Foundations of Solid Mechanics - COE-C2001 **Date of examination: 16/12/2021**

Name: Nguyen Xuan Binh, student ID: 887799 **Department of Civil Engineering**

1. For the given state of stress, as shown in **Fig. 1**, determine
   (a) the principal planes;
   (b) the principal stresses;
   (c) the maximum shearing stress and the corresponding normal stress.

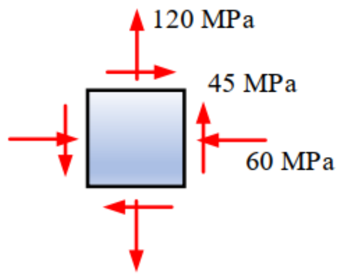


Fig. 1 for Q1

We have: $\sigma_x = -60$ MPa, $\sigma_y = 120$ MPa, $\tau_{xy} = -45$ MPa

a) Principle planes

We have: $\tan 2\alpha_p = -\dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = -\dfrac{2(-45)}{-60-120} = -\dfrac{1}{2}$

$$\Rightarrow \begin{cases} \alpha_{p1} = \frac{1}{2}\tan^{-1}\left(-\frac{1}{2}\right) \\ \alpha_{p2} = \frac{1}{2}\left(\tan^{-1}\left(-\frac{1}{2}\right) + 180\right) \end{cases} \Rightarrow \begin{cases} \alpha_{p1} = -13.28^\circ \\ \alpha_{p2} = 76.71^\circ \end{cases} \text{(answer)}$$

b) Principle stresses

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-60+120}{2} \pm \sqrt{\left(\frac{-60-120}{2}\right)^2 + (-45)^2}$$

$$\Rightarrow \sigma_{max} = 30 + 45\sqrt{5} = 130.623 \text{ MPa (answer)}$$

$$\sigma_{min} = 30 - 45\sqrt{5} = -70.623 \text{ MPa}$$

c) We have: $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\dfrac{\cancel{120}\,-60-120}{2}\right)^2 + (-45)^2} = 100.623$ MPa (ans)

Corresponding normal stress: $\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{-60+120}{2} = 30$ MPa (ans)

2. Shaft AC has a d diameter and shaft CD has a $2d$ diameter, and they are welded at section C and fixed at their ends A and D. Both are made of the same material with a shear modulus G. If they are subjected to a torque T at section B and a torque $2T$ at section C as shown in **Fig. 2**, determine the absolute maximum shear stress in the shaft.

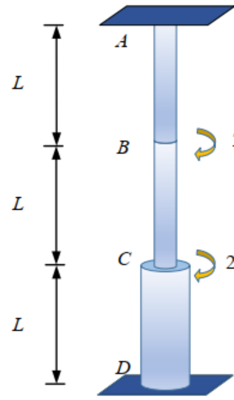
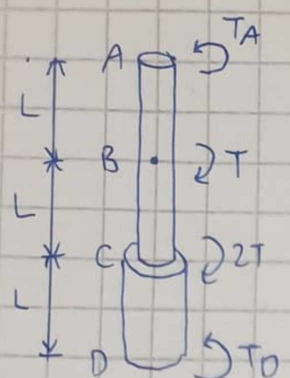


Fig. 2 for Q2



Equilibrium equations: $T_A - T_D = T_B + T_C = T + 2T = 3T$ (1)

Compabílity condition: $\phi_{A/D} = 0$ (Since A and D are fixed)

$$\Rightarrow \phi_{A/B} + \phi_{B/C} + \phi_{C/D} = 0$$

$$\Rightarrow \frac{T_{AB} L_{AB}}{J_{AB} G} + \frac{T_{BC} L_{BC}}{J_{BC} G} + \frac{T_{CD} L_{CD}}{J_{CD} G} = 0$$

$$\Rightarrow \frac{T_A L}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4 G} + \frac{(T_A - T) L}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4 G} + \frac{T_D L}{\frac{\pi}{2} d^4 G} = 0$$

$$\Rightarrow 16 T_A + 16(T_A - T) + T_D = 0 \quad (2)$$

From (1) and (2) $\Rightarrow T_{AB} = \frac{19}{33} T$, $T_{CD} = T_D = -\frac{80}{33} T$, $T_{BC} = T_{AB} - T = -\frac{14}{33} T$

The shear stress at each shaft:

$$\left.\begin{aligned}
\tau_{AB} &= \frac{T_{AB} \cdot c}{\frac{\pi}{2} c^4} = \frac{2 T_{AB}}{\pi c^3} = \frac{2 \cdot \frac{19}{33} T}{\pi \left(\frac{d}{2}\right)^3} = \frac{304 T}{33 \pi d^3} \\
\tau_{BC} &= \frac{T_{BC} \cdot c}{\frac{\pi}{2} c^4} = \frac{2 T_{BC}}{\pi c^3} = \frac{2 \cdot (14/33 \cdot T)}{\pi (d/2)^3} = \frac{224 T}{33 \pi d^3} \\
\tau_{CD} &= \frac{T_{CD} \cdot c}{\frac{\pi}{2} c^4} = \frac{2 T_{CD}}{\pi c^3} = \frac{2 \cdot (80/33 \cdot T)}{\pi d^3} = \frac{160 T}{33 \pi d^3}
\end{aligned}\right\} \Rightarrow \tau_{max} = \tau_{AB} = \frac{304 T}{33 \pi d^3}$$

(answer)

3. As shown in **Fig. 3**, if the bearing plates at A and B support only vertical forces, a uniform distributed load w = 5 kN/m and a concentrated load P = 15 kN are applied to the I shaped beam (the thickness of the flange is the same as that of web, which is 30 mm), determine
   (a) the maximum and minimum normal stress;
   (b) the maximum shear stress.

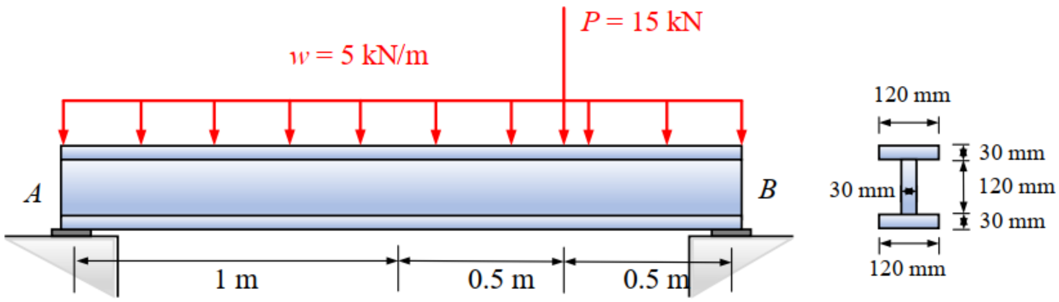
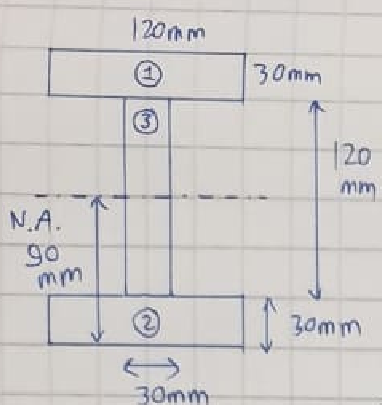


Fig. 3 for Q3



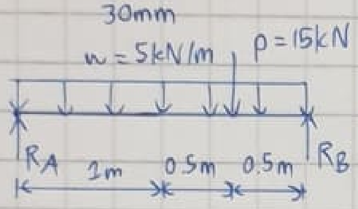
Since this cross section is symmetric along it x and y axis, its neutral axis will lie at the midde of the cross section like in the drawing

Moment of inertia: For (1) and (2) part

$$I_{x_{out}} = \frac{1}{12} 0.12 (30\times10^{-3})^3 + 0.075^2 \times (0.12\times0.03) = 2.052\times10^{-5}\, m^4$$

$$I_{x_{midde}} = 1/12 \cdot 0.03 \cdot 0.12^3 + 0^2 = 4.32\times10^{-6}\, m^4$$

$$\Rightarrow I = 2I_{x_{out}} + I_{x_{middle}} = 4.536\times10^{-5}\, m^4$$



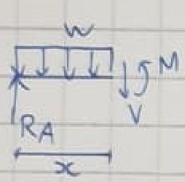
$$\uparrow + \Sigma F_y = 0 \Rightarrow R_A + R_B - P - w(1+0.5+0.5) = 0$$

$$\Rightarrow R_A + R_B = 25\, kN$$

$$\circlearrowleft + \Sigma M_A = 0 \Rightarrow P(1.5m) - R_B(2m) + 2w(1m) = 0$$

$$\Rightarrow \begin{cases} R_A = 8.75\, kN \\ R_B = 16.25\, kN \end{cases}$$

For $0 < x < 1.5m$

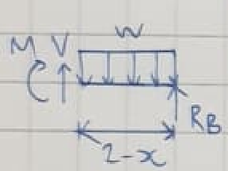


$$\uparrow + \Sigma F_y = 0 \Rightarrow R_A - V - wx = 0 \Rightarrow V = 8.75 - 5x \ (kN)$$

$$\circlearrowleft + \Sigma M = 0 \Rightarrow -M + R_A x - wx\frac{x}{2} = 0$$

$$\Rightarrow M = 8.75x - 2.5x^2 \ (kNm)$$

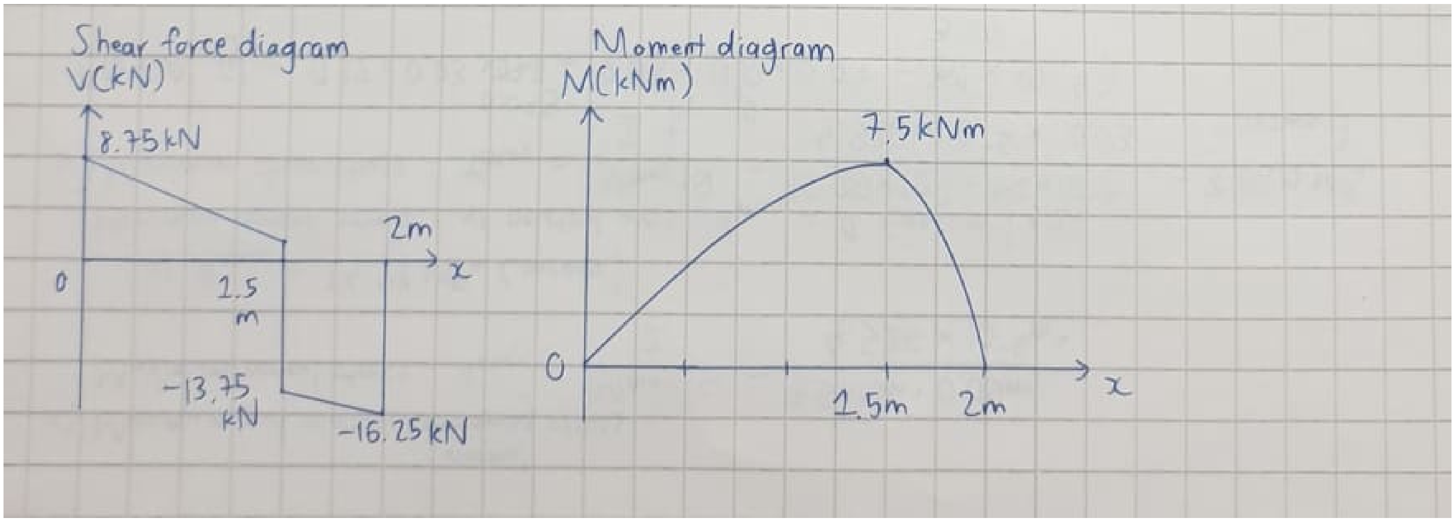
For $1.5m < x < 2m$



$$\uparrow + \Sigma F_y = 0 \Rightarrow \cancel{R_A}\, R_B + V - w(2-x) = 0 \Rightarrow V = -5x - 6.25 \ (kN)$$

$$\circlearrowleft + \Sigma M = 0 \Rightarrow M - R_B(2-x) + w(2-x)\frac{(2-x)}{2} = 0$$

$$\Rightarrow M = -2.5x^2 - 6.25x + 22.5 \ kNm$$



a) Maximum and minimum ~~shearing~~ normal stress

Maximum normal stress: $\sigma_{max} = \dfrac{|M_{max}|c}{I} = \dfrac{7.5\text{kNm} \cdot 0.09\text{m}}{4.536 \times 10^{-5}\text{m}^4}$

$\Rightarrow \sigma_{max} = 14.88$ MPa (answer)

Minimum normal stress: at neutral axis, $\sigma_{min} = 0$ (answer)

b) Maximum shear stress: $\tau_{max} = \dfrac{|V_{max}|Q}{I \cdot t} = \dfrac{16.25 \times 10^3 \times \cancel{8.1}\,3.24 \times 10^{-4}}{4.536 \times 10^{-5} \times 0.03} = 3.869$ MPa (answer)

$(Q = A' \cdot \bar{y}' = 0.12 \times 0.03 \times \cancel{075}\,0.075 + 0.03 \times 0.06 \times 0.03 = \cancel{8.1}\,3.24 \times 10^{-4})$

Foundations of Solid Mechanics - COE-C2001 | **Date of examination: 16/12/2021**

Name: Nguyen Xuan Binh, student ID: 887799 | **Department of Civil Engineering**

4. A gap $\delta = 3$ mm exists between the ends of the two rods shown in **Fig. 4**, and a force P is applied at the bottom end of rod A. Determine the normal stress in the rod A and B when (a) P=100 kN; (b) P=200 kN.

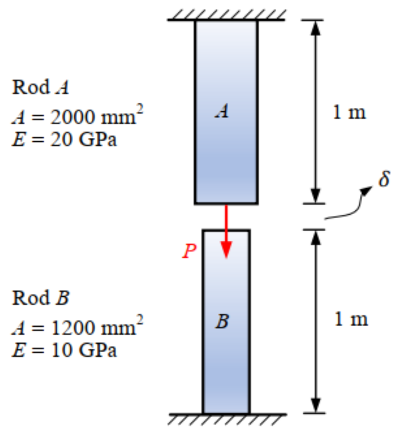
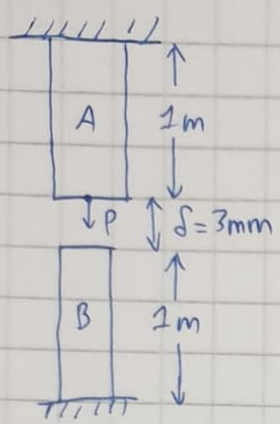


Fig. 4 for Q4



a) When $P = 100\,kN$

$$\Rightarrow \delta = \frac{PL}{EA} = \frac{100\times10^3\times1}{20\times10^9\times2000\times10^{-6}} = 2.5\,mm < \delta = 3mm$$

$\Rightarrow$ Normal stress in rod A: $\sigma = \dfrac{P}{A} = \dfrac{100\times10^3}{2000\times10^{-6}} = 50\,MPa$ (ans)

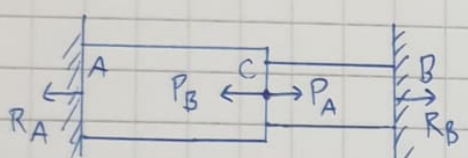
Normal stress in rod B: $\sigma = 0$

b) When $P = 200\,kN$

$$\Rightarrow \delta = \frac{PL}{EA} = \frac{200\times10^3\times1}{20\times10^9\times2000\times10^{-6}} = 5\,mm > \delta = 3\,mm$$

$\Rightarrow$ The bars touch each other

Vertical view



We have: $\delta_{AB} = \dfrac{P_A L_{AC}}{E_A A_A} + \dfrac{P_B L_{BC}}{E_B A_B}$

$$\Rightarrow 2.5\times10^{-8} P_A + \frac{25}{3}\times10^{-8} P_B = 0.003\,m \quad (1)$$

The bars apply on each other equal forces $\Rightarrow P_A = P_B$ (2)

$\Rightarrow$ From (1) & (2): $P_A = P_B = 27692\,N$

$$\Rightarrow \begin{cases} \sigma_A = P_A/A_A = 13.846\,MPa \\ \sigma_B = P_B/A_B = 23.076\,MPa \end{cases}$$ (answer)

5. Determine the reaction force at roller support C and draw the bending moment diagram of the beam shown in **Fig.5**.

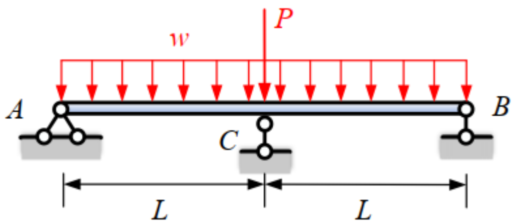
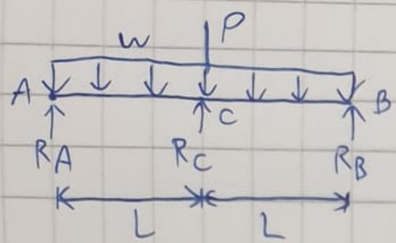


Fig. 5 for Q5

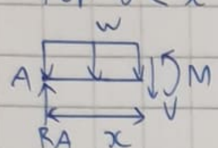


Equilibrium equation:

$\uparrow + \sum F_y = 0 \Rightarrow R_A + R_B + R_C = P + 2wL$

$\circlearrowleft + \sum M_C = 0 \Rightarrow R_A L = R_B L \Rightarrow R_A = R_B$

For $0 < x < L$:



$\sum M = 0 \Rightarrow M - R_A x + wx\frac{x}{2} = 0$

$\Rightarrow M = R_A x - w\frac{x^2}{2}$

$\Rightarrow EIy_1'' = -M_1$

$\Rightarrow EIy_1' = \frac{wx^3}{6} - \frac{R_a x^2}{2} + C_1$

$\Rightarrow EIy_1 = \frac{wx^4}{24} - \frac{R_a x^3}{6} + C_1 x + C_2$

Boundary conditions

$x = 0,\ y_1 = 0$

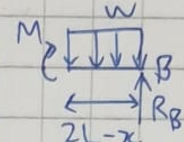
$x = 2L,\ y_2 = 0$

$x = L,\ y_1 = y_2 = 0$

Continuity conditions

$x = L,\ y_1' = y_2'$

For $L < x < 2L$



$\sum M = 0 \Rightarrow M - R_B x + w(2L-x)^2/2 = 0$

$\Rightarrow EIy_2'' = -M_2$

$\Rightarrow EIy_2' = 2wL^2x - wLx^2 + \frac{wx^3}{6} - \frac{1}{2}R_B x^2 + C_1$

$\Rightarrow EIy_2'' = wL^2x^2 - \frac{wLx^3}{3} + \frac{wx^4}{24} - \frac{R_B x^3}{6} + C_1 x^3 + C_2$

This is the best I can try

I hope I can score some few points