 4.144

4.144 An axial load P of magnitude 50 kN is applied as shown to a short section of a W150 × 24 rolled-steel member. Determine the largest distance a for which the maximum compressive stress does not exceed 90 MPa.



Add y- and z- axes.

For W 150 × 24 rolled-steel section

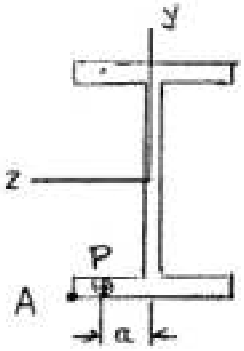
$A = 3060 \text{ mm}^2 = 3060 \times 10^{-6} \text{ m}^2$

$I_z = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$

$I_y = 1.83 \times 10^6 \text{ mm}^4 = 1.83 \times 10^{-6} \text{ m}^4$

$d = 160 \text{ mm}, \qquad b_f = 102 \text{ mm}$

$y_A = -\frac{d}{2} = -80 \text{ mm}, \quad z_A = \frac{b_f}{2} = 51 \text{ mm}$



$P = 50 \times 10^3 \text{ N}$

$M_z = -(50 \times 10^3)(75 \times 10^{-3}) = -3.75 \times 10^3 \text{ N}\cdot\text{m}$

$M_y = -Pa$

$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} \qquad \sigma_A = -90 \times 10^6 \text{ Pa}$$

$$M_y = \frac{I_y}{z_A}\left\{\frac{M_z y_A}{I_z} + \frac{P}{A} + \sigma_A\right\}$$

$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{\frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6)\right\}$$

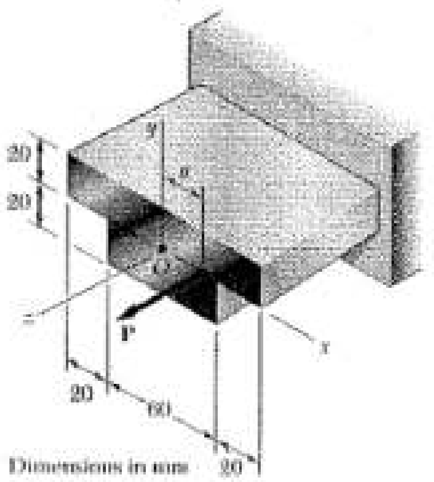
$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{+22.388 + 16.340 - 90\right\} \times 10^6$$

$$= -1.8398 \times 10^3 \text{ N}\cdot\text{m}$$

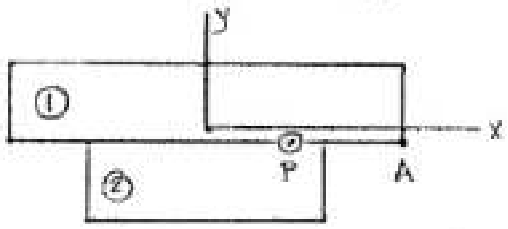
$$a = -\frac{M_y}{P} = -\frac{-1.8398 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-3} \text{ m} \qquad a = 36.8 \text{ mm}$$

Problem 4.145

4.145 A horizontal load **P** of magnitude 100 kN is applied to the beam shown. Determine the largest distance *a* for which the maximum tensile stress in the beam does not exceed 75 MPa.



Locate the centroid.



	A, mm²	$\bar{y}$, mm	$A\bar{y}$, mm³
①	2000	10	20×10^3
②	1200	−10	-12×10^3
Σ	3200		8×10^3

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8\times10^3}{3200} = 2.5 \text{ mm}$$

Move coordinate origin to the centroid.

Coordinates of load point: $x_P = a$, $y_P = -2.5$ mm

Bending couples $M_x = y_P P$ $M_y = -aP$

$$I_x = \tfrac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \tfrac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667\times10^6 \text{ mm}^4 = 0.40667\times10^{-6} \text{ m}^4$$

$$I_y = \tfrac{1}{12}(20)(100)^3 + \tfrac{1}{12}(20)(60)^3 = 2.0267\times10^6 \text{ mm}^4 = 2.0267\times10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$ $\sigma_A = 75\times10^6$ Pa, $P = 100\times10^3$ N

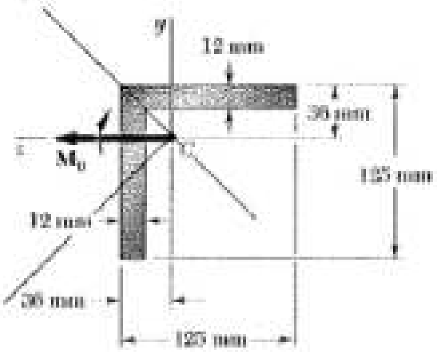
$$M_y = \frac{I_y}{x}\left\{\frac{P}{A} + \frac{M_x y}{I_x} - \sigma\right\}$$ For point A $x = 50$ mm, $y = -2.5$ mm

$$M_y = \frac{2.0267\times10^{-6}}{50\times10^{-3}}\left\{\frac{100\times10^3}{3200\times10^{-6}} + \frac{(-2.5)(100\times10^3)(-2.5\times10^{-3})}{0.40667\times10^{-6}} - 75\times10^6\right\}$$

$$= \frac{2.0267\times10^{-6}}{50\times10^{-3}}\left\{31.25 + 1.537 - 75\right\}\times10^6 = -1.7111\times10^3 \text{ N}\cdot\text{m}$$

$$a = -\frac{M_y}{P} = -\frac{(-1.7111\times10^3)}{100\times10^3} = 17.11\times10^{-3} \text{ m}$$ $a = 17.11$ mm ◄

Problem 4.146



4.146 A beam having the cross section shown is subjected to a couple M_0 that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 84 MPa. *Given:* $I_y = I_z = 4.7 \times 10^6$ mm^4, $A = 3064$ mm^2, $k_{min} = 25$ mm. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak_{min}^2$ and $I_{min} + I_{max} = I_y + I_z$.)

$$M_u = M_o \sin 45° = 0.70711\, M_o$$

$$M_v = M_o \cos 45° = 0.7071\, M_o$$

$$I_{min} = A k_{min}^2 = (3064)(25)^2 = 1.915 \times 10^6\ \text{mm}^4$$

$$I_{max} = I_y + I_z - I_{min} = (4.7 + 4.7 - 1.915)10^6 = 7.485 \times 10^6\ \text{mm}^4$$

$$u_B = y_B \cos 45° + z_B \sin 45° = -89 \cos 45° + 24 \sin 45° = -46\ \text{mm}$$

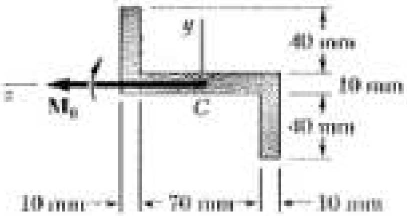
$$v_B = z_B \cos 45° - y_B \sin 45° = 24 \cos 45° - (-89) \sin 45° = 80\ \text{mm}.$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u} = -0.70711\, M_o \left[-\frac{u_B}{I_{min}} + \frac{v_B}{I_{max}} \right]$$

$$= 0.70711\, M_o \left[-\frac{(-0.046)}{1.915 \times 10^{-6}} + \frac{0.08}{7.485 \times 10^{-6}} \right] = 24543\, M_o$$

$$M_o = \frac{\sigma_B}{24543} = \frac{84 \times 10^6}{24543} = 3.42\ \text{kN m}.$$

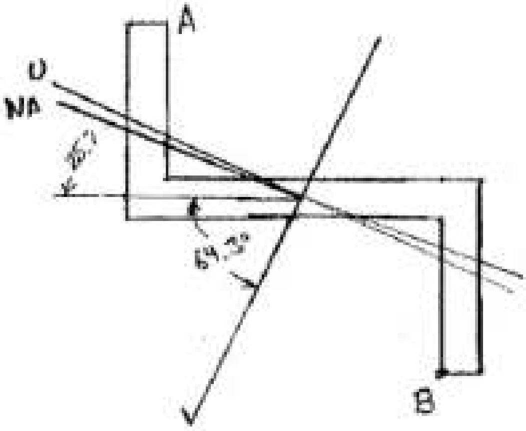
Problem 4.148



4.148 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes 25.7° and 64.3°.

$$I_v = I_{max} = 2.28 \times 10^6\ \text{mm}^4 = 2.28 \times 10^{-6}\ \text{m}^4$$

$$I_u = I_{min} = 0.23 \times 10^6\ \text{mm}^4 = 0.23 \times 10^{-6}\ \text{m}^4$$



$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan \varphi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3° = 20.597$$

$$\varphi = 87.22°$$

Points A and B are farthest from the neutral axis.

$$u_B = y_B \cos 64.3° + z_B \sin 64.3° = (-45)\cos 64.3° + (-35)\sin 64.3°$$

$$= -51.05\ \text{mm}$$

$$v_B = z_B \cos 64.3° - y_B \sin 64.3° = (-35)\cos 64.3° - (-45)\sin 64.3°$$

$$= +25.37\ \text{mm}$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u}$$

$$80 \times 10^6 = -\frac{(M_0 \cos 64.3°)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_0 \sin 64.3°)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$

$$= 93.81 \times 10^3\ M_0$$

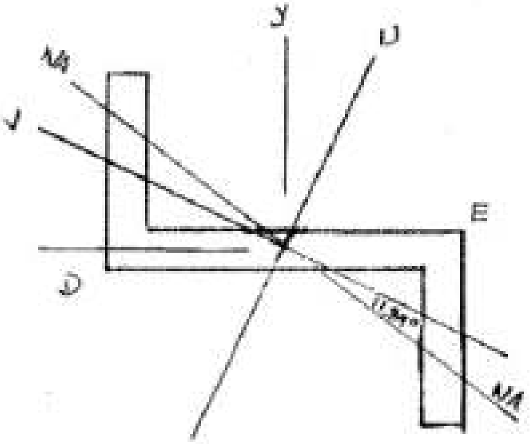
$$M_0 = \frac{80 \times 10^6}{109.1 \times 10^3}$$

$M_0 = 733\ \text{N}\cdot\text{m}$

Problem 4.149

4.149 Solve Prob. 4.148 assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

4.148 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes 25.7° and 64.3°.



$$I_v = I_{min} = 0.23 \times 10^{6}\ \text{mm}^4 = 0.23 \times 10^{-6}\ \text{m}^4$$

$$I_u = I_{max} = 2.28 \times 10^{6}\ \text{mm}^4 = 2.28 \times 10^{-6}\ \text{m}^4$$

$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta$$

$$= \frac{0.23\times10^{-6}}{2.28\times10^{-6}}\tan 64.3° = 0.20961$$

$$\varphi = 11.84°$$

Points D and E are farthest from the neutral axis.

$$u_D = y_D \cos 25.7° - z_D \sin 25.7° = (-5)\cos 25.7° - 45 \sin 25.7°$$
$$= -24.02\ \text{mm}$$

$$v_D = z_D \cos 25.7° + y_D \sin 25.7° = 45 \cos 25.7° + (-5)\sin 25.7°$$
$$= 38.38\ \text{mm}$$

$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -\frac{(M_0 \cos 64.3°)(-24.02\times10^{-3})}{0.23\times10^{-6}} + \frac{(M_0 \sin 64.3°)(38.38\times10^{-3})}{2.28\times10^{-6}}$$

$$80\times10^{6} = 60.48\times10^{3}\ M_0$$

$$M_0 = 1.323 \times 10^{3}\ \text{N}\cdot\text{m} \qquad M_0 = 1.323\ \text{kN}\cdot\text{m}$$

Problem 4.151

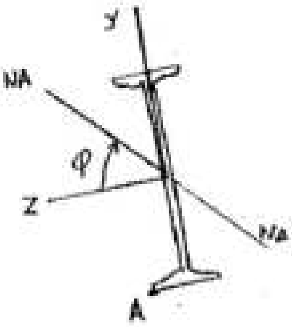
4.151 A couple $\mathbf{M}_0$ acting in a vertical plane is applied to a W310 × 23.8 rolled-steel beam, whose web forms an angle θ with the vertical. Denoting by σ_0 the maximum stress in the beam when $\theta = 0$, determine the angle of inclination θ of the beam for which the maximum stress is $2\sigma_0$.

For W 310×23.6 rolled steel section

$I_z = 42.7 \times 10^6\ \text{mm}^4 \quad I_y = 1.16 \times 10^6\ \text{mm}^4$

$d = 305\ \text{mm} \qquad b_f = 101\ \text{mm}$

$y_A = -\frac{d}{2} \qquad z_A = \frac{b_f}{2}$



$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{42.7}{1.16}\tan\theta = 36.81\ \tan\theta$

Point A is farthest from the neutral axis.

$M_y = M_0\sin\theta \qquad M_z = M_0\cos\theta$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_0 d}{2I_z}\cos\theta + \frac{M_0 b_f}{2I_y}\sin\theta$$

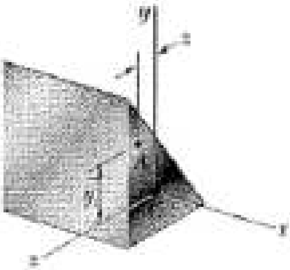
$$= \frac{M_0 d}{2I_z}\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right)$$

For $\theta = 0$ $\qquad \sigma_0 = \frac{M_0 d}{2I_z}$

$$\sigma_A = \sigma_0\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right) = 2\sigma_0$$

$$\tan\theta = \frac{I_y d}{I_z b_f} = \frac{(1.16)(305)}{(42.7)(101)} = 0.08204 \qquad \theta = 4.70°$$ ◄

Problem 4.152



4.152 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the vertical plane xy. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = -\frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_z$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_z the moment of the couple.

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1 and C_2 are constants.

$$M_y = \int z\sigma_A\, dA = C_1 \int yz\, dA + C_2 \int z^2 dA$$

$$= I_{yz} C_1 + I_y C_2 = 0$$

$$C_2 = -\frac{I_{yz}}{I_y} C_1$$

$$M_z = -\int y\sigma_A\, dz = -C_1 \int y^2 dA + C_2 \int yz\, dA$$

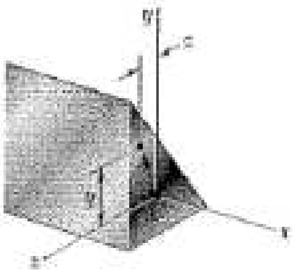
$$= -I_z C_1 - I_{yz} \frac{I_{yz}}{I_y} C_1$$

$$I_y M_z = -(I_y I_z - I_{yz}^2)\, C_1$$

$$C_1 = -\frac{I_y M_z}{I_y I_z - I_{yz}^2} \qquad C_2 = +\frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = -\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2} M_z$$

Problem 4.153



4.153 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the horizontal plane xz. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = \frac{zI_z - yI_{yz}}{I_y I_z - I_{yz}^2} M_y$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_y the moment of the couple.

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1 and C_2 are constants.

$$M_z = -\int y\sigma_A \, dA = -C_1 \int y^2 \, dA - C_2 \int yz \, dA$$

$$= -I_z C_1 - I_{yz} C_2 = 0$$

$$C_1 = -\frac{I_{yz}}{I_z} C_2$$

$$M_y = \int z\sigma_A \, dA = C_1 \int yz \, dA + C_2 \int z^2 \, dA$$

$$= I_{yz} C_1 + I_y C_2$$

$$-I_{yz} \frac{I_{yz}}{I_z} C_2 + I_y C_2$$

$$I_z M_y = (I_y I_z - I_{yz}^2) C_2$$

$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2} \qquad C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$

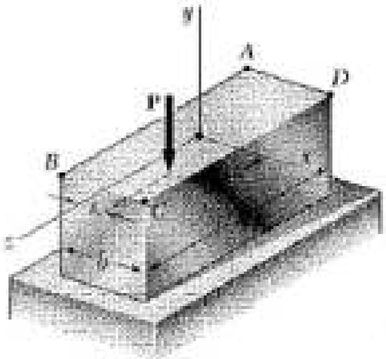
$$\sigma_A = \frac{I_z z - I_{yz} y}{I_y I_z - I_{yz}^2} M_y$$

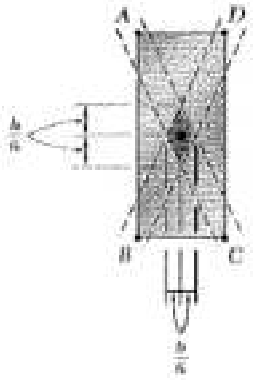
Problem 4.154

4.154 (*a*) Show that the stress at corner *A* of the prismatic member shown in Fig. P4.154*a* will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6}+\frac{z}{h/6}=1$$

(*b*) Further show that, if no tensile stress is to occur in the member, the force **P** must be applied at a point located within the area bounded by the line found in part *a* and three similar lines corresponding to the condition of zero stress at *B*, *C*, and *D*, respectively. This area, shown in Fig. 4.154*b*, is known as the *kern* of the cross section.





$$I_z = \frac{1}{12}hb^3 \qquad I_x = \frac{1}{12}bh^3 \qquad A = bh$$

$$z_A = -\frac{h}{2} \qquad x_A = -\frac{b}{2}$$

Let P be the load point.

$$M_z = -Px_P \qquad M_x = Pz_P$$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x}$$

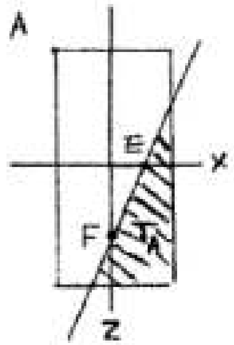
$$= -\frac{P}{bh} + \frac{(-Px_P)(-\frac{b}{2})}{\frac{1}{12}hb^3} - \frac{Pz_P(-\frac{h}{2})}{\frac{1}{12}bh^3}$$

$$= -\frac{P}{bh}\left[1 - \frac{x_P}{b/6} - \frac{z_P}{h/6}\right]$$

For $\sigma_A = 0$, $\quad 1 - \frac{x}{b/6} - \frac{z}{h/6} = 0, \quad \frac{x}{b/6} + \frac{z}{h/6} = 1$

At point E: $\quad z = 0 \quad \therefore \ x_E = b/6$

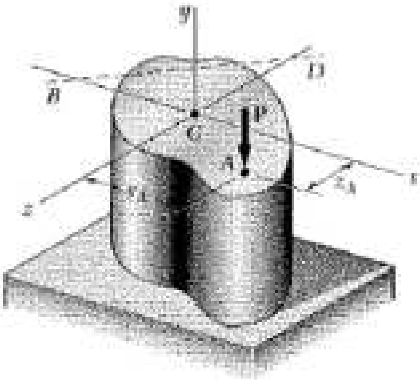
At point F: $\quad x = 0 \quad \therefore \ z_F = h/6$



If the line of action (x_P, z_P) lies within the portion marked T_A, a tensile will occur at corner A.

By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.

Problem 4.155



4.155 (*a*) Show that, if a vertical force **P** is applied at point *A* of the section shown, the equation of the neutral axis *BD* is

$$\left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

where k_z and k_x denote the radius of gyration of the cross section with respect to the *z* axis and the *x* axis, respectively. (*b*) Further show that, if a vertical force **Q** is applied at any point located on line *BD*, the stress at point *A* will be zero.

Definitions: $k_x^2 = \frac{I_x}{A}$, $k_z^2 = \frac{I_z}{A}$

(a) $M_x = Pz_A \qquad M_z = -Px_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{Px_A x_E}{Ak_z^2} - \frac{Pz_A z_E}{Ak_x^2}$$

$$= -\frac{P}{A}\left[1 + \left(\frac{x_A}{k_z^2}\right)x_E + \left(\frac{z_A}{k_x^2}\right)z_E\right] = 0 \text{ if } E \text{ lies on neutral axis.}$$

$$1 + \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = 0, \quad \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1 \quad \blacktriangleleft$$

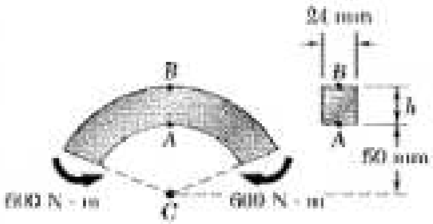
(b) $M_x = Pz_E \qquad M_z = -Px_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x} = -\frac{P}{A} - \frac{Px_E x_A}{Ak_z^2} - \frac{Pz_E z_A}{Ak_x^2}$$

$= 0$ by equation from Part (a). $\blacktriangleleft$

Problem 4.158

4.158 For the curved bar and loading shown, determine the stress at points A and B when $h = 55$ mm.



$h = 55 \text{ mm}, \quad r_1 = 50 \text{ mm}, \quad r_2 = 105 \text{ mm},$

$A = (24)(55) = 1.320 \times 10^3 \text{ mm}^2$

$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{55}{\ln \frac{105}{50}} = 74.13025 \text{ mm}$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 77.5 \text{ mm}$

$e = \bar{r} - R = 3.36975 \text{ mm}$

$y_A = 74.13025 - 50 = 24.13025 \text{ mm} \qquad r_A = 50 \text{ mm}$

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(600)(24.13025 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(50 \times 10^{-3})} = -65.1 \times 10^6 \text{ Pa}$$

$\sigma_A = -65.1 \text{ MPa}$ ◀

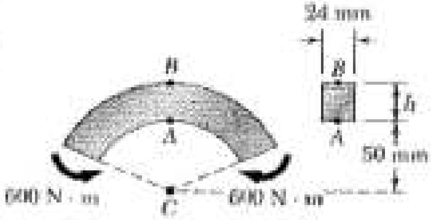
$y_B = 74.13025 - 105 = -30.86975 \text{ mm} \qquad r_B = 105 \text{ mm}$

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(600)(-30.8697 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(105 \times 10^{-3})} = 39.7 \times 10^6 \text{ Pa}$$

$\sigma_B = 39.7 \text{ MPa}$ ◀

Problem 4.159

4.159 For the curved bar and loading shown, determine the stress at point A when (a) $h = 50$ mm, (b) $h = 60$ mm.



(a) $h = 50$ mm, $r_1 = 50$ mm, $r_2 = 100$ mm

$$A = (24)(50) = 1.200\times10^3 \text{ mm}$$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{50}{\ln\frac{100}{50}} = 72.13475 \text{ mm}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 75 \text{ mm}$$

$$e = \bar{r} - R = 2.8652 \text{ mm}$$

$$y_A = 72.13475 - 50 = 22.13475 \text{ mm} \qquad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(22.13475\times10^{-3})}{(1.200\times10^{-3})(2.8652\times10^{-3})(50\times10^{-3})} = -77.3\times10^6 \text{ Pa}$$

$$\sigma_A = -77.3 \text{ MPa}$$

(b) $h = 60$ mm, $r_1 = 50$ mm, $r_2 = 110$ mm, $A = (24)(60) = 1.440\times10^3 \text{ mm}^2$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{60}{\ln\frac{110}{50}} = 76.09796 \text{ mm}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 80 \text{ mm} \qquad e = \bar{r} - R = 3.90204 \text{ mm}$$

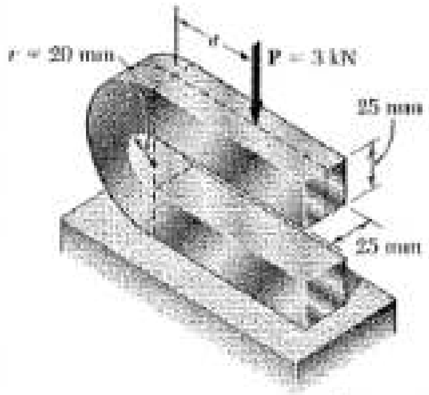
$$y_A = 76.09796 - 50 = 26.09796 \text{ mm} \qquad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(26.09796\times10^{-3})}{(1.440\times10^{-3})(3.90204\times10^{-3})(50\times10^{-3})} = -55.7\times10^6 \text{ Pa}$$

$$\sigma_A = -55.7 \text{ MPa}$$

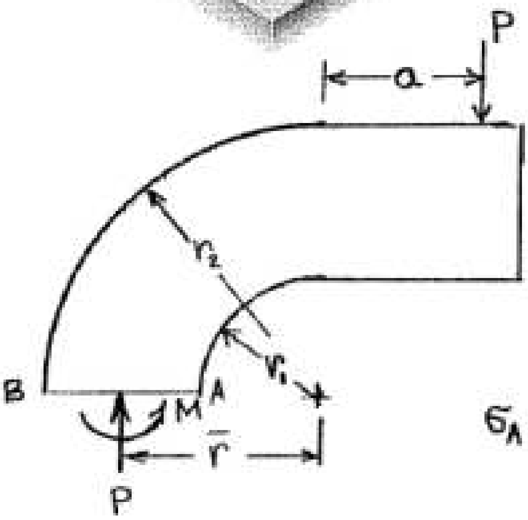
Problem 4.160

4.160 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance a from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$



For the rectangular section, the neutral axis for bending couple only lies

$$R = \frac{h}{\ln \frac{r_2}{r_1}} \qquad \text{Also} \quad e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \qquad \text{with } y_A = R - r_1$$

Thus, $\quad K = 1 + \frac{(a+\bar{r})(R-r_1)}{er_1}$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$$

$$R - r_1 = 10.8288 \text{ mm}$$

$$P = 3 \times 10^3 \text{ N-m} \qquad \sigma_A = -150 \times 10^6 \text{ Pa}$$

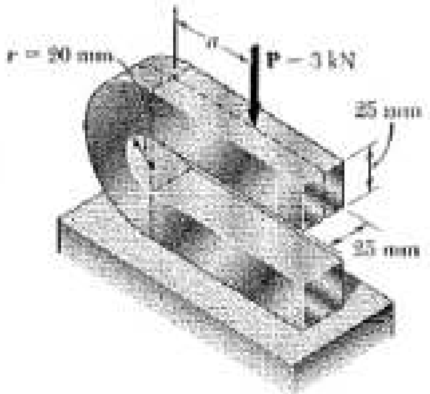
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-6})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)er_1}{R - r_1} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$$a = 93.37 - 32.5 \qquad a = 60.9 \text{ mm}$$

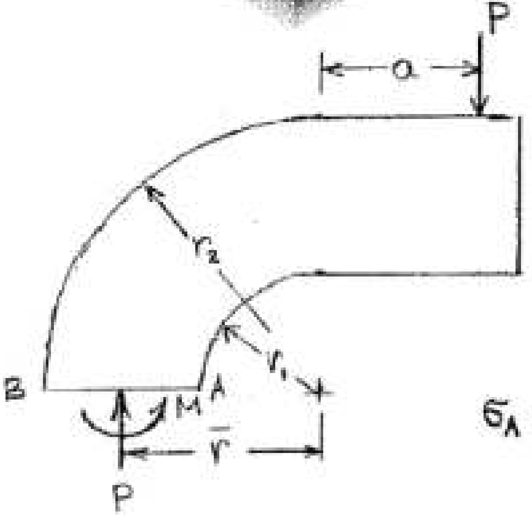
Problem 4.161

4.161 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance $a = 60$ mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$



For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \qquad \text{Also } e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \qquad \text{with } y_A = R - r_1$$

Thus, $\quad K = 1 + \dfrac{(a+\bar{r})(R-r_1)}{er_1}$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$$

$$a = 60 \text{ mm}, \quad a + \bar{r} = 92.5 \text{ mm}, \quad R - r_1 = 10.8288 \text{ mm}$$

$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968$$

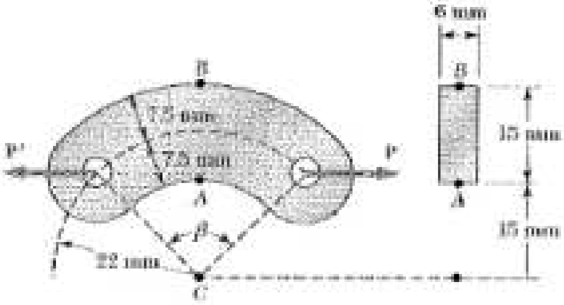
$$P = 3 \times 10^3 \text{ N}$$

$$\sigma_A = -\frac{KP}{A} = -\frac{(30.968)(3\times10^3)}{625\times10^{-6}} = -148.6 \times 10^6 \text{ Pa}$$

$$\sigma_A = -148.6 \text{ MPa}$$

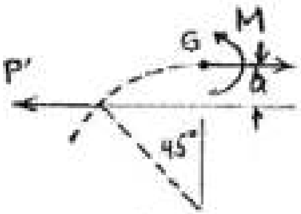
Problem 4.163

4.163 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 100 MPa, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.



Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$



The bending couple is $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}. \qquad \text{Also } e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \frac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 22\text{ mm}$, $r_1 = 15\text{ mm}$, $r_2 = 30\text{ mm}$, $h = 15\text{ mm}$, $b = 6\text{ mm}$

$A = (15)(6) = 90\text{ mm}^2$, $\qquad R = \frac{15}{\ln\frac{30}{15}} = 21.64\text{ mm}$

$e = 22 - 21.64 = 0.36\text{ mm}$, $\qquad y_A = 21.64 - 15 = 6.64\text{ mm}$

$a = 22(1 - \cos 45°) = 6.44\text{ mm}$

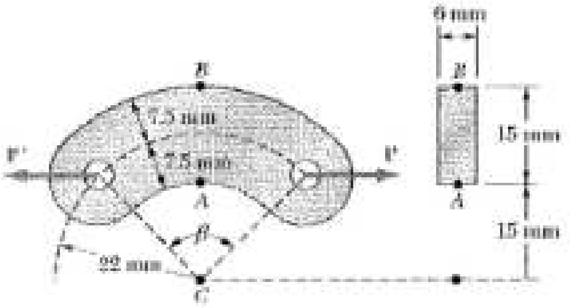
$$K = 1 + \frac{(6.44)(6.64)}{(0.36)(15)} = 8.92$$

$$P = \frac{(90\times10^{-6})(100\times10^{6})}{8.92} = 1009\text{ N} = 1\text{ kN}$$

Problem 4.164

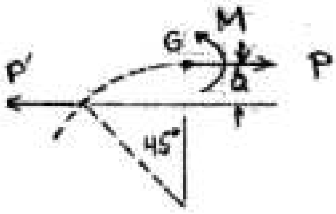
4.164 Solve Prob. 4.163, assuming that $\beta = 60°$.

4.163 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 100 MPa, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.



Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$



The bending couple is $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}. \qquad \text{Also } e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} + \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \frac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 22\text{mm}$, $r_1 = 15\text{mm}$, $r_2 = 30\text{mm}$, $h = 15\text{mm}$, $b = 6\text{mm}$

$$A = (6)(15) = 900\text{mm}^2, \qquad R = \frac{15}{\ln\frac{30}{15}} = 21.64\text{mm}$$

$$e = 22 - 21.64 = 0.36\text{mm} \qquad y_A = 21.64 - 15 = 6.64\text{mm}.$$

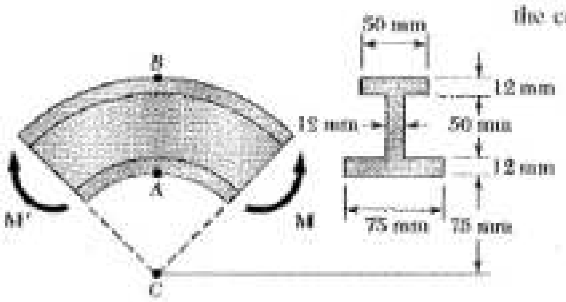
$$a = 22(1 - \cos 30°) = 2.95\text{mm}.$$

$$K = 1 + \frac{(2.95)(6.64)}{(0.36)(15)} = 4.627$$

$$P = \frac{(900\times10^{-6})(100\times10^{6})}{4.627} = 1945\text{ N} = 1.945\text{ kN}.$$

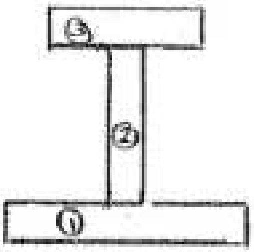
Problem 4.165

4.165 Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{\rho} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	part	b	h	A	$b \ln \frac{r_2}{r_1}$	$\bar{r}$	$A\bar{r}$
75	1	75	12	900	11.1315	81	72900
87	2	12	50	600	5.4489	112	67200
137	3	50	12	600	4.1983	143	85800
149	Σ			2100	20.7787		225900

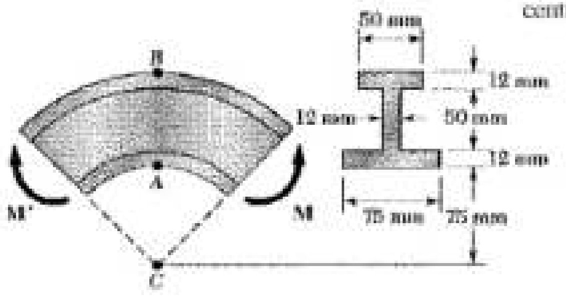
$R = \frac{2100}{20.7787} = 101.07$ mm, $\bar{r} = \frac{225900}{2100} = 107.57$

$e = \bar{r} - R = 6.5$ mm

6.5 mm

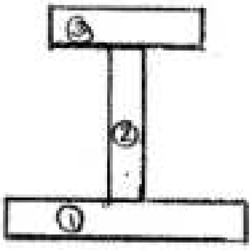
Problem 4.166

4.166 Three plates are welded together to form the curved beam shown. For $M = 900$ N · m, determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) the centroid of the cross section.



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r: 75, 87, 137, 149.

part	b	h	A	$b \ln \frac{r_2}{r_1}$	$\bar{r}$	$A\bar{r}$
①	75	12	900	11.1315	81	72900
②	12	50	600	5.4489	112	67200
③	50	12	600	4.1983	143	85800
Σ			2100	20.7787		225900

$R = \frac{2100}{20.7787} = 101.07$ mm , $\bar{r} = \frac{225900}{2100} = 107.57$ mm

$e = \bar{r} - R = 6.5$ mm $\qquad M = -900$ Nm.

(a) $y_A = R - r_1 = 101.07 - 75 = 26.07$ mm.

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-900)(0.02607)}{(2100\times10^{-6})(0.0065)(0.075)} = 22.9 \text{ MPa}$$ ◄

(b) $y_B = R - r_2 = 101.07 - 149 = -47.93$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(-900)(-0.04793)}{(2100\times10^{-6})(0.0065)(0.149)} = -21.2 \text{ MPa}$$ ◄

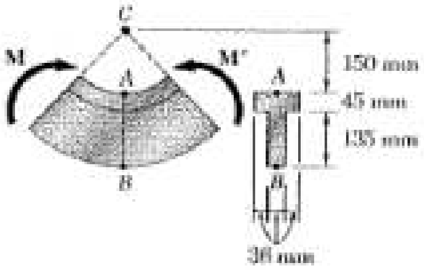
(c) $y_c = R - \bar{r} = -e$

$$\sigma_c = -\frac{M y_c}{A e \bar{r}} = -\frac{Me}{Ae\bar{r}} = -\frac{M}{A\bar{r}} = -\frac{-900}{(2100\times10^{-6})(0.10757)}$$

$$= 4 \text{ MPa}$$ ◄

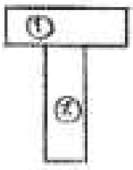
Problem 4.167

4.167 and 4.168 Knowing that $M = 20$ kN · m, determine the stress at (*a*) point *A*, (*b*) point *B*.



$$R = \frac{\Sigma A}{\Sigma \int \frac{dA}{r}} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm	Part	b, mm	h, mm	A, mm²	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm³
150							
	①	108	45	4860	28.3353	172.5	838.35×10^3
195							
	②	36	135	4860	18.9394	262.5	1275.75×10^3
330							
	Σ			9720	47.2747		2114.1×10^3

$R = \frac{9720}{47.2747} = 205.606$ mm $\qquad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5$ mm

$e = \bar{r} - R = 11.894$ mm $\qquad M = 20 \times 10^3$ N·m

(a) $y_A = R - r_1 = 205.606 - 150 = 55.606$ mm

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(20 \times 10^3)(55.606 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}$$

$= -64.1 \times 10^6$ Pa $\qquad \sigma_A = -64.1$ MPa ◄

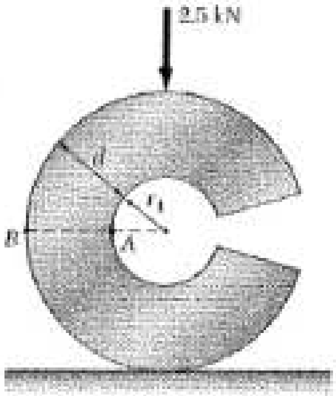
(b) $y_B = R - r_2 = 205.606 - 330 = -124.394$ mm

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(20 \times 10^3)(-124.394 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}$$

$= 65.2 \times 10^6$ Pa $\qquad \sigma_B = 65.2$ MPa ◄

Problem 4.169

4.169 The split ring shown has an inner radius $r_1 = 20$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



$c = \frac{1}{2}d = 16 \text{ mm} \qquad r_1 = 20 \text{ mm}, \quad r_2 = r_1 + d = 52 \text{ mm}$

$\bar{r} = r_1 + c = 36 \text{ mm}$

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[36 + \sqrt{36^2 - 16^2}\right]$

$= 34.1245 \text{ mm}$

$e = \bar{r} - R = 1.8755 \text{ mm}$

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$

$P = 2.5 \times 10^3 \text{ N} \qquad M = P\bar{r} = (2.5 \times 10^3)(36 \times 10^{-3}) = 90 \text{ N} \cdot \text{m}$

(a) Point A: $\quad y_A = R - r_1 = 34.1245 - 20 = 14.1245 \text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(14.1245 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(20 \times 10^{-3})}$$

$= -45.2 \times 10^6 \text{ Pa} \qquad \sigma_A = -45.2 \text{ MPa}$ ◄

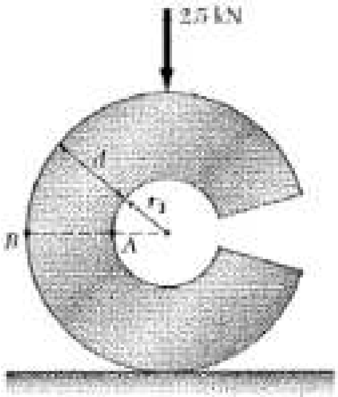
(b) Point B: $\quad y_B = R - r_2 = 34.1245 - 52 = -17.8755 \text{ mm}$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(-17.8755 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(52 \times 10^{-3})}$$

$= 17.40 \times 10^6 \text{ Pa} \qquad \sigma_B = 17.40 \text{ MPa}$ ◄

Problem 4.170

4.170 The split ring shown has an inner radius $r_1 = 16$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



$c = \frac{1}{2}d = 16$ mm, $r_1 = 16$ mm, $r_2 = r_1 + d = 48$ mm

$\bar{r} = r_1 + c = 32$ mm

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[32 + \sqrt{32^2 - 16^2}\right]$

$= 29.8564$ mm

$e = \bar{r} - R = 2.1436$ mm

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25\ \text{mm}^2 = 804.25\times10^{-6}\ \text{m}^2$

$P = 2.5\times10^3$ N $\qquad M = P\bar{r} = (2.5\times10^3)(32\times10^{-3}) = 80$ N·m

(a) Point A: $\quad y_A = R - r_1 = 29.8564 - 16 = 13.8564$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5\times10^3}{804.25\times10^{-6}} - \frac{(80)(13.8564\times10^{-3})}{(804.25\times10^{-6})(2.1436\times10^{-3})(16\times10^{-3})}$$

$= -43.3\times10^6$ Pa $\qquad \sigma_A = -43.3$ MPa ◄

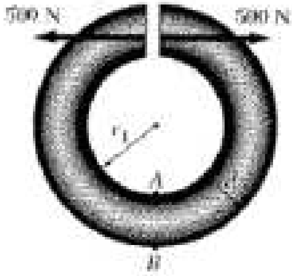
(b) Point B: $\quad y_B = R - r_2 = 29.8564 - 48 = -18.1436$ mm.

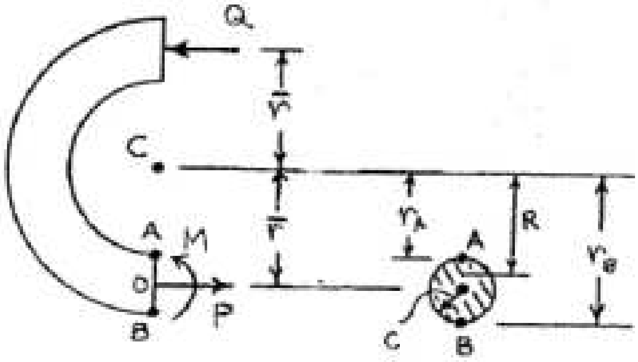
$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5\times10^6}{804.25\times10^{-6}} - \frac{(80)(-18.1436\times10^{-3})}{(804.25\times10^{-6})(2.1436\times10^{-3})(48\times10^{-3})}$$

$= 14.43\times10^6$ Pa $\qquad \sigma_B = 14.43$ MPa ◄

Problem 4.171

4.171 The split ring shown has an inner radius $r_1 = 20$ mm and a *circular* cross section of diameter $d = 15$ mm. Knowing that each of the 500-N forces is applied at the centroid of the cross section, determine the stress at (*a*) point *A*, (*b*) point *B*.





$r_A = r_1 = 20$ mm

$r_B = r_A + d = 20 + 15 = 35$ mm

$\bar{r} = \frac{1}{2}(r_A + r_B) = 27.5$ mm $\qquad c = \frac{1}{2}d = 7.5$ mm

$A = \pi c^2 = \pi(7.5)^2 = 176.7$ mm^2 $\qquad$ for solid circular section

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[27.5 + \sqrt{(27.5)^2 - (7.5)^2}\right] = 27$ mm

$e = \bar{r} - R = 27.5 - 27 = 0.5$ mm

$Q = 500$ N $\qquad \xrightarrow{+} \Sigma F_x = 0: \; P - Q = 0 \qquad P = 500$ N

$\circlearrowleft \Sigma M_0 = 0: \; 2\bar{r}Q + M = 0 \qquad M = -2\bar{r}Q = -(2)(0.0275)(500) = -27.5$ Nm

(a) $r = r_A = 20$ mm

$$\sigma_A = \frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = \frac{500}{176.7\times10^{-6}} + \frac{(-27.5)(0.02 - 0.027)}{(176.7\times10^{-6})(0.0005)(0.02)}$$
$$= 111.77\times10^6 \text{ Pa}$$

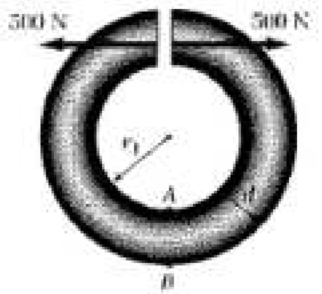
$\sigma_A = 111.8$ MPa ◄

(b) $r = r_B = 35$ mm

$$\sigma_B = \frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = \frac{500}{176.7\times10^{-6}} + \frac{(-27.5)(0.035 - 0.027)}{(176.7\times10^{-6})(0.0005)(0.035)}$$
$$= -68.32\times10^6$$

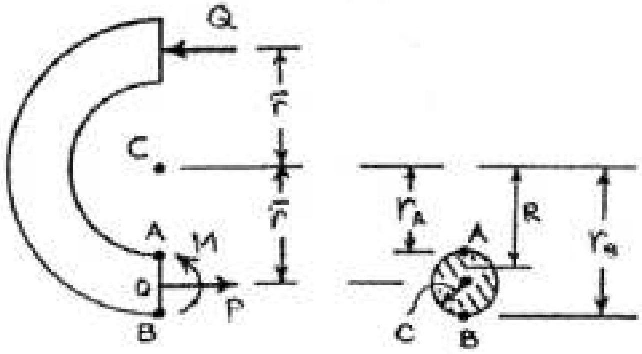
$\sigma_B = -68.3$ MPa ◄

Problem 4.172



4.172 Solve Prob. 4.171, assuming that the ring has an inner radius $r_1 = 15$ mm and a cross-sectional diameter $d = 20$ mm.

4.171 The split ring shown has an inner radius $r_1 = 20$ mm and a *circular* cross section of diameter $d = 15$ mm. Knowing that each of the 500-N forces is applied at the centroid of the cross section, determine the stress at (*a*) point *A*, (*b*) point *B*.



$r_A = r_1 = 15 \text{ mm}$

$r_B = r_A + d = 15 + 20 = 35 \text{ mm}$

$\bar{r} = \frac{1}{2}(r_A + r_B) = 25 \text{ mm}$ $\qquad c = \frac{1}{2}d = 10 \text{ mm}$

$A = \pi c^2 = \pi(10)^2 = 314.16 \text{ mm}^2$ $\qquad$ for solid circular section.

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[25 + \sqrt{(25)^2 - (10)^2}\right] = 24 \text{ mm}$

$e = \bar{r} - R = 25 - 24 = 1 \text{ mm}$

$Q = 500 \text{ N}$ $\qquad \overset{+}{\rightarrow}\Sigma F_x = 0 \qquad P - Q = 0 \qquad P = 500 \text{ N}$

$+\circlearrowleft \Sigma M_O = 0: \quad 2\bar{r}Q + M = 0 \qquad M = -2\bar{r}Q = -(2)(0.001)(500) = -1 \text{ Nm}$

(a) $r = r_A = 15 \text{ mm}$

$$\sigma_A = \frac{P}{A} + \frac{M(r_A - R)}{A e r_A} = \frac{500}{314.16\times10^{-6}} + \frac{(-1)(0.015 - 0.024)}{(314.16\times10^{-6})(0.001)(0.015)}$$

$$= 3.501\times10^6 \text{ Pa}$$

$\sigma_A = 3.5 \text{ MPa}$ ◄

(b) $r = r_B = 35 \text{ mm}$

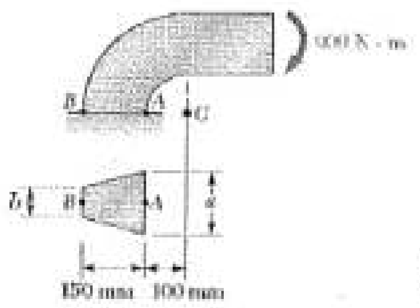
$$\sigma_B = \frac{P}{A} + \frac{M(r_B - R)}{A e r_B} = \frac{500}{314.16\times10^{-6}} + \frac{(-1)(0.035 - 0.024)}{(314.16\times10^{-6})(0.001)(0.035)}$$

$$= -0.59\times10^6$$

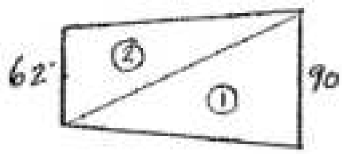
$\sigma_B = -0.6 \text{ MPa}$ ◄

Problem 4.175

4.175 Knowing that the machine component shown has a trapezoidal cross section with $a = 90$ mm and $b = 62$ mm, determine the stress at (*a*) point *A*, (*b*) point *B*.



Locate centroid



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	6750	150	1.0125×10^6
②	4650	200	0.93×10^6
Σ	11400		1942500

$$\bar{r} = \frac{1942500}{11400} = 170.4 \text{ mm}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(150)^2(90+62)}{[(90)(250) - (62)(100)]\ln\frac{250}{100} - (150)(90-62)} = 159.3 \text{ mm}$$

$e = \bar{r} - R = 11.1$ $\qquad M = 9 \text{ kN}$

(a) $y_A = R - r_1 = 159.3 - 100 = 59.3$ mm

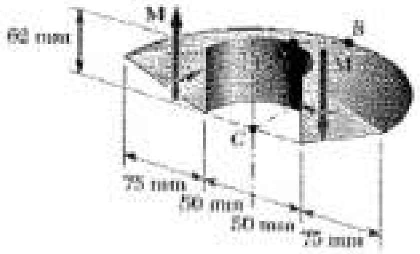
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(9000)(0.0593)}{(11400 \times 10^{-6})(0.0111)(0.1)} = -42.2 \text{ MPa}$$

(b) $y_B = R - r_2 = 159.3 - 250 = -90.7$ mm.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(9000)(-0.0907)}{(11400 \times 10^{-6})(0.0111)(0.25)} = 25.8 \text{ MPa}$$

Problem 4.177

4.177 and 4.178 Knowing that $M = 560$ N · m, determine the stress at (*a*) point *A*, (*b*) point *B*.



$A = \frac{1}{2}bh = \frac{1}{2}(62)(75) = 2325 \text{ mm}^2$

$\bar{r} = 50 + 25 = 76$ mm

$b_1 = 62$ mm, $r_1 = 50$ mm, $b_2 = 0$, $r_2 = 125$ mm

Use formula for trapezoid with $b_2 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(75)^2(62+0)}{[(62)(125)-(0)(50)]\ln\frac{125}{50} - 75(62-0)} = 71.14 \text{ mm}$$

$e = \bar{r} - R = 3.86$ mm $\qquad M = 560$ Nm

(a) $y_A = R - r_1 = 21.14$ mm.

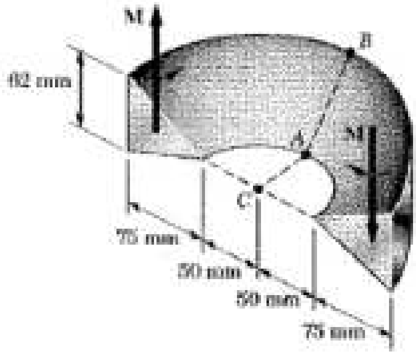
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(560)(0.02114)}{(2325\times10^{-6})(0.00386)(0.05)} = -26.4 \text{ MPa}$$

(b) $y_B = R - r_2 = -53.86$ mm

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(560)(-0.05386)}{(2325\times10^{-6})(0.00386)(0.125)} = 26.9 \text{ MPa}$$

Problem 4.178

4.177 and 4.178 Knowing that $M = 560$ N · m, determine the stress at (*a*) point *A*, (*b*) point *B*.



$A = \frac{1}{2}(62)(75) = 2325\ mm^2.$

$\bar{r} = 50 + 50 = 100\ mm$

$b_1 = 0,\ r_1 = 50\ mm,\ b_2 = 62\ mm\ \ r_2 = 125\ mm.$

Use formula for trapezoid with $b_1 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(75)^2(0+62)}{[(0)(125) - (62)(50)]\ln\frac{125}{50} - 75(0 - 62)} = 96.4\ mm$$

$e = \bar{r} - R = 3.6\ mm \qquad\qquad M = 560\ Nm.$

(a) $y_A = R - r_1 = 46.4\ mm$

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(560)(0.0464)}{(2325\times10^{-6})(0.0036)(0.05)} = -62.1\ MPa.$$

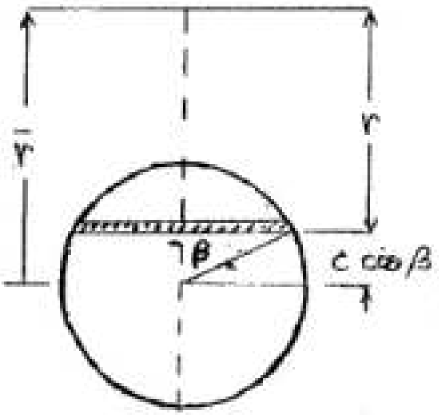
(b) $y_B = R - r_2 = -28.6\ mm$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(560)(-0.0286)}{(2325\times10^{-6})(0.0036)(0.125)} = 15.3\ MPa.$$

Problem 4.180

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

***4.180** A circular cross section.



Use polar coordinate β as shown.

width: $w = 2c\sin\beta$

$r = \bar{r} - c\cos\beta$

$dr = c\sin\beta \, d\beta$

$dA = w\,dr = 2c^2\sin^2\beta \, d\beta$

$$\int \frac{dA}{r} = \int_0^{\pi} \frac{2c^2\sin^2\beta}{\bar{r} - c\cos\beta} d\beta$$

$$\int \frac{dA}{r} = \int_0^{\pi} \frac{c^2(1-\cos^2\beta)}{\bar{r} - c\cos\beta} d\beta = 2\int_0^{\pi} \frac{\bar{r}^2 - c^2\cos^2\beta - (\bar{r}^2 - c^2)}{\bar{r} - c\cos\beta} d\beta$$

$$= 2\int_0^{\pi} (\bar{r} + c\cos\beta)\, d\beta - 2(\bar{r}^2 - c^2)\int_0^{\pi} \frac{dr}{\bar{r} - c\cos\beta}$$

$$= 2\bar{r}\beta\Big|_0^{\pi} + 2c\sin\beta\Big|_0^{\pi}$$

$$- 2(\bar{r}^2 - c^2)\frac{2}{\sqrt{\bar{r}^2 - c^2}} \tan^{-1}\frac{\sqrt{\bar{r}^2 - c^2}\,\tan(\frac{1}{2}\beta)}{\bar{r} + c}\Big|_0^{\pi}$$

$$= 2\bar{r}(\pi - 0) + 2c(0 - 0) - 4\sqrt{\bar{r}^2 - c^2}\cdot(\tfrac{\pi}{2} - 0)$$

$$2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\pi c^2}{2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\frac{c^2}{\bar{r} - \sqrt{\bar{r}^2 - c^2}} \cdot \frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}}$$

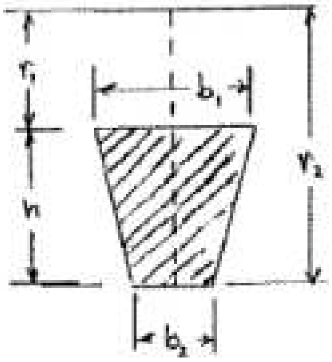
$$= \frac{1}{2}\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2}\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{c^2}$$

$$= \frac{1}{2}(\bar{r} + \sqrt{\bar{r}^2 - c^2})$$

Problem 4.181

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.181 A trapezoidal cross section.



The section width w varies linearly with r.

$$w = c_0 + c_1 r$$

$w = b_1$ at $r = r_1$ and $w = b_2$ at $r = r_2$

$$b_1 = c_0 + c_1 r_1$$

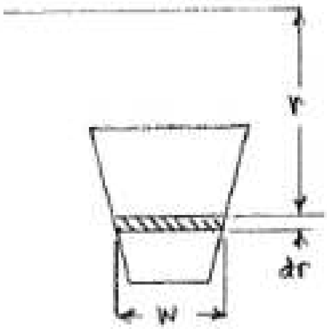
$$b_2 = c_0 + c_1 r_2$$

$$b_1 - b_2 = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1) c_0 = h c_0$$

$$c_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$



$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r}\, dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r}\, dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln\frac{r_2}{r_1} + c_1(r_2 - r_1)$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln\frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln\frac{r_2}{r_1} - (b_1 - b_2)$$

$$A = \tfrac{1}{2}(b_1 + b_2) h$$

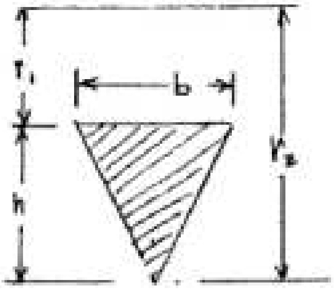
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

Problem 4.182

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.182 A triangular cross section.

The section width w varies linearly with r.



$$w = C_0 + C_1 r$$

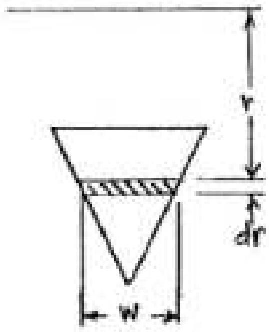
$$w = b \text{ at } r = r_1 \text{ and } w = 0 \text{ at } r = r_2$$

$$b = C_0 + C_1 r_1$$

$$0 = C_0 + C_1 r_2$$

$$b = C_1(r_1 - r_2) = -C_1 h$$

$$C_1 = -\frac{b}{h} \quad \text{and} \quad C_0 = -C_1 r_2 = \frac{b r_2}{h}$$



$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r}\,dr = \int_{r_1}^{r_2} \frac{C_0 + C_1 r}{r}\,dr$$

$$= C_0 \ln r \Big|_{r_1}^{r_2} + C_1 r \Big|_{r_1}^{r_2}$$

$$= C_0 \ln \frac{r_2}{r_1} + C_1(r_2 - r_1)$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)$$

$$A = \frac{1}{2} b h$$

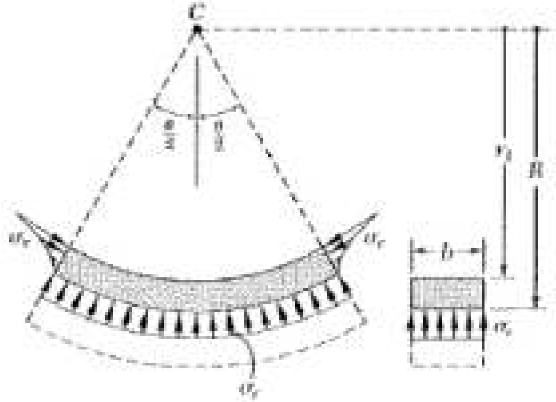
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} b h}{b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

Problem 4.183

*4.183 For a curved bar of rectangular cross section subjected to a bending couple **M**, show that the radial stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

and compute the value of σ_r for the curved bar of Examples 4.10 and 4.11. (*Hint:* consider the free-body diagram of the portion of the beam located above the neutral surface.)



At radial distance r

$$\sigma_\theta = \frac{M(r-R)}{Aer} = \frac{M}{Ae} - \frac{MR}{Aer}$$

For portion above the neutral axis, the resultant force is

$$H = \int \sigma_\theta \, dA = \int_{r_1}^{R} \sigma_\theta \, b \, dr$$

$$= \frac{Mb}{Ae}\int_{r_1}^{R} dr - \frac{MRb}{Ae}\int_{r_1}^{R}\frac{dr}{r}$$

$$= \frac{Mb}{Ae}(R - r_1) - \frac{MRb}{Ae}\ln\frac{R}{r_1} = \frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Resultant of σ_r:

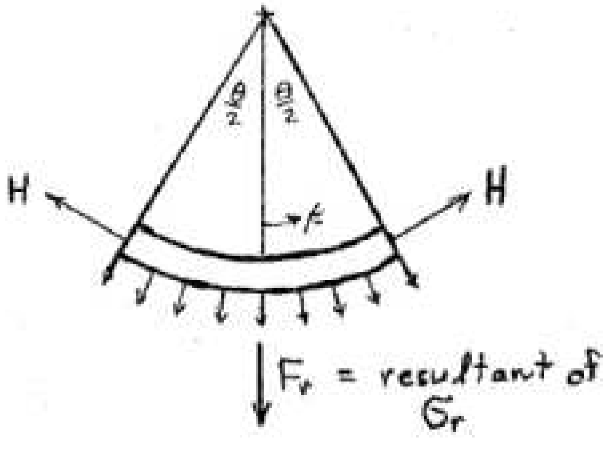
$$F_r = \int \sigma_r \cos\beta \, dA$$

$$= \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \sigma_r \cos\beta \, b \, R \, d\beta$$

$$= \sigma_r b R \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \cos\beta \, d\beta$$

$$= \sigma_r b R \sin\beta \Big|_{-\frac{\theta}{2}}^{\frac{\theta}{2}}$$

$$= 2\sigma_r b R \sin\frac{\theta}{2}$$



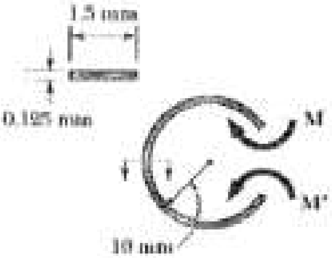
For equilibrium: $\quad F_r - 2H\sin\frac{\theta}{2} = 0$

$$2\sigma_r bR\sin\frac{\theta}{2} - 2\frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)\sin\frac{\theta}{2} = 0$$

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Problem 4.186

4.186 It is observed that a thin steel strip of 1.5 mm width can be bent into a circle of 10-mm diameter without any resulting permanent deformation. Knowing that $E = 200$ GPa, determine (*a*) the maximum stress in the bent strip, (*b*) the magnitude of the couples required to bend the strip.



$$I = \frac{1}{12} b h^3 = \frac{1}{12}(1.5)(0.125)^3 = 2.44 \times 10^{-4}$$

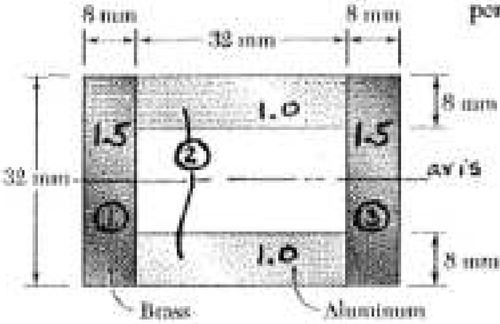
$$\rho = \frac{1}{2} D = \frac{1}{2}(20) = 10 \text{ mm}$$

$$c = \frac{1}{2} h = 0.0625 \text{ mm}$$

(a) $$\sigma_{max} = \frac{Ec}{\rho} = \frac{(200 \times 10^9)(0.0625 \times 10^{-3})}{0.01} = 1250 \text{ MPa}$$

(b) $$M = \frac{EI}{\rho} = \frac{(200 \times 10^9)(2.44 \times 10^{-16})}{0.01} = 0.00488 \text{ Nm.}$$

Problem 4.187



4.187 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the tranformed section,

$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12}(8)(32)^3 = 32.768 \times 10^3 \text{ mm}^4$

$I_2 = \frac{n_2}{12} b_2 (H_2^3 - h_2^3) = \frac{1.0}{12}(32)(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$

$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$

$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

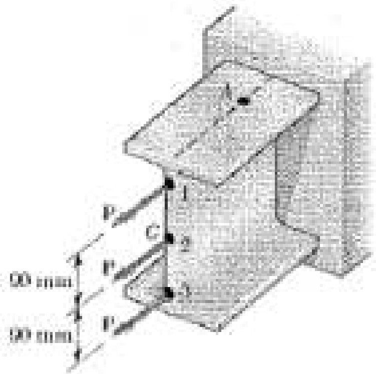
$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N}\cdot\text{m}$$

Choose the smaller value. $\qquad M = 887 \text{ N}\cdot\text{m}$ ◄

Problem 4.189



4.189 As many as three axial loads each of magnitude $P = 40$ kN can be applied to the end of a W200 × 31.3 rolled-steel shape. Determine the stress at point A, (*a*) for the loading shown, (*b*) if loads are applied at points 1 and 2 only.

For W200 × 31.3 Appendix C gives

$A = 4000 \text{ mm}^2 \quad d = 210 \text{ mm}, \quad I_x = 31.4 \times 10^6$

At point A $\quad y = \frac{1}{2} d = 105$ mm.

$$\sigma = \frac{F}{A} - \frac{My}{I}$$

(a) Centric loading: $F = 120$ kN, $M = 0$

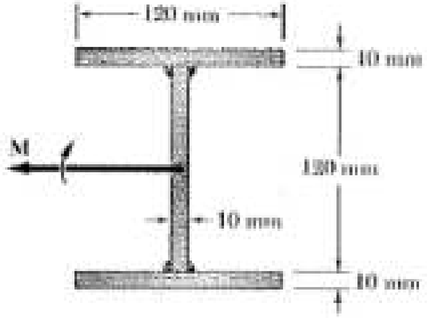
$$\sigma = \frac{120\,000}{4000 \times 10^{-6}} \qquad \sigma = 30 \text{ MPa.}$$

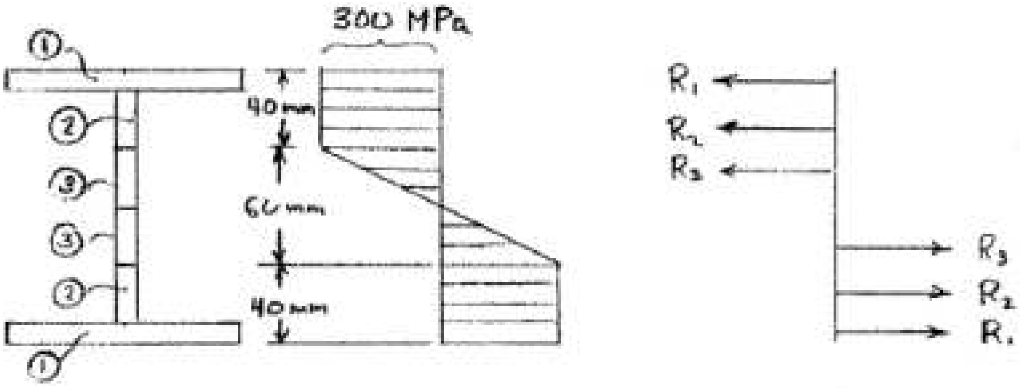
(b) Eccentric loading: $F = 2P = 80$ kN $\qquad M = -(40)(0.09) = 3.6$ kNm

$$\sigma = \frac{80 \times 10^3}{4 \times 10^{-3}} - \frac{(-3.6 \times 10^3)(0.105)}{31.4 \times 10^{-6}} \qquad \sigma = 32 \text{ MPa}$$

Problem 4.190

4.190 Three 120 × 10-mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (*a*) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (*b*) the corresponding radius of curvature of the beam.





$A_1 = (120)(10) = 1200 \text{ mm}^2$ $\qquad R_1 = \sigma_Y A_1 = (300\times10^6)(1200\times10^{-6}) = 360\times10^3$ N

$A_2 = (30)(10) = 300 \text{ mm}^2$ $\qquad R_2 = \sigma_Y A_2 = (300\times10^6)(300\times10^{-6}) = 90\times10^3$ N

$A_3 = (30)(10) = 300 \text{ mm}^2$ $\qquad R_3 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times10^6)(300\times10^{-6})$

$= 45\times10^3$ N

$y_1 = 65 \text{ mm} = 65\times10^{-3}$ m

$y_2 = 45 \text{ mm} = 45\times10^{-3}$ m

$y_3 = 20 \text{ mm} = 20\times10^{-3}$ m

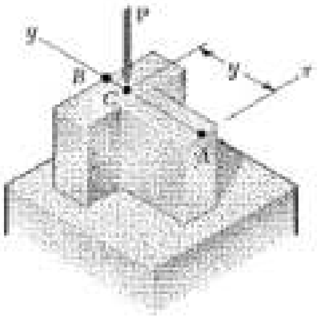
(a) $M = 2(R_1 y_1 + R_2 y_2 + R_3 y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$

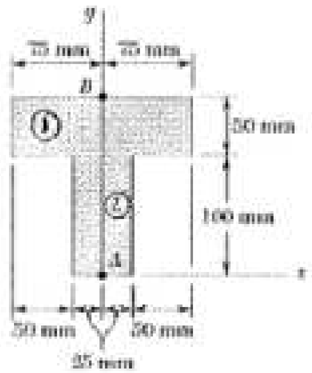
$= 56.7\times10^3$ N·m $= 56.7$ kN·m ◄

(b) $\dfrac{y_Y}{\rho} = \dfrac{\sigma_Y}{E}$ $\qquad \rho = \dfrac{E y_Y}{\sigma_Y} = \dfrac{(200\times10^9)(30\times10^{-3})}{300\times10^6} = 20$ m ◄

Problem 4.191

4.191 A vertical force **P** of magnitude 80 kN is applied at a point *C* located on the axis of symmetry of the cross section of a short column. Knowing that y = 125 mm, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the location of the neutral axis.





Locate centroid

Part	A, mm^2	$\bar{y}, mm$	$A\bar{y}, mm^3$
1	7500	125	937500
2	5000	50	250000
Σ	12500		1187500

$$\bar{y} = \frac{\Sigma A\bar{y}_i}{\Sigma A_i} = \frac{1187500}{12500} = 95 \text{ mm.}$$

Eccentricity of load $e = 125 - 95 = 30$ mm

$$I_1 = \frac{1}{12}(150)(50)^3 + (7500)(30)^2 = 8.3125 \times 10^6 \text{ mm}^4.$$

$$I_2 = \frac{1}{12}(50)(100)^3 + (5000)(45)^2 = 14.2917 \times 10^6 \text{ mm}^4.$$

$$I = I_1 + I_2 = 22.6 \times 10^6 \text{ mm}^4.$$

(a) Stress at A $\quad c_A = 95$ mm.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = \frac{80000}{12500 \times 10^{-6}} + \frac{(80000)(0.03)(0.095)}{22.6 \times 10^{-6}} = 3.69 \text{ MPa}$$

(b) Stress at B $\quad c_B = 150 - 95 = 55$ mm.

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = \frac{80000}{12500 \times 10^{-6}} - \frac{(80000)(0.03)(0.055)}{22.6 \times 10^{-6}} = -12.24 \text{ MPa}$$

(c) Location of neutral axis: $\quad \sigma = 0$

$$\sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \quad \frac{ea}{I} = \frac{1}{A}$$

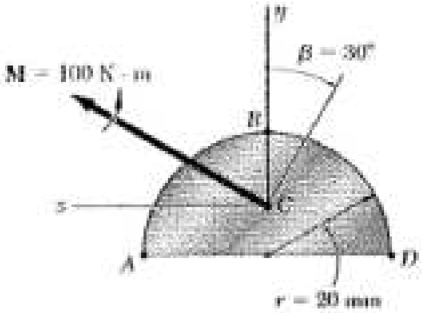
$$a = \frac{I}{Ae} = \frac{22.6 \times 10^6}{(12500)(30)} = 60.3 \text{ mm.}$$

Neutral axis lies 60.3 below centroid or 95 − 60.3 = 34.7 mm above point A.

Answer 34.7 mm from point A

Problem 4.193

4.193 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.



$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(20)^4 = 17.5611\times10^{3}\ \text{mm}^4 = 17.5611\times10^{-9}\ \text{m}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi(20)^4}{8} = 62.832\times10^{3}\ \text{mm}^4 = 62.832\times10^{-9}\ \text{m}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(20)}{3\pi} = -8.4883\ \text{mm}$$

$$y_B = 20 - 8.4883 = 11.5117\ \text{mm}$$

$$z_A = -z_D = 20\ \text{mm} \qquad z_B = 0$$

$$M_z = 100\cos 30° = 86.603\ \text{N}\cdot\text{m}$$

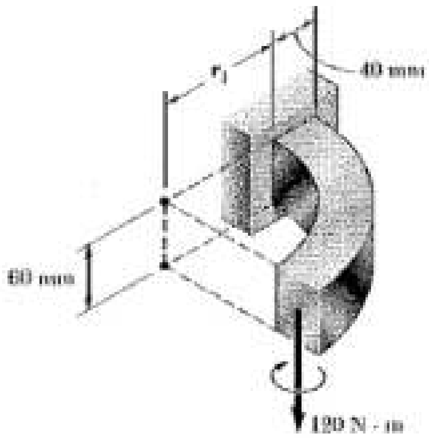
$$M_y = 100\sin 30° = 50\ \text{N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(20\times10^{-3})}{62.832\times10^{-9}}$$
$$= 57.8\times10^{6}\ \text{Pa} \qquad \sigma_A = 57.8\ \text{MPa}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(86.603)(11.5117\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(0)}{62.832\times10^{-9}}$$
$$= -56.8\times10^{6}\ \text{Pa} \qquad \sigma_B = -56.8\ \text{MPa}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(-20\times10^{-3})}{62.832\times10^{-9}}$$
$$= 25.9\times10^{6}\ \text{Pa} \qquad \sigma_c = 25.9\ \text{MPa}$$

Problem 4.195



4.195 The curved bar shown has a cross section of 40 × 60 mm and an inner radius $r_1 = 15$ mm. For the loading shown, determine the largest tensile and compressive stresses in the bar.

$h = 40$ mm, $r_1 = 15$ mm, $r_2 = 55$ mm

$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{40}{\ln \frac{55}{40}} = 30.786$ mm

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35$ mm

$e = \bar{r} - R = 4.214$ mm $\qquad \sigma = -\dfrac{My}{Aer}$

At $r = 15$ mm $\quad y = 30.786 - 15 = 15.786$ mm

$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^6 \text{ Pa}$$

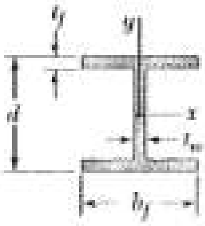
$= -12.49$ MPa ◄
(compression)

At $r = 55$ mm $\quad y = 30.786 - 55 = -24.214$ mm

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^6 \text{ Pa}$$

$= 5.22$ MPa ◄
(tension)

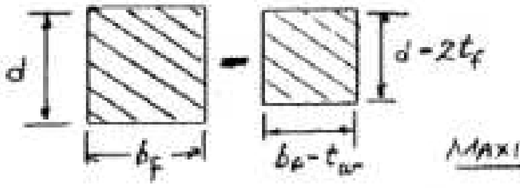
PROBLEM 4.C2



4.C2 A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength σ_Y and a modulus of elasticity E, is bent about the x axis. (*a*) Denoting by y_Y the half thickness of the elastic core, write a computer program to calculate the bending moment M and the radius of curvature ρ for values of y_Y from $\frac{1}{2}d$ to $\frac{1}{6}d$ using decrements equal to $\frac{1}{2}t_f$. Neglect the effect of fillets. (*b*) Use this program to solve Prob. 4.190

SOLUTION

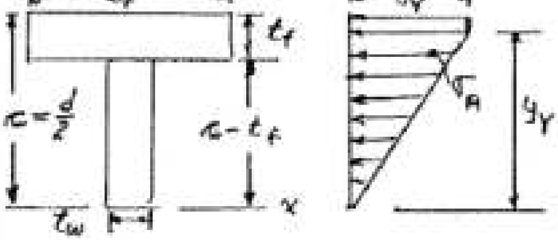
COMPUTE MOMENT OF INERTIA I_x



$$I_x = \frac{1}{12} b_f d^3 - \frac{1}{12}(b_f - t_w)(d-2t_f)^3$$

MAXIMUM ELASTIC MOMENT: $M_Y = \sigma_Y \frac{I_x}{(d/2)}$

FOR YIELDING IN THE FLANGES: (CONSIDER UPPER HALF OF CROSS SECTION) $c = \frac{d}{2}$

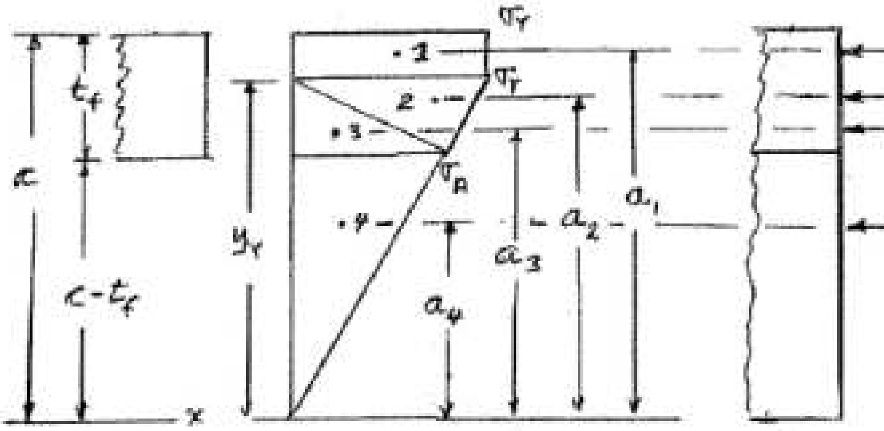


STRESS AT JUNCTION OF WEB AND FLANGE

$$\sigma_A = \frac{(d/2)-t_f}{y_Y}\sigma_Y$$

DETAIL OF STRESS DIAGRAM

RESULTANT FORCES



$R_1 = \sigma_Y [b_f(c-\sigma_Y)]$

$R_2 = \frac{1}{2}\sigma_Y b_f [y_Y - (c-t_f)]$

$R_3 = \frac{1}{2}\sigma_A b_f [y_Y - (c-t_f)]$

$R_3 = \frac{1}{2}\sigma_A t_w (c-t_f)$

$a_1 = \frac{1}{2}(c+y_Y)$

$a_2 = y_Y - \frac{1}{3}[y_Y - (c-t_f)]$

$a_3 = y_Y - \frac{2}{3}[y_Y - (c-t_f)]$

$a_4 = \frac{2}{3}(c-t_f)$

BENDING MOMENT

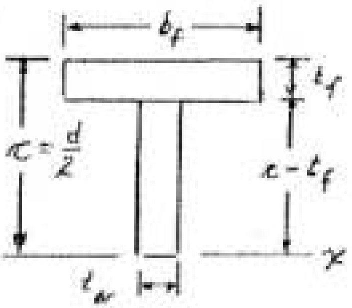
$$M = 2\sum_{n=1}^{4} R_n a_n$$

RADIUS OF CURVATURE

$$y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho; \quad \rho = \frac{y_Y E}{\sigma_Y}$$

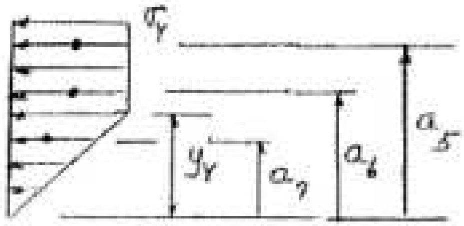
CONTINUED

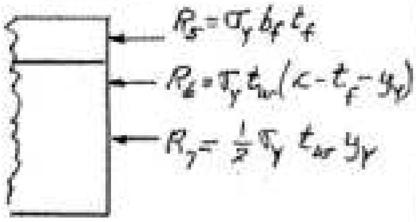
PROBLEM 4.C2 - CONTINUED



FOR YIELDING IN THE WEB (CONSIDER UPPER HALF OF CROSS SECTION)

$c = d/2$





$$a_5 = c - \tfrac{1}{2}t_f$$

$$a_6 = \tfrac{1}{2}[y_Y + (c - t_f)]$$

$$a_7 = \tfrac{2}{3}y_Y$$

BENDING MOMENT $\quad M = 2\sum_{n=5}^{7} R_n a_n$

RADIUS OF CURVATURE $\quad y_Y = \epsilon_Y \rho = \dfrac{\sigma_Y}{E}\rho \qquad \rho = \dfrac{y_Y E}{\sigma_Y}$

PROGRAM: KEY IN EXPRESSIONS FOR a_n and R_n FOR $n = 1$ TO 7.

FOR $y_Y = c$ TO $(c - t_f)$ AT $-t_f/2$ DECREMENTS

COMPUTE $M = 2\sum R_n a_n$ FOR $n = 1$ TO 4 AND $\rho = \dfrac{y_Y E}{\sigma_Y}$, THEN PRINT

FOR $y_Y = (c - t_w)$ TO $c/3$ AT $-t_f/2$ DECREMENTS

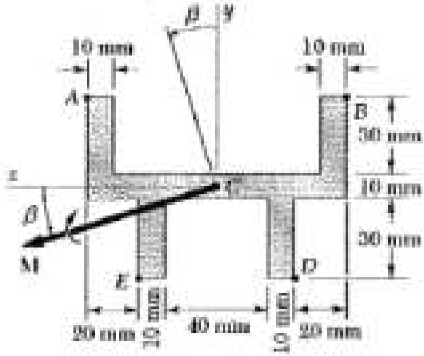
COMPUTE $M = 2\sum R_n a_n$ FOR $n = 5$ TO 7. AND $\rho = \dfrac{y_Y E}{\sigma_Y}$, THEN PRINT

INPUT NUMERICAL VALUES AND RUN PROGRAM

PROGRAM OUTPUT

```
 For a beam of Prob 4.190
Depth d = 140.00 mm                        Width of flange bf = 120.00 mm
Thickness of flange tf =  10.00 mm         Thickness of web tw =  10.00 mm

I = 0.000011600 m to the 4th
Yield strength of Steel sigmaY = 300 MPa
Yield Moment MY = 49.71 kip.in.

      yY(mm)          M(kN.m)     rho(m)
 For yielding still in the flange.
      70.000           49.71       46.67
      65.000           52.59       43.33
      60.000           54.00       40.00
 For yielding in the web
      60.000           54.00       40.00
      55.000           54.58       36.67
      50.000           55.10       33.33
      45.000           55.58       30.00
      40.000           56.00       26.67
      35.000           56.38       23.33
      30.000           56.70       20.00
      25.000           56.97       16.67
```

PROBLEM 4.C3



4.C3 A 900 N · m couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Noting that the centroid of the cross section is located at C and that the y and z axes are principal axes, write a computer program to calculate the stress at A, B, C, and D for values of β from 0 to 180° using 10° increments. (*Given:* $I_y = 2.59 \times 10^6$ mm^4 and $I_z = 0.62 \times 10^6$ mm^4.)

SOLUTION

INPUT COORDINATES OF A, B, C, D $(\times 10^{-3})$

$z_A = z(1) = 50$ $\quad y_A = y(1) = 35$

$z_B = z(2) = -50$ $\quad y_B = y(2) = 35$

$z_D = z(3) = -30$ $\quad y_D = y(3) = -35$

$z_E = z(4) = 30$ $\quad y_E = y(4) = -35$

COMPONENTS OF M.

$M_y = -M\sin\beta \qquad M_z = M\cos\beta$

Eq 4.55 page 273: $\sigma(n) = -\dfrac{M_z\, y(n)}{I_z} + \dfrac{M_y\, z(n)}{I_y}$

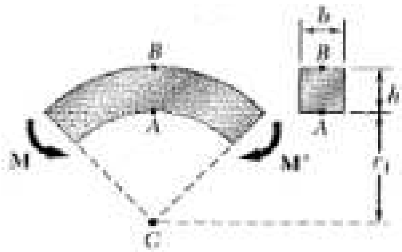
PROGRAM: FOR $\beta = 0$ TO 180° USING 10° INCREMENTS.
FOR $n = 1$ TO 4 USING UNIT INCREMENTS.
EVALUATE EQ 4.55 AND PRINT STRESSES
RETURN
RETURN

PROGRAM OUTPUT

```
Moment of couple M = 900 N.m
Moments of inertia: Iy = 2.59x10^-6 m^4      Iz = 0.62x10^-6 m^4

         Coordinates of points A, B, D, and E (x10^-3)
       Point A:  z(1) = 50:      y(1) =  35
       Point B:  z(2) =-50:      y(2) =  35
       Point D:  z(3) =-30:      y(3) = -35
       Point E:  z(4) = 30:      y(4) = -35
                 - - - Stress at Points - - -
```

beta deg	A MPa	B MPa	D MPa	E MPa
0	-52.161	-52.161	52.161	52.161
40	-51.340	-28.573	45.652	34.268
90	-17.713	17.713	8.853	-8.853
130	19.961	47.093	-26.746	-40.308
180	52.161	52.161	-52.161	-52.161

PROBLEM 4.C4



4.C4 Couples of moment $M = 2$ kN · m are applied as shown to a curved bar having a rectangular cross section with $h = 100$ mm and $b = 25$ mm. Write a computer program and use it to calculate the stresses at points A and B for values of the ratio r_1/h from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio r_1/h for which the maximum stress in the curved bar is 50 percent larger than the maximum stress in a straight bar of the same cross section.

SOLUTION INPUT: $h = 100$ mm, $b = 25$ mm, $M = 2$ kN·m

FOR STRAIGHT BAR: $\sigma_{STRAIGHT} = \frac{M}{S} = \frac{6M}{h^2 b} = 48$ MPa

FOLLOWING NOTATION OF SEC. 4.15, KEY IN THE FOLLOWING:

$r_2 = h + r_1$; $R = h/\ln(r_2 - r_1)$; $\bar{r} = r_1 + r_2$: $e = \bar{r} - R$; $A = bh = 2500$ (I)

STRESSES: $\sigma_A = \sigma_1 = M(r_1 - R)/(Aer_1)$ $\sigma_B = \sigma_2 = M(r_2 - R)/(Aer_2)$ (II)

SINCE $h = 100$ mm, FOR $r_1/h = 10$, $r_1 = 1000$ mm. ALSO $r_1/h = 10$, $r_1 = 100$

PROGRAM: FOR $r_1 = 1000$ TO 100 AT −100 DECREMENTS

USING EQUATIONS OF LINES I AND II EVALUATE $r_2, R, \bar{r}, e, \sigma_1$ AND σ_2

ALSO EVALUATE: ratio = $\sigma_1/\sigma_{STRAIGHT}$

RETURN AND REPEAT FOR $r_1 = 100$ TO 10 AT −10 DECREMENT

PROGRAM OUTPUT

```
M = Bending Moment = 2.  kN.m      h = 100.000 in.     A = 2500.00 mm^2
Stress in straight beam  =  48.00 MPa
```

r1 mm	rbar mm	R mm	e mm	sigma1 MPa	sigma2 MPa	r1/h -	ratio -
1000	1050	1049	0.794	-49.57	46.51	10.000	-1.033
900	950	949	0.878	-49.74	46.36	9.000	-1.036
800	850	849	0.981	-49.95	46.18	8.000	-1.041
700	750	749	1.112	-50.22	45.95	7.000	-1.046
600	650	649	1.284	-50.59	45.64	6.000	-1.054
500	550	548	1.518	-51.08	45.24	5.000	-1.064
400	450	448	1.858	-51.82	44.66	4.000	-1.080
300	350	348	2.394	-53.03	43.77	3.000	-1.105
200	250	247	3.370	-55.35	42.24	2.000	-1.153
100	150	144	5.730	-61.80	38.90	1.000	-1.288
100	150	144	5.730	-61.80	38.90	1.000	-1.288
90	140	134	6.170	-63.15	38.33	0.900	-1.316
80	130	123	6.685	-64.80	37.69	0.800	-1.350
70	120	113	7.299	-66.86	36.94	0.700	-1.393
60	110	102	8.045	-69.53	36.07	0.600	-1.449
50	100	91	8.976	-73.13	35.04	0.500	-1.523
40	90	80	10.176	-78.27	33.79	0.400	-1.631
30	80	68	11.803	-86.30	32.22	0.300	-1.798
20	70	56	14.189	-100.95	30.16	0.200	-2.103
10	60	42	18.297	-138.62	27.15	0.100	-2.888

Find r1/h for (sigma max)/(sigma straight) = 1.5

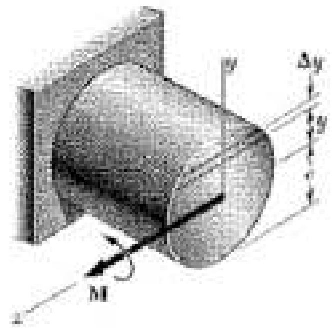
52.70	103	94	8.703	-72.036	35.34	0.527	-1.501
52.80	103	94	8.693	-71.998	35.35	0.528	-1.500
52.90	103	94	8.683	-71.959	35.36	0.529	-1.499

Ratio of stresses is 1.5 for r1 = 52.8 mm or r1/h = 0.529

[Note: The desired ratio r1/h is valid for any beam having a rectangular cross section.]

```
             Problem 4.7

Summary of Cross Section Dimensions
      Width (m)          Height (m)
        0.225               0.05
        0.075               0.15

Bending Moment = 60,000 Nm
Centroid is 0.075 m above lower edge
Centroidal Moment of Inertia is    79.  x 10^-6 m^4

Stress at top of beam     =  -94.103 MPa
Stress at bottom of beam =    56.462 MPa


             Problem 4.8

Summary of Cross Section Dimensions
      Width (m)          Height (m)
        0.1                0.025
        0.025              0.15
        0.2                0.025

Bending Moment = 60000 Nm
Centroid is    0.1194 m above lower edge
Centroidal Moment of Inertia is    60.59 x 10^-6 m^4

Stress at top of beam     =  -79.815 MPa
Stress at bottom of beam =  118.237 MPa


             PROBLEM 4.9

Summary of Cross Section Dimensions
     Width (mm)          Height (mm)
       50                 10
       20                 50

Bending Moment =    1500.0000 N.m
Centroid is    25.000 mm above lower edge
Centroidal Moment of Inertia is       512500 mm^4

Stress at top of beam     = -102.439 MPa
Stress at bottom of beam =    73.171 MPa
```

PROBLEM 4.C6



4.C6 A solid rod of radius $c = 30$ mm is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 290$ MPa. The rod is subjected to a couple of moment M that increases from zero to the maximum elastic moment M_Y and then to the plastic moment M_p. Denoting by y_Y the half thickness of the elastic core, write a computer program and use it to calculate the bending moment M and the radius of curvature ρ for values of y_Y from 30 mm to 0 using 5-mm decrements. (*Hint:* Divide the cross section into 80 horizontal elements of 1-mm height.)

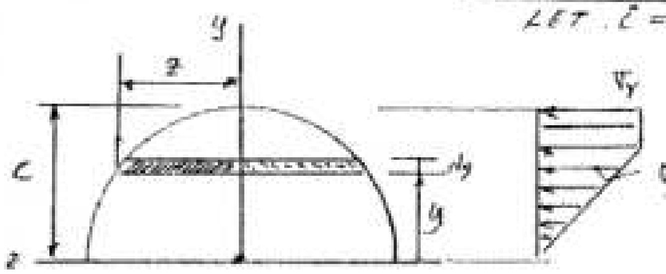
SOLUTION $M_Y = \sigma_Y \frac{\pi}{4} c^3 = (290\times10^6)\frac{\pi}{4}(0.03)^3 = 615$ MPa

$M_p = \sigma_Y \frac{4}{3} c^3 = (290\times10^6)\frac{4}{3}(0.03)^3 = 104.4$ kNm

CONSIDER TOP HALF OF ROD

LET i = NUMBER OF ELEMENTS IN TOP HALF

Δh = HEIGTH OF EACH ELEMENT: $\Delta h = \frac{c}{i}$



σ_E = STRESS ON ELEMENT

FOR $n = 0$ TO $i-1$ STEP 1

$y = n(\Delta h)$

$z = \left[c^2 - \{(n+0.5)\Delta h\}^2\right]$ ← z AT MIDHEIGHT OF ELEMENT

IF $y \geq y_Y$ GO TO 100

$\sigma_E = \sigma_Y \frac{(n+0.5)\Delta h}{\sigma_y}$ ← STRESS IN ELASTIC CORE

GO TO 200

100 $\sigma_E = \sigma_Y$ ← STRESS IN PLASTIC ZONE

200 $\Delta AREA = 2z(\Delta h)$

$\Delta FORCE = \sigma_E(\Delta AREA)$

$\Delta MOMENT = \Delta FORCE(n+.5)\Delta h$

$M = M + \Delta MOMENT$

$\rho = y_Y E/\sigma_Y$

PRINT y_Y, M, AND ρ.

NEXT

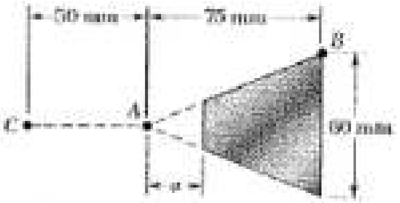
REPEAT FOR $y_Y = 30$mm TO $y_Y = 0$ AT 5mm DECREMENTS

PROGRAM OUTPUT

```
Radius of rod = 0.03 m
Yield point of steel = 290 MPa
Yield moment = 615 MPa          Plastic moment = 104.4 KNm
Number of elements in half of the rod = 40
```

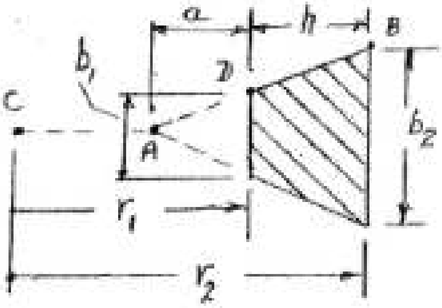
y_Y (mm)	P (mm x 10^3)
30	20.690
25	17.241
20	13.793
15	10.345
10	6.897
5	3.448
0	0

PROBLEM 4.C7



4.C7 The machine element of Prob. 4.178 is to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width a will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of a from 0 to 25 mm using 2.5 mm increments. Using appropriate smaller increments, determine the distance a for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

SOLUTION SEE FIG 4.79 PAGE 287



$M = 560\ \text{N m}$ $\quad r_2 = 125\ \text{mm}$ $\quad b_2 = 62\ \text{mm}$

FOR $a = 0$ TO 1.0 AT 0.1 INTERVALS

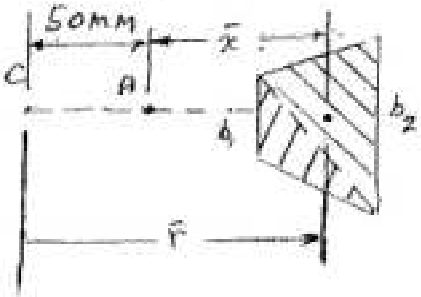
$h = 3 - a$

$r_1 = 2 + a$

$b_1 = b_2 (a/(h+a))$

$\text{AREA} = (b_1 + b_2)(h/2)$

$\bar{x} = a + \left[\frac{1}{2} b_1 h (h/3) + \frac{1}{2} b_2 h (2h/3)\right] / \text{AREA}$



$\bar{r} = r_2 - (h - \bar{x})$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$e = \bar{r} - R$

$\sigma_D = M(r_1 - R) / [\text{AREA}(e)(r_1)]$

$\sigma_B = M(r_2 - R) / [\text{AREA}(e)(r_2)]$

PRINT AND RETURN

PROGRAM OUTPUT

a mm	R mm	sigma D MPa	sigma B MPa	b1 mm	rbar mm	e mm
0	97.917	-58.656	14.489	0	101.600	3.683
5	98.273	-50.127	14.955	4.318	101.854	3.556
12.5	99.771	-45.288	16.990	10.668	102.870	3.023
20	102.057	-45.575	20.583	17.018	104.394	2.388
25	103.861	-48.022	23.782	21.082	105.918	1.981

Determination of the maximum compressive stress that is as small as possible

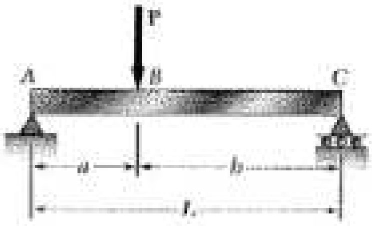
a mm	R mm	sigma D MPa	sigma B MPa	b1 mm	rbar mm	e mm
15.5	100.669	-44.9001	18.2200	1.321	10.338	0.277
15.625	100.660	-44.8992	18.2766	1.321	10.338	0.277
15.75	100.686	-44.8994	18.3345	1.321	10.338	0.277

Answer: When $a = 15.625$ mm the compressive stress is 44.9 MPa

Chapter 5

Problem 5.2

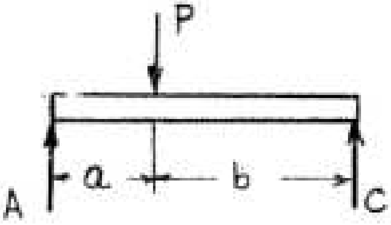
5.2 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.



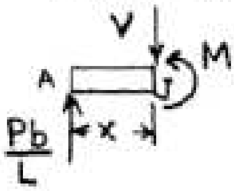
Reactions.

$\circlearrowleft \Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$

$\circlearrowleft \Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$



From A to B: $\quad 0 < x < a$

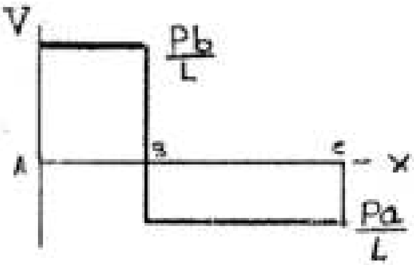


$+\uparrow \Sigma F_y = 0 \qquad \frac{Pb}{L} - V = 0$

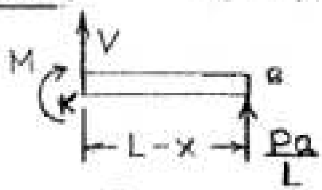
$V = \frac{Pb}{L}$ ◄

$\circlearrowleft \Sigma M_J = 0 \qquad M - \frac{Pb}{L}x = 0$

$M = \frac{Pbx}{L}$ ◄

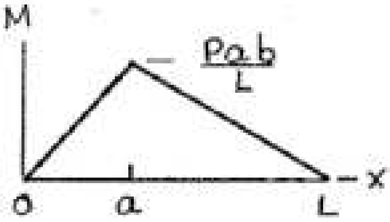


From B to C: $\quad a < x < L$



$+\uparrow \Sigma F_y = 0: \quad V + \frac{Pa}{L} = 0$

$V = -\frac{Pa}{L}$ ◄



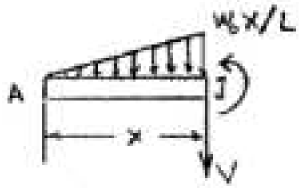
$\circlearrowleft \Sigma M_K = 0: \quad -M + \frac{Pa}{L}(L - x) = 0$

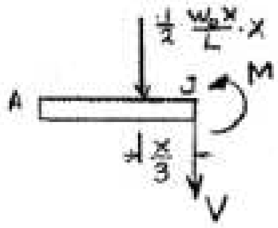
$M = \frac{Pa(L-x)}{L}$ ◄

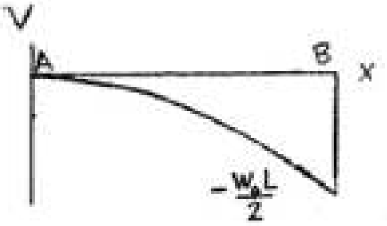
At section B: $\quad M = \frac{Pab}{L}$ ◄

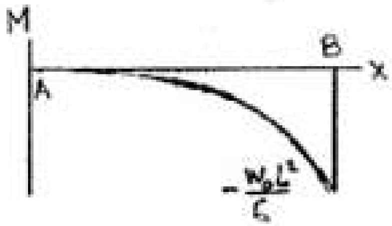
Problem 5.4

5.4 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.









$+\uparrow \Sigma F_y = 0 \qquad -\frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$

$$V = -\frac{w_0 x^2}{2L}$$

$\circlearrowleft \Sigma M_J = 0 \qquad \frac{1}{2}\frac{w_0 x}{L}\cdot x \cdot \frac{x}{3} + M = 0$

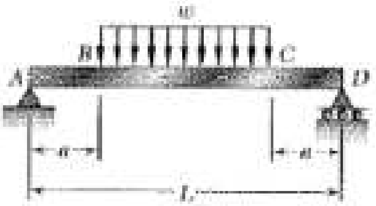
$$M = -\frac{w_0 x^3}{6L}$$

At $x = L$, $\quad V = -\frac{w_0 L}{2} \qquad |V|_{max} = \frac{w_0 L}{2}$

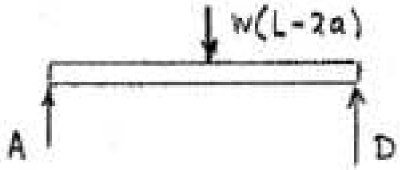
$M = -\frac{w_0 L^2}{6} \qquad |M|_{max} = \frac{w_0 L^2}{6}$

Problem 5.6

5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

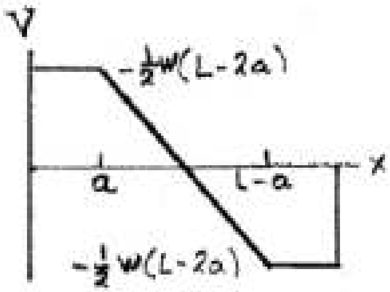


Calculate reactions after replacing distributed load by an equivalent concentrated load.

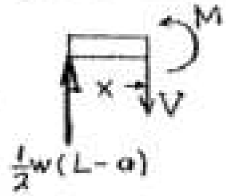


Reactions are

$$A = D = \tfrac{1}{2}w(L-2a)$$



From A to B: $\quad 0 < x < a$

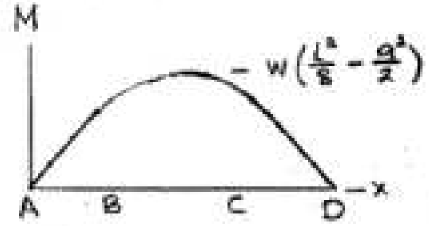


$+\uparrow \Sigma F_y = 0: \quad \tfrac{1}{2}w(L-2a) - V = 0$

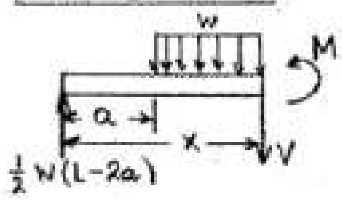
$$V = \tfrac{1}{2}w(L-2a)$$

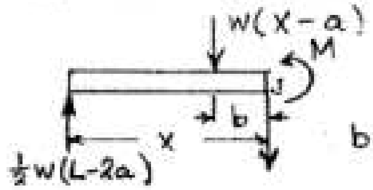
$\circlearrowleft \Sigma M = 0: \quad -\tfrac{1}{2}w(L-2a) + M = 0$

$$M = \tfrac{1}{2}w(L-2a)x$$



From B to C: $\quad a < x < L-a$





$b = \frac{x-a}{2}$

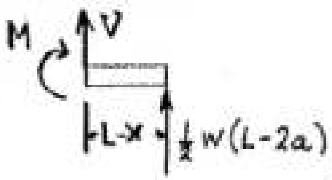
Place section cut at x. Replace distributed load by equiv. conc. load.

$+\uparrow \Sigma F_y = 0: \quad \tfrac{1}{2}w(L-2a) - w(x-a) - V = 0 \qquad V = w\left(\tfrac{L}{2} - x\right)$

$\circlearrowleft M_J = 0: \quad -\tfrac{1}{2}w(L-2a)x + w(x-a)\left(\tfrac{x-a}{2}\right) + M = 0$

$$M = \tfrac{1}{2}w\left[(L-2a)x - (x-a)^2\right]$$

From C to D: $\quad L-a < x < L$



$+\uparrow \Sigma F_y = 0: \quad V + \tfrac{1}{2}w(L-2a) = 0$

$$V = -\tfrac{w}{2}(L-2a)$$

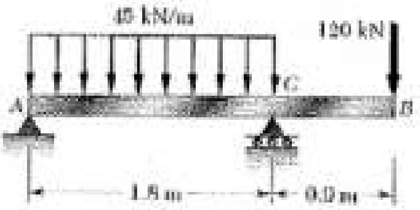
$\circlearrowleft \Sigma M_J = 0: \quad -M + \tfrac{1}{2}w(L-2a)(L-x) = 0$

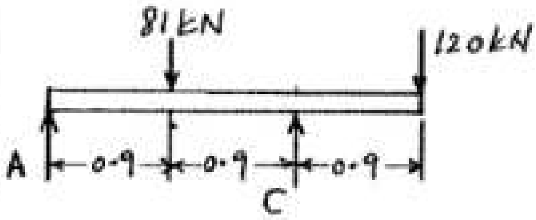
$$M = \tfrac{1}{2}w(L-2a)(L-x)$$

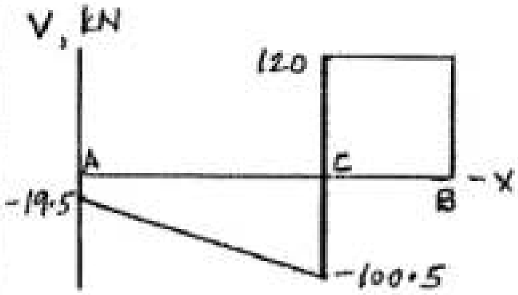
At $x = \frac{L}{2}$ $\qquad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$

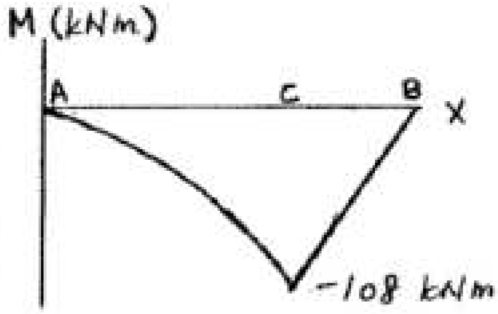
Problem 5.10

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.









Reactions

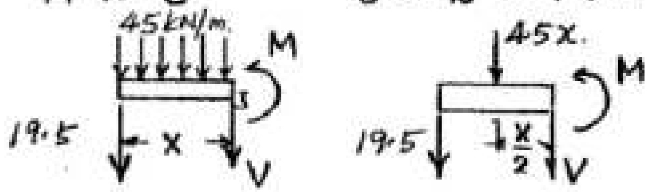
$\circlearrowleft \Sigma M_C = 0 \quad -1.8A + (0.9)(81) - (0.9)(120) = 0$

$A = -19.5\text{ kN}$ ie. $19.5 \downarrow$

$\circlearrowleft \Sigma M_A = 0 \quad 1.8C - (0.9)(81) - (2.7)(120) = 0$

$C = 220.5\text{ kN} \uparrow$

A to C $\qquad 0 < x < 1.8\text{ m}$



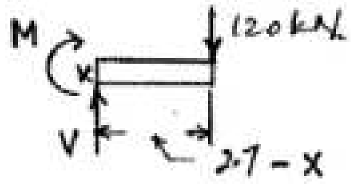
$+\uparrow \Sigma F_y = 0 \quad -19.5 - 45x - V = 0$

$V = -19.5 - 45x\text{ kN}$

$\circlearrowleft \Sigma M_J = 0 \quad -19.5x - (45x)\left(\frac{x}{2}\right) - M = 0$

$M = -19.5x - 22.5x^2\text{ kNm}$

C to B $\qquad 1.8\text{ m} < x < 2.7\text{ m}.$



$+\uparrow \Sigma F_y = 0, \qquad V - 120 = 0$

$V = 120\text{ kN}$

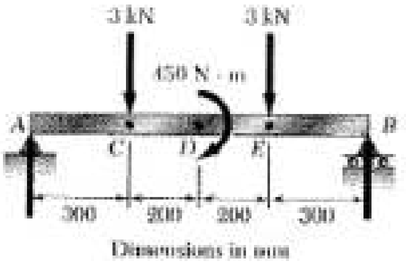
$\circlearrowleft \Sigma M_K = 0 \qquad -M - (2.7 - x)(120) = 0$

$M = 120x - 324\text{ kNm}.$

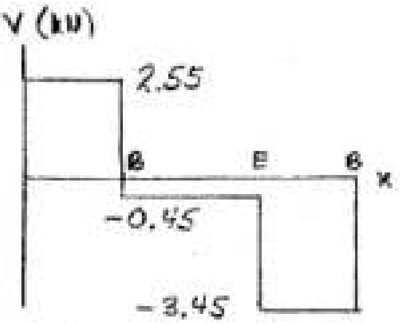
From the diagrams $\quad$ (a) $|V|_{max} = 120\text{ kN}$ ◄

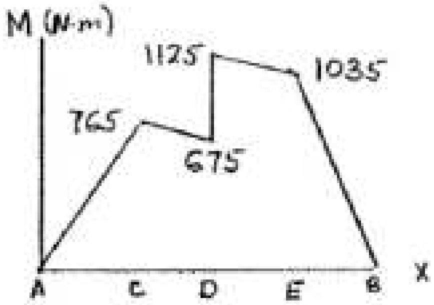
(b) $|M|_{max} = 108\text{ kNm}$ ◄

Problem 5.12



5.11 and 5.12 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.





$\circlearrowleft \Sigma M_B = 0: \quad (700)(3) - 450 + (300)(3) - 1000A = 0$

$A = 2.55 \text{ kN} \uparrow$

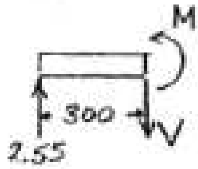
$\circlearrowleft \Sigma M_A = 0: \quad -(300)(3) - 450 - (700)(3) + 1000B = 0$

$B = 3.45 \text{ kN} \uparrow$

At A. $\quad V = 2.55 \text{ kN} \qquad M = 0$

A to C. $\quad V = 2.55 \text{ kN}$

At C.



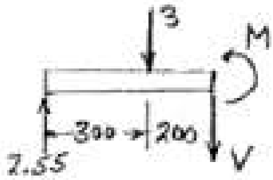
$\circlearrowleft \Sigma M_C = 0$

$-(300)(2.55) + M = 0$

$M = 765 \text{ N}\cdot\text{m}$

C to E. $\quad V = -0.45 \text{ N}\cdot\text{m}$

At D^-.

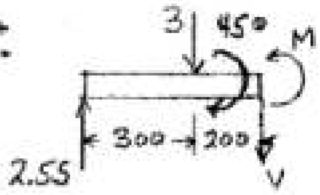


$\circlearrowleft \Sigma M_D = 0:$

$-(500)(2.55) + (200)(3) + M = 0$

$M = 675 \text{ N}\cdot\text{m}$

At D^+.



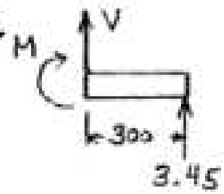
$\circlearrowleft \Sigma M_D = 0:$

$-(500)(2.55) + (200)(3) - 450 + M = 0$

$M = 1125 \text{ N}\cdot\text{m}$

E to B. $\quad V = -3.45 \text{ kN}$

At E.



$\circlearrowleft \Sigma M_E = 0:$

$-M + (300)(3.45) = 0$

$M = 1035 \text{ N}\cdot\text{m}$

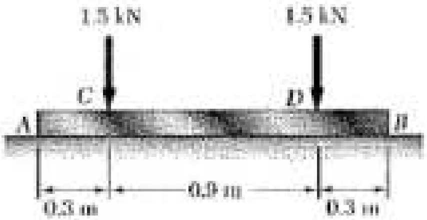
At B. $\quad V = 3.45 \text{ kN}, \qquad M = 0$

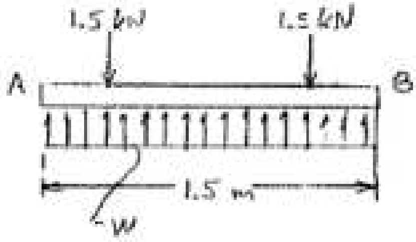
(a) Maximum $|V| = 3.45 \text{ kN}$ ◄

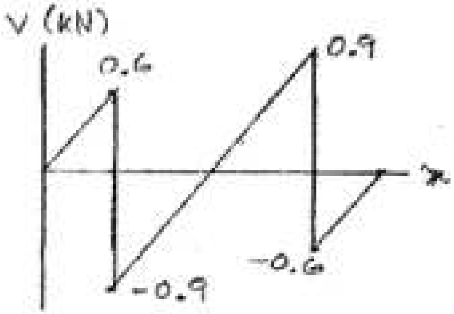
(b) Maximum $|M| = 1125 \text{ N}\cdot\text{m}$ ◄

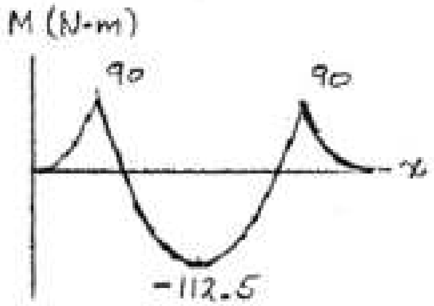
Problem 5.13

5.13 and *5.14* Assuming that the reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.







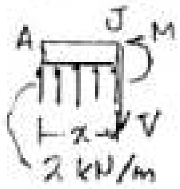


Over the whole beam.

$+\uparrow \Sigma F_y = 0: \quad 1.5w - 1.5 - 1.5 = 0$

$w = 2\ \text{kN/m}$

A to C. $\quad 0 \le x < 0.3\text{m}$



$+\uparrow \Sigma F_y = 0: \quad 2x - V = 0$

$V = (2x)\ \text{kN}$

$+\circlearrowleft M_J = 0: \ -(2x)(\frac{x}{2}) + M = 0$

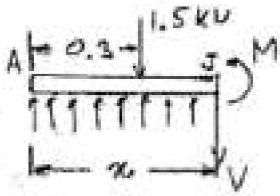
$M = (x^2)\ \text{kN·m}$

At C^-, $x = 0.3\text{m}$

$V = 0.6\ \text{kN}, \quad M = 0.090\ \text{kN·m}$

$= 90\ \text{N·m}$

C to D. $\quad 0.3\text{m} < x < 1.2\text{m}$



$+\uparrow \Sigma F_y = 0:$

$2x - 1.5 - V = 0$

$V = (2x - 1.5)\ \text{kN}$

$+\circlearrowleft \Sigma M_J = 0: \ -(2x)(\frac{x}{2}) + (1.5)(x - 0.3) + M = 0$

$M = (x^2 - 1.5x + 0.45)\ \text{kN·m}$

At the center of the beam: $x = 0.75\text{m}$

$V = 0 \qquad M = -0.1125\ \text{kN·m}$

$= -112.5\ \text{N·m}$

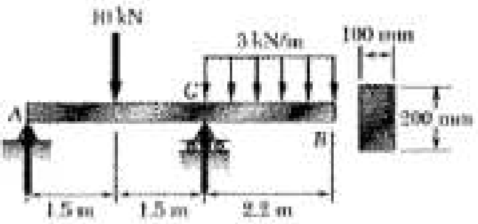
At C^+, $x = 0.3\text{m}, \quad V = -0.9\ \text{kN}$

(a) Maximum $|V| = 0.9\ \text{kN} = 900\ \text{N}$

(b) Maximum $|M| = 112.5\ \text{N·m}$

Problem 5.15

5.15 and *5.16* For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

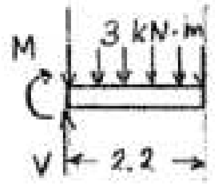


Using CB as a free body

$\circlearrowleft \Sigma M_C = 0$

$-M + (2.2)(3\times10^3)(1.1) = 0$

$M = 7.26\times10^3\ \text{N}\cdot\text{m}$



Section modulus for rectangle

$S = \frac{1}{6} b h^2$

$= \frac{1}{6}(100)(200)^2 = 666.7\times10^3\ \text{mm}^3$

$= 666.7\times10^{-6}\ \text{m}^3$

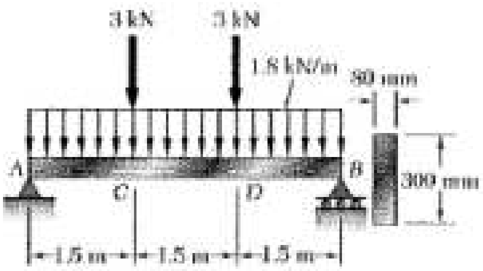
Normal stress

$\sigma = \dfrac{M}{S} = \dfrac{7.26\times10^3}{666.7\times10^{-6}} = 10.89\times10^6\ \text{Pa}$

$\sigma = 10.89\ \text{MPa}$ ◄

Problem 5.16

5.15 and *5.16* For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



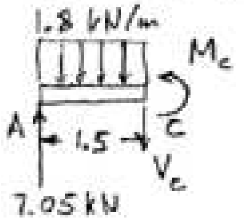
Reaction at A.

$+\circlearrowleft M_B = 0:$

$-4.5\,A + (3.0)(3) + (1.5)(3) + (1.8)(4.5)(2.25) = 0$

$A = 7.05\ \text{kN}\uparrow$

Use AC as free body.



$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (7.05)(1.5) + (1.8)(1.5)(0.75) = 0$

$M_C = 8.55\ \text{kN}\cdot\text{m} = 8.55\times10^3\ \text{N}\cdot\text{m}$

$I = \frac{1}{12} b h^3 = \frac{1}{12}(80)(300)^3 = 180\times10^6\ \text{mm}^4 = 180\times10^{-6}\ \text{m}^4$

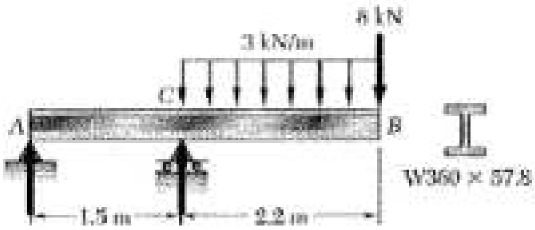
$c = \frac{1}{2}(300) = 150\ \text{mm} = 0.150\ \text{m}$

$\sigma = \dfrac{Mc}{I} = \dfrac{(8.55\times10^3)(0.150)}{180\times10^{-6}} = 7.125\times10^6\ \text{Pa}$

$\sigma = 7.13\ \text{MPa}$ ◄

Problem 5.19

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



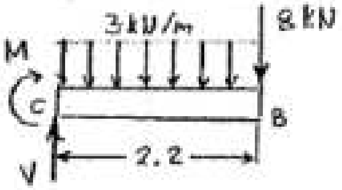
Use portion CB as free body.

$\circlearrowleft \Sigma M_C = 0:$

$-M - (1.1)(3)(2.2) - (2.2)(8) = 0$

$M = -24.86 \text{ kN}\cdot\text{m}$

$= -24.86 \times 10^3 \text{ N}\cdot\text{m}$



For W360 × 57.8, $S = 899 \times 10^3 \text{ mm}^3$
$= 899 \times 10^{-6} \text{ m}^3$

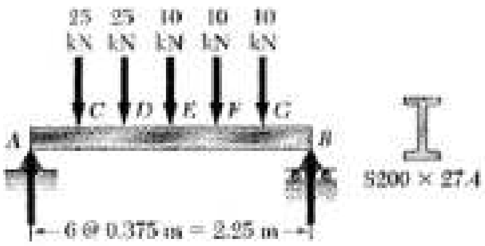
Normal stress. $\sigma = \dfrac{|M|}{S}$

$\sigma = \dfrac{24.86 \times 10^3}{899 \times 10^{-6}} = 27.7 \times 10^6 \text{ Pa}$

$\sigma = 27.7 \text{ MPa}$ ◄

Problem 5.20

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

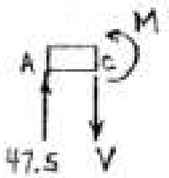


Use entire beam as free body.

$\circlearrowleft \Sigma M_B = 0:$

$2.25 A - (1.875)(25) - (1.5)(25) - (1.125)(10)$
$- (0.75)(10) - (0.375)(10) = 0$

$A = 47.5 \text{ kN}$



Use portion AC as free body.

$-(0.375)(47.5) + M = 0 \qquad M = 17.8125 \text{ kN}\cdot\text{m}$

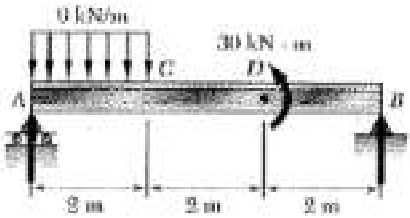
For S 200 × 27.4, $S = 235 \times 10^3 \text{ mm}^3 = 235 \times 10^{-6} \text{ m}^3$

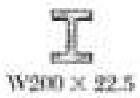
Normal stress. $\sigma = \dfrac{M}{S} = \dfrac{17.8125 \times 10^3}{235 \times 10^{-6}} = 75.8 \times 10^6 \text{ Pa}$

$\sigma = 75.8 \text{ MPa}$ ◄

Problem 5.23

5.22 and 5.23 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.





$\circlearrowleft \Sigma M_B = 0:$

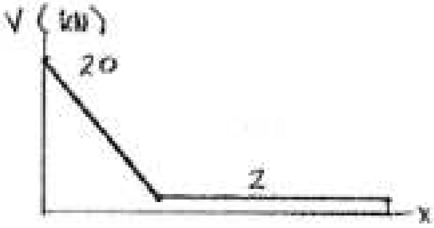
$-6A + (2)(9)(5) + 30 = 0$

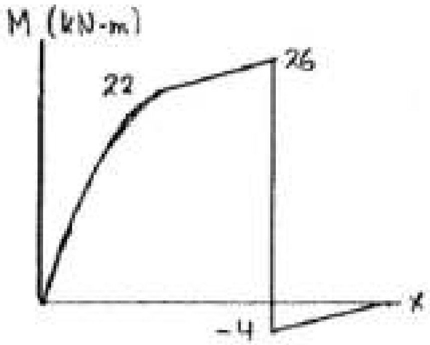
$A = 20 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0:$

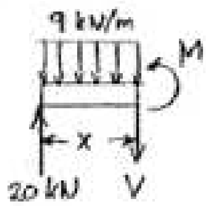
$-(2)(9)(1) + 30 + 6B = 0$

$B = -2 \text{ kN} \quad \text{i.e.} \quad 2 \text{ kN} \downarrow$





A to C, $\quad 0 < x < 2 \text{ m}$



$+\uparrow \Sigma F_y = 0 \qquad 20 - 9x - V = 0$

$V = 20 - 9x$

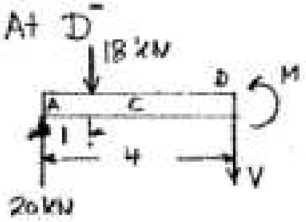
$\circlearrowleft \Sigma M_J = 0:$

$-20x + (9x)\frac{x}{2} + M = 0$

$M = 20x - 4.5x^2$

At C $\quad V = 2 \text{ kN} \qquad M = 22 \text{ kN·m}$

At D^-



$+\uparrow \Sigma F_y = 0:$

$20 - 18 - V = 0$

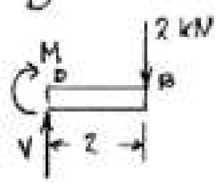
$V = 2 \text{ kN}$

$\circlearrowleft \Sigma M_D = 0:$

$-(4)(20) + (3)(18) + M = 0$

$M = 26 \text{ kN-m}$

At D^+



$+\uparrow \Sigma F_y = 0: \quad V - 2 = 0$

$V = 2 \text{ kN}$

$\circlearrowleft \Sigma M_D = 0:$

$-M - (2)(2) = 0$

$M = -4 \text{ kN·m}$

$\max |M| = 26 \text{ kN·m} = 26 \times 10^3 \text{ N·m}$

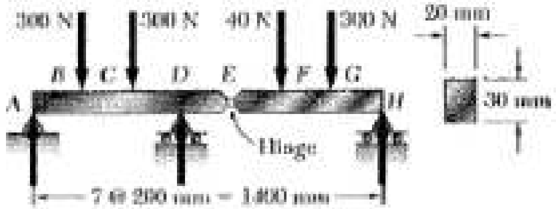
For rolled steel section $\quad W\ 200 \times 22.5, \quad S = 194 \times 10^3 \text{ mm}^3$

$= 194 \times 10^{-6} \text{ m}^3$

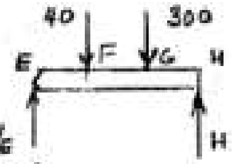
Normal stress, $\quad \sigma = \frac{|M|}{S} = \frac{26 \times 10^3}{194 \times 10^{-6}} = 134.0 \times 10^6 \text{ Pa} = 134.0 \text{ MPa}$ ◄

Problem 5.24

5.24 and 5.25 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



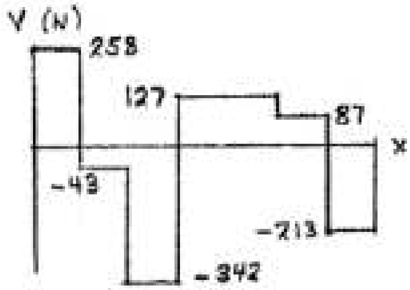
Free body EFGH.
Note that $M_E = 0$ due to hinge.



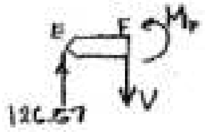
$\circlearrowleft \Sigma M_E = 0$:
$0.6\,H - (0.2)(40) - (0.4)(300) = 0$
$H = 213.33$ N

$+\uparrow \Sigma F_y = 0$: $\quad V_E - 40 - 300 + 213.33 = 0$
$V_E = 126.67$ N

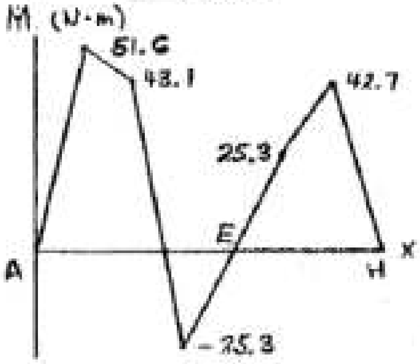
Shear: E to F. $\quad V = 126.67$ N·m
F to G. $\quad V = 86.67$ N·m
G to H. $\quad V = -213.33$ N·m



Bending moment at F.

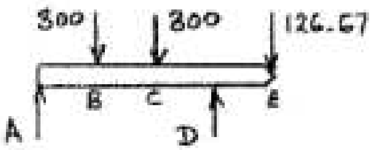


$\circlearrowleft \Sigma M_F = 0$:
$M_F - (0.2)(126.67) = 0$
$M_F = 25.33$ N·m



Bending moment at G.

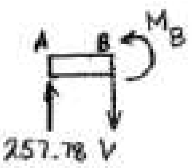
$\circlearrowleft \Sigma M_G = 0$
$-M_G + (0.2)(213.33) = 0$
$M_G = 42.67$ N·m



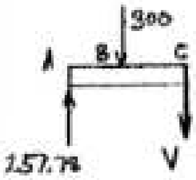
Free body ABCDE.

$+\circlearrowleft \Sigma M_B = 0$: $\quad -0.6\,A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$
$A = 257.78$ N

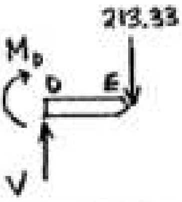
$+\circlearrowleft \Sigma M_A = 0$: $\quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6\,D = 0$
$D = 468.89$ N



Bending moment at B.
$\circlearrowleft \Sigma M_B = 0$:
$-(0.2)(257.78) + M_B = 0$
$M_B = 51.56$ N·m



Bending moment at C.
$+\circlearrowleft \Sigma M_C = 0$:
$-(0.4)(257.78) + (0.2)(300) + M_C = 0$
$M_C = 43.11$ N·m



Bending moment at D.
$+\circlearrowleft \Sigma M_D = 0$:
$-M_D - (0.2)(213.33) = 0$
$M_D = -25.33$ N·m

max $|M| = 51.56$ N·m

$S = \frac{1}{6} b h^2 = \frac{1}{6}(20)(30)^2 = 3 \times 10^3\ \text{mm}^3 = 3 \times 10^{-6}\ \text{m}^3$

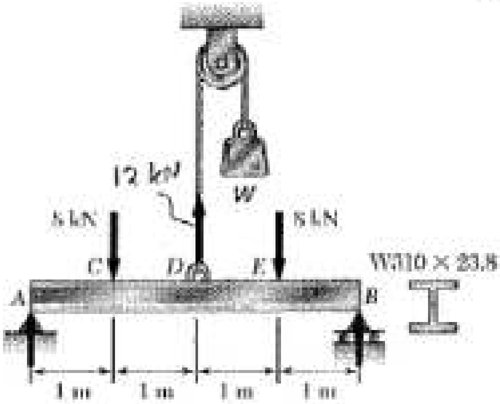
Normal stress.

$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6\ \text{Pa}$$

$\sigma = 17.19$ MPa

Problem 5.26

5.26 Knowing that $W = 12$ kN, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.



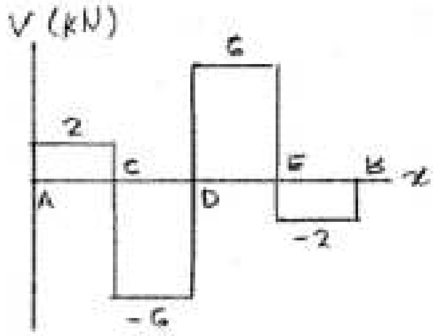
By symmetry, $A = B$

$+\uparrow \Sigma F_y = 0: \quad A - 8 + 12 - 8 + B = 0$

$A = B = 2$ kN

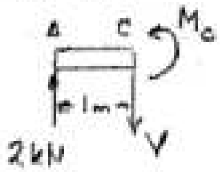
Shear:

A to C:	$V = 2$ kN
C^+ to D^-:	$V = -6$ kN
D^+ to E^-:	$V = 6$ kN
E^+ to B.	$V = -2$ kN



Bending moment:

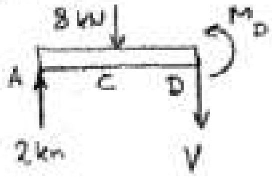
At C: $\quad +\circlearrowleft \Sigma M_C = 0:$



$M_C - (1)(2) = 0$

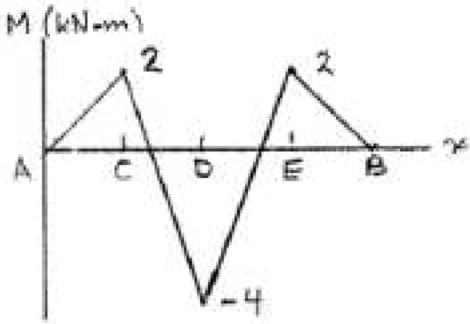
$M_C = 2$ kN·m

At D: $\quad + \Sigma M_D = 0:$



$M_D - (2)(2) + (8)(1) = 0$

$M_D = -4$ kN·m



By symmetry, $M = 2$ kN·m at E $\qquad M_E = 2$ kN·m

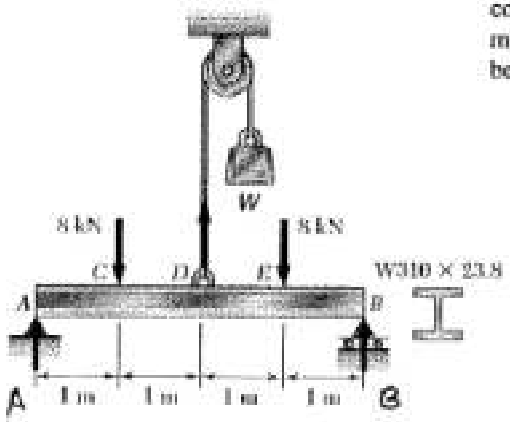
max $|M| = 4$ kN·m occurs at E

For W310 × 23.8, $\quad S_x = 280 \times 10^3\ \text{mm}^3 = 280 \times 10^{-6}\ \text{m}^3$

Normal stress: $\sigma_{max} = \dfrac{|M|_{max}}{S_x} = \dfrac{4 \times 10^3}{280 \times 10^{-6}} = 14.29 \times 10^6$ Pa

$\sigma_{max} = 14.29$ MPa

Problem 5.27



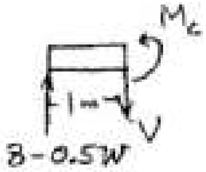
5.27 Determine (*a*) the magnitude of the counterweight W for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (*Hint:* Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.)

By symmetry, $A = B$

$+\uparrow \Sigma F_y = 0: \quad A - 8 + W - 8 + B = 0$

$A = B = 8 - 0.5W$

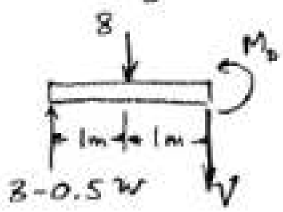
Bending moment at C. $+\circlearrowleft \Sigma M_C = 0:$



$-(8 - 0.5W)(1) + M_C = 0$

$M_C = (8 - 0.5W)$ kN·m

Bending moment at D.



$+\circlearrowleft \Sigma M_D = 0: \quad -(8 - 0.5W)(2) + (8)(1) + M_D = 0$

$M_D = (8 - W)$ kN·m

Equate. $\quad -M_D = M_C \qquad W - 8 = 8 - 0.5W$

(a) $\qquad W = 10.6667$ kN $\qquad W = 10.67$ kN ◄

$M_C = -2.6667$ kN·m $\qquad M_D = 2.6667$ kN·m $= 2.6667 \times 10^3$ N·m

$|M|_{max} = 2.6667$ kN·m

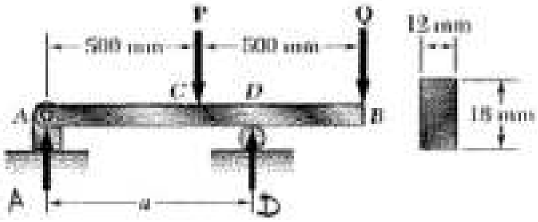
For W 310 × 23.8 rolled steel shape,

$S_x = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{ m}^3$

(b) $\quad \sigma_{max} = \dfrac{|M|_{max}}{S_x} = \dfrac{2.6667 \times 10^3}{280 \times 10^{-6}} = 9.52 \times 10^6$ Pa $\qquad \sigma_{max} = 9.52$ MPa ◄

Problem 5.28

5.28 Knowing that $P = Q = 480$ N, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

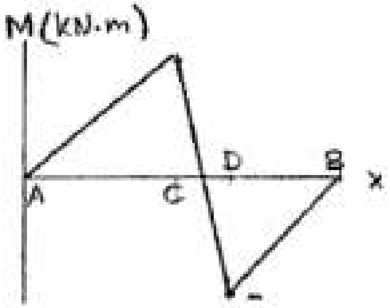


$P = 480$ N $Q = 480$ N

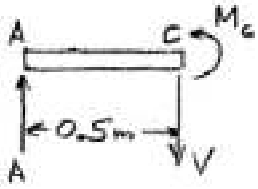
Reaction at A. $+\circlearrowleft \Sigma M_D = 0$:

$$-Aa + 480(a - 0.5) - 480(1 - a) = 0$$

$$A = \left(960 - \frac{720}{a}\right) \text{ N}$$



Bending moment at C. $+\circlearrowleft \Sigma M_C = 0$:

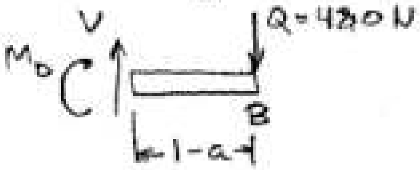


$$-0.5A + M_C = 0$$

$$M_C = 0.5A$$

$$= \left(480 - \frac{360}{a}\right) \text{ N}\cdot\text{m}$$

Bending moment at D. $+\circlearrowleft \Sigma M_D = 0$:



$$-M_D - 480(1 - a) = 0$$

$$M_D = -480(1 - a) \text{ N}\cdot\text{m}$$

(a) Equate. $-M_D = M_C$ $480(1 - a) = 480 - \frac{360}{a}$

$a = 0.86603$ m $a = 866$ mm ◄

$A = 128.62$ N $M_C = 64.31$ N·m $M_D = -64.31$ N·m

(b) For rectangular section, $S = \frac{1}{6}bh^2$

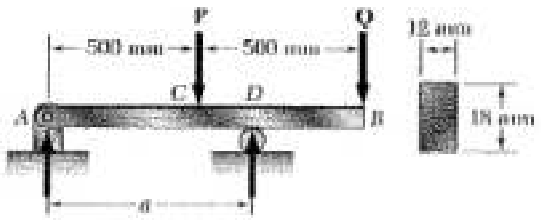
$$S = \frac{1}{6}(12)(18)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{ m}^3$$

$$\sigma_{max} = \frac{|M|_{max}}{S} = \frac{64.31}{648 \times 10^{-9}} = 99.2 \times 10^6 \text{ Pa}$$ $\sigma_{max} = 99.2$ MPa ◄

Problem 5.29

5.29 Solve Prob. 5.28, assuming that $P = 480$ N and $Q = 320$ N.

5.28 Knowing that $P = Q = 480$ N, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

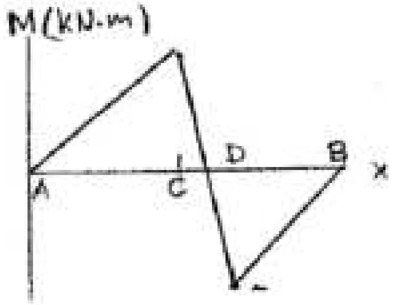


$P = 480$ N $\qquad Q = 320$ N

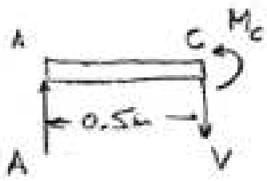
Reaction at A. $\circlearrowleft \Sigma M_D = 0:$

$$Aa + 480(a - 0.5) - 320(1 - a) = 0$$

$$A = \left(800 - \frac{560}{a}\right) \text{ N}$$



Bending moment at C. $\circlearrowleft \Sigma M_C = 0:$

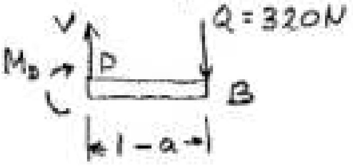


$$-0.5A + M_C = 0$$

$$M_C = 0.5A$$

$$= \left(400 - \frac{280}{a}\right) \text{ N} \cdot \text{m}$$

Bending moment at D. $\circlearrowleft \Sigma M_D = 0;$



$$-M_D - 320(1 - a) = 0$$

$$M_D = (-320 + 320a) \text{ N} \cdot \text{m}$$

(a) Equate. $-M_D = M_C \qquad 320 - 320a = 400 - \frac{280}{a}$

$$320a^2 + 80a - 280 = 0 \qquad a = 0.81873 \text{ m}, -1.06873 \text{ m}$$

Reject negative root. $\quad a = 819 \text{ mm}$ ◄

$A = 116.014$ N $\qquad M_C = 58.007$ N·m $\qquad M_D = -58.006$ N·m

(b) For rectangular section, $S = \frac{1}{6}bh^2$

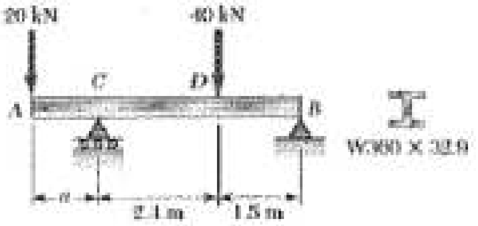
$$S = \frac{1}{6}(12)(18)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{ m}^3$$

$$\sigma_{max} = \frac{|M|_{max}}{S} = \frac{58.0065}{648 \times 10^{-9}} = 89.5 \times 10^6 \text{ Pa} \qquad \sigma_{max} = 89.5 \text{ MPa}$$ ◄

Problem 5.30

5.30 Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

(*Hint:* Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.)

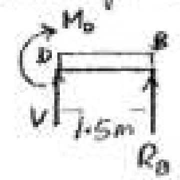


Reaction at B. $+\circlearrowleft \Sigma M_C = 0$

$$20a - (2.4)(40) + 3.9 R_B = 0$$

$$R_B = \frac{1}{3.9}(96 - 20a)$$

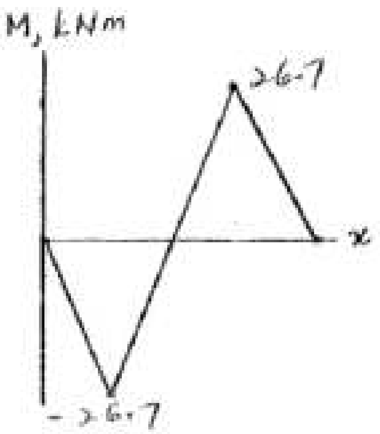
Bending moment at D.



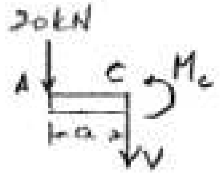
$+\circlearrowleft \Sigma M_D = 0$

$$-M_D + 1.5 R_B = 0$$

$$M_D = 1.5 R_B = \frac{1.5}{3.9}(96 - 20a)$$



Bending moment at C



$+\circlearrowleft M_C = 0$

$$20a + M_C = 0$$

$$M_C = -20a$$

Equate $\quad -M_C = M_D$

$$20a = \frac{1.5}{3.9}(96 - 20a)$$

$a = 1.33$ m $\qquad$ (a) $a = 1.33$ m

Then $\quad -M_C = M_D = (20)(1.33) = 26.7$ kN/m

$|M|_{max} = 26.7$ kN m.

For W360X32.9 rolled steel section $\quad S = 474 \times 10^3$ mm^3

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{26.7 \times 10^3}{474 \times 10^{-6}} = 56.3$ MPa $\qquad$ (b) 56.3 MPa

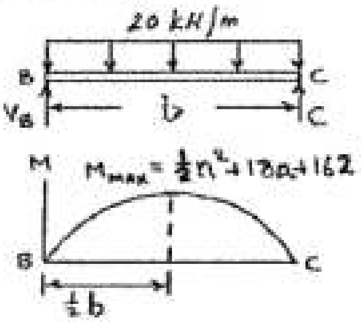
Problem 5.31

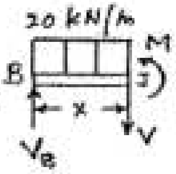
5.31 Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

For W 360 x 101, $S_x = 1690 \times 10^3 \text{ mm}^3$

Let $b = (5.4 - a)$ m

SEGMENT BC:





By symmetry, $V_B = C$

$+\uparrow \Sigma F_y = 0: \quad V_B + C - 20b = 0$

$V_B = 10b$

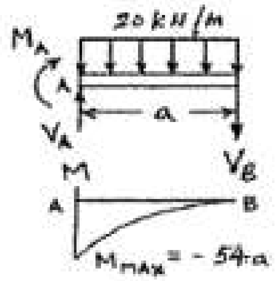
$+\circlearrowleft \Sigma M_J = 0: -V_B x + (20x)\left(\frac{x}{2}\right) + M = 0$

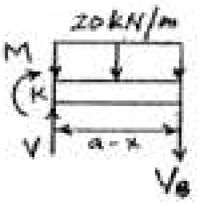
$M = V_B x - 10x^2 = 10bx - 10x^2 \text{ kN m}$

$\frac{dM}{dx} = 10b - 20x_m = 0 \qquad x_m = \frac{1}{2}b$

$M_{max} = 5b^2 - \frac{5}{2}b^2 = \frac{5}{2}b^2$

SEGMENT AB:





$+\circlearrowleft \Sigma M_K = 0:$

$-20(a-x)\left(\frac{a-x}{2}\right) - V_B(a-x) - M = 0$

$M = -10(a-x)^2 + 10b(a-x)$

$|M_{max}|$ occurs at $x = 0$.

$|M_{max}| = -10a^2 - 10ab$

$= -10a^2 - 10a(5.4 - a) = 54a$

(a) Equate the two values of $|M_{max}|$.

$54a = \frac{5}{2}b^2 = \frac{5}{2}(5.4 - a)^2 = 72.9 - 27a + \frac{5}{2}a^2$

$\frac{5}{2}a^2 - 81a + 72.9 = 0 \qquad a = \left[81 \pm \sqrt{(81)^2 - (4)\left(\frac{5}{2}\right)(72.9)}\right] / 5$

$a = (81 \pm 76.37)/5 = 0.926 \text{ m} \qquad\qquad a = 0.93 \text{ m}$ ◄

(b) $|M|_{max} = 54a = 50 \text{ kNm}$

$\sigma = \frac{|M|_{max}}{S_x} = \frac{50 \times 10^3}{1690 \times 10^{-6}} = 29.58 \text{ MPa}$

$\sigma_m = 29.6 \text{ MPa}$ ◄

Problem 5.32

5.32 A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 7860\ \mathrm{kg/m^3}$, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 28 MPa.

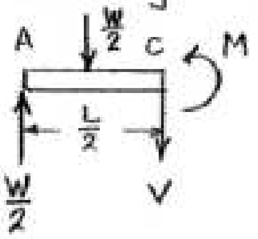
Let W = total weight

$$W = V\gamma = AL\gamma = \frac{\pi}{4} d^2 L \gamma$$

Reaction at A

$$A = \frac{1}{2} W$$

Bending moment at center of beam



$$\circlearrowleft + \Sigma M_C = 0$$

$$-\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32} d^2 L^2 \gamma$$

For circular cross section $(c = \frac{1}{2}d)$

$$I = \frac{\pi}{4} c^4, \qquad S = \frac{I}{c} = \frac{\pi}{4} c^3 = \frac{\pi}{32} d^3$$

Normal stress

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32} d^2 L^2 \gamma}{\frac{\pi}{32} d^3} = \frac{L^2 \gamma}{d}$$

Solving for d $\qquad d = \frac{L^2\gamma}{\sigma}$

Data: $L = 3\ \mathrm{m}$

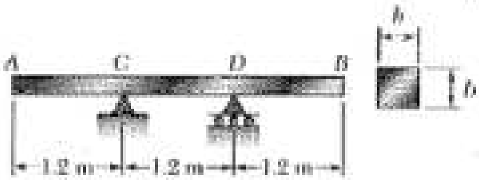
$\gamma = (7860 \times 9.81)\ \mathrm{N/m^3}$

$\sigma = 28\ \mathrm{MPa}.$

$$d = \frac{(3)^2(7860 \times 9.81)}{28 \times 10^6} = 0.0248\ \mathrm{m} = 24.8\ \mathrm{mm}.$$

Problem 5.33

5.33 A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860\ \text{kg/m}^3$, determine the dimension b for which the maximum normal stress due to bending is (*a*) 10 MPa, (*b*) 50 MPa.

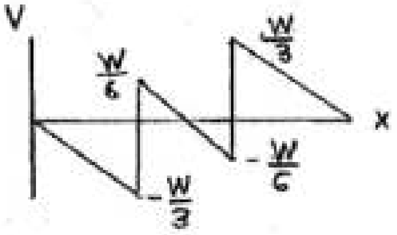


Weight density $\gamma = \rho g$

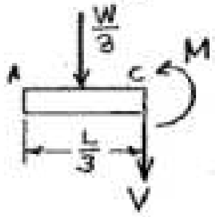
Let L = total length of beam.

$$W = V\gamma = AL\rho g = b^2 L \rho g$$

Reactions at C and D. $\quad C = D = \frac{W}{2}$



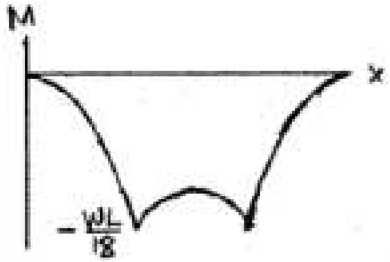
Bending moment at C.



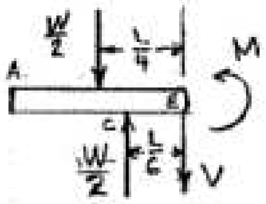
$\circlearrowleft \Sigma M_C = 0:$

$$\left(\frac{L}{6}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$



Bending moment at center of beam.



$\circlearrowleft \Sigma M_E = 0:$

$$\left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0$$

$$M = -\frac{WL}{24}$$

$$\max|M| = \frac{WL}{18} = \frac{b^2 L^2 \rho g}{18}$$

For a square section, $\quad S = \frac{1}{6} b^3$

Normal stress, $\quad \sigma = \frac{|M|}{S} = \frac{b^2 L^2 \rho g / 18}{b^3/6} = \frac{L^2 \rho g}{3b}$

Solve for b. $\quad b = \frac{L^2 \rho g}{3\sigma}$

Data: $L = 3.6$ m $\quad \rho = 7860\ \text{kg/m}^3 \quad g = 9.81\ \text{m/s}^2$

(a) $\sigma = 10 \times 10^6$ Pa $\qquad$ (b) $\sigma = 50 \times 10^6$ Pa

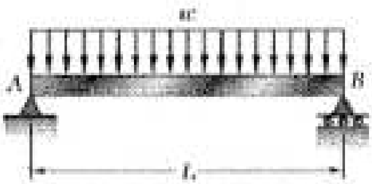
(a) $\quad b = \frac{(3.6)^2 (7860)(9.81)}{(3)(10 \times 10^6)} = 33.3 \times 10^{-3}$ m $\qquad b = 33.3$ mm ◄

(b) $\quad b = \frac{(3.6)^2 (7860)(9.81)}{(3)(50 \times 10^6)} = 6.66 \times 10^{-3}$ m $\qquad b = 6.66$ mm ◄

Problem 5.34

5.34 Using the method of Sec. 5.3, solve Prob. 5.1*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$\circlearrowleft \Sigma M_B = 0: \quad -AL + wL\cdot\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

$$\circlearrowleft \Sigma M_A = 0: \quad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = A - wx = \frac{wL}{2} - wx \blacktriangleleft$$

$$\frac{dM}{dx} = V$$

$$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{wL}{2} - wx\right)dx$$

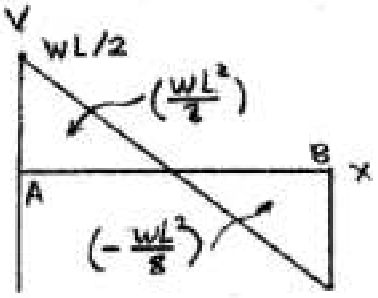
$$= \frac{wLx}{2} - \frac{wx^2}{2}$$

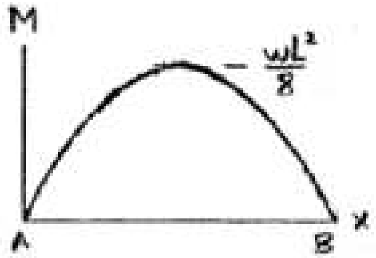
$$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w}{2}(Lx - x^2) \blacktriangleleft$$

Maximum M occurs at $x = \frac{L}{2}$, where

$$V = \frac{dM}{dx} = 0$$

$$M_{max} = \frac{wL^2}{8} \blacktriangleleft$$





Problem 5.36

5.36 Using the method of Sec. 5.3, solve Prob. 5.3*a*.

5.1 through 5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

At A^+: $V_A = -P$

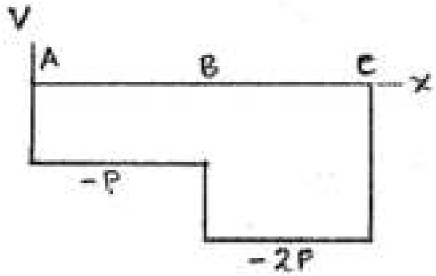
Over AB. $\frac{dV}{dx} = -w = 0$

$$\frac{dM}{dx} = V = V_A = -P$$

$$M = -Px + C$$

$M = 0$ at $x = 0$ $C_1 = 0$

$$M = -Px$$



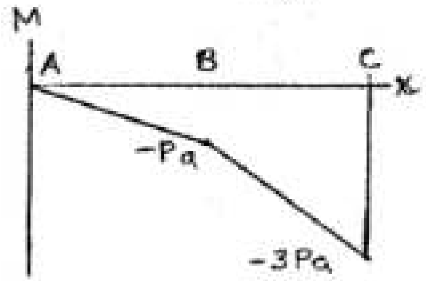
At point B. $x = a$ $M = -Pa$

At point B^+: $V = -P - P = -2P$

Over BC. $\frac{dV}{dx} = -w = 0$

$$\frac{dM}{dx} = V = -2P$$

$$M = -2Px + C_2$$



At B. $x = a$ $M = -Pa$

$-Pa = -2Pa + C_2$ $C_2 = Pa$

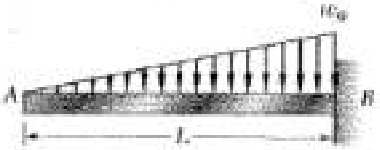
$$M = -2Px + Pa$$

At C. $x = 2a$ $M = -3Pa$

Problem 5.37

5.37 Using the method of Sec. 5.3, solve Prob. 5.4*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$w = w_0 \frac{x}{L}$$

$$V_A = 0, \qquad M_A = 0$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

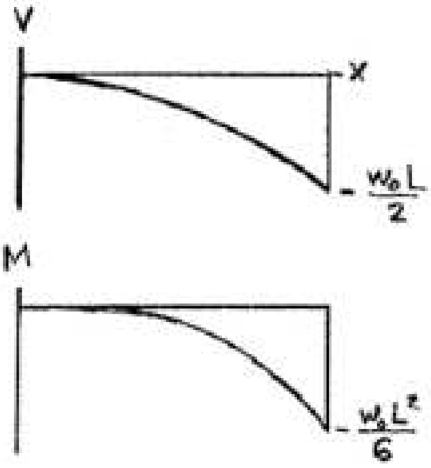
$$V - V_A = -\int_0^x \frac{w_0 x}{L} = -\frac{w_0 x^2}{2L}$$

$$V = -\frac{w_0 x^2}{2L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L}$$

$$M - M_A = \int_0^x V\,dx = -\int_0^x \frac{w_0 x^2}{2L}\,dx$$

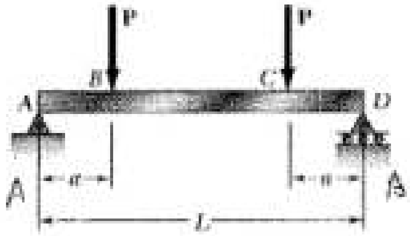
$$= -\frac{w_0 x^3}{6L}$$



Problem 5.38

5.38 Using the method of Sec. 5.3, solve Prob. 5.5*a*.

5.1 through 5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.



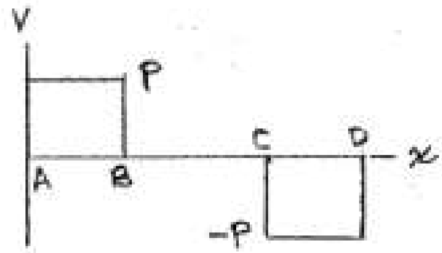
Reactions. By symmetry: $A = B$

$+\uparrow \Sigma F_y = 0: \quad A - P - P + B = 0$

$A = B = P$

Over each portion AB, BC, and CD $w = 0$.

$\frac{dV}{dx} = 0 \qquad V = \text{constant}.$

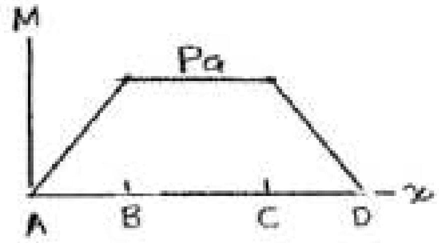


Shear diagram.

A to B: $V = P$

B to C: $V = P - P = 0$

C to D: $V = 0 - P = -P$



Bending moment diagram.

A to B: $\frac{dM}{dx} = V = P$

$M = \int P\,dx = Px + C_1$

$M = 0$ at $x = 0$. Thus $C_1 = 0$

$M = Px$

At B: $M = Pa$

B to C: $\frac{dM}{dx} = V = 0$

$M = Pa$

C to D: $\frac{dM}{dx} = V = -P$

$M = -\int P\,dx = -Px + C_2$

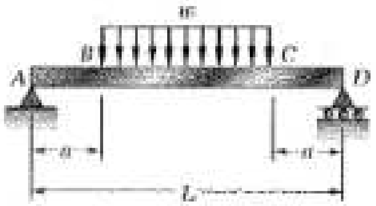
$M = 0$ at $x = L$. Thus $C_2 = PL$

$M = P(L - x)$

Problem 5.39

5.39 Using the method of Sec. 5.3, solve Prob. 5.6*a*

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



Reactions. $A = D = \frac{1}{2}w(L-2a)$

At A. $V_A = A = \frac{1}{2}w(L-2a)$, $M_A = 0$

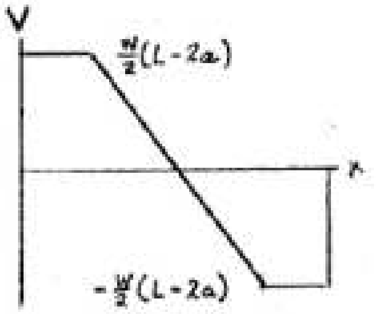
A to B. $0 < x < a$ $\quad w = 0$

$$V_B - V_A = -\int_0^a w\,dx = 0$$

$$V_B = V_A = \frac{1}{2}w(L-2a)$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{1}{2}w(L-2a)\,dx$$

$$M_B = \frac{1}{2}w(L-2a)a$$



B to C. $a < x < L-a$ $\quad w = w$

$$V - V_B = -\int_a^x w\,dx = -w(x-a)$$

$$V = \frac{1}{2}w(L-2a) - w(x-a) = \frac{1}{2}w(L-2x)$$

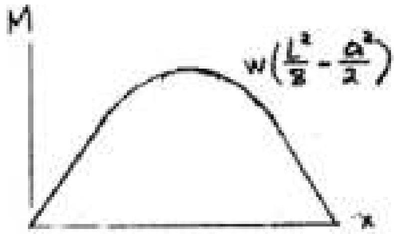
$$\frac{dM}{dx} = V = \frac{1}{2}w(L-2x)$$

$$M - M_B = \int_a^x V\,dx = \frac{1}{2}w(Lx - x^2)\Big|_a^x$$

$$= \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$M = \frac{1}{2}w(L-2a)a + \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$= \frac{1}{2}w(Lx - x^2 - a^2)$$



At C. $x = L-a$ $\quad V_C = -\frac{1}{2}w(L-2a)$ $\quad M_C = \frac{1}{2}(L-2a)a$

C to D. $V = V_C = -\frac{1}{2}w(L-2a)$

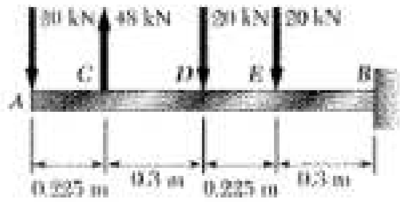
$M_D = 0$

At $x = \frac{L}{2}$, $\quad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$ ◄

Problem 5.40

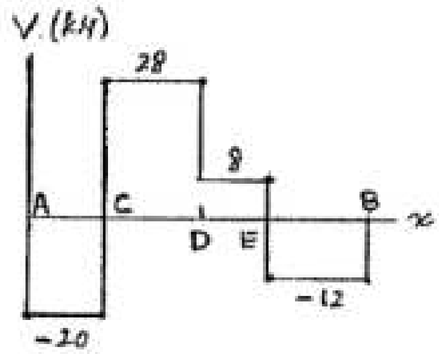
5.40 Using the method of Sec. 5.3, solve Prob. 5.7.

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Over portions AC, CD, DE, and EB,

$$\frac{dV}{dx} = -w = 0 \qquad V = \text{constant}$$



Shear diagram.

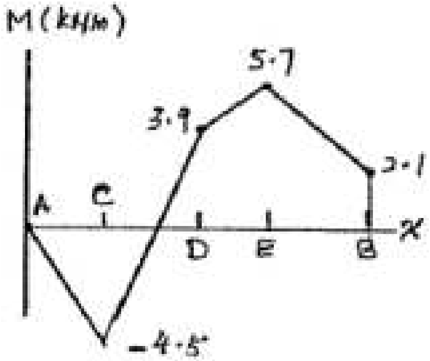
A to C: $V = -20$ kN

C to D: $V = -20 + 48 = 28$ kN

D to E: $V = 28 - 20 = 8$ kN

E to B: $V = 8 - 20 = -12$ kN

Draw the shear diagram as shown.



Areas of shear diagram.

A to C: $A_{AC} = (-20)(0.225) = -4.5$ kNm

C to D: $A_{CD} = (28)(0.3) = 8.4$ kNm

D to E: $A_{DE} = (8)(0.225) = 1.8$ kNm

E to B: $A_{EB} = (-12)(0.3) = -3.6$ kNm

Bending moments.

$M_A = 0$

$M_C = 0 - 4.5 = -4.5$ kNm

$M_D = -4.5 + 8.4 = 3.9$ kNm

$M_E = 3.9 + 1.8 = 5.7$ kNm

$M_B = 5.7 - 3.6 = 2.1$ kNm

Since V is constant over each of the portions AC, CD, DE and EB, draw the bending moment diagram with constant slopes over these portions.

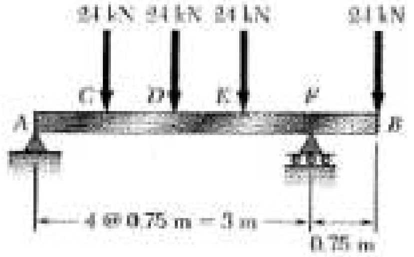
(a) $|V|_{max} = 28$ kN

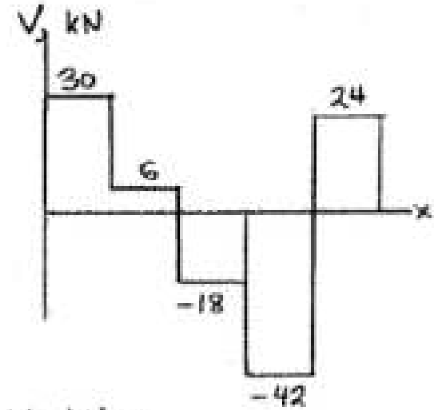
(b) $|M|_{max} = 5.7$ kNm

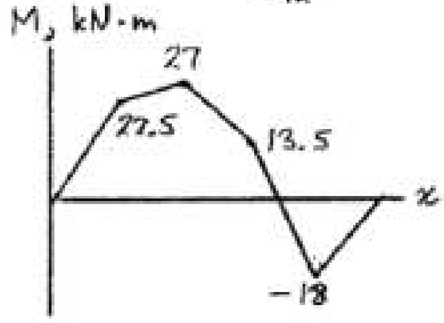
Problem 5.41

5.41 Using the method of Sec. 5.3, solve Prob. 5.8.

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.







Reactions at A and F.

$+\circlearrowleft \Sigma M_F = 0$:

$$-3R_A + (2.25)(24) + (1.50)(24) + (0.75)(24) - (0.75)(24) = 0$$

$$R_A = 30 \text{ kN} \uparrow$$

$+\circlearrowleft \Sigma M_A = 0$:

$$-(0.75)(24) - (1.50)(24) - (2.25)(24) + 3R_F - (3.75)(24) = 0$$

$$R_F = 66 \text{ kN} \uparrow$$

Shear diagram.

A to C. $V = 30$ kN

C to D. $V = 30 - 24 = 6$ kN

D to E. $V = 6 - 24 = -18$ kN

E to F. $V = -18 - 24 = -42$ kN

F to B. $V = -42 + 66 = 24$ kN

Areas of shear diagram.

A to C. $A_{AC} = (0.75)(30) = 22.5$ kN·m

C to D. $A_{CD} = (0.75)(6) = 4.5$ kN·m

D to E. $A_{DE} = (0.75)(-18) = -13.5$ kN·m

E to F. $A_{EF} = (0.75)(-42) = -31.5$ kN·m

F to B. $A_{FB} = (0.75)(24) = 18$ kN·m

Bending moments.

$M_A = 0$

$M_C = 0 + 22.5 = 22.5$ kN·m

$M_D = 22.5 + 4.5 = 27$ kN·m

$M_E = 27 - 13.5 = 13.5$ kN·m

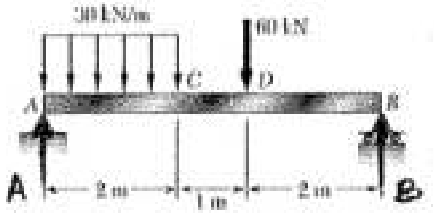
$M_F = 13.5 - 31.5 = -18$ kN·m

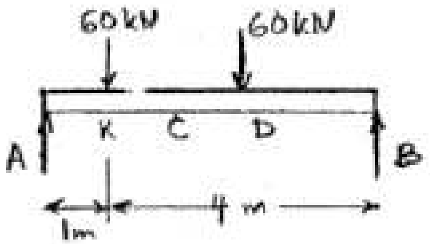
$M_B = -18 + 18 = 0$

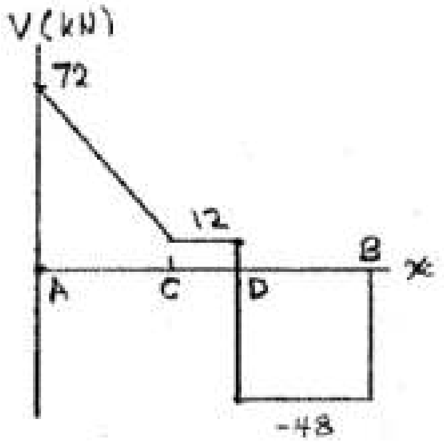
(a) $|V|_{max} = 42.0$ kN ◀

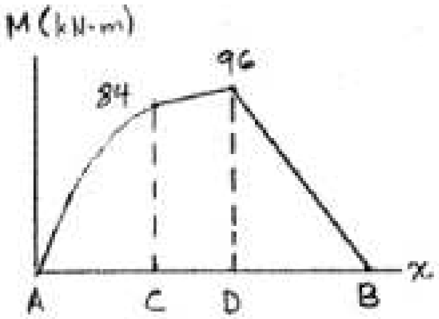
(b) $|M|_{max} = 27.0$ kN·m ◀

Problem 5.42









5.42 Using the method of Sec. 5.3, solve Prob. 5.9.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

Reactions A and B.

$+\circlearrowleft M_B = 0: \quad -5A + (2)(30)(4) + (60)(2) = 0$

$A = 72 \text{ kN} \uparrow$

$+\circlearrowleft M_A = 0: \quad 5B - (3)(60) - (1)(30)(2) = 0$

$B = 48 \text{ kN} \uparrow$

Check: $+\uparrow \Sigma F_y = 72 - (2)(30) - 60 + 12 = 0$

Shear diagram.

$V_A = 72 \text{ kN}$

A to C. $\quad 0 < x < 2\text{m} \quad w = 30 \text{ kN/m}$

$V_C - V_A = -\int_0^2 30\,dx = -60 \text{ kN}$

$V_C = 72 - 60 = 12 \text{ kN}$

C to D. $\quad V = V_C = 12 \text{ kN}$

D to B. $\quad V = V_C - 60 = -48 \text{ kN}$

Areas of shear diagram.

A to C. $\quad A_{AC} = \frac{1}{2}(72 + 12)(2) = 84 \text{ kN}\cdot\text{m}$

C to D. $\quad A_{CD} = (12)(1) = 12 \text{ kN}\cdot\text{m}$

D to B. $\quad A_{DB} = (-48)(2) = -96 \text{ kN-m}$

Bending moments: $\quad M_A = 0$

$M_C = 0 + 84 = 84 \text{ kN}$

$M_D = 84 + 12 = 96 \text{ kN}$

$M_B = 96 - 96 = 0$ as expected.

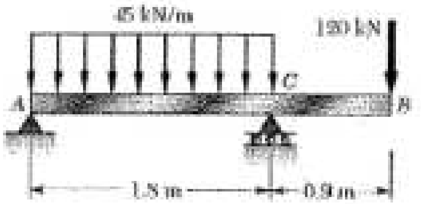
(a) $|V|_{max} = 72.0 \text{ kN}$ ◄

(b) $|M|_{max} = 96.0 \text{ kN-m}$ ◄

Problem 5.43

5.43 Using the method of Sec. 5.3, solve Prob. 5.10.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

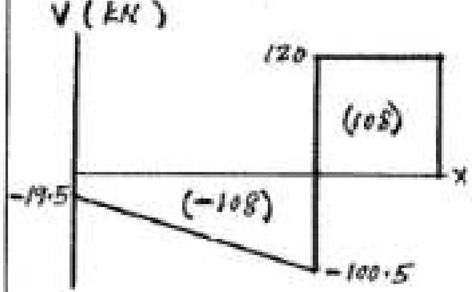


$\circlearrowleft \Sigma M_C = 0: \; -1.8A + (0.9)(81) - (0.9)(120) = 0$

$A = -19.5$ kN i.e. 19.5 kN $\downarrow$

$\circlearrowleft \Sigma M_A = 0: \; 1.8C - (0.9)(81) - (2.7)(120) = 0$

$C = 220.5$ kN $\uparrow$



<u>Shear diagram.</u>

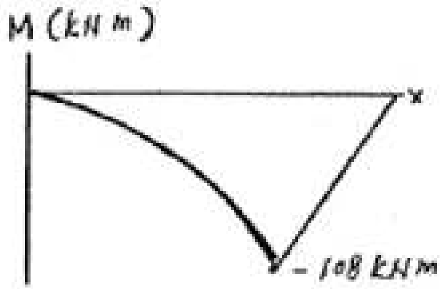
$V_A = -19.5$ kN

A to C. $\quad 0 < x < 1.8$ m $\qquad w = -45$ kN/m

$V_B - V_A = -\int_0^{\ell} w\,dx = -\int_0^{1.8} 45\,dx = -81$ kN

$V_B = -19.5 - 81 = -100.5$ kN

C to B. $\quad V = -100.5 + 220.5 = 120$ kN



<u>Areas of shear diagram.</u>

A to C. $\quad \int V dx = (\tfrac{1}{2})(-19.5 - 100.5)(1.8)$
$= -108$ kN m

C to B. $\quad \int V\,dx = (0.9)(120) = 108$ kN m

<u>Bending moments.</u>

$M_A = 0$

$M_C = M_A + \int V dx = 0 - 108 = -108$ kN m

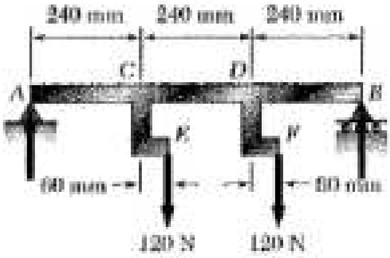
$M_B = M_C + \int V dx = -108 + 108 = 0$

Maximum $|V| = 120$ kN ◄

Maximum $|M| = 108$ kN m ◄

Problem 5.44

5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

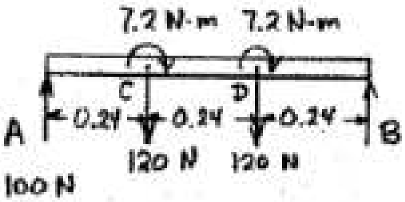


$\circlearrowleft \Sigma M_B = 0: \quad -0.72A + (0.48)(120) + (0.24)(120) - 7.2 - 7.2 = 0$

$A = 100 \text{ N} \uparrow$

$\circlearrowleft \Sigma M_A = 0: \quad -(0.24)(120) - (0.48)(120) - 7.2 - 7.2 + 0.72B = 0$

$B = 140 \text{ N} \uparrow$

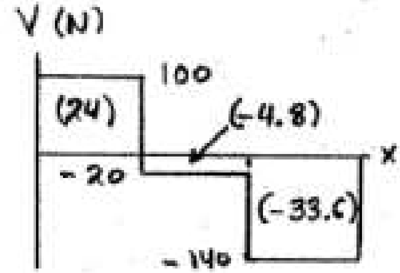


<u>Shear diagram.</u>

A to C. $V = 100 \text{ N}$

C to D. $V = 100 - 120 = -20 \text{ N}$

D to B. $V = -20 - 120 = -140 \text{ N}$

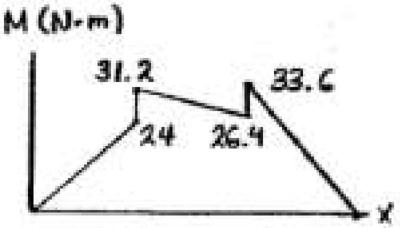


<u>Areas of shear diagram.</u>

A to C. $\int V\,dx = (0.24)(100) = 24 \text{ N·m}$

C to D. $\int V\,dx = (0.24)(-20) = -4.8 \text{ N·m}$

D to B. $\int V\,dx = (0.24)(-140) = -33.6 \text{ N·m}$



<u>Bending moments.</u>

$M_A = 0$

$M_C^- = 0 + 24 = 24 \text{ N·m}$

$M_C^+ = 24 + 7.2 = 31.2 \text{ N·m}$

$M_D^- = 31.2 - 4.8 = 26.4 \text{ N·m}$

$M_D^+ = 26.4 + 7.2 = 33.6 \text{ N·m}$

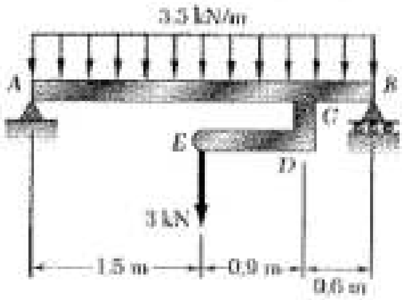
$M_B = 33.6 - 33.6 = 0$

(a) Maximum $|V| = 140 \text{ N}$ ◄

(b) Maximum $|M| = 33.6 \text{ N·m}$ ◄

Problem 5.45

5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Reaction at A.

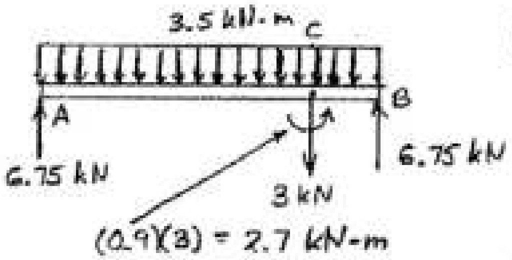
$+\circlearrowleft \Sigma M_B = 0:$

$-3.0\,A + (1.5)(3.0)(3.5) + (1.5)(3) = 0$

$A = 6.75 \text{ kN} \uparrow$

Reaction at B. $\quad B = 6.75 \text{ kN} \uparrow$

Beam ACB and loading. See sketch.



Areas of load diagram.

A to C. $\quad (2.4)(3.5) = 8.4 \text{ kN}$

C to B. $\quad (0.6)(3.5) = 2.1 \text{ kN}$

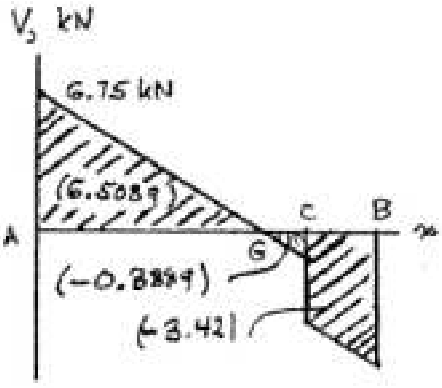
Shear diagram.

$V_A = 6.75 \text{ kN}$

$V_C^- = 6.75 - 8.4 = -1.65 \text{ kN}$

$V_C^+ = -1.65 - 3 = -4.65 \text{ kN}$

$V_B = -4.65 - 2.1 = -6.75 \text{ kN}$



Over A to C. $\quad V = 6.75 - 3.5x$

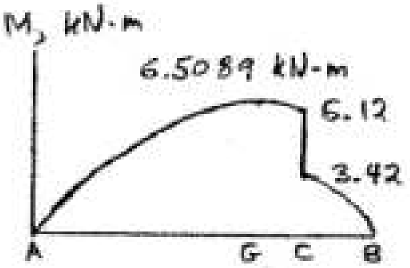
At G. $\quad V = 6.75 - 3.5x_G = 0 \quad x_G = 1.9286 \text{ m}$

Areas of shear diagram.

A to G. $\frac{1}{2}(1.9286)(6.75) = 6.5089 \text{ kN}\cdot\text{m}$

G to C. $\frac{1}{2}(0.4714)(-1.65) = -0.3889 \text{ kN}\cdot\text{m}$

C to B. $\frac{1}{2}(0.6)(-4.65 - 6.75) = -3.42 \text{ kN}\cdot\text{m}$



Bending moments.

$M_A = 0$

$M_G = 0 + 6.5089 = 6.5089 \text{ kN-m}$

$M_C^- = 6.5089 - 0.3889 = 6.12 \text{ kN-m}$

$M_C^+ = 6.12 - 2.7 = 3.42 \text{ kN-m}$

$M_B = 3.42 - 3.42 = 0$

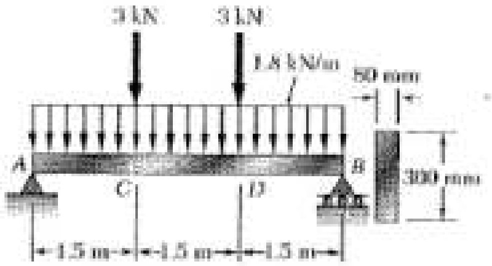
(a) $|V|_{max} = 6.75 \text{ kN}$ ◄

(b) $|M|_{max} = 6.51 \text{ kN}\cdot\text{m}$ ◄

Problem 5.47

5.47 Using the method of Sec. 5.3, solve Prob. 5.16.

5.15 and *5.16* For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.



By symmetry: $A = B$

$+\uparrow \Sigma F_y = 0: \quad A + B - 3 - 3 - (4.5)(1.8) = 0$

$A = B = 7.05 \text{ kN}$

<u>Shear diagram:</u> $V_A = 7.05 \text{ kN}$

A to C^-. $\quad w = 1.8 \text{ kN/m}$

At C^-. $\quad V = 7.05 - (1.8)(1.5) = 4.35 \text{ kN}$

At C^+. $\quad V = 4.35 - 3 = 1.35 \text{ kN}$

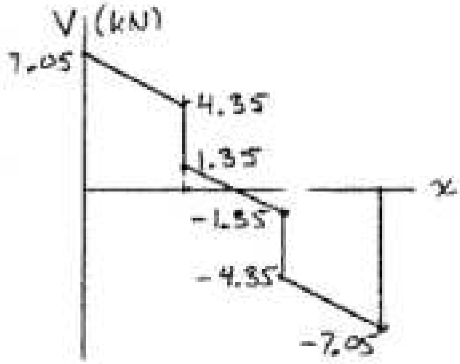
C^+ to D^-. $\quad w = 1.8 \text{ kN/m}$

At D^- $\quad V = 1.35 - (1.5)(1.8) = -1.35 \text{ kN}$

At D^+ $\quad V = -1.35 - 3 = -4.35 \text{ kN}$

D^+ to B. $\quad w = 1.8 \text{ kN}$

At B. $\quad V = -4.35 - (1.5)(1.8) = -7.05 \text{ kN}$



Draw the shear diagram.

$V = 0$ at point E, the midpoint of CD.

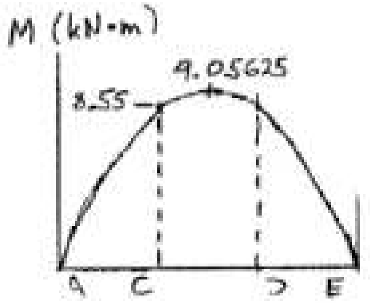
<u>Areas of the shear diagram.</u>

A to C $\quad \frac{1}{2}(7.05 + 4.35)(1.5) = 8.55 \text{ kN}\cdot\text{m}$

C to E $\quad \frac{1}{2}(1.35)(0.75) = 0.50625 \text{ kN}\cdot\text{m}$

E to D $\quad \frac{1}{2}(-1.35)(0.75) = -0.50625 \text{ kN}\cdot\text{m}$

D to B $\quad \frac{1}{2}(-4.35 - 7.05)(1.5) = -8.55 \text{ kN}\cdot\text{m}$



<u>Bending moments:</u> $M_A = 0$

$M_C = 0 + 8.55 = 8.55 \text{ kN}\cdot\text{m}$

$M_E = 8.55 + 0.50625 = 9.05625 \text{ kN}\cdot\text{m}$

$M_D = 9.05625 - 0.50625 = 8.55 \text{ kN}\cdot\text{m}$

$M_B = 8.55 - 8.55 = 0$

$M_C = 8.55 \times 10^3 \text{ N}\cdot\text{m}$

For a rectangular section

$S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(80)(300)^2$

$= 1.2 \times 10^6 \text{ mm}^3 = 1.2 \times 10^{-3} \text{ m}^3$

Maximum normal stress at C.

$\sigma = \frac{M_C}{S} = \frac{8.55 \times 10^3}{1.2 \times 10^{-3}}$

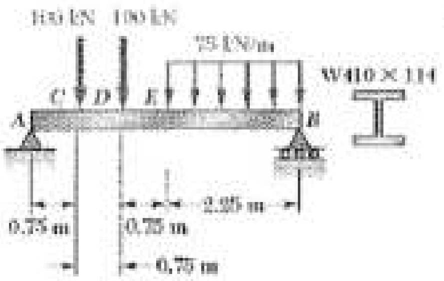
$= 7.125 \times 10^6 \text{ Pa}$

$\sigma = 7.13 \text{ MPa}$ ◄

Problem 5.48

5.48 Using the method of Sec. 5.3, solve Prob. 5.17.

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



$+\circlearrowleft \Sigma M_B = 0$

$-4.5A + (3.75)(100) + (3)(100) + (75)(2.25)(1.125) = 0$

$A = 192.2 \text{ kN}$

Shear A to C $\quad V = 192.2 \text{ kN}$

Area under shear curve A to C $\quad \int V dx = (0.75)(192.2)$
$= 144.15 \text{ kNm}$

$M_A = 0$

$M_C = 0 + 144.15 = 144.15 \text{ kNm}$

For W 410×114 rolled steel section $\quad S = 2200 \times 10^3 \text{ mm}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{144.15 \times 10^3}{2200 \times 10^{-6}} = 65.5 \text{ MPa}$

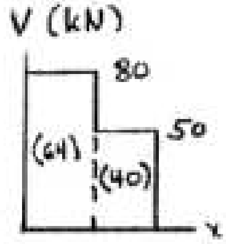
Problem 5.49

5.49 Using the method of Sec. 5.3, solve Prob. 5.18.

5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.

Reactions: By symmetry, $A = B$

$+\uparrow \Sigma F_y = 0: \qquad A = B = 80 \text{ kN}$



Shear diagram.

A to C. $V = 80$ kN

C to D. $V = 80 - 30 = 50$ kN

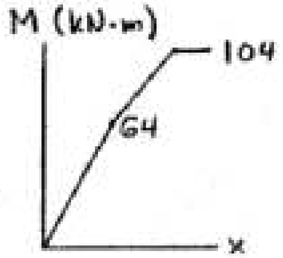
D to E. $V = 50 - 50 = 0$

Areas of shear diagram.

A to C. $\int V dx = (80)(0.8) = 64$ kN·m

C to D. $\int V dx = (50)(0.8) = 40$ kN·m

D to E. $\int V dx = 0$



Bending moments.

$M_A = 0$

$M_C = 0 + 64 = 64$ kN·m

$M_D = 64 + 40 = 104$ kN·m

$M_E = 104 + 0 = 104$ kN·m

$M_E = 104 \text{ kN·m} = 104 \times 10^3 \text{ N·m}$

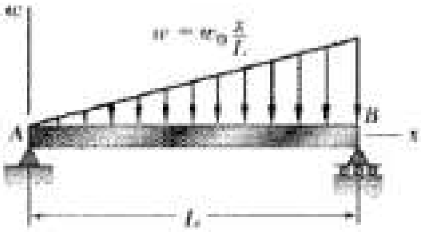
For W 310 × 52, $S = 748 \times 10^3 \text{ mm}^3 = 748 \times 10^{-6} \text{ m}^3$

Normal stress. $\sigma = \dfrac{|M|}{S} = \dfrac{104 \times 10^3}{748 \times 10^{-6}} = 139.0 \times 10^6 \text{ Pa}$

$\sigma = 139.0$ MPa ◄

Problem 5.50

5.50 and 5.51 Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.



$$\frac{dV}{dx} = -w = -w_0 \frac{x}{L}$$

$$V = -\frac{1}{2} w_0 \frac{x^2}{L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = -\frac{1}{6} w_0 L^2 + C_1 L \qquad C_1 = \frac{1}{6} w_0 L$

(a) $V = -\frac{1}{2} w_0 \frac{x^2}{L} + \frac{1}{6} w_0 L^2 = \frac{1}{6} w_0 (L^2 - 3x^2)/L$ ◄

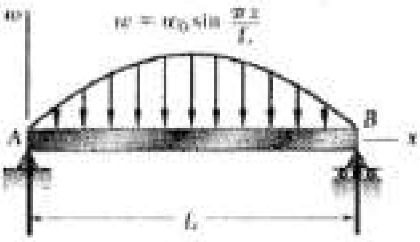
$M = -\frac{1}{6} w_0 \frac{x^3}{L} + \frac{1}{6} w_0 L x = \frac{1}{6} w_0 (Lx - x^3/L)$ ◄

(b) M_{max} occurs when $\frac{dM}{dx} = V = 0 \qquad L^2 - 3x_m^2 = 0$

$x_m = \frac{L}{\sqrt{3}} \qquad M_{max} = \frac{1}{6} w_0 \left(\frac{L^2}{\sqrt{3}} - \frac{L^2}{3\sqrt{3}} \right) \qquad M_{max} = 0.0642 w_0 L^2$ ◄

Problem 5.51

5.50 and 5.51 Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.



$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = 0 + C_1 L + 0 \qquad C_1 = 0$

(a) $V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$ ◄

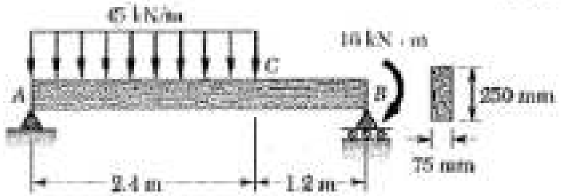
$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$ ◄

$\frac{dM}{dx} = V = 0$ at $x = \frac{L}{2}$

(b) $M_{max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2} \qquad M_{max} = \frac{w_0 L^2}{\pi^2}$ ◄

Problem 5.55

5.54 and 5.55 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$\circlearrowleft \Sigma M_B = 0$

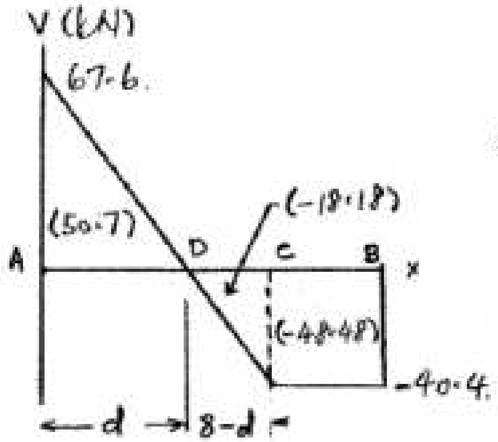
$-3.6A + (45)(2.4)(2.4) - 16 = 0$

$A = 67.6 \text{ kN}$

$\circlearrowright \Sigma M_A = 0$

$-(45)(2.4)(1.2) + 3.6B - 16 = 0$

$B = 40.4 \text{ kN}$



Shear: $V_A = 67.6 \text{ kN}$

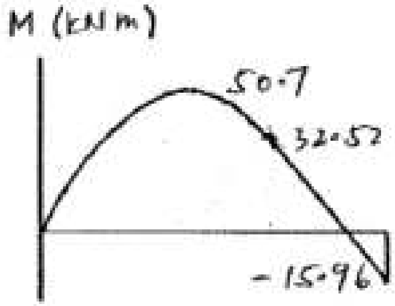
$V_C = 67.6 - (45)(2.4) = -40.4 \text{ kN}$

C to B $\quad V = -40.4 \text{ kN}$

Locate point D where $V = 0$

$\dfrac{d}{67.6} = \dfrac{2.4 - d}{40.4} \qquad 1.6d = 2.4$

$d = 1.5 \text{ m} \qquad 2.4 - d = 0.9 \text{ m}$



Areas under shear diagram

A to D $\quad \int V\,dx = (\tfrac{1}{2})(1.5)(67.6) = 50.7 \text{ kN m}$

D to C $\quad \int V\,dx = (\tfrac{1}{2})(0.9)(-40.4) = -18.18 \text{ kNm}$

C to B $\quad \int V\,dx = -(1.2)(40.4) = -48.48 \text{ kNm}$

Bending moments: $\quad M_A = 0$

$M_D = 0 + 50.7 = 50.7 \text{ kNm}$

$M_C = 50.7 - 18.18 = 32.52 \text{ kNm}$

$M_B = 32.52 - 48.48 = -15.96 \text{ kNm}$

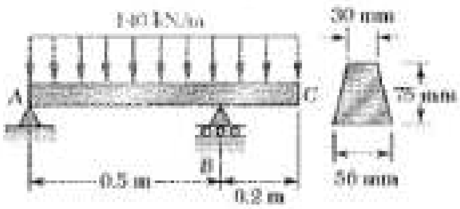
Maximum $|M| = 50.7 \text{ kNm}$.

For rectangular cross section $\quad S = \frac{1}{6}bh^2 = (\frac{1}{6})(75)(250)^2 = 781250 \text{ mm}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{50.7 \times 10^3}{781250 \times 10^{-9}} = 64.9 \text{ MPa}$

Problem 5.57

5.57 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

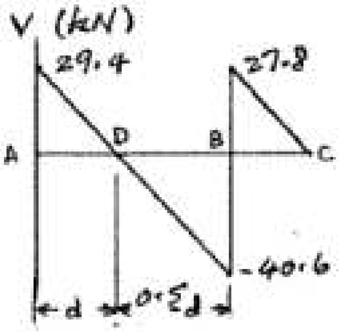


$\circlearrowleft \Sigma M_B = 0 \qquad -0.5A + (140)(0.7)(0.15) = 0$

$A = 29.4 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0 \qquad 0.5B - (140)(0.7)(0.35) = 0$

$B = 68.4 \text{ kN}$



Shear: $V_A = 29.4 \text{ kN}$

B^- $\quad V_{B^-} = 29.4 - (140)(0.5) = -40.6 \text{ kN}$.

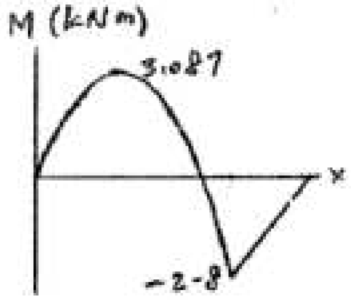
B^+ $\quad V_{B^+} = -40.6 + (68.4) = 27.8 \text{ kN}$

C $\quad V_C = 27.8 - (140)(0.2) \approx 0$

Locate point D where $V = 0$

$\frac{d}{29.4} = \frac{0.5-d}{40.6} \qquad 70.3d = 14.7$

$d = 0.21 \text{ m} \qquad 0.5 - d = 0.29 \text{ m}$.



Areas under shear diagram

A to D $\quad \int V dx = (\frac{1}{2})(29.4)(0.21) = 3.087 \text{ kNm}$

D to B $\quad \int V dx = (\frac{1}{2})(-40.6)(0.29) = -5.887 \text{ kNm}$.

B to C $\quad \int V dx = (\frac{1}{2})(27.8)(0.2) = 2.78 \text{ kNm}$.

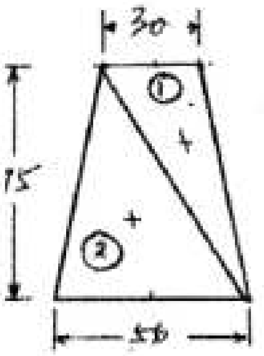
Bending moments: $\quad M_A = 0$

$M_D = 0 + 3.087 = 3.087 \text{ kNm}$

$M_B = 3.087 - 5.887 = -2.8 \text{ kNm}$

$M_C = -2.8 + 2.78 \approx 0$

Maximum $|M| = 3.087 \text{ kNm}$



Locate centroid of cross section

$\bar{Y} = \frac{108750}{3225} = 33.72 \text{ mm}$ from bottom

For each triangle $\bar{I} = \frac{1}{36} bh^3$

Moment of inertia

$I = \Sigma \bar{I} + \Sigma Ad^2$

$= 457849 + 1007813 = 1465662 \text{ mm}^4$

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, mm^3	d, mm	Ad^2 mm^4	$\bar{I}$, mm^4
1	1125	50	56250	16.28	298168	351563
2	2100	25	52500	8.72	159681	656250
Σ	3225		108750		457849	1007813

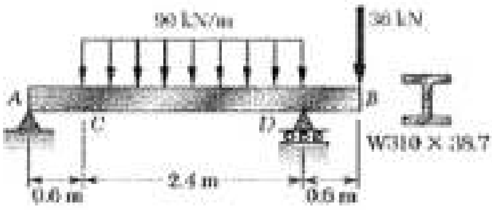
Normal stress

$\sigma = \frac{Mc}{I} = \frac{(3.087 \times 10^3)(41.28 \times 10^{-3})}{1465662 \times 10^{-12}}$

$= 86.9 \text{ MPa}$

Problem 5.58

5.58 and 5.59 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Reaction at A.

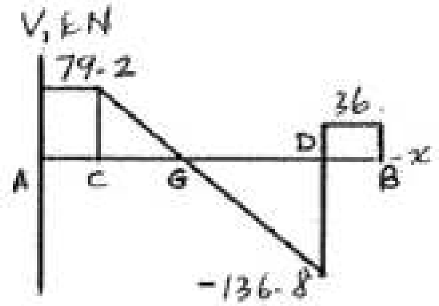
$+\circlearrowleft \Sigma M_D = 0$

$-3.0A + (90)(2.4)(1.2) - (36)(0.6) = 0 \qquad A = 79.2\ kN \uparrow$

Reaction at D.

$+\circlearrowright M_A = 0$

$-(90)(2.4)(1.8) + 3.0D - (36)(3.6) = 0 \qquad D = 172.8\ kN \uparrow$



Areas of load diagram.

C to D $\quad (90)(2.4) = 216\ kN$

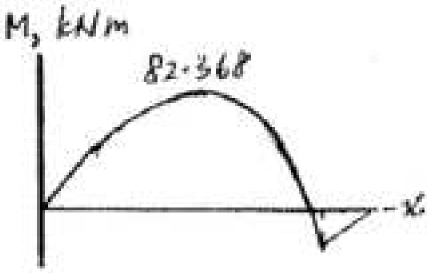
Shear diagram.

A to C $\qquad V = 79.2\ kN$

$V_D^- = 79.2 - 216 = -136.8\ kN$

$V_D^+ = -136.8 + 172.8 = 36\ kN$

D to B $\qquad V = 36\ kN$



Over CD $\quad V = 79.2 - 90(x - 0.6)$

At G $\quad V = 79.2 - 90(x_G - 0.6) = 0 \quad x_G - 0.6 = 0.88\ m$

Areas of shear diagram

A to C $\quad (0.6)(79.2) = 47.52\ kNm$

C to G $\quad \frac{1}{2}(0.88)(79.2) = 34.848\ kNm$

G to D $\quad \frac{1}{2}(1.52)(-136.8) = -103.968\ kNm$

D to B $\quad (36)(0.6) = 21.6\ kNm$

Bending moments. $\qquad M_A = 0$

$M_C = 0 + 47.52 = 47.52\ kNm$

$M_G = 47.52 + 34.848 = 82.368\ kNm$

$M_D = 82.368 - 103.968 = -21.6\ kNm$

$M_B = -21.6 + 21.6 = 0$

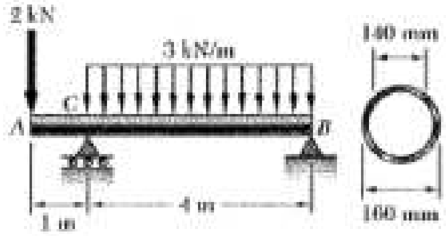
$|M|_{max} = M_G = 82.368\ kNm$

For W310×38.7 rolled steel section $\qquad S = 549 \times 10^3\ mm^3$

Normal stress $\qquad \sigma_{max} = \frac{|M|_{max}}{S} = \frac{82.368 \times 10^3}{549 \times 10^{-6}} \qquad \sigma_{max} = 150\ MPa$ ◀

Problem 5.59

5.58 and 5.59 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

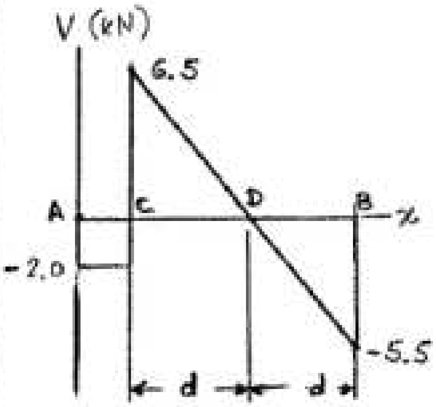


$+\circlearrowleft \Sigma M_C = 0: \ (2)(1) - (3)(4)(2) + 4B = 0$

$B = 5.5 \text{ kN}$

$+\circlearrowleft \Sigma M_B = 0: \ (5)(2) + (3)(4)(2) - 4C = 0$

$C = 8.5 \text{ kN}$



Shear: A to C: $V = -2 \text{ kN}$

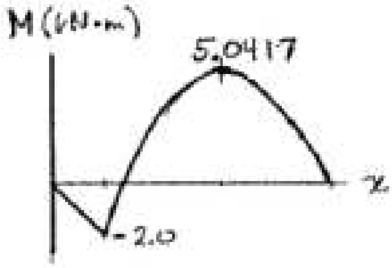
C^+: $V = -2 + 8.5 = 6.5 \text{ kN}$

D: $V = 6.5 - (3)(4) = -5.5 \text{ kN}$

Locate point D where $V = 0$.

$\frac{d}{6.5} = \frac{4-d}{5.5} \qquad 12d = 26$

$d = 2.1667 \text{ m} \qquad 4 - d = 3.8333 \text{ m}$



Areas of the shear diagram:

A to C. $\int V dx = (-2.0)(1) = -2.0 \text{ kN·m}$

C to D. $\int V dx = \frac{1}{2}(2.16667)(6.5) = 7.0417 \text{ kN·m}$

D to B. $\int V dx = \frac{1}{2}(3.83333)(-5.5) = -5.0417 \text{ kN·m}$

Bending moments: $M_A = 0$

$M_C = 0 - 2.0 = -2.0 \text{ kN·m}$

$M_D = -2.0 + 7.0417 = 5.0417 \text{ kN·m}$

$M_B = 5.0417 - 5.0417 = 0$

Maximum $|M| = 5.0417 \text{ kN·m} = 5.0417 \times 10^3 \text{ N·m}$

For pipe: $c_o = \frac{1}{2} d_o = \frac{1}{2}(160) = 80 \text{ mm}, \qquad c_i = \frac{1}{2} d_i = \frac{1}{2}(140) = 70 \text{ mm}$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = \frac{\pi}{4}[(80)^4 - (70)^4] = 13.3125 \times 10^6 \text{ mm}^4$

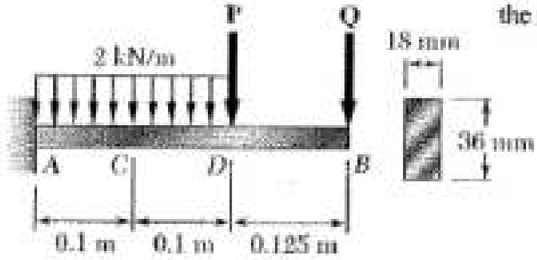
$S = \frac{I}{c_o} = \frac{13.3125 \times 10^6}{80} = 166.406 \times 10^3 \text{ mm}^3 = 166.406 \times 10^{-6} \text{ m}^3$

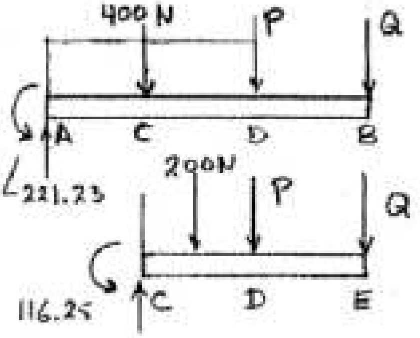
Normal stress: $\sigma = \frac{M}{S} = \frac{5.0417 \times 10^3}{166.406 \times 10^{-6}} = 30.3 \times 10^6 \text{ Pa}$

$\sigma = 30.3 \text{ MPa}$ ◀

Problem 5.62

*5.62 Beam *AB* supports a uniformly distributed load of 2 kN/m and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending in the bottom edge of the beam is –56.9 MPa at *A* and –29.9 MPa at *C*. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads **P** and **Q**.





$$I = \tfrac{1}{12}(18)(36)^3 = 69.984\times10^3\ \text{mm}^4$$

$$c = \tfrac{1}{2}d = 18\ \text{mm}$$

$$S = \frac{I}{c} = 3.888\times10^3\ \text{mm}^3 = 3.888\times10^{-6}\ \text{m}^3$$

At A, $M_A = S\sigma_A$

$$M_A = (3.888\times10^{-6})(-56.9) = -221.25\ \text{N·m}$$

At C, $M_C = S\sigma_C$

$$M_C = (3.888\times10^{-6})(-29.9) = -116.25\ \text{N·m}$$

$\circlearrowleft \Sigma M_A = 0:$

$$221.23 - (0.1)(400) - 0.2P - 0.325Q = 0$$

$$0.2P + 0.325Q = 181.25 \qquad (1)$$

$+\circlearrowleft M_C = 0:$

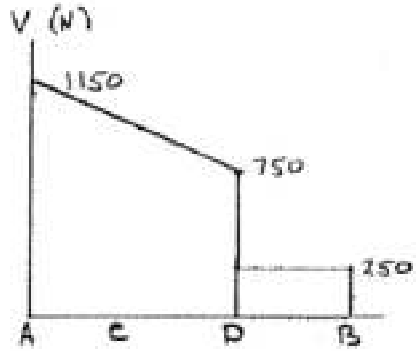
$$116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$$

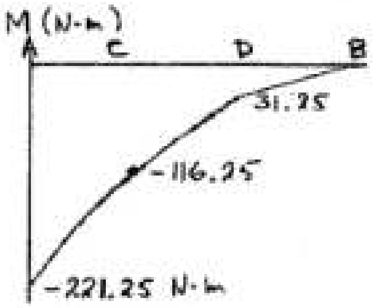
$$0.1P + 0.225Q = 106.25 \qquad (2)$$

Solving (1) and (2) simultaneously

$P = 500$ N

$Q = 250$ N





Reaction force at A

$$R_A - 400 - 500 - 250 = 0$$

$$R_A = 1150\ \text{N·m}$$

$V_A = 1150$ N $\qquad V_D = 250$

$M_A = -221.25$ N·m

$M_C = -116.25$ N·m

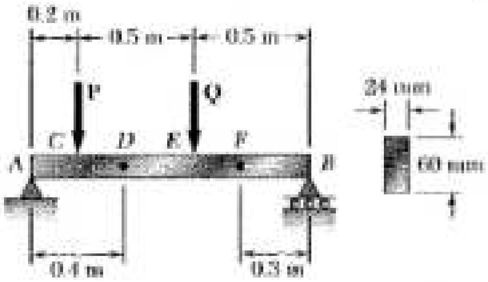
$M_D = -31.25$ N·m

$|V|_{max} = 1150$ N

$|M|_{max} = 221$ N·m

Problem 5.63

*5.63 The beam AB supports two concentrated loads $\mathbf{P}$ and $\mathbf{Q}$. The normal stress due to bending on the bottom edge of the beam is +55 MPa at D and +37.5 MPa at F. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending that occurs in the beam.



$$I = \frac{1}{12}(24)(60)^3 = 432\times10^3\ \text{mm}^4$$

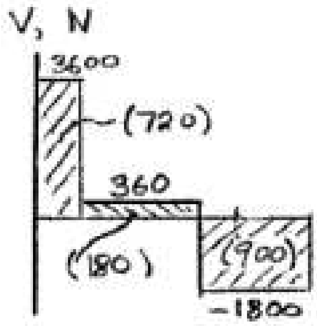
$$c = 30\ \text{mm}$$

$$S = \frac{I}{c} = 14.4\times10^3\ \text{mm}^3 = 14.4\times10^{-6}\ \text{m}^3$$

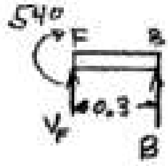
$$M = S\sigma$$

At D. $M_D = (14.4\times10^{-6})(55\times10^6) = 792\ \text{N}\cdot\text{m}$

At F. $M_F = (14.4\times10^{-6})(37.5\times10^6) = 540\ \text{N}\cdot\text{m}$



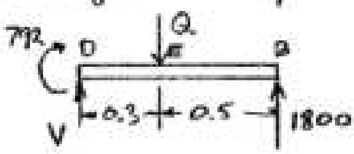
Using free body FB, $\circlearrowright\Sigma M_F = 0$:



$$-540 + 0.3B = 0$$

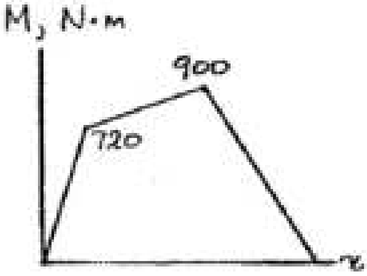
$$B = \frac{540}{0.3} = 1800\ \text{N}$$

Using free body DEFB, $+\circlearrowright\Sigma M_D = 0$:

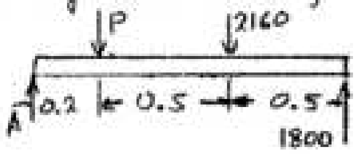


$$-792 - 0.3Q + (0.8)(1800) = 0$$

$$Q = 2160\ \text{N}$$



Using entire beam, $+\circlearrowright M_A = 0$:



$$-0.2P - (0.7)(2160) + (1.2)(1800) = 0$$

$$P = 3240\ \text{N}$$

$+\uparrow\Sigma F_y = 0$: $\quad A - 3240 - 2160 + 1800 = 0$

$$A = 3600\ \text{N}$$

<u>Shear diagram and its areas.</u>

A to C^-. $\quad V = 3600$ N

C^+ to E^-. $\quad V = 3600 - 3240 = 360$ N

E^+ to B. $\quad V = 360 - 2160 = -1800$ N

$A_{AC} = (0.2)(3600) = 720\ \text{N}\cdot\text{m}$

$A_{CE} = (0.5)(360) = 180\ \text{N}\cdot\text{m}$

$A_{EB} = (0.5)(-1800) = -900\ \text{N}\cdot\text{m}$

<u>Bending moments.</u>

$M_A = 0$

$M_C = 0 + 720 = 720\ \text{N}\cdot\text{m}$

$M_E = 720 + 180 = 900\ \text{N}\cdot\text{m}$

$M_B = 900 - 900 = 0$

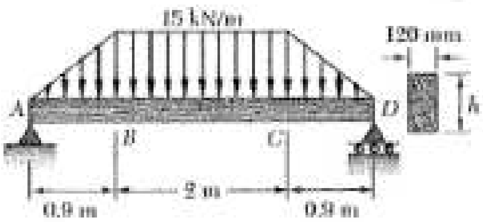
$|M|_{max} = 900\ \text{N}\cdot\text{m}$

$$\sigma_{max} = \frac{|M|_{max}}{S} = \frac{900}{14.4\times10^{-6}} = 62.5\times10^6\ \text{Pa}$$

(b) $\sigma_{max} = 62.5$ MPa

Problem 5.66

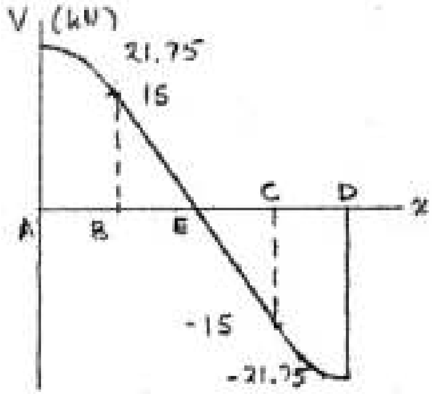
5.65 and 5.66 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



By symmetry, A = D.

$+\uparrow \Sigma F_y = 0: \quad A + \frac{1}{2}(0.9)(15) + (2)(15) + \frac{1}{2}(0.9) + D = 0$

$A = D = 21.75$ kN



Shear diagram: $V_A = 21.75$ kN

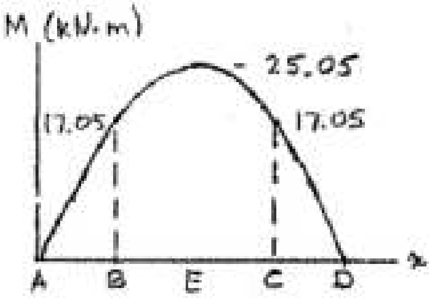
$V_B = 21.75 - \frac{1}{2}(0.9)(15) = 15.0$ kN

$V_C = 15.0 - (2)(15) = -15.0$ kN

$V_D = -15.0 - \frac{1}{2}(0.9)(15) = -21.75$ kN

Locate point E where V = 0.

By symmetry, E is the midpoint of BC.



Areas of shear diagram.

A to B. $(0.9)(15) + \frac{2}{3}(0.9)(21.75-15) = 17.55$ kN·m

B to E. $\frac{1}{2}(1)(15) = 7.5$ kN m

E to C. $\frac{1}{2}(1)(-15) = -7.5$ kN

C to D. By anti-symmetry -17.55 kN·m

Bending moments: $M_A = 0$

$M_B = 0 + 17.55 = 17.55$ kN·m

$M_E = 17.55 + 7.5 = 25.05$ kN·m

$M_C = 25.05 - 7.5 = 17.55$ kN·m

$M_D = 17.55 - 17.55 = 0$

$|M|_{max} = 25.05$ kN·m $= 25.05 \times 10^3$ N·m

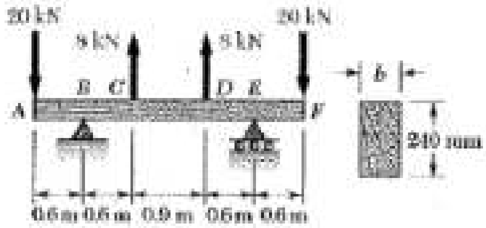
$\sigma = \frac{M}{S} \qquad S = \frac{M}{\sigma} = \frac{25.05 \times 10^3}{12 \times 10^6} = 2.0875 \times 10^{-3}\ m^3 = 2.0875 \times 10^6\ mm^3$

For a rectangular section, $S = \frac{1}{6} bh^2$

$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(2.0875 \times 10^6)}{120}} = 323$ mm $\qquad h = 323$ mm ◄

Problem 5.67

5.67 and 5.68 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



For equilibrium

$B = E = 12 \text{ kN}$

Shear diagram

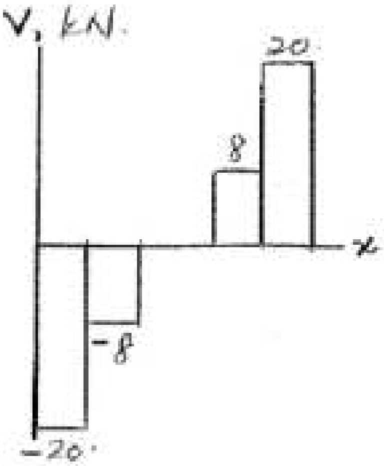
A to B^- $\quad V = -20 \text{ kN}$

B^+ to C^- $\quad V = -20 + 12 = -8$

C^+ to D^- $\quad V = -8 + 8 = 0$

D^+ to E^- $\quad V = 0 + 8 = 8 \text{ kN}$

E^+ to F $\quad V = 8 + 12 = 20 \text{ kN}$



Areas of shear diagram

A to B $\quad (0.6)(-20) = -12 \text{ kNm}$

B to C $\quad (0.6)(-8) = -4.8 \text{ kNm}$

C to D $\quad (0.9)(0) = 0$

D to E $\quad (0.6)(8) = 4.8 \text{ kNm}$

E to F $\quad (0.6)(20) = 12 \text{ kNm}$

Bending moments

$M_A = 0$

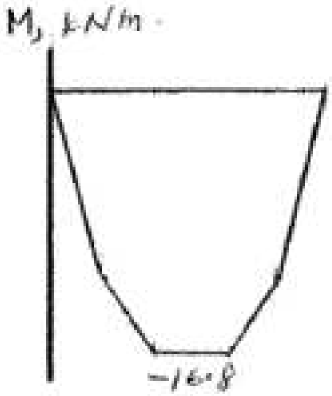
$M_B = 0 - 12 = -12 \text{ kNm}$

$M_C = -12 - 4.8 = -16.8 \text{ kNm}$

$M_D = -16.8 + 0 = -16.8 \text{ kNm}$

$M_E = -16.8 + 4.8 = -12$

$M_F = -12 + 12 = 0$



$|M|_{max} = 16.8 \text{ kNm}$

Required value for S

$$S = \frac{|M|_{max}}{\sigma_{all}} = \frac{16.8 \times 10^3}{12 \times 10^6} = 1400 \times 10^{-6} \text{ m}^3$$

For a rectangular section $\quad I = \frac{1}{12} bh^3, \quad c = \frac{1}{2} h$

$$S = \frac{I}{c} = \frac{bh^2}{6} = \frac{(b)(0.24)^2}{6} = 0.0096\, b$$

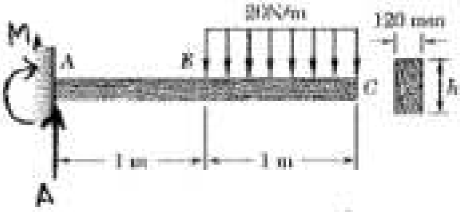
Equating the two expressions for S

$$0.0096\, b = 1400 \times 10^{-6} \qquad b = 146 \text{ mm}$$

$$= 0.146 \text{ m}$$

Problem 5.68

5.67 and 5.68 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

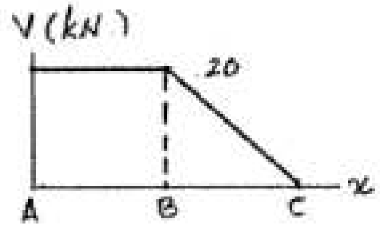


Reactions: $+\uparrow \Sigma F_y = 0: \quad A - (1)(20) = 0$

$A = 20kN$

$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (20)(1)(1.5) = 0$

$M_A = -30kNm$



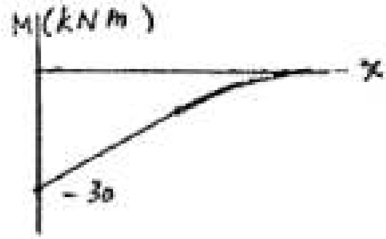
Shear diagram: $V_A = V_B = 20kN$

$V_C = 20 - (1)(20) = 0$

Areas of shear diagram.

A to B. $(1)(20) = 20kNm$

B to C. $\frac{1}{2}(1)(20) = 10kNm$



Bending moments: $M_A = -30kNm$

$M_B = -30 + 20 = -10kNm$

$M_C = -10 + 10 = 0$

$|M|_{max} = 30kNm$

$$\sigma_{max} = \frac{|M|_{max}}{S}$$

$$S = \frac{|M|_{max}}{\sigma_{max}} = \frac{30 \times 10^3}{12 \times 10^6} = 2.5 \times 10^{-3} m^3$$

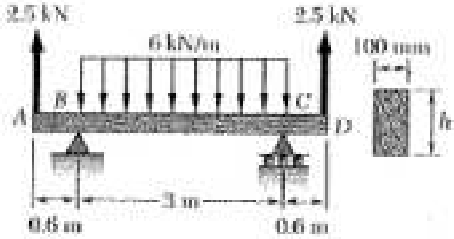
For a rectangular section, $S = \frac{1}{6} bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(2.5 \times 10^6)}{120}} = 353.6mm$$

$h = 354 \text{ mm}$ ◄

Problem 5.69

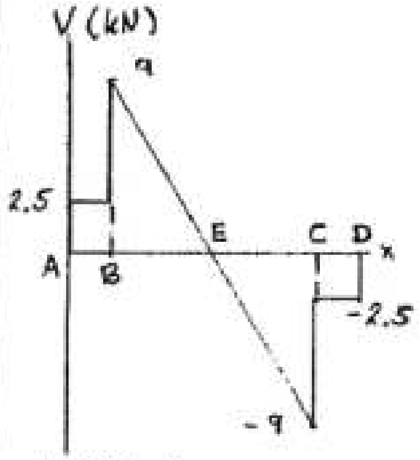
5.69 and 5.70 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



By symmetry, $B = C$

$+\uparrow \Sigma F_y = 0: \quad B + C + 2.5 + 2.5 - (3)(6) = 0$

$B = C = 6.5 \text{ kN}$



<u>Shear:</u> A to B. $V = 2.5 \text{ kN}$

$V_{B^+} = 2.5 + 6.5 = 9 \text{ kN}$

$V_{C^-} = 9 - (3)(6) = -9 \text{ kN}$

C to D. $V = -9 + 6.5 = -2.5 \text{ kN}$

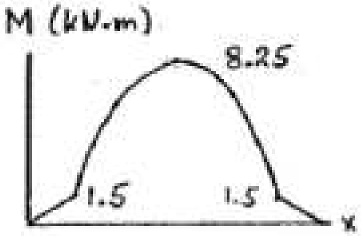
<u>Areas of the shear diagram.</u>

A to B. $\int V dx = (0.6)(2.5) = 1.5 \text{ kN}\cdot\text{m}$

B to E. $\int V dx = (\frac{1}{2})(1.5)(9) = 6.75 \text{ kN}\cdot\text{m}$

E to C. $\int V dx = -6.75 \text{ kN}\cdot\text{m}$

C to D. $\int V dx = -1.5 \text{ kN}\cdot\text{m}$



<u>Bending moments.</u> $M_A = 0$

$M_B = 0 + 1.5 = 1.5 \text{ kN}\cdot\text{m}$

$M_E = 1.5 + 6.75 = 8.25 \text{ kN}\cdot\text{m}$

$M_C = 8.25 - 6.75 = 1.5 \text{ kN}\cdot\text{m}$

$M_D = 1.5 - 1.5 = 0$

Maximum $|M| = 8.25 \text{ kN}\cdot\text{m} = 8.25 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$

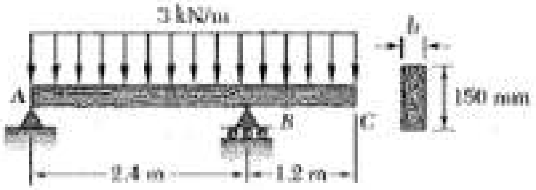
For a rectangular section, $S = \frac{1}{6} b h^2$

$687.5 \times 10^3 = (\frac{1}{6})(100) h^2 \qquad h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2$

$h = 203 \text{ mm}$ ◄

Problem 5.70

5.69 and 5.70 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

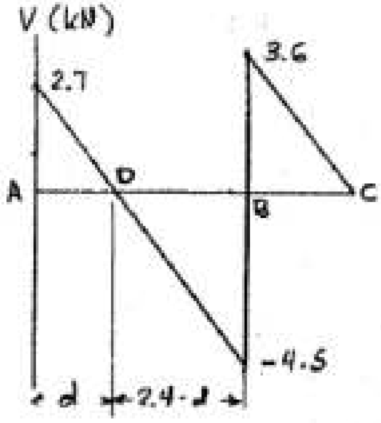


$+\circlearrowleft M_B = 0:$

$-2.4A + (0.6)(3.6)(3) = 0 \qquad A = 2.7 \text{ kN}$

$+\circlearrowleft M_A = 0:$

$-(1.8)(3.6)(3) + 2.4B = 0 \qquad B = 8.1 \text{ kN}$



<u>Shear:</u> $V_A = 2.7$ kN

$V_B^- = 2.7 - (2.4)(3) = -4.5$ kN

$V_B^+ = -4.5 + 8.1 = 3.6$ kN

$V_C = 3.6 - (1.2)(3) = 0$

Locate point D where $V = 0$.

$$\frac{d}{2.7} = \frac{2.4-d}{4.5} \qquad 7.2\,d = 6.48$$

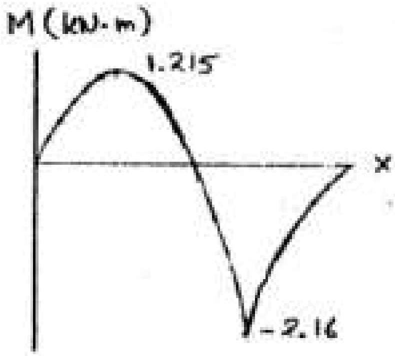
$$d = 0.9 \text{ m} \qquad 2.4 - d = 1.5 \text{ m}$$

<u>Areas of the shear diagram.</u>

A to D. $\int V dx = (\tfrac{1}{2})(0.9)(2.7) = 1.215$ kN·m

D to B. $\int V dx = (\tfrac{1}{2})(1.5)(-4.5) = -3.375$ kN·m

B to C. $\int V dx = (\tfrac{1}{2})(1.2)(3.6) = 2.16$ kN·m



<u>Bending moments:</u> $M_A = 0$

$M_D = 0 + 1.215 = 1.215$ kN·m

$M_B = 1.215 - 3.375 = -2.16$ kN·m

$M_C = -2.16 + 2.16 = 0$

Maximum $|M| = 2.16$ kN·m $= 2.16 \times 10^3$ N·m

$\sigma_{all} = 12$ MPa $= 12 \times 10^6$ Pa

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2.16 \times 10^3}{12 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

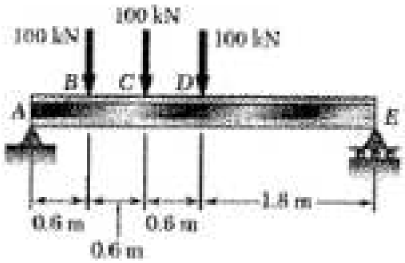
For rectangular section,

$$S = \tfrac{1}{6} b h^2 = \tfrac{1}{6} b (150)^2 = 180 \times 10^3$$

$$b = \frac{(6)(180 \times 10^3)}{150^2} \qquad b = 48.0 \text{ mm}$$

Problem 5.71

5.71 and 5.72 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.



$\circlearrowleft \Sigma M_E = 0 \qquad -3.6A + (3)(100) + (2.4)(100) + (1.8)(100) = 0$

$A = 200 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0 \qquad 3.6E - (1.8)(100) - (1.2)(100) - (0.6)(100) = 0$

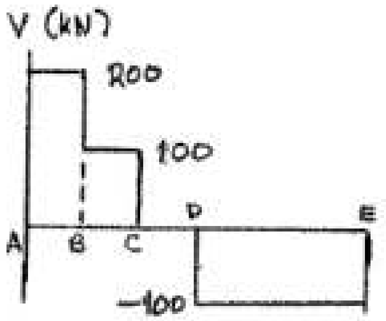
$E = 100 \text{ kN}$

Shear: A to B $\quad V = 200$ kN

B to C $\quad V = 200 - 100 = 100$ kN

C to D $\quad V = 100 - 100 = 0$

D to E $\quad V = 0 - 100 = -100$ kN



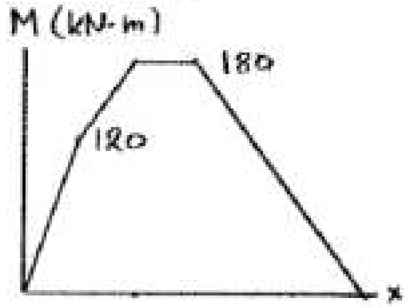
Areas under shear diagram

A to B $\quad \int V dx = (0.6)(200) = 120$ kN·m

B to C $\quad \int V dx = (0.6)(100) = 60$ kN·m

C to D $\quad \int V dx = 0$

D to E $\quad \int V dx = (1.8)(-100) = -162$ kN·m



Bending moments $\quad M_A = 0$

$M_B = 0 + 120 = 120$ kN·m

$M_C = 120 + 60 = 180$ kN·m

$M_D = 180 + 0 = 180$ kN·m

$M_E = 180 - 180 = 0$

Maximum $|M| = 180$ kN·m $= 180 \times 10^3$ N·m

$\sigma_{all} = 160$ MPa $= 160 \times 10^6$ Pa

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{180 \times 10^3}{160 \times 10^6} = 1.125 \times 10^{-3} \text{ m}^3 = 1125 \times 10^3 \text{ mm}^3$

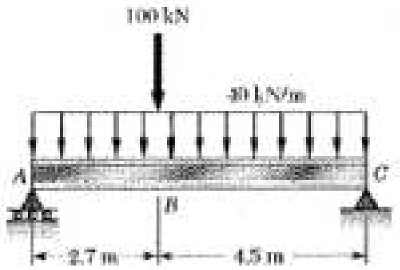
Shape	S (10^3 mm^3)
W 530 × 66	1340
W 460 × 74	1460
W 360 × 79	1280
W 250 × 101	1240

Lightest wide flange beam

W 530 × 66 @ 66 kg/m ◄

Problem 5.72

5.71 and 5.72 Knowing that the allowable stress for the steel used is 165 MPa, select the most economical wide-flange beam to support the loading shown.



$+\circlearrowleft \Sigma M_C = 0 \quad -7.2A + (40)(7.2)(3.6) + (100)(4.5) = 0$

$A = 206.5$ kN.

$+\circlearrowleft \Sigma M_A = 0 \quad 7.2C - (40)(7.2)(3.6) - (100)(2.7) = 0$

$C = 181.5$ kN

Shear: $V_A = 206.5$

$V_{B^-} = 206.5 - (40)(2.7) = 98.5$ kN.

$V_{B^+} = 98.5 - 100 = -1.5$ kN.

$V_C = -1.5 - (40)(4.5) = -181.5$

Areas under shear diagram

A to B $\int V dx = (\frac{1}{2})(2.7)(206.5 + 98.5) = 411.75$ kN m.

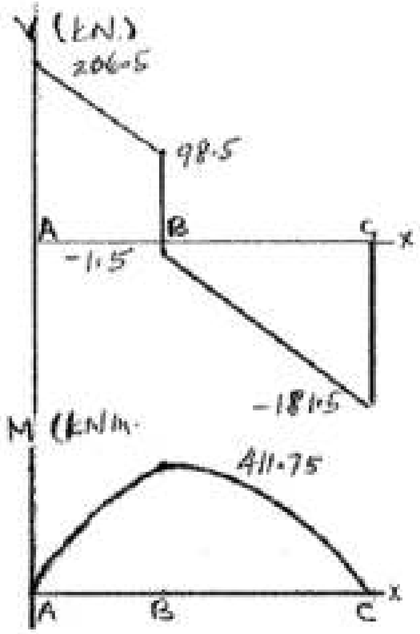
B to C $\int V dx = (\frac{1}{2})(4.5)(-1.5 - 181.5) = -411.75$ kN m

Bending moments: $M_A = 0$

$M_B = 0 + 411.75 = 411.75$ kNm.

$M_C = 411.75 - 411.75 = 0$

Maximum $|M| = 411.75$ kN m.



$\sigma_{all} = 165$ MPa.

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{411.75 \times 10^3}{165 \times 10^6} = 2.495 \times 10^{-3}\ m^3$

$= 2495 \times 10^3\ mm^3$.

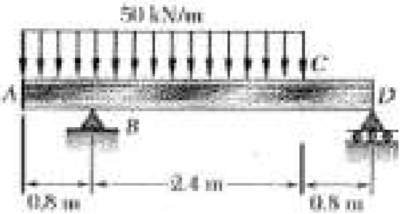
Shape	S (mm × 10³)
W690 × 125	3510
W610 × 101	2530 ←
W530 × 150	3720
W460 × 113	2400
W360 × 216	3800.

Lightest wide flange beam

W610×101 @ 101 kg/m.

Problem 5.73

5.73 and 5.74 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

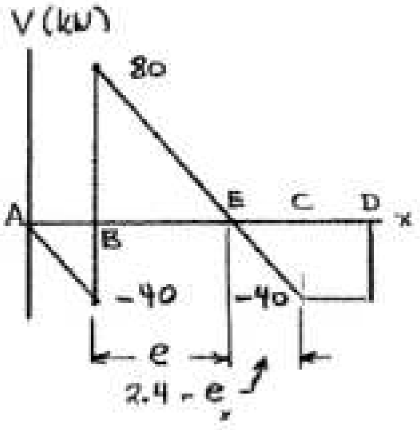


$\circlearrowleft \Sigma M_D = 0: \; -3.2B + (2.4)(3.2)(50) = 0$

$B = 120 \text{ kN}$

$\circlearrowleft \Sigma M_B = 0: \; 3.2D - (0.8)(3.2)(50) = 0$

$D = 40 \text{ kN}$



Shear: $V_A = 0$

$V_{B^-} = 0 - (0.8)(50) = -40 \text{ kN}$

$V_{B^+} = -40 + 120 = 80 \text{ kN}$

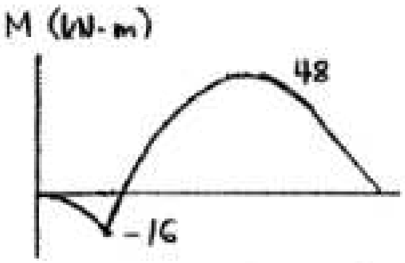
$V_C = 80 - (2.4)(50) = -40 \text{ kN}$

$V_D = -40 + 0 = -40 \text{ kN}$

Locate point E where $V = 0$.

$\frac{e}{80} = \frac{2.4 - e}{40} \qquad 120e = 192$

$e = 1.6 \text{ m} \qquad 2.4 - e = 0.8 \text{ m}$



Areas: A to B, $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN·m}$

B to E. $\int V dx = (\frac{1}{2})(1.6)(80) = 64 \text{ kN·m}$

E to C. $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN·m}$

C to D. $\int V dx = (0.8)(-40) = -32 \text{ kN·m}$

Bending moments: $M_A = 0$

$M_B = 0 - 16 = -16 \text{ kN·m}$

$M_E = -16 + 64 = 48 \text{ kN·m}$

$M_C = 48 - 16 = 32 \text{ kN·m}$

$M_D = 32 - 32 = 0$

Maximum $|M|$: $48 \text{ kN·m} = 48 \times 10^3 \text{ N·m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$

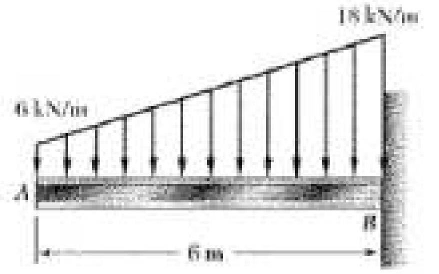
Shape	S (10^3 mm^3)
W 310 × 32.7	415
→ W 250 × 28.4	308
W 200 × 35.9	342

Lightest wide flange beam

W 250 × 28.4 @ 28.4 kg/m

Problem 5.74

5.73 and 5.74 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.



$$w = \left(6 + \frac{18-6}{6}x\right) = (6 + 2x)\ \text{kN/m}$$

$$\frac{dV}{dx} = -w = -6 - 2x$$

$$V = -6x - x^2 + C_1$$

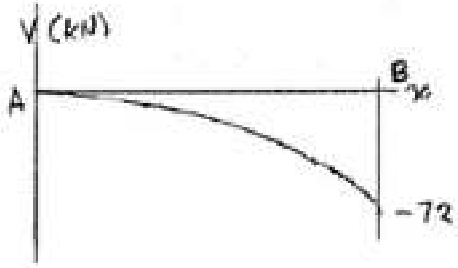
$$V = 0 \text{ at } x = 0. \qquad C_1 = 0$$

$$\frac{dM}{dx} = V = -6x - x^2$$

$$M = -3x^2 - \tfrac{1}{3}x^3 + C_2$$

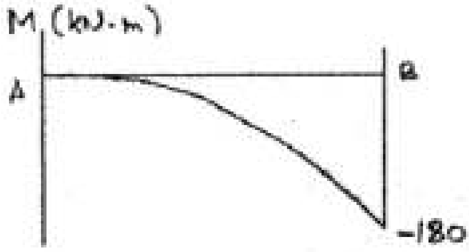
$$M = 0 \text{ at } x = 0 \qquad C_2 = 0$$

$$M = -3x^2 - \tfrac{1}{3}x^3$$



$|M|_{max}$ occurs at $x = 6$ m

$$|M|_{max} = \left|-(3)(6)^2 - \left(\tfrac{1}{3}\right)(6)^3\right| = 180\ \text{kN}\cdot\text{m} = 180 \times 10^3\ \text{N}\cdot\text{m}$$



$$\sigma_{all} = 160\ \text{MPa} = 160 \times 10^6\ \text{Pa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{180 \times 10^3}{160 \times 10^6} = 1.125 \times 10^{-3}\ \text{m}^3 = 1125 \times 10^3\ \text{mm}^3$$

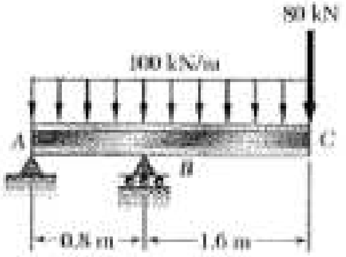
Shape	$S\ (10^3\ \text{mm}^3)$
W 530 × 66	1340 ←
W 460 × 74	1460
W 410 × 85	1510
W 360 × 79	1280
W 310 × 107	1590
W 250 × 101	1240

Lightest acceptable wide flange beam:

W 530 × 66 ◄

Problem 5.75

5.75 and 5.76 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

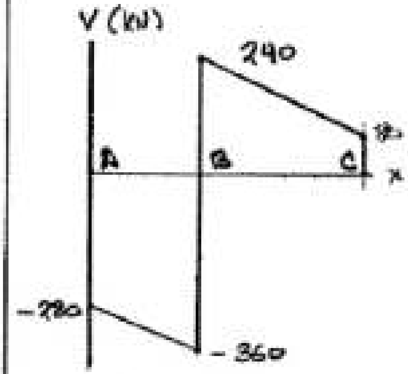


$+\circlearrowleft \Sigma M_B = 0 \qquad 0.8\,A - (0.4)(2.4)(100) - (1.6)(80) = 0$

$A = 280 \text{ kN} \downarrow$

$+\circlearrowleft \Sigma M_A = 0 \qquad 0.8\,B - (1.2)(2.4)(100) - (2.4)(80) = 0$

$B = 600 \text{ kN} \uparrow$



Shear: $V_A = -280$ kN

$V_{B^-} = -280 - (0.8)(100) = -360$ kN

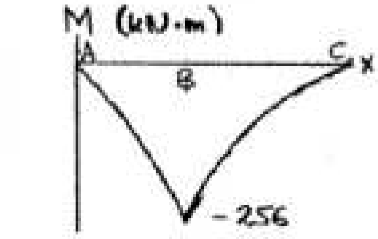
$V_{B^+} = -360 + 600 = 240$ kN

$V_C = 240 - (1.6)(100) = 80$ kN

Areas under shear diagram

A to B $\quad (\frac{1}{2})(0.8)(-280-360) = -256$ kN·m

B to C $\quad (\frac{1}{2})(1.6)(240+80) = 256$ kN·m



Bending moments: $\quad M_A = 0$

$M_B = 0 - 256 = -256$ kN·m

$M_C = -256 + 256 = 0$

Maximum $|M| = 256$ kN·m $= 256 \times 10^3$ N·m

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3} \text{ m}^3 = 1600 \times 10^3 \text{ mm}^3$$

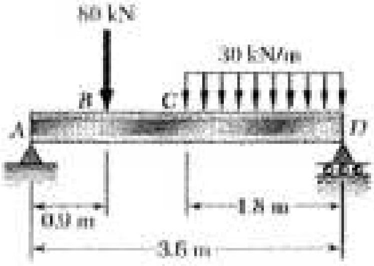
Shape	S (10^3 mm^3)
S 510 × 98.3	1950
S 460 × 104	1685

Lightest S-section

S 510 × 98.3 ◄

Problem 5.78

5.77 and 5.78 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

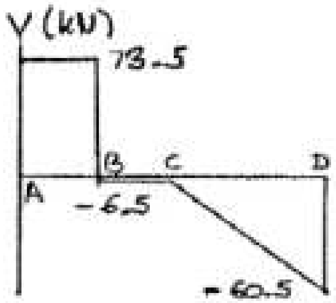


$+\circlearrowleft \Sigma M_D = 0: \; -3.6A + (2.7)(80) + (0.9)(1.8)(30) = 0$

$A = 73.5 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_A = 0: \; 3.6D - (0.9)(80) - (2.7)(1.8)(30) = 0$

$D = 60.5 \text{ kN} \uparrow$



Shear: A to B. $V = 73.5$ kN

B to C. $V = 73.5 - 80 = -6.5$ kN

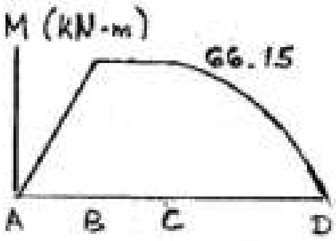
$V_D = -6.5 - (1.8)(30) = -60.5$ kN

Areas of the shear diagram.

A to B. $\int V dx = (0.9)(73.5) = 66.15$ kN·m

B to C. $\int V dx = (0.9)(-6.5) = -5.85$ kN·m

C to D. $(\frac{1}{2})(1.8)(-6.5 - 60.5) = -60.30$ kN·m



Bending moments: $M_A = 0$

$M_B = 0 + 66.15 = 66.15$ kN·m

$M_C = 66.15 - 5.85 = 60.30$ kN·m

$M_D = 66.30 - 66.30 = 0$

Maximum $|M| = 66.15$ kN·m $= 66.15 \times 10^3$ N·m

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$S_{min} = \dfrac{|M|}{\sigma_{all}} = \dfrac{66.15 \times 10^3}{160 \times 10^6} = 413.4 \times 10^{-6} \text{ m}^3 = 413.4 \times 10^3 \text{ mm}^3$

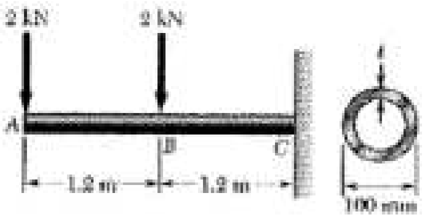
Shape	S (10^3 mm^3)
S 380 × 64	971
S 310 × 47.3	593
S 250 × 52	482

Lightest S-section

S 310 × 47.3 @ 47.3 kg/m

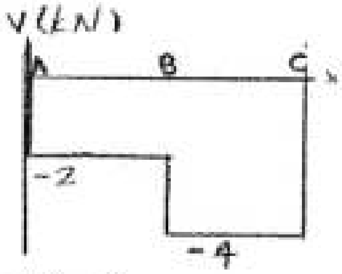
Problem 5.79

5.79 A steel pipe of 100-mm diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from 6 mm to 24 mm in 3-mm increments, and that the allowable normal stress for the steel used is 165 MPa, determine the minimum wall thickness t that can be used.



Shear: A to B $\quad V = -2\text{ kN}$

B to C $\quad V = -2 - 2 = -4\text{ kN}$



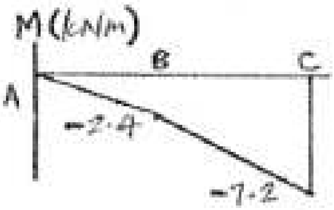
Areas: A to B $\quad -(1.2)(2) = -2.4\text{ kN m}$

B to C $\quad (1.2)(-4) = -4.8\text{ kN/m}$

Bending moments: $\quad M_A = 0$

$M_B = 0 - 2.4 = -2.4\text{ kNm}$

$M_C = -2.4 - 4.8 = -7.2\text{ kNm}$



Maximum $|M| = 7.2\text{ kNm}$

$\sigma_{all} = 165\text{ MPa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{7.2\times10^3}{165\times10^6} = 43.64\times10^{-6}\text{ m}^3 = 43.64\times10^3\text{ mm}^3$$

$I = \frac{\pi}{4}(c_2^4 - c_1^4) \qquad c = c_2 \qquad c_2 = \frac{1}{2}d = 50\text{ mm}$

$$S = \frac{I}{c} = \frac{\pi}{4}\,\frac{c_2^4 - c_1^4}{c_2} = \frac{\pi}{4}\,\frac{50^4 - c_1^4}{50} = 43.64\times10^3\text{ mm}^3$$

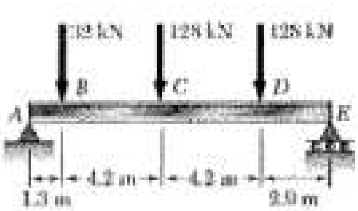
$c_1^4 = 3471791 \qquad c_1 = 43.17\text{ mm}$

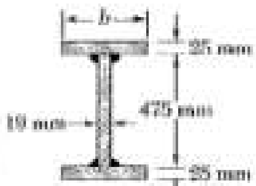
$t_{min} = c_2 - c_1 = 50 - 43.17 = 6.83\text{ mm}$

Using 3 mm increments for design $\quad t = 9\text{ mm}$ ◄

Problem 5.80

5.80 Three steel plates are welded together to form the beam shown. Knowing that the allowable normal stress for the steel used is 154 MPa, determine the minimum flange width b that can be used.





Reactions. $+\circlearrowleft \Sigma M_E = 0$:

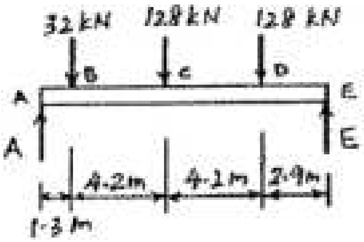
$$-12.6A + (11.3)(32) + (7.1)(128) + (2.9)(128) = 0$$

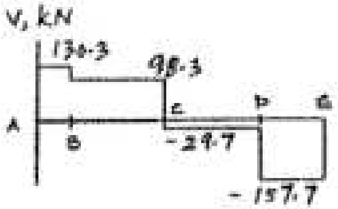
$$A = 130.3 \text{ kN} \uparrow$$

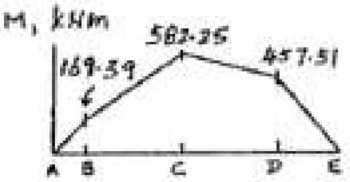
$+\circlearrowleft \Sigma M_A = 0$:

$$12.6E - (1.3)(32) - (5.5)(128) - (9.7)(128) = 0$$

$$E = 157.7 \text{ kN} \uparrow$$







Shear: A to B. 130.3 kN

B to C. $130.3 - 32 = 98.3$ kN

C to D. $98.3 - 128 = -29.7$ kN

D to E. $-29.7 - 128 = -157.7$ kN

Areas: A to B. $(1.3)(130.3) = 169.39$ kNm

B to C. $(4.2)(98.3) = 412.86$ kNm

C to D. $(4.2)(-29.7) = -124.74$ kNm

D to E. $(2.9)(-157.7) = -457.33$ kNm

Bending moments: $M_A = 0$

$M_B = 0 + 169.39 = 169.39$ kNm

$M_C = 169.39 + 412.86 = 582.25$ kNm

$M_D = 582.25 - 124.74 = 457.51$ kNm

$M_E = 457.51 - 457.33 = 0$

Maximum $|M| = 582.25$ kNm $\qquad \sigma_{all} = 154$ MPa

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{582.25 \times 10^3}{154 \times 10^6} = 3.7808 \times 10^{-3} \text{ m}^3 = 3.7808 \times 10^6 \text{ mm}^3$$

$$I = \frac{1}{12}(19)(475)^3 + 2\left[\frac{1}{12}(b)(25)^3 + (b)(25)(250)^2\right] = 169688802 + 3127604b$$

$$c = 237.5 + 25 = 262.5 \text{ mm}$$

$$S_{min} = \frac{I}{c} = 646433.5 + 11914.7b = 3.7808 \times 10^6 \qquad b = 263 \text{ mm}$$

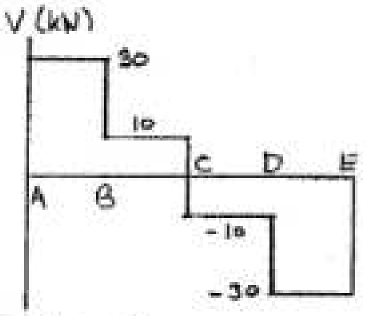
Problem 5.81

5.81 Two metric rolled-steel channels are to be welded along their edges and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 150 MPa, determine the most economical channels that can be used.

By symmetry, $A = E$

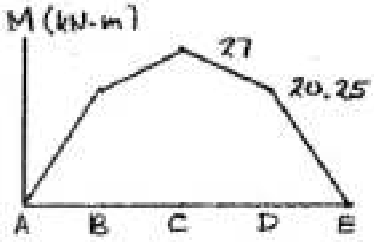
$+\uparrow \Sigma F_y = 0: \quad A + E - 20 - 20 - 20 = 0$

$A = E = 30 \text{ kN}$



<u>Shear:</u>
A to B. $V = 30$ kN
B to C. $V = 30 - 20 = 10$ kN
C to D. $V = 10 - 20 = -10$ kN
D to E. $V = -10 - 20 = -30$ kN

Areas:
A to B. $(0.675)(30) = 20.25$ kN-m
B to C. $(0.675)(10) = 6.75$ kN-m
C to D. $(0.675)(-10) = -6.75$ kN-m
D to E. $(0.675)(-30) = -20.25$ kN-m



<u>Bending moments:</u> $M_A = 0$

$M_B = 0 + 20.25 = 20.25$ kN·m
$M_C = 20.25 + 6.75 = 27$ kN·m
$M_D = 27 - 6.75 = 20.25$ kN·m
$M_E = 20.25 - 20.25 = 0$

Maximum $|M| = 27$ kN·m $= 27 \times 10^3$ N·m

$\sigma_{all} = 150$ MPa $= 150 \times 10^6$ Pa

For a section consisting of two channels,

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{27 \times 10^3}{150 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

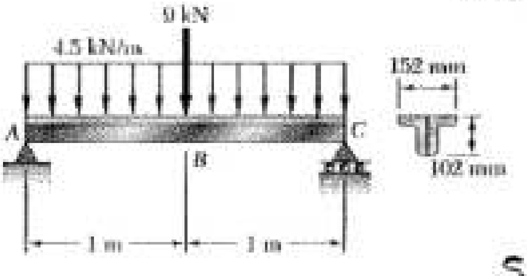
For each channel, $S_{min} = (\frac{1}{2})(180 \times 10^3) = 90 \times 10^3 \text{ mm}^3$

Shape	$S\ (10^3 \text{ mm}^3)$
C 180 × 14.6	99.2 ←
C 150 × 19.3	93.6

Lightest channel section
C 180 × 14.6

Problem 5.82

5.82 Two L102 × 76 rolled-steel angles are bolted together and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 140 MPa, determine the minimum angle thickness that can be used.



Reactions. By symmetry, $A = C$

$+\uparrow \Sigma F_y = 0: \quad A - (2)(4.5) - 9 + C = 0$

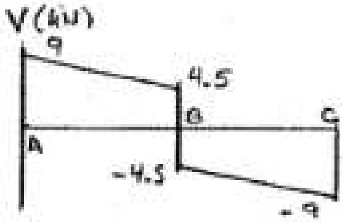
$A = C = 9 \text{ kN} \uparrow$

Shear: $V_A = 9$ kN

$V_B^- = 9 - (1)(4.5) = 4.5$ kN

$V_B^+ = 4.5 - 9 = -4.5$ kN

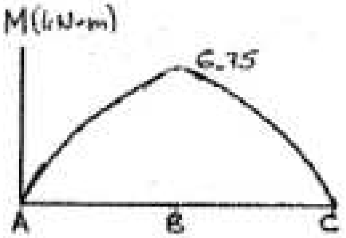
$V_C = -4.5 - (1)(4.5) = -9$ kN



Areas of shear diagram.

A to B. $\int V dx = \frac{1}{2}(1)(9 + 4.5) = 6.75$ kN·m

B to C. $\int V dx = \frac{1}{2}(1)(-9 - 4.5) = -6.75$ kN·m



Bending moments: $M_A = 0$

$M_B = 0 + 6.75 = 6.75$ kN·m

$M_C = 6.75 - 6.75 = 0$

Maximum $|M| = 6.75$ kN·m $= 6.75 \times 10^3$ N·m

$\sigma_{all} = 140$ MPa $= 140 \times 10^6$ Pa

For the section of two angles, $S_{min} = \dfrac{|M|}{\sigma_{all}} = \dfrac{6.75 \times 10^3}{140 \times 10^6} = 48.21 \times 10^{-6} \text{ m}^3$

$= 48.21 \times 10^3 \text{ mm}^3$

For each angle, $S_{min} = \frac{1}{2}(48.21) = 24.105 \times 10^3 \text{ mm}^3$

Shape	S (10^3 mm^3)
L 102 × 76 × 12.7	31.1 ←
L 102 × 76 × 9.5	24.0
L 102 × 76 × 6.4	16.6

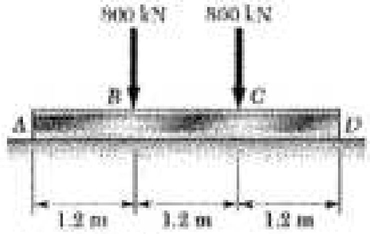
Lightest angle is

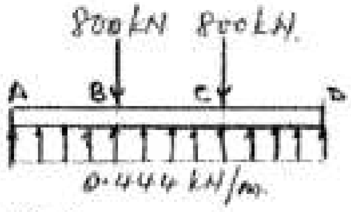
L 102 × 76 × 12.7 ◄

$t_{min} = 12.7$ mm ◄

Problem 5.84

5.84 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 165 MPa, select the most economical wide-flange beam to support the loading shown.





Distributed reaction $q = \frac{1600}{3.6} = 444.4$ kN/m

Shear: $V_A = 0$

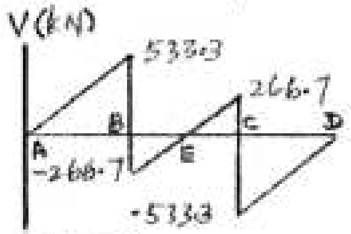
$V_{B^-} = 0 + (1.2)(444.4) = 533.3$ kN

$V_{B^+} = 533.3 - 0.8 = -266.7$ kN

$V_{C^-} = -0.267 + 1.2(444.4) = 266.7$ kN

$V_{C^+} = 266.7 - 0.8 = -533.3$ kN

$V_D = -533.3 + (1.2)(444.4) = 0$ kN

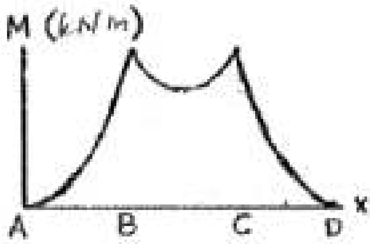


Areas: A to B $(\frac{1}{2})(1.2)(533.3) = 320$ kNm

B to E $(\frac{1}{2})(0.6)(-266.7) = -80$ kNm

E to C $(\frac{1}{2})(0.6)(266.7) = 80$ kNm

C to D $(\frac{1}{2})(1.2)(-533.3) = -320$ kNm.



Bending moments: $M_A = 0$

$M_B = 0 + 320 = 320$ kNm.

$M_E = 320 - 80 = 240$ kNm.

$M_C = 240 + 80 = 320$ kNm.

$M_D = 320 - 320 = 0$

Maximum $|M| = 320$ kNm

$\sigma_{all} = 165$ MPa

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{320000}{165 \times 10^6} = 1.939 \times 10^{-3}\ m^3 = 1939 \times 10^3\ mm^3$$

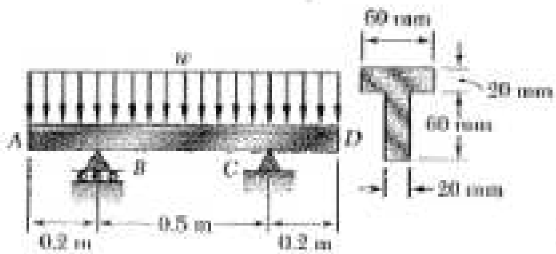
Shape	$S\ (\times 10^3\ mm^3)$
W530 × 92	2070 ←
W460 × 113	2400
W410 × 114	2200
W360 × 122	2010
W310 × 143	2150
W250 × 167	2080

Lightest W-shaped section

W530 × 92 ◄

Problem 5.85

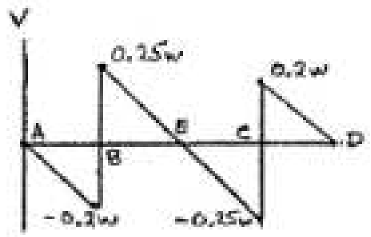
5.85 Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and −130 MPa in compression.



Reactions. By symmetry, $B = C$

$+\uparrow \Sigma F_y = 0: \quad B + C - 0.9w = 0$

$B = C = 0.45w \uparrow$



Shear: $V_A = 0$

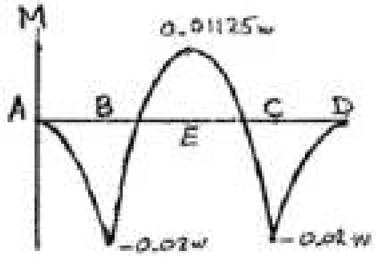
$V_B^- = 0 - 0.2w = -0.2w$

$V_B^+ = -0.2w + 0.45w = 0.25w$

$V_C^- = 0.25w - 0.5w = -0.25w$

$V_C^+ = -0.25w + 0.45w = 0.2w$

$V_D = 0.2w - 0.2w = 0$



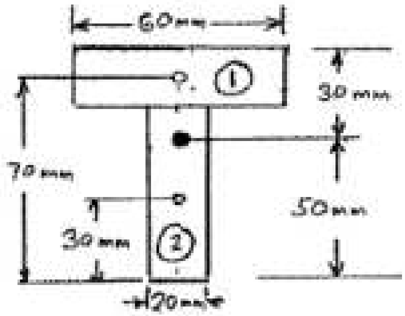
Areas: A to B. $\frac{1}{2}(0.2)(-0.2w) = -0.02w$

B to E. $\frac{1}{2}(0.25)(0.25w) = 0.03125w$

Bending moments: $M_A = 0$

$M_B = 0 - 0.02w = -0.02w$

$M_E = -0.02w + 0.03125w = 0.01125w$



Centroid and moment of inertia.

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}$ (10^3 mm^3)	d, mm	Ad^2 (10^3 mm^4)	$\bar{I}$ (10^3 mm^4)
1	1200	70	84	20	480	40
2	1200	30	36	20	480	360
Σ	2400		120		960	400

$\bar{Y} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$

$I = \Sigma Ad^2 + \Sigma \bar{I} = 1360 \times 10^3 \text{ mm}^4$

Top: $I/y = (1360 \times 10^3)/30 = 45.333 \times 10^3 \text{ mm}^3 = 45.333 \times 10^{-6} \text{ m}^3$

Bottom: $I/y = (1360 \times 10^3)/(-50) = -27.2 \times 10^3 \text{ mm}^3 = -27.2 \times 10^{-6} \text{ m}^3$

Bending moment limits $(M = -\sigma I/y)$ and load limits w.

Tension at B and C: $-0.02w = -(80 \times 10^6)(45.333 \times 10^{-6})$ $\quad w = 181.3 \times 10^3$ N/m

Compression at B and C: $-0.02w = -(-130 \times 10^6)(27.2 \times 10^{-6})$ $\quad w = 176.8 \times 10^3$ N/m

Tension at E: $0.01125w = -(80 \times 10^6)(27.2 \times 10^{-6})$ $\quad w = 193.4 \times 10^3$ N/m

Compression at E: $0.01125w = -(-130 \times 10)(45.333 \times 10^{-6})$ $\quad w = 523.8 \times 10^3$ N/m

The smallest allowable load controls. $\quad w = 176.8 \times 10^3$ N/m

$w = 176.8$ kN/m

Problem 5.87

5.87 Determine the allowable value of **P** for the loading shown, knowing that the allowable normal stress is +55 MPa in tension and −125 MPa in compression.

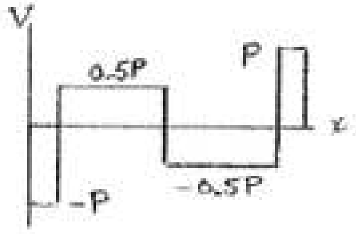
Reactions. $B = D = 1.5\,P \uparrow$

Shear diagram. A to B^- $V = -P$

B^+ to C^- $V = -P + 1.5\,P = 0.5P$

C^+ to D^- $V = 0.5P - P = -0.5P$

D^+ to E $V = -0.5P + 1.5P = P$

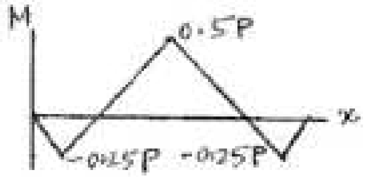


Areas. A to B $(0.25)(-P) = -0.25P$

B to C $(1.5)(0.5P) = 0.75P$

C to D $(1.5)(-0.5P) = -0.75P$

D to E $(0.25)(P) = 0.25P$



Bending moments. $M_A = 0$

$M_B = 0 - 0.25P = -0.25P$

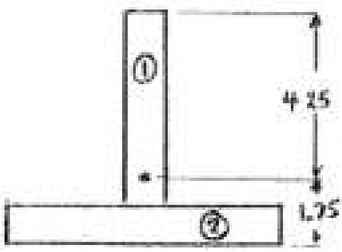
$M_C = -0.25P + 0.75P = 0.5P$

$M_D = 0.5P - 0.75P = -0.25P$

$M_E = -0.25P + 0.25P = 0$

Largest positive bending moment = $0.5P$

Largest negative bending moment = $-0.25P$



Centroid and moment of inertia.

Part	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³	d, mm	Ad^2 mm⁴	$\bar{I}$, mm⁴
①	3125	87.5	273437	43.75	5.981×10^6	4.069×10^6
②	4375	12.5	54687	31.25	4.273×10^6	0.228×10^6
Σ	7500		328124		10.254×10^6	4.3×10^6

Top: $y = 106.25$ mm

Bottom: $y = -43.75$ mm

$$\bar{Y} = \frac{328124}{7500} = 43.75 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 14.554 \times 10^6 \text{ mm}^4$$

$$\sigma = -\frac{My}{I}$$

Top, Tension $55 \times 10^6 = -\dfrac{(-0.25P)(0.10625)}{14.544 \times 10^{-6}}$ $P = 30.1$ kN

Top, Comp. $-125 \times 10^6 = -\dfrac{(0.5P)(0.10625)}{14.544 \times 10^{-6}}$ $P = 34.2$ kN

Bot. Tension $55 \times 10^6 = -\dfrac{(0.5P)(-0.04375)}{14.544 \times 10^{-6}}$ $P = 36.6$ kN

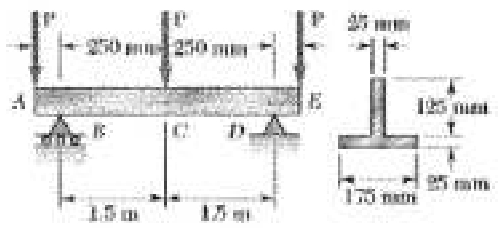
Bot. Comp $-125 \times 10^6 = -\dfrac{(-0.25P)(-0.04375)}{14.544 \times 10^{-6}}$ $P = 166.2$ kN

Smallest value of P is the allowable value $P = 30.1$ kN ◄

Problem 5.88

5.88 Solve Prob. 5.87, assuming that the T-shaped beam is inverted.

5.87 Determine the allowable value of **P** for the loading shown, knowing that the allowable normal stress is +55 MPa in tension and −125 MPa in compression.



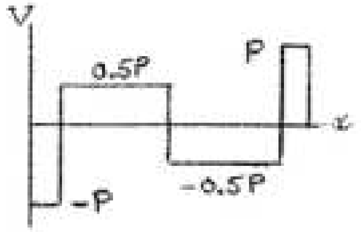
Reactions. $B = D = 1.5\,P \uparrow$

Shear diagram. A to B^- $V = -P$

B^+ to C^- $V = -P + 1.5\,P = 0.5P$

C^+ to D^- $V = 0.5P - P = -0.5P$

D^+ to E $V = -0.5P + 1.5P = P$

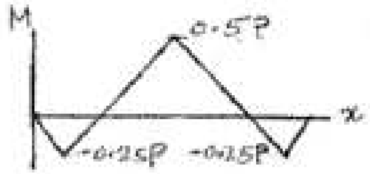


Areas. A to B $(0.25)(-P) = -0.25P$

B to C $(1.5)(0.5P) = 0.75P$

C to D $(1.5)(-0.5P) = -0.75P$

D to E $(0.25)(P) = 0.25P$



Bending moments. $M_A = 0$

$M_B = 0 - 0.25\,P = -0.25P$

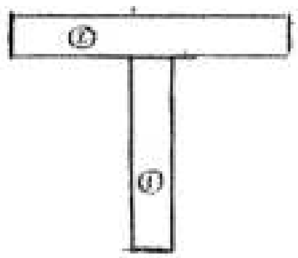
$M_C = -0.25P + 0.75P = 0.5\,P$

$M_D = 0.5\,P - 0.75P = -0.25P$

$M_E = -0.25\,P + 0.25\,P = 0$

Largest positive bending moment: $0.5\,P$

Largest negative bending moment: $-0.25P$



Centroid and moment of inertia.

Part	A, mm²	$\bar{y}_0$, mm	$A\bar{y}_0$, mm³	d, mm	Ad^2 mm⁴	$\bar{I}$, mm⁴
①	3125	62.5	195312	43.75	5.981×10^6	4.069×10^6
②	4375	137.5	601562	31.25	4.273×10^6	0.228×10^6
Σ	7500		796874		10.254×10^6	4.3×10^6

$\bar{Y} = \dfrac{796874}{7500} = 106.25$

$I = \Sigma Ad^2 + \Sigma \bar{I} = 14.544 \times 10^6\ \text{mm}^4$

Top: $y = 43.75$ mm

Bottom: $y = -106.25$ mm

$\sigma = -\dfrac{My}{I}$

Top, Tension $55 \times 10^6 = -\dfrac{(-0.25P)(0.04375)}{4.544 \times 10^{-6}}$ $P = 73.1$ kN

Top, Comp. $-125 \times 10^6 = -\dfrac{(0.5P)(0.04375)}{4.544 \times 10^{-6}}$ $P = 83.1$ kN

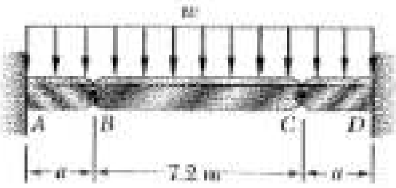
Bot. Tension $55 \times 10^6 = -\dfrac{(0.5\,P)(-0.10625)}{4.544 \times 10^{-6}}$ $P = 15.1$ kN

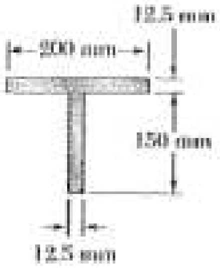
Bot. Comp $-125 \times 10^6 = -\dfrac{(-0.25P)(-0.10625)}{4.544 \times 10^{-6}}$ $P = 68.4$ kN

Smallest value of P is the allowable value $P = 15.1$ kN ◀

Problem 5.89

5.89 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of w if beam BC is not to be overstressed, (*b*) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.





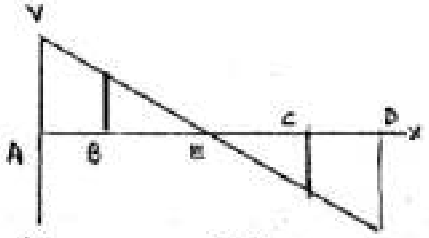
(a) $M_B = M_C = 0$

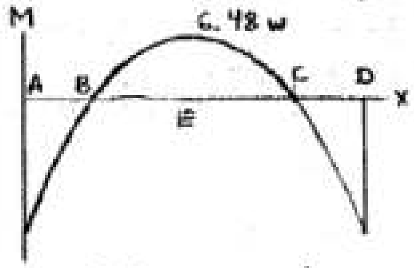
$V_B = -V_C = (\frac{1}{2})(7.2)w = 3.6\ w$

Area B to E of shear diagram,

$(\frac{1}{2})(3.6)(3.6\,w) = 6.48\ w$

$M_E = 0 + 6.48\ w = 6.48\ w$



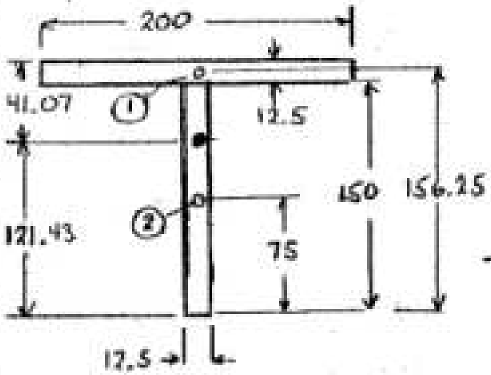


Centroid and moment of inertia

Part	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (mm^3)$	$d\ (mm)$	$Ad^2\ (mm^4)$	$\bar{I}\ (mm^4)$
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531250}{4375} = 121.43\ mm$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6\ mm^4$$



Location	y (mm)	$I/y\ (10^3\ mm^3)$
top	41.07	258.6
bottom	−121.43	−87.47

← also $(10^{-6}\ m^3)$

Bending moment limits: $M = -\sigma I/y$

Tension at E: $-(110\times10^6)(-87.47\times10^{-6}) = 9.622\times10^3$ N·m

Comp. at E: $-(-150\times10^6)(258.6\times10^{-6}) = 38.8\times10^3$ N·m

Tension at A & D: $-(110\times10^6)(258.6\times10^{-6}) = -28.45\times10^3$ N·m

Comp. at A & D: $-(-150\times10^6)(-87.47\times10^{-6}) = -13.121\times10^3$ N·m

(a) <u>Allowable load w.</u> $6.48\,w = 9.622\times10^3$ $w = 1.485\times10^3$ N/m

$w = 1.485$ kN/m ◄

Shear at A. $V_A = (a + 3.6)\,w$

Area A to B of shear diagram: $\frac{1}{2}a(V_A + V_B) = \frac{1}{2}a(a+7.2)w$

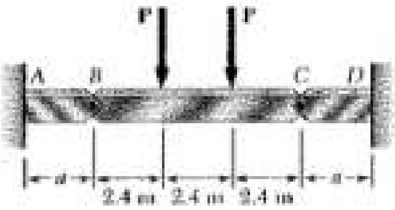
Bending moment at A (also D): $M_A = -\frac{1}{2}a(a+7.2)\,w$

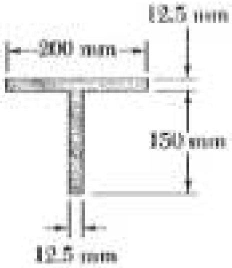
$-\frac{1}{2}a(a+7.2)(1.485\times10^3) = -13.121\times10^3$

(b) <u>Distance a.</u> $\frac{1}{2}a^2 + 3.6a - 8.837 = 0$ $a = 1.935$ m ◄

Problem 5.90

5.90 Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of **P** if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.





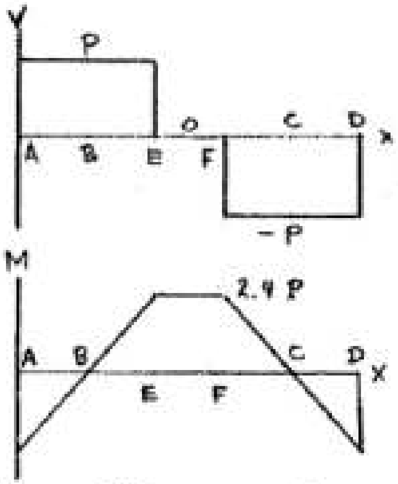
$M_B = M_C = 0$

$V_B = -V_C = P$

Area B to E of shear diagram.

$2.4\ P$

$M_E = 0 + 2.4\ P = 2.4\ P = M_F$

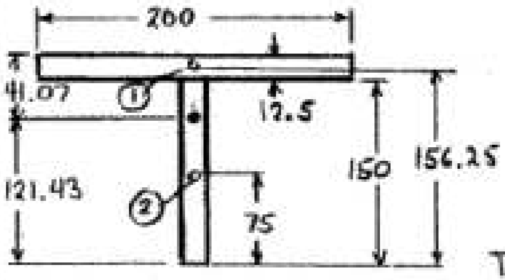


Centroid and moment of inertia.

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad^2 (mm⁴)	$\bar{I}$ (mm⁴)
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531250}{4375} = 121.43 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 10.621 \times 10^6 \text{ mm}^4$$



Location	y (mm)	I/y (10^3 mm³) ← also (10^{-6} m³)
top	41.07	258.6
bottom	−121.43	−87.47

Bending moment limits: $M = -\sigma I/y$

Tension at E & F: $-(110 \times 10^6)(-87.47 \times 10^{-6}) = 9.622 \times 10^3$ N·m

Comp. at E & F: $-(-150 \times 10^6)(258.6 \times 10^{-6}) = 38.8 \times 10^3$ N·m

Tension at A & D: $-(110 \times 10^6)(258.6 \times 10^{-6}) = -28.45 \times 10^3$ N·m

Comp. at A & D: $-(-150 \times 10^6)(-87.47 \times 10^{-6}) = -13.121 \times 10^3$ N·m

(a) Allowable load P. $2.4\ P = 9.622 \times 10^3$ $P = 4.01 \times 10^3$ N

$P = 4.01$ kN ◄

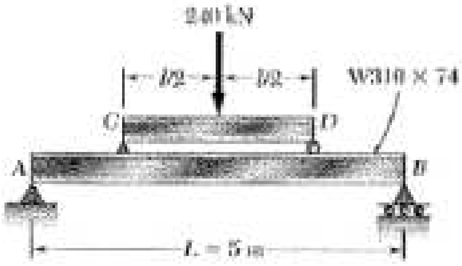
Shear at A $V_A = P$

Area A to B of shear diagram: $aV_A = aP$

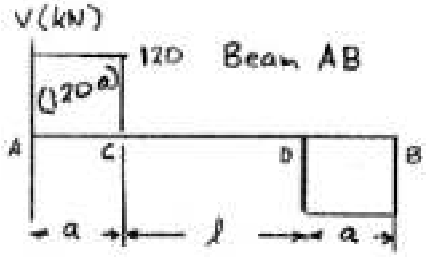
Bending moment at A. $M_A = -aP = -4.01 \times 10^3\ a$

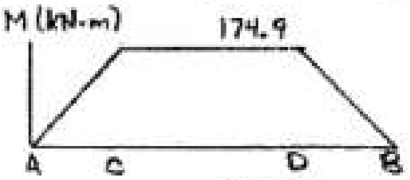
(b) Distance a. $-4.01 \times 10^3\ a = -13.121 \times 10^3$ $a = 3.27$ m ◄

Problem 5.91



5.91 A 240-kN load is to be supported at the center of the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (*a*) the smallest allowable length *l* of beam *CD* if the W310 × 74 beam *AB* is not to be overstressed, (*b*) the most economical W shape that can be used for beam *CD*. Neglect the weight of both beams.





For rolled steel section W 310 × 74 of beam AB

$S = 1060 \times 10^3 \text{ mm}^3 = 1060 \times 10^{-6} \text{ m}^3$

$\sigma_{all} = 165 \text{ MPa} = 165 \times 10^6 \text{ Pa}$

Allowable bending moment.

$M_{all} = S\sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3 \text{ N·m}$

$= 174.9 \text{ kN·m}$

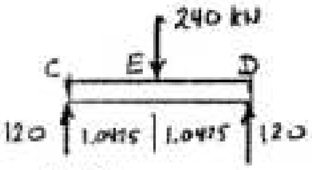
(a) Beam AB.

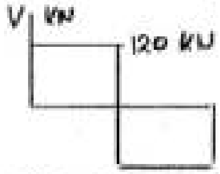
Area A to C of shear diagram = 120 a

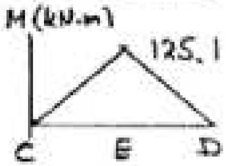
Bending moment at C   120 a

$120\,a = 174.9 \quad a = 1.4575 \text{ m}$

Geometry: $2a + l = 5 \qquad l = 5 - 2a = 2.085 \text{ m}$ ◄







(b) Beam CD   (midpoint E)

Area C to E of shear diagram

$= (1.0425)(120) = 125.1 \text{ kN·m}$

Bending moment at E.

$M = 125.1 \text{ kN·m} = 125.1 \times 10^3 \text{ N·m}$

$S_{min} = \dfrac{M}{\sigma_{all}} = \dfrac{125.1 \times 10^3}{165 \times 10^6} = 758.2 \times 10^{-6} \text{ m}^3$

$= 758.2 \times 10^3 \text{ mm}^3$

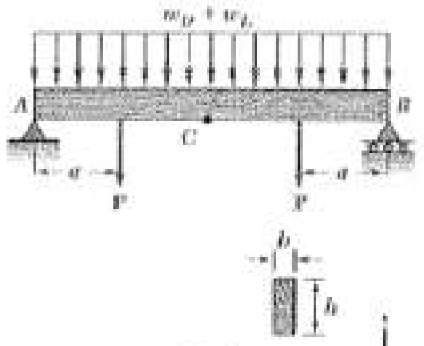
Shape	S (10^3 mm^3)
W 460 × 52	942
W 410 × 46.1	774 ←
W 360 × 57.8	899
W 310 × 60	851
W 250 × 67	809
W 200 × 86	853

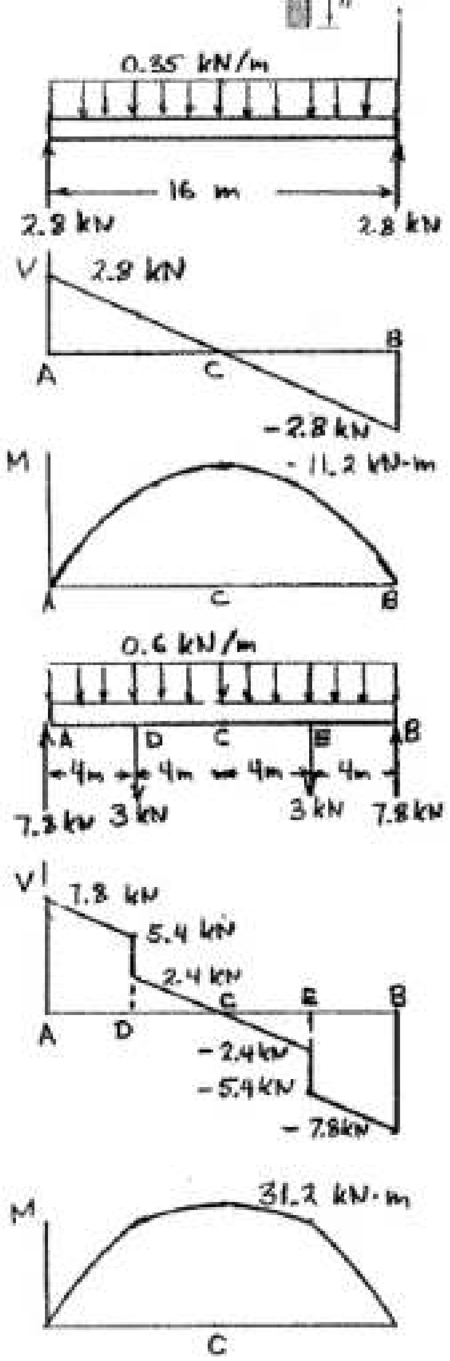
Answer: W 410 × 46.1 ◄

Problem 5.95

***5.95** Solve Prob. 5.94, assuming that the 6-kN concentrated load **P** applied to each beam is replaced by 3-kN concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ applied at a distance of 4 m from each end of the beams.

***5.94** A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.





$L = 16$ m, $a = 4$ m, $W_D = 350$ N/m $= 0.35$ kN/m

$W_L = 600$ N/m $= 0.6$ kN/m $\quad P = 3$ kN

Dead load: $\quad R_A = (\frac{1}{2})(16)(0.35) = 2.8$ kN

Area A to C of shear diagram

$(\frac{1}{2})(8)(2.8) = 11.2$ kN·m

Bending moment at C: 11.2 kN·m $= 11.2 \times 10^3$ N·m

Live load: $\quad R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8$ kN

Shear at D^- $\quad 7.8 - (4)(0.6) = 5.4$ kN

Shear at D^+ $\quad 5.4 - 3 = 2.4$ kN

Area A to D $\quad (\frac{1}{2})(4)(7.8 + 5.4) = 26.4$ kN·m

Area D to C $\quad (\frac{1}{2})(4)(2.4) = 4.8$ kN·m

Bending moment at C $= 26.4 + 4.8 = 31.2$ kN·m

$= 31.2 \times 10^3$ N·m

Design: $\quad \gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_U S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(31.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

$$= 1.408 \times 10^{-3} \text{ m}^3 = 1.408 \times 10^6 \text{ mm}^3$$

For a rectangular section $\quad S = \frac{1}{6} b h^2$

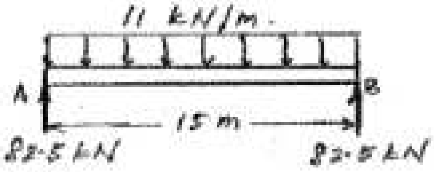
$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408 \times 10^6)}{75}}$$

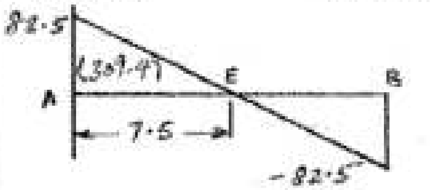
$h = 336$ mm ◄

Problem 5.96

*5.96 A bridge of length $L = 15$ m is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_U = 420$ MPa. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 11$ kN/m on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 4$ m from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 95 kN and 25 kN, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

$L = 15$ m
$a = 4$ m
$P_1 = 95$ kN
$P_2 = 25$ kN
$W = 11$ kN/m





Dead load: $R_A = R_B = (\frac{1}{2})(15)(11) = 82.5$ kN.

Area A to E of shear diagram $(\frac{1}{2})(7.5)(82.5) = 309.4$

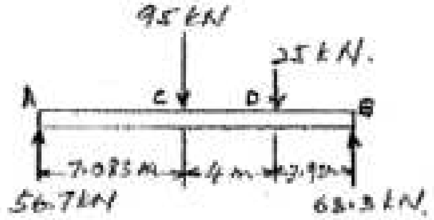
$M_{max} = 309.4$ kN·m at point E

Live load: $u = \dfrac{aP_2}{2(P_1 + P_2)} = \dfrac{(4)(25)}{(2)(120)} = 0.417$ m

$x = \frac{L}{2} - u = 7.5 - 0.417 = 7.083$ m
$x + a = 7.083 + 4 = 11.083$ m.
$L - x - a = 15 - 11.083 = 3.92$

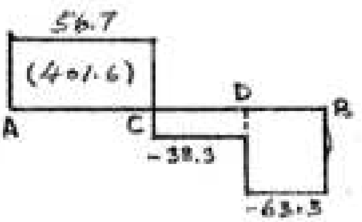
$\Sigma M_B = 0 \quad -15R_A + (7.92)(95) + (3.92)(25) = 0$
$R_A = 56.7$ kN.

Shear: A to C $V = 56.7$ kN
C to D $V = 56.7 - 95 = -38.3$
D to B $V = -63.3$



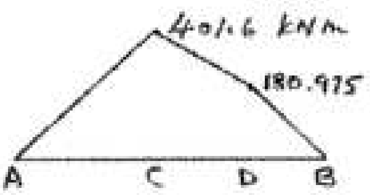
Area A to C $(7.083)(56.7) = 401.6$ kN·m

Bending moment: $M_L = 401.6$ kN·m.



Design: $\gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_{all} S_{min}$

$$S_{min} = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_{all}} = \frac{(1.25)(309.4 \times 10^3) + (1.75)(401.6 \times 10^3)}{(0.9)(420 \times 10^6)}$$
$$= 2882.4 \times 10^{-6} \text{ m}^3 = 2882.4 \times 10^3 \text{ mm}^3$$



Shape	S ($\times 10^3$ mm^3)
W 690 × 125	3510 ←
W 610 × 155	4220
W 530 × 150	3720
W 460 × 158	3340
W 360 × 216	3800

W690 × 125 ◄

Problem 5.97

$L = 15 m$
$a = 4 m$
$P_1 = 95 kN$
$P_2 = 25 kN$
$w = 11 kN/m$

*5.97 Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.96, determine how much heavier a truck could safely cross the bridge designed in that problem.

*5.96 A bridge of length $L = 15$ m is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_U = 420$ MPa. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 11$ kN/m on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 4$ m from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 95 kN and 25 kN, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/(P_1 + P_2)$.]

See solution to Problem 5.94 for calculation of the following:

$M_D = 309.4$ kNm $\quad M_L = 401.6$ kNm.

For rolled steel section W690x125 $\quad S = 3510 \times 10^3$ mm^3

Allowable live load moment M_L^*

$$\gamma_D M_D + \gamma_L M_L^* = \phi M_U = \phi \sigma_U S$$

$$M_L^* = \frac{\phi \sigma_U S - \gamma_D M_D}{\gamma_L} = \frac{(0.9)(420\times10^6)(3510\times10^{-6}) - (1.25)(309.4\times10^3)}{1.75}$$

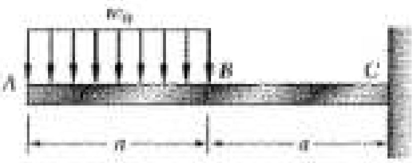
$$= 537.2 \text{ kNm}$$

Ratio $\frac{M_L^*}{M_L} = \frac{537.2}{401.6} = 1.338 = 1 + 0.338$

Increase $\quad 33.8\%$

Problem 5.98

5.98 through 5.100 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.



$$w = w_0 - w_0\langle x-a\rangle^0 = -\frac{dV}{dx}$$

(a) $$V = -w_0 x + w_0\langle x-a\rangle^1 = \frac{dM}{dx}$$

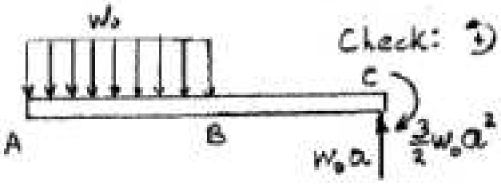
$$M = -\tfrac{1}{2}w_0 x^2 + \tfrac{1}{2}w_0\langle x-a\rangle^2$$

At point C $\quad x = 2a$

(b) $$M_C = -\tfrac{1}{2}w_0(2a)^2 + \tfrac{1}{2}w_0 a^2 = -\tfrac{3}{2}w_0 a^2$$

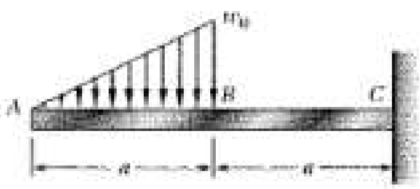
Check: $+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{3a}{2}\right)(w_0 a) + M_C = 0$

$$M_C = -\tfrac{3}{2}w_0 a^2$$



Problem 5.99

5.98 through 5.100 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.



$$w = \frac{w_0 x}{a} - w_0\langle x-a\rangle^0 - \frac{w_0}{a}\langle x-a\rangle^1 = -\frac{dV}{dx}$$

(a) $$V = -\frac{w_0 x^2}{2a} + w_0\langle x-a\rangle^1 + \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$

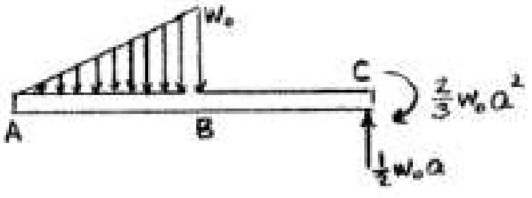
$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2}\langle x-a\rangle^2 + \frac{w_0}{6a}\langle x-a\rangle^3$$

At point C $\quad x = 2a$

(b) $$M_C = -\frac{w_0(2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a} \qquad M_C = -\tfrac{2}{3}w_0 a^2$$

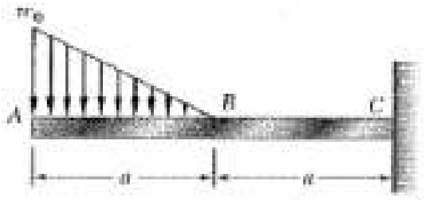
Check: $+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{4a}{3}\right)\left(\tfrac{1}{2}w_0 a\right) + M_C = 0$

$$M_C = -\tfrac{2}{3}w_0 a^2$$



Problem 5.100

5.98 through 5.100 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.



$$w = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a}\langle x-a\rangle^1 = -\frac{dV}{dx}$$

(a) $$V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a}\langle x-a\rangle^3$$

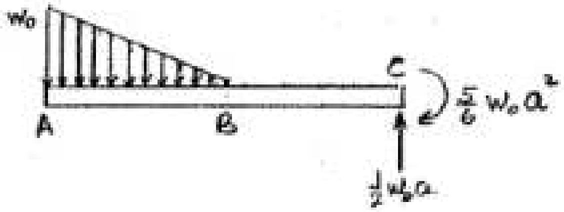
At point C $\quad x = 2a$

(b) $$M_C = -\frac{w_0(2a)^2}{2} + \frac{w_0(2a)^3}{6a} - \frac{w_0 a^3}{6a} \qquad M_C = -\frac{5}{6}w_0 a^2$$

Check:

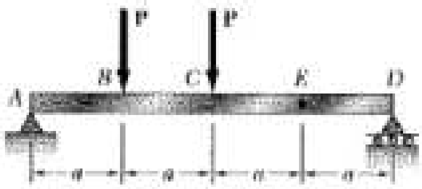
$+\circlearrowleft \Sigma M_C = 0: \quad (\frac{5}{3}a)(\frac{1}{2}w_0 a) + M_C = 0$

$M_C = -\frac{5}{6}w_0 a^2$



Problem 5.101

5.101 through 5.103 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.



$+\circlearrowleft \Sigma M_D = 0: \quad -4aA + 3aP + 2aP = 0$

$A = 1.25\ P$

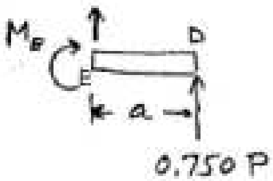
(a) $V = 1.25P - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$

$M = 1.25Px - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1$

(b) At point $E \quad x = 3a$

$M_E = 1.25P(3a) - P(2a) - P(a) = 0.750\ Pa$

Reaction: $+\uparrow \Sigma F_y = 0: \quad A - P - P + D = 0 \qquad D = 0.750P \uparrow$

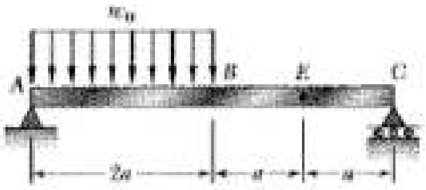


$+\circlearrowleft \Sigma M_E = 0 \qquad -M_E + 0.750Pa = 0$

$M_E = 0.750\ Pa$

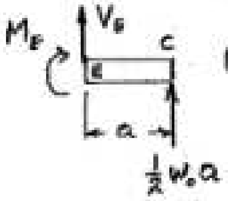
Problem 5.102

5.101 through *5.103* (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.



$\circlearrowleft \Sigma M_C = 0 \qquad -4aA + (3a)(2a w_0) = 0 \qquad A = \frac{3}{2} w_0 a$

$w = w_0 - w_0 \langle x - 2a \rangle^0 = -\frac{dV}{dx}$



(a) $V = -w_0 x + w_0 \langle x - 2a \rangle^1 + \frac{3}{2} w_0 a = \frac{dM}{dx}$

$M = -\frac{1}{2} w_0 x^2 + \frac{1}{2} w_0 \langle x - 2a \rangle^2 + \frac{3}{2} w_0 a x + 0$

At point E $\quad x = 3a$

(b) $M_E = -\frac{1}{2} w_0 (3a)^2 + \frac{1}{2} w_0 a^2 + \frac{3}{2} w_0 a (3a)$

$M_E = \frac{1}{2} w_0 a^2$

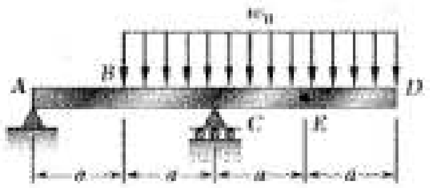
$\circlearrowleft \Sigma M_A = 0: \qquad 4aC - (a)(2a w_0) = 0 \qquad C = \frac{1}{2} w_0 a$

$\circlearrowleft \Sigma M_E = 0: \qquad -M_E + (a)(\frac{1}{2} w_0 a) = 0$

$M_E = \frac{1}{2} w_0 a^2$

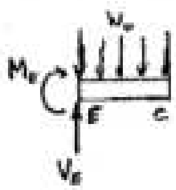
Problem 5.103

5.101 through *5.103* (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.



$+\circlearrowleft \Sigma M_C = 0 \qquad -2aA - (\frac{a}{2})(3a w_0) = 0 \qquad A = -\frac{3}{4} w_0 a$

$+\circlearrowleft \Sigma M_A = 0 \qquad 2aC + (\frac{5a}{2})(3a w_0) = 0 \qquad C = \frac{15}{4} w_0 a$



$w = w_0 \langle x - a \rangle^0 = -\frac{dV}{dx}$

(a) $V = -w_0 \langle x - a \rangle^1 - \frac{3}{4} w_0 a + \frac{15}{4} w_0 a \langle x - 2a \rangle^0 = \frac{dM}{dx}$

$M = -\frac{1}{2} w_0 \langle x - a \rangle^2 - \frac{3}{4} w_0 a x + \frac{15}{4} w_0 a \langle x - 2a \rangle^1 + 0$

At point E $\quad x = 3a$

(b) $M_E = -\frac{1}{2} w_0 (2a)^2 - \frac{3}{4} w_0 a (3a) + \frac{15}{4} w_0 a (a)$

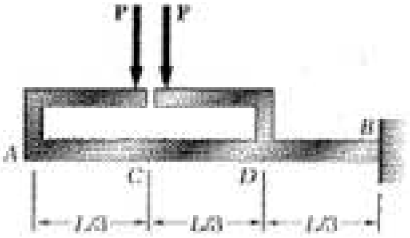
$M_E = -\frac{1}{2} w_0 a^2$

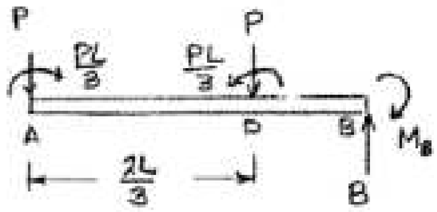
Check: $+\Sigma M_E = 0: \qquad -M_E - \frac{a}{2}(w_0 a) = 0$

$M_E = -\frac{1}{2} w_0 a^2$

Problem 5.104

5.104 (*a*) Using singularity functions, write the equations for the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *D*.





$$\text{(a)}\quad V = -P - P\langle x - \tfrac{2L}{3}\rangle^0 = \frac{dM}{dx}$$

$$M = -Px + \frac{PL}{3} - P\langle x - \tfrac{2L}{3}\rangle^1 - \frac{PL}{3}\langle x - \tfrac{2L}{3}\rangle^0$$

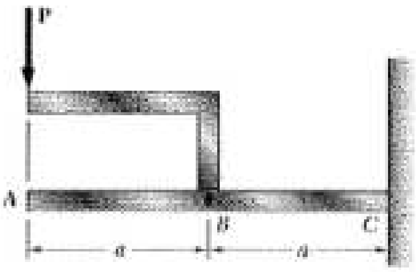
Just to the right of D $\quad x = \frac{2L}{3}$

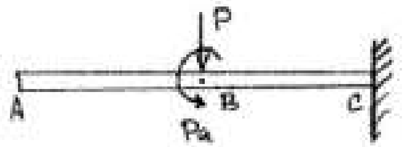
$$\text{(b)}\quad M_D^+ = -P\left(\frac{2L}{3}\right) + \frac{PL}{3} - P(0) - \frac{PL}{3}$$

$$M = -\frac{4PL}{3}$$

Problem 5.105

5.105 (*a*) Using singularity functions, write the equations for the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.





$$\text{(a)}\quad V = -P\langle x - a\rangle^0$$

$$\frac{dM}{dx} = -P\langle x - a\rangle^0$$

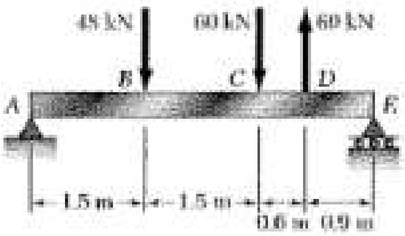
$$M = -P\langle x - a\rangle^1 - Pa\langle x - a\rangle^0$$

Just to the right of B $\quad x = a^+$

$$\text{(b)}\quad M = -0 - Pa \qquad M = -Pa$$

Problem 5.106

5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.



$$+\circlearrowleft \Sigma M_E = 0: \ -4.5 R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$$

$$R_A = 40 \text{ kN}$$

(a) $V = 40 - 48\langle x-1.5\rangle^0 - 60\langle x-3.0\rangle^0 + 60\langle x-3.6\rangle^0 \text{ kN}$ ◄

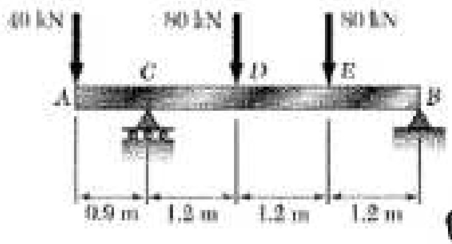
$M = 40x - 48\langle x-1.5\rangle^1 - 60\langle x-3.0\rangle^1 + 60\langle x-3.6\rangle^1 \text{ kN}\cdot\text{m}$ ◄

Pt.	x (m)	M (kN·m)
A	0	0
B	1.5	$(40)(1.5) = 60$ kN·m
C	3.0	$(40)(3.0) - (48)(1.5) = 48$ kN·m
D	3.6	$(40)(3.6) - (48)(2.1) - (60)(0.6) = 7.2$ kN·m
E	4.5	$(40)(4.5) - (48)(3.0) - (60)(1.5) + (60)(0.9) = 0$

(b) $M_{max} = 60$ kN·m ◄

Problem 5.107

5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.



$$+\circlearrowleft \Sigma M_B = 0: \ (40)(4.5) - 3.6C + (80)(2.4) + (80)(1.2) = 0$$

$$C = 130 \text{ kN} \uparrow$$

(a) $V = -40 + 130\langle x-0.9\rangle^0 - 80\langle x-2.1\rangle^0 - 80\langle x-3.3\rangle^0 \text{ kN}$ ◄

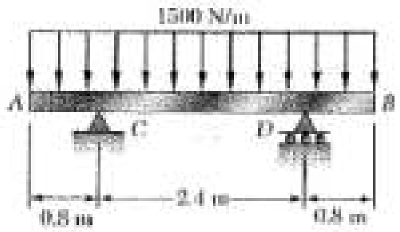
$M = -40x + 130\langle x-0.9\rangle^1 - 80\langle x-2.1\rangle^1 - 80\langle x-3.3\rangle^1 \text{ kN m}$ ◄

Point	x (m)	M (kN m)
A	0	0
C	0.9	$-(40)(0.9) = -36$
D	2.1	$-(40)(2.1) + (130)(1.2) = 72$
E	3.3	$-(40)(3.3) + (130)(2.4) - (80)(1.2) = 84$ ← maximum
B	4.5	$-(40)(4.5) + (130)(3.6) - (80)(2.4) - (80)(1.2) = 0$

(b) $|M|_{max} = 84$ kN m ◄

Problem 5.108

5.106 through 5.109 (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



$w = 1.5$ kN/m

By statics, $C = D = 3$ kN ↑

(a) $V = -1.5x + 3\langle x-0.8\rangle^0 + 3\langle x-3.2\rangle^0$ kN

$M = -0.75x^2 + 3\langle x-0.8\rangle^1 + 3\langle x-3.2\rangle^1$ kN·m

Locate point E where $V = 0$. Assume $x_C < x_E < x_D$

$0 = -1.5x_E + 3(x_E - 0.8) + 0 \qquad x_E = 2.0$ m

$M_C = -(0.75)(0.8)^2 + 0 + 0 = -0.480$ kN·m

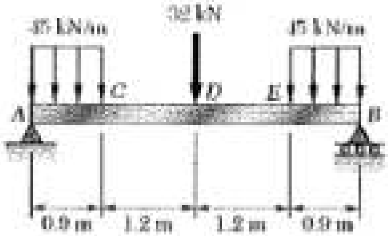
$M_E = -(0.75)(2.0)^2 + (3)(1.2) + 0 = 0.600$ kN·m

$M_D = -(0.75)(3.2)^2 + (3)(2.4) + 0 = -0.480$ kN·m

(b) $|M|_{max} = 0.600$ kN·m $\qquad$ 600 N·m

Problem 5.109

5.106 through 5.109 (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



From symmetry, $A = \frac{1}{2}[(32) + 2(45)(0.9)] = 56.5$ kN

$w = 45 - 45\langle x-0.9\rangle^0 + 45\langle x-3.3\rangle^0 = -\frac{dV}{dx}$

(a) $V = 56.5 - 45x + 45\langle x-0.9\rangle^1 - 32\langle x-2.1\rangle^0 - 45\langle x-3.3\rangle^1$ kN

$M = 56.5 - 22.5x^2 + 22.5\langle x-0.9\rangle^2 - 32\langle x-2.1\rangle^1 - 22.5\langle x-3.3\rangle^2$ kNm

$V_C = 56.5 - (45)(0.9) = 16$ kN

$V_D^- = 56.5 - (45)(2.1) + (45)(1.2) = 16$ kN $\qquad$ ← Changes sign

$V_D^+ = 56.5 - (45)(2.1) + (45)(1.2) - 32 = -16$ kN

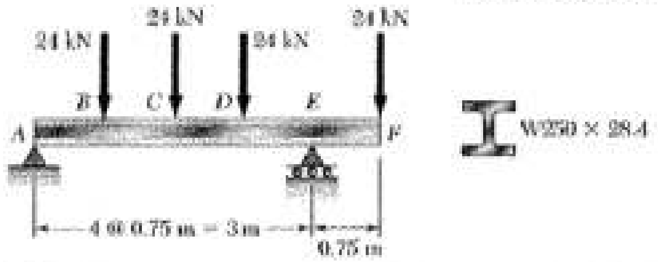
$V_E = 56.5 - (45)(3.3) + (45)(2.4) - 32 = -16$ kN

$V_B = 56.5 - (45)(4.2) + (45)(3.3) - 32 - (45)(0.9) = -56.5$ kN

(b) $|M|_{max} = M_D = (56.5)(2.1) - (22.5)(2.1)^2 + (22.5)(1.2)^2 - 0 - 0 = 51.825$ kNm

Problem 5.110

5.110 and *5.111* (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



$+\circlearrowleft \Sigma M_E = 0: \quad -3R_A + (2.25)(24) - (1.5)(24) - (0.75)(24) + (0.75)(24) = 0$

$R_A = 30 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0: \quad -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$

$R_E = 66 \text{ kN}$

(a) $V = 30 - 24\langle x-0.75\rangle^0 - 24\langle x-1.5\rangle^0 - 24\langle x-2.25\rangle^0 + 66\langle x-3\rangle^0 \text{ kN}$ ◄

$M = 30x - 24\langle x-0.75\rangle^1 - 24\langle x-1.5\rangle^1 - 24\langle x-2.25\rangle^1 + 66\langle x-3\rangle^1 \text{ kN}\cdot\text{m}$ ◄

Pt	x (m)	M (kN·m)
B	0.75	$(30)(0.75) = 22.5$ kN·m
C	1.5	$(30)(1.5) - (24)(0.75) = 27$ kN·m
D	2.25	$(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5$ kN·m
E	3.0	$(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18$ kN·m
F	3.75	$(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$ ✓

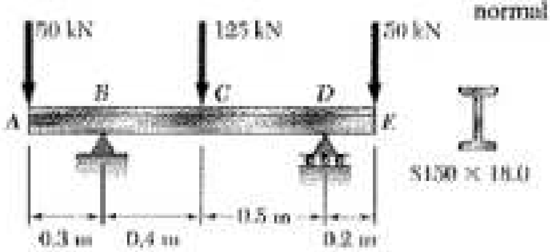
Maximum $|M| = 27 \text{ kN}\cdot\text{m} = 27 \times 10^3 \text{ N}\cdot\text{m}$

For rolled steel section W250 × 28.4, $S = 308 \times 10^3 \text{ mm}^3 = 308 \times 10^{-6} \text{ m}^3$

(b) <u>Normal stress.</u> $\sigma = \dfrac{|M|}{S} = \dfrac{27 \times 10^3}{308 \times 10^{-6}} = 87.7 \times 10^6 \text{ Pa} = 87.7 \text{ MPa}$ ◄

Problem 5.111

5.110 and 5.111 (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



$+\circlearrowleft \Sigma M_D = 0:$
$(1.2)(50) - 0.9B + (0.5)(125) - (0.2)(50) = 0$
$B = 125 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_B = 0:$
$(0.3)(50) - (0.4)(125) + 0.9D - (1.1)(50) = 0$
$D = 100 \text{ kN} \uparrow$

(a) $V = -50 + 125\langle x-0.3\rangle^0 - 125\langle x-0.7\rangle^0 + 100\langle x-1.2\rangle^0$ kN ◄

$M = -50x + 125\langle x-0.3\rangle^1 - 125\langle x-0.7\rangle^1 + 100\langle x-1.2\rangle^1$ kN·m ◄

Point	x (m)	M (kN·m)
B	0.3	$-(50)(0.3)+0-0+0 = -15$ kN·m
C	0.7	$-(50)(0.7)+(125)(0.4)-0+0 = 15$ kN·m
D	1.2	$-(50)(1.2)+(125)(0.9)-(125)(0.5)+0 = -10$ kN·m
E	1.4	$-(50)(1.4)+(125)(1.1)-(125)(0.7)+(100)(0.2) = 0$ (checks)

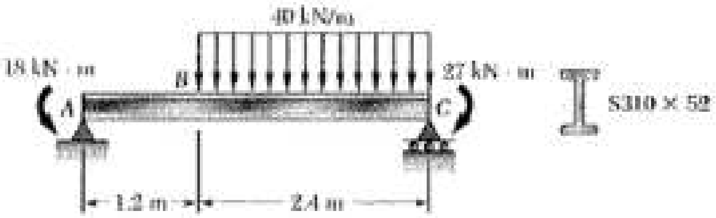
Maximum $|M| = 15$ kN·m $= 15\times10^3$ N·m

For S 150 × 18.0 rolled steel section, $S = 120\times10^3 \text{ mm}^3 = 120\times10^{-6} \text{ m}^3$

(b) Normal stress. $\sigma = \dfrac{|M|}{S} = \dfrac{15\times10^3}{120\times10^{-6}} = 125\times10^6$ Pa $\quad \sigma = 125.0$ MPa ◄

Problem 5.112

5.112 and 5.113 (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



$\circlearrowleft M_C = 0:$

$18 - 3.6 R_A + (1.2)(2.4)(40) - 27 = 0$

$R_A = 29.5 \text{ kN}$

$V = 29.5 - 40\langle x - 1.2\rangle^1 \text{ kN}$

Point D. $\quad V = 0 \qquad 29.5 - 40(x_D - 1.2) = 0$

$x_D = 1.9375 \text{ m}$

$M = -18 + 29.5x - 20\langle x - 1.2\rangle^2 \text{ kN}\cdot\text{m}$

$M_A = -18 \text{ kN}\cdot\text{m}$

$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN}\cdot\text{m}$

$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN}\cdot\text{m}$

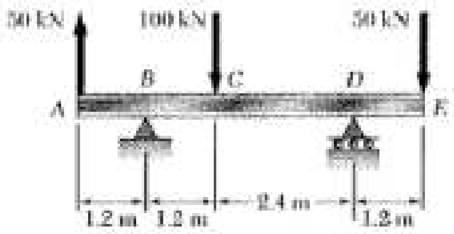
(a) Maximum $|M| = 28.278 \text{ kN}\cdot\text{m}$ at $x = 1.9375 \text{ m}$ ◀

For S310 × 52 rolled steel section, $S = 625 \times 10^3 \text{ mm}^3 = 625 \times 10^{-6} \text{ m}^3$

(b) Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{28.278 \times 10^3}{625 \times 10^{-6}} = 45.2 \times 10^6 \text{ Pa}$

$\sigma = 45.2 \text{ MPa}$ ◀

Problem 5.114



5.114 and 5.115 A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 165 MPa, find the most economical wide-flange shape that can be used.

$+\circlearrowleft \Sigma M_D = 0: \quad -(4.8)(50) - 3.6B + (2.4)(100) - (1.2)(50) = 0$

$B = -16.67\ kN \qquad B = 16.67\ kN \downarrow$

$+\circlearrowleft \Sigma M_B = 0: \quad -(1.2)(50) - (1.2)(100) + 3.6D - (4.8)(50) = 0$

$D = 116.67\ kN$

Check: $+\uparrow \Sigma F_y = 50 - 16.67 - 100 + 116.67 - 50 = 0 \quad \checkmark$

$V = 50 - 16.67\langle x-1.2\rangle^0 - 100\langle x-2.4\rangle^0 + 116.67\langle x-4.8\rangle^0$

$M = 50x - 16.67\langle x-1.2\rangle^1 - 100\langle x-2.4\rangle^0 + 116.67\langle x-4.8\rangle^0$

At A. $x = 0,\quad M = 0$

At B. $x = 1.2\ m,\quad M = (50)(1.2) = 60\ kNm$

At C. $x = 2.4\ m,\quad M = (50)(2.4) - (16.67)(1.2) = 100\ kNm$

At D. $x = 4.8\ m,\quad M = (50)(4.8) - (16.67)(3.6) - (100)(2.4) = -60\ kNm$

At E. $x = 6.0\ m,\quad M = (50)(6) - (16.67)(4.8) - (100)(3.6) + (116.67)(1.2) = 0$ (checks)

(a) $|M|_{max} = 100\ kN\ m$ at C

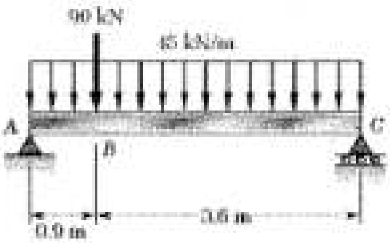
(b) $|M|_{max} = 100\ kNm \qquad \sigma_{all} = 165\ MPa$

$\sigma = \dfrac{|M|}{S} \qquad S_{min} = \dfrac{|M|_{max}}{\sigma} = \dfrac{100\,000}{165 \times 10^6} = 606.06 \times 10^{-6}\ m^3 = 606.06 \times 10^3\ mm^3$

From Appendix C, lightest wide flange shape is

W410 × 38.8

Problem 5.115



5.114 and 5.115 A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 165 MPa, find the most economical wide-flange shape that can be used.

$\circlearrowleft \Sigma M_C = 0 \quad -4.5 R_A + (45)(4.5)(2.25) + (90)(3.6) = 0$

$R_A = 173.25 \text{ kN}$

$w = 45 \text{ kN/m} = -\frac{dV}{dx}$

$V = 173.25 - 45x - 90\langle x - 0.9\rangle^0 \text{ kN}.$

Location of point D where $V = 0$. Assume $0.9 < x_D < 4.5$

$0 = 173.25 - 45 x_D - 90 \qquad x_D = 1.85 \text{ m}$

$M = 173.25x - 22.5x^2 - 90\langle x - 0.9\rangle^1 \text{ kNm}.$

At point D $(x = 1.85 \text{ m})$ $\quad M = (173.25)(1.85) - (22.5)(1.85)^2 - (90)(0.95)$

$= 158 \text{ kNm}$

(a) Maximum $|M| = 158 \text{ kNm}$ at $x = 1.85$ m.

$S_{min} = \frac{M}{\sigma_{all}} = \frac{158 \times 10^3}{165 \times 10^6} = 957.6 \times 10^{-6} \text{ m}^3 = 957.6 \times 10^3 \text{ mm}^3$

(b)

Shape	$S (\times 10^3 \text{ mm}^3)$
W530 × 66	1340
W460 × 74	1460
W410 × 60	1060 ←
W360 × 64	1030
W310 × 74	1060
W250 × 101	1240

Answer W410 × 60.

Problem 5.117

5.116 and 5.117 A timber beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth *h* varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

500 N/m = 0.5 kN/m

$+\circlearrowleft \Sigma M_C = 0:$

$-4R_A + (3.2)(1.6)(0.5) + (1.6)(\frac{1}{2})(2.4)(0.5) = 0$

$R_A = 0.880 \text{ kN} \uparrow$

$$w = 0.5 - \frac{0.5}{2.4}\langle x-1.6\rangle^1 = 0.5 - 0.20833\langle x-1.6\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.880 - 0.5x + 0.104167\langle x-1.6\rangle^2 \text{ kN}$$

$V_A = 0.880$ kN

$V_B = 0.880 - (0.5)(1.6) = 0.080$ kN

$V_C = 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520$ kN

(Sign change between V_B and V_C)

Locate point D (between B and C) where V = 0.

$$0 = 0.880 - 0.5x_D + 0.104167(x_D - 1.6)^2$$

$$0.104167x_D^2 - 0.83333x_D + 1.14667 = 0$$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)}$$

$$= 4.0 \pm 2.2342 = \cancel{6.2342}, \quad 1.7658 \text{ m}$$

$$M = 0.880x - 0.25x^2 + 0.347222\langle x-1.6\rangle^3 \text{ kN·m}$$

$$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.34722)(0.1658)^3 = 0.776 \text{ kN·m}$$

(a) $M_{max} = 0.776$ kN·m at $x = 1.7658$ m

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{0.776\times10^3}{12\times10^6} = 64.66\times10^{-6}\text{ m}^3 = 64.66\times10^3\text{ mm}^3$$

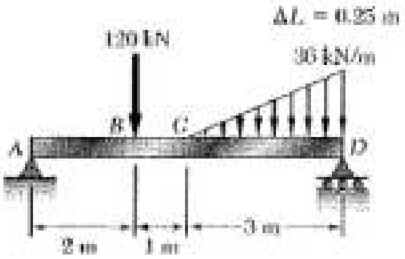
For a rectangular cross section, $S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}}$

$$h_{min} = \sqrt{\frac{(6)(64.66\times10^3)}{30}} = 113.7 \text{ mm}$$

(b) At next higher 10-mm increment $h = 120$ mm

Problem 5.119

5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.



$+\circlearrowleft \Sigma M_C = 0:$

$-6\,R_A + (4)(120) + (1)(\tfrac{1}{2})(3)(36) = 0$

$R_A = 89$ kN

$w = \frac{36}{3}\langle x-3\rangle^1 = 12\langle x-3\rangle^1$

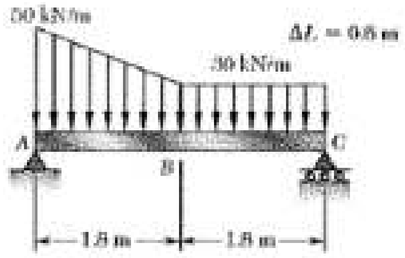
$V = 89 - 120\langle x-2\rangle^0 - 6\langle x-3\rangle^2$ kN ◄

$M = 89x - 120\langle x-2\rangle^1 - 2\langle x-3\rangle^3$ kN·m ◄

x m	V kN	M kN·m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	-31.0	178.0
2.3	-31.0	170.3
2.5	-31.0	162.5
2.8	-31.0	154.8
3.0	-31.0	147.0
3.3	-31.4	139.2
3.5	-32.5	131.3
3.8	-34.4	122.9
4.0	-37.0	114.0
4.3	-40.4	104.3
4.5	-44.5	93.8
4.8	-49.4	82.0
5.0	-55.0	69.0
5.3	-61.4	54.5
5.5	-68.5	38.3
5.8	-76.4	20.2
6.0	-85.0	-0.0

Problem 5.120

5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.



$\circlearrowleft \Sigma M_C = 0$

$-3.6 R_A + (30)(3.6)(1.8) + (20)(1.8)(\frac{1}{2})(3) = 0$

$R_A = 69 \text{ kN}$

$w = 50 - \frac{20}{1.8}x + \frac{20}{1.8}\langle x-1.8\rangle^1$

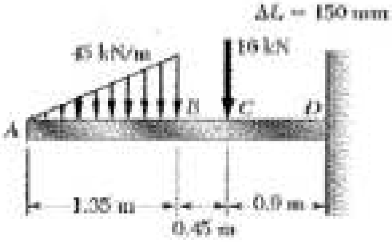
$= 50 - 11.11x + 11.11\langle x-1.8\rangle^1$

$V = 69 - 50x + 5.56x^2 - 5.56\langle x-1.8\rangle^2$ kN ◄

$M = 69x - 25x^2 + 1.852x^3 - 1.852\langle x-1.8\rangle^3$ kNm ◄

x m	V kN	M kNm.
0	69	0
0.6	41	32.8
1.2	17	50
1.8	−3	54
2.4	−20	46.8
3.0	−39	28.8
3.6	−57	0

Problem 5.121



5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

$$w = \frac{45}{1.35}x - 45\langle x-1.35\rangle^0 - \frac{45}{1.35}\langle x-1.35\rangle^1$$

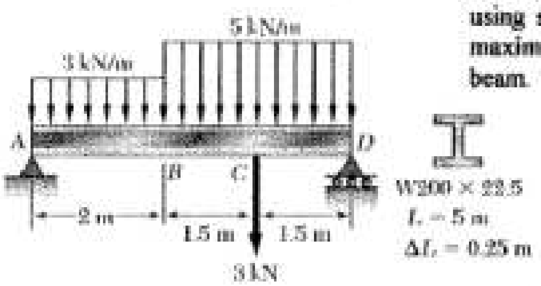
$$= 33.33x - 45\langle x-1.35\rangle^0 - 33.33\langle x-1.35\rangle^1 = -\frac{dV}{dx}$$

$$V = -16.67x^2 + 45\langle x-1.35\rangle^1 + 16.67\langle x-1.35\rangle^2 - 16\langle x-1.8\rangle^0$$

$$M = -5.56x^3 + 22.5\langle x-1.35\rangle^2 + 5.56\langle x-1.35\rangle^3 - 16\langle x-1.8\rangle^1$$

x m	V kN	M kNm.
0	0	0
0.6	−5.32	−1.2
1.35	−27	−13.74
2.1	−43	−42.04
2.7	−43	−71.19.

Problem 5.122



5.122 and 5.123 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

$\circlearrowleft \Sigma M_D = 0:$

$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$

$R_A = 10.2 \text{ kN}$

$w = 3 + 2\langle x-2 \rangle^0 \text{ kN/m} = -\frac{dV}{dx}$

$V = 10.2 - 3x - 2\langle x-2\rangle^1 - 3\langle x-3.5\rangle^0 \quad \text{kN}$ ◄

$M = 10.2x - 1.5x^2 - \langle x-2\rangle^2 - 3\langle x-3.5\rangle^1 \quad \text{kN·m}$ ◄

x m	V kN	M kN·m	sigma MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.72	24.4
0.75	7.95	6.81	35.1
1.00	7.20	8.70	44.8
1.25	6.45	10.41	53.6
1.50	5.70	11.92	61.5
1.75	4.95	13.26	68.3
2.00	4.20	14.40	74.2
2.25	2.95	15.29	78.8
2.50	1.70	15.88	81.8
2.75	0.45	16.14	83.2
3.00	-0.80	16.10	83.0
3.25	-2.05	15.74	81.2
3.50	-6.30	15.07	77.7
3.75	-7.55	13.34	68.8
4.00	-8.80	11.30	58.2
4.25	-10.05	8.94	46.1
4.50	-11.30	6.27	32.3
4.75	-12.55	3.29	17.0
5.00	-13.80	-0.00	-0.0
2.83	0.05	16.164	83.3
2.84	0.00	16.164	83.3 ←
2.85	-0.05	16.164	83.3

For rolled steel section W 200 × 22.5,

$S = 194 \times 10^3 \text{ mm}^3$

$= 194 \times 10^{-6} \text{ m}^3$

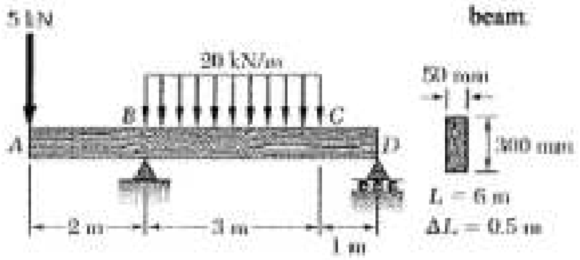
$\sigma_{max} = \frac{M_{max}}{S} = \frac{16.164 \times 10^3}{194 \times 10^{-6}}$

$= 83.3 \times 10^6 \text{ Pa}$

$= 83.3 \text{ MPa}$ ◄

Problem 5.123

5.122 and 5.123 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.



$+\circlearrowleft \Sigma M_D = 0:$

$-4R_B + (6)(5) + (2.5)(3)(20) = 0$

$R_B = 45 \text{ kN}$

$w = 20\langle x-2\rangle^0 - 20\langle x-5\rangle^0 \text{ kN/m} = -\dfrac{dV}{dx}$

$V = -5 + 45\langle x-2\rangle^0 - 20\langle x-2\rangle^1 + 20\langle x-5\rangle^1 \text{ kN}$ ◄

$M = -5x + 45\langle x-2\rangle^1 - 10\langle x-2\rangle^2 + 10\langle x-5\rangle^2 \text{ kN·m}$ ◄

x m	V kN	M kN·m	sigma MPa
0.00	-5	0.00	0.0
0.50	-5	-2.50	-3.3
1.00	-5	-5.00	-6.7
1.50	-5	-7.50	-10.0
2.00	40	-10.00	-13.3
2.50	30	7.50	10.0
3.00	20	20.00	26.7
3.50	10	27.50	36.7
4.00	0	30.00	40.0 ←
4.50	-10	27.50	36.7
5.00	-20	20.00	26.7
5.50	-20	10.00	13.3
6.00	-20	0.00	0.0

Maximum $|M| = 30 \text{ kN·m}$ at $x = 4.0 \text{ m}$

For rectangular cross section

$S = \frac{1}{6}bh^2 = (\frac{1}{6})(50)(300)^2$

$= 750 \times 10^3 \text{ mm}^3$

$= 750 \times 10^{-6} \text{ m}^3$

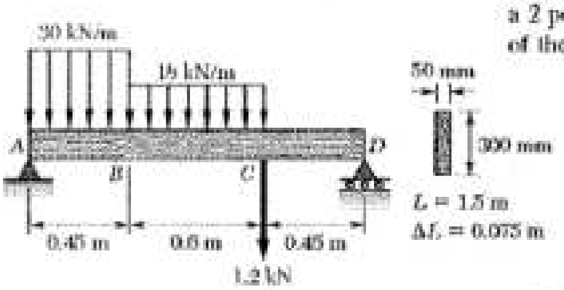
$\sigma_{max} = \dfrac{M_{max}}{S} = \dfrac{30 \times 10^3}{750 \times 10^{-6}}$

$= 40 \times 10^6 \text{ Pa}$

$\sigma_{max} = 40.0 \text{ MPa}$ ◄

Problem 5.124

5.124 and 5.125 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.



$$+\circlearrowleft \Sigma M_D = 0$$

$$-1.5R_A + (30)(0.45)(1.275) + (18)(0.6)(0.75) + (1.2)(0.45) = 0$$

$$R_A = 17.235 \text{ kN.}$$

$$w = 30 - 12\langle x - 0.45\rangle^0 - 18\langle x - 1.05\rangle^0 \text{ kN/m.}$$

$$V = 17.235 - 30x + 12\langle x - 0.45\rangle^1 + 18\langle x - 1.05\rangle^1 - 1.2\langle x - 1.05\rangle^0 \text{ kN}$$

$$M = 17.235x - 15x^2 + 6\langle x - 0.45\rangle^2 + 9\langle x - 1.05\rangle^2 - 1.2\langle x - 1.05\rangle^1 \text{ kNm}$$

x m	V kN	M kNm	sigma MPa
0	15.36	0	0
0.375	5.36	4.39	5.66
0.66	0	5.15	6.66
0.75	−1.44	5.09	6.56
1.125	−7.44	3.15	4.07
1.5	−7.44	0	0

Maximum $|M| = 5.15$ kNm

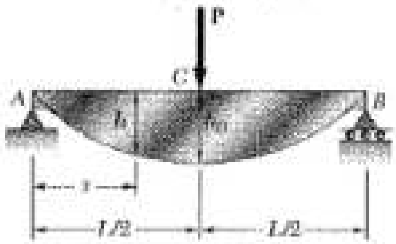
at $x = 0.66$m

Rectangular section
50 × 300 mm

$$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(50)(300)^2 = 750000 \text{ mm}^3$$

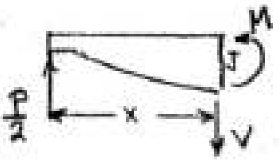
$$\sigma = \frac{M}{S} = \frac{5.15 \times 10^3}{750000 \times 10^{-9}} = 6.87 \text{ MPa.}$$

Problem 5.128



5.128 and 5.129 The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

$R_A = R_B = \frac{P}{2}\uparrow$



$+\circlearrowleft \Sigma M_J = 0:$

$-\frac{P}{2}x + M = 0$

$M = \frac{Px}{2} \qquad (0 < x < \frac{L}{2})$

$S = \frac{M}{\sigma_{all}} = \frac{Px}{2\sigma_{all}}$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{Px}{2\sigma_{all}} \qquad h = \sqrt{\frac{3Px}{\sigma_{all}b}}$

(a) At $x = \frac{L}{2}$, $h = h_0 = \sqrt{\frac{3PL}{2\sigma_{all}b}} \qquad h = h_0\sqrt{\frac{2x}{L}}, \quad 0 < x < \frac{L}{2}$ ◄

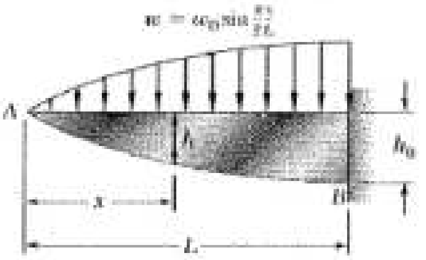
For $x > \frac{L}{2}$, replace x by $L - x$.

(b) Solving for P, $P = \frac{2\sigma_{all}bh_0^2}{3L} = \frac{(2)(72\times10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60\times10^3$ N

$P = 60$ kN ◄

Problem 5.130

5.130 **and 5.131** The beam *AB*, consisting of a cast-iron plate of uniform thickness *b* and length *L*, is to support the distributed load *w*(*x*) shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.



$$\frac{dV}{dx} = -w = -w_0 \sin\frac{\pi x}{2L}$$

$$V = \frac{2w_0 L}{\pi}\cos\frac{\pi x}{2L} + C_1$$

$$V = 0 \text{ at } x = 0 \quad \rightarrow \quad C_1 = -\frac{2w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi}\left(1 - \cos\frac{\pi x}{2L}\right)$$

$$M = -\frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right) \qquad |M| = \frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{L}\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{2w_0 L}{\pi\sigma_{all}}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating $\quad \frac{1}{6}bh^2 = \frac{2w_0 L}{\pi\sigma_{all}}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)$

$$h = \left\{\frac{12 w_0 L}{\pi\sigma_{all} b}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)\right\}^{1/2}$$

At $x = L \qquad h = h_0 = \left\{\frac{12 w_0 L^2}{\pi\sigma_{all} b}\left(1 - \frac{2}{\pi}\right)\right\}^{1/2} = 1.178\sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$

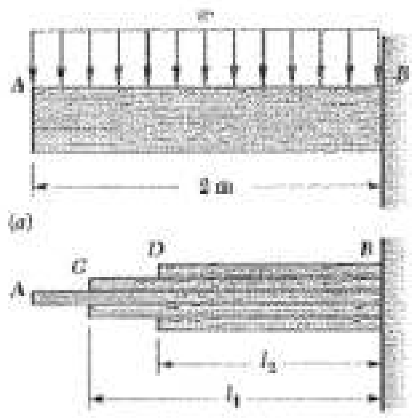
(a) $\quad h = h_0\left[\left(\frac{x}{L} - \frac{2}{\pi}\sin\frac{\pi x}{2L}\right) \Big/ \left(1 - \frac{2}{\pi}\right)\right]^{1/2} = 1.659\, h_0\left[\frac{x}{L} - \frac{2}{\pi}\sin\frac{\pi x}{2L}\right]^{1/2}$ ◄

Data: $\quad L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$

$\qquad w_0 = 300 \text{ kN/m} = 300\times10^3 \text{ N/m}, \quad \sigma_{all} = 200 \text{ MPa} = 200\times10^6 \text{ Pa}$

(b) $\quad h_0 = 1.178\sqrt{\frac{(300\times10^3)(0.75)^2}{(200\times10^6)(0.030)}} = 197.6\times10^{-3} \text{ m} \qquad h_0 = 197.6 \text{ mm}$ ◄

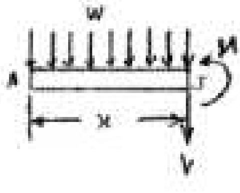
Problem 5.133



(a)

(b)

5.132 and 5.133 A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 250 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 50 × 250 mm cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the factor of safety as the original design.



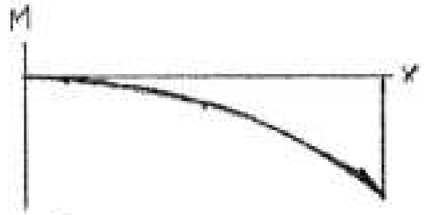
$\circlearrowright \Sigma M_J = 0 \quad wx\frac{x}{2} + M = 0$

$M = -\frac{wx^2}{2} \qquad |M| = \frac{wx^2}{2}$

At B $\quad |M|_B = |M|_{max}$

At C $\quad |M|_C = |M|_{max}(x_C/2)^2$

At D $\quad |M|_D = |M|_{max}(x_D/2)^2$



At B $\quad S_B = \frac{1}{6}bh^2 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$

A to C $\quad S_C = \frac{1}{6}bh^2 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$

C to D $\quad S_D = \frac{1}{6}bh^2 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

$\frac{|M|_C}{|M|_B} = \left(\frac{x_C}{2}\right)^2 = \frac{S_C}{S_B} = \frac{1}{25}$

$x_C = \frac{2}{\sqrt{25}} = 0.4\ m$

$l_1 = 2 - 0.4 = 1.6\ m$

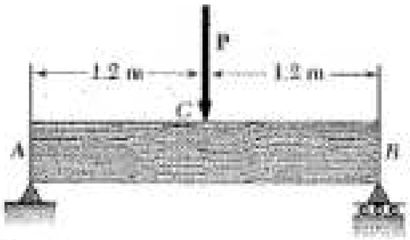
$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{2}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25}$

$x_D = \frac{2\sqrt{9}}{\sqrt{25}} = 1.2\ m$

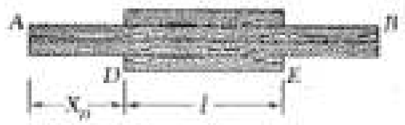
$l_2 = 2 - 1.2 = 0.8\ m$

Problem 5.134

5.134 and 5.135 A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.



(a)



(b)

$$R_A = R_B = \frac{P}{2}$$

$0 < x < \frac{L}{2}$

$\circlearrowright \Sigma M_J = 0: \quad -\frac{P}{2}x + M = 0$

$M = \frac{Px}{2}$ or $M = \frac{M_{max}x}{1.2}$

Bending moment diagram is two straight lines.

At C $\quad S_c = \frac{1}{6}bh_c^2 \quad M_c = M_{max}$

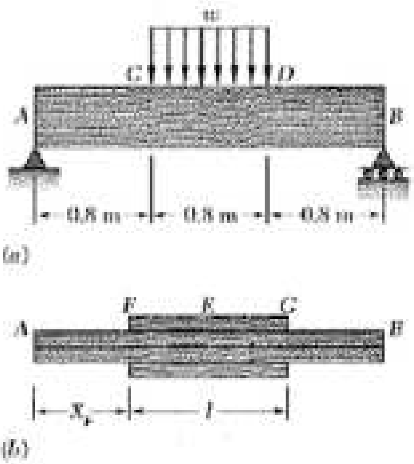
At D $\quad S_D = \frac{1}{6}bh_D^2 \quad M_D = \frac{M_{max}x_D}{1.2}$

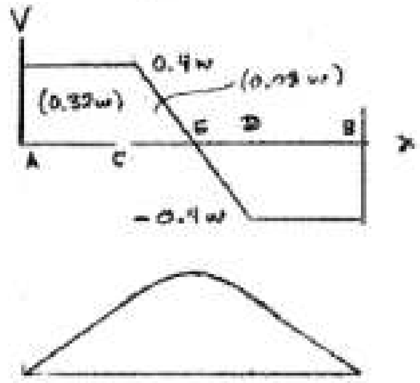
$$\frac{S_D}{S_c} = \frac{h_D^2}{h_c^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_D}{M_c} = \frac{x_D}{1.2} \qquad x_D = 0.3\text{ m}$$

$$\frac{l}{2} = 1.2 - x_D = 0.9 \qquad l = 1.800\text{ m}$$

Problem 5.135

5.134 and 5.135 A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.





$$R_A = R_B = \frac{0.8\,w}{2} = 0.4\,w$$

Shear: A to C $\quad V = 0.4\,w$

D to B $\quad V = -0.4\,w$

Areas: A to C $\quad (0.8)(0.4)w = 0.32\,w$

C to E $\quad (\tfrac{1}{2})(0.4)(0.4)w = 0.08\,w$

Bending moments.

At C $\quad M_C = 0.40\,w$

A to C $\quad M = 0.40\,wx$

At C $\quad S_c = \frac{1}{6}bh_c^2 \qquad M_c = M_{max} = 0.40\,w$

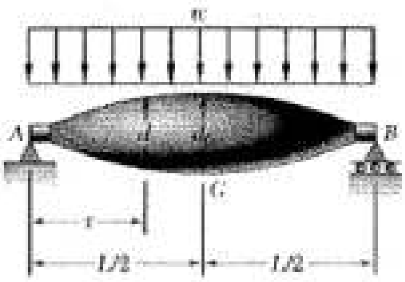
At F $\quad S_F = \frac{1}{6}bh_F^2 \qquad M_F = 0.40\,wx_F$

$$\frac{S_F}{S_c} = \frac{h_F^2}{h_c^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_F}{M_c} = \frac{0.40\,wx_F}{0.40\,w}$$

$x_F = 0.25\text{ m} \qquad \frac{l}{2} = 1.2 - x_F = 0.95\text{ m}$

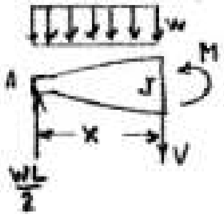
$l = 1.900\text{ m}$ ◀

Problem 5.137



5.136 and 5.137 A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

$$R_A = R_B = \frac{wL}{2}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

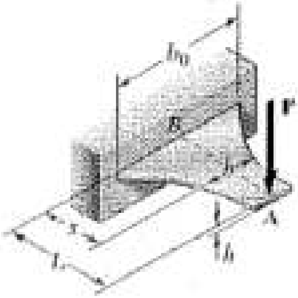
For a solid circular cross section $c = \frac{d}{2}$ $\quad I = \frac{\pi}{4}c^3$ $\quad S = \frac{I}{c} = \frac{\pi d^3}{32}$

Equating, $\frac{\pi d^3}{32} = \frac{wx(L-x)}{2\sigma_{all}}$ $\qquad d = \left\{\frac{16\,wx(L-x)}{\pi\sigma_{all}}\right\}^{1/3}$

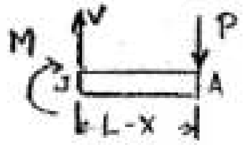
At $x = \frac{L}{2}$, $\quad d = d_0 = \left\{\frac{4wL^2}{\pi\sigma_{all}}\right\}^{1/3}$ $\qquad d = d_0\left\{4\frac{x}{L}\left(1-\frac{x}{L}\right)\right\}^{1/3}$ ◄

Problem 5.138

5.138 A cantilever beam *AB* consisting of a steel plate of uniform depth *h* and variable width *b* is to support the concentrated load **P** at point *A*. (*a*) Knowing that the beam is to be of constant strength, express *b* in terms of *x*, *L*, and b_0. (*b*) Determine the smallest allowable value of *h* if *L* = 300 mm, b_0 = 375 mm, *P* = 14.4 kN, and σ_{all} = 160 MPa.



$\circlearrowleft \Sigma M_J = 0: \quad -M - P(L-x) = 0 \qquad M = -P(L-x)$



$|M| = P(L-x)$

$S = \dfrac{|M|}{\sigma_{all}} = \dfrac{P(L-x)}{\sigma_{all}}$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \dfrac{P(L-x)}{\sigma_{all}} \qquad b = \dfrac{6P(L-x)}{\sigma_{all}h^2}$

At $x = 0$, $\quad b = b_0 = \dfrac{6PL}{\sigma_{all}h^2} \qquad b = b_0\left(1 - \dfrac{x}{L}\right)$ ◀

Solving for h, $\quad h = \sqrt{\dfrac{6PL}{\sigma_{all}b_0}}$

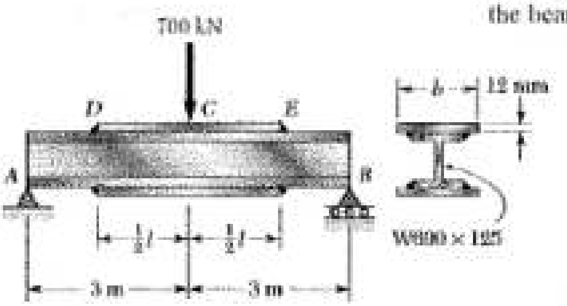
Data: $L = 300\text{ mm} = 0.300\text{ m}, \quad b_0 = 375\text{ mm} = 0.375\text{ m}$

$P = 14.4\text{ kN} = 14.4\times10^3\text{ N}\cdot\text{m} \qquad \sigma_{all} = 160\text{ MPa} = 160\times10^6\text{ Pa}$

$h = \sqrt{\dfrac{(6)(14.4\times10^3)(0.300)}{(160\times10^6)(0.375)}} = 20.8\times10^{-3}\text{ m} \qquad h = 20.8\text{ mm}$ ◀

Problem 5.141

5.141 Two cover plates, each 12 mm thick, are welded to a W690 × 125 beam as shown. Knowing that l = 3 m and b = 260 mm, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.

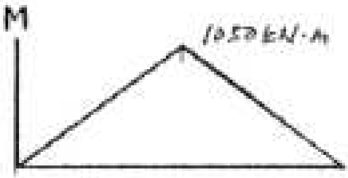


$R_A = R_B = 350\ kN$

$+\circlearrowleft \Sigma M_J = 0$

$-350x + M = 0$

$M = 350x \ kN \cdot m.$



At C $\quad x = 3m \quad M_C = 1050\ kNm$

At D $\quad x = 1.5\ m$

$M_D = 525\ kNm$

At center of beam $\quad I = I_{beam} + 2 I_{plate}$

$$I = (1190 \times 10^6) + 2\left\{(12)(260)\left(6 + \frac{678}{2}\right)^2 + \frac{1}{12}(260)(12)^3\right\}$$

$$= 1932.8 \times 10^6\ mm^4$$

$$c = \left(\frac{678}{2}\right) + 12 = 351\ mm.$$

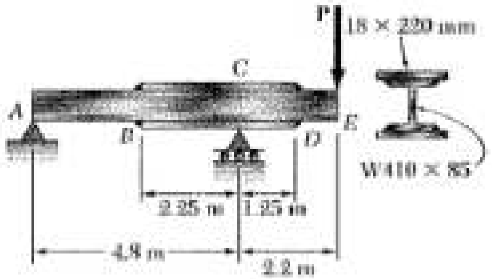
(a) Normal stress $\quad \sigma = \frac{Mc}{I} = \frac{(1050 \times 10^3)(0.351)}{1932.8 \times 10^{-6}} = 190.7\ MPa$ ◄

At point D $\quad S = 3510 \times 10^3\ mm^3 = 3510 \times 10^{-6}\ m^3.$

(b) Normal stress $\quad \sigma = \frac{M}{S} = \frac{525 \times 10^3}{3510 \times 10^{-6}} = 149.6\ MPa$ ◄

Problem 5.143

5.143 Knowing that $\sigma_{all} = 150$ MPa, determine the largest concentrated load **P** that can be applied at end *E* of the beam shown.



$+\circlearrowleft \Sigma M_C = 0: \; -4.8A - 2.2P = 0$

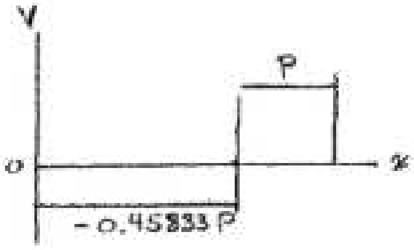
$A = -0.45833\,P \qquad A = 0.45833\,P \downarrow$

$+\circlearrowleft \Sigma M_A = 0: \; 4.8D - 7.0P = 0$

$D = 1.45833\,P \uparrow$

Shear: A to C. $\quad V = -0.45833\,P$

C to E. $\quad V = P$



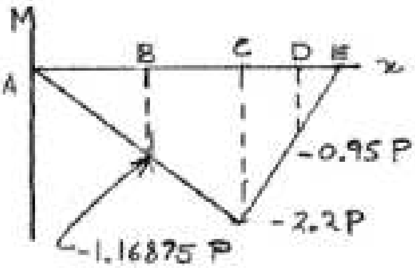
Bending moments: $M_A = 0$

$M_C = 0 + (4.8)(-0.45833\,P) = -2.2P$

$M_E = -2.2P + 2.2P = 0$

$M_B = \left(\frac{4.8 - 2.25}{4.8}\right)(-2.2P) = -1.16875\,P$

$M_D = \left(\frac{2.2 - 1.25}{2.2}\right)(-2.2P) = -0.95\,P$



$|M_D| < |M_B|$

For W410 × 85, $\quad S = 1510 \times 10^3\ \text{mm}^3 = 1510 \times 10^{-6}\ \text{m}^3$

Allowable value of P based on strength at B. $\quad \sigma = \frac{|M_B|}{S}$

$150 \times 10^6 = \frac{1.16875\,P}{1510 \times 10^{-6}} \qquad P = 193.8 \times 10^3\ \text{N}$

Section properties over portion BCD:

W410 × 85: $\quad d = 417$ mm $\quad \frac{1}{2}d = 208.5$ mm $\quad I_x = 315 \times 10^6\ \text{mm}^4$

Plate: $A = (18)(220) = 3960\ \text{mm}^2 \qquad d = 208.5 + (\frac{1}{2})(18) = 217.5$ mm

$= \frac{1}{12}(220)(18)^3 = 106.92 \times 10^3\ \text{mm}^4 \qquad Ad^2 = 187.333 \times 10^6\ \text{mm}^4$

$= \bar{I} + Ad^2 = 187.440 \times 10^6\ \text{mm}^4$

For section: $I = 315 \times 10^6 + (2)(187.440 \times 10^6) = 689.88 \times 10^6\ \text{mm}^4$

$c = 208.5 + 18 = 226.5$ mm

$S = \frac{I}{c} = \frac{689.88 \times 10^6}{226.5} = 3045.8 \times 10^3\ \text{mm}^3 = 3045.8 \times 10^{-6}\ \text{m}^3$

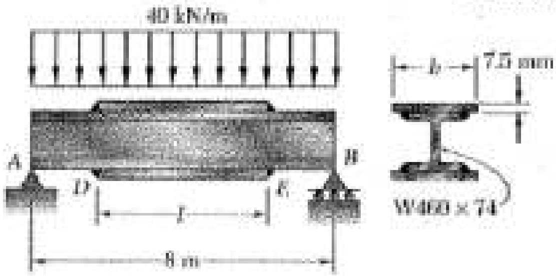
Allowable load based on strength at C. $\quad \sigma = \frac{|M_C|}{S}$

$150 \times 10^6 = \frac{2.2\,P}{3045.8 \times 10^{-6}} \qquad P = 207.7 \times 10^3\ \text{N}$

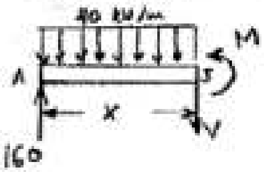
The smaller allowable load controls. $\quad P = 193.8 \times 10^3\ \text{N} = 193.8\ \text{kN}$ ◄

Problem 5.144

5.144 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $l = 5$ m and $b = 200$ mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



$R_A = R_B = 160 \text{ kN} \uparrow$



$+\circlearrowleft \Sigma M_J = 0:$

$-160x + (40x)\frac{x}{2} + M = 0$

$M = 160x - 20x^2 \text{ kN}\cdot\text{m}$

At center of beam $x = 4\text{m}$ $\quad M_c = 320 \text{ kN}\cdot\text{m}$

At D $\quad x = \frac{1}{2}(8 - l) = 1.5\text{ m}$ $\quad M_D = 195 \text{ kN}\cdot\text{m}$

At center of beam

$$I = I_{beam} + 2I_{plate}$$

$$= 333 \times 10^6 + 2\left\{(200)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(200)(7.5)^3\right\}$$

$$= 494.8 \times 10^6 \text{ mm}^4$$

$c = \frac{457}{2} + 7.5 = 236 \text{ mm}$ $\qquad S = \frac{I}{c} = 2097 \times 10^3 \text{ mm}^3 = 2097 \times 10^{-6} \text{ m}^3$

(a) Normal stress $\quad \sigma = \frac{M}{S} = \frac{320 \times 10^3}{2097 \times 10^{-6}} = 152.6 \times 10^6 \text{ Pa}$

$\sigma = 152.6 \text{ MPa}$ ◄

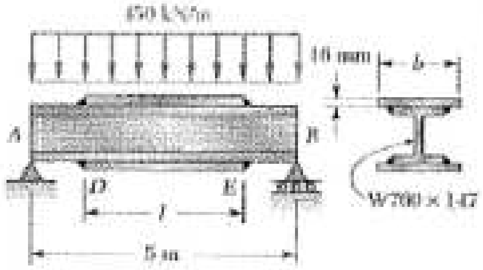
At D $\quad S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$

(b) Normal stress $\quad \sigma = \frac{M}{S} = \frac{195 \times 10^3}{1460 \times 10^{-6}} = 133.6 \times 10^6 \text{ Pa}$

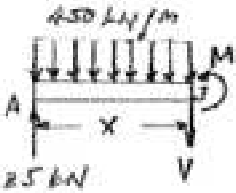
$\sigma = 133.6 \text{ MPa}$ ◄

Problem 5.146

5.146 Two cover plates, each 16 mm thick, are welded to a W760 × 147 beam as shown. Knowing that $l = 3$ m and $b = 300$ mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



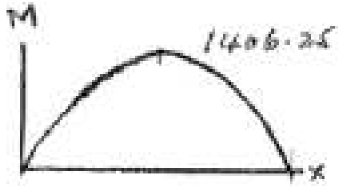
$R_A = R_B = 1125$ kN



$\circlearrowright \Sigma M_J = 0$

$-1125x + 450x \cdot \frac{x}{2} + M = 0$

$M = 1125x - 225x^2$ kNm.



At center of beam $x = 2.5$ m

$M_c = 1406.25$ kNm.

At point D, $x = \frac{1}{2}(5-3) = 1.0$ m

$M_D = 900$ kNm.

At center of beam $I = I_{beam} + 2I_{plate}$

$$I = (1660 \times 10^6) + 2\left\{(16)(300)\left(8 + \frac{753}{2}\right)^2 + \frac{1}{12}(300)(16)^3\right\} = 3.07926 \times 10^9 \text{ mm}^4$$

$c = \left(\frac{753}{2}\right) + 16 = 392.5$ mm

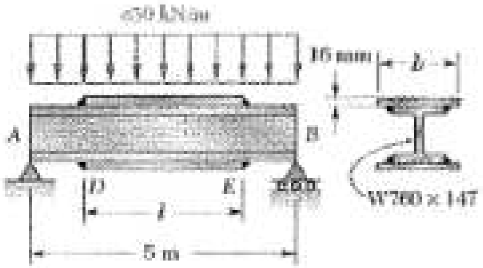
(a) Normal stress $\sigma = \dfrac{Mc}{I} = \dfrac{(1406.25 \times 10^3)(0.3925)}{3.07926 \times 10^{-3}} = 179.2$ MPa. ◄

At point D $S = 4410 \times 10^3$ mm^3.

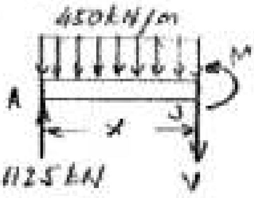
(b) Normal stress $\sigma = \dfrac{M}{S} = \dfrac{900 \times 10^3}{4410 \times 10^{-6}} = 204.1$ MPa. ◄

Problem 5.147

5.147 Two cover plates, each 16 mm thick, are welded to a W760 × 147 beam as shown. Knowing that $\sigma_{all} = 150$ MPa for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.



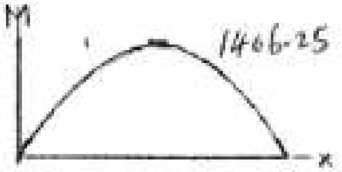
$R_A = R_B = 1125$ kN.



$\circlearrowright \Sigma M_J = 0$

$-1125\,x + 450\,x\,\frac{x}{2} + M = 0$

$M = 1125\,x - 225x^2$ kN·m



For W760×147 rolled steel section

$S = 4410 \times 10^3$ mm^3.

Allowable bending moment

$M_{all} = \sigma_{all} S = (150 \times 10^6)(4410 \times 10^{-6})$

$= 661.5$ kN·m

To locate points D and E, set $M = M_{all}$

$1125\,x - 225x^2 = 661.5 \qquad 225x^2 - 1125x + 661.5 = 0$

$$x = \frac{1125 \pm [(1125)^2 - (4)(225)(661.5)]^{1/2}}{(2)(225)} = 4.32 \text{ m}, \quad 0.68 \text{ m}.$$

(a) $\qquad l = x_E - x_D = 4.32 - 0.68 = 3.64$ m ◀

Center of beam $\qquad M = 1406.25$ kN·m

$$S = \frac{M}{\sigma_{all}} = \frac{1406.25 \times 10^3}{150 \times 10^6} = 9.375 \times 10^{-3}\ \text{m}^3 = 9.375 \times 10^6\ \text{mm}^3.$$

$c = 16 + \frac{753}{2} = 392.5$ mm.

Required moment of inertia $\qquad I = Sc = 3679.7 \times 10^6$ mm^4.

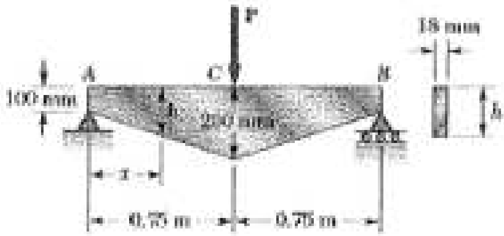
But $\quad I = I_{beam} + 2 I_{plate}$

$$3679.7 \times 10^6 = (1660 \times 10^6) + 2\left\{(b)(16)\left(8 + \frac{753}{2}\right)^2 + \frac{1}{12}(b)(16)^3\right\}$$

$$= 1660 \times 10^6 + 4.7316 \times 10^6\, b$$

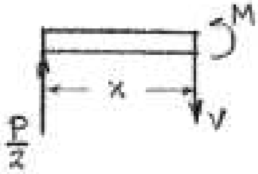
(b) $\qquad b = 426.9$ mm. ◀

Problem 5.148



5.148 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest concentrated load **P** that can be applied, knowing that $\sigma_{all} = 165$ MPa.

$$R_A = R_B = \frac{P}{2}$$



$$\circlearrowleft \Sigma M_J = 0$$

$$-\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \qquad \left(0 < x < \frac{L}{2}\right)$$

For a tapered beam $\quad h = a + kx$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\,\frac{(a+kx)^2 - x\cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\,\frac{a-kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Data: $\quad a = 100\text{ mm}, \quad k = \frac{200-100}{750} = \frac{2}{15}\text{ mm/mm}$

(a) $\quad x_m = \frac{100}{2/15} = 750\text{ mm}$

$$h_m = a + kx_m = 200\text{ mm}$$

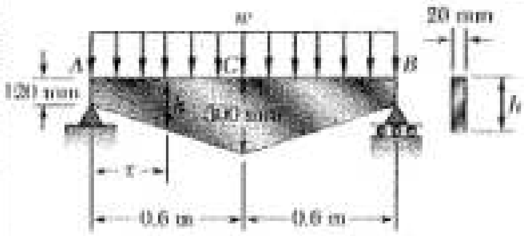
$$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)(18)(200)^2 = 120\times10^3\text{ mm}^3 = 120\times10^{-6}\text{ m}^3$$

$$M_m = \sigma_{all}S_m = (165\times10^6)(120\times10^{-6}) = 19.8\text{ kNm}$$

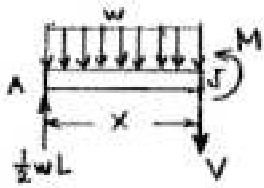
(b) $\quad P = \frac{2M_m}{x_m} = \frac{(2)(19.8)}{0.750} = 52.8\text{ kN}$

Problem 5.149

5.149 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load *w* that can be applied, knowing that $\sigma_{all} = 140$ MPa.



$R_A = R_B = \frac{1}{2}wL \uparrow \qquad L = 1.2\ \text{m}$

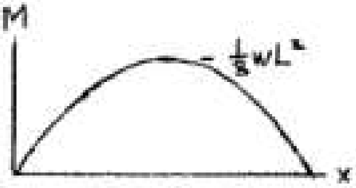


$\circlearrowleft \Sigma M_J = 0:$

$-\frac{1}{2}wL + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}x(L-x)$



For the tapered beam, $h = a + kx$

$a = 120\ \text{mm} \qquad k = \frac{300-120}{0.6} = 300\ \text{mm/m}$

For rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a+kL)x}{(a+kx)^3}\right\} = 0$$

(a) $x_m = \frac{aL}{2a+kL} = \frac{(120)(1.2)}{(2)(120)+(300)(1.2)} = 0.24\ \text{m}$ ◄

$h_m = a + kx_m = 120 + (300)(0.24) = 192\ \text{mm}$

$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88\times10^3\ \text{mm}^3 = 122.88\times10^{-6}\ \text{m}^3$

Allowable value of M_m: $M_m = S_m\sigma_{all} = (122.88\times10^{-6})(140\times10^6)$

$= 17.2032\times10^3\ \text{N}\cdot\text{m}$

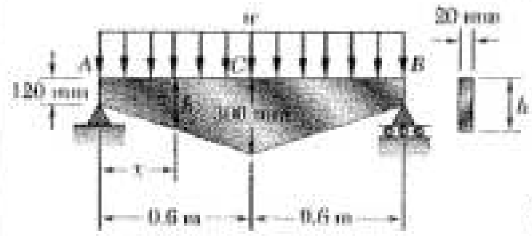
(b) Allowable value of w: $w = \frac{2M_m}{x_m(L-x_m)} = \frac{(2)(17.2032\times10^3)}{(0.24)(0.96)}$

$= 149.3\times10^3\ \text{N/m}$

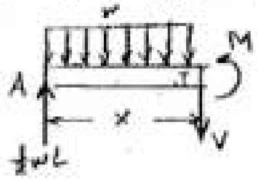
$w = 149.3\ \text{kN/m}$ ◄

Problem 5.150

5.150 For the tapered beam shown, knowing that $w = 160$ kN/m, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the corresponding value of the normal stress.



$R_A = R_B = \frac{1}{2}wL \uparrow$



$\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}x(L - x)$

where $w = 160$ kN/m and $L = 1.2$ m.

For the tapered beam, $h = a + kx$

$a = 120$ mm $\qquad k = \frac{300 - 120}{0.6} = 300$ mm/m

For a rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx - x^2)2(a+kx)k}{(a+kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx - x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - 2ax - kLx}{(a+kx)^3}\right\} = 0$$

(a) $\quad x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)}$ $\qquad x_m = 0.240$ m ◄

$h_m = a + kx_m = 120 + (300)(0.24) = 192$ mm

$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88 \times 10^3$ mm^3 $= 122.88 \times 10^{-6}$ m^3

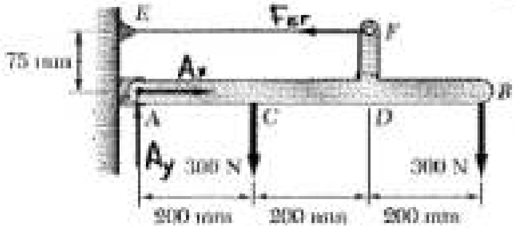
$M_m = \frac{w}{2}x_m(L - x_m) = \frac{160 \times 10^3}{2}(0.24)(0.96) = 18.432 \times 10^3$ N·m

(b) Maximum bending stress: $\quad \sigma_m = \frac{M_m}{S_m} = \frac{18.432 \times 10^3}{122.88 \times 10^{-6}} = 150 \times 10^6$ Pa

$\sigma_m = 150.0$ MPa ◄

Problem 5.153

5.153 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



$\overset{+}{\circlearrowleft}\Sigma M_A = 0:$

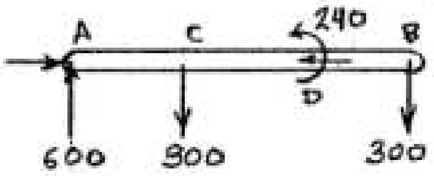
$0.075\, F_{EF} - (0.2)(300) - (0.6)(300) = 0$

$F_{EF} = 3.2\times10^3$ N

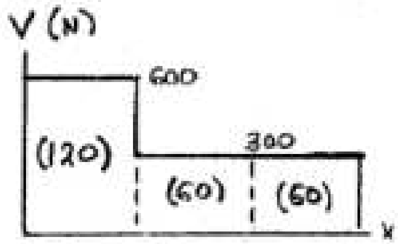
$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - F_{EF} = 0 \qquad A_x = 3.2\times10^3$ N

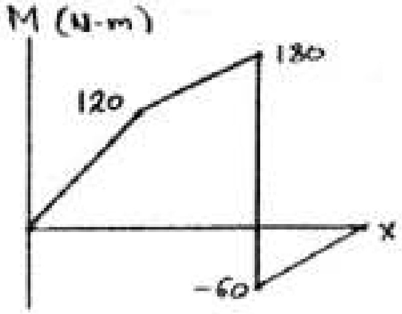
$+\uparrow \Sigma F_y = 0: \quad A_y - 300 - 300 = 0$

$A_y = 600$ N



Couple at D, $M_D = (0.075)(3.2\times10^3)$
$= 240$ N·m





Shear diagram.

A to C. $V = 600$ N

C to B. $V = 600 - 300 = 300$ N

Areas of the shear diagram.

A to C. $\int V dx = (0.2)(600) = 120$ N·m

C to D. $\int V dx = (0.2)(300) = 60$ N·m

D to B. $\int V dx = (0.2)(300) = 60$ N·m

Bending moments.

$M_A = 0$

$M_C = 0 + 120 = 120$ N·m

$M_D^- = 120 + 60 = 180$ N·m

$M_D^+ = 180 - 240 = -60$ N·m

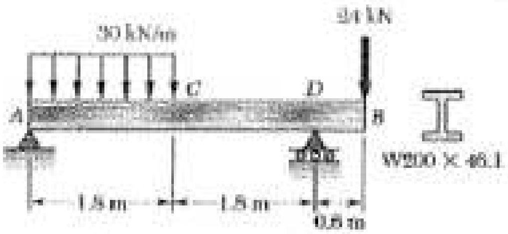
$M_B = -60 + 60 = 0$

(a) Maximum $|V| = 600$ N ◄

(b) Maximum $|M| = 180$ N·m ◄

Problem 5.154

5.154 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$+\circlearrowleft M_D = 0$

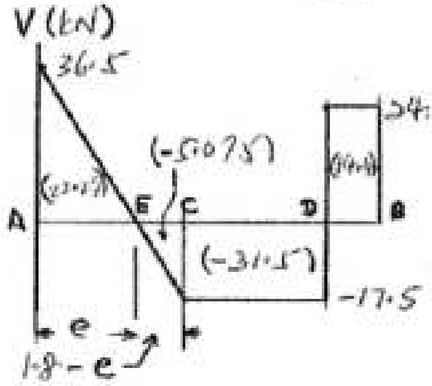
$-3.6A + (30)(1.8)(2.7) - (24)(0.6) = 0$

$A = 36.5$ kN.

$+\circlearrowleft M_A = 0$

$-(30)(1.8)(0.9) + 3.6D - (24)(4.2) = 0$

$D = 41.5$



Shear: $V_A = 36.5$ kN.

$V_C = 36.5 - (30)(1.8) = -17.5$ kN

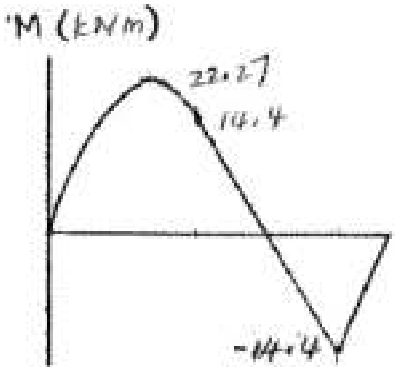
C to D $\quad V = -17.5$ kN.

D to B $\quad V = -17.5 + 41.5 = 24$ kN

Locate point E where $V = 0$

$\dfrac{e}{36.5} = \dfrac{1.8 - e}{17.5} \qquad 54e = 65.7$

$e = 1.22 \qquad 1.8 - e = 0.58$ mm.



Areas under shear diagram

A to E $\quad \int V dx = (\tfrac{1}{2})(1.22)(36.5) = 22.27$ kNm.

E to C $\quad \int V dx = (\tfrac{1}{2})-(0.58)(17.5) = -5.075$ kNm

C to D $\quad \int V dx = -(17.5)(1.8) = -31.5$ kNm.

D to B $\quad \int V dx = (24)(0.6) = 14.4$ kNm.

Bending moments: $\quad M_A = 0$

$M_E = 0 + 22.27 = 22.27$ kNm.

$M_C = 22.27 - 5.075 = 17.195$ kNm.

$M_D = 17.195 - 31.5 = -14.31$ kNm.

$M_B = -14.31 + 14.4 = 0$

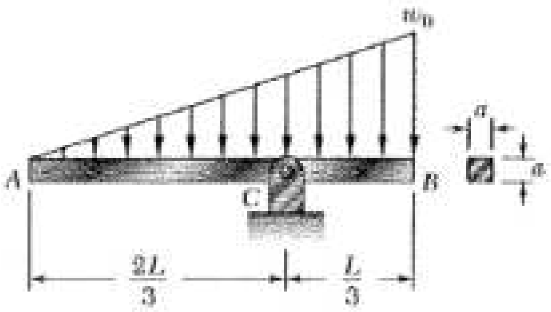
Maximum $|M| = 22.27$ kNm.

For W200×46.1 rolled steel section $\quad S = 448 \times 10^3\ mm^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{22.27 \times 10^3}{448 \times 10^{-6}} = 47.7$ MPa. ◄

Problem 5.155

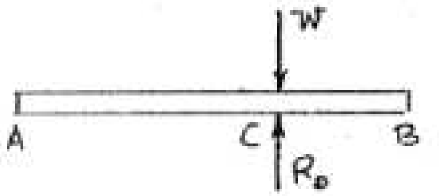
5.155 Beam AB, of length L and square cross section of side a, is supported by a pivot at C and loaded as shown. (*a*) Check that the beam is in equilibrium. (*b*) Show that the maximum stress due to bending occurs at C and is equal to $w_0L^2/(1.5a)^3$.



Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

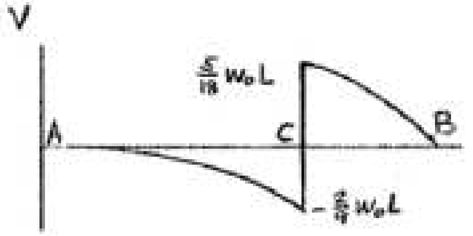
For the triangular distribution the centroid lies at $x = \frac{2L}{3}$.

$$W = \frac{1}{2} w_0 L$$



(a) $+\uparrow \Sigma F_y = 0: \quad R_D - W = 0 \quad R_D = \frac{1}{2} w_0 L$

$\circlearrowleft \Sigma M_C = 0: \quad 0 = 0 \quad$ equilibrium



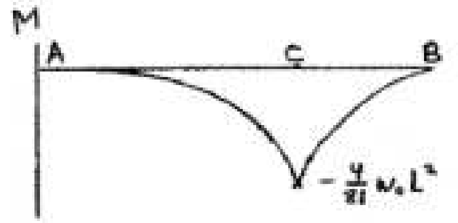
$V = 0, \; M = 0, \;$ at $x = 0$

$0 < x < \frac{2L}{3}$,

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L} + C_1 = -\frac{w_0 x^2}{2L}$$

$$M = -\frac{w_0 x^3}{6L} + C_2 = -\frac{w_0 x^3}{6L}$$



Just to the left of C,

$$V = -\frac{w_0 (2L/3)^2}{2L} = -\frac{2}{9} w_0 L$$

Just to the right of C,

$$V = -\frac{2}{9} w_0 L + R_D = \frac{5}{18} w_0 L$$

Note sign change. Maximum $|M|$ occurs at C.

$$M_C = -\frac{w_0 (2L/3)^3}{6L} = -\frac{4}{81} w_0 L^2$$

Maximum $|M| = \frac{4}{81} w_0 L^2$

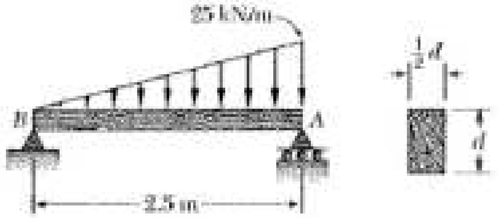
For square cross section, $\quad I = \frac{1}{12} a^4 \quad c = \frac{1}{2} a$

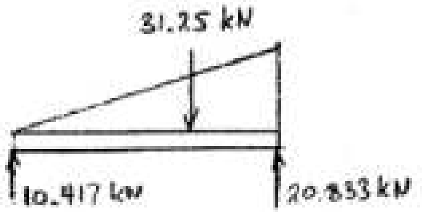
(b) $\quad \sigma_m = \frac{|M|_{max} c}{I} = \frac{4}{81} \frac{w_0 L^2 \, 6}{a^3} = \frac{8}{27} \frac{w_0 L^2}{a^3} = \left(\frac{2}{3}\right)^3 \frac{w_0 L^2}{a^3}$

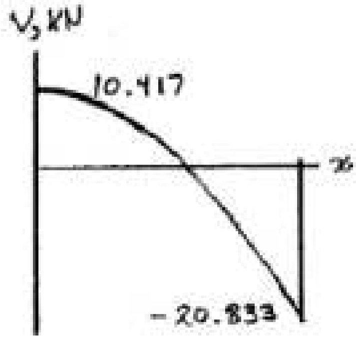
$$\sigma_m = \frac{w_0 L^2}{(1.5a)^3} \blacktriangleleft$$

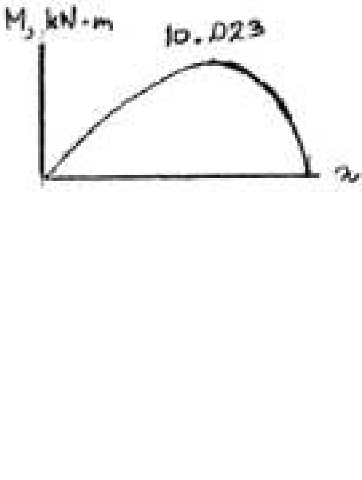
Problem 5.157

5.156 and 5.157 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.









Distributed load. $w = \frac{25}{2.5}x = 10x$ kN/m

$$\frac{dV}{dx} = -w = -10x$$

$$V = -5x^2 + C_1 = \frac{dM}{dx}$$

$$M = -\frac{5}{3}x^3 + C_1 x + C_2$$

$M = 0$ at $x = 0$. $\qquad C_2 = 0$

$M = 0$ at $x = 2.5$ m.

$$-\frac{5}{3}(2.5)^3 + C_1(2.5) = 0 \qquad C_1 = 10.417 \text{ kN}$$

$$V = -5x^2 + 10.417$$

$V = 0 \qquad -5x^2 + 10.417 = 0 \qquad x = 1.4434$ m

$$M_{max} = -\frac{5}{3}(1.4434)^3 + (10.417)(1.4434)$$

$$= 10.023 \text{ kN}\cdot\text{m} = 10.023 \times 10^3 \text{ N}\cdot\text{m}$$

Required $S = \dfrac{M_{max}}{\sigma_{all}} = \dfrac{10.023 \times 10^3}{12 \times 10^6} = 835.29 \times 10^{-6}\ \text{m}^3$

$$= 835.29 \times 10^3\ \text{mm}^3$$

For the rectangular section, $I = \frac{1}{12}\left(\frac{1}{2}d\right)(d)^3$

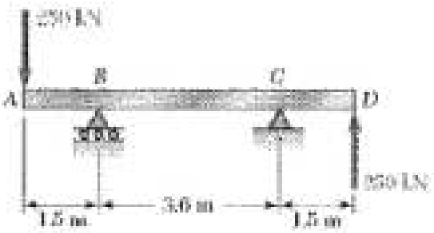
$c = \frac{1}{2}d \qquad S = \frac{I}{c} = \frac{d^3}{12}$

Equating expressions for S,

$$\frac{d^3}{12} = 835.29 \times 10^3 \qquad d = 216 \text{ mm}$$

Problem 5.158

5.158 Knowing that the allowable stress for the steel used is 165 MPa, select the most economical wide-flange beam to support the loading shown.



$+\circlearrowleft \Sigma M_C = 0$

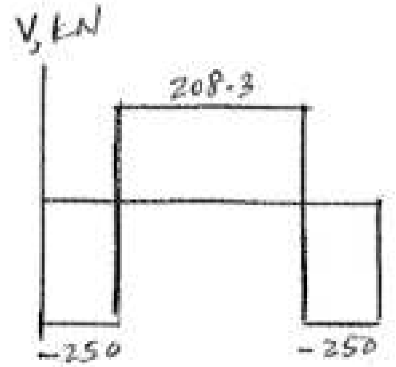
$(250)(5.1) - 3.6B + (1.5)(250) = 0$

$B = 458.3 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_B = 0$

$(250)(1.5) + 3.6C + (250)(5.1) = 0$

$C = -458.3 \text{ kN}$ or $C = 458.3 \text{ kN} \downarrow$



Shear diagram.

A to B^- $\quad V = -250$ kN

B^+ to C^- $\quad V = -250 + 458.3 = 208.3$ kN.

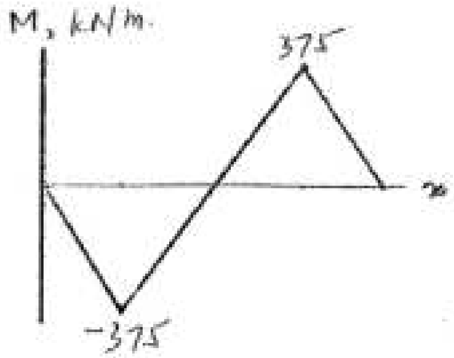
C^+ to D $\quad V = 208.3 - 458.3 = -250$ kN

Areas of shear diagram.

A to B $\quad (1.5)(-250) = -375$ kNm.

B to C $\quad (3.6)(208.3) = 749.88$ kNm

C to D $\quad (1.5)(-250) = -375$ kNm.



Bending moments. $\quad M_A = 0$

$M_B = 0 - 375 = -375$ kNm.

$M_C = -375 + 749.88 = 374.88$ kNm

$M_D = 374.88 - 375 \approx 0$

$|M|_{max} = 375$ kNm

Required S_{min}

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{375 \times 10^3}{165 \times 10^6} = 2272.7 \times 10^{-6} \text{ m}^3$$

$$= 2273 \times 10^{+3} \text{ mm}^3$$

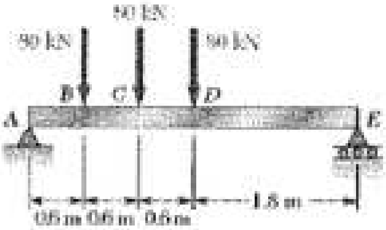
Shape	S (mm $\times 10^3$)
W690 × 125	3510
W610 × 101	2530 ←
W530 × 150	3720
W460 × 113	2400
W360 × 216	3800.

Lightest wide flange beam

W610 × 101 @ 101 kg/m.

Problem 5.160

5.160 (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



$+\sum M_E = 0 \quad -3.6A + (3)(80) + (2.4)(80) + (1.8)(80) = 0$

$A = 160 \text{ kN}.$

(a) $V = 160 - 80\langle x-0.6\rangle^0 - 80\langle x-1.2\rangle^0 - 80\langle x-1.8\rangle^0$ kN

$M = 160x - 80\langle x-0.6\rangle^1 - 80\langle x-1.2\rangle^1 - 80\langle x-1.8\rangle^1$ kN m.

Values of V

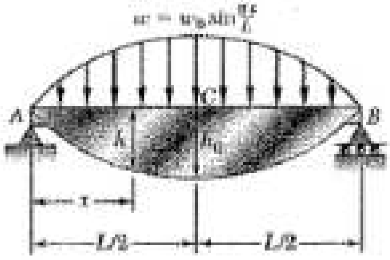
A to B	$V = 160$ kN
B to C	$V = 160 - 80 = 80$ kN
C to D	$V = 160 - 80 - 80 = 0$
D to E	$V = 160 - 80 - 80 - 80 = -80$ kN

Bending moment is constant and maximum over C to D.

(b) At C $\quad x = 1.2$ m $\quad M = (160)(1.2) - (80)(0.6) - 0 - 0 = 144$ kNm

Problem 5.161

5.161 The beam *AB*, consisting of an aluminum plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0 for portion *AC* of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm., $h_0 = 200$ mm, $b = 25$ mm., and $\sigma_{all} = 72$ MPa.



$$\frac{dV}{dx} = -w = -w_0 \sin\frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = \frac{w_0 L}{\pi}\cos\frac{\pi x}{L} + C_1$$

$$M = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L} + C_1 x + C_2$$

At A, $x = 0$ $M = 0$ $\quad 0 = 0 + 0 + C_2 \quad C_2 = 0$

At B, $x = L$ $M = 0$ $\quad 0 = \frac{w_0 L^2}{\pi^2}\sin\pi + C_1 L \quad C_1 = 0$

$$M = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L}$$

For constant strength $\quad S = \frac{|M|}{\sigma_{all}} = \frac{w_0 L^2}{\pi^2 \sigma_{all}}\sin\frac{\pi x}{L}$

For a rectangular section $\quad I = \frac{1}{12}bh^3,\ c = \frac{h}{2},\ S = \frac{I}{c} = \frac{1}{6}bh^2$

Equating the two expressions for S $\quad \frac{1}{6}bh^2 = \frac{w_0 L^2}{\pi^2\sigma_{all}}\sin\frac{\pi x}{L} \qquad (1)$

At $x = \frac{L}{2}$ $\quad h = h_0 \quad \frac{1}{6}bh_0^2 = \frac{w_0 L^2}{\pi^2 \sigma_{all}} \qquad (2)$

(a) Dividing Eq(1) by Eq. 2 $\quad \frac{h^2}{h_0^2} = \sin\frac{\pi x}{L} \qquad h = h_0\left(\sin\frac{\pi x}{L}\right)^{1/2}$ ◄

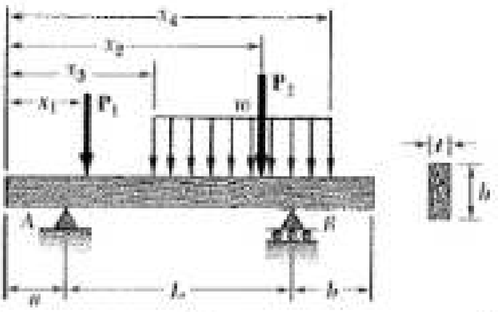
(b) Solving Eq. (1) for w_0 $\quad w_0 = \frac{\pi^2 \sigma_{all} b h_0^2}{6L^2}$

Data: $\sigma_{all} = 72\times10^6$ Pa, $L = 800$ mm $= 0.800$ m, $h_0 = 200$ mm $= 0.200$ m,
$b = 25$ mm $= 0.025$ m

$$w_0 = \frac{\pi^2(72\times10^6)(0.025)(0.200)^2}{(6)(0.800)^2} = 185.1\times10^3 \text{ N/m}$$

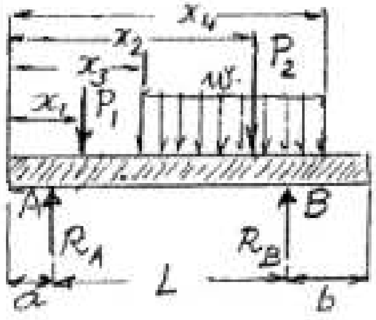
185.1 kN/m ◄

PROBLEM 5.C2



5.C2 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress in the beam will not exceed a given allowable value σ_{all}. Write a computer program that can be used to calculate at given intervals ΔL the shear, the bending moment, and the smallest acceptable value of the unknown dimension. Apply this program to solve the following problems, using the intervals ΔL indicated: (*a*) Prob. 5.65 ($\Delta L = 0.1$ m), (*b*) Prob. 5.69 ($\Delta L = 0.3$ m), (*c*) Prob. 5.70 ($\Delta L = 0.2$ m).

SOLUTION



REACTIONS AT A AND B

$$+\circlearrowleft \Sigma M_A = 0: \; R_B L - P_1(x_1 - a) - P_2(x_2 - a) - w(x_4 - x_3)\left(\frac{x_4 + x_3}{2} - a\right) = 0$$

$$R_B = \frac{1}{L}\left[P_1(x_1 - a) + P_2(x_2 - a) + \frac{1}{2} w(x_4 - x_3)(x_4 + x_3 - 2a)\right]$$

$$R_A = P_1 + P_2 + w(x_4 - x_3) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

SET $n = (a + b + L)/\Delta L$

FOR $i = 0$ TO n: $x = (\Delta L) i$

WE DEFINE: IF $x \geq a$ THEN STPA = 1 ELSE STPA = 0

IF $x \geq a + L$, THEN STPB = 1 ELSE, STB = 0

IF $x \geq x_1$ THEN STP1 = 1 ELSE STP1 = 0

IF $x \geq x_2$ THEN STP2 = 1 ELSE STP2 = 0

IF $x \geq x_3$ THEN STP3 = 1 ELSE STP3 = 0

IF $x \geq x_4$ THEN STP4 = 1 ELSE STP4 = 0

$$V = R_A \,\text{STPA} + R_B \,\text{STPB} - P_1 \,\text{STP1} - P_2 \,\text{STP2} - w(x - x_3)\,\text{STP3} + w(x - x_4)\,\text{STP4}$$

$$M = R_A (x - a)\,\text{STPA} + R_B (x - a - L)\,\text{STPB} - P_1 (x - x_1)\,\text{STP1} - P_2 (x - x_2)\,\text{STP2} - \frac{1}{2} w (x - x_3)^2\,\text{STP3} + \frac{1}{2} w (x - x_4)^2\,\text{STP4}$$

$$S_{min} = |M| / \sigma_{all}$$

IF UNKNOWN DIMENSION IS h:

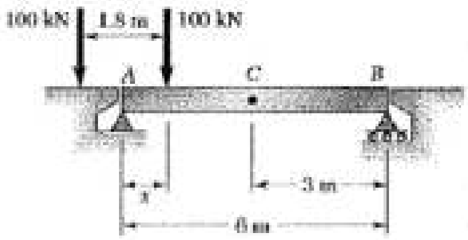
From $S = \frac{1}{6} t h^2$, we have $h = \sqrt{6S/t}$

IF UNKNOWN DIMENSION IS t:

From $S = \frac{1}{6} t h^2$, we have $t = 6S/h^2$

(CONTINUED)

PROBLEM 5.C4



5.C4 Two 100-kN loads are maintained 1.8 m apart as they are moved slowly across the 6-m beam AB. Write a computer program and use it to calculate the bending moment under each load and at the midpoint C of the beam for values of x from 0 to 7 m at intervals $\Delta x = 0.45$ m.

SOLUTION

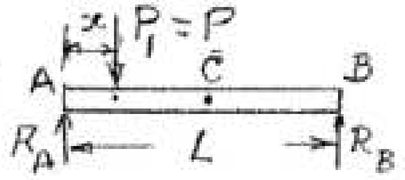
NOTATION: Length of beam $= L = 5.4$ m

Loads: $P_1 = P_2 = P = 100$ kN

Distance between loads $= d = 1.8$ m

We note that $d < L/2$

(1)



(1) FROM $x = 0$ TO $x = d$:

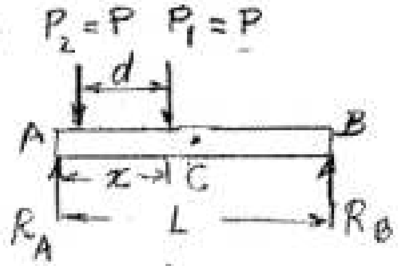
$$+\circlearrowleft \Sigma M_B = 0: \; P(L-x) - R_A L = 0.$$

$$R_A = P(L-x)/L$$

Under P_1: $M_1 = R_A x$

At C: $M_C = R_A\left(\frac{L}{2}\right) - P\left(\frac{L}{2} - x\right)$

(2A)



(2) FROM $x = d$ TO $x = L$:

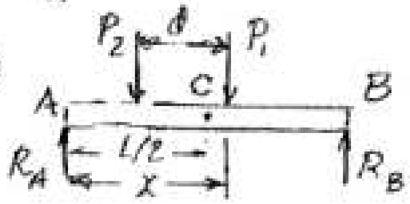
$$+\circlearrowleft \Sigma M_B = 0: \; P(L-x) + P(L-x+d) - R_A L = 0.$$

$$R_A = P(2L - 2x + d)/L$$

Under P_1: $M_1 = R_A x - Pd$

Under P_2: $M_2 = R_A(x-d)$

(2B)

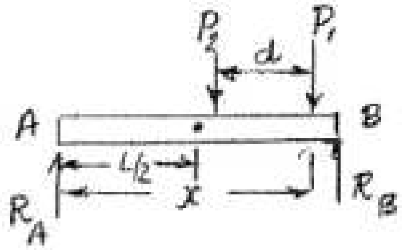


(2A) FROM $x = d$ TO $x = L/2$:

$$M_C = R_A\left(\frac{L}{2}\right) - P\left(\frac{L}{2} - x\right) - P\left(\frac{L}{2} - x + d\right)$$

$$= R_A(L/2) - P(L - 2x + d)$$

(2C)



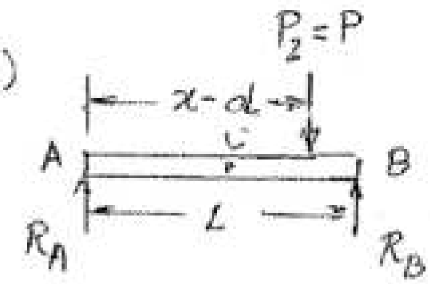
(2B) FROM $x = L/2$ TO $x = L/2 + d$:

$$M_C = R_A(L/2) - P\left(\frac{L}{2} - x + d\right)$$

(2C) FROM $x = L/2 + d$ TO $x = L$:

$$M_C = R_A L/2$$

(3)



(3) FROM $x = L$ TO $x = L + d$:

$$+\circlearrowleft \Sigma M_B = 0: \; P(L - x + d) - R_A L = 0$$

$$R_A = P(L - x + d)/L$$

Under P_2: $M_2 = R_A(x-d)$

At C: $M_C = R_A(L/2)$

(CONTINUED)

PROBLEM 5.C4 CONTINUED

PROGRAM OUTPUT

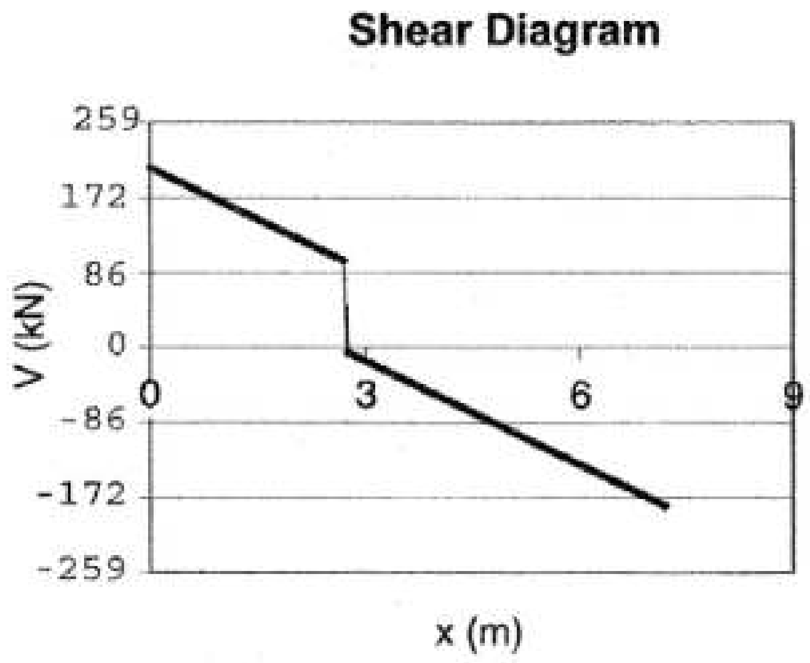
P = 100 kN, L = 5.4 m, D = 1.8 m

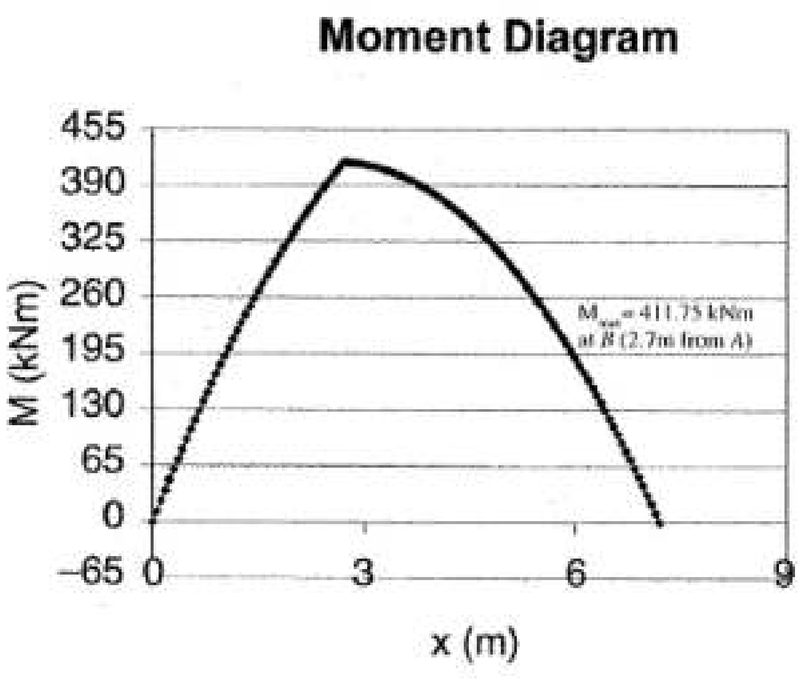
x m	MC kNm	M1 kNm	M2 kNm
0	0	0	0
0.9	50.85	84.75	0
1.8	101.7	135.6	0
2.7	203.4	203.4	135.6
3.6	203.4	203.4	203.4
4.5	203.4	135.6	203.4
5.4	101.7	0	135.6

PROGRAM OUTPUT

Problem 5.72

RA = 206.5 kN RB = 181.5 kN



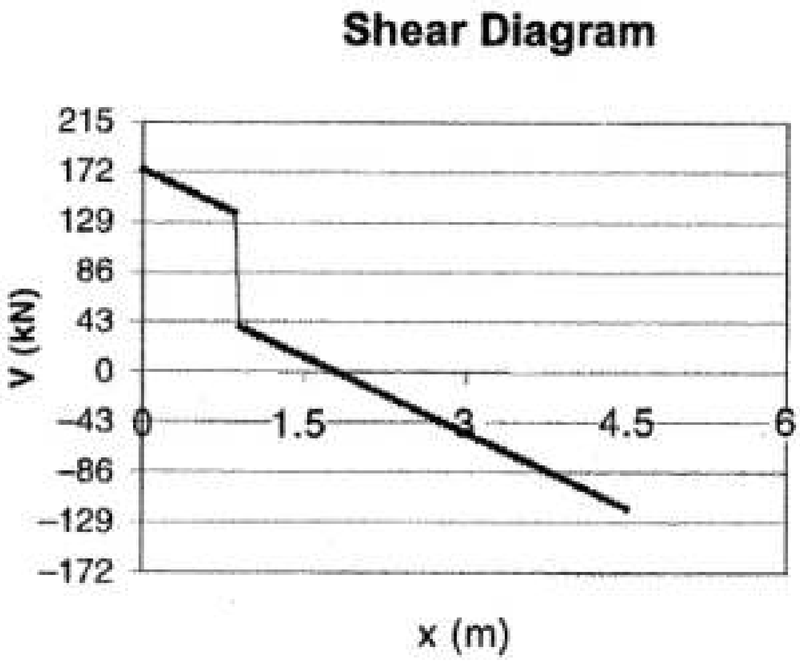


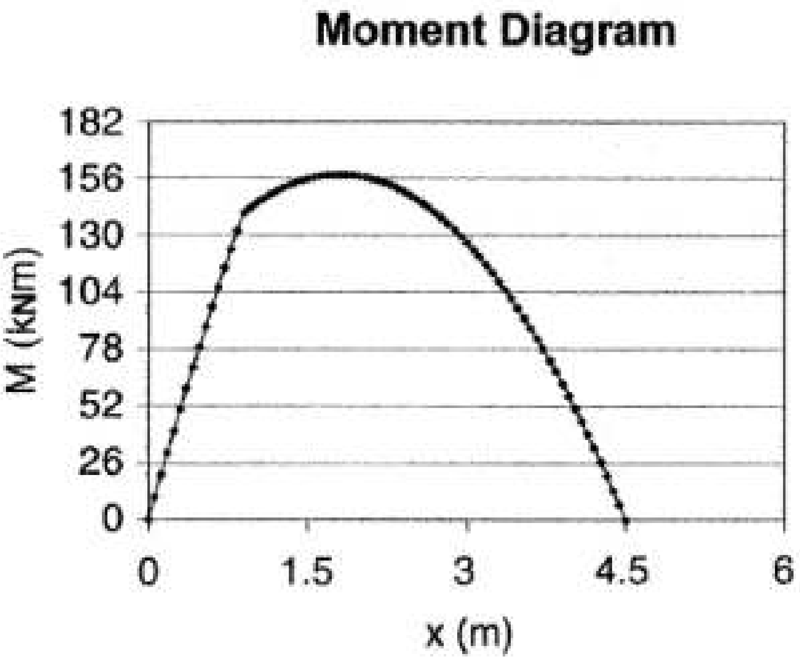
(CONTINUED)

PROGRAM OUTPUT FOR

Problem 5.115

RA = 173.25 kN RB = 119.25 kN

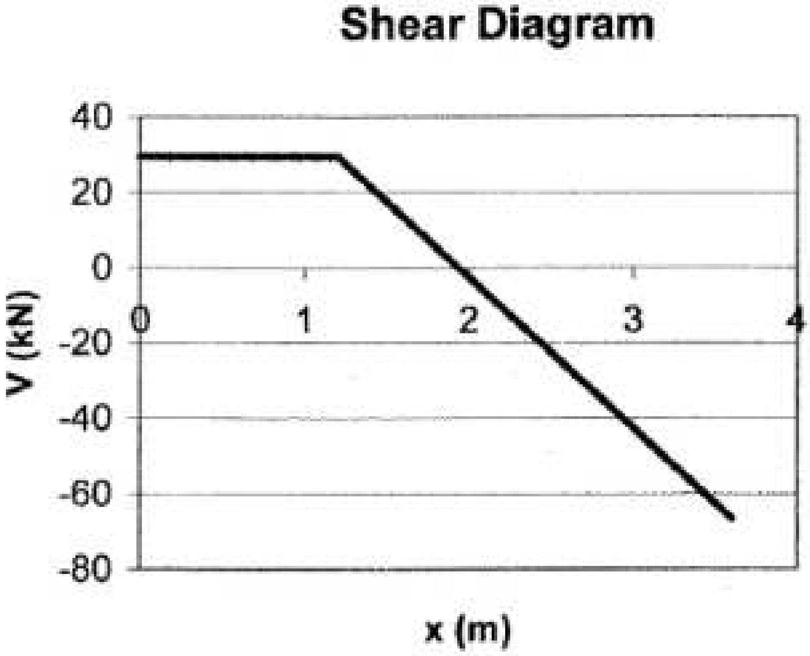


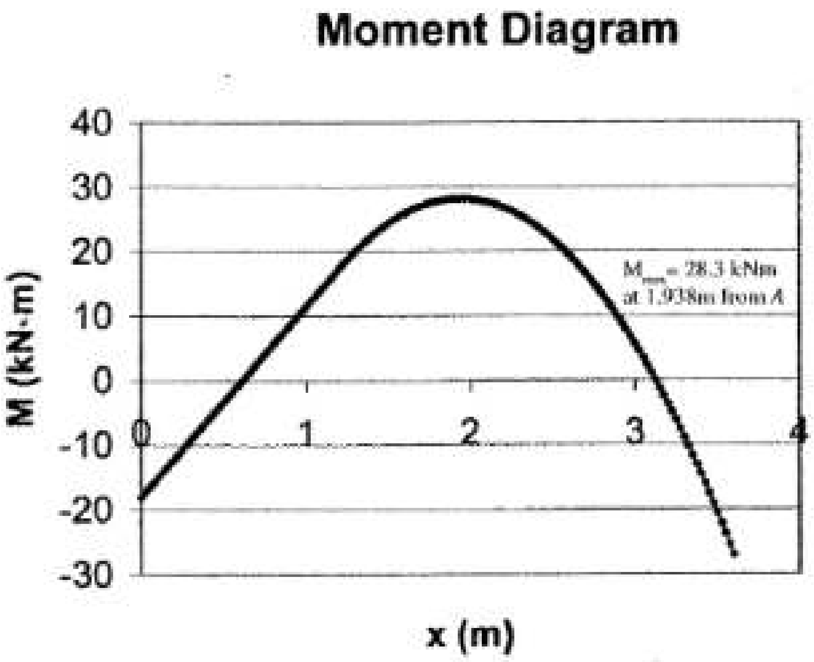


PROBLEM 5.C6 CONTINUED

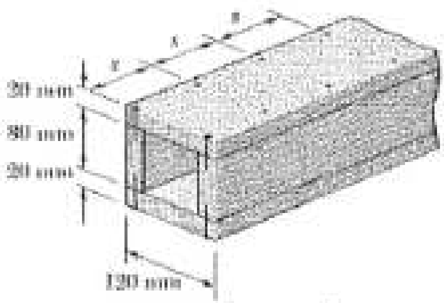
PROGRAM OUTPUT

```
Problem 5.112

RA =    29.50 kN    RB =     66.50 kN
```





Problem 6.1

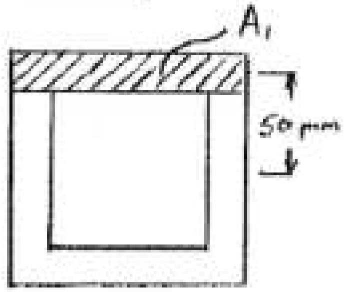


6.1 A square box beam is made of two 20 × 80-mm planks and two 20 × 120-mm planks nailed together as shown. Knowing that the spacing between the nails is $s = 50$ mm and that the allowable shearing force in each nail is 300 N, determine (*a*) the largest allowable vertical shear in the beam, (*b*) the corresponding maximum shearing stress in the beam.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(120)(120)^3 - \frac{1}{12}(80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4$$

$$= 13.8667 \times 10^{-6} \text{ m}^4$$



(a) $A_1 = (120)(20) = 2400 \text{ mm}^2$

$\bar{y}_1 = 50$ mm

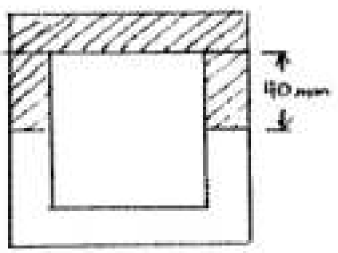
$$Q_1 = A_1 \bar{y}_1 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$$

$$q_{all} = \frac{2F_{nail}}{s} = \frac{(2)(300)}{50 \times 10^{-3}} = 12 \times 10^3 \text{ N}$$

$$q = \frac{VQ}{I}$$

$$V = \frac{qI}{Q} = \frac{(12 \times 10^3)(13.8667 \times 10^{-6})}{120 \times 10^{-6}}$$

$$= 1.38667 \times 10^3 \text{ N} = 1.387 \text{ kN}$$



(b) $Q = Q_1 + (2)(20)(40)(20)$

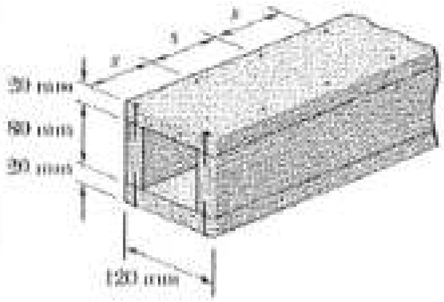
$$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3$$

$$= 152 \times 10^{-6} \text{ m}^3$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(1.38667 \times 10^3)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^{-3})}$$

$$= 380 \times 10^3 \text{ Pa} \qquad 380 \text{ kPa}$$

Problem 6.2

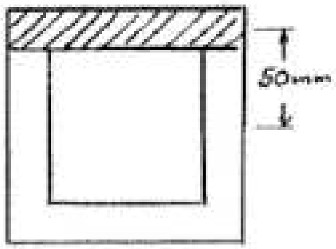


6.2 A square box beam is made of two 20 × 80-mm planks and two 20 × 120-mm planks nailed together as shown. Knowing that the spacing between the nails is $s = 30$ mm and that the vertical shear in the beam is $V = 1200$ N, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(120)(120)^3 - \frac{1}{12}(80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4$$

$$= 13.8667 \times 10^{-6} \text{ m}^4$$



(a) $A_1 = (120)(20) = 2400 \text{ mm}^2$

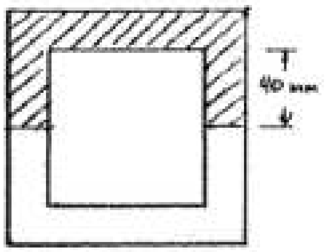
$\bar{y}_1 = 50 \text{ mm}$

$Q_1 = A_1 \bar{y}_1 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

$q = \frac{VQ}{I} = \frac{(1200)(120 \times 10^{-6})}{13.8667 \times 10^{-6}} = 10.385 \times 10^3 \text{ N/m}$

$qs = 2 F_{nail}$

$F_{nail} = \frac{qs}{2} = \frac{(10.385 \times 10^3)(30 \times 10^{-3})}{2} = 155.8 \text{ N}$ ◄



(b) $Q = Q_1 + (2)(20)(40)(20)$

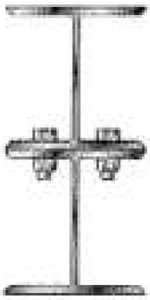
$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3$

$= 152 \times 10^{-6} \text{ m}^3$

$\tau_{max} = \frac{VQ}{It} = \frac{(1200)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^{-3})}$

$= 329 \times 10^3 \text{ Pa}$ $\qquad$ 329 kPa ◄

Problem 6.5



6.5 The composite beam shown is fabricated by connecting two W150 × 37.1 rolled-steel members, using bolts of 16-mm diameter spaced longitudinally every 150 mm. Knowing that the average allowable shearing stress in the bolts is 73 MPa, determine the largest allowable vertical shear in the beam.

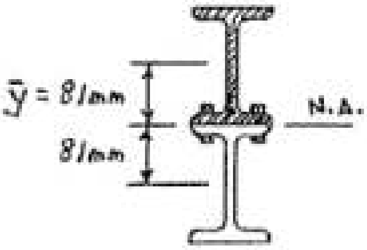
W150x37.1: $A = 4370\ mm^2$, $d = 162\ mm$, $I_x = 22.2 \times 10^6\ mm^4$

$\bar{y} = \frac{1}{2}d = 81\ mm$

Composite: $I = 2[22.2 \times 10^6 + (4370)(81)^2]$

$= 101.743 \times 10^6\ mm^4$

$Q = A\bar{y} = (4370)(81) = 353970\ mm^3$



Bolts: $d = 16mm$ $\tau_{all} = 73\ MPa$, $s = 150mm$.

$A_{bolt} = \frac{\pi}{4}(16)^2 = 201.06\ mm^2$

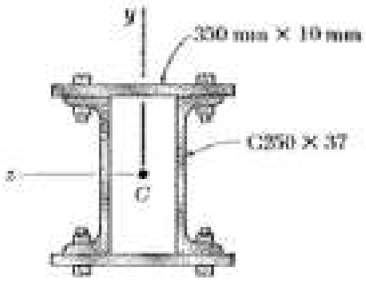
$F_{bolt} = \tau_{all} A_{bolt} = (73 \times 10^6)(201.06 \times 10^{-6}) = 14677\ N$

$q = \frac{2F_{bolt}}{s} = \frac{(2)(14677)}{150} = 195.69\ N/mm$

Shear: $q = \frac{VQ}{I}$ $V = \frac{Iq}{Q} = \frac{(101.743 \times 10^6)(195.69)}{353970}$ $V = 56.2\ kN$

Problem 6.6

6.6 A column is fabricated by connecting the rolled-steel members shown by bolts of 18-mm diameter spaced longitudinally every 125 mm. Determine the average shearing stress in the bolts caused by a shearing force of 120 kN parallel to the y axis.



Calculate moment of inertia.

Part	A (mm^2)	d (mm)	Ad^2 (mm^4)	$\bar{I}$ (mm^4)
Top plate	3500	* 132	60.984×10^6	29167
C250×37	4750	0		37.9×10^6
C250×37	4750	0		37.9×10^6
Bot. plate	3500	* 132	60.984×10^6	29167
Σ			121.968×10^6	75.858×10^6

$$* \ d = \frac{254}{2} + \frac{10}{2} = 132 \text{ mm} = \bar{y}_1$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 121.968 \times 10^6 + 75.858 \times 10^6 = 197.83 \times 10^6 \text{ mm}^4$$

$$Q = A_{plate} \bar{y}_1 = (3500)(132) = 462000 \text{ mm}^3$$

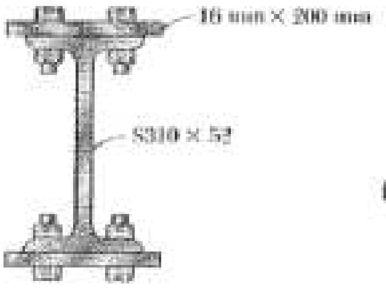
$$q = \frac{VQ}{I} = \frac{(120)(462000 \times 10^{-9})}{197.83 \times 10^{-6}} = 280.24 \text{ kN/m}$$

$$F_{bolt} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(280.24)(0.125) = 17.515 \text{ kN}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18)^2 = 254.67 \text{ mm}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{17515}{254.67 \times 10^{-6}} = 68.8 \text{ MPa}$$

Problem 6.7



6.7 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16 × 200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

Calculate moment of inertia.

Part	A (mm²)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
Top plate	3200	*160.5	82.43	0.07
S310×52	6650	0		95.3
Bot. plate	3200	*160.5	82.43	0.07
Σ			164.86	95.44

$$* \ d = \frac{305}{2} + \frac{16}{2} = 160.5 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 260.3 \times 10^6 \text{ mm}^4 = 260.3 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate}\, d_{plate} = (3200)(160.5) = 513.6 \times 10^3 \text{ mm}^3 = 513.6 \times 10^{-6} \text{ m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

$$F_{bolt} = \tau_{all} A_{bolt} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3 \text{ N}$$

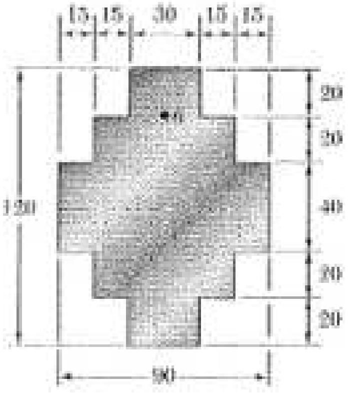
$$qs = 2F_{bolt} \qquad q = \frac{2F_{bolt}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^{-6})(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3 \text{ N}$$

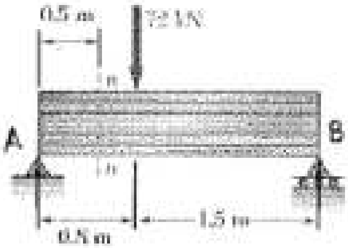
$$V = 193.5 \text{ kN}$$

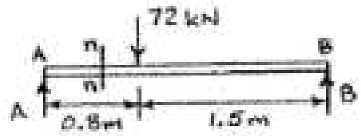
Problem 6.9

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



Dimensions in mm.





$+\circlearrowleft \Sigma M_B = 0:$

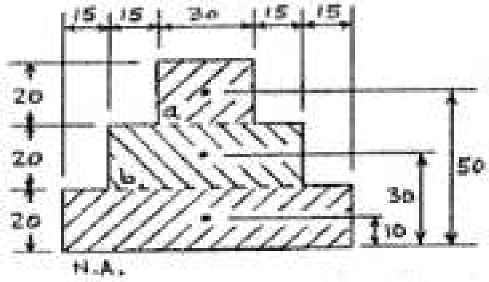
$-2.3A + (1.5)(72) = 0$

$A = 46.957 \text{ kN} \uparrow$

At section n-n, $\quad V = A = 46.957 \text{ kN}$

Calculate moment of inertia. $\quad I = 2\left[\frac{1}{12}(15)(40)^3\right] + 2\left[\frac{1}{12}(15)(80)^3\right] + \frac{1}{12}(30)(120^3)$

$= 5.76 \times 10^6 \text{ mm}^4 = 5.76 \times 10^{-6} \text{ m}^4$



<u>At a</u>: $\quad t_a = 30 \text{ mm} = 0.030 \text{ m}$

$Q_a = (30 \times 20)(50) = 30 \times 10^3 \text{ mm}^3$

$= 30 \times 10^{-6} \text{ m}^3$

$\tau_a = \frac{VQ_a}{It_a} = \frac{(46.957 \times 10^3)(30 \times 10^{-6})}{(5.76 \times 10^{-6})(0.030)}$

$= 8.15 \times 10^6 \text{ Pa} = 8.15 \text{ MPa}$

<u>At b</u>: $\quad t_b = 60 \text{ mm} = 0.060 \text{ m}$

$Q_b = Q_a + (60 \times 20)(30) = 30 \times 10^3 + 36 \times 10^3 = 66 \times 10^3 \text{ mm}^3 = 66 \times 10^{-6} \text{ m}^4$

$\tau_b = \frac{VQ_b}{It_b} = \frac{(46.957 \times 10^3)(66 \times 10^{-6})}{(5.76 \times 10^{-6})(0.060)} = 8.97 \times 10^6 \text{ Pa} = 8.97 \text{ MPa}$

<u>At NA</u>: $\quad t_{NA} = 90 \text{ mm} = 0.090 \text{ m}$

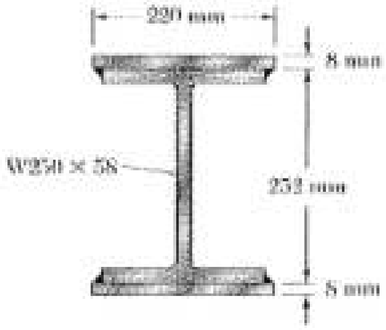
$Q_{NA} = Q_b + (90 \times 20)(10) = 66 \times 10^3 + 18 \times 10^3 = 84 \times 10^3 \text{ mm}^3 = 84 \times 10^{-6} \text{ m}^3$

$\tau_{NA} = \frac{VQ_{NA}}{It_{NA}} = \frac{(46.957 \times 10^3)(84 \times 10^{-6})}{(5.76 \times 10^{-6})(0.090)} = 7.61 \times 10^6 \text{ Pa} = 7.61 \text{ MPa}$

(a) τ_{max} occurs at b. $\qquad \tau_{max} = 8.97 \text{ MPa}$ ◄

(b) $\tau_a = 8.15 \text{ MPa}$ ◄

Problem 6.14



6.14 Solve Prob. 6.13, assuming that the two steel plates are (*a*) replaced by 8 × 220-mm plates, (*b*) removed.

6.13 Two steel plates of 12 × 220-mm rectangular cross section are welded to the W250 × 58 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

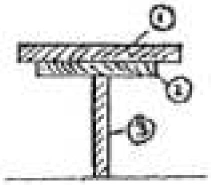
(a) Calculate moment of inertia.

Part	A (mm²)	d (mm)	Ad² (10⁶ mm⁴)	$\bar{I}$ (10⁶ mm⁴)
Top plate	1760	* 130	29.744	0.0094
W 250×58	7420	0	0	87.3
Bot. plate	1760	* 130	29.744	0.0094
Σ			59.488	87.312

$$* \; d = \frac{252}{2} + \frac{8}{2}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 146.807 \times 10^6 \text{ mm}^4 = 146.807 \times 10^{-6} \text{ m}^4$$

τ_{max} occurs at neutral axis. $t = 8.0 \text{ mm} = 8.0 \times 10^{-3} \text{ m}$



Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10³ mm³)
① Top plate	1760	130	228.8
② Top flange	2740.5	119.25	326.805
③ Half web	900	56.25	50.625
Σ			606.23

Dimensions in mm: ① 8 × 220 ; ② 13.5 × 203 ; ③ 8.0 × 112.5

$$Q = \Sigma A\bar{y} = 606.23 \times 10^3 \text{ mm}^3 = 606.23 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} \qquad V = \frac{It\tau}{Q} = \frac{(146.807\times10^{-6})(8.0\times10^{-3})(90\times10^{6})}{606.23\times10^{-6}}$$

$$= 174.4 \times 10^3 \text{ N} \qquad V = 174.4 \text{ kN}$$

(b) With plates removed.

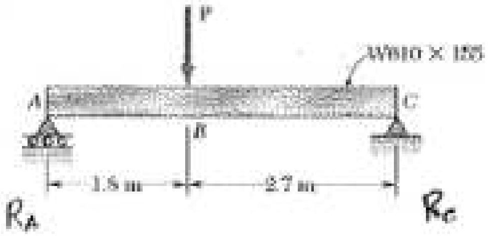
$$I = 87.3 \times 10^6 \text{ mm}^4 = 87.3 \times 10^{-6} \text{ m}^4 \qquad t = 8.0 \times 10^{-3} \text{ m}$$

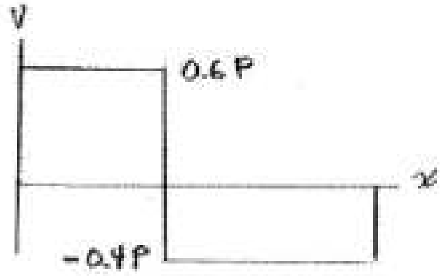
$$Q = (326.805 + 50.625) \times 10^3 = 377.43 \times 10^3 \text{ mm}^3 = 377.43 \times 10^{-6} \text{ m}^3$$

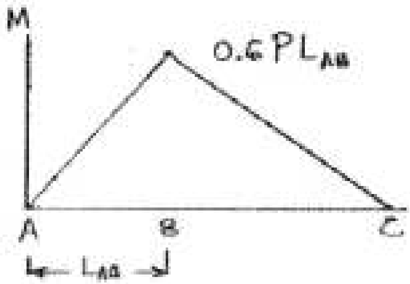
$$V = \frac{It\tau}{Q} = \frac{(87.3\times10^{-6})(8.0\times10^{-3})(90\times10^{6})}{377.43\times10^{-6}}$$

$$= 166.5 \times 10^3 \text{ N} \qquad V = 166.5 \text{ kN}$$

Problem 6.15







6.15 For the wide-flange beam with the loading shown, determine the largest load **P** that can be applied, knowing that the maximum normal stress is 165 MPa and the largest shearing stress using the approximation $\tau_m = V/A_{web}$ is 100 MPa.

$$+\circlearrowleft \Sigma M_C = 0: \quad -4.5R_A + 2.7P = 0$$

$$R_A = 0.6P$$

Draw shear and bending moment diagrams.

$$|V|_{max} = 0.6P \qquad |M|_{max} = 0.6PL_{AB}$$

$$L_{AB} = 1.8 \text{ m}$$

Bending. For W610 × 155, $S = 4220 \times 10^3 \text{ mm}^3$

$$S = \frac{|M|_{max}}{\sigma_{all}} = \frac{0.6PL_{AB}}{\sigma_{all}}$$

$$P = \frac{\sigma_{all}S}{0.6L_{AB}} = \frac{(165)(4220 \times 10^3)}{(0.6)(1800)} = 644.7 \text{ kN} \approx 645 \text{ kN}$$

Shear. $A_{web} = d t_w$

$$= (611)(12.7)$$

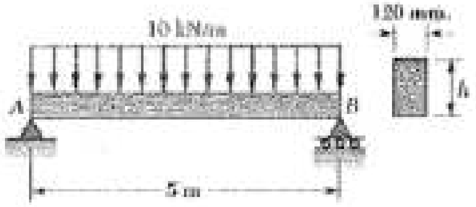
$$= 7759.7 \text{ mm}^2$$

$$\tau = \frac{|V|_{max}}{A_{web}} = \frac{0.6P}{A_{web}}$$

$$P = \frac{\tau A_{web}}{0.6} = \frac{(100)(7759.7)}{0.6} = 1293 \text{ kN}$$

The smaller value of P is the allowable value. $P = 645 \text{ kN}$ ◄

Problem 6.18



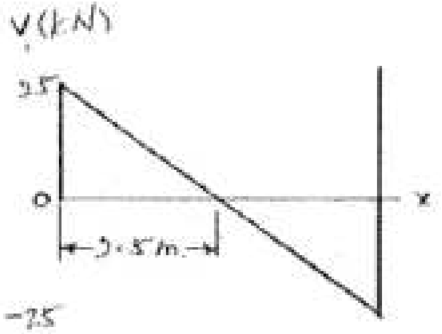
6.18 For the beam and loading shown, determine the minimum required depth h, knowing that for the grade of timber used, $\sigma_{all} = 12$ MPa and $\tau_{all} = 0.9$ MPa.

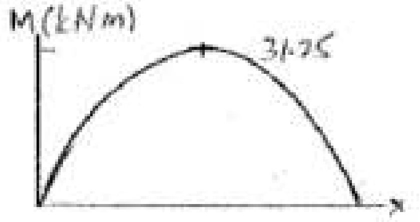
Total load $(10)(5) = 50$ kN

Reaction at A $R_A = 25$ kN

$V_{max} = 25$ kN.

$M_{max} = \frac{1}{2}(2.5)(25) = 31.25$ kNm



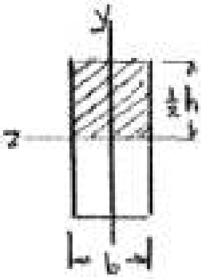


Bending. $S = \frac{1}{6}bh^2$ for rectangular section.

$$S = \frac{M_{max}}{\sigma_{all}} = \frac{31.25 \times 10^6}{12} = 2604167 \text{ mm}^3$$

$$h = \sqrt{\frac{6S}{b}} = \left[\frac{(6)(2604167)}{120}\right]^{1/2} = 360.8 \text{ mm}$$

Shear. $I = \frac{1}{12}bh^3$ for rectangular section.



$A = \frac{1}{2}bh$

$\bar{y} = \frac{1}{4}h$

$Q = A\bar{y} = \frac{1}{8}bh^2$

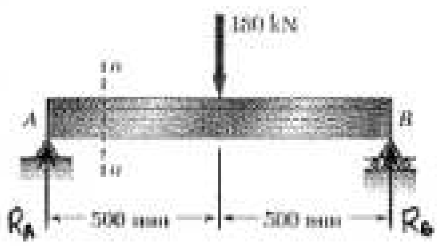
$$\tau_{max} = \frac{VQ}{Ib} = \frac{3V_{max}}{2bh}$$

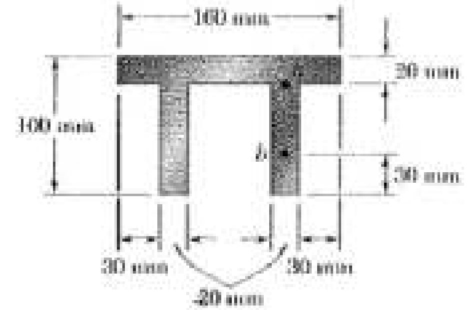
$$h_1 = \frac{3V_{max}}{2b\,\tau_{all}} = \frac{(3)(25 \times 10^3)}{(2)(120)(0.9)} = 347.2 \text{ mm}$$

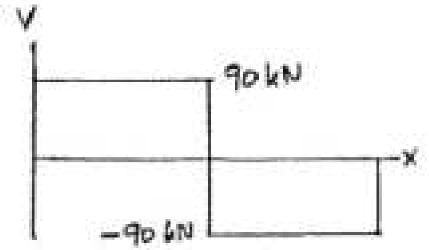
The larger value of h is the minimum required depth. $h = 360.8$ mm ◄

Problem 6.21

6.21 and 6.22 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

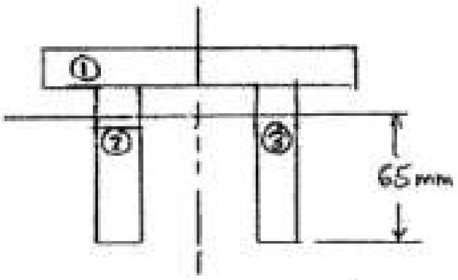






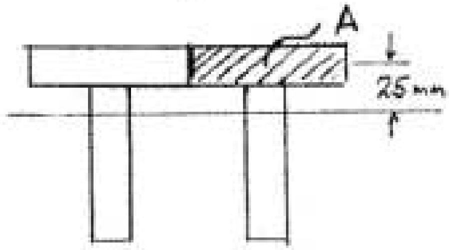
Draw the shear diagram. $|V|_{max} = 90$ kN

Part	$A(mm^2)$	$\bar{y}(mm)$	$A\bar{y}\,(10^3 mm^3)$	$d(mm)$	$Ad^2\,(10^6 mm^4)$	$\bar{I}\,(10^6 mm^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	−25	1.000	0.8533
③	1600	40	64	−25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416\times10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = (4.000 + 1.8133)\times10^6 \text{ mm}^4$$
$$= 5.8133\times10^6 \text{ mm}^4 = 5.8133\times10^{-6} \text{ m}^4$$



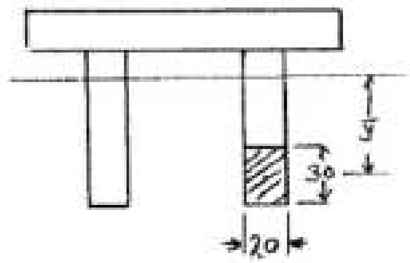
(a) $A = (80)(20) = 1600 \text{ mm}^2$

$\bar{y} = 25$ mm

$Q_a = A\bar{y} = 40\times10^3 \text{ mm}^3 = 40\times10^{-6} \text{ m}^3$

$$\tau_a = \frac{VQ_a}{It} = \frac{(90\times10^3)(40\times10^{-6})}{(5.8133\times10^{-6})(20\times10^{-3})} = 31.0\times10^6 \text{ Pa}$$

$\tau_a = 31.0$ MPa ◄



(b) $A = (30)(20) = 600 \text{ mm}^2$

$\bar{y} = 65 - 15 = 50$ mm

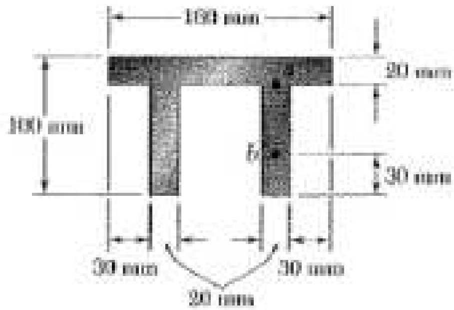
$Q_b = A\bar{y} = 30\times10^3 \text{ mm}^3 = 30\times10^{-6} \text{ m}^3$

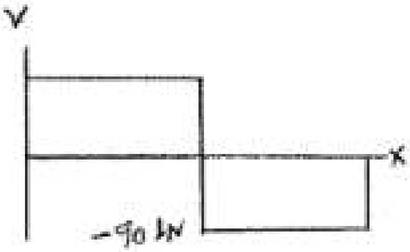
$$\tau_b = \frac{VQ_b}{It} = \frac{(90\times10^3)(30\times10^{-6})}{(5.8133\times10^{-6})(20\times10^{-3})} = 23.2\times10^6 \text{ Pa}$$

$\tau_b = 23.2$ MPa ◄

Problem 6.23

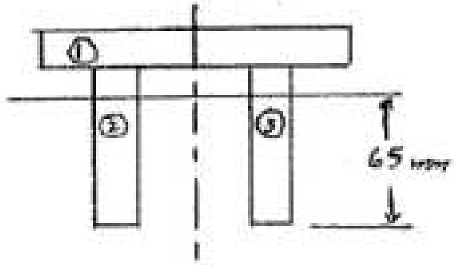
6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section *n-n*.





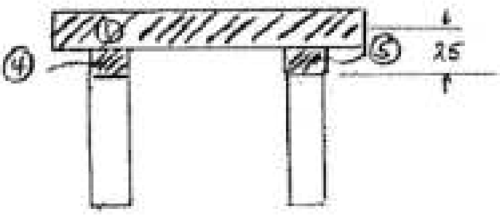
Draw the shear diagram. $|V|_{max} = 90$ kN

Part	A (mm^2)	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm^3)	d (mm)	Ad^2 (10^6 mm^4)	$\bar{I}$ (10^6 mm^4)
1	3200	90	288	25	2.000	0.1067
2	1600	40	64	-25	1.000	0.8533
3	1600	40	64	-25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416 \times 10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = (4.000 + 1.8133) \times 10^6 \text{ mm}^4$$
$$= 5.8133 \times 10^6 \text{ mm}^4 = 5.8133 \times 10^{-6} \text{ m}^4$$



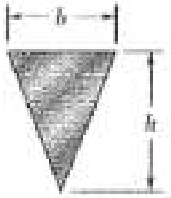
Part	A (mm^2)	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm^3)
1	3200	25	80
4	300	7.5	2.25
5	300	7.5	2.25
Σ			84.5

$$Q = \Sigma A\bar{y} = 84.5 \times 10^3 \text{ mm}^3 = 84.5 \times 10^{-6} \text{ m}^3$$

$$t = (2)(20) = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(90 \times 10^3)(84.5 \times 10^{-6})}{(5.8133 \times 10^{-6})(40 \times 10^{-3})} = 32.7 \times 10^6 \text{ Pa} \qquad \tau_m = 32.7 \text{ MPa}$$

Problem 6.25

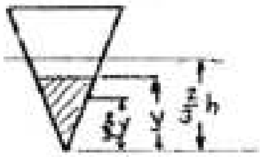


6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

$$A = \frac{1}{2}bh \qquad I = \frac{1}{36}bh^3$$



For a cut at location y,

$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = \frac{2}{3}h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h-y)$$

$$t(y) = \frac{by}{h}$$

$$\tau(y) = \frac{VQ}{It} = \frac{V\frac{by^2}{3}(h-y)}{\left(\frac{1}{36}bh^3\right)\frac{by}{h}} = \frac{12\,Vy(h-y)}{bh^3} = \frac{12V}{bh^3}(hy-y^2)$$

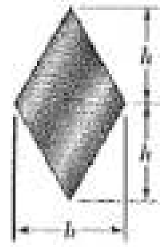
(a) To find location of maximum of τ, set $\frac{d\tau}{dy} = 0$.

$$\frac{d\tau}{dy} = \frac{12V}{bh^3}(h-2y_m) = 0 \qquad y_m = \frac{1}{2}h \quad \text{i.e. at mid-height}$$

(b) $$\tau_m = \frac{12V}{bh^3}(hy_m - y_m^2) = \frac{12V}{bh^3}\left[\frac{1}{2}h^2 - \left(\frac{1}{2}h\right)^2\right] = \frac{3V}{bh^2} = \frac{3}{2}\frac{V}{A}$$

$$k = \frac{3}{2} = 1.500$$

Problem 6.26

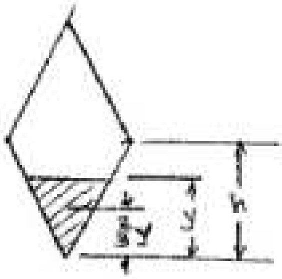


6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

$$A = 2\left(\tfrac{1}{2}bh\right) = bh \qquad I = 2\left(\tfrac{1}{12}bh^3\right) = \tfrac{1}{6}bh^3$$



For a cut at location y, where $y \le h$

$$A(y) = \tfrac{1}{2}\left(\tfrac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = h - \tfrac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{2} - \frac{by^3}{3h}$$

$$t(y) = \frac{by}{h}$$

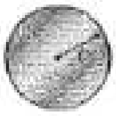
$$\tau(y) = \frac{VQ}{It} = V\frac{6}{bh^3}\cdot\frac{h}{by}\cdot\frac{by^2}{2} - \frac{by^3}{3h} = \frac{V}{bh}\left[3\left(\frac{y}{h}\right) - 2\left(\frac{y}{h}\right)^2\right]$$

(a) To find the location of maximum of τ, set $\frac{d\tau}{dy} = 0$.

$$\frac{d\tau}{dy} = \frac{V}{bh^2}\left[3 - 4\frac{y_m}{h}\right] = 0 \qquad \frac{y_m}{h} = \frac{3}{4}$$ i.e. $\pm\frac{1}{4}h$ from neutral axis. ◄

(b) $$\tau(y_m) = \frac{V}{bh}\left[3\left(\tfrac{3}{4}\right) - 2\left(\tfrac{3}{4}\right)^2\right] = \frac{9}{8}\frac{V}{bh} = 1.125\frac{V}{A} \qquad k = 1.125$$ ◄

Problem 6.27

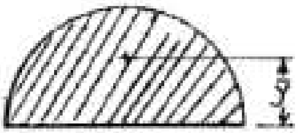


6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

$I = \frac{\pi}{4}c^4$ and $A = \pi c^2$



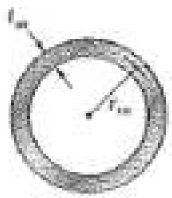
For semicircle, $A_s = \frac{\pi}{2}c^2$ $\quad \bar{y} = \frac{4c}{3\pi}$

$$Q = A_s\bar{y} = \frac{\pi}{2}c^2 \cdot \frac{4c}{3\pi} = \frac{2}{3}c^3$$

(a) τ_{max} occurs at center where $t = 2c$. ◄

(b) $\tau_{max} = \frac{VQ}{It} = \frac{V \cdot \frac{2}{3}c^3}{\frac{\pi}{4}c^4 \cdot 2c} = \frac{4V}{3\pi c^2} = \frac{4}{3}\frac{V}{A}$ $\qquad k = \frac{4}{3} = 1.333$ ◄

Problem 6.28



6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

For a thin walled circular section, $\quad A = 2\pi r_m t_m$

$J = Ar_m^2 = 2\pi r_m^3 t_m, \qquad I = \frac{1}{2}J = \pi r_m^3 t_m$

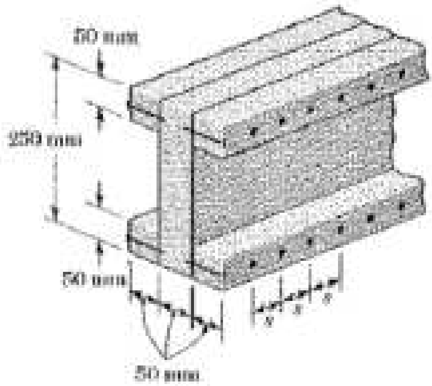
For a semicircular arc, $\quad \bar{y} = \frac{2r_m}{\pi}$

$A_s = \pi r_m t_m \qquad Q = A_s\bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$

(a) $t = 2t_m$ at neutral axis where maximum occurs. ◄

(b) $\tau_{max} = \frac{VQ}{It} = \frac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \frac{V}{\pi r_m t_m} = \frac{2V}{A}$ $\qquad k = 2.00$ ◄

Problem 6.29



6.29 The built-up timber beam is subjected to a vertical shear of 5 kN. Knowing that the allowable shearing force in the nails is 300 N, determine the largest permissible spacing s of the nails.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

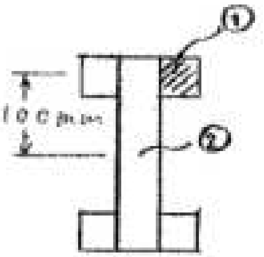
$$= \frac{1}{12}(50)(50)^3 + (50)(50)(100)^2 = 25520833 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(50)(250)^3 = 65104167 \text{ mm}^4$$

$$I = 4I_1 + I_2 = 167187499 \text{ mm}^4$$

$$Q = Q_1 = A_1 \bar{y}_1 = (50)(50)(100) = 250000 \text{ mm}^3$$

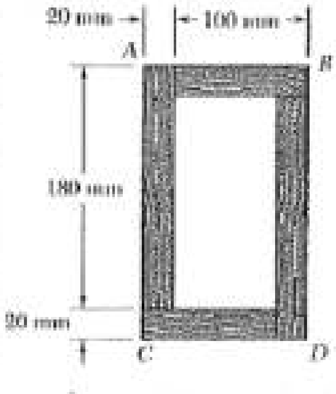
$$q = \frac{VQ}{I} = \frac{(5000)(250000)}{167187499} = 7.48 \text{ N/mm}$$



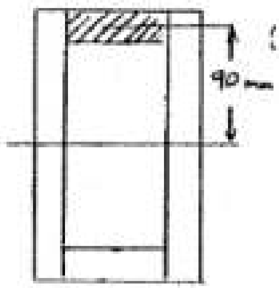
$$F_{nail} = qs \qquad s = \frac{F_{nail}}{q} = \frac{300}{7.48} = 40.1 \text{ mm}$$

Problem 6.30

6.30 Two 20 × 100-mm and two 20 × 180-mm boards are glued together as shown to form a 120 × 200-mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.



$$I = \frac{1}{12}(120)(200)^3 - \frac{1}{12}(80)(160)^3 = 52.693 \times 10^6 \ \text{mm}^4$$
$$= 52.693 \times 10^{-6} \ \text{m}^4$$

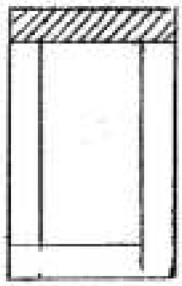


(a) $Q_A = (80)(20)(90) = 144 \times 10^3 \ \text{mm}^3$
$= 144 \times 10^{-6} \ \text{m}^3$

$t_A = (2)(20) = 40 \ \text{mm} = 0.040 \ \text{m}$

$$\tau_A = \frac{VQ_A}{It_A} = \frac{(3.5\times10^3)(144\times10^{-6})}{(52.693\times10^{-6})(0.040)}$$

$= 239 \times 10^3 \ \text{Pa}$ $\qquad \tau_A = 239 \ \text{kPa}$ ◄



(b) $Q_B = (120)(20)(90) = 216 \times 10^3 \ \text{mm}^3 = 216 \times 10^{-6} \ \text{m}^3$

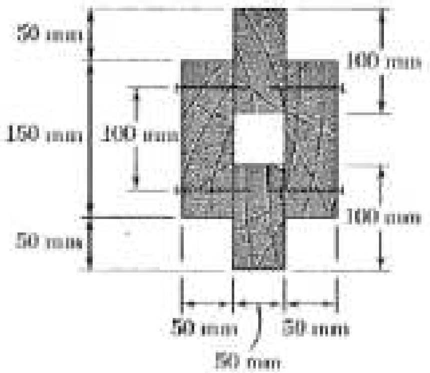
$t_B = (2)(20) = 40 \ \text{mm} = 0.040 \ \text{m}$

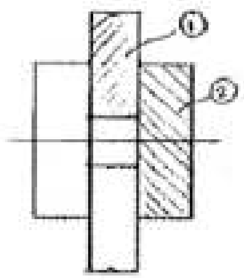
$$\tau_B = \frac{VQ_B}{It_B} = \frac{(3.5\times10^3)(216\times10^{-6})}{(52.693\times10^{-6})(0.040)} = 359 \times 10^3 \ \text{Pa}$$

$\tau_B = 359 \ \text{kPa}$ ◄

Problem 6.31

6.31 The built-up timber beam is subjected to a 6-kN vertical shear. Knowing that the longitudinal spacing of the nails is $s = 60$ mm and that each nail is 90 mm long, determine the shearing force in each nail.





$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$
$= \frac{1}{12}(50)(100)^3 + (50)(100)(75)^2$
$= 32.292 \times 10^6 \ \text{mm}^4$

$I_2 = \frac{1}{12} b h^3 = \frac{1}{12}(50)(150)^3$
$= 14.0625 \times 10^6 \ \text{mm}^4$

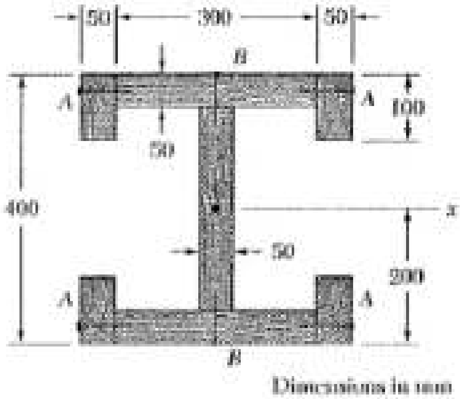
$I = 2I_1 + 2I_2 = 92.71 \times 10^6 \ \text{mm}^4 = 92.71 \times 10^{-6} \ \text{m}^4$

$Q = Q_1 = A_1 \bar{y}_1 = (50)(100)(75) = 375 \times 10^3 \ \text{mm}^3 = 375 \times 10^{-6} \ \text{m}^3$

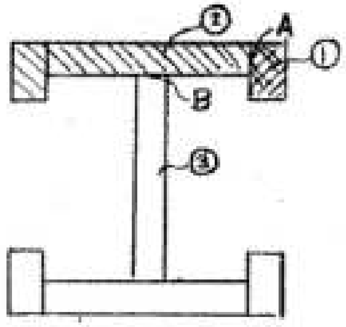
$q = \frac{VQ}{I} = \frac{(6\times10^3)(375\times10^{-6})}{92.71\times10^{-6}} = 24.27 \times 10^3 \ \text{N/m}$ $\qquad s = 60 \ \text{mm} = 60\times10^{-3} \ \text{m}$

$2F_{nail} = qs$ $\qquad F_{nail} = \frac{1}{2} qs = \frac{1}{2}(24.27\times10^3)(60\times10^{-3}) = 728 \ \text{N}$ ◄

Problem 6.32



6.32 The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (*a*) at A, (*b*) at B. (*Given*: $I_x = 1.504 \times 10^9\ \text{mm}^4$.)



$I_x = 1.504 \times 10^9\ \text{mm}^4 = 1504 \times 10^{-6}\ \text{m}^4$

$S_A = 60\ \text{mm} = 0.060\ \text{m}$

$S_B = 25\ \text{mm} = 0.025\ \text{m}$

(a) $Q_A = Q_1 = A_1\bar{y}_1 = (50)(100)(150) = 750 \times 10^3\ \text{mm}^3$
$= 750 \times 10^{-6}\ \text{m}^3$

$F_A = q_A S_A$

$= \dfrac{VQ_1 S_A}{I} = \dfrac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}}$

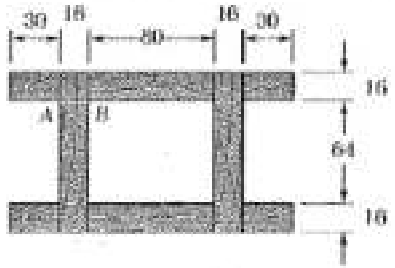
$= 239\ \text{N}$ ◄

(b) $Q_2 = A_2\bar{y}_2 = (300)(50)(175) = 2625 \times 10^3\ \text{mm}^3$

$Q_B = 2Q_1 + Q_2 = 4125 \times 10^3\ \text{mm}^3$
$= 4125 \times 10^{-6}\ \text{m}^3$

$F_B = q_B S_B = \dfrac{VQ_B S_B}{I} = \dfrac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-6}}$ $\qquad F_B = 549\ \text{N}$ ◄

Problem 6.33



Dimensions in millimeters

6.33 The built-up beam was made by gluing together several wooden planks. Knowing that the beam is subjected to a 5-kN shear, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

$b_1 = 16 \text{ mm}, \quad h_1 = 16 + 64 + 16 = 96 \text{ mm}$

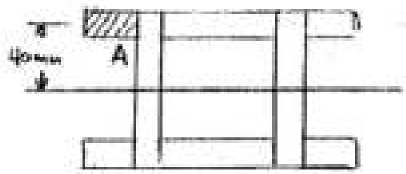
$I_1 = \frac{1}{12} b_1 h_1^3 = 1.17965 \times 10^6 \text{ mm}^4$

$b_2 = 30 + 80 + 30 = 140 \text{ mm}, \quad h_2 = 16 \text{ mm}$

$A_2 = b_2 h_2 = 2240 \text{ mm}^2 \qquad d_2 = \frac{64}{2} + \frac{16}{2} = 40 \text{ mm}$

$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = 3.63179 \times 10^6 \text{ mm}^4$

$I = 2I_1 + 2I_2 = 9.6229 \times 10^6 \text{ mm}^4 = 9.6229 \times 10^{-6} \text{ m}^4$



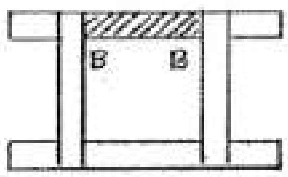
(a) $A_A = (30)(16) = 480 \text{ mm}^2, \quad \bar{y}_A = 40 \text{ mm}$

$Q_A = A_A \bar{y}_A = 19.2 \times 10^3 \text{ mm}^3 = 19.2 \times 10^{-6} \text{ m}^3$

$t_A = 16 \text{ mm} = 0.016 \text{ m}$

$\tau_A = \frac{VQ_A}{It} = \frac{(5 \times 10^3)(19.2 \times 10^{-6})}{(9.6229 \times 10^{-6})(0.016)}$

$= 624 \times 10^3 \text{ Pa} \qquad \tau_A = 624 \text{ kPa}$



(b) $A_B = (80)(16) = 1280 \text{ mm}^2, \quad \bar{y}_B = 40 \text{ mm}$

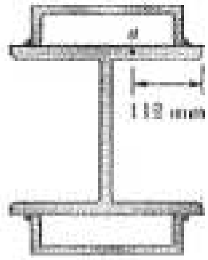
$Q_B = A_B \bar{y}_B = 51.2 \times 10^3 \text{ mm}^3 = 51.2 \times 10^{-6} \text{ m}^3$

$t_B = (2)(16) = 32 \text{ mm} = 0.032 \text{ m}$

$\tau_B = \frac{VQ_B}{It_B} = \frac{(5 \times 10^3)(51.2 \times 10^{-6})}{(9.6229 \times 10^{-6})(0.032)} = 831 \times 10^3 \text{ Pa} \qquad \tau_B = 831 \text{ kPa}$

Problem 6.34

6.34 The composite beam shown is made by welding C200 × 17.1 rolled-steel channels to the flanges of a W250 × 80 wide-flange rolled-steel shape. Knowing that the beam is subjected to a vertical shear of 200 kN, determine (*a*) the horizontal shearing force per meter at each weld, (*b*) the shearing stress at point *a* of the flange of the wide-flange shape.



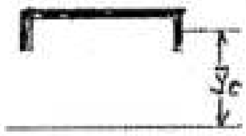
For W250 × 80: $d = 256\text{ mm}$, $t_f = 15.6\text{ mm}$, $I_x = 126\times10^6\text{ mm}^4$

For C200 × 17.1: $A = 2170\text{ mm}^2$, $b_f = 57\text{ mm}$, $t_f = 9.9\text{ mm}$,

$I_y = 0.538\times10^6\text{ mm}^4$, $\bar{x} = 14.4\text{ mm}$

For the channel in the composite beam,

$$\bar{y}_c = \frac{256}{2} + 57 - 14.4 = 170.6\text{ mm}$$



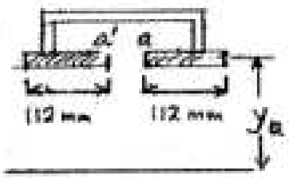
For the composite beam,

$$I = 126\times10^6 + 2\left[0.538\times10^6 + (2170)(170.6)^2\right]$$
$$= 253.39\times10^6\text{ mm}^4 = 253.39\times10^{-6}\text{ m}^4$$

(a) For the two welds,

$$Q_w = A\bar{y}_c = (2170)(170.6)$$
$$= 370.20\times10^3\text{ mm}^3 = 370.20\times10^{-6}\text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(200\times10^3)(370.20\times10^{-6})}{253.39\times10^{-6}}$$
$$= 292\times10^3\text{ N/m}$$



For one weld, $\frac{q}{2} = 146.1\times10^3\text{ N/m}$

<u>Shearing force per meter of weld</u>: 146.1 kN/m ◄

(b) For cuts at a and a' together,

$A_a = 2(112)(15.6) = 3494.4\text{ mm}^2$ $\quad \bar{y}_a = \frac{256}{2} - \frac{15.6}{2} = 120.2\text{ mm}$

$Q_a = 370.20\times10^3 + (3494.4)(120.2) = 790.23\times10^3\text{ mm}^3 = 790.23\times10^{-6}\text{ m}^3$

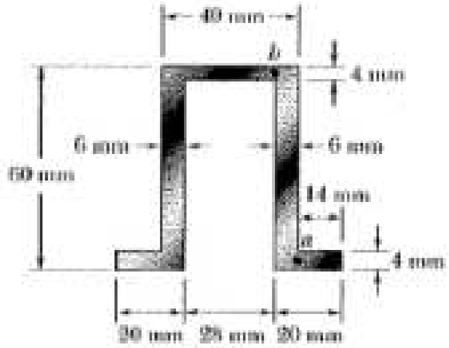
Since there are cuts at a and a', $t = 2t_f = (2)(15.6) = 31.2\text{ mm} = 0.0312\text{ m}$.

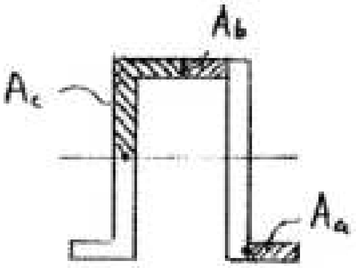
$$\tau_a = \frac{VQ_a}{It} = \frac{(200\times10^3)(790.23\times10^{-6})}{(253.39\times10^{-6})(0.0312)} = 19.99\times10^6\text{ Pa}$$

$\tau_a = 19.99\text{ MPa}$ ◄

Problem 6.35

6.35 Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress at (*a*) point *a*, (*b*) point *b*.





Neutral axis lies 30 mm above bottom.

$$\tau_c = \frac{VQ_c}{It}, \quad \tau_a = \frac{VQ_a}{It_a}, \quad \tau_b = \frac{VQ_b}{It_b}$$

$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a}, \quad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$

$Q_c = (6)(30)(15) + (14)(4)(28) = 4260 \text{ mm}^3$

$t_c = 6 \text{ mm}$

$Q_a = (14)(4)(28) = 1568 \text{ mm}^3$

$t_a = 4 \text{ mm}$

$Q_b = (14)(4)(28) = 1568 \text{ mm}^3$

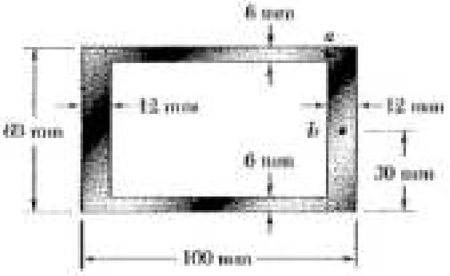
$t_b = 4 \text{ mm}$

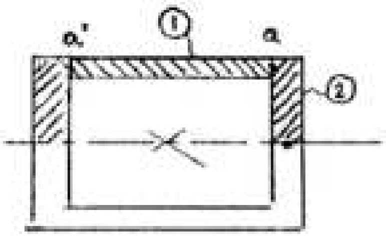
$\tau_c = 75 \text{ MPa}$

$$\tau_a = \frac{Q_a}{Q_c} \cdot \frac{t_c}{t_a} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

$$\tau_b = \frac{Q_b}{Q_c} \cdot \frac{t_c}{t_b} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

Problem 6.36





6.36 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 40 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

$$I = \frac{1}{12}(100)(60)^3 - \frac{1}{12}(100-25)(60-12)^3$$

$$= 1108800 \text{ mm}^4$$

(a) $Q_a = A_1 \bar{y}_1 = (100-25)(6)(30 - \frac{6}{2})$

$= 12150 \text{ mm}^3$

$t_a = (2)(6) = 12 \text{ mm}$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(40\times10^3)(12150\times10^{-9})}{(1108800\times10^{-12})(0.012)}$$

$\tau_a = 36.5 \text{ MPa}$

(b) $Q_b = 12150 + (2)(12)(30)(\frac{30}{2})$

$= 22950 \text{ mm}^3$

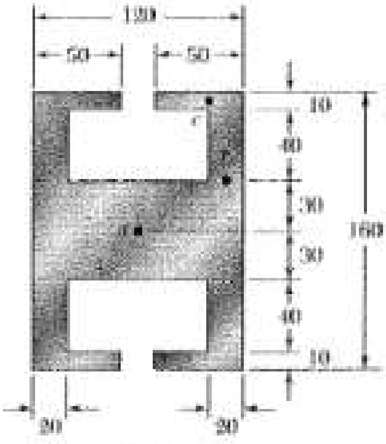
$t_b = (2)(12) = 24 \text{ mm}$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(40\times10^3)(22950\times10^{-9})}{(1108800\times10^{-12})(0.024)}$$

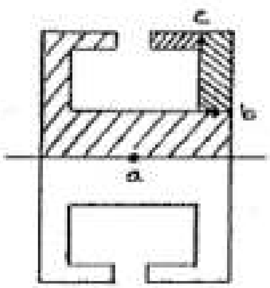
$\tau_b = 34.5 \text{ MPa}$

Problem 6.39

6.39 Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa an extruded beam having the cross section shown, determine the shearing stress at the three points indicated.



Dimensions in mm



$\tau = \dfrac{VQ}{It}$ $\qquad \tau$ is proportional to Q/t.

Point c: $\quad Q_c = (30)(10)(75) = 22.5 \times 10^3 \text{ mm}^3$

$t_c = 10 \text{ mm}$

$Q_c/t_c = 2250 \text{ mm}^2$

Point b: $\quad Q_b = Q_c + (20)(50)(55) = 77.5 \times 10^3 \text{ mm}^3$

$t_b = 20 \text{ mm}$

$Q_b/t_b = 3875 \text{ mm}^2$

Point a: $\quad Q_a = 2Q_b + (120)(30)(15) = 209 \times 10^3 \text{ mm}^3$

$t_a = 120 \text{ mm}$

$Q_a/t_a = 1741.67 \text{ mm}^2$

$(Q/t)_m$ occurs at b. $\qquad \tau_m = \tau_b = 75 \text{ MPa}$

$$\frac{\tau_a}{Q_a/t_a} = \frac{\tau_b}{Q_b/t_b} = \frac{\tau_c}{Q_c/t_c}$$

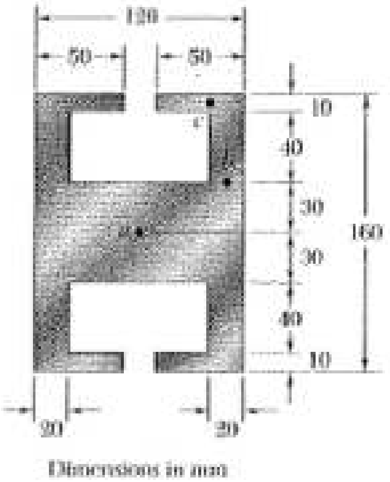
$$\frac{\tau_a}{1741.67 \text{ mm}^2} = \frac{75 \text{ MPa}}{3875 \text{ mm}^2} = \frac{\tau_c}{2250 \text{ mm}^2}$$

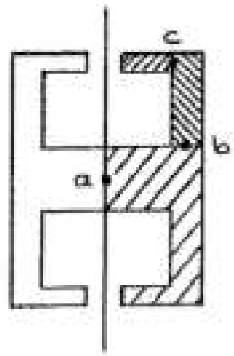
$\tau_a = 33.7 \text{ MPa}$ ◄

$\tau_b = 75.0 \text{ MPa}$ ◄

$\tau_c = 43.5 \text{ MPa}$ ◄

Problem 6.40





6.40 Solve Prob. 6.39 assuming that the beam is subjected to a horizontal shear **V**.

6.39 Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa an extruded beam having the cross section shown, determine the shearing stress at the three points indicated.

$$\tau = \frac{VQ}{It} \qquad \tau \text{ is proportional to } Q/t$$

Point c: $Q_c = (10)(30)(25) = 7.5 \times 10^3 \text{ mm}^3$

$t_c = 10 \text{ mm}$

$Q_c/t_c = 750 \text{ mm}^2$

Point b: $Q_b = Q_c + (50)(20)(50) = 57.5 \times 10^3 \text{ mm}^3$

$t_b = 20 \text{ mm}$

$Q_b/t_b = 2875 \text{ mm}^2$

Point a: $Q_a = 2Q_b + (60)(60)(30) = 223 \times 10^3 \text{ mm}^3$

$t_a = 60 \text{ mm}$

$Q_a/t_a = 3716.7 \text{ mm}^2$

$(Q/t)_m$ occurs at a. $\qquad \tau_m = \tau_a = 75 \text{ MPa}$

$$\frac{\tau_a}{Q_a/t_a} = \frac{\tau_b}{Q_b/t_b} = \frac{\tau_c}{Q_c/t_c}$$

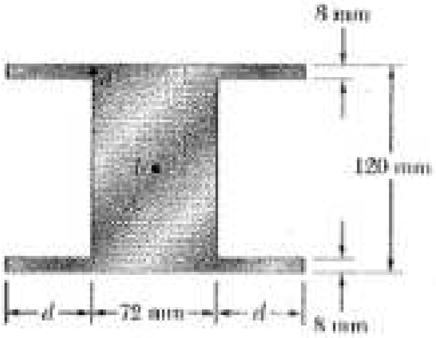
$$\frac{75 \text{ MPa}}{3716.7 \text{ mm}^2} = \frac{\tau_b}{2875 \text{ mm}^2} = \frac{\tau_c}{750 \text{ mm}^2}$$

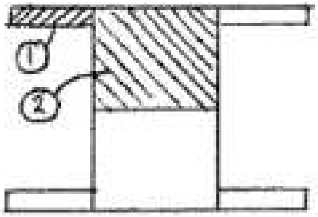
$\tau_a = 75.0 \text{ MPa}$ ◄

$\tau_b = 58.0 \text{ MPa}$ ◄

$\tau_c = 15.13 \text{ MPa}$ ◄

Problem 6.42





6.42 The vertical shear is 25 kN in a beam having the cross section shown. Determine (*a*) the distance *d* for which $\tau_a = \tau_b$, (*b*) the corresponding shearing stress at points *a* and *b*.

$Q_1 = (d)(8)(56) = 448\,d \text{ mm}^3$

$Q_2 = (72)(60)(30) = 129.6 \times 10^3 \text{ mm}^3$

$Q_a = Q_1 = 448\,d \qquad t_a = 8 \text{ mm}$

$Q_b = 2Q_1 + Q_2 = 896\,d + 129.6 \times 10^3 \text{ mm}^3$

$t_b = 72 \text{ mm}$

$\tau_a = \dfrac{VQ_a}{It_a} \qquad \tau_b = \dfrac{VQ_b}{It_b}$

Since $\tau_a = \tau_b \qquad \dfrac{Q_a}{t_a} = \dfrac{Q_b}{t_b}$

(a) $\dfrac{448\,d}{8} = \dfrac{896\,d + 129.4 \times 10^3}{72} \qquad d = 41.3 \text{ mm}$ ◄

(b) $I_1 = \frac{1}{12}(d)(8)^3 + (d)(8)(56)^2 = 25{,}131\,d = 1.03696 \times 10^6 \text{ mm}^4$

$I_2 = \frac{1}{3}(72)(60)^3 = 5.184 \times 10^6 \text{ mm}^4$

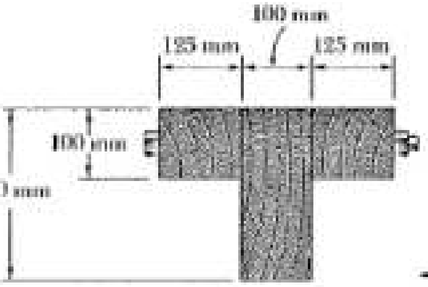
$I = 4I_1 + 2I_2 = 14.5158 \times 10^6 \text{ mm}^4 = 14.5158 \times 10^{-6} \text{ m}^4$

$Q_a = 448\,d = (448)(41.263) = 18.4857 \times 10^3 \text{ mm}^3 = 18.4857 \times 10^{-6} \text{ m}^3$

$\tau_a = \dfrac{VQ_a}{It_a} = \dfrac{(25 \times 10^3)(18.4857 \times 10^{-6})}{(14.5158 \times 10^{-6})(8 \times 10^{-3})} = 3.98 \times 10^6 \text{ Pa}$

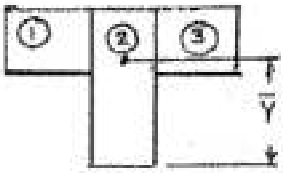
$\tau_a = 3.98 \text{ MPa}$ ◄

Problem 6.43



6.43 Three planks are connected as shown by bolts of 14-mm diameter spaced every 150 mm along the longitudinal axis of the beam. For a vertical shear of 10 kN, determine the average shearing stress in the bolts.

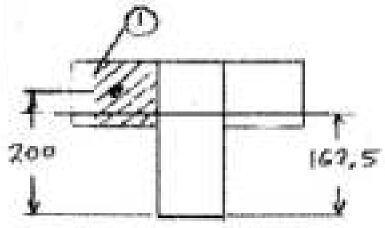
Locate neutral axis and compute moment of inertia.



PART	$A\,(mm^2)$	$\bar{y}\,(mm)$	$A\bar{y}\,(mm^3)$	$d\,(mm)$	$Ad^2\,(mm^4)$	$\bar{I}\,(mm^4)$
①	12 500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
②	25 000	125	3.125×10^6	37.5	35.156×10^6	130.208×10^6
③	12 500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
Σ	50 000		8.125×10^6		70.312×10^6	151.041×10^6

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.125\times10^6}{50\times10^3} = 162.5\text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 221.35\times10^6\text{ mm}^4 = 221.35\times10^{-6}\text{ m}^4$$



$$Q = A_1\bar{y}_1 = (12500)(37.5) = 468.75\times10^3\text{ mm}^3 = 468.75\times10^{-6}\text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(10\times10^3)(468.75\times10^{-6})}{221.35\times10^{-6}} = 21.177\times10^3\text{ N}$$

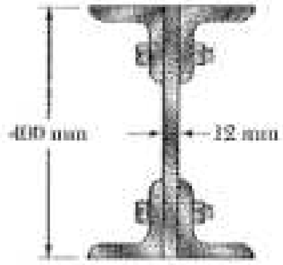
$$F_{bolt} = qs = (21.177\times10^3)(150\times10^{-3}) = 3.1765\times10^3\text{ N}$$

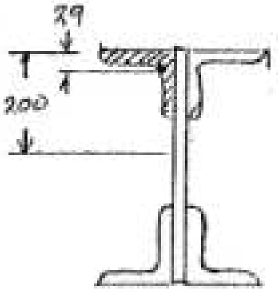
$$A_{bolt} = \frac{\pi}{4}(14)^2 = 153.938\text{ mm}^2 = 153.938\times10^{-6}\text{ m}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{3.1765\times10^3}{153.938\times10^{-6}} = 20.6\times10^6\text{ Pa}$$

$$\tau_{bolt} = 20.6\text{ MPa}$$

Problem 6.45





6.45 Four L102 × 102 × 9.5 steel angle shapes and a 12 × 400-mm steel plate are bolted together to form a beam with the cross section shown. The bolts are of 22-mm diameter and are spaced longitudinally every 120 mm. Knowing that the beam is subjected to a vertical shear of 240 kN, determine the average shearing stress in each bolt.

Angle: $A = 1850 \text{ mm}^2, \quad \bar{I} = 1.83 \times 10^6 \text{ mm}^4, \quad y = 29 \text{ mm}$

$d = 200 - 29 = 171 \text{ mm}$

$I_a = \bar{I} + Ad^2 = 55.926 \times 10^6 \text{ mm}^4$

Plate: $I_p = \frac{1}{12}(12)(400)^3 = 64 \times 10^6 \text{ mm}^4$

$I = 4I_a + I_p = 287.7 \times 10^6 \text{ mm}^4 = 287.7 \times 10^{-6} \text{ m}^4$

$Q = (1850)(171) = 316.35 \times 10^3 \text{ mm}^3 = 316.35 \times 10^{-6} \text{ m}^3$

$$q = \frac{VQ}{I} = \frac{(240 \times 10^3)(316.35 \times 10^{-6})}{287.7 \times 10^{-6}} = 263.9 \times 10^3 \text{ N/m}$$

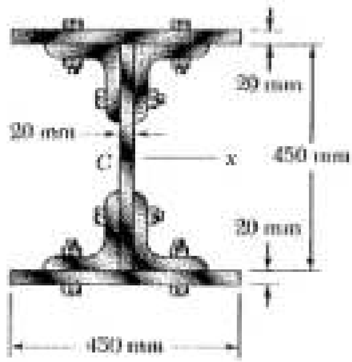
$F_{bolt} = qs = (263.9 \times 10^3)(120 \times 10^{-3}) = 31.668 \times 10^3 \text{ N}$

$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(22)^2 = 380.13 \text{ mm}^2 = 380.13 \times 10^{-6} \text{ m}^2$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{31.668 \times 10^3}{380.13 \times 10^{-6}} = 83.3 \times 10^6 \text{ Pa}$$

$\tau_{bolt} = 83.3 \text{ MPa}$

Problem 6.46



6.46 Two 20 × 450-mm steel plates are bolted to four L 152 × 152 × 19.0 angles to form a beam with the cross section shown. The bolts have a 22-mm diameter and are spaced longitudinally every 125 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shear in the beam. (*Given:* $I_x = 1896 \times 10^6$ mm^4).

Flange: $I_f = \frac{1}{12}(450)(20)^3 + (450)(20)(235)^2 = 497.3 \times 10^6 \text{ mm}^4$

Web: $I_w = \frac{1}{12}(20)(450)^3 = 151.9 \times 10^6 \text{ mm}^4$

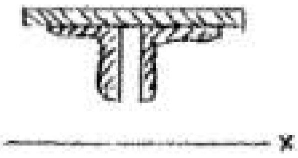
Angle: $\bar{I} = 11.6 \times 10^6 \text{ m}^4$, $A = 5420 \text{ mm}^2$

$y = 44.9$ mm $\quad d = 225 - 44.9 = 180.1$ mm

$I_a = \bar{I} + Ad^2$

$= 11.6 \times 10^6 + (5420)(180.1)^2 = 187.4 \times 10^6 \text{ mm}^4$

$I = 2I_f + I_w + 4I_a = 1896 \times 10^6 \text{ mm}^4 = 1896 \times 10^{-6} \text{ m}^4$



$Q_f = (450)(20)(235) = 2115 \times 10^3 \text{ mm}^3$

$Q_a = (5420)(180.1) = 976 \times 10^3 \text{ mm}^3$

$Q = Q_f + 2Q_a = 4067 \times 10^3 \text{ mm}^3 = 4067 \times 10^{-6} \text{ m}^3$

$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(22)^2 = 380.1 \text{ mm}^2 = 380.1 \times 10^{-6} \text{ m}^2$

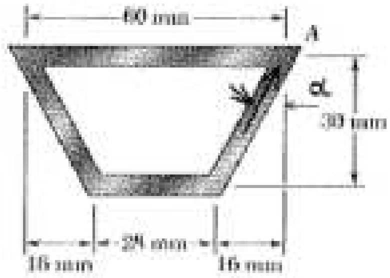
$F_{bolt} = 2\tau_{bolt} A_b = (2)(90 \times 10^6)(380.1 \times 10^{-6}) = 68.42 \times 10^3 \text{ N}$

$q_{all} = \frac{F_{bolt}}{s} = \frac{68.42 \times 10^3}{0.125} = 547.36 \times 10^3 \text{ N/m}$

$q = \frac{VQ}{I} \qquad V_{all} = \frac{I q_{all}}{Q} = \frac{(1896 \times 10^{-6})(547.36 \times 10^3)}{4067 \times 10^{-6}} = 255 \times 10^3 \text{ N}$

$= 255 \text{ kN}$ ◄

Problem 6.48



6.48 An extruded beam has the cross section shown and a uniform wall thickness of 3 mm. For a vertical shear of 10 kN, determine *(a)* the shearing stress at point *A*, *(b)* the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.

$\tan\alpha = \frac{16}{30}$ $\quad \alpha = 28.07°$

Side: $A = (3\sec\alpha)(30) = 102 \text{ mm}^2$

$\bar{I} = \frac{1}{12}(3\sec\alpha)(30)^3 = 7.6498 \times 10^3 \text{ mm}^4$

Part	A (mm²)	$\bar{y}_0$ (mm)	$A\bar{y}$ (10^3 mm³)	d (mm)	Ad^2 (10^3 mm⁴)	$\bar{I}$ (10^3 mm⁴)
Top	180	30	5.4	11.932	25.627	neglect
Side	102	15	1.53	3.077	0.966	7.6498
Side	102	15	1.53	3.077	0.966	7.6498
Bot	84	0	0	18.077	27.449	neglect
Σ	468		8.46		55.008	15.2996

$$\bar{Y}_0 = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.46 \times 10^3}{468} = 18.077 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 70.31 \times 10^3 \text{ mm}^4 = 70.31 \times 10^{-9} \text{ m}^4$$

(a) $\quad Q_A = (180)(11.932) = 2.14776 \times 10^3 \text{ mm}^3 = 2.14776 \times 10^{-6} \text{ m}^3$

$t = (2)(3 \times 10^{-3}) = 6 \times 10^{-3} \text{ m}$

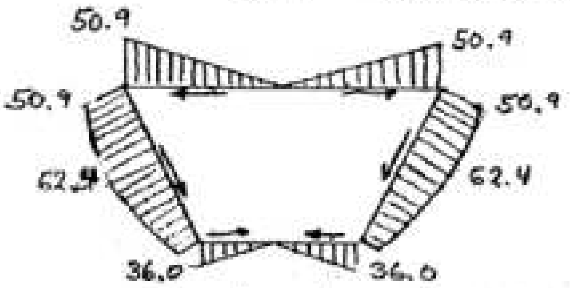
$$\tau_A = \frac{VQ}{It} = \frac{(10 \times 10^3)(2.14776 \times 10^{-6})}{(70.31 \times 10^{-9})(6 \times 10^{-3})} = 50.9 \times 10^6 \text{ Pa} \qquad \tau_A = 50.9 \text{ MPa}$$

(b)

$$Q_m = Q_A + (2)(3\sec\alpha)(11.932)(\tfrac{1}{2} \times 11.932)$$
$$= 2.14776 \times 10^3 + 484.06 = 2.6318 \times 10^3 \text{ mm}^3$$
$$= 2.6318 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau_m = \frac{VQ_m}{It} = \frac{(10 \times 10^3)(2.6318 \times 10^{-6})}{(70.31 \times 10^{-9})(6 \times 10^{-3})} = 62.4 \times 10^6 \text{ Pa} \qquad \tau_m = 62.4 \text{ MPa}$$



Shearing stress (MPa)

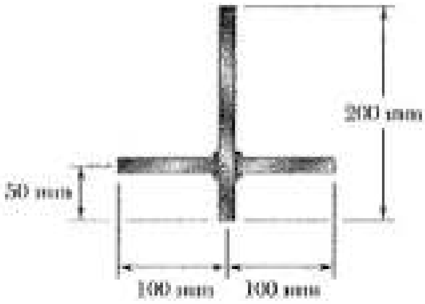
$$Q_B = (28)(3)(18.077) = 1.51847 \times 10^3 \text{ mm}^3$$

$$\tau_B = \frac{Q_B}{Q_A}\tau_A = \frac{1.51847 \times 10^3}{2.14776 \times 10^3}(50.9)$$

$$= 36.0 \text{ MPa}$$

Problem 6.49

6.49 Three plates, each 12 mm thick, are welded together to form the section shown. For a vertical shear of 100 kN, determine the shear flow through the welded surfaces and sketch the shear flow in the cross section.



Locate neutral axis.

$$\Sigma A = (12)(200) + (2)(94)(12) = 4656 \text{ mm}^2$$

$$\Sigma A\bar{y} = (12)(200)(100) + (2)(94)(12)(50) = 352.8 \times 10^3 \text{ mm}^3$$

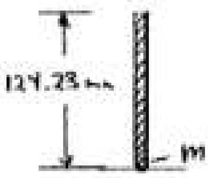
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 75.77 \text{ mm}$$

$$I = \tfrac{1}{12}(12)(200)^3 + (12)(200)(24.23)^2 + 2\left[\tfrac{1}{12}(94)(12)^3 + (94)(12)(25.77)^2\right] = 10.934 \times 10^6 \text{ mm}^4 = 10.934 \times 10^{-6} \text{ m}^4$$

$$Q = (94)(12)(25.77) = 29.07 \times 10^3 \text{ mm}^3 = 29.07 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(100 \times 10^3)(29.07 \times 10^{-6})}{10.934 \times 10^{-6}} = 266 \times 10^3 \text{ N/m} = 266 \text{ kN/m}$$

The maximum shear flow in the cross section occurs at the neutral axis.

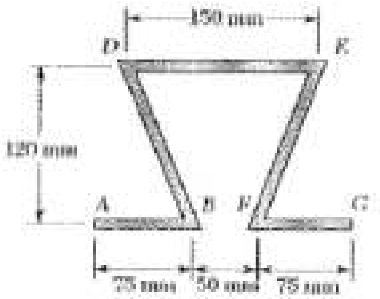


$$Q_m = (12)(124.33)\left(\frac{124.33}{2}\right) = 92.748 \times 10^3 \text{ mm}^3 = 92.748 \times 10^{-6} \text{ m}^3$$

$$q_m = \frac{VQ_m}{I} = \frac{(100 \times 10^3)(92.748 \times 10^{-6})}{(10.934 \times 10^{-6})} = 848 \times 10^3 \text{ N/m}$$

$$q_m = 848 \text{ kN/m}$$

Problem 6.50



6.50 A plate of thickness t is bent as shown and then used as a beam. For a vertical shear of 2.4 kN, determine (*a*) the thickness t for which the maximum shearing stress is 2 MPa, (*b*) the corresponding shearing stress at point E. Also sketch the shear flow in the cross section.

$$L_{BD} = L_{EF} = \sqrt{120^2 + 50^2} = 130 \text{ mm}.$$

Neutral axis lies at 60 mm above AB.

Calculate I.

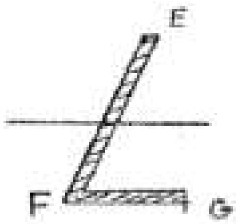
$$I_{AB} = (75t)(60)^2 = 270000t$$
$$I_{BD} = \tfrac{1}{12}(130t)(120)^2 = 156000t$$
$$I_{DE} = (150t)(60)^2 = 540000t$$
$$I_{EF} = I_{DB} = 156000t$$
$$I_{FG} = I_{AB} = 270000t$$

$$I = \Sigma I = 1392000t$$

(a) At point C: $\quad Q_C = Q_{AB} + Q_{BC} = (75t)(60) + (65t)(30) = 6450t$

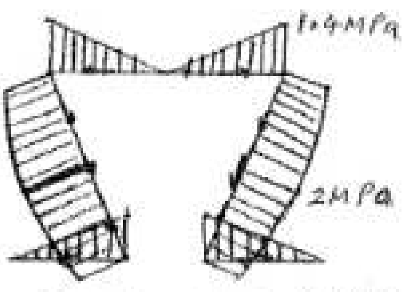
$$\tau = \frac{VQ_C}{It} \quad \therefore \quad t = \frac{VQ}{\tau I} = \frac{(2400)(6450t)}{(2)(1392000t)} = 5.56 \text{ mm}$$

(b) $I = (1392000)(5.56) = 7739520 \text{ mm}^4$



$$Q_E = Q_{EF} + Q_{FG}$$
$$= 0 + (75)(5.56)(60) = 25020 \text{ mm}^3$$

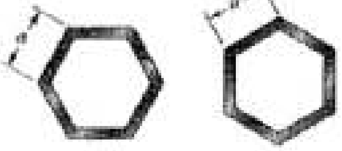
$$\tau_E = \frac{VQ_E}{It} = \frac{(2400)(25020)}{(7739520)(5.56)} = 1.4 \text{ MPa}$$



Shearing stress (MPa)

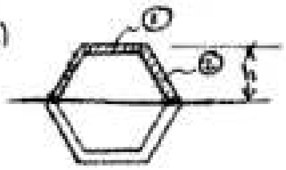
Problem 6.51

6.51 and 6.52 An extruded beam has a uniform wall thickness t. Denoting by V the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{max} = k(V/A)$ and determine the constant k for each of the two orientations shown.



(a) (b)

(a)



$$h = \frac{\sqrt{3}}{2}a$$

$$A_1 = A_2 = at$$

$$I_1 = A_1 h^2 = a t h^2 = \frac{3}{4}a^3 t$$

$$I_2 = \frac{1}{3}A_2 h^2 = \frac{1}{3}at\frac{3}{4}a^2 = \frac{1}{4}a^3 t$$

$$I = 2I_1 + 4I_2 = \frac{5}{2}a^3 t$$

$$Q_1 = A_1 h = \frac{\sqrt{3}}{2}a^2 t$$

$$Q_2 = A_2 \frac{h}{2} = \frac{\sqrt{3}}{4}a^2 t$$

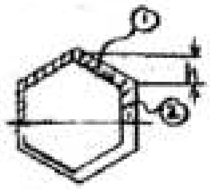
$$Q_m = Q_1 + 2Q_2 = \sqrt{3}a^2 t$$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V\sqrt{3}a^2 t}{(\frac{5}{2}a^3 t)2t} = \frac{\sqrt{3}}{5}\frac{V}{at}$$

$$= \frac{6\sqrt{3}}{5}\frac{V}{6at} = \frac{6\sqrt{3}}{5}\frac{V}{A} = k\frac{V}{A}$$

$$k = \frac{6\sqrt{3}}{5} = 2.08$$

(b)



$$h = \frac{a}{2}$$

$$A_1 = at \qquad A_2 = \frac{1}{2}at$$

$$I_1 = \bar{I}_1 + A_1 d^2$$

$$= \frac{1}{12}at h^2 + at\left(\frac{a}{2}+\frac{h}{2}\right)^2$$

$$= \frac{1}{48}a^3 t + \frac{9}{16}a^3 t = \frac{7}{12}a^3 t$$

$$I_2 = \frac{1}{3}t\left(\frac{a}{2}\right)^3 = \frac{1}{24}a^3 t$$

$$I = 4I_1 + 4I_2 = \frac{5}{2}a^3 t$$

$$Q_1 = at\left(\frac{a}{2}+\frac{h}{2}\right) = \frac{3}{4}a^2 t$$

$$Q_2 = \left(\frac{1}{2}at\right)\left(\frac{a}{4}\right) = \frac{1}{8}a^2 t$$

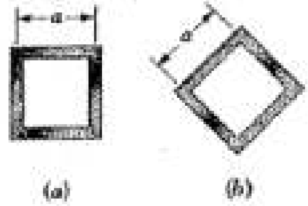
$$Q = 2Q_1 + 2Q_2 = \frac{7}{4}a^3 t$$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V\cdot\frac{7}{4}a^2 t}{(\frac{5}{2}a^3 t)(2t)}$$

$$= \frac{7}{20}\frac{V}{at} = \frac{42}{20}\frac{V}{6at} = \frac{21}{10}\frac{V}{A}$$

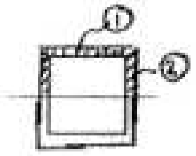
$$= k\frac{V}{A} \qquad k = \frac{21}{10} = 2.10$$

Problem 6.52



6.51 and 6.52 An extruded beam has a uniform wall thickness t. Denoting by **V** the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{max} = k(V/A)$ and determine the constant k for each of the two orientations shown.

(a)



$$I_1 = (at)\left(\frac{a}{2}\right)^2 = \frac{1}{4}a^3t$$

$$I_2 = \frac{1}{3}t\left(\frac{a}{2}\right)^3 = \frac{1}{24}a^3t$$

$$I = 2I_1 + 4I_2 = \frac{2}{3}a^3t$$

$$Q_1 = (at)\left(\frac{a}{2}\right) = \frac{1}{2}a^2t$$

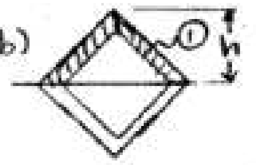
$$Q_2 = \left(\frac{1}{2}at\right)\left(\frac{a}{4}\right) = \frac{1}{8}a^2t$$

$$Q = Q_1 + 2Q_2 = \frac{3}{4}a^2t$$

$$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V(\frac{3}{4}a^2t)}{(\frac{2}{3}a^3t)(2t)} = \frac{9}{16}\frac{V}{at} = \frac{9}{4}\frac{V}{4at} = \frac{9}{4}\frac{V}{A}$$

$$= k\frac{V}{A} \quad \therefore \quad k = \frac{9}{4} = 2.25$$

(b)



$$h = \frac{1}{2}\sqrt{2}\,a$$

$$I_1 = \frac{1}{3}A_1h^2 = \left(\frac{1}{3}at\right)\left(\frac{\sqrt{2}}{2}a\right)^2 = \frac{1}{6}a^3t$$

$$I = 4I_1 = \frac{2}{3}a^3t$$

$$Q_1 = at\left(\frac{h}{2}\right) = \frac{1}{4}\sqrt{2}\,a^2t$$

$$Q = 2Q_1 = \frac{1}{2}\sqrt{2}\,a^2t$$

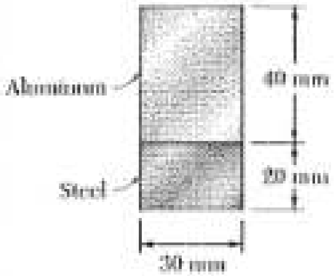
$$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V(\frac{1}{2}\sqrt{2}\,a^2t)}{(\frac{2}{3}a^3t)(2t)}$$

$$= \frac{3\sqrt{2}}{8}\frac{V}{at} = \frac{3\sqrt{2}}{2}\frac{V}{4at}$$

$$= \frac{3\sqrt{2}}{2}\frac{V}{A} = k\frac{V}{A}$$

$$k = \frac{3\sqrt{2}}{2} = 2.12$$

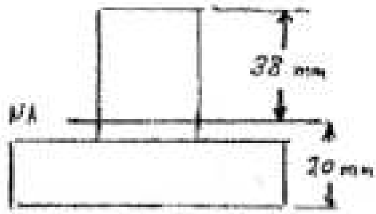
Problem 6.56



6.56 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 20 kN and that the modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint*: Use the method indicated in Prob. 6.55.)

$n = 1$ in aluminum

$n = \frac{210}{70} = 3.0$ in steel



Locate centroid and compute moment of inertia of transformed section.

Part	A, mm²	nA, mm²	$\bar{y}$, mm	$nA\bar{y}$, mm³	d, mm	nAd^2, mm⁴	$n\bar{I}$, mm⁴
Alum.	1200	1200	40	48 000	18	388.8×10^3	160×10^3
Steel.	600	1800	10	18 000	12	259.2×10^3	60×10^3
		3000		66000		648×10^3	220×10^3

$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma nA} = \frac{66000}{3000} = 22 \text{ mm}$$

$$I = \Sigma n\bar{I} + \Sigma nAd^2 = 868 \times 10^3 \text{ mm}^4 = 868 \times 10^{-9} \text{ m}^4$$

(a) At the bonded surface, $Q = (30)(40)(18) = 21.6 \times 10^3 \text{ mm}^3 = 21.6 \times 10^{-6} \text{ m}^3$

$$\tau = \frac{VQ}{It} = \frac{(20 \times 10^3)(21.6 \times 10^{-6})}{(868 \times 10^{-9})(0.030)} = 16.59 \times 10^6 \text{ Pa}$$

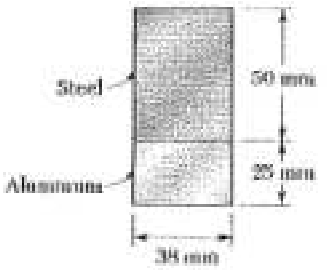
$$\tau = 16.59 \text{ MPa}$$

(b) At the neutral axis, $Q = (30)(38)(\frac{38}{2}) = 21.66 \times 10^3 \text{ mm}^3 = 21.66 \times 10^{-6} \text{ m}^3$

$$\tau = \frac{VQ}{It} = \frac{(20 \times 10^3)(21.66 \times 10^{-6})}{(868 \times 10^{-9})(0.030)} = 16.64 \times 10^6 \text{ Pa}$$

$$\tau = 16.64 \text{ MPa}$$

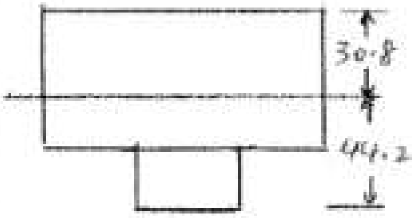
Problem 6.57



6.57 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 16 kN and that the modulus of elasticity is 200 GPa for the steel and 73 GPa for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

$n = 1$ in aluminum $\qquad n = \frac{200}{73} = 2.74$ in steel

Part	nA (mm²)	$\bar{y}$ (mm)	$nA\bar{y}$ (mm³)	d (mm)	nAd^2 (mm⁴)	$n\bar{I}$ (mm⁴)
Steel	5206	50	260300	5.8	175130	1084583
Alum.	950	12.5	11875	31.7	954646	49479
Σ	6156		272175		1129776	1134062



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{272175}{6156} = 44.2 \text{ mm}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 2263838 \text{ mm}^4$$

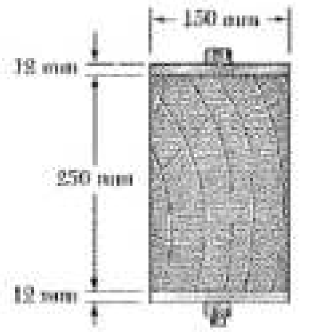
(a) At the bonded surface: $\quad Q = (950)(31.7) = 30115 \text{ mm}^3$

$$\tau = \frac{VQ}{It} = \frac{(16000)(30115)}{(2263838)(38)} = 5.6 \text{ MPa}$$

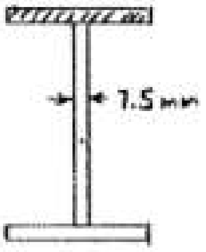
(b) At the neutral axis: $\quad Q = (2.74)(38)(30.8)\left(\frac{30.8}{2}\right) = 49386 \text{ mm}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{(16000)(49386)}{(2263838)(38)} = 9.2 \text{ MPa}$$

Problem 6.58



6.58 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (*a*) the average shearing stress in the bolts, (*b*) the shearing stress at the center of the cross section. (*Hint*: Use the method indicated in Prob. 6.55.)



Let $E_{ref} = E_s = 200\ \text{GPa}$

$$n_s = 1 \qquad n_w = \frac{E_w}{E_s} = \frac{10\ \text{GPa}}{200\ \text{GPa}} = \frac{1}{20}$$

Widths of transformed section:

$$b_s = 150\ \text{mm} \qquad b_w = \left(\tfrac{1}{20}\right)(150) = 7.5\ \text{mm}$$

$$I = 2\left[\tfrac{1}{12}(150)(12)^3 + (150)(12)(125+6)^2\right]$$
$$\tfrac{1}{12}(7.5)(250)^3$$
$$= 2\left[0.0216\times10^6 + 30.890\times10^6\right] + 9.766\times10^6$$
$$71.589\times10^6\ \text{mm}^4 = 71.589\times10^{-6}\ \text{m}^4$$

(a) $$Q_1 = (150)(12)(125+6) = 235.8\times10^3\ \text{mm}^3$$
$$= 235.8\times10^{-6}\ \text{m}^3$$

$$q = \frac{VQ_1}{I} = \frac{(4\times10^3)(235.8\times10^{-6})}{71.589\times10^{-6}} = 13.175\times10^3\ \text{N/m}$$

$$F_{bolt} = qs = (23.187\times10^3)(200\times10^{-3}) = 2.635\times10^3\ \text{N}$$

$$A_{bolt} = \frac{\pi}{4}d_{bolt}^2 = \left(\frac{\pi}{4}\right)(12)^2 = 113.1\ \text{mm}^2 = 113.1\times10^{-6}\ \text{m}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.635\times10^3}{113.1\times10^{-6}} = 23.3\times10^6\ \text{Pa} = 23.3\ \text{MPa}$$ ◄

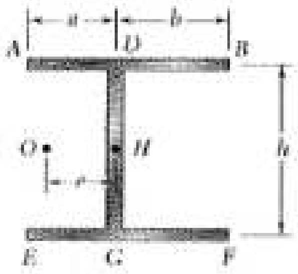
(b) $$Q_2 = Q_1 + (7.5)(125)(62.5) = 235.8\times10^3 + 58.594\times10^3 = 294.4\times10^3\ \text{mm}^3$$
$$= 294.4\times10^{-6}\ \text{m}^3$$

$$t = 150\ \text{mm} = 150\times10^{-3}\ \text{m}$$

$$\tau_c = \frac{VQ_2}{It} = \frac{(4\times10^3)(294.4\times10^{-6})}{(71.589\times10^{-6})(150\times10^{-3})} = 109.7\times10^3\ \text{Pa} = 109.7\ \text{kPa}$$ ◄

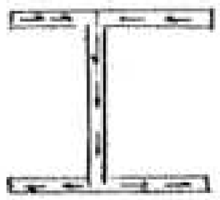
Problem 6.61

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^2$$

$$I_{DG} = \frac{1}{12}th^3 \qquad I = \Sigma I = \frac{1}{12}t(6a+6b+h)h^2$$

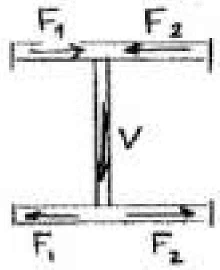


Part AD: $\quad Q = tx\frac{h}{2} = \frac{1}{2}thx$

$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_1 = \int \tau\, dA = \int_0^a \frac{Vhx}{2I}t\,dx = \frac{Vht}{2I}\int_0^a x\,dx$$

$$= \frac{Vht}{2I}\,\frac{x^2}{2}\Big|_0^a = \frac{Vhta^2}{4I}$$

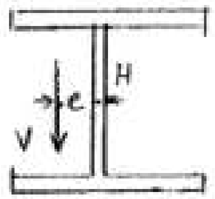


Part BD: $\quad Q = tx\frac{h}{2} = \frac{1}{2}thx$

$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_2 = \int \tau\, dA = \int_0^b \frac{Vhx}{2I}t\,dx = \frac{Vht}{2I}\int_0^b x^2\,dx$$

$$= \frac{Vht}{2I}\,\frac{x^2}{2}\Big|_0^b = \frac{Vhtb^2}{4I}$$



$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H$:

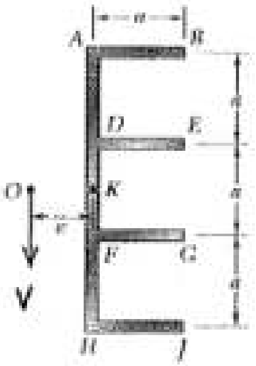
$$Ve = F_2h - F_1h = \frac{Vh^2t(b^2-a^2)}{4I}$$

$$= \frac{Vh^2t(b^2-a^2)}{4\cdot\frac{1}{12}t(6a+6b+h)h^2} = \frac{3V(b^2-a^2)}{6a+6b+h}$$

$$e = \frac{3(b^2-a^2)}{6(a+b)+h}$$

Problem 6.63

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

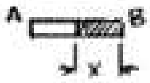


$$I_{AB} = I_{HJ} = at\left(\frac{3a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{9}{4}ta^3$$

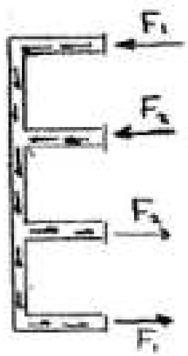
$$I_{DE} = I_{FG} = at\left(\frac{a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{1}{4}ta^3$$

$$I_{AH} = \frac{1}{12}t(3a)^3 = \frac{9}{4}ta^3 \qquad I = \Sigma I = \frac{29}{4}ta^3$$

<u>Part AB:</u> $\quad A = tx \quad \bar{y} = \frac{3a}{2} \quad Q = \frac{3}{2}atx$

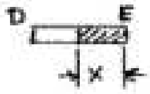


$$\tau = \frac{VQ}{It} = \frac{V\cdot\frac{3}{2}atx}{\frac{29}{4}ta^3t} = \frac{6Vx}{29a^2t}$$



$$F_1 = \int \tau\, dA = \int_0^a \frac{6Vx}{29a^2t}\,t\,dx = \frac{6V}{29a^2}\int_0^a x\,dx = \frac{3}{29}V$$

<u>Part DE:</u> $\quad A = tx \quad \bar{y} = \frac{a}{2} \quad Q = \frac{1}{2}atx$



$$\tau = \frac{VQ}{It} = \frac{V\cdot\frac{1}{2}atx}{\frac{29}{4}ta^3t} = \frac{2Vx}{29a^2t}$$

$$F_2 = \int \tau\, dA = \int_0^a \frac{2Vx}{29a^2t}\,t\,dx = \frac{2V}{29a^2}\int_0^a x\,dx = \frac{1}{29}V$$

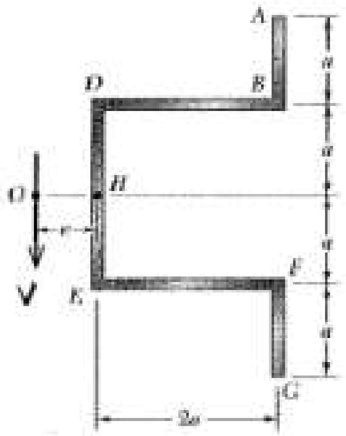
$$\circlearrowright \Sigma M_K = \circlearrowright \Sigma M_K:$$

$$Ve = F_1(3a) + F_2(a) = \frac{9}{29}Va + \frac{1}{29}Va = \frac{10}{29}Va$$

$$e = \frac{10}{29}a = 0.345\,a$$

Problem 6.64

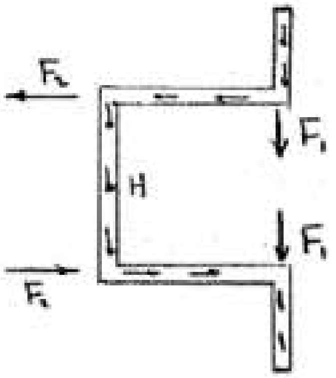
6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = I_{FG} = \frac{1}{12}ta^3 + (ta)\left(\frac{3a}{2}\right)^2 = \frac{7}{3}ta^3$$

$$I_{DB} = I_{EF} = (2at)a^2 + \frac{1}{12}(2a)t^3 \approx 2a^3t$$

$$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3 \qquad I = \Sigma I = \frac{28}{3}ta^3$$



Part AB: $A = t(2a - y) \quad \bar{y} = \frac{2a+y}{2}$

$$Q = A\bar{y} = \frac{1}{2}t(2a-y)(2a+y) = \frac{1}{2}t(4a^2 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}(4a^2 - y^2)$$

$$F_1 = \int \tau\, dA = \int_a^{2a} \frac{V}{2I}(4a^2 - y^2)t\,dy$$

$$= \frac{Vt}{2I}\left(4a^2y - \frac{y^3}{3}\right)\Big|_a^{2a} = \frac{Vta^3}{2I}\left[(4)(2) - \frac{(2)^3}{3} - (4)(1) + \left(\frac{1}{3}\right)\right]$$

$$= \frac{5}{6}\frac{Vta^3}{I} = \frac{5}{56}V$$

Part DB:

$$Q = (ta)\frac{3a}{2} + txa = ta\left(\frac{3a}{2} + x\right)$$

$$\tau = \frac{VQ}{It} = \frac{Va}{I}\left(\frac{3a}{2} + x\right)$$

$$F_2 = \int \tau\, dA = \int_0^{2a} \frac{Va}{I}\left(\frac{3a}{2} + x\right)t\,dx = \frac{Vta}{I}\int_0^{2a}\left(\frac{3a}{2} + x\right)dx$$

$$= \frac{Vta}{I}\left(\frac{3ax}{2} + \frac{x^2}{2}\right)\Big|_0^{2a} = \frac{Vta^3}{I}\left[\frac{(3)(2)}{2} + \frac{(2)^2}{2}\right]$$

$$= 5\frac{Vta^3}{I} = \frac{15}{28}V$$

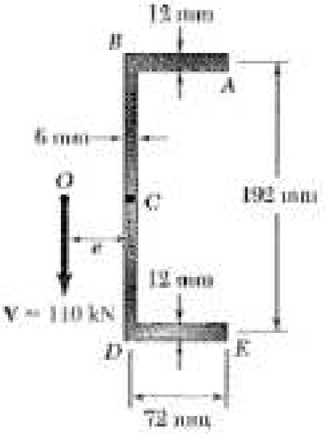
$+\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H$:

$$Ve = F_2(2a) - 2F_1(2a) = \frac{30}{28}Va - \frac{20}{56}Va = \frac{5}{7}Va$$

$$e = \frac{5}{7}a = 0.714\,a$$

Problem 6.67

6.67 and 6.68 For an extruded beam having the cross section shown, determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.



$$I = 2\left[\left(\tfrac{1}{12}\right)(72)(12)^3 + (72)(12)\left(\tfrac{192}{2}\right)^2\right] + \tfrac{1}{12}(6)(192)^3$$
$$= 19.4849 \times 10^6 \text{ mm}^4 = 19.4849 \times 10^{-6} \text{ m}^4$$

Part AB: $\quad A = 12x \qquad Q = A\bar{y} = (12x)\left(\tfrac{192}{2}\right) = 1152x$

$$q = \frac{VQ}{I} = \frac{1152\,Vx}{I}$$

$x = 0$ at point A. $\quad x = l_{AB} = 72$ mm at point B.

$$F_1 = \int_{x_A}^{x_B} q\,dx = \int_0^{72} \frac{1152\,Vx}{I}\,dx = \frac{1152\,V}{I}\,\frac{(72)^2}{2}$$
$$= \frac{(576)(72)^2}{19.4849 \times 10^6}\,V = 0.153246\,V$$

$$+\circlearrowleft \Sigma M_c = +\circlearrowleft M_c: \quad Ve = (0.153246\,V)(192)$$

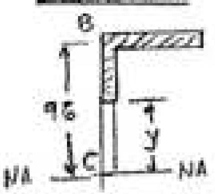
(a) $\quad e = 29.423$ mm $\qquad\qquad e = 29.4$ mm ◄

(b) Point A: $\quad x = 0 \qquad Q = 0, \quad q = 0 \qquad \tau_A = 0$ ◄

Point B in Part AB. $\quad x = 72$ mm

$$Q = (1152)(72) = 82.944 \times 10^3 \text{ mm}^3 = 82.944 \times 10^{-6} \text{ m}^3$$
$$t = 12 \text{ mm} = 0.012 \text{ m}$$
$$\tau_B = \frac{VQ}{It} = \frac{(110 \times 10^3)(82.944 \times 10^{-6})}{(19.4849 \times 10^{-6})(0.012)}$$
$$= 39.0 \times 10^6 \text{ Pa} \qquad \tau_B = 39.0 \text{ MPa in AB} ◄$$

Part BD:



Point B: $\quad y = 96$ mm $\qquad Q = 82.944 \times 10^3 \text{ mm}^3 = 82.944 \times 10^{-6} \text{ m}^3$

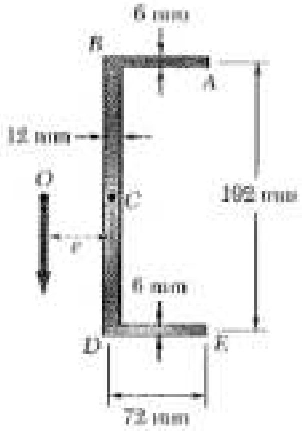
$$t = 6 \text{ mm} = 0.006 \text{ m}$$
$$\tau_B = \frac{VQ}{It} = \frac{(110 \times 10^3)(82.944 \times 10^{-6})}{(19.4849 \times 10^{-6})(0.006)}$$
$$= 78.0 \times 10^6 \text{ Pa} \qquad \tau_B = 78.0 \text{ MPa in BD} ◄$$

Point C: $\quad y = 0, \quad t = 6 \text{ mm} = 0.006$ m

$$Q = 82.944 \times 10^3 + (6)(96)\left(\tfrac{96}{2}\right) = 110.592 \times 10^3 \text{ mm}^3 = 110.592 \times 10^{-6} \text{ m}^3$$
$$\tau = \frac{VQ}{It} = \frac{(110 \times 10^3)(110.512 \times 10^{-6})}{(19.4849 \times 10^{-6})(0.006)} = 104.1 \times 10^6 \text{ Pa} \qquad \tau_c = 104.1 \text{ MPa} ◄$$

Problem 6.68

6.67 and 6.68 For an extruded beam having the cross section shown, determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.



$$I = 2\left[\left(\tfrac{1}{12}\right)(72)(6)^3 + (72)(6)\left(\tfrac{192}{2}\right)^2\right] + \tfrac{1}{12}(12)(192)^3$$
$$= 15.0431\times10^6 \text{ mm}^3 = 15.0431\times10^{-6} \text{ m}^4$$

Part AB: $\quad A = 6x \qquad Q = A\bar{y} = (6x)\left(\frac{192}{2}\right) = 576\,x$

$$q = \frac{VQ}{I} = \frac{576\,Vx}{I}$$

$x = 0$ at point A. $\quad x = l_{AB} = 72$ mm at point B

$$F_1 = \int_{x_A}^{x_B} q\,dx = \int_0^{72} \frac{576\,Vx}{I}\,dx = \frac{576\,V}{I}\,\frac{(72)^2}{2}$$
$$= \frac{(288)(72)^2}{15.0431\times10^6}\,V = 0.099247\ V$$

$+\circlearrowleft M_c = +\circlearrowleft M_c: \quad Ve = (0.099247)V\,(192)$

(a) $e = 19.0555$ mm $\qquad\qquad e = 19.06$ mm ◄

(b) Point A: $x = 0 \quad Q = 0, \quad q = 0 \quad \tau_A = 0$ ◄

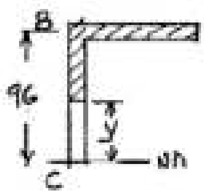
Point B in Part AB: $x = 72$ mm

$Q = (576)(72) = 41.472\times10^3 \text{ mm}^3 = 41.472\times10^{-6} \text{ m}^3$

$t = 6 \text{ mm} = 0.006 \text{ m}$

$$\tau_B = \frac{VQ}{It} = \frac{(110\times10^3)(41.472\times10^{-6})}{(15.0431\times10^{-6})(0.006)}$$
$$= 50.5\times10^6 \text{ Pa} \qquad\qquad \tau_B = 50.5 \text{ MPa} ◄$$

Part BD:



Point B: $y = 96$ mm $\qquad Q = 41.472\times10^3 \text{ mm}^3 = 41.472\times10^{-6} \text{ m}^3$

$t = 12 \text{ mm} = 0.012 \text{ m}$

$$\tau_B = \frac{VQ}{It} = \frac{(110\times10^3)(41.472\times10^{-6})}{(15.0431\times10^{-6})(0.012)}$$
$$= 25.271\times10^6 \text{ Pa} \qquad\qquad \tau_B = 25.3 \text{ MPa} ◄$$

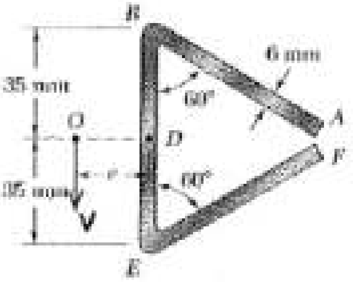
Point C: $y = 0 \qquad t = 0.012$ m

$$Q = 41.472\times10^3 + (12)(96)\left(\tfrac{96}{2}\right) = 96.768\times10^3 \text{ mm}^3 = 96.768\times10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(110\times10^3)(96.768\times10^{-6})}{(15.0431\times10^{-6})(0.012)} = 58.967\times10^6 \text{ Pa} \qquad \tau_c = 59.0 \text{ MPa} ◄$$

Problem 6.70

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

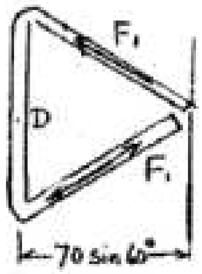


$$I_{DB} = \tfrac{1}{3}(6)(35)^3 = 85.75 \times 10^3 \text{ mm}^4$$

$$L_{AB} = 70 \text{ mm} \qquad A_{AB} = (70)(6) = 420 \text{ mm}^2$$

$$I_{AB} = \tfrac{1}{3} A_{AB} h^2 = (\tfrac{1}{3})(420)(35)^2 = 171.5 \times 10^3 \text{ mm}^4$$

$$I = (2)(85.75 \times 10^3) + (2)(171.5 \times 10^3) = 514.5 \times 10^3 \text{ mm}^4$$



Part AB: $A = ts = 6s$

$$\bar{y} = \tfrac{1}{2} s \sin 30° = \tfrac{1}{4} s$$

$$Q = A\bar{y} = \tfrac{3}{2} s^2$$

$$\tau = \frac{VQ}{It} = \frac{3Vs^2}{It}$$

$$F_1 = \int \tau \, dA = \int_0^{70} \frac{3Vs^2}{2It} t\, ds = \frac{3V}{I}\int_0^{70} s^2\, ds$$

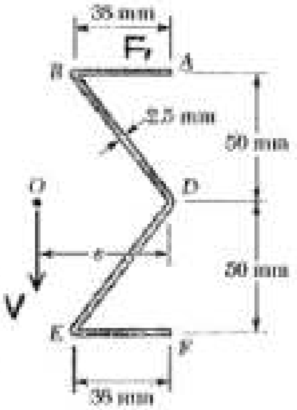
$$= \frac{(3)(70)^3}{(2)(3)I} V = \tfrac{1}{3} V$$

$$\circlearrowleft \Sigma M_O = \circlearrowleft \Sigma M_O: \quad Ve = 2[F_1 \cos 60°)(70 \sin 60°)]$$
$$= 20.2\ V$$

Dividing by V, $\qquad e = 20.2$ mm ◄

Problem 6.71

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



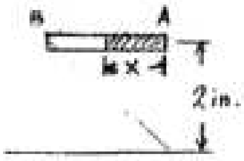
$I_{AB} = \frac{1}{12}(38)(2.5)^3 + (38)(2.5)(50)^2 = 237549\ mm^3$

$L_{BD} = \sqrt{38^2 + 50^2} = 62.8\ mm \qquad A_{BD} = (62.8)(2.5) = 157\ mm^2$

$I_{BD} = \frac{1}{3} A_{BD} h^2 = \frac{1}{3}(157)(50)^2 = 130833\ mm^4$

$I = 2 I_{AB} + 2 I_{BD} = 736766\ mm^4$

Part AB:



$A(x) = tx = 2.5x, \quad \bar{y} = 50\ mm.$

$Q(x) = A(x)\bar{y} = 125x\ mm^3$

$q(x) = \frac{VQ(x)}{I} = \frac{125Vx}{I}$

$F_1 = \int_0^{38} q(x)dx = \frac{125V}{I}\int_0^{38} x\,dx$

$= \frac{(125)(38)^2}{2}\frac{V}{I} = 90250\frac{V}{I}$

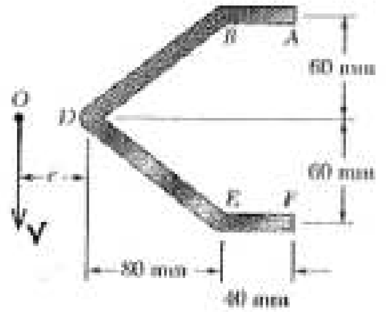
Likewise, by symmetry, in part EF: $F_1 = 90250\frac{V}{I}$

$+\circlearrowleft \Sigma M_D = +\circlearrowleft \Sigma M_D: \quad Ve = 100F_1 = 9025000\frac{V}{I} = 12.25V$

Dividing by V, $\qquad e = 12.25\ mm$

Problem 6.72

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

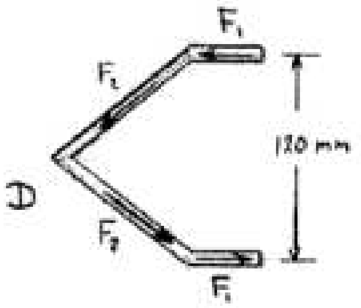


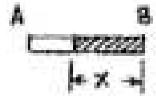
$$I_{AB} = (40t)(60)^2 = 144 \times 10^3\, t$$

$$L_{DB} = \sqrt{80^2 + 60^2} = 100 \text{ mm} \qquad A_{DB} = 100\, t$$

$$I_{DB} = \tfrac{1}{3} A_{DB} h^2 = \tfrac{1}{3}(100t)(60)^2 = 120 \times 10^3\, t$$

$$I = 2I_{AB} + 2I_{DB} = 528 \times 10^3\, t$$





Part AB: $A = tx \qquad \bar{y} = 60 \text{ mm}$

$$Q = A\bar{y} = 60\,tx \quad \text{mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{V(60tx)}{It} = \frac{60Vx}{I}$$

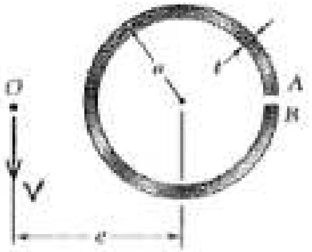
$$F_1 = \int \tau\, dA = \int_0^{40} \frac{60Vx}{I}\, t\, dx = \frac{60Vt}{I}\int_0^{40} x\, dx$$

$$= \frac{60Vt}{I}\left.\frac{x^2}{2}\right|_0^{30} = \frac{(60)(30)^2 V\cancel{t}}{(2)(528 \times 10^3)\cancel{t}} = 0.051136\, V$$

$$\circlearrowright \Sigma M_D = \circlearrowright \Sigma M_D: \quad Ve = (0.051136\, V)(120) \qquad e = (0.051136)(120) = 6.14 \text{ mm}$$

Problem 6.73

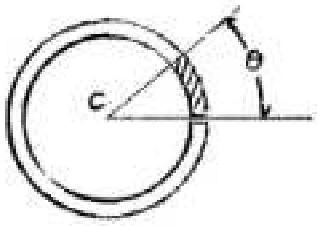
6.69 through 6.74 Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.



For whole cross section, $A = 2\pi a t$

$J = Aa^2 = 2\pi a^3 t \qquad I = \frac{1}{2} J = \pi a^3 t$

Use polar coordinate θ for partial cross section.



$A = st = a\theta t \qquad s = \text{arc length}$

$\bar{r} = a \frac{\sin\alpha}{\alpha} \quad \text{where} \quad \alpha = \frac{1}{2}\theta$

$\bar{y} = \bar{r} \sin\alpha = a \frac{\sin^2\alpha}{\alpha}$

$Q = A\bar{y} = a\theta t \, a \frac{\sin^2\alpha}{\alpha} = a^2 t \; 2\sin^2\alpha$

$= a^2 t \; 2\sin^2\frac{\theta}{2} = a^2 t (1 - \cos\theta)$

$\tau = \frac{VQ}{It} = \frac{Va^2}{I}(1-\cos\theta)$

$M_c = \int a\tau \, dA = \int_0^{2\pi} \frac{Va^3}{I}(1-\cos\theta)\, t\, a\, d\theta = \frac{Va^4 t}{I}(\theta - \sin\theta)\Big|_0^{2\pi}$

$= \frac{2\pi V a^4 t}{\pi a^3 t} = 2aV$

But $M_c = Ve$, hence $e = 2a$ ◄

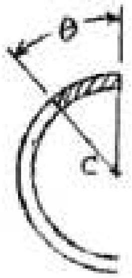
Problem 6.74

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

For a thin-walled hollow circular cross section $A = 2\pi a t$

$J = a^2 A = 2\pi a^3 t \qquad I = \frac{1}{2}J = \pi a^3 t$

For the half-pipe section $I = \frac{\pi}{2}a^3 t$



Use polar coordinate θ for partial cross section.

$A = st = a\theta t \qquad s = \text{arc length}$

$\bar{r} = a\frac{\sin\alpha}{\alpha}$ where $\alpha = \frac{\theta}{2}$

$\bar{y} = \bar{r}\cos\alpha = a\frac{\sin\alpha\cos\alpha}{\alpha}$

$Q = A\bar{y} = a\theta t\, a\frac{\sin\alpha\cos\alpha}{\alpha} = a^2 t\,(2\sin\alpha\cos\alpha)$

$= a^2 t \sin 2\alpha = a^2 t \sin\theta$

$\tau = \frac{VQ}{It} = \frac{Va^2}{I}\sin\theta$

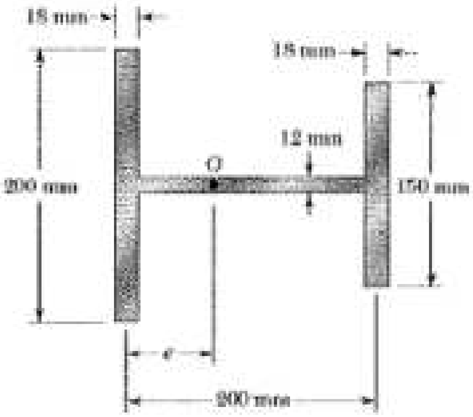
$M_H = \int a\tau\, dA = \int_0^{\pi} a\frac{Va^2}{I}\sin\theta\, t\, a\, d\theta = \frac{Va^4 t}{I}\left.-\cos\theta\right|_0^{\pi}$

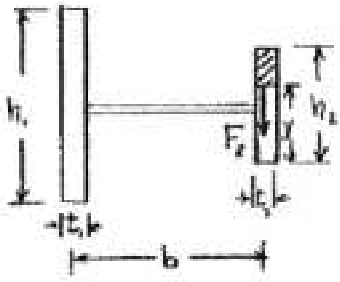
$= 2\frac{Va^4 t}{I} = \frac{4}{\pi}Va$

But $M_H = Ve$, hence $e = \frac{4}{\pi}a = 1.273\ a$ ◄

Problem 6.75

6.75 and 6.76 A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.





$$I = \frac{1}{12} t_1 h_1^3 + \frac{1}{12} t_2 h_2^3$$

Right flange:

$$A = \left(\tfrac{1}{2}h_2 - y\right)t_2$$

$$\bar{y} = \tfrac{1}{2}\left(\tfrac{1}{2}h_2 + y\right)t_2$$

$$Q = A\bar{y} = \tfrac{1}{2}\left(\tfrac{1}{2}h_2 - y\right)\left(\tfrac{1}{2}h_2 + y\right)t_2 = \tfrac{1}{2}\left(\tfrac{1}{4}h_2^2 - y^2\right)t_2$$

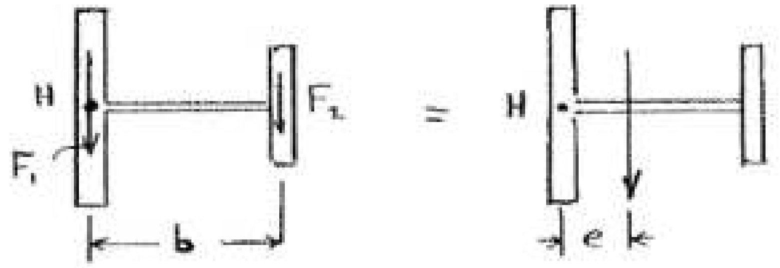
$$\tau = \frac{VQ}{It_2} = \frac{V}{2It_2}\left(\tfrac{1}{4}h_2^2 - y^2\right)t_2$$

$$F_2 = \int \tau\, dA = \int_{-h_2/2}^{h_2/2} \frac{Vt_2}{2It_2}\left(\tfrac{1}{4}h_2^2 - y^2\right)t_2\, dy = \frac{Vt_2}{2I}\left(\tfrac{1}{4}h_2^2 y - \frac{y^3}{3}\right)\Bigg|_{-h_2/2}^{h_2/2}$$

$$= \frac{Vt_2}{2I}\left\{\tfrac{1}{4}h_2^2\frac{h_2}{2} - \tfrac{1}{3}\left(\frac{h_2}{2}\right)^3 + \tfrac{1}{4}h_2^2\frac{h_2}{2} - \tfrac{1}{3}\left(\frac{h_2}{2}\right)^3\right\} = \frac{Vt_2h_2^3}{12I} = \frac{V\,t_2h_2^3}{t_1h_1^3 + t_2h_2^3}$$

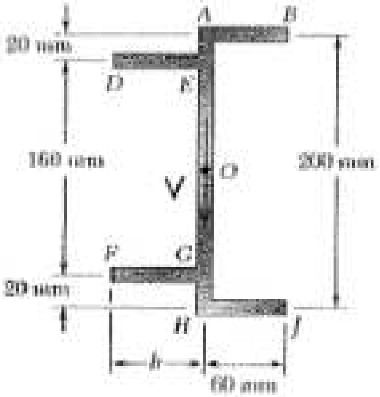
$$\circlearrowright M_H = \circlearrowright M_H: \quad -Ve = -F_2 b = -V\frac{t_2h_2^3 b}{t_1h_1^3 + t_2h_2^3}$$

$$e = \frac{t_2h_2^3 b}{t_1h_1^3 + t_2h_2^3} = \frac{(18)(150)^3(200)}{(18)(200)^3 + (18)(150)^3} = 59.3 \text{ mm} \blacktriangleleft$$

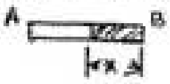


Problem 6.78

6.77 and 6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



Part AB:

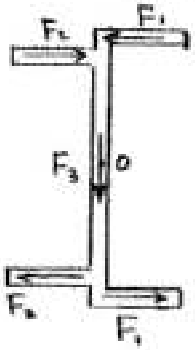


$A = tx, \quad \bar{y} = 100 \text{ mm}, \quad Q = 100\, tx$

$q = \frac{VQ}{I} = \frac{100\, Vtx}{I}$

$F_1 = \int_{x_A}^{x_B} q\, dx = 100 \frac{Vt}{I} \int_0^{60} x\, dx$

$= \frac{(100)(60)^2}{2} \frac{Vt}{I} = 180 \times 10^3 \frac{Vt}{I}$



Part DE:

$A = tx, \quad \bar{y} = 80 \text{ mm}, \quad Q = 80\, tx$

$q = \frac{VQ}{I} = \frac{80\, Vtx}{I}$

$F_2 = \int_{x_D}^{x} q\, dx = 80 \frac{Vt}{I} \int_0^{a} x\, dx$

$= (40\, a^2) \frac{Vt}{I}$

$+\circlearrowleft \Sigma M_O = 0: \quad (200)(180 \times 10^3)\frac{Vt}{I} - (160)(40 a^2)\frac{Vt}{I} = 0$

$a^2 = \frac{(200)(180 \times 10^3)}{(160)(40)} = 5625 \text{ mm}^2 \qquad a = 75.0 \text{ mm}$ ◄

Problem 6.79

6.79 For the angle shape and loading of Sample Prob. 6.6, check that $\int q\, dz = 0$ along the horizontal leg of the angle and $\int q\, dy = P$ along its vertical leg.

Refer to Sample Prob. 6.6.

Along horizontal leg: $\tau_f = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3}(a^2 - 4az + 3z^2)$

$\int q\, dz = \int_0^a \tau_f t\, dz = \frac{3P}{4a^3}\int_0^a (a^2 - 4az + 3z^2)\, dz = \frac{3P}{4a^3}\left(a^2 z - 4a\frac{z^2}{2} + 3\frac{z^3}{3}\right)\Big|_0^a$

$= \frac{3P}{4a^3}(a^3 - 2a^3 + a^3) = 0$ ◄

Along vertical leg: $\tau_e = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$

$\int q\, dy = \int_0^a \tau_e t\, dy = \frac{3P}{4a^3}\int_0^a (a^2 + 4ay - 5y^2)\, dy = \frac{3P}{4a^3}\left(a^2 y + 4a\frac{y^2}{2} - 5\frac{y^3}{3}\right)\Big|_0^a$

$= \frac{3P}{4a^3}\left(a^3 + 2a^3 - \frac{5}{3}a^3\right) = \frac{3P}{4a^3} \cdot \frac{4}{3}a^3 = P$ ◄

Problem 6.80

6.80 For the angle shape and loading of Sample Prob. 6.6, (*a*) determine the points where the shearing stress is maximum and the corresponding values of the stress, (*b*) verify that the points obtained are located on the neutral axis corresponding to the given loading.

Refer to Sample Prob. 6.6.

(a) Along vertical leg: $\tau_e = \dfrac{3P(a-y)(a+5y)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$

$$\frac{d\tau_e}{dy} = \frac{3P}{4ta^3}(4a - 10y) = 0 \qquad y = \tfrac{2}{5}a$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 + (4a)(\tfrac{2}{5}a) - (5)(\tfrac{2}{5}a)^2\right] = \frac{3P}{4ta^3}(\tfrac{9}{5}a^2) = \frac{27}{20}\frac{P}{ta}$$

Along horizontal leg: $\tau_f = \dfrac{3P(a-z)(a-3z)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 - 4az + 3z^2)$

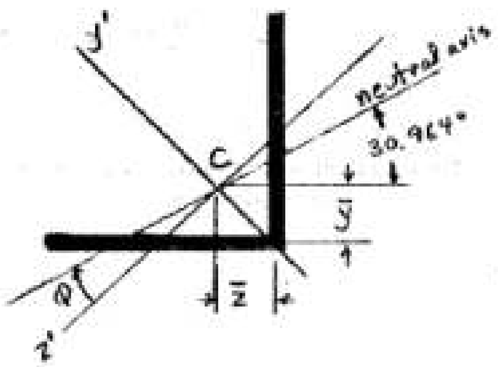
$$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3}(-4a + 6z) = 0 \qquad z = \tfrac{2}{3}a$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 - (4a)(\tfrac{2}{3}a) + (3)(\tfrac{2}{3}a)^2\right] = \frac{3P}{4ta^3}(-\tfrac{5}{3}a^2) = -\frac{1}{4}\frac{P}{ta}$$

At the corner: $y = 0$, $z = 0$ $\qquad \tau = \dfrac{3}{4}\dfrac{P}{ta}$

(b) $I_{y'} = \frac{1}{3}ta^3 \qquad I_{z'} = \frac{1}{12}ta^3 \qquad \theta = 45°$

$$\tan\varphi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1}{4} \qquad \varphi = 14.036°$$



$$\theta - \varphi = 45 - 14.036 = 30.964°$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

$$\bar{z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

Neutral axis intersects vertical leg at

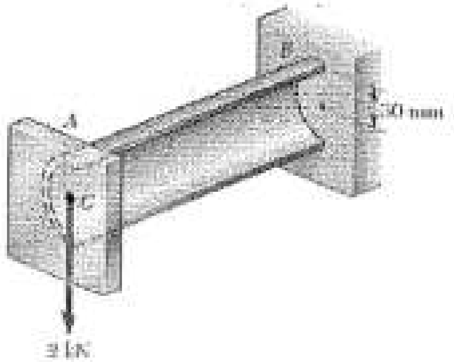
$$y = \bar{y} + \bar{z}\tan 30.964° = \left(\tfrac{1}{4} + \tfrac{1}{4}\tan 30.964°\right)a = 0.400a = \tfrac{2}{5}a$$

Neutral axis intersects horizontal leg at

$$z = \bar{z} + \bar{y}\tan(45° + \varphi) = \left(\tfrac{1}{4} + \tfrac{1}{4}\tan 59.036°\right)a = 0.6667a = \tfrac{2}{3}a$$

Problem 6.81

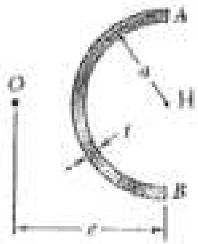
*6.81 The cantilever beam AB, consisting of half of a thin-walled pipe of 30-mm mean radius and 10-mm wall thickness, is subjected to a 2-kN vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (a) the equivalent force-couple system at the shear center of the cross section, (b) the maximum shearing stress in the beam. (*Hint:* The shear center O of this cross section was shown in Prob. 6.73 to be located twice as far from its vertical diameter as its centroid C.)



From the solution to Problem 6.73,

$$I = \frac{\pi}{2}a^3 t \qquad Q = a^2 t \sin\theta$$

$$e = \frac{4}{\pi}a \qquad Q_{max} = a^2 t$$



For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi}a$$

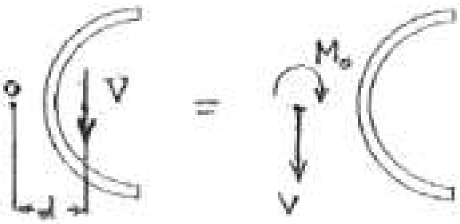
At each section of the beam, the shearing force V is equal to P. Its line of action passes through the centroid C. The moment arm of its moment about the shear center O is

$$d = e - \bar{x} = \frac{4}{\pi}a - \frac{2}{\pi}a = \frac{2}{\pi}a$$

(a) Equivalent force-couple system at O.

$$V = P \qquad M_o = Vd = \frac{2}{\pi}Pa$$

Data: $P = 2\ kN$ $\qquad a = 30\ mm$



$$M_o = \left(\frac{2}{\pi}\right)(2000)(30)$$

$V = 2\ kN$ ◄

$M_o = 38.2\ N\,m$ ◄

(b) Shearing stresses

(1) Due to V. $\qquad \tau_V = \frac{V Q_{max}}{I t}$

$$\tau_V = \frac{(P)(a^2 t)}{(\frac{\pi}{2}a^3 t)(t)} = \frac{2P}{\pi a t} = \frac{(2)(2000)}{\pi(0.03)(0.01)} = 4.24\ MPa$$

(2) Due to M_o, the torque.

For a long rectangular section of length l and width t the shearing stress due to torque M_o is

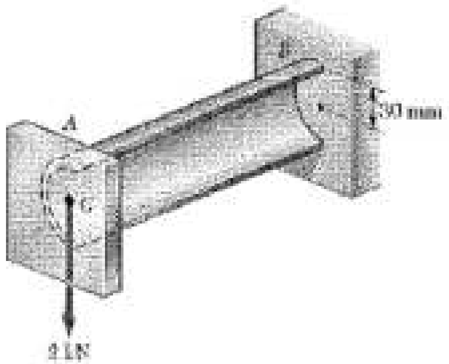
$$\tau_M = \frac{M_o}{c_1 l t^2} \qquad \text{where} \qquad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

Data: $l = \pi a = \pi(30) = 94.25\ mm$ $\qquad t = 10\ mm$ $\qquad c_1 = 0.311$

$$\tau_M = \frac{38200}{(0.311)(94.25)(10)^2} = 13.03\ MPa$$

By superposition $\qquad \tau = \tau_V + \tau_M = 4.24 + 13.03 = 17.3\ MPa$

Problem 6.82



***6.82** Solve Prob. 6.81, assuming that the thickness of the beam is reduced to 6 mm.

***6.81** The cantilever beam AB, consisting of half of a thin-walled pipe of 30-mm mean radius and 10-mm wall thickness, is subjected to a 2-kN vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum shearing stress in the beam. (*Hint:* The shear center O of this cross section was shown in Prob. 6.74 to be located twice as far from its vertical diameter as its centroid C.)

From the solution to Problem 6.73

$$I = \frac{\pi}{2}a^3 t \qquad Q = a^2 t \sin\theta$$

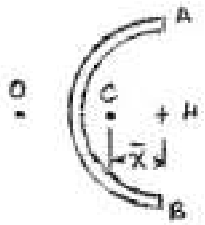
$$e = \frac{4}{\pi}a \qquad Q_{max} = a^2 t$$

For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi}a$$

At each section of the beam, the shearing force V is equal to P. Its line of action passes through the centroid C. The moment arm of its moment about the shear center O is

$$d = e - \bar{x} = \frac{4}{\pi}a - \frac{2}{\pi}a = \frac{2}{\pi}a$$



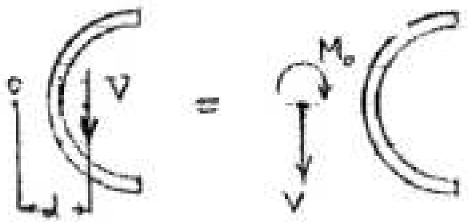
(a) Equivalent force-couple system at O.

$$V = P \qquad M_o = Vd = \frac{2}{\pi}Pa$$

Data: $P = 2\,kN \qquad a = 30\,mm$

$V = 2\,kN$ ◄

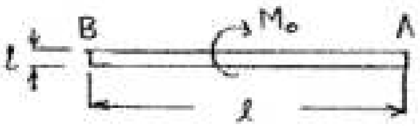
$M_o = 38.2\,N{\cdot}m$ ◄



(b) Shearing stresses

(1) Due to V. $\quad \tau_v = \dfrac{V Q_{max}}{It}$

$$\tau_v = \frac{(P)(a^2 t)}{(\frac{\pi}{2}a^3 t)(t)} = \frac{2P}{\pi a t} = \frac{(2)(2000)}{\pi(30)(6)} = 7.07\,MPa$$



(2) Due to M_o, the torque.

For a long rectangular section of length l and width t the shearing stress due to torque M_o is

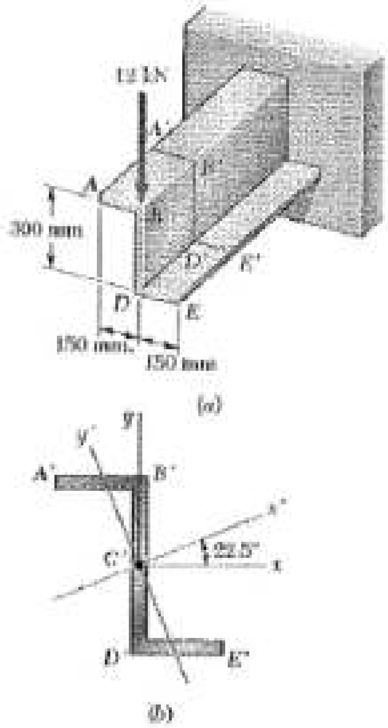
$$\tau_M = \frac{M_o}{c_1 l t^2} \qquad \text{where} \quad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

Data: $l = \pi a = \pi(30) = 94.25\,mm \qquad t = 6\,mm \qquad c_1 = 0.31996$

$$\tau_M = \frac{38200}{(0.31996)(94.25)(36)} = 35.19\,MPa$$

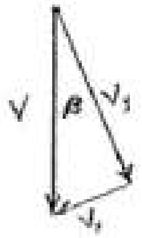
By superposition $\quad \tau = \tau_v + \tau_M = 7.07 + 35.19 = 42.3\,MPa$

Problem 6.83



*6.83 The cantilever beam shown consists of a Z shape of 6-mm thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 69.2 \times 10^6\ \text{mm}^4$ and are $I_{y'} = 5.7 \times 10^6\ \text{mm}^4$.

$V = 12\ kN \qquad \beta = 22.5°$



$V_{x'} = V \sin\beta \qquad V_{y'} = V\cos\beta$

In upper horizontal leg use coordinate x $(-150\,mm \le x \le 0)$

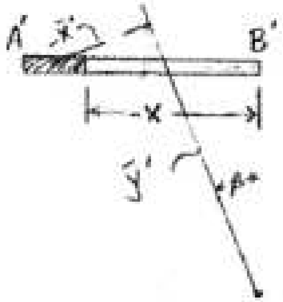
$A = 6(150 + x)\ mm$

$\bar{x} = \frac{1}{2}(-150 + x)\ mm$

$\bar{y} = 150\ mm$

$\bar{x}' = \bar{x}\cos\beta + \bar{y}\sin\beta$

$\bar{y}' = \bar{y}\cos\beta - \bar{x}\sin\beta$



Due to $V_{x'}$: $\quad \tau_1 = \dfrac{V_{x'} A \bar{x}'}{I_{y'} t}$

$$\tau_1 = \frac{(V\sin\beta)(6)(150+x)\left[\frac{1}{2}(-150+x)\cos\beta + 150\sin\beta\right]}{(5.7\times10^6)(6)}$$

$$= 8.056\times10^{-4}(150+x)(-11.9 + 0.46194x)$$

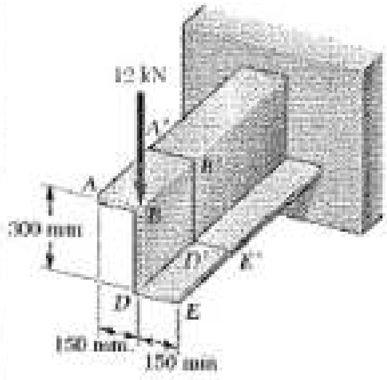
Due to $V_{y'}$: $\quad \tau_2 = \dfrac{V_{y'} A \bar{y}'}{I_{x'} t} = \dfrac{(V\cos\beta)(6)(150+x)\left[150\cos\beta - \frac{1}{2}(-150+x)\sin\beta\right]}{(69.2\times10^6)(6)}$

$$= 1.602\times10^{-4}(150+x)\left[167.3 - 0.19134x\right]$$

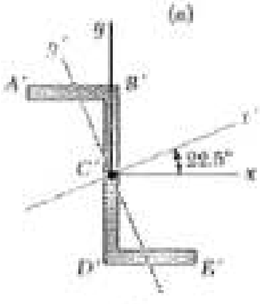
Total: $\tau_1 + \tau_2 = (150 + x)\left[0.0172 + 0.0003415x\right]$

x (m)	-150	-125	-100	-75	-50	-25	0
τ (MPa)	0	-0.643	-0.848	-0.631	0.0125	1.083	2.58

Problem 6.84



(a)

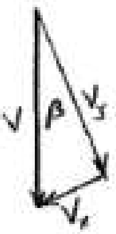


(b)

***6.84** For the cantilever beam and loading of Prob. 6.83, determine the distribution of the shearing stress along line $B'D'$ in the vertical web of the Z shape.

***6.83** The cantilever beam shown consists of a Z shape of 6-mm thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 69.2 \times 10^6\ \text{mm}^4$ and are $I_{y'} = 5.7 \times 10^6\ \text{mm}^4$.

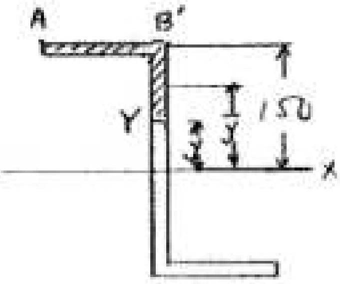
$V = 12\ \text{kN} \qquad \beta = 22.5°$



$V_{x'} = V \sin\beta \qquad V_{y'} = V\cos\beta$

For part AB' $\quad A = (6)(150) = 900\ \text{mm}^2$

$\bar{x} = -75\ \text{mm}, \ \bar{y} = 150\ \text{mm}$



For part B'Y

$A = 6(150 - y)$

$\bar{x} = 0 \qquad \bar{y} = \frac{1}{2}(150 + y)$

$x' = x\cos\beta + y\sin\beta$

$y' = y\cos\beta - x\sin\beta$

Due to $V_{x'}$: $\quad \tau_1 = \dfrac{V_{x'}(A_{AB}\bar{x}'_{AB} + A_{BY}\bar{x}'_{BY})}{I_{y'}\, t}$

$$\tau_1 = \frac{(V\sin\beta)\left[(900)(-75\cos\beta + 150\sin\beta) + 6(150-y)\frac{1}{2}(150+y)\sin\beta\right]}{(5.7\times10^6)(6)}$$

$$= \frac{(V\sin\beta)\left[-10700 + 25831 - 1.148y^2\right]}{34.2\times10^6} = 2.0317 - 0.0001541\,y^2$$

Due to $V_{y'}$: $\quad \tau_2 = \dfrac{V_{y'}(A_{AB}\bar{y}'_{AB} + A_{BY}\bar{y}')}{I_{x'}\, t}$

$$\tau_2 = \frac{(V\cos\beta)\left[(900)(150\cos\beta + 75\sin\beta) + 6(150-y)\frac{1}{2}(150+y)\cos\beta\right]}{(69.2\times10^6)(6)}$$

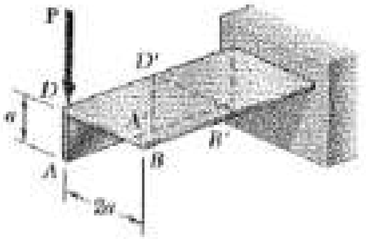
$$= \frac{(V\cos\beta)\left[187086 + 62361 - 2.772y^2\right]}{(69.2\times10^6)(6)} = 6.66 - 7.402\times10^{-5}\,y^2$$

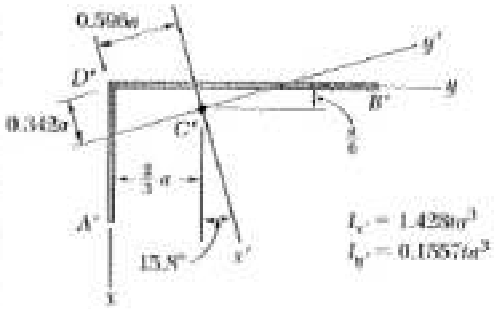
Total: $\quad \tau_1 + \tau_2 = 8.6917 - 2.2812\times10^{-4}\,y^2$

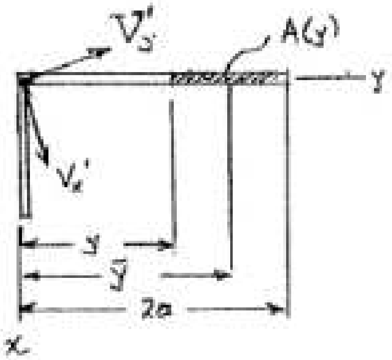
y (mm)	0	± 50	± 100	± 150
τ (MPa)	8.692	8.121	6.41	3.56

Problem 6.85

*6.85 Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.







$\beta = 15.8°\qquad V_x' = P\cos\beta \qquad V_y' = -P\sin\beta$

$A(y) = (2a - y)t \qquad \bar{y} = \frac{1}{2}(2a + y),\ \bar{x} = 0$

Coordinate transformation.

$y' = (y - \frac{2}{3}a)\cos\beta - (x - \frac{1}{6}a)\sin\beta$

$x' = (x - \frac{1}{6}a)\cos\beta + (y - \frac{2}{3}a)\sin\beta$

In particular,

$\bar{y}' = (\bar{y} - \frac{2}{3}a)\cos\beta - (\bar{x} - \frac{1}{6}a)\sin\beta$

$= (\frac{1}{2}y + \frac{1}{3}a)\cos\beta - (-\frac{1}{6}a)\sin\beta$

$= 0.48111\,y + 0.36612\,a$

$\bar{x}' = (\bar{x} - \frac{1}{6}a)\cos\beta + (\bar{y} - \frac{2}{3}a)\sin\beta$

$= (-\frac{1}{6}a)\cos\beta + (\frac{1}{2}y + \frac{1}{3}a)\sin\beta$

$= 0.13614\,y - 0.06961\,a$

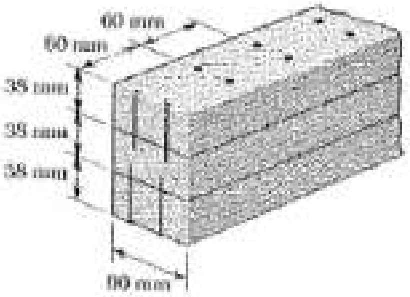
$$+\rightarrow \quad \tau = \frac{V_x' A \bar{x}'}{I_y' t} + \frac{V_y' A \bar{y}'}{I_x' t}$$

$$= \frac{(P\cos\beta)(2a-y)(t)(0.13614y - 0.06961a)}{(0.1557\ ta^3)(t)} + \frac{(-P\sin\beta)(2a-y)(0.48111y + 0.36612a)}{(1.428\ a^3 t)(t)}$$

$$= \frac{P(2a-y)(0.750y - 0.500a)}{ta^3}$$

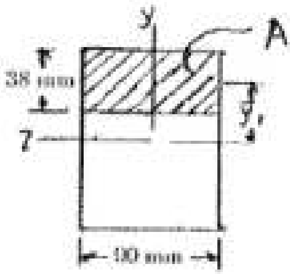
$y\ (a)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$\rightarrow \tau\ (\frac{P}{at})$	-1.000	-0.417	0	0.250	0.333	0.250	0

Problem 6.89



6.89 Three boards, each of 38 × 90-mm rectangular cross section, are nailed together to form a beam that is subjected to a vertical shear of 1 kN. Knowing that the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(90)(114)^3 = 11.112 \times 10^6 \text{ mm}^4$$



$A = (90)(38) = 3420 \text{ mm}^2$

$\bar{y}_1 = 38 \text{mm}.$

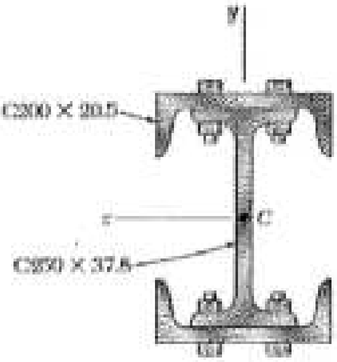
$Q = A\bar{y}_1 = 129960 \text{ mm}^3$

$$q = \frac{VQ}{I} = \frac{(1)(129960 \times 10^{-9})}{(11.112) \times 10^{-6}} = 11.7 \text{ kN/m}$$

$$qs = 2 F_{nail} \qquad F_{nail} = \frac{qs}{2} = \frac{(11.7)(0.06)}{2} = 351 N$$

Problem 6.90

6.90 A column is fabricated by connecting the rolled-steel members shown by bolts of 18-mm diameter spaced longitudinally every 125 mm. Determine the average shearing stress in the bolts caused by a shearing force of 120 kN parallel to the y axis.



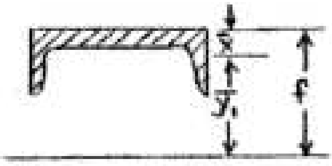
Geometry

$$f = \left(\frac{d}{2}\right)_s + (t_w)_c$$

$$= \frac{254}{2} + 7.7 = 134.7 \text{ mm}$$

$$\bar{x} = 13.9 \text{ mm}$$

$$\bar{y}_1 = f - \bar{x} = 134.7 - 13.9 = 120.8 \text{ mm}$$



Determine moment of inertia.

Part	A (mm²)	d (mm)	A d² (mm⁴)	$\bar{I}$ (mm⁴)
C200×20.5	2660	120.8	38.816×10^6	0.625×10^6
S250×37.8	7.46	0	0	51.1×10^6
C200×20.5	2660	120.8	38.816×10^6	0.625×10^6
Σ			77.63×10^6	52.35×10^6

$$I = \Sigma A d^2 + \Sigma \bar{I} = (77.63 + 52.35)10^6 = 130\times10^6$$

$$Q = A\bar{y}_1 = (2660)(120.8) = 321328 \text{ mm}^3.$$

$$q = \frac{VQ}{I} = \frac{(120)(321328\times10^{-9})}{130\times10^{-6}} = 296.6 \text{ kN/m}.$$

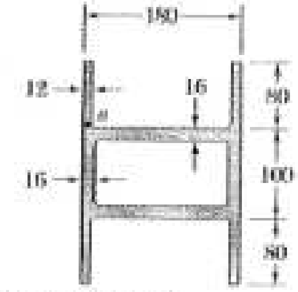
$$F_{bolt} = \tfrac{1}{2} q s = (\tfrac{1}{2})(296.6)(0.125) = 18.54 \text{ kN}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2.$$

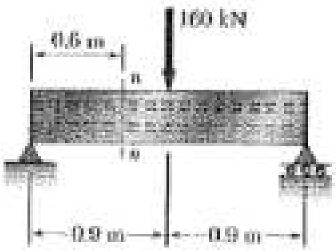
$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{18540}{254.47\times10^{-6}} = 72.9 \text{ MPa.} \blacktriangleleft$$

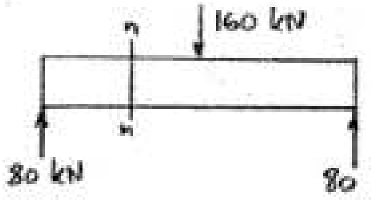
Problem 6.91

6.91 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

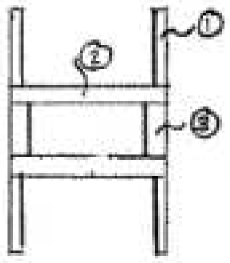


Dimensions in mm





At section n-n $\quad V = 80$ kN



Consider cross section as composed of rectangles of types ①, ②, and ③.

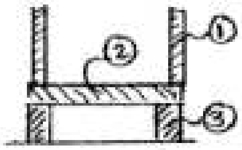
$$I_1 = \frac{1}{12}(12)(80)^3 + (12)(80)(90)^2 = 8.288 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(180)(16)^3 + (180)(16)(42)^2 = 5.14176 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(16)(68)^3 = 419.24 \times 10^3 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + 2I_1 = 44.274 \times 10^6 \text{ mm}^4$$
$$= 44.274 \times 10^{-6} \text{ m}^4$$

(a) Calculate Q at neutral axis.



$$Q_1 = (12)(80)(90) = 86.4 \times 10^3 \text{ mm}^4$$
$$Q_2 = (180)(16)(42) = 120.96 \times 10^3 \text{ mm}^4$$
$$Q_3 = (16)(34)(17) = 9.248 \times 10^3 \text{ mm}^4$$

$$Q = 2Q_1 + Q_2 + 2Q_3 = 312.256 \times 10^3 \text{ mm}^3 = 312.256 \times 10^{-6} \text{ m}^3$$

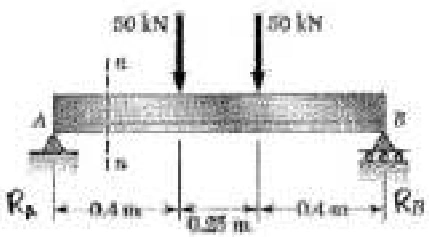
$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(312.256 \times 10^{-6})}{(44.274 \times 10^{-6})(2 \times 16 \times 10^{-3})} = 17.63 \times 10^6 \text{ Pa} \qquad 17.63 \text{ MPa} \blacktriangleleft$$

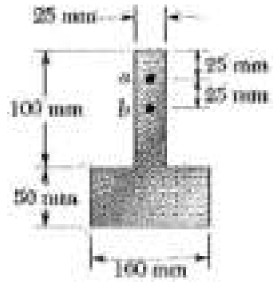
(b) At point a, $\quad Q = Q_1 = 86.4 \times 10^3 \text{ mm}^4 = 86.4 \times 10^{-6} \text{ m}^4$

$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(86.4 \times 10^{-6})}{(44.274 \times 10^{-6})(12 \times 10^{-3})} = 13.01 \times 10^6 \text{ Pa} \qquad 13.01 \text{ MPa} \blacktriangleleft$$

Problem 6.92

6.92 For the beam and loading shown, consider section n-n and determine the shearing stress at (a) point a, (b) point b.

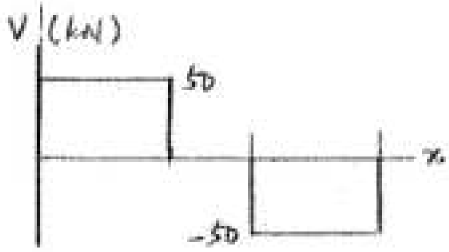




$R_A = R_B = 50\ \text{kN}$

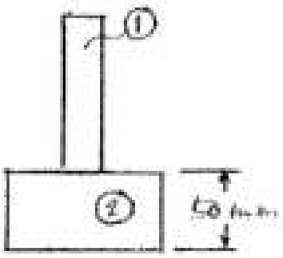
Draw shear diagram.

$V = 50\ \text{kN}$



Determine section properties.

Part	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (mm^3)$	$d\ (mm)$	$Ad^2\ (mm^4)$	$\bar{I}\ (mm^4)$
1	2500	100	250000	50	6250000	2083333
2	5000	25	125000	-25	3125000	1041667
Σ	7500		375000		9375000	3125000



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{375000}{7500} = 50\ mm$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 12.5 \times 10^6\ mm^4$$

(a) $A = 625\ mm^2$ $\quad \bar{y} = 87.5\ mm$ $\quad Q_a = A\bar{y} = 54687.5\ mm^3$

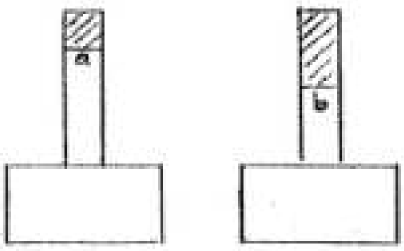
$t = 25\ mm$

$$\tau_a = \frac{VQ_a}{It} = \frac{(50000)(54687.5)}{(12.5\times10^6)(25)} = 8.75\ MPa$$

(b) $A = 1250\ mm^2$ $\quad \bar{y} = 75\ mm$ $\quad Q_b = A\bar{y} = 93750\ mm^3$

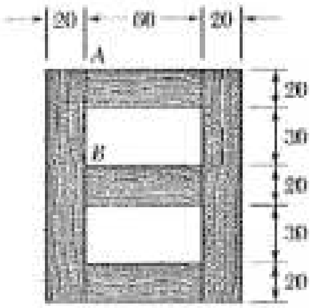
$t = 25\ mm$

$$\tau_b = \frac{VQ_b}{It} = \frac{(50000)(93750)}{(12.5\times10^6)(25)} = 15\ MPa$$



(a) (b)

Problem 6.94



Dimensions in mm

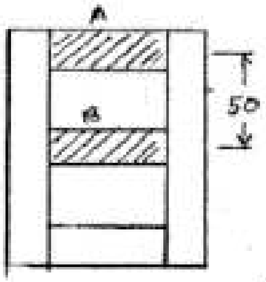
6.94 Several planks are glued together to form the box beam shown. Knowing that the beam is subjected to a vertical shear of 3 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

$$I_A = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(60)(20)^3 + (60)(20)(50)^2$$
$$= 3.04 \times 10^6 \text{ mm}^4$$

$$I_B = \frac{1}{12}bh^3 = \frac{1}{12}(60)(20)^3 = 0.04 \times 10^6 \text{ mm}^4$$

$$I_C = \frac{1}{12}bh^3 = \frac{1}{12}(20)(120)^3 = 2.88 \times 10^6 \text{ mm}^4$$

$$I = 2I_A + I_B + 2I_C = 11.88 \times 10^6 \text{ mm}^4$$
$$= 11.88 \times 10^{-6} \text{ m}^4$$



$$Q_A = A\bar{y} = (60)(20)(50) = 60 \times 10^3 \text{ mm}^3 = 60 \times 10^{-6} \text{ m}^3$$

$$t = 20 \text{ mm} + 20 \text{ mm} = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

(a) $$\tau_A = \frac{VQ}{It} = \frac{(3 \times 10^3)(60 \times 10^{-6})}{(11.88 \times 10^{-6})(40 \times 10^{-3})} = 379 \times 10^3 \text{ Pa}$$

$$\tau_A = 379 \text{ kPa}$$

$Q_B = 0$ (b) $$\tau_B = \frac{VQ_B}{It} = 0$$

$$\tau_B = 0$$

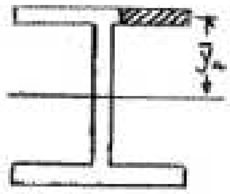
Problem 6.95

6.95 Knowing that a W360 × 122 rolled-steel beam is subjected to a 250-kN vertical shear, determine the shearing stress (*a*) at point *A*, (*b*) at the centroid *C* of the section.

For W360 × 122, $d = 363$ mm, $b_f = 257$ mm, $t_f = 21.70$ mm, $t_w = 13.0$ mm

$I = 365 \times 10^6\ \text{mm}^4 = 365 \times 10^{-6}\ \text{m}^4$

(a)



$A_a = (105)(21.70) = 2278.5\ \text{mm}^2$

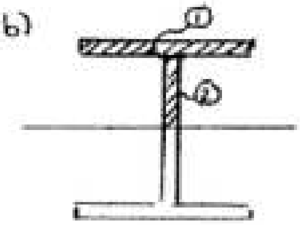
$\bar{y}_a = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65$ mm

$Q_a = A_a \bar{y}_a = 388.8 \times 10^3\ \text{mm}^3 = 388.8 \times 10^{-6}\ \text{m}^3$

$t_a = t_f = 21.70$ mm $= 21.7 \times 10^{-3}$ m

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(250 \times 10^3)(388.8 \times 10^{-6})}{(365 \times 10^{-6})(21.7 \times 10^{-3})} = 12.27 \times 10^6\ \text{Pa} = 12.27\ \text{MPa}$$ ◄

(b)



$A_1 = b_f t_f = (257)(21.70) = 5577\ \text{mm}^2$

$\bar{y}_1 = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65$ mm

$A_2 = t_w\left(\frac{d}{2} - t_f\right) = (13.0)(159.8) = 2077\ \text{mm}^2$

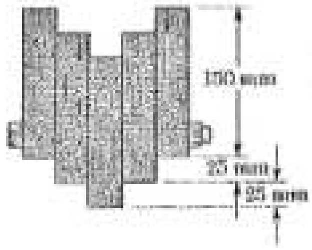
$\bar{y}_2 = \frac{1}{2}\left(\frac{d}{2} - t_f\right) = 79.9$ mm

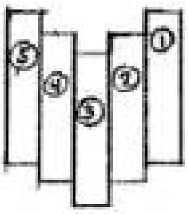
$Q_c = \Sigma A\bar{y} = (5577)(170.65) + (2077)(79.9) = 1117.7 \times 10^3\ \text{mm}^3$
$= 1117.7 \times 10^{-6}\ \text{m}^3$

$t_c = t_w = 13.0$ mm $= 13 \times 10^{-3}$ m

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(250 \times 10^3)(1117.7 \times 10^{-6})}{(365 \times 10^{-6})(13 \times 10^{-3})} = 58.9 \times 10^6\ \text{Pa} = 58.9\ \text{MPa}$$ ◄

Problem 6.96





6.96 A beam consists of five planks of 38 × 150-mm cross section connected by steel bolts with a longitudinal spacing of 220 mm. Knowing that the shear in the beam is vertical and equal to 8 kN and that the allowable average shearing stress in each bolt is 50 MPa, determine the smallest permissible bolt diameter that may be used.

Part	$A\ (mm^2)$	$\bar{y}_0\ (mm)$	$A\bar{y}_0\ (mm^3)$	$\bar{y}\ (mm)$	$A\bar{y}^2\ (mm^4)$	$\bar{I}\ mm^4$
1	5700	125	712500	20	2280000	10687500
2	5700	100	570000	−5	142500	10687500
3	5700	75	427500	−30	5130000	10687500
4	5700	100	570000	−5	142500	10687500
5	5700	125	712500	20	2280000	10687500
Σ	28500		2992500		9.975×10^6	53.4375×10^6

$$\bar{Y}_0 = \frac{\Sigma A y}{\Sigma A} = \frac{2992500}{28500} = 105 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 63.4125\times10^6$$

Between 1 and 2: $\quad Q_{12} = Q_1 = A\bar{y}_1 = (5700)(20) = 114000 \text{ mm}^3$

Between 2 and 3: $\quad Q_{23} = Q_1 + A\bar{y}_2 = 114000 + (5700)(-5) = 85500 \text{ mm}^3$

$q = \dfrac{VQ}{I}$ $\quad$ Maximum q is based on $Q_{12} = 114000 \text{ mm}^3$

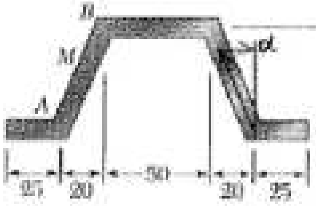
$$q = \frac{(8)(114000)}{63.4125\times10^6} = 0.01438 \text{ kN/mm}$$

$$F_{bolt} = qs = (0.01438)(220) = 3.164 \text{ kN}$$

$\tau_{bolt} = \dfrac{F_{bolt}}{A_{bolt}}$ $\qquad A_{bolt} = \dfrac{F_{bolt}}{\tau_{bolt}} = \dfrac{3164}{50} = 63.28 \text{ mm}^2$

$A_{bolt} = \dfrac{\pi}{4} d_{bolt}^2$ $\qquad d_{bolt} = \sqrt{\dfrac{4A_{bolt}}{\pi}} = \sqrt{\dfrac{(4)(63.28)}{\pi}} = 9 \text{ mm}$ ◄

Problem 6.97



Dimensions in mm

6.97 A plate of 4-mm thickness is bent as shown and then used as a beam. For a vertical shear of 12 kN, determine (*a*) the shearing stress at point *A*, (*b*) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.

$\tan\alpha = \frac{20}{48}$ $\qquad \alpha = 22.62°$

Slanted side: $A_S = (4\sec\alpha)(48) = 208\ \text{mm}^2$

$\bar{I}_S = \frac{1}{12}(4\sec\alpha)(48)^3 = 39.936\times10^3\ \text{mm}^4$

Top: $I_T = \frac{1}{12}(50)(4)^3 + (50)(4)(24)^2 = 115.46\times10^3\ \text{mm}^4$

Bottom: $I_B = I_T = 115.46\times10^3\ \text{mm}^4$

$I = 2\bar{I}_S + I_T + I_B = 310.8\times10^3\ \text{mm}^4 = 310.8\times10^{-9}\ \text{m}^4$

(a) $Q_A = (25)(4)(24) = 2.4\times10^3\ \text{mm}^3 = 2.4\times10^{-6}\ \text{m}^3$

$t = 4\ \text{mm} = 4\times10^{-3}\ \text{m}$

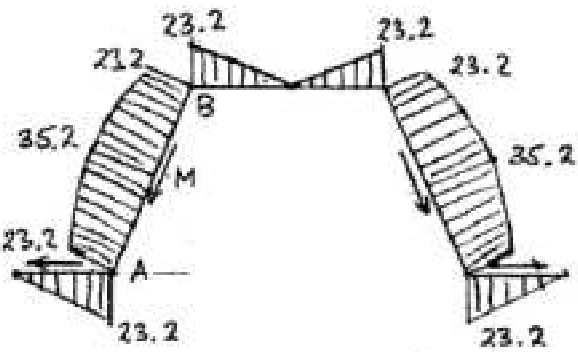
$$\tau_A = \frac{VQ_A}{It} = \frac{(12\times10^3)(2.4\times10^{-6})}{(310.8\times10^{-9})(4\times10^{-3})} = 23.2\times10^6\ \text{Pa} = 23.2\ \text{MPa}$$

(b) Maximum shearing occurs at point M, 24 mm above the bottom

$Q_M = Q_A + (4\sec\alpha)(24)(12) = 2.4\times10^3 + 1.248\times10^3 = 3.648\times10^3\ \text{mm}^3$
$= 3.648\times10^{-6}\ \text{m}^3$

$$\tau_M = \frac{VQ_M}{It} = \frac{(12\times10^3)(3.648\times10^{-6})}{(310.8\times10^{-9})(4\times10^{-3})} = 35.2\times10^6\ \text{Pa} = 35.2\ \text{MPa}$$

$Q_B = Q_A \qquad \tau_B = \tau_A = 23.2\ \text{MPa}$



Shearing stress (MPa)

Problem 6.98 continued

Just above D and just below F:

$$Q = 8.1\times10^3 + (6)(30)(30) = 13.5\times10^3\ mm^3 = 13.5\times10^{-6}\ m^3$$

$$t = 6\times10^{-3}\ m$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^3)(13.5\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 68.5\times10^6\ Pa = 68.5\ MPa$$

Just to right of D and just to the right of F:

$$Q = (30)(4)(15) = 1.8\times10^3\ mm^3 = 1.8\times10^{-6}\ m^3 \qquad t = 4\times10^{-3}\ m$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^3)(1.8\times10^{-6})}{(1.14882\times10^{-6})(4\times10^{-3})} = 13.71\times10^6\ Pa = 13.71\ MPa$$

Just below D and just above F:

$$Q = 13.5\times10^3 + 1.8\times10^3 = 15.3\times10^3\ mm^3 = 15.3\times10^{-6}\ m^3$$

$$t = 6\times10^{-3}\ m$$

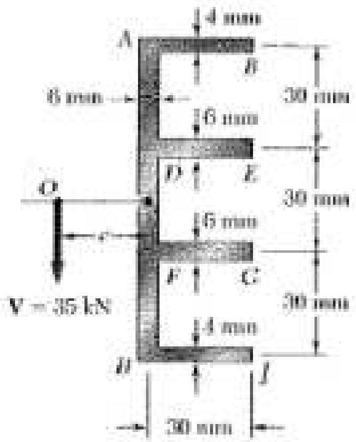
$$\tau = \frac{VQ}{It} = \frac{(35\times10^3)(15.3\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 77.7\times10^6\ Pa = 77.7\ MPa$$

At K: $Q = 15.3\times10^3 + (6)(15)(7.5) = 15.975\times10^3\ mm^3 = 15.975\times10^{-6}\ m^3$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^3)(15.975\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 81.1\times10^6\ Pa = 81.1\ MPa$$

Problem 6.99

6.98 and 6.99 For an extruded beam having the cross section shown, determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.



$I_z = 0.933 \times 10^6\ \text{mm}^4$

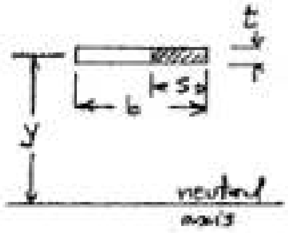
$I_{AB} = I_{HJ} = \frac{1}{12}(30)(4)^3 + (30)(4)(45)^2 = 0.24316 \times 10^6\ \text{mm}^4$

$I_{DE} = I_{FG} = \frac{1}{12}(30)(6)^3 + (30)(6)(15)^2 = 0.04104 \times 10^6\ \text{mm}^4$

$I_{AH} = \frac{1}{12}(6)(90)^3 = 0.3645 \times 10^6\ \text{mm}^3$

$I = \Sigma I = 0.9329 \times 10^6\ \text{mm}^4$

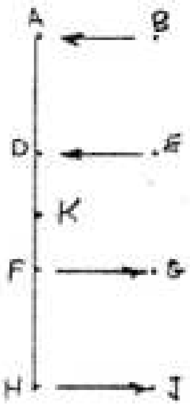
(a) For a typical flange, $A(s) = ts$



$Q(s) = yts$

$q(s) = \frac{VQ(s)}{I} = \frac{Vyts}{I}$

$F = \int_0^b q(s)\,ds = \frac{Vytb^2}{2I}$



Flange AB: $F_{AB} = \frac{V(45)(4)(30)^2}{(2)(0.9329 \times 10^6)} = 0.086826\ V \leftarrow$

Flange DE: $F_{DE} = \frac{V(15)(6)(30)^2}{(2)(0.9329 \times 10^6)} = 0.043413\ V \leftarrow$

Flange FG: $F_{FG} = 0.043413\ V \rightarrow$

Flange HJ: $F_{HJ} = 0.086826\ V \rightarrow$

$+\circlearrowright \Sigma M_K = +\circlearrowright \Sigma M_K$:

$Ve = 45 F_{AB} + 15 F_{DE} + 15 F_{FG} + 45 F_{HJ} = 9.1167\ V$

Dividing by V, $e = 9.12\ \text{mm}$ ◄

(b) Calculation of shearing stresses.

$V = 35 \times 10^3\ \text{N}$ $I = 0.9329 \times 10^{-6}\ \text{m}^4$

At B, E, G, and J: $\tau = 0$ ◄

At A and H: $Q = (30)(4)(45) = 5.4 \times 10^3\ \text{mm}^3 = 5.4 \times 10^{-6}\ \text{m}^3$

$q = \frac{VQ}{I}\ \frac{(35 \times 10^3)(5.4 \times 10^{-6})}{0.9329 \times 10^{-6}} = 202.59 \times 10^3\ \text{N/m}$

Just to the right of A and H: $t = 4 \times 10^{-3}\ \text{m}$

$\tau = \frac{q}{t} = \frac{202.59 \times 10^3}{4 \times 10^{-3}} = 50.6 \times 10^6\ \text{Pa} = 50.6\ \text{MPa}$ ◄

Continued on next page.

Problem 6.99 continued

Just below A and just above H: $t = 6\times10^{-3}$ m

$$\tau = \frac{q}{t} = \frac{202.59\times10^{3}}{6\times10^{-3}} = 33.8\times10^{6}\ \text{Pa} = 33.8\ \text{MPa}$$

Just above D and just below F: $t = 6\times10^{-3}$ m

$$Q = 5.4\times10^{3} + (6)(30)(30) = 10.8\times10^{3}\ \text{mm}^3 = 10.8\times10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(10.8\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-3})} = 67.5\times10^{6}\ \text{Pa} = 67.5\ \text{MPa}$$

Just to the right of D and E: $t = 6\times10^{-3}$ m

$$Q = (30)(6)(15) = 2.7\times10^{3}\ \text{mm}^3 = 2.7\times10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(2.7\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-3})} = 16.88\times10^{6}\ \text{Pa} = 16.88\ \text{MPa}$$

Just below D and just above F: $t = 6\times10^{-3}$

$$Q = 10.8\times10^{3} + 2.7\times10^{3} = 13.5\times10^{3}\ \text{mm}^3 = 13.5\times10^{-6}\ \text{m}^3$$

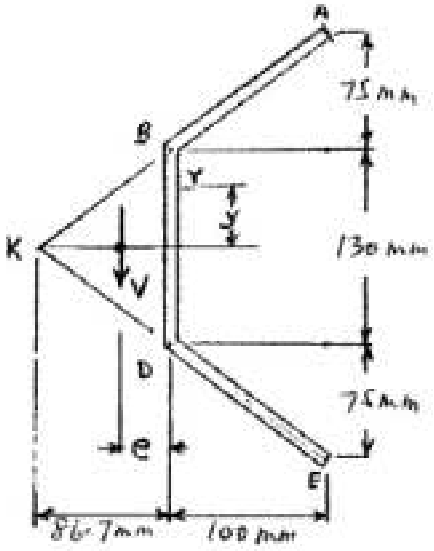
$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(13.5\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-3})} = 84.4\times10^{6}\ \text{Pa} = 84.4\ \text{MPa}$$

At K: $Q = 13.5\times10^{3} + (6)(15)(7.5) = 14.175\times10^{3}\ \text{mm}^3 = 14.175\times10^{-6}\ \text{m}^3$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(14.175\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-3})} = 88.6\times10^{6}\ \text{Pa} = 88.6\ \text{MPa}$$

Problem 6.100

6.100 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$L_{AB} = \sqrt{100^2 + 75^2} = 125 \text{ mm} \qquad A_{AB} = 125t$$

$$I_{AB} = \frac{1}{12}A_{AB}h^2 + A_{AB}d^2 = \frac{1}{12}(125t)(75)^2 + (125t)(100)^2$$

$$= 1308594\,t \text{ mm}^4$$

$$I_{BD} = \frac{1}{12}(t)(130)^3 = 183083\,t \text{ mm}^4$$

$$I = 2I_{AB} + I_{BD} = 2.8\times10^6\,t \text{ mm}^4$$

In part BD: $\quad Q = Q_{AB} + Q_{BY}$

$$Q = (125t)(100) + (65-y)t\left(\tfrac{1}{2}\right)(65+y)$$

$$= 12500t + 2112.5t - \tfrac{1}{2}ty^2$$

$$= \left(14612.5 - \tfrac{1}{2}y^2\right)t$$

$$\tau = \frac{VQ}{It} \qquad F_{BD} = \int \tau\, dA = \int_{-65}^{65} \frac{V\left(14612.5 - \frac{1}{2}y^2\right)t}{I\,t}\cdot t\,dy$$

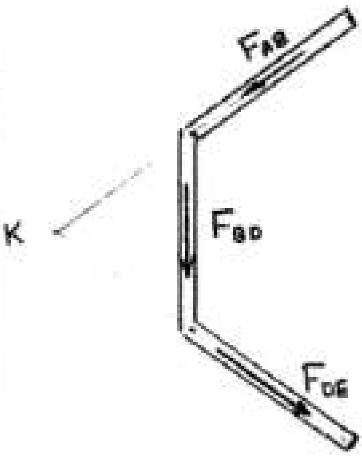
$$= \frac{Vt}{I}\int_{-65}^{65}\left(14612.5 - \tfrac{1}{2}y^2\right)dy = \frac{Vt}{I}\left[14612.5y - \tfrac{1}{6}y^3\right]_{-65}^{65}$$

$$= \frac{Vt}{I}\cdot 2\left[(14612.5)(65) - \frac{(65)^3}{6}\right] = \frac{Vt(1899623)}{2.8\times10^6\,t}$$

$$= 0.678\,V$$

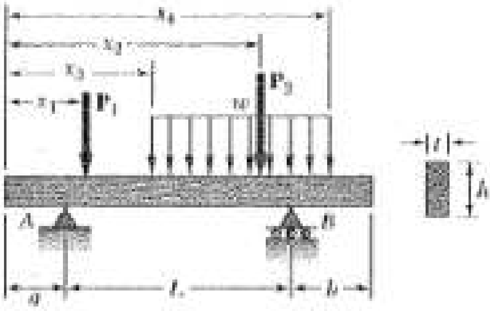
$$\circlearrowleft\Sigma M_K = \circlearrowleft\Sigma M_K: \qquad -V(86.7 - e) = -86.7(0.678\,V)$$

$$e = 86.7[1 - 0.678] = 27.9 \text{ mm} \blacktriangleleft$$



Note that the lines of action of F_{AB} and F_{DE} pass through point K. Thus, these forces have zero moment about point K.

PROBLEM 6.C1



6.C1 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress and the maximum shearing stress in the beam will not exceed given allowable values σ_{all} and τ_{all}. Measuring x from end A, write a computer program to calculate for successive cross sections, from $x = 0$ to $x = L$ and using given increments Δx, the shear, the bending moment, and the smallest value of the unknown dimension that satisfies in that section (1) the allowable normal stress requirement, (2) the allowable shearing stress requirement. Use this program to design the beams of uniform cross section of the following problems, assuming $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa, and using the increments indicated: (*a*) Prob. 5.65 ($\Delta x = 0.1$ m), (*b*) Prob. 5.157 ($\Delta x = 0.2$ m).

SOLUTION

See solution of P 5.C2 for the determination of R_A, R_B, $V(x)$, and $M(x)$

We recall that

$$V(x) = R_A\,STPA + R_B\,STPB - P_1\,STP1 - P_2\,STP2 - w(x-x_3)\,STP3 + w(x-x_4)\,STP4$$

$$M(x) = R_A(x-a)\,STPA + R_B(x-a-L)\,STPB - P_1(x-x_1)\,STP1 - P_2(x-x_2)\,STP2 - \tfrac{1}{2}w(x-x_3)^2\,STP3 + \tfrac{1}{2}w(x-x_4)^2\,STP4$$

where STPA, STPB, STP1, STP2, STP3, and STP4 are step functions defined in P5.C2.

(1) TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

If unknown dimension is h:

$S_{min} = |M|/\sigma_{all}$. From $S = \frac{1}{6}th^2$, we have $h_\sigma = h = \sqrt{6S/t}$

If unknown dimension is t:

$S_{min} = |M|/\sigma_{all}$. From $S = \frac{1}{6}th^2$, we have $t_\sigma = t = 6S/h^2$

(2) TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

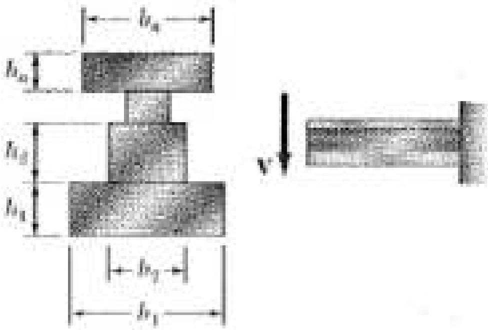
We use Eq. (6.10), page 378: $\tau_{max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{V}{th}$

If unknown dimension is h: $h_\tau = h = \dfrac{3V}{2t\tau_{all}}$

If unknown dimension is t: $t_\tau = t = \dfrac{3V}{2h\tau_{all}}$

(CONTINUED)

PROBLEM 6.C3



6.C3 A beam having the cross section shown is subjected to a vertical shear **V**. Write a computer program that can be used to calculate the shearing stress along the line between any two adjacent rectangular areas forming the cross section. Use this program to solve (*a*) Prob. 6.10, (*b*) Prob. 6.12, (*c*) Prob. 6.21.

SOLUTION

1. Enter V and the number n of rectangles.
2. For $i = 1$ to n, enter the dimensions b_i and h_i.
3. Determine the area $A_i = b_i h_i$ of each rectangle.
4. Determine the elevation of the centroid of each rectangle:

$$\bar{y}_i = \sum_{k=1}^{i} h_k - 0.5 h_i$$

and the elevation $\bar{y}$ of the centroid of the entire section:

$$\bar{y} = \left(\sum_i A_i \bar{y}_i\right) / \left(\sum_i A_i\right)$$

5. Determine the centroidal moment of inertia of the entire section:

$$I = \sum_i \left[\frac{1}{12} b_i h_i^3 + A_i (\bar{y}_i - \bar{y})^2\right]$$

6. For each surface separating two rectangles i and $i+1$, determine Q_i of the area below that surface:

$$Q_i = \sum_{k=1}^{i} = A_k (\bar{y}_k - \bar{y})$$

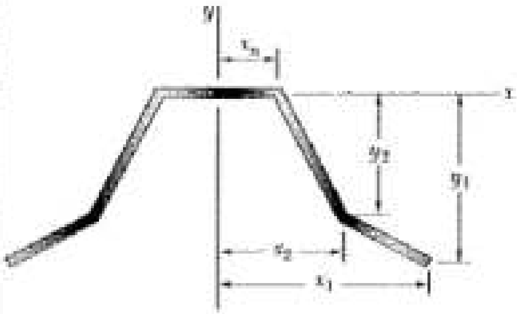
7. Select for t_i the smaller of b_i and b_{i+1}.

The shearing stress on the surface between the rectangles i and $i+1$ is

$$\tau_i = \frac{V Q_i}{I t_i}$$

(CONTINUED)

PROBLEM 6.C4



6.C4 A plate of uniform thickness t is bent as shown into a shape with a vertical plane of symmetry and is then used as a beam. Write a computer program that can be used to determine the distribution of shearing stresses caused by a vertical shear **V**. Use this program (a) to solve Prob. 6.47, (b) to find the shearing stress at a point E for the shape and load of Prob. 6.50, assuming a thickness $t = 6$ mm.

SOLUTION

For each element on the right-hand side, we compute (for $i=1$ to n):

Length of element $= L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$

Area of element $= A_i = t L_i$ where $t = 6$ mm

Distance from x axis to centroid of element $= \bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$

Distance from x axis to centroid of section:

$$\bar{y} = (\Sigma A_i \bar{y}_i)/\Sigma A_i$$

Note that $y_n = 0$ and that $x_{n+1} = y_{n+1} = 0$

Moment of inertia of section about centroidal axis:

$$\bar{I} = 2\Sigma A_i \left[\frac{1}{12}(y_i - y_{i+1})^2 + (\bar{y}_i - \bar{y})^2\right]$$

Computation of Q at point P where stress is desired

$Q = \Sigma A_i(\bar{y}_i - \bar{y})$ where sum extends to the areas located between one end of section and point P.

Shearing stress at P:

$$\tau = \frac{VQ}{It}$$

NOTE: τ_{max} occurs on neutral axis, i.e., for $y_P = \bar{y}$.

PROGRAM OUTPUTS

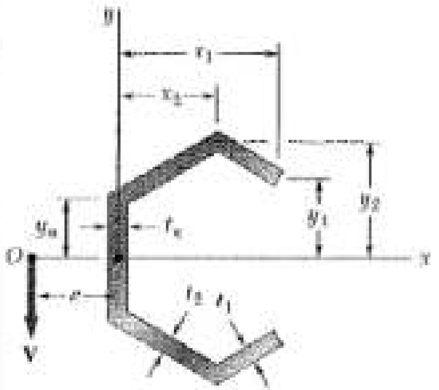
Part (a):

$I = 0.2 \times 10^6$ mm^4
Taumax = 14.06 MPa
TauB = 12.5 MPa

Part (b):

$I = 8.352 \times 10^6$ mm^4
TauE = 1.293 MPa

PROBLEM 6.C5



6.C5 The cross section of an extruded beam is symmetric with respect to the x axis and consists of several straight segments as shown. Write a computer program that can be used to determine (*a*) the location of the shear center O, (*b*) the distribution of shearing stresses caused by a vertical force applied at O. Use this program to solve Probs. 6.66 and 6.70.

SOLUTION

SINCE SECTION IS SYMMETRIC WITH x AXIS, COMPUTATIONS WILL BE DONE FOR TOP HALF.

FOR $i = 1$ TO $n+1$ (NOTE: $n+1$ IS THE ORIGIN)

ENTER t_i, x_i, y_i

COMPUTE LENGTH OF EACH SEGMENT

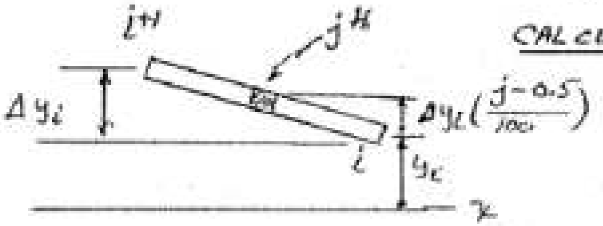
FOR $i = 1$ TO n

$\Delta x_i = x_{i+1} - x_i$

$\Delta y_i = y_{i+1} - y_i$

$L = (\Delta x_i^2 + \Delta y_i^2)^{1/2}$

CALCULATE MOMENT OF INERTIA I_x



CONSIDER EACH SEGMENT AS MADE OF 100 EQUAL PARTS

FOR $i = 1$ TO n

$\Delta AREA = L_i t_i / 100$

FOR $j = 1$ TO 100

$y = y_i + \Delta y_i (j - 0.5)/100$

$\Delta I = (\Delta AREA) y^2$

$I_x = I_x + \Delta I$

SINCE ONLY TOP HALF WAS USED

$I_x = 2 I_x$

CALCULATE SHEARING STRESS AT ENDS OF SEGMENTS AND SHEAR FORCES IN SEGMENTS

FOR $i = 1$ TO n

$\Delta AREA = L_i t_i / 100$, $\tau_{new} = \tau_{nextE}$

FOR $j = 1$ TO 100

$y = y_i + \Delta y_i (j - 0.5)/100$

$\Delta Q = (\Delta AREA) y$

$\tau_{old} = \tau_{new}$, $Q = Q + \Delta Q$

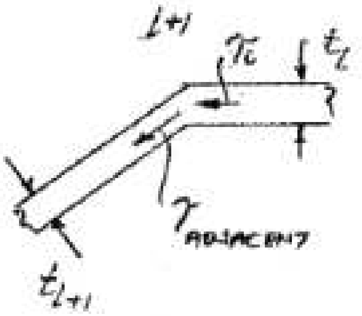
$\tau_{new} = VQ / I_x t_i$

$\tau_{ave} = 0.5(\tau_{old} + \tau_{new})$

$\tau = \tau + \tau_{ave}$

CONTINUED

PROBLEM 6.C5 - CONTINUED



$\text{FORCE}_i = \tau(\Delta\text{AREA})$

$\tau_i = VQ/I_x t_i$

$(\tau_{ADJACENT})_i = VQ/I_x t_{i+1}$

$Q_i = Q$

$\tau_{next} = (\tau_{ADJACENT})_i$

LOCATION OF SHEAR CENTER

CALCULATE MOMENT OF SHEAR FORCES ABOUT ORIGIN

FOR $i = 1$ TO n

$(F_x)_i = \text{FORCE}_i\,(\Delta x_i)/L_i$

$(F_y)_i = \text{FORCE}_i\,(\Delta y_i)/L_i$

$\text{MOMENT}_i = -(F_x)_i\, y_i + (F_y)_i\, x_i$

$\text{MOMENT} = \text{MOMENT} + \text{MOMENT}_i$

FOR WHOLE SECTION $\text{MOMENT} = 2(\text{MOMENT})$

SHEAR CENTER IS AT

$e = \text{MOMENT}/V$

PROGRAM OUTPUT

Prob. 6.66

	T(K) mm	X(K) mm	Y(K) mm	L(K) mm
1	3	100	.00	75
2	3	100	75	100
3	3	.00	75	75
4	3	.00	.00	

Moment of inertia: Ix = $5.063 \times 10^6\ mm^4$ Shear = 2 kN

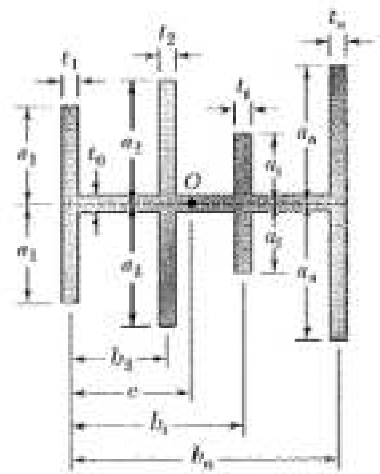
Junction of segments	Q mm**3	Tau Before MPa	Tau After MPa	Force in segment kN
1 and 2	8435.7	6.66	6.66	0.499
2 and 3	30937.5	24.44	24.44	4.640
3 and 4	39375	31.11	31.11	6.461

Moment of shear forces about origin: M = 800 kNm
+ counterclockwise

Distance from origin to shear center: e = 66.667 mm

CONTINUED

PROBLEM 6.C6



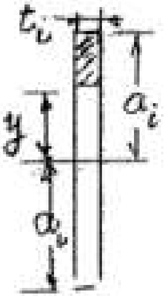
6.C6 A thin-walled beam has the cross section shown. Write a computer program that can be used to determine the location of the shear center O of the cross section. Use the program to solve Prob. 6.75.

SOLUTION

Distribution of shearing stresses in element i

Let V = shear in cross section

I = Centroidal moment of inertia of section

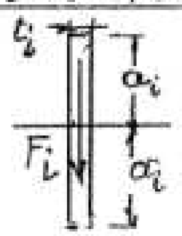


We have for shaded area

$$Q = A\bar{y} = t_i(a_i - y)\,\frac{a_i + y}{2} = \frac{1}{2}t_i(a_i^2 - y^2)$$

$$\tau = \frac{QV}{It_i} = \frac{V}{2I}(a_i^2 - y^2)$$

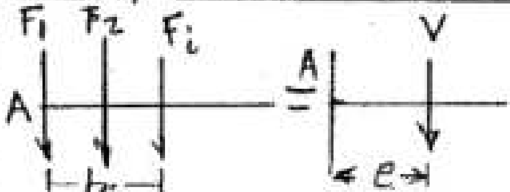
Force exerted on element i



$$F_i = \int_{-a_i}^{a_i} \tau(t_i\,dy) = \frac{Vt_i}{2I}\int_{-a_i}^{a_i}(a_i^2 - y^2)\,dy$$

$$= \frac{Vt_i}{I}\int_0^{a_i}(a_i^2 - y^2)\,dy = \frac{Vt_i}{I}\left(a_i^3 - \frac{1}{3}a_i^3\right) = \frac{2}{3}\frac{V}{I}t_i a_i^2$$

The system of the forces F_i must be equivalent to V at shear center.



$$\Sigma F_i = \Sigma F: \quad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 = V \qquad (1)$$

$$\Sigma M_A = \Sigma M_A: \quad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 b_i = eV \qquad (2)$$

Divide (2) by (1): $\quad e = \dfrac{\Sigma t_i a_i^3 b_i}{\Sigma t_i a_i^3}$ ◄

PROGRAM OUTPUT:

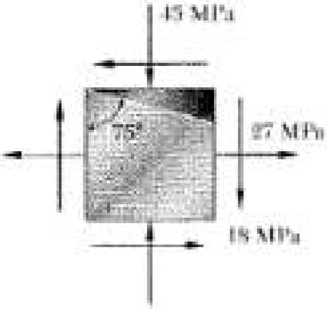
```
Problem 6.75

For element 1:
t = 18mm, a = 100mm, b = 0

For element 2:
t = 18mm, a = 75mm, b= 200mm

Answer: e = 59.34 mm
```
◄

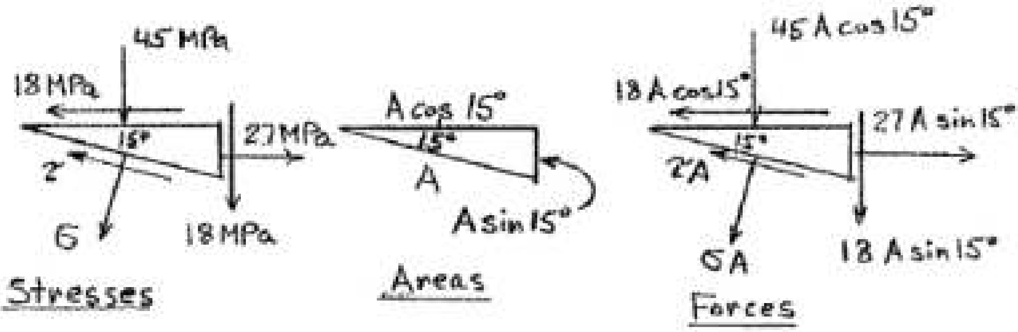
Problem Solutions

By
Dean Updike

Problem 7.1

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.





$+\swarrow \Sigma F = 0:$

$$\sigma A + 18 A \cos 15^\circ \sin 15^\circ + 45 A \cos 15^\circ \cos 15^\circ - 27 A \sin 15^\circ \sin 15^\circ + 18 A \sin 15^\circ \cos 15^\circ = 0$$

$$\sigma = -18 \cos 15^\circ \sin 15^\circ - 45 \cos^2 15^\circ + 27 \sin^2 15^\circ - 18 \sin 15^\circ \cos 15^\circ$$

$$\sigma = -49.2 \text{ MPa}$$

$+\nwarrow \Sigma F = 0:$

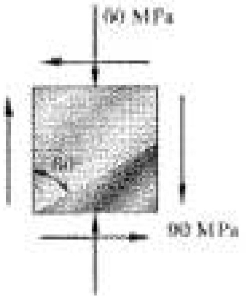
$$\tau A + 18 A \cos 15^\circ \cos 15^\circ - 45 A \cos 15^\circ \sin 15^\circ - 27 A \sin 15^\circ \cos 15^\circ - 18 A \sin 15^\circ \sin 15^\circ = 0$$

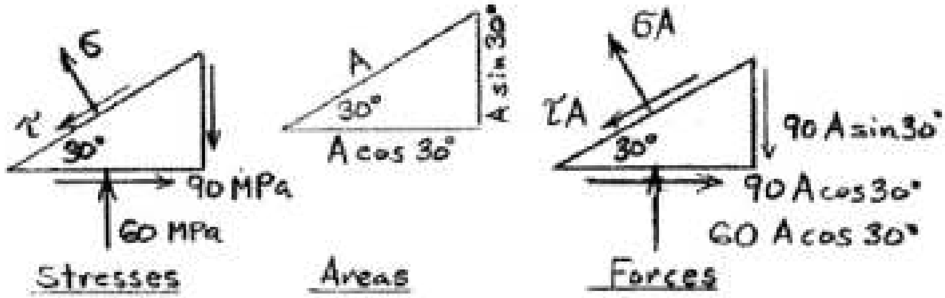
$$\tau = -18 (\cos^2 15^\circ - \sin^2 15^\circ) + (45 + 27) \cos 15^\circ \sin 15^\circ$$

$$\tau = 2.41 \text{ MPa}$$

Problem 7.2

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.





$+\nwarrow \Sigma F = 0$

$$\sigma A - 90 A \sin 30^\circ \cos 30^\circ - 90 A \cos 30^\circ \sin 30^\circ + 60 A \cos 30^\circ \cos 30^\circ = 0$$

$$\sigma = 180 \sin 30^\circ \cos 30^\circ - 60 \cos^2 30^\circ = 32.9 \text{ MPa}$$

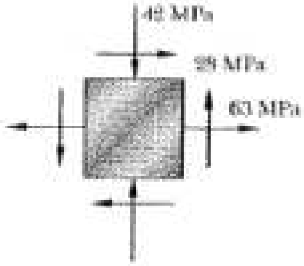
$+\swarrow \Sigma F = 0$

$$\tau A + 90 A \sin 30^\circ \sin 30^\circ - 90 A \cos 30^\circ \cos 30^\circ - 60 A \cos 30^\circ \sin 30^\circ = 0$$

$$\tau = 90 (\cos^2 30^\circ - \sin^2 30^\circ) + 60 \cos 30^\circ \sin 30^\circ = 71.0 \text{ MPa} \swarrow$$

Problem 7.7

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



$\sigma_x = 63 \text{ MPa} \qquad \sigma_y = -42 \text{ MPa} \qquad \tau_{xy} = 28 \text{ MPa}$

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(28)}{63+42} = 0.5333$

$2\theta_p = 28.07° \qquad \theta_p = 14.04°, \ 104.04°$ ◄

(b) $\sigma_{max,min} = \dfrac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \dfrac{63-42}{2} \pm \sqrt{\left(\dfrac{63+42}{2}\right)^2 + 28^2}$

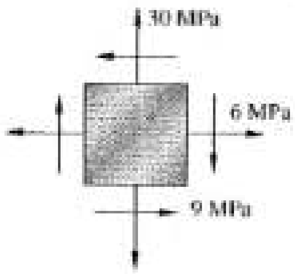
$= 10.5 \pm 59.5$ ◄

$\sigma_{max} = 70 \text{ MPa}$ ◄

$\sigma_{min} = -49 \text{ MPa}$

Problem 7.8

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



$\sigma_x = 6 \text{ MPa} \qquad \sigma_y = 30 \text{ MPa} \qquad \tau_{xy} = -9 \text{ MPa}$

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(-9)}{6-30} = -0.750$

$2\theta_p = 36.87° \qquad \theta_p = 18.4°, \ 108.4°$ ◄

(b) $\sigma_{max,min} = \dfrac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

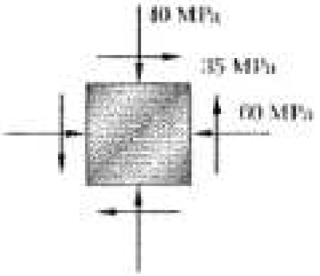
$= \dfrac{6+30}{2} \pm \sqrt{\left(\dfrac{6-30}{2}\right)^2 + (-9)^2}$

$= 18 \pm 15 \qquad \sigma_{max} = 33.0 \text{ MPa}$ ◄

$\sigma_{min} = 3.00 \text{ MPa}$ ◄

Problem 7.9

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.



$\sigma_x = -60$ MPa $\qquad \sigma_y = -40$ MPa $\qquad \tau_{xy} = 35$ MPa

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{-60+40}{(2)(35)} = 0.2857$

$2\theta_s = 15.95°$ $\qquad \theta_s = 8.0°,\ 98.0°$ ◄

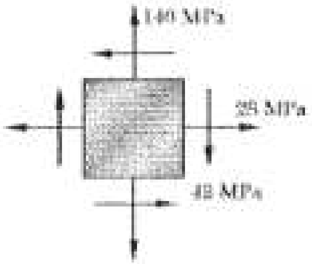
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-60+40}{2}\right)^2 + (35)^2} \qquad \tau_{max} = 36.4 \text{ MPa} ◄$$

(b) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{-60-40}{2}$ $\qquad \sigma' = -50.0$ MPa ◄

Problem 7.10

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.



$\sigma_x = 28$ MPa $\qquad \sigma_y = 140$ MPa $\qquad \tau_{xy} = -42$ MPa

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{28-140}{(2)(-42)} = -1.3333$

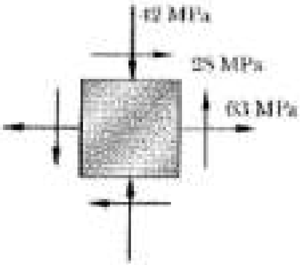
$2\theta_s = -53.13°$ $\qquad \theta_s = -26.57°,\ 63.43°$ ◄

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \sqrt{\left(\frac{28-140}{2}\right)^2 + (-42)^2} = 70 \text{ MPa} ◄$$

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{28+140}{2} = 84$ MPa ◄

Problem 7.11



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

$\sigma_x = 63\text{ MPa} \qquad \sigma_y = -42\text{ MPa} \qquad \tau_{xy} = 28\text{ MPa}$

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{63+42}{(2)(28)} = -1.875$

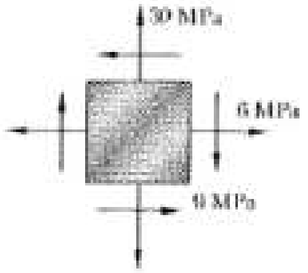
$2\theta_s = -61.93° \qquad \theta_s = -30.96°,\ 59.04°$ ◄

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{63+42}{2}\right)^2 + (28)^2} = 59.5\text{ MPa}$ ◄

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{63-42}{2} = 10.5\text{ MPa}$ ◄

Problem 7.12



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

$\sigma_x = 6\text{ MPa} \qquad \sigma_y = 30\text{ MPa} \qquad \tau_{xy} = -9\text{ MPa}$

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{6-30}{(2)(-9)} = -1.33333$

$2\theta_s = -53.13° \qquad \theta_s = -26.6°,\ 63.4°$ ◄

$\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

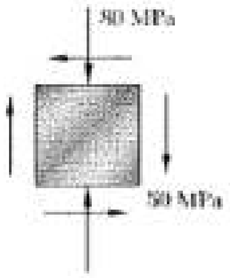
$= \sqrt{\left(\dfrac{6-30}{2}\right)^2 + (-9)^2} \qquad \tau_{max} = 15.00\text{ MPa}$ ◄

(b) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2}$

$= \dfrac{6+30}{2} \qquad \sigma' = 18.00\text{ MPa}$ ◄

Problem 7.13

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$$\sigma_x = 0 \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = -50 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \qquad \frac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\underline{\theta = -25^\circ}$ $\qquad 2\theta = -50^\circ$

$\sigma_{x'} = -40 + 40\cos(-50^\circ) - 50\sin(-50^\circ)$ $\qquad \sigma_{x'} = 24.0 \text{ MPa}$

$\tau_{x'y'} = -40\sin(-50^\circ) - 50\cos(-50^\circ)$ $\qquad \tau_{x'y'} = -1.5 \text{ MPa}$

$\sigma_{y'} = -40 - 40\cos(-50^\circ) + 50\sin(-50^\circ)$ $\qquad \sigma_{y'} = -104.0 \text{ MPa}$

(b) $\underline{\theta = 10^\circ}$ $\qquad 2\theta = 20^\circ$

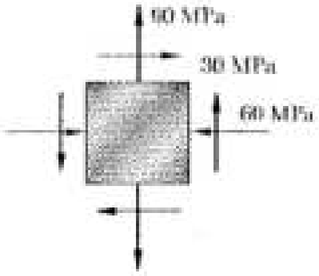
$\sigma_{x'} = -40 + 40\cos(20^\circ) - 50\sin(20^\circ)$ $\qquad \sigma_{x'} = -19.5 \text{ MPa}$

$\tau_{x'y'} = -40\sin(20^\circ) - 50\cos(20^\circ)$ $\qquad \tau_{x'y'} = -60.7 \text{ MPa}$

$\sigma_{y'} = -40 - 40\cos(20^\circ) + 50\sin(20^\circ)$ $\qquad \sigma_{y'} = -60.5 \text{ MPa}$

Problem 7.15

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$$\sigma_x = -60 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \tau_{xy} = 30 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa} \qquad \frac{\sigma_x - \sigma_y}{2} = -75 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\underline{\theta = -25^\circ}$ $\qquad 2\theta = -50^\circ$

$\sigma_{x'} = 15 - 75\cos(-50^\circ) + 30\sin(-50^\circ)$ $\qquad \sigma_{x'} = -56.2 \text{ MPa}$ ◀

$\tau_{x'y'} = +75\sin(-50^\circ) + 30\cos(-50^\circ)$ $\qquad \tau_{x'y'} = -38.2 \text{ MPa}$ ◀

$\sigma_{y'} = 15 + 75\cos(-50^\circ) - 30\sin(-50^\circ)$ $\qquad \sigma_{y'} = 86.2 \text{ MPa}$ ◀

(b) $\underline{\theta = 10^\circ}$ $\qquad 2\theta = 20^\circ$

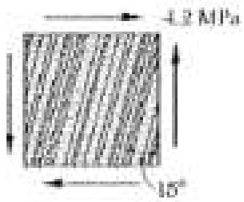
$\sigma_{x'} = 15 - 75\cos(20^\circ) + 30\sin(20^\circ)$ $\qquad \sigma_{x'} = -45.2 \text{ MPa}$ ◀

$\tau_{x'y'} = +75\sin(20^\circ) + 30\cos(20^\circ)$ $\qquad \tau_{x'y'} = 53.8 \text{ MPa}$ ◀

$\sigma_{y'} = 15 + 75\cos(20^\circ) - 30\sin(20^\circ)$ $\qquad \sigma_{y'} = 75.2 \text{ MPa}$ ◀

Problem 7.17

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 4.2\ \text{MPa}$

$\theta = -15° \qquad 2\theta = -30°$

(a) $\tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= -0 + 4.2\cos(-30°)$

$= 3.64\ \text{MPa}$ ◄

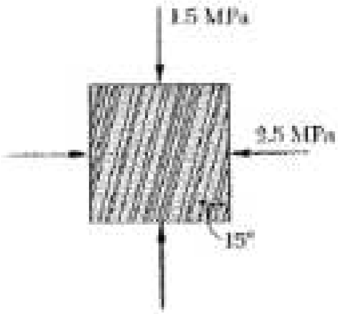
(b) $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$= 0 + 0 + 4.2\sin(-30°)$

$= -2.1\ \text{MPa}$ ◄

Problem 7.18

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$\sigma_x = -2.5\ \text{MPa} \qquad \sigma_y = -1.5\ \text{MPa} \qquad \tau_{xy} = 0$

$\theta = -15° \qquad 2\theta = -30°$

(a) $\tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= -\dfrac{-2.5 - (-1.5)}{2}\sin(-30°) + 0$

$\tau_{x'y'} = -0.250\ \text{MPa}$ ◄

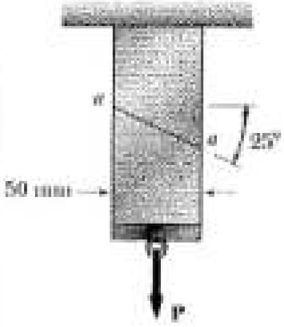
(b) $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

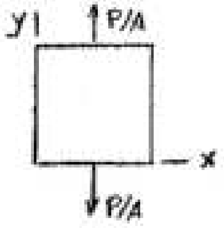
$= \dfrac{-2.5 + (-1.5)}{2} + \dfrac{-2.5 - (-1.5)}{2}\cos(-30°) + 0$

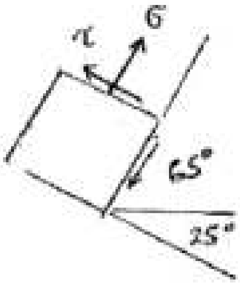
$\sigma_{x'} = -2.43\ \text{MPa}$ ◄

Problem 7.19

7.19 Two members of uniform cross section 50 × 80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are σ = 800 kPa and τ = 600 kPa, determine the largest centric load **P** that can be applied.







For plane a-a $\theta = 65°$.

$$\sigma_x = 0, \quad \tau_{xy} = 0, \quad \sigma_y = \frac{P}{A}$$

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta = 0 + \frac{P}{A}\sin^2 65° + 0$$

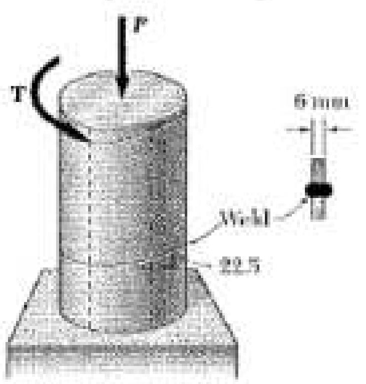
$$P = \frac{A\sigma}{\sin^2 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(800\times10^{3})}{\sin^2 65°} = 3.90\times10^{3}\ \text{N}$$

$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) = \frac{P}{A}\sin 65°\cos 65° + 0$$

$$P = \frac{A\tau}{\sin 65°\cos 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(600\times10^{3})}{\sin 65°\cos 65°} = 6.27\times10^{3}\ \text{N}$$

Allowable value of P is the smaller one. $P = 3.70$ kN ◄

Problem 7.20



7.20 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 160-kN axial force **P** and an 800 N · m torque **T**, each directed as shown, are applied to the pipe, determine σ and τ in directions, respectively, normal and tangential to the weld.

$$d_2 = 0.3\,m, \quad c_2 = \tfrac{1}{2} d_2 = 0.15\,m, \quad t = 0.006\,m$$

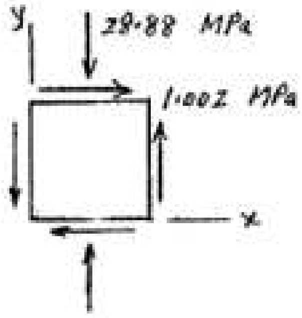
$$c_1 = c_2 - t = 0.144$$

$$A = \pi (c_2^2 - c_1^2) = \pi (0.15^2 - 0.144^2) = 5541.8 \times 10^{-6}\, m^2$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.15^4 - 0.144^4) = 119.8 \times 10^{-6}\, m^4$$

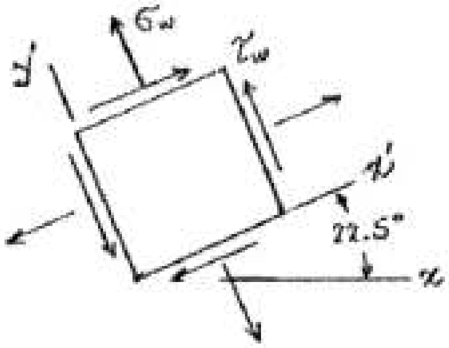
<u>Stresses</u>

$$\sigma = -\frac{P}{A} = -\frac{160 \times 10^3}{5541 \times 10^{-6}} = -28.88 \text{ MPa}$$



$$\tau = \frac{T c_2}{J} = \frac{(800)(0.15)}{119.8 \times 10^{-6}} = 1.002 \text{ MPa}$$

$$\sigma_x = 0, \quad \sigma_y = -28.88 \text{ MPa}, \quad \tau_{xy} = 1.002 \text{ MPa}$$



Choose the x' and y' axes respectively tangential and normal to the weld.

Then, $\sigma_w = \sigma_{y'}$ and $\tau_w = \tau_{x'y'}$

$$\theta = 22.5°$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{(-28.88)}{2} - \frac{[-(-28.88)]}{2}\cos 45° - 1.002 \sin 45°$$

$$= -25.36 \text{ MPa}$$

$\sigma_w = -25.4$ MPa ◄

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

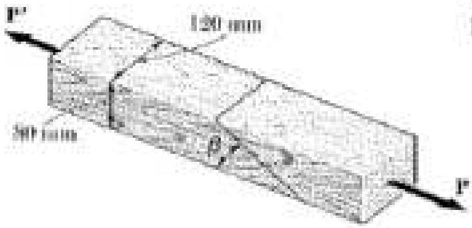
$$= -\frac{[-(-28.88)]}{2}\sin 45° + 1.002 \cos 45°$$

$$= -9.5 \text{ MPa}$$

$\tau_w = -9.5$ MPa ◄

Problem 7.21

7.21 Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25°$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (*a*) the in-plane shearing stress parallel to the splice, (*b*) the normal stress perpendicular to the splice.



Forces

Areas

$A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$

(b) $+\nearrow \Sigma F_{y'} = 0: \quad N - P\sin\beta = 0 \qquad N = P\sin\beta = (10\times10^3)\sin 25° = 4.226\times10^3 \text{ N}$

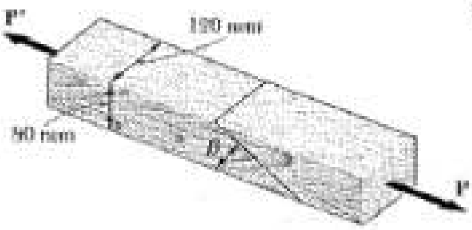
$$\sigma = \frac{N}{A/\sin\beta} = \frac{(4.226\times10^3)\sin 25°}{9.6\times10^{-3}} = 186.0\times10^3 \text{ Pa} = 186.0 \text{ kPa}$$

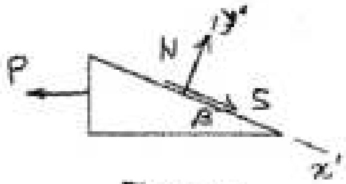
(a) $+\searrow \Sigma F_{x'} = 0: \quad S - P\cos\beta = 0 \qquad S = P\cos\beta = (10\times10^3)\cos 25° = 9.063\times10^3 \text{ N}$

$$\tau = \frac{N}{A/\sin\beta} = \frac{(9.063\times10^3)\sin 25°}{9.6\times10^{-3}} = 399\times10^3 \text{ Pa} = 399 \text{ kPa}$$

Problem 7.22

7.22 Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22°$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice) and 600 kPa in shear (parallel to the splice), determine the largest centric load **P** that can be applied.





Forces

Areas

$A = (80)(120) = 9.6\times10^3 \text{ mm}^2 = 9.6\times10^{-3} \text{ m}^2$

$$N_{all} = \sigma_{all} A/\sin\beta = \frac{(400\times10^3)(9.6\times10^{-3})}{\sin 22°} = 10.251\times10^3 \text{ N}$$

$$+\nearrow \Sigma F_{y'} = 0: \quad N - P\sin\beta = 0 \qquad P = \frac{N}{\sin\beta} = \frac{10.251\times10^3}{\sin 22°} = 27.4\times10^3 \text{ N}$$

$$S_{all} = \tau_{all} A/\sin\beta = \frac{(600\times10^3)(9.6\times10^{-3})}{\sin 22°} = 15.376\times10^3 \text{ N}$$

$$+\searrow \Sigma F_{x'} = 0: \quad S - P\cos\beta = 0 \qquad P = \frac{S}{\cos\beta} = \frac{15.376\times10^3}{\cos 22°} = 16.58\times10^3 \text{ N}$$

The smaller value for P governs. $\qquad P = 16.58 \text{ kN}$

Problem 7.24 continued

Properties of section. (Circle) $c = \frac{1}{2}d = 30\text{ mm} = 30\times10^{-3}\text{ m}$

$$A = \pi c^2 = \pi(30)^2 = 2.8274\times10^{3}\text{ mm}^2 = 2.8274\times10^{-3}\text{ m}^2$$

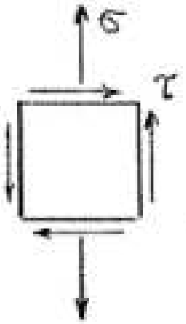
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(30)^4 = 636.17\times10^{3}\text{ mm}^4 = 636.17\times10^{-9}\text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30)^4 = 1.27235\times10^{6}\text{ mm}^4 = 1.27235\times10^{-6}\text{ m}^4$$

(Semicircle) $Q = \frac{2}{3}c^3 = \frac{2}{3}(30)^3 = 18\times10^{3}\text{ mm}^3 = 18\times10^{-6}\text{ m}^3$

$$t = d = 60\text{ mm} = 60\times10^{-3}\text{ m}$$

Stresses at K.



$$\sigma = \frac{F_y}{A} + \frac{M_z c}{I} = \frac{-18\times10^{3}}{2.8274\times10^{-3}} + \frac{(-2.7\times10^{3})(30\times10^{-3})}{636.17\times10^{-9}}$$

$$= -133.69\times10^{6}\text{ Pa} = -133.69\text{ MPa}$$

$$\tau = -\frac{F_z Q}{It} + \frac{M_y c}{J} = -\frac{(-7.5\times10^{3})(18\times10^{-6})}{(636.17\times10^{-9})(60\times10^{-3})} + \frac{(1.125\times10^{3})(30\times10^{-3})}{1.27235\times10^{-6}}$$

$$= 3.5368\times10^{6} + 26.526\times10^{6}$$

$$= 30.06\times10^{6}\text{ Pa} = 30.06\text{ MPa}$$

Principal stresses $\quad \sigma_{max,min} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$\sigma_{max,min} = \frac{0-133.69}{2} \pm \sqrt{\left(\frac{0+133.69}{2}\right)^2 + (30.06)^2}$$

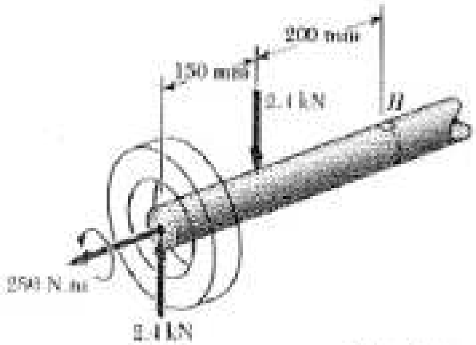
$$= -66.845 \pm 73.293\text{ MPa}$$

$\sigma_{max} = 6.45\text{ MPa}$ ◄

$\sigma_{min} = -140.2\text{ MPa}$ ◄

Maximum shearing stress $\quad \tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \tau_{max} = 73.3\text{ MPa}$ ◄

Problem 7.25



7.25 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 30 mm, determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.

$$c = \frac{1}{2}d = \frac{1}{2}(30) = 15 \text{ mm}$$

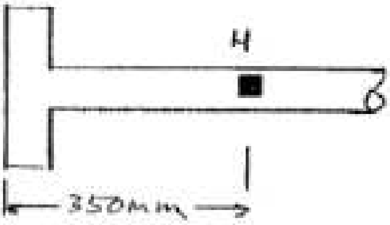
Torsion: $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

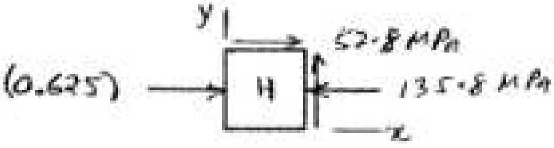
$$\tau = \frac{(2)(280)}{\pi(15)^3} = 52.8 \text{ kPa}$$

Bending: $I = \frac{\pi}{4}c^4 = 39761 \text{ mm}^4$

$M = (150)(2400) = 360000$ N mm $\quad \sigma = -\frac{My}{I} = -\frac{(36\times10^4)(15)}{39761} = -135.8$ MPa

Top view





Stresses

$\sigma_x = -135.8$ MPa $\qquad \sigma_y = 0 \qquad \tau_{xy} = 52.8$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -67.9$ MPa

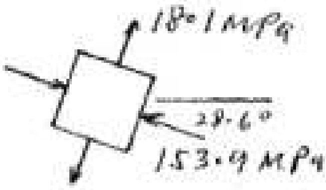
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-67.9)^2 + (52.8)^2} = 86$ MPa

(a) $\sigma_1 = \sigma_{ave} + R = -67.9 + 86 = 18.1$

$\sigma_2 = \sigma_{ave} - R = -67.9 - 86 = -153.9$ MPa

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(52.8)}{-67.9} = -1.5552$

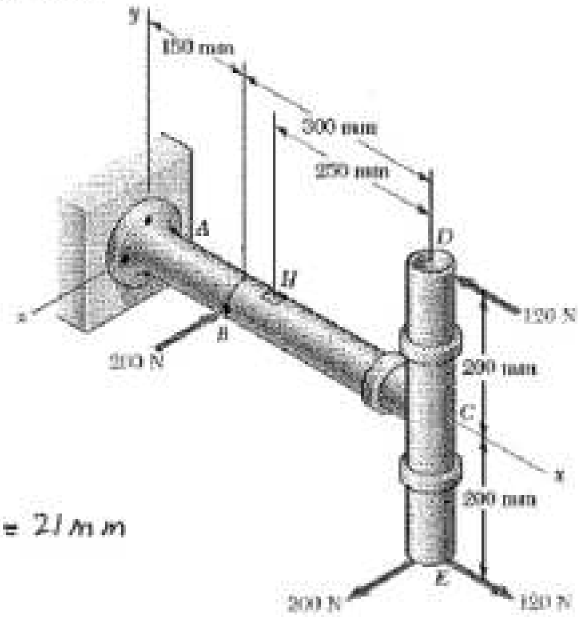
$\theta_p = -28.6°$ and $61.4°$



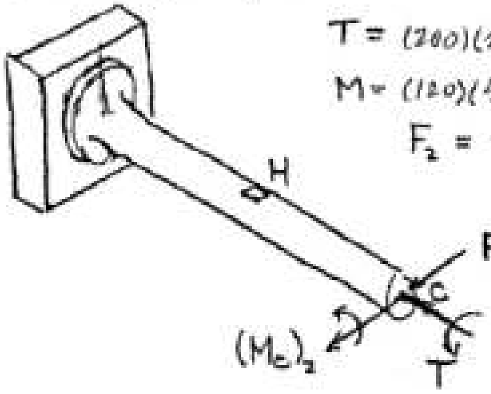
(b) $\tau_{max} = R = 86$ MPa

Problem 7.26

7.26 Several forces are applied to the pipe assembly shown. Knowing that the inner and outer diameters of the pipe are equal to 38 mm and 42 mm, respectively, determine (*a*) the principal planes and the principal stresses at point *H* located at the top of the outside surface of the pipe, (*b*) the maximum shearing stress at the same point.



Replace forces on pipe DCE by an equivalent force-couple system at C.



$T = (200)(200) = 40000$ N mm

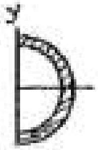
$M = (120)(400)\ 48000$ N mm

$F_z = 200$ N

Cross section. $c_1 = \frac{d_i}{2} = 19$ mm $\quad c_2 = \frac{d_o}{2} = 21$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 100782$ mm^4

$I = \frac{1}{2}J = 50391$ mm^4



$Q_y = \frac{2}{3}(c_2^3 - c_1^3) = \frac{2}{3}(21^3 - 19^3) = 1601$ mm^3

$t = c_2 - c_1 = 2$ mm

At the section containing element H

$T = 40000$ Nmm, $\quad M_z = 48000$ Nmm, $\quad V_z = 200$ N

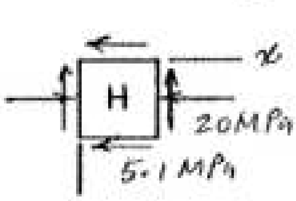
Stresses.

Torsion: $\tau_{zx} = -\frac{Tc}{J} = -\frac{(40000)(21)}{100782} = -8.3$ MPa

Bending: $\sigma_x = -\frac{My}{I} = -\frac{(48000)(21)}{50391} = -20$ MPa

Transverse Shear: $\tau_{zx} = \frac{VQ}{I(2t)} = \frac{(200)(1601)}{(50391)(2)} = 3.2$ MPa

Total: $\sigma_z = 0$, $\quad \sigma_x = -20$ MPa, $\quad \tau_{zx} = -8.3 + 3.2 = -5.1$ MPa

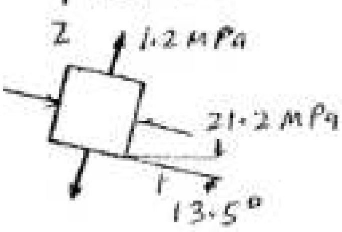


(a) $\tan 2\theta_p = \frac{2\tau_{zx}}{\sigma_z - \sigma_x} = \frac{(2)(-5.1)}{0+20} = -0.51$

$\theta_p = -13.5°$ and $76.5°$ from z-axis. ◀

$\sigma_{ave} = \frac{1}{2}(\sigma_z + \sigma_x) = \frac{1}{2}(0 - 20) = -10$ MPa

$R = \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} = \sqrt{10^2 + 5.1^2} = 11.2$ MPa



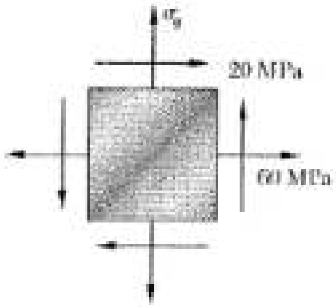
$\sigma_1 = \sigma_{ave} + R = 1.2$ MPa at $13.5°$ ↻ from z-axis. ◀

$\sigma_2 = \sigma_{ave} - R = -21.2$ MPa at $76.5°$ ↻ from z-axis. ◀

(b) $\tau_{max} = R$ $\qquad \tau_{max} = 11.2$ MPa ◀

Problem 7.27

7.27 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.



$\sigma_x = 60$ MPa, $\sigma_y = ?$, $\tau_{xy} = 20$ MPa

Let $u = \dfrac{\sigma_x - \sigma_y}{2}$. Then $\sigma_y = \sigma_x - 2u$

$R = \sqrt{u^2 + \tau_{xy}^2} = 75$ MPa

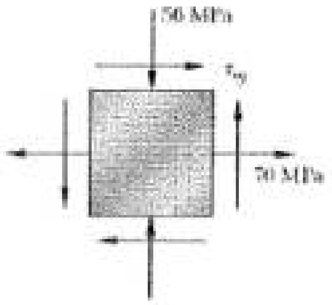
$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{75^2 - 20^2} = 72.284$ MPa

$\sigma_y = \sigma_x - 2u = 60 \mp (2)(72.284) = -84.6$ MPa or 205 MPa

Largest value of σ_y is required. $\sigma_y = 205$ MPa

Problem 7.28

7.28 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 84 MPa, (*b*) the corresponding principal stresses.



$\sigma_x = 70$ MPa, $\sigma_y = -56$ MPa, $\tau_{xy} = ?$

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{70-(-56)}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{63^2 + \tau_{xy}^2} = 84 \text{ MPa}$$

(a) $\tau_{xy} = \sqrt{84^2 - 63^2} = 55.6$ MPa

(b) $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 7$ MPa

$\sigma_a = \sigma_{ave} + R = 7 + 84 = 91$ MPa

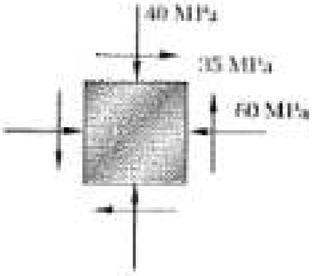
$\sigma_b = \sigma_{ave} - R = 7 - 84 = -77$ MPa

Problem 7.31

7.31 Solve Probs. 7.5 and 7.9, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.



$\sigma_x = -60\text{ MPa}, \qquad \sigma_y = -40\text{ MPa}, \qquad \tau_{xy} = 35\text{ MPa}$

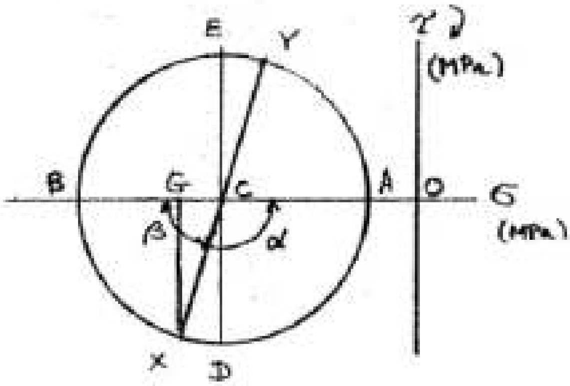
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -50\text{ MPa}$

Plotted points for Mohr's circle.

X: $(\sigma_x, -\tau_{xy}) = (-60\text{ MPa}, -35\text{ MPa})$

Y: $(\sigma_y, \tau_{xy}) = (-40\text{ MPa}, 35\text{ MPa})$

C: $(\sigma_{ave}, 0) = (-50\text{ MPa}, 0)$



(a) $\tan\beta = \dfrac{GX}{CG} = \dfrac{35}{10} = 3.500$

$\beta = 74.05°$

$\theta_B = -\frac{1}{2}\beta = -37.03°$ ◄

$\alpha = 180° - \beta = 105.95°$

$\theta_A = \frac{1}{2}\alpha = 52.97°$ ◄

$R = \sqrt{\overline{CG}^2 + \overline{GX}^2} = \sqrt{10^2 + 35^2} = 36.4\text{ MPa}$

(b) $\sigma_{min} = \sigma_{ave} - R = -50 - 36.4$ $\qquad \sigma_{min} = -86.4\text{ MPa}$ ◄

$\sigma_{max} = \sigma_{ave} + R = -50 + 36.4$ $\qquad \sigma_{max} = -13.6\text{ MPa}$ ◄

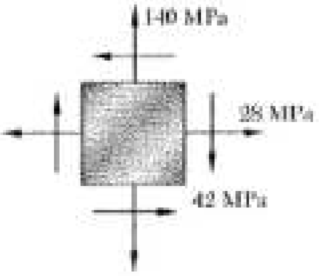
(a′) $\theta_D = \theta_B + 45° = 7.97°$ $\qquad \theta_d = 8.0°$ ◄

$\theta_E = \theta_A + 45° = 97.97°$ $\qquad \theta_e = 98.0°$ ◄

$\tau_{max} = R = 36.4\text{ MPa}$ $\qquad \tau_{max} = 36.4\text{ MPa}$ ◄

(b′) $\sigma' = \sigma_{ave} = -50\text{ MPa}$ $\qquad \sigma' = -50.0\text{ MPa}$ ◄

Problem 7.32



7.32 Solve Probs. 7.6 and 7.10, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

$\sigma_x = 28 \text{ MPa}, \quad \sigma_y = -140 \text{ MPa}, \quad \tau_{xy} = -42 \text{ MPa}.$

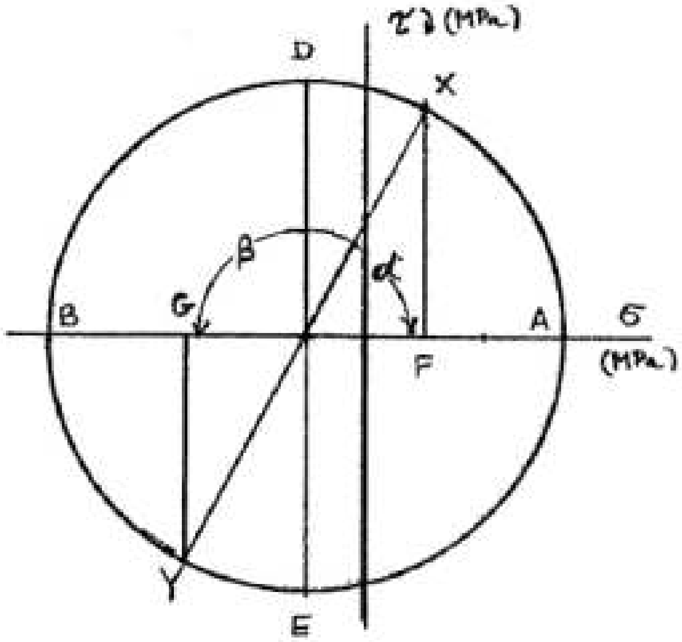
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -56 \text{ MPa}$$

Plotted points for Mohr's circle.

$X: (\sigma_x, -\tau_{xy}) = (28 \text{ MPa}, 42 \text{ MPa})$

$Y: (\sigma_y, \tau_{xy}) = (-140 \text{ MPa}, -42 \text{ MPa})$

$C: (\sigma_{ave}, 0) = (-56 \text{ MPa}, 0)$



(a) $\tan\alpha = \frac{FX}{CF} = \frac{42}{84} = 0.5$

$\alpha = 26.57°$

$\theta_a = -\frac{1}{2}\alpha = -13.29°$ ◀

$\beta = 180° - \alpha = 153.43°$

$\theta_b = \frac{1}{2}\beta = 76.72°$ ◀

$R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(84)^2 + (42)^2} = 93.9 \text{ MPa}$

(b) $\sigma_a = \sigma_{max} = \sigma_{ave} + R = -56 + 93.9$ $\qquad \sigma_{max} = 37.9 \text{ MPa}$ ◀

$\sigma_{min} = \sigma_{min} = \sigma_{ave} - R = -56 - 93.9$ $\qquad \sigma_{min} = -149.9 \text{ MPa}$ ◀

(a') $\theta_d = \theta_a + 45° = 31.71°$ $\qquad \theta_d = 31.7°$ ◀

$\theta_e = \theta_b + 45° = 121.72°$ $\qquad \theta_e = 121.7°$ ◀

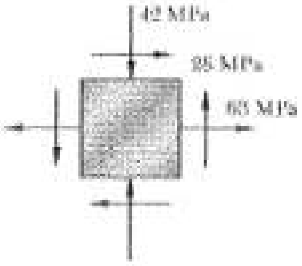
$\tau_{max} = R$ $\qquad \tau_{max} = 93.9 \text{ MPa}$ ◀

(b') $\sigma' = \sigma_{ave}$ $\qquad \sigma' = -56 \text{ MPa}$ ◀

Problem 7.33

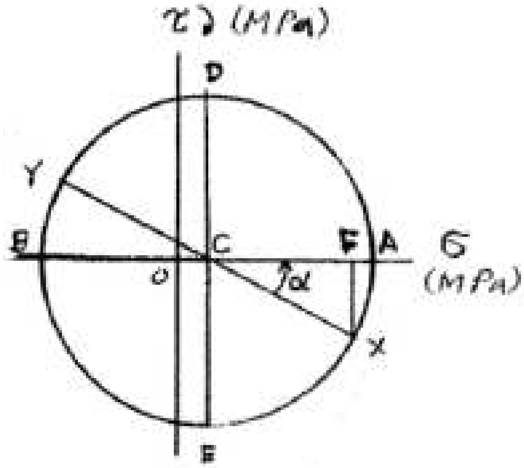
7.33 Solve Prob. 7.11, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.



$\sigma_x = 63\,MPa \qquad \sigma_y = -42\,MPa \qquad \tau_{xy} = 28\,MPa$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 10.5\ MPa$



Points

X: $(\sigma_x, -\tau_{xy}) = (63 MPa, 28 MPa)$

Y: $(\sigma_y, \tau_{xy}) = (-42 MPa, 28 MPa)$

C: $(\sigma_{ave}, 0) = (10.5 MPa, 0)$

$\tan\alpha = \dfrac{FX}{CF} = \dfrac{28}{52.5} = 0.5333$

$\alpha = 28.07°$

$\theta_a = \frac{1}{2}\alpha = 14.04°$

$\theta_d = \theta_a + 45° = 59.04°$ ◄

$\theta_e = \theta_a - 45° = -30.96°$ ◄

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{52.5^2 + 28^2} = 59.5\ MPa$

$\tau_{max} = R = 59.5\ MPa$ ◄

$\sigma' = \sigma_{ave} = 10.5\ MPa$ ◄

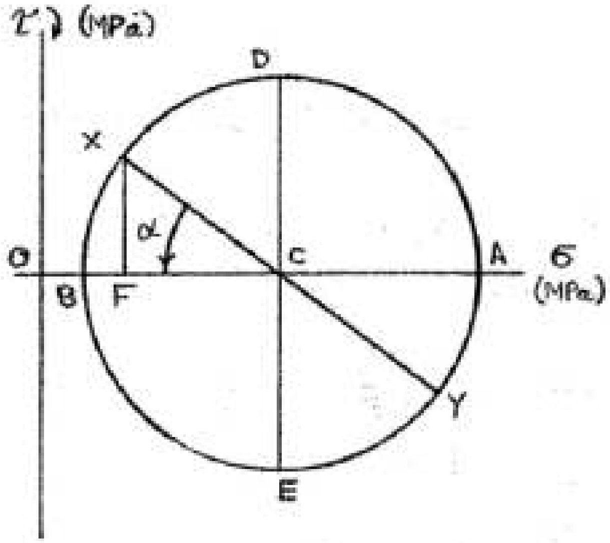
Problem 7.34

7.34 Solve Prob. 7.12, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

$\sigma_x = 6\text{ MPa} \qquad \sigma_y = 30\text{ MPa} \qquad \tau_{xy} = -9\text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(6+36) = 18\text{ MPa}$



Plotted points for Mohr's circle.

$X: (\sigma_x, -\tau_{xy}) = (6\text{ MPa}, 9\text{ MPa})$

$Y: (\sigma_y, \tau_{xy}) = (30\text{ MPa}, -9\text{ MPa})$

$C: (\sigma_{ave}, 0) = (18\text{ MPa}, 0)$

$\tan\alpha = \frac{FX}{FC} = \frac{9}{12} = 0.75$

$\alpha = 36.87°$

$\theta_b = \frac{1}{2}\alpha = 18.43°$

(a) $\theta_d = \theta_b - 45° \qquad\qquad \theta_d = -26.6°$ ◄

$\theta_e = \theta_b + 45° \qquad\qquad \theta_e = 63.4°$ ◄

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{12^2 + 9^2} = 15\text{ MPa}$

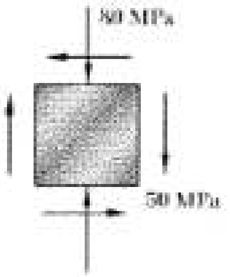
$\tau_{max}(\text{in-plane}) = R \qquad\qquad \tau_{max}(\text{in-plane}) = 15.00\text{ MPa}$ ◄

(b) $\sigma' = \sigma_{ave} \qquad\qquad \sigma' = 18.00\text{ MPa}$ ◄

Problem 7.36

7.36 Solve Prob. 7.14, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$\sigma_x = 0 \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = -50 \text{ MPa}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$

Points

X: (0, 50 MPa)

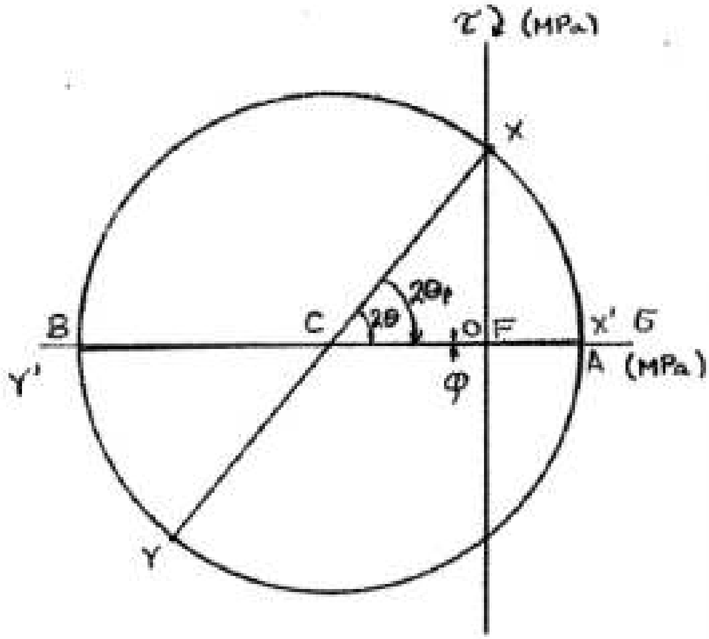
Y: (-80 MPa, -50 MPa)

C: (-40 MPa, 0)

$\tan 2\theta_p = \dfrac{FX}{CF} = \dfrac{50}{40} = 1.25$

$2\theta_p = 51.34°$

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{40^2 + 50^2}$
$= 64.03 \text{ MPa}$



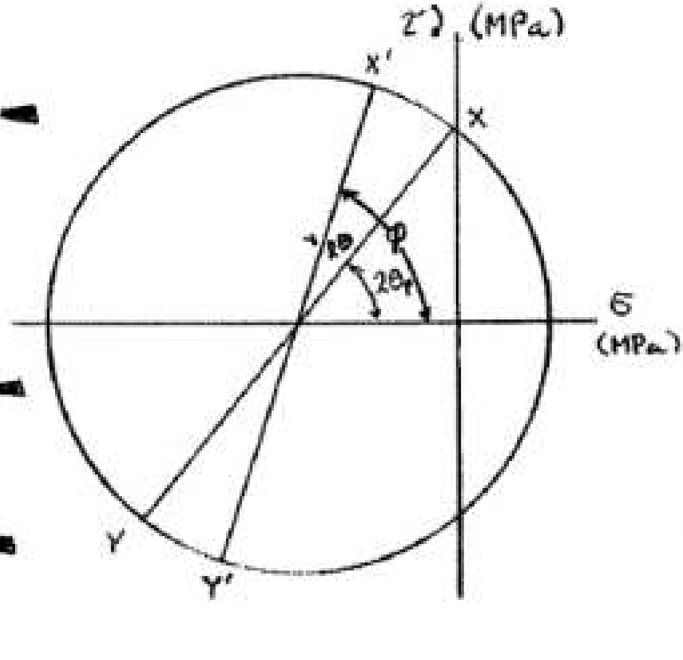
(a) $\theta = 25°$ ↻ $\quad 2\theta = 50°$ ↻

$\varphi = 51.34° - 50° = 1.34°$ ↺

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 24.0 \text{ MPa}$ ◄

$\tau_{x'y'} = -R\sin\varphi = -1.5 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -104.0 \text{ MPa}$ ◄



(b) $\theta = 10°$ ↺ $\quad 2\theta = 20°$ ↺

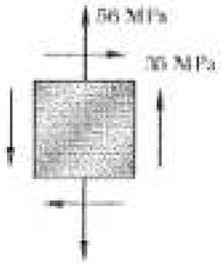
$\varphi = 51.34° + 20° = 71.34°$

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = -19.5 \text{ MPa}$ ◄

$\tau_{x'y'} = -R\sin\varphi = -60.7 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -60.5 \text{ MPa}$ ◄

Problem 7.38

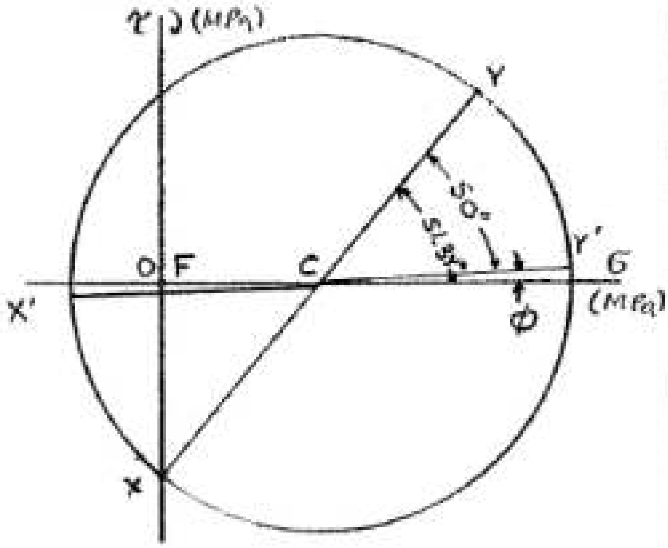


7.38 Solve Prob. 7.16, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated (*a*) 25° clockwise, (*b*) 10° counterclockwise.

$\sigma_x = 0 \qquad \sigma_y = 56\text{ MPa} \qquad \tau_{xy} = 35\text{ MPa}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 28\text{ MPa}$



Points:
X: (0, −35 MPa)
Y: (56 MPa, 35 MPa)
C: (28 MPa, 0)

$\tan 2\theta_p = \dfrac{FX}{FC} = \dfrac{35}{28} = 1.25$

$2\theta_p = 51.34°$

$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{28^2 + 35^2}$
$= 44.8\text{ MPa}$

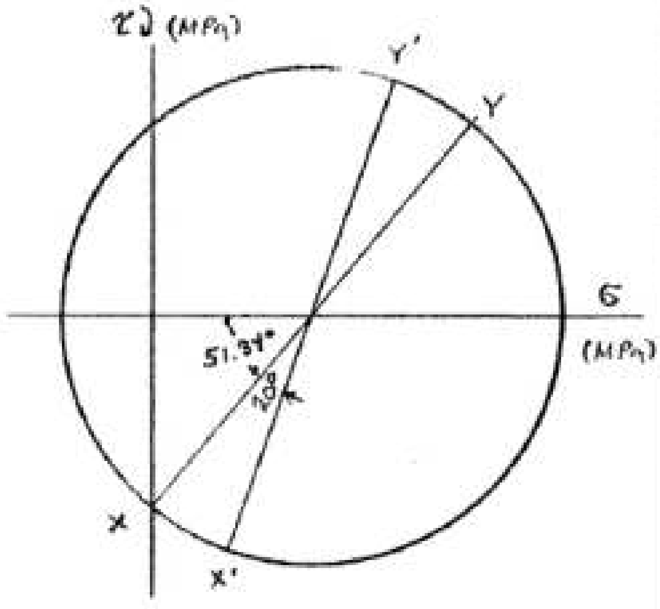
(a) $\theta = 25°$ ↻ $\qquad 2\theta = 50°$ ↻

$\varphi = 51.34° - 50° = 1.34°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -16.8\text{ MPa}$ ◄

$\tau_{x'y'} = R\sin\varphi = 1.05\text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 72.8\text{ MPa}$ ◄



(b) $\theta = 10°$ ↺ $\qquad 2\theta = 20°$ ↺

$\varphi = 51.34° + 20° = 71.34°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = 13.7\text{ MPa}$ ◄

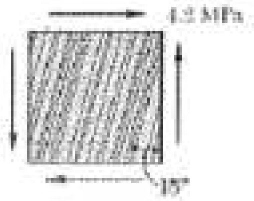
$\tau_{x'y'} = R\sin\varphi = 42.3\text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 42.4\text{ MPa}$ ◄

Problem 7.39

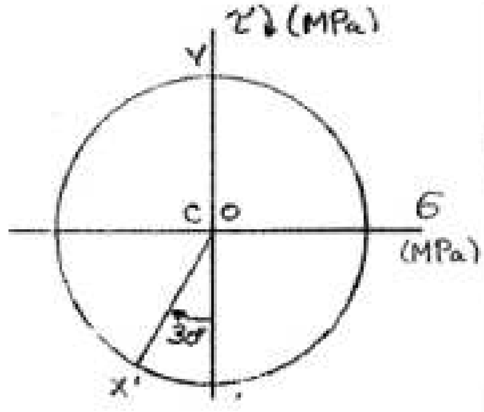
7.39 Solve Prob. 7.17, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = 2.8 \text{ MPa}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 0$



Points

X: $(\sigma_x, -\tau_{xy}) = (0, -4.2 \text{ MPa})$

Y: $(\sigma_y, \tau_{xy}) = (0, 4.2 \text{ MPa})$

C: $(\sigma_{ave}, 0) = (0, 0)$

$\theta = -15° \qquad 2\theta = -30°$

$\overline{CX} = R = 4.2 \text{ MPa}$

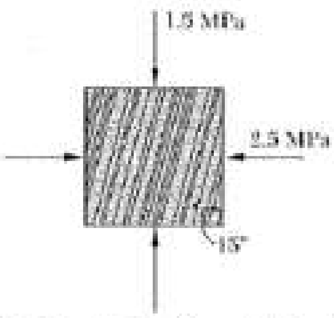
(a) $\tau_{x'y'} = R\cos 30° = 600\cos 30° = 3.6 \text{ MPa}$ ◄

(b) $\sigma_{x'} = \sigma_{ave} - R\sin 30° = -600\sin 30° = -2.1 \text{ MPa}$ ◄

Problem 7.40

7.40 Solve Prob. 7.18, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$\sigma_x = -2.5 \text{ MPa} \qquad \sigma_y = -1.5 \text{ MPa} \qquad \tau_{xy} = 0$

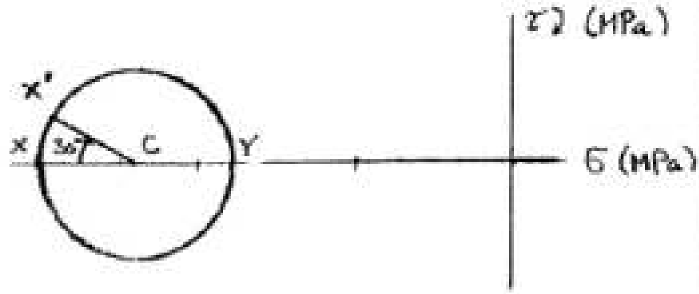
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -2.0 \text{ MPa}$

<u>Plotted points for Mohr's circle.</u>

X: $(\sigma_x, -\tau_{xy}) = (-2.5 \text{ MPa}, 0)$

Y: $(\sigma_y, \tau_{xy}) = (-1.5 \text{ MPa}, 0)$

C: $(\sigma_{ave}, 0) = (-2.0 \text{ MPa}, 0)$



$\underline{\theta = -15°}, \qquad 2\theta = -30°$

$\overline{CX} = 0.5 \text{ MPa} \qquad R = 0.5 \text{ MPa}$

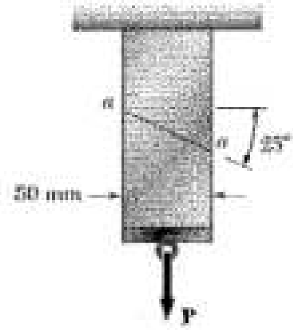
(a) $\tau_{x'y'} = -\overline{CX}'\sin 30 = -R\sin 30° = -0.5\sin 30° \qquad \tau_{x'y'} = -0.25 \text{ MPa}$ ◄

(b) $\sigma_{x'} = \sigma_{ave} - \overline{CX}'\cos 30° = -2.0 - 0.5\cos 30° = \qquad \sigma_{x'}\ -2.43 \text{ MPa}$ ◄

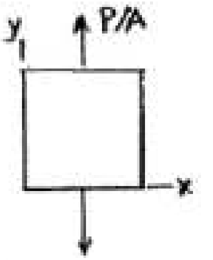
Problem 7.41

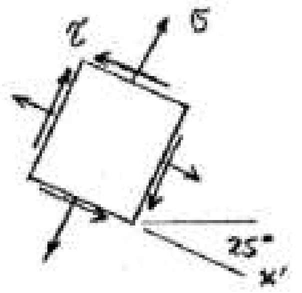
7.41 Solve Prob. 7.19, using Mohr's circle.

7.19 Two members of uniform cross section 50 × 80 mm are glued together along plane *a-a*, that forms an angle or 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

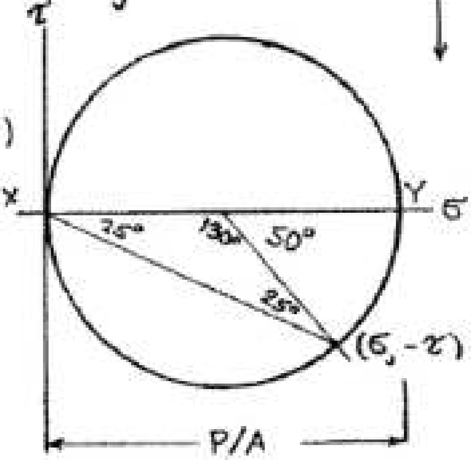


$\sigma_x = 0$
$\tau_{xy} = 0$
$\sigma_y = P/A$





$A = (50\times10^{-3})(80\times10^{-3})$
$= 4\times10^{-3}\ \text{m}^2$



$$\sigma = \frac{P}{2A}(1+\cos 50°)$$

$$P = \frac{2A\sigma}{1+\cos 50°}$$

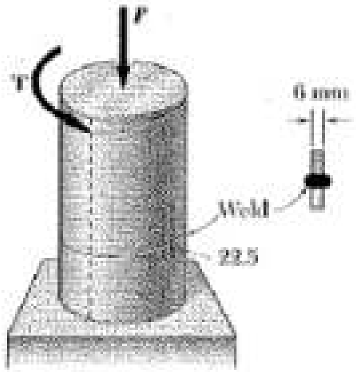
$$P \le \frac{(2)(4\times10^{-3})(800\times10^{3})}{1+\cos 50°}$$

$$P \le 3.90\times10^{3}\ \text{N}$$

$$\tau = \frac{P}{2A}\sin 50° \qquad P = \frac{2A\tau}{\sin 50°} \le \frac{(2)(4\times10^{-3})(600\times10^{3})}{\sin 50°} = 6.27\times10^{3}\ \text{N}$$

Choosing the smaller value, $P = 3.90\ \text{kN}$

Problem 7.42



7.42 Solve Prob. 7.20, using Mohr's circle.

7.20 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 160-kN axial force **P** and an 800 N · m torque **T**, each directed as shown, are applied to the pipe, determine σ and τ in directions, respectively, normal and tangential to the weld.

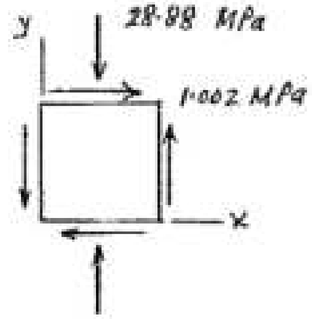
$d_2 = 0.3\ m \qquad c_2 = \frac{1}{2} d_2 = 0.15\ m, \quad t = 0.006\ m$

$c_1 = c_2 - t = 0.144\ m$

$A = \pi(c_2^2 - c_1^2) = \pi(0.15^2 - 0.144^2) = 5541 \times 10^{-6}\ m^2$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.15^4 - 0.144^4) = 119.8 \times 10^{-6}\ m^4$

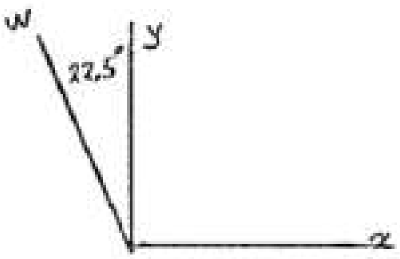
Stresses

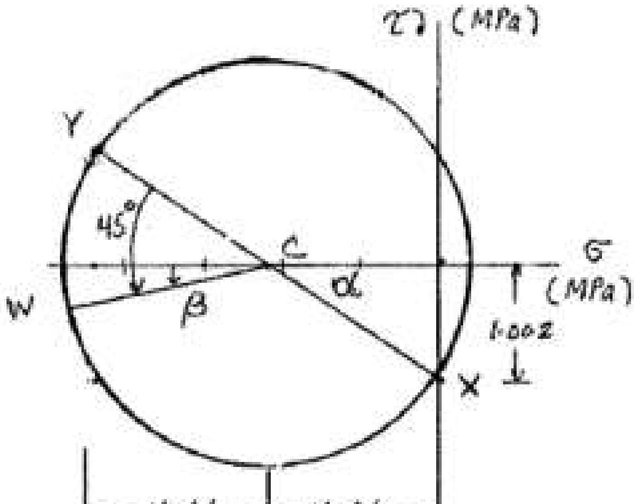


$\sigma = -\frac{P}{A} = -\frac{160 \times 10^3}{5541 \times 10^{-6}} = -28.88\ MPa$

$\tau = \frac{Tc_2}{J} = \frac{(800)(0.15)}{119.8 \times 10^{-6}} = 1.002\ MPa$

$\sigma_x = 0, \quad \sigma_y = -28.88\ MPa, \qquad \tau_{xy} = 1.002\ MPa$





Draw the Mohr's circle.

X: $(0, -1.002\ MPa)$

Y: $(-28.88\ MPa, 1.002\ MPa)$

C: $(-14.44, 0)$

$\tan\alpha = \frac{1.002}{14.44} = 0.06939 \qquad \alpha = 4.0° \qquad \beta = (2)(22.5°) - \alpha = 41°$

$R = \sqrt{(14.44)^2 + (1.002)^2} = 14.47\ MPa$

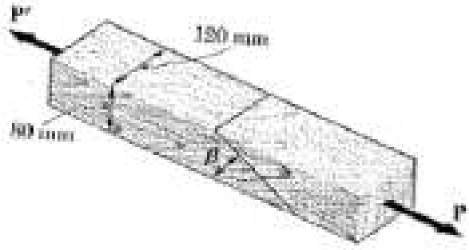
$\sigma_w = -14.44 - 14.47 \cos 41° \qquad \sigma_w = -25.4\ MPa$

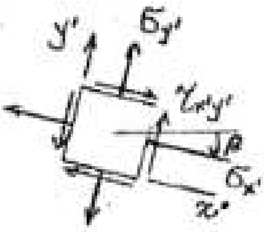
$\tau_w = -14.47 \sin 41° \qquad \tau_w = -9.5\ MPa$

Problem 7.43

7.43 Solve Prob. 7.21, using Mohr's circle.

7.21 Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25°$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (*a*) the in-plane shearing stress parallel to the splice, (*b*) the normal stress perpendicular to the splice.





$\sigma_x = \frac{P}{A}$, $\sigma_y = 0$, $\tau_{xy} = 0$

<u>Plotted points for Mohr's circle.</u>

X: $(P/A, 0)$, Y: $(0,0)$

C: $(P/2A, 0)$

$R = \overline{CX} = \frac{P}{2A}$

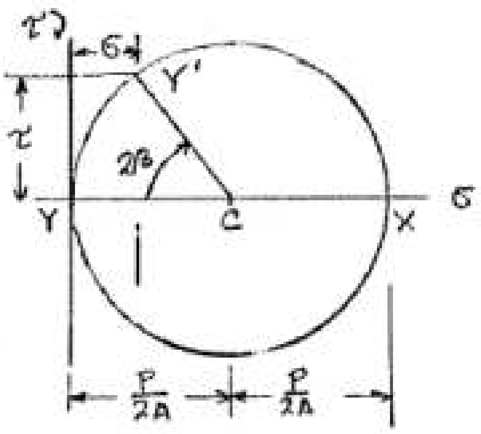
Coordinates of point Y':

$\sigma = \frac{P}{2A}(1 - \cos 2\beta)$ $\quad \tau = \frac{P}{2A}\sin 2\beta$

<u>Data:</u> $A = (80)(120) = 9.6 \times 10^3\ \text{mm}^2 = 9.6 \times 10^{-3}\ \text{m}^2$

(a) $\tau = \frac{(10\times10^3)\sin 50°}{(2)(9.6\times10^{-3})} = 399\times10^3\ \text{Pa} = 399\ \text{kPa}$ ◄

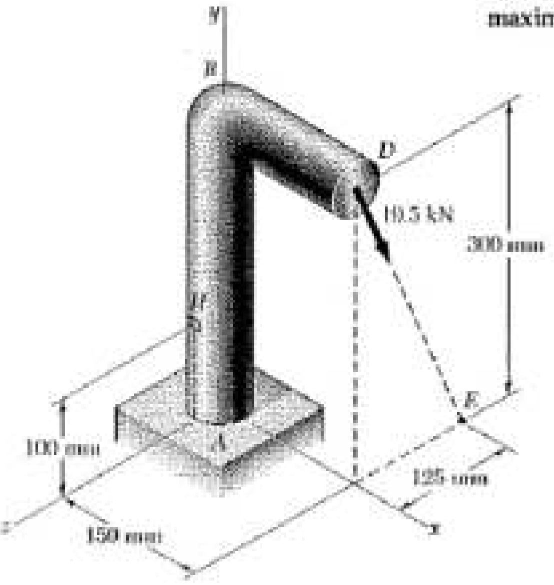
(b) $\sigma = \frac{(10\times10^3)(1-\cos 50°)}{(2)(9.6\times10^{-3})} = 186.0\times10^3\ \text{Pa} = 186.0\ \text{kPa}$ ◄



Problem 7.45

7.45 Solve Prob. 7.23, using Mohr's circle.

7.23 A 19.5-kN force is applied at point *D* of the cast-iron post shown. Knowing that the post has a diameter of 60 mm, determine the principal stresses and the maximum shearing stress at point *H*.



$$l_{DE} = \sqrt{(300)^2 + (125)^2} = 325 \text{ mm}$$

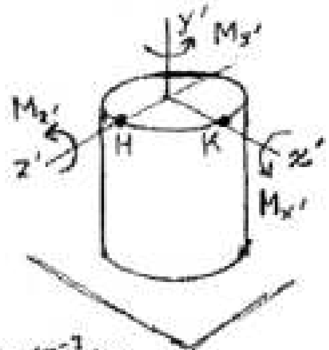
Resolve the 19.5 kN force F at point D into x, y, and z components.

$$F_x = 0$$

$$F_y = -\frac{300}{325}(19.5) = -18 \text{ kN} = -18\times10^3 \text{ N}$$

$$F_z = -\frac{125}{325}(19.5) = -7.5 \text{ kN} = -7.5\times10^3 \text{ N}$$

Determine the force-couple system at the point on the y-axis where it intersects the plane containing elements H and K.



$F_{x'} = 0$, $F_{y'} = -18\times10^3$ N, $F_{z'} = -7.5\times10^3$ N

$M_{x'} = -(7.5 \text{ kN})(300 \text{mm} - 100 \text{ mm}) = -1.5\times10^3$ N·m

$M_{y'} = (7.5 \text{ kN})(150 \text{ mm}) = 1.125\times10^3$ N·m

$M_{z'} = -(18 \text{ kN})(150 \text{ mm}) = -2.7\times10^3$ N·m

Properties of section. (Circle) $c = \frac{1}{2}d = 30 \text{ mm} = 30\times10^{-3}$ m

$$A = \pi c^2 = \pi(30)^2 = 2.8274\times10^3 \text{ mm}^2 = 2.8274\times10^{-3} \text{ m}^2$$

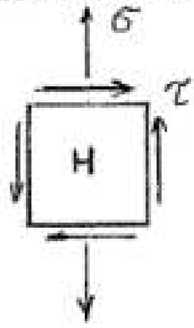
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(30)^4 = 636.17\times10^3 \text{ mm}^4 = 636.17\times10^{-9} \text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30)^4 = 1.27235\times10^6 \text{ mm}^4 = 1.27235\times10^{-6} \text{ m}^4$$

(Semicircle) $Q = \frac{2}{3}c^3 = \frac{2}{3}(30)^3 = 18\times10^3 \text{ mm}^3 = 18\times10^{-6} \text{ m}^3$

$$t = d = 60 \text{ mm} = 60\times10^{-3} \text{ m}$$

Stresses at H.



$$\sigma = \frac{F_y}{A} - \frac{M_x c}{I} = \frac{-18\times10^3}{2.8274\times10^{-3}} - \frac{(-1.5\times10^3)(30\times10^{-3})}{636.17\times10^{-9}}$$

$$= 64.370\times10^6 \text{ Pa} = 64.37 \text{ MPa}$$

$$\tau = \frac{F_z Q}{It} + \frac{M_y c}{J} = 0 + \frac{(1.125\times10^3)(30\times10^{-3})}{1.27235\times10^{-6}}$$

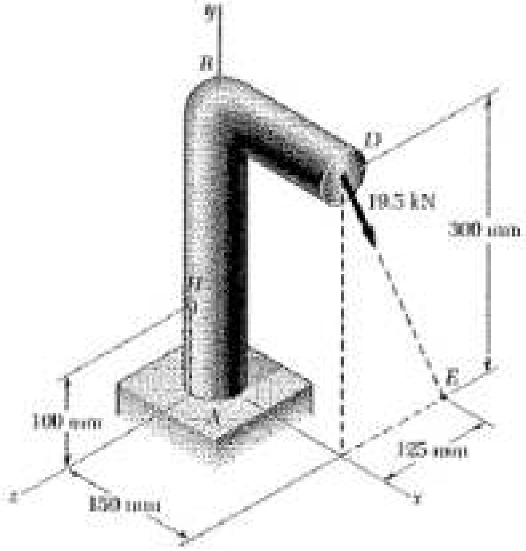
$$= 26.526\times10^6 \text{ Pa} = 26.526 \text{ MPa}$$

continued after Prob. 7.46

Problem 7.46

7.46 Solve Prob. 7.24, using Mohr's circle.

7.24 A 19.5-kN force is applied at point *D* of the cast-iron post shown. Knowing that the post has a diameter of 60 mm, determine the principal stresses and the maximum shearing stress at point *K*.



$$l_{DE} = \sqrt{(300)^2 + (125)^2} = 325 \text{ mm}$$

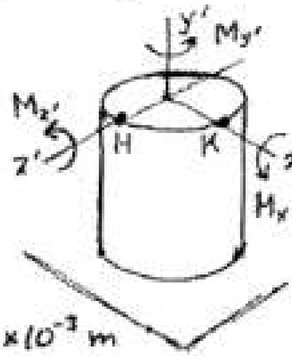
Resolve the 19.5 kN force F at point D into x, y, and z components.

$$F_x = 0$$

$$F_y = -\frac{300}{325}(19.5) = -18 \text{ kN} = -18\times10^3 \text{ N}$$

$$F_z = -\frac{125}{325}(19.5) = -7.5 \text{ kN} = -7.5\times10^3 \text{ N}$$

Determine the force-couple system at the point on the y-axis where it intersects the plane containing elements H and K.



$F_{x'} = 0, \quad F_{y'} = -18\times10^3 \text{ N}, \quad F_{z'} = -7.5\times10^3 \text{ N}$

$M_{x'} = -(7.5 \text{ kN})(300\text{mm} - 100 \text{ mm}) = +1.5\times10^3 \text{ N}\cdot\text{m}$

$M_{y'} = (7.5 \text{ kN})(150 \text{ mm}) = 1.125\times10^3 \text{ N}\cdot\text{m}$

$M_{z'} = -(18 \text{ kN})(150 \text{ mm}) = -2.7\times10^3 \text{ N}\cdot\text{m}$

Properties of section. (Circle) $c = \frac{1}{2}d = 30 \text{ mm} = 30\times10^{-3} \text{ m}$

$A = \pi c^2 = \pi(30)^2 = 2.8274\times10^3 \text{ mm}^2 = 2.8274\times10^{-3} \text{ m}^4$

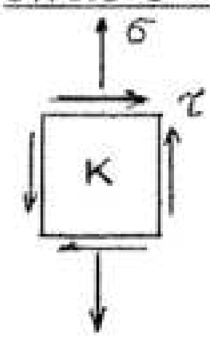
$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(30)^4 = 636.17\times10^3 \text{ mm}^4 = 636.17\times10^{-9} \text{ m}^4$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30)^4 = 1.27235\times10^6 \text{ mm}^4 = 1.27235\times10^{-6} \text{ m}^4$

(Semicircle) $Q = \frac{2}{3}c^3 = \frac{2}{3}(30)^3 = 18\times10^3 \text{ mm}^3 = 18\times10^{-6} \text{ m}^3$

$t = d = 60 \text{ mm} = 60\times10^{-3} \text{ m}$

Stresses at K.



$$\sigma = \frac{F_y}{A} + \frac{M_z c}{I} = \frac{-18\times10^3}{2.8874\times10^{-3}} + \frac{(-2.7\times10^3)(30\times10^{-3})}{636.17\times10^{-9}}$$

$$= -133.69\times10^6 \text{ Pa} = -133.69 \text{ MPa}$$

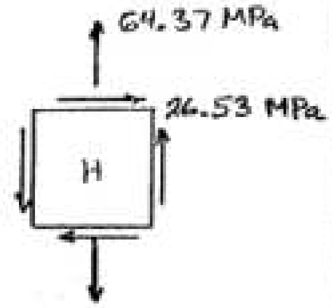
$$\tau = -\frac{F_z Q}{It} + \frac{M_y c}{J} = -\frac{(-7.5\times10^3)(18\times10^{-6})}{(636.17\times10^{-9})(60\times10^{-3})} + \frac{(1.125\times10^3)(30\times10^{-3})}{1.27235\times10^{-6}}$$

$$= 3.5368\times10^6 + 26.526\times10^6 \text{ Pa}$$

$$= 30.06\times10^6 \text{ Pa} = 30.06 \text{ MPa}$$

continued

Problem 7.45 continued

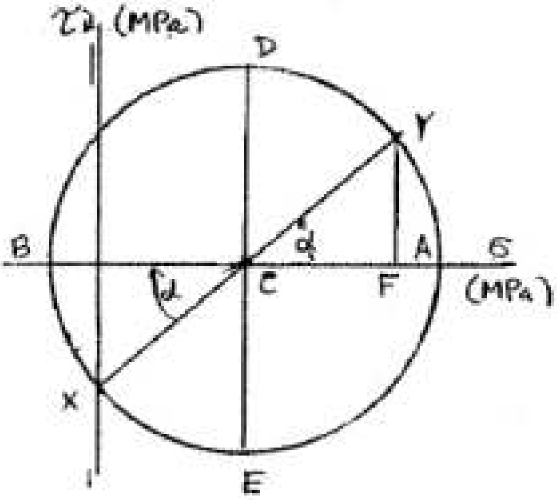


Plotted points for Mohr's circle.

X: $(\sigma_x, -\tau_{xy}) = (0, -26.53\text{ MPa})$

Y: $(\sigma_y, \tau_{xy}) = (64.37\text{ MPa}, 26.53\text{ MPa})$

C: $(\sigma_{ave}, 0) = (32.19\text{ MPa}, 0)$



$R = \sqrt{\overline{CF}^2 + \overline{FY}^2}$

$= \sqrt{32.19^2 + 26.53^2} = 41.71\text{ MPa}$

$\tan \alpha = \frac{FY}{CF} = \frac{26.53}{32.19} = 0.8242$

$\alpha = 39.49°$ $\quad \theta_b = 19.7°$

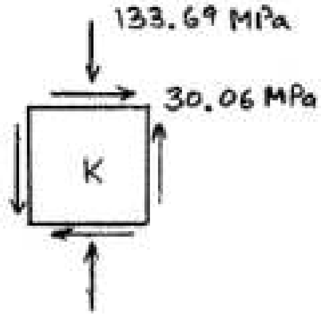
$\theta_a = 70.3°$

$\sigma_a = \sigma_{ave} + R \qquad \sigma_a = 73.9\text{ MPa}$ ◄

$\sigma_b = \sigma_{ave} - R \qquad \sigma_b = -9.5\text{ MPa}$ ◄

$\tau_{max} = ? \qquad \tau_{max} = 41.7\text{ MPa}$ ◄

Problem 7.46 continued

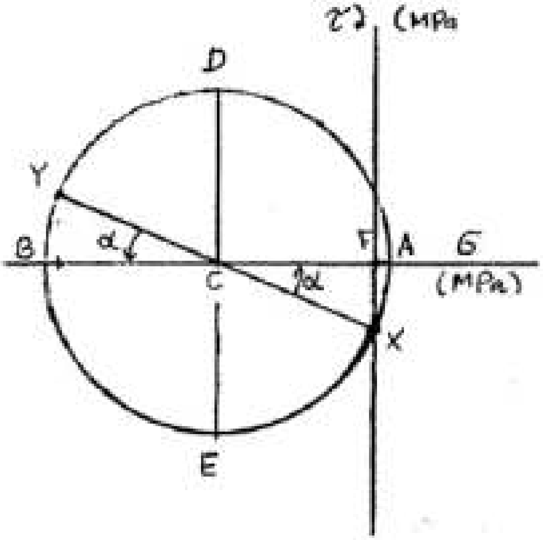


Plotted points for Mohr's circle.

X: $(\sigma_x, -\tau_{xy}) = (0, -30.06\text{ MPa})$

Y: $(\sigma_y, \tau_{xy}) = (-133.69\text{ MPa}, 30.06\text{ MPa})$

C: $(\sigma_{ave}, 0) = (-66.85\text{ MPa}, 0)$



$R = \sqrt{\overline{CF}^2 + \overline{FX}^2}$

$= \sqrt{66.85^2 + 30.06^2} = 73.3\text{ MPa}$

$\tan \alpha = \frac{FX}{CF} = \frac{30.06}{66.85} = 0.4497$

$\alpha = 24.2°$ $\quad \theta_a = 12.1°$

$\theta_b = 177.9°$

$\sigma_a = \sigma_{ave} + R \qquad \sigma_a = 6.45\text{ MPa}$ ◄

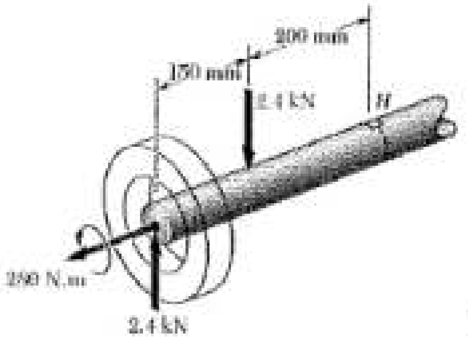
$\sigma_b = \sigma_{ave} - R \qquad \sigma_b = -140.2\text{ MPa}$ ◄

$\tau_{max} = R \qquad \tau_{max} = 73.3\text{ MPa}$ ◄

Problem 7.47

7.47 Solve Prob. 7.25, using Mohr's circle.

7.25 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 30 mm, determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.



$c = \frac{1}{2}d = \frac{1}{2}(30) = 15$ mm

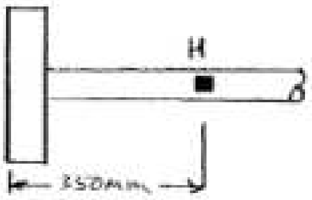
Torsion: $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$= \frac{(2)(280)}{\pi(15)^3} = 52.8$ MPa

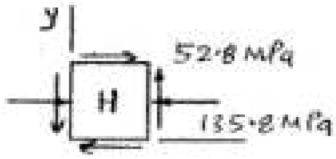
$I = \frac{\pi}{4}c^4 = 39761$ mm^4

$M = (150)(2400) = 360000$ N mm $\qquad \sigma = -\frac{My}{I} = -\frac{(36\times10^4)(15)}{39761} = -135.8$ MPa

Top view

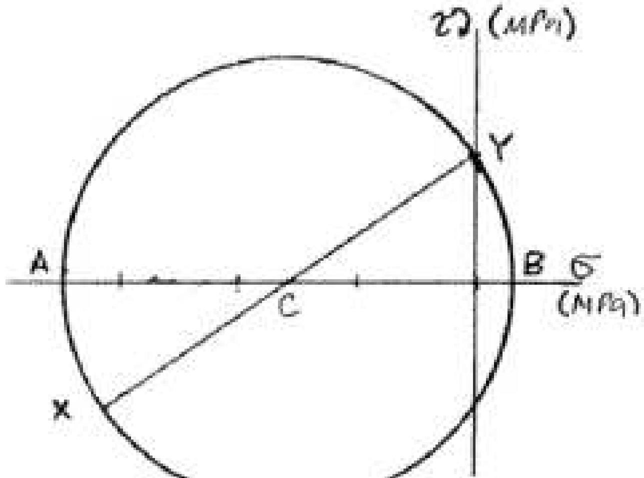


Stresses



$\sigma_x = -135.8$ MPa, $\quad \sigma_y = 0$, $\quad \tau_{xy} = 52.8$ MPa

Plotted points. $\quad X: (-135.8, -52.8)$; $\; Y: (0, 52.8)$; $\; C(-67.9, 0)$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -67.9$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\frac{-135.8}{2}\right)^2 + (52.8)^2}$

$= 86$ MPa

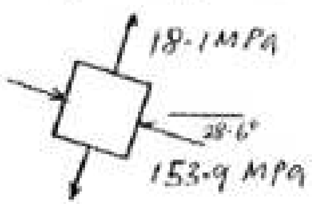
$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(52.8)}{-135.8}$

$= -1.5552$

(a) $\qquad \theta_a = 28.6°$, $\quad \theta_b = 61.4°$ ◄

$\sigma_a = \sigma_{ave} - R = -67.9 - 86 = -153.9$ MPa ◄

$\sigma_b = \sigma_{ave} + R = -67.9 + 86 = 18.1$ MPa ◄

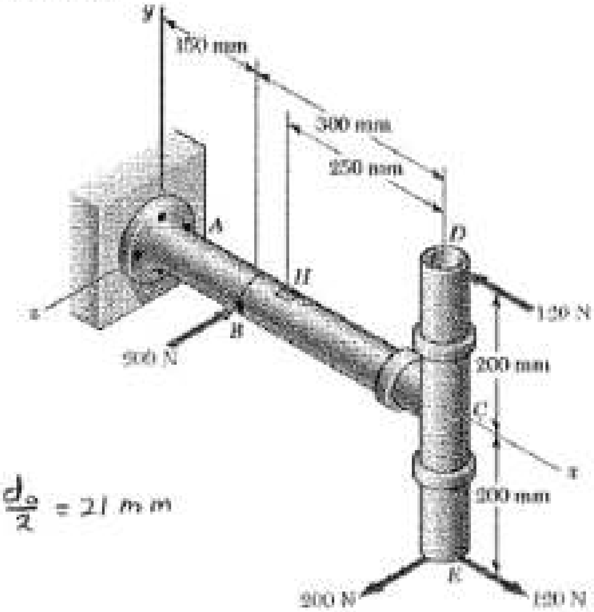


(b) $\tau_{max} = R = 86$ MPa ◄

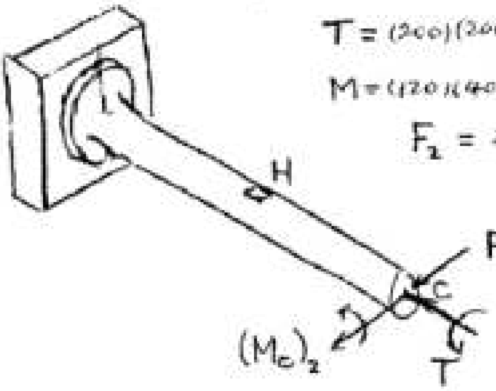
Problem 7.48

7.48 Solve Prob. 7.26, using Mohr's circle.

7.26 Several forces are applied to the pipe assembly shown. Knowing that the inner and outer diameters of the pipe are equal to 38 mm and 42 mm, respectively, determine (*a*) the principal planes and the principal stresses at point *H* located at the top of the outside surface of the pipe, (*b*) the maximum shearing stress at the same point.



Replace forces on pipe DCE by an equivalent force-couple system at C.



$T = (200)(200) = 40000$ Nmm

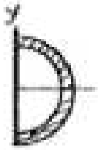
$M = (120)(400) = 48000$ Nmm

$F_z = 200$ N

Cross section. $c_1 = \frac{d_i}{2} = 19$ mm $\qquad c_2 = \frac{d_o}{2} = 21$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 100782$ mm^4

$I = \frac{1}{2}J = 50391$ mm^4



$Q_y = \frac{2}{3}(c_2^3 - c_1^3) = \frac{2}{3}(21^3 - 19^3) = 1601$ mm^3

$t = c_2 - c_1 = 2$ mm

At the section containing element H

$T = 40000$ Nmm, $\quad M_z = 48000$ Nmm, $\quad V_z = 200$ N

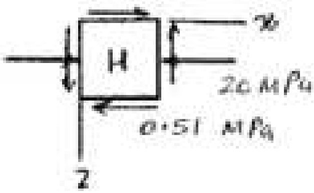
Stresses.

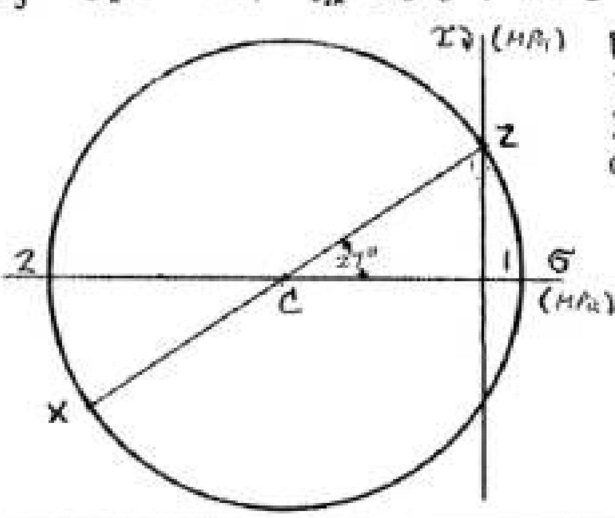
Torsion: $\tau_{zx} = -\frac{Tc}{J} = -\frac{(40000)(21)}{100782} = -8.3$ MPa

Bending: $\sigma_x = -\frac{M_z y}{I} = -\frac{(48000)(21)}{50391} = -20$ MPa

Transverse Shear: $\tau_{zx} = \frac{V_z Q_y}{I(2t)} = \frac{(200)(1601)}{(50391)(2)} = 3.2$ MPa

Total: $\sigma_z = 0, \quad \sigma_x = -20, \quad \tau_{zx} = -8.3 + 3.2 = -5.1$ MPa





Plotted points
Z: (0, 5.1)
X: (−20, −5.1)
C: (−10, 0)

$\theta_1 = 13.5°$ ◄

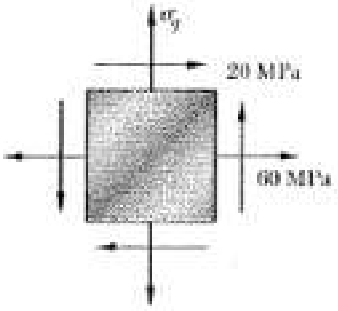
$\theta_2 = 76.5°$ ◄

$\sigma_1 = 1.2$ MPa ◄

$\sigma_2 = -21.2$ MPa ◄

$\tau_{max} = R = 11.2$ MPa ◄

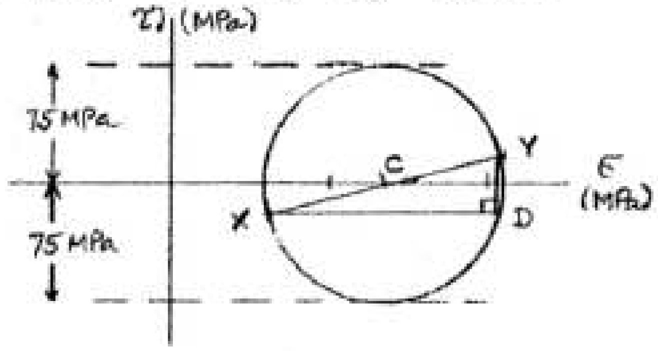
Problem 7.49



7.49 Solve Prob. 7.27, using Mohr's circle.

7.27 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.

$\sigma_x = 60\text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20\text{ MPa}$



Given: $\tau_{max} = R \quad 75\text{ MPa}$

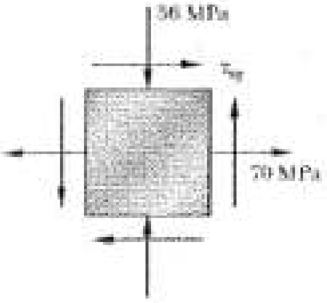
$\overline{XY} = 2R = 150\text{ MPa}$

$\overline{DY} = (2)(\tau_{xy}) = 40\text{ MPa}$

$\overline{XD} = \sqrt{\overline{XY}^2 - \overline{DY}^2} = \sqrt{150^2 - 40^2} = 144.6\text{ MPa}$

$\sigma_y = \sigma_x + \overline{XD} = 60 + 144.6$ $\qquad\qquad \sigma_y = 205\text{ MPa}$ ◄

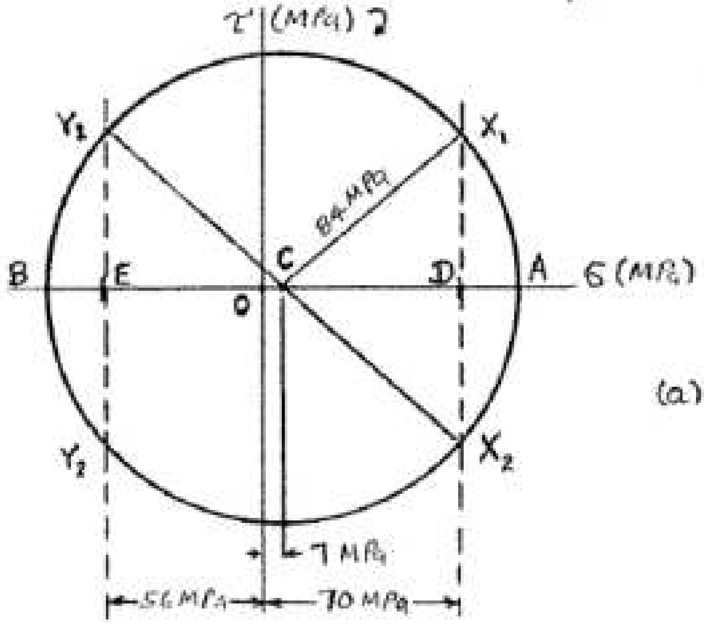
Problem 7.50



7.50 Solve Prob. 7.28, using Mohr's circle.

7.28 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 84 MPa, (*b*) the corresponding principal stresses.

The center of the Mohr's circle lies at point C with coordinates $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{70-56}{2}, 0\right)$ $= (7.0, 0)$. The radius of the circle is $\tau_{max(in\text{-}plane)} = 84\,\text{MPa}$



The stress point $(\sigma_x, -\tau_{xy})$ lie along the line X_1X_2 of th Mohr circle diagram. The extreme points with $R \leq 84\,\text{MPa}$ are X_1 and X_2.

(a) The largest allowable value of τ_{xy} is obtained from triangle CDX_1

$$\overline{DX_1}^2 = \overline{DX_2}^2 = \sqrt{\overline{CX_1}^2 - \overline{CD}^2}$$

$$\tau_{xy} = \sqrt{84^2 - 63^2} = 55.6\,\text{MPa}$$

(b) The principal stresses are

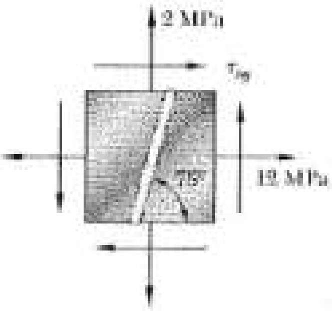
$$\sigma_a = 7 + 84 = 91\,\text{MPa}$$

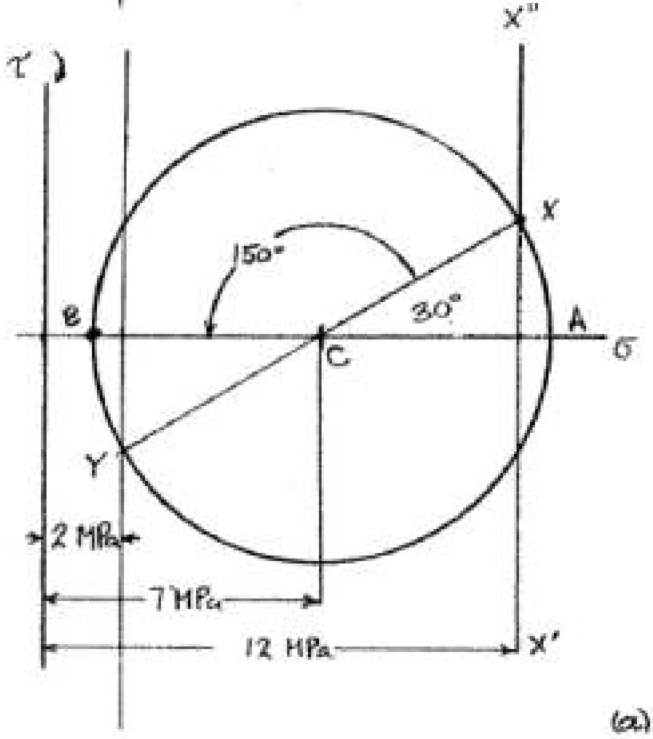
$$\sigma_b = 7 - 84 = -77\,\text{MPa}$$

Problem 7.51

7.51 Solve Prob. 7.29, using Mohr's circle.

7.29 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.





Point X of Mohr's circle must lie on X'X'' so that $\sigma_x = 12$ MPa. Likewise, point Y lies on line Y'Y'' so that $\sigma_y = 2$ MPa. The coordinates of C are

$$\frac{2+12}{2},\ 0 = (7\ \text{MPa}, 0).$$

Counterclockwise rotation through 150° brings line CX to CB, where $\tau = 0$.

$$R = \frac{\sigma_x - \sigma_y}{2}\sec 30°$$

$$= \frac{12-2}{2}\sec 30°$$

$$= 5.77\ \text{MPa}$$

(a) $$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2}\tan 30°$$

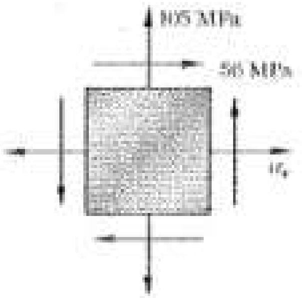
$$= -\frac{12-2}{2}\tan 30°$$

$$\tau_{xy} = -2.89\ \text{MPa} ◄$$

(b) $\sigma_a = \sigma_{ave} + R = 7 + 5.77$ $\qquad \sigma_a = 12.77$ MPa ◄

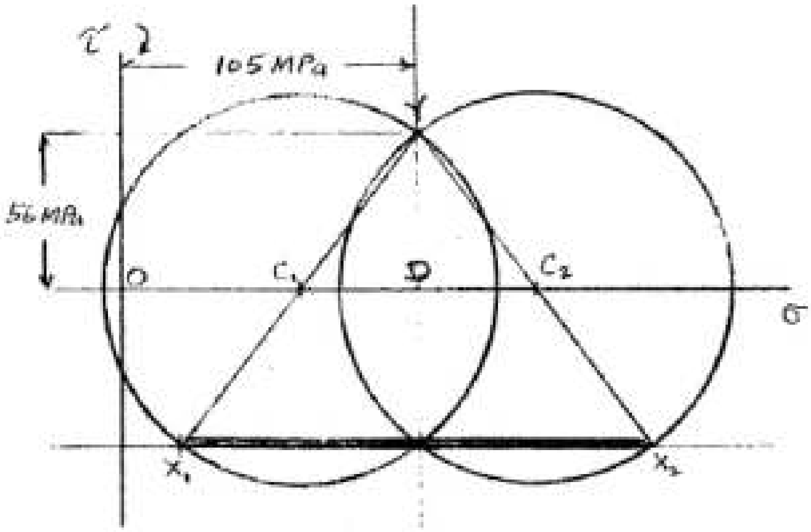
$\sigma_b = \sigma_{ave} - R = 7 - 5.77$ $\qquad \sigma_b = 1.23$ MPa ◄

Problem 7.52



7.52 Solve Prob. 7.30, using Mohr's circle.

7.30 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 70 MPa.



For the Mohr's circle point Y lies at (105 MPa, 56 MPa).

The radius of limiting circles is $R = 70$ MPa

Let C_1 be the location of the left most limiting circle and C_2 be that of the right most one.

$\overline{C_1Y} = 70$ MPa

$\overline{C_2Y} = 70$ MPa

Noting right triangles C_1DY and C_2DY

$\overline{C_1D}^2 + \overline{DY}^2 = \overline{C_1Y}^2 \qquad \overline{C_1D}^2 + 56^2 = 70^2 \qquad \overline{C_1D} = 42$ MPa

Coordinates of point C_1 are $(0, 105 - 42) = (0, 63\text{ MPa})$

Likewise, coordinates of point C_2 are $= (0, 105 + 42) = (0, 147\text{ MPa})$

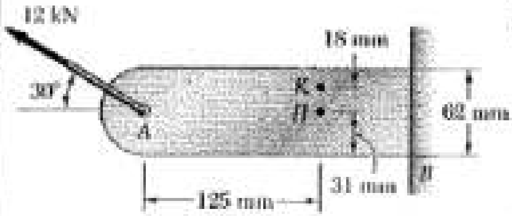
Coordinates of point X_1 $(133 - 42, -56) = (21\text{ MPa}, -56\text{ MPa})$

Coordinates of point X_2 $(144 + 42, -56) = (186\text{ MPa}, -56\text{ MPa})$

The point $(\sigma_x, -\tau_{xy})$ must lie on the line X_1X_2

Thus $\qquad 21\text{ MPa} \le \sigma_x \le 186\text{ MPa}$ ◄

Problem 7.53

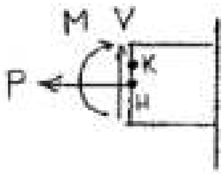


7.53 Knowing that the bracket AB has a uniform thickness of 15 mm, determine (a) the principal planes and principal stresses at point H, (b) the maximum shearing stress at point H.

Resolve the 12 kN force F at point A into x and y components.

$F_x = -F\cos 30° = -(12)\cos 30° = -10.39$ kN

$F_y = F\sin 30° = (12)\sin 30° = 6$ kN



At the section containing points H and K,

$P = -F_x = 10.39$ kN , $V = F_y = 6$ kN

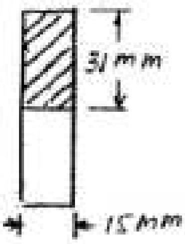
$M = (0.125)(6000) = 750$ N m

Section properties. $t = 0.015$ m , $A = (0.015)(0.062) = 93\times10^{-5}$ m^2

$I = \frac{1}{12}(0.015)(0.062)^3 = 297\times10^{-9}$ m^4 , $c = 0.031$ m

At point H, $y = 0$,

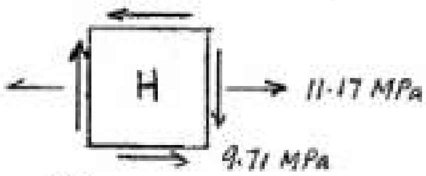
$Q = (0.015)(0.031)\left(\frac{0.031}{2}\right) = 7.207\times10^{-6}$ m^3



Stresses at point H.

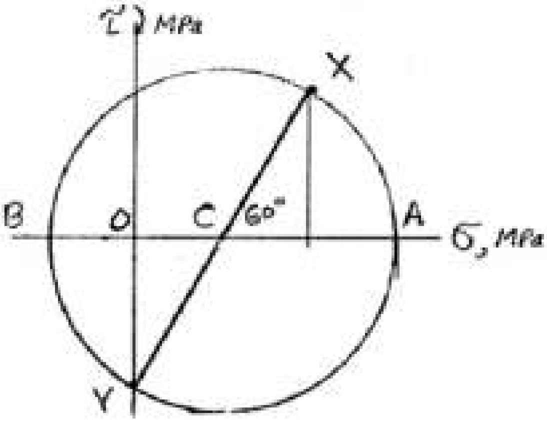
$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{10.39\times10^3}{93\times10^{-5}} + 0 = 11.17 \text{ MPa}$$

$$\tau = \frac{VQ}{It} = \frac{(6000)(7.207\times10^{-6})}{(297\times10^{-9})(0.015)} = 9.71 \text{ MPa}$$



$\sigma_x = 11.17$ MPa , $\sigma_y = 0$, $\tau_{xy} = -9.71$ MPa

Mohr's circle



X: $(\sigma_x, -\tau_{xy}) = (11.17 \text{ MPa}, 9.71 \text{ MPa})$

Y: $(\sigma_y, \tau_{xy}) = (0, -9.71 \text{ MPa})$

C: $(\sigma_{ave}, 0) = (5.585 \text{ MPa}, 0)$

$$\frac{\sigma_x - \sigma_y}{2} = 5.585$$

$$R = \sqrt{5.585^2 + 9.71^2} = 11.2 \text{ MPa}$$

$\tan 2\theta_p = \frac{9.71}{5.585} = 1.7386$ $2\theta_p = 60.0°$ ↻ $\theta_p = 30°$ ↻

(a) $\sigma_{max} = \sigma_{ave} + R = 5.585 + 11.2$ $\sigma_{max} = 16.8$ MPa at $30.0°$ ↻ ◄

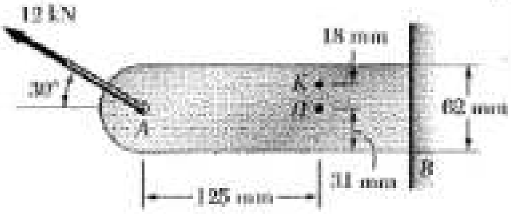
$\sigma_{min} = \sigma_{ave} - R = 5.585 - 11.2$ $\sigma_{min} = -5.6$ MPa at $60.0°$ ↺ ◄

(b) $\tau_{max} = R = 11.2$ MPa $\tau_{max} = 11.2$ MPa ◄

Problem 7.54

7.54 Solve Prob. 7.53, considering point K.

7.53 Knowing that the bracket AB has a uniform thickness of 15 mm, determine (a) the principal planes and principal stresses at point H, (b) the maximum shearing stress at point H.



Resolve the 12 kN force F at point A into x and y components.

$F_x = -F\cos 30° = -(12)\cos 30° = -10.39$ kN

$F_y = F\sin 30° = (12)\sin 30° = 6$ kN

At the section containing points H and K,

$P = -F_x = 10.39$ kN, $V = F_y = 6$ kN

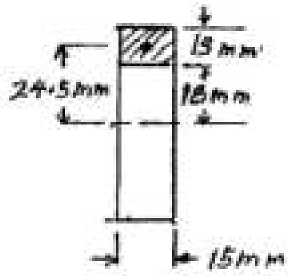
$M = (0.125)(600) = 750$ N m

Section properties. $t = 0.015$ m, $A = (0.015)(0.062) = 93\times10^{-5}\ m^2$

$I = \frac{1}{12}(0.015)(0.062)^3 = 297\times10^{-9}\ m^4$, $c = 0.031$ m

At point K, $y = 0.018$ m

$Q = (0.015)(0.013)(0.0245) = 4.777\times10^{-6}\ m^3$

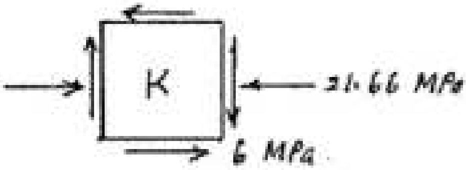


Stresses at point K.

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{10390}{93\times10^{-5}} - \frac{(750)(0.018)}{297\times10^{-9}} = -34.28\ \text{MPa}$$

$$\tau = \frac{VQ}{It} = \frac{(6000)(4.777\times10^{-6})}{(297\times10^{-9})(0.015)} = 6.43\ \text{MPa}$$

$\sigma_x = -21.66$ MPa, $\sigma_y = 0$, $\tau_{xy} = -6.43$ MPa

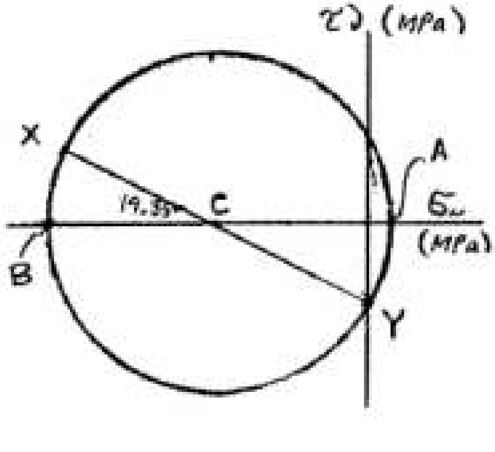


Mohr's circle

$X: (\sigma_x, -\tau_{xy}) = (-34.28$ MPa, 6.43 MPa$)$

$Y: (\sigma_y, \tau_{xy}) = (0, -6.43$ MPa$)$

$C: (\sigma_{ave}, 0) = (-17.14$ MPa$, 0)$



$\frac{\sigma_x - \sigma_y}{2} = -17.14$ MPa

$R = \sqrt{(17.14)^2 + (6.43)^2} = 18.31$ MPa

$\tan 2\theta_p = \frac{6.43}{17.14} = 0.3751 \qquad 2\theta_p = 20.56°$

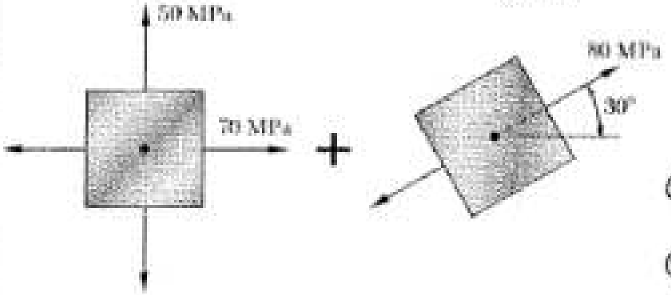
(a) $\sigma_{max} = \sigma_{ave} + R = -17.14 + 18.31$ $\qquad$ $\sigma_{max} = 1.17$ MPa at 79.7°

(b) $\sigma_{min} = \sigma_{ave} - R = -17.14 - 18.31$ $\qquad$ $\sigma_{min} = -35.45$ MPa at 10.3°

$\tau_{max} = R = 18.31$ MPa $\qquad$ $\tau_{max} = 18.3$ MPa

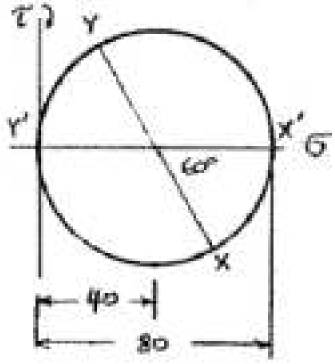
Problem 7.55

7.55 through 7.58 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



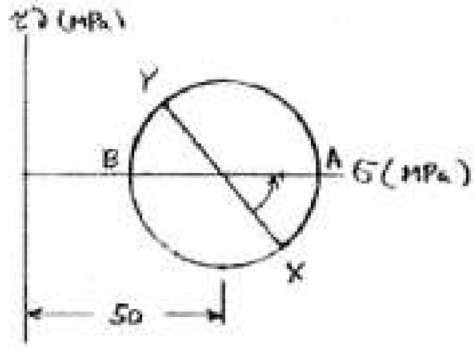
Mohr's circle for 2nd stress state

$\sigma_x = 40 + 40\cos 60°$
$= 60$ MPa
$\sigma_y = 40 - 40\cos 60°$
$= 20$ MPa
$\tau_{xy} = 40\sin 60°$
$= 34.64$ MPa



Resultant stresses

$\sigma_x = 70 + 60 = 130$ MPa
$\sigma_y = 50 + 20 = 70$ MPa
$\tau_{xy} = 0 + 34.64 = 34.64$ MPa



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(130 + 70) = 100$ MPa

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(34.64)}{130 - 70} = 1.1547$

$2\theta_p = 49.11°$ $\quad \theta_a = 24.6° \quad \theta_b = 114.6°$ ◄

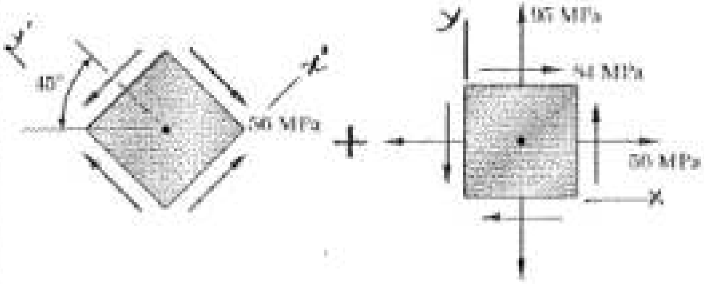
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 45.8$ MPa

$\sigma_a = \sigma_{ave} + R$ $\qquad \sigma_a = 145.8$ MPa ◄

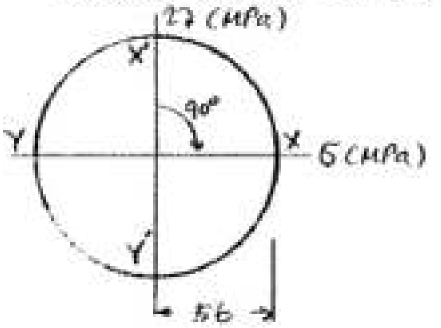
$\sigma_b = \sigma_{ave} - R$ $\qquad \sigma_b = 54.2$ MPa ◄

Problem 7.56

7.55 through 7.58 Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.



Mohr's circle for 1st stress state.



$\sigma_x = 56$ MPa

$\sigma_y = -56$ MPa

$\tau_{xy} = 0$

Resultant stresses

$\sigma_x = 56 + 56 = 112$ MPa

$\sigma_y = -56 + 98 = 42$ MPa

$\tau_{xy} = 84 + 0 = 84$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 77$ MPa

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(84)}{70} = 2.4$

$2\theta_p = 67.38°$ $\quad \theta_a = 33.69°$ ◀

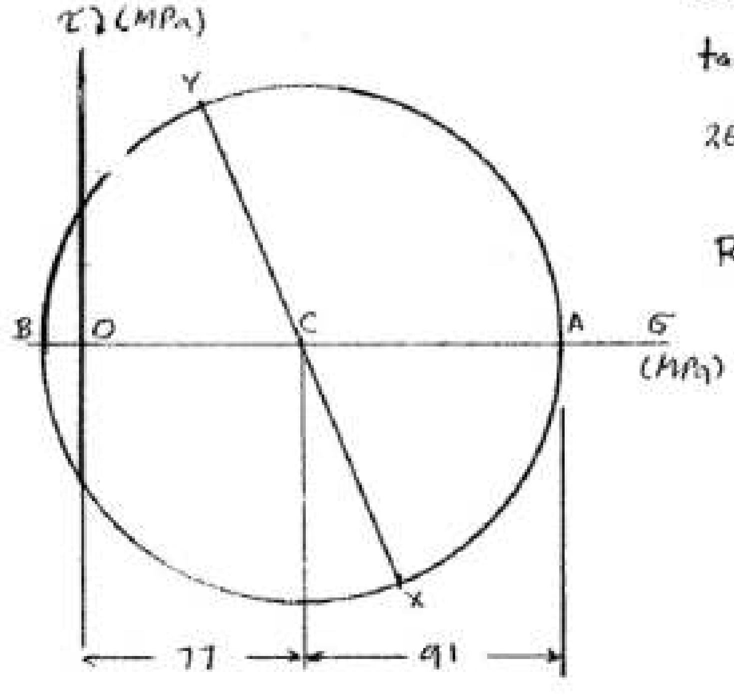
$\theta_b = 123.69°$ ◀

$R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{35^2 - 84^2} = 91$ MPa

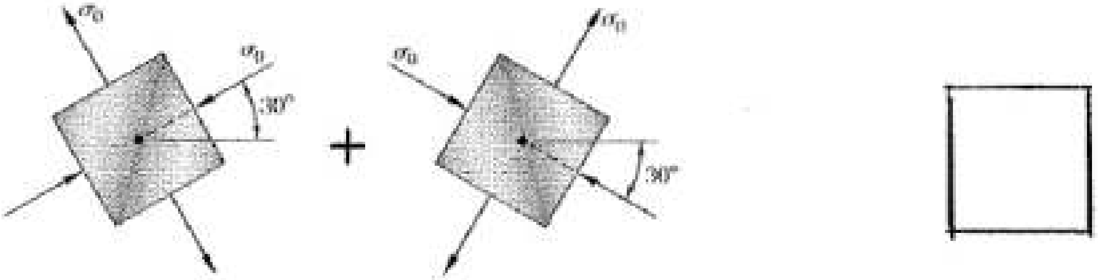
$\sigma_a = \sigma_{ave} + R = 168$ MPa ◀

$\sigma_b = \sigma_{ave} - R = -14$ MPa ◀

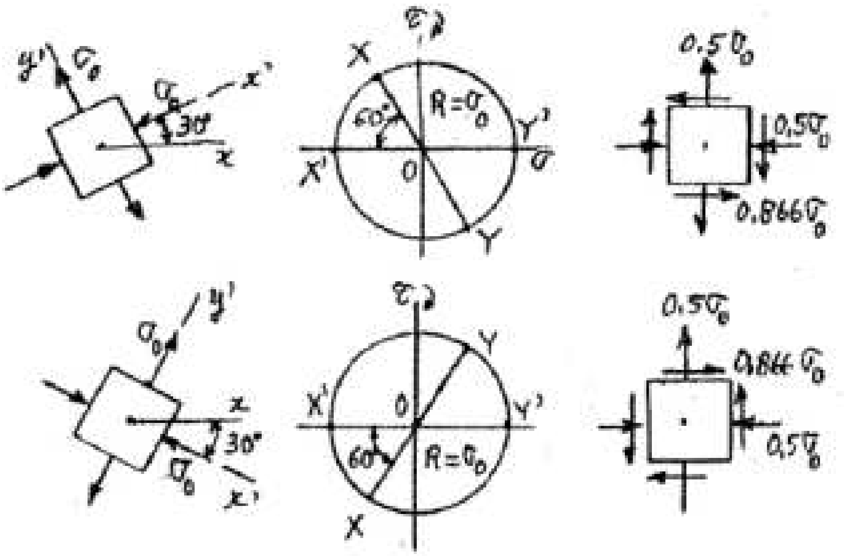


Problem 7.57

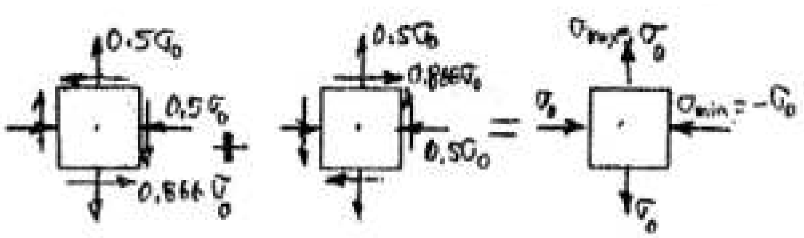
7.55 through 7.58 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



Express each state of stress in terms of components acting on the element shown at the right.



Add like components of the two states of stress.



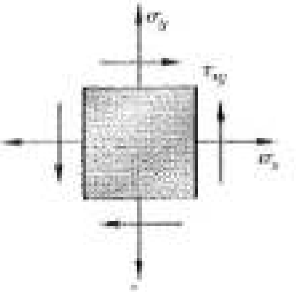
$\theta_p = 0$ and $90°$

$\sigma_{max} = \sigma_0$

$\sigma_{min} = -\sigma_0$

Problem 7.63

7.63 For the state of stress shown it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 98$ MPa, $\sigma_y = 63$ MPa, and $\sigma_{min} = 35$ MPa. Determine (*a*) the orientation of the principal planes, (*b*) the principal stress σ_{max}, (*c*) the maximum in-plane shearing stress.



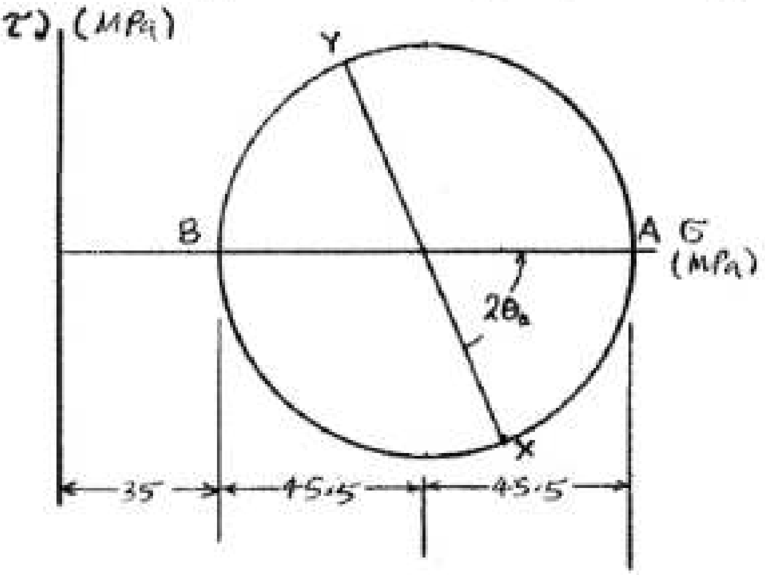
$$\sigma_x = 98\text{ MPa},\quad \sigma_y = 63\text{ MPa},\quad \sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 80.5\text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R \quad \therefore \quad R = \sigma_{ave} - \sigma_{min} = 80.5 - 35 = 45.5\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \pm\sqrt{45.5^2 - 17.5^2} = \pm 42\text{ MPa}$$

But it is given that τ_{xy} is positive, thus $\tau_{xy} = +42$ MPa



(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(6)}{5} = 2.4$

$2\theta_p = 67.38°$

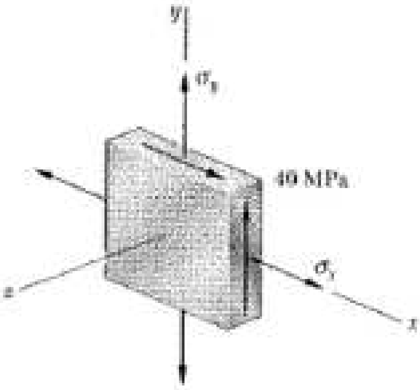
$\theta_a = 33.69°$ ◄

$\theta_b = 123.69°$ ◄

(b) $\sigma_{max} = \sigma_{ave} + R = 126$ MPa ◄

(c) $\tau_{max\,(in\text{-}plane)} = R = 45.5$ MPa ◄

Problem 7.66



7.66 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 0$ and $\sigma_y = 60$ MPa, (*b*) $\sigma_x = 105$ MPa and $\sigma_y = 45$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

(a) $\sigma_x = 0$, $\sigma_y = 60$ MPa, $\tau_{xy} = 40$ MPa

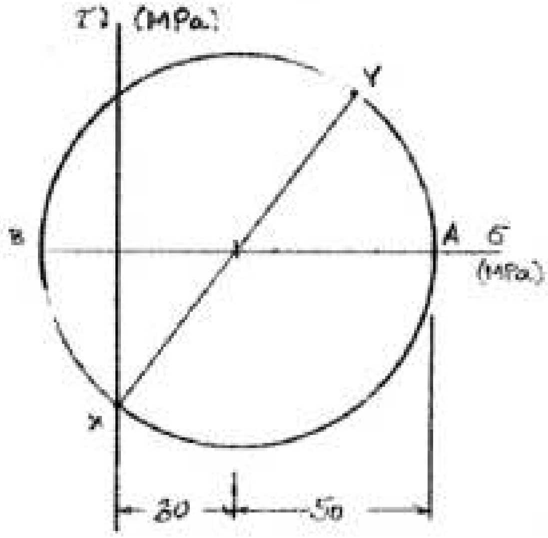
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 30$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-30)^2 + 40^2}$

$= 50$ MPa



$\sigma_a = \sigma_{ave} + R = 80$ MPa (max)

$\sigma_b = \sigma_{ave} - R = -20$ MPa (min)

$\sigma_c = 0$

$\sigma_{max} = 80$ MPa $\qquad \sigma_{min} = -20$ MPa

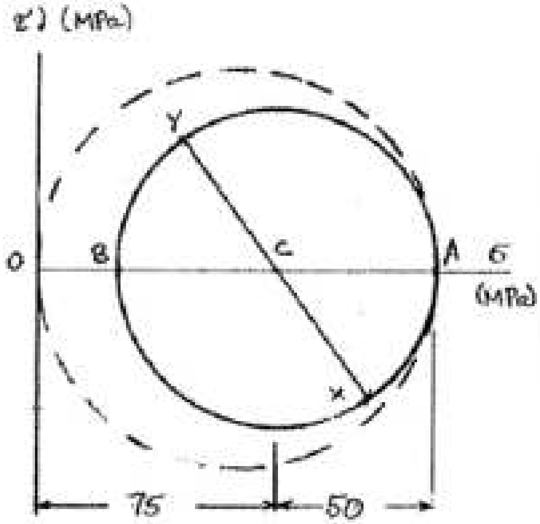
$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 50$ MPa ◄

(b) $\sigma_x = 105$ MPa $\quad \sigma_y = 45$ MPa $\quad \tau_{xy} = 40$ MPa

$\sigma_{ave} = 75$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-30)^2 + 40^2} = 50$ MPa



$\sigma_a = \sigma_{ave} + R = 125$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 25$ MPa

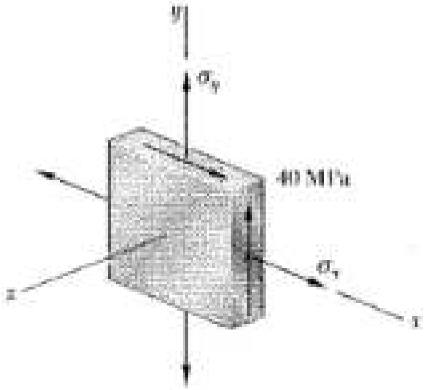
$\sigma_c = 0$ (min)

$\sigma_{max} = 125$ MPa, $\quad \sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 62.5$ MPa ◄

Problem 7.67

7.67 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 30$ MPa and $\sigma_y = 90$ MPa, (*b*) $\sigma_x = 70$ MPa and $\sigma_y = 10$ MPa.. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)



(a) $\sigma_x = 30$ MPa $\quad \sigma_y = 90$ MPa $\quad \tau_{xy} = 40$ MPa

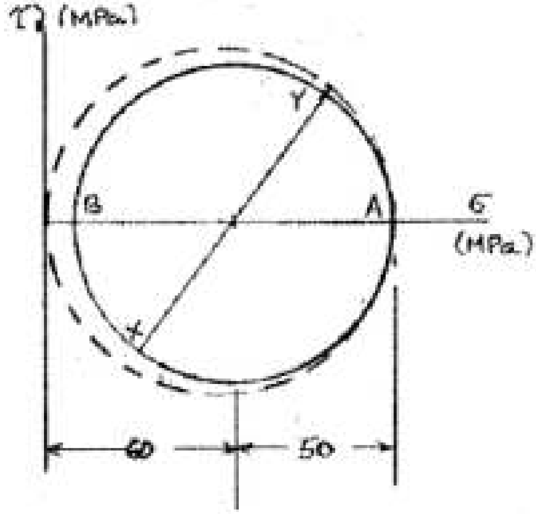
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 60$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{30^2 + 40^2}$

$= 50$ MPa



$\sigma_a = \sigma_{ave} + R = 60 + 50 = 110$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 60 - 50 = 10$ MPa

$\sigma_c = 0$ (min)

$\tau_{max(in\text{-}plane)} = R = 50$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 55$ MPa ◄

(b) $\sigma_x = 70$ MPa $\quad \sigma_y = 10$ MPa $\quad \tau_{xy} = 40$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 40$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{30^2 + 40^2} = 50$ MPa

$\sigma_a = \sigma_{ave} + R = 90$ MPa (max)

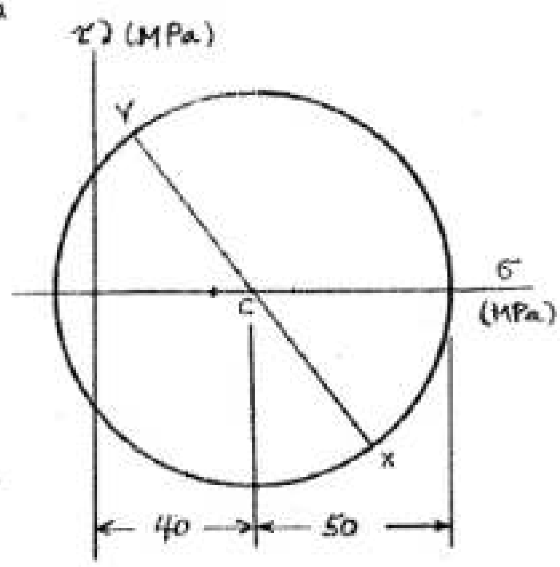
$\sigma_b = \sigma_{ave} - R = -10$ MPa (min)

$\sigma_c = 0$

$\sigma_{max} = 90$ MPa

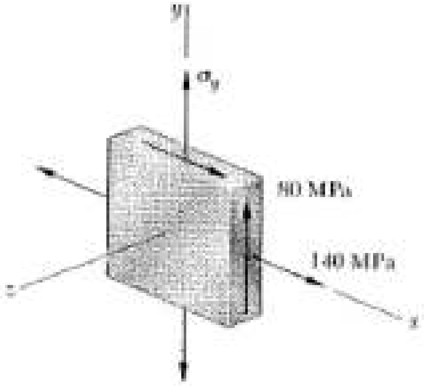
$\sigma_{min} = -10$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 50$ MPa ◄



Problem 7.68

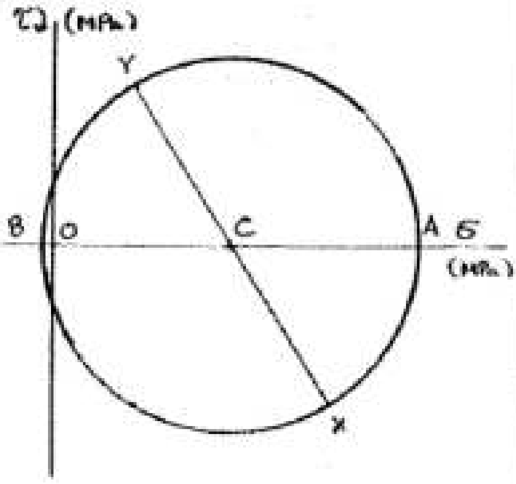
7.68 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 40$ MPa, (*b*) $\sigma_y = 120$ MPa. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)



(a) $\sigma_x = 140$ MPa, $\sigma_y = 40$ MPa, $\tau_{xy} = 80$ MPa

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{50^2 + 80^2} = 94.34 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 184.34$ MPa (max)

$\sigma_b = \sigma_{ave} - R = -4.34$ MPa (min)

$\sigma_c = 0$

$$\tau_{max(in\text{-}plane)} = \tfrac{1}{2}(\sigma_a - \sigma_b) = R = 94.34 \text{ MPa}$$

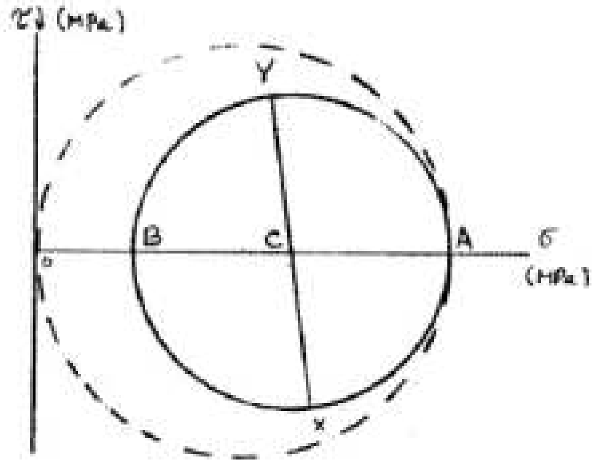
$$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = \tfrac{1}{2}(\sigma_a - \sigma_b) = 94.3 \text{ MPa}$$

$\tau_{max} = 94.3$ MPa ◄

(b) $\sigma_x = 140$ MPa, $\sigma_y = 120$ MPa, $\tau_{xy} = 80$ MPa

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 130 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{10^2 + 80^2} = 80.62 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 210.62$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 49.38$ MPa

$\sigma_c = 0$ (min)

$\sigma_{max} = \sigma_a = 210.62$ MPa

$\sigma_{min} = \sigma_c = 0$

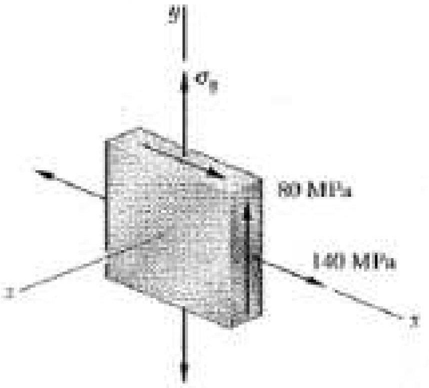
$\tau_{max(in\text{-}plane)} = R = 86.62$ MPa

$$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 105.3 \text{ MPa}$$

$\tau_{max} = 105.3$ MPa ◄

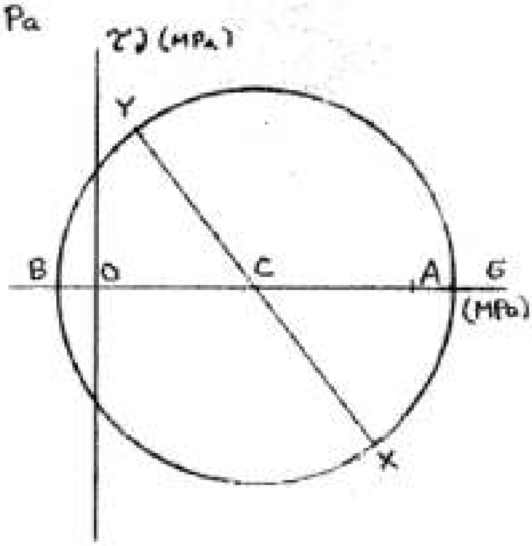
Problem 7.69

7.69 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 20$ MPa, (*b*) $\sigma_y = 140$ MPa. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)



(a) $\sigma_x = 140$ MPa, $\sigma_y = 20$ MPa

$\tau_{xy} = 80$ MPa



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$$
$$= 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \sqrt{60^2 + 80^2} = 100 \text{ MPa}$$

$\sigma_a = \sigma_{ave} + R = 80 + 100 = 180$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 80 - 100 = -20$ MPa (min)

$\sigma_c = 0$

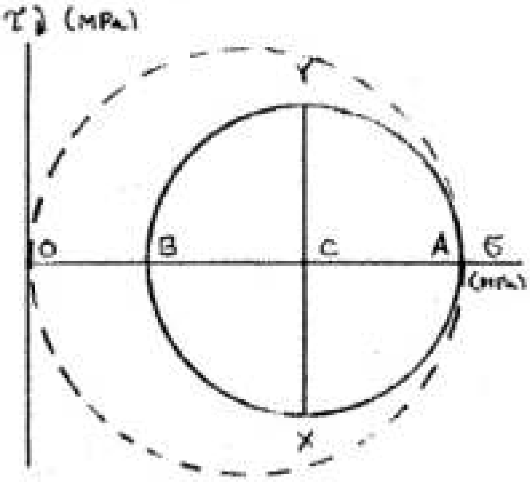
$\tau_{max(in\text{-}plane)} = \frac{1}{2}(\sigma_a - \sigma_b) = 100$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 100$ MPa

$\tau_{max} = 100$ MPa ◄

(b) $\sigma_x = 140$ MPa, $\sigma_y = 140$ MPa

$\tau_{xy} = 80$ MPa



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 140 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \sqrt{0 + 80^2} = 80 \text{ MPa}$$

$\sigma_a = \sigma_{ave} + R = 220$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 60$ MPa

$\sigma_c = 0$ (min)

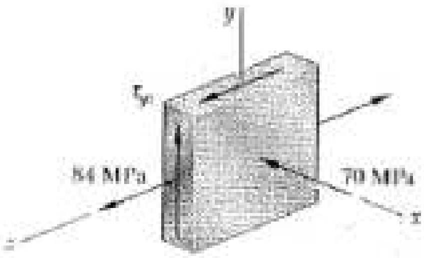
$\tau_{max(in\text{-}plane)} = \frac{1}{2}(\sigma_a - \sigma_b) = 80$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 110$ MPa

$\tau_{max} = 110$ MPa ◄

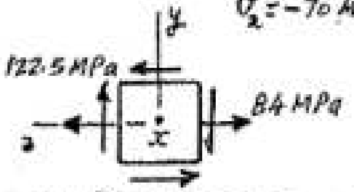
Problem 7.72

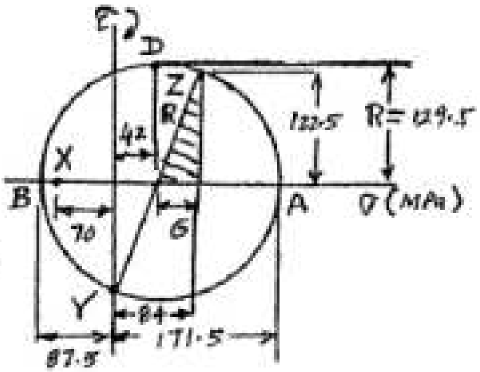
7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (a) $\tau_{yz} = 122.5$ MPa, (b) $\tau_{yz} = 56$ MPa, (c) $\tau_{yz} = 0$.



(a) $\tau_{yz} = 122.5$ MPa

$\sigma_z = -70$ MPa





$R = \sqrt{(42)^2 + (122.5)^2} = 129.5$

$\sigma_A = 42 + 129.5 = 171.5$

$\sigma_B = 42 - 129.5 = -87.5$

$\sigma_{max} = \sigma_A = 171.5$ MPa

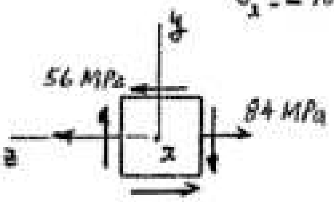
$\sigma_{min} = \sigma_B = -87.5$ MPa

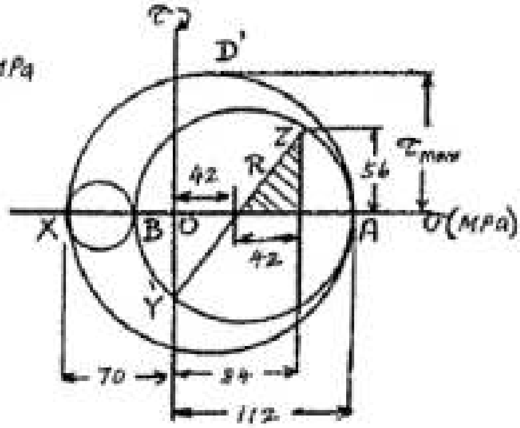
$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$

$\tau_{max} = 129.5$ MPa ◄

(b) $\tau_{yz} = 56$ MPa

$\sigma_z = -70$ MPa





$R = \sqrt{(42)^2 + (56)^2} = 70$

$\sigma_A = 42 + 70 = 112$

$\sigma_B = 42 - 70 = -28$

$\sigma_{max} = \sigma_A = 112$ MPa

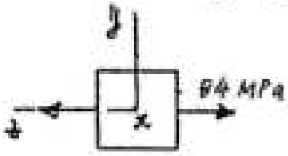
$\sigma_{min} = \sigma_x = -70$ MPa

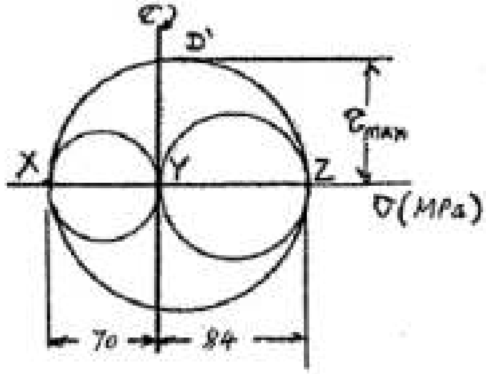
$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$

$\tau_{max} = 91$ MPa ◄

(c) $\tau_{yz} = 0$

$\sigma_z = -70$ MPa





$\sigma_{max} = \sigma_z = 84$ MPa

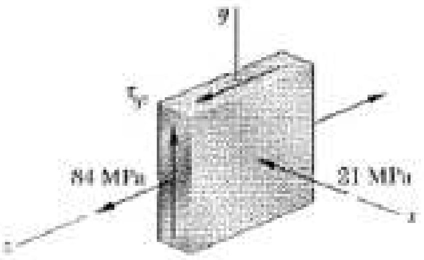
$\sigma_{min} = \sigma_x = -70$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$

$\tau_{max} = 77$ MPa ◄

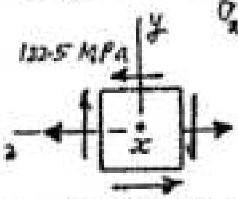
Problem 7.73

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (*a*) $\tau_{yz} = 122.5$ MPa, (*b*) $\tau_{yz} = 56$ MPa, (*c*) $\tau_{yz} = 0$.



(a) $\tau_{yz} = 122.5$ MPa

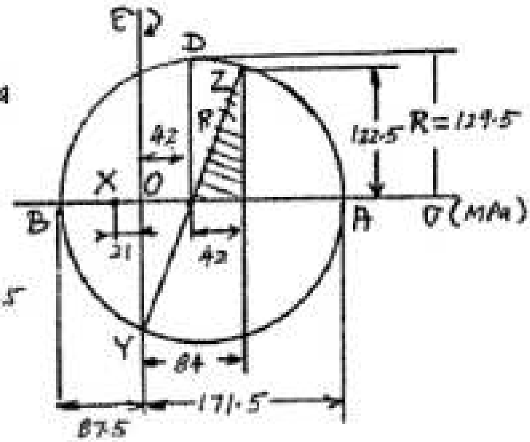
$\sigma_x = -21$ MPa



$R = \sqrt{(42)^2 + (122.5)^2} = 129.5$

$\sigma_A = 42 + 129.5 = 171.5$

$\sigma_B = 42 - 129.5 = -87.5$

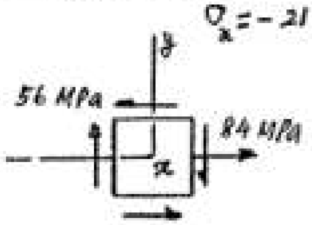


$\sigma_{max} = \sigma_A = 171.5$ MPa $\qquad \sigma_{min} = \sigma_B = -87.5$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\qquad \tau_{max} = 129.5$ MPa ◄

(b) $\tau_{yz} = 56$ MPa

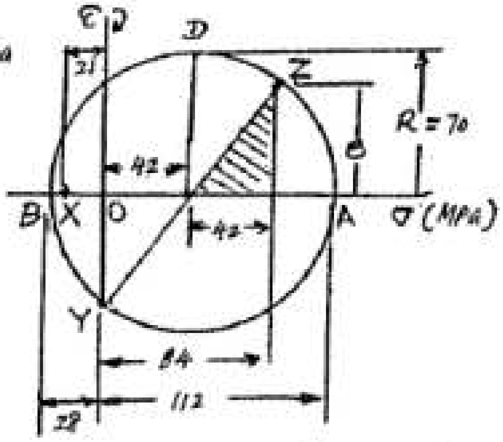
$\sigma_x = -21$ MPa



$R = \sqrt{(42)^2 + (56)^2} = 70$

$\sigma_A = 42 + 70 = 112$

$\sigma_B = 42 - 70 = -28$

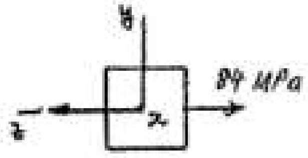


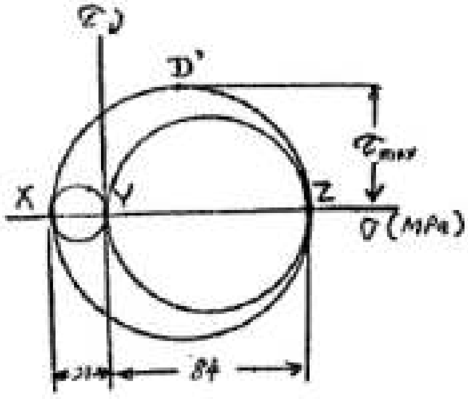
$\sigma_{max} = \sigma_A = 112$ MPa $\qquad \sigma_{min} = \sigma_B = -28$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\qquad \tau_{max} = 70$ MPa ◄

(c) $\tau_{yz} = 0$

$\sigma_x = -21$ MPa



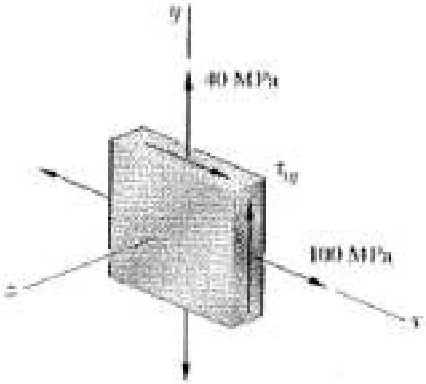


$\sigma_{max} = \sigma_z = 84$ MPa $\qquad \sigma_{min} = \sigma_x = -21$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\qquad \tau_{max} = 52.5$ MPa ◄

Problem 7.74

7.74 For the state of plane stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 60 MPa, (*b*) 78 MPa.



$\sigma_x = 100$ MPa, $\sigma_y = 40$ MPa, $\sigma_z = 0$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 70$ MPa

(a) $\underline{\tau_{max} = 60 \text{ MPa}}$

If σ_z is σ_{min}, then $\sigma_{max} = \sigma_{min} + 2\tau_{max}$.

$\sigma_{max} = 0 + (2)(60) = 120$ MPa

$\sigma_{max} = \sigma_{ave} + R$

$R = \sigma_{max} - \sigma_{ave} = 120 - 70 = 50$ MPa

$\sigma_b = \sigma_{max} - 2R = 20 \text{ MPa} > 0$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2} = 50$ MPa

$\tau_{xy} = \sqrt{50^2 - 30^2}$ $\qquad \tau_{xy} = 40$ MPa ◄

(b) $\underline{\tau_{max} = 78 \text{ MPa}}$

If σ_z is σ_{min}, then $\sigma_{max} = \sigma_{min} + 2\tau_{max} = 0 + (2)(78) = 156$ MPa.

$\sigma_{max} = \sigma_{ave} + R$

$R = \sigma_{max} - \sigma_{ave} = 156 - 70 = 86 \text{ MPa} > \tau_{max} = 78$ MPa

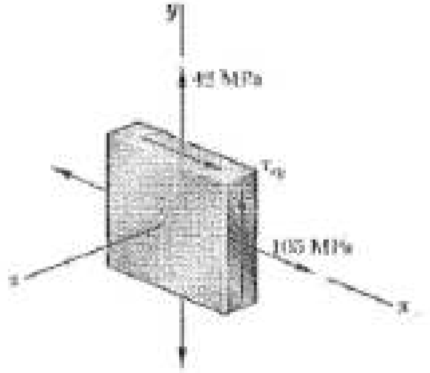
Set $R = \tau_{max} = 78$ MPa. $\quad \sigma_{min} = \sigma_{ave} - R = -8 \text{ MPa} < 0$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2}$

$\tau_{xy} = \sqrt{78^2 - 30^2}$ $\qquad \tau_{xy} = 72$ MPa ◄

Problem 7.75

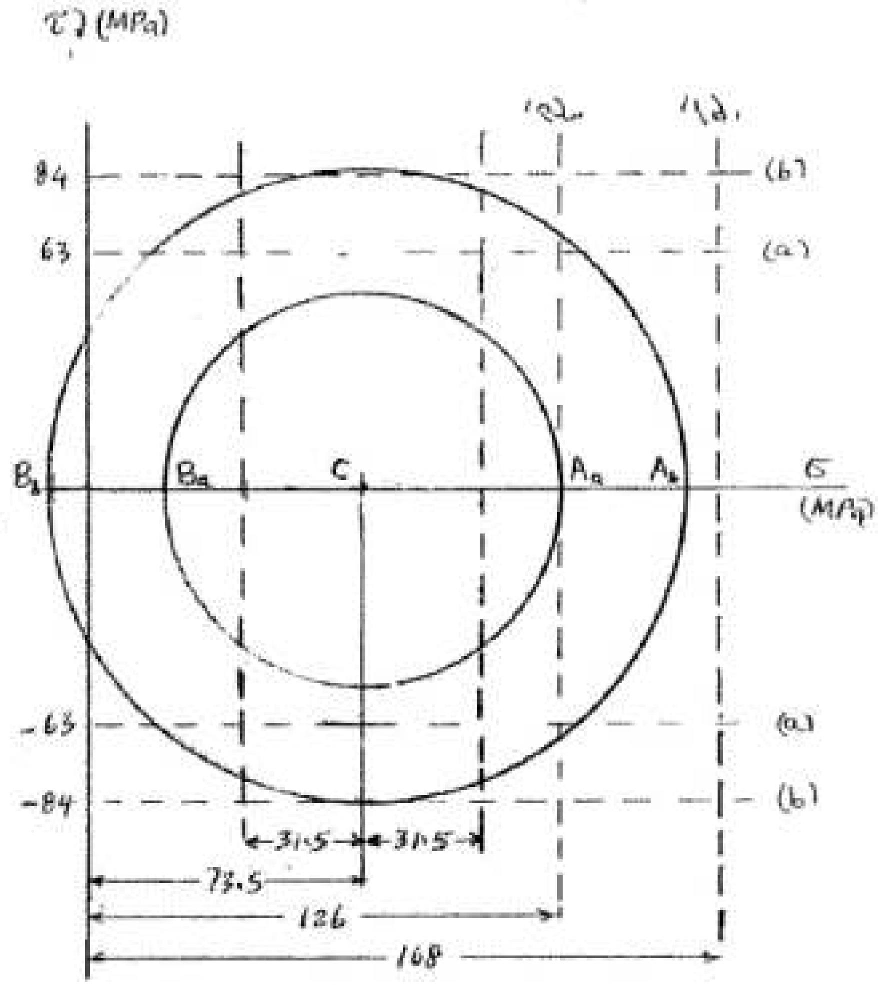
7.75 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 63 MPa, (*b*) 84 MPa.



$\sigma_x = 105\ MPa \qquad \sigma_y = 42\ MPa$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 73.5\ MPa$

$U = \frac{\sigma_x - \sigma_y}{2} = 31.5\ MPa$



(a) For $\tau_{max} = 63\ MPa$

Center of Mohrs circle lies at point C. Lines marked (a) show the limits on τ_{max}. Limit on σ_{max} is $\sigma_{max} = 2\tau_{max} = 126\ MPa$. For the Mohr's cirle $\sigma_a = \sigma_{max}$ corresponds to point A_a.

$R = \sigma_a - \sigma_{ave}$
$= 126 - 73.5 = 52.5\ MPa$

$R = \sqrt{U^2 + \tau_{xy}^2}$

$\tau_{xy} = \pm\sqrt{R^2 - U^2}$
$= \pm\sqrt{52.5^2 - 31.5^2}$
$= \pm\ 42\ MPa$

(b) For $\tau_{max} = 84\ MPa$

Center of Mohr's circle lies at point C. $\quad R = 84\ MPa$

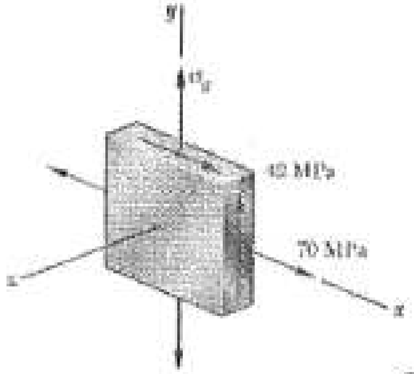
$\tau_{xy} = \pm\sqrt{R^2 - U^2} = \pm\ 78.7\ MPa$

Checking $\quad \sigma_a = 73.5 + 84 = 157.5\ MPa \qquad \sigma_b = 73.5 - 84 = -10.5\ MPa$
$\sigma_c = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 84\ MPa \quad$ O.K

Problem 7.76

7.76 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 52.5 MPa.



$\sigma_x = 70$ MPa, $\tau_{xy} = 42$ MPa, $\tau_{max} = 52.5$ MPa

Let $u = \dfrac{\sigma_y - \sigma_x}{2}$ $\qquad \sigma_y = 2u + \sigma_x$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$

$R = \sqrt{u^2 + \tau_{xy}^2}$ $\qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$

Case 1 $\qquad \tau_{max} = R = 52.5$ MPa, $u = \pm 31.5$ MPa

(1a) $u = +31.5$ MPa $\qquad \sigma_y = 2u + \sigma_x = 133$ MPa $\qquad$ reject

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 101.5$ MPa, $\sigma_a = \sigma_{ave} + R = 154$ MPa, $\sigma_b = \sigma_{ave} - R = 49$ MPa

$\sigma_{max} = 154$ MPa, $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 77$ MPa $\neq 52.5$ MPa

(1b) $u = -31.5$ MPa $\qquad \sigma_y = 2u + \sigma_x = 7$ MPa ◄

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 38.5$ MPa, $\sigma_a = \sigma_{ave} + R = 91$ MPa, $\sigma_b = \sigma_{ave} - R = -14$ MPa

$\sigma_{max} = 91$ MPa, $\sigma_{min} = -14$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 52.5$ MPa OK.

Case 2 $\qquad$ Assume $\sigma_{min} = 0$ $\qquad \sigma_{max} = 2\tau_{max} = 105$ MPa $= \sigma_a$

$\sigma_a = \sigma_{ave} + R = \sigma_x + u + \sqrt{u^2 + \tau_{xy}^2}$

$\sigma_a - \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$

$(\sigma_a - \sigma_x - u)^2 = u^2 + \tau_{xy}^2$

$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)u + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$

$2u = \dfrac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \dfrac{(105 - 70)^2 - 42^2}{105 - 70} = -15.4$ MPa

$u = -7.7$ MPa $\qquad \sigma_y = 2u + \sigma_x = 54.6$ MPa ◄

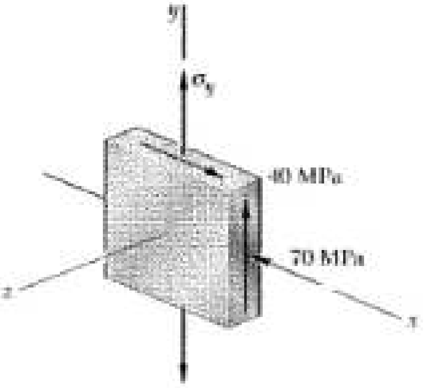
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 62.3$ MPa $\qquad R = \sqrt{u^2 + \tau_{xy}^2} = 42.7$ MPa

$\sigma_a = \sigma_{ave} + R = 105$ MPa ✓ $\qquad \sigma_b = \sigma_{ave} - R = 19.6$ MPa

$\sigma_{max} = 105$ MPa, $\sigma_{min} = 0$ $\qquad \tau_{max} = 52.5$ MPa ✓

Problem 7.77

7.77 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 75 MPa.



$\sigma_x = -70$ MPa, $\tau_{xy} = 40$ MPa

Let $u = \frac{\sigma_y - \sigma_x}{2}$ $\qquad \sigma_y = 2u + \sigma_x$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$

$R = \sqrt{u^2 + \tau_{xy}^2} \qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$

Case (1) $\quad \tau_{max} = R = 75$ MPa, $\quad u = \pm\sqrt{75^2 - 40^2} = \pm 63.44$ MPa

(1a) $u = +63.44$ MPa $\quad \sigma_y = 2u + \sigma_x = 56.9$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.56$ MPa

$\sigma_a = \sigma_{ave} + R = 68.44$ MPa, $\quad \sigma_b = \sigma_{ave} - R = -81.56$ MPa

$\sigma_c = 0 \quad \sigma_{max} = 68.44$ MPa, $\sigma_{min} = -81.56$ MPa $\quad \tau_{max} = 75$ MPa

(1b) $u = -63.44$ MPa $\quad \sigma_y = 2u + \sigma_x = -196.88$ MPa (reject)

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -133.44$ MPa $\quad \sigma_a = \sigma_{ave} + R = -58.44$ MPa

$\sigma_b = \sigma_{ave} - R = -208.44$ MPa, $\quad \sigma_c = 0$, $\quad \sigma_{max} = 0$

$\sigma_{min} = -208.44$ MPa, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 104.22$ MPa $\neq 75$ MPa

$\sigma_y = 56.9$ MPa ◄

Case (2) $\quad$ Assume $\sigma_{max} = 0$, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 75$ MPa

$\sigma_{min} = -150$ MPa $= \sigma_b$

$\sigma_b = \sigma_{ave} - R = \sigma_x + u - \sqrt{u^2 + \tau_{xy}^2}$

$\sqrt{u^2 + \tau_{xy}^2} = -\sigma_x + u - \sigma_b$

$\cancel{u^2} + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)u + \cancel{u^2}$

$2u = \frac{\tau_{xy}^2 - (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} \qquad \frac{(40)^2 - (-70+150)^2}{-70+150} = -160$ MPa

$u = -30$ MPa $\qquad \sigma_y = 2u + \sigma_x = -130$ MPa

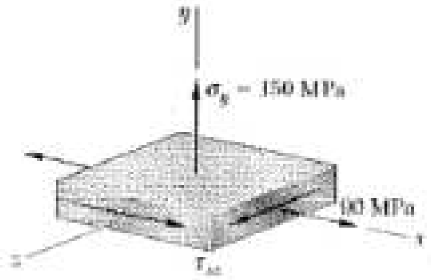
$R = \sqrt{u^2 + \tau_{xy}^2} = 50$ MPa

$\sigma_a = \sigma_b + 2R = -150 + 100 = -50$ MPa $\qquad$ O.K.

$\sigma_y = -130.0$ MPa ◄

Problem 7.78

7.78 For the state of stress shown, determine the range of values of τ_{xz} for which the maximum shearing stress is equal to or less than 90 MPa.



$\sigma_x = 90$ MPa, $\sigma_z = 0$, $\sigma_y = 150$ MPa

For Mohr's circle of stresses in zx-plane,

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 45$ MPa

$U = \dfrac{\sigma_x - \sigma_z}{2} = 45$

Assume $\sigma_{max} = \sigma_y = 150$ MPa.

$\sigma_{min} = \sigma_b = \sigma_{max} - 2\tau_{max}$

$= 150 - (2)(90) = -30$ MPa

$R = \sigma_{ave} - \sigma_b$

$= 45 - (-30) = 75$ MPa

$\sigma_a = \sigma_{ave} + R$

$= 45 + 75 = 120 \text{ MPa} < \sigma_y$

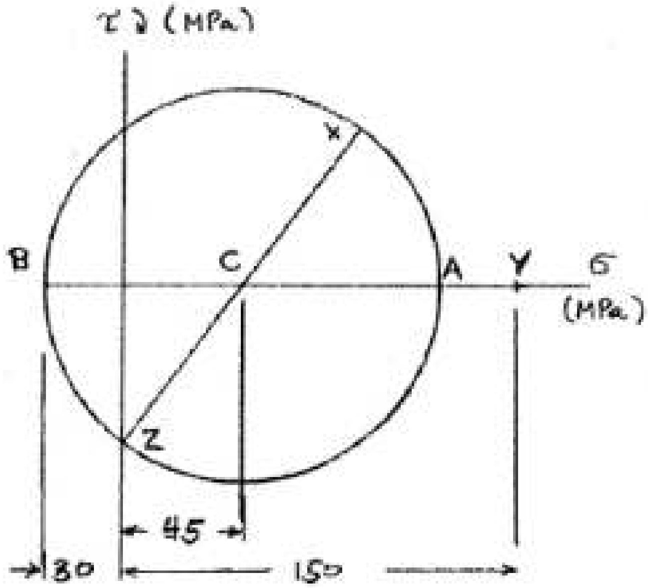
O.K

$R = \sqrt{U^2 + \tau_{xz}^2}$

$\tau_{xz} = \pm\sqrt{R^2 - U^2}$

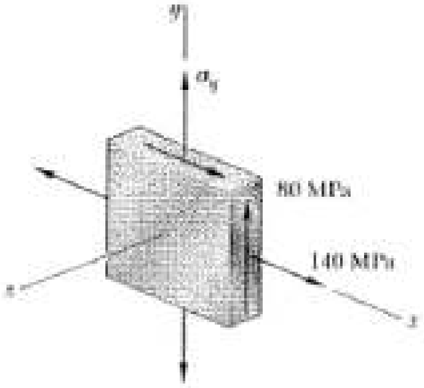
$= \pm\sqrt{75^2 - 45^2} = \pm 60$ MPa

$-60 \text{ MPa} \le \tau_{xz} \le 60 \text{ MPa}$ ◀



Problem 7.80

*7.80 For the state of stress of Prob. 7.69, determine (a) the value of σ_y for which the maximum shearing stress is as small as possible, (b) the corresponding value of the shearing stress.



Let $u = \frac{\sigma_x - \sigma_y}{2}$ $\quad \sigma_y = \sigma_x - 2u$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - u$

$R = \sqrt{u^2 + \tau_{xy}^2}$

$\sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$

$\sigma_b = \sigma_{ave} - R = \sigma_x - u - \sqrt{u^2 + \tau_{xy}^2}$

Assume τ_{max} is the in-plane shearing stress, $\quad \tau_{max} = R$

Then, $\tau_{max(in\text{-}plane)}$ is minimum if $u = 0$.

$\sigma_y = \sigma_x - 2u = \sigma_x = 140$ MPa, $\quad \sigma_{ave} = \sigma_x - u = 140$ MPa

$R = |\tau_{xy}| = 80$ MPa

$\sigma_a = \sigma_{ave} + R = 140 + 80 = 220$ MPa

$\sigma_b = \sigma_{ave} - R = 140 - 80 = 60$ MPa

$\sigma_{max} = 220$ MPa, $\sigma_{min} = 0$, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 110$ MPa

Assumption is incorrect.

Assume $\sigma_{max} = \sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$

$\sigma_{min} = 0 \qquad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}\sigma_a$

$\frac{d\sigma_a}{du} = -1 + \frac{u}{\sqrt{u^2 + \tau_{xy}^2}} \neq 0$ (no minimum)

Optimum value for u occurs when $\tau_{max(out\text{-}of\text{-}plane)} = \tau_{max(in\text{-}plane)}$

$\frac{1}{2}(\sigma_a + R) = R$ or $\sigma_a = R$ or $\sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$

$(\sigma_x - u)^2 = \sigma_x^2 - 2u\sigma_x + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$

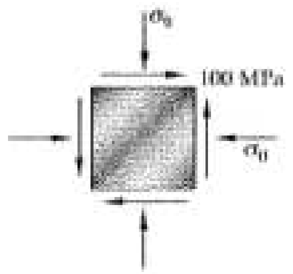
$2u = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.3$ MPa $\qquad u = 47.14$ MPa

(a) $\sigma_y = \sigma_x - 2u = 140 - 94.3$ $\qquad \sigma_y = 45.7$ MPa ◄

(b) $R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 92.9$ MPa $\qquad \tau_{max} = 92.9$ MPa ◄

Problem 7.81

7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.



$\sigma_{ave} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$

(a) $\underline{\sigma_0 = 200 \text{ MPa}} \qquad \sigma_{ave} = -200 \text{ MPa}$

$\sigma_a = \sigma_{ave} + R = -100 \text{ MPa}, \qquad \sigma_b = \sigma_{ave} - R = -300 \text{ MPa}$

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 264.56 \text{ MPa} < 325 \text{ MPa}$ (No yielding)

$F.S. = \frac{325}{264.56} \qquad F.S. = 1.228$ ◄

(b) $\underline{\sigma_0 = 240 \text{ MPa}} \qquad \sigma_{ave} = -240 \text{ MPa}$

$\sigma_a = \sigma_{ave} + R = -140 \text{ MPa}, \qquad \sigma_b = \sigma_{ave} - R = -340 \text{ MPa}$

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 295.97 \text{ MPa} < 325 \text{ MPa}$ (No yielding)

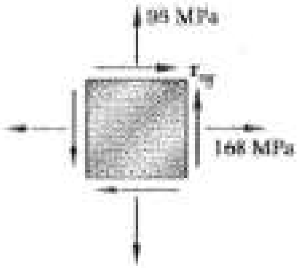
$F.S. = \frac{325}{295.97} \qquad F.S. = 1.098$ ◄

(c) $\underline{\sigma_0 = 280 \text{ MPa}} \qquad \sigma_{ave} = -280 \text{ MPa}$

$\sigma_a = \sigma_{ave} + R = -180 \text{ MPa}, \qquad \sigma_b = \sigma_{ave} - R = -380 \text{ MPa}$

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 329.24 \text{ MPa} > 325 \text{ MPa}$ (Yielding occurs)

Problem 7.83



7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 210$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 42$ MPa, (*b*) $\tau_{xy} = 84$ MPa, (*c*) $\tau_{xy} = 98$ MPa. If yield does not occur, determine the corresponding factor of safety.

$\sigma_x = 168$ MPa $\qquad \sigma_y = 98$ MPa $\qquad \sigma_z = 0$

For stresses in xy-plane $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 133$ MPa

$\frac{\sigma_x - \sigma_y}{2} = 35$ MPa

(a) $\tau_{xy} = 6$ MPa $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (42)^2} = 54.7$ MPa

$\sigma_a = \sigma_{ave} + R = 187.7$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = 78.3$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 163.3$ MPa < 210 MPa (No yielding)

F.S. $= \frac{210}{163.3} = 1.286$

(b) $\tau_{xy} = 84$ MPa $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (84)^2} = 91$ MPa

$\sigma_a = \sigma_{ave} + R = 224$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = 42$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 206.2$ MPa < 210 MPa (No yielding)

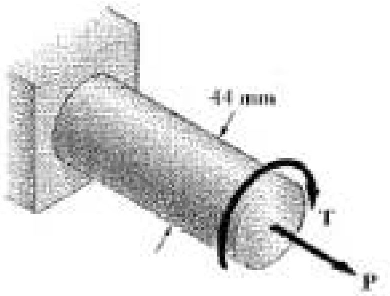
F.S. $= \frac{210}{206.2} = 1.018$

(c) $\tau_{xy} = 98$ MPa $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (98)^2} = 104.1$ MPa

$\sigma_a = \sigma_{ave} + R = 237.1$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = 28.9$ MPa

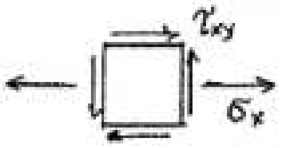
$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 224$ MPa > 210 MPa (Yielding occurs)

Problem 7.85



7.85 The 44-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the force **P** for which yield occurs when $T = 1500$ N·m.

Let the x-axis lie along the shaft axis.



$\sigma_y = 0, \quad \sigma_z = 0$

$\sigma_x = \dfrac{P}{A}, \quad \tau_{xy} = \dfrac{Tc}{J}$

Section properties: $\quad c = \frac{1}{2}d = 0.022$ m

$A = \pi c^2 = \pi(0.022)^2 = 152.05 \times 10^{-5}$ m^2, $\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.022)^4 = 367.97 \times 10^{-9}$ m^4

From torsion, $\quad \tau_{xy} = \dfrac{Tc}{J} = \dfrac{(1500)(0.022)}{367.97 \times 10^{-9}} = 89.68$ MPa.

From Mohr's circle, $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} =$$

$$\sigma_a = \sigma_{ave} + R = \frac{1}{2}\sigma_x + \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \frac{1}{2}\sigma_x - \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Maximum shearing stress criterion.

$$\sigma_{max} = \sigma_a \qquad \sigma_{min} = \sigma_b$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = \sigma_a - \sigma_b = 2\sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} = \sigma_Y$$

$$\sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y \qquad \sigma_x = \sqrt{\sigma_Y^2 - 4\tau_{xy}^2}$$

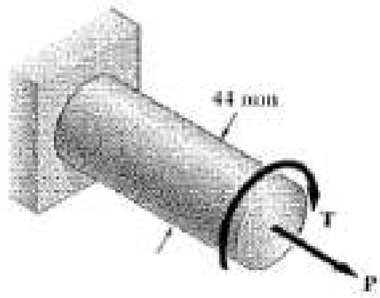
$$\sigma_x = \sqrt{(250)^2 - 4(89.68)^2} = 174.16 \text{ MPa}$$

$P = \sigma_x A = (174.16 \times 10^6)(152.05 \times 10^{-5}) \qquad P = 264.8$ kN ◄

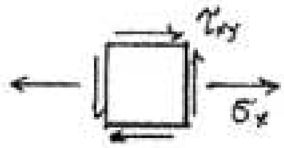
Problem 7.86

7.86 Solve Prob. 7.85, using the maximum-distortion-energy criterion.

7.85 The 44-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the force **P** for which yield occurs when $T = 1500$ N·m.



Let the x-axis lie along the shaft axis.



$$\sigma_y = 0, \quad \sigma_z = 0$$

$$\sigma_x = \frac{P}{A}, \quad \tau_{xy} = \frac{Tc}{J}$$

Section properties: $c = \frac{1}{2}d = 0.022$ m

$$A = \pi c^2 = \pi(0.022)^2 = 152.05\times10^{-5}\ \text{m}^2, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.022)^4 = 367.97\times10^{-9}\ \text{m}^4$$

From torsion, $$\tau_{xy} = \frac{Tc}{J} = \frac{(1500)(0.022)}{367.97\times10^{-9}} = 89.68\ \text{MPa}.$$

From Mohr's circle, $$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R =$$

Distortion energy criterion

$$\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = \sigma_Y^2$$

$$(\sigma_{ave} + R)^2 + (\sigma_{ave} - R)^2 - (\sigma_{ave} + R)(\sigma_{ave} - R) = \sigma_Y^2$$

$$\sigma_{ave}^2 + 3R^2 = \sigma_Y^2$$

$$\left(\frac{\sigma_x}{2}\right)^2 + 3\left[\frac{\sigma_x^2}{4} + \tau_{xy}^2\right] = \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2$$

$$\sigma_x = \sqrt{\sigma_Y^2 - 3\tau_{xy}^2}$$

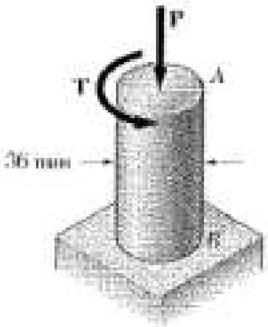
$$\sigma_x = \sqrt{(250)^2 - (3)(89.68)^2} = 195.89\ \text{MPa}$$

$$P = \sigma_x A = (195.89\times10^6)(152.05\times10^{-5}) \qquad P = 297.9\ \text{kN}$$

Problem 7.88

7.88 Solve Prob. 7.87 using the maximum-distortion-energy criterion.

7.87 The 36-mm-diameter shaft is made of a grade of steel with a 250-MPa tensile yield stress. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 200$ kN.



$P = 200 \text{ kN} = 200\times10^3 \text{ N} \qquad c = \frac{1}{2}d = 18 \text{ mm} = 18\times10^{-3} \text{ m}$

$A = \pi c^2 = \pi(18\times10^{-3})^2 = 1.01788\times10^{-3} \text{ m}^2$

$\sigma_y = -\frac{P}{A} = -\frac{200\times10^3}{1.01788\times10^{-3}} = 196.488\times10^6 \text{ Pa}$

$= 196.448 \text{ MPa}$

$\sigma_x = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x+\sigma_y) = \frac{1}{2}\sigma_y = 98.244 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(98.244)^2 + \tau_{xy}^2}$

$\sigma_a = \sigma_{ave} + R \qquad \sigma_b = \sigma_{ave} - R$

<u>Distortion energy criterion.</u>

$\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = \sigma_Y^2$

$(\sigma_{ave}+R)^2 + (\sigma_{ave}-R)^2 - (\sigma_{ave}+R)(\sigma_{ave}-R) = \sigma_Y^2$

$\sigma_{ave}^2 + 3R^2 = \sigma_Y^2$

$(98.244)^2 + (3)\left[(98.244)^2 + \tau_{xy}^2\right] = (250)^2$

$\tau_{xy} = 89.242 \text{ MPa}$

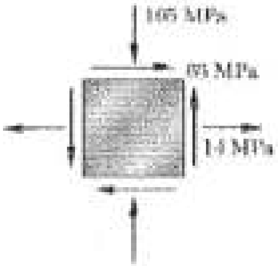
<u>Torsion</u> $\qquad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(18\times10^{-3})^4 = 164.846\times10^{-9} \text{ m}^4$

$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c} = \frac{(164.846\times10^{-9})(89.242\times10^6)}{18\times10^{-3}}$

$= 818 \text{ N}\cdot\text{m} \qquad\qquad T = 818 \text{ N}\cdot\text{m}$ ◄

Problem 7.89

7.89 and 7.90 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 210$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.



$\sigma_x = 14 \text{ MPa} \qquad \sigma_y = -105 \text{ MPa} \qquad \tau_{xy} = 63 \text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -45.5 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{59.5^2 + 63^2} = 86.7 \text{ MPa}$

$\sigma_a = \sigma_{ave} + R = 41.2 \text{ MPa}$

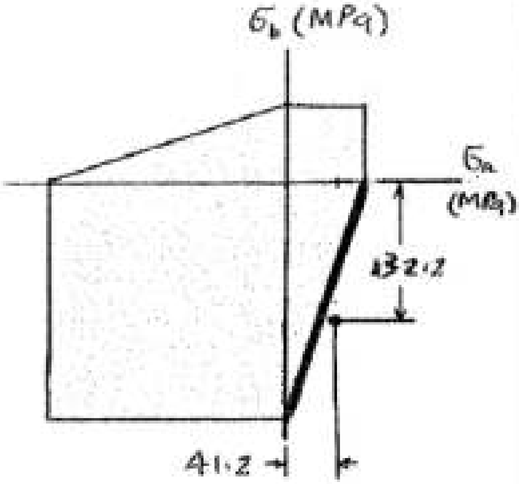
$\sigma_b = \sigma_{ave} - R = -132.2 \text{ MPa}$

Equation of 4th quadrant of boundary

$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

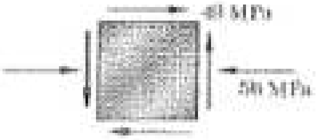
$\frac{41.2}{70} - \frac{(-132.2)}{210} = 1.218 > 1$

Rupture will occur.



Problem 7.90

7.89 and 7.90 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 210$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.



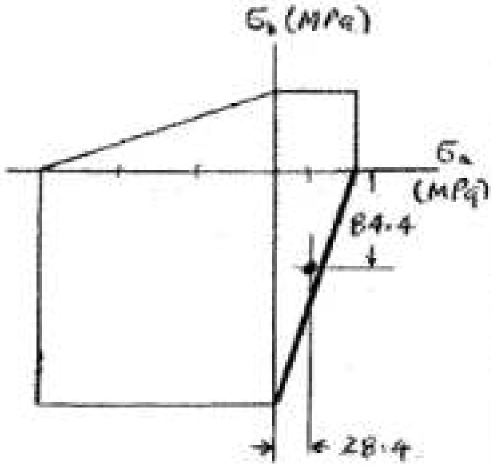
$\sigma_x = -56\text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = 49\text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -28\text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{28^2 + 49^2} = 56.4\text{ MPa}$

$\sigma_a = \sigma_{ave} + R = -28 + 56.4 = 28.4\text{ MPa}$

$\sigma_b = \sigma_{ave} - R = -28 - 56.4 = -84.4\text{ MPa}$



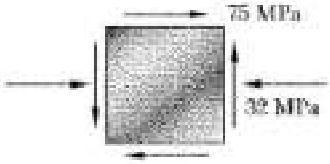
Equation of 4 th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{28.4}{70} - \frac{(-84.4)}{210} = 0.808 < 1$$

(No rupture)

Problem 7.91



7.91 and 7.92 The state of plane stress shown is expected in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

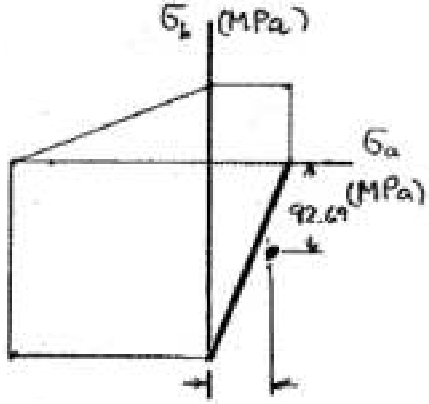
$\sigma_x = -32$ MPa, $\sigma_y = 0$, $\tau_{xy} = 75$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -16$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(16)^2 + (75)^2} = 76.69$ MPa

$\sigma_a = \sigma_{ave} + R = -16 + 76.69 = 60.69$ MPa

$\sigma_b = \sigma_{ave} - R = -16 - 76.69 = -92.69$ MPa



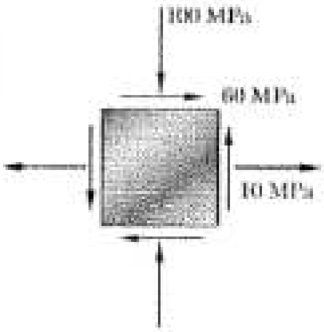
Equation of 4th quadrant of boundary.

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{60.69}{80} - \frac{(-92.69)}{200} = 1.222 > 1$$

Rupture will occur. ◄

Problem 7.92



7.91 and 7.92 The state of plane stress shown is expected in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

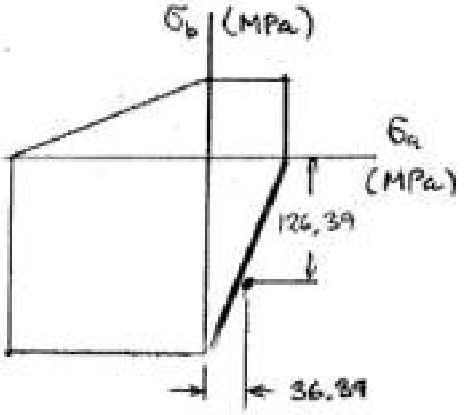
$\sigma_x = 10$ MPa, $\sigma_y = -100$ MPa, $\tau_{xy} = 60$ MPa

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{10 - 100}{2} = -45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(55)^2 + (60)^2} = 81.39 \text{ MPa}$$

$\sigma_a = \sigma_{ave} + R = -45 + 81.39 = 36.39$ MPa

$\sigma_b = \sigma_{ave} - R = -45 - 81.39 = -126.39$ MPa



Equation of 4th quadrant of boundary.

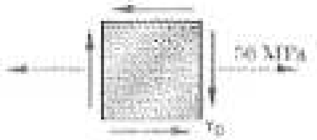
$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{36.37}{80} - \frac{(-126.39)}{200} = 1.087 > 1$$

Rupture will occur.

Problem 7.93

7.93 The state of plane stress shown will occur at a critical point in an aluminum casting that is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 170$ MPa. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.



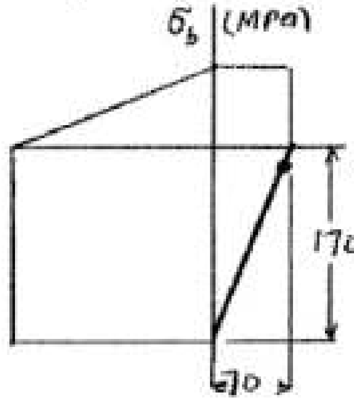
$$\sigma_x = 56\text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = \tau_0$$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 28\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{28^2 + \tau_0^2}, \qquad \tau_0 = \pm\sqrt{R^2 - 4^2}$$

$$\sigma_a = \sigma_{ave} + R = (28 + R)\text{ MPa} \qquad \sigma_b = \sigma_{ave} - R = (28 - R)\text{ MPa}$$

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is



$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

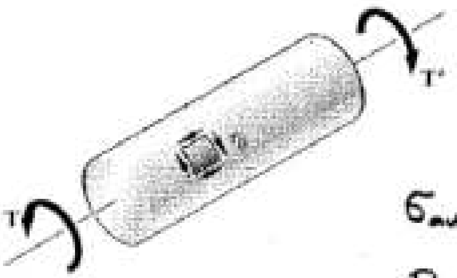
$$\frac{28 + R}{70} - \frac{28 - R}{170} = 1$$

$$\left(\frac{1}{70} + \frac{1}{170}\right)R = 1 - \frac{28}{70} + \frac{28}{170}$$

$$R = 37.9\text{ MPa} \qquad \tau_0 = \pm\sqrt{37.9^2 - 28^2} = \pm 25.5\text{ MPa}$$

Problem 7.95

7.95 The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 175$ MPa. Knowing that the magnitude T of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress τ_0 that should be expected at rupture.



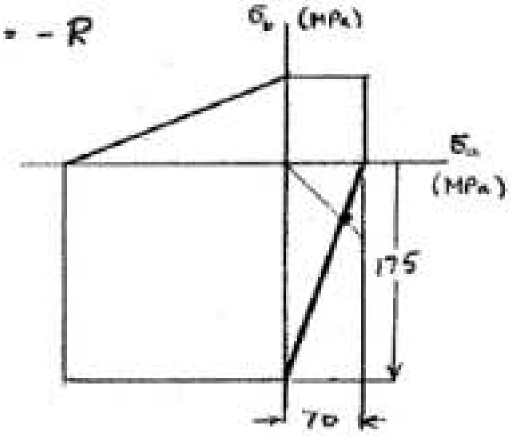
$$\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = -\tau_0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_{xy}^2} = |\tau_{xy}|$$

$$\sigma_a = \sigma_{ave} + R = R \qquad \sigma_b = \sigma_{ave} - R = -R$$

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is



$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

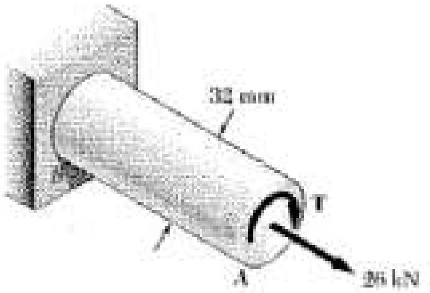
$$\frac{R}{70} - \frac{-R}{175} = 1$$

$$\left(\frac{1}{70} + \frac{1}{175}\right) R = 1$$

$$R = 50 \text{ MPa} \qquad \tau_0 = R \qquad \tau_0 = 50.0 \text{ MPa}$$

Problem 7.96

7.96 The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 60$ MPa and $\sigma_{UC} = 120$ MPa. Using Mohr's criterion, determine the magnitude of the torque **T** for which failure should be expected.



$$P = 26 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

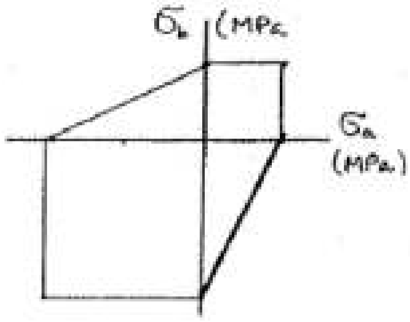
$$\sigma_x = \frac{P}{A} = \frac{26 \times 10^3}{804.25 \times 10^{-6}} = 32.328 \times 10^6 \text{ Pa} = 32.328 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(32.328 + 0) = 16.164 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(32.328 - 0) = 16.164 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 16.164 + R \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 16.164 - R \text{ MPa}$$

Since $|\sigma_{ave}| < R$, stress point lies in the 4th quadrant. Equation of the 4th quadrant is



$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{16.164 + R}{60} - \frac{16.164 - R}{120} = 1$$

$$\left(\frac{1}{60} + \frac{1}{120}\right)R = 1 - \frac{16.164}{60} + \frac{16.164}{120} \qquad R = 34.612 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{34.612^2 - 16.164^2} = 30.606 \text{ MPa}$$

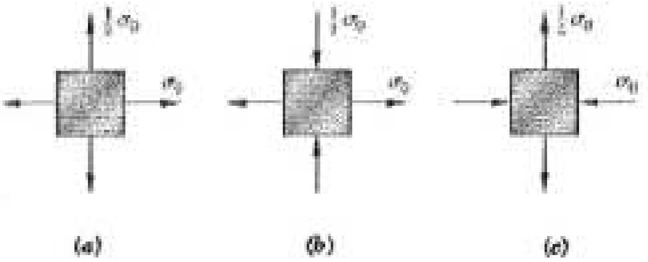
$$= 30.606 \times 10^6 \text{ Pa}$$

For torsion $\quad \tau_{xy} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad$ where $\quad c = \frac{1}{2}d = 16 \text{ mm} = 16 \times 10^{-3}$ m

$$T = \frac{\pi}{2}c^3\tau_{xy} = \frac{\pi}{2}(16 \times 10^{-3})^3(30.606 \times 10^6) \qquad T = 196.9 \text{ N}\cdot\text{m}$$

Problem 7.97

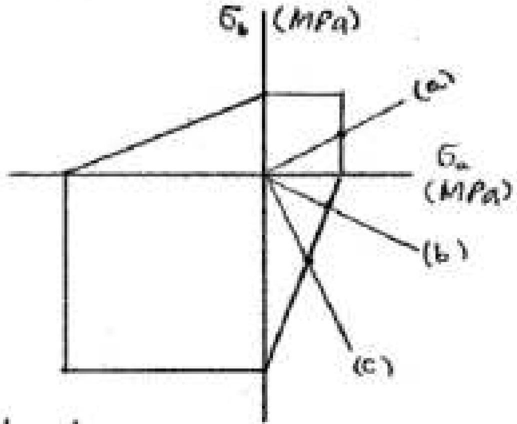
7.97 A machine component is made of a grade of cast iron for which $\sigma_{UT} = 56$ MPa and $\sigma_{UC} = 140$ MPa. For each of the states of stress shown, and using Mohr's criterion, determine the normal stress σ_0 at which rupture of the component should be expected.



(a) $\sigma_a = \sigma_0, \quad \sigma_b = \frac{1}{2}\sigma_0$

Stress point lies in 1st quadrant.

$\sigma_a = \sigma_0 = \sigma_{UT} = 56 \text{ MPa}$ ◄



(b) $\sigma_a = \sigma_0, \quad \sigma_b = -\frac{1}{2}\sigma_0$

Stress point lies in 4th quadrant.
Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_0}{56} - \frac{-\frac{1}{2}\sigma_0}{140} = 1 \qquad \sigma_0 = 46.7 \text{ MPa} \blacktriangleleft$$

(c) $\sigma_a = \frac{1}{2}\sigma_0, \quad \sigma_b = -\sigma_0, \quad$ 4th quadrant

$$\frac{\frac{1}{2}\sigma_0}{56} - \frac{-\sigma_0}{140} = 1 \qquad \sigma_0 = 62.2 \text{ MPa} \blacktriangleleft$$

Problem 7.98

7.98 A basketball has a 300-mm outer diameter and a 3-mm wall thickness. Determine the normal stress in the wall when the basketball is inflated to a 120-kPa gage pressure.

$r = \frac{1}{2}d - t = 147 \text{ mm} = 147 \times 10^{-3} \text{ m} \qquad p = 120 \times 10^3 \text{ Pa}$

$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(120 \times 10^3)(147 \times 10^{-3})}{(2)(3 \times 10^{-3})} = 2.94 \times 10^6 \text{ Pa} \qquad \sigma = 2.94 \text{ MPa}$ ◄

Problem 7.99

7.99 A spherical pressure vessel of 1.2-m outer diameter is to be fabricated from a steel having an ultimate stress $\sigma_U = 450$ MPa. Knowing that a factor of safety of 4.0 is desired and that the gage pressure can reach 3 MPa, determine the smallest wall thickness that should be used.

$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{450}{4} = 112.5 \text{ MPa} \qquad r = \frac{1}{2}d - t = (0.6 - t) \text{ m}$

$\sigma_{all} = \frac{pr}{2t} \qquad 2\sigma_{all} t = pr$

$(2)(112.5)t = 3(0.6 - t) \qquad 228t = 1.8$

$t = 7.89 \times 10^{-3} \text{ m} \qquad t_{min} = 7.89 \text{ mm}$ ◄

Problem 7.100

7.100 A spherical gas container made of steel has a 6-m outer diameter and a wall thickness of 9 mm. Knowing that the internal pressure is 500 kPa, determine the maximum normal stress and the maximum shearing stress in the container.

$r = \frac{1}{2}d - t = \frac{6}{2} - 9 \times 10 = 2.991 \text{ m} \qquad t = 9 \times 10^{-3} \text{ m}$

$p = 500 \times 10^3 \text{ Pa}$

$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(500 \times 10^3)(2.991)}{(2)(9 \times 10^{-3})} = 83.1 \times 10^6 \text{ Pa}$

$\sigma_{max} = \sigma_1 \qquad \sigma_{max} = 83.1 \text{ MPa}$ ◄

$\sigma_{min} \approx -p = 0.5 \times 10^6 \text{ Pa}$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 41.8 \times 10^6 \text{ Pa} \qquad \tau_{max} = 41.8 \text{ MPa}$ ◄

Problem 7.101

7.101 A spherical pressure vessel has an outer diameter of 3 m and a wall thickness of 12 mm. Knowing that for the steel used $\sigma_{all} = 80$ MPa, $E = 200$ GPa, and $\nu = 0.29$, determine (*a*) the allowable gage pressure, (*b*) the corresponding increase in the diameter of the vessel.

$r = \frac{1}{2}d - t = \frac{1}{2}(3) - 12 \times 10^{-3} = 1.488$ m $\qquad \sigma_1 = \sigma_2 = \sigma_{all} = 80 \times 10^6$ Pa

(a) $\sigma_1 = \sigma_2 = \frac{pr}{2t} \qquad p = \frac{2t\sigma_1}{r} = \frac{(2)(12\times10^{-3})(80\times10^6)}{1.488}$

$p = 1.290 \times 10^6$ Pa $\qquad p = 1.290$ MPa ◄

(b) $\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1 = \frac{1-0.29}{200\times10^9}(80\times10^6) = 284\times10^{-6}$

$\Delta d = d\varepsilon_1 = (3)(284\times10^{-6}) = 852\times10^{-6}$ m $\qquad \Delta d = 0.852$ mm ◄

Problem 7.102

7.102 A spherical gas container having an outer diameter of 4.5 m and a wall thickness of 22 mm is made of a steel for which $E = 200$ GPa and $\nu = 0.29$. Knowing that the gage pressure in the container is increased from zero to 1.7 MPa, determine (*a*) the maximum normal stress in the container, (*b*) the increase in the diameter of the container.

$r = \frac{1}{2}d - t = \frac{1}{2}(4500) - 22 = 2228$ mm $\qquad t = 22$ mm

(a) $\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(1.7)(2228)}{(2)(22)} = 86.1$ MPa $\qquad 86.1$ MPa ◄

(b) $\varepsilon_1 = \varepsilon_2 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1$

$= \frac{1-0.29}{200\times10^3}(86.1) = 305.66\times10^{-6}$

$\Delta d = (d)(\varepsilon_1) = (4500)(305.66\times10^{-6}) = 1.38$ mm ◄

Problem 7.103

7.103 The maximum gage pressure is known to be 10 MPa in a spherical steel pressure vessel having a 200-mm outer diameter and a 6-mm wall thickness. Knowing that the ultimate stress in the steel used is $\sigma_U = 400$ Mpa, determine the factor of safety with respect to tensile failure.

$r = \frac{1}{2}d - t = \frac{1}{2}(200) - 6 = 94$ mm $= 94\times10^{-3}$ m $\qquad t = 6\times10^{-3}$ m

$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(10\times10^6)(94\times10^{-3})}{(2)(6\times10^{-3})} = 78.333\times10^6$ Pa

$F.S. = \frac{\sigma_U}{\sigma_{max}} = \frac{450\times10^6}{78.333\times10^6}$ $\qquad F.S. = 5.11$ ◄

Problem 7.106

7.106 A standard-weight steel pipe of 300-mm nominal diameter carries water under a pressure of 2.8 MPa. (*a*) Knowing that the outside diameter is 320 mm and the wall thickness is 10 mm, determine the maximum tensile stress in the pipe. (*b*) Solve part *a*, assuming an extra-strong pipe is used, of 320 mm outside diameter and 12-mm wall thickness.

(a) $d_o = 0.32\ m \qquad t = 0.01\ m \qquad r = \frac{1}{2}d_o - t = 0.15\ m$

$$\sigma = \frac{pr}{t} = \frac{(2.8)(0.15)}{0.01} = 42\ MPa \qquad \sigma = 42\ MPa$$

(b) $d_o = 0.32\ m \qquad t = 0.012\ m \qquad r = \frac{1}{2}d_o - t = 0.148\ m$

$$\sigma = \frac{pr}{t} = \frac{(2.8)(0.148)}{0.012} = 34.5\ MPa \qquad \sigma = 34.5\ MPa.$$

Problem 7.107

7.107 A storage tank contains liquified propane under a pressure of 1.5 MPa at a temperature of 38°C. Knowing that the tank has an outer diameter of 320 mm and a wall thickness of 3 mm, determine the maximum normal stress and the maximum shearing stress in the tank.

$r = \frac{1}{2}d - t = \frac{1}{2}(320) - 3 = 157\ mm \qquad t = 3\ mm$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5)(157)}{3} = 78.5\ MPa$$

$$\sigma_{max} = \sigma_1 = 78.5\ MPa \qquad \sigma_{max} = 78.5\ MPa$$

$$\sigma_{min} \approx -p = -1.5\ MPa$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 40\ MPa \qquad \tau_{max} = 40\ MPa$$

Problem 7.108

7.108 The bulk storage tank shown in Fig. 7.49 has an outer diameter of 3.5 m and a wall thickness of 20 mm. At a time when the internal pressure of the tank is 1.2 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

$r = \frac{1}{2}d - t = \frac{1}{2}(3.5) - 20\times10^{-3} = 1.73\ m \qquad t = 20\times10^{-3}\ m$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2\times10^6)(1.73)}{20\times10^{-3}} = 103.8\times10^6\ Pa$$

$$\sigma_{max} = \sigma_1 = 103.8\times10^6\ Pa$$

$$\sigma_{max} = 103.8\ MPa$$

$$\sigma_{min} = -p \approx 1.2\times10^6\ Pa$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 52.5\times10^6\ Pa \qquad \tau_{max} = 52.5\ MPa$$

Problem 7.109

7.109 Determine the largest internal pressure that can be applied to a cylindrical tank of 1.75-m outer diameter and 16-mm wall thickness if the ultimate normal stress of the steel used is 450 MPa and a factor of safety of 5.0 is desired.

$$\sigma_1 = \frac{\sigma_U}{F.S.} = \frac{450 \text{ MPa}}{5} = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa}$$

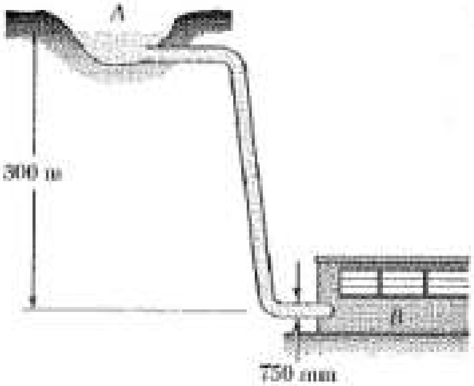
$$r = \frac{1}{2}d - t = \frac{1.75}{2} - 16 \times 10^{-3} = 0.859 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} \qquad p = \frac{t\sigma_1}{r} = \frac{(16 \times 10^{-3})(90 \times 10^6)}{0.859} = 1.676 \times 10^6 \text{ Pa}$$

$$\sigma_1 = 1.676 \text{ MPa}$$

Problem 7.110

7.110 A steel penstock has a 750-mm outer diameter, a 12-mm wall thickness, and connects a reservoir at *A* with a generating station at *B*. Knowing that the density of water is 1000 kg/m³, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.



$$r = \frac{1}{2}d - t = \frac{1}{2}(750) - 12 = 363 \text{ mm} = 363 \times 10^{-3} \text{ m}$$

$$t = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

$$p = \rho g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(300 \text{ m}) = 2.943 \times 10^6 \text{ Pa}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(2.943 \times 10^6)(363 \times 10^{-3})}{12 \times 10^{-3}} = 89.0 \times 10^6 \text{ Pa}$$

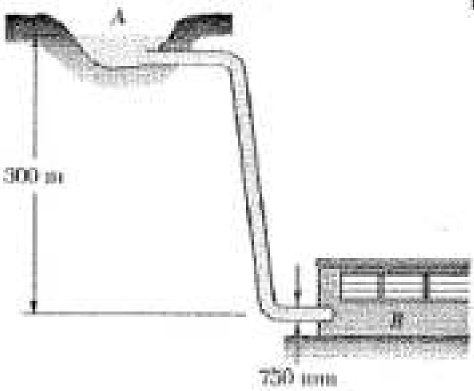
$$\sigma_{max} = \sigma_1 \qquad \sigma_{max} = 89.0 \text{ MPa}$$

$$\sigma_{min} = -p \approx 0$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) \qquad \tau_{max} = 44.5 \text{ MPa}$$

Problem 7.111

7.111 A steel penstock has a 750-mm outer diameter and connects a reservoir at *A* with a generating station at *B*. Knowing that the density of water is 1000 kg/m³ and that the allowable normal stress in the steel is 85 MPa, determine the smallest thickness that can be used for the penstock.



$$p = \rho g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(300 \text{ m}) = 2.943 \times 10^6 \text{ Pa}$$

$$\sigma_1 = 85 \text{ MPa} = 85 \times 10^6 \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(750 \times 10^{-3}) - t = 0.375 - t$$

$$\sigma_1 = \frac{pr}{t}$$

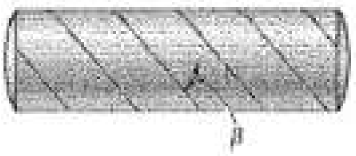
$$85 \times 10^6 = \frac{(2.943 \times 10^6)(0.375 - t)}{t}$$

$$(87.943 \times 10^6)\,t = 1.103625 \times 10^6 \qquad t = 12.549 \times 10^{-3} \text{ m}$$

$$t = 12.55 \text{ mm}$$

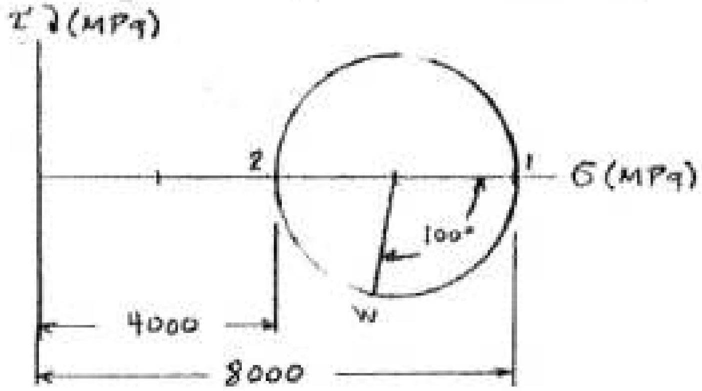
Problem 7.112

7.112 The steel pressure tank shown has a 750-mm inner diameter and a 9-mm wall thickness. Knowing that the butt welded seams form an angle $\beta = 50°$ with the longitudinal axis of the tank and that the gage pressure in the tank is 1.4 MPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.



$$r = \frac{1}{2}d = 375 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.4)(375)}{9} = 58.3 \text{ MPa} \qquad \sigma_2 = \frac{1}{2}\sigma_1 = 29.15 \text{ MPa}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 43.725 \text{ MPa}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 14.575 \text{ MPa}$$

(a) $\sigma_w = \sigma_{ave} + R\cos 100°$

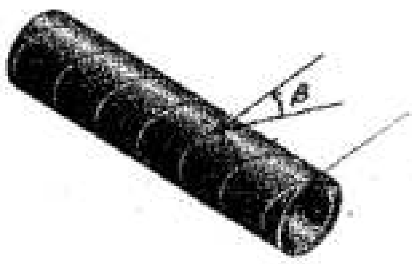
$= 41.2 \text{ MPa}$ ◄

(b) $\tau_w = R\sin 100°$

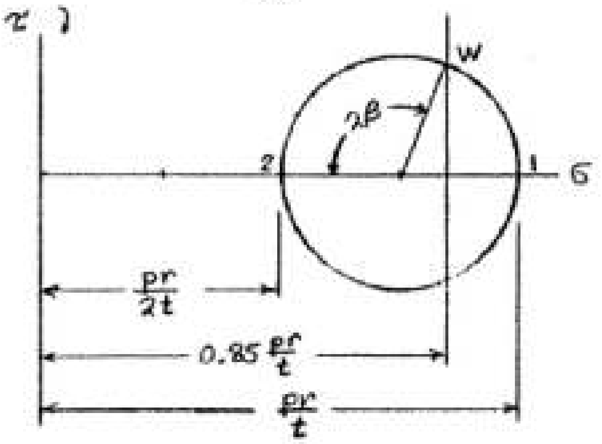
$= 14.4 \text{ MPa}$ ◄

Problem 7.113

7.113 The pressurized tank shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not be larger than 85 percent of the maximum stress in the tank.



$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} - R\cos 2\beta$$

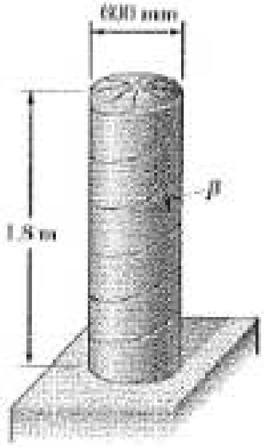
$$0.85\frac{pr}{t} = \left(\frac{3}{4} - \frac{1}{4}\cos 2\beta\right)\frac{pr}{t}$$

$$\cos 2\beta = -4\left(0.85 - \frac{3}{4}\right) = -0.4$$

$2\beta = 113.6°$ $\qquad \beta = 56.8°$ ◄

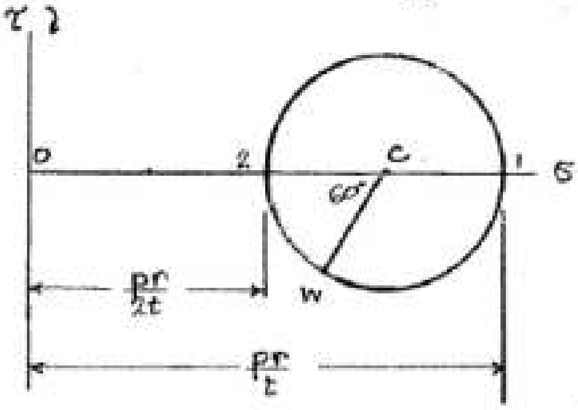
Problem 7.114

7.114 The cylindrical portion of the compressed air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.



$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 6 = 292 \text{ mm}$

$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{1}{2}\frac{pr}{t}$



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

$\sigma_w = \sigma_{ave} + R\cos 60°$

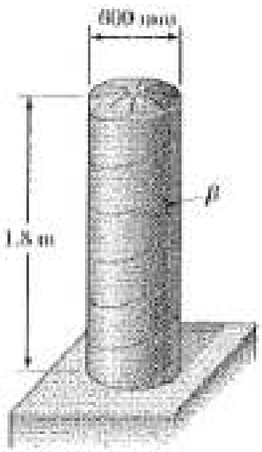
$= \frac{5}{8}\frac{pr}{t}$

$p = \frac{8}{5}\frac{\sigma_w t}{r}$

$p = \frac{8}{5}\frac{(75)(8)}{292} = 3.29 \text{ MPa}$ ◄

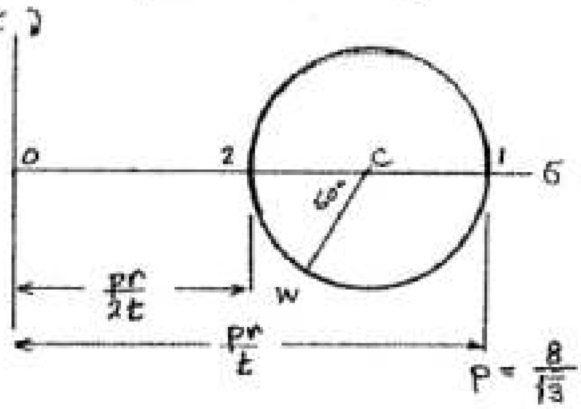
Problem 7.115

7.115 The cylindrical portion of the compressed air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Determine the gage pressure that will cause a shearing stress parallel to the weld of 30 MPa.



$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 8 = 292 \text{ mm}$

$\sigma_1 = \frac{pr}{t}, \quad \sigma_2 = \frac{1}{2}\frac{pr}{t}$



$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

$\tau_w = R\sin 60°$

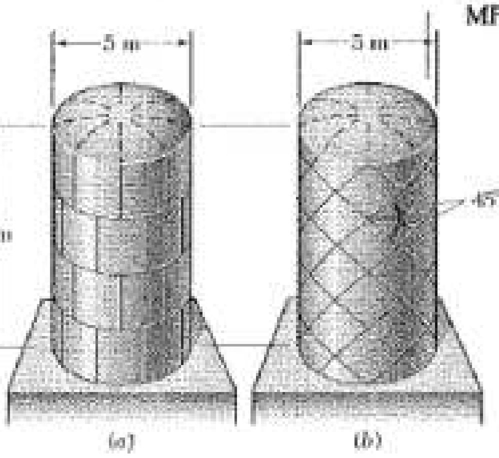
$= \frac{\sqrt{3}}{8}\frac{pr}{t}$

$p = \frac{8}{\sqrt{3}}\frac{\tau_w t}{R}$

$p = \frac{8}{\sqrt{3}}\frac{(30)(8)}{294} = 3.80 \text{ MPa}$ ◄

Problem 7.116

7.116 Square plates, each of 16-mm thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed air tank. Knowing that the allowable normal stress perpendicular to the weld is 65 MPa, determine the largest allowable gage pressure in each case.



$r = \frac{1}{2}d - t = \frac{1}{2}(5) - 16\times10^{-3} = 2.484 \text{ m}$

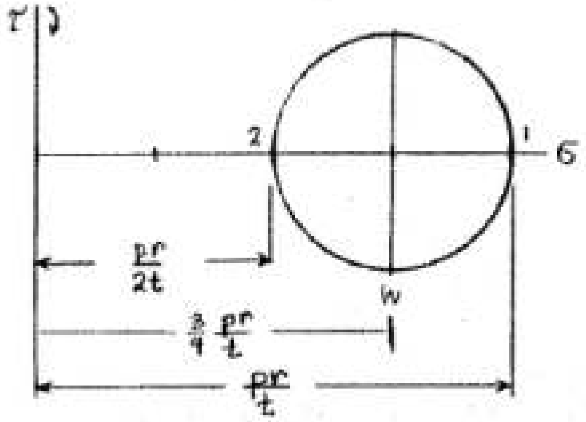
$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$

(a) $\sigma_1 = 65 \text{ MPa} = 65\times10^6 \text{ Pa}$

$p = \frac{\sigma_1 t}{r} = \frac{(65\times10^6)(16\times10^{-3})}{2.484} = 419\times10^3 \text{ Pa}$

$p = 419 \text{ kPa}$ ◄

(b)



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

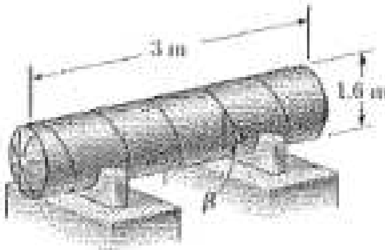
$\beta = \pm 45°$

$\sigma_w = \sigma_{ave} + R\cos\beta$

$= \frac{3}{4}\frac{pr}{t}$

$p = \frac{4\sigma_w t}{3r} = \frac{(4)(65\times10^6)(16\times10^{-3})}{(3)(2.484)} = 558\times10^3 \text{ Pa} \qquad p = 558 \text{ kPa}$ ◄

Problem 7.117

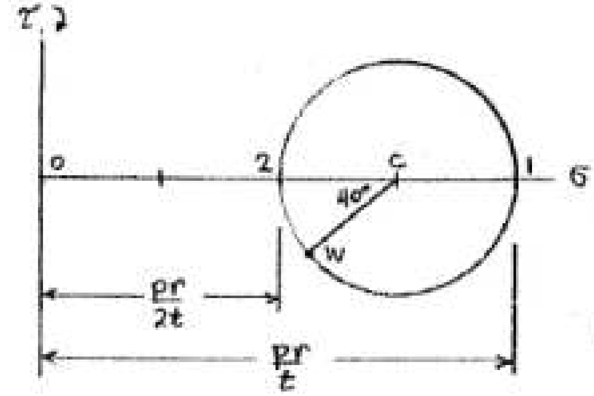


7.117 The pressure tank shown has an 8-mm wall thickness and butt welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

$d = 1.6 \text{ m} \qquad t = 8\times10^{-3} \text{ m} \qquad r = \frac{1}{2}d - t = 0.792 \text{ m}$

$$\sigma_1 = \frac{pr}{t} = \frac{(600\times10^3)(0.792)}{8\times10^{-3}} = 59.4\times10^6 \text{ Pa}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(600\times10^3)(0.792)}{(2)(8\times10^{-3})} = 29.7\times10^6 \text{ Pa}$$



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_1+\sigma_2) = 44.55\times10^6 \text{ Pa}$$

$$R = \tfrac{1}{2}(\sigma_1-\sigma_2) = 14.85\times10^6 \text{ Pa}$$

(a) $\sigma_w = \sigma_{ave} - R\cos 40° = 33.17\times10^6 \text{ Pa}$

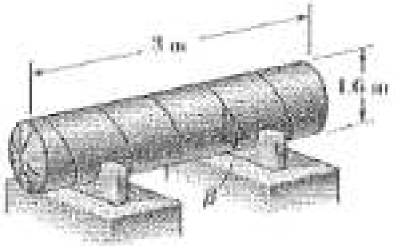
$\sigma_w = 33.2 \text{ MPa}$ ◄

(b) $\tau_w = R\sin 40° = 9.55\times10^6 \text{ Pa}$

$\tau_w = 9.55 \text{ MPa}$ ◄

Problem 7.118

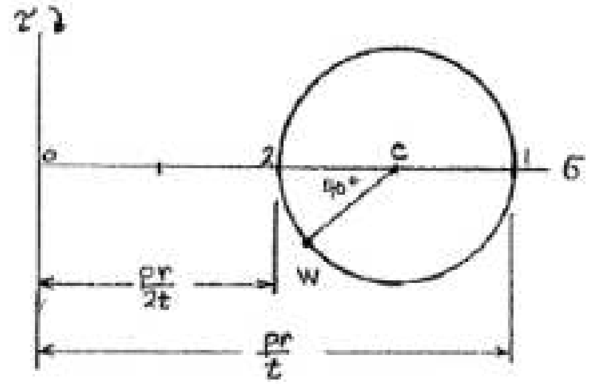
7.118 For the tank of Prob. 7.117, determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 120 MPa and the allowable shearing stress parallel to the weld is 80 MPa.



7.117 The pressure tank shown has a 8-mm wall thickness and butt welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

$d = 1.6 \text{ m}, \quad t = 8\times10^{-3} \text{ m}, \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_1+\sigma_2) = \tfrac{3}{4}\frac{pr}{t}$$

$$R = \tfrac{1}{2}(\sigma_1-\sigma_2) = \tfrac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} - R\cos 40° = \left(\tfrac{3}{4} - \tfrac{1}{4}\cos 40°\right)\frac{pr}{t} = 0.5585\frac{pr}{t}$$

$$p = \frac{\sigma_w t}{0.5585\, r} = \frac{(120\times10^6)(8\times10^{-3})}{(0.5585)(0.792)} = 2.17\times10^6 \text{ Pa} = 2.17 \text{ MPa}$$

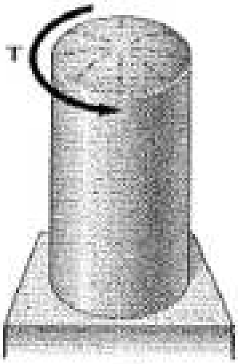
$$\tau_w = R\sin 40° = \left(\tfrac{1}{4}\sin 40°\right)\frac{pr}{t} = 0.1607\frac{pr}{t}$$

$$p = \frac{\tau_w t}{0.1607\, r} = \frac{(80\times10^6)(8\times10^{-3})}{(0.1607)(0.792)} = 5.03\times10^6 \text{ Pa} = 5.06 \text{ MPa}$$

The largest allowabe pressure is the smaller value. $\quad p = 2.17 \text{ MPa}$ ◄

Problem 7.120

7.120 A torque of magnitude $T = 12$ kN · m is applied to the end of a tank containing compressed air under a pressure of 8 MPa. Knowing that the tank has a 180-mm inner diameter and a 12-mm wall thickness, determine the maximum normal stress and the maximum shearing stress in the tank.



$d = 180$ mm $\quad r = \frac{1}{2}d = 90$ mm $\quad t = 12$ mm

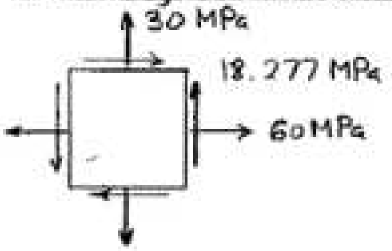
Torsion: $c_1 = 90$ mm $\quad c_2 = 90 + 12 = 102$ mm

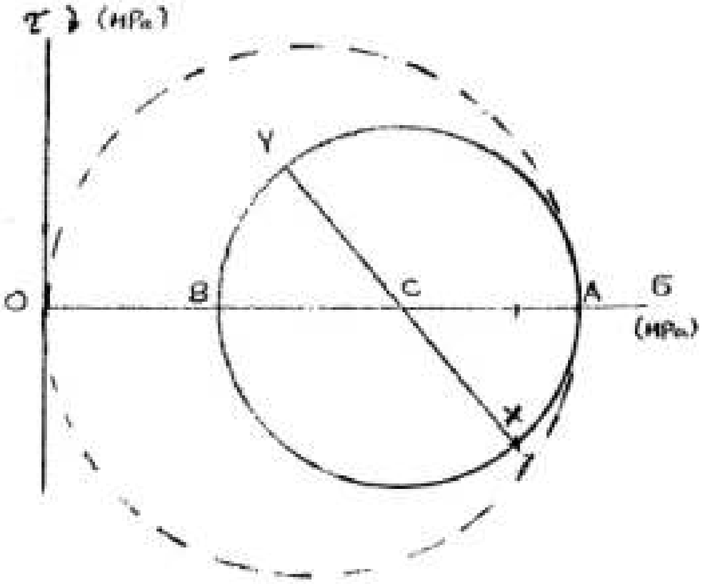
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12 \times 10^3)(102 \times 10^{-3})}{66.968 \times 10^{-6}} = 18.277 \text{ MPa}$$

Pressure: $\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60$ MPa $\qquad \sigma_2 = \frac{pr}{2t} = 30$ MPa

Summary of stresses: $\qquad \sigma_x = 60$ MPa, $\sigma_y = 30$ MPa, $\tau_{xy} = 18.277$ MPa





$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64$ MPa

$\sigma_a = \sigma_{ave} + R = 68.64$ MPa

$\sigma_b = \sigma_{ave} - R = 21.36$ MPa

$\sigma_c \approx 0$

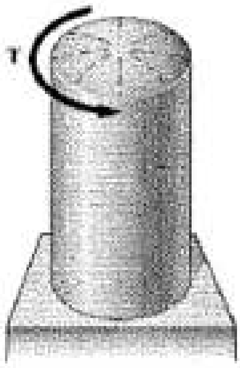
$\sigma_{max} = 68.6$ MPa ◄

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) \qquad\qquad \tau_{max} = 34.3$ MPa ◄

Problem 7.121

7.121 The tank shown has a 180-mm inner diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude T of the applied torque for which the maximum normal stress is 75 MPa.



$r = \frac{1}{2}d = (\frac{1}{2})(180) = 90$ mm $\qquad t = 12$ mm

$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60$ MPa $\qquad \sigma_2 = \frac{pr}{2t} = 30$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 45$ MPa

$\sigma_{max} = 75$ MPa $\qquad R = \sigma_{max} - \sigma_{ave} = 30$ MPa

$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$

$\tau_{xy} = \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98$ MPa

$= 25.98 \times 10^6$ Pa

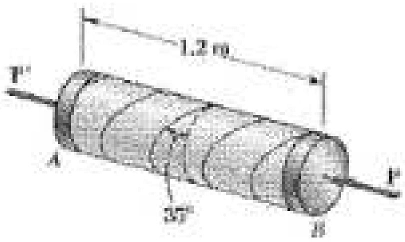
Torsion: $\quad c_1 = 90$ mm $\qquad c_2 = 90 + 12 = 102$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968 \times 10^6\ \text{mm}^4 = 66.968 \times 10^{-6}\ \text{m}^4$

$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3$ N·m

$T = 17.06$ kN·m ◄

Problem 7.122



7.122 A pressure vessel of 250-mm inner diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe *AB* and is equipped with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 40-kN centric axial forces **P** and **P'** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

$r = \frac{1}{2}d = \frac{1}{2}(250) = 125 \text{ mm} \qquad t = 6 \text{ mm}$

$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$

$\sigma_2 = \frac{pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$

$r_o = r + t = 125 + 6 = 131 \text{ mm}$

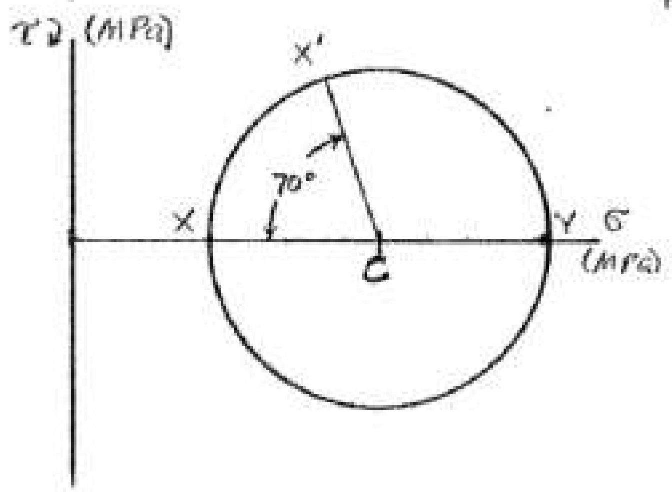
$A = \pi(r_o^2 - r^2) = \pi(131^2 - 125^2) = 4825 \text{ mm}^2$

$\sigma = -\frac{P}{A} = -\frac{40000}{4825} = -8.29 \text{ MPa}$

Total stresses. Longitudinal $\sigma_x = 20.83 - 8.29 = 15.54 \text{ MPa}$

Circumferential $\sigma_y = 41.67 \text{ MPa}$

Shear $\tau_{xy} = 0$



Plotted points for Mohr's circle.

X: (15.54, 0)

Y: (41.67, 0)

C: (28.605, 0)

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.605 \text{ MPa}$

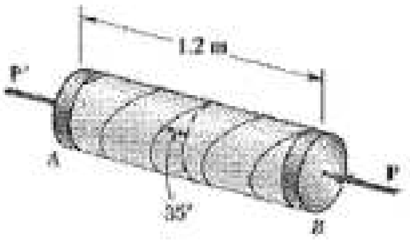
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left[\frac{(15.54 - 41.67)}{2}\right]^2 + 0} = 13.065 \text{ MPa}$

(a) $\sigma_{x'} = \sigma_{ave} + R\cos 70° = 28.605 - 13.065\cos 70° = 24.1 \text{ MPa}$ ◄

(b) $|\tau_{xy}| = R\sin 70° = 13.065 \sin 70° = 12.3 \text{ MPa}$ ◄

Problem 7.123



7.123 Solve Prob. 7.122, assuming that the magnitude P of the two forces is increased to 120 kN.

7.122 A pressure vessel of 250-mm inner diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 40-kN centric axial forces **P** and **P'** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

$r = \frac{1}{2}d = \frac{1}{2}(250) = 125 \text{ mm} \qquad t = 6 \text{ mm}$

$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$

$\sigma_2 = \frac{pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$

$r_o = r + t = 125 + 6 = 131 \text{ mm}$

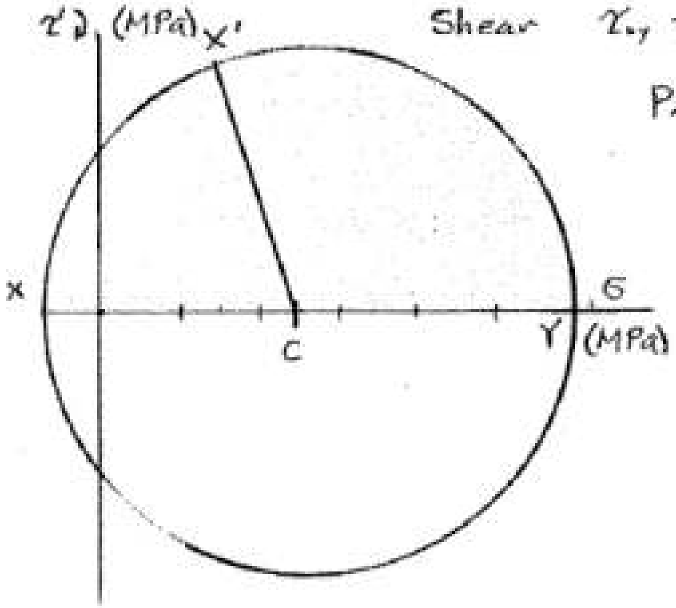
$A = \pi(r_o^2 - r^2) = \pi(131^2 - 125^2) = 4825 \text{ mm}^2$

$\sigma = -\frac{P}{A} = -\frac{120 \times 10^3}{4825} = -24.87$

Total stresses. Longitudinal $\sigma_x = 20.83 - 24.87 = -4.04 \text{ MPa}$

Circumferential $\sigma_y = 41.67 \text{ MPa}$

Shear $\tau_{xy} = 0$



Plotted points for Mohr's circle.

$X: (-4.04, 0)$

$Y: (41.67, 0)$

$C: (18.815, 0)$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 18.815 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\frac{-4.04 - 41.67}{2}\right)^2 + 0} = 22.855 \text{ MPa}$

(a) $\sigma_{x'} = \sigma_{ave} - R\cos 70° = 18.815 - 22.855 \cos 70° = 11 \text{ MPa}$ ◄

(b) $|\tau_{x'y'}| = R \sin 70° = 22.855 \sin 70° = 21.5 \text{ MPa}$ ◄

Problem 7.124 continued

Principal stresses. $\sigma_a = 77.4$ MPa , $\sigma_b = 32.3$ MPa ◄

The 3rd principal stress is the radial stress. $\sigma_z \approx 0$ ◄

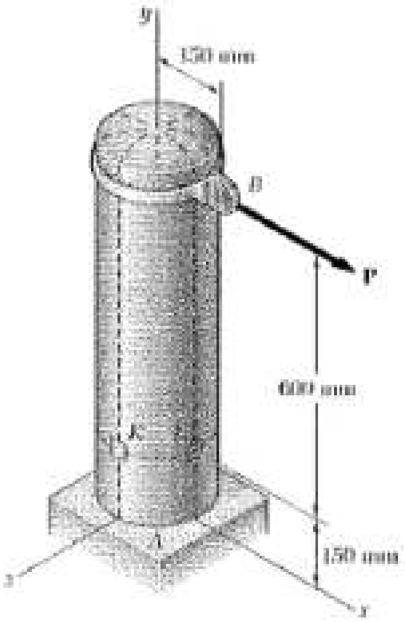
$\sigma_{max} = 77.4$ MPa , $\sigma_{min} = 0$

Maximum shearing stress. $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\tau_{max} = 38.7$ MPa ◄

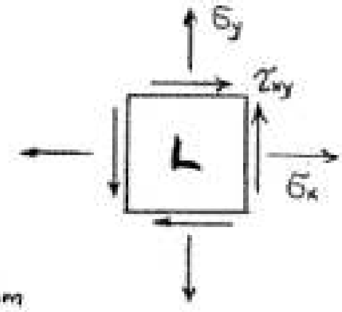
Problem 7.125

7.125 In Prob. 7.124, determine the maximum normal stress and the maximum shearing stress at point *L*.

7.124 The compressed-air tank *AB* has a 250-mm outside diameter and an 8-mm wall thickness. It is fitted with a collar by which a 40-kN force **P** is applied at *B* in the horizontal direction. Knowing that the gage pressure inside the tank is 5 MPa, determine the maximum normal stress and the maximum shearing stress at point *K*.



Consider element at point L.



Stresses due to internal pressure.

$p = 5$ MPa $= 5 \times 10^6$ Pa

$r = \frac{1}{2}d - t = \frac{250}{2} - 8 = 117$ mm

$\sigma_x = \frac{pr}{t} = \frac{(5 \times 10^6)(117 \times 10^{-3})}{8 \times 10^{-3}} = 73.125$ MPa

$\sigma_y = \frac{pr}{2t} = \frac{(5 \times 10^6)(117 \times 10^{-3})}{(2)(8 \times 10^{-3})} = 36.563$ MPa

Stress due to bending moment. $M = (40\text{ kN})(600\text{ mm}) = 24000$ N·m

$c_2 = \frac{1}{2}d = 125$ mm $\quad c_1 = c_2 - t = 125 - 8 = 117$ mm

$I = \frac{\pi}{4}(c_2^4 - c_1^4) = \frac{\pi}{4}(125^4 - 117^4) = 44.573 \times 10^6\ \text{mm}^4 = 44.573 \times 10^{-6}\ \text{m}^4$

$\sigma_y = -\frac{Mc}{I} = -\frac{(24000)(125 \times 10^{-3})}{44.573 \times 10^{-6}} = -67.305$ MPa

Stress due to transverse shear. Point L lies in a plane of symmetry.

$\tau_{xy} = 0$

Total stresses. $\sigma_x = 73.125$ MPa , $\sigma_y = -30.742$ MPa, $\tau_{xy} = 0$

Principal stresses. Since $\tau_{xy} = 0$, σ_x and σ_y are principal stresses.

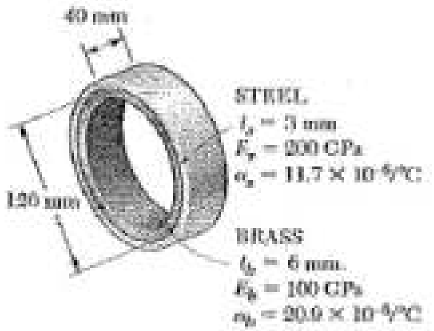
The 3rd principal stress is in the radial direction. $\sigma_z \approx 0$.

$\sigma_{max} = 73.125$ MPa, $\sigma_{min} = 0$ / $\sigma_a = 73.1$ MPa, $\sigma_b = -30.7$ MPa, $\sigma_z = 0$ ◄

Maximum shearing stress $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\tau_{max} = 51.9$ MPa ◄

Problem 7.126

7.126 A brass ring of 126-mm outer diameter and 6-mm thickness fits exactly inside a steel ring of 126-mm inner diameter and 3-mm thickness when the temperature of both rings is 10°C. Knowing that the temperature of both rings is then raised to 52°C, determine *(a)* the tensile stress in the steel ring, *(b)* the corresponding pressure exerted by the brass ring on the steel ring.



Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p, $\quad \sigma_s = \dfrac{pr}{t_s} \qquad (1)$

Corresponding strain $\quad \varepsilon_{sp} = \dfrac{\sigma_s}{E_s} = \dfrac{pr}{E_s t_s}$

Strain due to temperature change $\quad \varepsilon_{sT} = \alpha_s \Delta T$

Total strain $\quad \varepsilon_s = \dfrac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p, $\quad \sigma_b = -\dfrac{pr}{t_b}$

Corresponding strains $\quad \varepsilon_{bp} = -\dfrac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b $\quad \dfrac{pr}{E_s t_s} + \alpha_s \Delta T = -\dfrac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s)\Delta T \qquad (2)$$

Data: $\quad \Delta T = 52°C - 10°C = 42°C$

$r = \frac{1}{2} d = \frac{1}{2}(126) = 63$ mm.

From Eq. (2) $\left[\dfrac{63 \times 10^{-3}}{(200 \times 10^9)(0.003)} + \dfrac{63 \times 10^{-3}}{(100 \times 10^9)(0.006)} \right] p = (20.9 - 11.7)(10^{-6})(38)$

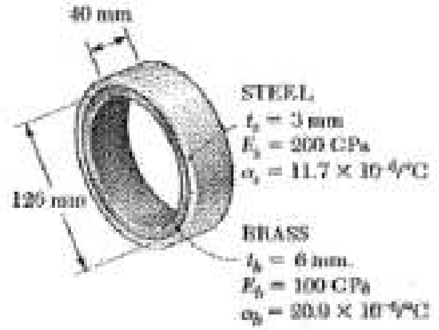
$2.1 \times 10^{-10}\, p = 3.496 \times 10^{-4} \qquad p = 1.67$ MPa

From Eq. (1) $\quad \sigma_s = \dfrac{pr}{t_s} = \dfrac{(1.67)(63)}{3} = 35.1$ MPa

(a) $\sigma_s = 35.1$ MPa ◄

(b) $p = 1.67$ MPa ◄

Problem 7.127



7.127 Solve Prob. 7.126, assuming that the brass ring is 3 mm thick and the steel ring is 6 mm thick.

7.126 A brass ring of 126-mm outer diameter and 6-mm thickness fits exactly inside a steel ring of 126-mm inner diameter and 3-mm thickness when the temperature of both rings is 10°C. Knowing that the temperature of both rings is then raised to 52°C, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p, $\quad \sigma_s = \dfrac{pr}{t_s}$ (1)

Corresponding strain $\quad \varepsilon_{sp} = \dfrac{\sigma_s}{E_s} = \dfrac{pr}{E_s t_s}$

Strain due to temperature change $\quad \varepsilon_{sT} = \alpha_s \Delta T$

Total strain $\quad \varepsilon_s = \dfrac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p, $\quad \sigma_b = -\dfrac{pr}{t_b}$

Corresponding strains $\quad \varepsilon_{bp} = -\dfrac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b $\quad \dfrac{pr}{E_s t_s} + \alpha_s \Delta T = -\dfrac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s)\Delta T \qquad (2)$$

Data: $\Delta T = 52°C - 10°C = 42°C$

$r = \frac{1}{2} d = \frac{1}{2}(126) = 63$ mm.

From Eq. (2) $\left[\dfrac{63 \times 10^{-3}}{(200 \times 10^9)(0.006)} + \dfrac{63 \times 10^{-3}}{(100 \times 10^9)(0.003)} \right] p = (20.9 - 11.7)(10^{-6})38.$

$2.625 \times 10^{-10}\, p = 3.496 \times 10^{-4} \qquad p = 1.33$ MPa

From Eq. (1) $\quad \sigma_s = \dfrac{pr}{t_s} = \dfrac{(1.33)(63)}{6} = 13.3$ MPa

(a) $\sigma_s = 14$ MPa ◄

(b) $p = 1.33$ MPa ◄

Problem 7.130

7.128 through 7.131 For the given state of plane strain, use the method of Sec. 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$\underline{\varepsilon_x = 350\mu,\ \varepsilon_y = 0,\ \gamma_{xy} = +120\mu,\ \theta = 15^\circ}$ ↻ $= -15^\circ$

$\frac{\varepsilon_x + \varepsilon_y}{2} = 175\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = 175\mu \qquad \frac{\gamma_{xy}}{2} = 60\mu$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 175 + 175\cos(-30^\circ) + 60\sin(-30^\circ) \qquad \varepsilon_{x'} = 297\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin(-30^\circ)$$

$$= 175 - 175\cos(-30^\circ) - 60\sin(-30^\circ) \qquad \varepsilon_{y'} = 53.4\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(350 - 0)\sin(-30^\circ) + 120\cos(-30^\circ) \qquad \gamma_{x'y'} = 279\mu$$

Problem 7.131

7.128 through 7.131 For the given state of plane strain, use the method of Sec. 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$\underline{\varepsilon_x = 0,\ \varepsilon_y = +320\mu,\ \gamma_{xy} = -100\mu,\ \theta = 30^\circ}$ ↺ $= +30^\circ$

$\frac{\varepsilon_x + \varepsilon_y}{2} = 160\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -160\mu$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 160 - 160\cos 60^\circ - \frac{100}{2}\sin 60^\circ \qquad \varepsilon_{x'} = +36.7\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 160 + 160\cos 60^\circ + \frac{100}{2}\sin 60^\circ \qquad \varepsilon_{y'} = +283\mu$$

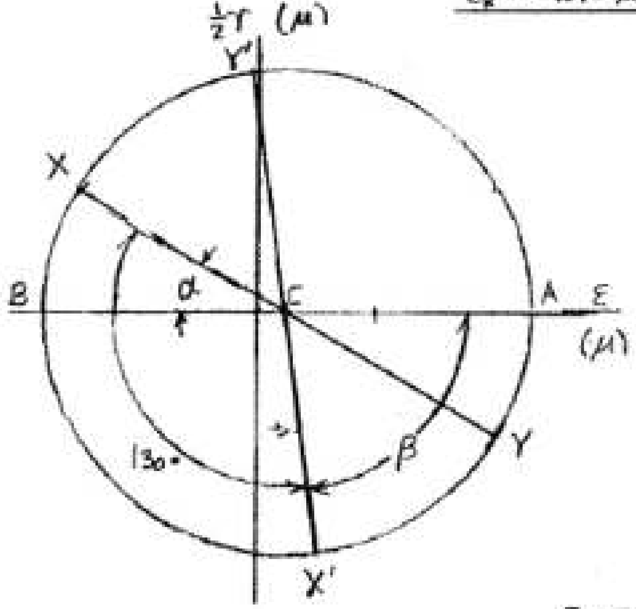
$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(0 - 320)\sin 60^\circ - 100\cos 60^\circ \qquad \gamma_{x'y'} = +227\mu$$

Problem 7.132

7.132 through 7.135 For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$\varepsilon_x = -240\mu,\ \varepsilon_y = 320\mu,\ \gamma_{xy} = -330\mu,\ \theta = 65^\circ$ ↺



$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 40\mu$

Plotted points for Mohr's circle:

X: $(-240\mu,\ 165\mu)$

Y: $(320\mu,\ -165\mu)$

C: $(40\mu,\ 0)$

$\tan\alpha = \frac{165}{280} = 0.58929 \qquad \alpha = 30.5^\circ$

$\beta = 180^\circ - 130^\circ + \alpha = 80.5^\circ$

$R = \sqrt{(280)^2 + (165)^2} = 325\mu$

$\varepsilon_{x'} = \varepsilon_{ave} + R\cos\beta \qquad \varepsilon_{x'} = 93.6\mu$ ◀

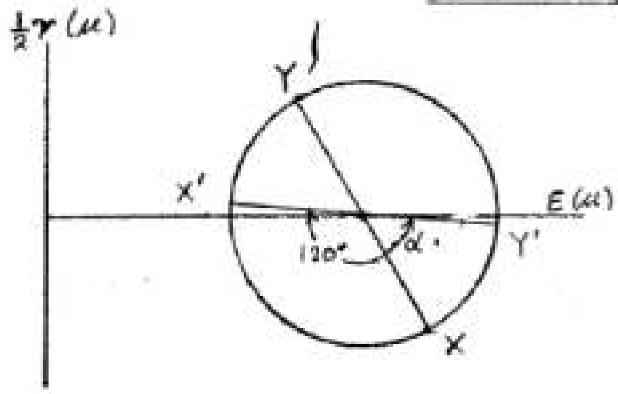
$\varepsilon_{y'} = \varepsilon_{ave} - R\cos\beta \qquad \varepsilon_{y'} = -13.6\mu$ ◀

$\frac{1}{2}\gamma_{x'y'} = R\sin\beta = 320.5\mu \qquad \gamma_{x'y'} = 641\mu$ ◀

Problem 7.133

7.132 through 7.135 For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$\varepsilon_x = +240\mu,\ \varepsilon_y = +160\mu,\ \gamma_{xy} = +150\mu,\ \theta = 60^\circ$ ↻



Plotted points for Mohr's circle:

X: $(+240\mu,\ -75\mu)$

Y: $(+160\mu,\ 75\mu)$

C: $(+200\mu,\ 0)$

$\tan\alpha = \frac{75}{40} = 1.875 \qquad \alpha = 61.93^\circ$

$R = \sqrt{(40\mu)^2 + (75\mu)^2} = 85\mu$

$\beta = 2\theta - \alpha = -120^\circ - 61.93^\circ = -181.93^\circ$

$\varepsilon_{x'} = \varepsilon_{ave} + R\cos\beta = 200\mu + (85\mu)\cos(-181.93^\circ)$

$\varepsilon_{x'} = 115.0\mu$ ◀

$\varepsilon_{y'} = \varepsilon_{ave} - R\cos\beta = 200\mu - (85\mu)\cos(-181.93^\circ)$

$\varepsilon_{y'} = 285\mu$ ◀

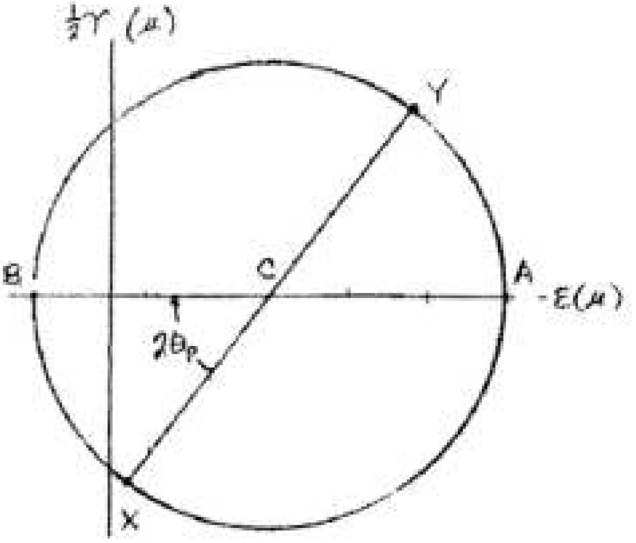
$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = -85\sin(-181.93) = -2.86\mu$

$\gamma_{x'y'} = -5.72\mu$ ◀

Problem 7.136

7.136 through 7.139 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$$\varepsilon_x = +30\,\mu, \quad \varepsilon_y = +570\,\mu, \quad \gamma_{xy} = +720\,\mu$$



Plotted points for Mohr's circle:

X : $(30\,\mu,\ -360\,\mu)$

Y : $(570\,\mu,\ +360\,\mu)$

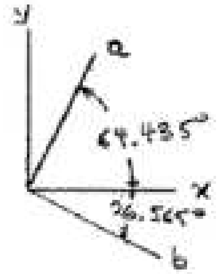
C : $(300\,\mu,\ 0)$

$$\tan 2\theta_p = \frac{-360}{270} = -1.3333$$

$$2\theta_p = -53.13°$$

(a) $\theta_b = -26.565°$ ◄

$\theta_a = 64.435°$ ◄



$$R = \sqrt{(270\,\mu)^2 + (360\,\mu)^2} = 450\,\mu$$

$\varepsilon_a = \varepsilon_{ave} + R = 300\,\mu + 450\,\mu$ $\qquad \varepsilon_a = 750\,\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = 300\,\mu - 450\,\mu$ $\qquad \varepsilon_b = -150\,\mu$ ◄

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 900\,\mu$ ◄

$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{1/3}{2/3}(750\,\mu - 150\,\mu)$ $\qquad \varepsilon_c = -300\,\mu$ ◄

$\varepsilon_{max} = \varepsilon_a = 750\,\mu, \qquad \varepsilon_{min} = \varepsilon_c = -300\,\mu$

(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 750\,\mu - (-300\,\mu)$ $\qquad \gamma_{max} = 1050\,\mu$ ◄

Problem 7.137

7.136 through 7.139 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

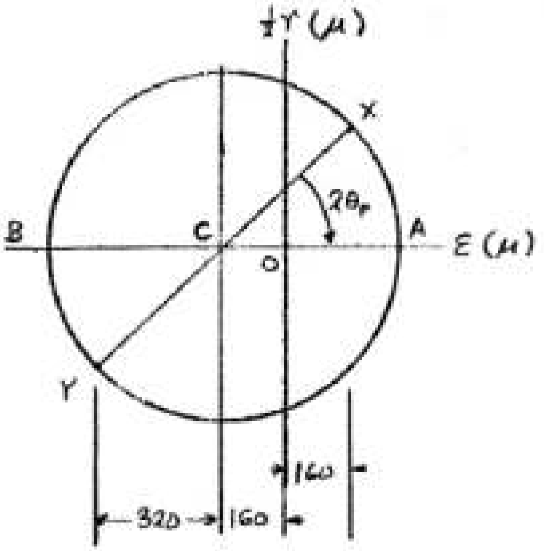
$\varepsilon_x = +160\,\mu, \quad \varepsilon_y = -480\,\mu, \quad \gamma_{xy} = -600\,\mu$

(a) For Mohr's circle of strain, plot points

X: $(160\,\mu,\ 300\,\mu)$

Y: $(-480\,\mu,\ -300\,\mu)$

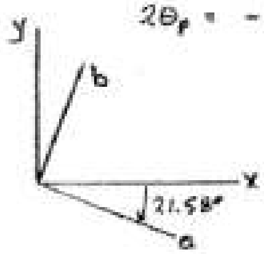
C: $(-160\,\mu,\ 0)$



(a) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{-300}{320} = -0.9375$

$2\theta_p = -43.15° \qquad \theta_p = -21.58°$

and $-21.58 + 90 = 68.42°$



$\theta_a = -21.58°$ ◄

$\theta_b = 68.42°$ ◄

$R = \sqrt{(320\,\mu)^2 + (300\,\mu)^2} = 438.6\,\mu$

$\varepsilon_a = \varepsilon_{ave} + R = -160\,\mu + 438.6\,\mu \qquad \varepsilon_a = +278.6\,\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = -160\,\mu - 438.6\,\mu \qquad \varepsilon_b = -598.6\,\mu$ ◄

(b) $\frac{1}{2}\gamma_{(max,\ in\text{-}plane)} = R \qquad \gamma_{(max,\ in\text{-}plane)} = 2R = 877\,\mu$ ◄

(c) $\varepsilon_c = -\dfrac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\dfrac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\dfrac{1/3}{2/3}(160\,\mu - 480\,\mu)$

$\varepsilon_c = 160.0\,\mu$ ◄

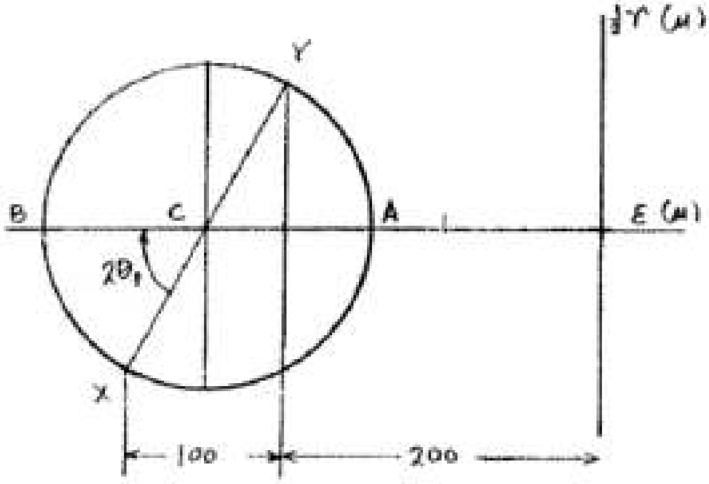
$\varepsilon_{max} = 278.6\,\mu \qquad \varepsilon_{min} = -598.6\,\mu$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 278.6\,\mu + 598.6\,\mu \qquad \gamma_{max} = 877\,\mu$ ◄

Problem 7.138

7.136 through 7.139 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$\underline{\epsilon_x = -600\,\mu, \quad \epsilon_y = -400\,\mu, \quad \gamma_{xy} = +350\,\mu}$



Plotted points for Mohr's circle:

X: $(-600\,\mu, -175\,\mu)$

Y: $(-400\,\mu, +175\,\mu)$

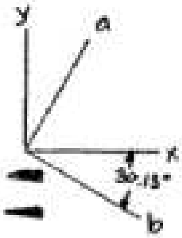
C: $(-500\,\mu, 0)$

$\tan 2\theta_p = -\frac{175}{100}$

$2\theta_p = -60.26$

$\theta_b = -30.13°$

$\theta_a = 59.87°$



$R = \sqrt{(100\,\mu)^2 + (175\,\mu)^2}$
$= 201.6\,\mu$

(a) $\epsilon_a = \epsilon_{ave} + R = -500\mu + 201.6\mu$ $\qquad \epsilon_a = -298\,\mu$

$\epsilon_b = \epsilon_{ave} - R = -500\mu - 201.6\mu$ $\qquad \epsilon_b = -702\,\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 403\,\mu$

$\epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu}(\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3}(-600\mu - 400\mu)$

$\epsilon_c = 500\,\mu$

$\epsilon_{max} = 500\,\mu \qquad \epsilon_{min} = -702\,\mu$

(c) $\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 500\,\mu + 702\,\mu$ $\qquad \gamma_{max} = 1202\,\mu$

Problem 7.139

7.136 through 7.139 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

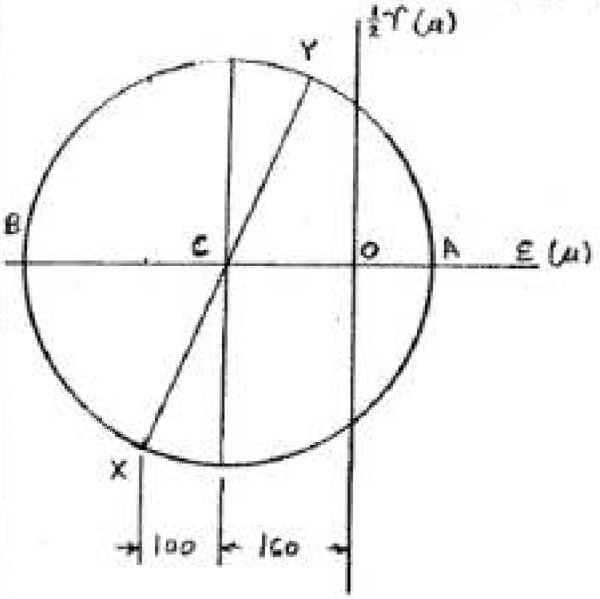
$\varepsilon_x = -260\mu, \quad \varepsilon_y = -60\mu, \quad \gamma_{xy} = +480\mu$

For Mohr's circle of strain plot points

X: $(-260\mu, -240\mu)$

Y: $(-60\mu, 240\mu)$

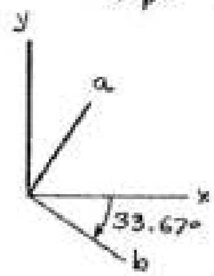
C: $(-160\mu, 0)$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{480}{-260+60} = -2.4$$

$2\theta_p = -67.38°$ $\qquad \theta_b = -33.67°$ ◄

$\theta_a = 56.31°$ ◄



$$R = \sqrt{(100\mu)^2 + (240\mu)^2}$$

$R = 260\mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = -160\mu + 260\mu$ $\qquad \varepsilon_a = 100\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = -160\mu - 260\mu$ $\qquad \varepsilon_b = -420\mu$ ◄

(b) $\frac{1}{2}\gamma_{max\,(in\text{-}plane)} = R \qquad \gamma_{max\,(in\text{-}plane)} = 2R \qquad \gamma_{max(in\text{-}plane)} = 520\mu$ ◄

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-260 - 60)$$
$$= 160\mu$$

$\varepsilon_{max} = 160\mu \qquad \varepsilon_{min} = -420\mu$

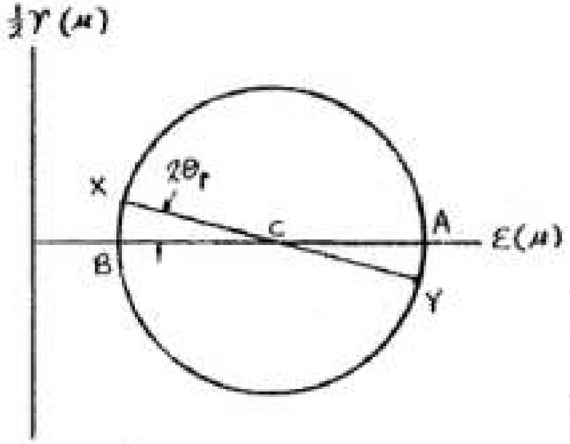
(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 160\mu + 420\mu$ $\qquad \gamma_{max} = 580\mu$ ◄

Problem 7.142

7.140 through 7.143 For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$\varepsilon_x = +60\,\mu, \quad \varepsilon_y = +240\,\mu, \quad \gamma_{xy} = -50\,\mu$

Plotted points: X: $(60\,\mu,\ 25\,\mu)$ Y: $(240\,\mu,\ -25\,\mu)$ C: $(150\,\mu,\ 0)$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-50}{60-240} = 0.277778$$

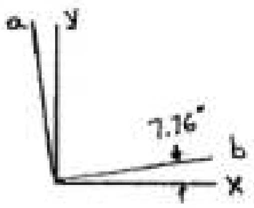
$2\theta_p = 15.52°$ $\theta_b = 7.76°$ ◄ $\theta_a = 97.76°$ ◄

$R = \sqrt{(90\,\mu)^2 + (25\,\mu)^2} = 93.4\ \mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 150\,\mu + 93.4\,\mu = 243.4\,\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = 150\,\mu - 93.4\,\mu = 56.6\,\mu$ ◄

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 186.8\,\mu$ ◄



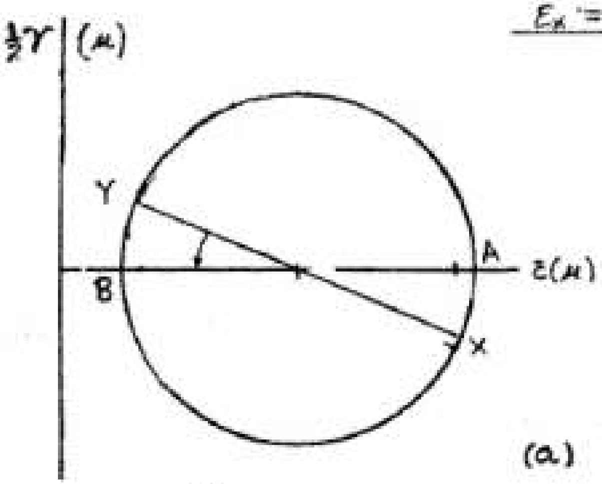
(c) $\varepsilon_c = 0$ $\varepsilon_{max} = 243.4\,\mu$ $\varepsilon_{min} = 0$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 243.4$ ◄

Problem 7.143

7.140 through 7.143 For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$\varepsilon_x = +300\,\mu, \quad \varepsilon_y = +60\,\mu, \quad \gamma_{xy} = +100\,\mu$



X: $(300\,\mu,\ -50\,\mu)$ Y: $(60\,\mu,\ 50\,\mu)$

C: $(180\,\mu,\ 0)$

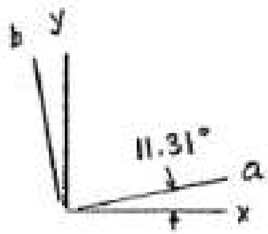
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{100}{300-60} = 22.62°$$

$\theta_a = 11.31°$ $\theta_b = 101.31°$ ◄

$R = \sqrt{(120\,\mu)^2 + (50\,\mu)^2} = 130\ \mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 180\,\mu + 130\,\mu = 310\,\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = 180\,\mu - 130\,\mu = 50\,\mu$ ◄



(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 260\,\mu$ ◄

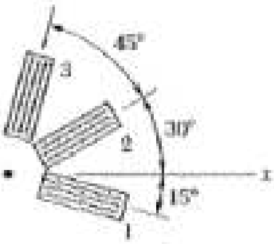
(c) $\varepsilon_c = 0$ $\varepsilon_{max} = 310\,\mu$ $\varepsilon_{min} = 0$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 310\,\mu$ ◄

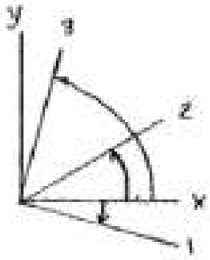
Problem 7.144

7.144 Determine the strain ϵ_x knowing that the following strains have been determined by use of the rosette shown:

$\epsilon_1 = +480\mu$ $\epsilon_2 = -120\mu$ $\epsilon_3 = +80\mu$



$\theta_1 = -15°$
$\theta_2 = 30°$
$\theta_3 = 75°$



$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 = \varepsilon_1$$

$$0.9330\,\varepsilon_x + 0.06699\,\varepsilon_y - 0.25\,\gamma_{xy} = 480\,\mu \qquad (1)$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 = \varepsilon_2$$

$$0.75\,\varepsilon_x + 0.25\,\varepsilon_y + 0.4330\,\gamma_{xy} = -120\,\mu \qquad (2)$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3 = \varepsilon_3$$

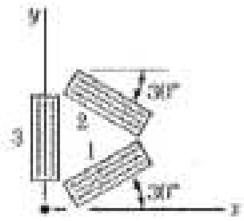
$$0.06699\,\varepsilon_x + 0.9330\,\varepsilon_y + 0.25\,\gamma_{xy} = 80\,\mu \qquad (3)$$

Solving (1), (2), and (3) simultaneously,

$\varepsilon_x = 253\,\mu$ $\varepsilon_y = \overline{3}07\,\mu$ $\gamma_{xy} = -892\,\mu$

$\varepsilon_x = 253\,\mu$ ◄

Problem 7.145

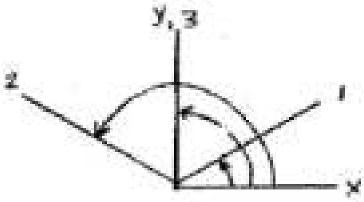


7.145 The strains determined by the use of the rosette shown during the test of a machine element are

$\epsilon_1 = +600\mu \qquad \epsilon_2 = +450\mu \qquad \epsilon_3 = -75\mu$

Determine (*a*) the in-plane principal strains, (*b*) the in-plane maximum shearing strain.

$\theta_1 = 30°$
$\theta_2 = 150°$
$\theta_3 = 90°$



$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 = \varepsilon_1$$
$$0.75\,\varepsilon_x + 0.25\,\varepsilon_y + 0.43301\,\gamma_{xy} = 600\mu \quad (1)$$
$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 = \varepsilon_2$$
$$0.75\,\varepsilon_x + 0.25\,\varepsilon_y - 0.43301\,\gamma_{xy} = 450\mu \quad (2)$$
$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3 = \varepsilon_3$$
$$0 + \varepsilon_y \qquad 0 = -75\mu \quad (3)$$

Solving (1), (2), and (3) simultaneously,

$$\varepsilon_x = 725\,\mu, \quad \varepsilon_y = -75\,\mu, \quad \gamma_{xy} = 173.21\,\mu$$

$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = 325\,\mu$$

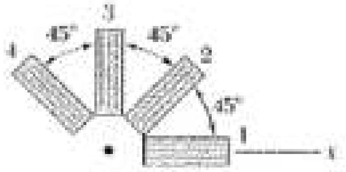
$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725 + 75}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 407.3\,\mu$$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 734\,\mu \qquad \varepsilon_a = 734\,\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = -84.3\,\mu \qquad \varepsilon_b = -84.3\,\mu$ ◄

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 819\,\mu \qquad \gamma_{max} = 819\,\mu$ ◄

Problem 7.146



7.146 The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$\epsilon_1 = +420\ \mu$ $\epsilon_2 = -45\ \mu$ $\epsilon_4 = +165\ \mu$

(*a*) What should be the reading of gage 3? (*b*) Determine the principal strains and the maximum in-plane shearing strain.

(a) Gages 2 and 4 are 90° apart $\quad \varepsilon_{ave} = \frac{1}{2}(\varepsilon_2 + \varepsilon_4)$

$$\varepsilon_{ave} = \frac{1}{2}(-45\mu + 165\mu) = 60\mu$$

Gages 1 and 3 are also 90° apart $\quad \varepsilon_{ave} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$

$$\varepsilon_3 = 2\varepsilon_{ave} - \varepsilon_1 = (2)(60\mu) - 420\mu = -300\mu$$

(b) $\varepsilon_x = \varepsilon_1 = 420\mu \qquad \varepsilon_y = \varepsilon_3 = -300\mu$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_x - \varepsilon_y = (2)(-45\mu) - 420\mu + 300\mu$$

$$= -210\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420\mu + 300\mu}{2}\right)^2 + \left(\frac{-210\mu}{2}\right)^2}$$

$$= 375\mu$$

$$\varepsilon_a = \varepsilon_{ave} + R = 60\mu + 375\mu = 435\mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = 60\mu - 375\mu = -315\mu$$

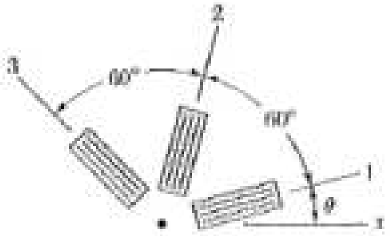
$$\gamma_{max(in\text{-}plane)} = 2R = 750\mu$$

Problem 7.148

7.148 Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 3\epsilon_{avg}$$

where ϵ_{avg} is the abscissa of the center of the corresponding Mohr's circle.



$$\varepsilon_1 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

$$\begin{aligned}\varepsilon_2 &= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 120°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 120°)\\
&= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 120°\cos 2\theta - \sin 120°\sin 2\theta)\\
&\qquad + \frac{\gamma_{xy}}{2}(\cos 120°\sin 2\theta + \sin 120°\cos 2\theta)\\
&= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\tfrac{1}{2}\cos 2\theta - \tfrac{\sqrt{3}}{2}\sin 2\theta\right)\\
&\qquad + \frac{\gamma_{xy}}{2}\left(-\tfrac{1}{2}\sin 2\theta + \tfrac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (2)\end{aligned}$$

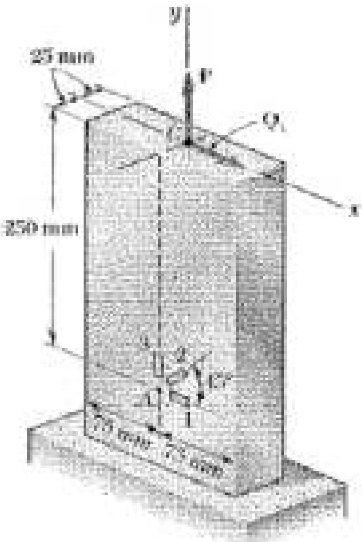
$$\begin{aligned}\varepsilon_3 &= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 240°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 240°)\\
&= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 240°\cos 2\theta - \sin 240°\sin 2\theta)\\
&\qquad + \frac{\gamma_{xy}}{2}(\cos 240°\sin 2\theta + \sin 240°\cos 2\theta)\\
&= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\tfrac{1}{2}\cos 2\theta + \tfrac{\sqrt{3}}{2}\sin 2\theta\right)\\
&\qquad + \frac{\gamma_{xy}}{2}\left(-\tfrac{1}{2}\sin 2\theta - \tfrac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (3)\end{aligned}$$

Adding (1), (2), and (3),

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{ave} + 0 + 0$$

$$3\varepsilon_{ave} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad ◀$$

Problem 7.151



7.151 Solve Prob. 7.150, assuming that the rosette at point A indicates the following strains:

$$\epsilon_1 = -60 \times 10^{-6} \text{ mm/mm} \qquad \epsilon_2 = +410 \times 10^{-6} \text{ mm/mm}$$
$$\epsilon_3 = +200 \times 10^{-6} \text{ mm/mm}$$

7.150 A centric axial force **P** and a horizontal force **Q** are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$$\epsilon_1 = -75 \times 10^{-6} \text{ mm/mm} \qquad \epsilon_2 = +300 \times 10^{-6} \text{ mm/mm}$$
$$\epsilon_3 = +250 \times 10^{-6} \text{ mm/mm}$$

Knowing that $E = 200$ GPa and $\nu = 0.30$, determine the magnitudes of **P** and **Q**.

$\varepsilon_x = \varepsilon_1 = -60 \times 10^{-6} \qquad \varepsilon_y = \varepsilon_3 = 200 \times 10^{-6}$

$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 680 \times 10^{-6}$

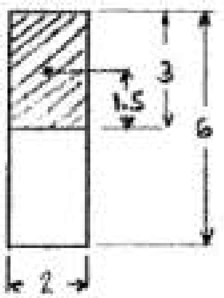
$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{200 \times 10^9}{1-0.3^2}[-60 + (0.3)(200)](10^{-6})$
$= 0$

$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{200 \times 10^9}{1-0.3^2}[200 + (0.3)(-60)](10^{-6})$
$= 40 \text{ MPa}$

$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (50)(150)(40)$
$= 300 \text{ kN}$

$G = \frac{E}{2(1+\nu)} = \frac{200 \times 10^9}{(2)(1.3)} = 76.92 \text{ GPa}$

$\tau_{xy} = G\gamma_{xy} = (76.92 \times 10^9)(680)(10^{-6}) = 52.306 \text{ MPa}$



$I = \frac{1}{12}bh^3 = \frac{1}{12}(50)(150)^3 = 14062500 \text{ mm}^4$

$Q = A\bar{y} = (75)(50)(\frac{75}{2}) = 140625 \text{ mm}^3 \quad t = 50 \text{ mm}$

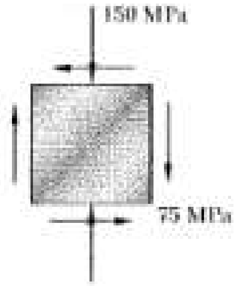
$\tau_{xy} = \frac{VQ}{It}$

$V = \frac{It\tau_{xy}}{Q} = \frac{(14062500)(50)(52.306)}{140625} = 261.53 \text{ kN}$

$Q_x = V = 261.5 \text{ kN}$

Problem 7.156

7.156 The given state of plane strain is known to exist on the surface of a machine component. Knowing that $E = 200$ GPa and $G = 77.2$ GPa, determine the direction and magnitude of the three principal strains (*a*) by determining the corresponding state of strain [use Eq. (2.43) and Eq. (2.38)] and then using Mohr's circle for strain, (*b*) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.



(a) $\sigma_x = 0$, $\sigma_y = -150 \times 10^6$ Pa, $\tau_{xy} = -75 \times 10^6$ Pa

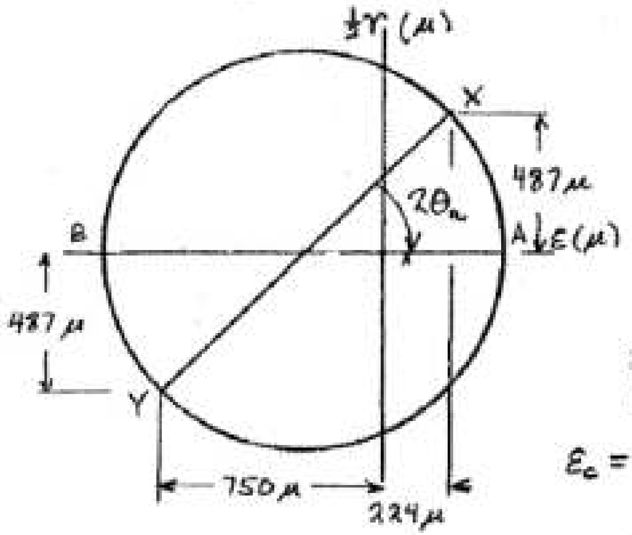
$E = 200 \times 10^9$ Pa $\quad G = 77 \times 10^9$ Pa

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = 0.2987$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{200 \times 10^9}\left[0 + (0.2987)(150 \times 10^6)\right] = 224\ \mu$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{200 \times 10^9}\left[(-150 \times 10^6) - 0\right] = -750\ \mu$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-75 \times 10^6}{77 \times 10^9} = -974\ \mu \qquad \frac{\gamma_{xy}}{2} = -487.0\ \mu$$



$$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = -263\ \mu$$

$$\varepsilon_x - \varepsilon_y = 974\ \mu$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-974}{974} = -1.000$$

$$2\theta_a = -45.0° \qquad \theta_a = -22.5°$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 689\ \mu$$

$$\varepsilon_a = \varepsilon_{ave} + R \qquad \varepsilon_a = 426\ \mu$$

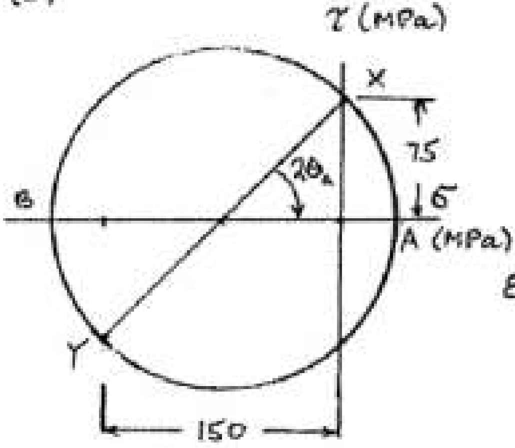
$$\varepsilon_b = \varepsilon_{ave} - R \qquad \varepsilon_b = -952\ \mu$$

$$\varepsilon_c = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 150 \times 10^6)}{200 \times 10^9}$$

$$= -224\ \mu$$

(b)

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -75\ \text{MPa}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0 + 150}{2}\right)^2 + 75^2}$$

$$= 106.07\ \text{MPa}$$

$$\sigma_a = \sigma_{ave} + R = 31.07\ \text{MPa}$$

$$\sigma_b = \sigma_{ave} - R = -181.07\ \text{MPa}$$

$$\varepsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_b)$$

$$= \frac{1}{200 \times 10^9}\left[31.07 \times 10^6 - (0.2987)(-181.07 \times 10^6)\right]$$

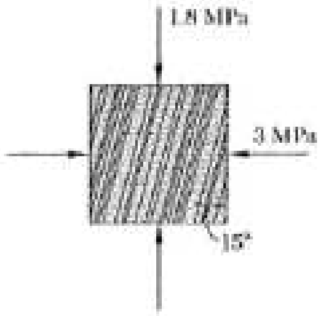
$$= 426 \times 10^{-6} \qquad \varepsilon_a = 426\ \mu$$

$$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.000 \qquad 2\theta_a = -45°$$

$$\theta_a = -22.5°$$

Problem 7.158

7.158 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$\sigma_x = -3\text{ MPa} \qquad \sigma_y = -1.8\text{ MPa} \qquad \tau_{xy} = 0$

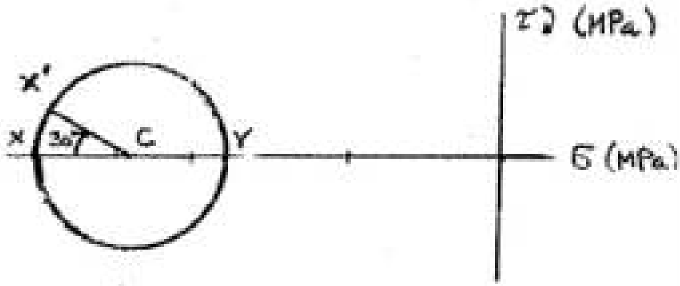
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -2.4\text{ MPa}$

Plotted points for Mohr's circle:

X: $(\sigma_x, -\tau_{xy}) = (-3\text{ MPa}, 0)$

Y: $(\sigma_y, \tau_{xy}) = (-1.8\text{ MPa}, 0)$

C: $(\sigma_{ave}, 0) = (-2.4\text{ MPa}, 0)$



$\theta = -15° \qquad 2\theta = -30°$

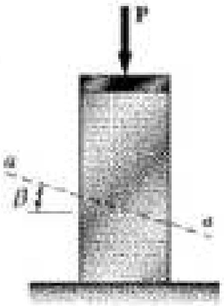
$\overline{CX} = 0.6\text{ MPa} \qquad R = 0.6\text{ MPa}$

(a) $\tau_{x'y'} = -\overline{CX}'\sin 30 = -R\sin 30° = -0.6\sin 30° \qquad \tau_{x'y'} = -0.30\text{ MPa}$

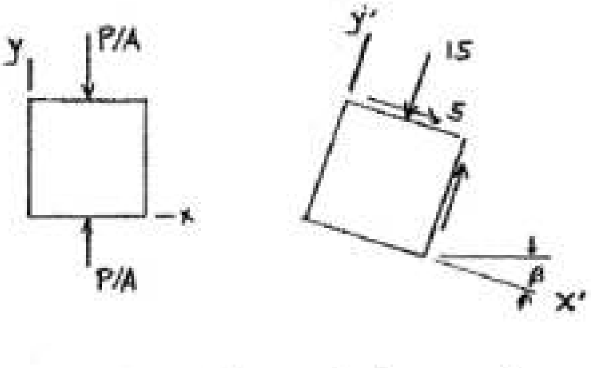
(b) $\sigma_{x'} = \sigma_{ave} - \overline{CX}'\cos 30° = -2.4 - 0.6\cos 30° \qquad \sigma_{x'} = -2.92\text{ MPa}$

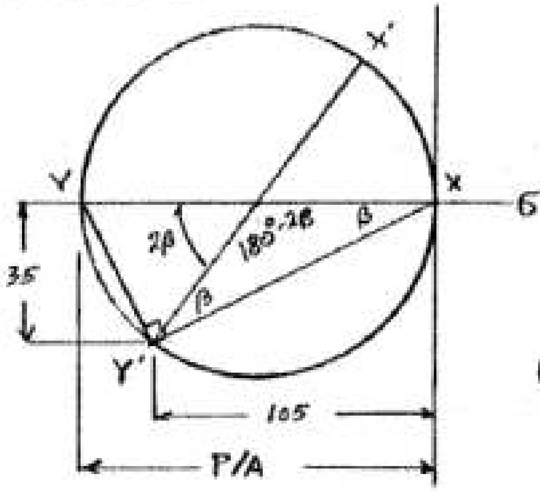
Problem 7.159

7.159 The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -105$ MPa and $\tau = 35$ MPa, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.



$\sigma_x = 0$
$\tau_{xy} = 0$
$\sigma_y = -P/A$





(a) From the Mohr's circle,

$\tan\beta = \dfrac{35}{105} = 0.3333 \qquad \beta = 18.4°$

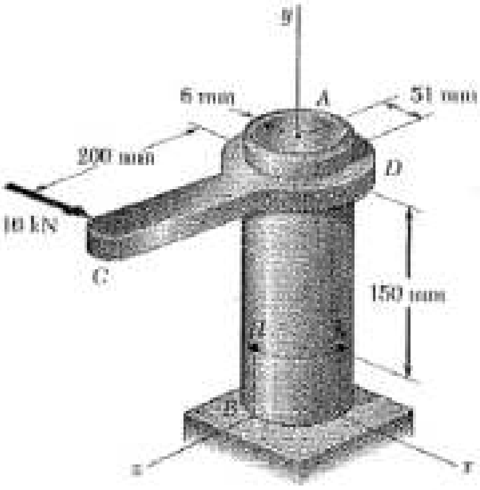
$-\sigma = \dfrac{P}{2A} + \dfrac{P}{2A}\cos 2\beta$

(b) $\dfrac{P}{A} = \dfrac{2(-\sigma)}{1+\cos 2\beta} = \dfrac{(2)(105)}{1+\cos 2\beta}$

$\dfrac{P}{A} = 116.6\text{ MPa}$

Problem 7.160

7.160 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *H*.



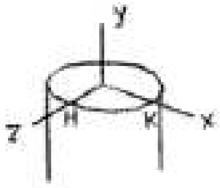
$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4$
$= 4.1855 \times 10^{-6} \text{ m}^4$

$I = \frac{1}{2}J = 2.0927 \text{ m}^4$

Force-couple system at center of tube in the plane containing points H and K.

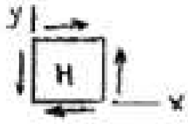
$F_x = 10 \times 10^3 \text{ N}$
$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N·m}$
$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N·m}$



Torsion

$T = M_y = 2000 \text{ N·m}$
$c = r_o = 51 \times 10^{-3} \text{ m}$

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \times 10^6 \text{ Pa}$



Transverse Shear

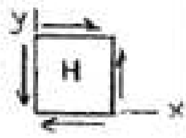
For semicircle $\quad A = \frac{\pi}{2}r^2 \quad \bar{y} = \frac{4}{3\pi}r$

$Q = A\bar{y} = \frac{2}{3}r^3$

$Q = Q_o - Q_i = \frac{2}{3}r_o^3 - \frac{2}{3}r_i^3 = 27.684 \times 10^3 \text{ mm}^3$
$= 27.684 \times 10^{-6} \text{ m}^3$

$V = F_x = 10 \times 10^3 \text{ N} \qquad t = (2)(6 \text{ mm}) = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$

$\tau_{xy} = \frac{VQ}{It} = \frac{(10 \times 10^3)(27.684 \times 10^{-6})}{(2.0927 \times 10^{-6})(12 \times 10^{-3})} = 11.02 \times 10^6 \text{ Pa}$



Bending: Point H lies on neutral axis. $\sigma_y = 0$

Total stresses at point H: $\sigma_x = 0, \; \sigma_y = 0$
$\tau_{xy} = 24.37 \times 10^6 + 11.02 \times 10^6 = 35.39 \times 10^6 \text{ Pa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 35.39 \times 10^6 \text{ Pa}$

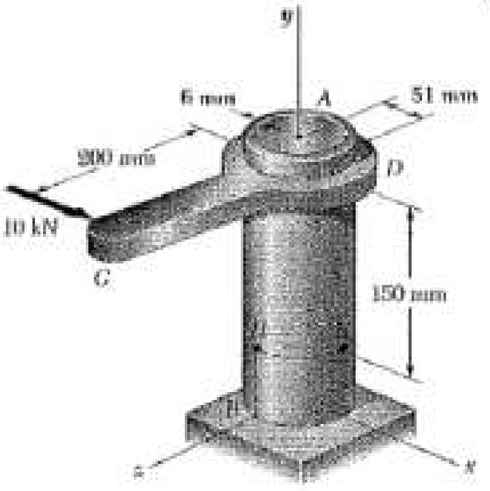
$\sigma_{max} = \sigma_{ave} + R = 35.39 \times 10^6 \text{ Pa} \qquad \sigma_{max} = 35.4 \text{ MPa}$ ◄

$\sigma_{min} = \sigma_{ave} - R = -35.39 \times 10^6 \text{ Pa} \qquad \sigma_{min} = -35.4 \text{ MPa}$ ◄

$\tau_{max} = R \qquad \tau_{max} = 35.4 \text{ MPa}$ ◄

Problem 7.161

7.161 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.



$r_o = \frac{d_o}{2} = \frac{102}{2} = 51$ mm $\qquad r_i = r_o - t = 45$ mm

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6\ \text{mm}^4 = 4.1855 \times 10^{-6}\ \text{m}^4$

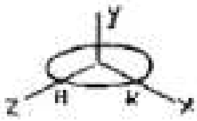
$I = \frac{1}{2}J = 2.0927 \times 10^{-6}\ \text{m}^4$

Force-couple system at center of tube in the plane containing points H and K

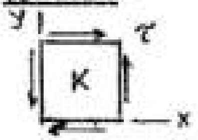
$F_x = 10$ kN $= 10 \times 10^3$ N

$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000$ N·m

$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500$ N·m



Torsion:



At point K, place local x-axis in negative global z-direction

$T = M_y = 2000$ N·m $\qquad c = r_o = 51 \times 10^{-3}$ m

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \times 10^6\ \text{Pa} = 24.37\ \text{MPa}$$

Transverse Shear: Stress due to transverse shear $V = F_x$ is zero at pt. K.

Bending: $$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56 \times 10^6\ \text{Pa} = 36.56\ \text{MPa}$$

Point K lies on compression side of neutral axis: $\sigma_y = -36.56$ MPa

Total stresses at point K: $\quad \sigma_x = 0 \quad \sigma_y = -36.56$ MPa, $\tau_{xy} = 24.37$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46$ MPa

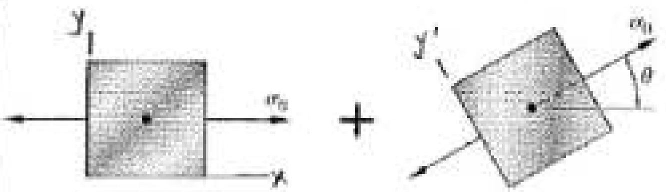
$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 \qquad \sigma_{max} = 12.18$ MPa ◄

$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 \qquad \sigma_{min} = -48.7$ MPa ◄

$\tau_{max} = R \qquad \tau_{max} = 30.5$ MPa ◄

Problem 7.162

7.162 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

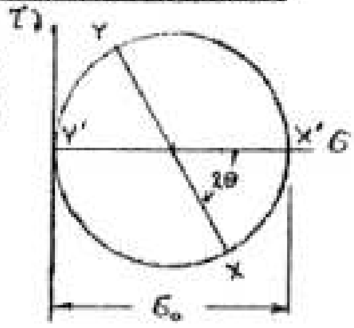


Mohr's circle for 2nd stress state.

$$\sigma_x = \tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = \tfrac{1}{2}\sigma_0 \sin 2\theta$$



Resultant stresses

$$\sigma_x = \sigma_0 + \tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0 \cos 2\theta = \tfrac{3}{2}\sigma_0 + \tfrac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = 0 + \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0 \cos 2\theta = \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0 \cos 2\theta$$

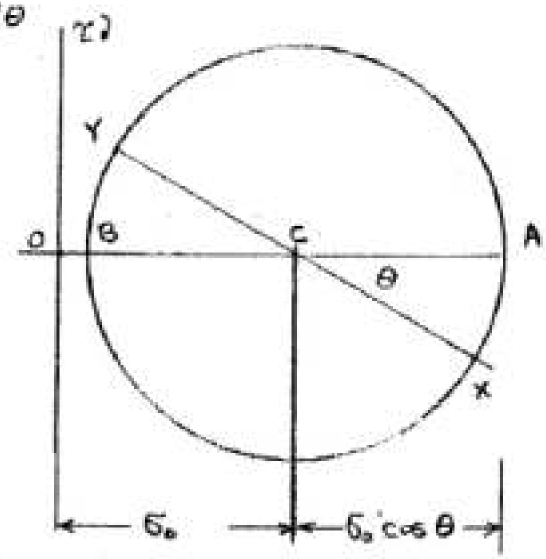
$$\tau_{xy} = 0 + \tfrac{1}{2}\sigma_0 \sin 2\theta = \tfrac{1}{2}\sigma_0 \sin 2\theta$$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \sigma_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0 \sin 2\theta}{\sigma_0 + \sigma_0 \cos 2\theta}$$

$$= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$$

$$\theta_p = \tfrac{1}{2}\theta \quad \blacktriangleleft$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} =$$

$$= \sqrt{\left(\tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0 \cos 2\theta\right)^2 + \left(\tfrac{1}{2}\sigma_0 \sin 2\theta\right)^2}$$

$$= \tfrac{1}{2}\sigma_0 \sqrt{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}$$

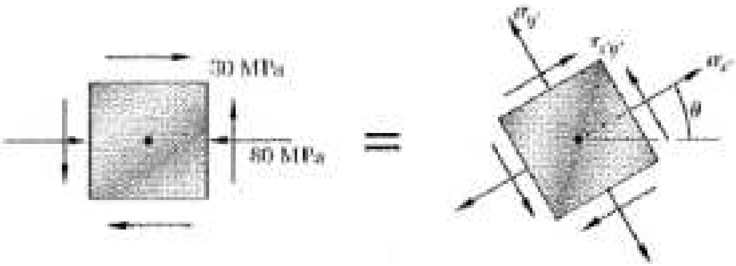
$$= \frac{\sqrt{2}}{2}\sigma_0 \sqrt{1 + \cos 2\theta} = \sigma_0 |\cos\theta|$$

$$\sigma_a = \sigma_{ave} + R = \sigma_0 + \sigma_0 \cos\theta \quad \blacktriangleleft$$

$$\sigma_b = \sigma_{ave} - R = \sigma_0 - \sigma_0 \cos\theta \quad \blacktriangleleft$$

Problem 7.163

7.163 For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress $\tau_{x'y'}$ is equal to or less than 40 MPa.



$\sigma_x = -80$ MPa, $\sigma_y = 0$

$\tau_{xy} = 30$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -40$ MPa

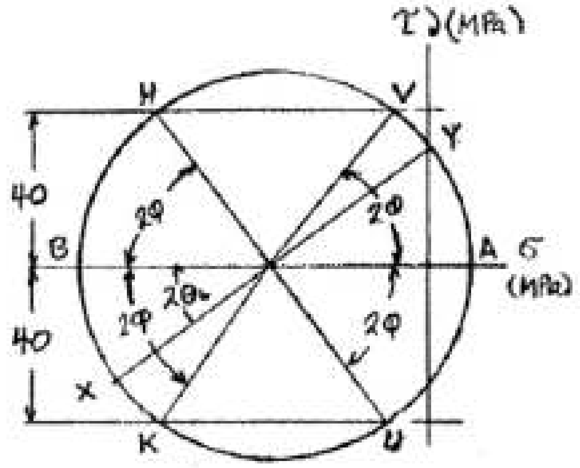
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-40)^2 + 30^2} = 50$ MPa

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(30)}{-80} = -0.75$

$2\theta_p = -36.870°$

$\theta_b = -18.435°$



$|\tau_{x'y'}| \leq 40$ MPa for states of stress corresponding to arcs HBK and UAV of Mohr's circle. The angle φ is calculated from

$R \sin 2\varphi = 40$

$\sin 2\varphi = \frac{40}{50} = 0.8$

$2\varphi = 53.130°$ $\qquad \varphi = 26.565°$

$\theta_H = \theta_b - \varphi = -18.435° - 26.565° = -45°$

$\theta_K = \theta_b + \varphi = -18.435 + 26.565° = 8.13°$

$\theta_U = \theta_H + 90° = 45°$

$\theta_V = \theta_K + 90° = 98.13°$

<u>Permissible ranges of θ</u>: $\quad \theta_H \leq \theta \leq \theta_K$

$-45° \leq \theta \leq 8.13°$

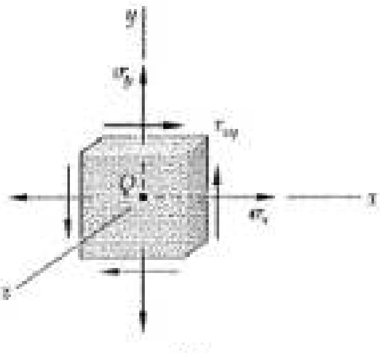
$\theta_U \leq \theta \leq \theta_V$

$45° \leq \theta \leq 98.13°$

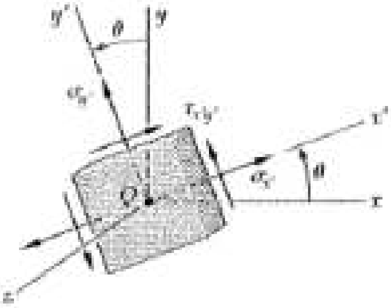
Also, $\quad 135° \leq \theta \leq 188.13°$

$225° \leq \theta \leq 278.13°$

PROBLEM 7.C1



(a)



(b)

7.C1 A state of plane stress is defined by the stress components σ_x, σ_y, and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to calculate the stress components $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ associated with the element after it has rotated through an angle θ about the *z* axis (Fig. P7.C1*b*). (*b*) Use this program to solve Probs. 7.13 through 7.16.

SOLUTION PROGRAM FOLLOWING EQUATIONS

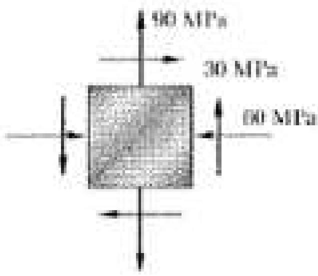
EQ (7.5), p427: $\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

EQ (7.7), p427: $\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

EQ (7.6), p427: $\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

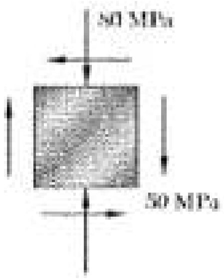
ENTER σ_x, σ_y, τ_{xy} AND θ

PRINT VALUES OBTAINED FOR $\sigma_{x'}$, $\sigma_{y'}$ and $\tau_{x'y'}$




```
Problem 7.13a

Sigma x  = -60 MPa
Sigma y  =  90 MPa
Tau xy   =  30 MPa

Rotation of element
(+ counterclockwise)
theta = -25 degrees

     Sigma x'  =  -56.19 MPa
     Sigma y'  =   86.19 MPa
     Tau x'y'  =  -38.17 MPa
```

```
Problem 7.13b

Sigma x  = -60 MPa
Sigma y  =  90 MPa
Tau xy   =  30 MPa

Rotation of element
(+ counterclockwise)
theta =  10 degrees

     Sigma x'  =  -45.22 MPa
     Sigma y'  =   75.22 MPa
     Tau x'y'  =   53.84 MPa
```



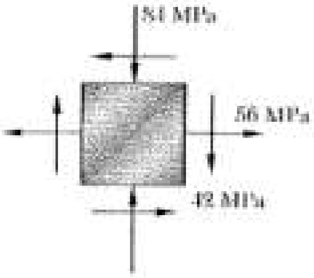

```
Problem 7.14a

  Sigma x  =   0 MPa
  Sigma y  = -80 MPa
  Tau xy   = -50 MPa

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x' =    24.01 MPa
  Sigma y' =  -104.01 MPa
  Tau x'y' =    -1.50 MPa
```

```
Problem 7.14b

  Sigma x  =   0 MPa
  Sigma y  = -80 MPa
  Tau xy   = -50 MPa

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x' = -19.51 MPa
  Sigma y' = -60.49 MPa
  Tau x'y' = -60.67 MPa
```

CONTINUED

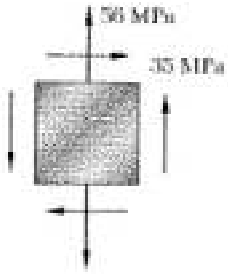
PROBLEM 7.C1 - CONTINUED

PROGRAM OUTPUT




```
Problem 7.15a

  Sigma x  = 56 MPa
  Sigma y  = -84 MPa
  Tau xy   = -42 MPa

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x'  =  63.1 MPa
  Sigma y'  =  -91.1 MPa
  Tau x'y'  =  26.6 MPa
```

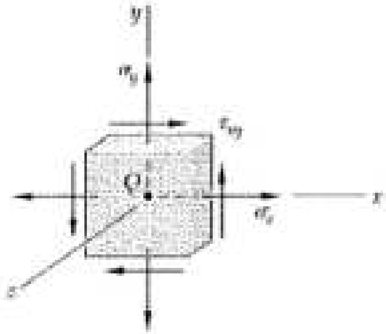
```
Problem 7.15b

  Sigma x  = 56 MPa
  Sigma y  = -84 MPa
  Tau xy   = -42 MPa

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x'  =  37.4 MPa
  Sigma y'  =  -65.4 MPa
  Tau x'y'  =  -63.4 MPa
```




```
Problem 7.16a

Sigma x  =   0 MPa
Sigma y  =   56 MPa
Tau xy   =   35 MPa

Rotation of element
(+ counterclockwise)
theta = -25 degrees

     Sigma x'  =  -16.8 MPa
     Sigma y'  =  72.8 MPa
     Tau x'y'  =  1.05 MPa
```

```
Problem 7.16b

Sigma x  =   0 MPa
Sigma y  =   56 MPa
Tau xy   =   35 MPa

Rotation of element
(+ counterclockwise)
theta =  10 degrees

     Sigma x'  =  13.7 MPa
     Sigma y'  =  42.4 MPa
     Tau x'y'  =  41.5 MPa
```

PROBLEM 7.C2



7.C2 A state of plane stress is defined by the stress components σ_x, σ_y, and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to calculate the principal axes, the principal stresses, the maximum in-plane shearing stress, and the maximum shearing stress. (*b*) Use this program to solve Probs. 7.5, 7.9, 7.68 and 7.69.

SOLUTION PROGRAM FOLLOWING EQUATIONS

EQ.(7.10) $\quad \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} \; : \; R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

EQ.(7.14) $\quad \sigma_{max} = \sigma_{ave} + R$

$\sigma_{min} = \sigma_{ave} - R$

EQ.(7.12) $\quad \theta_p = \tan^{-1} \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

EQ.(7.15) $\quad \theta_s = \tan^{-1} - \dfrac{\sigma_x - \sigma_y}{2\tau_{xy}}$

SHEARING STRESS IF $\sigma_{max} > 0$ and $\sigma_{min} < 0$:

THEN $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = R$

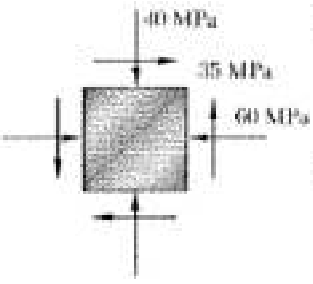
IF $\sigma_{max} > 0$ and $\sigma_{min} > 0$:

THEN $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_{max}$

IF $\sigma_{max} < 0$ and $\sigma_{min} < 0$:

THEN $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}|\sigma_{min}|$

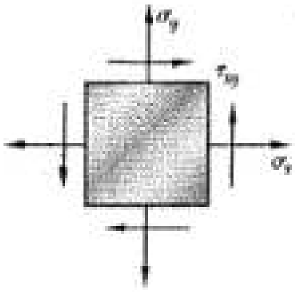
PROGRAM OUTPUT




```
Problems 7.5 and 7.9

  Sigma x = -60.00 MPa
  Sigma y = -40.00 MPa
  Tau xy  =  35.00 MPa

Angle between xy axes and principal axes
            ( + counterclockwise )
      Theta p = -37.03 deg.  and  52.97 deg.
  Sigma max = -13.60 MPa
  Sigma min = -86.40 MPa
Angle between xy axis and planes of maximum in-plane shearing stress
            ( + counterclockwise )
      Theta s =   7.97 deg.  and  97.97 deg.
  Tau max (in plane) =  36.40 MPa
  Tau max   =  43.20 MPa
```

CONTINUED

PROBLEM 7.C3



7.C3 (*a*) Write a computer program that, for a given state of plane stress and a given yield strength of a ductile material, can be used to determine whether the material will yield. The program should use both the maximum shearing-strength criterion and the maximum-distortion-energy criterion. It should also print the values of the principal stresses and, if the material does no yield, calculate the factor of safety. (*b*) Use this program to solve Probs. 7.81, 7.82 and 7.165.

SOLUTION

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \quad ; \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$
$$\sigma_b = \sigma_{ave} - R$$

MAXIMUM-SHEARING-STRESS CRITERION $\quad \tau_Y = \frac{1}{2}\sigma_Y$

IF σ_a AND σ_B HAVE SAME SIGN, $\tau_{max} = \frac{1}{2}\sigma_a$

IF $\tau_{max} > \tau_Y$, YIELDING OCCURS

IF $\tau_{max} < \tau_Y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\tau_Y}{\tau_{max}}$$

MAXIUM- DISTORTION- ENERGY CRITERION

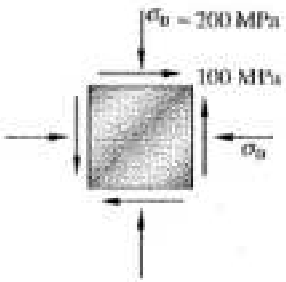
COMPUTE $\quad \text{RADICAL} = \sqrt{\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2}$

IF RADICAL $> \sigma_Y$, YIELDING OCCURS

IF RADICAL $< \sigma_Y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\sigma_Y}{\text{RADICAL}}$$

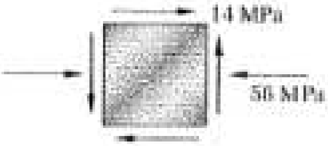
PROGRAM OUTPUT




```
Problems 7.81a and 7.82a     Yield strength = 325 MPa
                       Sigma x = -200.00 MPa
                       Sigma y = -200.00 MPa
                       Tau xy  =  100.00 MPa
              Sigma max = -100.00 MPa
              Sigma min = -300.00 MPa
          Using the maximum-shearing-stress criterion:
            Material will not yield
            F.S. =  1.083
          Using the maximum-distortion-energy criterion:
            Material will not yield
            F.S. =  1.228
```

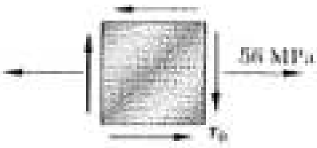
CONTINUED

PROBLEM 7.C4 - CONTINUED




```
Problem 7.90          Sigma x =   -56 MPa
                      Sigma y =   0.00 MPa
                       Tau xy =   49 MPa
Ultimate strength in tension      = 70 MPa
Ultimate strength in compression  = 210 MPa

  Sigma max = Sigma a    28.42 MPa
  Sigma min = Sigma b   -84.42 MPa
Rupture will not occur
```

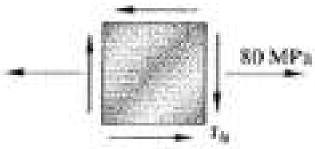
TO CHECK ANSWERS TO FOLLOWING PROBLEMS, WE CHECK FOR RUPTURE USING GIVEN ANSWERS AND AN ADJACENT VALUE.



ANSWER!
RUPTURE OCCURS FOR $\tau_0 = 25.7$ MPa

```
Problem 7.93     Sigma x =  56 MPa
                 Sigma y =  0.00 MPa
                  Tau xy =  25.7 MPa
Ultimate strength in tension      = 70 MPa
Ultimate strength in compression  = 175 MPa
  Sigma max = Sigma a      66.01 MPa
  Sigma min = Sigma b     -10.01 MPa
     Rupture will not occur
```

```
                 Sigma x =  56 MPa
                 Sigma y =  0.00 MPa
                  Tau xy =  25.7 MPa
Ultimate strength in tension      = 70 MPa
Ultimate strength in compression  = 175 MPa
  Sigma max = Sigma a      66.08 MPa
  Sigma min = Sigma b     -10.08 MPa
     Rupture will occur
```



ANSWER!
RUPTURE OCCURS FOR $\tau_0 = 49.1$ MPa

```
Problem 7.94     Sigma x = -80.00 MPa
                 Sigma y =   0.00 MPa
                  Tau xy =  49.10 MPa
Ultimate strength in tension      =  75 MPa
Ultimate strength in compression  = 150 MPa
  Sigma max = Sigma a =    23.33 MPa
  Sigma min = Sigma b =  -103.33 MPa
     Rupture will not occur
```

```
                 Sigma x = -80.00 MPa
                 Sigma y =   0.00 MPa
                  Tau xy =  49.20 MPa
Ultimate strength in tension      =  75 MPa
Ultimate strength in compression  = 150 MPa
  Sigma max = Sigma a =    23.41 MPa
  Sigma min = Sigma b =  -103.41 MPa
     Rupture will occur
```

PROBLEM 7.C6

7.C6 A state of strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy} associated with the x and y axes. (a) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.136 through 7.139.

SOLUTION PROGRAM FOLLOWING EQUATIONS

EQ(7.50) $\epsilon_{ave} = \frac{\epsilon_x + \epsilon_y}{2}$ $R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

EQ.(7.51) $\epsilon_{max} = \epsilon_{ave} + R$ $\epsilon_{min} = \epsilon_{ave} - R$

EQ(7.52) $\theta_p = \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max(in\text{-}plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET $\epsilon_a = \epsilon_{max}$
$\epsilon_b = \epsilon_{min}$

CALCULATE $\epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b)$

IF $\epsilon_a > \epsilon_b > \epsilon_c$: $\gamma_{out\text{-}of\text{-}plane} = \epsilon_a - \epsilon_c$

IF $\epsilon_a > \epsilon_c > \epsilon_b$: $\gamma_{out\text{-}of\text{-}plane} = \epsilon_a - \epsilon_b = 2R$

IF $\epsilon_c > \epsilon_a > \epsilon_b$: $\gamma_{out\text{-}of\text{-}plane} = \epsilon_c - \epsilon_b$

PROGRAM PRINTOUT

```
Problem 7.136       Epsilon x =   30 micro meters
                    Epsilon y =  570 micro meters
                    Gamma xy  =  720 micro radians
                    nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p    =    -26.57 degrees
        Epsilon a =    750.00 micro meters
        Epsilon b =   -150.00 micro meters
        Epsilon c =   -300.00 micro meters

        Gamma max (in plane) =  900.00 micro radians
        Gamma max = 1050.00 micro radians
```

CONTINUED

PROBLEM 7.C7

7.C7 A state of plane strain is defined by the strain components ε_x, ε_y, and γ_{xy} measured at a point. (*a*) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the magnitude of the shearing strain. (*b*) Use this program to solve Probs. 7.140 through 7.143.

SOLUTION

PROGRAM FOLLOWING EQUATIONS

EQ(7.50) $\varepsilon_{ave} = \frac{\varepsilon_x + \varepsilon_y}{2}$ $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

EQ(7.51) $\varepsilon_{max} = \varepsilon_{ave} + R$ $\quad \varepsilon_{min} = \varepsilon_{ave} - R$

EQ(7.52) $\theta_p = \tan^{-1} \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$\gamma_{xy\,(in\text{-}plane)} = 2R$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET $\varepsilon_a = \varepsilon_{max}$

$\varepsilon_b = \varepsilon_{min}$

$\varepsilon_c = 0$ (PLAIN STRAIN)

IF $\varepsilon_a > \varepsilon_b > \varepsilon_c$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \varepsilon_a - \varepsilon_c$

IF $\varepsilon_a > \varepsilon_c > \varepsilon_b$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \varepsilon_a - \varepsilon_b = 2R$

IF $\varepsilon_c > \varepsilon_a > \varepsilon_b$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \varepsilon_c - \varepsilon_b$

PROGRAM PRINTOUT

```
Problem 7.140        Epsilon x =  400 micro meters
                     Epsilon y =  200 micro meters
                     Gamma xy  =  375 micro radians
                     nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =    30.96 and -59.04 degrees
       Epsilon a =   512.50 micro meters
       Epsilon b =    87.50 micro meters
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  425.00 micro radians
       Gamma max =  512.50 micro radians
```

CONTINUED

```
Problem 7.141        Epsilon x = -180 micro meters
                     Epsilon y = -260 micro meters
                     Gamma xy  =  315 micro radians
                     nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p    =    37.87 and -52.13 degrees
       Epsilon a =   -57.50 micro meters
       Epsilon b =  -382.50 micro meters
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  325.00 micro radians
       Gamma max =  382.50 micro radians



Problem 7.142        Epsilon x =   60 micro meters
                     Epsilon y =  240 micro meters
                     Gamma xy  =  -50 micro radians
                     nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p    =     7.76 and -82.24 degrees
       Epsilon a =   243.41 micro meters
       Epsilon b =    56.59 micro meters
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  186.82 micro radians
       Gamma max =  243.41 micro radians



Problem 7.143        Epsilon x =  300 micro meters
                     Epsilon y =   60 micro meters
                     Gamma xy  =  100 micro radians
                     nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p    =    11.31 and -78.69 degrees
       Epsilon a =   310.00 micro meters
       Epsilon b =    50.00 micro meters
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  260.00 micro radians
       Gamma max =  310.00 micro radians
```

PROBLEM 7.C8

7.C8 A rosette consisting of three gages forming, respectively, angles of θ_1, θ_2, and θ_3 with the x axis is attached to the free surface of a machine component made of a material with a given Poisson's ratio v. (*a*) Write a computer program that, for given readings ϵ_1, ϵ_2, and ϵ_3 of the gages, can be used to calculate the strain components associated with the x and y axes and the determine the orientation and magnitude of the three principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs. 7.144, 7.145, 7.146 and 7.169.

SOLUTION

FOR $n = 1$ TO 3, ENTER θ_n and ϵ_n

ENTER: $NU = \nu$

SOLVE EQS. (7.60) FOR ϵ_x, ϵ_y, AND γ_{xy} USING METHOD OF DETERMINATES OR ANY OTHER METHOD,

ENTER $\epsilon_{ave} = \dfrac{\epsilon_x + \epsilon_y}{2}$; $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \gamma_{xy}^2}$

$$\epsilon_a = \epsilon_{max} = \epsilon_{ave} + R$$

$$\epsilon_b = \epsilon_{min} = \epsilon_{avg} - R$$

$$\epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b)$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max\,(in-plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN, AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN,

IF $\epsilon_c < \epsilon_b$: $\gamma_{OUT-OF-PLANE} = \epsilon_a - \epsilon_c$

IF $\epsilon_c > \epsilon_a$: $\gamma_{OUT-OF-PLANE} = \epsilon_c - \epsilon_b$

OTHERWISE: $\gamma_{OUT-OF-PLANE} = 2R$

PROGRAM OUTPUT

Problem 7.144

```
Gage    theta       epsilon
        degrees     micro meters

 1       -15          480
 2        30         -120
 3        75           80

Epsilon x =    253.21 micro meters
Epsilon y =    306.79 micro meters
Gamma xy  =   -892.82 micro radians

Epsilon a =    727.21 micro meters
Epsilon b =   -167.21 micro meters
Gamma max (in plane) =   894.43 micro radians
```

CONTINUED

Problem 7.145

Gage	theta degrees	epsilon micro meters
1	30	600
2	-30	450
3	90	-75

```
Epsilon x =   725.000 micro meters
Epsilon y =   -75.000 micro meters
Gamma xy  =   173.205 micro radians

Epsilon a =   734.268 micro meters
Epsilon b =   -84.268 micro meters
Gamma max (in plane) =  818.535 micro radians
```

Problem 7.146

OBSERVE THAT GAGE 3 IS ORIENTATED ALONG THE y AXIS. THEREFORE
ENTER θ_4 AND ϵ_4 AS θ_3 AND ϵ_3,
THE VALUE OF ϵ_y THAT IS OBTAINED
IS ALSO THE EXPECTED READING OF GAGE 3.

Gage	theta degrees	epsilon mm/mm
1	0	420
2	45	-45
4	135	165

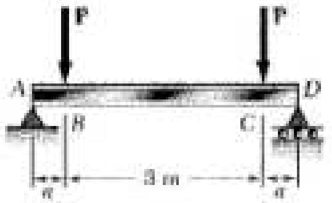
```
Epsilon x =   420.00 mm/mm
Epsilon y =  -300.00 mm/mm
Gamma xy  =  -210.00 micro radians

Epsilon a =   435.00 mm/mm
Epsilon b =  -315.00 mm/mm
Gamma max (in plane) =  750.00 micro radians
```

Problem 7.169

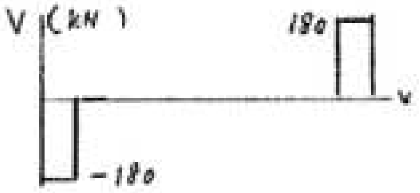
Gage	theta degrees	epsilon mm/mm
1	45	-50
2	-45	360
3	0	315

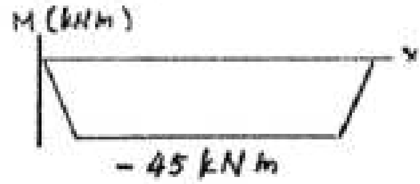
```
Epsilon x =   315.000  mm/mm
Epsilon y =    -5.000  mm/mm
Gamma xy  =  -410.000 micro radians

Epsilon a =   415.048  mm/mm
Epsilon b =  -105.048  mm/mm
Gamma max (in plane) =  520.096 micro radians
```

Problem 8.1



8.1 A W250 × 101 rolled-steel beam supports a load **P** as shown. Knowing that $P = 180$ kN, $a = 0.25$ m, and $\sigma_{all} = 126$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.





$|V|_{max} = 180$ kN

$|M|_{max} = (180)(0.25) = 45$ kN m

For W250 × 101 rolled steel section,

$d = 264$ mm $\quad b_f = 257$ mm $\quad t_f = 19.6$ mm

$t_w = 11.9$ mm $\quad I_x = 164 \times 10^6$ mm^4 $\quad S_x = 1240 \times 10^3$ mm^3

$c = \frac{1}{2}d = 132$ mm $\quad y_b = c - t_f = 112.4$ mm

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{45 \times 10^3}{1240 \times 10^{-6}}$ $\qquad \sigma_m = 36.3$ MPa

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \left(\dfrac{112.4}{132}\right)(36.3) = 30.9$ MPa

$A_f = b_f t_f = 5037$ mm^2

$\bar{y}_f = \frac{1}{2}(c + y_b) = 122.2$ mm

$Q_b = A_f \bar{y}_f = 615521$ mm^3

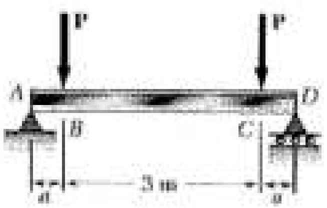
$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(180 \times 10^3)(615521 \times 10^{-9})}{(164 \times 10^{-6})(0.0119)} = 56.77$ MPa

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 58.83$ MPa

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 74.29$ MPa $\qquad \sigma_{max} = 74.3$ MPa

(c) Since $\sigma_{max} < \sigma_{all}$ (= 126 MPa), W250 × 101 is <u>acceptable</u>.

Problem 8.2

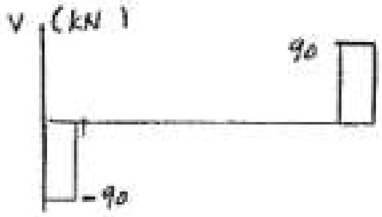


8.2 Solve Prob. 8.1, assuming that $P = 90$ kN, $a = 0.5$ m.

8.1 A W250 × 101 rolled-steel beam supports a load **P** as shown. Knowing that $P = 180$ kN, $a = 0.25$ m, and $\sigma_{all} = 126$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

$|V|_{max} = 90$ kN

$|M|_{max} = (90 \times 0.5) = 45$ kNm

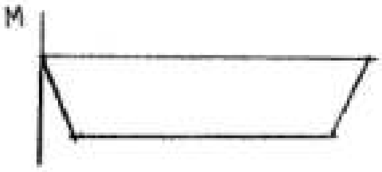


For W250 × 101 rolled steel section,

$d = 264$ mm $\quad b_f = 257$ mm $\quad t_f = 19.6$ mm

$t_w = 11.9$ mm $\quad I_x = 164 \times 10^6$ mm^4 $\quad S_x = 1240 \times 10^3$ mm^3

$c = \frac{1}{2}d = 132$ mm $\quad y_b = c - t_f = 112.4$ mm



(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{45 \times 10^3}{1240 \times 10^{-6}}$ $\qquad \sigma_m = 36.3$ MPa

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \left(\dfrac{112.4}{132}\right)(36.3) = 30.9$ MPa

$A_f = b_f t_f = 5037$ mm^2

$\bar{y}_f = \frac{1}{2}(c + y_b) = 122.2$ mm

$Q_b = A_f \bar{y}_f = 615521$ mm^3

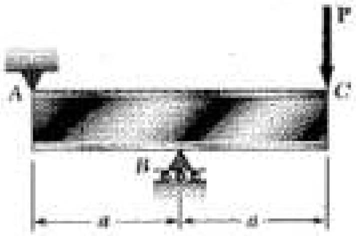
$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(45 \times 10^3)(615521 \times 10^{-9})}{(164 \times 10^{-6})(0.119)} = 4.12$ MPa

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 15.99$ MPa

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 31.44$ MPa $\qquad \sigma_{max} = 31.4$ MPa

(c) Since $\sigma_{max} < \sigma_{all}$ (= 126 MPa), W 250 × 101 is <u>acceptable</u>.

Problem 8.3

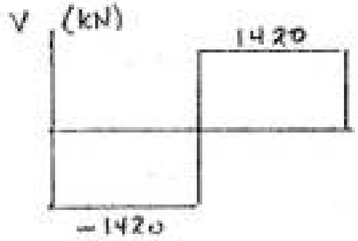


8.3 An overhanging W920 × 446 rolled-steel beam supports a load **P** as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{all} = 100$ MPa, determine *(a)* the maximum value of the normal stress σ_m in the beam, *(b)* the maximum value of the principal stress σ_{max} at the junction of the flange and web, *(c)* whether the specified shape is acceptable as far as these two stresses are concerned.

$|V|_{max} = 700 \text{ kN} = 700 \times 10^3 \text{ N}$

$|M|_{max} = (700 \times 10^3)(2.5) = 1.75 \times 10^6 \text{ N}\cdot\text{m}$

For W920×446 rolled steel beam

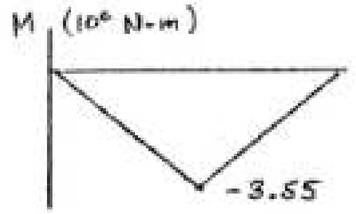


$d = 933$ mm $\quad b_f = 423$ mm $\quad t_f = 42.70$ mm

$t_w = 24.0$ mm $\quad I_x = 8470 \times 10^6 \text{ mm}^4 \quad S_x = 18200 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2} d = 466.5$ mm $\quad y_b = c - t_f = 423.8$ mm

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{1.75 \times 10^6}{18200 \times 10^{-6}} \qquad \sigma_m = 96.2 \text{ MPa}$ ◄



$\sigma_b = \dfrac{y_b}{c} \sigma_m = \dfrac{423.8}{466.5}(96.2) = 87.4$ MPa

$A_f = b_f t_f = 18.062 \times 10^3 \text{ mm}^2$

$\bar{y}_f = \frac{1}{2}(c + y_b) = 445.15$ mm

$Q_b = A_f \bar{y}_f = 8040.3 \times 10^3 \text{ mm}^3 = 8040.3 \times 10^{-6} \text{ m}^3$

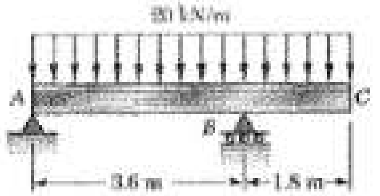
$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(700 \times 10^3)(8040.3 \times 10^{-6})}{(8470 \times 10^{-6})(24.0 \times 10^{-3})} = 27.7$ MPa

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{87.4}{2}\right)^2 + 27.7^2} = 51.7$ MPa

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 95.4$ MPa $\qquad \sigma_{max} = 95.4$ MPa ◄

(c) Since 51.7 MPa < σ_{all}(= 100 MPa), W920×446 is <u>acceptable</u> ◄

Problem 8.5



8.5 (*a*) Knowing that $\sigma_{all} = 165$ MPa and $\tau_{all} = 100$ MPa, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

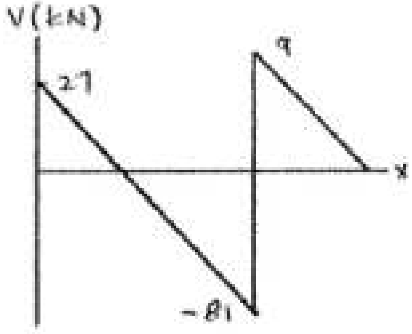
$\circlearrowright \Sigma M_B = 0 \quad -3.6\, R_A + (20)(5.4)(0.9) = 0 \quad R_A = 27 \text{ kN} \uparrow$

$\circlearrowright \Sigma M_A = 0 \quad 3.6\, R_B + (20)(5.4)(2.7) = 0 \quad R_B = 81 \text{ kN} \uparrow$

$|V|_{max} = 81 \text{ kN}.$

$|M|_{max} = 32.4 \text{ kNm}.$

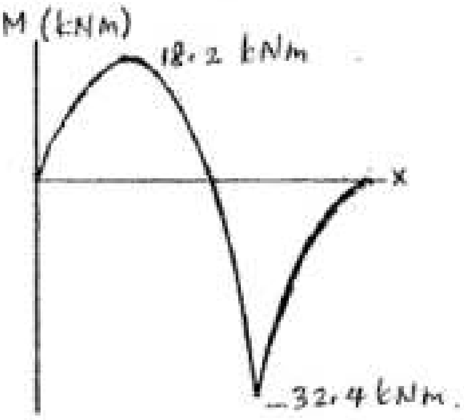
$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{32.4 \times 10^6}{165} = 196.4 \times 10^3 \text{ mm}^3.$$



Shape	S ($\times 10^3$ mm^3)
W 310×23.8	280
→ W 250×22.3	229
W 200×26.6	249
W W150×29.8	219

(a) Use W 250×22.3

$d = 254$ mm
$t_f = 6.9$ mm
$t_w = 5.8$ mm



$$\text{(b)} \quad \sigma_m = \frac{|M|_{max}}{S} = \frac{32.4 \times 10^6}{229 \times 10^3} = 141.5 \text{ MPa}$$

$$\tau_m = \frac{|V|_{max}}{d\, t_w} = \frac{81000}{(254)(5.8)} = 55 \text{ MPa.}$$

$$c = \tfrac{1}{2} d = \frac{254}{2} = 127 \text{ mm.}$$

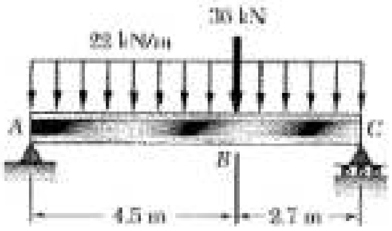
$$y_b = c - t_f = 127 - 6.9 = 120.1 \text{ mm.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{120.1}{127}\right)(141.5) = 133.8 \text{ MPa.}$$

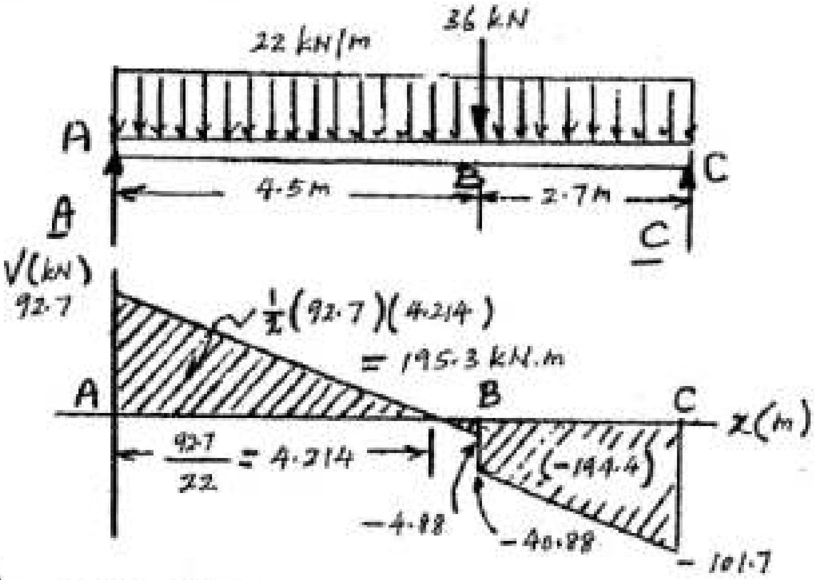
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{133.8}{2}\right)^2 + (55)^2} = 86.6 \text{ MPa.}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{133.8}{2} + 86.6 = 153.5 \text{ MPa}$$

Problem 8.6



8.5 and 8.6 (*a*) Knowing that $\sigma_{all} = 165$ MPa and $\tau_{all} = 100$ MPa, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$+\circlearrowleft \Sigma M_C = 0$:

$(22 \times 7.2)(3.6) + (36)(2.7) - A(7.2) = 0$

$A = 92.7$ kN

$B = 101.7$ kN

$|V|_{max} = 101.7$ kN $\qquad |M|_{max} = 195.3$ kN m

Shape	$S(10^3 mm^3)$
W 530x66	1340
W 460x74	1460
W 410x85	1510
W 360x79	1280
W 310x107	1590
W 250x101	1240

Section Modulus

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{195.3 \times 10^6}{165} = 1183.64 \times 10^3 \ mm^3$$

(a) Select W 530 x 66 ◄

$S = 1340 \times 10^3 \ mm^3 \qquad d = 525$ mm

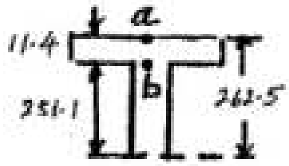
$t_f = 11.4$ mm $\qquad t_w = 8.9$ mm

Stress σ_m: $\quad \sigma_m = \frac{|M|_{max}}{S} = \frac{195.3 \times 10^3}{1340 \times 10^{-6}} \qquad \sigma_m = 145.7$ MPa ◄

Shearing stress τ_m: $\quad \tau_m = \frac{|V|_{max}}{A_{web}} = \frac{|V|_{max}}{t_w d} = \frac{101.7 \times 10^3}{(0.0089)(0.525)}$

$\tau_m = 21.8$ MPa ◄

Max. Normal Stress in Web at B



$$\sigma_a = \frac{|M_B|}{S} = \frac{194.4 \times 10^3}{1340 \times 10^{-6}} = 145.1 \text{ MPa}$$

$$\sigma_b = (145.1)(251.1/262.5) = 138.8 \text{ MPa}$$

$$\tau_b = \frac{40.88 \times 10^3}{(0.0089)(0.525)} = 26.3 \text{ kPa}$$

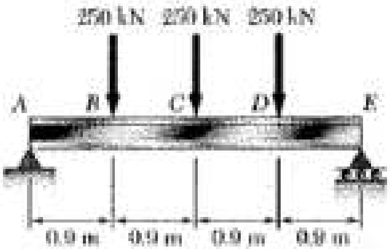
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{\left(\frac{138.8}{2}\right)^2 + (0.0263)^2} = 69.4 \text{ MPa}$$

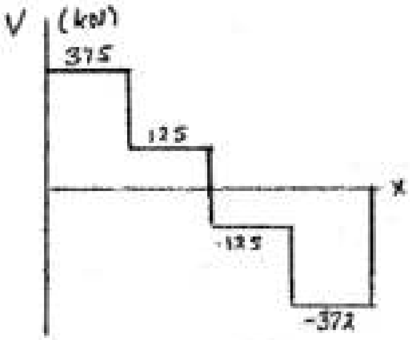
$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{138.8}{2} + 69.4$$

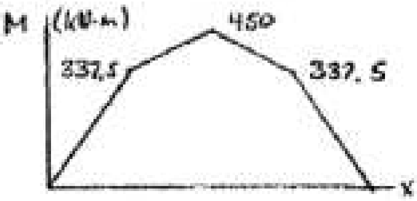
$\sigma_{max} = 138.8$ MPa ◄

Problem 8.7

8.7 and 8.8 (*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.







Reactions: $R_A = 375$ kN ↑ , $R_E = 375$ kN ↑

$|V|_{max} = 375$ kN

$|M|_{max} = 450$ kN·m

$|V|$ at point C : 125 kN

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{450 \times 10^3}{160 \times 10^6} = 2.8125 \times 10^{-3}\ m^3 = 2812.5 \times 10^3\ mm^3$$

Shape	S_x (10^3 mm^3)
W 840 × 176	5890
W 760 × 147	4410
→ W 690 × 125	3510
W 610 × 155	4220
W 530 × 150	3720
W 460 × 158	3340
W 360 × 216	3800

(a) Use W 690 × 125. ◄

$d = 678$ mm

$t_f = 16.30$ mm

$t_w = 11.7$ mm

(b) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450 \times 10^3}{3510 \times 10^{-6}} = 128.2 \times 10^6\ Pa \qquad \sigma_m = 128.2\ MPa ◄$$

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{d\,t_w} = \frac{375 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} \qquad \tau_m = 47.3\ MPa ◄$$

At point C,
$$\tau_w = \frac{V}{A_w} = \frac{125 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 15.76 \times 10^6\ Pa = 15.76\ MPa$$

$$c = \tfrac{1}{2} d = \frac{678}{2} = 339\ mm \qquad y_b = c - t_f = 339 - 16.30 = 322.7\ mm$$

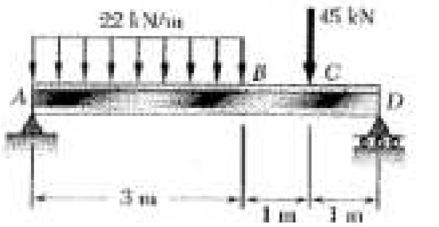
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(128.2) = 122.0\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(61.0)^2 + (15.76)^2} = 63.0\ MPa$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 61.0 + 63.0 \qquad \sigma_{max} = 124.0\ MPa ◄$$

Problem 8.8

8.7 and 8.8 (*a*) Knowing that σ_{all} = 160MPa and τ_{all} = 100 MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$\circlearrowleft \Sigma M_D = 0: \quad -5R_A + (3.5)(3)(22) + (1)(45) = 0$

$R_A = 55.2$ kN ↑ $\qquad R_D = 55.8$ kN ↑

Draw shear and bending moment diagrams.

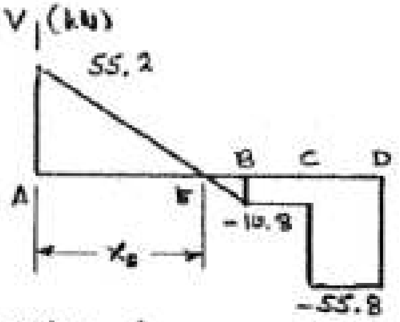
Locate point where $V = 0$ and M_E.

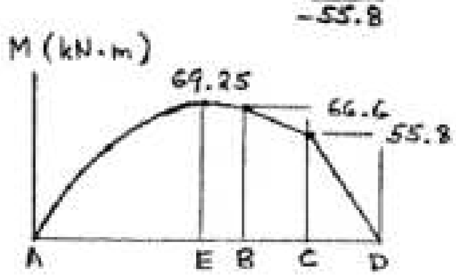
$55.2 - 22x_E = 0 \qquad x_E = 2.5091$ m.

$M_E = \frac{1}{2}(55.2)(2.5091) = 69.25$ kN-m

$$S_{min} = \frac{|M_{max}|}{\sigma_{all}} = \frac{69.25\times10^3}{160\times10^6} = 432.8\times10^{-6}\ m^3$$

$= 432.8\times10^3\ mm^3$





Shape	S (10^3 mm^3)
W 410 × 38.8	637
→ W 360 × 32.9	474
W 310 × 38.7	549
W 250 × 44.8	535
W 200 × 46.1	448

(a) Use W 360 × 32.9 ◄

For W 360 × 32.9 $\qquad S = 474\times10^3\ mm^3 = 474\times10^{-6}\ m^3$

$A_{web} = d\,t_w = (349)(5.8) = 2024.2\ mm^2 = 2024.2\times10^{-6}\ m^2$

At point E: $\quad \sigma_m = \frac{M_E}{S} = \frac{69.25\times10^3}{474\times10^{-6}}$ $\qquad \sigma_m = 146.1$ MPa ◄

At point C: $\quad \sigma_m = \frac{M_C}{S} = \frac{55.8\times10^3}{474\times10^{-6}}$ $\qquad \sigma_m = 117.7$ MPa

$\tau_m = \frac{|V|}{A_{web}} = \frac{55.8\times10^3}{2024.2\times10^{-6}}$ $\qquad \tau_m = 27.6$ MPa ◄

$c = \frac{1}{2}d = \frac{1}{2}(349) = 174.5$ mm $\qquad y_b = c - t_f = 166$ mm

$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{166}{174.5}\right)(117.7) = 112.0$ MPa

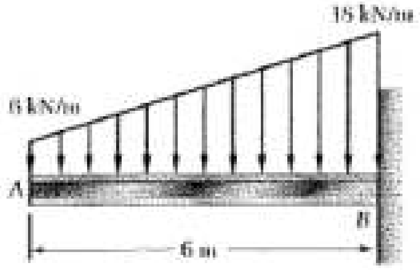
$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = 62.4$ MPa

$\sigma_{max} = \frac{\sigma_b}{2} + R$ $\qquad \sigma_{max} = 118.4$ MPa ◄

Problem 8.9

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

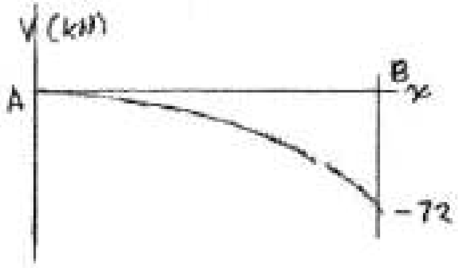
8.9 Loading of Prob. 5.74 and selected W530 × 66 shape.



From Problem 5.74 $\sigma_{all} = 160$ MPa

$|M_{max}| = 180$ kN·m at section $\underline{B}$.

$|V| = 72$ kN at section $\underline{B}$

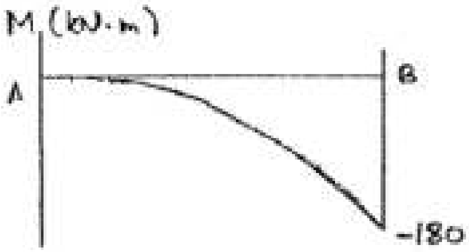


For W530 × 66 rolled steel section

$d = 525$ mm $\quad b_f = 165$ mm $\quad t_f = 11.40$ mm

$t_w = 8.9$ mm $\quad I = 351\times10^6$ mm^4 $S = 1340\times10^3$ mm^3

$c = \frac{1}{2}d = 262.5$ mm



(a) $|M|_{max} = 180\times10^3$ N·m $\quad S = 1340\times10^{-6}$ m^3

$$\sigma_m = \frac{|M|_{max}}{S} = 134.33\times10^6 \qquad \sigma_m = 134.3 \text{ MPa} \blacktriangleleft$$

(b) $y_b = c - t_f = 251.1$ mm $\quad \bar{y} = \frac{1}{2}(c + y_b) = 256.8$ mm

$A_f = b_f t_f = 1881$ mm^2

At section B $\qquad V = 72\times10^3$ N

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It} = \frac{(72\times10^3)(1881\times10^{-6})(256.8\times10^{-3})}{(351\times10^{-6})(8.9\times10^{-3})}$$

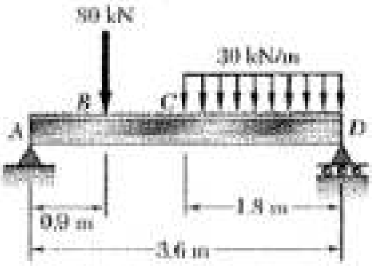
$$= 11.523\times10^6 = 11.523 \text{ MPa}$$

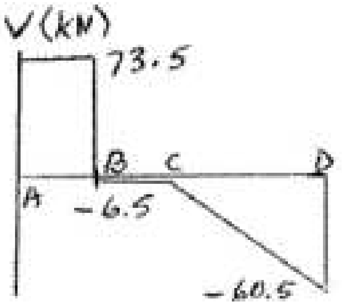
$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{251.1}{262.5}(134.33) = 128.50 \text{ MPa}$$

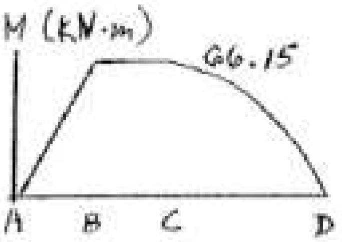
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 65.226 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{128.50}{2} + 65.226 \qquad \sigma_{max} = 129.5 \text{ MPa} \blacktriangleleft$$

Problem 8.11







8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.11 Loading of Prob. 5.78 and selected S310 × 47.3 shape.

From Problem 5.78 $\quad \sigma_{all} = 160$ MPa

$|M|_{max} = 66.15$ kN·m at section B.

$|V| = 73.5$ kN at B

For S 310 × 47.3 rolled steel section

$d = 305$ mm $\quad b_f = 127$ mm $\quad t_f = 13.8$ mm

$t_w = 8.9$ mm $\quad I = 90.6 \times 10^6$ mm^4 $\quad S = 593 \times 10^3$ mm^3

$c = \frac{1}{2}d = 152.5$ mm

(a) $\quad \sigma_m = \dfrac{|M|_{max}}{S} = \dfrac{66.15 \times 10^3}{593 \times 10^{-6}} \qquad \sigma_m = 111.6$ MPa ◄

$y_b = c - t_f = 138.7$ mm

$\sigma_b = \dfrac{y_b}{c}\sigma_m = 101.5$ MPa

$A_f = b_f t_f = 1752.6$ mm^2

$\bar{y} = \frac{1}{2}(c + y_b) = 145.6$ mm

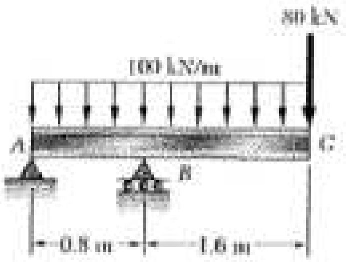
$Q = A_f \bar{y} = 255.18 \times 10^3$ mm^3 $= 255.18 \times 10^{-6}$ m^3

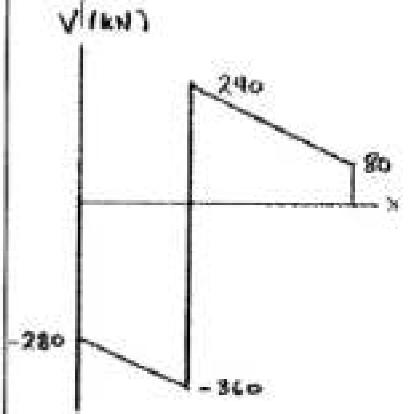
$$\tau_b = \frac{VQ}{I t_w} = \frac{(73.5 \times 10^3)(255.18 \times 10^{-6})}{(90.6 \times 10^{-6})(8.9 \times 10^{-3})} = 23.26 \text{ MPa}$$

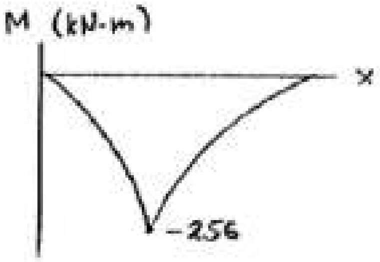
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{50.75^2 + 23.26^2} = 55.83 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 106.6 \text{ MPa} \qquad \sigma_{max} = 106.6 \text{ MPa} ◄$$

Problem 8.12







8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.12 Loading of Prob. 5.75 and selected S20 × 66 shape.

From Problem 5.75 $\quad \sigma_{all} = 160$ MPa

$|M|_{max} = 256$ kN·m at point B

$|V| = 360$ kN at B

For S 20 × 66 rolled steel section

$d = 508$ mm, $b_f = 159$ mm, $t_f = 20.2$ mm

$t_w = 12.8$ mm, $I_x = 495 \times 10^6$ mm^4, $S_x = 1950 \times 10^3$ mm^3

$c = \frac{1}{2}d = 254$ mm

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{256 \times 10^3}{1950 \times 10^{-6}} = 131.3$ MPa

$y_b = c - t_f = 233.8$

$\sigma_b = \dfrac{y_b}{c}\sigma_m = 120.9$ MPa $\qquad \dfrac{\sigma_b}{2} = 60.45$ MPa

$A_f = b_f t_f = 3212$ mm^2

$\bar{y} = \frac{1}{2}(c + y_b) = 243.9$ mm

$Q = A_f \bar{y} = 783.4 \times 10^3$ mm^3

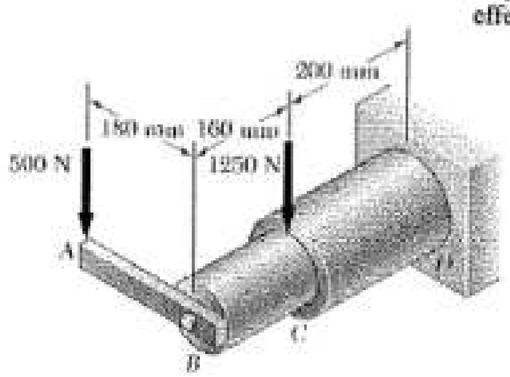
$$\tau_b = \frac{VQ}{I t_w} = \frac{(360 \times 10^3)(783.4 \times 10^{-6})}{(495 \times 10^{-6})(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$$

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 60.45 + 75.06 = 135.5$ MPa

Problem 8.16

8.16 Knowing that rods *BC* and *CD* are of diameter 24 mm and 36 mm, respectively, determine the maximum shearing stress in each rod. Neglect the effect of fillets and of stress concentrations.



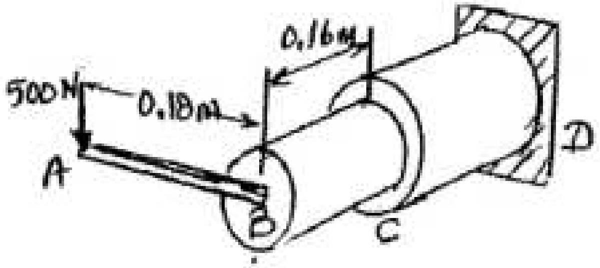
Over BC: $c = \frac{1}{2}d = 12\text{ mm} = 0.012\text{ m}$

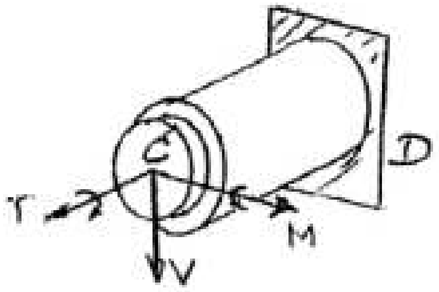
Over CD: $c = \frac{1}{2}d = 18\text{ mm} = 0.018\text{ m}$

$$\tau = \frac{\sqrt{M^2+T^2}\,c}{J} \quad \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{c^3}$$

Bending Moments and Torques

Just to the left of C:



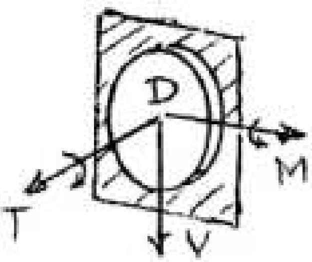


$M = (500)(0.16) = 80\text{ N}\cdot\text{m}$

$T = (500)(0.18) = 90\text{ N}\cdot\text{m}$

$\sqrt{M^2+T^2} = 120.416\text{ N}\cdot\text{m}$

Just to the left of D:

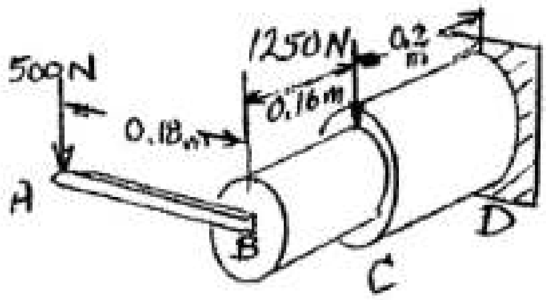


$T = 90\text{ N}\cdot\text{m}$

$M = (500)(0.36) + (1250)(0.2)$

$= 430\text{ N}\cdot\text{m}$

$\sqrt{M^2+T^2} = 439.32\text{ N}\cdot\text{m}$



Maximum shearing stress in portion BC.

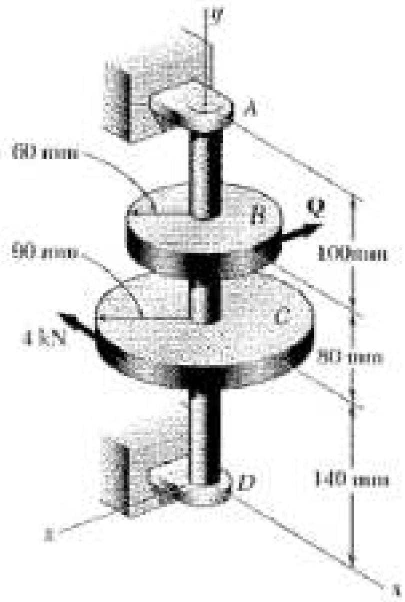
$$\tau_{max} = \frac{(2)(120.416)}{\pi(0.012)^3} = 44.36\times10^6\text{ Pa} \qquad \tau_{max} = 44.4\text{ MPa} ◄$$

Maximum shearing stress in portion CD.

$$\tau_{max} = \frac{(2)(439.32)}{\pi(0.018)^3} = 47.96\times10^6\text{ Pa} \qquad \tau_{max} = 48.0\text{ MPa} ◄$$

Problem 8.18

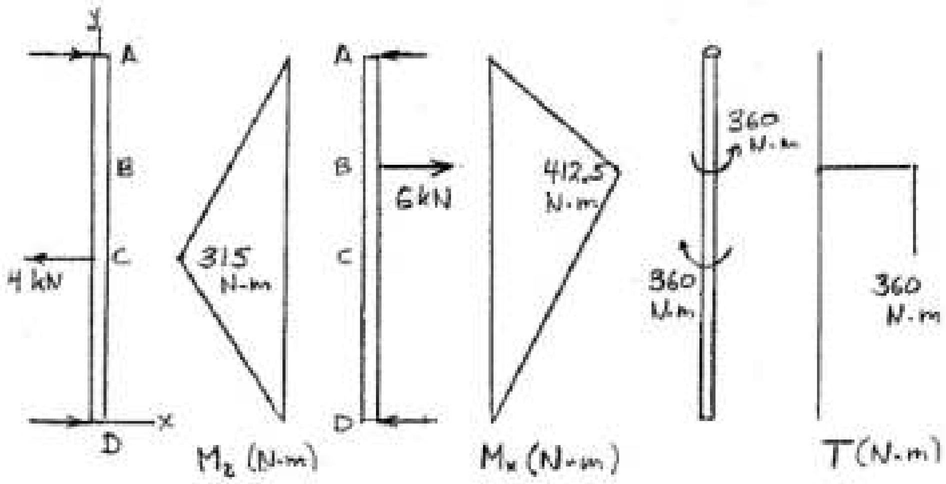
8.18 The 4-kN force is parallel to the x axis, and the force **Q** is parallel to the z axis. The shaft AD is hollow. Knowing that the inner diameter is half the outer diameter and that $\tau_{all} = 60$ MPa, determine the smallest permissible outer diameter of the shaft.



$$\Sigma M_y = 0: \quad 60\times10^{-3}\,Q - (90\times10^{-3})(4\times10^{3}) = 0$$

$$Q = 6\times10^{3}\text{ N} = 6\text{ kN}$$

Bending moment and torque diagrams.



In xy plane $(M_z)_{max} =$ 315 N·m at C.

In yz plane $(M_x)_{max} =$ 412.5 N·m at B.

About z-axis $T_{max} =$ 360 N·m between B and C.

At B; $M_z = \left(\frac{100}{180}\right)(315) = 175$ N·m

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{175^2 + 412.5^2 + 360^2} = 574.79\text{ N·m}$$

At C: $M_x = \left(\frac{140}{220}\right)(412.5) = 262.5$ N·m

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{315^2 + 262.5^2 + 360^2} = 545.65\text{ N·m}$$

Largest value is 574.79 N·m

$$\tau_{max} = \frac{max\sqrt{M_x^2 + M_z^2 + T^2}}{J}\,c$$

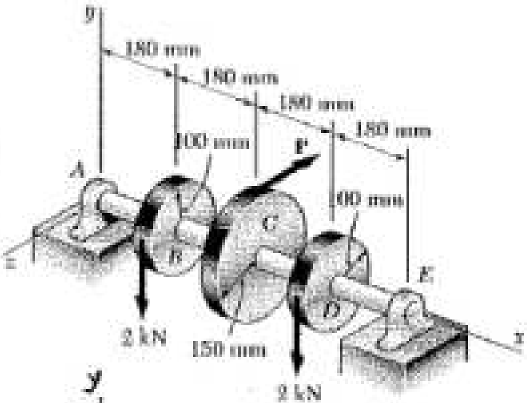
$$\frac{J}{c} = \frac{max\sqrt{M_x^2 + M_z^2 + T^2}}{\tau_{max}} = \frac{574.79}{60\times10^{6}} = 9.5798\times10^{-6}\text{ m}^3 = 9.5798\times10^{3}\text{ mm}^3$$

$$\frac{J}{c} = \frac{\frac{\pi}{2}(c_o^4 - c_i^4)}{c_o} = \frac{\pi}{2}c_o^3\left(1 - \frac{c_i^4}{c_o^4}\right) = \frac{\pi}{2}c_o^3\left[1 - \left(\tfrac{1}{2}\right)^4\right] = 1.47262\,c_o^3$$

$1.47262\,c_o^3 = 9.5798\times10^{3}$ $\qquad c_o = 18.67$ mm

$d_o = 2c_o$ $\qquad d_o = 37.3$ mm ◄

Problem 8.19



8.19 The two 2-kN forces are vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{all} = 55$ MPa, determine the smallest permissible diameter of the solid shaft AE.

$\Sigma M_x = 0 \qquad (2)(100) - 150P + (2)(100) = 0$

$P = 2.67$ kN

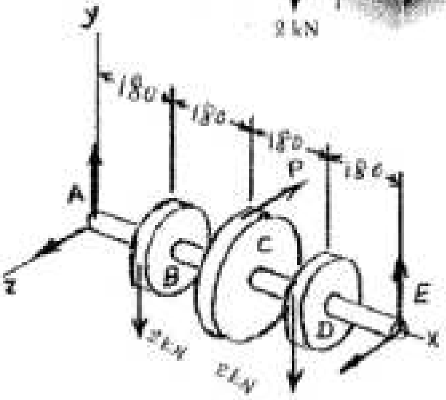
Torques:

AB: $T = 0$

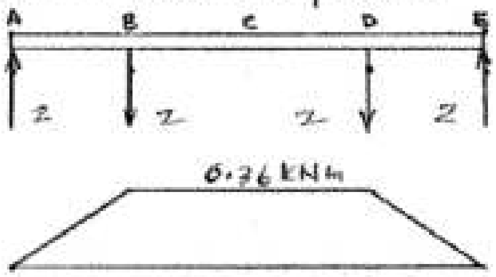
BC: $T = -(2)(0.1) = -0.2$ kNm

CD: $T = (2)(0.1) = 0.2$ kNm

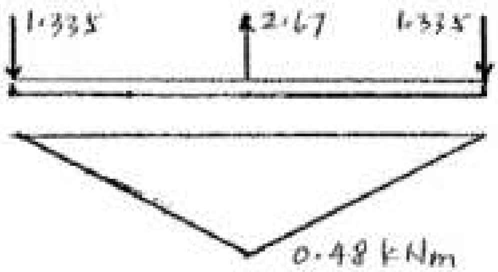
DE: $T = 0$



Forces in vertical plane:



Forces in horizontal plane:



Critical sections are either side of disk C.

$T = 0.2$ kNm $\qquad M_z = 0.36$ kNm

$M_y = 0.48$ kNm

$$\tau_{all} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}} = \frac{\sqrt{0.48^2 + 0.36^2 + 0.2^2}}{55\times10^3} = 11.5\times10^{-6}\ m^3$$

$c = 0.0194$ m $\qquad d = 2c = 38.8$ mm ◄

Problem 8.21

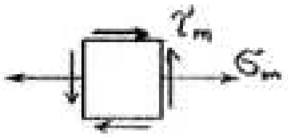
8.21 Using the notation of Sec. 8.3 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a circular shaft can be expressed as follows:

$$\sigma_{max} = \frac{c}{J}\left[(M_y^2 + M_z^2)^{1/2} + (M_y^2 + M_z^2 + T^2)^{1/2}\right]_{max}$$

Maximum bending stress $\quad \sigma_m = \dfrac{|M|c}{I} = \dfrac{\sqrt{M_y^2 + M_z^2}\,c}{I}$

Maximum torsional stress $\quad \tau_m = \dfrac{Tc}{J}$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{2I} = \frac{c}{J}\sqrt{M_y^2 + M_z^2}$$



Using Mohr's circle,

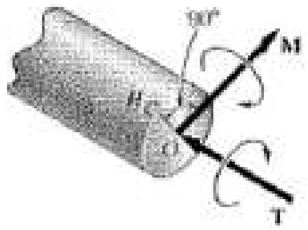
$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2}(M_y^2 + M_z^2) + \frac{T^2c^2}{J^2}}$$

$$= \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

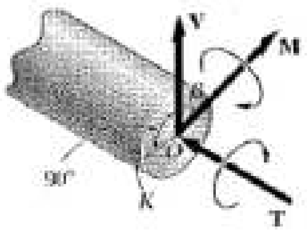
$$\sigma_{max} = \frac{\sigma_m}{2} + R = \frac{c}{J}\sqrt{M_y^2 + M_z^2} + \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$= \frac{c}{J}\left[(M_y^2 + M_z^2)^{\frac{1}{2}} + (M_y^2 + M_z^2 + T^2)^{\frac{1}{2}}\right]$$

Problem 8.22



(a)



(b)

8.22 It was stated in Sec. 8.3 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point H (Fig. P8.22*a*) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point K (Fig. P8.22*b*), where the effect of the shear V is greatest, can be expressed as

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + (\tfrac{2}{3}cV + T)^2}$$

where β is the angle between the vectors **V** and **M**. It is clear that the effect of the shear V cannot be ignored when $\tau_K \geq \tau_H$. (*Hint*: Only the component of **M** along **V** contributes to the shearing stress at K.)

Shearing stress at point K.

Due to V: For a semicircle $Q = \frac{2}{3}c^3$

For a circle cut across its diameter $t = d = 2c$

For a circular section $I = \frac{1}{2}J$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)(\frac{2}{3}c^3)}{(\frac{1}{2}J)(2c)} = \frac{2}{3}\frac{Vc^2}{J}$$

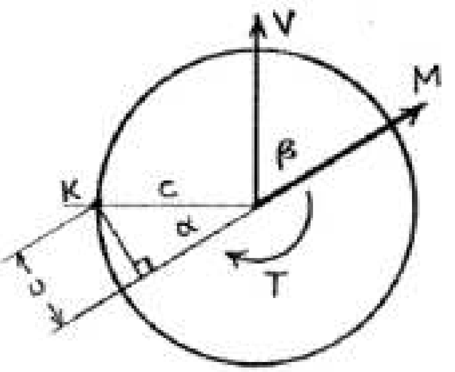
Due to T: $\tau_{xy} = \frac{Tc}{J}$

Since these shearing stresses have the same orientation,

$$\tau_{xy} = \frac{c}{J}\left(\tfrac{2}{3}Vc + T\right)$$

<u>Bending stress at point K:</u> $\sigma_x = \frac{Mu}{I} = \frac{2Mu}{J}$

where u is distance between point K and the neutral axis.



cross section

$$u = c\sin\alpha = c\sin(\tfrac{\pi}{2} - \beta) = c\cos\beta$$

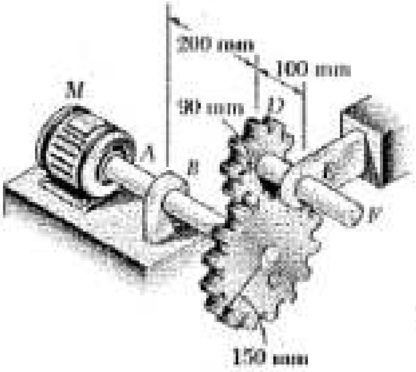
$$\sigma_x = \frac{2Mc\cos\beta}{J}$$

Using Mohr's circle.

$$\tau_K = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{c}{J}\sqrt{(M\cos\beta)^2 + (\tfrac{2}{3}Vc + T)^2}$$

Problem 8.23



8.23 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 15 kW from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 50$ MPa, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$

(a) Shaft ABC: $T = \frac{P}{2\pi f} = \frac{30}{(2\pi)(4)} = 1.194 \text{ kN m}$

Gear C $\quad F_{CD} = \frac{T}{r_C} = \frac{1.194}{0.15} = 7.96 \text{ kN}$

Bending moment at B: $M_B = (0.2)(7.96) = 1.592 \text{ kN}$

$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$

$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(1.194)^2 + (1.592)^2}}{50 \times 10^3} = 39.8 \times 10^{-6} \text{ m}^3$

$c = 0.0294 \text{ m} \qquad d = 2c = 58.8 \text{ mm}$ ◄

(b) Shaft DEF: $T = r_D F_{CD} = (0.09)(7.96) = 0.7164 \text{ kN m}$

Bending moment at E: $M_E = (0.1)(7.96) = 0.796 \text{ kNm}$

$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$

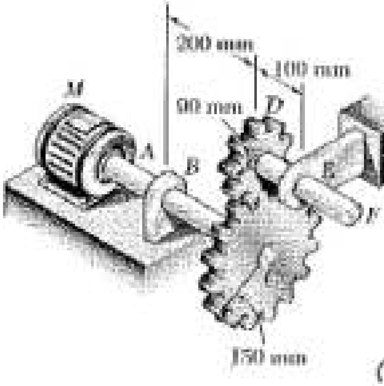
$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(0.796)^2 + (0.7164)^2}}{50 \times 10^3} = 21.418 \times 10^{-6} \text{ m}^3$

$c = 0.0239 \text{ m} \qquad d = 2c = 47.8 \text{ mm}$ ◄

Problem 8.24

8.24 Solve Prob. 8.23, assuming that the motor rotates at 360 rpm.

8.23 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 15 kW from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 50$ MPa, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.



$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

(a) Shaft ABC: $T = \frac{P}{2\pi f} = \frac{30}{(2\pi)(6)} = 0.796 \text{ kNm}$

Gear C $\quad F_{CD} = \frac{T}{r_C} = \frac{0.796}{0.15} = 5.31 \text{ kN}$

Bending moment at B: $\quad M_B = (0.2)(5.31) = 1.062 \text{ kN m}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{1.062^2 + 0.796^2}}{50\times10^3} = 26.544\times10^{-6} \text{ m}^3$$

$c = 0.0257$ m $\qquad d = 2c = 51.4$ mm

(b) Shaft DEF: $\quad T = r_D F_{CD} = (0.09)(5.31) = 0.478 \text{ kNm}$

Bending moment at E: $\quad M_E = (0.1)(5.31) = 0.531 \text{ kNm}$

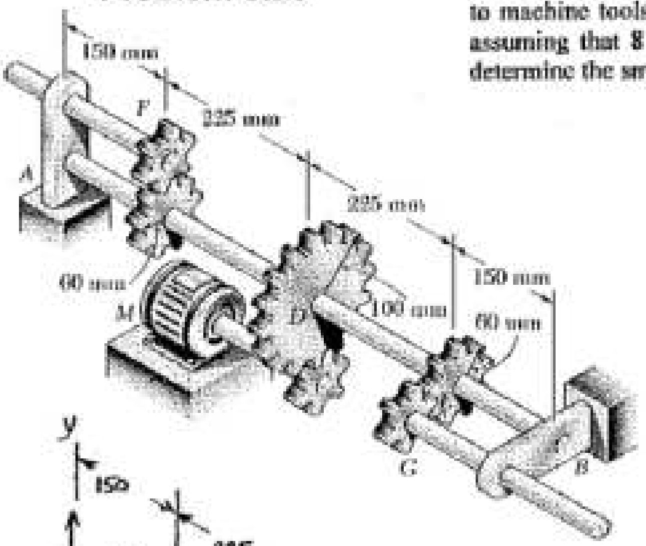
$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

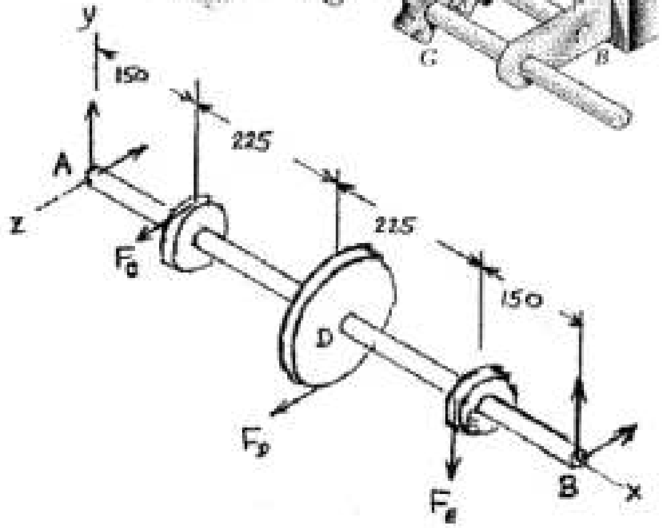
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{0.531^2 + 0.478^2}}{50\times10^3} = 14.29\times10^{-6}$$

$c = 0.0209$ m $\qquad d = 2c = 41.8$ mm

Problem 8.25

8.25 The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G. Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G, determine the smallest permissible diameter of shaft AB.





$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D:

$$T_D = \frac{P}{2\pi f} = \frac{20\times10^3}{(2\pi)(7.5)} = 424.41 \text{ N}\cdot\text{m}$$

Torques on gears C and E:

$$T_C = \frac{8}{20} T_D = 169.76 \text{ N}\cdot\text{m}$$

$$T_E = \frac{12}{20} T_D = 254.65 \text{ N}\cdot\text{m}$$

Forces on gears:

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100\times10^{-3}} = 4244 \text{ N}$$

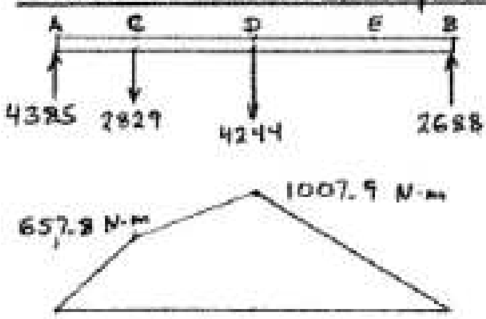
$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60\times10^{-3}} = 2829 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60\times10^{-3}} = 4244 \text{ N}$$

Torques in various parts

AC: $T = 0$
CD: $T = 169.76$ N·m
DE: $T = 254.65$ N·m
EB: $T = 0$

Forces in horizontal plane.

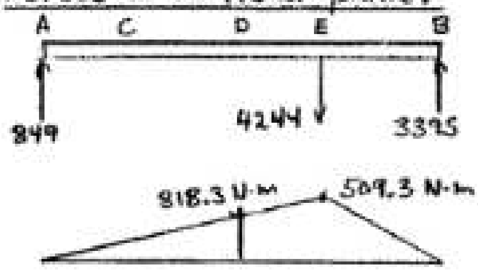


Critical point lies just the right of D.

$T = 254.65$ N·m
$M_y = 1007.9$ N·m
$M_z = 318.3$ N·m

$$\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max} = 1087.2 \text{ N}\cdot\text{m}$$

Forces in vertical plane.



$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}$$

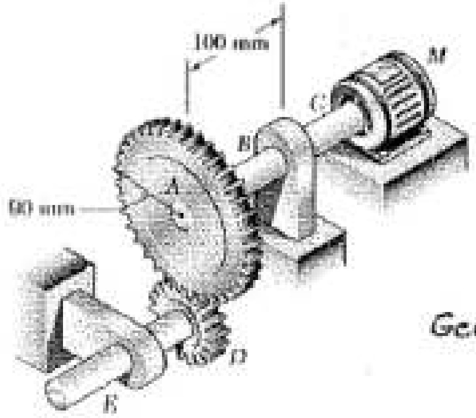
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2+M_z^2+T^2}\right)_{max}}{\tau_{all}} = \frac{1087.2}{55\times10^6}$$

$$= 19.767\times10^{-6} \text{ m}$$

$$c = 23.26\times10^{-3} \text{ m}$$

$$d = 2c = 46.5\times10^{-3} \text{ m} \qquad d = 46.5 \text{ mm}$$

Problem 8.27

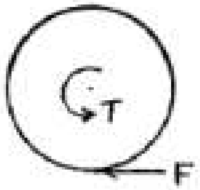


8.27 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A.



$$Fr_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B: $M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}}$$

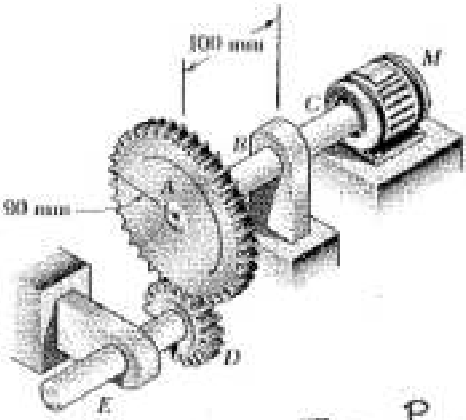
$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{(2)\sqrt{442.1^2 + 397.89^2}}{\pi(60 \times 10^6)} = 6.3108 \times 10^{-6} \text{ m}^3$$

$c = 18.479 \times 10^{-3}$ m $\qquad d = 2c = 37.0 \times 10^{-3}$ m $\qquad d = 37.0$ mm ◄

Problem 8.28

8.28 Assuming that shaft ABC of Prob. 8.27 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.

8.27 The solid shaft ABC and the gears shown are used to transmit 10 kW from the motor M to a machine tool connected to gear D. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft ABC.



$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A.

$$F r_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B: $\quad M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$

$$\tau_{all} = \frac{c_o}{J}\sqrt{M^2 + T^2} \qquad c_o = \tfrac{1}{2} d_o = 25 \times 10^{-3} \text{ m}$$

$$\frac{J}{c_o} = \frac{\pi}{2}\frac{(c_o^4 - c_i^4)}{c_o} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$c_i^4 = c_o^4 - \frac{2c_o\sqrt{M^2 + T^2}}{\pi \tau_{all}} = (25 \times 10^{-3})^4 - \frac{(2)(25 \times 10^{-3})\sqrt{442.1^2 + 397.89^2}}{\pi (60 \times 10^6)}$$

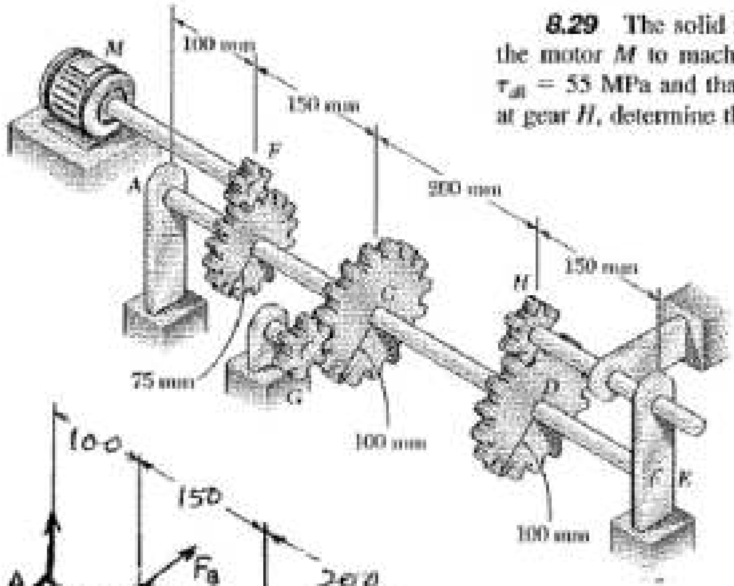
$$= 390.625 \times 10^{-9} - 157.772 \times 10^{-9} = 232.85 \times 10^{-9}$$

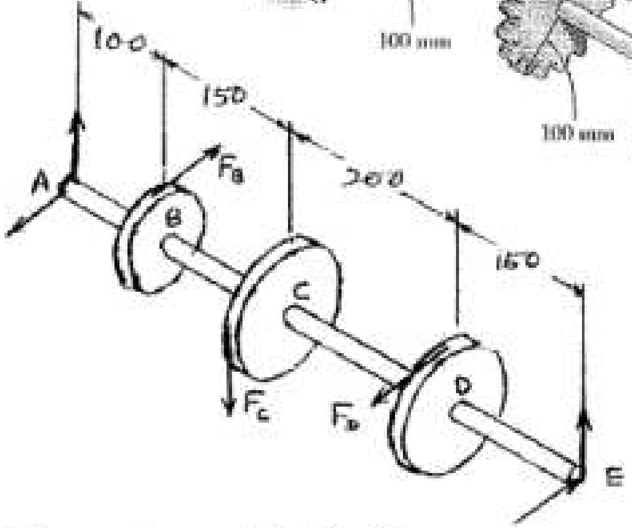
$$c_i = 21.967 \times 10^{-3} \text{ m} \qquad d_i = 2c_i = 43.93 \times 10^{-3} \text{ m} \qquad d = 43.9 \text{ mm} \blacktriangleleft$$

Problem 8.30

8.30 Solve Prob. 8.29, assuming that 22 kW is taken off at gear G and 22 kW is taken off at gear H.

8.29 The solid shaft AE rotates at 600 rpm and transmits 45 kW from the motor M to machine tools connected to gears G and H. Knowing that $\tau_{all} = 55$ MPa and that 30 kW is taken off at gear G and 15 kW is taken off at gear H, determine the smallest permissible diameter of shaft AE.





$f = \dfrac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$

Torque on gear B

$T_B = \dfrac{P}{2\pi f} = \dfrac{45000}{2\pi(10)}$

$= 716.2 \text{ Nm}$

Torques on gears C and D

$T_C = \dfrac{22}{45} T_B = 350 \text{ Nm}$

$T_D = \dfrac{22}{45} T_B = 350 \text{ Nm}$

Shaft torques

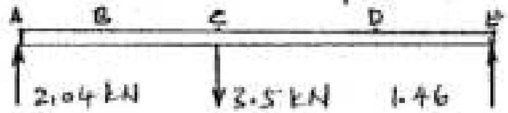
AB: $T_{AB} = 0$

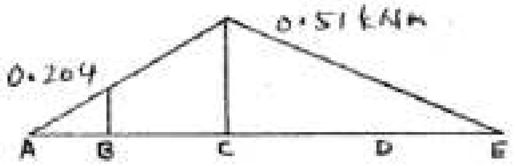
BC: $T_{BC} = 716.2 \text{ Nm}$

CD: $T_{CD} = 350 \text{ Nm}$

DE: $T_{DE} = 0$

Forces in vertical plane





Gear forces

$F_B = \dfrac{T_B}{r_B} = \dfrac{716.2}{0.075} = 9.55 \text{ kN}$

$F_C = \dfrac{T_C}{r_C} = \dfrac{350}{0.1} = 3.5 \text{ kN}$

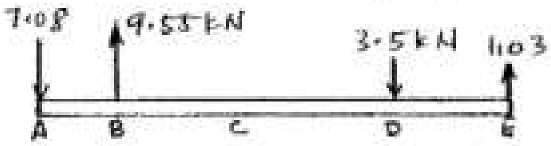
$F_D = \dfrac{T_D}{r_D} = \dfrac{350}{0.1} = 3.5 \text{ kN}$

At B^+ $\sqrt{M_z^2 + M_y^2 + T^2}$

$= \sqrt{0.204^2 + 0.708^2 + 0.7162^2}$

$= 1.0275 \text{ kNm}$ (maximum)

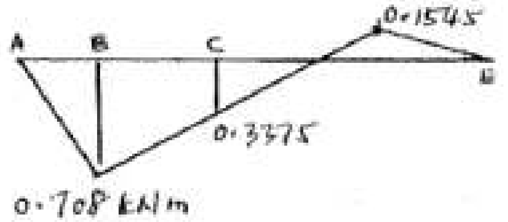
Forces in horizontal plane



At C^- $\sqrt{M_z^2 + M_y^2 + T^2}$

$= \sqrt{0.51^2 + 0.3375^2 + 0.7162^2}$

$= 0.942 \text{ kNm}$



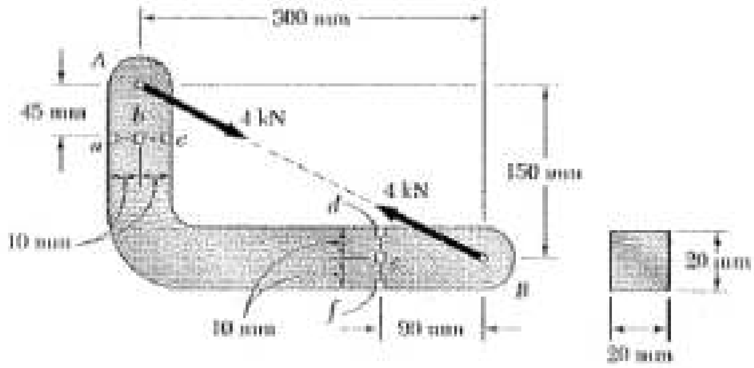
$\tau_{all} = \dfrac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{max}$

$\dfrac{J}{c} = \dfrac{\pi}{2} c^3 = \dfrac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{all}} = \dfrac{1.0275}{55 \times 10^3} = 18.682 \times 10^{-6} \text{ m}^3$

$c = 0.0228$ $\qquad d = 2c = 45.6 \text{ mm}$

Problem 8.32

8.32 Two 4-kN forces are applied to an L-shaped machine element AB as shown. Determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.

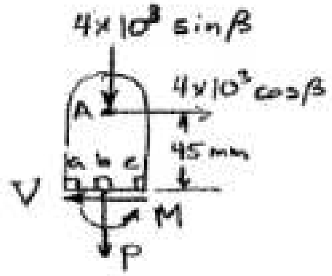


Let β be the slope angle of line AB.

$\tan\beta = \frac{150}{300}$ $\beta = 26.565°$

Draw free body sketch of the portion of the machine element lying above section $\underline{abc}$.

$P = -(4\times10^3)\sin\beta$
$= -1.78885\times10^3$ N



$V = (4\times10^3)\cos\beta = 3.5777\times10^3$ N

$M = (45\times10^{-3})(4\times10^3)\cos\beta = 160.997$ N·m

Section properties: $A = (20)(20) = 400\text{ mm}^2 = 400\times10^{-6}\text{ m}^2$

$I = \frac{1}{12}(20)(20)^3 = 13.3333\times10^3\text{ mm}^4 = 13.3333\times10^{-9}\text{ m}^4$

(a) Point $\underline{a}$. $\sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{1.78885\times10^3}{400\times10^{-6}} - \frac{(160.997)(-10\times10^{-3})}{13.3333\times10^{-9}}$

$= 116.3$ MPa

$\tau = 0$

(b) Point $\underline{b}$. $\sigma = \frac{P}{A} = -\frac{1.78885\times10^3}{400\times10^{-6}} = -4.47$ MPa

$Q = (20)(10)(5) = 1000\text{ mm}^3 = 10^{-6}\text{ m}^3$

$\tau = \frac{VQ}{It} = \frac{(3.5777)(10^{-6})}{(13.3333\times10^{-6})(20\times10^{-3})} = 13.42$ MPa

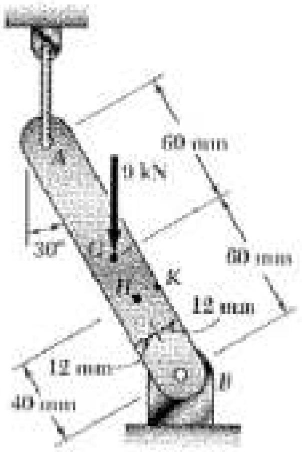
(c) Point $\underline{c}$. $\sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{1.78885\times10^3}{400\times10^{-6}} - \frac{(160.997)(10\times10^{-3})}{13.3333\times10^{-9}}$

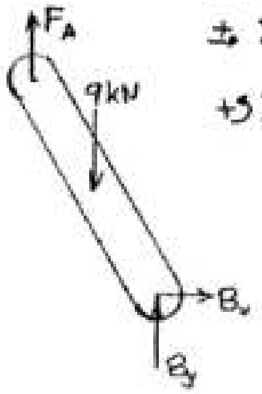
$= -125.2$ MPa

$\tau = 0$

Problem 8.34

8.34 through 8.36 Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point H, (*b*) point K.

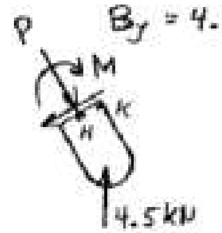




$\xrightarrow{+} \Sigma F_x = 0: \qquad B_x = 0$

$+\circlearrowleft \Sigma M_A = 0: \qquad B_y(120 \sin 30° - 9(60 \sin 30°) = 0$

$B_y = 4.5 \text{ kN}$



At the section containing points H and K,

$P = 4.5 \cos 30° = 3.897 \text{ kN} \qquad V = 4.5 \sin 30° = 2.25 \text{ kN}$

$M = (4.5 \times 10^3)(40 \times 10^{-3} \sin 30°) = 90 \text{ N·m.}$

$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$

(a) <u>At point H</u>: $\sigma_x = -\frac{P}{A} = -\frac{3897 \times 10^3}{240 \times 10^{-6}} = \qquad \sigma = -16.24 \text{ MPa}$ ◄

$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}} \qquad \tau = 14.06 \text{ MPa}$ ◄

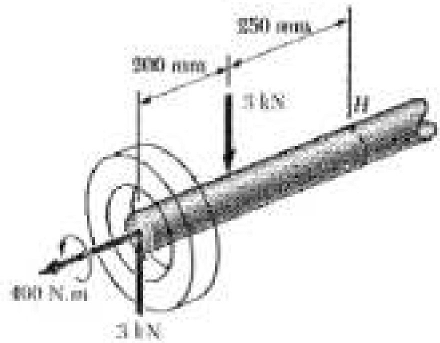
(b) <u>At point K</u>: $\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$

$\sigma = 110.0 \text{ MPa}$ ◄

$\tau = 0$ ◄

Problem 8.37

8.37 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 36 mm, determine the normal and shearing stresses at point H located on the top of the axle.



The bending moment causing normal stress at point H is

$M = (0.2)(3) = 0.6 \text{ kNm}.$

$c = \frac{1}{2}d = 18 \text{ mm}.$

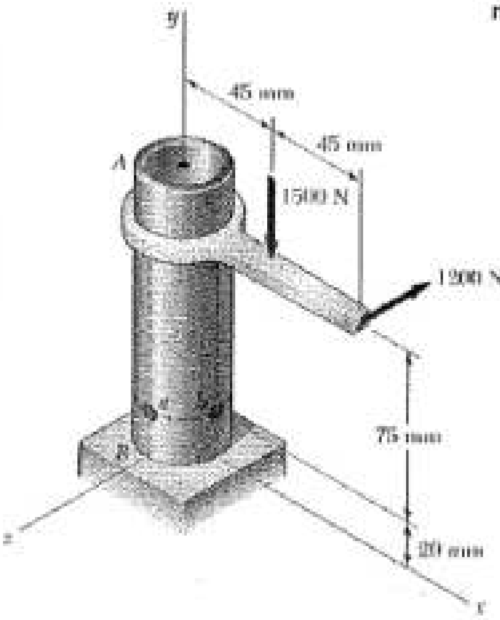
$I = \frac{\pi}{4}c^4 = 82448 \text{ mm}^4, \quad J = 2I = 164896 \text{ mm}^4$

Normal stress at H: $\sigma_H = -\frac{Mc}{I} = -\frac{(0.6\times10^6)(18)}{82448} = -131 \text{ MPa}.$ ◄

At the section containing point H, $V = 0$, $T = 400 \text{Nm}$.

$\tau_H = \frac{Tc}{J} = \frac{(400000)(18)}{164896} = 43.7 \text{ MPa}$ ◄

Problem 8.40



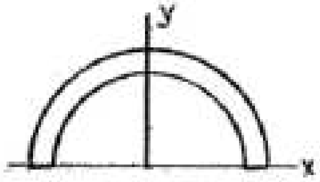
8.40 Two forces are applied to the pipe *AB* as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

$c_o = \frac{d_o}{2} = 21 \text{ mm}, \quad c_i = \frac{d_i}{2} = 17.5 \text{ mm}$

$A = \pi(c_o^2 - c_i^2) = 423.33 \text{ mm}^2$

$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 158.166 \times 10^3 \text{ mm}^4$

$I = \frac{1}{2} J = 79.083 \times 10^3 \text{ mm}^4$



For semicircle with semicircular cutout

$Q = \frac{2}{3}(c_o^3 - c_i^3)$

$Q = 2.6011 \times 10^3 \text{ mm}^3$

At the section containing points a and b

$P = -1500 \text{ N} \qquad V_z = -1200 \text{ N} \qquad V_x = 0$

$M_z = -(45 \times 10^{-3})(1500) = -67.5 \text{ N}\cdot\text{m}$

$M_x = -(75 \times 10^{-3})(1200) = -90 \text{ N}\cdot\text{m}$

$T = (90 \times 10^{-3})(1200) = 108 \text{ N}\cdot\text{m}$

(a) $\sigma = \frac{P}{A} - \frac{M_x c}{I} = \frac{-1500}{423.33 \times 10^{-6}} - \frac{(-90)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$ $\qquad \sigma = 20.4 \text{ MPa}$ ◄

$\tau = \frac{Tc}{J} + \frac{V_x Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + 0$ $\qquad \tau = 14.34 \text{ MPa}$ ◄

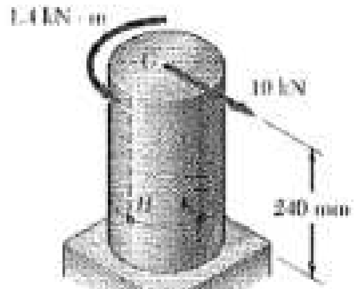
(b) $\sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} + \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$ $\qquad \sigma = -21.5 \text{ MPa}$ ◄

$\tau = \frac{Tc}{J} + \frac{|V_z| Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + \frac{(1200)(2.6011 \times 10^{-6})}{(79.083 \times 10^{-9})(7 \times 10^{-3})}$

$\tau = 19.98 \text{ MPa}$ ◄

Problem 8.41

8.41 A 10-kN force and a 1.4-kN · m couple are applied at the top of the 65-mm diameter cast-iron post shown. Determine the principal stresses and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.



At the section containing points H and K,

$$V = 10 \text{ kN} = 10\times10^3 \text{ N}$$
$$M = (10\times10^3)(240\times10^{-3}) = 2.4\times10^3 \text{ N}\cdot\text{m}$$
$$T = 1.4\times10^3 \text{ N}\cdot\text{m}$$
$$c = \tfrac{1}{2}d = 32.5 \text{ mm} = 0.0325 \text{ m}$$
$$J = \tfrac{\pi}{2}c^4 = 1.75248\times10^{-6} \text{ m}^4$$
$$I = \tfrac{1}{2}J = 0.87624\times10^{-6} \text{ m}^4$$

For a semicircle, $Q = \frac{2}{3}c^3 = \frac{2}{3}(0.0325)^3 = 22.885\times10^{-6} \text{ m}^3$

(a) Stresses at point H. H lies on the neutral axis: $\sigma = 0$

Due to torque: $\tau = \dfrac{Tc}{J} = \dfrac{(1.4\times10^3)(0.0325)}{1.75248\times10^{-6}} = 25.963$ MPa

Due to shear: $\tau = \dfrac{VQ}{It} = \dfrac{(10\times10^3)(22.885\times10^{-6})}{(0.87624\times10^{-6})(0.065)} = 4.018$ MPa

Total at H: $\tau = 30.0$ MPa

$\sigma_c = 0$, $R = 30.0$ MPa

$\sigma_{max} = \sigma_c + R$ $\qquad \sigma_{max} = 30.0$ MPa ◄

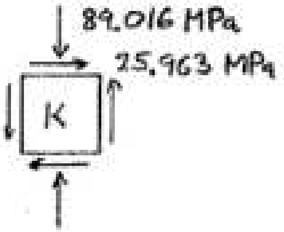
$\sigma_{min} = \sigma_c - R$ $\qquad \sigma_{min} = -30.0$ MPa ◄

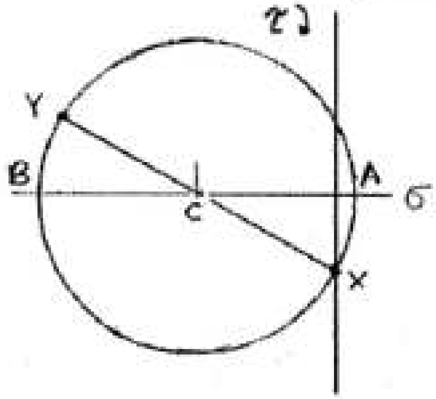
$\tau_{max} = R$ $\qquad \tau_{max} = 30.0$ MPa ◄

(b) Stresses at point K. Due to shear: $\tau = 0$

Due to torque: $\tau = \dfrac{Tc}{J} = 25.963$ MPa

Due to bending: $\sigma = -\dfrac{Mc}{I} = -\dfrac{(2.4\times10^3)(0.0325)}{(0.87624\times10^{-6})} = -89.016$ MPa





$\sigma_c = \dfrac{-89.016}{2} = -44.508$ MPa

$R = \sqrt{\left(\dfrac{89.016}{2}\right)^2 + (25.963)^2}$

$= 51.527$ MPa

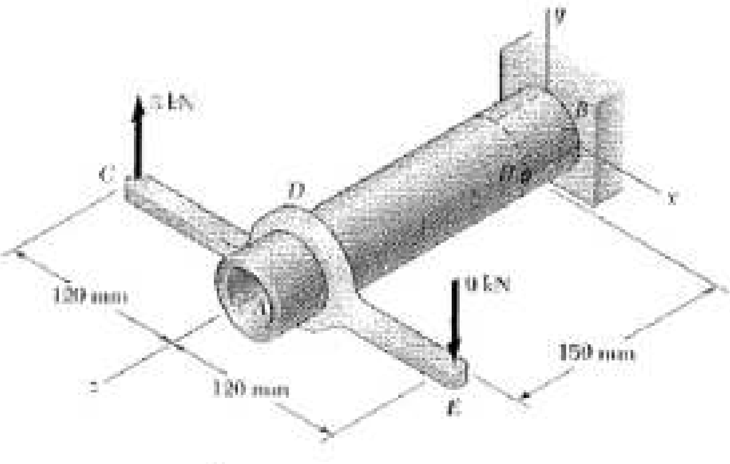
$\sigma_{max} = \sigma_c + R$ $\qquad \sigma_{max} = 7.02$ MPa ◄

$\sigma_{min} = \sigma_c - R$ $\qquad \sigma_{min} = -96.0$ MPa ◄

$\tau_{max} = R$ $\qquad \tau_{max} = 51.5$ MPa ◄

Problem 8.44

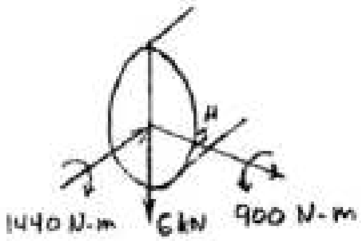
8.44 The steel pipe *AB* has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that the arm *CDE* is rigidly attached to the pipe, determine the principal stresses, principal planes, and the maximum shearing stress at point *H*.



Replace the forces at C and E by an equivalent force-couple system at D.

$F_D = 9 - 3 = 6 \text{ kN} \downarrow$

$T_D = (9\times10^3)(120\times10^{-3}) + (3\times10^3)(120\times10^{-3}) = 1440 \text{ N}\cdot\text{m}$



At the section containing point H,

$P = 0$, $V = 6 \text{ kN}$, $T = 1440 \text{ N}\cdot\text{m}$

$M = (6\times10^3)(150\times10^{-3}) = 900 \text{ N}\cdot\text{m}$

Section properties: $d_o = 72$ mm $\quad c_o = \frac{1}{2}d_o = 36$ mm $\quad c_i = c_o - t = 31$ mm

$A = \pi(c_o^2 - c_i^2) = 1.0524\times10^3 \text{ mm}^2 = 1.0524\times10^{-3} \text{ m}^2$

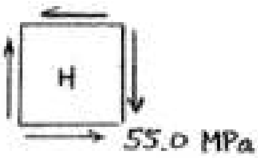
$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84\times10^{3} \text{ mm}^4 = 593.84\times10^{-9} \text{ m}^4$

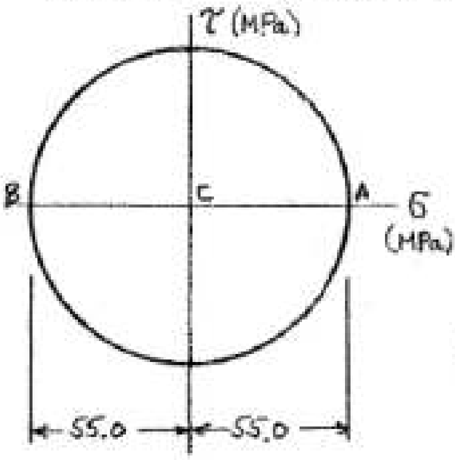
$J = 2I = 1.1877\times10^{-6} \text{ m}^4$

For half-pipe, $Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243\times10^3 \text{ mm}^3 = 11.243\times10^{-6} \text{ m}^3$

At point H. Point H lies on the neutral axis of bending. $\sigma_H = 0$.

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} + \frac{(6\times10^3)(11.243\times10^{-6})}{(593.84\times10^{-9})(10\times10^{-3})} = 55.0 \text{ MPa}$$





Use Mohr's circle.

$\sigma_c = 0$

$R = 55.0$ MPa

$\sigma_a = \sigma_c + R \qquad \sigma_a = 55.0 \text{ MPa}$

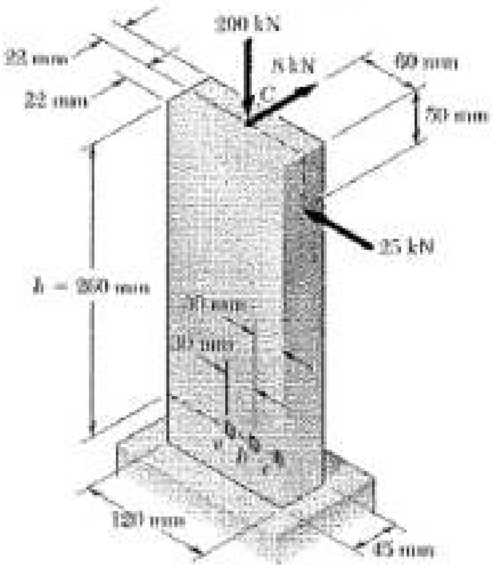
$\sigma_b = \sigma_c - R \qquad \sigma_b = -55.0 \text{ MPa}$

$\theta_a = -45^\circ, \quad \theta_b = +45^\circ$

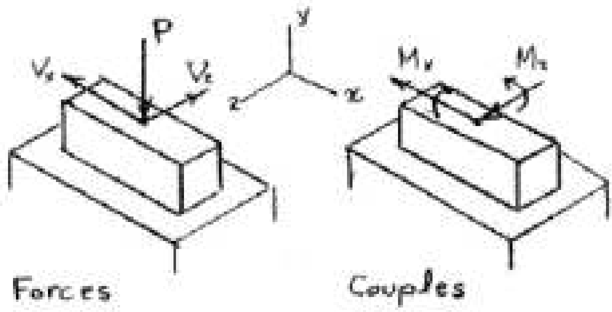
$\tau_{max} = R \qquad \tau_{max} = 55.0 \text{ MPa}$

Problem 8.45

8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.



Calculate forces and couples at section containing points $\underline{a}$, $\underline{b}$, and $\underline{c}$. $h = 260$ mm



$P = 200\,kN \qquad V_x = 25\,kN \qquad V_z = 8\,kN$

$M_z = (260 - 50)(25) = 5250\ kN \cdot mm$

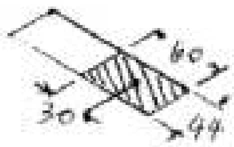
$M_x = (260)(8) = 2080\ kN \cdot mm$

Section properties. $A = (120)(44) = 5280\ mm^2$

$I_x = \frac{1}{12}(120)(44)^3 = 851840 \qquad I_z = \frac{1}{12}(44)(120)^3 = 6.336 \times 10^6\ mm^4$

Stresses $\qquad \sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$

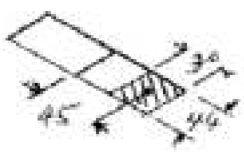
(a) Point $\underline{a}$: $\quad x = 0, \quad z = 22\,mm, \quad Q = (44)(60)\left(\frac{60}{2}\right) = 79200\ mm^3$



$\sigma = -\frac{200 \times 10^3}{5280} + 0 + \frac{(2.08 \times 10^6)(22)}{0.851840 \times 10^6} = 15.8\ MPa$ ◄

$\tau = \frac{(25 \times 10^3)(79200)}{(6.336 \times 10^6)(44)} = 7.1\ MPa$ ◄

(b) Point $\underline{b}$: $\quad x = 30\,mm, \quad z = 22\,mm, \quad Q = (44)(30)(45) = 59400\ mm^3$



$\sigma = -\frac{200 \times 10^3}{5280} + \frac{(5.25 \times 10^6)(30)}{6.336 \times 10^6} + \frac{(2.08 \times 10^6)(22)}{0.85184 \times 10^6} = 40.7\ MPa$ ◄

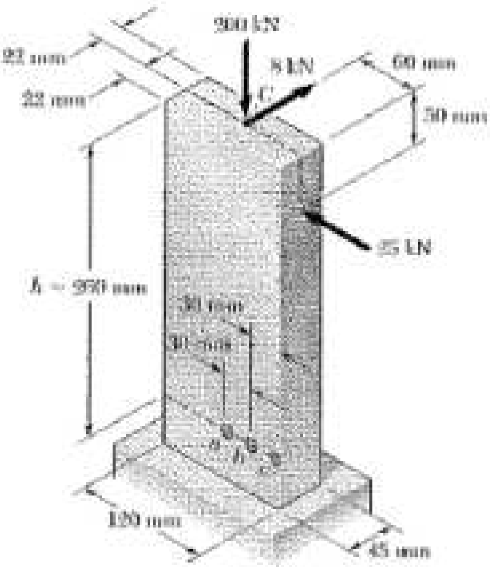
$\tau = \frac{(25 \times 10^3)(59400)}{[(6.336 \times 10^6)(44)]} = 5.3\ MPa$ ◄

(c) Point $\underline{c}$: $\quad x = 60\,mm, \quad z = 22\,mm, \quad Q = 0$

$\sigma = -\frac{200 \times 10^3}{5280} + \frac{(5.25 \times 10^6)(60)}{6.336 \times 10^6} + \frac{(2.08 \times 10^6)(22)}{0.85184 \times 10^6} = 65.6\ MPa$ ◄

$\tau = 0$ ◄

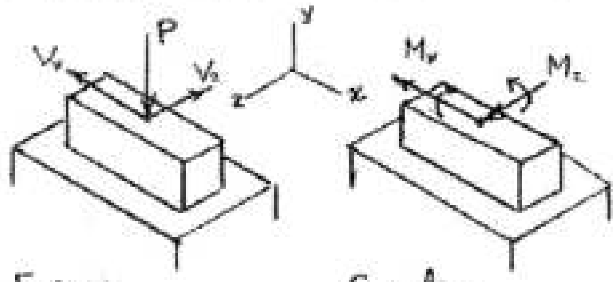
Problem 8.46



8.46 Solve Prob. 8.45, assuming that $h = 300$ mm.

8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

Calculate forces and couples at section containing points $\underline{a}$, $\underline{b}$, and $\underline{c}$. $h = 300$ mm.



$P = 200$ kN $\qquad V_x = 25$ kN $\qquad V_z = 8$ kN

$M_z = (300 - 50)(25) = 6250$ kN·mm

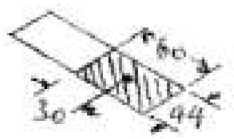
$M_x = (300)(8) = 2400$ kN·mm

Section properties. $A = (120)(44) = 5280$ mm^2

$I_x = \frac{1}{12}(120)(44)^3 = 851840$ mm^4 $\qquad I_z = \frac{1}{12}(44)(120)^3 = 6.336 \times 10^6$ mm^4

Stresses $\qquad \sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$

(a) Point $\underline{a}$: $\quad x = 0, \quad z = 22$ mm, $\quad Q = (44)(60)(30) = 79200$ mm^3



$$\sigma = -\frac{200 \times 10^3}{5280} + 0 + \frac{(2.4 \times 10^6)(22)}{0.85184 \times 10^6} = 24.1 \text{ MPa}$$ ◀

$$\tau = \frac{(25 \times 10^3)(79200)}{(6.336 \times 10^6)(44)} = 7.1 \text{ MPa}$$ ◀

(b) Point $\underline{b}$: $\quad x = 30$ mm $\quad z = 22$ mm, $\quad Q = (30)(44)(45) = 59400$ mm^3

$$\sigma = -\frac{200 \times 10^3}{5280} + \frac{(6.25 \times 10^6)(30)}{6.336 \times 10^6} + \frac{(2.4 \times 10^6)(22)}{0.85184 \times 10^6} = 53.7 \text{ MPa}$$ ◀

$$\tau = \frac{(25 \times 10^3)(59400)}{(6.336 \times 10^6)(44)} = 5.3 \text{ MPa}$$ ◀

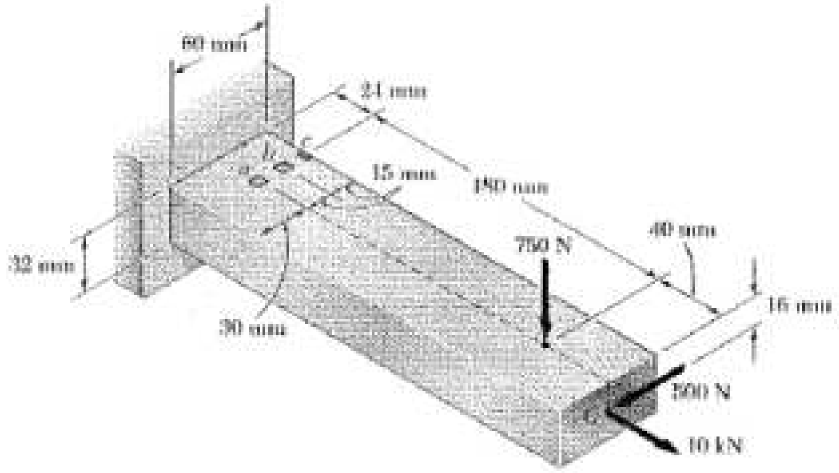
(c) Point $\underline{c}$: $\quad x = 60$ mm, $\quad z = 22$ mm, $\quad Q = 0$

$$\sigma = -\frac{200 \times 10^3}{5280} + \frac{(6.25 \times 10^6)(60)}{(6.336 \times 10^6)} + \frac{(2.4 \times 10^6)(22)}{0.85184 \times 10^6} = 83.3 \text{ MPa}$$ ◀

$$\tau = 0$$ ◀

Problem 8.47

8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.



$A = (60)(32) = 1920\ \text{mm}^2$
$= 1920 \times 10^{-6}\ \text{m}^2$

$I_z = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3\ \text{mm}^4$
$= 163.84 \times 10^{-9}\ \text{m}^4$

$I_y = \frac{1}{12}(32)(60)^3 = 576 \times 10^3\ \text{mm}^4$
$= 576 \times 10^{-9}\ \text{m}^4$

At the section containing points a, b, and c

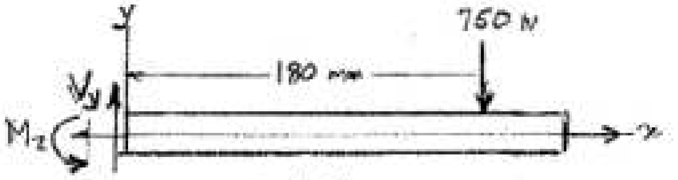
$P = 10\ \text{kN}$

$V_y = 750\ \text{N}, \quad V_z = 500\ \text{N}$

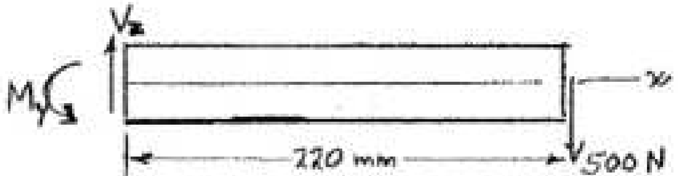
$M_z = (180 \times 10^{-3})(750)$
$= 135\ \text{N}\cdot\text{m}$

$M_y = (220 \times 10^{-3})(500)$
$= 110\ \text{N}\cdot\text{m}$

$T = 0$



Side View

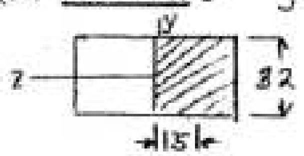


Top View

$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$

$$\tau = \frac{V_z Q}{I_y t}$$

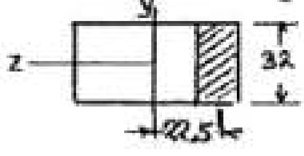
(a) Point a. $y = 16\ \text{mm}, \ z = 0, \quad Q = A\bar{z} = (32)(30)(15) = 14.4 \times 10^3\ \text{mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - 0 \qquad \sigma = 18.39\ \text{MPa}$$

$$\tau = \frac{(500)(14.4 \times 10^{-6})}{(576 \times 10^{-9})(32 \times 10^{-3})} \qquad \tau = 0.391\ \text{MPa}$$

(b) Point b. $y = 16\ \text{mm}, \ z = -15\ \text{mm}, \quad Q = A\bar{z} = (32)(15)(22.5) = 10.8 \times 10^3\ \text{mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-15 \times 10^{-3})}{576 \times 10^{-9}}$$

$\sigma = 21.3\ \text{MPa}$

$$\tau = \frac{(500)(10.8 \times 10^{-6})}{(576 \times 10^{-9})(32 \times 10^{-3})} \qquad \tau \ \ 0.293\ \text{MPa}$$

(c) Point c. $y = 16\ \text{mm}, \ z = -30\ \text{mm}, \quad Q = 0$

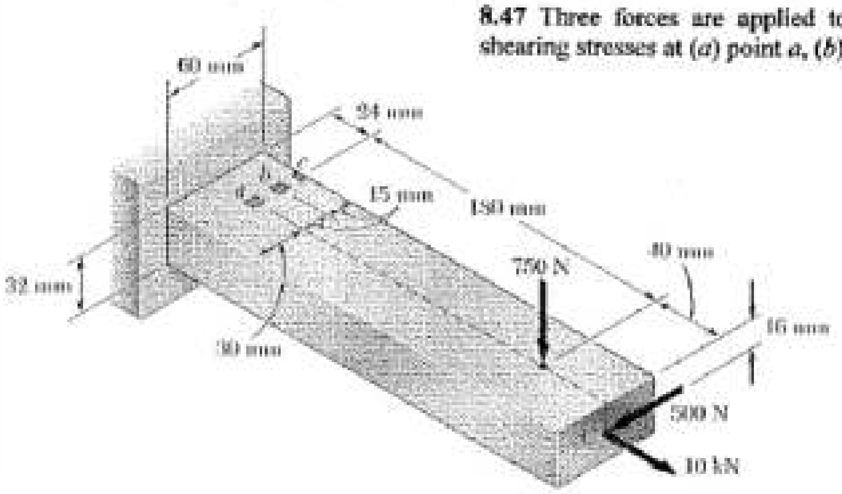
$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-30 \times 10^{-3})}{576 \times 10^{-9}} \qquad \sigma = 24.1\ \text{MPa}$$

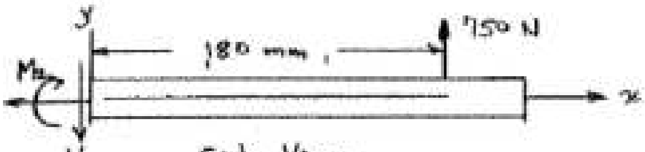
$\tau = 0$

Problem 8.48

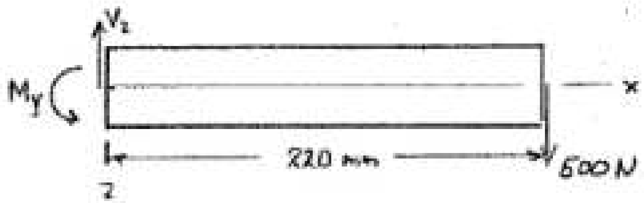
8.48 Solve Prob. 8.47, assuming that the 750-N force is directed vertically upward.

8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.





Side View



Top View

$A = (60)(32) = 1920\ \text{mm}^2 = 1920\times10^{-6}\ \text{m}^2$

$I_z = \frac{1}{12}(60)(32)^3 = 163.84\times10^3\ \text{mm}^4 = 163.84\times10^{-9}\ \text{m}^4$

$I_y = \frac{1}{12}(32)(60)^3 = 576\times10^3\ \text{mm}^4 = 576\times10^{-9}\ \text{m}^4$

At the section containing points $\underline{a}$, $\underline{b}$, and $\underline{c}$

$P = 10\ \text{kN}$, $T = 0$

$V_y = 750\ \text{N}$, $V_z = 500\ \text{N}$

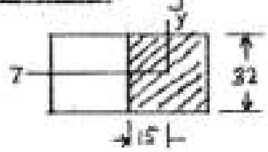
$M_z = (180\times10^{-3})(750) = 135\ \text{N}\cdot\text{m}$

$M_y = (220\times10^{-3})(500) = 110\ \text{N}\cdot\text{m}$

$\sigma = \frac{P}{A} - \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$

$\tau = \frac{V_z Q}{I_z t}$

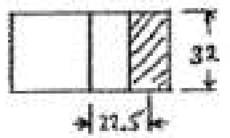
(a) Point a. $y = 16\ \text{mm}$, $z = 0$, $Q = A\bar{z} = (32)(30)(15) = 14.4\times10^3\ \text{mm}^3$



$\sigma = \frac{10\times10^3}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - 0$ $\quad \sigma = -7.98\ \text{MPa}$ ◄

$\tau = \frac{(500)(14.4\times10^{-6})}{(163.84\times10^{-9})(32\times10^{-3})}$ $\quad \tau = 0.391\ \text{MPa}$ ◄

(b) Point b. $y = 16\ \text{mm}$, $z = -15\ \text{mm}$, $Q = A\bar{z} = (32)(15)(22.5) = 10.8\times10^3\ \text{mm}^3$



$\sigma = \frac{10\times10^3}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-15\times10^{-3})}{576\times10^{-9}}$ $\quad \sigma = -5.11\ \text{MPa}$ ◄

$\tau = \frac{(500)(10.8\times10^{-6})}{(163.84\times10^{-9})(32\times10^{-3})}$ $\quad \tau = 0.293\ \text{MPa}$ ◄

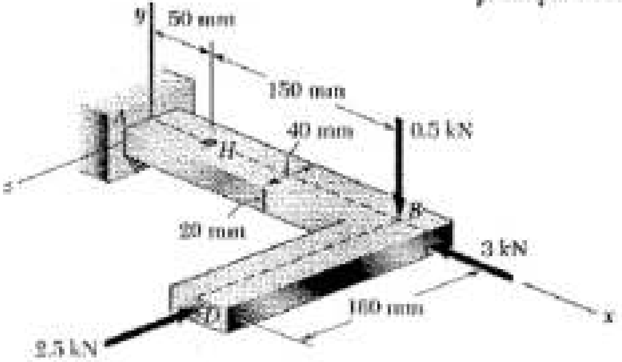
(c) Point c. $y = 16\ \text{mm}$, $z = -30\ \text{mm}$, $Q = 0$

$\sigma = \frac{10\times10^3}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-30\times10^{-3})}{576\times10^{-9}}$ $\quad \sigma = -2.25\ \text{MPa}$ ◄

$\tau = 0$ ◄

Problem 8.49

8.49 Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20 × 40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

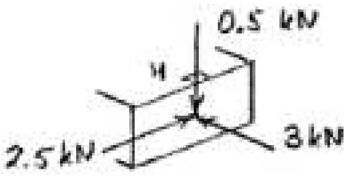


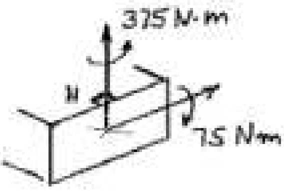
Equivalent force-couple system at section containing point H:

$F_x = -3\text{ kN}, \quad F_y = -0.5\text{ kN}, \quad F_z = -2.5\text{ kN}$

$M_x = 0, \quad M_y = (0.150)(2500) = 375\text{ N-m}$

$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$





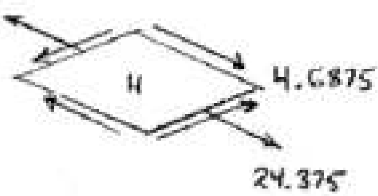
$A = (20)(40) = 800\text{ mm}^2 = 800 \times 10^{-6}\text{ m}^2$

$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3\text{ mm}^4 = 26.667 \times 10^{-9}\text{ m}^4$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{2500}{800 \times 10^{-6}} = 4.6875\text{ MPa}$$

Use Mohr's circle.



$\sigma_c = \frac{1}{2}\sigma_H = 12.1875\text{ MPa}$

$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2} = 13.0579\text{ MPa}$

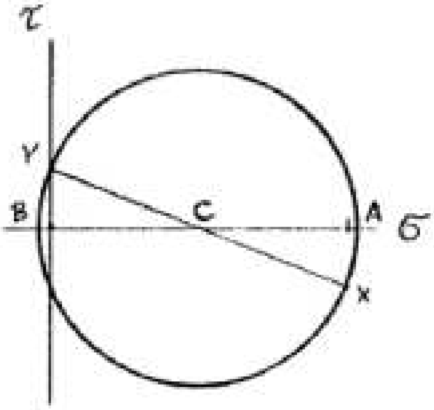
$\sigma_a = \sigma_c + R \qquad \sigma_a = 25.2\text{ MPa}$ ◄

$\sigma_b = \sigma_c - R \qquad \sigma_b = -0.87\text{ MPa}$ ◄

$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$

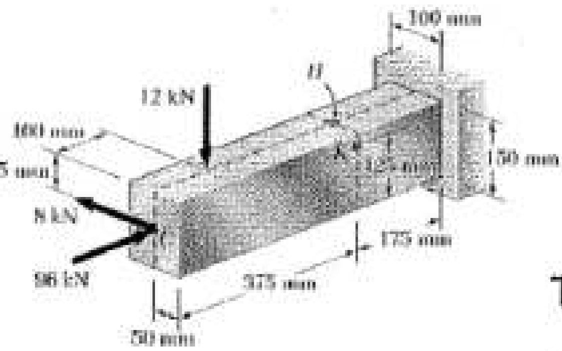
$\theta_a = 10.5°, \quad \theta_b = 100.5°$

$\tau_{max} = R \qquad \tau_{max} = 13.06\text{ MPa}$ ◄



Problem 8.51

8.51 Three forces are applied to the cantilever beam shown. Determine the principal stresses and the maximum shearing stress at point H.



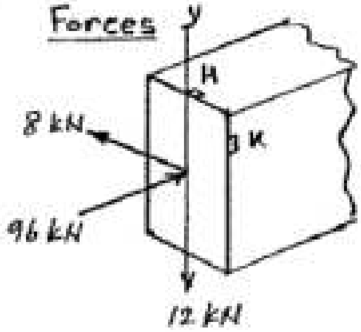
At the section containing points H and K, the axial and shearing forces are:

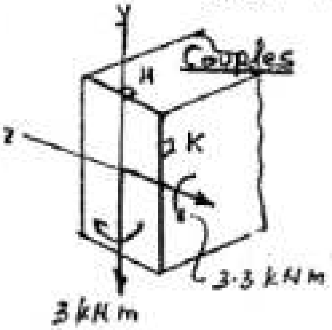
$P = 96\ kN,\qquad V = \begin{matrix} 12\ kN & \text{vertical} \\ 8\ kN & \text{horizontal} \end{matrix}$

The bending moment components are:

about horizontal axis: $M = (0.375 - 0.1)(12000) = 3.3\ kN\,m$

about vertical axis $M = (0.375)(8000) = 3\ kN\,m$





Section properties

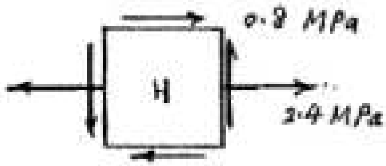
$A = (0.1)(0.15) = 0.015\ m^2$

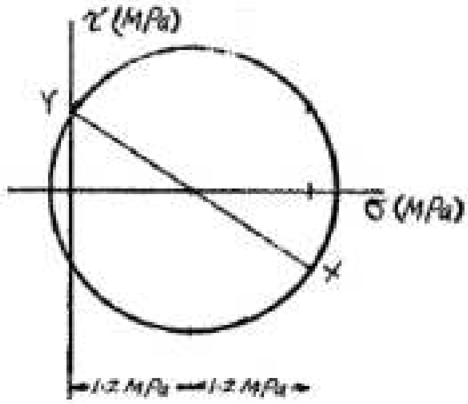
$I_z = \frac{1}{12}(0.1)(0.15)^3 = 28.125 \times 10^{-6}\ m^4$

$I_y = \frac{1}{12}(0.15)(0.1)^3 = 12.5 \times 10^{-6}\ m^4$

At point H; $\sigma = -\frac{P}{A} + \frac{Mc}{I} = -\frac{96000}{0.015} + \frac{(3300)(0.075)}{28.125 \times 10^{-6}}$ $\qquad \sigma = 2.4\ MPa$

$\tau = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{12000}{0.015}$ $\qquad \tau = 0.8\ MPa$





$\sigma_c = \frac{2.4}{2} = 1.2\ MPa$

$R = \sqrt{\left(\frac{2.4}{2}\right)^2 + (0.8)^2} = 1.44\ MPa$

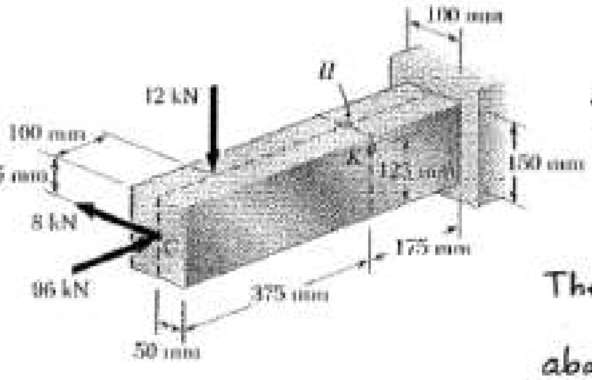
$\sigma_a = \sigma_c + R \qquad \sigma_a = 2.64\ MPa$ ◄

$\sigma_b = \sigma_c - R \qquad \sigma_b = -0.24\ MPa$ ◄

$\tau_{max} = R \qquad \tau_{max} = 1.44\ MPa$ ◄

Problem 8.52

8.52 For the beam and loading of Prob. 8.51, determine the principal stresses and maximum shearing stress at point K.



At the section containing points H and K the axial and shearing forces are

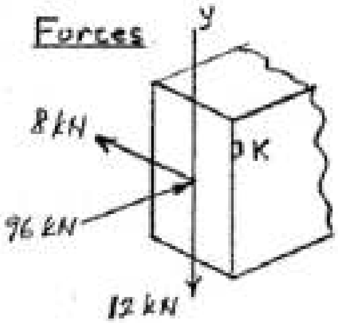
$P = 96$ kN, $V = $ 12 kN vertical, 8 kN horizontal

The bending moment components are

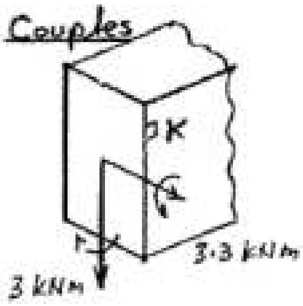
about horizontal axis: $M = (0.375 - 0.1)(12000) = 3300$ Nm

about vertical axis: $M = (0.375)(8000) = 3000$ Nm

Forces



Couples



Section properties

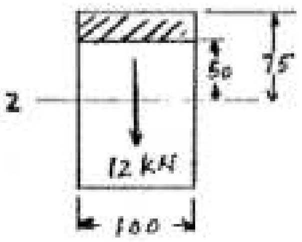
$A = (0.1)(0.15) = 0.015 \text{ m}^2$

$I_z = \frac{1}{12}(0.1)(0.15)^3 = 28.125 \times 10^{-6} \text{ m}^4$

$I_y = \frac{1}{12}(0.15)(0.1)^3 = 12.5 \times 10^{-6} \text{ m}^4$

At point K:

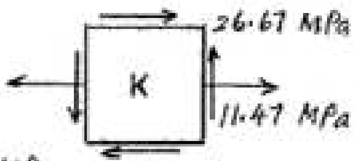
$$\sigma = -\frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = -\frac{96000}{0.015} - \frac{(-3300)(0.05)}{28.125 \times 10^{-6}} + \frac{(-3000)(-0.05)}{12.5 \times 10^{-6}} = 11.47 \text{ MPa}$$

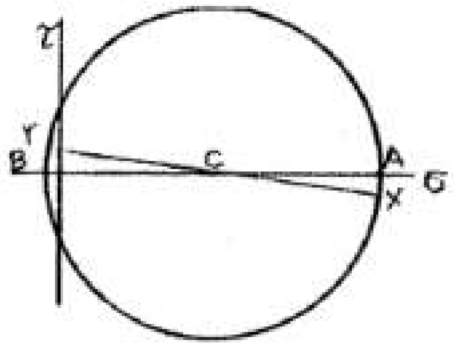


$A^* = (0.025)(0.1) = 0.0025 \text{ m}^2$ $\quad \bar{y} = 62.5$ mm

$Q = A^* \bar{y} = (0.1)(0.0625) = 0.00625 \text{ m}^3$

$\tau = \frac{VQ}{It} = \frac{(12000)(0.00625)}{(28.125 \times 10^{-6})(0.1)} = 26.67$ MPa





$\sigma_c = \frac{11.47}{2} = 5.735$ MPa

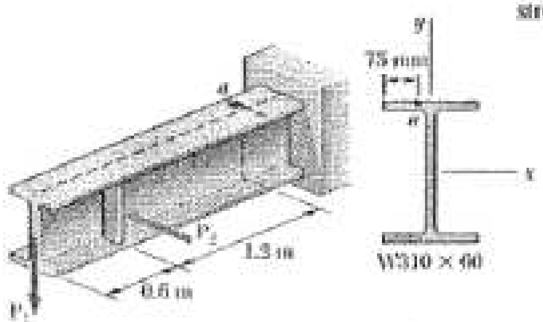
$R = \sqrt{\left(\frac{11.47}{2}\right)^2 + (26.67)^2} = 27.28$ MPa

$\sigma_a = \sigma_c + R$ $\qquad \sigma_a = 33.02$ MPa

$\sigma_b = \sigma_c - R$ $\qquad \sigma_b = -21.55$ MPa

$\tau_{max} = R$ $\qquad \tau_{max} = 27.28$ MPa

Problem 8.55



8.55 Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $\mathbf{P}_1$ = 20 kN and $\mathbf{P}_2$ = 12 kN, determine the principal stresses and the maximum shearing stress at point a.

At the section containing point $\underline{a}$

$M_x = (20)(1.8) = 36\ kNm$

$M_y = -(12)(1.2) = -14.4\ kNm$

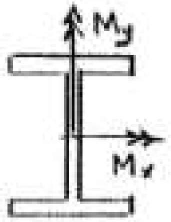
$V_x = -12kN \qquad V_y = -20\ kN.$

For W310X60 rolled steel section

$d = 303\ mm \qquad b_f = 203\ mm \qquad t_f = 13.1\ mm \qquad t_w = 7.5\ mm$

$I_x = 129\times10^6\ mm^4 \quad I_y = 18.3\times10^6\ mm^4$

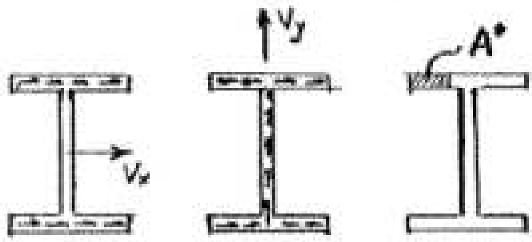
Normal stress at point $\underline{a}$. $\quad x = -\frac{b_f}{2} + 75 = -26.5\ mm \quad y = \frac{d}{2} = 151.5\ mm.$



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(36\times10^6)(151.5)}{129\times10^6} - \frac{(-14.4\times10^6)(-26.5)}{18.3\times10^6}$$

$$= 42.28 - 20.85 = 21.43\ MPa.$$

Shearing stress at point $\underline{a}$.



$$\tau_{xz} = \frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$

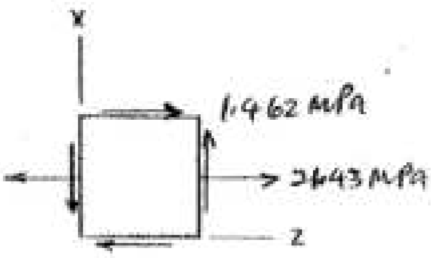
$A^* = (75)(13.1) = 982.5\ mm^2.$

$\bar{x} = -\frac{203}{2} + \frac{75}{2} = -64\ mm$

$\bar{y} = \frac{303}{2} - \frac{13.1}{2} = 144.95\ mm,$

$$\tau_{xz} = \frac{(-12\times10^3)(982.5)(-64)}{(18.3\times10^6)(13.1)} - \frac{(-20\times10^3)(982.5)(144.95)}{(129\times10^6)(13.1)}$$

$$= 3.1475 - 1.6855 = 1.462\ MPa.$$



$\sigma_{ave} = \frac{21.43+0}{2} = 10.715\ MPa.$

$R = \sqrt{\left(\frac{21.43-0}{2}\right)^2 + 1.462^2} = 10.814\ MPa$

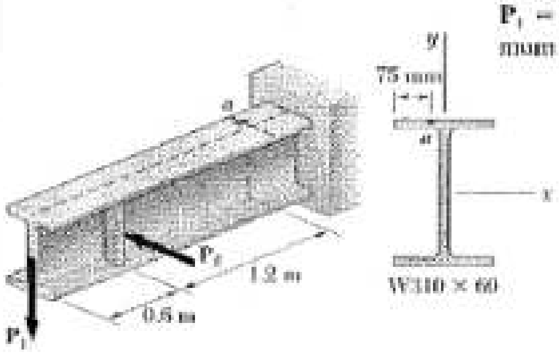
$\sigma_{max} = \sigma_{ave} + R = 10.715 + 10.814 = 21.5\ MPa$ ◄

$\sigma_{min} = \sigma_{ave} - R = 10.715 - 10.814 = -0.099\ MPa$ ◄

$\tau_{max} = R = 10.8\ MPa.$ ◄

Problem 8.56

8.56 Two forces P_1 and P_2 are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $P_1 = 20$ kN and $P_2 = 12$ kN, determine the principal stresses and the maximum shearing stress at point b.



At the section containing point $\underline{b}$,

$M_x = (20)(1.8) = 36$ kNm

$M_y = -(12)(1.2) = -14.4$ kNm

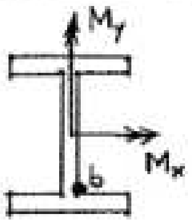
$V_x = -12$ kN $\qquad V_y = -20$ kN

For W 310 × 60 rolled steel section

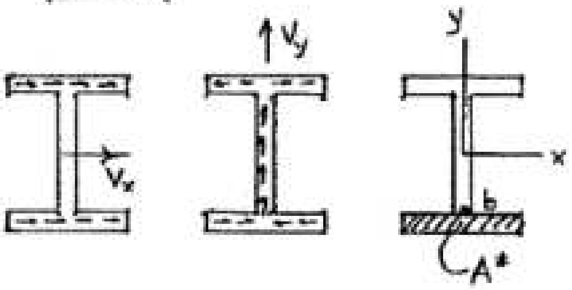
$d = 303$ mm $\qquad b_f = 203$ mm $\qquad t_f = 13.1$ mm $\qquad t_w = 7.5$ mm

$I_x = 129 \times 10^6$ mm^4 $\qquad I_y = 18.3 \times 10^6$ mm^4

Normal stress at point b $\qquad x \approx 0 \qquad y = -\frac{1}{2}d + t_f = -164.6$ mm



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(36000)(-164.6 \times 10^{-3})}{129 \times 10^{-6}} - 0$$
$$= -45.93 \text{ MPa}$$



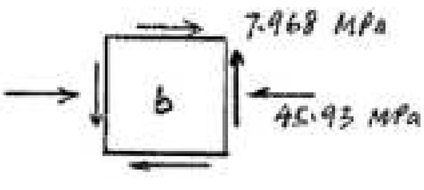
Shearing stress at point b

$$\tau_{yz} = \frac{V_y A^* \bar{y}}{I_x t_w}$$

$A^* = A_f = b_f t_f = 2659.3$ mm^2

$\bar{x} = 0$, $\bar{y} = -\frac{1}{2}d + \frac{1}{2}t_f = -144.95$ mm

$$\tau_{yz} = \frac{(-20000)(2659.3 \times 10^{-6})(-0.14495)}{(129 \times 10^{-6})(0.0075)} = 7.968 \text{ MPa}$$



$$\sigma_c = -\frac{45.93}{2} = -22.965 \text{ MPa}$$

$$R = \sqrt{\left(\frac{45.93}{2}\right)^2 + (7.968)^2} = 24.308 \text{ MPa}$$

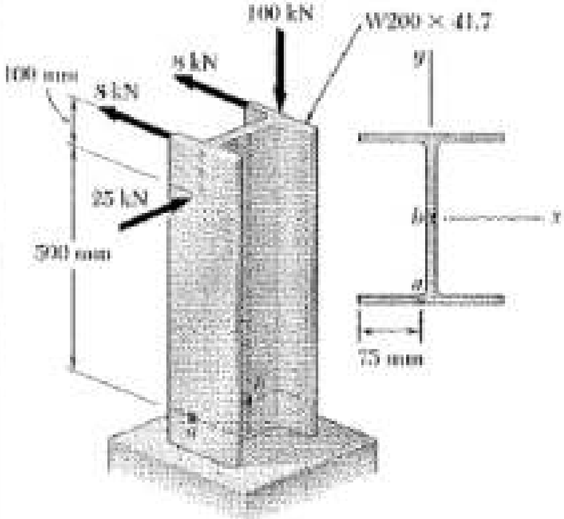
$\sigma_{max} = \sigma_c + R \qquad \sigma_{max} = 1.34$ MPa

$\sigma_{min} = \sigma_c - R \qquad \sigma_{min} = -47.27$ MPa

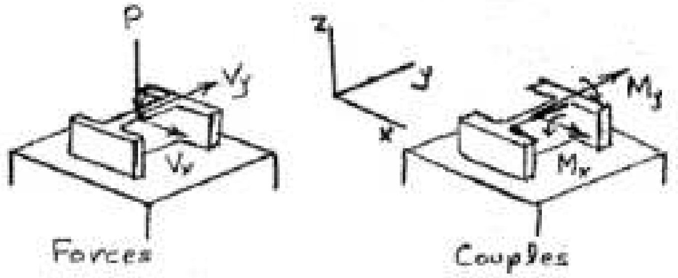
$\tau_{max} = R \qquad \tau_{max} = 24.31$ MPa

Problem 8.57

8.57 Four forces are applied to a W 200 × 41.7 rolled beam as shown. Determine the principal stresses and maximum shearing stress at point *a*.



Calculate forces and couples at section containing point $\underline{a}$.



$P = 100 \text{ kN} \qquad V_x = -16 \text{ kN} \qquad V_y = 25 \text{ kN}$

$$M_x = -(500\times10^{-3})(25\times10^{3}) - \left(\frac{205}{2}\times10^{-3}\right)(100\times10^{3}) = -22.75\times10^{3} \text{ N}\cdot\text{m}$$

$$M_y = (500+100)(10^{-3})(16\times10^{3}) = 9.6\times10^{3} \text{ N}\cdot\text{m}$$

Section properties.

$A = 5310 \text{ mm}^2 \qquad d = 205 \text{ mm}$

$b_f = 166 \text{ mm} \qquad t_f = 11.8 \text{ mm}$

$t_w = 7.2 \text{ mm}$

$I_x = 40.9\times10^{6} \text{ mm}^4 = 40.9\times10^{-6} \text{ m}^4, \qquad I_y = 9.01\times10^{6} \text{ mm}^4 = 9.01\times10^{-6} \text{ m}^4$

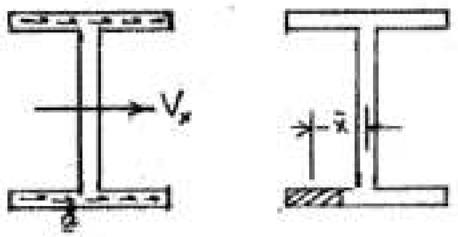
Point $\underline{a}$: $\quad x_a = -\frac{166}{2} + 75 = -8 \text{ mm}, \qquad y_a = -\frac{205}{2} = -102.5 \text{ mm}$

$$\sigma_a = -\frac{P}{A} + \frac{M_x y_a}{I_x} + \frac{M_y x_a}{I_y}$$

$$= -\frac{100\times10^{3}}{5310\times10^{-6}} + \frac{(-22.75\times10^{3})(-102.5\times10^{-3})}{40.9\times10^{-6}} + \frac{(9.6\times10^{3})(-8\times10^{-3})}{9.01\times10^{-6}}$$

$$= -18.83\times10^{6} + 57.01\times10^{6} - 8.52\times10^{6} = 29.66 \text{ MPa}$$

Shearing stress at point $\underline{a}$ due to V_x



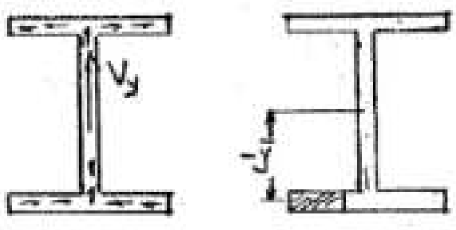
$A = (75)(11.8) = 885 \text{ mm}^2$

$\bar{x} = -\frac{166}{2} + \frac{75}{2} = -45.5 \text{ mm}$

$Q = -A\bar{x} = 40.2675\times10^{3} \text{ mm}^3$

$$\tau_{xz} = \frac{V_x Q}{I_y t} = \frac{(-16\times10^{3})(40.2675\times10^{-6})}{(9.01\times10^{-6})(11.8\times10^{-3})} = -6.060 \text{ MPa}$$

Shearing stress at point $\underline{a}$ due to V_y



$A = (75)(11.8) = 885 \text{ mm}^2$

$\bar{y} = -\frac{205}{2} + \frac{11.8}{2} = -96.6 \text{ mm}$

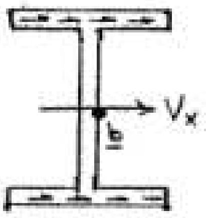
$Q = -A\bar{y} = -85.491\times10^{3} \text{ mm}^3$

$$\tau_{yz} = -\frac{V_y Q}{I_x t} = -\frac{(25\times10^{3})(-85.491\times10^{-6})}{(40.9\times10^{-6})(11.8\times10^{-3})} = 4.428 \text{ MPa}$$

continued

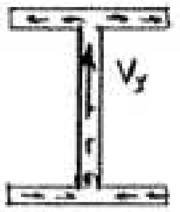
Problem 8.58 continued

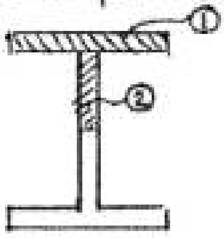
Shearing stress at point $\underline{b}$ due to V_x.



$\tau_{xz} = 0$

Shearing stress at point $\underline{b}$ due to V_y.

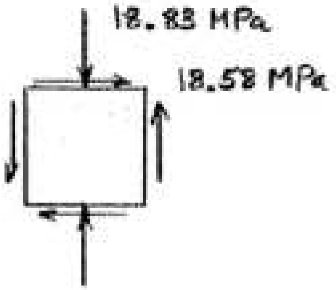




	$A\,(mm^2)$	$\bar{y}\,(mm)$	$A\bar{y}\,(10^3 mm^3)$
①	1958.8	96.6	189.220
②	653.04	45.35	29.615
Σ			218.835

$$Q = \Sigma A\bar{y} = 218.835 \times 10^3\, mm^3 = 218.835 \times 10^{-6}\, m^3$$

$$\tau_b = \frac{V_y Q}{I_x t_w} = \frac{(25\times10^3)(218.835\times10^{-6})}{(40.9\times10^{-6})(7.2\times10^{-3})} = 18.58\ MPa$$



$$\sigma_{ave} = \frac{-18.83+0}{2} = -9.415\ MPa$$

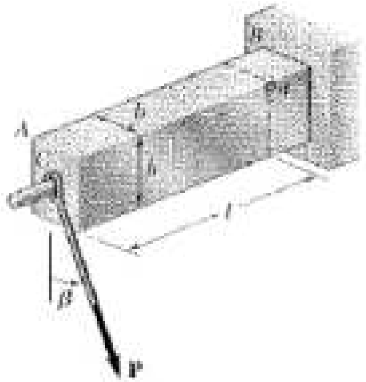
$$R = \sqrt{\left(\frac{-18.83-0}{2}\right)^2 + (18.58)^2} = 20.829\ MPa$$

$\sigma_{max} = \sigma_{ave} + R$ $\quad\quad$ $\sigma_{max} = 11.41\ MPa$ ◄

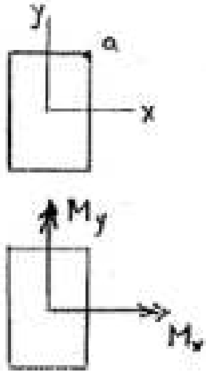
$\sigma_{min} = \sigma_{ave} - R$ $\quad\quad$ $\sigma_{min} = -30.2\ MPa$ ◄

$\tau_{max} = R$ $\quad\quad$ $\tau_{max} = 20.8\ MPa$ ◄

Problem 8.59



8.59 A force **P** is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that **P** acts in a direction perpendicular to the longitudinal axis of the beam, determine (*a*) the normal stress at point *a* in terms of *P*, *b*, *h*, *l*, and β, (*b*) the values of β for which the normal stress at *a* is zero.



$I_x = \frac{1}{12} b h^3 \qquad I_y = \frac{1}{12} h b^3$

$$\sigma = \frac{M_x (h/2)}{I_x} - \frac{M_y (b/2)}{I_y}$$

$$= \frac{6 M_x}{b h^2} - \frac{6 M_y}{h b^2}$$

$\vec{P} = P \sin\beta \, \vec{i} - P \cos\beta \, \vec{j} \qquad \vec{r} = l \, \vec{k}$

$\vec{M} = \vec{r} \times \vec{P} = l \vec{k} \times (P \sin\beta \, \vec{i} - P \cos\beta \, \vec{j}) = P l \cos\beta \, \vec{i} + P l \sin\beta \, \vec{j}$

$M_x = P l \cos\beta \qquad M_y = P l \sin\beta$

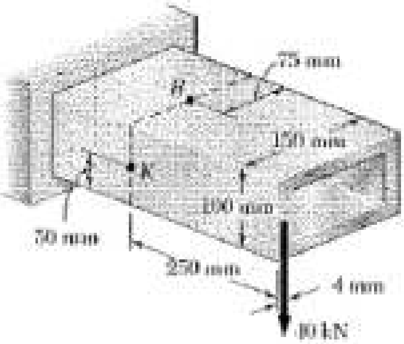
(a) $\sigma = \dfrac{6 P l \cos\beta}{b h^2} - \dfrac{6 P l \sin\beta}{h b^2} = \dfrac{6 P l}{b h}\left[\dfrac{\cos\beta}{h} - \dfrac{\sin\beta}{b}\right]$ ◄

(b) $\sigma = 0 \qquad \dfrac{\cos\beta}{h} - \dfrac{\sin\beta}{b} = 0 \qquad \tan\beta = \dfrac{b}{h}$

$\beta = \tan^{-1}\left(\dfrac{b}{h}\right)$ ◄

Problem 8.62

*8.62 Knowing that the structural tube shown has a uniform wall thickness of 8 mm, determine the principal stresses, principal planes, and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.



At the section containing points H and K

$V = 40\text{ kN} \qquad M = (40)(0.25) = 10\text{ kN m}$

$T = (40)(0.075 - 0.004) = 2.84\text{ kN m.}$

Torsion:

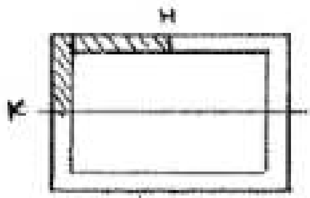
$\mathcal{a} = (142)(92) = 13064\text{ mm}^2$

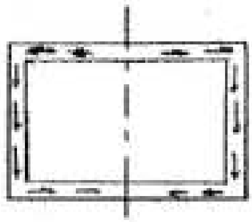
$\tau = \dfrac{T}{2t\mathcal{a}} = \dfrac{(2.84\times10^6)}{(2)(8)(13064)} = 13.59\text{ MPa}$

Transverse shear:

$Q_H = 0$

$Q_K = (75)(50)(25) - (67)(42)(21)$
$= 34656\text{ mm}^3.$



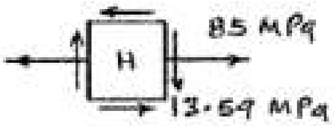


$I = \frac{1}{12}(150)(100)^3 - \frac{1}{12}(134)(84)^3 = 5.88\times10^6\text{ mm}^4$

$\tau_H = 0 \qquad \tau_K = \dfrac{VQ_K}{It} = \dfrac{(40\times10^3)(34656)}{(5.88\times10^6)(8)} = 29.47\text{ MPa.}$

Bending: $\sigma_H = \dfrac{Mc}{I} = \dfrac{(10\times10^6)(50)}{5.88\times10^6} = 85\text{ MPa}, \qquad \sigma_K = 0$

(a) Point H:



$\sigma_c = \dfrac{85}{2} = 42.5\text{ MPa.}$

$R = \sqrt{\left(\dfrac{85}{2}\right)^2 + (13.59)^2} = 44.62\text{ MPa.}$

$\sigma_{max} = \sigma_c + R = 87.1\text{ MPa}$

$\sigma_{min} = \sigma_c - R = -2.1\text{ MPa}$

$\tan 2\theta_p = \dfrac{2\tau}{\sigma} = -0.32 \qquad \theta_p = -8.9^\circ,\ 81.1^\circ$

$\tau_{max} = R = 44.6\text{ MPa.}$

(b) Point K: $\sigma = 0 \qquad \tau = 13.59 + 29.47 = 43.06\text{ MPa.}$

$\sigma_{max} = 43.1\text{ MPa.}$

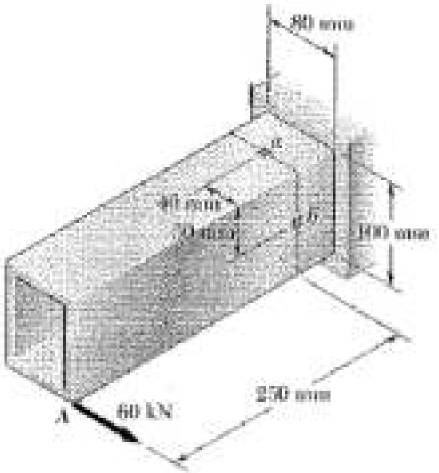
$\sigma_{min} = -43.1\text{ MPa}$

$\theta_p = \pm 45^\circ$

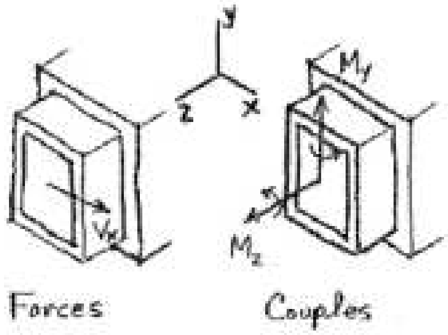
$\tau_{max} = 43.1\text{ MPa.}$

Problem 8.63

*8.63 The structural tube shown has a uniform wall thickness of 8 mm. Knowing that the 60-kN load is applied 4 mm above the base of the tube, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



Calculate forces and couples at section containing points $\underline{a}$ and $\underline{b}$.



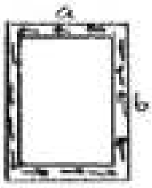
$V_x = 60 \text{ kN}$

$M_z = (50-4)(60) = 2760 \text{ kN}\cdot\text{mm}$

$M_y = (60)(250) = 15000 \text{ kN}\cdot\text{mm}$

Shearing stresses due to torque $T = M_z$

$\mathcal{a} = [80-(2)(4)][100-(2)(4)] = 6624 \text{ mm}^2$

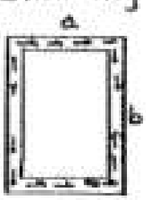


$q = \dfrac{M_z}{2\mathcal{a}} = \dfrac{2760\times10^3}{(2)(6624)} = 208.3 \text{ N/mm}$

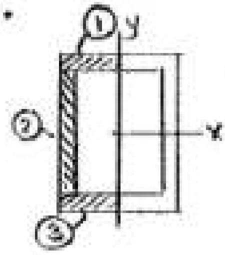
At point $\underline{a}$. $t = 8 \text{ mm}$ $\qquad \tau_a = \dfrac{q}{t} = \dfrac{208.3}{8} = 26.04 \text{ MPa} \leftarrow$

At point $\underline{b}$. $t = 8 \text{ mm}$ $\qquad \tau_b = \dfrac{q}{t} = \dfrac{208.3}{8} = 26.04 \text{ MPa} \uparrow$

Shearing stresses due to V_x.



At point $\underline{a}$



Part	A (mm²)	$\bar{x}$ (mm)	$A\bar{x}$ (mm³)
1	320	−20	−6400
2	672	36	24192
3	320	−20	−6400
Σ			−36992

$Q = |\Sigma A\bar{x}| = 36992 \text{ mm}^3 \qquad t = (2)(8) = 16 \text{ mm}.$

$I_y = \frac{1}{12}(100)(80)^3 - \frac{1}{12}(84)(64)^3 = 2.4317\times10^6 \text{ mm}^4.$

$\tau_a = \dfrac{V_x Q}{I_y t} = \dfrac{(60000)(36992)}{(2.4317\times10^6)(16)} = 57.05 \text{ MPa}.$

At point $\underline{b}$ $\qquad \tau_b = 0$

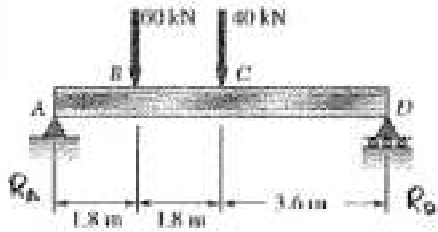
Combined shearing stresses.

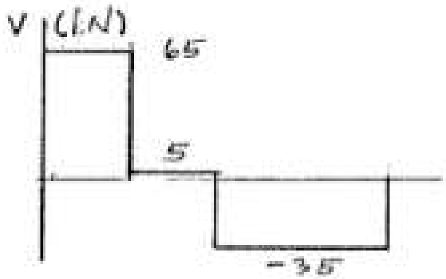
At point $\underline{a}$. $\quad \tau_a = 26.04 \leftarrow + 57.05 \rightarrow = 31 \text{ MPa} \rightarrow$ ◄

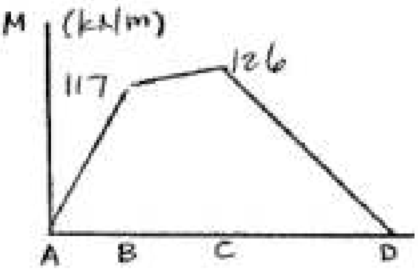
At point $\underline{b}$. $\quad \tau_b = 26.04 \uparrow + 0 = 26 \text{ MPa} \uparrow$ ◄

Problem 8.65

8.65 (*a*) Knowing that $\sigma_{all} = 165$ MPa and $\tau_{all} = 100$ MPa, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.







$+\circlearrowleft \Sigma M_D = 0: \; -7.2R_A + (60)(5.4) + (40)(3.6) = 0$

$R_A = 65 \text{ kN} \uparrow$

$|V|_{max} = 65 \text{ kN}$

$|M|_{max} = 126 \text{ kN m}$

$S_{min} = \dfrac{|M|_{max}}{\sigma_{all}} = \dfrac{126 \times 10^3}{165 \times 10^6} = 763.6 \times 10^{-6} \text{ m}^3$

$= 763.6 \times 10^3 \text{ mm}^3$

Shape	$S (\times 10^3 \text{ mm}^3)$
W 460×52	942
→ W 410×46.1	774
W 360×57.8	899
W 310×60	851
W 250×67	809
W 200×86	853

(a) Use W 410×46.1 ◄

$d = 403$ mm

$t_f = 11.2$ mm

$t_w = 7$ mm

(a) Use W 410×46.1 with $S = 774 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2}d = 201.5 \qquad y_b = c - t_f = 190.3 \text{ m}$

At C: $M = (35)(3.6) = 126 \text{ kN m} \qquad \sigma_m = \dfrac{M}{S} = \dfrac{126 \times 10^6}{774 \times 10^3} = 162.8 \text{ MPa}$ ◄

$|V| = 35 \text{ kN} \qquad \tau_m = \dfrac{V}{dt_w} = \dfrac{35000}{(403)(7)} = 12.4 \text{ MPa}$

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \left(\dfrac{190.3}{201.5}\right)(162.8) = 153.8 \text{ MPa}$

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\dfrac{153.8}{2}\right)^2 + 12.4^2} = 77.9 \text{ MPa}$

$\sigma_{max} = \dfrac{\sigma_b}{2} + R = 154.8 \text{ MPa}$ ◄

At B: $M = (65)(1.8) = 117 \text{ kNm} \qquad \sigma_m = \dfrac{117 \times 10^6}{774 \times 10^3} = 151.1 \text{ MPa}$

$|V| = 65 \text{ kN} \qquad \tau_m = \dfrac{V}{dt_w} = \dfrac{65000}{(403)(7)} = 23 \text{ MPa}$ ◄

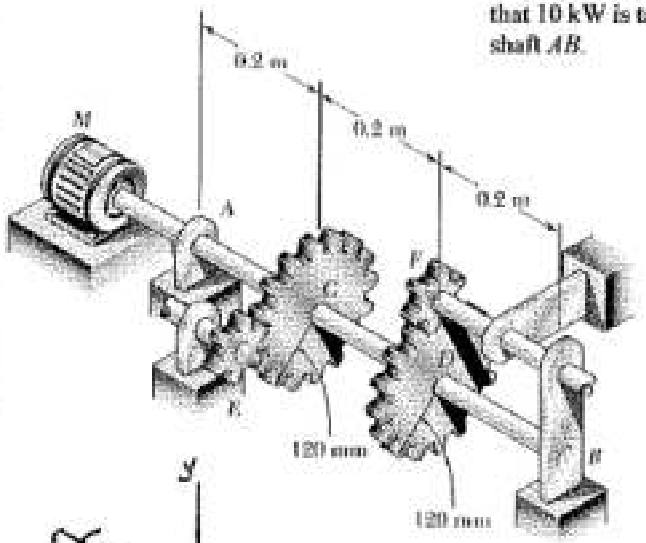
$\sigma_b = \dfrac{y_b}{c}\sigma_m = \left(\dfrac{190.3}{201.5}\right)(151.1) = 142.7 \text{ MPa}$

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_m^2} = 75 \text{ MPa}$

$\sigma_{max} = \dfrac{\sigma_b}{2} + R = 146.4 \text{ MPa}$

Problem 8.67

8.67 The solid shaft *AB* rotates at 360 rpm and transmits 20 kW from the motor *M* to machine tools connected to gears *E* and *F*. Knowing that $\tau_{all} = 45$ MPa and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft *AB*.



$$f = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

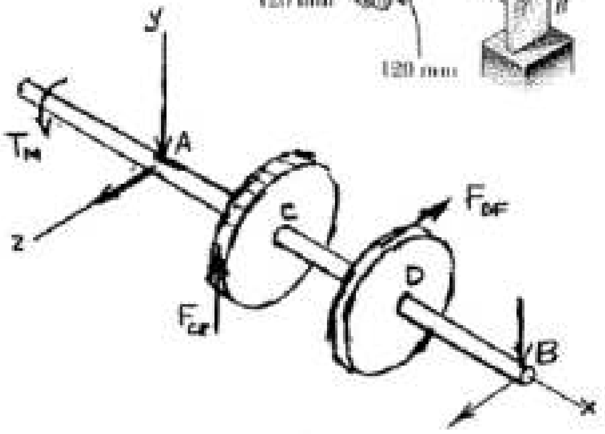
$$T_M = \frac{\mathcal{P}_M}{2\pi f} = \frac{20\times10^3}{2\pi(6)} = 530.5 \text{ N}\cdot\text{m}$$

$$T_C = \frac{\mathcal{P}_C}{2\pi f} = \frac{10\times10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

$$T_D = \frac{\mathcal{P}_D}{2\pi f} = \frac{10\times10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

$$F_{CE} = \frac{T_C}{r_C} = \frac{265.26}{120\times10^{-3}} = 2.210\times10^3 \text{ N}$$

$$F_{DF} = \frac{T_D}{r_D} = \frac{265.26}{120\times10^{-3}} = 2.210\times10^3 \text{ N}$$

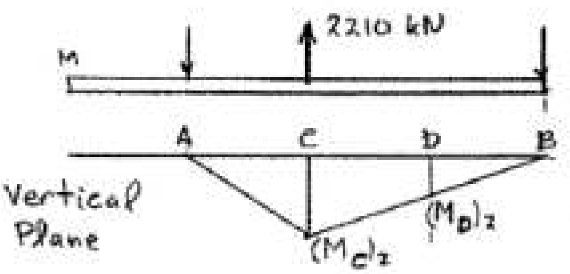


$$(M_C)_z = \frac{(0.2)(0.4)(2.210\times10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

$$(M_D)_z = \tfrac{1}{2}(M_C)_z = 147.37 \text{ N}\cdot\text{m}$$

$$(M_D)_y = \frac{(0.4)(0.2)(2.210\times10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

$$(M_C)_y = \tfrac{1}{2}(M_D)_y = 147.37 \text{ N}\cdot\text{m}$$

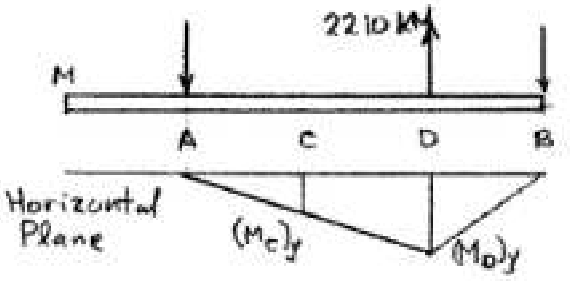


Torques in shaft.

$T_{MAC} = 530.5$ N·m

$T_{CD} = 265.26$ N·m

$T_{DB} = 0$



Just to the left of gear C.

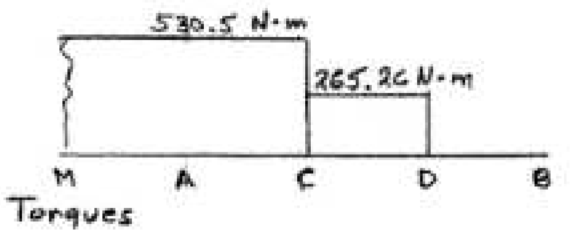
$$\max\sqrt{M_y^2 + M_z^2 + T^2} = 624.5 \text{ N}\cdot\text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\max\sqrt{M_y^2+M_z^2+T^2}}{\tau_{max}}$$

$$= \frac{624.5}{45\times10^6} = 13.878\times10^{-6} \text{ m}^3$$

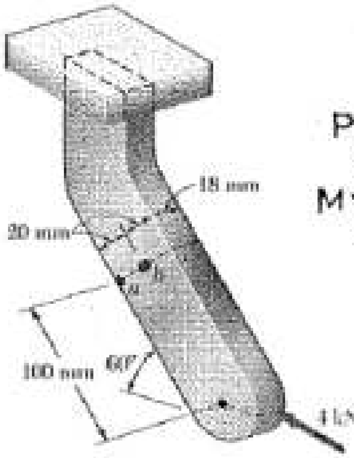
$$c = 20.67\times10^{-3} \text{ m} = 20.67 \text{ mm}$$

$$d = 2c \qquad d = 41.3 \text{ mm}$$

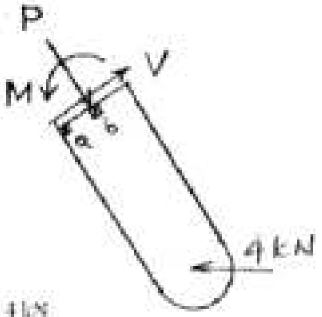


Problem 8.68

8.68 For the bracket and loading shown, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.



Draw free body diagram of portion below section ab.



From statics.

$P = 4000 \cos 60° = 2000$ N

$V = 4000 \sin 60° = 3464$ N

$M = (0.1)(4000) \sin 60°$

$= 346.4$ Nm

Section properties.

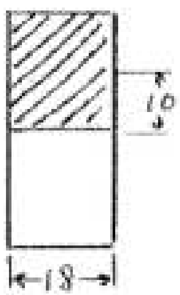
$A = (18)(40) = 720 \text{ mm}^2$

$I = \frac{1}{12}(18)(40)^3 = 96000 \text{ mm}^4$

(a) Point $\underline{a}$ $\quad \sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2000}{720} - \frac{(346400)(20)}{96000} = -74.9$ MPa ◀

$\tau = 0$ ◀

(b) Point $\underline{b}$ $\quad \sigma = -\frac{P}{A} = -\frac{2000}{720} = -2.8$ MPa ◀



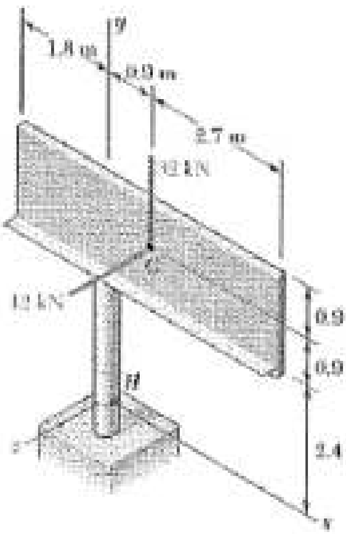
$A = (20)(18) = 360 \text{ mm}^2$

$\bar{y} = 10$ mm

$Q = A\bar{y} = (360)(10) = 3600 \text{ mm}^3$

$\tau = \frac{VQ}{It} = \frac{(3464)(3600)}{(96000)(18)} = 7.2$ MPa ◀

Problem 8.69



8.69 The billboard shown weights 32 kN and is supported by a structural tube that has a 380-mm outer diameter and a 12-mm wall thickness. At a time when the resultant of the wind pressure is 12 kN located at the center C of the billboard, determine the normal and shearing stresses at point H.

At section containing point H,

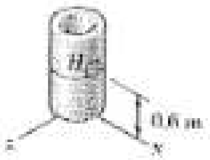
$P = 32\text{ kN}$ (compression)

$T = (12)(0.9) = 10.8\text{ kN m}$

$M_x = -(12)(3.3) = -39.6\text{ kN m}$

$M_z = -(0.9)(32) = -28.8\text{ kN m}$

$V = 12\text{ kN}$



Section properties.

$d_o = 380\text{ mm}$ $\quad c_o = \frac{1}{2}d_o = 190\text{ mm}$ $\quad c_i = c_o - t = 178\text{ mm}$

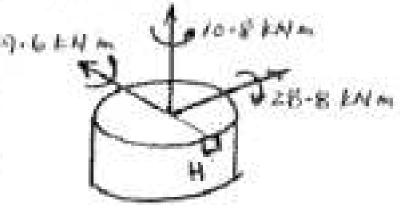
$A = \pi(c_o^2 - c_i^2) = 13873\text{ mm}^2$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 235.1\times10^6\text{ mm}^4$

$J = 2I = 470.2\times10^6\text{ mm}^4$

$Q = \frac{2}{3}(c_o^3 - c_i^3) = 812832\text{ mm}^3$

Couples

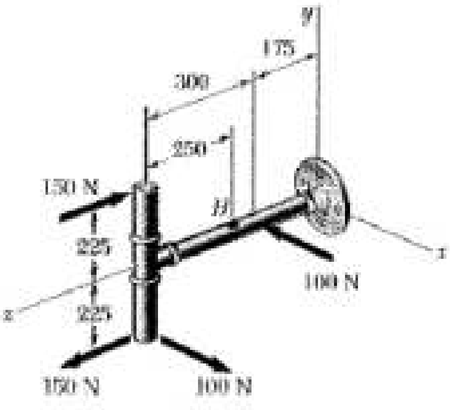


$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{32\times10^3}{13873} - \frac{(28.8\times10^6)(190)}{235.1\times10^6} = -2.307 - 23.275 = -25.6\text{ MPa}$$

$$\tau = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(10.8\times10^6)(190)}{470.2\times10^6} + \frac{(12\times10^3)(812832)}{(235.1\times10^6)(24)} = 4.364 + 1.729 = 6.1\text{ MPa}$$

Problem 8.70

8.70 Several forces are applied to the pipe assembly shown. Knowing that each section of pipe has inner and outer diameters equal to 36 and 42 mm, respectively, determine the normal and shearing stresses at point H located at the top of the outer surface of the pipe.



Dimensions in mm

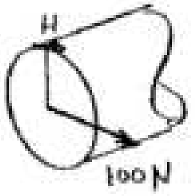
At the section containing point H,

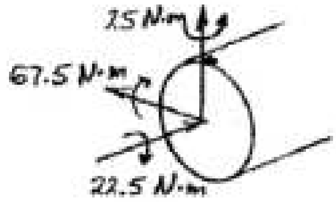
$$P = 0, \quad V_x = 100 \text{ N}, \quad V_y = 0$$

$$M_x = -(0.450)(150) = -67.5 \text{ N·m}$$

$$M_y = (0.250)(100) = 25 \text{ N·m}$$

$$M_z = -(0.225)(100) = -22.5 \text{ N·m}$$





$$d_o = 42 \text{ mm} \qquad d_i = 32 \text{ mm}$$

$$c_o = 21 \text{ mm} \qquad c_i = 18 \text{ mm}$$

$$t = c_o - c_i = 3 \text{ mm}$$

$$A = \pi(c_o^2 - c_i^2) = 367.57 \text{ mm}^2 = 367.57\times10^{-6} \text{ m}^2$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 70.30\times10^3 \text{ mm}^4 = 70.30\times10^{-9} \text{ m}^4, \quad J = 2I = 140.59\times10^{-9} \text{ m}^4$$

For half-pipe, $Q = \frac{2}{3}(c_o^3 - c_i^3) = 2.286\times10^3 \text{ mm}^3 = 2.286\times10^{-6} \text{ m}^3$

$$\sigma_H = \frac{M_x y}{I_x} = \frac{(-67.5)(21\times10^{-3})}{70.30\times10^{-9}} \qquad \sigma = -20.2 \text{ MPa} \blacktriangleleft$$

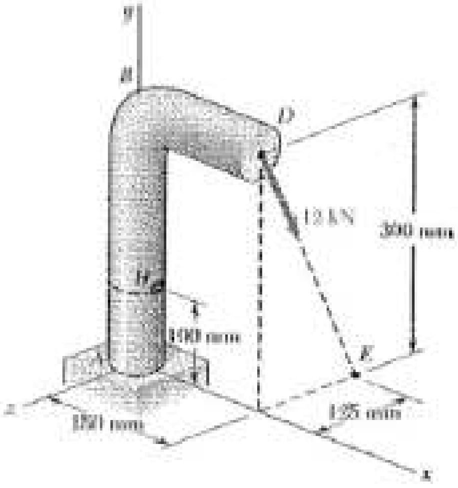
Due to torque: $(\tau_H)_T = \dfrac{Tc}{J} = \dfrac{(22.5)(21\times10^{-3})}{140.59\times10^{-9}} = 3.36 \text{ MPa}$

Due to shear: $(\tau_H)_V = \dfrac{VQ}{It} = \dfrac{(100)(2.286\times10^{-6})}{(70.30\times10^{-9})(6\times10^{-3})} = 0.54 \text{ MPa}$

Net: $\tau_H = 3.36 - 0.54 \qquad \tau = 2.82 \text{ MPa} \blacktriangleleft$

Problem 8.73

8.73 A 12-kN force is applied as shown to the 60-mm-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.



$DE = \sqrt{125^2 + 300^2} = 325$ mm

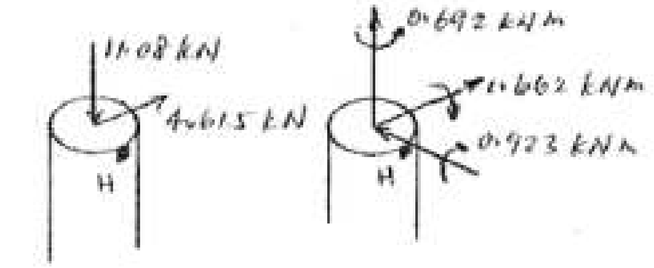
At point D $\qquad F_x = 0$

$F_y = -\left(\frac{300}{325}\right)(12) = -11.08$ kN

$F_z = -\left(\frac{125}{325}\right)(12) = -4.615$ kN

Moment of equivalent force-couple system at C, the centroid of the section containing point H

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.15 & 0.2 & 0 \\ 0 & -11.08 & -4.615 \end{vmatrix} = 0.923\,\vec{i} + 0.692\,\vec{j} - 1.662\,\vec{k} \ \text{kN}\cdot\text{m}$$



Section properties

$d = 60$ mm $\qquad c = \frac{1}{2}d = 30$ mm

$A = \pi c^2 = 2827$ mm^2

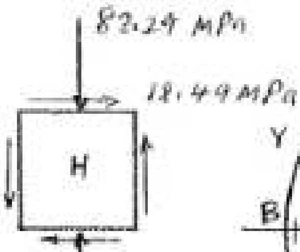
$I = \frac{\pi}{4}c^4 = 0.6362 \times 10^6$ mm^4

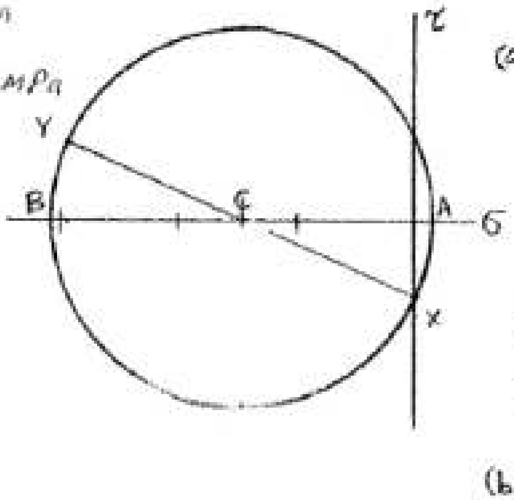
$J = 2I = 1.2724 \times 10^6$ mm^4

For a semicircle $\qquad Q = \frac{2}{3}c^3 = 18 \times 10^3$ mm^3

At point H $\quad \sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{(11.08 \times 10^3)}{(2827)} - \frac{(1.662 \times 10^6)(30)}{(0.6362 \times 10^6)} = -82.29$ MPa

$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.692 \times 10^6)(30)}{1.2724 \times 10^6} + \frac{(4.615 \times 10^3)(18 \times 10^3)}{(0.6362 \times 10^6)(60)} = 18.49$ MPa





(a) $\sigma_c = \frac{\sigma_H}{2} = -41.145$ MPa

$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 45.109$ MPa

$\sigma_a = \sigma_c + R = 4$ MPa ◄

$\sigma_b = \sigma_c - R = -86.3$ MPa ◄

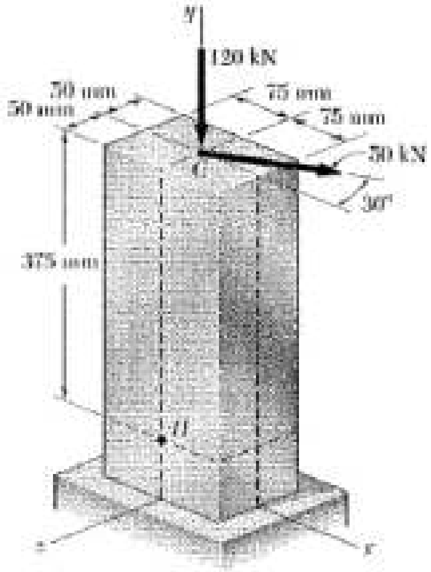
$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4494$

$\theta_a = 12.1°$, $\theta_b = 102.1°$ ◄

(b) $\tau_{max} = R = 45.1$ MPa ◄

Problem 8.74

8.74 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point *H*.



Components of force at point C.

$F_x = 50 \cos 30° = 43.301$ kN

$F_z = -50 \sin 30° = -25$ kN, $\quad F_y = -120$ kN

Section forces and couples at the section containing points H and K.

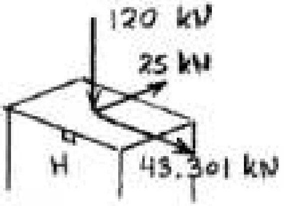
$P = 120$ kN (compression)

$V_x = 43.301$ kN, $\quad V_z = -25$ kN

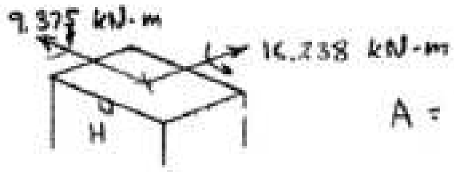
$M_x = -(25)(0.375) = -9.375$ kN-m

$M_y = 0$

$M_z = -(43.301)(0.375) = -16.238$ kN-m



Forces



Couples

$A = (100)(150) = 15 \times 10^3$ mm^2
$= 15 \times 10^{-3}$ m^2

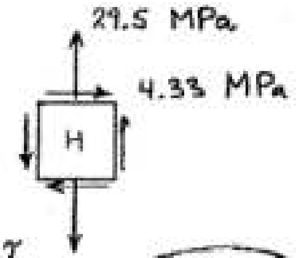
$I_x = \frac{1}{12}(150)(100)^3 = 12.5 \times 10^6$ mm^4
$= 12.5 \times 10^{-6}$ m^4

Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120 \times 10^3)}{15 \times 10^{-3}} - \frac{(-9.375 \times 10^3)(50 \times 10^{-3})}{12.5 \times 10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_x}{A} = \frac{3}{2}\,\frac{43.301 \times 10^3}{15 \times 10^{-3}} = 4.33 \text{ MPa}$$

Use Mohr's circle.



$\sigma_c = \frac{1}{2}\sigma_H = 14.75$ MPa

$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37$ MPa

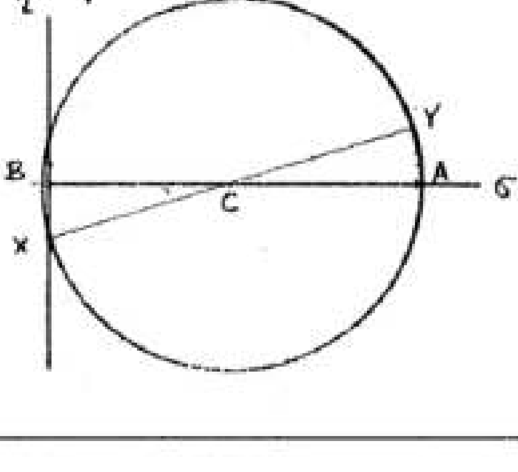
$\sigma_a = \sigma_c + R = \quad \sigma_a = 30.1$ MPa ◄

$\sigma_b = \sigma_c - R = \quad \sigma_b = -0.62$ MPa ◄

$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$

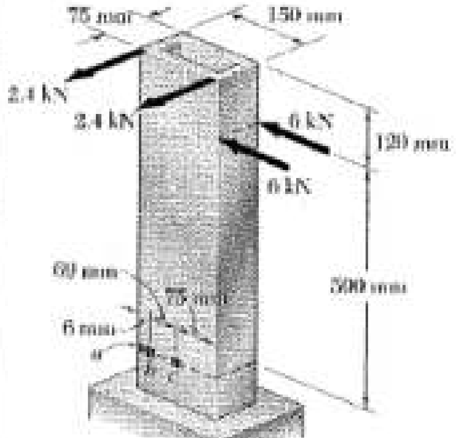
$\theta_a = -8.2° \quad \theta_b = 81.8°$ ◄

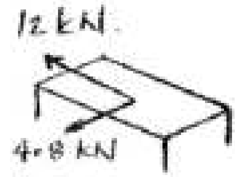
$\tau_{max} = R \quad \tau_{max} = 15.37$ MPa ◄



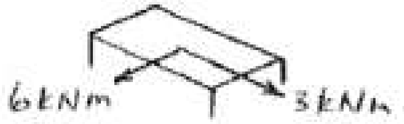
Problem 8.75

8.75 Knowing that the structural tube shown has a uniform wall thickness of 6 mm, determine the normal and shearing stresses at the three points indicated.

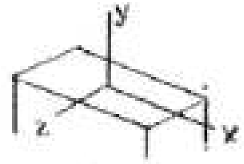




Shear



Bending moment



$b_o = 150$ mm $\quad b_i = b_o - 2t = 138$ mm

$h_o = 75$ mm $\quad h_i = h_o - 2t = 63$ mm.

$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 2.4 \times 10^6 \text{ mm}^4 \qquad I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 7.3 \times 10^6 \text{ mm}^4.$

Normal stresses

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x}$$

$$\frac{(6\times10^6)(-75)}{(7.3\times10^6)} - \frac{(3\times10^6)(37.5)}{2.4\times10^6} = -108.5 \text{ MPa}$$

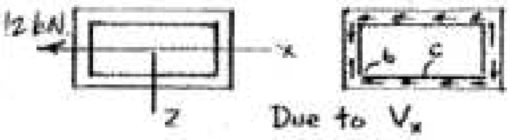
$$\frac{(6\times10^6)(69)}{(7.3\times10^6)} - \frac{(3\times10^6)(37.5)}{2.4\times10^6} = -103.6 \text{ MPa}$$

$$\frac{(6\times10^6)(0)}{(7.3\times10^6)} - \frac{(3\times10^6)(37.5)}{2.4\times10^6} = -46.9 \text{ MPa}$$

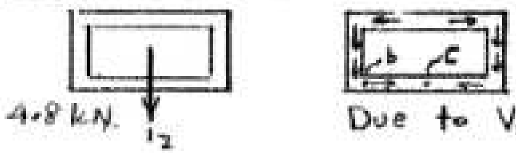
Shearing stresses

Point $\underline{a}$ is an outside corner; $\tau_a = 0$

Direction of shearing stresses

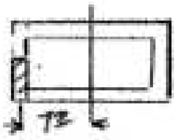


Due to V_x



Due to V_z

At point $\underline{b}$



$Q_{zb} = (37.5)(6)(72) = 16200 \text{ mm}^3$

$$\tau_{b,V_x} = \frac{V_x Q_z}{I_z t} = \frac{(12000)(16200)}{(7.3\times10^6)(6)} = 4.44 \text{ MPa}$$

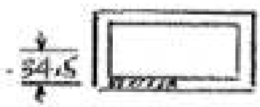
At point $\underline{c}$

$Q_{zc} = Q_{zb} + (69)(6)(34.5)$

$= 30483 \text{ mm}^3$

$$\tau_{c,V_x} = \frac{V_x Q_z}{I_z t} = \frac{(12000)(30483)}{(7.3\times10^6)(6)} = 8.4 \text{ MPa}$$

At point $\underline{b}$



$Q_{xb} = (69)(6)(34.5) = 14283 \text{ mm}^3.$

$$\tau_{b,V_z} = \frac{V_z Q_x}{I_x t} = \frac{(4800)(14283)}{(2.4\times10^6)(6)} = 4.76 \text{ MPa.}$$

At point $\underline{c}$ (symmetry axis)

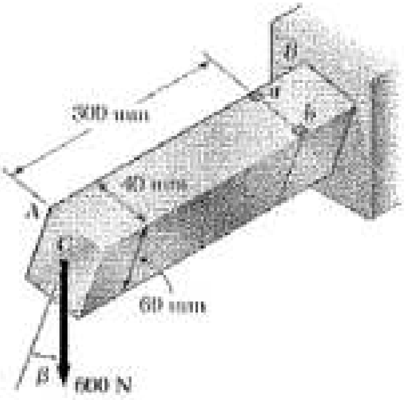
$\tau_{c,V_z} = 0$

Net shearing stress at points $\underline{b}$ and $\underline{c}$

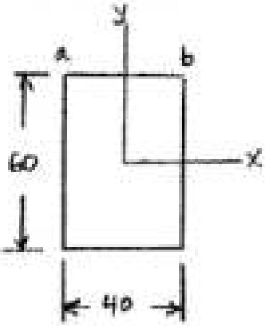
$\tau_b = 4.76 - 4.44 = 0.32$ MPa

$\tau_c = 8.4$ MPa

Problem 8.76

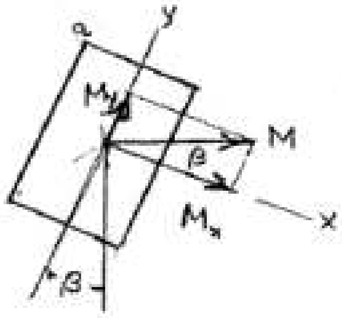


8.76 The cantilever beam *AB* will be installed so that the 60-mm side forms an angle β between 0 and 90° with the vertical. Knowing that the 600-N vertical force is applied at the center of the free end of the beam, determine the normal stress at point *a* when (*a*) $\beta = 0$, (*b*) $\beta = 90°$. (*c*) Also, determine the value of β for which the normal stress at point *a* is a maximum and the corresponding value of that stress.



$$S_x = \frac{1}{6}(40)(60)^2 = 24\times10^3\ \text{mm}^3 = 24\times10^{-6}\ \text{m}^3$$

$$S_y = \frac{1}{6}(60)(40)^2 = 16\times10^3\ \text{mm}^3 = 16\times10^{-3}\ \text{m}^3$$



$$M = P\ell = (600)(300\times10^{-3}) = 180\ \text{N}\cdot\text{m}$$

$$M_x = M\cos\beta = 180\cos\beta$$

$$M_y = M\sin\beta = 180\sin\beta$$

$$\sigma_a = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{180\cos\beta}{24\times10^{-6}} + \frac{180\sin\beta}{16\times10^{-6}}$$

$$= (7.5\times10^6)(\cos\beta + \tfrac{3}{2}\sin\beta)\ \text{Pa}$$

$$= 7.5(\cos\beta + \tfrac{3}{2}\sin\beta)\ \text{MPa}$$

(a) $\underline{\beta = 0}$ $\qquad \sigma_a = 7.50\ \text{MPa}$ $\qquad \sigma = 7.50\ \text{MPa}$ ◄

(b) $\underline{\beta = 90°}$ $\qquad \sigma_a = 11.25\ \text{MPa}$ $\qquad \sigma = 11.25\ \text{MPa}$ ◄

(c) $$\frac{d\sigma_a}{d\beta} = 7.5(-\sin\beta + \tfrac{3}{2}\cos\beta) = 0$$

$\sin\beta = \frac{3}{2}\cos\beta \qquad \tan\beta = \frac{3}{2}$ $\qquad \beta = 56.3°$ ◄

$\sigma_a = 7.5(\cos 56.3° + \frac{3}{2}\sin 56.3°)$ $\qquad \sigma = 13.52\ \text{MPa}$ ◄

PROGRAM OUTPUTS

For x/c = 2 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta°
1.0	0.000	1.000	0.00
0.8	-0.010	0.810	6.34
0.6	-0.040	0.640	14.04
0.4	-0.090	0.490	23.20
0.2	-0.160	0.360	33.69
0.0	-0.250	0.250	45.00
-0.2	-0.360	0.160	-33.69
-0.4	-0.490	0.090	-23.20
-0.6	-0.640	0.040	-14.04
-0.8	-0.810	0.010	-6.34
-1.0	-1.000	0.000	-0.00

For x/c = 8 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta°
1.0	0.000	1.000	0.00
0.8	-0.001	0.801	1.61
0.6	-0.003	0.603	3.80
0.4	-0.007	0.407	7.35
0.2	-0.017	0.217	15.48
0.0	-0.062	0.063	45.00
-0.2	-0.217	0.017	-15.48
-0.4	-0.407	0.007	-7.35
-0.6	-0.603	0.003	-3.80
-0.8	-0.801	0.001	-1.61
-1.0	-1.000	0.000	-0.00

To check that $\sigma_{max} > \sigma_m$ if $x \le 0.544c$, we run the program for $x/c = 0.544$ and for $x/c = 0.545$ and observe that σ_{max}/σ_m exceeds 1 for several values of y/c in the first case, but does not exceed 1 in the second case.

For x/c = 0.544 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta°
0.30	-0.700	0.9997	39.92
0.31	-0.690	1.0001	39.72
0.32	-0.680	1.0004	39.51
0.33	-0.670	1.0005	39.30
0.34	-0.660	1.0005	39.09
0.35	-0.650	1.0003	38.88
0.36	-0.640	1.0000	38.66
0.37	-0.630	0.9996	38.44
0.38	-0.619	0.9990	38.21
0.39	-0.608	0.9983	37.98
0.40	-0.598	0.9975	37.74

For x/c = 0.545 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta°
0.30	-0.698	0.9982	39.91
0.31	-0.689	0.9986	39.71
0.32	-0.679	0.9989	39.50
0.33	-0.669	0.9990	39.29
0.34	-0.659	0.9990	39.08
0.35	-0.649	0.9988	38.87
0.36	-0.639	0.9986	38.65
0.37	-0.628	0.9982	38.42
0.38	-0.618	0.9976	38.20
0.39	-0.607	0.9970	37.96
0.40	-0.596	0.9962	37.73

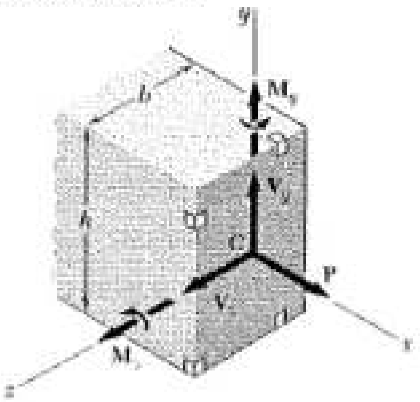
PROBLEM 8.C4 CONTINUED

PROGRAM OUTPUTS

```
Problem 8.25

Omega = 600 rpm
Number of Gears: 3
Length of shaft = 600 mm
Tau =  55 MPa
For Gear 1
Power input =  45 kW
Radius of gear= 75 mm
Distance from A in mm =  100 mm.
FY=  0
FZ = 9.55 kN
For Gear 2
Power input =  -30 kW
Radius of gear= 100 mm
Distance from A in inches =  250 mm.
FY=  1.78 kN
FZ = 0
For Gear 3
Power input = -15 kW
Radius of gear= 100 mm.
Distance from A in inches = 450 mm
FY=        0
FZ =       -2.31 kN
AY= -2.79 kN., AZ= -7.36 kN
BY= -2 kN   , BZ= 0.2 kN.
Just to the left of Gear 1
MY= -0.736 kNm.
MZ= 0.03 kNm.
T=  0.000 kNm.
Diameter must be at least  41.8 mm
Just to the right of Gear 1
T=  716.2 kNm
Diameter must be at least  46.2 mm
Just to the left of Gear 2
MY=  -0.4075 kNm.
MZ=   0.701 kNm.
T=  716.25 kNm.
Diameter must be at least  46.4 mm
Just to the right of Gear 2
T=  238.7 kNm
Diameter must be at least  42.7 mm.
Just to the left of Gear 3
MY=  0.0299 kNm
MZ=  0.298 kNm
T=  238.72 kNm.
Diameter must be at least  32.9 mm.
Just to the right of Gear 3
T=  0.0000 kNm
Diameter must be at least 30.3 mm.
```

```
Problem  8.29

Omega = 450 rpm
Number of Gears: 3
Length of shaft =  750 mm
Tau =  55 MPa
For Gear 1
Power input =  -8.00 kW
Radius of gear= 60 mm
Distance from A in mm = 150
For Gear 2
Power input =  20.00 kW
Radius of gear=100 mm
Distance from A in mm = 375
For Gear 3
Power input = -12.00 kW
Radius of gear= 60 mm
Distance from A in mm = 600
AY= -0.849 kN, AZ=   4.386
BY= -3.395 kN, BZ=   2.688
Just to the left of Gear 1
MY= 657.84 Nm
MZ= 127.32 Nm
T=   0.00 Nm
Diameter must be at least  39.59 mm
Just to the right of Gear 1
T=-169.77 Nm
Diameter must be at least  40.00 mm
Just to the left of Gear 2
MY=1007.98 Nm
MZ= 318.31 Nm
T=-169.77 Nm
Diameter must be at least  46.28 mm
Just to the right of Gear 2
T= 254.65 Nm
Diameter must be at least  46.52 mm
Just to the left of Gear 3
MY= 403.19 Nm
MZ= 509.30 Nm
T= 254.65 Nm
Diameter must be at least  40.13 mm
Just to the right of Gear 3
T=   0.00 Nm
Diameter must be at least  39.18 mm
```

PROBLEM 8.C5



8.C5 Write a computer program that can be used to calculate the normal and shearing stresses at points with given coordinates *y* and *z* located on the surface of a machine part having a rectangular cross section. The internal forces are known to be equivalent to the force-couple system shown. Write the program so that the loads and dimensions can be expressed in either SI or U.S. customary units. Use this program to solve (*a*) Prob. 8.45b, (*b*) Prob. 8.47a.

SOLUTION

ENTER: b AND h

PROGRAM: $A = bh \quad I_y = b^3 h/12 \quad I_z = b h^3/12$

FOR POINT ON SURFACE, ENTER y AND z

NOTE y AND z MUST SATISFY ONE OF FOLLOWING:

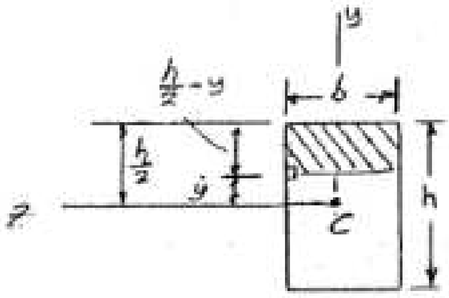
$$y^2 = h^2/4 \text{ AND } z^2 \le b^2/4 \qquad (1)$$

OR

$$z^2 = b^2/4 \text{ AND } y^2 \le h^2/4 \qquad (2)$$

IF EITHER (1) OR (2) ARE SATISFIED, COMPUTE

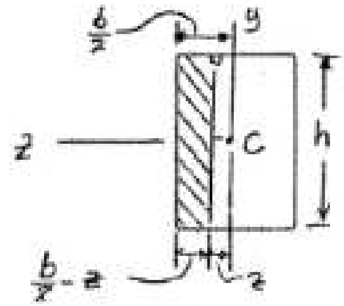
$$\sigma = \frac{P}{A} + \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$



IF $z^2 = b^2/4$, THEN POINT IS ON VERTICAL SURFACE AND

$$Q_z = b\left(\frac{h}{2} - y\right)\left(\frac{h}{2} + y\right)\frac{1}{2} = b\left(\frac{h^2}{8} - \frac{y^2}{2}\right)$$

$$\tau = \frac{V_y Q_z}{I_z b}$$

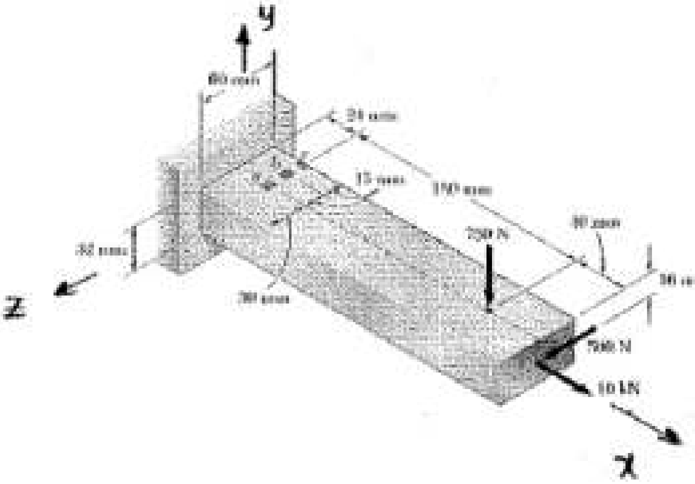


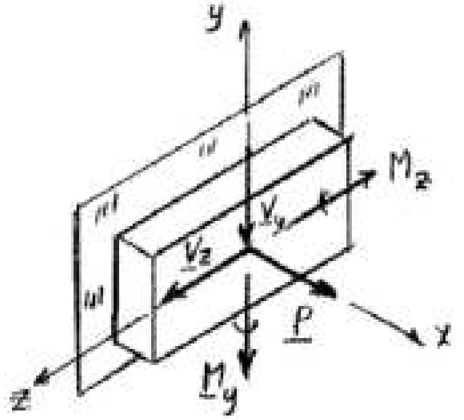
IF $y^2 = h^2/4$, THE POINT IS ON HORIZONTAL SURFACE, AND

$$Q_y = h\left(\frac{b}{2} - z\right)\left(\frac{b}{2} + z\right)\frac{1}{2} = h\left(\frac{b^2}{8} - \frac{z^2}{2}\right)$$

$$\tau = \frac{V_z Q_y}{I_y h}$$

PROBLEM 8.C5 CONTINUED



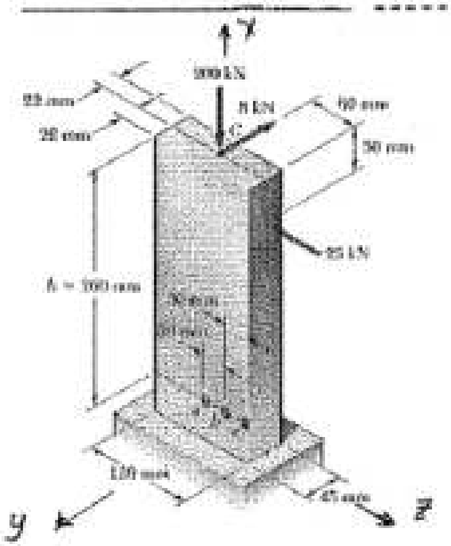


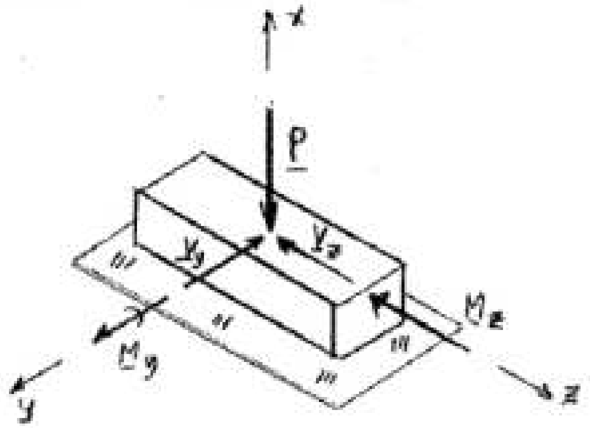
FORCE-COUPLE SYSTEM

$P = 10\ kN \quad V_y = -750\ N \quad V_z = 500\ N$

$M_y = (500\ N)(220\ mm) = 110\ N\cdot m \quad M_z = (750\ N)(180 mm) = 135\ N\cdot m$

```
Problem 8.45 a          Force-Couple at Centroid
                    P =    10000.00 N
                   MY =      110.00 N·m      MZ =    -135.00 N·m
                   VY =      750.00 N        VZ =     500.00 N
                  + + + + + + + + + + + + + + + + + + + + + + + + + + +
                 At point of coordinates:       y  =  16.00 mm    z =   0.00 mm
                        sigma =  18.392 MPa
                          tau =   0.391 MPa
```





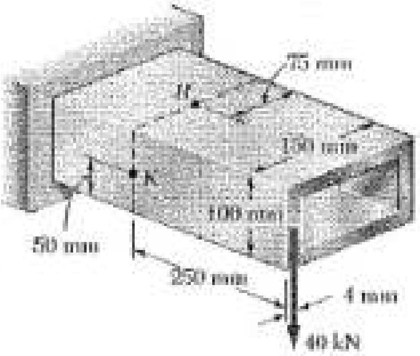
FORCE-COUPLE SYSTEM $\quad P = -200\ kN \quad V_z = -25\ kN \quad V_y = -8\ kN$

$M_y = (25\ kN)(0.262 m) = 5.25\ kNm$

$M_z = (8\ kN)(0.26 m) = 2.08\ kNm$

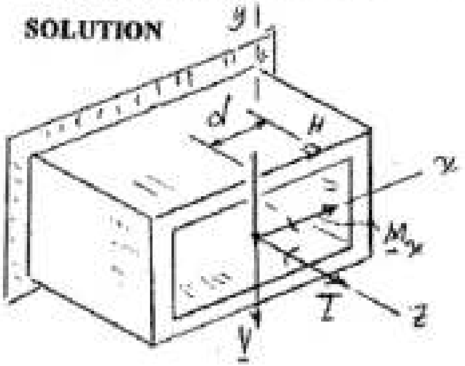
```
Problem 8.47 b          Force-Couple at Centroid
                    P =        -200 kN.
                   MY =      5.25 kNm               MZ =      -2.08 kNm
                   VY =       -8 kN                 VZ =      -25 kN
                  + + + + + + + + + + + + + + + + + + + + + + + + + + +
                 At point of coordinates:       y  =  45 mm       z =   30 mm
                        sigma =  40.7 MPa
                          tau =  -5.3 MPa
```

PROBLEM 8.C7



*8.C7 The structural tube shown has a uniform wall thickness of 8 mm. A 40-kN force is applied to a bar (not shown) that is welded to the end of the tube. Write a computer program that can be used to determine, for any given value of c, the principal stresses, principal planes, and maximum shearing stress at point H for values of d from −75 mm to −75 mm, using 25-mm increments. Use this program to solve Prob. 8.62a.

SOLUTION



FORCE-COUPLE SYSTEM

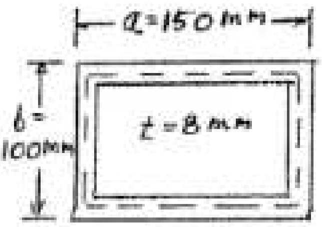
ENTER:

$V = 40\text{kN}\downarrow$

$M_x = (40\text{kN})(0.25\text{m})$

$= 10\text{kNm}$

$T = (40\text{kN})$

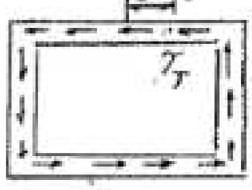


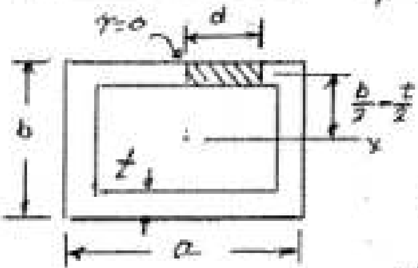
AREA ENCLOSED

$\mathcal{A} = (a-t)(b-t)$

$\tau_T = \dfrac{T}{2t\mathcal{A}} = \dfrac{Tc}{2t\mathcal{A}}$

τ_T = SHEARING STRESS DUE TO TORSION



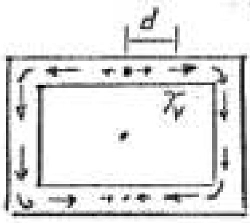


$Q = dt\left(\frac{b}{2} - \frac{t}{2}\right)$

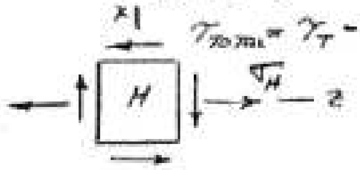
$I = ab^3/12 - (a-2t)(b-2t)^3/12$

$\tau_V = \dfrac{VQ}{It}$

τ_V = SHEARING STRESS DUE TO V



$\tau_{TOTAL} = \tau_T - \tau_V$



BENDING: $\sigma_H = \dfrac{M_x\left(\frac{b}{2}\right)}{I}$

PRINCIPAL STRESSES

$\sigma_{ave} = \frac{1}{2}\sigma_H$; $R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{TOTAL}^2}$

$\sigma_{max} = \sigma_{ave} + R$; $\sigma_{min} = \sigma_{ave} - R$; $\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{TOTAL}}{\sigma_{ave}}\right)$; $\tau_{max} = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{TOTAL}^2}$

```
Rectangular tube of uniform thickness t = 8 mm
Outside dimensions
  Horizontal width a = 150 mm
  Vertical depth   b = 100 mm
Vertical load    P = 40 kN; line of action at x = -c
Find normal and shearing streses at
     Point H (x = d, y = b/2)
```

Problem 8.64a Program Output for Value of c = 71 mm.

d mm	sigma MPa	tauV MPa	tauT MPa	tauTotal MPa	sigmaMax MPa	sigmaMin MPa	tauMax MPa	theta p degrees
−75	85.03	−23.36	−13.59	−36.96	98.08	−13.95	55.97	−123.78
−50	85.03	−15.60	−13.59	−29.12	93.32	−9.11	51.21	−107.11
−25	85.03	−7.77	−13.59	−21.36	89.31	−5.09	47.20	−85.56
0	85.03	0	−13.59	−13.59	87.12	−2.12	44.62	−8.87
25	85.03	7.77	−13.59	−5.76	84.55	−0.402	42.51	−26.04
50	85.03	15.60	−13.59	2.01	84.22	−0.667	42.18	9.11
75	85.03	23.36	−13.59	9.77	85.29	−1.14	43.25	43.25

Chapter 9

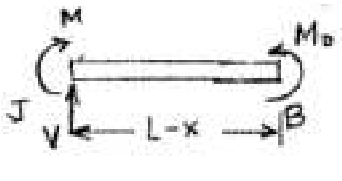
Problem 9.1

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

$$+\circlearrowleft \Sigma M_J = 0: \quad M_0 - M = 0 \qquad M = M_0$$

$[x=0, y=0]$

$[x=0, \frac{dy}{dx}=0]$



$$EI\frac{d^2y}{dx^2} = M_0$$

$$EI\frac{dy}{dx} = M_0 x + C_1$$

$$EI\,y = \frac{1}{2}M_0x^2 + C_1x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \qquad 0 = 0 + C_1 \qquad C_1 = 0$$

$$EIy = \frac{1}{2}M_0x^2 + C_2$$

$$[x=0, y=0] \qquad 0 = 0 + C_2 \qquad C_2 = 0$$

(a) Elastic curve. $\quad y = \dfrac{M_0x^2}{2EI}$ ◄

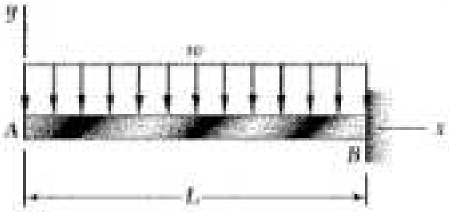
$\dfrac{dy}{dx} = \dfrac{M_0x}{EI}$

(b) y @ $x = L$. $\quad y_B = \dfrac{M_0L^2}{2EI} \qquad y_B = \dfrac{M_0L^2}{2EI} \uparrow$ ◄

(c) $\dfrac{dy}{dx}$ @ $x = L$. $\quad \dfrac{dy}{dx}\Big|_B = \dfrac{M_0L}{EI} \qquad \theta_B = \dfrac{M_0L}{EI} \measuredangle$ ◄

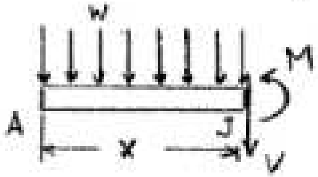
Problem 9.3

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



$[x = L,\ y = 0]$

$[x = L,\ \frac{dy}{dx} = 0]$



$$+\circlearrowleft \Sigma M_J = 0: \quad (wx)\frac{x}{2} + M = 0$$

$$M = -\frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$[x = L,\ \frac{dy}{dx} = 0]$ $\qquad 0 = -\frac{1}{6}wL^3 + C_1$

$$C_1 = \frac{1}{6}wL^3$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$[x = L,\ y = 0]$ $\qquad 0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$

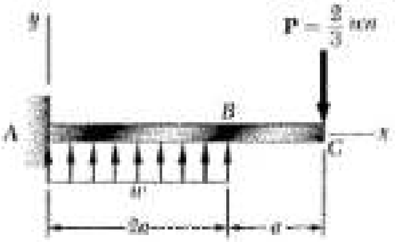
$$C_2 = \left(\frac{1}{24} - \frac{1}{6}\right)wL^4 = -\frac{3}{24}wL^4$$

(a) Elastic curve. $\qquad y = -\frac{w}{24EI}(x^4 - 4L^3x + 3L^4)$ ◀

(b) y @ $x = 0$: $\qquad y_A = -\frac{3wL^4}{24EI} = -\frac{wL^4}{8EI} \qquad y_A = \frac{wL^4}{8EI}\downarrow$ ◀

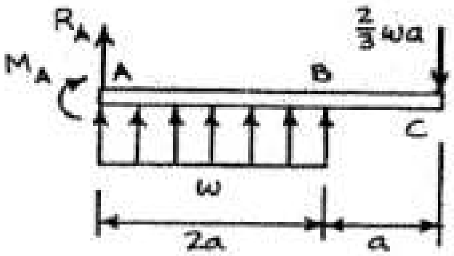
(c) $\frac{dy}{dx}$ @ $x = 0$: $\qquad \left.\frac{dy}{dx}\right|_A = \frac{wL^3}{6EI} \qquad \theta_A = \frac{wL^3}{6EI}\angle$ ◀

Problem 9.5



9.5 and 9.6 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

FBD ABC:



USING ABC AS A FREE BODY,

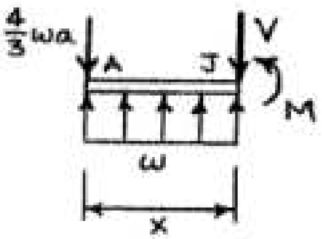
$$+\uparrow \Sigma F_y = 0: \quad R_A + 2wa - \frac{2}{3}wa = 0$$

$$R_A = -\frac{4}{3}wa = \frac{4}{3}wa \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + (2wa)(a) - \left(\frac{2}{3}wa\right)(3a) = 0$$

$$M_A = 0$$

FBD AJ:



USING AJ AS A FREE BODY,

$$+\circlearrowleft \Sigma M_J = 0: \quad M + \left(\frac{4}{3}wa\right)(x) - (wx)\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{2}{3}wax^2 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right]: \quad 0 = 0 - 0 + C_1 \qquad \therefore C_1 = 0$$

$$EIy = \frac{1}{24}wx^4 - \frac{2}{9}wax^3 + C_2$$

$$[x = 0, y = 0]: \quad 0 = 0 - 0 + C_2 \qquad \therefore C_2 = 0$$

(a) ELASTIC CURVE OVER AB: $\quad y = \frac{w}{72EI}(3x^4 - 16ax^3)$ ◄

$$\frac{dy}{dx} = \frac{w}{6EI}(x^3 - 4ax^2)$$

(b) y AT x = 2a: $\quad y_B = -\frac{10wa^4}{9EI} \qquad y_B = \frac{10wa^4}{9EI}\downarrow$ ◄

(c) $\frac{dy}{dx}$ AT x = 2a: $\quad \left(\frac{dy}{dx}\right)_B = -\frac{4wa^3}{3EI} \qquad \theta_B = \frac{4wa^3}{3EI}$ ◄

Problem 9.6

9.5 and 9.6 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

$[x = 0, y = 0]$

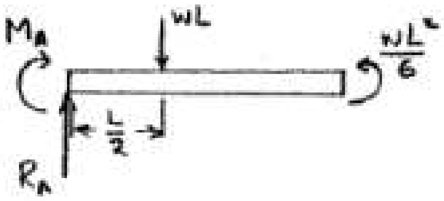
$[x = 0, \frac{dy}{dx} = 0]$

Using ABC as a free body,

$+\uparrow \Sigma F_y = 0: \quad R_A - wL = 0 \qquad R_A = wL$

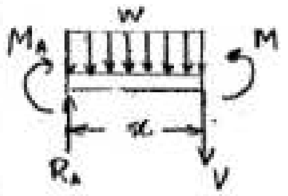
$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (wL)(\frac{L}{2}) + \frac{wL^2}{6} = 0$

$M_A = -\frac{1}{3} wL^2$



Using AJ as a free body, (Portion AB only)

$+\circlearrowleft M_J = 0: \quad M + (wx)(\frac{x}{2}) - R_A x - M_A = 0$



$$M = -\frac{1}{2}wx^2 + R_A x + M_A$$

$$= -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{2}wLx^2 - \frac{1}{3}wLx + C_1$$

$[x = 0, \frac{dy}{dx} = 0]: \quad -0 + 0 - 0 + C_1 = 0 \qquad C_1 = 0$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wLx^3 - \frac{1}{6}wLx^2 + C_2$$

$[x = 0, y = 0] \quad -0 + 0 - 0 + C_2 = 0 \qquad C_2 = 0$

(a) Elastic curve over AB. $\quad y = \frac{w}{24EI}(-x^4 + 4Lx^3 - 4L^2x^2)$ ◄

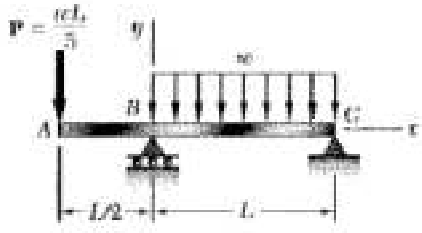
$\frac{dy}{dx} = \frac{w}{6EI}(-x^3 + 2Lx^2 - L^2x)$

(b) y at $x = L$: $\quad y_B = -\frac{wL^4}{24EI} \qquad y_B = \frac{wL^4}{24EI} \downarrow$ ◄

(c) $\frac{dy}{dx}$ at $x = L$: $\quad \frac{dy}{dx}\Big|_B = 0 \qquad \theta_B = 0$ ◄

Problem 9.7

9.7 For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *BC* of the beam, (*b*) the deflection at midspan, (*c*) the slope at *B*.

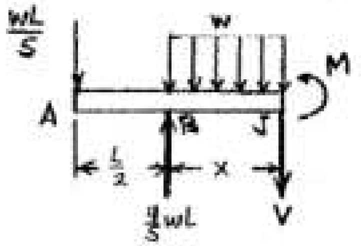


$x = -\frac{L}{2}$ $\begin{bmatrix} x=0 \\ y=0 \end{bmatrix}$ $\begin{bmatrix} x=L \\ y=0 \end{bmatrix}$

Using ABC as a free body,

$$\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{wL}{5}\right)\left(\frac{3L}{2}\right) - R_B L + (wL)\left(\frac{L}{2}\right) = 0$$

$$R_B = \frac{4}{5} wL$$



For portion BC only, $(0 < x < L)$

$$\circlearrowleft \Sigma M_J = 0: \quad \frac{wL}{5}\left(\frac{L}{2} + x\right) - \frac{4}{5} wLx + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{3}{5} wLx - \frac{1}{2} wx^2 - \frac{1}{10} wL^2$$

$$EI \frac{d^2y}{dx^2} = \frac{3}{5} wLx - \frac{1}{2} wx^2 - \frac{1}{10} wL^2$$

$$EI \frac{dy}{dx} = \frac{3}{10} wLx^2 - \frac{1}{6} wx^3 - \frac{1}{10} wL^2 x + C_1$$

$$EI\, y = \frac{1}{10} wLx^3 - \frac{1}{24} wx^4 - \frac{1}{20} wL^2 x^2 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 - 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L,\ y = 0]$ $\quad 0 = \left(\frac{1}{10} - \frac{1}{24} - \frac{1}{20}\right) wL^4 + C_1 L + 0 \qquad C_1 = -\frac{1}{120} wL^3$

(a) <u>Elastic curve.</u>

$$y = \frac{w}{EI}\left(\frac{1}{10} Lx^3 - \frac{1}{24} x^4 - \frac{1}{20} L^2 x^2 - \frac{1}{120} L^3 x\right) \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{3}{10} Lx^2 - \frac{1}{6} x^3 - \frac{1}{10} L^2 x - \frac{1}{120} L^3\right)$$

(b) <u>y @ $x = \frac{L}{2}$:</u>

$$y_M = \frac{w}{EI}\left\{\frac{1}{10} L\left(\frac{L}{2}\right)^3 - \frac{1}{24}\left(\frac{L}{2}\right)^4 - \frac{1}{20} L^2\left(\frac{L}{2}\right)^2 - \frac{1}{120} L^3\left(\frac{L}{2}\right)\right\}$$

$$= \frac{wL^4}{EI}\left\{\frac{1}{80} - \frac{1}{384} - \frac{1}{80} - \frac{1}{240}\right\} = -\frac{13\,wL^4}{1920\,EI}$$

$$y_M = \frac{13\,wL^4}{1920\,EI} \downarrow \blacktriangleleft$$

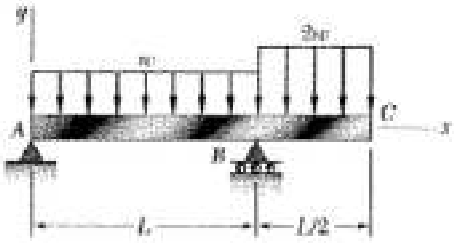
(c) <u>$\frac{dy}{dx}$ @ $x = 0$:</u>

$$\left.\frac{dy}{dx}\right|_B = \frac{w}{EI}\left(0 - 0 - 0 - \frac{1}{120} L^3\right) = -\frac{wL^3}{120\,EI}$$

$$\theta_B = \frac{wL^3}{120\,EI} \blacktriangleleft$$

Problem 9.8

9.8 For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the slope at *A*, (*c*) the slope at *B*.

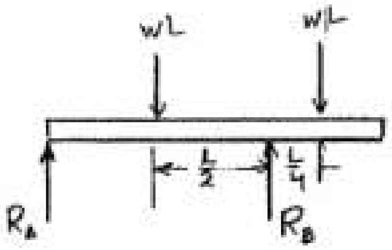


[x=0, y=0]   [x=L, y=0]

Using free body ABC,

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + (wL)\left(\tfrac{L}{2}\right) - (wL)\left(\tfrac{L}{4}\right) = 0$$

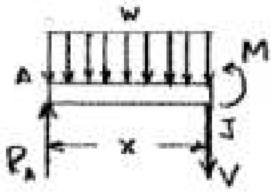
$$R_A = \tfrac{1}{4} wL$$



For portion AB, $(0 < x < L)$

$$+\circlearrowleft \Sigma M_J = 0 \qquad M - R_A x + (wx)\left(\tfrac{x}{2}\right) = 0$$

$$M = \tfrac{1}{4} wLx - \tfrac{1}{2} wx^2$$



$$EI \frac{d^2y}{dx^2} = \tfrac{1}{4} wLx - \tfrac{1}{2} wx^2$$

$$EI \frac{dy}{dx} = \tfrac{1}{8} wLx^2 - \tfrac{1}{6} wx^3 + C_1$$

$$EI\, y = \tfrac{1}{24} wLx^3 - \tfrac{1}{24} wx^4 + C_1 x + C_2$$

$[x=0, y=0]: \quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0] \quad 0 = \tfrac{1}{24} wL^4 - \tfrac{1}{24} wL^4 + C_1 L + 0 = 0 \qquad C_1 = 0$

(a) Elastic curve $(0 \le x \le L)$. $\qquad y = \dfrac{w}{24EI}(Lx^3 - x^4)$ ◄

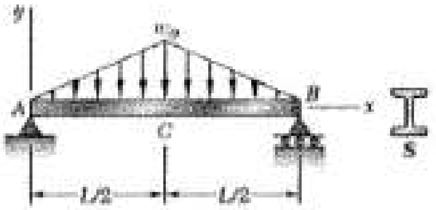
$$\frac{dy}{dx} = \frac{w}{24EI}(3Lx^2 - 4x^3)$$

(b) $\dfrac{dy}{dx}$ at $x = 0$. $\qquad \left.\dfrac{dy}{dx}\right|_A = 0 \qquad \theta_A = 0$ ◄

(c) $\dfrac{dy}{dx}$ at $x = L$. $\qquad \left.\dfrac{dy}{dx}\right|_B = -\dfrac{wL^3}{24EI} \qquad \theta_B = \dfrac{wL^3}{24EI}$ ◄

Problem 9.9

9.9 Knowing that beam AB is an S200 × 27.4 rolled shape and that $w_0 = 60$ kN/m, $L = 2.7$ m, and $E = 200$ GPa, determine (*a*) the slope at A, (*b*) the deflection at C.



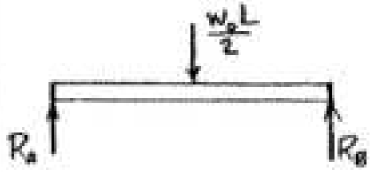
$[x = 0, y = 0]$ $[x = L, y = 0]$

$[x = \frac{L}{2}, \frac{dy}{dx} = 0]$

Use symmetry boundary condition at C.

Using free body ACB and symmetry

$$R_A = R_B = \frac{1}{4} w_0 L$$

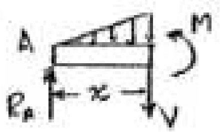


For $0 < x < \frac{L}{2}$ $\quad w = \frac{2 w_0 x}{L}$

$$\frac{dV}{dx} = -w = -\frac{2 w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{L} + R_A = \frac{w_0}{L}\left(\frac{1}{4}L^2 - x^2\right)$$

$$M = \frac{w_0}{L}\left(\frac{1}{4}L^2 x - \frac{1}{3}x^3\right) + C_M$$



But $M = 0$ at $x = 0$; hence $C_M = 0$

$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L}\left(\frac{1}{4}L^2 x - \frac{1}{3}x^3\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L}\left(\frac{1}{8}L^2 x^2 - \frac{1}{12}x^4\right) + C_1$$

$[x = \frac{L}{2}, \frac{dy}{dx} = 0]$ $\quad 0 = \frac{w_0}{L}\left(\frac{1}{32}L^4 - \frac{1}{192}L^4\right) + C_1 = 0 \qquad C_1 = -\frac{5}{192} w_0 L^3$

$$EIy = \frac{w_0}{L}\left(\frac{1}{24}L^2 x^3 - \frac{1}{120}x^5\right) - \frac{5}{192} w_0 L^3 x + C_2$$

$[x = 0, y = 0]$ $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

Elastic curve.

$$y = \frac{w_0}{EIL}\left(\frac{1}{24}L^2 x^3 - \frac{1}{60}x^5 - \frac{5}{192}L^4 x\right)$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{8}L^2 x^2 - \frac{1}{12}x^4 - \frac{5}{192}L^4\right)$$

Data: $w_0 = 60$ kN/m $\quad E = 200$ GPa, $\quad I = 23.9 \times 10^6$ mm^4

$EI = (200 \times 10^9)(23.9 \times 10^{-6}) = 4.78 \times 10^6$ N m^2

$L = 2.7$ m

(a) Slope at $x = 0$. $\quad \frac{dy}{dx} = \frac{60000}{(4.78 \times 10^6)(2.7)}\left[-\left(\frac{5}{192}\right)(2.7)^4\right] = -6.434 \times 10^{-3}$

$\theta_A = 6.43 \times 10^{-3}$ rad ◄

(b) Deflection at $x = 1.35$ m

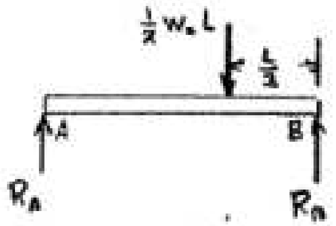
$$y = \frac{60000}{(4.78 \times 10^6)(2.7)}\left[\left(\frac{1}{24}\right)(2.7)^2(1.35)^3 - \frac{1}{60}(1.35)^5 - \frac{5}{192}(2.7)^4(1.35)\right] = -0.0056 \times 10^{-3} \text{ m}$$

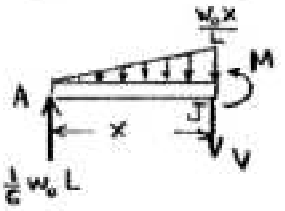
$= 5.6$ mm ↓ ◄

Problem 9.11

9.11 For the beam and loading shown, (*a*) express the magnitude and location of the maximum deflection in terms of w_0, L, E, and I. (*b*) Calculate the value of the maximum deflection, assuming that beam AB is a W460 × 74 rolled shape and that w_0 = 60 kN/m, L = 6 m, and E = 200 GPa.

$[x=0, y=0]$ $[x=L, y=0]$





Using entire beam as a free body,

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\tfrac{1}{2}w_0 L\right)\left(\tfrac{L}{3}\right) = 0$$

$$R_A = \tfrac{1}{6}w_0 L$$

Using AJ as a free body, $+\circlearrowleft \Sigma M_J = 0$:

$$-\tfrac{1}{6}w_0 L x + \left(\tfrac{1}{2}\frac{w_0 x^2}{L}\right)\left(\tfrac{x}{3}\right) + M = 0$$

$$M = \tfrac{1}{6}w_0 L x - \tfrac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{d^2y}{dx^2} = \tfrac{1}{6}w_0 L x - \tfrac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{dy}{dx} = \tfrac{1}{12}w_0 L x^2 - \tfrac{1}{24}\frac{w_0}{L}x^4 + C_1$$

$$EI\,y = \tfrac{1}{36}w_0 L x^3 - \tfrac{1}{120}\frac{w_0}{L}x^5 + C_1 x + C_2$$

$[x=0, y=0]$ $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0]$ $\quad 0 = \frac{1}{36}w_0 L^4 - \frac{1}{120}w_0 L^4 + C_1 L + 0 \qquad C_1 = -\frac{7\,w_0 L^3}{360}$

Elastic curve.

$$y = \frac{w_0}{EI}\left\{\tfrac{1}{36}Lx^3 - \tfrac{1}{120}\frac{x^5}{L} - \tfrac{7}{360}L^3 x\right\}$$

$$\frac{dy}{dx} = \frac{w_0}{EI}\left\{\tfrac{1}{12}Lx^2 - \tfrac{1}{24}\frac{x^4}{L} - \tfrac{7}{360}L^3\right\}$$

To find location of maximum deflection set $\frac{dy}{dx} = 0$.

$$15x_m^4 - 30L^2x_m^2 + 7L^4 = 0 \qquad x_m^2 = \frac{30L^2 - \sqrt{900L^4 - 420L^4}}{30}$$

$$x_m^2 = \left(1 - \sqrt{\tfrac{8}{15}}\right)L^2 = 0.2697\,L^2 \qquad x_m = 0.5193L$$

$$y_m = \frac{w_0}{EI}\left\{\tfrac{1}{36}L(0.5193L)^3 - \tfrac{1}{120}\frac{(0.5193\,L^5)}{L} - \tfrac{7}{360}L^3(0.5193\,L)\right\}$$

$$= -0.00652\frac{w_0L^4}{EI} \quad \text{or} \quad 0.00652\frac{w_0L^4}{EI}\downarrow$$

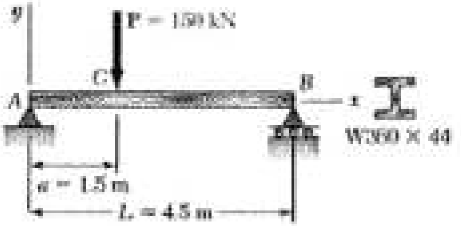
Data: $w_0 = 60$ kN/m $= 60\times10^3$ N/m $\qquad L = 6$ m

For W460 × 74, $\quad I = 333\times10^6\ \text{mm}^4 = 333\times10^{-6}\ \text{m}^4$

$$y_m = \frac{(0.00652)(60\times10^3)(6)^4}{(200\times10^9)(333\times10^{-6})} = 7.61\times10^{-3}\ \text{m} \qquad y_m = 7.61\ \text{mm}\downarrow$$

Problem 9.13

9.13 For the beam and loading shown, determine the deflection at point C. Use $E = 200$ GPa.



$[x=0,\ y=0]$ $\quad [x=L,\ y=0]$

$[x=a,\ y=y]$

$[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}]$

Let $b = L - a$

Reactions: $R_A = \frac{Pb}{L} \uparrow$, $R_B = \frac{Pa}{L} \uparrow$

Bending moments

$0 < x < a \qquad M = \frac{Pb}{L}x$

$a < x < L \qquad M = \frac{P}{L}[bx - L(x-a)]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left(\frac{1}{2}bx^2\right) + C_1 \qquad (1)$$

$$EI\,y = \frac{P}{L}\left(\frac{1}{6}bx^3\right) + C_1x + C_2 \qquad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x-a)]$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left[\frac{1}{2}bx^2 - \frac{1}{2}L(x-a)^2\right] + C_3 \qquad (3)$$

$$EI\,y = \frac{P}{L}\left[\frac{1}{6}bx^3 - \frac{1}{6}L(x-a)^3\right] + C_3x + C_4 \qquad (4)$$

$[x=0,\ y=0]$ Eq (2) $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}]$ Eqs. (1) and (3) $\quad \frac{P}{L}\left(\frac{1}{2}ba^2\right) + C_1 = \frac{P}{L}\left[\frac{1}{2}ba^2 + 0\right] + C_3 \;\therefore\; C_3 = C_1$

$[x=a,\ y=y]$ Eqs (2) and (4) $\quad \frac{P}{L}\left(\frac{1}{6}ba^3\right) + C_1a + C_2$

$$= \frac{P}{L}\left[\frac{1}{6}ba^3 + 0\right] + C_1a + C_4 \qquad C_4 = C_2 = 0$$

$[x=L,\ y=0]$ Eq. (4) $\quad \frac{P}{L}\left[\frac{1}{6}bL^3 - \frac{1}{6}L(L-a)^3\right] + C_3L = 0$

$$C_1 = C_3 = \frac{P}{L}\left[\frac{1}{6}(L-a)^3 - \frac{1}{6}bL^2\right] = \frac{P}{L}\left(\frac{1}{6}b^3 - \frac{1}{6}bL^2\right)$$

Make $x = a$ in Eq (2).

$$y_C = \frac{P}{EIL}\left[\frac{1}{6}ba^3 + \frac{1}{6}b^3a - \frac{1}{6}bL^2a\right] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

Data: $P = 150$ kN, $\quad E = 200$ GPa

$L = 4.5$ m, $\quad a = 1.5$ m, $\quad b = 3$ m

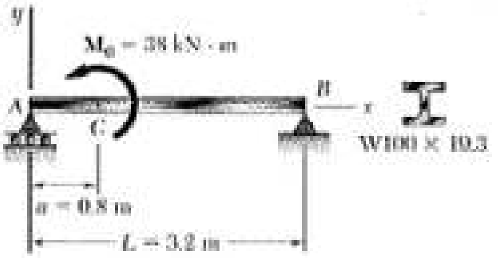
$I = 122 \times 10^{-6}\ \text{m}^4$, $\quad EI = 24.4 \times 10^6\ \text{N m}^2$

$$y_C = \frac{150 \times 10^3}{(24.4\times10^6)(4.5)}\left[(3)(1.5)^3 + (3)^3(1.5) - (4.5)^2(1.5)(3)\right]$$

$$= -9.22 \times 10^{-3}\ \text{m} \qquad y_C = 9.2\ \text{mm} \downarrow$$

Problem 9.14

9.14 For the beam and loading shown, determine the deflection at point C. Use $E = 200$ GPa.



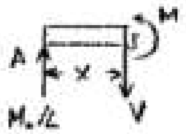
$[x=0, y=0]$ $[x=L, y=0]$

$[x=a, y=y]$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$

Reactions: $R_A = M_0/L \uparrow$, $R_B = M_0/L \downarrow$

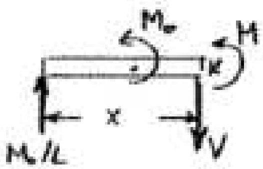
$0 < x < a$: $\circlearrowleft \Sigma M_J = 0$:



$$-\frac{M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}x$$

$a < x < L$: $+\circlearrowleft M_K = 0$:



$$-\frac{M_0}{L}x + M_0 + M = 0$$

$$M = \frac{M_0}{L}(x-L)$$

<u>$0 < x < a$</u>

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}x$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\tfrac{1}{2}x^2\right) + C_1 \qquad (1)$$

$$EI\,y = \frac{M_0}{L}\left(\tfrac{1}{6}x^3\right) + C_1x + C_2 \qquad (2)$$

<u>$a < x < L$</u>

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(x-L)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\tfrac{1}{2}x^2 - Lx\right) + C_3 \qquad (3)$$

$$EI\,y = \frac{M_0}{L}\left(\tfrac{1}{6}x^3 - \tfrac{1}{2}Lx^2\right) + C_3x + C_4 \qquad (4)$$

$[x=0, y=0]$ Eq. (2): $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ Eqs. (1) & (3): $\quad \frac{M_0}{L}\left(\tfrac{1}{2}a^2\right) + C_1 = \frac{M_0}{L}\left(\tfrac{1}{2}a^2 - La\right) + C_3$

$$C_3 = C_1 + M_0a$$

$[x=a, y=y]$ Eqs (2) & (4) $\quad \frac{M_0}{L}\left(\tfrac{1}{6}a^3\right) + \cancel{C_1a} = \frac{M_0}{L}\left(\tfrac{1}{6}a^3 - \tfrac{1}{2}La^2\right) + (\cancel{C_1} + M_0a)a + C_4$

$$C_4 = -\tfrac{1}{2}M_0a^2$$

$[x=L, y=0]$ Eq (4) $\quad \frac{M_0}{L}\left(\tfrac{1}{6}L^3 - \tfrac{1}{2}L^3\right) + (C_1 + M_0a)L - \tfrac{1}{2}M_0a^2 = 0$

$$C_1 = \frac{M_0}{L}\left(\tfrac{1}{3}L^2 + \tfrac{1}{2}a^2 - aL\right)$$

<u>Elastic curve for $0 < x < a$.</u> $\quad y = \frac{M_0}{EIL}\left[\tfrac{1}{6}x^3 + \left(\tfrac{1}{3}L^2 + \tfrac{1}{2}a^2 - aL\right)x\right]$

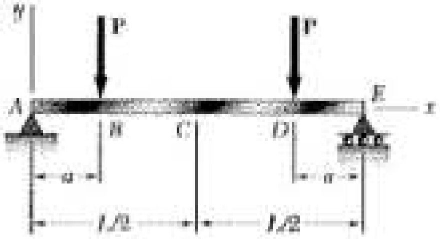
Make $x = a$. $\quad y_C = \frac{M_0}{EIL}\left[\tfrac{1}{6}a^3 + \tfrac{1}{3}L^2a + \tfrac{1}{2}a^3 - a^2L\right] = \frac{M_0}{EIL}\left[\tfrac{2}{3}a^3 + \tfrac{1}{3}L^2a - La^2\right]$

<u>Data:</u> $E = 200\times10^9$ Pa, $I = 4.77\times10^6\ \text{mm}^4 = 4.77\times10^{-6}\ \text{m}$, $M_0 = 38\times10^3$ N·m

$$y_C = \frac{(38\times10^3)\left[(2)(0.8)^3/3 + (3.2)^2(0.8)/3 - (3.2)(0.8)^2\right]}{(200\times10^9)(4.77\times10^{-6})(3.2)} = 12.75\times10^{-3}\ \text{m}$$

$$y_C = 12.75\ \text{mm} \uparrow$$

Problem 9.16



9.16 Knowing that beam *AE* is an S200 × 27.4 rolled shape and that $P = 17.5$ kN, $L = 2.5$ m, $a = 0.8$ m and $E = 200$ GPa, determine (*a*) the equation of the elastic curve for portion *BD*, (*b*) the deflection at the center *C* of the beam.

Consider portion ABC only. Apply symmetry about C.

Reactions: $R_A = R_E = P$

Boundary conditions: $[x=0,\ y=0]$, $[x=a,\ y=y]$, $[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}]$, $[x=\frac{L}{2},\ \frac{dy}{dx}=0]$

$0 < x < a$	$a < x < L-a$
$EI\frac{d^2y}{dx^2} = M = Px$	$EI\frac{d^2y}{dx^2} = M = Pa$
$EI\frac{dy}{dx} = \frac{1}{2}Px^2 + C_1 \qquad (1)$	$EI\frac{dy}{dx} = Pax + C_3$
$EIy = \frac{1}{6}Px^3 + C_1x + C_2 \qquad (2)$	$EIy = \frac{1}{2}Pax^2 + C_3x + C_4$
$[x=0,\ y=0] \rightarrow C_2 = 0$	$[x=\frac{L}{2},\ \frac{dy}{dx}=0] \rightarrow C_3 = -\frac{1}{2}PaL$

$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}] \qquad \frac{1}{2}Pa^2 + C_1 = Pa^2 - \frac{1}{2}PaL \qquad C_1 = \frac{1}{2}Pa^2 - \frac{1}{2}PaL$

$[x=\frac{L}{2},\ y=y] \qquad \frac{1}{6}Pa^3 + (\frac{1}{2}Pa^2 - \frac{1}{2}PaL)a = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L + C_4$

$$C_4 = \frac{1}{6}Pa^3$$

(a) Elasic curve for portion BD.

$$y = \frac{1}{EI}\left(\frac{1}{2}Pax^2 + C_3x + C_4\right)$$

$$= \frac{P}{EI}\left(\frac{1}{2}ax^2 - \frac{1}{2}aLx + \frac{1}{6}a^3\right)$$

For deflection at C. set $x = \frac{L}{2}$.

$$y_C = \frac{P}{EI}\left(\frac{1}{8}aL^2 - \frac{1}{4}aL^2 + \frac{1}{6}a^3\right) = -\frac{Pa}{EI}\left(\frac{1}{8}L^2 - \frac{1}{6}a^2\right)$$

Data: $I = 23.9\times10^6\ \text{mm}^4 = 23.9\times10^{-6}\ \text{m}^4$, $E = 200\times10^9$ Pa

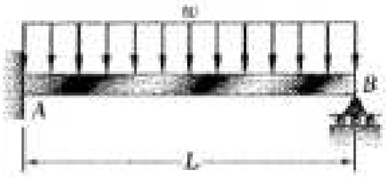
$P = 17.5\times10^3$ N, $L = 2.5$ m, $a = 0.8$ m

(b) $$y_C = -\frac{(17.5\times10^3)(0.8)}{(200\times10^9)(23.9\times10^{-6})}\left\{\frac{2.5^2}{8} - \frac{0.8^2}{6}\right\} = -1.976\times10^{-3}\ \text{m}$$

$$y_C = 1.976\ \text{mm}\downarrow$$

Problem 9.19

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.

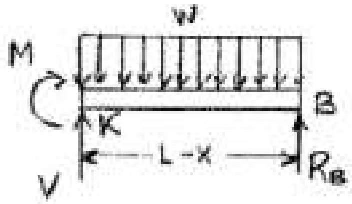


$[x=0, y=0]$ $[x=0, y=0]$
$[x=0, \frac{dy}{dx}=0]$

Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body KB,



$+\circlearrowleft M_K = 0: \quad R_B(L-x) - w(L-x)\left(\frac{L-x}{2}\right) - M = 0$

$M = R_B(L-x) - \frac{1}{2}w(L-x)^2$

$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{1}{2}w(L-x)^2$

$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L-x)^2 + \frac{1}{6}w(L-x)^3 + C_1$

$[x=0, \frac{dy}{dx}=0]: \quad 0 = -\frac{1}{2}R_BL^2 + \frac{1}{6}wL^3 + C_1 \qquad C_1 = \frac{1}{2}R_BL^2 - \frac{1}{6}wL^3$

$EIy = \frac{1}{6}R_B(L-x)^3 - \frac{1}{24}w(L-x)^4 + C_1x + C_2$

$[x=0, y=0]: \quad 0 = \frac{1}{6}R_BL^3 - \frac{1}{24}wL^4 + C_2 \qquad C_2 = -\frac{1}{6}R_BL^3 + \frac{1}{24}wL^4$

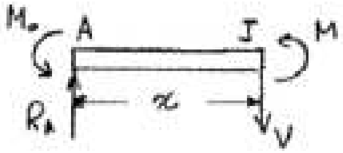
$[x=L, y=0] \quad 0 = 0 - 0 + C_1L + C_2$

$\frac{1}{2}R_BL^3 - \frac{1}{6}wL^4 - \frac{1}{6}R_BL^3 + \frac{1}{24}wL^4 = 0 \qquad R_B = \frac{3}{8}wL \uparrow$ ◄

Problem 9.20

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.

$[x = 0, y = 0]$ $[x = L, y = 0]$ $[x = L, \frac{dy}{dx} = 0]$



Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body AJ,

$$+\circlearrowleft \Sigma M_J = 0: \quad M_0 - R_A x + M = 0$$

$$M = R_A x - M_0$$

$$EI \frac{d^2y}{dx^2} = R_A x - M_0$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_0 x + C_1$$

$[x = L, \frac{dy}{dx} = 0]$ $\quad 0 = \frac{1}{2} R_A L^2 - M_0 L + C_1 \qquad C_1 = M_0 L - \frac{1}{2} R_A L^2$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_0 x^2 + C_1 x + C_2$$

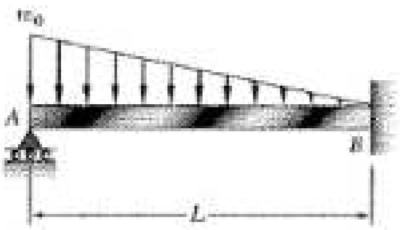
$[x = 0, y = 0]$ $\rightarrow$ $C_2 = 0$

$[x = L, y = 0]$ $\quad 0 = \frac{1}{6} R_A L^3 - \frac{1}{2} M_0 L^2 + (M_0 L - \frac{1}{2} R_A L^2) L + 0.$

$$R_A = \frac{3}{2} \frac{M_0}{L} \uparrow$$

Problem 9.22

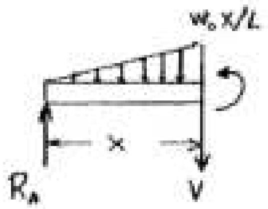
9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



Reactions are statically indeterminate.

Boundary conditions are shown at left.

$[x=0, y=0]$ $\quad$ $[x=L, y=0]$ $\quad$ $[x=L, \frac{dy}{dx}=0]$



$$w = \frac{w_0}{L}(L-x)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L-x)$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}\left(Lx - \frac{1}{2}x^2\right) + R_A$$

$$M = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L}\left(\frac{1}{6}Lx^3 - \frac{1}{24}x^4\right) + \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = -\frac{w_0}{L}\left(\frac{1}{24}Lx^4 - \frac{1}{120}x^5\right) + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x=0, y=0]$ $\qquad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, \frac{dy}{dx}=0]$ $\qquad -\frac{w_0}{L}\left(\frac{1}{6}L^4 - \frac{1}{24}L^4\right) + \frac{1}{2}R_A L^2 + C_1 = 0$

$$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$$

$[x=L, y=0]$ $\qquad -\frac{w_0}{L}\left(\frac{1}{24}L^4 - \frac{1}{120}L^4\right) + \frac{1}{6}R_A L^3 + \left(\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2\right)L = 0$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{8} - \frac{1}{24} + \frac{1}{120}\right)w_0 L$$

$$\frac{1}{3}R_A = \frac{11}{120}w_0 L \qquad\qquad R_A = \frac{11}{40}w_0 L \uparrow \blacktriangleleft$$

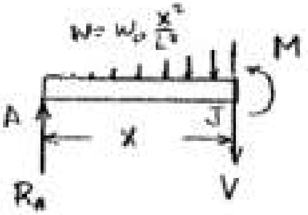
Problem 9.24

9.24 For the beam shown determine the reaction at the roller support when $w_0 = 65$ kN/m.

Reactions are statically indeterminate.

Boundary conditions are shown at left.

$[x=0,\ y=0]$ $\quad$ $[x=L,\ y=0]$ $\quad$ $[x=L,\ \frac{dy}{dx}=0]$



$$w = w_0 \frac{x^2}{L^2}$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2}x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2}\frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L^2}\frac{x^5}{60} + \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = -\frac{w_0}{L^2}\frac{x^6}{360} + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x=0,\ y=0]$ $\quad 0 = 0 + 0 + 0 + C_2 \quad\quad C_2 = 0$

$[x=L,\ \frac{dy}{dx}=0]$ $\quad -\frac{1}{60}w_0 L^3 + \frac{1}{2}R_A L^2 + C_1 = 0 \quad\quad C_1 = \frac{1}{60}w_0 L^3 - \frac{1}{2}R_A L^2$

$[x=L,\ y=0]$ $\quad -\frac{1}{360}w_0 L^4 + \frac{1}{6}R_A L^3 + \left(\frac{1}{60}w_0 L^3 - \frac{1}{2}R_A L^2\right)L = 0$

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A = \left(\frac{1}{60}-\frac{1}{360}\right)w_0 L$$

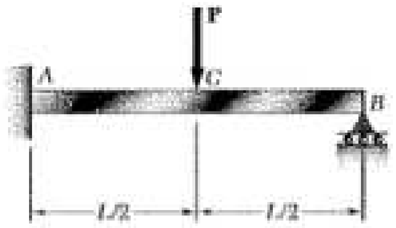
$$\frac{1}{3}R_A = \frac{1}{72}w_0 L \quad\quad R_A = \frac{1}{18}w_0 L$$

Data: $w_0 = 65$ kN/m, $\quad L = 4$ m

$$R_A = \frac{1}{18}(65)(4) = 14.44 \text{ kN} \uparrow$$ ◄

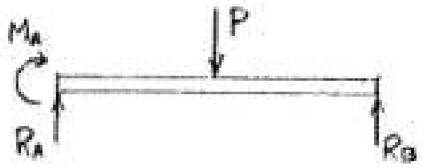
Problem 9.25

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



$[x=0, y=0]$ $\qquad [x=L, y=0]$

$[x=0, \frac{dy}{dx}=0]$



Reactions are statically indeterminate.

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = P - R_B$

$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + \frac{1}{2}PL - R_B L = 0$

$M_A = R_B L - \frac{1}{2}PL$

$\underline{0 < x < \frac{1}{2}L:} \quad M = M_A + R_A x$

$$EI\frac{d^2y}{dx^2} = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$\underline{\frac{1}{2}L < x < L:} \quad M = M_A + R_A x - P(x - \frac{1}{2}L)$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P(x - \frac{1}{2}L)$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P(x - \frac{1}{2}L)^2 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P(x - \frac{L}{2})^3 + C_3 x + C_4$$

$[x=0, \frac{dy}{dx}=0] \qquad 0 + 0 + C_1 = 0 \qquad\qquad C_1 = 0$

$[x=0, y=0] \qquad 0 + 0 + 0 + C_2 = 0 \qquad\qquad C_2 = 0$

$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \qquad \frac{1}{2}M_A L + \frac{1}{8}R_A L^2 + 0 = \frac{1}{2}M_A + \frac{1}{8}R_A L - 0 + C_3 \qquad\qquad C_3 = 0$

$[x=\frac{L}{2}, y=y] \qquad \frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 + 0 + 0$

$\qquad = \frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 - 0 + 0 + C_4 \qquad\qquad C_4 = 0$

$[x=L, y=0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{48}PL^3 + 0 + 0 = 0$

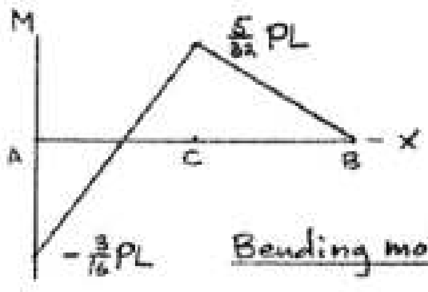
$\qquad \frac{1}{2}(R_B L - \frac{1}{2}P)L^3 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0 \qquad\qquad R_B = \frac{5}{16}P\uparrow$ ◄

$R_A = P - \frac{5}{16}P \qquad\qquad R_A = \frac{7}{16}P\uparrow$

$M_A = \frac{5}{16}PL - \frac{1}{2}PL \qquad\qquad M_A = -\frac{3}{16}PL$ ◄

$M_C = R_B(\frac{L}{2}) = (\frac{5}{16}P)(\frac{L}{2}) \qquad\qquad M_C = \frac{5}{32}PL$ ◄

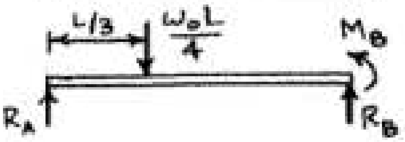
$M_B = 0$ ◄



Bending moment diagram

Problem 9.27

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



REACTIONS ARE STATICALLY INDETERMINATE.

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - \frac{w_0 L}{4} = 0 \qquad R_B = \frac{w_0 L}{4} - R_A$

$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{w_0 L}{4}\right)\left(\frac{2L}{3}\right) + M_B = 0 \qquad M_B = R_A L - \frac{w_0 L^2}{6}$

$0 \le x \le \frac{L}{2}: \quad w = \frac{2w_0}{L}x \qquad V = R_A - \frac{w_0}{L}x^2 \qquad M = R_A x - \frac{w_0}{3L}x^3$

$$EI\frac{d^2y}{dx^2} = R_A x - \frac{w_0}{3L}x^3$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{12}\frac{w_0}{L}x^4 + C_1$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{60}\frac{w_0}{L}x^5 + C_1 x + C_2$$

$\frac{L}{2} \le x \le L: \quad M = R_A x - \frac{w_0 L}{4}\left(x - \frac{L}{3}\right)$

$$EI\frac{d^2y}{dx^2} = R_A x - w_0 L\left(\frac{1}{4}x - \frac{1}{12}L\right)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - w_0 L\left(\frac{1}{8}x^2 - \frac{1}{12}Lx\right) + C_3$$

$$EI\,y = \frac{1}{6}R_A x^3 - w_0 L\left(\frac{1}{24}x^3 - \frac{1}{24}Lx^2\right) + C_3 x + C_4$$

$[x = 0,\ y = 0]: \quad 0 = 0 - 0 + 0 + C_2 \qquad \therefore C_2 = 0$

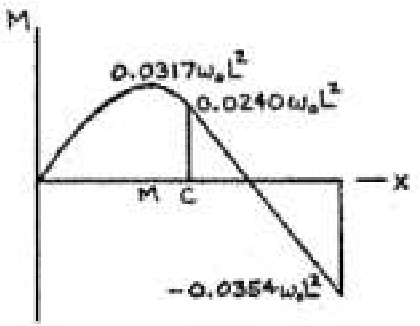
$[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}]: \quad \cancel{\frac{1}{8}R_A L^2} - \frac{1}{192}w_0 L^3 + C_1 = \cancel{\frac{1}{8}R_A L^2} + \frac{1}{96}w_0 L^3 + C_3 \qquad \therefore C_3 = C_1 - \frac{1}{64}w_0 L^3$

$[x = \frac{L}{2},\ y = y]: \quad \cancel{\frac{1}{48}R_A L^3} - \frac{1}{1920}w_0 L^4 + \cancel{\frac{1}{2}C_1 L} + 0$
$\quad = \cancel{\frac{1}{48}R_A L^3} - w_0 L\left(\frac{1}{192}L^3 - \frac{1}{96}L^3\right) + (\cancel{C_1} - \frac{1}{64}w_0 L^3)\left(\frac{L}{2}\right) + C_4 \qquad \therefore C_4 = \frac{1}{480}w_0 L^4$

$[x = L,\ \frac{dy}{dx} = 0]: \quad \frac{1}{2}R_A L^2 - w_0 L\left(\frac{1}{8}L^2 - \frac{1}{12}L^2\right) + C_3 = 0 \qquad \therefore C_3 = -\frac{1}{2}R_A L^2 + \frac{1}{24}w_0 L^3$

$[x = L,\ y = 0]: \quad \frac{1}{6}R_A L^3 - w_0 L\left(\frac{1}{24}L^3 - \frac{1}{24}L^3\right) + \left(-\frac{1}{2}R_A L^2 + \frac{1}{24}w_0 L^3\right)(L) + \frac{1}{480}w_0 L^4 = 0$

$R_A = \frac{21}{160}w_0 L \uparrow$ ◄



$R_B = \frac{w_0 L}{4} - \frac{21}{160}w_0 L \qquad R_B = \frac{19}{160}w_0 L \uparrow$

$M_B = \frac{21}{160}w_0 L^2 - \frac{w_0 L^2}{6} \qquad M_B = -\frac{17}{480}w_0 L^2 = -0.0354\, w_0 L^2$ ◄

OVER $0 < x < \frac{L}{2}, \quad V = R_A - \frac{w_0}{L}x^2 = \frac{21}{160}w_0 L - \frac{w_0}{L}x^2$

$V = 0$ AT $x = x_m = 0.36228L$

$M = \frac{21}{160}w_0 L x - \frac{w_0}{3L}x^3 \qquad M_A = M(x=0) = 0$

$M_C = M(x = \frac{L}{2}) = 0.0240 w_0 L^2 \qquad M_m = M(x_m = 0.36228L) = 0.0317 w_0 L^2$ ◄

Problem 9.28

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

$[x=0, y=0]$ $\quad [x=L, y=0]$

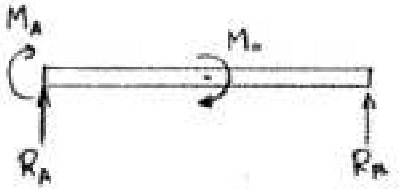
$[x=0, \frac{dy}{dx}=0]$

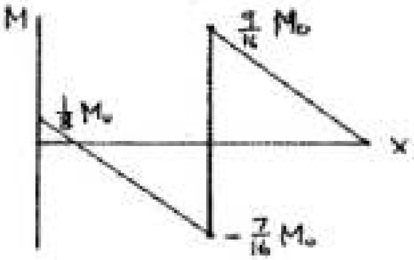
Reactions are statically indeterminate.

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B = 0 \qquad R_A = -R_B$

$\circlearrowleft \Sigma M_A = 0: \quad -M_A - M_0 + R_B L = 0$

$M_A = R_B L - M_0$





$0 < x < \frac{L}{2}$

$M = R_A x + M_A = -M_0 + R_B L - R_B x$

$EI\frac{d^2y}{dx^2} = -M_0 + R_B(L-x)$

$EI\frac{dy}{dx} = -M_0 x + R_B(Lx - \frac{1}{2}x^2) + C_1$

$EIy = -\frac{1}{2}M_0 x^2 + R_B(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + C_1 x + C_2$

$\frac{L}{2} < x < L \qquad M = R_B(L-x)$

$EI\frac{d^2y}{dx^2} = R_B(L-x)$

$EI\frac{dy}{dx} = R_B(Lx - \frac{1}{2}x^2) + C_3$

$EIy = R_B(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + C_3 x + C_4$

$[x=0, \frac{dy}{dx}=0] \qquad 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x=0, y=0] \qquad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \qquad -M_0\frac{L}{2} + \cancel{R_B(\frac{1}{2}L^2 - \frac{1}{6}L^2)} = \cancel{R_B(\frac{1}{2}L^2 - \frac{1}{6}L^2)} + C_3 \qquad C_3 = -\frac{M_0L}{2}$

$[x=\frac{L}{2}, y=y] \qquad -\frac{1}{2}M_0(\frac{L}{2})^2 + \cancel{R_B(\frac{1}{8}L^3 - \frac{1}{48}L^3)} = \cancel{R_B(\frac{1}{8}L^3 - \frac{1}{48}L^3)} + C_3\frac{L}{2} + C_4$

$C_4 = -\frac{1}{8}M_0L^2 - \frac{1}{2}C_3L = (-\frac{1}{8} + \frac{1}{4})M_0L^2 = \frac{1}{8}M_0L^2$

$[x=L, y=0] \qquad R_B(\frac{1}{2}L^3 - \frac{1}{6}L^3) - \frac{M_0L}{2}L + \frac{1}{8}M_0L^2 = 0$

$(\frac{1}{2} - \frac{1}{6})R_BL^3 = (\frac{1}{2} - \frac{1}{8})M_0L^2 \qquad \frac{1}{3}R_B = \frac{3}{8}\frac{M_0}{L}$

$R_B = \frac{9}{8}\frac{M_0}{L}\uparrow$ ◄

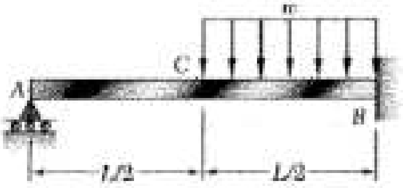
$M_A = \frac{9}{8}M_0 - M_0 = \frac{1}{8}M_0 \qquad M_A = \frac{1}{8}M_0$ ◄

$M_{C^-} = -M_0 + \frac{9}{8}\frac{M_0}{L}\frac{L}{2} = -\frac{7}{16}M_0 \qquad M_{C^-} = -\frac{7}{16}M_0$ ◄

$M_{C^+} = R_B(L - \frac{L}{2}) = \frac{9}{8}\frac{M_0}{L}(\frac{L}{2}) = \frac{9}{16}M_0 \qquad M_{C^+} = \frac{9}{16}M_0$ ◄

Problem 9.29

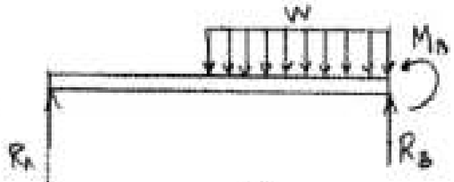
9.29 and 9.30 Determine the reaction at the roller support and the deflection at point C.

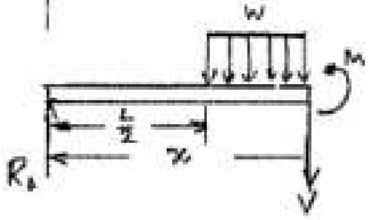


[x=0, y=0] [x=L, y=0] [x=L, $\frac{dy}{dx}=0$]

[x=$\frac{L}{2}$, y=y]

[x=$\frac{L}{2}$, $\frac{dy}{dx}=\frac{dy}{dx}$]





Reactions are statically indeterminate.

$0 < x \le \frac{L}{2}$ $M = R_A x$

$EI \frac{d^2y}{dx^2} = R_A x$

$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$

$EI\, y = \frac{1}{6} R_A x^3 + C_1 x + C_2$

$\frac{L}{2} \le x < L$

$M = R_A x - w(x-\frac{L}{2})\frac{1}{2}(x-\frac{L}{2}) = R_A x - \frac{1}{2}w(x-\frac{L}{2})^2$

$EI \frac{d^2y}{dx^2} = R_A x - \frac{1}{2}w(x-\frac{L}{2})^2$

$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}w(x-\frac{L}{2})^3 + C_3$

$EI\, y = \frac{1}{6}R_A x^3 - \frac{1}{24}w(x-\frac{L}{2})^4 + C_3 x + C_4$

[x=0, y=0] $0 + 0 + C_2 = 0$ $C_2 = 0$

[x=$\frac{L}{2}$, $\frac{dy}{dx}=\frac{dy}{dx}$] $\frac{1}{8}R_A L^2 + C_1 = \frac{1}{8}R_A x^2 + C_3$ $C_3 = C_1$

[x=$\frac{L}{2}$, y=y] $\frac{1}{48}R_A L^3 + \frac{1}{2}C_1 L + 0 = \frac{1}{48}R_A x^3 - 0 + \frac{1}{2}C_1 L + C_4$

$C_4 = 0$

[x=L, $\frac{dy}{dx}=0$] $\frac{1}{2}R_A L^2 - \frac{1}{24}wL^3 + C_3 = 0$ $C_3 = -(\frac{1}{2}R_A L^2 - \frac{1}{48}wL^3)$

[x=L, y=0] $\frac{1}{6}R_A L^3 - \frac{1}{384}wL^4 - (\frac{1}{2}R_A - \frac{1}{48}wL^2)L + 0$

$R_A = \frac{7}{128}wL \uparrow$

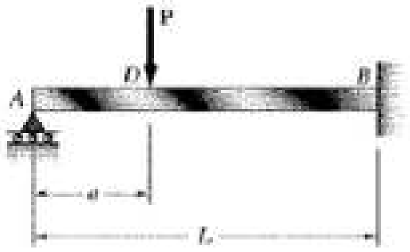
$C_1 = C_3 = -(\frac{7}{256}wL^3 - \frac{1}{48}wL^3) = -\frac{5}{768}wL^3$

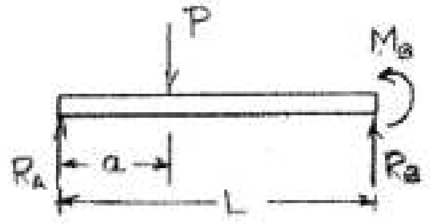
At $x = \frac{L}{2}$ $EI\, y_C = \frac{1}{6}R_A(\frac{L}{2})^3 + (-\frac{5}{768}wL^3)\frac{L}{2} + 0 = -\frac{13}{6144}wL^4$

$y_C = \frac{13}{6144}\frac{wL^4}{EI} \downarrow$

Problem 9.31

9.31 and 9.32 Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.





$0 \le x \le a$:

$$M = R_A x$$

$$EI\frac{d^2y}{dx^2} = M = R_A x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$a \le x \le L$:

$$M = R_A x - P(x-a)$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - P(x-a)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{2}P(x-a)^2 + C_3$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{6}P(x-a)^3 + C_3 x + C_4$$

$[x=0,\ y=0]$: $\quad 0 + 0 + C_2 = 0 \qquad \therefore C_2 = 0$

$\left[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}\right]$: $\quad \frac{1}{2}R_A a^2 + C_1 = \frac{1}{2}R_A a^2 - 0 + C_3 \qquad \therefore C_1 = C_3$

$[x=a,\ y=y]$: $\quad \frac{1}{6}R_A a^3 + C_1 a + 0 = \frac{1}{6}R_A a^3 - 0 + C_1 a + C_4 \qquad \therefore C_4 = 0$

$\left[x=L,\ \frac{dy}{dx}=0\right]$: $\quad \frac{1}{2}R_A L^2 - \frac{1}{2}P(L-a)^2 + C_3 = 0 \qquad \therefore C_3 = \frac{1}{2}P(L-a)^2 - \frac{1}{2}R_A L^2$

$[x=L,\ y=0]$: $\quad \frac{1}{6}R_A L^3 - \frac{1}{6}P(L-a)^3 + \left[\frac{1}{2}P(L-a)^2 - \frac{1}{2}R_A L^2\right](L) + 0 = 0$

$$R_A = \frac{P}{2L^3}(2L^3 - 3aL^2 + a^3) = \frac{P}{2L^3}\left(2L^3 - L^3 + \frac{L^3}{9}\right) \qquad R_A = \frac{14}{27}P \uparrow$$

<u>DEFLECTION AT D.</u> (y AT $x = a = \frac{L}{3}$)

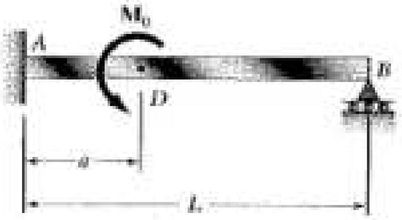
$$y_D = \frac{1}{EI}\left\{\frac{1}{6}R_A\left(\frac{L}{3}\right)^3 + C_1\left(\frac{L}{3}\right)\right\}$$

$$= \frac{1}{EI}\left\{\frac{1}{6}\left(\frac{14}{27}P\right)\left(\frac{L}{3}\right)^3 + \left[\frac{1}{2}P\left(L-\frac{L}{3}\right)^2 - \frac{1}{2}\left(\frac{14}{27}P\right)L^2\right]\left(\frac{L}{3}\right)\right\} = -\frac{20}{2187}\frac{PL^3}{EI}$$

$$y_D = \frac{20}{2187}\frac{PL^3}{EI} \downarrow$$

Problem 9.32

9.31 and 9.32 Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.



$+\uparrow \Sigma F_y = 0: \quad R_A + R_B = 0 \qquad R_A = -R_B$

$+\circlearrowleft \Sigma M_A = 0: \quad M_0 - M_A + R_B L = 0 \qquad M_A = R_B L + M_0$

$0 \le x \le a: \quad M = M_A + R_A x$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$a \le x \le L: \quad M = M_A + R_A x - M_0$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - M_0$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - M_0 x + C_3$$

$$EI\,y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{2}M_0 x^2 + C_3 x + C_4$$

$\left[x=0, \frac{dy}{dx}=0\right]: \quad 0 + 0 + C_1 = 0 \qquad \therefore C_1 = 0$

$[x=0, y=0]: \quad 0+0+0+C_2 = 0 \qquad \therefore C_2 = 0$

$\left[x=a, \frac{dy}{dx}=\frac{dy}{dx}\right]: \quad M_A a + \frac{1}{2}R_A a^2 = M_A a + \frac{1}{2}R_A a^2 - M_0 a + C_3 \qquad \therefore C_3 = M_0 a$

$[x=a, y=y]: \quad \frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 = \frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 - \frac{1}{2}M_0 a^2 + (M_0 a)(a) + C_4 \qquad \therefore C_4 = -\frac{1}{2}M_0 a^2$

$[x=L, y=0]: \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{2}M_0 L^2 + (M_0 a)(L) - \frac{1}{2}M_0 a^2 = 0$

$$\frac{1}{2}(R_B L + M_0)L^2 + \frac{1}{6}(-R_B)L^3 - \frac{1}{2}M_0 L^2 + M_0 a L - \frac{1}{2}M_0 a^2 = 0$$

$$R_B = \frac{3M_0 a}{2L^3}(a - 2L) = \frac{3M_0}{2L^3}\left(\frac{L}{3}\right)\left(\frac{L}{3} - 2L\right) = -\frac{5M_0}{6L} \qquad R_B = \frac{5M_0}{6L}\downarrow \blacktriangleleft$$

<u>DEFLECTION AT D.</u> (y AT $x = a = \frac{L}{3}$)

$$y_D = \frac{1}{EI}\left\{\frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3\right\} = \frac{1}{EI}\left\{\frac{1}{2}\left(-\frac{5M_0}{6L}L + M_0\right)\left(\frac{L}{3}\right)^2 + \frac{1}{6}\left(+\frac{5M_0}{6L}\right)\left(\frac{L}{3}\right)^3\right\}$$

$$= \frac{7M_0L^2}{486EI} \qquad y_D = \frac{7M_0L^2}{486EI}\uparrow \blacktriangleleft$$

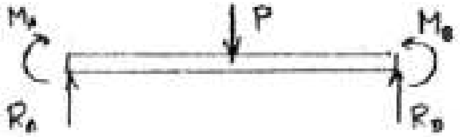
Problem 9.33

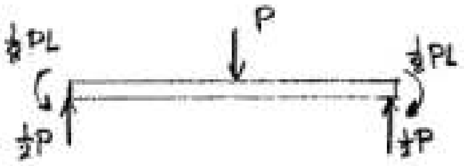
9.33 and 9.34 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

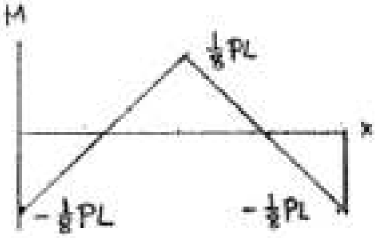
$[x=0, y=0]$

$[x=0, \frac{dy}{dx}=0]$

$[x=\frac{L}{2}, \frac{dy}{dx}=0]$







By symmetry, $R_A = R_B$ and $\frac{dy}{dx} = 0$ at $x = \frac{L}{2}$.

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = R_B = \frac{1}{2}P$ ◄

Moment reaction is statically indeterminate.

<u>$0 < x < \frac{L}{2}$</u> $\qquad M = M_A + R_A x = M_A + \frac{1}{2}Px$

$$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}Px$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{4}Px^2 + C_1$$

$[x=0, \frac{dy}{dx}=0] \qquad 0 - 0 + C_1 = 0 \qquad C_1 = 0$

$[x=\frac{L}{2}, \frac{dy}{dx}=0] \qquad M_A\frac{L}{2} + \frac{1}{4}P\left(\frac{L}{2}\right)^2 + 0 = 0$

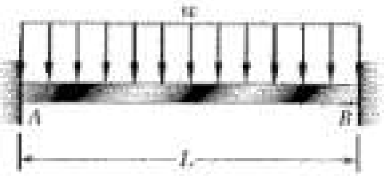
$M_A = -\frac{1}{8}PL \qquad M_A = \frac{1}{8}PL$ ↺ ◄

By symmetry, $M_B = M_A = \frac{1}{8}PL$ ↻ ◄

$M_C = M_A + \frac{1}{2}P\frac{L}{2} = -\frac{1}{8}PL + \frac{1}{4}PL = \frac{1}{8}PL$ ◄

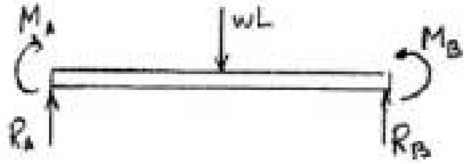
Problem 9.34

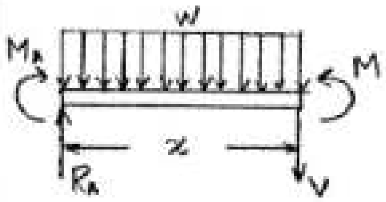
9.33 and 9.34 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.



$[x=0, y=0]$ $\quad$ $[x=L, y=0]$

$\left[x=0, \frac{dy}{dx}=0\right]$ $\quad$ $\left[x=L, \frac{dy}{dx}=0\right]$





Reactions are statically indeterminate.

By symmetry, $R_B = R_A$; $M_B = M_A$

$$\frac{dy}{dx} = 0 \quad \text{at } x = \frac{L}{2}$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wL = 0$$

$$R_B = R_A = \frac{1}{2}wL \uparrow \blacktriangleleft$$

Over entire beam, $M = M_A + R_A x - \frac{1}{2}wx^2$

$$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}wLx - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{4}wLx^2 - \frac{1}{6}wx^3 + C_1$$

$$\left[x=0, \frac{dy}{dx}=0\right] \quad 0 + 0 - 0 + C_1 = 0$$

$$C_1 = 0$$

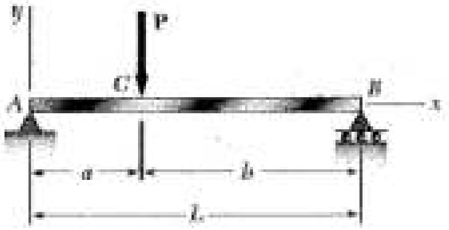
$$\left[x=\frac{L}{2}, \frac{dy}{dx}=0\right] \quad \frac{1}{2}M_A L + \frac{1}{16}wL^3 - \frac{1}{48}wL^3 + 0 = 0$$

$$M_A = -\frac{1}{12}wL^2 \blacktriangleleft$$

$$M = -\frac{1}{12}wL^2 + \frac{1}{2}wLx - \frac{1}{2}wx^2 \qquad M = w\cdot[6x(L-x) - L^2]/12 \blacktriangleleft$$

Problem 9.35

9.35 and 9.36 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.



$[x=0, M=0]$ $[x=L, M=0]$

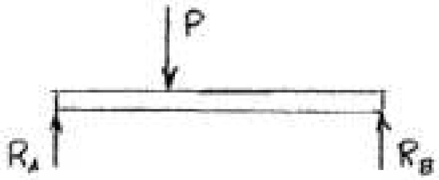
$[x=0, y=0]$ $[x=L, y=0]$

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + Pb = 0 \qquad R_A = \frac{Pb}{L}$$

$$\frac{dM}{dx} = V = R_A - P\langle x-a\rangle^0 = \frac{Pb}{L} - P\langle x-a\rangle^0$$

$$M = \frac{Pb}{L}x - P\langle x-a\rangle^1 + \cancel{M_A}$$

$$EI\frac{d^2y}{dx^2} = \frac{Pb}{L}x - P\langle x-a\rangle$$



$$EI\frac{dy}{dx} = \frac{Pb}{2L}x^2 - \frac{1}{2}P\langle x-a\rangle^2 + C_1$$

$$EIy = \frac{Pb}{6L}x^3 - \frac{1}{6}P\langle x-a\rangle^3 + C_1x + C_2$$

$[x=0, y=0]$ $\qquad C_2 = 0$

$[x=L, y=0]$ $\qquad \frac{Pb}{6L}L^3 - \frac{1}{6}P(L-a)^3 + C_1L = 0$

$$C_1 = -\frac{1}{6}\frac{P}{L}(bL^2 - b^3) = -\frac{1}{6}\frac{Pb}{L}(L^2-b^2)$$

(a) Elastic curve.

$$y = \frac{P}{EI}\left\{\frac{b}{6L}x^3 - \frac{1}{6}\langle x-a\rangle^3 - \frac{1}{6}\frac{b}{L}(L^2-b^2)x\right\}$$

$$y = \frac{P}{6EIL}\left\{bx^3 - L\langle x-a\rangle^3 - b(L^2-b^2)x\right\} \blacktriangleleft$$

(b) Slope at end A.

$$EI\left.\frac{dy}{dx}\right|_{x=0} = C_1 = -\frac{Pb}{6L}(L^2-b^2)$$

$$\theta_A = -\frac{Pb}{6EIL}(L^2-b^2) \qquad \theta_A = \frac{Pb}{6EIL}(L^2-b^2) \measuredangle \blacktriangleleft$$

(c) Deflection at C.

$$EIy_C = \frac{Pb}{6L}a^3 + C_1a = \frac{Pba^3}{6L} - \frac{Pb}{6L}(L^2-b^2)a$$

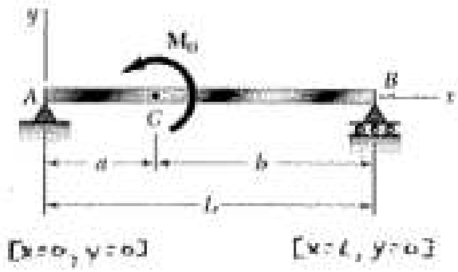
$$= \frac{Pba}{6L}(a^2 - L^2 - b^2)$$

$$y_C = -\frac{Pab}{6EIL}(L^2 - a^2 - b^2) = -\frac{Pab}{6EIL}\{a^2 + 2ab + b^2 - a^2 - b^2\}$$

$$= -\frac{Pa^2b^2}{3EIL} \qquad y_C = \frac{Pa^2b^2}{3EIL}\downarrow \blacktriangleleft$$

Problem 9.36

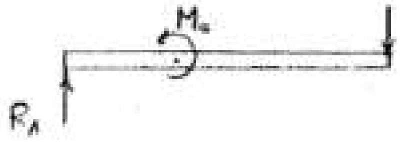
9.35 and 9.36 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.



Reactions $R_A = \frac{M_0}{L}\uparrow$, $R_B = \frac{M_0}{L}\downarrow$

$0 < x < a$ $M = R_A x$

$a < x < L$ $M = R_A x - M_0$



Using singularity functions,

$$EI\frac{d^2y}{dx^2} = M = R_A x - M_0\langle x-a\rangle^0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\langle x-a\rangle^1 + C_1$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\langle x-a\rangle^2 + C_1 x + C_2$$

[x=0, y=0] $0 = 0 - 0 + 0 + C_2$ $C_2 = 0$

[x=L, y=0] $\frac{1}{6}R_A L^3 - \frac{1}{2}M_0(L-a)^2 + C_1 L + 0 = 0$

$$C_1 L = -\frac{1}{6}\frac{M_0}{L}L^3 + \frac{1}{2}M_0 b^2 \qquad C_1 = \frac{M_0}{6L}(3b^2 - L^2)$$

(a) Elastic curve. $y = \frac{1}{EI}\left\{\frac{1}{6}\frac{M_0}{L}x^3 - \frac{1}{2}M_0\langle x-a\rangle^2 + \frac{M_0}{6L}(3b^2-L^2)x\right\}$

$$= \frac{M_0}{6EIL}\left\{x^3 - 3L\langle x-a\rangle^2 + (3b^2-L^2)x\right\}$$

$$\frac{dy}{dx} = \frac{M_0}{6EIL}\left\{3x^2 - 6L\langle x-a\rangle^1 + (3b^2-L^2)\right\}$$

(b) Slope at A. $\left(\frac{dy}{dx}\text{ at } x=0\right)$

$$\theta_A = \frac{M_0}{6EIL}\left\{0 - 0 + 3b^2 - L^2\right\} \qquad \theta_A = \frac{M_0}{6EIL}(3b^2-L^2)$$

(c) Deflection at C. (y at x = a)

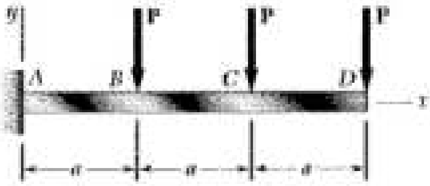
$$y_C = \frac{M_0}{6EIL}\left\{a^3 - 0 + (3b^2-L^2)a\right\} = \frac{M_0 a}{6EIL}\left\{a^2 + 3b^2 - (a+b)^2\right\}$$

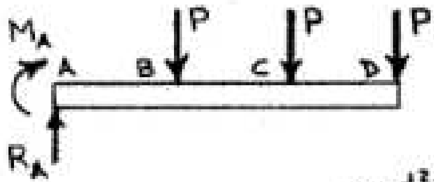
$$= \frac{M_0 a}{6EIL}\left\{\cancel{a^2} + 3b^2 - \cancel{a^2} - 2ab - b^2\right\} = \frac{M_0 a}{6EIL}\left\{2b^2 - 2ab\right\}$$

$$y_C = \frac{M_0 ab}{3EIL}(b-a)\uparrow$$

Problem 9.37

9.37 and 9.38 For the beam and loading shown, determine the deflection at (*a*) point *B*, (*b*) point *C*, (*c*) point *D*.





$$+\uparrow \Sigma F_y = 0: \quad R_A - P - P - P = 0 \qquad R_A = 3P$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - Pa - P(2a) - P(3a) = 0 \qquad M_A = -6Pa$$

$$\frac{dM}{dx} = V = 3P - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = 3Px - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 - 6Pa$$

$$EI\frac{dy}{dx} = \frac{3}{2}Px^2 - \frac{1}{2}P\langle x-a\rangle^2 - \frac{1}{2}P\langle x-2a\rangle^2 - 6Pax + C_1$$

$$\left[x=0,\ \frac{dy}{dx}=0\right]: \quad 0 - 0 - 0 - 0 + C_1 = 0 \qquad \therefore C_1 = 0$$

$$EIy = \frac{1}{2}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-2a\rangle^3 - 3Pax^2 + C_2$$

$$[x=0,\ y=0]: \quad 0 - 0 - 0 - 0 + C_2 = 0 \qquad \therefore C_2 = 0$$

ELASTIC CURVE: $y = \dfrac{P}{6EI}\left[3x^3 - \langle x-a\rangle^3 - \langle x-2a\rangle^3 - 18ax^2\right]$

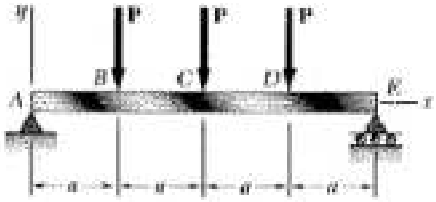
(a) $x = a$: $\quad y_B = \dfrac{Pa^3}{6EI}[3 - 0 - 0 - 18] \qquad y_B = \dfrac{5Pa^3}{2EI}\downarrow$ ◄

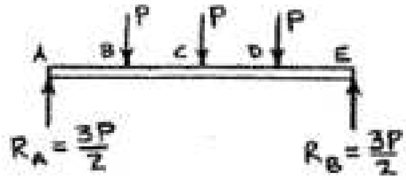
(b) $x = 2a$: $\quad y_C = \dfrac{Pa^3}{6EI}[24 - 1 - 0 - 72] \qquad y_C = \dfrac{49Pa^3}{6EI}\downarrow$ ◄

(c) $x = 3a$: $\quad y_D = \dfrac{Pa^3}{6EI}[81 - 8 - 1 - 162] \qquad y_D = \dfrac{15Pa^3}{EI}\downarrow$ ◄

Problem 9.38

9.37 and 9.38 For the beam and loading shown, determine the deflection at (*a*) point *B*, (*b*) point *C*, (*c*) point *D*.





$$\frac{dM}{dx} = V = \frac{3P}{2} - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0 - P\langle x-3a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = \frac{3P}{2}x - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 - P\langle x-3a\rangle^1$$

$$EI\frac{dy}{dx} = \frac{3P}{4}x^2 - \frac{1}{2}P\langle x-a\rangle^2 - \frac{1}{2}P\langle x-2a\rangle^2 - \frac{1}{2}P\langle x-3a\rangle^2 + C_1$$

$$EI\,y = \frac{P}{4}x^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}\langle x-2a\rangle^3 - \frac{1}{6}P\langle x-3a\rangle^3 + C_1x + C_2$$

$[x=0, y=0]$: $\quad 0 - 0 - 0 - 0 + 0 + C_2 = 0 \qquad \therefore C_2 = 0$

$[x=4a, y=0]$: $\quad 16Pa^3 - \frac{9}{2}Pa^3 - \frac{4}{3}Pa^3 - \frac{1}{6}Pa^3 + 4aC_1 = 0$

$$\therefore C_1 = -\frac{5}{2}Pa^2$$

ELASTIC CURVE:

$$y = \frac{P}{12EI}\left[3x^3 - 2\langle x-a\rangle^3 - 2\langle x-2a\rangle^3 - 2\langle x-3a\rangle^3 - 30a^2x\right]$$

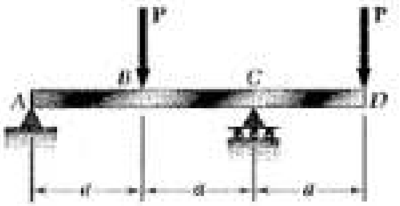
(a) $x = a$: $\quad y_B = \frac{Pa^3}{12EI}[3 - 0 - 0 - 0 - 30] \qquad y_B = \frac{9Pa^3}{4EI}\downarrow$ ◄

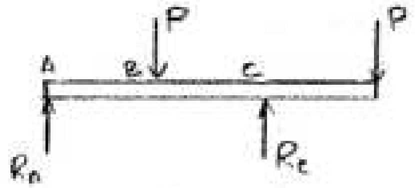
(b) $x = 2a$: $\quad y_C = \frac{Pa^3}{12EI}[24 - 2 - 0 - 0 - 60] \qquad y_C = \frac{19Pa^3}{6EI}\downarrow$ ◄

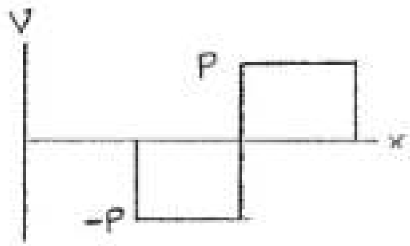
(c) $x = 3a$: $\quad y_D = \frac{Pa^3}{12EI}[81 - 16 - 2 - 0 - 90] \qquad y_D = \frac{9Pa^3}{4EI}\downarrow$ ◄

Problem 9.39

9.39 and 9.40 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *B*, (*c*) the deflection at end *D*.







$+\circlearrowleft \Sigma M_C = 0: \quad -2aR_A + aP - aP = 0 \qquad R_A = 0$

$+\uparrow \Sigma F_y = 0: \quad R_A + R_C - P - P = 0 \qquad R_C = 2P\uparrow$

$$\frac{dM}{dx} = V = -P\langle x-a\rangle^0 + 2P\langle x-2a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = -P\langle x-a\rangle^1 + 2P\langle x-2a\rangle^1$$

$$EI\frac{dy}{dx} = -\frac{1}{2}P\langle x-a\rangle^2 + P\langle x-2a\rangle^2 + C_1$$

$$EIy = -\frac{1}{6}P\langle x-a\rangle^3 + \frac{1}{3}P\langle x-2a\rangle^3 + C_1x + C_2$$

$[x=0,\ y=0] \quad -0+0+0+C_2 = 0 \qquad C_2 = 0$

$[x=2a,\ y=0] \quad -\frac{1}{6}Pa^3 + 0 + 0 + C_1(2a) + 0 = 0$

$$C_1 = \frac{1}{12}Pa^2$$

$$EIy = -\frac{1}{6}P\langle x-a\rangle^3 + \frac{1}{3}P\langle x-2a\rangle^3 + \frac{1}{12}Pa^2x$$

Elastic curve. $\quad y = \frac{P}{EI}\left\{-\frac{1}{6}\langle x-a\rangle^3 + \frac{1}{3}\langle x-2a\rangle^3 + \frac{1}{12}a^2x\right\}$

$$\frac{dy}{dx} = \frac{P}{EI}\left\{-\frac{1}{2}\langle x-a\rangle^2 + \langle x-2a\rangle^2 + \frac{1}{12}a^2\right\}$$

(a) Slope at end A. $\left(\frac{dy}{dx} \text{ at } x=0\right)$

$$\left.\frac{dy}{dx}\right|_A = \frac{P}{EI}\left\{-0+0+\frac{1}{12}a^2\right\} = \frac{Pa^2}{12EI} \qquad \theta_A = \frac{Pa^2}{12EI} \measuredangle$$

(b) Deflection at point B. $(y \text{ at } x=a)$

$$y_B = \frac{P}{EI}\left\{-0+0+\frac{1}{12}a^3\right\} = \frac{Pa^3}{12EI} \qquad y_B = \frac{Pa^3}{12EI}\uparrow$$

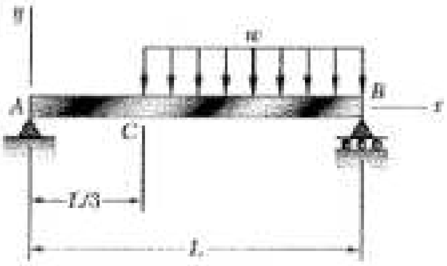
(c) Deflection at end D. $(y \text{ at } x=3a)$

$$y_D = \frac{P}{EI}\left\{-\frac{1}{6}(2a)^3 + \frac{1}{3}a^3 + \left(\frac{1}{12}a^2\right)(3x)\right\} = -\frac{3Pa^3}{4EI}$$

$$y_D = \frac{3Pa^3}{4EI}\downarrow$$

Problem 9.42

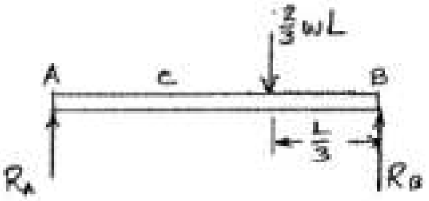
9.42 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at point *A*, (*c*) the deflection at point *C*.



Use free body ACB with the distributed load replaced by equivalent concentrated load.

$$+\circlearrowleft M_B = 0: \quad -R_A L + \left(\tfrac{2}{3}wL\right)\left(\tfrac{L}{3}\right) = 0$$

$$R_A = \tfrac{2}{9}wL$$



$$\frac{dV}{dx} = -w = -w\langle x - \tfrac{L}{3}\rangle^0$$

$$\frac{dM}{dx} = V = \tfrac{2}{9}wL - w\langle x - \tfrac{L}{3}\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = \tfrac{2}{9}wLx - \tfrac{1}{2}w\langle x - \tfrac{L}{3}\rangle^2$$

$$EI\frac{dy}{dx} = \tfrac{1}{9}wLx^2 - \tfrac{1}{6}w\langle x - \tfrac{L}{3}\rangle^3 + C_1$$

$$EIy = \tfrac{1}{27}wLx^3 - \tfrac{1}{24}w\langle x - \tfrac{L}{3}\rangle^4 + C_1x + C_2$$

$[x = 0, y = 0]$ $\qquad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = L, y = 0]$ $\qquad \left(\tfrac{1}{27}wL\right)L^3 - \tfrac{1}{24}\left(\tfrac{2L}{3}\right)^4 + C_1L + 0 = 0$

$$C_1 = -\tfrac{7}{243}wL^3$$

$$EIy = \tfrac{1}{27}wLx^3 - \tfrac{1}{24}w\langle x - \tfrac{L}{3}\rangle^4 - \tfrac{7}{243}wL^3x$$

(a) Elastic curve.

$$y = \frac{w}{EI}\left\{\tfrac{1}{27}Lx^3 - \tfrac{1}{24}\langle x - \tfrac{L}{3}\rangle^4 - \tfrac{7}{243}L^3x\right\} \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w}{EI}\left\{\tfrac{1}{9}Lx^2 - \tfrac{1}{6}\langle x - \tfrac{L}{3}\rangle^3 - \tfrac{7}{243}L^3\right\}$$

(b) Slope at point A. $\quad \left(\frac{dy}{dx} \text{ at } x = 0\right)$

$$\left.\frac{dy}{dx}\right|_A = \frac{w}{EI}\left\{0 - 0 - \tfrac{7}{243}L^3\right\} = -\frac{7wL^3}{243EI} \qquad \theta_A = \frac{7wL^3}{243EI} \blacktriangleleft$$

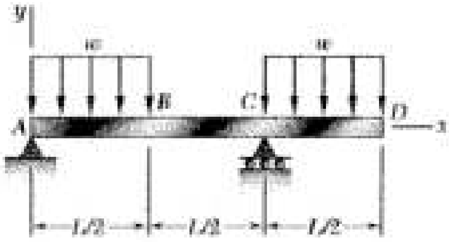
(c) Deflection at point C. $\quad \left(y \text{ at } x = \tfrac{L}{3}\right)$

$$y_C = \frac{w}{EI}\left\{\left(\tfrac{1}{27}L\right)\left(\tfrac{L}{3}\right)^3 - 0 - \left(\tfrac{7}{243}L^3\right)\left(\tfrac{L}{3}\right)\right\} = -\frac{2wL^4}{243EI}$$

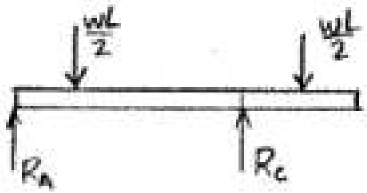
$$y_C = \frac{2wL^4}{243EI} \downarrow \blacktriangleleft$$

Problem 9.43

9.43 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at point *B*, (*c*) the deflection at point *D*.



Use free body ABCD with the distributed loads replaced by equivalent concentrated loads.



$$+\circlearrowleft \Sigma M_C = 0: \quad -R_A L + \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$R_A = \frac{1}{4}wL$$

$$+\circlearrowleft \Sigma M_A = 0: \quad R_C L - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{5L}{4}\right) = 0$$

$$R_C = \frac{3}{4}wL$$

$$\frac{dV}{dx} = -w = -w + w\langle x - \tfrac{L}{2}\rangle^0 - w\langle x - L\rangle^0$$

Integrating and adding terms to account for the reactions,

$$\frac{dM}{dx} = V = -wx + w\langle x - \tfrac{L}{2}\rangle^1 - w\langle x - L\rangle^1 + R_A + R_C\langle x - L\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2 + \frac{1}{2}w\langle x - \tfrac{L}{2}\rangle^2 - \frac{1}{2}w\langle x - L\rangle^2 + R_A x + R_C\langle x - L\rangle^1$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}w\langle x - \tfrac{L}{2}\rangle^3 - \frac{1}{6}w\langle x - L\rangle^3 + \frac{1}{2}R_A x^2 + \frac{1}{2}R_C\langle x - L\rangle^2 + C_1$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\langle x - \tfrac{L}{2}\rangle^4 - \frac{1}{24}w\langle x - L\rangle^4 + \frac{1}{6}R_A x^3 + \frac{1}{6}R_C\langle x - L\rangle^3 + C_1 x + C_2$$

$[x = 0, y = 0]$ $\quad -0 + 0 - 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = L, y = 0]$

$$-\frac{1}{24}wL^4 + \frac{1}{24}w\left(\frac{L}{2}\right)^4 - 0 + \frac{1}{6}\left(\frac{wL}{4}\right)L^3 + 0 + C_1 L + 0 = 0 \qquad C_1 = -\frac{1}{384}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\langle x - \tfrac{L}{2}\rangle^4 + \frac{1}{24}w\langle x - L\rangle^4 + \frac{1}{6}\left(\frac{wL}{4}\right)x^3 + \frac{1}{6}\left(\frac{3wL}{4}\right)\langle x - L\rangle^3 - \frac{1}{384}wL^3 x$$

(a) Elastic curve.

$$y = \frac{w}{24EI}\left\{-x^4 + \langle x - \tfrac{L}{2}\rangle^4 - \langle x - L\rangle^4 + Lx^3 + 3L\langle x - L\rangle^3 - \frac{1}{16}L^3 x\right\} \blacktriangleleft$$

(b) Deflection at B. $\quad$ (y at $x = \frac{L}{2}$)

$$y_B = \frac{w}{24EI}\left\{-\left(\frac{L}{2}\right)^4 + 0 - 0 + (L)\left(\frac{L}{2}\right)^3 + 0 - \left(\frac{1}{16}L^3\right)\left(\frac{L}{2}\right)\right\} \qquad y_B = \frac{wL^4}{768EI} \uparrow \blacktriangleleft$$

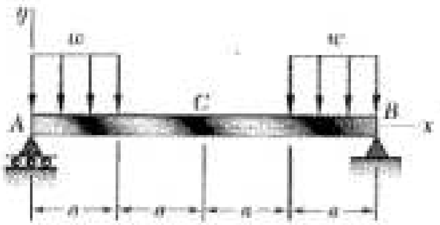
(c) Deflection at D. $\quad$ (y at $x = \frac{3L}{2}$)

$$y_D = \frac{w}{24EI}\left\{-\left(\frac{3L}{2}\right)^4 + L^4 - \left(\frac{L}{2}\right)^4 + (L)\left(\frac{3L}{2}\right)^3 + (3L)\left(\frac{L}{2}\right)^3 - \left(\frac{1}{16}L\right)\left(\frac{3L}{2}\right)\right\}$$

$$= -\frac{5wL^4}{256EI} \qquad y_D = \frac{5wL^4}{256EI} \downarrow \blacktriangleleft$$

Problem 9.44

9.44 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.



BY SYMMETRY, $R_A = R_B = wa$

$$w(x) = w - w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w + w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = V = R_A - wx + w\langle x-a\rangle^1 - w\langle x-3a\rangle^1$$

$$M = M_A + R_A x - \tfrac{1}{2}wx^2 + \tfrac{1}{2}w\langle x-a\rangle^2 - \tfrac{1}{2}w\langle x-3a\rangle^2 \quad (\text{WITH } M_A = 0)$$

$$EI\frac{d^2y}{dx^2} = M = wax - \tfrac{1}{2}wx^2 + \tfrac{1}{2}w\langle x-a\rangle^2 - \tfrac{1}{2}w\langle x-3a\rangle^2$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}wax^2 - \tfrac{1}{6}wx^3 + \tfrac{1}{6}w\langle x-a\rangle^3 - \tfrac{1}{6}w\langle x-3a\rangle^3 + C_1$$

$$EIy = \tfrac{1}{6}wax^3 - \tfrac{1}{24}wx^4 + \tfrac{1}{24}w\langle x-a\rangle^4 - \tfrac{1}{24}w\langle x-3a\rangle^4 + C_1x + C_2$$

$[x=0,\ y=0]$: $0 - 0 + 0 - 0 + 0 + C_2 = 0 \qquad \therefore C_2 = 0$

$[x=4a,\ y=0]$: $\tfrac{1}{6}wa(4a)^3 - \tfrac{1}{24}w(4a)^4 + \tfrac{1}{24}w(3a)^4 - \tfrac{1}{24}w(a)^4 + C_1(4a) = 0$

$$\therefore C_1 = -\tfrac{5}{6}wa^3$$

(a) EQUATION OF ELASTIC CURVE.

$$y = \frac{w}{EI}\left[\tfrac{1}{6}ax^3 - \tfrac{1}{24}x^4 + \tfrac{1}{24}\langle x-a\rangle^4 - \tfrac{1}{24}\langle x-3a\rangle^4 - \tfrac{5}{6}a^3x\right] \blacktriangleleft$$

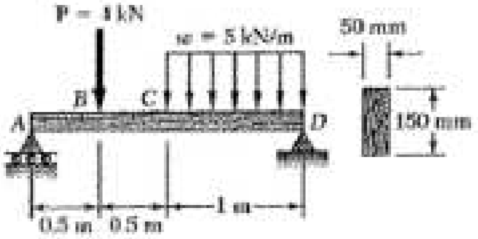
(b) DEFLECTION AT C. (y AT $x = 2a$)

$$y_C = \frac{wa^4}{EI}\left[\tfrac{1}{6}(2)^3 - \tfrac{1}{24}(2)^4 + \tfrac{1}{24}(1)^4 - 0 - \tfrac{5}{6}(2)\right]$$

$$= -\frac{23}{24}\frac{wa^4}{EI} \qquad y_C = \frac{23}{24}\frac{wa^4}{EI}\downarrow \blacktriangleleft$$

Problem 9.45

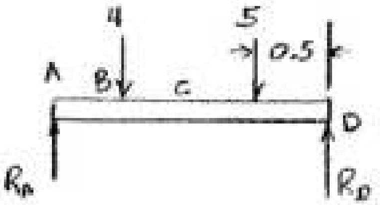
9.45 For the timber beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 12$ GPa.



Units: Forces in kN, lengths in meters.

$$I = \frac{1}{12}(50)(150)^3 = 14.0625 \times 10^6 \text{ mm}^4 = 14.0625 \times 10^{-6} \text{ m}^4$$

$$EI = (12 \times 10^9)(14.0625 \times 10^{-6}) = 168.75 \times 10^3 \text{ N}\cdot\text{m}^2 = 168.75 \text{ kN}\cdot\text{m}^2$$



$$+\circlearrowleft M_D = 0: \quad -2R_A + (1.5)(4) + (0.5)(5) = 0$$

$$R_A = 4.25 \text{ kN}$$

$$w(x) = 5\langle x-1\rangle^0 \qquad \text{kN}\cdot\text{m}$$

$$\frac{dV}{dx} = -w = -5\langle x-1\rangle^0 \qquad \text{kN/m}$$

$$\frac{dM}{dx} = V = -5\langle x-1\rangle^1 + 4.25 - 4\langle x-0.5\rangle^0 \qquad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{5}{2}\langle x-1\rangle^2 + 4.25x - 4\langle x-0.5\rangle^1 \qquad \text{kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = -\frac{5}{6}\langle x-1\rangle^3 + 2.125x^2 - 2\langle x-0.5\rangle^2 + C_1 \qquad \text{kN}\cdot\text{m}^2$$

$$EIy = -\frac{5}{24}\langle x-1\rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5\rangle^3 + C_1x + C_2 \qquad \text{kN}\cdot\text{m}^3$$

$$[x=0,\ y=0] \qquad -0+0-0+0+C_2 = 0 \qquad C_2 = 0$$

$$[x=2\text{m},\ y=0] \qquad -\left(\frac{5}{24}\right)(1)^4 + \left(\frac{2.125}{3}\right)(2)^3 - \left(\frac{2}{3}\right)(1.5)^3 + 2C_1 = 0$$

$$C_1 = -1.60417 \text{ kN}\cdot\text{m}^2$$

(a) Slope at end A. $\left(\frac{dy}{dx} \text{ at } x=0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = -0+0-0+C_1$$

$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-1.60417}{168.75} = -9.51 \times 10^{-3} \qquad \theta_A = 9.51 \times 10^{-3} \text{ rad} \circlearrowright$$

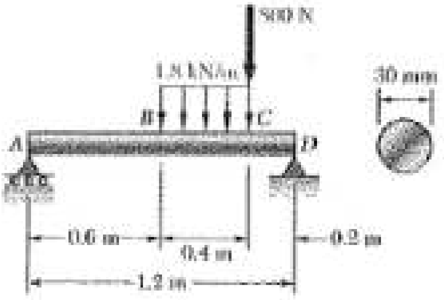
(b) Deflection at midpoint C. $(y \text{ at } x = 1 \text{ m})$

$$EIy_C = -0 + \left(\frac{2.125}{3}\right)(1)^3 - \left(\frac{2}{3}\right)(0.5)^3 + (-1.60417)(1) = -979.17 \times 10^{-3} \text{ kN}\cdot\text{m}^3$$

$$y_C = \frac{-979.17 \times 10^{-3}}{168.75} = -5.80 \times 10^{-3} \text{ m} \qquad y_C = 5.80 \text{ mm} \downarrow$$

Problem 9.46

9.46 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.



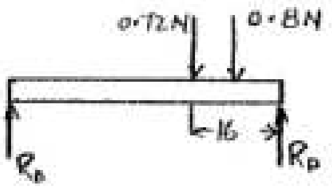
Units: Forces in kN Lengths in metres

$c = \frac{1}{2}d = (\frac{1}{2})(30) = 15$ mm

$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(15)^4 = 39761$ mm^4

$EI = (200\times10^6)(39761\times10^{-12}) = 7.95$ kN m^2

Use entire beam ABCD as free body.



$+\circlearrowleft \Sigma M_D = 0: \ -1.2R_A + (0.72)(0.4) + (0.8)(0.2) = 0$

$R_A = 0.373$ kN $\uparrow$

$w(x) = 1.8\langle x-0.6\rangle^0 - 1.8\langle x-1\rangle^0$ kN/m

$\frac{dV}{dx} = -w = -1.8\langle x-0.6\rangle^0 + 1.8\langle x-1\rangle^0$ kN/m

$\frac{dM}{dx} = V = -1.8\langle x-0.6\rangle^1 + 1.8\langle x-1\rangle^1 + 0.373 - 0.8\langle x-1\rangle^0$ kN m

$EI\frac{d^2y}{dx^2} = M = -0.9\langle x-0.6\rangle^2 + 0.9\langle x-1\rangle^2 + 0.373x - 0.8\langle x-1\rangle^1$ kN.m

$EI\frac{dy}{dx} = -0.3\langle x-0.6\rangle^3 + 0.3\langle x-1\rangle^3 + 0.1865x^2 - 0.4\langle x-1\rangle^2 + C_1$ kN m^2

$EIy = -0.075\langle x-0.6\rangle^4 + 0.075\langle x-1\rangle^4 + 0.0622x^3 - 0.133\langle x-1\rangle^3 + C_1x + C_2$ kN m^3

$[x=0, y=0]\ -0+0+0+0+C_2 = 0$ $\qquad C_2 = 0$

$[x=1.2, y=0]\ -0.075(0.6)^4 + 0.075(0.2)^4 + 0.0622(1.2)^3 - 0.133(0.2)^3 + 1.2C_1 = 0$

$C_1 = -0.0807$ kN m^2

(a) Slope at end A. $(\frac{dy}{dx}$ at $x = 0)$

$EI\left(\frac{dy}{dx}\right)_A = -0 + 0 + 0 + C_1$

$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-0.0807}{7.95} = -0.01015$ $\qquad \theta_A = 0.01015$ rad $\measuredangle$ ◄

(b) Deflection at point B. (y at $x = 0.6$ m)

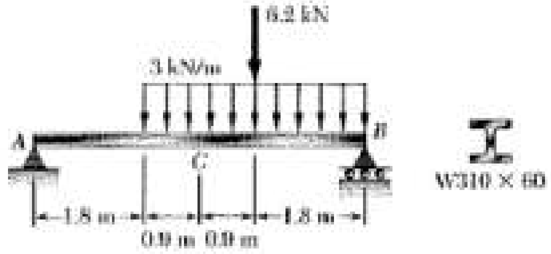
$EIy_B = -0+0+0.0622(0.6)^3 - 0 + (-0.0807)(0.6)$

$= -0.035$ kN.m^3

$y_B = -\frac{0.035}{7.95} = 4.402\times10^{-3}$ m $\qquad y_B = 4.4$ mm $\downarrow$ ◄

Problem 9.47

9.47 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.

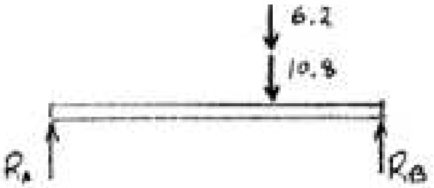


Units: Forces in kN, lengths in meters.

$+\circlearrowleft \Sigma M_B = 0:$

$-5.4 R_A - (1.8)(6.2 + 10.8) = 0$

$R_A = 5.6667 \text{ kN}$



$w(x) = 3\langle x - 1.8\rangle^0$

$\frac{dV}{dx} = -w(x) = -3\langle x - 1.8\rangle^0$

$\frac{dM}{dx} = V = 5.6667 - 3\langle x - 1.8\rangle^1 - 6.2\langle x - 3.6\rangle^0$

$$EI\frac{d^2y}{dx^2} = M = 5.6667x - \frac{3}{2}\langle x - 1.8\rangle^2 - 6.2\langle x - 3.6\rangle^1 \quad \text{kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}\langle x - 1.8\rangle^3 - 3.1\langle x - 3.6\rangle^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EI\,y = 0.9444x^3 - \frac{1}{8}\langle x - 1.8\rangle^4 - 1.0333\langle x - 3.6\rangle^3 + C_1 x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$[x = 0,\ y = 0]$ $\quad 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = 5.4,\ y = 0]$ $\quad (0.9444)(5.4)^3 - \frac{1}{8}(3.6)^4 - 1.0333(1.8)^3 + C_1(5.4) + 0 = 0$

$C_1 = -22.535 \text{ kN}\cdot\text{m}^2$

Data: $E = 200 \times 10^9$ Pa, $\quad I = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6 \text{ N}\cdot\text{m}^2 = 25.8 \times 10^3 \text{ kN}\cdot\text{m}^2$

(a) Slope at A. $\left(\frac{dy}{dx} \text{ at } x = 0\right)$

$EI\frac{dy}{dx} = 0 - 0 - 0 - 22.535 \quad \text{kN}\cdot\text{m}^2$

$\theta_A = -\frac{22.535}{25.8 \times 10^3} = -873 \times 10^{-6} \qquad \theta_A = 0.873 \times 10^{-3} \text{ rad} \circlearrowright$ ◄

(b) Deflection at C. $(y \text{ at } x = 2.7 \text{ m})$

$EI\,y_C = (0.9444)(2.7)^3 - \frac{1}{8}(0.9)^4 - 0 - (22.535)(2.7) + 0$

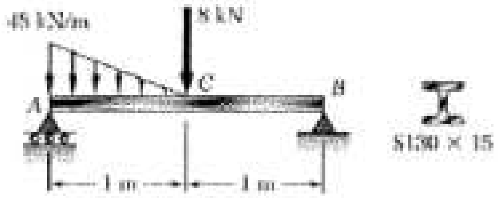
$= -42.337 \text{ kN}\cdot\text{m}^3$

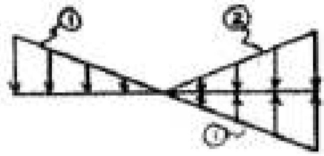
$y_C = -\frac{42.337}{25.8 \times 10^3} = -1.641 \times 10^{-3} \text{ m}$

$y_C = 1.641 \text{ mm} \downarrow$ ◄

Problem 9.48

9.48 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.

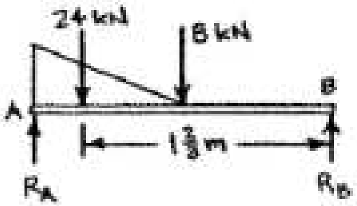




DISTRIBUTED LOADS: ① $w_1(x) = w_0 - kx$ ② $w_2 = k\langle x-1\rangle^1$

$\circlearrowleft \Sigma M_B = 0$: $-2R_A + (24)(1\frac{2}{3}) + (8)(1) = 0$ $\quad R_A = 24 \text{ kN} \uparrow$

$w(x) = w_0 - kx + k\langle x-1\rangle^1 = 48 - 48x + 48\langle x-1\rangle^1$



$$\frac{dV}{dx} = -w = -48 + 48x - 48\langle x-1\rangle^1 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = 24 - 48x + 24x^2 - 24\langle x-1\rangle^2 - 8\langle x-1\rangle^0 \quad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = 24x - 24x^2 + 8x^3 - 8\langle x-1\rangle^3 - 8\langle x-1\rangle^1 \quad \text{kN·m}$$

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2\langle x-1\rangle^4 - 4\langle x-1\rangle^2 + C_1 \quad \text{kN·m}^2$$

$$EI\,y = 4x^3 - 2x^4 + \tfrac{2}{5}x^5 - \tfrac{2}{5}\langle x-1\rangle^5 - \tfrac{4}{3}\langle x-1\rangle^3 + C_1x + C_2 \quad \text{kN·m}^3$$

$[x=0,\ y=0]$: $0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0$ $\qquad \therefore C_2 = 0$

$[x=2,\ y=0]$: $4(2)^3 - 2(2)^4 + \frac{2}{5}(2)^5 - \frac{2}{5}(1)^5 - \frac{4}{3}(1)^3 + C_1(2) = 0$ $\qquad \therefore C_1 = -\frac{83}{15}$ kN·m²

DATA: $E = 200(10^6)$ kN/m² $\qquad I = 5.07(10^6)\ \text{mm}^4 = 5.07(10^{-6})\ \text{m}^4$

$$EI = (200\times10^6)(5.07\times10^{-6}) = 1014 \text{ kN·m}^2$$

(a) SLOPE AT A. $\left(\frac{dy}{dx} \text{ AT } x = 0\right)$

$$EI\theta_A = 0 - 0 + 0 - 0 - 0 - \frac{83}{15} \quad \text{kN·m}^2$$

$$\theta_A = -\frac{83/15}{1014} = -5.4569\times10^{-3}\text{ rad}$$

5.46×10^{-3} rad ◄

(b) DEFLECTION AT C. (y AT $x = 1$ m)

$$EI\,y_C = 4(1)^3 - 2(1)^4 + \tfrac{2}{5}(1)^5 - 0 - 0 - \tfrac{83}{15}(1) = -3.1333 \text{ kN·m}^3$$

$$y_C = -\frac{3.1333}{1014} = -3.0900\times10^{-3}\text{ m}$$

3.09 mm ↓ ◄

Problem 9.50

9.49 and 9.50 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *C*.

$[x=0, y=0]$ $\quad [x=L, \frac{dy}{dx}=0]$ $\quad [x=L, y=0]$

For $0 \leq x \leq \frac{L}{2}$, $\quad M = R_A x$

For $\frac{L}{2} \leq x \leq L$, $\quad M = R_A x - M_0$

Then $EI\frac{d^2y}{dx^2} = M = R_A x - M_0\langle x - \frac{L}{2}\rangle^0$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\langle x - \frac{L}{2}\rangle^1 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\langle x - \frac{L}{2}\rangle^2 + C_1 x + C_2$$

$[x=0, y=0]$ $\quad 0 - 0 + 0 + C_2 = 0$ $\quad C_2 = 0$

$[x=L, \frac{dy}{dx}=0]$ $\quad \frac{1}{2}R_A L^2 - M_0(\frac{L}{2}) + C_1 = 0$ $\quad C_1 = \frac{1}{2}(M_0 L - R_A L^2)$

$[x=L, y=0]$ $\quad \frac{1}{6}R_A L^3 - \frac{1}{2}M_0(\frac{L}{2})^2 + \frac{1}{2}(M_0 L - R_A L^2)L + 0 = 0$

$$-\frac{1}{3}R_A L^3 + \frac{3}{8}M_0 L^2 = 0$$

(a) Reaction at A. $\quad M_A = 0, \quad R_A = \frac{9M_0}{8L} \uparrow$ ◄

$$C_1 = \frac{1}{2}[M_0 L - (\frac{9M_0}{8L})(L^2)] = -\frac{1}{16}M_0 L$$

$$EIy = \frac{1}{6}(\frac{9M_0}{8L})x^3 - \frac{1}{2}M_0\langle x - \frac{L}{2}\rangle^2 - \frac{1}{16}M_0 L x + 0$$

Elastic curve. $\quad y = \frac{M_0}{EIL}\{\frac{9}{8}x^3 - \frac{1}{2}L\langle x - \frac{L}{2}\rangle^2 - \frac{1}{16}L^2 x\}$

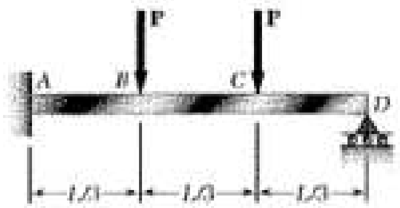
(b) Deflection at point C. $\quad$ (y at $x = \frac{L}{2}$)

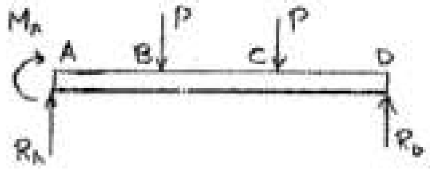
$$y_C = \frac{M_0}{EIL}\{(\frac{1}{6})(\frac{9}{8})(\frac{L}{2})^3 - 0 - (\frac{1}{16}L^2)(\frac{L}{2})\} = -\frac{M_0 L^2}{128\,EI}$$

$$y_C = \frac{M_0 L^2}{128\,EI} \downarrow$$ ◄

Problem 9.51

9.51 and 9.52 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.





$+\uparrow \Sigma F_y = 0: \quad R_A - P - P + R_D = 0 \qquad R_A = 2P - R_D$

$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \frac{PL}{3} - \frac{2PL}{3} + R_D L = 0$

$M_A = R_D L - PL$

$\frac{dM}{dx} = V = R_A - P\langle x - \frac{L}{3}\rangle^0 - P\langle x - \frac{2L}{3}\rangle^0$

$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\langle x - \frac{L}{3}\rangle^1 - P\langle x - \frac{2L}{3}\rangle^1$

$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\langle x - \frac{L}{3}\rangle^2 - \frac{1}{2}P\langle x - \frac{2L}{3}\rangle^2 + C_1$

$[x = 0, \frac{dy}{dx} = 0] \quad 0 + 0 - 0 - 0 + C_1 = 0 \qquad C_1 = 0$

$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\langle x - \frac{L}{3}\rangle^3 - \frac{1}{6}P\langle x - \frac{2L}{3}\rangle^3 + C_2$

$[x = 0, y = 0] \quad 0 + 0 - 0 - 0 + C_2 = 0 \qquad C_2 = 0$

$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\langle x - \frac{L}{3}\rangle^3 - \frac{1}{6}P\langle x - \frac{2L}{3}\rangle^3$

$[x = L, y = 0] \quad \frac{1}{2}(R_D L - PL)L^2 + \frac{1}{6}(2P - R_D)L^3 - \frac{1}{6}P(\frac{2L}{3})^3 - \frac{1}{6}P(\frac{L}{3})^3 = 0$

$\frac{1}{3}R_D L^3 - \frac{2}{9}PL^3 = 0$

(a) Reaction at D. $\qquad R_D = \frac{2}{3}P\uparrow$ ◄

$M_A = \frac{2}{3}PL - PL = -\frac{1}{3}PL \qquad R_A = 2P - \frac{2}{3}P = \frac{4}{3}P$

$EIy = \frac{1}{2}(-\frac{1}{3}PL)x^2 + \frac{1}{6}(\frac{4}{3}P)x^3 - \frac{1}{6}P\langle x - \frac{L}{3}\rangle^3 - \frac{1}{6}\langle x - \frac{2L}{3}\rangle^3$

Elastic curve.

$y = \frac{P}{EI}\left\{ -\frac{1}{6}Lx^2 + \frac{2}{9}x^3 - \frac{1}{6}\langle x - \frac{L}{3}\rangle^3 - \frac{1}{6}\langle x - \frac{2L}{3}\rangle^3 \right\}$

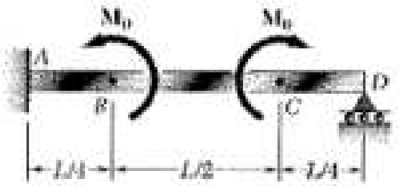
(b) Deflection at B. $\quad$ (y at $x = \frac{L}{3}$)

$y_B = \frac{P}{EI}\left\{ -(\frac{1}{6}L)(\frac{L}{3})^2 + \frac{2}{9}(\frac{L}{3})^3 - 0 - 0 \right\} = -\frac{5PL^3}{486EI}$

$y_B = \frac{5PL^3}{486EI}\downarrow$ ◄

Problem 9.52

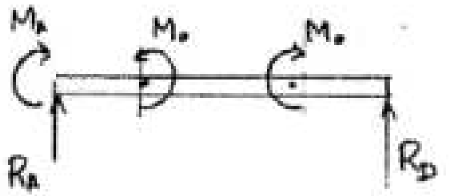
9.51 and 9.52 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_D = 0 \qquad R_A = -R_D$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + M_0 - M_0 + RL = 0$$

$$M_A = R_D L$$



$$M(x) = M_A + R_A x - M_0\langle x - \tfrac{L}{4}\rangle^0 + M_0\langle x - \tfrac{3L}{4}\rangle^0$$

$$EI\frac{d^2y}{dx^2} = R_D L - R_D x - M_0\langle x - \tfrac{L}{4}\rangle^0 + M_0\langle x - \tfrac{3L}{4}\rangle^0$$

$$EI\frac{dy}{dx} = R_D L x - \tfrac{1}{2}R_D x^2 - M_0\langle x - \tfrac{L}{4}\rangle^1 + M_0\langle x - \tfrac{3L}{4}\rangle^1 + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \qquad 0 - 0 - 0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$EIy = \tfrac{1}{2}R_D L x^2 - \tfrac{1}{6}R_D x^3 - \tfrac{1}{2}M_0\langle x - \tfrac{L}{4}\rangle^2 + \tfrac{1}{2}M_0\langle x - \tfrac{3L}{4}\rangle^2 + C_2$$

$$[x = 0,\ y = 0] \qquad 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = L,\ y = 0] \qquad \tfrac{1}{2}R_D L^3 - \tfrac{1}{6}R_D L^3 - \tfrac{1}{2}M_0\left(\tfrac{3L}{4}\right)^2 + \tfrac{1}{2}M_0\left(\tfrac{L}{4}\right)^2 = 0$$

(a) <u>Reaction at D.</u>

$$R_D = \frac{3M_0}{4L}\uparrow \blacktriangleleft$$

$$EIy = \tfrac{1}{2}\left(\tfrac{3M_0}{4L}\right)Lx^2 - \tfrac{1}{6}\left(\tfrac{3M_0}{4L}\right)x^3 - \tfrac{1}{2}M_0\langle x - \tfrac{L}{4}\rangle^2 + \tfrac{1}{2}M_0\langle x - \tfrac{3L}{4}\rangle^2$$

<u>Elastic curve.</u>

$$y = \frac{M_0}{EIL}\left\{\tfrac{3}{8}Lx^2 - \tfrac{1}{8}x^3 - \tfrac{1}{2}L\langle x - \tfrac{L}{4}\rangle^2 + \tfrac{1}{2}L\langle x - \tfrac{3L}{8}\rangle^2\right\}$$

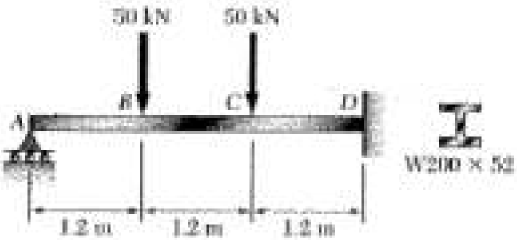
(b) <u>Deflection at point B.</u> $\quad$ (y at $x = \frac{L}{4}$)

$$y_B = \frac{M_0}{EIL}\left\{\left(\tfrac{3}{8}L\right)\left(\tfrac{L}{4}\right)^2 - \left(\tfrac{1}{8}\right)\left(\tfrac{L}{4}\right)^3 - 0 + 0\right\}$$

$$y_B = \frac{11\,M_0L^2}{512\,EI}\uparrow \blacktriangleleft$$

Problem 9.54

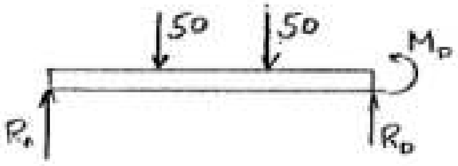
9.54 For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.



Units: Forces in kN, lengths in meters.

$$I = 52.7\times10^{6}\ \text{mm}^4 = 52.7\times10^{-6}\ \text{m}^4$$

$$EI = (200\times10^{9})(52.7\times10^{-6}) = 10.54\times10^{6}\ \text{N}\cdot\text{m}^2$$
$$= 10540\ \text{kN}\cdot\text{m}^2$$



$$\frac{dM}{dx} = V = R_A - 50\langle x-1.2\rangle^0 - 50\langle x-2.4\rangle^0 \quad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - 50\langle x-1.2\rangle^1 - 50\langle x-2.4\rangle^1 \ \text{kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - 25\langle x-1.2\rangle^2 - 25\langle x-2.4\rangle^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{25}{3}\langle x-1.2\rangle^3 - \frac{25}{3}\langle x-2.4\rangle^3 + C_1 x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$[x=0,\ y=0]$ $\qquad 0-0-0+0+C_2=0 \qquad C_2 = 0$

$[x=3.6,\ \frac{dy}{dx}=0]$ $\qquad \frac{1}{2}R_A(3.6)^2 - (25)(2.4)^2 - (25)(1.2)^2 + C_1 = 0$

$$C_1 = 180 - 6.48R_A \quad \text{kN}\cdot\text{m}^2$$

$[x=3.6,\ y=0]$ $\qquad \frac{1}{6}R_A(3.6)^3 - \frac{25}{3}(2.4)^3 - \frac{25}{3}(1.2)^3 + (180-6.48R_A)(3.6) = 0$

$$-15.552R_A + 518.4 = 0$$

(a) Reaction at A. $\qquad R_A = \frac{100}{3}$ kN $\qquad R_A = 33.3\ \text{kN}\uparrow$ ◄

$$C_1 = 180 - (6.48)(33.3333) = -36 \quad \text{kN}\cdot\text{m}^2$$

$$EIy = \frac{1}{6}\left(\frac{100}{3}\right)x^3 - \frac{25}{3}\langle x-1.2\rangle^3 - \frac{25}{3}\langle x-2.4\rangle^3 - 36x \quad \text{kN}\cdot\text{m}^3$$

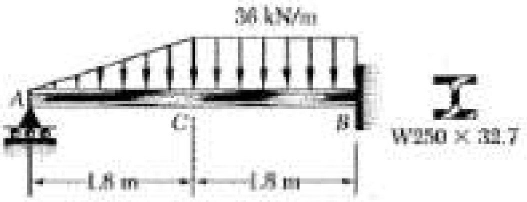
(b) Deflection at point B. $\qquad$ (y at $x = 1.2$ m)

$$EIy_B = \left(\frac{1}{6}\right)\left(\frac{100}{3}\right)(1.2)^3 - 0 - 0 - (36)(1.2) = -33.6 \quad \text{kN}\cdot\text{m}^3$$

$$y_B = \frac{-33.6}{10540} = -3.19\times10^{-3}\ \text{m} \qquad y_B = 3.19\ \text{mm}\downarrow$$ ◄

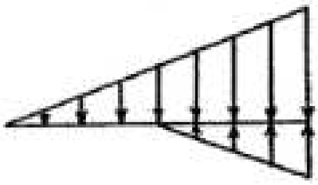
Problem 9.55

9.55 and 9.56 For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.



$$k = \frac{36 \text{ kN/m}}{1.8 \text{ m}} = 20 \text{ kN/m}^2$$

$$w(x) = 20x - 20\langle x - 1.8\rangle^1 \text{ kN/m}$$



$$\frac{dV}{dx} = -w(x) = -20x + 20\langle x - 1.8\rangle^1 \text{ kN/m}$$

$$\frac{dM}{dx} = V = R_A - 10x^2 + 10\langle x - 1.8\rangle^2 \text{ kN}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{10}{3}x^3 + \frac{10}{3}\langle x - 1.8\rangle^3 \text{ kN m}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{5}{6}x^4 + \frac{5}{6}\langle x - 1.8\rangle^4 + C_1 \text{ kN m}^2$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{6}x^5 + \frac{1}{6}\langle x - 1.8\rangle^5 + C_1 x + C_2 \text{ kN m}^3$$

$[x = 0, y = 0]$: $\quad 0 - 0 + 0 + 0 + C_2 = 0 \qquad \therefore C_2 = 0$

$[x = 3.6 \text{ m}, \frac{dy}{dx} = 0]$: $\quad \frac{1}{2}R_A(3.6)^2 - \frac{5}{6}(3.6)^4 + \frac{5}{6}(1.8)^4 + C_1 = 0$

$$\therefore C_1 = 131.22 - 6.48 R_A \text{ kN m}^2$$

$[x = 3.6 \text{ m}, y = 0]$: $\quad \frac{1}{6}R_A(3.6)^3 - \frac{1}{6}(3.6)^5 + \frac{1}{6}(1.8)^5 + (131.22 - 6.48 R_A)(3.6) = 0$

$$(7.776 - 23.328)R_A = -374.76 \qquad R_A = 24.097 \text{ kN}$$

(a) <u>REACTION AT A.</u> $\qquad R_A = 24.1 \text{ kN} \uparrow$ ◀

$$C_1 = 131.22 - 6.48(24.097) = -24.929 \text{ kN m}^2$$

DATA: $E = 200 \times 10^6 \text{ kPa} \qquad I = 48.9 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^6)(48.9 \times 10^{-6}) = 9780 \text{ kN m}^2$$

(b) <u>DEFLECTION AT C.</u> (y AT $x = 1.8$ m)

$$EI\,y_C = \frac{1}{6}(24.097)(1.8)^3 - \frac{1}{6}(1.8)^5 + 0 - 24.929(1.8)$$

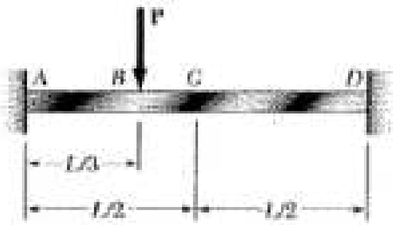
$$= -24.599 \text{ kN m}^3$$

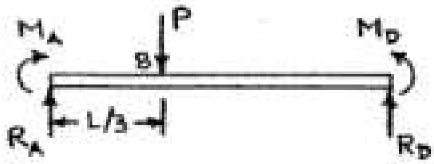
$$y_C = -\frac{24.599}{9780} = -2.515 \times 10^{-3} \text{ m}$$

$$y_C = 2.5 \text{ mm} \downarrow$$ ◀

Problem 9.58

9.57 and 9.58 For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at midpoint *C*.





$$\frac{dM}{dx} = V = R_A - P\langle x - \tfrac{L}{3}\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\langle x - \tfrac{L}{3}\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - \tfrac{1}{2}P\langle x - \tfrac{L}{3}\rangle^2 + C_1$$

$$EIy = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{6}P\langle x - \tfrac{L}{3}\rangle^3 + C_1 x + C_2$$

$[x=0,\ \frac{dy}{dx}=0]:\quad 0 + 0 - 0 + C_1 = 0 \qquad \therefore C_1 = 0$

$[x=0,\ y=0]:\quad 0 + 0 - 0 + C_2 = 0 \qquad \therefore C_2 = 0$

$[x=L,\ \frac{dy}{dx}=0]:\quad M_A L + \tfrac{1}{2}R_A L^2 - \tfrac{1}{2}P\left(\tfrac{2L}{3}\right)^2 = 0 \qquad (1)$

$[x=L,\ y=0]:\quad \tfrac{1}{2}M_A L^2 + \tfrac{1}{6}R_A L^3 - \tfrac{1}{6}P\left(\tfrac{2L}{3}\right)^3 = 0 \qquad (2)$

(a) SOLVING EQ'S (1) & (2) SIMULTANEOUSLY,

$R_A = \frac{20}{27}P \qquad M_A = -\frac{4}{27}PL$

$R_A = \frac{20}{27}P \uparrow$ ◄

$M_A = \frac{4}{27}PL \circlearrowright$ ◄

ELASTIC CURVE.

$$y = \frac{P}{EI}\left[-\tfrac{2}{27}Lx^2 + \tfrac{10}{81}x^3 - \tfrac{1}{6}\langle x - \tfrac{L}{3}\rangle^3\right]$$

(b) DEFLECTION AT MIDPOINT C. (y AT $x = \frac{L}{2}$)

$$y_C = \frac{P}{EI}\left[-\tfrac{2}{27}L\left(\tfrac{L}{2}\right)^2 + \tfrac{10}{81}\left(\tfrac{L}{2}\right)^3 - \tfrac{1}{6}\left(\tfrac{L}{6}\right)^3\right]$$

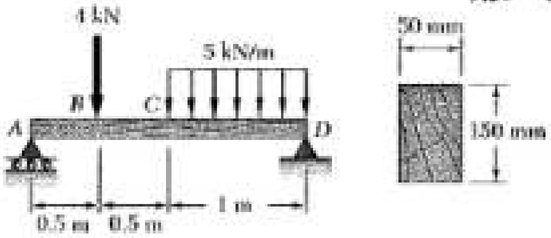
$$= -\frac{5PL^3}{1296EI}$$

$y_C = \frac{5PL^3}{1296EI} \downarrow$ ◄

Problem 9.59

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.59 Beam and loading of Prob. 9.45.



See solution to Prob. 9.45 for the derivation of the equations used in the following.

$EI = 168.75 \text{ kN}\cdot\text{m}^2 \qquad C_1 = -1.60417 \text{ kN}\cdot\text{m}^2 \qquad C_2 = 0$

$$EI\frac{dy}{dx} = -\frac{5}{6}\langle x-1\rangle^3 + 2.125x^2 - 2\langle x-0.5\rangle^2 + C_1 \qquad \text{kN}\cdot\text{m}^2$$

$$EIy = -\frac{5}{24}\langle x-1\rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5\rangle^3 + C_1x + C_2 \qquad \text{kN}\cdot\text{m}^3$$

Compute slope at C. ($\frac{dy}{dx}$ at $x = 1$ m)

$$EI\left(\frac{dy}{dx}\right)_C = 0 + (2.125)(1)^2 - 2(0.5)^2 - 1.60417 = 20.83\times10^{-3} \text{ kN}\cdot\text{m}^2$$

Since the slope at C is positive, the largest deflection occurs in portion BC, where

$$EI\frac{dy}{dx} = 2.125x^2 - 2(x-0.5)^2 - 1.60417$$

$$EIy = \frac{2.125}{3}x^3 - \frac{2}{3}(x-0.5)^3 - 1.60417x$$

To find the location of the largest downward deflection, set $\frac{dy}{dx} = 0$.

$$2.125x_m^2 - 2(x_m^2 - x_m + 0.25) - 1.60417$$
$$= 0.125x_m^2 + 2x_m - 2.10417 = 0$$

$$x = 1.0521 - 0.0625x^2$$

Solve by iteration. $x_m = 1,\ 0.989,\ 0.991,$ $\qquad x_m = 0.991 \text{ m}$ ◄

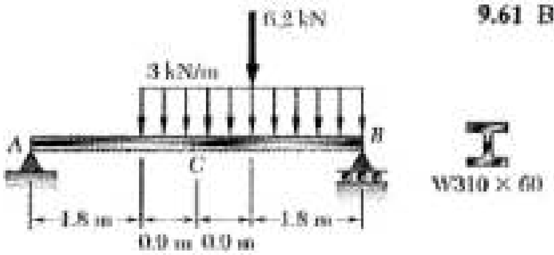
$$EIy_m = \left(\frac{2.125}{3}\right)(0.991)^3 - \frac{2}{3}(0.991-0.5)^3 - (1.60417)(0.991)$$
$$= -0.97927 \text{ kN}\cdot\text{m}^3$$

$$y_m = \frac{-0.97927}{168.75} = -5.80\times10^{-3} \text{ m} \qquad y_m = 5.80 \text{ mm} \downarrow$$ ◄

Problem 9.61

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.61 Beam and loading of Prob. 9.47.



See solution to Prob. 9.47 for the derivation of the equations used in the following.

$EI = 25.8 \times 10^3 \text{ kN}\cdot\text{m}^2$

$EI\frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}\langle x-1.8\rangle^2 - 3.1\langle x-3.6\rangle^2 - 22.535$

$EI\,y = 0.9444x^3 - \frac{1}{6}\langle x-1.8\rangle^3 - 1.03333\langle x-3.6\rangle^3 - 22.535x$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $1.8 < x_m < 3.6$

$EI\frac{dy}{dx} = 2.8333x_m^2 - \frac{1}{2}(x_m - 1.8)^2 - 22.535 = 0$

Solving by iteration: $x_m = 3,\ 2.86,\ 2.855$ $\qquad x_m = 2.855$ m ◄

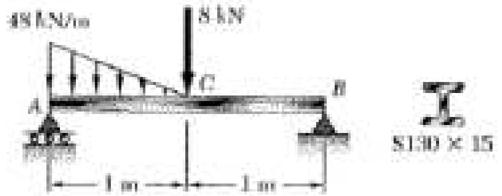
$df/dx = 15.8,\ 15.15$

$EI\,y_m = 0.9444x_m^3 - \frac{1}{6}(x_m - 1.8)^3 - 22.535x_m$

$= (0.9444)(2.855)^3 - \frac{1}{6}(2.855-1.8)^3 - (22.535)(2.855) = -42.507 \text{ kN}\cdot\text{m}^3$

$y_m = -\frac{42.507}{25.8 \times 10^3} = -1.648 \times 10^{-3}$ m $\qquad y_m = 1.648$ mm ↓ ◄

Problem 9.62



9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.62 Beam and loading of Prob. 9.48.

SEE SOLUTION TO PROB. 9.48 FOR THE DERIVATION OF THE EQUATIONS USED IN THE FOLLOWING.

$EI = 1014 \text{ kN}\cdot\text{m}^2$

$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2\langle x-1\rangle^4 - 4\langle x-1\rangle^2 - \frac{83}{15} \text{ kN}\cdot\text{m}^2$

$EI\,y = 4x^3 - 2x^4 + \frac{2}{5}x^5 - \frac{2}{5}\langle x-1\rangle^5 - \frac{4}{3}\langle x-1\rangle^3 - \frac{83}{15}x \text{ kN}\cdot\text{m}^3$

TO FIND LOCATION OF MAXIMUM $|y|$, SET $\frac{dy}{dx} = 0$.
ASSUME $0 < x_m < 1$ m

$$EI\frac{dy}{dx} = 12x_m^2 - 8x_m^3 + 2x_m^4 - \frac{83}{15} = 0$$

SOLVING: $x_m = 0.94166$ m $\qquad x_m = 0.942$ m ◄

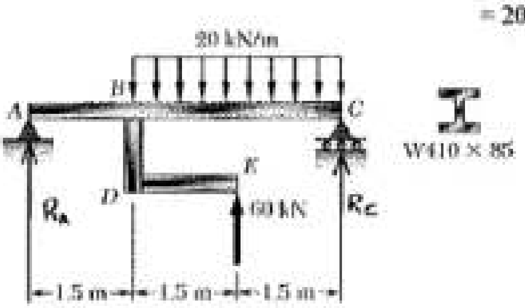
$EI\,y_m = 4(0.94166)^3 - 2(0.94166)^4 + \frac{2}{5}(0.94166)^5 - \frac{83}{15}(0.94166)$

$= -3.1469 \text{ kN}\cdot\text{m}^3$

$y_m = -\frac{3.1469}{1014} = -3.1035 \times 10^{-3}$ m $\qquad y_m = 3.10$ mm ↓ ◄

Problem 9.63

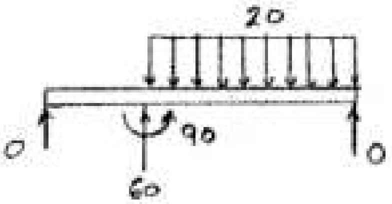
9.63 The rigid bar *BDE* is welded at point *B* to the rolled-steel beam *AC*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.



$$+\circlearrowleft \Sigma M_C = 0: \quad -4.5R_A + (20)(3)(1.5) - (60)(1.5) = 0$$

$$R_A = 0$$

Units: Forces in kN; lengths in m.



$$EI\frac{d^2y}{dx^2} = M = 60\langle x-1.5\rangle^1 - 90\langle x-1.5\rangle^0 - \tfrac{1}{2}(20)\langle x-1.5\rangle^2$$

$$EI\frac{dy}{dx} = 30\langle x-1.5\rangle^2 - 90\langle x-1.5\rangle^1 - \left(\tfrac{1}{6}\right)(20)\langle x-1.5\rangle^3 + C_1$$

$$EI\,y = 10\langle x-1.5\rangle^3 - 45\langle x-1.5\rangle^2 - \tfrac{1}{24}(20)\langle x-1.5\rangle^4 + C_1x + C_2$$

$[x=0,\ y=0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=4.5,\ y=0]$ $\quad (10)(3)^3 - (45)(3)^2 - \tfrac{1}{24}(20)(3)^4 + 4.5C_1 + 0 = 0$

$$C_1 = 45 \text{ kN}\cdot\text{m}^2$$

Data: $E = 200\times10^9$ Pa, $I = 315\times10^6 \text{ mm}^4 = 315\times10^{-6} \text{ m}^4$

$$EI = (200\times10^9)(315\times10^{-6}) = 63\times10^6 \text{ N}\cdot\text{m}^2 = 63000 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A. $\left(\frac{dy}{dx} \text{ at } x = 0\right)$

$$EI\,\theta_A = C_1 = 45 \text{ kN}\cdot\text{m}^2$$

$$\theta_A = \frac{45}{63000} = 0.714\times10^{-3} \text{ rad} \qquad \theta_A = 0.714\times10^{-3} \text{ rad} \angle$$ ◄

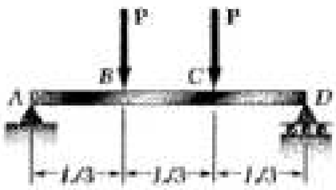
(b) Deflection at B. $(y \text{ at } x = 1.5)$

$$EI\,y_B = (C_1)(1.5) = (45)(1.5) = 67.5 \text{ kN}\cdot\text{m}^3$$

$$y_B = \frac{67.5}{63000} = 1.071\times10^{-3} \text{ m} = 1.071 \text{ mm} \uparrow$$ ◄

Problem 9.65

9.65 through 9.68 For the beam and loading shown, determine (*a*) the deflection at *C*, (*b*) the slope at end *A*.



LOADING I: LOAD AT B. CASE 5 OF APPENDIX D.

$$a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad x = \frac{2L}{3}$$

FOR $x > a$, REPLACE x BY $L - x$ & INTERCHANGE a & b

$$y = \frac{Pa}{6EIL}\left[(L-x)^3 - (L^2 - a^2)(L-x)\right]$$

$$y_C = \frac{P(L/3)}{6EIL}\left[\left(L - \frac{2L}{3}\right)^3 - \left(L^2 - \frac{L^2}{9}\right)\left(L - \frac{2L}{3}\right)\right] = -\frac{7}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(2L/3)(L^2 - 4L^2/9)}{6EIL} = -\frac{5}{81}\frac{PL^2}{EI}$$

LOADING II: LOAD AT C. CASE 5 OF APPENDIX D.

$$a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad x = \frac{2L}{3}$$

$$y_C = \frac{Pb}{6EIL}\left[x^3 - (L^2 - b^2)x\right] = \frac{P(L/3)}{6EIL}\left[\frac{8L^3}{27} - \left(L^2 - \frac{L^2}{9}\right)\left(\frac{2L}{3}\right)\right]$$

$$= -\frac{8}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(L/3)(L^2 - L^2/9)}{6EIL} = -\frac{4}{81}\frac{PL^2}{EI}$$

(a) DEFLECTION AT C. $\quad y_C = -\frac{7}{486}\frac{PL^3}{EI} - \frac{8}{486}\frac{PL^3}{EI} = -\frac{15}{486}\frac{PL^3}{EI}$

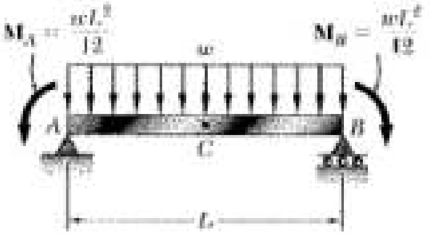
$$y_C = \frac{5}{162}\frac{PL^3}{EI} \downarrow \quad ◄$$

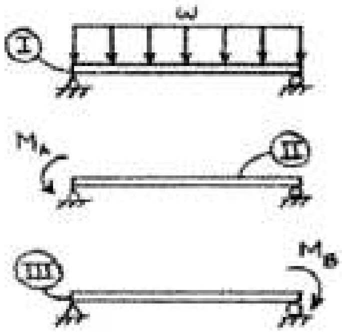
(b) SLOPE AT A. $\quad \theta_A = -\frac{5}{81}\frac{PL^2}{EI} - \frac{4}{81}\frac{PL^2}{EI} = -\frac{1}{9}\frac{PL^2}{EI}$

$$\theta_A = \frac{1}{9}\frac{PL^2}{EI} \quad ◄$$

Problem 9.68

9.65 through 9.68 For the beam and loading shown, determine (*a*) the deflection at C, (*b*) the slope at end *A*.





LOADING I: CASE 6 IN APPENDIX D.

$$y_C = -\frac{5wL^4}{384EI} \qquad \theta_A = -\frac{wL^3}{24EI}$$

LOADING II: CASE 7 IN APPENDIX D.

$$y_C = -\frac{M_A}{6EIL}\left[\left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right)\right] = \frac{1}{16}\frac{M_A L^2}{EI} \qquad \theta_A = \frac{M_A L}{3EI}$$

WITH $M_A = \frac{wL^2}{12}$ $\qquad y_C = \frac{1}{192}\frac{wL^4}{EI} \qquad \theta_A = \frac{1}{36}\frac{wL^3}{EI}$

LOADING III: CASE 7 IN APPENDIX D.

$y_C = \frac{1}{16}\frac{M_B L^3}{EI}$ (USING LOADING II RESULT) $\qquad \theta_A = \frac{M_B L}{6EI}$

WITH $M_B = \frac{wL^2}{12}$ $\qquad y_C = \frac{1}{192}\frac{wL^4}{EI} \qquad \theta_A = \frac{1}{72}\frac{wL^3}{EI}$

(a) DEFLECTION AT C.

$$y_C = -\frac{5}{384}\frac{wL^4}{EI} + \frac{1}{192}\frac{wL^4}{EI} + \frac{1}{192}\frac{wL^4}{EI} = -\frac{1}{384}\frac{wL^4}{EI}$$

$$y_C = \frac{1}{384}\frac{wL^4}{EI} \downarrow$$

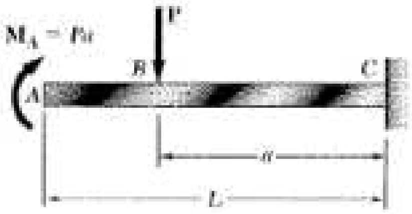
(b) SLOPE AT A.

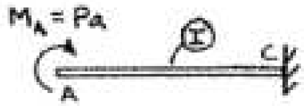
$$\theta_A = -\frac{1}{24}\frac{wL^3}{EI} + \frac{1}{36}\frac{wL^3}{EI} + \frac{1}{72}\frac{wL^3}{EI} = 0$$

$$\theta_A = 0$$

Problem 9.70

9.69 and 9.70 For the cantilever beam and loading shown, determine the slope and deflection at the free end.



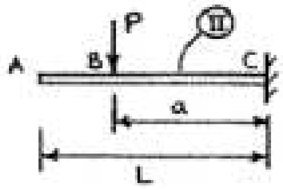


LOADING I: M_A AT A. CASE 3 OF APPENDIX D.

$$\theta_A' = -\frac{M_A L}{EI} \qquad y_A' = \frac{M_A L^2}{2EI}$$

WITH $M_A = Pa$

$$\theta_A' = -\frac{PaL}{EI} \qquad y_A' = \frac{PaL^2}{2EI}$$



LOADING II: P DOWNWARD AT B.

CASE 1 OF APPENDIX D APPLIED TO PORTION BC.

$$\theta_B'' = \frac{Pa^2}{2EI} \qquad y_B'' = -\frac{Pa^3}{3EI}$$

AB REMAINS STRAIGHT

$$\theta_A'' = \theta_B'' = \frac{Pa^2}{2EI}$$

$$y_A'' = y_B'' - (L-a)\theta_B''$$

$$= -\frac{Pa^3}{3EI} - (L-a)\frac{Pa^2}{2EI} = -\frac{Pa^2 L}{2EI} + \frac{Pa^3}{6EI}$$

BY SUPERPOSITION,

$$\theta_A = \theta_A' + \theta_A'' = -\frac{PaL}{EI} + \frac{Pa^2}{2EI} = -\frac{Pa}{2EI}(2L-a)$$

$$\frac{Pa}{2EI}(2L-a) \quad ◄$$

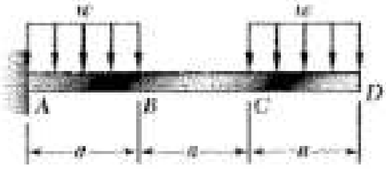
$$y_A = y_A' + y_A'' = \frac{PaL^2}{2EI} - \frac{Pa^2 L}{2EI} + \frac{Pa^3}{6EI}$$

$$= \frac{Pa}{6EI}(3L^2 - 3aL + a^2)$$

$$\frac{Pa}{6EI}(3L^2 - 3aL + a^2) \uparrow \quad ◄$$

Problem 9.71

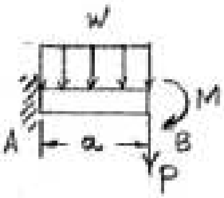
9.71 and 9.72 For the cantilever beam and loading shown, determine the slope and deflection at point B.



Consider portion AB as a cantilever beam subjected to the three loadings shown.

By statics: $\quad P = wa$

$$M = (wa)(a + \tfrac{a}{2}) = \tfrac{3}{2} wa^2$$



Slope at B. $\quad \theta_B = (\theta_B)_w + (\theta_B)_P + (\theta_B)_M$

Case 2 of App. D. $\quad (\theta_B)_w = -\dfrac{wa^3}{6EI}$

Case 1 of App. D. $\quad (\theta_B)_P = -\dfrac{(wa)a^2}{2EI} = -\dfrac{wa^3}{2EI}$

Case 3 of App. D $\quad (\theta_B)_M = -\dfrac{(\frac{3}{2}wa^2)a}{EI} = -\dfrac{3wa^3}{2EI}$

$$\theta_B = -\frac{13wa^3}{6EI} \qquad \qquad \theta_B = \frac{13wa^3}{6EI} \measuredangle \blacktriangleleft$$

Deflection at B. $\quad y_B = (y_B)_w + (y_B)_P + (y_B)_M$

Case 2 of App. D. $\quad (y_B)_w = -\dfrac{wa^4}{8EI}$

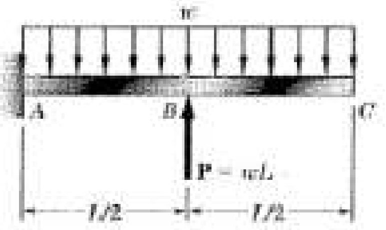
Case 1 of App. D. $\quad (y_B)_P = -\dfrac{(wa)a^3}{3EI} = -\dfrac{wa^4}{3EI}$

Case 3 of App. D. $\quad (y_B)_M = -\dfrac{(\frac{3}{2}wa^2)a}{2EI} = -\dfrac{3wa^4}{4EI}$

$$y_B = -\frac{29wa^4}{24EI} \qquad \qquad y_B = \frac{29wa^4}{24EI} \downarrow \blacktriangleleft$$

Problem 9.72

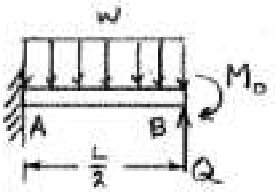
9.71 and 9.72 For the cantilever beam and loading shown, determine the slope and deflection at point *B*.



Consider portion AB as a cantilever beam subjected to the three loadings shown.

By statics: $Q = P - \frac{wL}{2} = \frac{wL}{2}$

$$M_0 = \left(\frac{wL}{2}\right)\left(\frac{L}{2}\cdot\frac{L}{2}\right) = \frac{wL^2}{8}$$



Slope at B. $\theta_B = (\theta_B)_w + (\theta_B)_Q + (\theta_B)_M$

Case 2 of App. D. $(\theta_B)_w = -\frac{w}{6EI}\left(\frac{L}{2}\right)^3 = -\frac{wL^3}{48EI}$

Case 1 of App. D. $(\theta_B)_Q = \frac{+(wL/2)}{2EI}\left(\frac{L}{2}\right)^2 = \frac{wL^3}{16EI}$

Case 3 of App. D. $(\theta_B)_M = -\frac{(wL^2/8)}{EI}\left(\frac{L}{2}\right) = -\frac{wL^3}{16EI}$

$\theta_B = -\frac{wL^3}{48EI}$ $\qquad\qquad$ $\theta_B = \frac{wL^3}{48EI}$ ◄

Deflection at B. $y_B = (y_B)_w + (y_B)_Q + (y_B)_M$

Case 2 of App. D. $(y_B)_w = -\frac{w}{8EI}\left(\frac{L}{2}\right)^4 = -\frac{wL^4}{128EI}$

Case 1 of App. D. $(y_B)_Q = +\frac{(wL/2)}{3EI}\left(\frac{L}{2}\right)^3 = \frac{wL^4}{48EI}$

Case 3 of App. D $(y_B)_M = -\frac{(wL^2/8)}{2EI}\left(\frac{L}{2}\right)^2 = -\frac{wL^4}{64EI}$

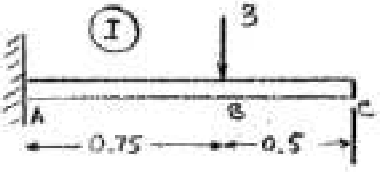
$y_B = -\frac{wL^4}{384EI}$ $\qquad\qquad$ $y_B = \frac{wL^4}{384EI} \downarrow$ ◄

Problem 9.73

9.73 For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 200$ GPa.

Units: Forces in kN, lengths in m.

Loading I: Concentrated load at B.



Case 1 of Appendix D applied to portion AB.

$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion BC remains straight.

$$\theta_C' = \theta_B' = -\frac{0.84375}{EI}$$

$$y_C' = y_B' - (0.5)\theta_B' = -\frac{0.84375}{EI}$$

Loading II: Concentrated load at C. Case 1 of Appendix D.

$$\theta_A'' = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y_A'' = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition,
$$\theta_A = \theta_A' + \theta_A'' = -\frac{3.1875}{EI}$$

$$y_A = y_A' + y_A'' = -\frac{2.796875}{EI}$$

Data: $E = 200 \times 10^9$ Pa, $I = 2.53 \times 10^6\ \text{mm}^4 = 2.53 \times 10^{-6}\ \text{m}^4$

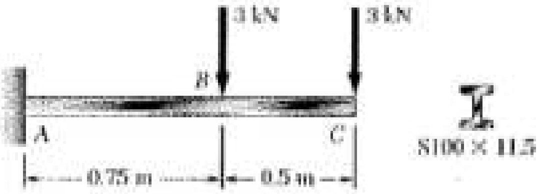
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3\ \text{N}\cdot\text{m}^2 = 506\ \text{kN}\cdot\text{m}^2$$

Slope at C. $\theta_C = -\frac{3.1875}{506} = -6.30 \times 10^{-3}$ rad $= 6.30 \times 10^{-3}$ rad ◄

Deflection at C. $y_C = -\frac{2.796875}{506} = -5.53 \times 10^{-3}$ m $= 5.53$ mm ↓

Problem 9.74

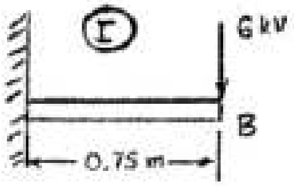
9.74 For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200$ GPa.

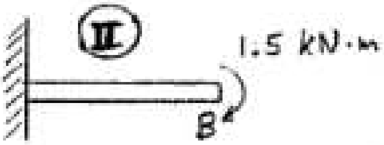


Units: Forces in kN, lengths in m.

The slope and deflection at B depend only on the deformation of portion AB.

Reduce the force at C to an equivalent force-couple system at B and add the force already at B to obtain the loadings I and II shown.





Loading I: Case 1 of Appendix D.

$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(6)(0.75)^2}{2EI} = -\frac{1.6875}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(6)(0.75)^3}{3EI} = -\frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix D.

$$\theta_B'' = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y_B'' = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{EI} = -\frac{0.421875}{EI}$$

By superposition,

$$\theta_B = \theta_B' + \theta_B'' = -\frac{2.8125}{EI}$$

$$y_B = y_B' + y_B'' = -\frac{1.265625}{EI}$$

Data: $E = 200 \times 10^9$ Pa, $I = 2.53 \times 10^6\ \text{mm}^4 = 2.53 \times 10^{-6}\ \text{m}^4$

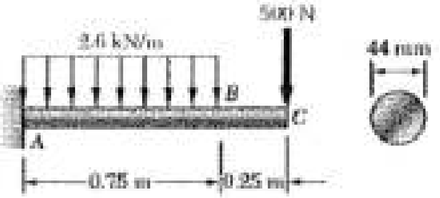
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3\ \text{N}\cdot\text{m}^2 = 506\ \text{kN}\cdot\text{m}^2$$

Slope at B. $\theta_B = -\dfrac{2.8125}{506} = -5.56 \times 10^{-3}$ rad $= 5.56 \times 10^{-3}$ rad ↻ ◄

Deflection at B. $y_B = -\dfrac{1.265625}{506} = -2.50 \times 10^{-3}$ m $=$ 2.50 mm ↓ ◄

Problem 9.75

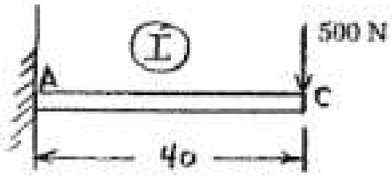
9.75 For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 200$ GPa.



$$c = \tfrac{1}{2}d = \tfrac{1}{2}(44) = 22 \text{ mm}$$

$$I = \tfrac{\pi}{4}c^4 = \tfrac{\pi}{4}(22)^4 = 183984 \text{ mm}^4$$

$$EI = (200 \times 10^9)(183984 \times 10^{-12}) = 36.8 \text{ kN}\cdot\text{m}^2$$

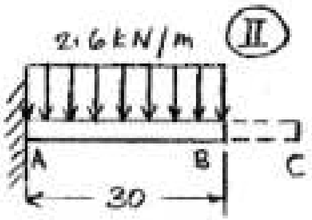


Loading I. Case 1 of Appendix D.

$P = 0.5$ kN, $L = 1$ m

$$(y_C)_1 = -\frac{PL^3}{3EI} = -\frac{(0.5)(1)^3}{(3)(36.8)} = -4.529 \times 10^{-3} \text{ m}$$

$$(\theta_C)_1 = -\frac{PL^2}{2EI} = -\frac{(0.5)(1)^2}{(2)(36.8)} = -6.793 \times 10^{-3}$$



Loading II. Treat portion AB as a cantilever beam. (Case 2)

$w = 2.6$ kN/m, $L = 0.75$ m

$$(y_B)_2 = -\frac{wL^4}{8EI} = -\frac{(2.6)(0.75)^4}{(8)(36.8)} = -2.794 \times 10^{-3} \text{ m}$$

$$(\theta_B)_2 = -\frac{wL^3}{6EI} = -\frac{(2.6)(0.75)^3}{(6)(36.8)} = -4.968 \times 10^{-3}$$

Portion BC remains straight for loading II.

$L_{BC} = 0.25$ m.

$$(y_C)_2 = (y_B)_2 + L_{BC}(\theta_B)_2 = -4.036 \times 10^{-3} \text{ m}$$

$$(\theta_C)_2 = (\theta_B)_2 = -4.968 \times 10^{-3}$$

Slope at end C. By superposition, $\theta_C = (\theta_C)_1 + (\theta_C)_2$

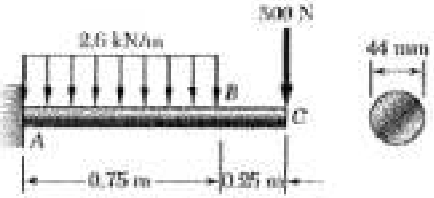
$\theta_C = -11.761 \times 10^{-3}$ $\qquad\qquad$ $\theta_C = 11.76 \times 10^{-3}$ rad ◄

Deflection at end C. By superposition, $y_C = (y_C)_1 + (y_C)_2$

$y_C = -8.565 \times 10^{-3}$ m $\qquad\qquad$ $y_C = 8.57$ mm ↓ ◄

Problem 9.76

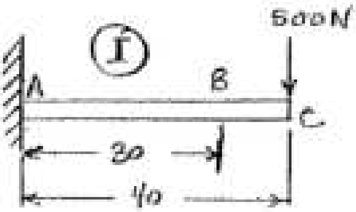
9.76 For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200$ GPa.



$$c = \tfrac{1}{2}d = \tfrac{1}{2}(44) = 22 \text{ mm}$$

$$I = \tfrac{\pi}{4}c^4 = \tfrac{\pi}{4}(22)^4 = 183984 \text{ mm}^4$$

$$EI = (200\times10^6)(183984\times10^{-6}) = 36.8 \text{ kN}\,\text{m}^2$$



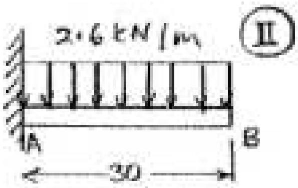
Loading I: Case 1 of Appendix D.

$$P = 0.5 \text{ kN} \qquad L = 1 \text{ m} \qquad x = 0.75 \text{ m}$$

$$y_1 = \frac{P}{6EI}(x^3 - 3Lx^2)$$

$$\theta_1 = \frac{dy}{dx} = \frac{P}{2EI}(x^2 - 2Lx)$$

$$(y_B)_1 = \frac{0.5}{(6)(36.8)}\left[(0.75)^3 - (3)(1)(0.75)^2\right] = -2.866\times10^{-3} \text{ mm}$$



$$(\theta_B)_1 = \frac{0.5}{(2)(36.8)}\left[(0.75)^2 - (2)(1)(0.75)\right] = -6.369\times10^{-3}$$

Loading II. Case 2 of Appendix D.

$$w = 2.6 \text{ kN/m}, \quad L = 0.75 \text{ m}$$

$$(y_B)_2 = -\frac{wL^4}{8EI} = -\frac{(2.6)(0.75)^4}{(8)(36.8)} = -2.794\times10^{-3} \text{ m}$$

$$(\theta_B)_2 = -\frac{wL^3}{6EI} = -\frac{(2.6)(0.75)^3}{(6)(36.8)} = -4.968\times10^{-3}$$

Slope at point B. $\qquad \theta_B = (\theta_B)_1 + (\theta_B)_2$

$$\theta_B = -11.337\times10^{-3}$$

$$\theta_B = 11.34\times10^{-3} \text{ rad}$$

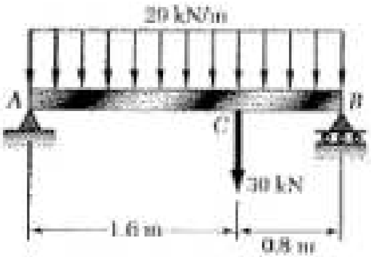
Deflection at point B. $\qquad y_B = (y_B)_1 + (y_B)_2$

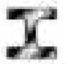
$$y_B = -5.66\times10^{-3} \text{ m}$$

$$y_B = 5.66 \text{ mm} \downarrow$$

Problem 9.78

9.77 and 9.78 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

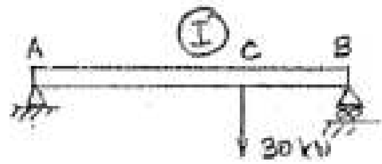



W150 × 24

<u>Units:</u> Forces in kN. Lengths in meters.

For W150×24 $\quad I = 13.4 \times 10^6\ \text{mm}^4 = 13.4 \times 10^{-6}\ \text{m}^4$

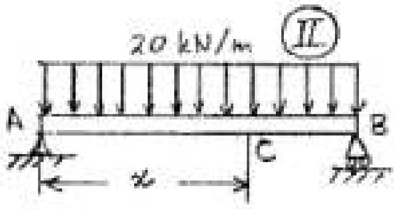
$$EI = (200 \times 10^9)(13.4 \times 10^{-6}) = 2.68 \times 10^6\ \text{N}\cdot\text{m} = 2680\ \text{kN}\cdot\text{m}^2$$



<u>Loading I:</u> Case 5 of Appendix D.

$P = 30$ kN, $L = 2.4$ m, $a = 1.6$ m, $b = 0.8$ m

$$(\theta_A)_1 = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(30)(0.8)(2.4^2 - 0.8^2)}{(6)(2680)(2.4)} = -3.1841 \times 10^{-3}$$



$$(y_C)_1 = -\frac{Pa^2b^2}{3EIL} = -\frac{(30)(1.6)^2(0.8)^2}{(3)(2680)(2.4)} = -2.54726 \times 10^{-3}\ \text{m}$$

<u>Loading II:</u> Case 6 of Appendix D.

$w = 20$ kN/m, $L = 2.4$ m, $x = 1.6$ m at point C.

$$(\theta_A)_2 = -\frac{wL^3}{24EI} = -\frac{(20)(2.4)^3}{(24)(2680)} = -4.2985 \times 10^{-3}$$

$$y_2 = -\frac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$$

$$(y_C)_2 = -\frac{20}{(24)(2680)}\left[(1.6)^4 - (2)(2.4)(1.6)^3 + (2.4)^3(1.6)\right] = -2.80199 \times 10^{-3}\ \text{m}$$

(a) <u>Slope at end A.</u> $\quad \theta_A = (\theta_A)_1 + (\theta_A)_2$

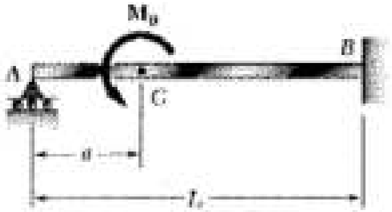
$\theta_A = -7.4826 \times 10^{-3}$ $\qquad\qquad \theta_A = 7.48 \times 10^{-3}$ rad ◄

(b) <u>Deflection at point C.</u> $\quad y_C = (y_C)_1 + (y_C)_2$

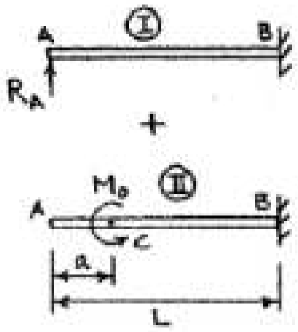
$y_C = -5.34925 \times 10^{-3}$ m $\qquad\qquad y_C = 5.35$ mm ↓ ◄

Problem 9.79

9.79 and 9.80 For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.



CONSIDER R_A AS REDUNDANT AND REPLACE LOADING SYSTEM BY I AND II.



LOADING I. (CASE 1 OF APPENDIX D.) $\quad y_A' = \dfrac{R_A L^3}{3EI}$

LOADING II. PORTION BC. (CASE 3 OF APPENDIX D.)

$$y_C'' = -\frac{M_0(L-a)^2}{2EI} \qquad \theta_C'' = \frac{M_0(L-a)}{EI}$$

PORTION AC IS STRAIGHT. $\quad y_A'' = y_C'' - (a)\theta_C$

$$= -\frac{M_0(L-a)^2}{2EI} - \frac{aM_0(L-a)}{EI}$$

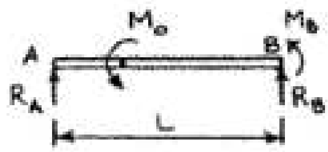
(a) SUPERPOSITION AND CONSTRAINT: $\quad y_A = y_A' + y_A'' = 0$

$$\frac{R_A L^3}{3EI} - \frac{M_0(L-a)^2}{2EI} - \frac{aM_0(L-a)}{EI} = 0$$

$$\tfrac{2}{3}R_A L^3 - M_0(L-a)(L-a+2a) = 0$$

$$\tfrac{2}{3}R_A L^3 = M_0(L^2-a^2) \qquad R_A = \frac{3M_0}{2L^3}(L^2-a^2)\uparrow$$

(b) $\quad +\uparrow \Sigma F_y = 0: \; R_A + R_B = 0 \quad R_B = -R_A \qquad R_B = \dfrac{3M_0}{2L^3}(L^2-a^2)\downarrow$

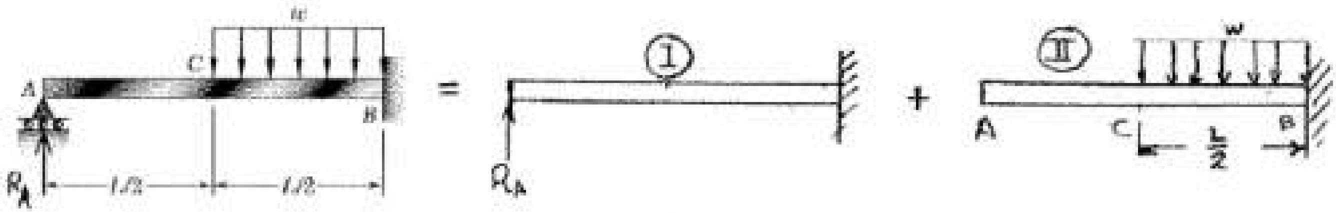


$+\circlearrowleft \Sigma M_B = 0: \; M_B + M_0 - R_A L = 0 \qquad M_B + M_0 - \frac{3}{2}\frac{M_0}{L^2}(L^2-a^2) = 0$

$M_B = \dfrac{3M_0}{2L^2}\left(L^2 - a^2 - \tfrac{2}{3}L^2\right) \qquad M_B = \dfrac{M_0}{2L^2}(L^2-3a^2)\circlearrowleft$

Prolem 9.80

9.79 and 9.80 For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.



Consider R_A as redundant and replace loading system by I and II.

Loading I. (Case 1 of Appendix D) $\quad (y_A)_1 = \frac{R_A L^3}{3EI}$

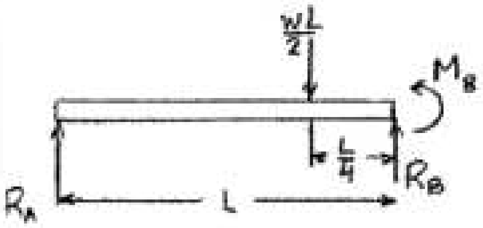
Loading II. Portion CB. (Case 2 of Appendix D)

$$(y_B)_2 = -\frac{w(L/2)^4}{8EI} = -\frac{wL^4}{128EI}, \quad (\theta_B)_2 = \frac{w(L/2)^3}{6EI} = \frac{wL^3}{48EI}$$

Portion AC is straight. $\quad (y_A)_2 = (y_B)_2 - \frac{L}{2}(\theta_B)_2 = -\frac{7wL^4}{384EI}$

Superposition and constraint. $\quad y_A = (y_A)_1 + (y_A)_2 = 0$

(a) $\quad \frac{R_A L^3}{3EI} - \frac{7wL^4}{384EI} = 0 \qquad R_A = \frac{7wL}{128} \uparrow$ ◀



Using entire beam as a free body,

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - \frac{wL}{2} = 0$

$R_B = \frac{wL}{2} - \frac{7wL}{128} \qquad R_B = \frac{57wL}{128} \uparrow$ ◀

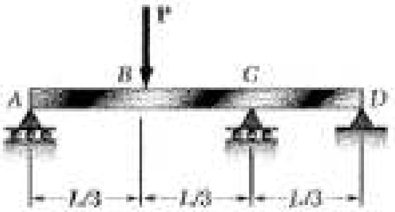
$+\circlearrowleft \Sigma M_B = 0: \quad M_B - R_A L - \frac{wL}{2}\cdot\frac{L}{4} = 0$

$M_B = R_A L - \frac{wL^2}{8} = -\frac{9wL^2}{128}$

$M_B = \frac{9wL^2}{128} \circlearrowright$ ◀

Problem 9.81

9.81 and 9.82 For the uniform beam shown, determine the reaction at each of the three supports.



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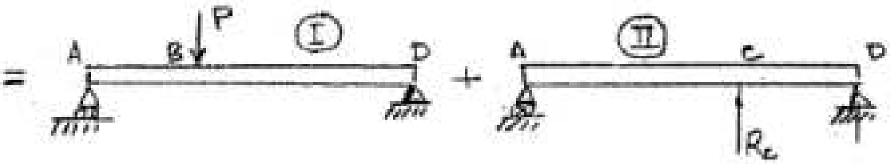


Consider R_C as redundant and replace loading system by I and II.

Loading I. (Case 5 of Appendix D) $a = \frac{2L}{3}$, $b = \frac{L}{3}$, $x = \frac{L}{3}$ at C.

$$(y_C)_1 = \frac{Pb}{6EIL}\left[x^3 - (L^2 - b^2)x\right] = \frac{P(L/3)}{6EIL}\left[\left(\frac{L}{3}\right)^3 - \left\{L^2 - \left(\frac{L}{3}\right)^2\right\}\frac{L}{3}\right]$$

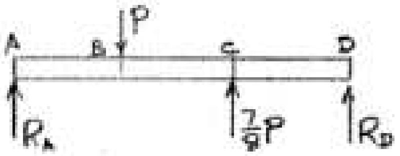
$$= -\frac{7PL^3}{486EI}$$

Loading II. (Case 5 of Appendix D) $a = \frac{2L}{3}$, $b = \frac{L}{3}$

$$(y_C)_2 = \frac{R_C a^2 b^2}{3EIL} = \frac{R_C (L/3)^2 (2L/3)^2}{3EIL} = \frac{4R_C L^3}{243EI}$$

Superposition and constraint. $y_C = (y_C)_1 + (y_C)_2 = 0$

$$-\frac{7PL^3}{486EI} + \frac{4R_C}{243EI} = 0 \qquad R_C = \frac{7}{8}P \uparrow$$



$+\circlearrowleft \Sigma M_D = 0$:

$$-R_A L + P\left(\frac{2L}{3}\right) - \left(\frac{7}{8}P\right)\left(\frac{L}{3}\right) = 0$$

$$R_A = \frac{3}{8}P \uparrow$$

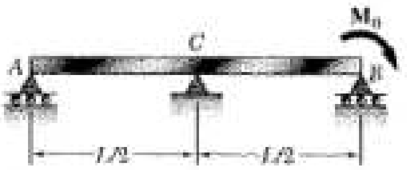
$+\uparrow \Sigma F_y = 0$: $R_A + R_D - P + \frac{7}{8}P = 0$

$$R_D = P - \frac{7}{8}P - \frac{3}{8}P = -\frac{1}{4}P$$

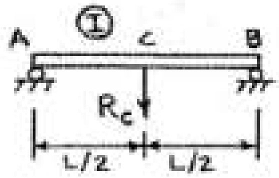
$$R_D = \frac{1}{4}P \downarrow$$

Problem 9.82

9.81 and 9.82 For the uniform beam shown, determine the reaction at each of the three supports.



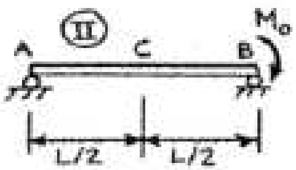
CONSIDER R_C AS REDUNDANT AND REPLACE LOADING SYSTEM BY I AND II.



LOADING I. (CASE 4 OF APPENDIX D.) $\quad y_C' = -\dfrac{R_C L^3}{48EI}$

LOADING II. (CASE 7 OF APPENDIX D.)

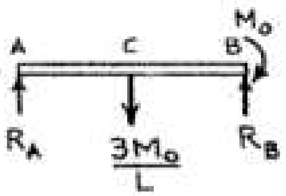
$$y_C'' = -\frac{M_0}{6EIL}\left[\left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right)\right] = \frac{M_0L^2}{16EI}$$



SUPERPOSITION AND CONSTRAINT.

$$y_C = y_C' + y_C'' = 0$$

$$-\frac{R_C L^3}{48EI} + \frac{M_0 L^2}{16EI} = 0 \qquad R_C = \frac{3M_0}{L} \downarrow$$



$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{3M_0}{L}\right)\left(\frac{L}{2}\right) - M_0 = 0$$

$$R_A = \frac{M_0}{2L} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{M_0}{2L} - \frac{3M_0}{L} + R_B = 0$$

$$R_B = \frac{5M_0}{2L} \uparrow$$

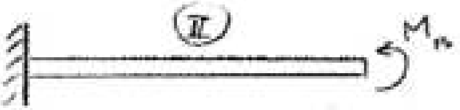
Problem 9.83

9.83 and 9.84 For the beam shown, determine the reaction at *B*.

Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.

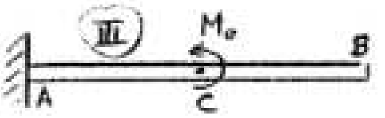
Loading I. Case 1 of Appendix D.

$$(y_B)_I = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_I = -\frac{R_B L^2}{2EI}$$



Loading II. Case 3 of Appendix D.

$$(y_B)_{II} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{II} = \frac{M_B L}{EI}$$



Loading III. Case 3 applied to portion AC.

$$(y_C)_{III} = \frac{M_0 (L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$(\theta_C)_{III} = \frac{M_0 (L/2)}{EI} = \frac{M_0 L}{2EI}$$

Portion CB remains straight.

$$(y_B)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{3}{8}\frac{M_0 L^2}{EI}$$

$$(\theta_B)_{III} = (\theta_C)_{III} = \frac{1}{2}\frac{M_0 L^2}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$-\frac{L^3}{3EI}R_B + \frac{L^2}{2EI}M_B + \frac{3}{8}\frac{M_0 L^2}{EI} = 0 \qquad (1)$$

$$\theta_B = (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0$$

$$-\frac{L^2}{2EI}R_B + \frac{L}{EI}M_B + \frac{1}{2}\frac{M_0 L}{EI} = 0 \qquad (2)$$

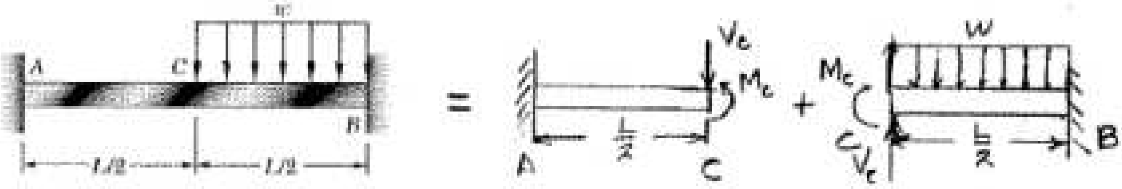
Solving (1) and (2) simultaneously,

$$R_B = \frac{3}{2}\frac{M_0}{L} \downarrow \quad \blacktriangleleft$$

$$M_B = \frac{1}{4}M_0 \circlearrowleft \quad \blacktriangleleft$$

Problem 9.84

9.83 and 9.84 For the beam shown, determine the reaction at B.



Portion AC: Superposition of Cases 3 and 1 of Appendix D.

$$y_c = \frac{M_c(L/2)^2}{2EI} - \frac{V_c(L/2)^3}{3EI} = \frac{M_cL^2}{8EI} - \frac{V_cL^3}{24EI}$$

$$\theta_c = \frac{M_c(L/2)}{EI} - \frac{V_c(L/2)^2}{2EI} = \frac{M_cL}{2EI} - \frac{V_cL^3}{8EI}$$

Portion CB: Superposition of Cases 3, 1, and 2 of Appendix D.

$$y_c = \frac{M_c(L/2)^2}{2EI} + \frac{V_c(L/2)^3}{3EI} - \frac{w(L/2)^4}{8EI}$$
$$= \frac{M_cL^2}{8EI} + \frac{V_cL^3}{24EI} - \frac{wL^4}{128EI}$$

$$\theta_c = -\frac{M_c(L/2)}{EI} - \frac{V_c(L/2)^3}{2EI} + \frac{w(L/2)^3}{6EI}$$
$$= -\frac{M_cL}{2EI} - \frac{V_cL^3}{8EI} + \frac{wL^3}{48EI}$$

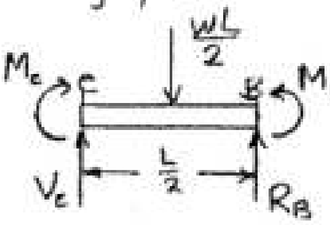
Matching expressions for y_c,

$$\cancel{\frac{M_cL^2}{8EI}} - \frac{V_cL^3}{24EI} = \cancel{\frac{M_cL^2}{8EI}} + \frac{V_cL^3}{24EI} - \frac{wL^4}{128EI} \qquad V_c = \frac{3}{32}wL$$

Matching expressions for θ_c,

$$\frac{M_cL}{2EI} - \cancel{\frac{V_cL^3}{8EI}} = -\frac{M_cL}{2EI} - \cancel{\frac{V_cL^3}{8EI}} + \frac{wL^3}{48EI} \qquad M_c = \frac{1}{48}wL^2$$

Using portion CB as a free body,



$+\uparrow\Sigma F_y = 0: \quad R_B + V_c - \frac{wL}{2} = 0$

$R_B = \frac{wL}{2} - \frac{3}{32}wL \qquad R_B = \frac{13}{32}wL\uparrow$ ◄

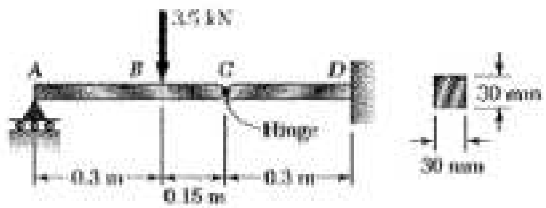
$\circlearrowleft \Sigma M_B = 0: \quad M_B - M_c - V_c\frac{L}{2} + \frac{wL}{2}\cdot\frac{L}{4} = 0$

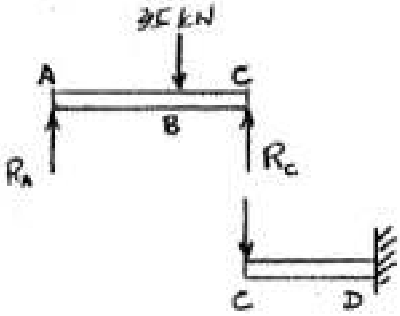
$M_B = \frac{1}{48}wL^2 + \left(\frac{3}{32}wL\right)\left(\frac{L}{2}\right) - \frac{wL^2}{8}$

$M_B = -\frac{11}{192}wL^2 \qquad M_B = \frac{11}{192}wL^2 \circlearrowright$ ◄

Problem 9.87

9.87 The two beams shown have the same cross section and are joined by a hinge at C. For the loading shown, determine (*a*) the slope at point A, (*b*) the deflection at point B. Use $E = 200$ GPa.





Using free body ABC,

$$\circlearrowright \Sigma M_A = 0: \quad 0.45 R_c - (0.3)(35) = 0$$

$$R_c = 2.33 \text{ kN.}$$

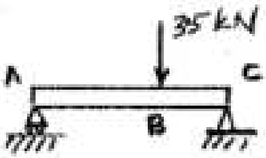
$$E = 200 \text{ GPa}$$

$$I = \tfrac{1}{12} bh^3 = \tfrac{1}{12}(30)(30)^3 = 67500 \text{ mm}^4$$

$$EI = (200 \times 10^9)(67500 \times 10^{-12}) = 13.5 \text{ kN m}^2$$

Using cantilever beam CD with load R_c,

Case 1 of Appendix D.



$$y_c = -\frac{R_c L_{CD}^3}{3EI} = -\frac{(2.33)(0.3)^3}{(3)(13.5)} = 0.001555 \text{ m}$$

Calculation of θ_A' and y_B' assuming that point C does not move.

Case 5 of Appendix D $\qquad P = 35$ kN $\quad L = 0.45$ m, $a = 0.3$ m, $b = 0.15$ m

$$\theta_A' = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(35)(0.15)(0.45^2 - 0.15^2)}{(6)(13.5)(0.45)} = -0.00259 \text{ rad.}$$

$$y_B' = -\frac{Pb^2a^2}{3EIL} = -\frac{(35)(0.15)^2(0.3)^2}{(3)(13.5)(0.45)} = -0.000389 \text{ m}$$

Additional slope and deflection due to movement of point C.

$$\theta_A'' = \frac{y_c}{L_{AC}} = -\frac{0.001555}{0.45} = -0.003456 \text{ rad.}$$

$$y_B'' = \frac{a}{L} y_c = -\frac{(0.3)(.001555)}{0.45} = -0.0010367 \text{ mm.}$$

(a) Slope at A. $\quad \theta_A = \theta_A' + \theta_A'' = -0.00259 - 0.003456$

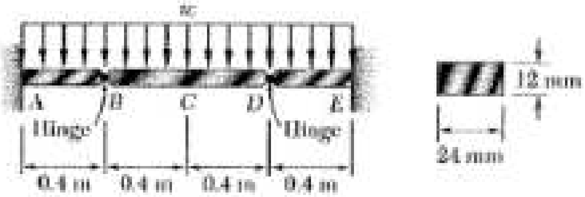
$$= -0.006046 \text{ rad} = 0.00605 \text{ rad}$$

(b) Deflection at B. $\quad y_B = y_B' + y_B'' = -0.000389 - 0.0010367$

$$= -1.4257 \times 10^{-3} \text{ m} = 1.43 \text{ mm} \downarrow$$

Problem 9.88

9.88 A central beam *BD* is joined at hinges to two cantilever beams *AB* and *DE*. All beams have the cross section shown. For the loading shown, determine the largest *w* so that the deflection at *C* does not exceed 3 mm. Use *E* = 200 GPa.



Let $a = 0.4$ m

Cantilever beams AB and CD.

Cases 1 and 2 of Appendix D. $\quad y_C = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = \frac{11}{24}\frac{wa^4}{EI}$

Beam BCD, with L = 0.8 m, assuming that points B and D do not move.

Case 6 of Appendix D.

$$y_C' = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points B and D.

$$y_C'' = y_B = y_D = -\frac{11}{24}\frac{wa^4}{EI}$$

Total deflection at C. $\quad y_C = y_C' + y_C''$

$$y_C = -\frac{w}{EI}\left\{\frac{5L^4}{384} + \frac{11a^4}{24}\right\}$$

Data: $E = 200\times10^9$ Pa, $\quad I = \frac{1}{12}(24)(12)^3 = 3.456\times10^3 \text{ mm}^4 = 3.456\times10^{-9} \text{ m}^4$

$$EI = (200\times10^9)(3.456\times10^{-9}) = 691.2 \text{ N}\cdot\text{m}^2$$

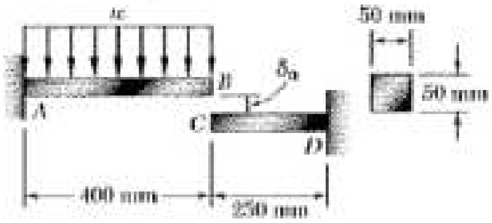
$$y_C = -3\times10^{-3} \text{ m}$$

$$-3\times10^{-3} = -\frac{w}{691.2}\left\{\frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24}\right\} = -24.69\times10^{-6}\, w$$

$w = 121.5$ N/m ◄

Problem 9.89

9.89 Before the uniformly distributed load w is applied, a gap, $\delta_0 = 1.2$ mm, exists between the ends of the cantilever bars AB and CD. Knowing that $E = 105$ GPa and $w = 30$ kN/m, determine (*a*) the reaction at A, (*b*) the reaction at D.



$I = \frac{1}{12}(50)(50)^3 = 520.833 \times 10^3\ \text{mm}^4 = 520.833 \times 10^{-9}\ \text{m}^4$

$EI = (105 \times 10^9)(520.833 \times 10^{-9}) = 54.6875 \times 10^3\ \text{N}\cdot\text{m}^2$

$= 54.6875\ \text{kN}\cdot\text{m}^2$

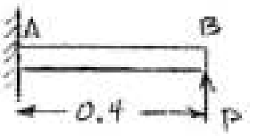
Units: Forces in kN. Lengths in meters.

Compute deflection at B due to w. Case 8 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(30)(0.400)^4}{(8)(54.6875)} = -1.75543 \times 10^{-3} = -1.7553\ \text{mm}$$

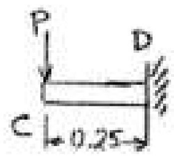
The displacement is more than δ_0, the gap closes.

Let P be the contact force between points B and C.



Compute deflection of B due to P. Use Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(0.4)^3}{(3)(54.6875)} = 390.095 \times 10^{-6}\ P\ \text{m}$$



Compute deflection of C due to P.

$$y_C = -\frac{PL^3}{3EI} = -\frac{P(0.25)^3}{(3)(54.6875)} = -95.238 \times 10^{-6}\ P\ \text{m}$$

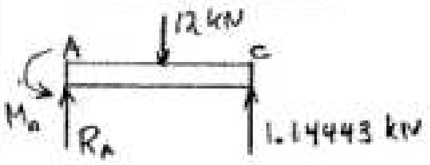
Displacement condition. $\quad y_B + \delta_0 = y_C$

Using superposition, $\quad (y_B)_1 + (y_B)_2 - \delta_0 = y_C$

$$-1.75543 \times 10^{-3} + 390.095 \times 10^{-6} P + 1.2 \times 10^{-3} = -95.238 \times 10^{-6} P$$

$$485.333 \times 10^{-6} P = 0.55543 \times 10^{-3} \qquad P = 1.14443\ \text{kN}.$$

(a) Reaction at A.



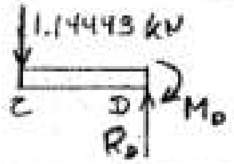
$+\uparrow \Sigma F_y = 0: \quad R_A - 12 + 1.14443 = 0$

$R_A = 10.86\ \text{kN} \uparrow$ ◀

$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (0.2)(12) + (0.4)(1.14443) = 0$

$M_A = 1.942\ \text{kN}\cdot\text{m} \circlearrowleft$ ◀

(b) Reaction at D.



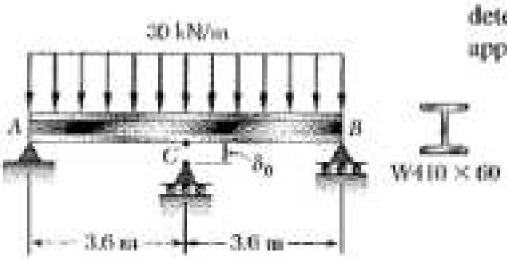
$+\uparrow \Sigma F_y = 0: \quad R_D - 1.14443 = 0$

$R_D = 1.144\ \text{kN} \uparrow$ ◀

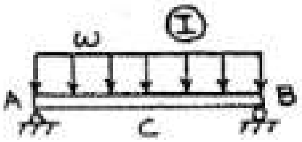
$+\circlearrowleft \Sigma M_D = 0: \quad -M_D + (0.25)(1.14443) = 0$

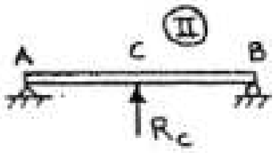
$M_D = 0.286\ \text{kN}\cdot\text{m} \circlearrowright$ ◀

Problem 9.90



9.90 Before the 30 kN/m load is applied, a gap, $\delta_0 = 20$ mm, exists between the W410 × 60 beam and the support at C. Knowing that $E = 200$ GPa, determine the reaction at each support after the uniformly distributed load is applied.





DATA: $\delta_0 = 0.02$ m

$E = 200 \times 10^6$ kPa $\qquad I = 216 \times 10^{-6}$ m^4

$EI = 43.2 \times 10^3$ kN m^2

LOADING I. CASE 6 OF APPENDIX D.

$$y_C' = -\frac{5wL^4}{384EI} = -\frac{5(30)(7.2)^4}{384(43.2 \times 10^3)}$$

$$= -24.3 \times 10^{-3} \text{ m}$$

LOADING II. CASE 4 OF APPENDIX D.

$$y_C'' = \frac{R_C L^3}{48EI} = \frac{R_C (7.2)^3}{48(43.2 \times 10^3)}$$

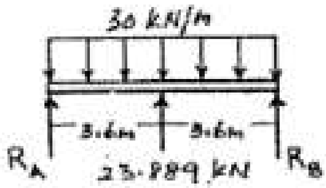
$$= 0.18 \times 10^{-3} R_C$$

DEFLECTION AT C.

$$y_C = y_C' + y_C'' = -\delta_0$$

$$-24.3 \times 10^{-3} + 0.18 \times 10^{-3} R_C = -20 \times 10^{-3}$$

$R_C = 23.889$ kN $\qquad R_C = 23.9$ kN ↑ ◄



$+↺ \Sigma M_B = 0: (30)(7.2)(3.6) - R_A(7.2) - (23.889)(3.6) = 0$

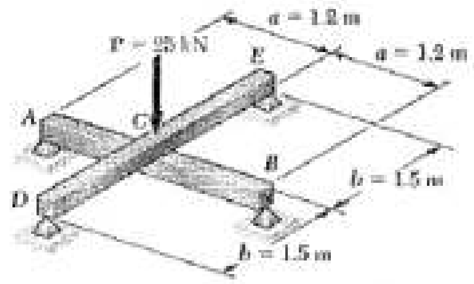
$R_A = 96.055$ $\qquad R_A = 96.1$ kN ↑ ◄

$+↑ \Sigma F_y = 0: 96.055 - 30(7.2) + 23.889 + R_B = 0.$

$R_B = 96.1$ kN ↑ ◄

Problem 9.91

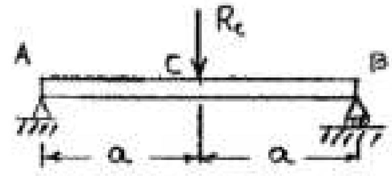
9.91 For the loading shown, and knowing that beams AB and DE have the same flexural rigidity, determine the reaction (*a*) at B, (*b*) at E.



Units: Forces in kN; lengths in m.

For beam ACB, using Case 4 of Appendix D.

$$(y_C)_1 = -\frac{R_C(2a)^3}{48EI}$$

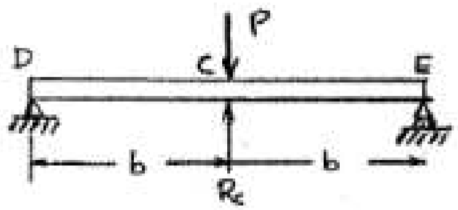


For beam DCE, using Case 4 of Appendix D.

$$(y_C)_2 = \frac{(R_C - P)(2b)^3}{48EI}$$

Matching deflections at C,

$$-\frac{R_C(2a)^3}{48EI} = \frac{(R_C - P)(2b)^3}{48EI}$$



$$R_C = \frac{Pb^3}{a^3 + b^3} = \frac{(25)(1.5)^3}{1.2^3 + 1.5^3} = 16.53 \text{ kN}$$

$$P - R_C = 25 - 16.53 = 8.47 \text{ kN}$$

Using free body ACB, $\circlearrowleft M_A = 0$: $2aR_B - aR_C = 0$

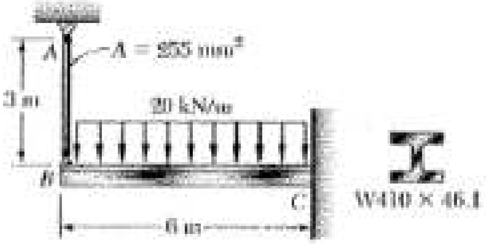
(a) $R_B = \frac{1}{2}R_C = 8.265 \text{ kN} \uparrow$ ◄

Using free body DCE, $\circlearrowleft M_D = 0$: $2bR_E - b(P - R_C) = 0$

(b) $R_E = \frac{1}{2}(P - R_C) = 4.235 \text{ kN} \uparrow$ ◄

Problem 9.92

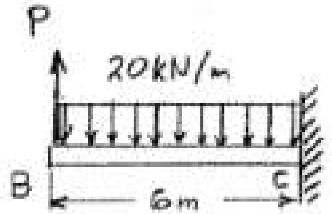
9.92 The cantilever beam BC is attached to the steel cable AB as shown. Knowing that the cable is initially taut, determine the tension in the cable caused by the distributed load shown. Use $E = 200$ GPa.



Let P be the tension developed in member AB and δ_B be the elongation of that member.

Cable AB: $A = 255\ \text{mm}^2 = 255 \times 10^{-6}\ \text{m}^2$

$$\delta_B = \frac{PL}{EA} = \frac{(P)(3)}{(200\times10^9)(255\times10^{-6})} = 58.82 \cdot 10^{-9}\, P$$



Beam BC: $I = 156 \times 10^6\ \text{mm}^4 = 156 \times 10^{-6}\ \text{m}^4$

$$EI = (200\times10^9)(156\times10^{-6}) = 31.2\times10^6\ \text{N}\cdot\text{m}^2$$

Loading I. 20 kN/m downward.

Refer to Case 2 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(20\times10^3)(6)^4}{(8)(31.2\times10^6)} = -103.846\times10^{-3}\ \text{m}$$

Loading II. Upward force P at point B.

Refer to Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(6)^3}{(3)(31.2\times10^6)} = 2.3077\times10^{-6}\,P$$

By superposition, $y_B = (y_B)_1 + (y_B)_2$

Also, matching the deflection at B, $y_B = -\delta_B$

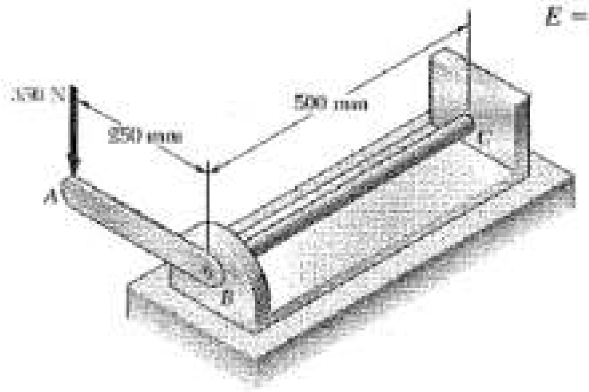
$$-103.846\times10^{-3} + 2.3077\times10^{-6}\,P = -58.82\cdot10^{-9}\,P$$

$$2.3666\times10^{-6}\,P = -103.846\times10^{-3} \qquad P = 43.9\times10^3\ \text{N}$$

$$P = 43.9\ \text{kN}$$

Problem 9.93

9.93 A 22-mm-diameter rod BC is attached to the lever AB and to the fixed support at C. Lever AB has a uniform cross section 10 mm thick and 25 mm deep. For the loading shown, determine the deflection of point A. Use E = 200 GPa and G = 77 GPa.



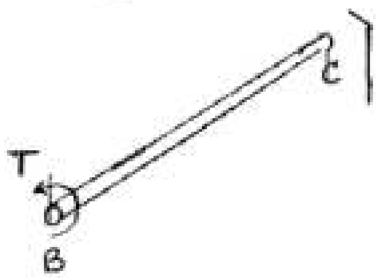
Deformation of rod BC. (Torsion)

$$c = \tfrac{1}{2}d = \tfrac{1}{2}(22) = 11 \text{ mm}$$

$$J = \tfrac{\pi}{2}c^4 = 22998 \text{ mm}^4$$

$$T = Pa = (350)(0.25) = 87.5 \text{ N m}$$

$$L = 0.5 \text{ m}$$



$$\varphi_B = \frac{TL}{GJ} = \frac{(87.5)(0.5)}{(77\times10^9)(22998\times10^{-12})}$$

$$= 0.0247 \text{ rad}$$

Deflection of point A assuming lever AB to be rigid.

$$(y_A)_1 = a\varphi_B = (0.25)(0.0247)$$

$$= 0.006175 \text{ m} \downarrow$$

Additional deflection due to bending of lever AB.
Refer to Case 1 of Appendix D.

$$I = \tfrac{1}{12}(10)(25)^3 = 13021 \text{ mm}^4$$

$$(y_A) = \frac{PL^3}{3EI} = \frac{(350)(0.25)^3}{(3)(200\times10^9)(13021\times10^{-12})}$$

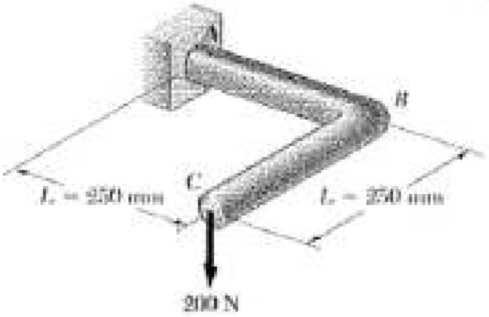
$$= 2.1\times10^{-3} \text{ m} \downarrow$$

Total deflection at point A.

$$y_A = (y_A)_1 + (y_A)_2 = 8.28 \text{ mm} \downarrow$$

Problem 9.94

9.94 A 16-mm-diameter rod has been bent into the shape shown. Determine the deflection of end C after the 200-N force is applied. Use $E = 200$ GPa and $G = 80$ GPa.



LET $200\text{ N} = P$.

CONSIDER TORSION OF ROD AB.

$$\phi_B = \frac{TL}{JG} = \frac{(PL)L}{JG} = \frac{PL^2}{JG}$$

$$y_C' = -L\phi_B = -\frac{PL^3}{JG}$$

CONSIDER BENDING OF AB. (CASE 1 OF APPENDIX D.)

$$y_C'' = y_B = -\frac{PL^3}{3EI}$$

CONSIDER BENDING OF BC. (CASE 1 OF APPENDIX D.)

$$y_C''' = -\frac{PL^3}{3EI}$$

SUPERPOSITION:

$$y_C = y_C' + y_C'' + y_C''' = -\frac{PL^3}{JG} - \frac{PL^3}{3EI} - \frac{PL^3}{3EI} = -\frac{PL^3}{EI}\left(\frac{EI}{JG} + \frac{2}{3}\right)$$

DATA: $G = 80(10^9)\text{ Pa}$ $\quad J = \frac{1}{2}\pi(0.008)^4 = 6.4340(10^{-9})\text{ m}^4$

$E = 200(10^9)\text{ Pa}$ $\quad I = \frac{1}{2}J = 3.2170(10^{-9})\text{ m}^4$

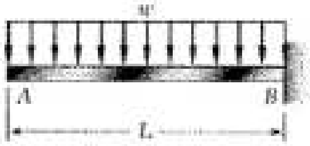
$EI = 643.40\text{ N}\cdot\text{m}^2$ $\quad JG = 514.72\text{ N}\cdot\text{m}^2$

$$y_C = -\frac{(200)(0.25)^3}{643.40}\left(\frac{643.40}{514.72} + \frac{2}{3}\right) = -9.3093(10^{-3})\text{ m}$$

$y_C = 9.31\text{ mm}\downarrow$ ◄

Problem 9.97

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

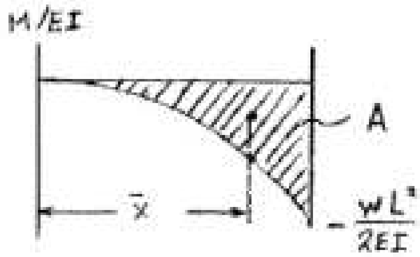


Place reference tangent at B. $\theta_B = 0$

Draw $\frac{M}{EI}$ curve as parabola.

$$A = -\frac{1}{3}\left(\frac{wL^2}{2EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\bar{x} = L - \frac{1}{4}L = \frac{3}{4}L$$



By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{6}\frac{wL^3}{EI} = \frac{1}{6}\frac{wL^3}{EI}$$

By second moment-area theorem,

$$t_{A/B} = \bar{x}A = \left(\frac{3}{4}L\right)\left(-\frac{1}{6}\frac{wL^3}{EI}\right) = -\frac{1}{8}\frac{wL^4}{EI}$$

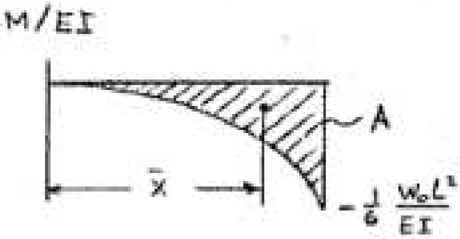
$$y_A = t_{A/B} = -\frac{1}{8}\frac{wL^4}{EI}$$

(a) $\theta_A = \frac{wL^3}{6EI}$ ◄

(b) $y_A = \frac{wL^4}{8EI} \downarrow$ ◄

Problem 9.98

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.



Place reference tangent at B. $\theta_B = 0$

$$\circlearrowleft \Sigma M_B = 0: \quad \left(\tfrac{1}{2}w_0L\right)\tfrac{L}{3} + M_B = 0$$

$$M_B = -\tfrac{1}{6}w_0L^2$$

Draw $\frac{M}{EI}$ curve as cubic parabola.

$$A = -\tfrac{1}{4}\left(\tfrac{1}{6}\tfrac{w_0L^2}{EI}\right)L = -\tfrac{1}{24}\tfrac{w_0L^3}{EI}$$

$$\bar{x} = L - \tfrac{1}{5}L = \tfrac{4}{5}L$$

By first moment-area theorem,

$$\theta_{B/A} = A = -\tfrac{1}{24}\tfrac{w_0L^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \tfrac{1}{24}\tfrac{w_0L^3}{EI} = \tfrac{1}{24}\tfrac{w_0L^3}{EI}$$

By second moment-area theorem,

$$t_{A/B} = \bar{x}A = \left(\tfrac{4}{5}L\right)\left(-\tfrac{1}{24}\tfrac{w_0L^3}{EI}\right) = -\tfrac{1}{30}\tfrac{w_0L^4}{EI}$$

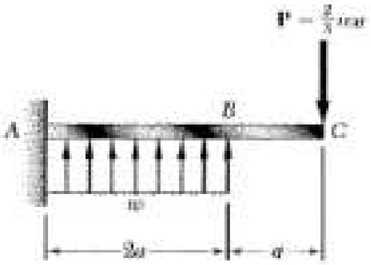
$$y_A = t_{A/B} = -\tfrac{1}{30}\tfrac{w_0L^4}{EI}$$

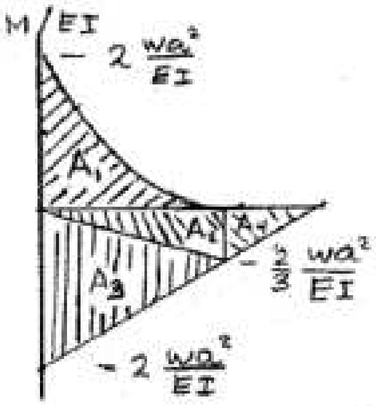
(a) $\theta_A = \dfrac{w_0L^3}{24EI}$ ◄

(b) $y_A = \dfrac{w_0L^4}{30EI} \downarrow$ ◄

Problem 9.99

9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (*a*) the slope and deflection at (*a*) point *B*, (*b*) point *C*.





Draw $\frac{M}{EI}$ diagram by parts and divide into areas A_1, A_2, A_3, and A_4.

$$A_1 = \frac{1}{3}(2a)\left(2\frac{wa^2}{EI}\right) = \frac{4}{3}\frac{wa^3}{EI}$$

$$A_2 = \frac{1}{2}(2a)\left(-\frac{2wa^2}{3EI}\right) = -\frac{2}{3}\frac{wa^3}{EI}$$

$$A_3 = \frac{1}{2}(2a)\left(-\frac{2wa^2}{EI}\right) = -2\frac{wa^3}{EI}$$

$$A_4 = \frac{1}{2}(a)\left(-\frac{2}{3}\frac{wa^2}{EI}\right) = -\frac{1}{3}\frac{wa^3}{EI}$$

(a) Slope at B.

$$\theta_B = A_1 + A_2 + A_3 = -\frac{4}{3}\frac{wa^3}{EI} \qquad \frac{4wa^3}{3EI} \measuredangle$$

Deflection at B. $\quad y_B = t_{B/A}$

$$y_B = t_{B/A} = A_1\left(\frac{3}{4}\cdot 2a\right) + A_2\left(\frac{1}{3}\cdot 2a\right) + A_3\left(\frac{2}{3}\cdot 2a\right)$$

$$= 2\frac{wa^4}{EI} - \frac{4}{9}\frac{wa^4}{EI} - \frac{8}{3}\frac{wa^4}{EI} = -\frac{10}{9}\frac{wa^4}{EI}$$

$$y_B = \frac{10}{9}\frac{wa^4}{EI}\downarrow$$

(b) Slope at C.

$$\theta_C = \theta_B + A_4 = -\frac{4}{3}\frac{wa^3}{EI} - \frac{1}{3}\frac{wa^3}{EI} = -\frac{5}{3}\frac{wa^3}{EI} \qquad \theta_A = \frac{5}{3}\frac{wa^3}{EI}\measuredangle$$

Deflection at C.

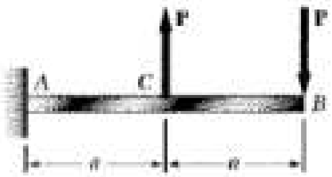
$$y_C = y_B + \theta_B a + t_{C/B}$$

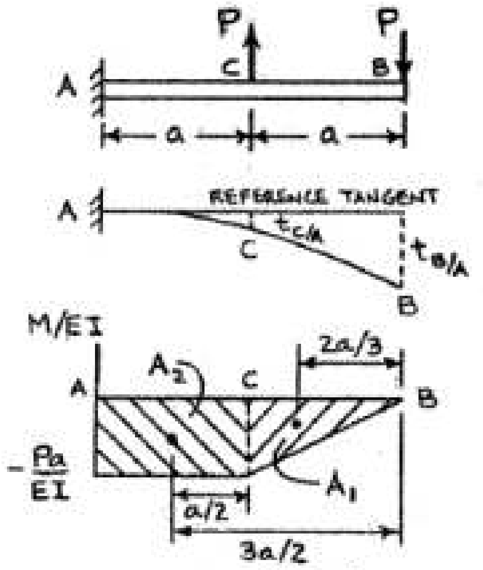
$$= -\frac{10}{9}\frac{wa^4}{EI} + \left(-\frac{4}{3}\frac{wa^3}{EI}\right)(a) + \left(-\frac{1}{3}\frac{wa^3}{EI}\right)\left(\frac{2a}{3}\right) \qquad -\frac{8}{3}\frac{wa^4}{EI}$$

$$y_A = \frac{8}{3}\frac{wa^4}{EI}\downarrow$$

Problem 9.100

9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (*a*) the slope and deflection at (*a*) point *B*, (*b*) point *C*.





$$A_1 = \tfrac{1}{2}(a)\left(-\frac{Pa}{EI}\right) = -\frac{Pa^2}{2EI}$$

$$A_2 = (a)\left(-\frac{Pa}{EI}\right) = -\frac{Pa^2}{EI}$$

(a) AT POINT B

$$\theta_B = \theta_{B/A} = A_1 + A_2 = -\frac{Pa^2}{2EI} - \frac{Pa^2}{EI} = -\frac{3Pa^2}{2EI}$$

$$\theta_B = \frac{3Pa^2}{2EI}$$

$$y_B = t_{B/A} = A_1\left(\frac{2a}{3}\right) + A_2\left(\frac{3a}{2}\right)$$

$$= \left(-\frac{Pa^2}{2EI}\right)\left(\frac{2a}{3}\right) + \left(-\frac{Pa^2}{EI}\right)\left(\frac{3a}{2}\right) = -\frac{11Pa^3}{6EI}$$

$$y_B = \frac{11Pa^3}{6EI} \downarrow$$

(b) AT POINT C

$$\theta_C = \theta_{C/A} = A_2 = -\frac{Pa^2}{EI}$$

$$\theta_C = \frac{Pa^2}{EI}$$

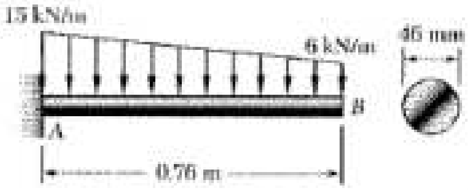
$$y_C = t_{C/A} = A_2\left(\frac{a}{2}\right)$$

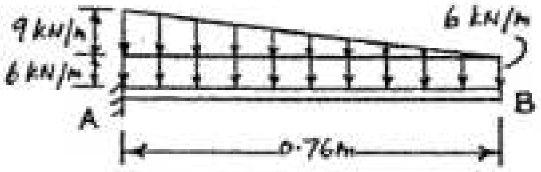
$$= \left(-\frac{Pa^2}{EI}\right)\left(\frac{a}{2}\right) = -\frac{Pa^3}{2EI}$$

$$y_C = \frac{Pa^3}{2EI} \downarrow$$

Problem 9.101

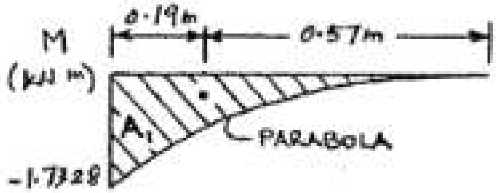
9.101 For the cantilever beam and loading shown, determine (*a*) the slope at point *B*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.





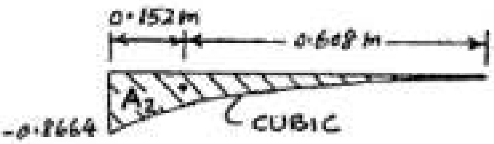
$$I = \frac{\pi}{4}(0.023)^4 = 219.8 \times 10^{-9}\ \text{m}^4$$

$$EI = (200 \times 10^6)(219.8 \times 10^{-9}) = 43.96\ \text{kN m}^2$$



$$M_A = -\frac{wL^2}{2} = -\frac{(6)(0.76)^2}{2} = -1.7328\ \text{kNm}$$

$$A_1 = \frac{1}{3}(-1.7328)(0.76) = -0.439\ \text{kNm}^2$$



$$M_A = -\frac{wL^2}{6} = -\frac{(9)(0.76)^2}{6} = -0.8664\ \text{kNm}$$

$$A_2 = \frac{1}{4}(-0.8664)(0.76) = -0.1646\ \text{kNm}^2$$

(a) SLOPE AT B

$$EI\,\theta_{B/A} = A_1 + A_2 = -0.439 - 0.1646 = -0.6036\ \text{kN m}^2$$

$$\theta_B = \theta_{B/A} = \frac{-0.6036}{43.96} = -13.7307 \times 10^{-3}\ \text{rad}$$

$$\theta_B = 13.73 \times 10^{-3}\ \text{rad}$$ ◄

(b) DEFLECTION AT B

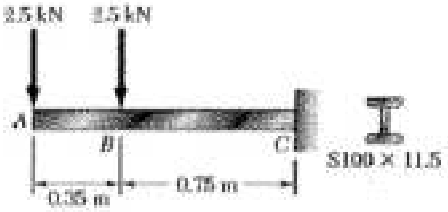
$$EI\,y_B = EI\,t_{B/A} = (-0.439)(0.57) + (-0.1646)(0.608) = -0.3503\ \text{kN m}^3$$

$$y_B = \frac{-0.3503}{43.96} = -0.00797\ \text{m}$$

$$y_B = 7.97\ \text{mm} \downarrow$$ ◄

Problem 9.103

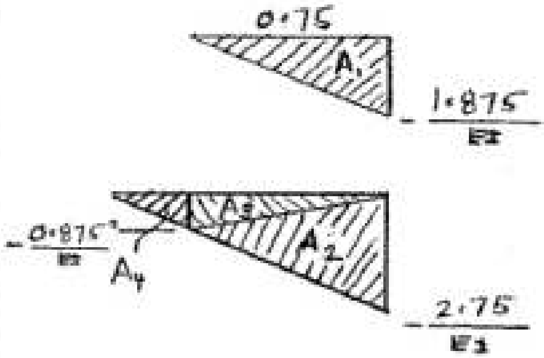
9.103 For the cantilever beam and loading shown, determine the slope and deflection at (*a*) end *A*, (*b*) point *B*. Use $E = 200$ GPa.



$I = 2.53 \times 10^6 \text{ mm}^4$

$EI = (200 \times 10^6)(2.53 \times 10^{-6}) = 506 \text{ kN m}^2$

Draw $\frac{M}{EI}$ diagram by parts.



$A_1 = -\frac{1}{2}(0.75)\frac{1.875}{EI} = -1.39 \times 10^{-3}$

$A_2 = -\frac{1}{2}(0.75)\frac{2.75}{EI} = -2.038 \times 10^{-3}$

$A_3 = -\frac{1}{2}(0.75)\frac{0.875}{EI} = -0.648 \times 10^{-3}$

$A_4 = -\frac{1}{2}(0.35)\frac{0.875}{EI} = -0.324 \times 10^{-3}$

Place reference tangent at C.

$\theta_c = 0 \qquad y_c = 0$

(a) Slope at A. $\quad \theta_A = -\theta_{C/A} = -A_1 - A_2 - A_3 - A_4 = 4.4 \times 10^{-3}$ ◀

Deflection at A. $\quad y_A = t_{A/C}$

$y_A = A_1(0.35+0.5) + A_2(0.35+0.5) + A_3(0.35+0.25) + A_4(0.23) = -3.38 \text{ mm}$ ◀

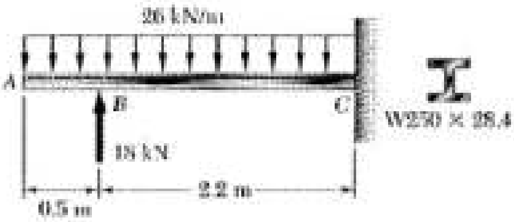
(b) Slope at B. $\quad \theta_B = -\theta_{B/A} = -A_1 + A_2 - A_3 = 4.08 \times 10^{-3}$ ◀

Deflection at B. $\quad y_B = t_{B/C}$

$y_B = A_1(0.5) + (A_2(0.5) + A_3(0.23) = -1.86 \times 10^{-3} \text{ m} = -1.86 \text{ mm}$ ◀

Problem 9.104

9.104 For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.



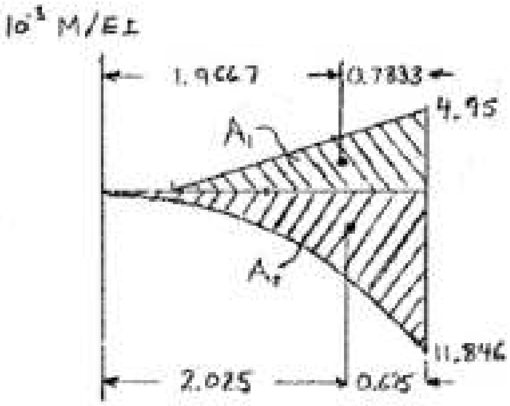
Units: Forces in kN, lengths in m.

$E = 200 \times 10^9$ Pa

$I = 40.0 \times 10^6 \text{ mm}^4 = 40.0 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(40.0 \times 10^{-6}) = 8.00 \times 10^6 \text{ N}\cdot\text{m}^2$

$= 8000 \text{ kN}\cdot\text{m}^2$



Draw M/EI diagram by parts.

$\frac{M_1}{EI} = \frac{(18)(2.2)}{8000} = 4.95 \times 10^{-3} \text{ m}^{-1}$

$A_1 = \frac{1}{2}(4.95 \times 10^{-3})(2.2) = 5.445 \times 10^{-3}$

$\bar{x}_1 = \frac{1}{3}(2.2) = 0.7333 \text{ m}$

$\frac{M_2}{EI} = -\frac{(26)(2.7)^2}{(2)(8000)} = -11.846 \times 10^{-3} \text{ m}^{-1}$

$A_2 = \frac{1}{3}(-11.846 \times 10^{-3})(2.7) = -10.662 \times 10^{-3}$

$\bar{x}_2 = \frac{1}{4}(2.7) = 0.675 \text{ m}$

Draw reference tangent at C.

$\theta_C = \theta_A + \theta_{C/A} = \theta_A + A_1 + A_2 = 0$

$\theta_A = -A_1 - A_2 = -5.445 \times 10^{-3} + 10.662 \times 10^{-3}$

$= 5.22 \times 10^{-3} \text{ rad}$

(a) Slope at A. $\qquad \theta_A = 5.22 \times 10^{-3}$ ⦣ ◄

$y_A = y_C - \theta_C L + t_{A/C}$

$= 0 - 0 + A_1\bar{x}_1 + A_2\bar{x}_2$

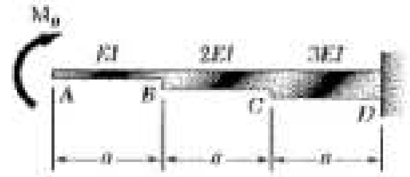
$= 0 - 0 + (5.445 \times 10^{-3})(1.9667) - (10.662 \times 10^{-3})(2.025)$

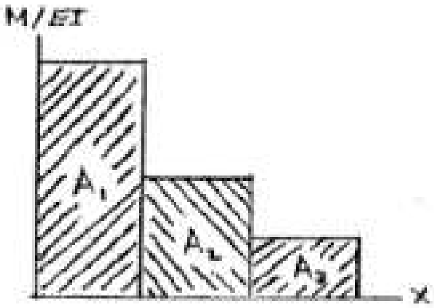
$= -10.881 \times 10^{-3} \text{ m}$

(b) Deflection at A. $\qquad y_A = 10.88 \text{ mm} \downarrow$ ◄

Problem 9.105

9.105 For the cantilever beam and loading shown, determine the deflection and slope at end A caused by the moment $\mathbf{M}_0$.





Draw $\frac{M}{EI}$ diagram.

$$A_1 = +\frac{M_0 a}{EI}$$

$$A_2 = +\frac{M_0 a}{2EI}$$

$$A_3 = +\frac{M_0 a}{3EI}$$

$$\theta_D = 0, \quad y_D = 0$$

Place reference tangent at D.

<u>Deflection at A.</u> $\quad y_A = t_{A/D}$

$$y_A = A_1\left(\tfrac{1}{2}a\right) + A_2\left(\tfrac{3}{2}a\right) + A_3\left(\tfrac{5}{2}a\right) = \frac{25M_0a^2}{12EI} \uparrow$$

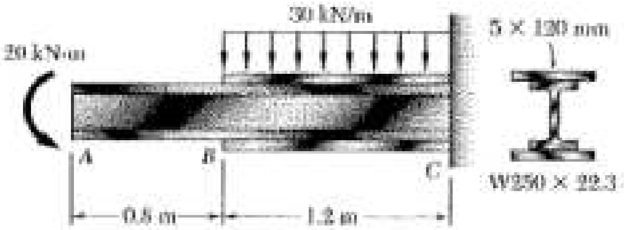
<u>Slope at A.</u> $\quad \theta_A = -\theta_{D/A}$

$$\theta_A = -A_1 - A_2 - A_3 = -\frac{11M_0a}{6EI}$$

$$\theta_A = \frac{11M_0a}{6EI} \measuredangle$$

Problem 9.107

9.107 Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200$ GPa, determine the (*a*) slope at end *A*, (*b*) the deflection at end *A*.



Units: Forces in kN, lengths in m.

$E = 200 \times 10^9$ Pa

From A to B: $I = 28.9 \times 10^6 \text{ mm}^4 = 28.9 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6 \text{ N}\cdot\text{m}^2 = 5780 \text{ kN}\cdot\text{m}^2$

From B to C: $I = I_w + 2A_p d^2 + 2\bar{I}_p$

$A_p = 5 \times 120 = 600 \text{ mm}^2$

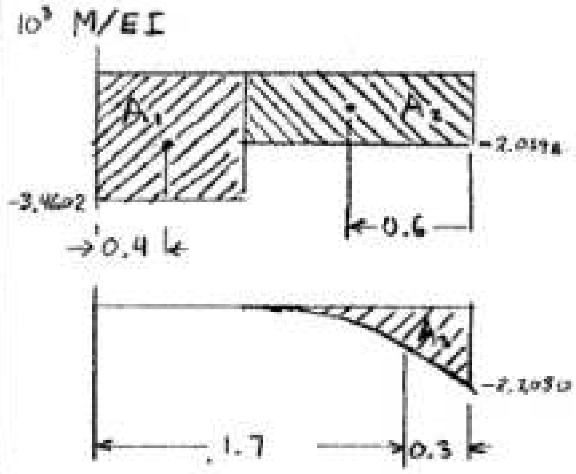
$d = \frac{254}{2} + \frac{5}{2} = 129.5 \text{ mm}$

$Ad^2 = 10.062 \times 10^6 \text{ mm}^4$

$\bar{I}_p = \frac{1}{12}(120)(5)^3 = 0.00125 \times 10^6 \text{ mm}^4$

$I = [28.9 + (2)(10.062) + (2)(0.00125)] \times 10^6 \text{ mm}^4 = 49.03 \times 10^6 \text{ mm}^4 = 49.03 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(49.03 \times 10^{-6}) = 9.805 \times 10^6 \text{ N}\cdot\text{m}^2 = 9805 \text{ kN}\cdot\text{m}^2$



Draw M/EI diagram by parts.

A to B: $\frac{M_1}{EI} = -\frac{20}{5780} = -3.4602 \times 10^{-3} \text{ m}^{-1}$

B to C: $\frac{M_2}{EI} = -\frac{20}{9805} = -2.0398 \times 10^{-3} \text{ m}^{-1}$

$\frac{M_3}{EI} = -\frac{(30)(1.2)^2}{(2)(9805)} = -2.2030 \times 10^{-3} \text{ m}^{-1}$

$A_1 = (-3.4602 \times 10^{-3})(0.8) = -2.7682 \times 10^{-3}$

$A_2 = (-2.0398 \times 10^{-3})(1.2) = -2.4478 \times 10^{-3}$

$A_3 = \frac{1}{3}(-2.2030 \times 10^{-3})(1.2) = -0.8812 \times 10^{-3}$

Place reference tangent at C. $\theta_C = 0$

(a) Slope at A. $\theta_A = \theta_C - \theta_{A/C} = 0 - (A_1 + A_2 + A_3) = 6.10 \times 10^{-3}$ rad ∡ ◄

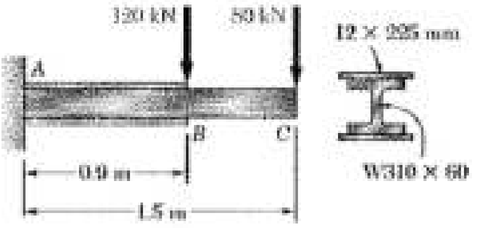
(b) Deflection at A.

$y_A = t_{A/C} = (-2.7682 \times 10^{-3})(0.4) + (-2.4478 \times 10^{-3})(1.4) + (-0.8812 \times 10^{-3})(1.7)$

$= -6.03 \times 10^{-3} \text{ m} = 6.03 \text{ mm} \downarrow$ ◄

Problem 9.108

9.108 Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200$ GPa, determine the slope and deflection at end C.



Use units of kN and m

Over portion BC $\quad I = 129 \times 10^{-6}\ m^4$

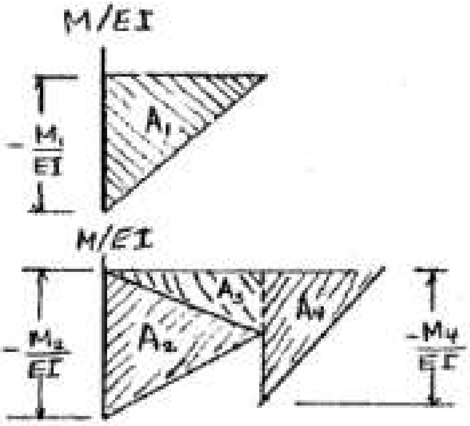
$$EI = (200 \times 10^6)(129 \times 10^{-6}) = 25800\ kN\ m^2$$

Portion AB

	A(mm²)	d(mm)	Ad²(mm⁴)	$\bar{I}$(mm⁴)
Top plate	2700	145.5	57.16×10⁶	32400
W310 x 60		0	0	129×10⁶
Bot. plate	2700	145.5	57.16×10⁶	32400
Σ			114.32×10⁶	129.065×10⁶

$$I = (10^6)(114.32 + 129.065) = 243.385 \times 10^6\ mm^4$$

$$EI = 48677\ kN\ m^2$$



Draw $\frac{M}{EI}$ diagram.

$$\frac{M_1}{EI} = -\frac{(120)(0.9)}{48677} = -2.219 \times 10^{-3}\ m^{-1}$$

$$\frac{M_2}{EI} = -\frac{(80)(1.5)}{48677} = -2.465 \times 10^{-3}\ m^{-1}$$

$$\frac{M_4}{EI} = -\frac{(80)(0.6)}{25800} = -1.86 \times 10^{-3}\ m^{-1}$$

$$A_1 = (\tfrac{1}{2})(0.9)(-0.002219) = -0.9986 \times 10^{-3}$$

$$A_2 = (\tfrac{1}{2})(0.9)(0.002465) = -1.10925 \times 10^{-3}$$

$$A_3 = A_2(\tfrac{0.6}{1.5}) = -0.4437 \times 10^{-3}$$

$$A_4 = (\tfrac{1}{2})(0.6)(-0.00186) = -0.558 \times 10^{-3}$$

Place reference tangent at A.

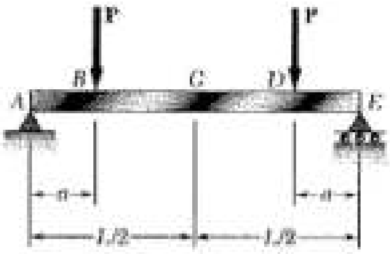
Slope at C. $\quad \theta_C = \theta_{C/A} = A_1 + A_2 + A_3 + A_4 = -3.11 \times 10^{-3}$ ◄

Deflection at C. $\quad y_C = t_{C/A}$

$$y_C = (1.2)(A_1) + (1.2)(A_2) + (0.9)(A_3) + (0.4)(A_4) = -3.152 \times 10^{-3}\ m = 3.15\ mm \downarrow$$ ◄

Problem 9.109

9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



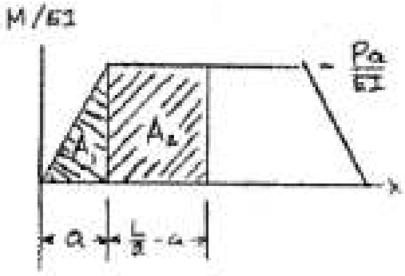
Symmetric beam with symmetric loading.

$\theta_C = 0$ Place reference tangent at C.

Draw $\frac{M}{EI}$ diagram.

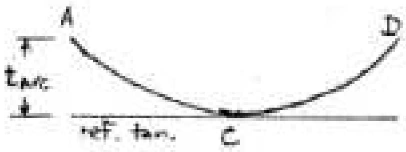
$$A_1 = \frac{1}{2} a \left(\frac{Pa}{EI}\right) = \frac{Pa^2}{2EI}$$

$$A_2 = \left(\frac{L}{2} - a\right)\left(\frac{Pa}{EI}\right) = \frac{Pa(L-2a)}{2EI}$$



<u>Slope at end A.</u> $\theta_A = -\theta_{C/A}$

$$\theta_A = A_1 + A_2 = -\frac{Pa(L-a)}{2EI}$$ ◄



<u>Deflection at C.</u> $y_C = -t_{A/C}$

$$t_{A/C} = \left(\frac{2}{3}a\right)A_1 + \left[\frac{1}{2}\left(\frac{L}{2}+a\right)\right]A_2$$

$$= \frac{2}{3}\frac{Pa^3}{2EI} + \left(\frac{L+2a}{4}\right)\frac{Pa(L-2a)}{2EI}$$

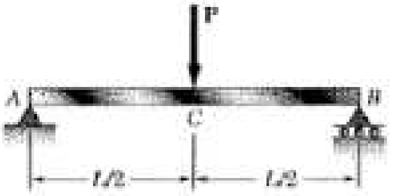
$$= \frac{Pa}{EI}\left\{\frac{1}{3}a^2 + \frac{1}{8}(L^2 - 4a^2)\right\}$$

$$= \frac{Pa}{24EI}(3L^2 - 4a^2)$$

$$y_C = -\frac{Pa}{24EI}(3L^2 - 4a^2)$$ ◄

Problem 9.110

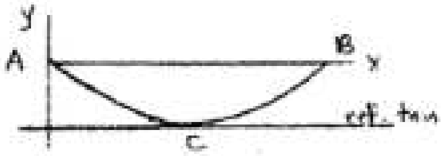
9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



Symmetric beam and loading.

Place reference tangent at C.

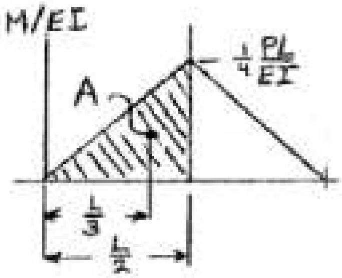
$\theta_C = 0 \quad , \quad y_C = -t_{A/C}$



Reactions $R_A = R_B = \frac{1}{2}P$

Bending moment at C. $M_C = \frac{1}{4}PL$

$A = \frac{1}{2}\left(\frac{1}{4}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{16}\frac{PL^2}{EI}$



(a) <u>Slope at A.</u> $\theta_A = \theta_C - \theta_{C/A}$

$\theta_A = 0 - \frac{1}{16}\frac{PL^2}{EI} = -\frac{1}{16}\frac{PL^2}{EI}$ ◄

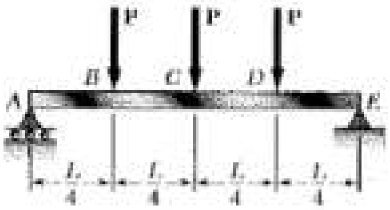
(b) <u>Deflection at C.</u>

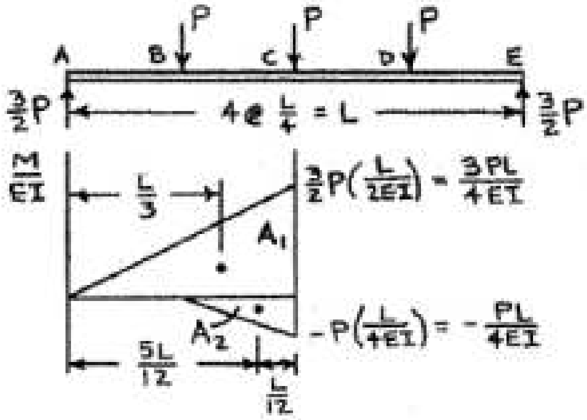
$y_C = -t_{A/C} = -A\left(\frac{L}{3}\right) = -\left(\frac{1}{16}\frac{PL^2}{EI}\right)\left(\frac{L}{3}\right)$

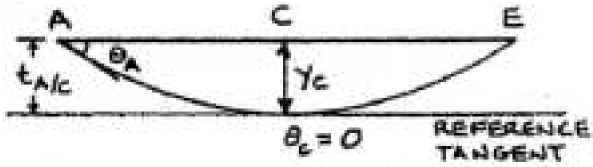
$y_C = \frac{1}{48}\frac{PL^3}{EI}$ ◄

Problem 9.111

9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.







$$A_1 = \frac{1}{2}\left(\frac{3PL}{4EI}\right)\left(\frac{L}{2}\right) = \frac{3PL^2}{16EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{PL}{4EI}\right)\left(\frac{L}{4}\right) = -\frac{PL^2}{32EI}$$

(a) <u>SLOPE AT A</u>: $\theta_C = \theta_A + \theta_{C/A}$; $\theta_A = 0 - \theta_{C/A}$

$$\theta_A = -\theta_{C/A} = -(A_1 + A_2) = -\left[\frac{3PL^2}{16EI} - \frac{PL^2}{32EI}\right] = -\frac{5PL^2}{32EI}$$

$$\theta_A = \frac{5PL^2}{32EI} \;◄$$

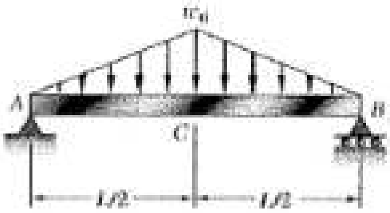
(b) <u>DEFLECTION AT C</u>:

$$t_{A/C} = A_1\left(\frac{L}{3}\right) + A_2\left(\frac{5L}{12}\right) = \left(\frac{3PL^2}{16EI}\right)\left(\frac{L}{3}\right) + \left(-\frac{PL^2}{32EI}\right)\left(\frac{5L}{12}\right) = \frac{19PL^3}{384EI}$$

$$y_C = -t_{A/C} = -\frac{19PL^3}{384EI} \qquad y_C = \frac{19PL^3}{384EI} \downarrow \;◄$$

Problem 9.113

9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

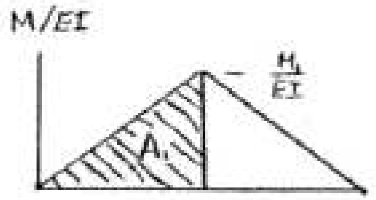


Symmetric beam and loading.

$\theta_C = 0$ Place reference tangent at C.

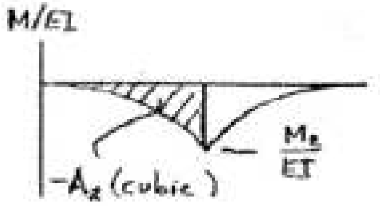
Reactions. $R_A = R_B = \frac{w_0 L}{4}$

Draw $\frac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = \frac{R_A L}{2} = \frac{w_0 L^2}{8EI}$$

$$A_1 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{M_1}{EI}\right) = \frac{w_0 L^3}{32EI}$$



$$\frac{M_2}{EI} = \frac{1}{EI}\cdot\frac{1}{2}\left(\frac{w_0 L}{2}\right)\left(\frac{L}{3}\cdot\frac{L}{2}\right) = -\frac{w_0 L^2}{24EI}$$

$$A_2 = \frac{1}{4}\left(\frac{L}{2}\right)\left(-\frac{w_0 L^2}{24EI}\right) = -\frac{w_0 L^3}{192EI}$$

Slope at A. $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 = \left(-\frac{1}{32} + \frac{1}{192}\right)\frac{w_0 L^3}{EI}$$

$$\theta_A = \frac{5 w_0 L^3}{192EI}$$

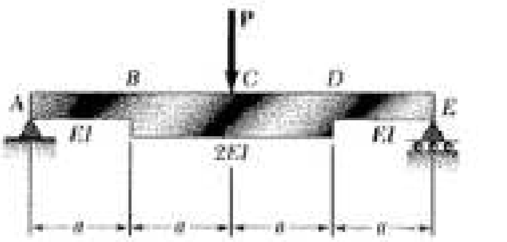
Deflection at C. $y_C = t_{A/C}$

$$t_{A/C} = A_1\left[\left(\frac{2}{3}\right)\left(\frac{L}{2}\right)\right] + A_2\left[\left(\frac{4}{5}\right)\left(\frac{L}{2}\right)\right] = \left[\left(\frac{1}{3}\right)\left(\frac{1}{32}\right) - \left(\frac{2}{5}\right)\left(\frac{1}{192}\right)\right]\frac{w_0 L^4}{EI} = \frac{w_0 L^4}{120EI}$$

$$y_C = \frac{w_0 L^4}{120EI} \downarrow$$

Problem 9.115

9.115 and 9.116 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



Symmetric beam and loading. $R_A = R_E = \frac{1}{2}P$

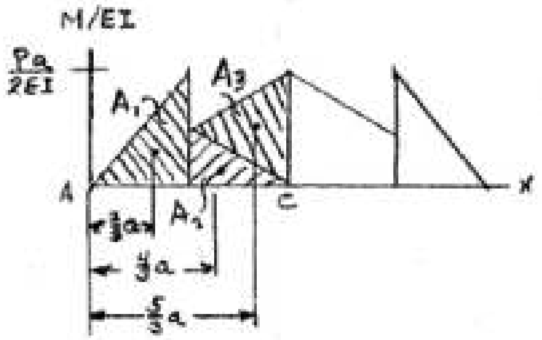
$$M_{max} = (\tfrac{1}{2}P)(2a) = Pa$$

Draw M and $\frac{M}{EI}$ diagrams.

$$A_1 = \tfrac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

$$A_2 = \tfrac{1}{2}\left(\frac{Pa}{4EI}\right)a = \frac{1}{8}\frac{Pa^2}{EI}$$

$$A_3 = \tfrac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$



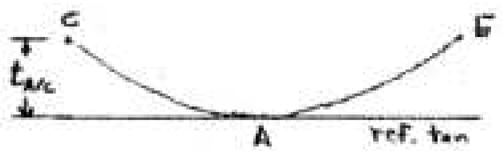
Place reference tangent at C. $\theta_C = 0$

(a) Slope at A.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{5}{8}\frac{Pa^2}{EI} \qquad \theta_A = \frac{5Pa^2}{8EI} \measuredangle$$

(b) Deflection at C.



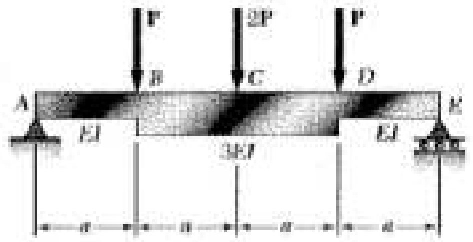
$$|y_C| = t_{A/C} = A_1(\tfrac{2}{3}a) + A_2(\tfrac{4}{3}a) + A_3(\tfrac{5}{3}a)$$

$$\frac{1}{6}\frac{Pa^3}{EI} + \frac{1}{6}\frac{Pa^3}{EI} + \frac{5}{12}\frac{Pa^3}{EI} = \frac{3}{4}\frac{Pa^3}{EI}$$

$$y_C = \frac{3Pa^3}{4EI} \downarrow$$

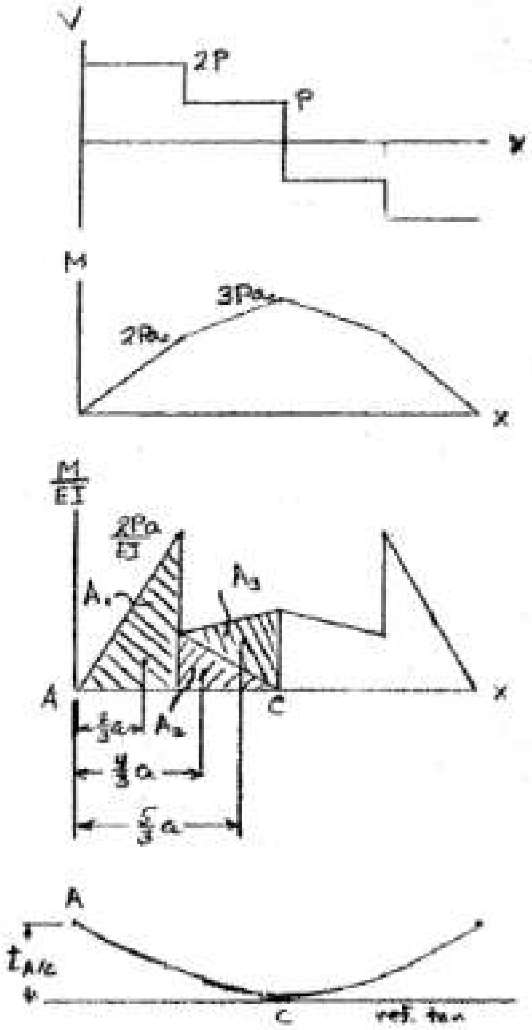
Problem 9.116

9.115 and 9.116 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



Symmetric beam and loading. $R_A = R_E = 2P$.

Draw V, M, and $\frac{M}{EI}$ diagrams.



$$A_1 = \frac{1}{2}\left(\frac{2Pa}{EI}\right)a = \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{2}{3}\frac{Pa}{EI}\right)a = \frac{1}{3}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{EI}\right)a = \frac{1}{2}\frac{Pa^2}{EI}$$

Place reference tangent at C. $\theta_C = 0$

(a) <u>Slope at A.</u>

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{11}{6}\frac{Pa^2}{EI} \qquad \theta_A = \frac{11 Pa^2}{6EI} ↘ \blacktriangleleft$$

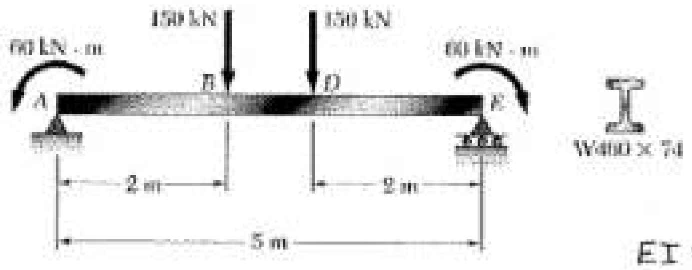
(b) <u>Deflection at C.</u>

$$|y_C| = t_{A/C} = A_1\left(\tfrac{2}{3}a\right) + A_2\left(\tfrac{4}{3}a\right) + A_3\left(\tfrac{5}{3}a\right)$$

$$= \frac{35}{18}\frac{Pa^3}{EI} \qquad y_C = \frac{35 Pa^3}{18 EI} \downarrow \blacktriangleleft$$

Problem 9.119

9.118 and 9.119 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.

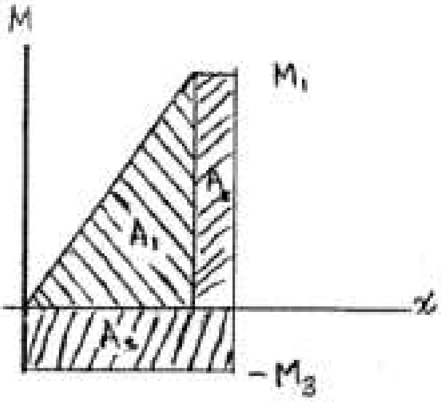


Use units of kN and m.

For W460 × 74

$I = 333 \times 10^6 \text{ mm}^4 = 333 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(333 \times 10^{-6})$
$= 66.6 \times 10^6 \text{ N}\cdot\text{m}^2 = 66600 \text{ kN}\cdot\text{m}^2$



Symmetric beam and loading. Place reference tangent at midpoint C where $\theta_C = 0$.

Reactions. $R_A = R_E = 150$ kN ↑

Draw bending moment diagram of left half of beam by parts

$M_1 = (2)(150) = 300$ kN·m

$A_1 = (\frac{1}{2})(2)(300) = 300$ kN·m²

$A_2 = (0.5)(300) = 150$ kN·m²

$M_3 = -60$ kN·m

$A_3 = (2.5)(-60) = -150$ kN·m²

(a) Slope at end A. $\theta_A = -\theta_{C/A}$

$$\theta_A = \frac{1}{EI}\left\{-A_1 - A_2 - A_3\right\} = \frac{-300 - 150 + 150}{66600}$$

$= -4.50 \times 10^{-3}$ rad $\qquad \theta_A = 4.50 \times 10^{-3}$ rad ∢ ◄

(b) Deflection at midpoint C. $\qquad y_C = -t_{A/C}$

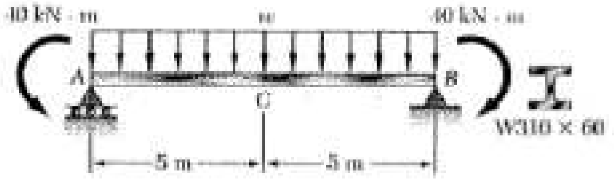
$$t_{A/C} = \frac{1}{EI}\left\{\left(\frac{2}{3}\cdot 2\right)A_1 + \left(2 + \frac{0.5}{2}\right)A_2 + \left(\frac{2.5}{2}\right)A_3\right\}$$

$$= \frac{400 + 337.5 - 187.5}{66600} = 8.26 \times 10^{-3} \text{ m}$$

$y_C = -8.26 \times 10^{-3}$ m $\qquad y_C = 8.26$ mm ↓ ◄

Problem 9.121

9.121 For the beam and loading shown and knowing that $w = 8$ kN/m, determine (*a*) the slope at end *A*, (*b*) the deflection at midpoint *C*. Use $E = 200$ GPa.

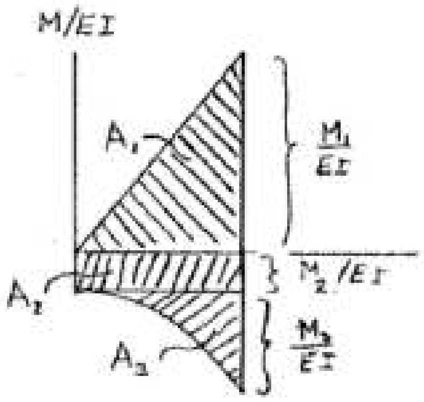


$E = 200 \times 10^9$ Pa

$I = 129 \times 10^6\ mm^4 = 129 \times 10^{-6}\ m^4$

$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6\ N \cdot m^2$

$= 25800\ kN \cdot m^2$



Symmetrical beam and loading.

$R_A = R_B = \frac{1}{2}(8)(10) = 40$ kN

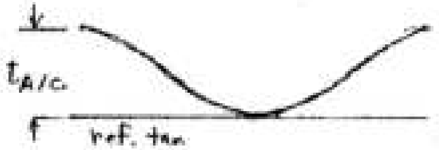
Bending moment

$M = 40x - 40 - \frac{1}{2}(8)x^2$

At $x = 5$

$M = 200 - 40 - 100$

Draw $\frac{M}{EI}$ diagram by parts.



$\frac{M_1}{EI} = \frac{200}{25800} = 7.7519 \times 10^{-3}\ m^{-1}$

$\frac{M_2}{EI} = \frac{-40}{25800} = -1.5504 \times 10^{-3}\ m^{-1}$

$\frac{M_3}{EI} = \frac{-100}{25800} = -3.8760 \times 10^{-3}\ m^{-1}$

$A_1 = \frac{1}{2}(7.7519 \times 10^{-3})(5) = 19.380 \times 10^{-3}$ $\qquad \bar{x}_1 = (\frac{2}{3})(5) = 3.3333$ m

$A_2 = -(1.5504)(5) = -7.7520 \times 10^{-3}$ $\qquad \bar{x}_2 = (\frac{1}{2})(5) = 2.5$ m

$A_3 = -\frac{1}{3}(3.8760)(5) = -6.4600 \times 10^{-3}$ $\qquad \bar{x}_3 = (\frac{3}{4})(5) = 3.75$ m

Place reference tangent at C. $\theta_C = 0$

(a) <u>Slope at A.</u> $\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$\theta_A = -(19.380 \times 10^{-3} - 7.7520 \times 10^{-3} - 6.4600 \times 10^{-3}) = -5.17 \times 10^{-3}$

$\theta_A = 5.17 \times 10^{-3}$ rad ⤹ ◄

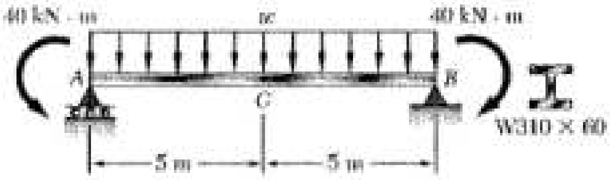
(b) <u>Deflection at C.</u> $|y_C| = t_{A/C}$

$= (19.380 \times 10^{-3})(3.3333) - (7.7520 \times 10^{-3})(2.5) - (6.4600 \times 10^{-3})(3.75)$

$= 21.0 \times 10^{-3}$ m $\qquad y_C = 21.0$ mm ↓ ◄

Problem 9.122

9.122 For the beam and loading shown, determine the value of w for which the deflection is zero at the midpoint C of the beam. Use $E = 200$ GPa.



Symmetric beam and loading.

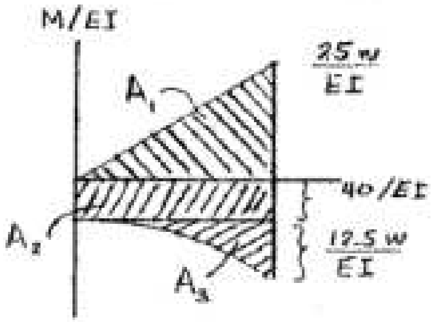
$R_A = R_B = 5w$ (w in kN/m)

Bending moment in kN·m.

$M = 5wx - 40 - \frac{1}{2}wx^2$

At $x = 5$ m

$M = 25w - 40 - 12.5w$

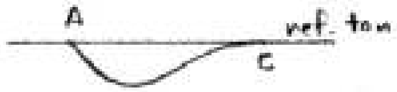


Draw $\frac{M}{EI}$ diagram by parts.

$A_1 = \frac{1}{2}\left(\frac{25w}{EI}\right)(5) = \frac{62.5w}{EI}$

$A_2 = -\frac{(40)(5)}{EI} = -\frac{200}{EI}$

$A_3 = -\frac{1}{3}\left(\frac{12.5w}{EI}\right)(5) = -\frac{20.833w}{EI}$



$\bar{x}_1 = \frac{2}{3}(5) = 3.3333$ m

$\bar{x}_2 = \frac{1}{2}(5) = 2.5$ m

$\bar{x}_3 = \frac{3}{4}(5) = 3.75$ m

Place reference tangent at C.

Deflection at C is zero. $t_{A/C} = y_A - y_C = 0$

$$A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 = 0$$

$$\left(\frac{62.5w}{EI}\right)(3.3333) - \left(\frac{200}{EI}\right)(2.5) - \left(\frac{20.833w}{EI}\right)(3.75) = 0$$

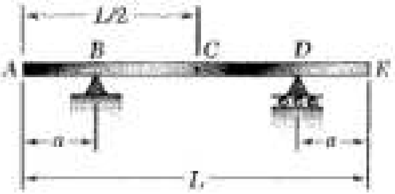
$$\frac{130.21w}{EI} - \frac{500}{EI} = 0$$

$$w = \frac{500}{130.21} = 3.84 \text{ kN/m}$$

$w = 3.84$ kN/m ◀

Problem 9.124

*9.124 A uniform rod AE is to be supported at two points B and D. Determine the distance a from the ends of the rod to the points of support, if the downward deflections of points A, C, and E are to be equal.



Let w = weight per unit length of rod.

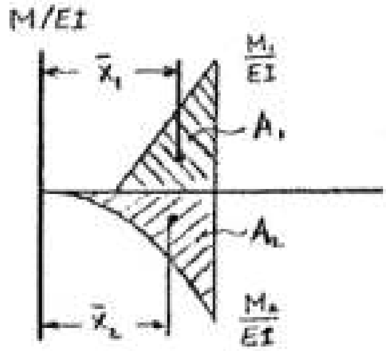
Symmetric beam and loading.

$$R_B = R_D = \frac{1}{2}wL$$

Bending moment:

Over AB $\quad M = -\frac{1}{2}wx^2$

Over BCD $\quad M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$



Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2}\frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

$$\bar{x}_1 = a + \frac{2}{3}\left(\frac{L}{2}-a\right) = \frac{1}{3}(L+a)$$

$$\bar{x}_2 = \frac{L}{2} - \frac{1}{4}\left(\frac{L}{2}\right) = \frac{3}{8}L$$

Place reference tangent at C.

$$y_c - y_c = t_{A/C} = 0$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 = 0$$

$$\frac{1}{16}\frac{wL(L-2a)^2}{EI}\frac{1}{3}(L+a) - \frac{1}{48}\frac{wL^3}{EI}\frac{3}{8}L = 0$$

Let $u = a/L$. Divide by $\frac{wL^4}{48EI}$.

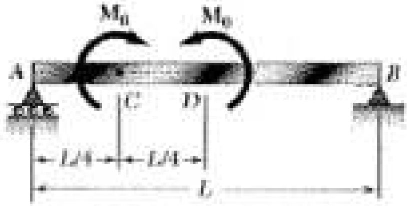
$$(1-2u)^2(1+u) - \frac{3}{8} = 0$$

$$4u^3 - 3u + \frac{5}{8} = 0$$

Solving for u, $\quad u = 0.22315 \qquad \frac{a}{L} = 0.223 \qquad a = 0.223\,L$ ◄

Problem 9.125

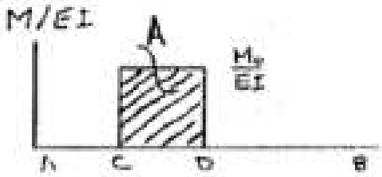
9.125 through 9.128 For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.



From Statics $R_A = R_B = 0$.

Draw $\frac{M}{EI}$ diagram.

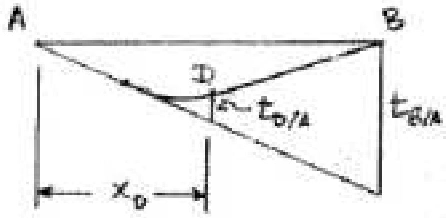
$$A = \left(\frac{M_0}{EI}\right)\left(\frac{L}{4}\right) = \frac{1}{4}\frac{M_0 L}{EI}$$



Place reference tangent at A.

$$t_{B/A} = A\left(\frac{L}{2} + \frac{L}{8}\right) = \frac{5}{32}\frac{M_0 L^2}{EI}$$

$$t_{D/A} = A\left(\frac{L}{8}\right) = \frac{1}{32}\frac{M_0 L^2}{EI}$$



(a) <u>Deflection at D.</u>

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = t_{D/A} - \frac{1}{2} t_{B/A}$$

$$= \frac{1}{32}\frac{M_0 L^2}{EI} - \frac{5}{64}\frac{M_0 L^2}{EI} = -\frac{3}{64}\frac{M_0 L^2}{EI}$$

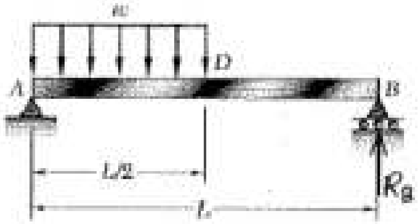
$$y_D = \frac{3 M_0 L^2}{64 EI} \downarrow$$

(b) <u>Slope at A.</u>

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{5}{32}\frac{M_0 L}{EI} \qquad \theta_A = \frac{5 M_0 L}{32 EI}$$

Problem 9.127

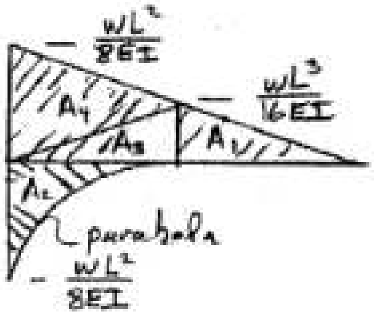
9.125 through *9.128* For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.



$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - \frac{wL}{2}\left(\frac{L}{4}\right) = 0$$

$$R_B = \frac{1}{8} w \uparrow$$

Draw $\frac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = \frac{R_B L}{EI} = \frac{wL^2}{8EI}$$

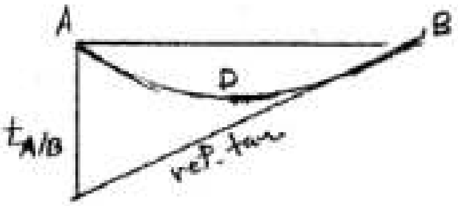
$$\frac{M_2}{EI} = -\frac{wL^3}{8EI}$$

$$A_1 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_2 = \left(\frac{1}{3}\right)\left(\frac{L}{2}\right)\frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

$$A_3 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_4 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{8EI} = \frac{wL^3}{32EI}$$



(a) Deflection at D.

Place reference tangent at B.

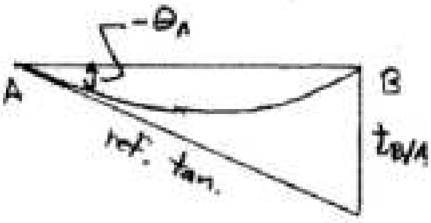
$$y_D = t_{D/B} - \frac{1}{2} t_{A/B}$$

$$t_{D/A} = \left(\frac{1}{3}\cdot\frac{L}{2}\right)A_1 = \frac{wL^4}{384EI}$$

$$t_{B/A} = \frac{L}{3}(A_1 + A_3 + A_4) + \left(\frac{L}{4}\cdot\frac{L}{2}\right)A_2 = \frac{7wL^4}{384EI}$$

$$y_D = \frac{wL^4}{384EI} - \frac{1}{2}\cdot\frac{7wL^4}{384EI} = -\frac{5wL^4}{768EI}$$

$$y_D = \frac{5wL^4}{768EI} \downarrow \blacktriangleleft$$



(b) Slope at A. Place reference tangent at A.

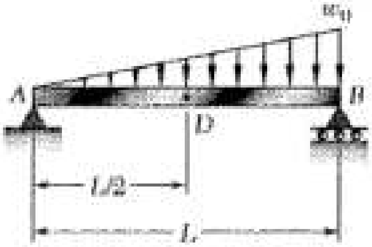
$$\theta_A = -\frac{1}{L} t_{B/A} = -\left(\frac{1}{L}\right)\left\{\left(\frac{2L}{3}\right)(A_1 + A_3 + A_4) + \left(L - \frac{1}{4}\cdot\frac{L}{2}\right)A_2\right\}$$

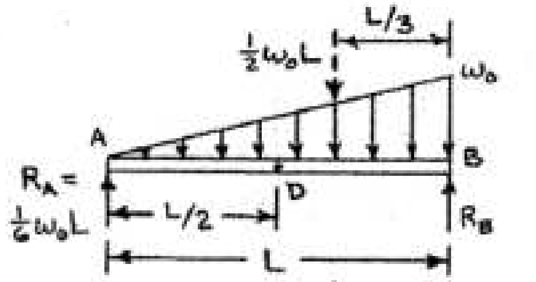
$$= -\frac{3wL^3}{128EI}$$

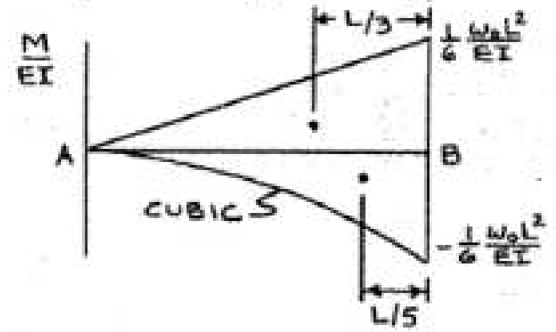
$$\theta_A = \frac{3wL^3}{128EI} \circlearrowright \blacktriangleleft$$

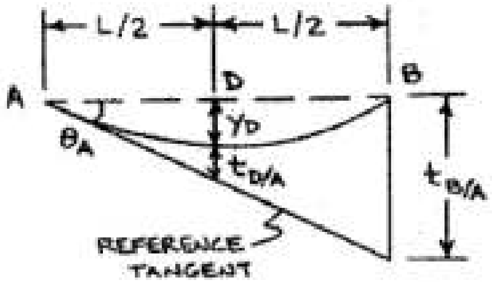
Problem 9.128

9.125 through 9.128 For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

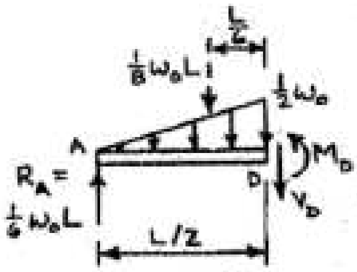


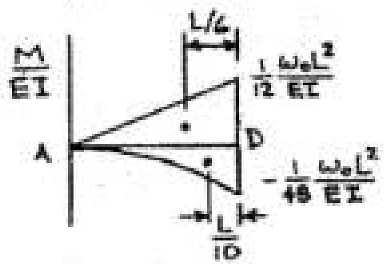






$$t_{B/A} = \frac{1}{2}\left(\frac{1}{6}\frac{w_0L^2}{EI}\right)(L)\left(\frac{L}{3}\right) + \frac{1}{4}\left(-\frac{1}{6}\frac{w_0L^2}{EI}\right)(L)\left(\frac{L}{5}\right)$$
$$= \frac{7w_0L^4}{360EI}$$





$$t_{D/A} = \frac{1}{2}\left(\frac{1}{12}\frac{w_0L^2}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) + \frac{1}{4}\left(-\frac{1}{48}\frac{w_0L^2}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{10}\right)$$
$$= \frac{37w_0L^4}{11520EI}$$

(a) DEFLECTION AT D:

$$y_D = \frac{1}{2}t_{B/A} - t_{D/A} = \frac{1}{2}\left(\frac{7w_0L^4}{360EI}\right) - \frac{37w_0L^4}{11520EI} = \frac{75w_0L^4}{11520EI}$$

$$y_D = \frac{5w_0L^4}{768EI} \downarrow$$

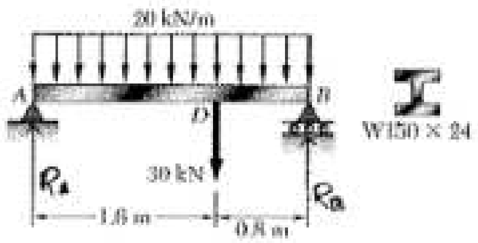
(b) SLOPE AT A:

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{7w_0L^3}{360EI}$$

$$\theta_A = \frac{7w_0L^3}{360EI}$$

Problem 9.131

9.131 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.



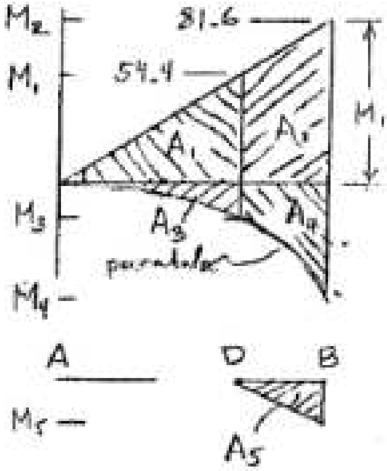
Units: Forces in kN. Lengths in meters.

For W150×24 $\quad I = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(13.4 \times 10^{-6}) = 2.68 \times 10^6 \text{ N}\cdot\text{m}^2 = 2680 \text{ kN}\cdot\text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -2.4 R_A + (0.8)(30) + (1.2)(2.4)(20) = 0$$

$$R_A = 34 \text{ kN} \uparrow$$



Draw bending moment diagram by parts.

$$M_1 = (1.6)(34) = 54.4 \text{ kN}\cdot\text{m}$$
$$M_2 = (2.4)(34) = 81.6 \text{ kN}\cdot\text{m}$$
$$M_3 = -\tfrac{1}{2}(20)(1.6)^2 = -25.6 \text{ kN}\cdot\text{m}$$
$$M_4 = -\tfrac{1}{2}(20)(2.4)^2 = -57.6 \text{ kN}\cdot\text{m}$$
$$M_5 = -(0.8)(30) = -24 \text{ kN}\cdot\text{m}$$

$$A_1 = \tfrac{1}{2}(1.6)(54.4) = 43.52 \text{ kN}\cdot\text{m}^2$$
$$A_1 + A_2 = \tfrac{1}{2}(2.4)(81.6) = 97.92 \text{ kN}\cdot\text{m}^2$$
$$A_3 = \tfrac{1}{3}(1.6)(-25.6) = -13.6533 \text{ kN}\cdot\text{m}^2$$
$$A_3 + A_4 = \tfrac{1}{3}(2.4)(-57.6) = -46.08 \text{ kN}\cdot\text{m}^2$$
$$A_5 = \tfrac{1}{2}(0.8)(-24) = -9.6 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A. $\quad$ Place reference tangent at A. $\quad \theta_A = -\frac{1}{L} t_{B/A}$

$$t_{B/A} = \frac{1}{EI}\left\{(A_1 + A_2)(\tfrac{1}{3})(2.4) + (A_3 + A_4)(\tfrac{1}{4})(2.4) + A_5(\tfrac{1}{3})(0.8)\right\}$$
$$= \frac{48.128}{2680} = 17.9582 \times 10^{-3} \text{ m}$$

$$\theta_A = -\frac{17.9582 \times 10^{-3}}{2.4} = -7.48258 \times 10^{-3} \qquad \theta_A = 7.48 \times 10^{-3} \text{ rad} \,\measuredangle$$

(b) Deflection at point D. $\quad y_D = t_{D/A} + \theta_A x_D$

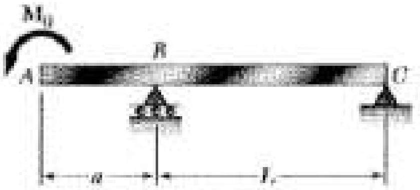
$$t_{D/A} = \frac{1}{EI}\left\{A_1(\tfrac{1}{3})(1.6) + A_3(\tfrac{1}{4})(1.6)\right\} = \frac{17.7493}{2680} = 6.62289 \times 10^{-3} \text{ m}$$

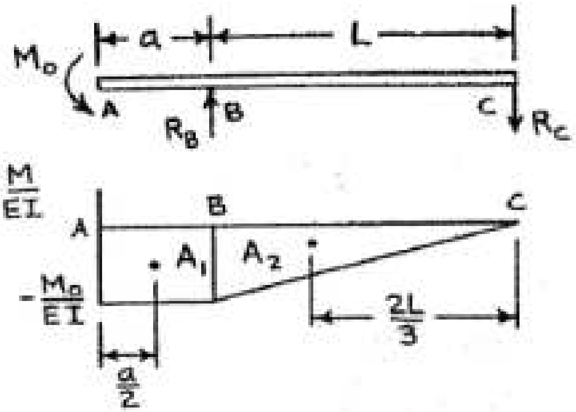
$$y_D = 6.62289 \times 10^{-3} + (-7.48258 \times 10^{-3})(1.6) = -5.3492 \times 10^{-3} \text{ m}$$

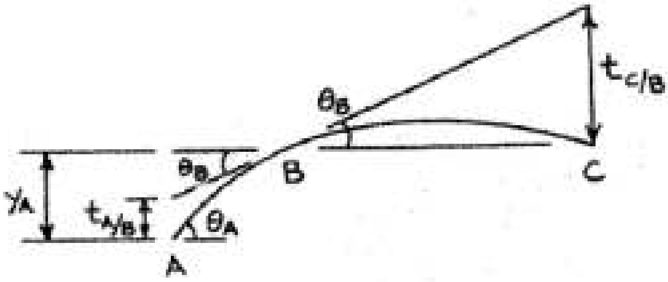
$$y_D = 5.35 \text{ mm} \downarrow$$

Problem 9.132

9.132 For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*.







$$A_1 = -\frac{M_0 a}{EI} \qquad A_2 = -\frac{M_0 L}{2EI}$$

$$t_{C/B} = A_2\left(\frac{2L}{3}\right) = \left(-\frac{M_0 L}{2EI}\right)\left(\frac{2L}{3}\right) = -\frac{M_0 L^2}{3EI}$$

(a) <u>SLOPE AT A</u>:

$$\theta_B = \frac{t_{C/B}}{L} = \frac{M_0 L}{3EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1$$

$$\frac{M_0 L}{3EI} = \theta_A - \frac{M_0 a}{EI} \qquad \theta_A = \frac{M_0}{3EI}(L + 3a) \measuredangle$$

(b) <u>DEFLECTION AT A</u>:

$$t_{A/B} = A_1\left(\frac{a}{2}\right) = -\frac{M_0 a^2}{2EI}$$

$$y_A = \frac{a}{L} t_{C/B} + t_{A/B}$$

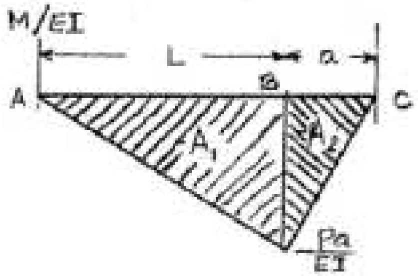
$$= \frac{a}{L}\left(-\frac{M_0 L^2}{3EI}\right) - \frac{M_0 a^2}{2EI} \qquad y_A = \frac{M_0 a}{6EI}(2L + 3a) \downarrow$$

Problem 9.133

9.133 For the beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *C*.

$$A_1 = -\frac{PaL}{2EI}$$

$$A_2 = -\frac{Pa^2}{2EI}$$



$$t_{A/B} = A_1\left(\tfrac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

$$\theta_B = \frac{t_{A/B}}{L} = -\frac{PaL}{3EI}$$

(a) Slope at C. $\quad \theta_C = \theta_B + \theta_{C/B}$

$$\theta_C = -\frac{PaL}{3EI} - \frac{Pa^2}{2EI}$$

$$= -\frac{Pa(2L+3a)}{6EI}$$

$$\theta_C = \frac{Pa(2L+3a)}{6EI}$$

Deflection at point C.

$$y_C = a\theta_B + t_{C/B}$$

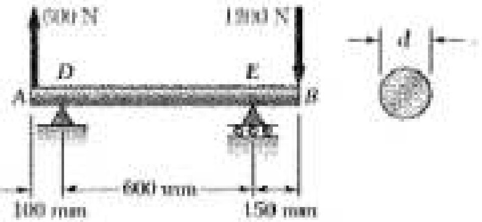
$$y_C = -\frac{Pa^2L}{3EI} + \left(-\frac{Pa^2}{2EI}\right)\left(\tfrac{2}{3}a\right)$$

$$= -\frac{Pa^2(L+a)}{3EI}$$

$$y_C = \frac{Pa^2(L+a)}{3EI} \downarrow$$

Problem 9.134

9.134 Knowing that the beam AB is made of a solid steel rod of diameter $d = 18$ mm, determine for the loading shown (*a*) the slope at point D, (*b*) the deflection at point A. Use $E = 200$ GPa.

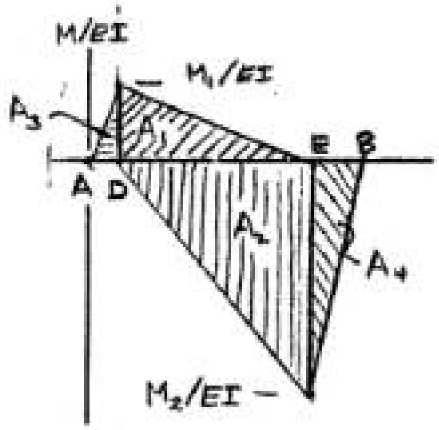


Units: Forces in kN; lengths in m.

$$c = \frac{d}{2} = \frac{1}{2}(18) = 9 \text{ mm.}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(9)^4 = 5153 \text{ mm}^4$$

$$EI = (200\times10^6)(5153\times10^{-12}) = 1.0306 \text{ kN m}^2.$$



Draw $\frac{M}{EI}$ diagram by parts by considering the bending moment diagram due to each of the applied loads.

$$\frac{M_1}{EI} = \frac{(0.6)(0.1)}{1.0306} = 0.0582 \text{ m}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(1.2)(0.15)}{1.0306} = -0.1747 \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(0.6)(0.0582) = 0.01746$$

$$A_2 = \frac{1}{2}(0.6)(-0.1747) = -0.05241$$

$$A_3 = \frac{1}{2}(0.1)(0.0582) = 0.00291$$

Place reference tangent at D.

(a) Slope at point D. $\quad y_E = y_D + L\theta_D + t_{E/D} \qquad \theta_D = -t_{E/D}/L$

$$t_{E/A} = 0.4A_1 + 0.2A_2 = -3.498\times10^{-3}$$

$$\theta_D = \frac{-0.003498}{0.6} = -5.83\times10^{-3} \qquad \theta_D = 5.83\times10^{-3} \text{ rad} \measuredangle$$

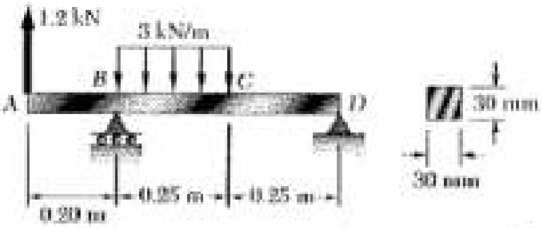
(b) Deflection at A. $\quad y_A = y_D - a\theta_D + t_{A/D} = t_{A/D} - a\theta_D$

$$y_A = A_3\left(\tfrac{2}{3}\right)(0.1) - (0.1)(0.00583) = -0.389\times10^{-3} \text{ m}$$

$$y_A = 0.39 \text{ mm} \downarrow$$

Problem 9.135

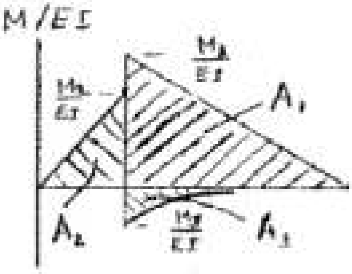
9.135 Knowing that the beam AD is made of a solid steel bar, determine the (*a*) slope at point B, (*b*) the deflection at point A. Use $E = 200$ GPa.



$E = 200 \times 10^9$ Pa

$I = \frac{1}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$
$= 67.5 \times 10^{-9} \text{ m}^4$

$EI = (200 \times 10^9)(67.5 \times 10^{-9}) = 13500 \text{ N} \cdot \text{m}^2$
$= 13.5 \text{ kN} \cdot \text{m}^2$



$\circlearrowright \Sigma M_B = 0: \quad -(0.2)(1.2) - (3)(0.25)(0.125) + .5R_D = 0$

$R_D = 0.6675 \text{ kN}$

Draw $\frac{M}{EI}$ diagram by parts.

$M_1 = (0.6675)(0.5) = 0.33375 \text{ kN} \cdot \text{m}$

$M_2 = (1.2)(0.2) = 0.240 \text{ kN} \cdot \text{m}$

$M_3 = -\frac{1}{2}(3)(0.25)^2 = -0.09375 \text{ kN} \cdot \text{m}$

$A_1 = \frac{1}{2}(0.33375)(0.5)/EI = 0.0834375/EI$

$A_2 = \frac{1}{2}(0.240)(0.2)/EI = 0.024/EI$

$A_3 = \frac{1}{3}(-0.09375)(0.25)/EI = -0.0078125/EI$

Place reference tangent at B.

$t_{D/B} = A_1\left(\frac{2}{3} \cdot 0.5\right) + A_3\left(\frac{3}{4}(0.25) + 0.25\right) = 0.024395/EI$

(a) <u>Slope at B.</u> $\quad \theta_B = -\frac{t_{D/B}}{L} = -\frac{0.024395}{0.5\,EI} = -\frac{0.048789}{EI}$

$= -3.6140 \times 10^{-3}$ $\qquad \theta_B = 3.61 \times 10^{-3} \text{ rad} \circlearrowright$ ◄

$t_{A/B} = A_2\left(\frac{2}{3}(0.20)\right) = 0.0032/EI = 0.23704 \times 10^{-3} \text{ m}$

(b) <u>Deflection at A.</u>

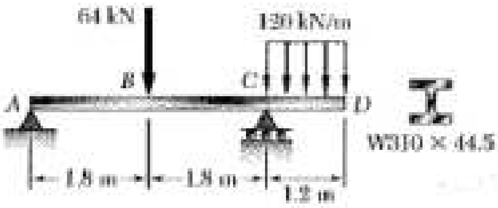
$y_A = t_{A/B} - L_{AB}\theta_B$

$= 0.23704 \times 10^{-3} - (0.2)(-3.6140 \times 10^{-3}) = 0.960 \times 10^{-3} \text{ m}$

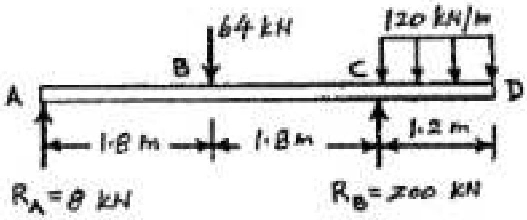
$y_A = 0.960 \text{ mm} \uparrow$ ◄

Problem 9.136

9.136 For the beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.



FREE BODY AD:

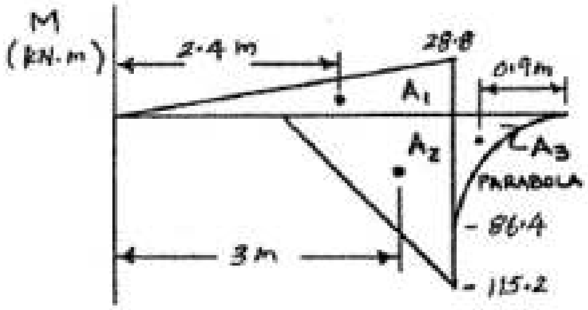


$$+\circlearrowleft \Sigma M_C = 0: \quad (64)(1.8) - (144)(0.6) - 3.6 R_A = 0$$

$$R_A = 8 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 8 - 64 + R_B - 144 = 0$$

$$R_B = 200 \text{ kN} \uparrow$$



FOR W310×44.5 : $I = 99.2 \times 10^{-6}\ m^4$

$$EI = (200 \times 10^6)(99.2 \times 10^{-6}) = 19840 \text{ kN·m}^2$$

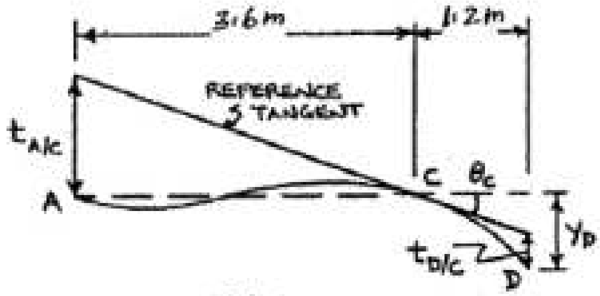
(a) SLOPE AT C:

$$A_1 = \tfrac{1}{2}(28.8)(3.6) = 51.84 \text{ kN m}^2$$

$$A_2 = \tfrac{1}{2}(-115.2)(1.8) = -103.68 \text{ kN m}^2$$

$$EI\, t_{A/C} = A_1(2.4\text{m}) + A_2(3\text{m})$$

$$= (51.84)(2.4) + (-103.68)(3) = -186.624 \text{ kNm}^3$$



$$t_{A/C} = -\frac{186.624}{19840} = -9.406 \times 10^{-3}\text{ m}$$

$$\theta_C = \frac{t_{A/C}}{L} = -\frac{9.406 \times 10^{-3}\text{ m}}{3.6 \text{ m}}$$

$$\theta_C = 2.61 \times 10^{-3} \text{ rad} \circlearrowright$$ ◄

(b) DEFLECTION AT D:

$$EI\, t_{D/C} = A_3(0.9\text{m}) = \tfrac{1}{3}(-86.4)(1.2)(0.9) = -31.1 \text{ kNm}$$

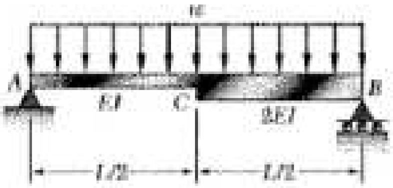
$$t_{D/C} = -\frac{31.1}{19840} = -1.5675 \times 10^{-3}\text{ m}$$

$$y_D = t_{D/C} + \frac{1.2}{3.6} t_{A/C} = -1.5675 \times 10^{-3} + \tfrac{1}{3}(-9.406 \times 10^{-3}) = -4.703 \times 10^{-3}\text{ m}$$

$$y_C = 4.7 \text{ mm} \downarrow$$ ◄

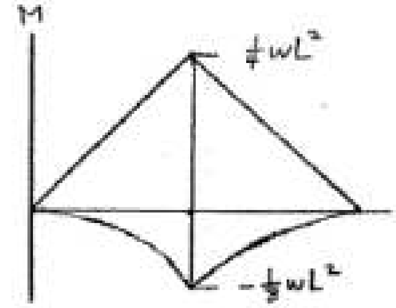
Problem 9.138

9.138 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the slope at end *B*, (*c*) the deflection at the midpoint *C*.



Reactions: $R_A = R_B = \frac{1}{2}wL$

Draw bending moment and M/EI diagrams by parts as shown.



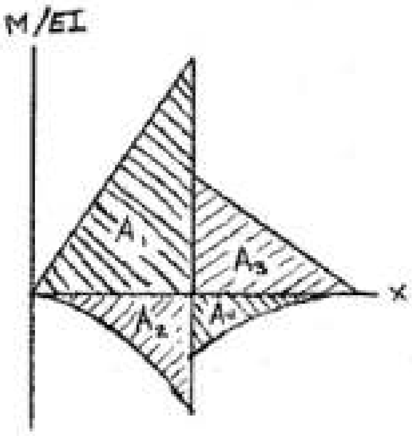
$$A_1 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{4EI} = \frac{wL^3}{16EI}$$

$$A_2 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

$$A_3 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = \frac{wL^3}{32EI}$$

$$A_4 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{16EI} = -\frac{wL^3}{96EI}$$

Place reference tangent at A.



(a) Slope at end A. $\qquad y_B = y_A + L\theta_A + t_{B/A}$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \left(\frac{L}{2}+\frac{L}{6}\right)A_1 + \left(\frac{L}{2}+\frac{L}{8}\right)A_2 + \frac{L}{3}A_3 + \frac{3L}{8}A_4$$

$$= \frac{wL^4}{EI}\left(\frac{1}{24} - \frac{5}{384} + \frac{1}{96} - \frac{1}{256}\right) = \frac{9wL^4}{256EI}$$

$$\theta_A = -\frac{9wL^4}{256EI}\cdot\frac{1}{L} = -\frac{9wL^3}{256EI} \qquad \theta_A = \frac{9wL^3}{256EI} ◄$$

(b) Slope at end B. $\qquad \theta_B = \theta_A + \theta_{B/A} = -\frac{9wL^3}{256EI} + A_1 + A_2 + A_3 + A_4$

$$\theta_B = \frac{7wL^3}{256EI} \qquad \theta_B = \frac{7wL^3}{256EI} ◄$$

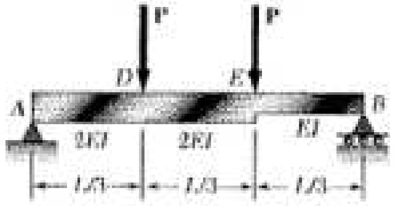
(c) Deflection at midpoint C. $\qquad y_C = y_C + \frac{L}{2}\theta_A + t_{C/A}$

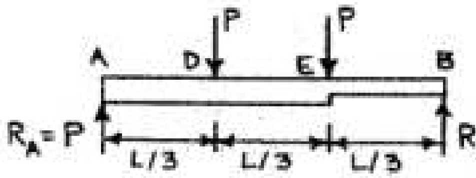
$$t_{C/A} = \left(\frac{L}{6}\right)A_1 + \left(\frac{L}{8}\right)A_2 = \frac{wL^4}{128EI}$$

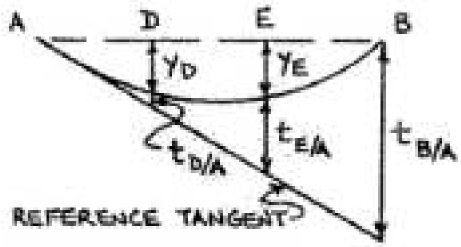
$$y_C = 0 + \left(\frac{L}{2}\right)\left(-\frac{9wL^3}{256EI}\right) + \frac{wL^4}{128EI} = -\frac{5wL^4}{512EI} \qquad y_C = \frac{5wL^4}{512EI}\downarrow ◄$$

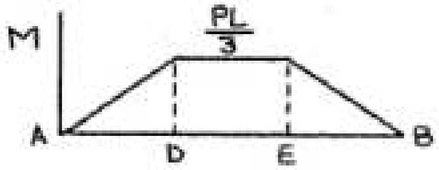
Problem 9.139

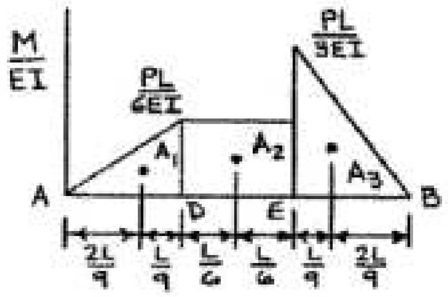
9.139 For the beam and loading shown, determine the deflection (*a*) at point *D*, (*b*) at point *E*.











$$A_1 = \frac{1}{2}\left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{36EI}$$

$$A_2 = \left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{18EI}$$

$$A_3 = \frac{1}{2}\left(\frac{PL}{3EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{18EI}$$

$$t_{D/A} = A_1\left(\frac{L}{9}\right) = \left(\frac{PL^2}{36EI}\right)\left(\frac{L}{9}\right) = \frac{PL^3}{324EI}$$

$$t_{E/A} = A_1\left(\frac{L}{9} + \frac{L}{3}\right) + A_2\left(\frac{L}{6}\right)$$

$$\left(\frac{PL^3}{36EI}\right)\left(\frac{4L}{9}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{L}{6}\right) = \frac{7PL^3}{324EI}$$

$$t_{B/A} = A_1\left(\frac{7L}{9}\right) + A_2\left(\frac{L}{2}\right) + A_3\left(\frac{2L}{9}\right)$$

$$= \left(\frac{PL^3}{36EI}\right)\left(\frac{7L}{9}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{L}{2}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{2L}{9}\right) = \frac{5PL^3}{81EI}$$

(a) <u>DEFLECTION AT D:</u>

$$y_D = \frac{1}{3}t_{B/A} - t_{D/A} = \frac{1}{3}\left(\frac{5PL^3}{81EI}\right) - \frac{PL^3}{324EI} = \frac{17PL^3}{972EI}$$

$$y_D = \frac{17PL^3}{972EI} \downarrow$$ ◄

(b) <u>DEFLECTION AT E:</u>

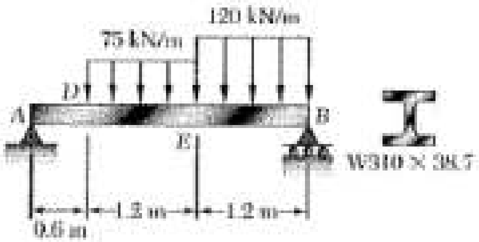
$$y_E = \frac{2}{3}t_{B/A} - t_{E/A} = \frac{2}{3}\left(\frac{5PL^3}{81EI}\right) - \frac{7PL^3}{324EI} = \frac{19PL^3}{972EI}$$

$$y_E = \frac{19PL^3}{972EI} \downarrow$$ ◄

Problem 9.143

9.140 through 9.143 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.143 Beam and loading of Prob. 9.130.



From the solution to Problem 9.130

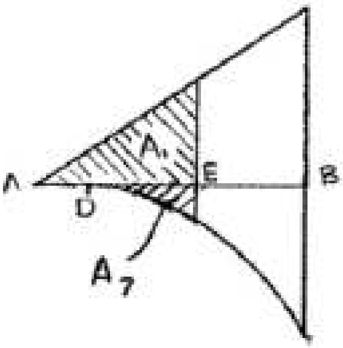
$EI = 17020 \text{ kN m}^2$

$R_A = 82.8 \text{ kN}$

$A_1 = 134 \text{ kN m}^2$

$A_3 = -21.6 \text{ kN m}^2$

$\theta_A = -0.005183$



Slope at E. $\theta_E = \theta_A + \theta_{E/A}$

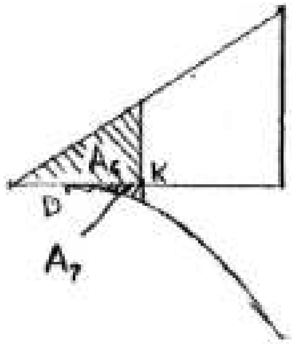
$$\theta_{E/A} = \frac{1}{EI}\left\{A_1 + A_3\right\} = \frac{278.767}{41083} = 6.604 \times 10^{-3}$$

$$\theta_E = 1.421 \times 10^{-3}$$

Since $\theta_E > 0$, the point K of zero slope lies to the left of point E. Let x_K be the coordinate of point K.

$$A_6 = \frac{1}{2} R_A x_K^2 = 41.4 x_K^2$$

$$A_7 = -\frac{1}{6}(75)(x_K - 0.6)^3$$



$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + \frac{1}{EI}\left\{A_6 + A_7\right\} = 0$$

$$A_6 + A_7 + EI\theta_A = 0$$

$$f(x_K) = 41.4 x_K^2 - \frac{75}{6}(x_K - 0.6)^3 - 88.2 = 0$$

$$\frac{df}{dx_K} = 82.8 x_K - 37.5(x_K - 0.6)^2$$

Solve for x_K by iteration.

$$x_K = (x_K)_0 - \frac{f}{df/dx_K}$$

x_K	1.5	1.5442	1.54575
f	−4.1625	−0.00157	0.145
df/dx_K	93.825	94.428	

$x_K = 1.545$ m.

$A_6 = 98.823 \text{ kN m}^2$, $\quad A_7 = -10.55 \text{ kN m}^2$.

Maximum deflection. $\quad y_A \pm y_K + t_{A/K} = 0 \quad y_K = -t_{A/K}$

$$\bar{x}_6 = \frac{2}{3}x_K, \quad \bar{x}_7 = 0.6 + \frac{3}{4}(x_K - 0.6) = \frac{3x_K + 0.6}{4}$$

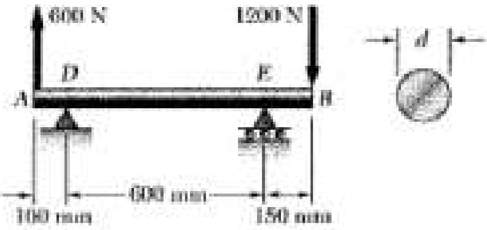
$x_K = 1.545$ m. ◄

$$y_7 = -\frac{1}{EI}\left\{A_6\bar{x}_6 + A_7\bar{x}_7\right\} = -\frac{87.98}{17020} = -0.005169 \text{ m}$$

$y_K = 5.2$ mm ↓ ◄

Problem 9.144

9.144 For the beam and loading of Prob. 9.134, determine the largest upward deflection in span *DE*.



Units: Forces in kN; lengths in m.

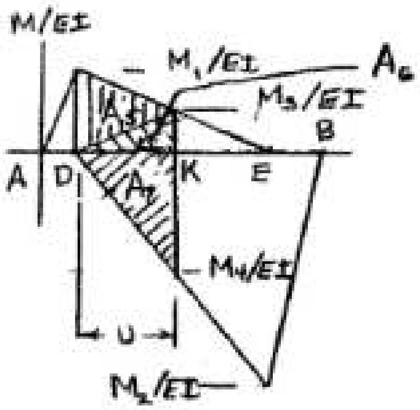
From the solution to Problem 9.134

$EI = 1.0306 \text{ kN m}^2$

$\frac{M_1}{EI} = 0.0582 \text{ m}^{-1}.$

$\frac{M_2}{EI} = -0.1747 \text{ m}^{-1}.$

$\theta_D = 5.83 \times 10^{-3}.$



Location of maximum deflection.

$\frac{M_3}{EI} = \frac{M_1}{EI}\left(1 - \frac{u}{0.6}\right)$

$\frac{M_4}{EI} = \frac{M_2}{EI}\frac{u}{0.6}$

$A_5 = \frac{1}{2}\frac{M_1}{EI} \cdot u = 0.0291\, u$

$A_6 = \frac{1}{2}\frac{M_3}{EI}\left(\frac{u}{0.6}\right) = 0.0291\left(1 - \frac{u}{0.6}\right)u$

$A_7 = \frac{1}{2}\frac{M_4}{EI}\frac{u}{0.6} = -0.0873\left(\frac{u}{0.6}\right)u$

$\theta_K = \theta_D + A_5 + A_6 + A_7 = 0$ Multiply by 10^3.

$0.00583 + 0.0291\, u + 0.0291\left(1 - \frac{u}{0.6}\right)u - (0.0873)\frac{u}{0.6}u = 0$

$0.00583 + 0.0582\, u - 0.097\, u^2 = 0$

$u = 0.3897 \text{ m} = 381 \text{ mm}.$

$A_5 = 0.0111, \quad A_6 = 0.00465, \quad A_7 = -0.0211$

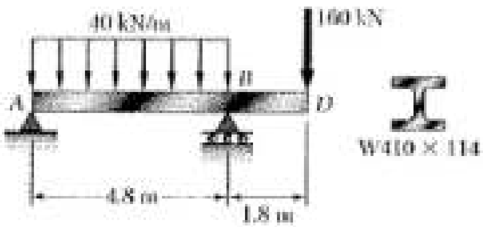
Maximum deflection in portion DE. $\quad y_D = y_K + t_{D/K} = 0$

$y_K = -t_{D/K} = -\left\{A_5\left(\frac{u}{3}\right) + A_6\left(\frac{2u}{3}\right) + A_7\left(\frac{2u}{3}\right)\right\}$

$= -\{-0.00292\} = 2.9 \text{ mm} \uparrow$ ◄

Problem 9.145

9.145 For the beam and loading of Prob. 9.137, determine the largest upward deflection in span *AB*.



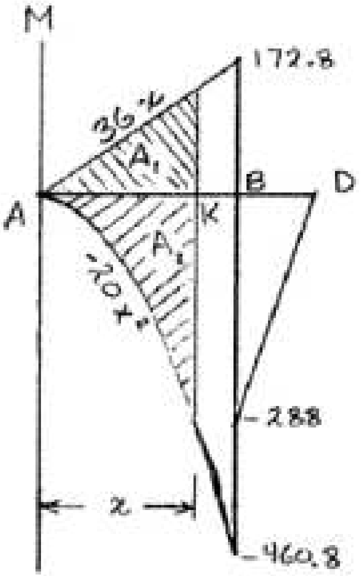
Units: Forces in kN. Lengths in meters.

$I = 462 \times 10^6\ \text{mm}^4 = 462 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(462 \times 10^{-6})$
$= 92.4 \times 10^6\ \text{N}\cdot\text{m}^2 = 92400\ \text{kN}\cdot\text{m}$

$+\circlearrowleft M_B = 0: \quad -4.8 R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$

$R_A = 36\ \text{kN}$



$A_1 = \frac{1}{2}x(36x) = 18x^2$

$A_2 = \frac{1}{3}x(-20x^2) = -\frac{20}{3}x^3$

Place reference tangent at A.

$y_B = y_A + L\theta_A + t_{B/A} = 0$

$\theta_A = -\dfrac{t_{B/A}}{L}$

$(A_1)_B = (18)(4.8)^2 = 414.72\ \text{kN}\cdot\text{m}^2$

$(A_2)_B = (\frac{20}{3})(4.8)^3 = -737.28\ \text{kN}\cdot\text{m}^2$

$$\theta_A = -\frac{1}{EIL}\left\{(A_1)_B(\tfrac{1}{3})(4.8) + (A_2)_B(\tfrac{1}{4})(4.8)\right\}$$
$$= -\frac{-221.184}{(92400)(4.8)} = 0.49870 \times 10^{-3}$$

Locate point K of maximum deflection. $\quad \theta_K = \theta_A + \theta_{K/A} = 0$

$EI\theta_A + A_1 + A_2 = 0$

$f = 46.08 + 18x_K^2 - \frac{20}{3}x_K^3 = 0 \qquad \dfrac{df}{dx} = 36x_K - 20x_K^2$

Solve by iteration.

$x_K = (x_K)_0 - \dfrac{f}{df/dx}$

x_K	3	3.39	3.327	3.3251	3.32514
dF/dx	−72	−107.8	−101.6	−101.42	
f	28.08	−6.78	−0.188	0.005	

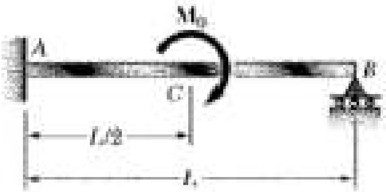
Place reference tangent at K. $\quad y_A = y_K + t_{A/K}$

$$y_A - y_K = -t_{A/K} = -\frac{1}{EI}\left\{(A_1)(\tfrac{2}{3}x_K) + A_2(\tfrac{3}{4}x_K)\right\} = -\frac{1}{EI}\left\{12x_K^3 - 5x_K^4\right\}$$
$$= -\frac{170.064}{92400} = -1.841 \times 10^{-3}\ \text{m}$$

$y_K = 1.841\ \text{mm}$

Problem 9.146

9.146 through 1.149 For the beam and loading shown, determine the reaction at the roller support.

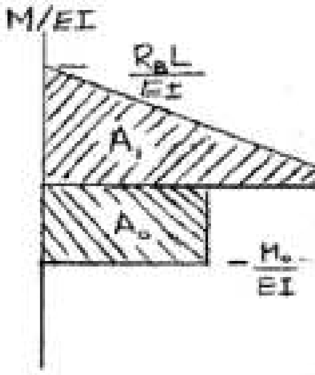


Remove support B and treat R_B as redundant.

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = \frac{L}{2} \cdot \frac{M_0 L}{EI} = \frac{M_0 L^2}{2EI}$$



Place reference tangent at A.

$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

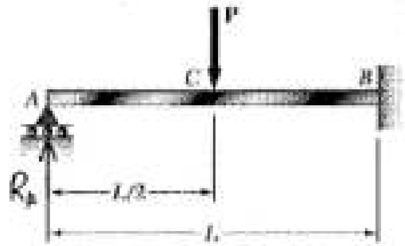
$$t_{B/A} = 0$$

$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{4}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{3M_0 L^2}{8EI} = 0 \qquad R_A = \frac{9}{8}\frac{M_0}{L} \uparrow$$

Problem 9.147

9.146 through 1.149 For the beam and loading shown, determine the reaction at the roller support.

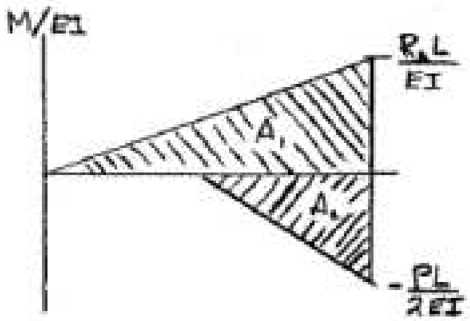


Remove support A and treat R_A as redundant.

Draw the $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = -\frac{1}{2}\frac{L}{2}\frac{PL}{2} = -\frac{PL^2}{8EI}$$



Place reference tangent at B.

$$y_A = y_B - \theta_B L + t_{A/B} = 0$$

$$t_{A/B} = 0$$

$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{3}\right) = 0$$

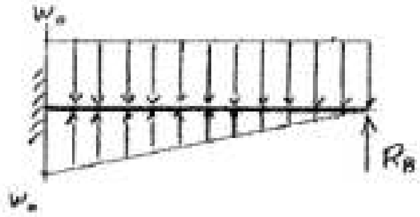
$$\frac{R_A L^3}{3EI} - \frac{5PL^3}{48EI} = 0 \qquad R_A = \frac{5}{16}P \uparrow$$

Problem 9.148

9.146 through 1.149 For the beam and loading shown, determine the reaction at the roller support.

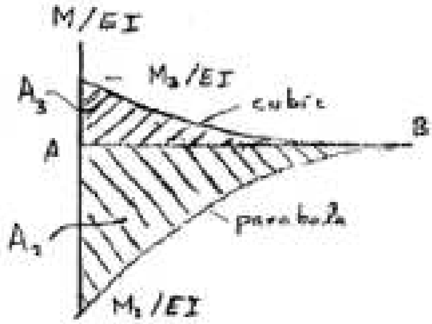
Remove support B and treat R_B as redundant.

Replace loading by equivalent shown at left.



Draw M/EI diagram for load w_0 and R_B.

Use parts as shown.



$$A_1 = \frac{1}{2}\left(\frac{R_B L}{EI}\right)(L) = \frac{1}{2}\frac{R_B L^2}{EI}$$

$$M_2 = -\frac{1}{2} w_0 L^2$$

$$A_2 = \frac{1}{3}\left(-\frac{1}{2}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{6}\frac{w_0 L^3}{EI}$$

$$M_3 = \frac{1}{6}\frac{w_0}{L} L^3 = \frac{1}{6} w_0 L^2$$

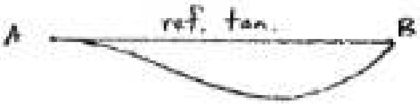
$$A_3 = \frac{1}{4}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = \frac{1}{24}\frac{w_0 L^3}{EI}$$

Place reference tangent at A.

$$t_{B/A} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{3}{4}L\right) + A_3\left(\frac{4}{5}L\right)$$

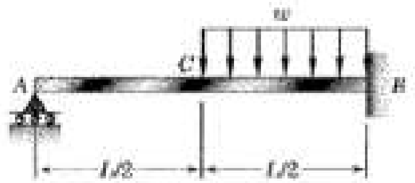
$$= \frac{1}{3}\frac{R_B L^3}{EI} - \frac{1}{8}\frac{w_0 L^4}{EI} + \frac{1}{30}\frac{w_0 L^4}{EI} = 0$$

$$R_B = \frac{11}{40} w_0 L \uparrow \qquad R_B = 0.275\, w_0 L \uparrow$$



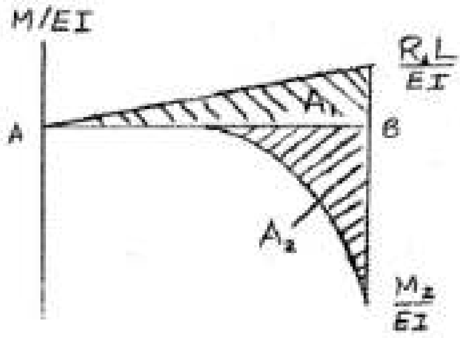
Problem 9.149

9.146 through 1.149 For the beam and loading shown, determine the reaction at the roller support.



Remove support A and treat R_A as redundant.

Draw M/EI diagram for loads R_A and w.

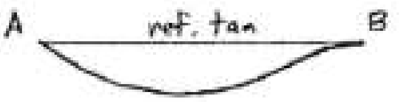


$$M_2 = -\frac{1}{2} w \left(\frac{L}{2}\right)^2 = -\frac{1}{8} wL^2$$

$$A_1 = \frac{1}{2}\left(\frac{R_A L}{EI}\right) L = \frac{1}{2}\frac{R_A L^2}{EI}$$

$$A_2 = \frac{1}{3}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

Place reference tangent at B.

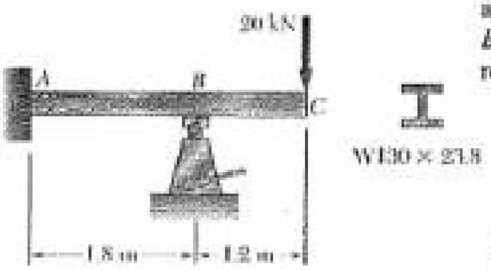


$$t_{A/B} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{L}{2} + \frac{3}{4}\frac{L}{2}\right)$$

$$= \frac{1}{3}\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI} = 0$$

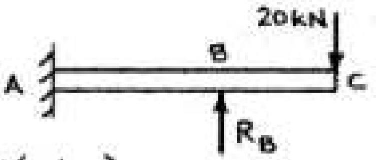
$$R_A = \frac{7}{128} wL \qquad R_A = \frac{7}{128} wL \uparrow$$

Problem 9.152



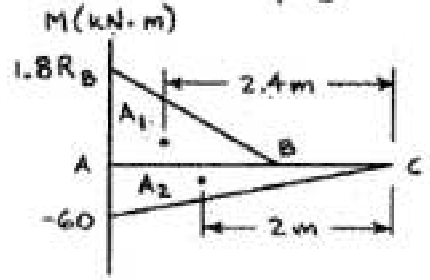
9.152 A hydraulic jack can be used to raise point *B* of the cantilever beam *ABC*. The beam was originally straight, horizontal, and unloaded. A 20-kN load was then applied at point *C*, causing this point to move down. Determine (*a*) how much point *B* should be raised to return point *C* to its original position, (*b*) the final value of the reaction at *B*. Use *E* = 200 GPa.

FOR W130 × 23.8 $\quad I_x = 8.80 \times 10^6\ \text{mm}^4$



$EI = (200 \times 10^6\ \text{kPa})(8.80 \times 10^{-6}\ \text{m}^4) = 1760\ \text{kN}\cdot\text{m}^2$

LET R_B BE THE JACK FORCE IN kN.



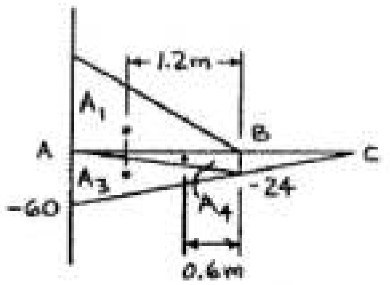
$A_1 = \frac{1}{2}(1.8R_B)(1.8) = 1.62R_B$

$A_2 = \frac{1}{2}(-60)(3) = -90\ \text{kN}\cdot\text{m}^2$

$EIt_{C/A} = (2.4)A_1 + (2)A_2$

$0 = (2.4)(1.62R_B) + (2)(-90)$

$R_B = 46.296\ \text{kN}$



$A_1 = 75\ \text{kN}\cdot\text{m}^2$

$A_3 = \frac{1}{2}(-60)(1.8) = -54\ \text{kN}\cdot\text{m}^2$

$A_4 = \frac{1}{2}(-24)(1.8) = -21.6\ \text{kN}\cdot\text{m}^2$

$EIt_{B/A} = (1.2)A_1 + (1.2)A_3 + (0.6)A_4$

$= (1.2)(75) + (1.2)(-54) + (0.6)(-21.6)$

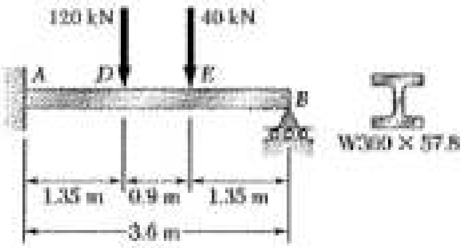
$= 12.24\ \text{kN}\cdot\text{m}^2$

(a) $y_B = t_{B/A} = \dfrac{EIt_{B/A}}{EI} = \dfrac{12.24}{1760} = 6.9545 \times 10^{-3}\ \text{m}$ $\qquad y_B = 6.95\ \text{mm} \uparrow$ ◄

(b) $\qquad R_B = 46.3\ \text{kN} \uparrow$ ◄

Problem 9.153

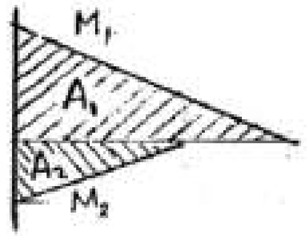
9.152 and 9.153 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Units. Forces in kN. Lengths in m.

Let R_B be the redundant reaction.
Remove support B and add load R_B.

Draw bending moment diagram by parts.



$M_1 = 3.6 R_B$ kNm

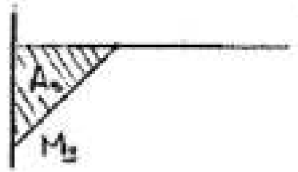
$M_2 = -(1.35 + 0.9)(40) = -90$ kNm

$M_3 = -(1.35)(120) = -162$ kNm

$A_1 = \frac{1}{2}(3.6)(3.6 R_B) = R_B\, 6.48$ kNm²

$A_2 = \frac{1}{2}(2.25)(-90) = -101.25$ kNm²

$A_3 = \frac{1}{2}(1.35)(-162) = -109.35$ kNm²

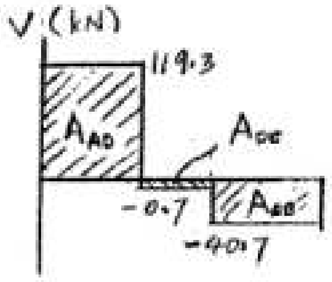


$y_B = y_A + 3.6\,\theta_A + t_{B/A} = 0$

$t_{B/A} = 0$

$$t_{B/A} = \frac{1}{EI}\left\{(6.48 R_B)(2.4) + (-101.25)(2.85) + (-109.35)(3.15)\right\} = 0$$

$15.552 R_B - 633.015 = 0 \qquad R_B = 40.7$ kN ↑

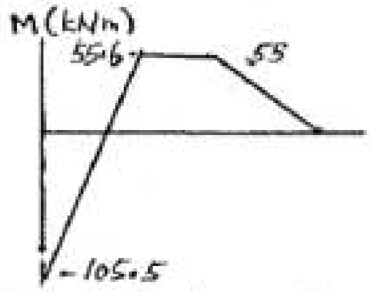


Draw shear diagram working from right to left.

B to E $\qquad V = -R_B = -40.7$ kN

E to D $\qquad V = -40.7 + 40 = -0.7$ kN

D to A $\qquad V = -0.7 + 120 = 119.3$ kN



Areas of shear diagram.

$A_{AD} = (1.35)(119.3) = 161.055$ kNm

$A_{DE} = (0.9)(-0.7) = -0.63$ kNm

$A_{EB} = (1.35)(-40.7) = 54.945$ kNm

Bending moments.

$M_A = M_1 + M_2 + M_3 = -105.5$ kNm

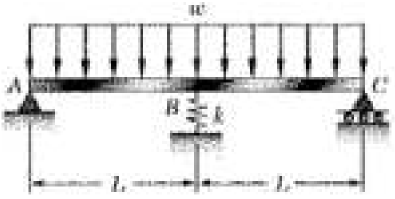
$M_D = M_A + A_{AD} = 55.6$ kNm

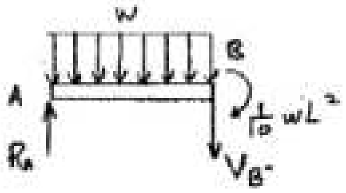
$M_E = M_D + A_{DE} = 55$ kNm

$M_B = M_E + A_{EB} = 0$

Problem 9.155

9.155 For the beam and loading shown, determine the spring constant k for which the bending moment at B is $M_B = -wL^2/10$.



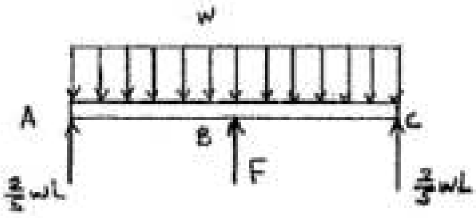


Using free body AB,

$+\circlearrowleft M_B = 0$:

$$-R_A L + (wL)\left(\frac{L}{2}\right) - \frac{1}{10}wL^2 = 0$$

$$R_A = \frac{2}{5}wL \uparrow$$

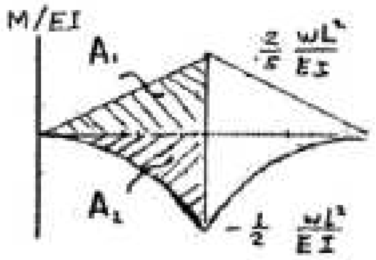


Symmetric beam and loading. $R_C = R_A$

Using free body ABC, $+\uparrow \Sigma F_y = 0$:

$$\frac{2}{5}wL + F + \frac{2}{5}wL - 2wL = 0$$

$$F = \frac{6}{5}wL$$

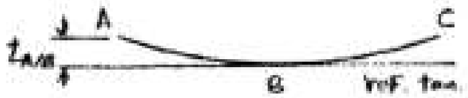


Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{2}{5}\frac{wL^2}{EI}\right)L = \frac{1}{5}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$



$$y_B = -t_{A/B}$$

$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

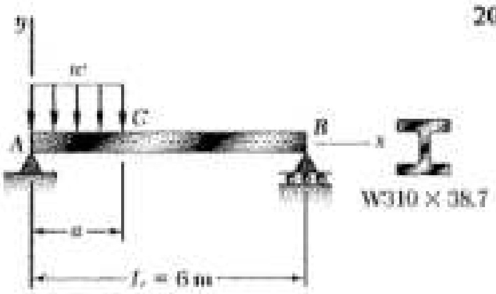
$$= -\frac{1}{120}\frac{wL^4}{EI}$$

$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{6}{5}wL}{\frac{1}{120}\frac{wL^4}{EI}} \qquad k = 144\frac{EI}{L^3}$$

Problem 9.157

9.157 For the beam and loading shown, knowing that $a = 2$ m, $w = 50$ kN/m, and $E = 200$ GPa, determine (*a*) the slope at support *A*, (*b*) the deflection at point *C*.

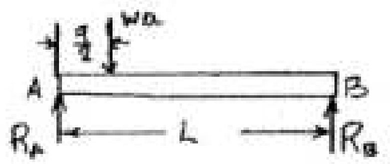


$[x=0, y=0]$ $[x=L, y=0]$

$[x=a, y=y]$

$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$

Using ACB as a free body and noting that $L = 3a$,



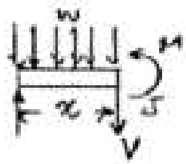
$+\circlearrowright \Sigma M_A = 0:$

$R_B L - (wa)(\frac{a}{2}) = 0$

$R_B = (wa)\frac{a}{2L} = \frac{1}{6} wa$

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wa = 0 \qquad R_A = \frac{5}{6} wa$

$0 \le x \le a$



$+\circlearrowright \Sigma M_J = 0:$

$M - R_A x + (wx)(\frac{x}{2}) = 0$

$M = R_A x - \frac{1}{2} w x^2$

$EI \frac{d^2y}{dx^2} = R_A x - \frac{1}{2} w x^2$

$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} w x^3 + C_1$

$EI y = \frac{1}{6} R_A x^3 - \frac{1}{24} w x^4 + C_1 x + C_2$

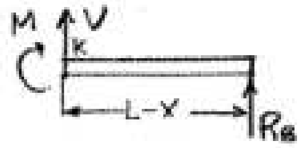
$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2$

$C_2 = 0$

$EI y = \frac{1}{6} R_A x^3 - \frac{1}{24} w x^4 + C_1 x$

$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} w x^3 + C_1$

$a \le x \le L$



$+\circlearrowright \Sigma M_K = 0$

$-M + R_B(L-x) = 0$

$M = R_B(L-x)$

$EI \frac{d^2y}{dx^2} = R_B(L-x)$

$EI \frac{dy}{dx} = -\frac{1}{2} R_B (L-x)^2 + C_3$

$EI y = \frac{1}{6} R_B (L-x)^3 + C_3 x + C_4$

$[x=L, y=0] \quad 0 = 0 + C_3 L + C_4$

$C_4 = -C_3 L$

$EI y = \frac{1}{6} R_B (L-x)^3 - C_3 (L-x)$

$EI \frac{dy}{dx} = -\frac{1}{2} R_B (L-x)^2 + C_3$

$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \qquad \frac{1}{2} R_A a^2 - \frac{1}{6} w a^3 + C_1 = -\frac{1}{2} R_B (2a)^2 + C_3$

$C_3 = C_1 + \frac{1}{2} R_A a^2 - \frac{1}{6} w a^3 + \frac{1}{2} R_B (2a)^2 = C_1 + \frac{7}{12} w a^3$

$[x=a, y=y] \qquad \frac{1}{6} R_A a^3 - \frac{1}{24} w a^4 + C_1 a = \frac{1}{6} R_B (2a)^3 - (C_1 + \frac{7}{12} w a^3)(2a)$

$3C_1 a = -\frac{1}{6} R_A a^3 + \frac{1}{24} w a^4 + \frac{1}{6} R_B (2a)^3 - \frac{7}{12} w a^3 (2a) = -\frac{25}{24} w a^4$

$C_1 = -\frac{25}{72} w a^3$

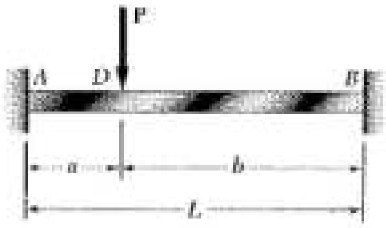
For $0 \le x \le a$, $\quad EI y = \frac{5}{36} w a x^3 - \frac{1}{24} w x^4 - \frac{25}{72} w a^3 x$

$EI \frac{dy}{dx} = \frac{5}{12} w a x^2 - \frac{1}{6} w x^3 - \frac{25}{72} w a^3$

continued

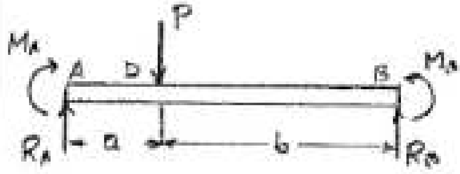
Problem 9.159

9.159 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.



$[x=0, y=0]$ $\quad$ $[x=L, y=0]$

$[x=0, \frac{dy}{dx}=0]$ $\quad$ $[x=L, \frac{dy}{dx}=0]$



Reactions are statically indeterminate.

$0 \le x \le a$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x=0, \frac{dy}{dx}=0] \quad \longrightarrow \quad C_1 = 0$

$[x=0, y=0] \quad \longrightarrow \quad C_2 = 0$

$a \le x \le L$

$$EI\frac{d^2y}{dx^2} = M_A + R_A x - P(x-a)$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P(x-a)^2 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P(x-a)^3 + C_3 x + C_4$$

$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad M_A a + \frac{1}{2}R_A a^2 = M_A a + \frac{1}{2}R_A a^2 - 0 + C_3 \quad C_3 = 0$

$[x=a, y=y] \quad \frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 = \frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 - 0 + 0 + C_4 \quad C_4 = 0$

$[x=L, \frac{dy}{dx}=0] \quad M_A L + \frac{1}{2}R_A L^2 - \frac{1}{2}Pb^2 = 0 \quad (1)$

$[x=L, y=0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}Pb^3 = 0 \quad (2)$

Solving (1) and (2) simultaneously,

$$M_A = -\frac{Pb^2(L-b)}{L^2} = -\frac{Pab^2}{L^2} \blacktriangleleft$$

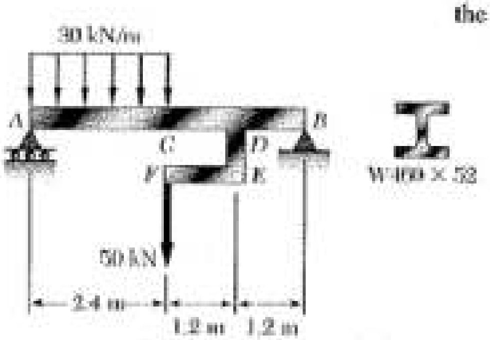
$$R_A = \frac{Pb^2}{L^2} - \frac{2M_A}{L} = \frac{Pb^2}{L^2}\left(1 + 2\frac{a}{L}\right) = \frac{Pb^2(3a+b)}{L^3} \uparrow \blacktriangleleft$$

$$M_B = M_A + R_A L - Pb = -\frac{Pab^2}{L^2} + \frac{Pb^2(3a+b)(a+b)}{L^3} - Pb = -\frac{Pba^2}{L^2} \blacktriangleleft$$

$$M_D = M_A + R_A a = -\frac{Pab^2}{L^2} + \frac{Pb^2(3a+b)a}{L^3} = \frac{2Pa^2b^2}{L^3} \blacktriangleleft$$

Problem 9.161

9.161 The rigid bar *DEF* is welded at point *D* to the rolled-steel beam *AB*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 200$ GPa.

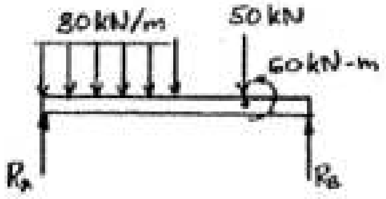


<u>Units:</u> Forces in kN; lengths in meters.

$+\circlearrowleft \Sigma M_B = 0:$

$-4.8 R_A + (30)(2.4)(3.6) + (50)(2.4) = 0$

$R_A = 79 \text{ kN} \uparrow$



$I = 212 \times 10^6 \text{ mm}^4 = 212 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(212 \times 10^{-6}) = 42.4 \times 10^6 \text{ N}\cdot\text{m}^2$
$= 42400 \text{ kN}\cdot\text{m}^2$

$w(x) = 30 - 30\langle x-2.4\rangle^0$

$\frac{dV}{dx} = -w = -30 + 30\langle x-2.4\rangle^0 \qquad \text{kN/m}$

$\frac{dM}{dx} = V = 79 - 30x + 30\langle x-2.4\rangle^1 - 50\langle x-3.6\rangle^0 \qquad \text{kN}$

$EI\frac{d^2y}{dx^2} = M = 79x - 15x^2 + 15\langle x-2.4\rangle^2 - 50\langle x-3.6\rangle^1 - 60\langle x-3.6\rangle^0 \qquad \text{kN}\cdot\text{m}$

$EI\frac{dy}{dx} = \frac{79}{2}x^2 - 5x^3 + 5\langle x-2.4\rangle^3 - 25\langle x-3.6\rangle^2 - 60\langle x-3.6\rangle^1 + C_1 \qquad \text{kN}\cdot\text{m}^2$

$EIy = \frac{79}{6}x^3 - \frac{5}{4}x^4 + \frac{5}{4}\langle x-2.4\rangle^4 - \frac{25}{3}\langle x-3.6\rangle^3 - 30\langle x-3.6\rangle^2 + C_1x + C_2 \qquad \text{kN}\cdot\text{m}^3$

$[x=0,\ y=0] \qquad 0-0+0-0-0+0+C_2 = 0 \qquad C_2 = 0$

$[x=4.8,\ y=0] \qquad (\frac{79}{6})(4.8)^3 - (\frac{5}{4})(4.8)^4 + (\frac{5}{4})(2.4)^4 - (\frac{25}{3})(1.2)^3 - (30)(1.2)^2 + 4.8C_1 = 0$

$C_1 = -161.76 \text{ kN}\cdot\text{m}^2$

(a) <u>Slope at point A.</u> ($\frac{dy}{dx}$ at $x=0$)

$EI\left(\frac{dy}{dx}\right)_A = 0-0+0-0-0-161.76 \qquad = -161.76 \text{ kN}\cdot\text{m}^2$

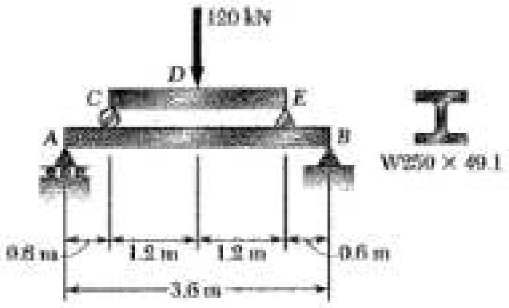
$\left(\frac{dy}{dx}\right)_A = \frac{-161.76}{42400} = -3.82 \times 10^{-3} \qquad \theta_A = 3.82 \times 10^{-3} \text{ rad} \searrow$ ◄

(b) <u>Deflection at midpoint C.</u> (y at $x = 2.4$)

$EIy_C = (\frac{79}{6})(2.4)^3 - (\frac{5}{4})(2.4)^4 + 0 - 0 - 0 - (161.76)(2.4) + 0 = -247.68 \text{ kN}\cdot\text{m}^3$

$y_C = \frac{-247.68}{42400} = -5.84 \times 10^{-3} \text{ m} \qquad y_C = 5.84 \text{ mm} \downarrow$ ◄

Problem 9.162



9.162 Beam CE rests on beam AB, as shown. Knowing that a W250 × 49.1 rolled-steel shape is used for each beam, determine for the loading shown the deflection at point D. Use $E = 200$ GPa.

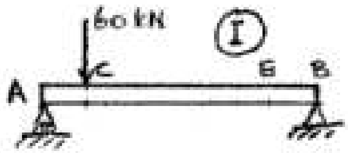
For W250×49.1 $\quad I = 70.6 \times 10^{6}\ \text{mm}^4$

$EI = (200\times10^{6})(70.6\times10^{-6})$

$= 14120\ \text{kN m}^2$

Beam AB: 60 kN downward loads at C and E.

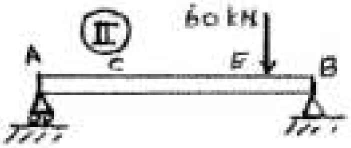
Refer to Case 5 of Appendix D.



Loading I. $\quad (y_C)_1 = -\dfrac{Pa^2b^2}{3EIL}$

with $a = 0.6$ m, $\quad b = 3$ m, $\quad L = 3.6$ m

$$(y_C)_1 = -\frac{(60)(0.6)^2(3)^2}{(3)(14120)(3.6)} = -1.275\times10^{-3}\ \text{m}$$



Loading II. $\quad (y_C)_2 = \dfrac{Pb\left[x^3 - (L^2 - b^2)x\right]}{6EIL}$

with $b = 0.6$ m, $x = 0.6$ m, $\quad L = 3.6$ m

$$(y_C)_2 = \frac{(60)(0.6)\left[0.6^3 - (3.6^2 - 0.6^2)(0.6)\right]}{(6)(14120)(3.6)} = -0.0867\times10^{-3}\ \text{m}$$

$y_C = (y_C)_1 + (y_C)_2 = -1.362\times10^{-3}$ m

By symmetry $\quad y_E = y_C$

Beam CDE. $\quad$ 120 kN downward load at D.

Refer to Case 4 of Appendix D.

$y_{D/C} = -\dfrac{PL^3}{48EI}$

with $P = 120$ kN and $L = 2.4$ m

$$y_{D/C} = -\frac{(120)(2.4)^3}{(48)(14120)} = -2.448\times10^{-3}\ \text{m}$$

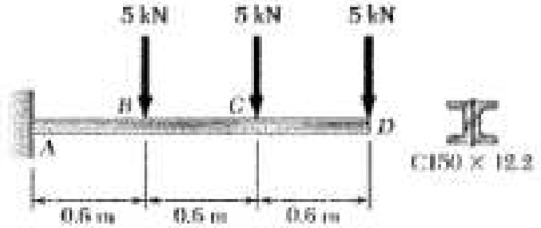
Total deflection at D.

$y_D = y_C + y_{D/C} = -3.81\times10^{-3}$ m

$= 3.81$ mm $\downarrow$ ◄

Problem 9.164

9.164 Two C150 × 12.2 channels are welded back to back and loaded as shown. Knowing that $E = 200$ GPa, determine (*a*) the slope at point *D*, (*b*) the deflection at point *D*.



Units: Forces in kN; lengths in m

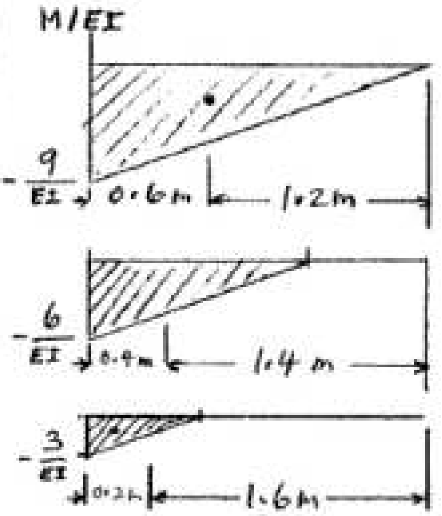
$E = 200$ GPa

$I = (2)(5.35 \times 10^{-6}) = 10.7 \times 10^{-6}\ m^4$

$EI = (200 \times 10^{6})(10.7 \times 10^{-6})$

$= 2140$ kN m²

Draw $\frac{M}{EI}$ diagram by parts.



$\frac{M_1}{EI} = \frac{(5)(1.8)}{EI} = -\frac{9}{EI}\ m^{-1}$

$A_1 = \frac{1}{2}\left(\frac{9}{EI}\right)(1.8) = -\frac{8.1}{EI}$

$\bar{x}_1 = \frac{1}{3}(1.8) = 0.6$ m

$\frac{M_2}{EI} = -\frac{(5)(1.2)}{EI} = -\frac{6}{EI}\ m^{-1}$

$A_2 = \frac{1}{2}\left(-\frac{6}{EI}\right)(1.2) = -\frac{3.6}{EI}$

$\bar{x}_2 = \frac{1}{3}(1.2) = 0.4$ m

$\frac{M_3}{EI} = -\frac{(5)(0.6)}{EI} = -\frac{3}{EI}\ m^{-1}$

$A_3 = \frac{1}{2}\left(-\frac{3}{EI}\right)(0.6) = -\frac{0.9}{EI}$

$\bar{x}_3 = \frac{1}{3}(0.6) = 0.2$ m

Place reference tangent at A. $\theta_A = 0$

$\theta_{D/A} = A_1 + A_2 + A_3 = -\frac{12.6}{EI} = -\frac{12.6}{2140} = -5.89 \times 10^{-3}$ rad.

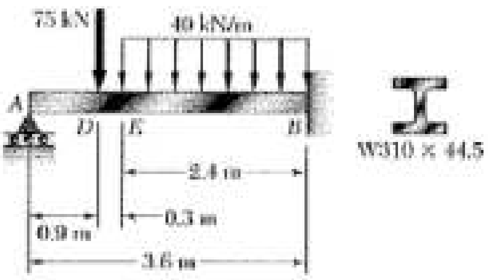
$\theta_D = \theta_A + \theta_{D/A} = 5.89 \times 10^{-3}$ rad. ◄

$t_{D/A} = \left(-\frac{8.1}{EI}\right)(1.2) + \left(-\frac{3.6}{EI}\right)(1.4) + \left(-\frac{3}{EI}\right)(1.6) = -\frac{19.56}{EI} = -9.14 \times 10^{-3}$ m

$y_D = t_{D/A} = 9.14$ mm ↓ ◄

Problem 9.167

9.167 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.

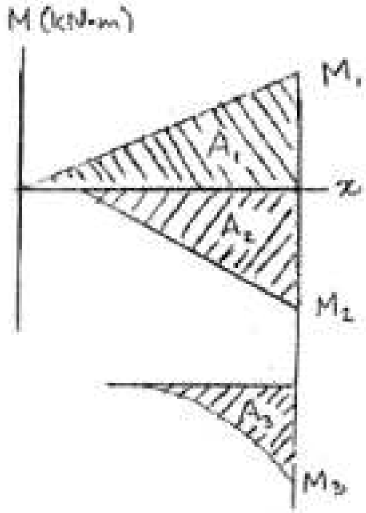


Units: Forces in kN. Lengths in meters.

Let R_A be the redundant reaction.

Remove support at A and add reaction $R_A\uparrow$.

Draw bending moment diagram by parts.



$M_1 = 3.6\,R_A$ kN·m

$M_2 = -(75)(0.3 + 2.4) = -202.5$ kN·m

$M_3 = -\frac{1}{2}(40)(2.4)^2 = -115.2$ kN·m

$A_1 = \frac{1}{2}(3.6)(3.6R_A) = 6.48$ kN·m²

$A_2 = \frac{1}{2}(2.7)(-202.5) = -273.375$ kN·m²

$A_3 = \frac{1}{3}(2.4)(-115.2) = -92.16$ kN·m²

Place reference tangent at B, where

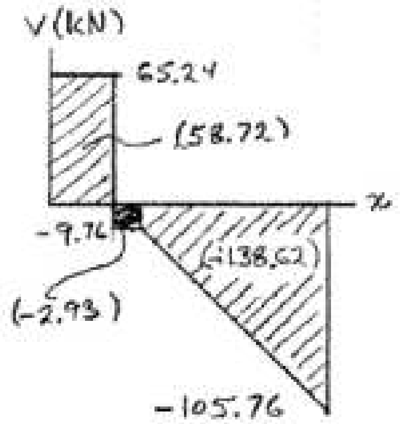
$\theta_B = 0$ and $y_B = 0$

Then, $y_A = t_{A/B} = 0$

$$t_{A/B} = \frac{1}{EI}\left\{\left(\tfrac{2}{3}\cdot 3.6\right)A_1 + \left(0.9 + \tfrac{2}{3}\cdot 2.7\right)A_2 + \left(0.9 + 0.3 + \tfrac{3}{4}\cdot 2.4\right)A_3\right\}$$

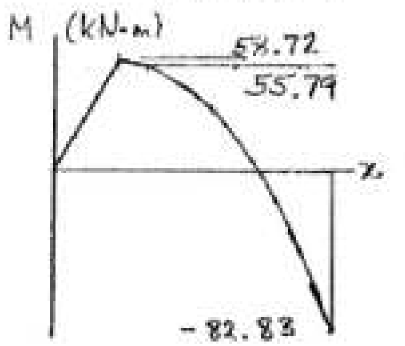
$$= \frac{1}{EI}\left\{15.552\,R_A - 1014.5925\right\} = 0$$

$R_A = 65.24$ kN $\uparrow$ ◄



Draw shear diagram.

A to D	$V = R_A = 65.24$ kN
D to E	$V = 65.24 - 75 = -9.76$ kN
E to B	$V = -9.76 - 40(x - 1.2)$ kN
At B	$V_B = -105.76$ kN.



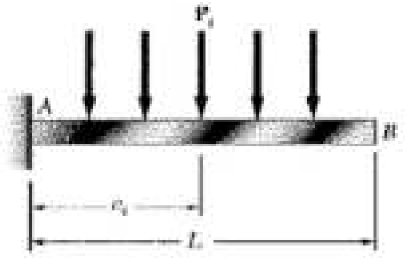
Bending moment diagram. $M_A = 0$

$M_D = M_A + 58.72 = 58.72$ kN·m

$M_E = 58.72 - 2.93 = 55.79$ kN·m

$M_B = 55.79 - 138.62 = -82.83$ kN·m

PROBLEM 9.C1



9.C1 Several concentrate loads can be applied to the cantilever beam *AB*. Write a computer program to calculate the slope and deflection of beam *AB* from $x = 0$ to $x = L$, using given increments Δx. Apply this program with increments $\Delta x = 50$ mm to the beam and loading of Probs. 9.73 and Prob. 9.74.

SOLUTION

FOR EACH LOAD, ENTER

P_i, c_i

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS

$R_A = R_A + P_i$

$M_A = M_A - P_i c_i$

COMPUTE SLOPE AND DEFLECTION

USE METHOD OF INTEGRATION STARTING WITH $x = 0$ AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A:

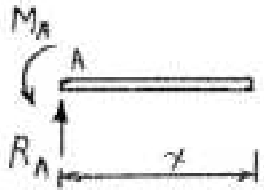
$\theta = (1/EI)(R_A x^2/2.0 + M_A x)$

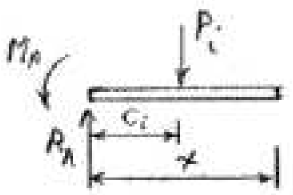
$y = (1/EI)(R_A x^3/6.0 + M_A x^2/2.0)$

(2) DUE TO EACH LOAD WITH $c_i < x$:

$\theta = -(1/EI)(P_i/2.0)(x - c_i)^2$

$y = -(1/EI)(P_i/6.0)(x - c_i)^3$



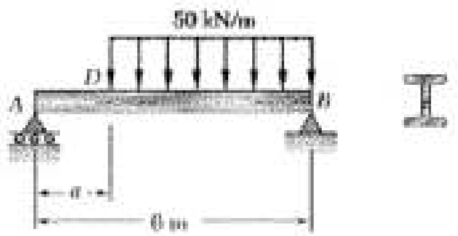


AT $x = 0$, $y = \frac{dy}{dx} = 0$

$\therefore$ THE CONSTANTS OF INTEGRATION EQUAL ZERO

CONTINUED

PROBLEM 9.C2



9.C2 The 6-m beam *AB* consists of a W530 × 92 rolled-steel shape and supports a 50-kN/m distributed load as shown. Write a computer program and use it to calculate for values of *a* from 0 to 6 m, using 0.3-m increments, (*a*) the slope and deflection at *D*, (*b*) the location and magnitude of the maximum deflection. Use *E* = 200 GPa.

SOLUTION

ENTER LOAD w, LENGTH L, a

COMPUTE REACTION AT A

$$R_A = w(L-a)^2/(2.0L)$$

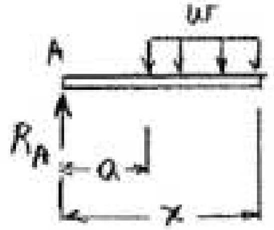
COMPUTE SLOPE AND DEFLECTION AT D

USING SINGULARITY FUNCTIONS:

$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_A L^2$$

$$\theta = (1/EI)(R_A a^2/2.0 + C_1)$$

$$y = (1/EI)(R_A a^3/6.0 + C_1 a)$$



$$EI\frac{d^2y}{dx^2} = R_A x - \frac{w}{2}\langle x-a\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{w}{6}\langle x-a\rangle^3 + C_1$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{w}{24}\langle x-a\rangle^4 + C_1 x + C_2$$

FROM BOUNDARY CONDITIONS:

$$C_2 = 0$$

$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_A L^2$$

COMPUTE LOCATION AND MAGNITUDE OF MAXIMUM DEFLECTION

MAXIMUM y AT $\theta = 0$:

$$0 = \frac{1}{2}R_A x^2 - \frac{w}{6}\langle x-a\rangle^3 + C_1$$

IF $x_{max} \le a$

$$\frac{1}{2}R_A x^2 + C_1 = 0$$

$$x_{max} = \sqrt{\frac{-2.0\,C_1}{R_A}}$$

$$y_{max} = \frac{1}{6}R_A x_{max}^3 + C_1 x_{max}$$

ASSUME $x < A$:

$$x_{max} = (-2.0\,C_1/R_A)^{\frac{1}{2}}$$

IF $x_{max} < a$, THEN

$$y_{max} = (1/EI)\left(\frac{1}{6}R_A x_{max}^3 + C_1 x_{max}\right)$$

IF $x_{max} > a$, THEN

BEGIN WITH $x = a$

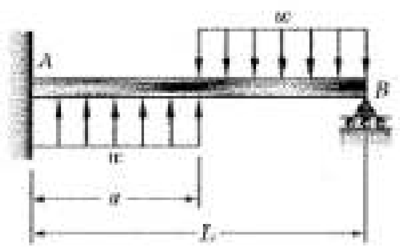
$$\theta = (1/EI)\left(\frac{1}{2}R_A x - \frac{1}{6}(x-a)^3 + C_1\right)$$

INCREASE x BY SMALL AMOUNT UNTIL θ IS APPROXIMATELY 0

$$y_{max} = (1/EI)\left(\frac{1}{6}R_A x^3 - \frac{w}{24}(x-a)^4 + C_1 x\right)$$

CONTINUED

PROBLEM 9.C3



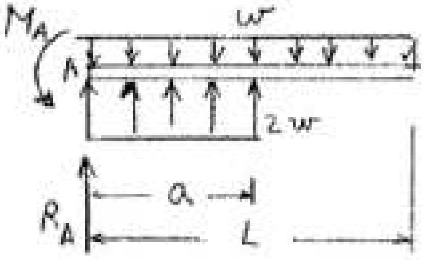
9.C3 The cantilever beam AB caries the distributed loads shown. Write a computer program to calculate the slope and deflection of beam AB from $x = 0$ to $x = L$ using given increments Δx. Apply this program with increments $\Delta x = 100$ mm, assuming that $L = 2.4$ m, $w = 36$ kN/m, and (*a*) $a = 0.6$ m, (*b*) $a = 1.2$ m, (*c*) $a = 1.8$ m. Use $E = 200$ GPa.

SOLUTION

ENTER w, a, L

COMPUTE REACTION AT A

$$R_A = wL - 2.0wa$$
$$M_A = \frac{1}{2}wL^2 - \frac{1}{2}wa^2$$



COMPUTE SLOPE AND DEFLECTION

USE EQUATION OF ELASTIC CURVE

STARTING WITH $x=0$ AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTIONS AT A

$$\theta = (1/EI)\left(\frac{1}{2}R_A x^2 + M_A x\right)$$
$$y = (1/EI)\left(\frac{1}{6}R_A x^3 + \frac{1}{2}M_A x\right)$$

(2) DUE TO LOAD w

$$\theta = -(1/EI)\left(\frac{1}{6}wx^3\right)$$
$$y = -(1/EI)\left(\frac{1}{24}wx^4\right)$$

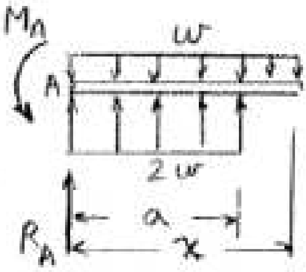
(3) DUE TO LOAD $2w$

IF $x \le a$

$$\theta = (1/EI)\left(\frac{1}{3}wx^3\right)$$
$$y = (1/EI)\left(\frac{1}{12}wx^4\right)$$

IF $x > a$

$$\theta = (1/EI)\left(\frac{1}{3}wx^3 - \frac{1}{3}w(x-a)^3\right)$$
$$y = (1/EI)\left(\frac{1}{12}wx^4 - \frac{1}{12}w(x-a)^4\right)$$



AT $x = 0$, $y = \frac{dy}{dx} = 0$

$\therefore$ THE CONSTANTS OF INTEGRATION ARE ZERO

CONTINUED

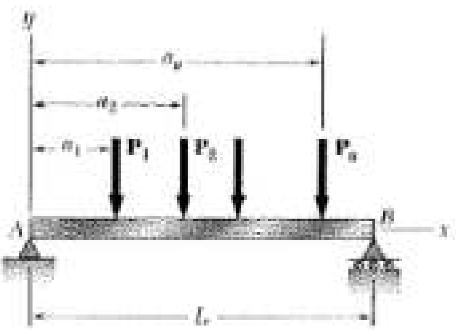
PROBLEM 9.C3 CONTINUED

PROGRAM OUTPUT

```
Problem 9.C3 (a) a = 0.6 m

 At A: Force = 43.2 kN  Couple = -90.7 kN·m

        x          slope         deflection
        m         radians            m

      .00        .000000         .000000
      .10       -.000905        -.000046
      .20       -.001762        -.000179
      .30       -.002567        -.000396
      .40       -.003318        -.000691
      .50       -.004009        -.001058
      .60       -.004638        -.001491
      .70       -.005202        -.001983
      .80       -.005703        -.002529
      .90       -.006145        -.003122
     1.00       -.006533        -.003756
     1.10       -.006868        -.004427
     1.20       -.007156        -.005128
     1.30       -.007399        -.005856
     1.40       -.007602        -.006607
     1.50       -.007769        -.007376
     1.60       -.007902        -.008160
     1.70       -.008006        -.008955
     1.80       -.008083        -.009760
     1.90       -.008139        -.010571
     2.00       -.008177        -.011387
     2.10       -.008199        -.012206
     2.20       -.008211        -.013027
     2.30       -.008215        -.013848
     2.40       -.008216        -.014669

Problem 9.C3 (b) a = 1.2 m

 At A: Force = 0.0 kN  Couple = -51.8 kN·m

        x          slope         deflection
        m         radians            m

      .00        .000000         .000000
      .10       -.000529        -.000026
      .20       -.001055        -.000106
      .30       -.001574        -.000237
      .40       -.002081        -.000420
      .50       -.002574        -.000653
      .60       -.003048        -.000934
      .70       -.003500        -.001262
      .80       -.003926        -.001633
      .90       -.004323        -.002046
     1.00       -.004687        -.002497
     1.10       -.005014        -.002982
     1.20       -.005301        -.003498
     1.30       -.005544        -.004041
     1.40       -.005747        -.004606
     1.50       -.005913        -.005189
     1.60       -.006047        -.005787
     1.70       -.006150        -.006398
     1.80       -.006228        -.007017
     1.90       -.006284        -.007642
     2.00       -.006321        -.008273
     2.10       -.006344        -.008906
     2.20       -.006356        -.009541
     2.30       -.006360        -.010177
     2.40       -.006361        -.010813
```

CONTINUED

PROBLEM 9.C4



9.C4 The simple beam AB is of constant flexural rigidity EI and carries several concentrated loads as shown. Using the *Method of Integration*, write a computer program that can be used to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx. Apply this program to the beam and loading of (*a*) Prob. 9.13 with $\Delta x = 0.3$ m, (*b*) Prob. 9.16 with $\Delta x = 0.05$ m, (*c*) Prob. 9.129 $\Delta x = 0.25$ m.

SOLUTION

FOR EACH LOAD, ENTER P_i, a_i

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS:

$M_A = M_A + P_i a_i$

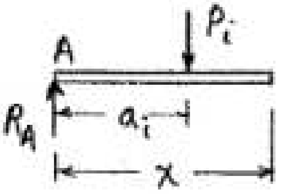
$LOAD = LOAD + P_i$

THEN:

$R_B = M_A / L$

$R_A = LOAD - R_B$

FOR LOAD P_i:



FOR $x < a_i$

$$EI \frac{d^2y}{dx^2} = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI\, y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

FOR $x > a_i$

$$EI \frac{d^2y}{dx^2} = R_A x - P_i (x - a_i)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{2} P_i (x - a_i)^2 + C_3$$

$$EI\, y = \frac{1}{6} R_A x^3 - \frac{1}{6} (x - a_i)^3 + C_3 x + C_4$$

FROM BOUNDARY CONDITIONS

$$C_2 = C_4 = 0$$

$$C_1 = C_3 = \frac{P_i}{6L} (L - a_i)^3 - \frac{1}{6} R_A L^2$$

NOTE: R_A FOR LOAD P_i

COMPUTE SLOPE AND DEFLECTION

STARTING WITH $x = 0$ AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A

$$\theta = (1/EI)\left(\frac{1}{2} R_A x^2\right)$$

$$y = (1/EI)\left(\frac{1}{6} R_A x^3\right)$$

(2) DUE TO LOADS – CONSTANT PART

$$CONST_1 = -\frac{1}{6} R_A L^2$$

FOR 1 TO NUMBER LOADS

$$CONST_2 = \frac{1}{6L} P_i (L - a_i)^3 + CONST_2$$

THEN, TOTAL CONTRIBUTION FOR CONSTANT

$$CONST = (1/EI)(CONST_1 + CONST_2)$$

(3) DUE TO LOADS – REMAINING PART

IF $x \le a_i$

$$\theta = (1/EI)\left(\frac{1}{2.0} R_A x^2\right)$$

$$y = (1/EI)\left(\frac{1}{6.0} R_A x^3\right)$$

IF $x > a_i$

$$\theta = (1/EI)\left(\frac{1}{2.0} R_A x^2 - \frac{1}{2.0} P_i (x - a_i)^2\right)$$

$$y = (1/EI)\left(\frac{1}{6.0} R_A x^3 - \frac{1}{6.0} P_i (x - a_i)^3\right)$$

CONTINUED

PROBLEM 9.C4 CONTINUED

PROGRAM OUTPUT

Problem 9.13

x m	theta rad*10**3	y mm
0	-7.6845	0
1.5	-3.0738	-9.221
2.4	1.6291	-9.747
3.0	3.8423	-8.068
4.5	6.1477	0

Problem 9.129

x m	theta rad*10**3	y mm	
.000	-8.703	.000	(a)
.250	-8.615	-2.168	
.500	-8.351	-4.293	
.750	-7.911	-6.329	
1.000	-7.296	-8.234	
1.250	-6.505	-9.962	
1.500	-5.538	-11.472	
1.750	-4.483	-12.724	
2.000	-3.428	-13.713	
2.250	-2.373	-14.438	
2.500	-1.319	-14.900	
2.750	-.264	-15.098	
3.000	.791	-15.032	(b)
3.250	1.802	-14.706	
3.500	2.725	-14.138	
3.750	3.560	-13.350	
4.000	4.307	-12.365	
4.250	4.967	-11.204	
4.500	5.538	-9.889	
4.750	6.021	-8.442	
5.000	6.417	-6.886	
5.250	6.725	-5.241	
5.500	6.944	-3.531	
5.750	7.076	-1.776	
6.000	7.120	.000	

Problem 9.16

x m	theta rad*10**3	y mm
.000	-2.490	.000
.050	-2.485	-.124
.100	-2.471	-.248
.150	-2.448	-.371
.200	-2.416	-.493
.250	-2.375	-.613
.300	-2.325	-.730
.350	-2.265	-.845
.400	-2.197	-.957
.450	-2.119	-1.065
.500	-2.032	-1.168
.550	-1.936	-1.268
.600	-1.831	-1.362
.650	-1.716	-1.451
.700	-1.593	-1.533
.750	-1.460	-1.610
.800	-1.318	-1.679
.850	-1.172	-1.741
.900	-1.025	-1.796
.950	-.879	-1.844
1.000	-.732	-1.884
1.050	-.586	-1.917
1.100	-.439	-1.943
1.150	-.293	-1.961
1.200	-.146	-1.972
1.250	.000	-1.976
1.300	.146	-1.972
1.350	.293	-1.961
1.400	.439	-1.943
1.450	.586	-1.917
1.500	.732	-1.884
1.550	.879	-1.844
1.600	1.025	-1.796
1.650	1.172	-1.741
1.700	1.318	-1.679
1.750	1.460	-1.610
1.800	1.593	-1.533
1.850	1.716	-1.451
1.900	1.831	-1.362
1.950	1.936	-1.268
2.000	2.032	-1.168
2.050	2.119	-1.065
2.100	2.197	-.957
2.150	2.265	-.845
2.200	2.325	-.730
2.250	2.375	-.613
2.300	2.416	-.493
2.350	2.448	-.371
2.400	2.471	-.248
2.450	2.485	-.124
2.500	2.490	.000

PROBLEM 9.C5 CONTINUED

PROGRAM OUTPUT

Problem 9.C5 (a)

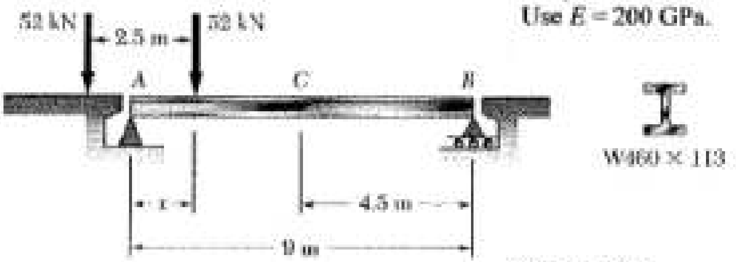
a m	theta B rad*10^-3	y at B mm
0	-3.019	-8.15
0.6	-2.007	-4.7125
1.2	-1.216	-2.3825
1.8	-0.613	-0.935
2.4	-0.168	-0.1675
3.0	0.152	0.0925
3.6	0.377	0

Problem 9.C5 (b)

a m	theta B rad*10^-3	y at B mm
.0	-.728	-1.6389
.2	-.624	-1.3324
.4	-.529	-1.0663
.6	-.442	-.8374
.8	-.364	-.6426
1.0	-.293	-.4789
1.2	-.230	-.3435
1.4	-.174	-.2338
1.6	-.124	-.1472
1.8	-.079	-.0813
2.0	-.040	-.0337
2.2	-.006	-.0024
2.4	.023	.0149
2.6	.049	.0198
2.8	.072	.0143

PROBLEM 9.C7

9.C7 Two 52 kN loads are maintained 2.5 m apart as they are moved slowly across beam *AB*. Write a computer program to calculate the deflection at the midpoint *C* of the beam for values of *x* from 0 to 9 m, using 0.5-m increments. Use $E = 200$ GPa.



SOLUTION

ENTER LOAD P, BEAM LENGTH L AND SPACE BETWEEN LOADS D