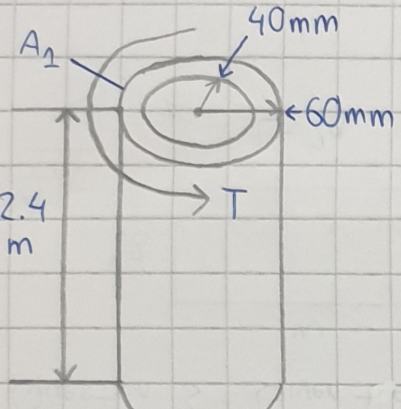
Nguyen Xuan Binh 887799 Assignment Week 2

Question 1:



a) Determine the torque T that causes a maximum shearing stress of 70 MPa in the hollow cylindrical steel shaft

Since this steel shaft is hollow inside, its polar moment of inertia is: $J = \frac{\pi}{2}(c_0^4 - c_i^4) = \frac{\pi}{2}(0.06^4 - 0.04^4)$

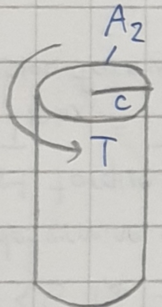
$= 1.6336 \times 10^{-5}\ m^4$

Torsion formula: $\tau = \frac{T\rho}{J} \Rightarrow T = \frac{\tau J}{\rho}$ or $T = \frac{\tau_{max} \cdot J}{c}$

$\Rightarrow T = \frac{70 \cdot 10^6 \times 1.6336 \cdot 10^{-5}}{0.06} = 19058.6\ Nm$

$\approx 19.058\ kN$ (answer)

b) Determine τ_{max} with torque in (a) in solid cylindrical shaft with same crosssectional area



We have same cross sectional area: $A_1 = A_2$

$\Rightarrow \pi(c_2^2 - c_1^2) = \pi c^2 \Rightarrow c_2^2 - c_1^2 = c^2 \Rightarrow c^2 = 0.06^2 - 0.04^2$

$\Rightarrow c^2 = 0.002 m \Rightarrow c = 0.04472 m$

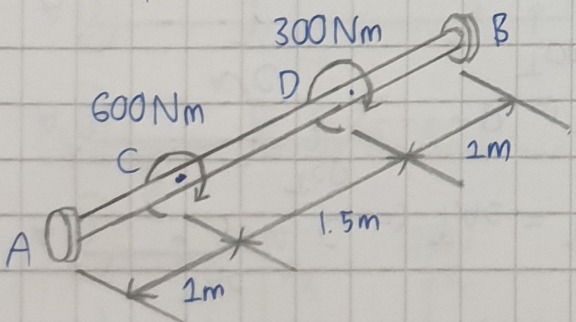
No holes inside $\Rightarrow J = \frac{\pi}{2} c^4 = \frac{\pi}{2} 0.04472^4$

$\Rightarrow J = 6.2824 \times 10^{-6}\ m^4$

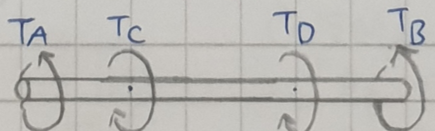
Torsion formula: $\tau_{max} = \frac{T \cdot c}{J} = \frac{19058.6 \times 0.04472}{6.2824 \times 10^{-6}}$

$\Rightarrow \tau_{max} = 135664808 \approx 135,6648\ MPa$ (answer)

Question 2: Steel shaft has diameter $d = 100mm$, fixed at ends A and B. Determine absolute maximum shear stress



Directions of torque at A and B: since C and D torque are in same direction, the torques at A and B should be reverse



where $L_{AC} = L_{DB} = 1m$

$L_{CD} = 1.5m$

Equilibrium equation: $\Sigma M_x = 0$

$\Rightarrow T_A + T_B - T_C - T_D = 0 \Rightarrow T_A + T_B = 600 + 300 = 900\ Nm$

Since the shaft is fixed at A and B, according to compatibility condition, angle of twist between A and B is unchanged

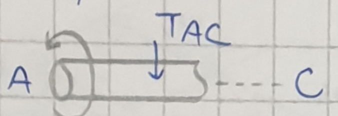
$\Rightarrow \phi_{A/B} = 0$

And we have $\phi_{A/B} = \phi_{A/C} + \phi_{C/D} + \phi_{D/B}$ $\Bigg\} \Rightarrow \phi_{A/C} + \phi_{C/D} + \phi_{D/B} = 0$

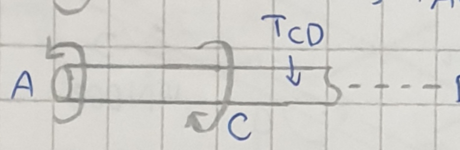
$\Rightarrow \dfrac{1}{JG}\left(T_{AC}L_{AC} + T_{CD}L_{CD} + T_{DB}L_{DB}\right) = 0$

Or $T_{AC}L_{AC} + T_{CD}L_{CD} + T_{DB}L_{DB} = 0$

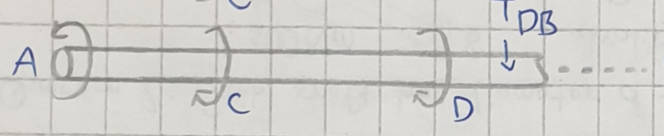
Analysis:



$\Rightarrow T_{AC} = T_A$



$\Rightarrow T_{CD} = T_A - T_C = T_A - 600\,Nm$



$\Rightarrow T_{DB} = T_A - T_C - T_D = T_A - 900\,Nm$

$\Rightarrow T_{AC}L_{AC} + T_{CD}L_{CD} + T_{DB}L_{DB} = T_A L_{AC} + (T_A - 600)L_{CD} + (T_A - 900)L_{DB} = 0$

$\Rightarrow T_A \cdot 1m + (T_A - 600)\,1.5m + (T_A - 900)\cdot 1m = 0$

$\Rightarrow 3.5\,T_A - 1800\,Nm = 0 \Rightarrow T_A \approx 514.28\ Nm$

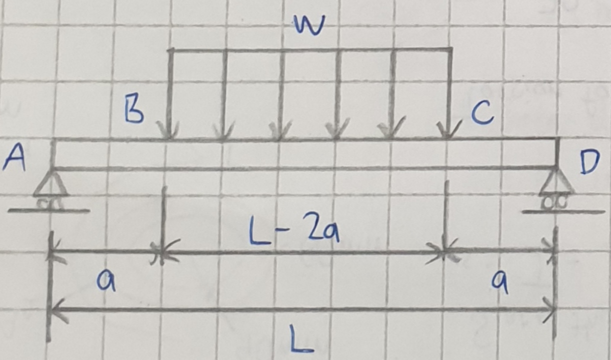
To determine maximum shear stress, we need to find the shear stress of the section with largest torque. From analysis above, we see that $T_{AC} > T_{CD} > T_{DB}$

$\Rightarrow T_{max} = T_{AC} = 514.28\,Nm$

Torsion formula: $\tau_{max} = \dfrac{T_{max}\cdot c}{J} = \dfrac{514.28\,Nm \cdot 0.05}{\frac{\pi}{2}(0.05)^4} = 2619206.5\ Pa$

$\Rightarrow \tau_{max} \approx 2.6192\ MPa$ (answer)

Question 3: Given the beam and loading, draw the shear and bending moment diagram



Support reactions: R_A, R_D

Moment equilibrium: $\circlearrowleft M_A = 0$

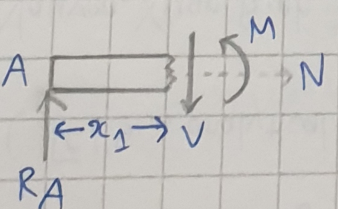
$\Rightarrow w \cdot \dfrac{1}{2}(L-2a)\cdot L - R_D \cdot L = 0$

$\Rightarrow \dfrac{1}{2}w(L-2a) = R_D$

Same calculation for $R_A \Rightarrow R_A = R_D = \dfrac{1}{2}w(L-2a)$

From A to B: $0 \le x_1 < a$

We have:



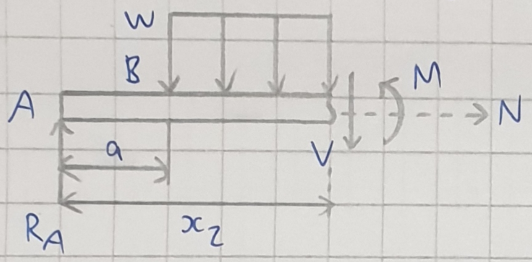
$+\uparrow \Sigma F_y = 0 \Rightarrow R_A - V = 0$

$\Rightarrow V = \dfrac{1}{2}w(L-2a)$

$\circlearrowleft + \Sigma M = 0 \Rightarrow M - \dfrac{1}{2}w(L-2a)x_1 = 0$

$\Rightarrow M = \dfrac{1}{2}w(L-2a)x_1$

From B to C : $a \leq x_2 \leq L-a$



We have:

$+\uparrow \Sigma F_y = 0 \Rightarrow R_A - V - w(x_2 - a) = 0$

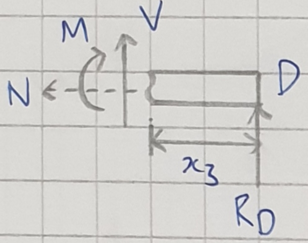
$\Rightarrow V = R_A - w(x_2 - a) = \frac{1}{2}w(L-2a) - w(x_2 - a)$

$\Rightarrow V = w\left(\frac{1}{2}L - x_2\right)$

$\circlearrowleft + \Sigma M = 0 \Rightarrow M - R_A x_2 + w \cdot (x_2 - a)\frac{1}{2}(x_2 - a) = 0$

$\Rightarrow M = \frac{1}{2}w(L-2a)x_2 - \frac{1}{2}w(x_2-a)^2 = \frac{1}{2}w(-x_2^2 + Lx_2 - a^2)$

From C to D : $L-a < x_3 \leq L$. For simplicity, we calculate from the D side



We have:

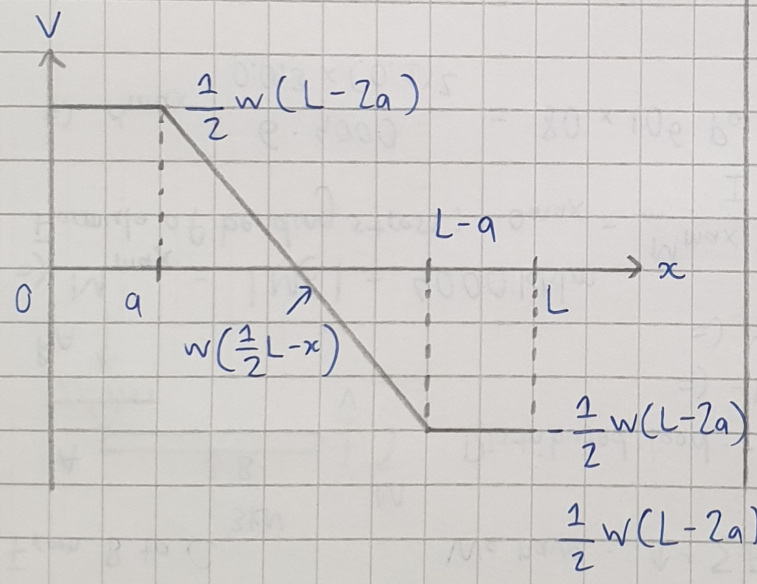
$+\uparrow \Sigma F_y = 0 \Rightarrow R_D + V = 0$

$\Rightarrow V = -\frac{1}{2}w(L-2a)$

$\circlearrowright + \Sigma M = 0 \Rightarrow M - R_D \cancel{x_3} (L - x_3) = 0$

$\Rightarrow M = \frac{1}{2}w(L-2a)(L-x_3)$

$\Rightarrow$ The shear diagram



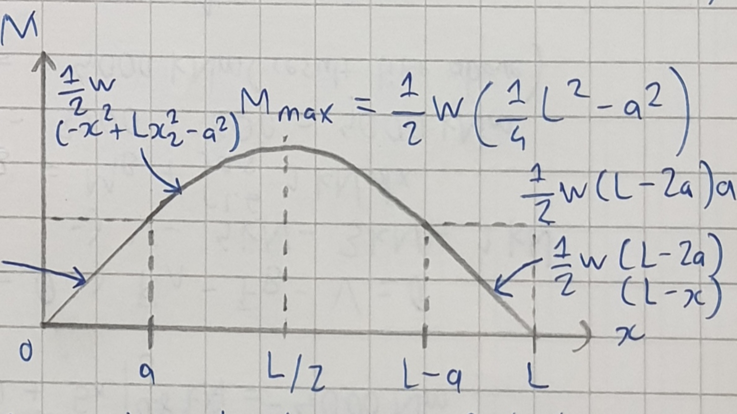
The moment diagram

For x_2: $M = \frac{1}{2}w(-x_2^2 + Lx_2 - a^2)$

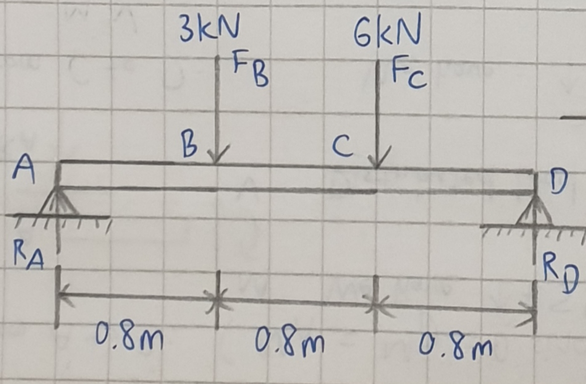
$\Rightarrow M = -1/2\, w\left(x_2^2 - 2\left(\frac{1}{2}L\right)x_2 + \left(\frac{1}{2}L\right)^2 - \left(\frac{1}{2}L\right)^2 + a^2\right)$

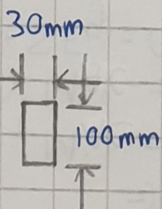
$\Rightarrow M = -1/2\, w\left(\left[x_2 - \frac{1}{2}L\right]^2 + a^2 - \frac{1}{4}L^2\right)$

$\Rightarrow M_{max} : x_2 = \frac{L}{2} \Rightarrow M_{max} = \frac{1}{2}w\left(\frac{1}{4}L^2 - a^2\right)$



Question 4: Given the beam and loading, determine maximum bending stress of the beam





Moment equilibrium at A:

$\circlearrowright + \Sigma M_A = 0$

$\Rightarrow F_B \cdot 0{,}8 + F_C \cdot 1{,}6 - R_D \cdot 2{,}4 = 0$

$\Rightarrow 2{,}4 R_D = 3kN \cdot 0.8 + 6kN \cdot 1.6$

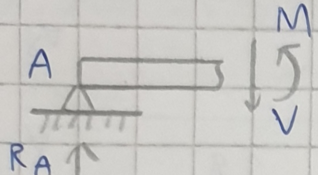
$\Rightarrow R_D = 5kN$

Also: $\uparrow + F_y = 0 \Rightarrow R_A + R_D - F_B - F_C = 0$

$\Rightarrow R_A = 3 + 6 - 5 = 4kN$

Cross sectional area: $0.03 \times 0.1 = 3 \times 10^{-3}\ m^2$

From A to B: ($M_A = M_D = 0$ since they are fixed)



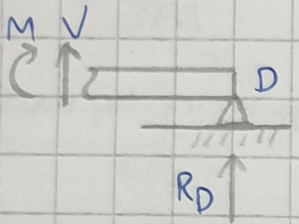
We have $\uparrow + \sum F_y = 0 \Rightarrow R_A - V = 0$

$\Rightarrow V = R_A = 4kN$

Distributed load: $M_B = M_A + \int_0^{0.8m} 4kN\,dx$

$\Rightarrow M_B = 0 + 4x\big|_0^{0.8}\ kN = 3200\ Nm$

From C to D:



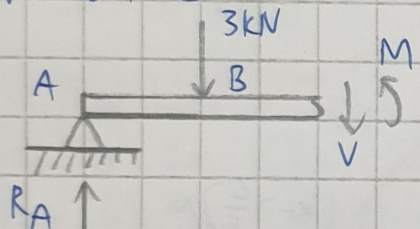
We have: $\uparrow + \sum F_y = 0 \Rightarrow R_D + V = 0$

$\Rightarrow V = -R_D = -5kN$

Distributed load: $M_C = M_D + \int_0^{0.8m} -5kN\,dx$

$\Rightarrow M_C = 0 + 5x\big|_0^{0.8}\ kN = -4000\ Nm$

From B to C



We have: $\uparrow + \sum F_y = 0 \Rightarrow R_A - F_B - V = 0$

$\Rightarrow V = 4kN - 3kN = 1kN$

Distributed load: $-M_C = M_B + \int_{0.8}^{1.6} 1kN\,dx$

$\Rightarrow -M_C = 3200 + 800 = 4000\ Nm$

$\Rightarrow M_C = -4000\ Nm$ (result like above)

$\Rightarrow M_{max} = |M_C| = 4000\ Nm$

Formula of bending stress: $\sigma_{max} = \dfrac{M_{max} \cdot c}{I} = \dfrac{|M_C| \cdot \frac{1}{2}h}{\frac{1}{12}bh^3} = \dfrac{6|M_C|}{bh^2}$

$\Rightarrow \sigma_{max} = \dfrac{6 \cdot 4000}{0.03 \times (0.1)^2} = 80 \times 10^6\ Pa = 80\ MPa$ (answer)