Nguyen Xuan Binh 887799 Assignment Week 6

Question 1: The rigid bar AD is attached to two springs of constant k and is in equilibrium in the position shown in Fig.1. Knowing that the equal and opposite loads P and P` remain horizontal, determine the magnitude P_{cr} of the critical load for the system. (40 points)

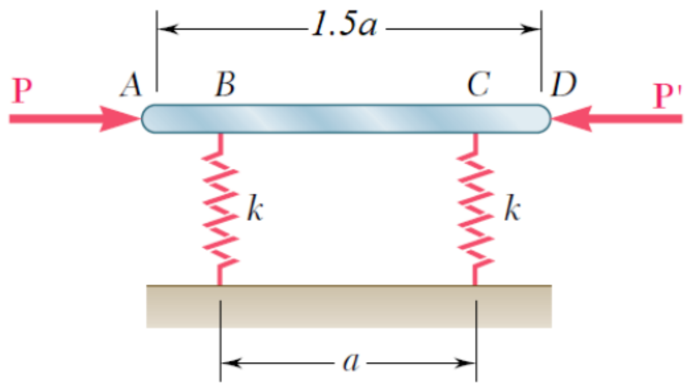
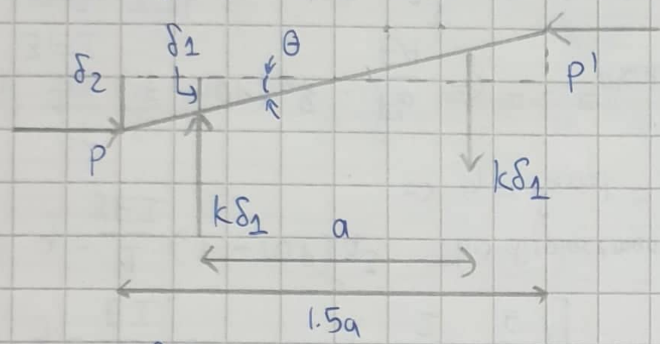


Fig.1

The rigid bar is in equilibrium: $\Sigma F_x = 0 \Rightarrow P = P'$

Over the critical load, the bar starts to rotate

Rotation angle is θ



$\Rightarrow \delta_1 = \frac{a}{2}\tan\theta, \quad \delta_2 = \frac{1.5a}{2}\tan\theta$

Moment equilibrium around the center of the bar: $\circlearrowleft + \Sigma M = 0$

$\Rightarrow 2 \times (P \cdot \delta_2) = 2 \times (k\delta_1 \cdot a/2)$

$\Rightarrow 2P\delta_2 = k\delta_1 a$

$\Rightarrow P = \frac{k\delta_1 a}{2\delta_2}$

$$\Rightarrow P_{cr} = \frac{k \cdot a/2 \cdot \tan\theta \cdot a}{2 \cdot \frac{1.5a}{2}\tan\theta} = \frac{ak}{3} \text{ (answer)}$$

Question 2: A frame as shown in Fig.2 is loaded with uniformly distributed design load q_d and a point design load F_d (a safety factor is already considered). Square-section woods are used for the frame structure (E=13 GPa, σ_{all} =12 MPa)

(a) Draw the diagrams for shear, bending moment and axial force, respectively.

(b) Considering the axial force only, determine the size of the column section if the column is to safely support the loads. (60 points)

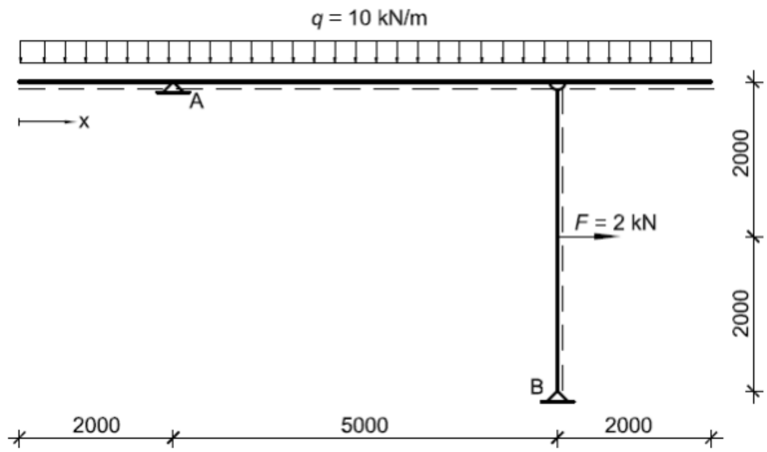


Fig.2

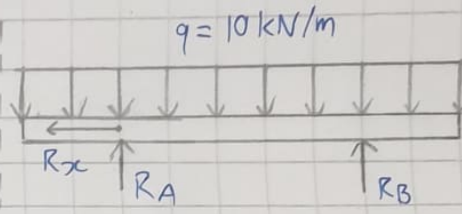
From the drawing, equilibrium equation is

$\uparrow + \sum F_y = 0 \Rightarrow R_A + R_B - 10\ kN/m \times 9m = 0$

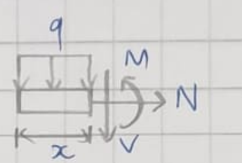
$\circlearrowleft + \sum M_A = 0 \Rightarrow -90kN \cdot 2.5m - R_B \cdot 5m = 0$

$\overset{+}{\leftarrow} \sum F_x = 0 \Rightarrow 2R_x - F = 0$

$\Rightarrow R_A = R_B = 45kN$, $R_x = 1kN$



- For $0 \leq x < 2m$

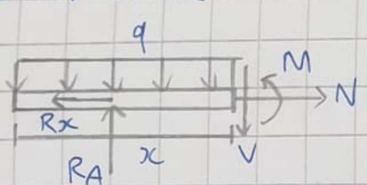


$\uparrow + \sum F_y = 0 \Rightarrow V = -10x\ kN$

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow N = 0$

$\circlearrowleft + \sum M = 0 \Rightarrow M = -5x^2\ kNm$

- For $2m \leq x < 7m$



$\uparrow + \sum F_y = 0 \Rightarrow -V - qx + R_A = 0$

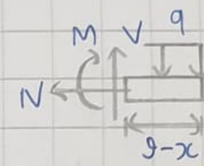
$\Rightarrow V = 45 - 10x\ (kN)$

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow N = 1kN$

$\circlearrowleft + \sum M = 0 \Rightarrow M + 10x(x/2) - R_A(x-2) = 0$

$\Rightarrow M = -5x^2 + 45x - 90\ (kNm)$

- For $7m \leq x < 9m$



$\uparrow + \sum F_y = 0 \Rightarrow V - q(9-x) = 0$

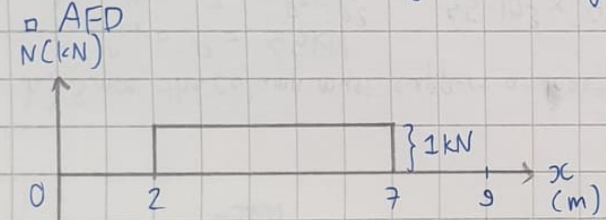
$\Rightarrow V = 90 - 10x\ (kN)$

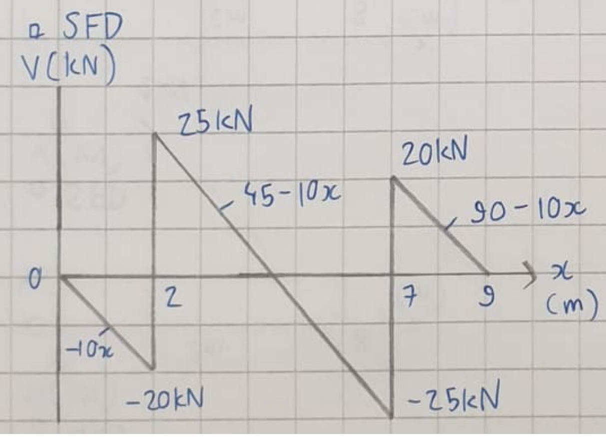
$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow N = 0$

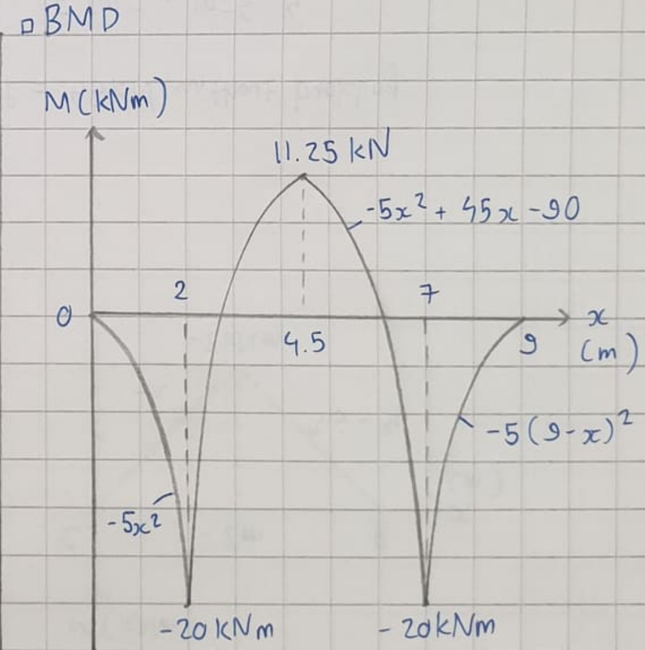
$\circlearrowright + \sum M = 0 \Rightarrow M + q(9-x)^2 \frac{1}{2} = 0$

$\Rightarrow M = -5(9-x)^2\ (kNm)$

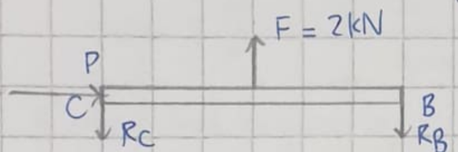
a) The axial, shear and bending moment diagram of the frame





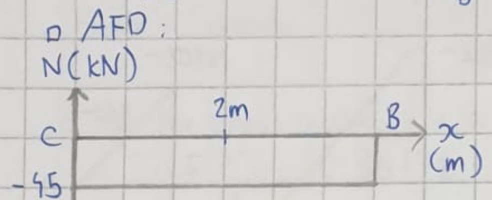


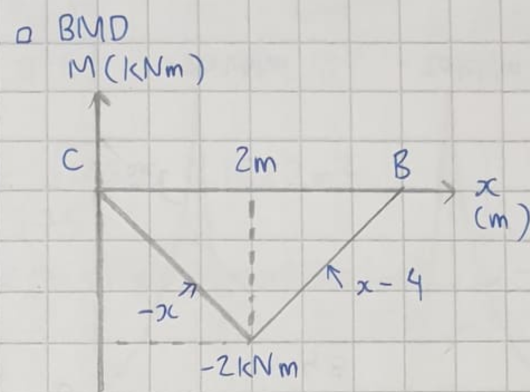
AFD, SFD, BMD of the supporting column. Name intersection of frame and column as C

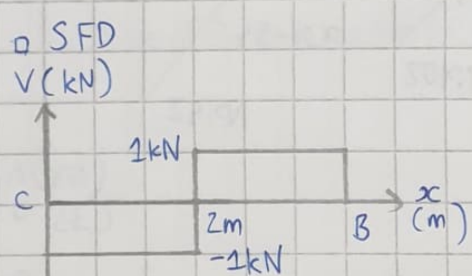


We have $R_C = R_B = 1$ kN due to symmetry

$P = R_{B(previous)} = 45$ kN







b) Since the column must support at least $P = 45kN$ without buckling

$\Rightarrow P_{cr} = P = 45kN$

We have $I = \dfrac{P_{cr}\,\ell^2}{\pi^2 E} = \dfrac{45 \cdot 10^3 \times 4^2}{\pi^2 \times 13 \cdot 10^9} = 5.6116 \times 10^{-6} m^4$

Size of the cross section: $I = \dfrac{a^4}{12} = \dfrac{A^2}{12} \Rightarrow A = \sqrt{12 I} = 8.206 \times 10^{-3} m^2$

Check if the cross section area can withstand the allowable stress

$$\sigma = \frac{P}{A} = \frac{45000}{8.206 \times 10^{-3}} = 5.483 \text{ MPa}, \quad \sigma_{allow} = 12 \text{ MPa}$$

$\Rightarrow A = 8.206 \times 10^{-3} m^2$ of the column can safely support the frame