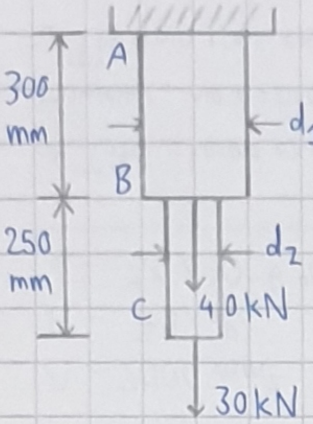
Nguyen Xuan Binh 887799 Assignment Week 1

Question 1 & 2: Two solid cylindrical rods AB and BC are welded together at B and loaded as shown

Q1: Knowing that the average normal stress must not exceed 200 MPa in rod AB and 170 MPa in rod BC, determine the smallest allowable values of d_1 and d_2

Q2: Knowing that $d_1 = 30$ mm and $d_2 = 20$ mm, find the average normal stress at the midsection of (a) rod AB, (b) rod BC



According to figure, $P_{BC} = 30$ kN and $P_{AB} = 30 + 40 = 70$ kN

Question 1: $\sigma_{AB} = 200$ MPa, $\sigma_{BC} = 170$ MPa

We have: $\sigma_{AB} = \frac{P_{AB}}{A_{AB}} \Rightarrow \frac{\pi}{4} d_1^2 = P_{AB} / \sigma_{AB}$

$= 70 \times 10^3 / 200 \times 10^6$

$= 3,5 \times 10^{-4}$

$$\Rightarrow d_1 = \sqrt{\frac{4 \times 3,5 \times 10^{-4}}{\pi}} = 0.0211 \text{ (m)} = 2.11 \text{ cm (answer)}$$

Similarly: $\sigma_{BC} = \frac{P_{BC}}{A_{BC}} \Rightarrow \frac{\pi}{4} d_2^2 = P_{BC} / \sigma_{BC}$

$= 30 \times 10^3 / 170 \times 10^6$

$= 3/17 \times 10^{-3}$

$$\Rightarrow d_2 = \sqrt{\frac{4 \times 3 \times 10^{-3}}{17\pi}} = 0.01498 \text{ (m)} = 1.5 \text{ cm (answer)}$$

Question 2: $d_1 = 30$ mm, $d_2 = 20$ mm, $P_{AB} = 70$ kN, $P_{BC} = 30$ kN

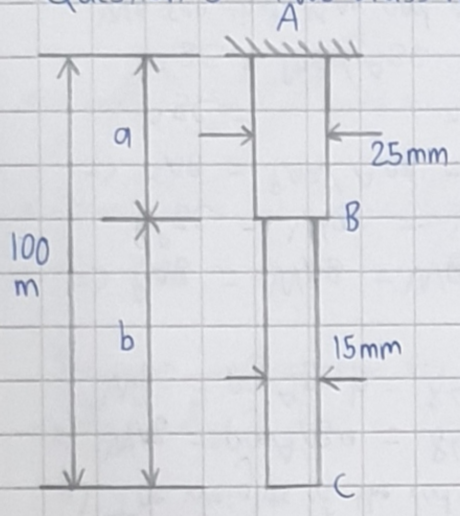
a) Rod AB: $A_{AB} = \frac{\pi d_1^2}{4} = \frac{\pi}{4} (3 \times 10^{-2})^2 = 7.0685 \times 10^{-4} (m^2)$

$$\Rightarrow \sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{70 \times 10^3}{7.0685 \times 10^{-4}} = 99030912 \text{ Pa} \approx 99.03 \text{ MPa (answer)}$$

b) Rod BC: $A_{BC} = \frac{\pi d_2^2}{4} = \frac{\pi}{4} (2 \times 10^{-2})^2 = 3.1415 \times 10^{-4} (m^2)$

$$\Rightarrow \sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{30 \times 10^3}{3.1415 \times 10^{-4}} = 95495782 \text{ Pa} \approx 95.49 \text{ MPa (answer)}$$

Question 3: Two brass rods AB and BC of uniform diameter are brazed together at B to form a non-uniform rod of total length 100m which will be suspended from a support at A as shown. We have $\rho_{brass} = 8470$ kg/m3



Determine

a) Length of rod AB for which maximum normal stress in ABC is min

b) Corresponding value of max normal stress

Since there are no external forces on the rod ⇒ weight of the rods is the only body force that affects the rod's normal stress

a) The weights of the rods are

$W_{AB} = \rho g A_{AB} a = 8470\ kg/m^3 \cdot 9.81 \cdot \frac{\pi}{4}(25 \times 10^{-3})^2 a = 40.787a$

$W_{BC} = \rho g A_{BC} b = 8470\ kg/m^3 \cdot 9.81 \cdot \frac{\pi}{4}(15 \times 10^{-3})^2 a = 14.683b$

$\Rightarrow P_{AB} = W_{AB} + W_{BC} = 40.787a + 14.683b$

$P_{BC} = W_{BC} = \cancel{W_{BC}} = 14.683b$

$\Rightarrow \sigma_{AB} = P_{AB}/A_{AB} = 14.683b / \frac{\pi}{4}(25 \times 10^{-3})^2 + 8470 \cdot 9.81a$

$= 83090.7a + 29911.9b$

$\sigma_{BC} = P_{BC}/A_{BC} = \rho g b = 83090.7b$

We see that σ_{AB} and σ_{BC} are in reverse proportion $\Rightarrow$ to make $\sigma_{ABC\ max}$ to be at its minimum $\Rightarrow \sigma_{AB} = \sigma_{BC} \Rightarrow 83090.7a + 29911.9b = 83090.7b$

$\Rightarrow 83090.7(100 - b) + 29911.9b = 83090.7b$

$\Rightarrow b \approx 60.975\ (m) \Rightarrow a = 100 - b \approx 39{,}025\ (m)$

$\Rightarrow$ Length of rod AB to minimize $\sigma_{ABC\ max}$ is $a = 39{,}025\ (m)$ (answer)

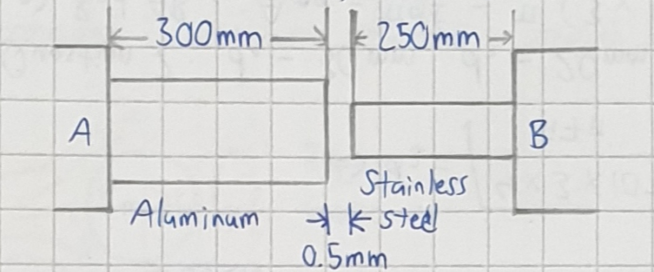
b) Corresponding value of max normal stress: $\sigma_{AB} = \sigma_{BC} = 83090.7 \times 60.975$

$\approx 5{,}066\ MPa$ (answer)

Question 4: At room temperature ($20°C$) a 0,5 mm gap exists between ends of the rod as shown. At $150°C$, determine:

a) Normal stress in aluminum rod

b) Change in length of aluminium rod



Analysis: when temperature rises, the two bars will expand / elongate towards each other.

Thermal expansion formula: $\delta_T = \alpha \Delta T \cdot L$

Let A and S subscript for aluminum and stainless ~~stell~~ steel

□ Aluminum expansion

Aluminum	Stainless steel
$A = 2000\ mm^2$	$A = 800\ mm^2$
$E = 75\ GPa$	$E = 190\ GPa$
$\alpha = 23 \times 10^{-6}\ °C$	$\alpha = 17.3 \times 10^{-6}\ °C$

a) $\delta_{TA} = \alpha_A \Delta T L_A = 23 \times 10^{-6} \cdot (150 - 20) \times 0.3$

$= 0.897\ mm$

$\delta_{TS} = \alpha_S \Delta T L_S = 17.3 \times 10^{-6} \cdot 130 \times 0.25 = 0.562\ mm$

$\Rightarrow$ Total thermo expansion: $\delta_T = \delta_{TA} + \delta_{TS} = 0.897 + 0.562 = 1.459\ mm$

However, we notice that $\delta_T >$ gap $= 0.5\ mm$ while the other ends of the rods are fixed

$\Rightarrow$ The two rods expansion make them come into contact and impose on each other stress

$\Rightarrow \delta_T - \delta_P = gap \Rightarrow \delta_P = \delta_T - gap = 1.459 - 0.5 = 0.959 = \delta_{PA} + \delta_{PS}$

* $\delta_P = \delta_{PA} + \delta_{PB} = \frac{P_A L_A}{E_A A_A} + \frac{P_S L_S}{E_A A_S}$ The two bars act on each other opposite, equal forces $\Rightarrow P_A = P_S$

$\Rightarrow P_A\left(\frac{L_A}{E_A A_A} + \frac{P_S L_S}{E_A A_S}\right) = \delta_P \Rightarrow P_A\left(\frac{300 \cdot 10^{-3}}{75 \times 10^9 \times 2000 \times 10^{-6}} + \frac{250 \times 10^{-3}}{190 \times 10^9 \times 800 \times 10^{-6}}\right)$

$= 0.959\ mm$

$\Rightarrow P_A = 263119\ N = P_S \approx 263\ kN$

The normal stress in the aluminum rod ~~is~~ is compressive stress since the rods are compressed ~~afth~~ after the 0.5 mm gap is filled

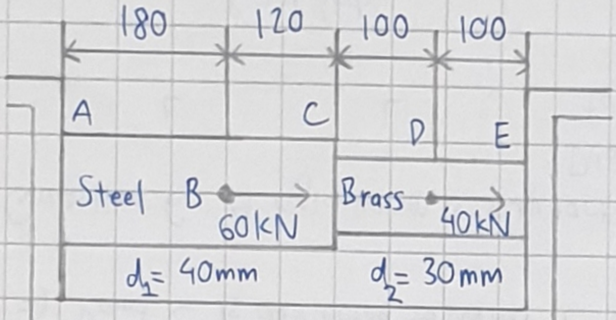
$\Rightarrow \sigma_A = -\dfrac{P_A}{A_A} = -\dfrac{263119}{2000\times10^{-6}} = -131559500\ Pa \approx -131.559\ MPa$ (answer)

b) Analysis: the actual change in length in the rod is the difference between force ~~and~~ compression and thermal expansion

$\Rightarrow$ Change in length of aluminium rod: $\delta_A = \delta_{T_A} - \delta_{P_A} = \alpha_A \Delta T L_A - \dfrac{P_A L_A}{E_A A_A}$

$\Rightarrow \delta_A = 0.897_n \times 10^{-3} - \dfrac{263119 \times 300 \times 10^{-3}}{75\times10^{9} \times 2000\times10^{-6}} = 0.3707\ mm$ (answer)

Question 5: Two cylindrical rods of steel and brass are joined at C and restrained by rigid supports at A and E. Determine



a) Reactions at A and E

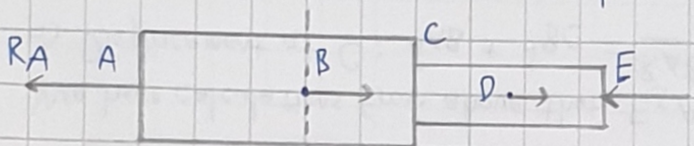
b) Deflection of point C

We have: $A_{steel} (A_S) = \dfrac{\pi}{4} d_1^2 = \dfrac{\pi}{4} 0.04^2 = 1.256\times10^{-3}\ m^2$

$A_{brass} (A_{br}) = \dfrac{\pi}{4} d_2^2 = \dfrac{\pi}{4} 0.03^2 = 0.7068\times10^{-3}\ m^2$

$E_S = 200\ GPa,\ E_b = 105\ GPa$

a) Lets assume that resultant force at point A and E are as follows



Since we need to find the pressure in each component, we start analyzing from A to E

* From A to B: A is fixed and B has a force away from A $\Rightarrow$ AB is in tension and pressure in AB causes resultant force $R_A \Rightarrow P_{AB} = R_A \Rightarrow$ Elongation: $\delta_{AB} = \dfrac{P_{AB} l_{AB}}{E_S A_S} = \dfrac{R_A l_{AB}}{E_S A_S}$

* From B to C: We have $F_B > F_D$ and both F_B, F_D are in same direction and E is fixed $\Rightarrow$ BC is in compression and $P_{BC} = R_A - F_B \Rightarrow \delta_{BC} = \dfrac{(R_A - F_B) l_{BC}}{E_S A_S}$

* From C to D: same as above but in different rod $\Rightarrow P_{CD} = R_A - F_B \Rightarrow \delta_{CD} = \dfrac{(R_A - F_B) l_{CD}}{E_{B_r} A_{B_r}}$

* From D to E: In compression as both F_B and F_D are exerting force on $\Rightarrow P_{DE} = R_A - F_B - F_D \Rightarrow \delta_{DE} = \dfrac{(R_A - F_B - F_D) l_{DE}}{E_{Br} A_{Br}}$

However, we know that the rods are fixed at the ends, so sum of all displacements should be 0

$\Rightarrow \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 0$

$$\Rightarrow R_A\left(\frac{l_{AB}+l_{BC}}{E_S A_S}+\frac{l_{CD}+l_{DE}}{E_{Br}A_{Br}}\right)=\frac{F_B l_{BC}}{E_S A_S}+\frac{F_B l_{CD}+(F_B+F_D)l_{DE}}{E_{Br}A_{Br}}$$

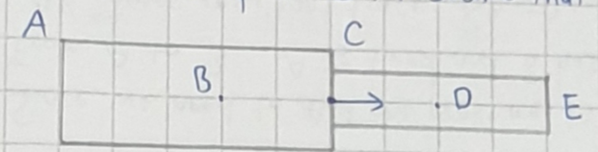
$$\Rightarrow R_A\left(\frac{0.18+0.12}{200\times10^9\times1.256\times10^{-3}}+\frac{0.1+0.1}{105\times10^9\times0.7068\times10^{-3}}\right)=\frac{60\times10^3\times0.12}{200\times10^9\times1.256\times10^{-3}}+\frac{60\times10^3\times0.1+(100\times10^3)\times0.1}{105\times10^9\times0.7068\times10^{-3}}$$

$\Rightarrow 3.8883\times10^{-9}R_A = 2.4422\times10^{-4} \Rightarrow R_A \approx 62809\,N \approx 62.8\,kN$ (answer)
(Direction like drawing)

Resultant forces of A and E should equal the forces

$\Rightarrow R_A + R_E = F_B + F_D \Rightarrow R_E = F_B + F_D - R_A = (100000 - 62809)$
$= 37191\,N \approx 37.19\,kN$ (answer)
(Direction like drawing)

b) Deflection at point C: Assume that point C moves towards E



$\Rightarrow |\delta_{AB}+\delta_{BC}| = |\delta_{CD}+\delta_{DE}|$
and $\delta_{AB}+\delta_{BC}$ is positive and $\delta_{CD}+\delta_{DE}$ is negative

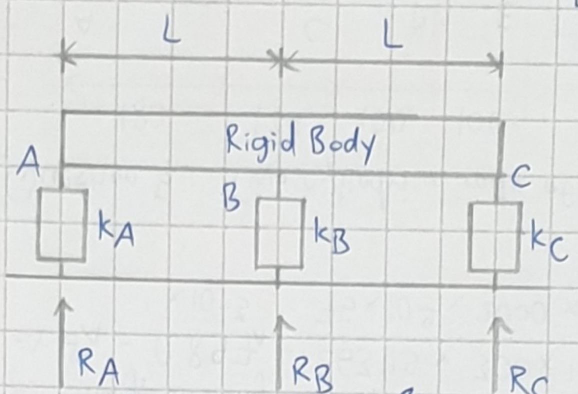
We have calculations from above that $E_S A_S = 251.32\times10^6$ and $E_{Br}A_{Br} = 74.22\times10^6$

$\Rightarrow$ Displacement at C: $\delta_{AB}+\delta_{BC} = \frac{R_A l_{AB} + (R_A - F_B)l_{BC}}{E_S A_S} = 4.632\times10^{-5}\,m$ (positive)

$\delta_{CD}+\delta_{DE} = \frac{(R_A - F_B)l_{CD} + (R_A - F_B - F_D)l_{DE}}{E_{Br}A_{Br}} = -4.632\times10^{-5}\,m$ (negative)

$\Rightarrow$ Point C is deflected towards E at $4.632\times10^{-5}\,m$ (answer)

Question 6: A rigid beam is supported by 3 springs, where $k_A = 2k_B = 4k_C$. Dead weight of rigid beam is $5wL$. Determine reaction forces R_A, R_B, R_C



The rigid beam is static

$\Rightarrow \uparrow\Sigma F_y = 0: R_A + R_B + R_C - \cancel{2}5wL = 0$ (1)

$\circlearrowright \Sigma M_B = 0: -R_A L + R_C L = 0 \Rightarrow -R_A + R_C = 0$ (2)

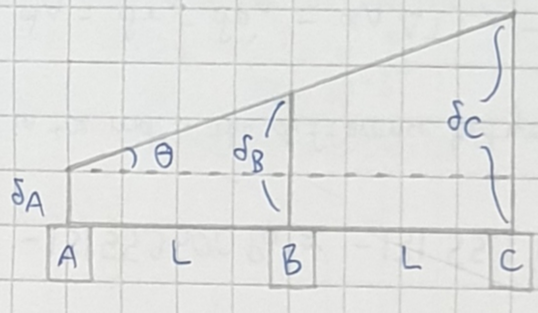
We have $k_A > k_B > k_C \Rightarrow \delta_A < \delta_B < \delta_C$

$\Rightarrow$ Diagram of before and after rigid beam is placed

$$\Rightarrow \theta = \frac{\delta_B - \delta_A}{L} = \frac{\delta_C - \delta_{\cancel{A}B}}{L} = \frac{\delta_C - \delta_A}{2L}$$

$$\Rightarrow \frac{R_B/k_B - R_A/k_A}{L} = \frac{R_C/k_C - R_{\cancel{A}B}/k_{\cancel{A}B}}{L}$$

$$\Rightarrow \frac{R_B}{k_B} - \frac{R_A}{k_A} = \frac{R_C}{k_C} - \frac{R_B}{k_B}$$



$\Rightarrow$ Let $k_A = 4$, $k_B = 2$, $k_C = 1$ (Because $k_A = 2k_B = 4k_C$)

$$\Rightarrow \frac{R_B}{2} - \frac{R_A}{4} = \frac{R_C}{1} - \frac{R_B}{2} \Rightarrow -R_A + 4R_B - 4R_C = 0 \quad (3)$$

From equation (1) (2) (3) $\Rightarrow$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & 4 & -4 \end{bmatrix} \begin{bmatrix} R_A \\ R_B \\ R_C \end{bmatrix} = \begin{bmatrix} 5wL \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} R_A \\ R_B \\ R_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & 4 & -4 \end{bmatrix}^{-1} \begin{bmatrix} 5wL \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 20wL/13 \\ 25wL/13 \\ 20wL/13 \end{bmatrix} \Rightarrow R_A = R_C = \frac{20wL}{13}, \; R_B = \frac{25wL}{13} \text{ (answer)}$$