Nguyen Xuan Binh 887799 Midterm report

Question 1: Determine the maximum normal and minimum normal stress in the bracket (Fig. 1) at section a-a when the load is applied at x = 250mm. (Note: Maximum and minimum denote "maximum positive value" and "maximum negative value", respectively). (6 points)

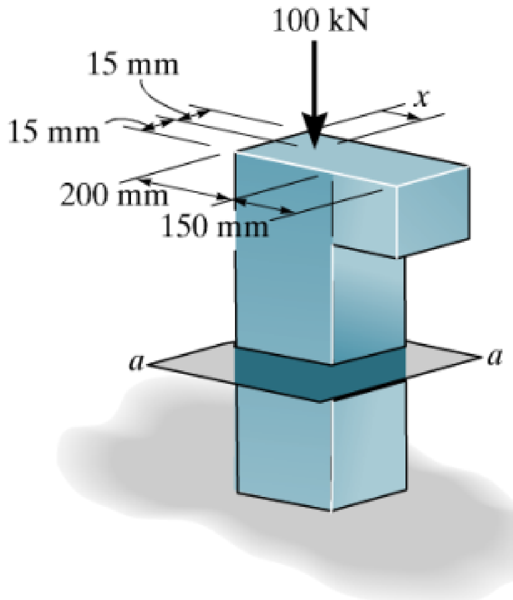
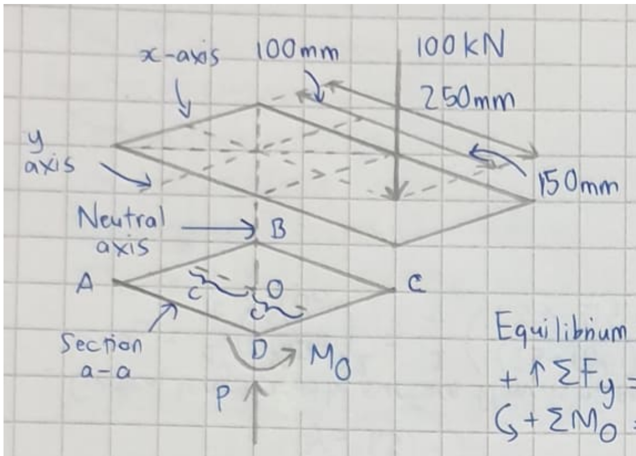


Fig. 1. for Q1



Cross sectional area:

$A = 200 \times 30 = 6000\,mm^2 = 6 \times 10^{-3}\,m^2$

Moment of inertia of the cross section with respect to the y axis

$I_y = \frac{1}{12} \times 0.03 \times (0,2)^3 = 2 \times 10^{-5}\,m^4$

Equilibrium equation:

$+\uparrow \Sigma F_y = 0 \Rightarrow \underbrace{P - 100\,kN}_{=0} \Rightarrow P = 100\,kN$

$\circlearrowleft + \Sigma M_O = 0 \Rightarrow M_O - 100\,kN \cdot (0.15) = 0$

$\Rightarrow M_O = 15\,kNm$

Normal stress: $\sigma_N = \frac{P}{A} = \frac{100 \times 10^3}{6 \times 10^{-3}} \approx 16.667 \times 10^6\,Pa = 16.667\,MPa$

Stress due to bending moment: $\sigma_B = \frac{M_O \cdot c}{I_y} = \frac{15 \times 10^3 \cdot 0.1}{2 \times 10^{-5}} = 75 \times 10^6\,Pa = 75\,MPa$

For edge AB on section a-a: $\sigma = -\sigma_N + \sigma_B = 58.333\,MPa$ (tensile stress)

For edge CD on section a-a: $\sigma = -\sigma_N - \sigma_B = -91.667\,MPa$ (compression stress)

$\Rightarrow \sigma_{min} = -91.667\,MPa$ (compression), $\sigma_{max} = 58.333\,MPa$ (tension) (answer)

Question 2: The assembly shown in Fig. 2 consists of two 10-mm diameter red brass C83400 copper rods AB and CD, a 15-mm diameter 304 stainless steel rod EF, and a rigid bar G. (a) If P = 5 kN, determine the horizontal displacement of end F of rod EF, (b) If the horizontal displacement of end F of rod EF is 0.45 mm, determine the magnitude of P (6 points)

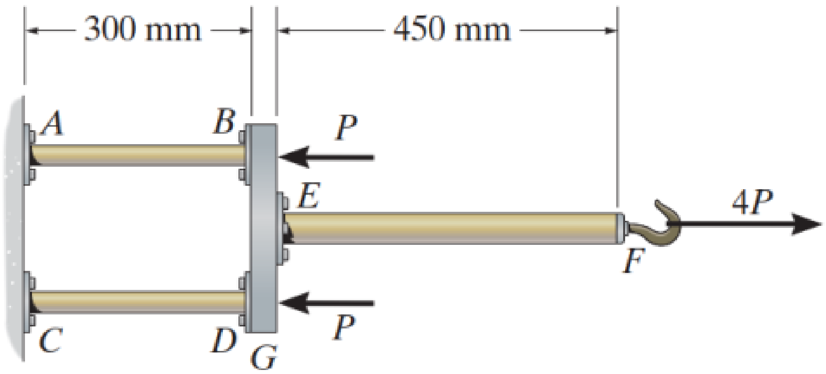


Fig. 2. for Q2

Cross sectional area of red brass rods: $A_b = \frac{\pi}{4} d^2 = 78.539 \times 10^{-6}\ m^2 = A_{AB} = A_{CD}$

Cross sectional area of stainless steel rod: $A_s = \frac{\pi}{4} d^2 = 176.714 \times 10^{-6}\ m^2 = A_{EF}$

Since rod AB and CD are the same physically, their total displacement will be the same

$\Rightarrow \delta_{AB} = \delta_{CD}$. The total displacement will be: $\Sigma\delta = \delta_{AB} + \delta_{EF}$

$\Rightarrow \Sigma\delta = \frac{P_{AB} L_{AB}}{A_{AB} E_{AB}} + \frac{P_{EF} L_{EF}}{A_{EF} E_{EF}} = \frac{P \cdot L_{AB}}{A_{AB} E_{brass}} + \frac{4P \cdot L_{EF}}{A_{EF} E_{steel}}$

a) $P = 5kN \Rightarrow \Sigma\delta = ?$ (E_{brass} and E_{steel} can be found with their model specification)

$\Sigma\delta = \frac{5 \times 10^3 \cdot 0.3}{78.539 \times 10^{-6} \times 101 \times 10^9} + \frac{4 \times 5 \times 10^3 \times 0.45}{176.714 \times 10^{-6} \times 193 \times 10^9} = 4.529 \times 10^{-4}\ m$

$\Rightarrow \Sigma\delta \approx 4.529\ mm$ (answer)

b) If $\Sigma\delta = 0.45 mm$, $P = ?$

$\Rightarrow 0.45 \times 10^{-3} = \frac{P \times 0.3}{78.539 \times 10^{-6} \times 101 \times 10^9} + \frac{4P \times 0.45}{176.714 \times 10^{-6} \times 193 \times 10^9}$

$\Rightarrow 0.45 \times 10^{-3} = 9.05963 \times 10^{-8} \times P \Rightarrow P \approx 4967 N = 4.967\ kN$ (answer)

Question 3: The assembly is made of stainless steel and consists of a solid rod 25 mm in diameter fixed to the inside of a tube using a rigid disk at B, as shown in Fig. 3. The shear modulus of stainless steel is 77 GPa. The tube has an outer diameter of 45 mm and wall thickness of 5 mm. Determine the angle of twist at D. (6 points)

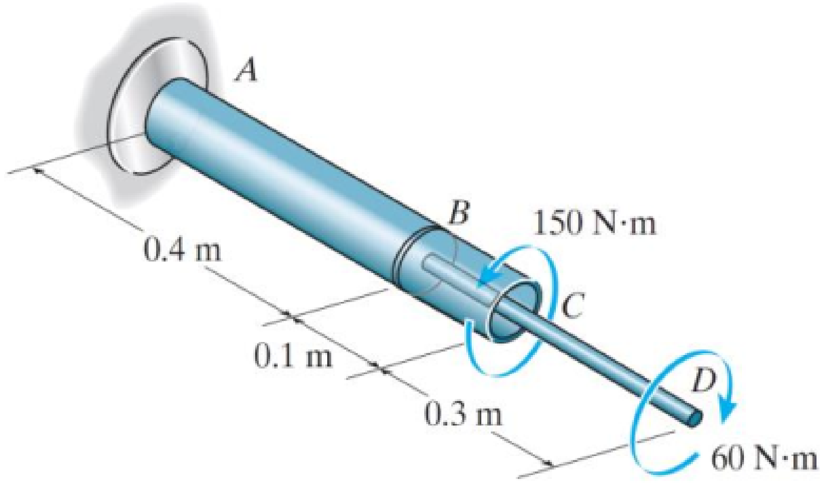


Fig. 3. for Q3

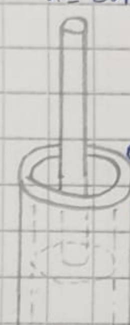
Analysis: total amount of torsion in the assembly should be 0, as A is fixed

Let positive direction the same direction of the torque at D

$\Rightarrow T_{AB} + T_{BD} - T_{BC} = 0 \Rightarrow T_{AB} = 150\,Nm - 60\,Nm = 90\,Nm$

Angle of twist at D is the angle of twist of rod AB and BD

$\Rightarrow \phi_D = \phi_{A/B} + \phi_{B/D}$



The polar moment of inertia:

$J_{AB} = \frac{\pi}{2}\left(c^4_{\substack{out\\tube}} - c^4_{\substack{in\\tube}}\right) = \frac{\pi}{2}\left(0.0225^4 - 0.0175^4\right)$

$= 2.5525 \times 10^{-7}\ m^4$

$J_{BD} = \frac{\pi}{2} c^4_{rod} = \frac{\pi}{2}(0.0125^4) = 3.8349 \times 10^{-8}\ m^4$

$\Rightarrow c_{rod} = 12.5mm$

$c_{\substack{out\\tube}} = 22.5mm$

$c_{\substack{in\\tube}} = 17.5mm$

Angle of twist at D:

$\phi_D = \phi_{A/B} + \phi_{B/D} = \frac{T_{AB}L_{AB}}{J_{AB}G} + \frac{T_{BD}L_{BD}}{J_{BD}G}$

$\Rightarrow \phi_D = \frac{90 \times 0.4}{2.5525 \times 10^{-7} \times 77 \times 10^9} + \frac{-60 \times 0.4}{3.8349 \times 10^{-8} \times 77 \times 10^9}$

$\Rightarrow \phi_D \approx -6.296 \times 10^{-3}\ rad = -0.3607^\circ$ or angle of twist at D is 0.36° with direction of torque at BC

Question 4: Two cylindrical rods are joined at B and restrained by rigid supports at A and D. Two loads are applied at B and C respectively, as shown in Fig. 4. The Young's modulus and cross-sectional area of two rods are shown in the figure. Determine (10 points)

(a) the reactions at A and D;

(b) the deflection of point B.

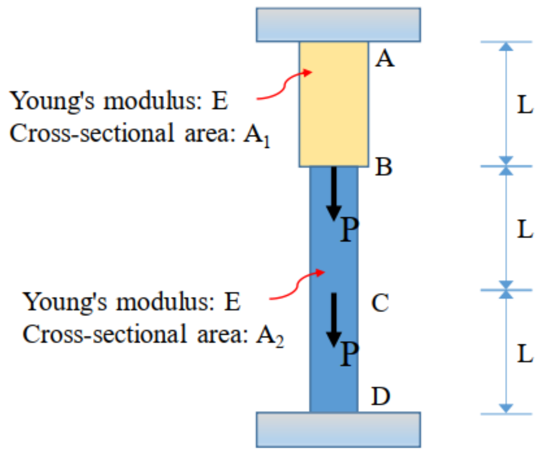
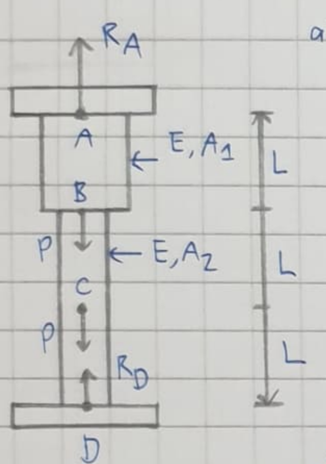


Fig. 4. for Q4



a) We have: $\delta_{AB} = \frac{R_A L}{EA_1}$, $\delta_{BC} = \frac{(R_A - P)L}{EA_2}$, $\delta_{CD} = \frac{(R_A - 2P)L}{EA_2}$

The rods are fixed at A and D. Directions of R_A and R_D are like in the drawing

$\Rightarrow \delta_{AB} + \delta_{BC} + \delta_{CD} = 0$

$\Rightarrow \frac{R_A L}{EA_1} + \frac{(R_A - P)L}{EA_2} + \frac{(R_A - 2P)L}{EA_2} = 0$

Multiply both side with E and divide both side by L

$\Rightarrow \frac{R_A}{A_1} + \frac{2R_A - 3P}{A_2} = 0 \Rightarrow R_A A_2 + (2R_A - 3P)A_1 = 0$

$\Rightarrow R_A A_2 + 2R_A A_1 = 3PA_1$

$\Rightarrow R_A(A_2 + 2A_1) = 3PA_1 \Rightarrow R_A = \frac{3PA_1}{2A_1 + A_2}$ (answer)

We have: $R_A + R_D = 2P \Rightarrow R_D = \frac{2P(2A_1 + A_2)}{2A_1 + A_2} - \frac{3PA_1}{2A_1 + A_2} = \frac{P(A_1 + 2A_2)}{2A_1 + A_2}$ (answer)

b) Deflection of point B: $\delta_{AB} = \frac{R_A L}{EA_1} = \frac{3PA_1}{2A_1 + A_2} \cdot \frac{L}{EA_1} = \frac{3PL}{E(2A_1 + A_2)}$ (answer)

Direction of deflection of point B is the same direction as force P

Question 5: The three suspender bars are made of steel and have equal cross-sectional areas of 450 mm^2. Determine the average normal stress in each bar if the rigid beam is subjected to the loading shown in Fig. 5. (8 points)

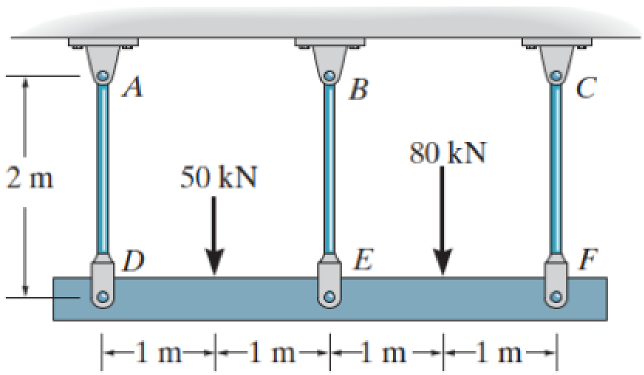
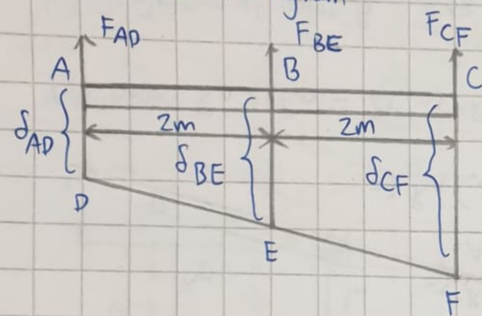


Fig. 5. for Q5

Analysis: the suspended bars are deflected when the loads are applied. Since 50kN < 80 kN, the amount of deflection will be greater in rod CF

$\Rightarrow \delta_{CF} > \delta_{BE} > \delta_{AD}$. Since the rigid beam is fixed, deflections don't occur physically

Deflection diagram



We have: $\dfrac{\delta_{BE}-\delta_{AD}}{2m} = \dfrac{\delta_{CF}-\delta_{BE}}{2m}$

$\Rightarrow \delta_{BE}-\delta_{AD} = \delta_{CF}-\delta_{BE}$

$\Rightarrow 2\delta_{BE} = \delta_{AD}+\delta_{CF}$

$\Rightarrow 2\dfrac{F_{BE}\cdot 2m}{AE} = \dfrac{F_{AD}\cdot 2m}{AE} + \dfrac{F_{CF}\cdot 2m}{AE}$

$\Rightarrow 2F_{BE} = F_{AD} + F_{CF}$ (1)

Rigid beam is fixed

$\Rightarrow \uparrow + \Sigma F_y = 0 \Rightarrow F_{AD} + F_{BE} + F_{CF} - 50\text{ kN} - 80\text{ kN} = 0$

$\Rightarrow F_{AD} + F_{BE} + F_{CF} = 130\text{ kN}$ (2)

$\Rightarrow \circlearrowleft + \Sigma M_E = 0 \Rightarrow F_{AD}\cdot(2m) - 50\text{kN}\cdot(1m) + 80\text{kN}(1m) - F_{CF}(2m) = 0$

$\Rightarrow 2F_{AD} - 2F_{CF} = -30\text{kN}$ (3)

From (1)(2)(3)

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} F_{AD} \\ F_{BE} \\ F_{CF} \end{bmatrix} = \begin{bmatrix} 0 \\ 130000 \\ -30000 \end{bmatrix} \Rightarrow \begin{bmatrix} F_{AD} \\ F_{BE} \\ F_{CF} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 107500 \\ 130000 \\ 152500 \end{bmatrix} \text{(N)}$$

Average normal stress in each bar: $\sigma = F/A$

$$\begin{bmatrix} \sigma_{AD} \\ \sigma_{BE} \\ \sigma_{CF} \end{bmatrix} = \frac{1}{3\times 450\times 10^{-6}} \begin{bmatrix} 107500 \\ 130000 \\ 152500 \end{bmatrix} \Rightarrow \begin{cases} \sigma_{AD} = 79.629\text{ MPa} \\ \sigma_{BE} = 96.296\text{ MPa} \\ \sigma_{CF} = 112.962\text{ MPa} \end{cases}$$ (answer)

Question 6: As shown in Fig. 6, The magnesium tube is bonded to a steel rod. If the allowable shear stresses for the magnesium and steel are (τall)mg = 45 MPa and (τall)st = 75 MPa, respectively, determine the maximum allowable torque that can be applied at A. Also, find the corresponding angle of twist of end A. (6 points)

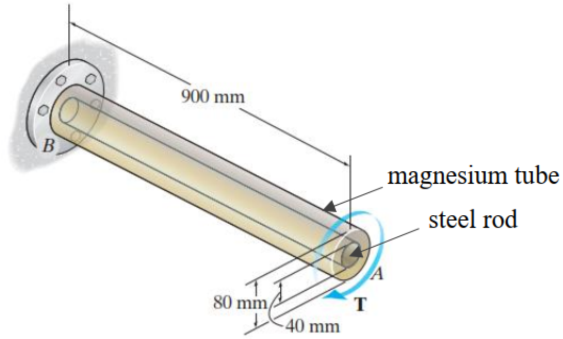
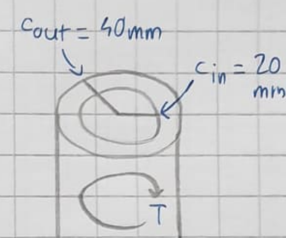


Fig. 6. for Q6

Let mg denotes magnesium tube and st denotes steel rod. Since both magnesium tube and steel rod are unified into one structure

$$\Rightarrow \begin{cases} T_{mg} + T_{st} = T \\ \phi_{st/A} = \phi_{mg/A} \end{cases}$$



We have:

- $(\tau_{all})_{mg} = \dfrac{T_{mg} \cdot c_{out}}{J_{mg}} = \dfrac{T_{mg} \cdot c_{out}}{\frac{\pi}{2}(c_{out}^4 - c_{in}^4)}$
$= T_{mg} \dfrac{0.04}{\frac{\pi}{2}(0.04^4 - 0.02^4)} = T_{mg} \cdot 10610.329 \quad (1)$

- $(\tau_{all})_{st} = \dfrac{T_{st} \cdot c_{in}}{J_{st}} = \dfrac{T_{st} \cdot c_{in}}{\frac{\pi}{2} c_{in}^4} = T_{st} \dfrac{0.02}{\frac{\pi}{2}(0.02^4)} = T_{st} \times 79577.471 \quad (2)$

- $\phi_{st/A} = \phi_{mg/A} \Rightarrow \dfrac{T_{st} L}{J_{st} G_{st}} = \dfrac{T_{mg} L}{J_{mg} G_{mg}} \Rightarrow \dfrac{T_{st}}{\frac{\pi}{2}(0.02^4) \times 75 \times 10^9} = \dfrac{T_{mg}}{\frac{\pi}{2}(0.04^4 - 0.02^4) \times 18 \times 10^9}$

$\Rightarrow 5.30516 \times 10^{-5}\, T_{st} = 1.47365 \times 10^{-5}\, T_{mg}$

$\Rightarrow T_{st} = 0.27776\, T_{mg}$
Also: $T_{mg} + T_{st} = T$
$\Rightarrow \begin{cases} T_{st} = 0.2173T \quad (3) \\ T_{mg} = 0.7826T \quad (4) \end{cases}$

From (1)(4) $\Rightarrow (\tau_{all})_{mg} = 0.7826 \cdot 10610.329\,T = 8303.6434\,T$

$\Rightarrow T = \dfrac{45 \times 10^6}{8303.6434} = 5419.3 \text{ Nm}$

From (2)(3) $\Rightarrow (\tau_{all})_{st} = 0.2173 \cdot 79577.471\,T = 17292.184\,T$

$\Rightarrow T = \dfrac{75 \times 10^6}{17292.184} = 4337.2 \text{ Nm}$

$\Rightarrow$ Maximum allowable torque: $T = 4337.2$ Nm corresponding to steel rod (answer)

$\Rightarrow T_{st} = 0.2173T = 0.2173 \times 4337.2 = 942.473$ Nm

Corresponding angle of twist at end A:

$\phi_A = \phi_{st/A} = 5.30516 \times 10^{-5}\, T_{st} \times L = 5.30516 \times 10^{-5} \cdot 942.473 \cdot 0.9$

$\Rightarrow \phi_A = 0.04499$ rad $= 2.577°$ (answer) - Direction like torque applied at A

Question 7: As shown in Fig. 7, the built-up beam is subjected to an internal moment of M = 75 kN·m. Determine the amount of this internal moment resisted by plate A. (10 points)

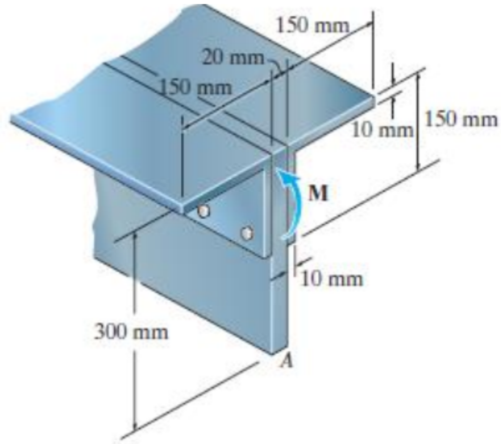
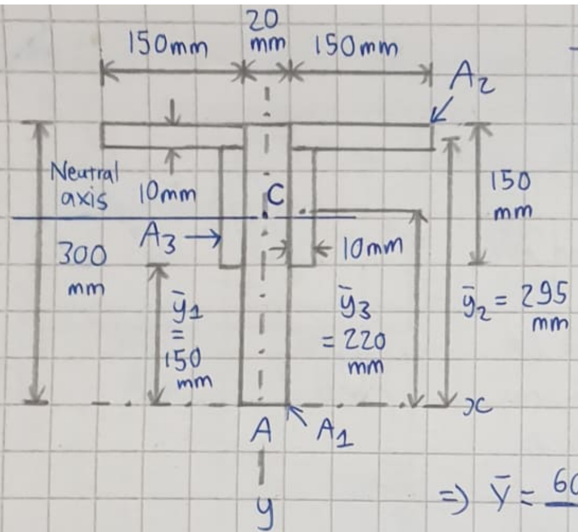


Fig. 7. for Q7



The shape is divided into two rectangles like the drawing. The centroid of the plate A is expected to lie on y-axis as it is symmetry axis $\Rightarrow \bar{X} = 0$. The ordinate Y of the centroid is

$$\bar{Y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{A_1\bar{y}_1 + 2A_2\bar{y}_2 + 2A_3\bar{y}_3}{A_1 + 2A_2 + 2A_3}$$

We have: $A_1 = 20 \times 300 = 6000\ mm^2$

$A_2 = 10 \times 150 = 1500\ mm^2$

$A_3 = 10 \times 140 = 1400\ mm^2$

$$\Rightarrow \bar{Y} = \frac{6000 \times 150 + 2 \times 1500 \times 295 + 2 \times 1400 \times 220}{6000 + 2 \times 1500 + 2 \times 1400}$$

$\Rightarrow \bar{Y} = 203.47\ mm = 0.20347\ m$ (centroid located in the drawing)

Now we need to compute moment of inertia around the neutral axis

- Rectangular Area A_1

$$(\bar{I}_{x'})_1 = \frac{1}{12} \cdot 0.02 \cdot 0.3^3 = 4.5 \times 10^{-5}\ m^4$$

$$\Rightarrow (I_x)_1 = (\bar{I}_{x'})_1 + A_1 d_1^2 = 4.5 \times 10^{-5} + 6 \times 10^{-3} \times (0.20347 - 0.15)^2$$
$$= 6.2154 \times 10^{-5}\ m^4$$

- Rectangular Area A_2

$$(I_x)_2 = \frac{1}{12} bh^3 + A_2 d_2^2 = \frac{1}{12} \cdot 0.15 \cdot (0.01)^3 + 1.5 \times 10^{-3} (0.20347 - 0.295)^2$$
$$= 1.2579 \times 10^{-5}\ m^4$$

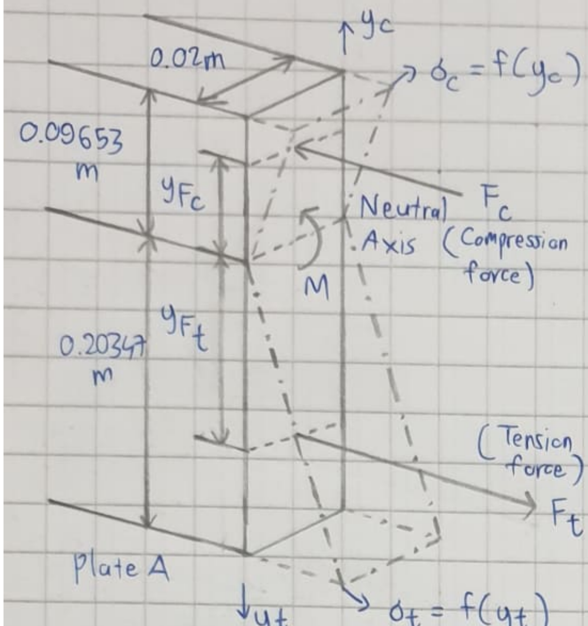
- Rectangular Area A_3

$$(I_x)_3 = \frac{1}{12} b^3 h + A_3 d_3^2 = \frac{1}{12} \cdot (0.14)^3 \cdot 0.01 + 1.4 \times 10^{-3} (0.20347 - 0.220)^2$$
$$= 2.6692 \times 10^{-6}\ m^4$$

$\Rightarrow$ Moment of inertia of the entire area with respect to the neutral axis

$$\bar{I}_x = (I_x)_1 + 2(I_x)_2 + 2(I_x)_3$$
$$= 6.2154 \times 10^{-5} + 2 \times 1.2579 \times 10^{-5} + 2 \times 2.6692 \times 10^{-6}$$
$$= 9.265 \times 10^{-5} \, m^4$$

Diagram of moment ~~atta~~ on plate A



We have: $\sigma_c = f(y_c) = \dfrac{M}{I} y_c \; (y_c \in [0; 0.09653])$

$\circ$ $dF_c = \sigma_c dA_c \Rightarrow dF_c = \dfrac{M}{I} y_c \; 0.02 \, dy_c$

$$\Rightarrow F_c = \int dF_c = \int_0^{0.09653} \frac{M}{I} 0.02 \, y_c \, dy_c$$

$$F_c = 0.02 \int_0^{0.09653} \frac{75 \times 10^3}{9.265 \times 10^{-5}} y_c \, dy_c$$

$\Rightarrow F_c = 75429.36 \, N$ (upper compression)

Position of F_c from neutral axis

$$y_{F_c} = \frac{2}{3} \cdot (0.09653) = 0.0643533 \, m$$

Similarly, we can compute F_t

$\sigma_t = f(y_t) = \dfrac{M}{I} y_t \; (y_t \in [0; 0.20347])$

$$\Rightarrow F_t = 0.02 \int_0^{0.20347} \frac{75 \times 10^3}{9.265 \times 10^{-5}} y_t \, dy_t = 335132.5 \, N \text{ (below tension)}$$

Position of F_t from neutral axis: $y_{F_t} = \dfrac{2}{3} \cdot (0.20347) = 0.135646 \, m$

Finally, the amount of internal moment resisted by plate A is

$$M_{internal} = F_c \cdot y_{F_c} + F_t \cdot y_{F_t} = 75429.36 \times 0.0643533 + 335132.5 \times 0.135646$$

$\Rightarrow M_{internal} = 50313.5 \, Nm \approx 50.31 \, kNm$ (answer)

Question 8: Determine the maximum normal stress and shear stress developed in the beam shown in Fig. 8. E = 100 GPa. (12 points)

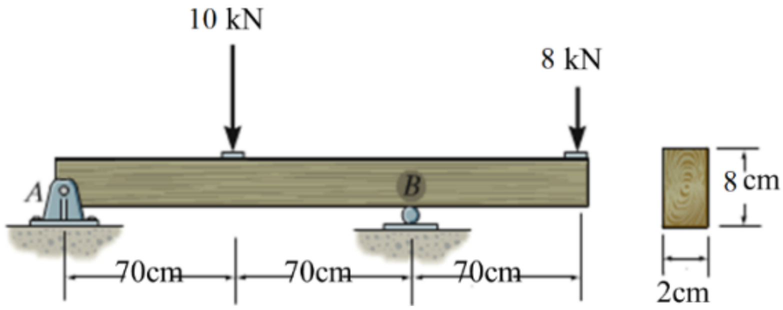


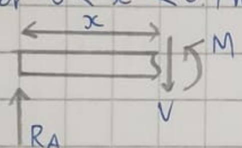
Fig. 8. for Q8

The beam is fixed

$$\Rightarrow \begin{cases} \uparrow + \sum F_y = 0 \rightarrow R_A + R_B - 10kN - 8kN = 0 \\ \circlearrowleft + \sum M_A = 0 \rightarrow -10kN \cdot 0.7 + R_B \cdot 1.4 - 8kN \cdot 2.1 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} R_A + R_B = 18kN \\ 1.4 R_B - 23.8 = 0 \end{cases} \Rightarrow \begin{cases} R_A = 1\ kN \\ R_B = 17kN \end{cases}$$

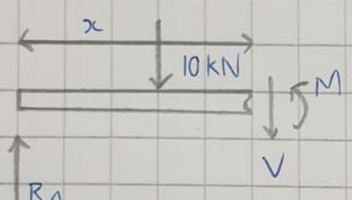
For $0 < x < 0.7m$



$\uparrow + \sum F_y = 0 \Rightarrow R_A - V = 0 \Rightarrow V = 1\ kN$

$\circlearrowleft + \sum M = 0 \Rightarrow M - R_A x = 0 \Rightarrow M = x\ kNm$

For $0.7 < x < 1.4m$

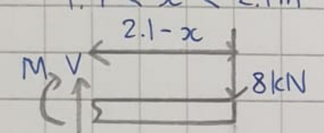


$\uparrow + \sum F_y = 0 \Rightarrow R_A - 10kN - V = 0 \Rightarrow V = -9kN$

$\circlearrowleft + \sum M = 0 \Rightarrow M + 10kN(x - 0.7m) - R_A \cdot x = 0$

$\Rightarrow M = 7 - 9x\ (kNm)$

For $1.4 < x < 2.1m$

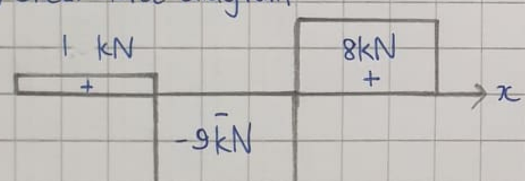


$\uparrow + \sum F_y = 0 \Rightarrow V - 8kN = 0 \Rightarrow V = 8kN$

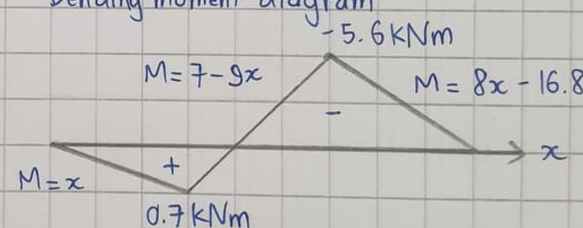
$\circlearrowleft + \sum M = 0 \Rightarrow -M - 8kN(2.1 - x) = 0$

$\Rightarrow M = 8x - 16.8\ (kNm)$

$\Rightarrow$ Shear force diagram



Bending moment diagram



$\Rightarrow$ Maximum normal stress: $\sigma_{max} = \dfrac{M_{max} \cdot y}{I} = \dfrac{5.6 \times \frac{1}{2}(0.08)}{\frac{1}{12}(0.02)(0.08)^3} = 262.5\ kPa$ (answer)

Maximum shear stress: $\tau_{max} = \dfrac{V_{max} Q}{I \cdot t} = \dfrac{9 \times 0.02 \cdot 0.04 \cdot \frac{1}{2} 0.04}{\frac{1}{12}(0.02)(0.08)^3 \times 0.02} = 8.4375\ kPa$ (answer)

Question 9: A timber beam AB of length L and rectangular cross section carries a uniformly distributed load w and is supported as shown in the Fig. 9. (a) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to 2h/L, where h and L are the depth and the length of the beam, respectively, (b) Determine the depth h and the width b of the beam, knowing that L = 4 m, w = 10 kN/m, τ_m = 1.28 MPa, and σ_m = 9.8 MPa. (15 points)

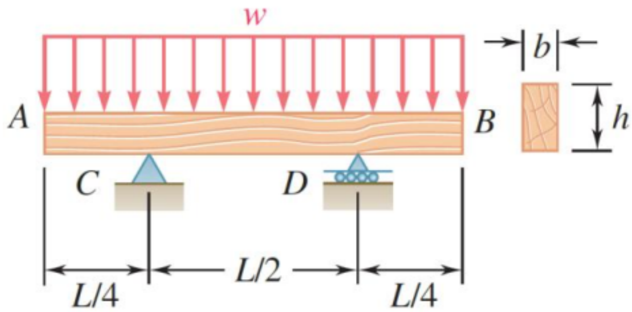
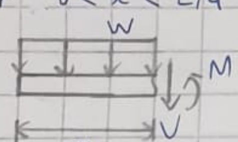


Fig. 9. for Q9

The beam is fixed

$$\Rightarrow \begin{cases} \uparrow + \Sigma F_y = 0 \\ \circlearrowleft + \Sigma M_C = 0 \end{cases} \Rightarrow \begin{cases} R_C + R_D = wL \\ -wL \cdot L/4 + R_D \cdot L/2 = 0 \end{cases} \Rightarrow \begin{cases} R_C = \frac{1}{2} wL \\ R_D = \frac{1}{2} wL \end{cases}$$

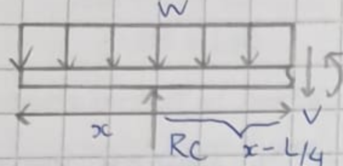
For $0 < x < L/4$



$$\uparrow + \Sigma F_y = 0 \Rightarrow -w \cdot x - V = 0 \Rightarrow V = -wx$$

$$\circlearrowleft + \Sigma M = 0 \Rightarrow M + wx \cdot \frac{x}{2} = 0 \Rightarrow M = -\frac{1}{2} wx^2$$

For $L/4 < x < 3L/4$

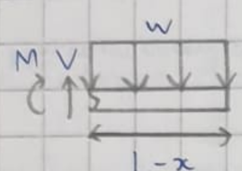


$$\uparrow + \Sigma F_y = 0 \Rightarrow -w \cdot x + R_C - V = 0 \Rightarrow V = -wx + \frac{1}{2} wL$$

$$\circlearrowleft + \Sigma M = 0 \Rightarrow M - R_C(x - \tfrac{L}{4}) + wx \cdot \frac{x}{2} = 0$$

$$\Rightarrow M = \frac{1}{2} wLx - \frac{1}{2} wx^2 - \frac{1}{8} wL^2$$

For $3L/4 < x < L$

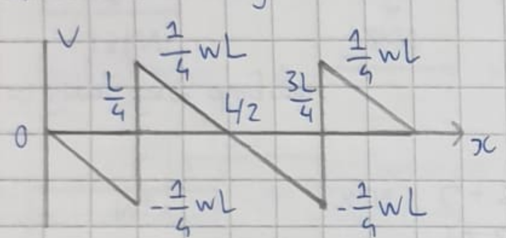


$$\uparrow + \Sigma F_y = 0 \Rightarrow V - w(L-x) = 0 \Rightarrow V = wL - wx$$

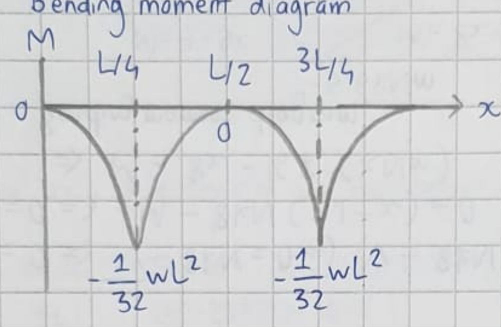
$$\circlearrowleft + \Sigma M = 0 \Rightarrow -M - w(L-x)\frac{L-x}{2} = 0$$

$$\Rightarrow M = -\frac{1}{2} w(L-x)^2$$

$\Rightarrow$ Shear force diagram



Bending moment diagram

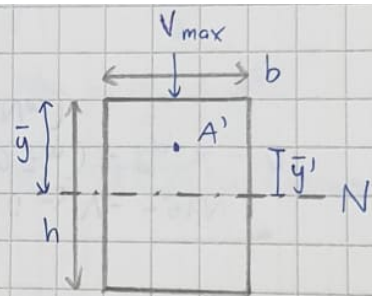


V_{max}

$\Rightarrow V_{max} = \frac{1}{4} wL$, $M_{max} = \frac{1}{32} wL^2$

$$\Rightarrow \tau_{max} = \frac{VQ}{It} = \frac{V_{max} A'\bar{y}'}{It} = \frac{\frac{1}{4} wL \cdot \frac{1}{2} bh \cdot \frac{1}{4} h}{\frac{1}{12} bh^3 \cdot b} = \frac{3}{8} \frac{wL}{bh}$$

$$\sigma_{max} = \frac{M \cdot \bar{y}}{I} = \frac{\frac{1}{32} wL^2 \cdot \frac{h}{2}}{1/12\, bh^3} = \frac{3}{16} \cdot \frac{wL^2}{bh^2}$$



a) Prove that $\frac{\tau_{max}}{\sigma_{max}} = \frac{2h}{L}$?

We have: $\frac{\tau_{max}}{\sigma_{max}} = \frac{3}{8} \frac{wL}{bh} \times \frac{16}{3} \frac{bh^2}{wL^2} = \frac{2h}{L}$ (answer)

b) Let $L = 4m$, $w = 10 kN/m$, $\tau_{max} = 1.28 MPa$, $\sigma_{max} = 9.8 MPa$, find b, h

From (a) $\Rightarrow h = \frac{L\tau_{max}}{2\sigma_{max}} = \frac{4 \times 1.28}{2 \times 9.8} = 0.2612 m$ (answer)

Also: $\tau_{max} = \frac{3}{8} \frac{wL}{bh} \Rightarrow b = \frac{3wL}{8h\tau_{max}} = \frac{3 \times 10 \cdot 10^3 \times 4}{8 \times 0.2612 \times 1.28 \cdot 10^6}$

$\Rightarrow b = 0.0448 m$ (answer)

Question 10: The beam shown in Fig. 10 is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple M that can be applied to the beam. (6 points)

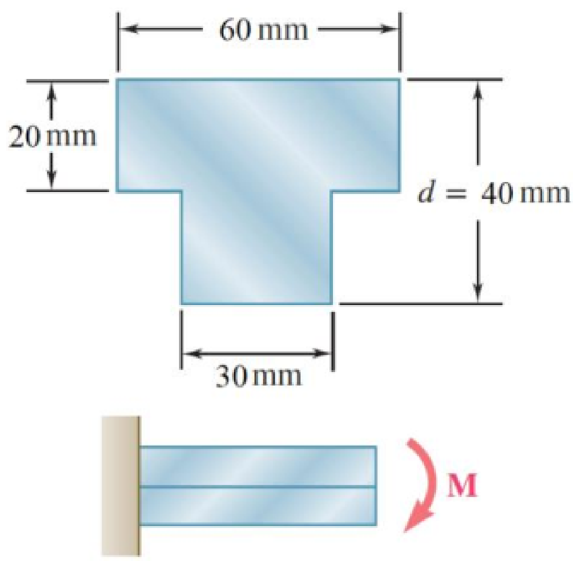
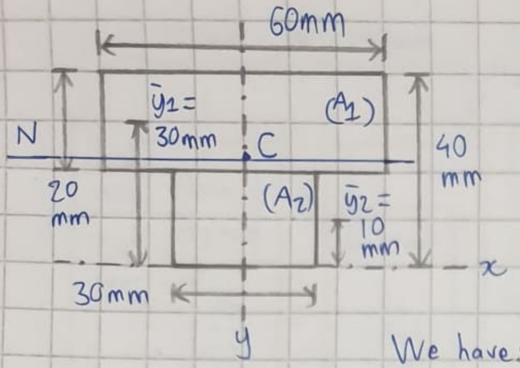


Fig. 10. for Q10



The cross section is divided into A_1 and A_2 like the drawing. The centroid of the cross section is expected to lie on y-axis as it is symmetry axis

$\Rightarrow \bar{X} = 0$. The ordinate of the centroid is

$$\bar{Y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2}$$

We have: $A_1 = 60 \times 20 = 1200\ mm^2 = 12 \times 10^{-4}\ m^2$

$A_2 = 30 \times 20 = 600\ mm^2 = 6 \times 10^{-4}\ m^2$

$\Rightarrow \bar{y} = \dfrac{1200 \times 30 + 600 \times 10}{1200 + 600} = 23.333\ mm$ $\Rightarrow$ Centroid located like in the drawing

Compute moment of inertia around the neutral axis

- Rectangular Area A_1: $(I_x)_1 = \frac{1}{12} bh^3 + A_1 d_1^2$

$\Rightarrow (I_x)_1 = \frac{1}{12} \cdot 60 \cdot (20)^3 + 1200 \cdot (30 - 23.333)^2 = 93338.666\ mm^4$

- Rectangular Area A_2: $(I_x)_2 = \frac{1}{12} bh^3 + A_2 d_2^2$

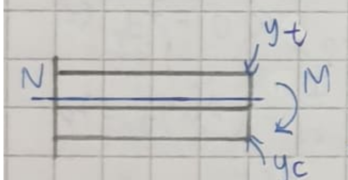
$\Rightarrow (I_x)_2 = \frac{1}{2} \cdot 30 \cdot (20)^3 + 600 \cdot (10 - 23.333)^2 = 126661.333\ mm^4$

$\Rightarrow$ Total moment of inertia around neutral axis

$\bar{I}_x = (I_x)_1 + (I_x)_2 = 93338.666 + 126661.333 = 219999.999\ mm^4$

$\approx 22 \times 10^4\ mm^4$

Bending stress: $\sigma = \dfrac{M \cdot y}{I} \Rightarrow M = \dfrac{\sigma I}{y}$



$\Rightarrow$ Part above neutral axis is in tension and part below neutral axis is in compression

$\Rightarrow$ Tension: $M_t = \dfrac{\sigma_t I}{y_t} = \dfrac{24 \times 10^6 \cdot 22 \times 10^4 \times 10^{-12}}{(40 - 23.333) \times 10^{-3}} = 316.79\ Nm$

Compression: $M_c = \dfrac{\sigma_c I}{y_c} = \dfrac{30 \times 10^6 \cdot 22 \times 10^4 \times 10^{-12}}{23.333 \times 10^{-3}} = 282.86\ Nm \Rightarrow M_c < M_t$

$\Rightarrow$ Largest couple M that can be applied to the beam: $M_{max} = M_c = 282.86\ Nm$ (answer)

Question 11: For the simply supported beam shown in Fig. 11, determine the greatest magnitude of P that can be applied to the beam, where $\sigma_{allow} = 25$ MPa, $\tau_{allow} = 2.5$ MPa. (15 points)

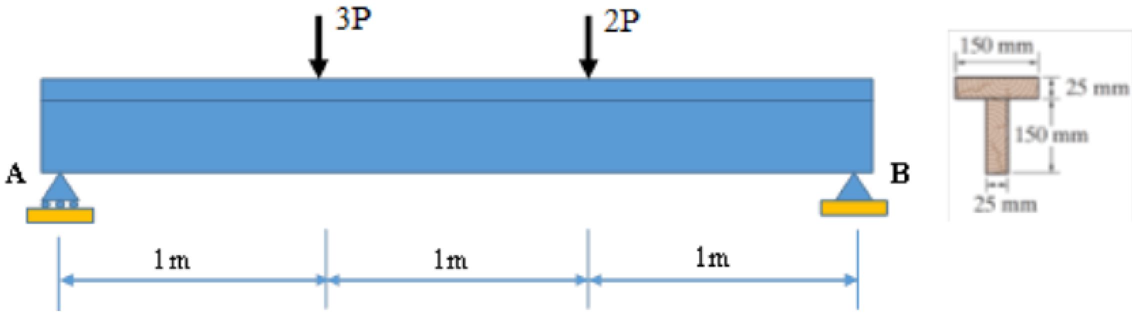
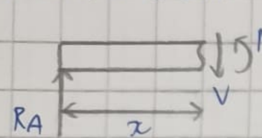


Fig. 11. for Q11

The beam is fixed

$$\Rightarrow \begin{cases} \uparrow + \sum F_y = 0 \\ \circlearrowleft + \sum M_A = 0 \end{cases} \Rightarrow \begin{cases} R_A + R_B = 5P \\ -3P \cdot 1m - 2P \cdot 2m + R_B \cdot 3m = 0 \end{cases} \Rightarrow \begin{cases} R_A = 8/3\,P \\ R_B = 7/3\,P \end{cases}$$

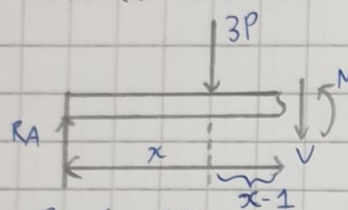
For $0 < x < 1m$



$\uparrow + \sum F_y = 0 \Rightarrow R_A - V = 0 \Rightarrow V = 8/3\,P$

$\circlearrowleft + \sum M = 0 \Rightarrow M - R_A x = 0 \Rightarrow M = \frac{8}{3} P x$

For $1m < x < 2m$

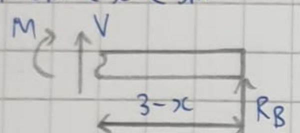


$\uparrow + \sum F_y = 0 \Rightarrow R_A - 3P - V = 0 \Rightarrow V = -1/3\,P$

$\circlearrowleft + \sum M = 0 \Rightarrow M + 3P(x-1) - R_A x = 0$

$\Rightarrow M = -\frac{1}{3} P x + 3P$

For $2m < x < 3m$

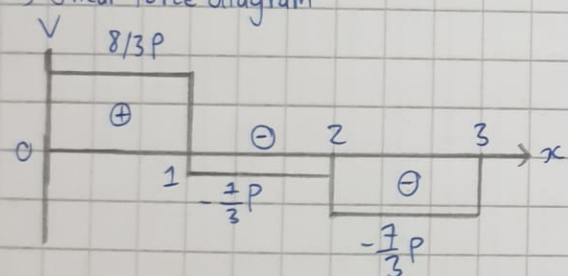


$\uparrow + \sum F_y = 0 \Rightarrow R_B + V = 0 \Rightarrow V = -7/3\,P$

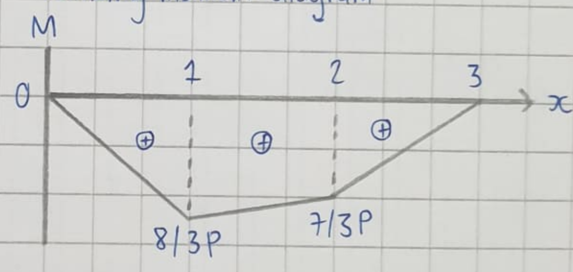
$\circlearrowleft + \sum M = 0 \Rightarrow -M + R_B(3-x) = 0$

$\Rightarrow M = \left(7 - \frac{7}{3} x\right) P$

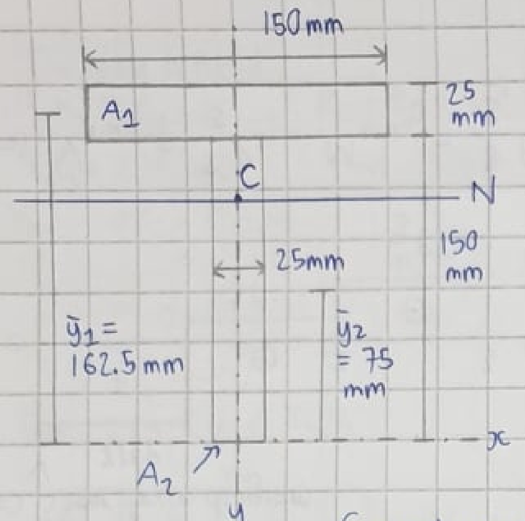
$\Rightarrow$ Shear force diagram



Bending moment diagram



$\Rightarrow V_{max} = \frac{8}{3} P$, $M_{max} = \frac{8}{3} P$



Cross section is symmetical $\Rightarrow \bar{X} = 0$

We have: $A_1 = A_2 = 150 \times 25 = 3750\,mm^2$
$= 3.75 \times 10^{-3}\,m^2$

Coordinate of $\bar{y}$: $\bar{y} = \dfrac{\Sigma A_i \bar{y}_i}{\Sigma A_i}$

$$\Rightarrow \bar{y} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2} = \frac{3750 \cdot 162.5 + 3750 \cdot 75}{3750 + 3750}$$

$\Rightarrow \bar{y} = 118.75\,mm \Rightarrow$ centroid located like in the drawing

Compute moment of inertia around the neutral axis

- Rectangular area A_1: $(I_x)_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(0.15)(0.025)^3 + 3.75 \times 10^{-3} \times (0.1625 - 0.11875)^2$
$\Rightarrow (I_x)_1 = 7.373 \times 10^{-6}\,m^4$
- Rectangular area A_2: $(I_x)_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(0.025)(0.15)^3 + 3.75 \times 10^{-3} \times (0.075 - 0.11875)^2$
$\Rightarrow (I_x)_2 = 1.4208 \times 10^{-5}\,m^4$

Total moment of inertia around neutral axis

$$I_x = (I_x)_1 + (I_x)_2 = 7.373 \times 10^{-6} + 1.4208 \times 10^{-5} = 2.1582 \times 10^{-5}\,m^2$$

Maximum P allowed with regard to σ_{allow}

$$M_{allow} = \frac{\sigma_{allow} I_x}{y_{max}} \Rightarrow \frac{8}{3} P_{max} = \frac{25 \times 10^6 \times 2.1582 \times 10^{-5}}{0.11875} \Rightarrow P_{max} = 1703.84\,N$$

Maximum shear stress will occur at the neutral axis

$$\Rightarrow Q = A' \cdot \bar{y}' = (0.11875 \times 0.025) \times \frac{1}{2} 0.11875 = 1.76269 \times 10^{-4}\,m^3$$

Maximum P allowed with regard to τ_{allow}

$$V_{allow} = \frac{\tau_{allow} I t}{Q} \Rightarrow \frac{8}{3} P_{max} = \frac{2.5 \times 10^6 \cdot 2.1582 \times 10^{-5} \cdot 0.025}{1.76269 \times 10^{-4}}$$

$\Rightarrow P_{max} = 2869.63\,N$. Since $P_{\sigma\,max} < P_{\tau\,max}$

$\Rightarrow$ Greatest magnitude of P that can be applied to the beam: $P_{max} = 1703.84\,N$ (answer)