Nguyen Xuan Binh 887799 Assignment Week 4

Question 1: For the given state of stress as shown in Fig.1, determine (a) the principal planes, (b) the principal stresses, (c) the maximum shearing stress and the corresponding normal stress. (25 points)

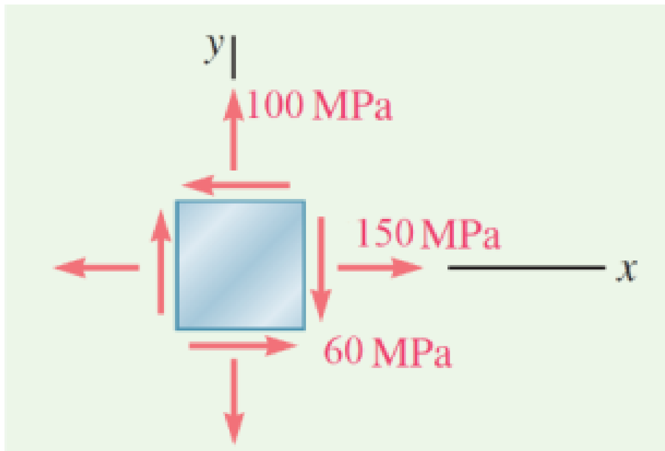


Figure 1

According to the figure, we have : $\sigma_x = 150$ MPa , $\sigma_y = 100$ MPa, $\tau_{xy} = 60$ MPa

a) The principal planes

$$\tan(2\alpha_p) = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -\frac{2(60)}{150-100} = -2.4$$

$$\Rightarrow \begin{cases} 2\alpha_p = -67.38^\circ \\ 2\alpha_p = 112.62^\circ \end{cases} \Rightarrow \begin{cases} \alpha_p = -33.69^\circ \\ \alpha_p = 56.31^\circ \end{cases} \text{(answer)}$$

b) The principle stresses

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 125 \pm 65$$

$$\Rightarrow \begin{cases} \sigma_{max} = 125 + 65 = 190 \text{ MPa (answer)} \\ \sigma_{min} = 125 - 65 = 60 \text{ MPa} \end{cases}$$

c) Maximum stress and corresponding normal stress

We have $(\sigma_\alpha - \sigma_{ave})^2 + \tau_\alpha^2 = R^2 \Rightarrow \tau_{max}$ achieved when $\sigma_\alpha = \sigma_{ave}$

$$\Rightarrow \tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{25^2 + 60^2} = 65 \text{ MPa (answer)}$$

Corresponding normal stress : $\sigma_\alpha = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{150 + 100}{2} = 125$ MPa (answer)

Question 2: Determine the normal stress and shear stress acting on the inclined plane AB in Fig.2. Solve the problem using the stress transformation equations. Show the result on the sectioned element. (25 points)

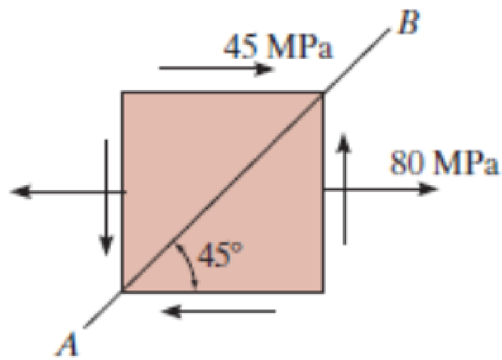
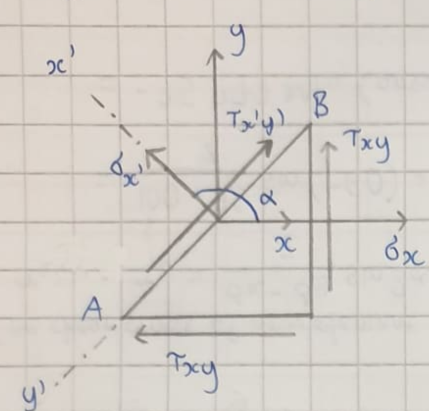


Figure 2

According to the figure, we have $\sigma_x = 80$ MPa, $\sigma_y = 0$, $\tau_{xy} = -45$ MPa

Diagram of stresses acting on the plane



α is the angle from x-axis to the normal axis x' to the principal plane

$\Rightarrow \alpha = 45 + 90 = 135°$

Normal stress acting on the inclined plane AB is given by

$$\sigma_{x'} = \sigma_\alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha - \tau_{xy}\sin 2\alpha$$

$$= \frac{80+0}{2} + \frac{80-0}{2}\cos 270 - (-45)\sin 270$$

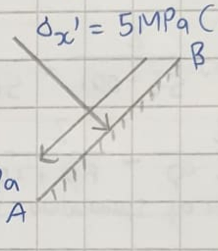
$$= -5 \text{ MPa (answer)}$$

The shear stress acting on plane AB is given by

$$\tau_{x'y'} = \tau_\alpha = \frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau_{xy}\cos 2\alpha = \frac{80-0}{2}\sin 270 + (-45)\cos 270$$

$\Rightarrow \tau_{x'y'} = -40$ MPa

$\Rightarrow$ Normal and shear stress acting on plane



Question 3: For the given state as shown in Fig.3, determine the equivalent state of stress on an element at the same point oriented 30° clockwise with respect to the element shown. Sketch the results on the element. (25 points)

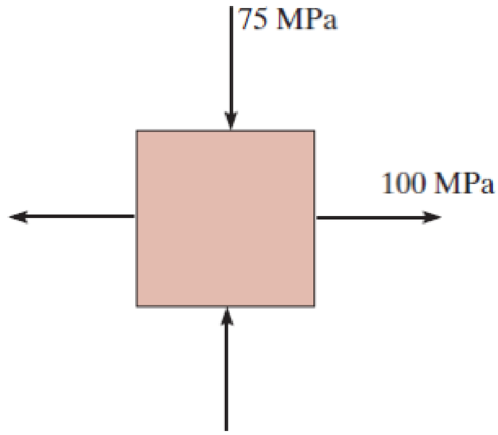
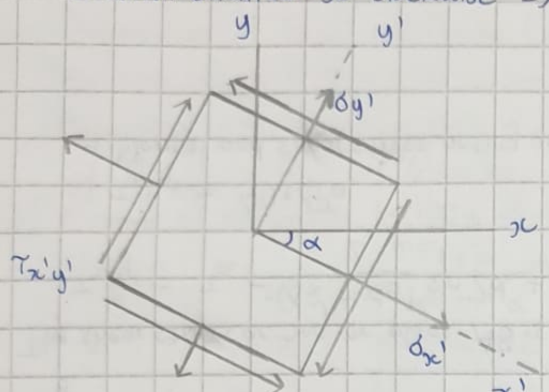


Figure 3

According to the figure, we have $\sigma_x = 100\,MPa$, $\sigma_y = -75\,MPa$, $\tau_{xy} = 0$

Element oriented 30° clockwise $\Rightarrow \alpha = -30^0$



Normal stress components to new element

$$\sigma_{x'} = \sigma_\alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha - \tau_{xy}\sin 2\alpha$$

$$= \frac{100-75}{2} + \frac{100+75}{2}\cos(-60) - 0\sin(-60)$$

$$= 56.25\,MPa \text{ (answer)}$$

$$\sigma_{y'} = \sigma_{\alpha+90^0} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$= \frac{100-75}{2} - \frac{100+75}{2}\cos(-60) = -31.25\,MPa$$

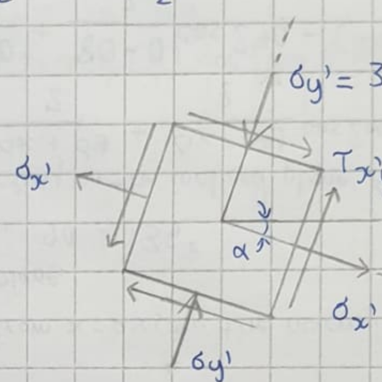
(answer)

The shear stress of new element

$$\tau_{x'y'} = \tau_\alpha = \frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau_{xy}\cos 2\alpha$$

$$= \frac{100+75}{2}\sin(-60) + 0$$

$$= -75.777\,MPa \text{ (answer)}$$



$\sigma_{y'} = 31.25\,MPa$ (Compression)

$\tau_{x'y'} = 75.777\,MPa$

$\sigma_{x'} = 56.25\,MPa$ (Tension)

Question 4: For the given state as shown in Fig.4, determine the equivalent state of stress on an element at the same point oriented 60° clockwise with respect to the element shown. Sketch the results on the element. (25 points)

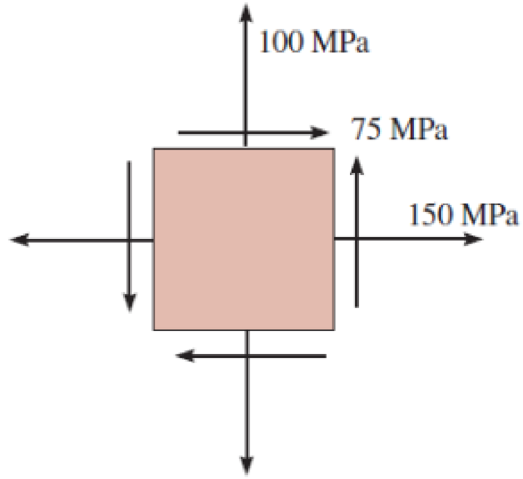
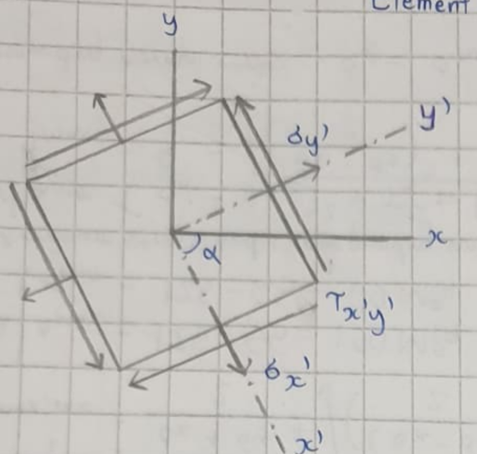


Figure 4

According to the figure, we have $\sigma_x = 150$ MPa, $\sigma_y = 100$ MPa, $\tau_{xy} = -75$ MPa

Element oriented 60° clockwise $\Rightarrow \alpha = -60°$



Normal stress components to new element

$$\sigma_{x'} = \sigma_\alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha - \tau_{xy}\sin 2\alpha$$

$$= \frac{150 + 100}{2} + \frac{150 - 100}{2}\cos(-120) - (-75)\sin(-120)$$

$= 47.548$ MPa (answer)

$$\sigma_{y'} = \sigma_{\alpha + 90°} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$= \left(\frac{150 + 100}{2}\right) - \left(\frac{150 - 100}{2}\right)\cos(-120) - 75\sin(-120)$$

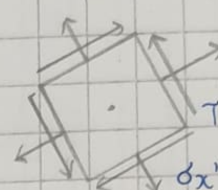
$= 202.45$ MPa (answer)

The shear stress of new element

$$\tau_{x'y'} = \tau_\alpha = \frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau_{xy}\cos 2\alpha$$

$$= \frac{150 - 100}{2}\sin(-120) + (-75)\cos(-120)$$

$= 15.849$ MPa (answer)



$\sigma_{y'} = 202.45$ MPa (Tension)

$\tau_{x'y'} = 15.849$ MPa

$\sigma_{x'} = 47.548$ MPa (Tension)