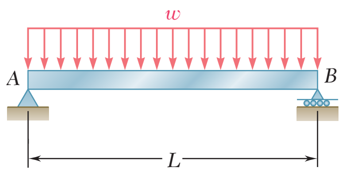
Chapter 5 - Solution Manual-Beer Johnston - Mechanics of Materials 7th c2015

Mechanics of Solids (Ghulam Ishaq Khan Institute of Engineering Sciences and Technology)

CHAPTER 5

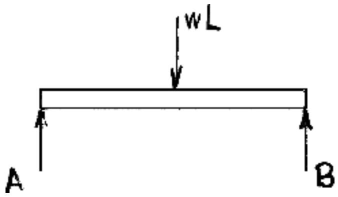


PROBLEM 5.1

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

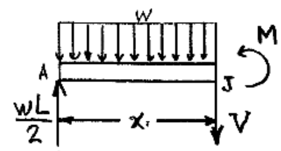
Reactions:



$$+\circlearrowleft \Sigma M_B = 0: \quad -AL + wL \cdot \frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad BL - wL \cdot \frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

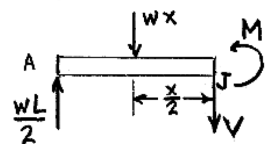
Free body diagram for determining reactions:



Over whole beam, $\qquad 0 < x < L$

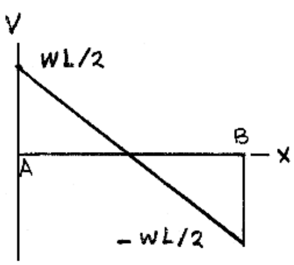
Place section at x.

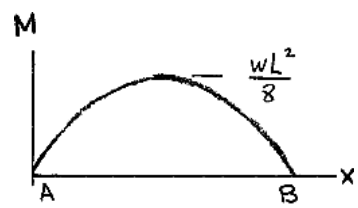
Replace distributed load by equivalent concentrated load.



$$+\uparrow \Sigma F_y = 0: \quad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$





$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

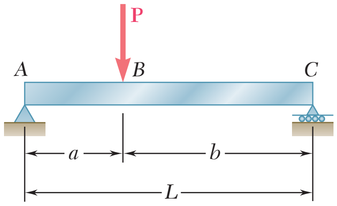
$$M = \frac{w}{2}(Lx - x^2)$$

$$M = \frac{w}{2}x(L - x) \blacktriangleleft$$

Maximum bending moment occurs at $x = \dfrac{L}{2}$.

$$M_{\max} = \frac{wL^2}{8} \blacktriangleleft$$

PROBLEM 5.2



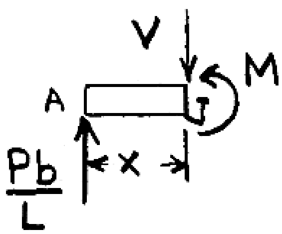
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Reactions:

$$\circlearrowleft + \Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$$

$$\circlearrowleft + \Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$$



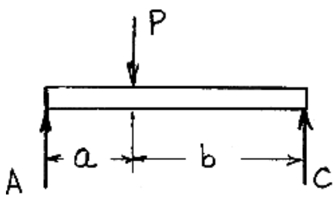
From A to B, $\quad 0 < x < a$

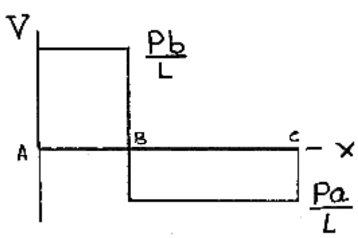
$$+\uparrow \Sigma F_y = 0: \quad \frac{Pb}{L} - V = 0$$

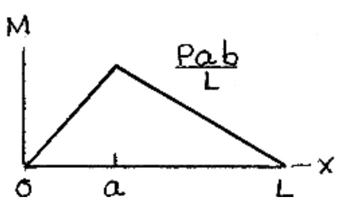
$$V = \frac{Pb}{L} \blacktriangleleft$$

$$\circlearrowleft + \Sigma M_J = 0: \quad M - \frac{Pb}{L}x = 0$$

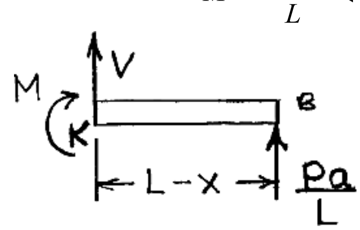
$$M = \frac{Pbx}{L} \blacktriangleleft$$







From B to C, $\quad a < x < L$



$$+\uparrow \Sigma F_y = 0: \quad V + \frac{Pa}{L} = 0$$

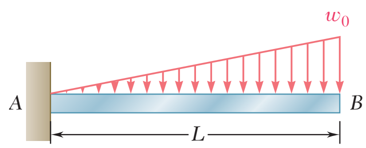
$$V = -\frac{Pa}{L} \blacktriangleleft$$

$$\circlearrowleft + \Sigma M_K = 0: \quad -M + \frac{Pa}{L}(L - x) = 0$$

$$M = \frac{Pa(L - x)}{L} \blacktriangleleft$$

At section B, $\quad M = \frac{Pab}{L^2} \blacktriangleleft$

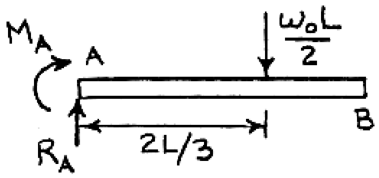
PROBLEM 5.3



For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.

SOLUTION

Free body diagram for determining reactions.

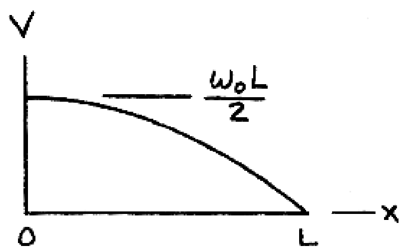


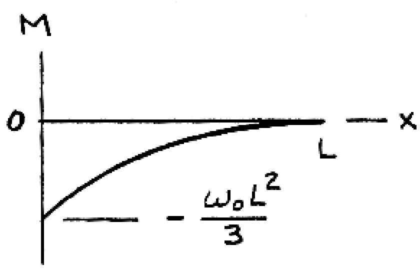
Reactions:

$$+\uparrow \Sigma F_y = 0: \quad R_A - \frac{w_0 L}{2} = 0 \qquad R_A = \frac{w_0 L}{2} \uparrow$$

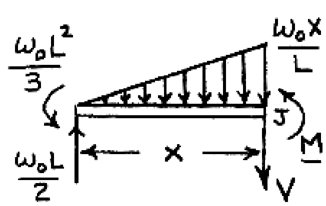
$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \left(\frac{w_0 L}{2}\right)\left(\frac{2L}{3}\right) = 0$$

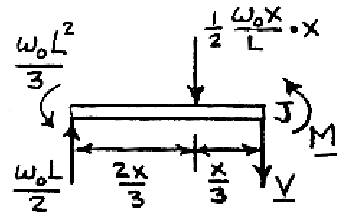
$$M_A = -\frac{w_0 L^2}{3} = \frac{w_0 L^2}{3} \circlearrowright$$





Use portion to left of the section as the free body.





Replace distributed load with equivalent concentrated load.

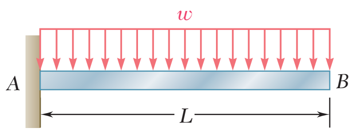
$$+\uparrow \Sigma F_y = 0: \quad \frac{w_0 L}{2} - \frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$$

$$V = \frac{w_0 L}{2} - \frac{w_0 x^2}{2L} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0:$$

$$\frac{w_0 L^2}{3} - \left(\frac{w_0 L}{2}\right)(x) + \left(\frac{1}{2}\frac{w_0 x}{L}\cdot x\right)\left(\frac{x}{3}\right) + M = 0$$

$$M = -\frac{w_0 L^2}{3} + \frac{w_0 L x}{2} - \frac{w_0 x^3}{6L} \blacktriangleleft$$

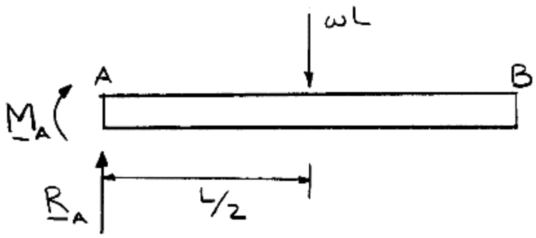


PROBLEM 5.4

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Free body diagram for determining reactions.



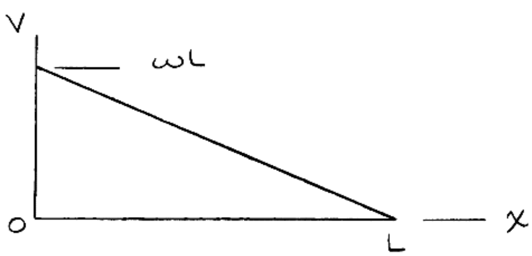
Reactions:

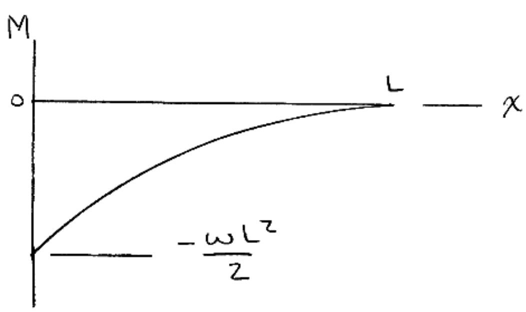
$$+\uparrow \Sigma F_y = 0: \quad R_A - wL = 0$$

$$R_A = wL \uparrow$$

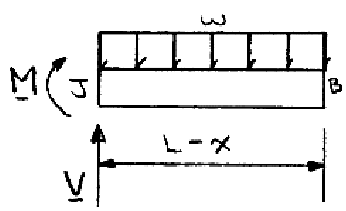
$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (wL)\left(\frac{L}{2}\right) = 0$$

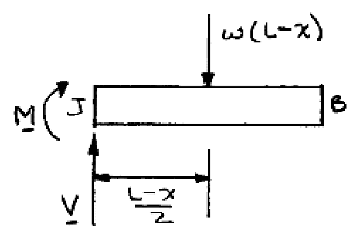
$$M_A = \frac{w_0 L^2}{2} \circlearrowleft$$





Use portion to the right of the section as the free body.





Replace distributed load by equivalent concentrated load.

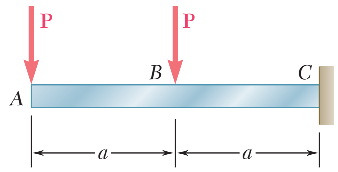
$$+\uparrow \Sigma F_y = 0: \quad V - w(L - x) = 0$$

$$V = w(L - x) \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - w(L - x)\left(\frac{L - x}{2}\right) = 0$$

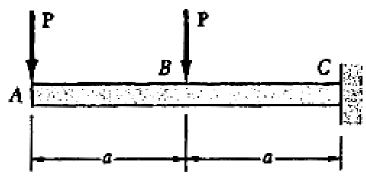
$$M = -\frac{w}{2}(L - x)^2 \blacktriangleleft$$

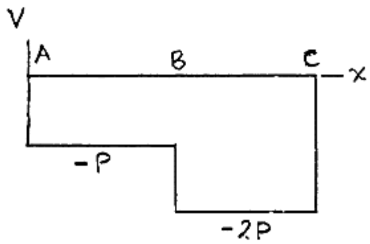
PROBLEM 5.5

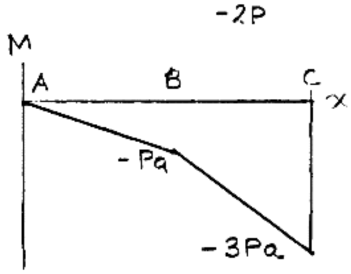


For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

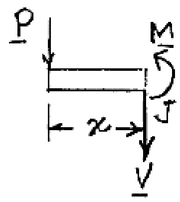
SOLUTION







<u>From *A* to *B*:</u> $0 < x < a$



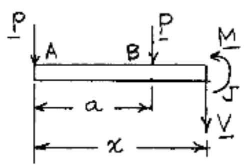
$+\uparrow \Sigma F_y = 0: \quad -P - V = 0$

$$V = -P \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0: \quad Px + M = 0$

$$M = -Px \blacktriangleleft$$

<u>From *B* to *C*:</u> $a < x \le 2a$

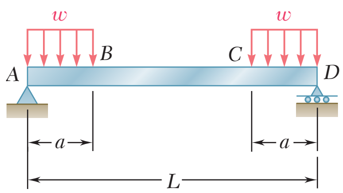


$+\uparrow \Sigma F_y = 0: \quad -P - P - V = 0$

$$V = -2P \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0: \quad Px + P(x - a) + M = 0$

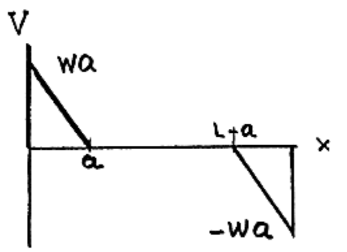
$$M = -2Px + Pa \blacktriangleleft$$

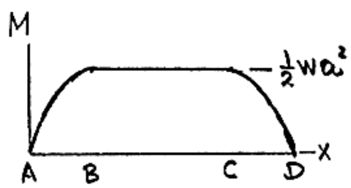


PROBLEM 5.6

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

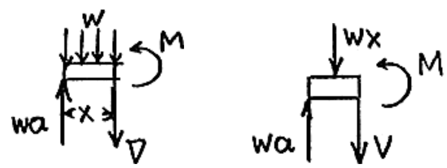
SOLUTION





Reactions: $A = D = wa$

From A to B, $0 < x < a$

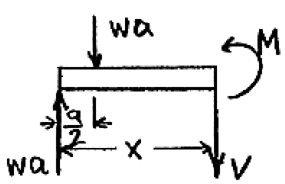


$$+\!\uparrow \Sigma F_y = 0: \quad wa - wx - V = 0$$

$$V = w(a - x) \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -wax + (wx)\frac{x}{2} + M = 0$$

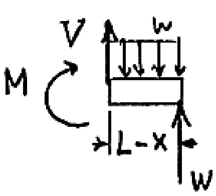
$$M = w\left(ax - \frac{x^2}{2}\right) \blacktriangleleft$$



From B to C, $a < x < L - a$

$$\Sigma F_y = 0: \quad wa - wa - V = 0$$

$$V = 0 \blacktriangleleft$$

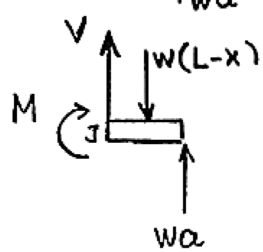


$$+\circlearrowleft \Sigma M_J = 0: \quad -wax + wa\left(x - \frac{a}{2}\right) + M = 0 \qquad M = \frac{1}{2}wa^2 \blacktriangleleft$$

From C to D, $L - a < x < L$

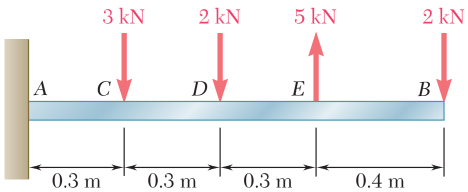
$$+\!\uparrow \Sigma F_y = 0: \quad V - w(L - x) + wa = 0$$

$$V = w(L - x - a) \blacktriangleleft$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -M - w(L - x)\left(\frac{L - x}{2}\right) + wa(L - x) = 0$$

$$M = w\left[a(L - x) + \frac{1}{2}(L - x)^2\right] \blacktriangleleft$$

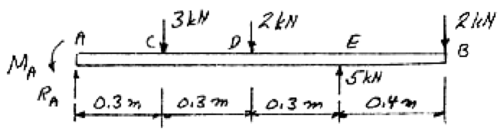


PROBLEM 5.7

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Origin at A:



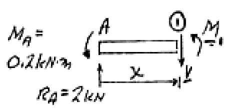
Reaction at A:

$$+\uparrow \Sigma F_y = 0: \quad R_A - 3 - 2 + 5 - 2 = 0 \qquad R_A = 2 \text{ kN}\uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (3 \text{ kN})(0.3 \text{ m}) - (2 \text{ kN})(0.6 \text{ m}) + (5 \text{ kN})(0.9 \text{ m}) - (2 \text{ kN})(1.3 \text{ m}) = 0$$

$$M_A = 0.2 \text{ kN} \cdot \text{m} \circlearrowleft \blacktriangleleft$$

From A to C:

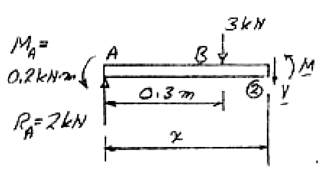


$$\Sigma F_y = 0: \quad V = 2 \text{ kN}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad 0.2 \text{ kN} \cdot \text{m} - (2 \text{ kN})x + M = 0$$

$$M = -0.2 + 2x$$

From C to D:



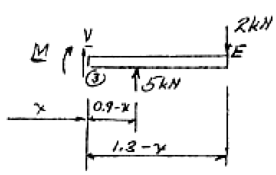
$$+\uparrow \Sigma F_y = 0: \quad 2 - 3 - V = 0$$

$$V = -1 \text{ kN}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad +0.2 \text{ kN} \cdot \text{m} - (2 \text{ kN})x + (3 \text{ kN})(x - 0.3) + M = 0$$

$$M = 0.7 - x$$

From D to E:



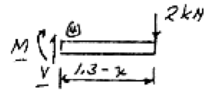
$$\Sigma F_y = 0: \quad V + 5 - 2 = 0 \qquad V = -3 \text{ kN}$$

$$\circlearrowright \Sigma M_3 = 0: \quad -M + 5(0.9 - x) - (2)(1.3 - x) = 0$$

$$M = +1.9 - 3x$$

PROBLEM 5.7 *(Continued)*

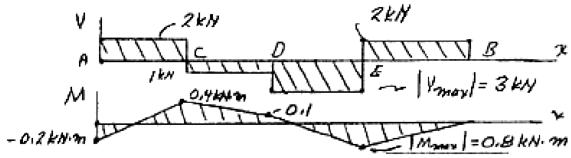
<u>From E to B</u>:



$$\Sigma F_y = 0: \quad V = 2 \text{ kN}$$

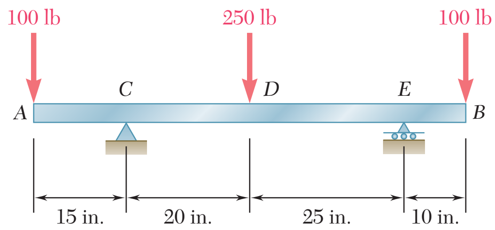
$$\Sigma M_4 = 0: \quad -M - 2(1.3 - x) = 0$$

$$M = -2.6 + 2x$$



(*a*) $|V|_{\max} = 3.00 \text{ kN}$ ◀

(*b*) $|M|_{\max} = 0.800 \text{ kN} \cdot \text{m}$ ◀

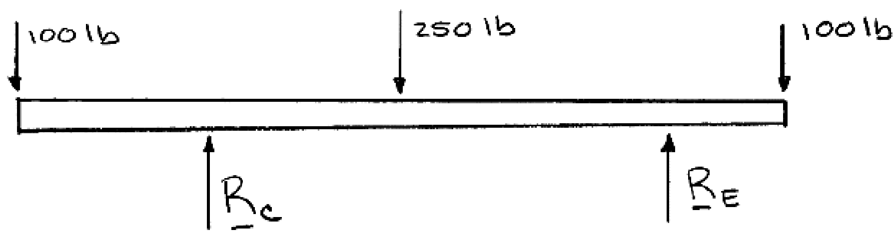


PROBLEM 5.8

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Reactions:



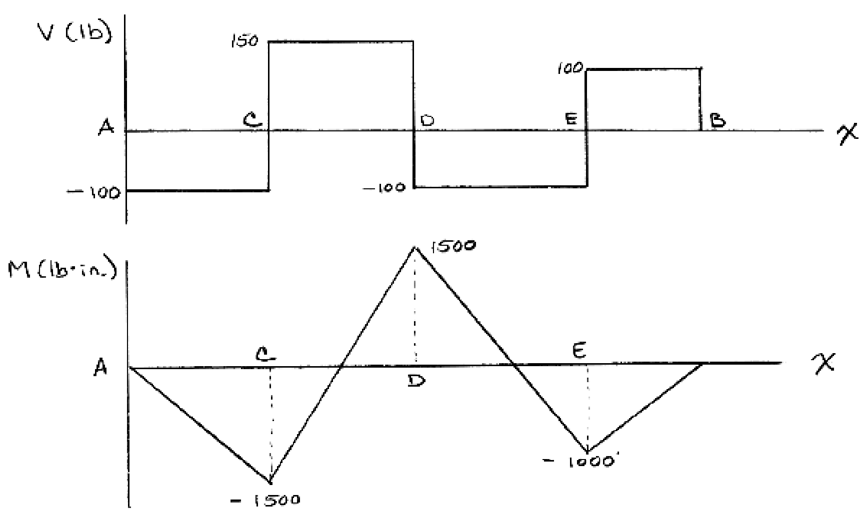
$$+\circlearrowleft \Sigma M_C = 0: \quad R_E(45 \text{ in.}) + 100 \text{ lb}(15 \text{ in.}) - 250 \text{ lb}(20 \text{ in.}) - 100 \text{ lb}(55 \text{ in.}) = 0$$

$$R_E = 200 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad R_C + 200 \text{ lb} - 100 \text{ lb} - 250 \text{ lb} - 100 \text{ lb} = 0$$

$$R_C = 250 \text{ lb}$$

At any point, V is the sum of the loads and reactions to the left (assuming $+\uparrow$) and M the sum of their moments about that point (assuming $\circlearrowright +$).



(*a*) $V_{max} = 150.0$ lb ◀

(*b*) $M_{max} = 1500$ lb · in. ◀

PROBLEM 5.8 *(Continued)*

<u>Detailed computations of moments</u>:

$$M_A = 0$$

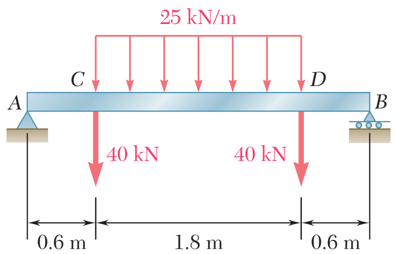
$$M_C = -(100 \text{ lb})(15 \text{ in.}) = -1500 \text{ lb} \cdot \text{in.}$$

$$M_D = -(100 \text{ lb})(35 \text{ in.}) + (250 \text{ lb})(20 \text{ in.}) = +1500 \text{ lb} \cdot \text{in.}$$

$$M_E = -(100 \text{ lb})(60 \text{ in.}) + (250 \text{ lb})(45 \text{ in.}) - (250 \text{ lb})(25 \text{ in.}) = -1000 \text{ lb} \cdot \text{in.}$$

$$M_B = -(100 \text{ lb})(70 \text{ in.}) + (250 \text{ lb})(55 \text{ in.}) - (250 \text{ lb})(35 \text{ in.}) + (200 \text{ lb})(10 \text{ in.}) = 0$$

(Checks)



PROBLEM 5.9

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

The distributed load is replaced with an equivalent concentrated load of 45 kN to compute the reactions.

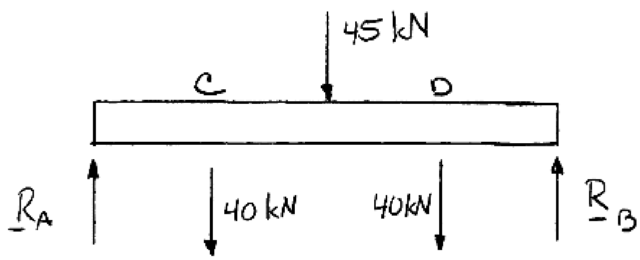
$$(25\text{ kN/m})(1.8\text{ m}) = 45\text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(40\text{ kN})(0.6\text{ m}) - 45\text{ kN}(1.5\text{ m}) - 40\text{ kN}(2.4\text{ m}) + R_B(3.0\text{ m}) = 0$$

$$R_B = 62.5\text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + 62.5\text{ kN} - 40\text{ kN} - 45\text{ kN} - 40\text{ kN} = 0$$

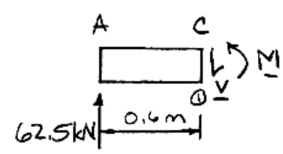
$$R_A = 62.5\text{ kN}$$



At *C*:

$$+\uparrow \Sigma F_y = 0: \quad V = 62.5\text{ kN}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad M = (62.5\text{ kN})(0.6\text{ m}) = 37.5\text{ kN}\cdot\text{m}$$



At centerline of the beam:

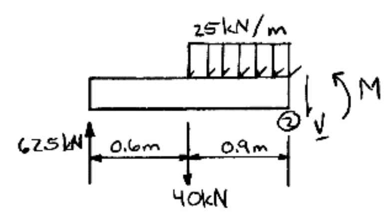
$$+\uparrow \Sigma F_y = 0: \quad 62.5\text{ kN} - 40\text{ kN} - (25\text{ kN/m})(0.9\text{ m}) - V = 0$$

$$V = 0$$

$$+\circlearrowleft \Sigma M_2 = 0:$$

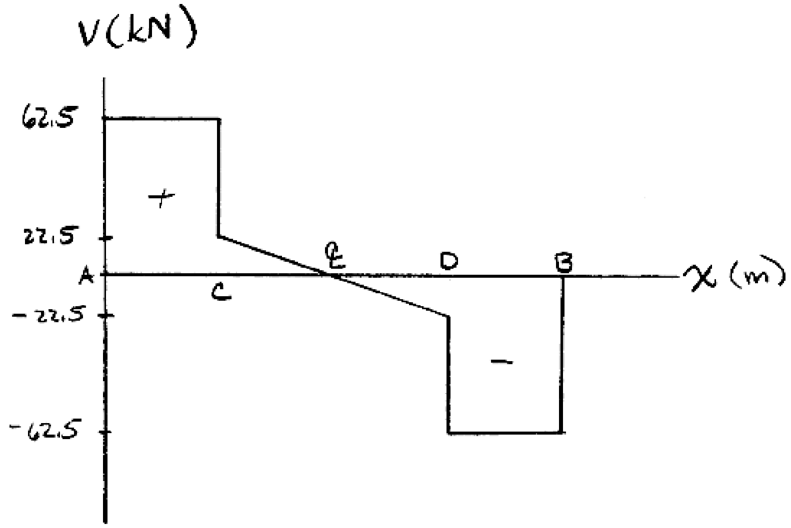
$$M - (62.5\text{ kN})(1.5\text{ m}) + (40\text{ kN})(0.9\text{ m}) + (25\text{ kN/m})(0.9\text{ m})(0.45\text{ m}) = 0$$

$$M = 47.625\text{ kN}\cdot\text{m}$$

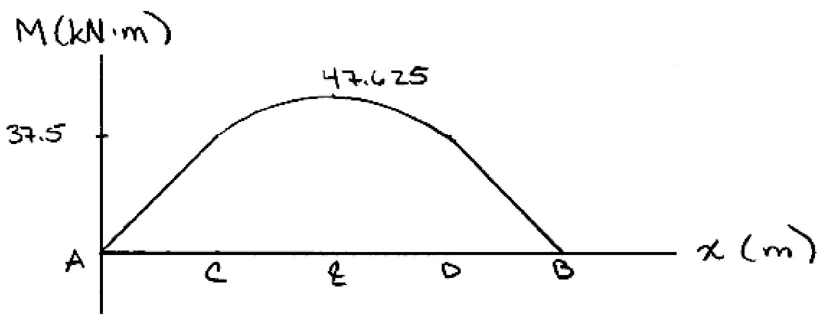


PROBLEM 5.9 *(Continued)*

Shear and bending-moment diagrams:



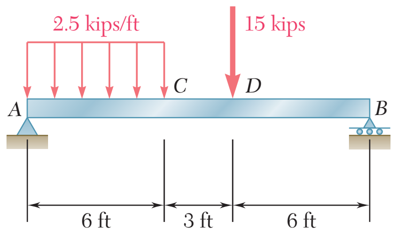
(*a*) $|V|_{max} = 62.5 \text{ kN}$ ◀



(*b*) $|M|_{max} = 47.6 \text{ kN} \cdot \text{m}$ ◀

From A to C and D to B, V is uniform; therefore M is linear.

From C to D, V is linear; therefore M is parabolic.



PROBLEM 5.10

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$+\circlearrowleft \Sigma M_B = 0: \quad -15R_A + (12)(6)(2.5) + (6)(15) = 0$

$R_A = 18 \text{ kips}$

$+\circlearrowleft \Sigma M_A = 0: \quad 15R_B - (3)(6)(2.5) - (9)(15) = 0$

$R_B = 12 \text{ kips}$

Shear:

$V_A = 18 \text{ kips}$

$V_C = 18 - (6)(2.5) = 3 \text{ kips}$

C to D: $\quad V = 3 \text{ kips}$

D to B: $\quad V = 3 - 15 = -12 \text{ kips}$

Areas under shear diagram:

A to C: $\quad \int V\,dx = \left(\frac{1}{2}\right)(6)(18+3) = 63 \text{ kip} \cdot \text{ft}$

C to D: $\quad \int V\,dx = (3)(3) = 9 \text{ kip} \cdot \text{ft}$

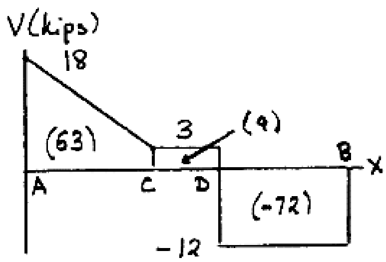
D to B: $\quad \int V\,dx = (6)(-12) = -72 \text{ kip} \cdot \text{ft}$

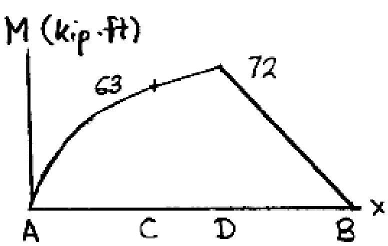
Bending moments: $\quad M_A = 0$

$M_C = 0 + 63 = 63 \text{ kip} \cdot \text{ft}$

$M_D = 63 + 9 = 72 \text{ kip} \cdot \text{ft}$

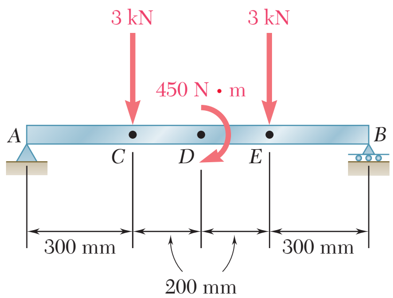
$M_B = 72 - 72 = 0$





$|V|_{max} = 18.00 \text{ kips}$ ◀

$|M|_{max} = 72.0 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 5.11

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$+\circlearrowleft \Sigma M_B = 0: \quad (700)(3) - 450 + (300)(3) - 1000A = 0$

$$A = 2.55 \text{ kN} \uparrow$$

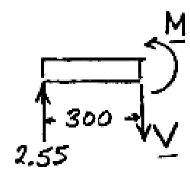
$+\circlearrowleft \Sigma M_A = 0: \quad -(300)(3) - 450 - (700)(3) + 1000B = 0$

$$B = 3.45 \text{ kN} \uparrow$$

At A: $V = 2.55$ kN $\quad M = 0$

A to C: $V = 2.55$ kN

At C:



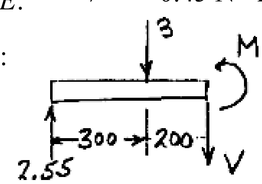
$+\circlearrowleft \Sigma M_C = 0:$

$-(300)(2.55) + M = 0$

$M = 765$ N · m

C to E: $V = -0.45$ N · m

At D:

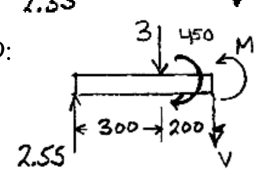


$+\circlearrowleft \Sigma M_D = 0:$

$-(500)(2.55) + (200)(3) + M = 0$

$M = 675$ N · m

At D:



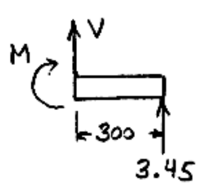
$+\circlearrowleft \Sigma M_D = 0:$

$-(500)(2.55) + (200)(3) - 450 + M = 0$

$M = 1125$ N · m

E to B: $V = -3.45$ kN

At E:

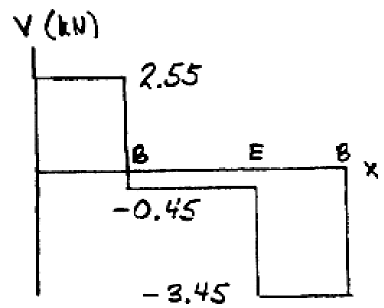


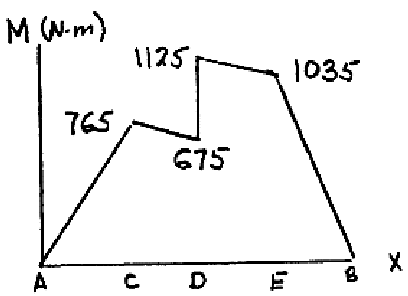
$+\circlearrowleft \Sigma M_E = 0:$

$-M + (300)(3.45) = 0$

$M = 1035$ N · m

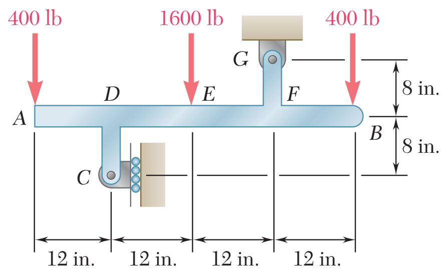
At B: $V = 3.45$ kN, $\quad M = 0$





(*a*) $|V|_{max} = 3.45$ kN ◀

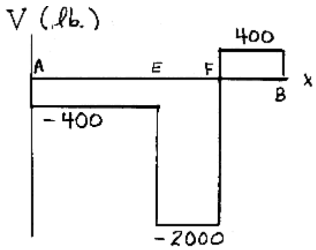
(*b*) $|M|_{max} = 1125$ N · m ◀



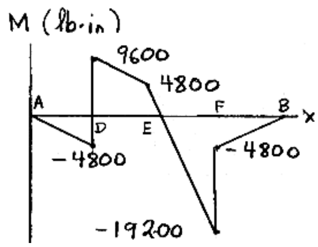
PROBLEM 5.12

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



$$+\circlearrowleft \Sigma M_G = 0: \quad -16C + (36)(400) + (12)(1600)$$
$$-(12)(400) = 0 \qquad C = 1800 \text{ lb}$$
$$\xrightarrow{+} \Sigma F_x = 0: \quad -C + G_x = 0 \qquad G_x = 1800 \text{ lb}$$
$$+\uparrow \Sigma F_y = 0: \quad -400 - 1600 + G_y - 400 = 0 \qquad G_y = 2400 \text{ lb}$$

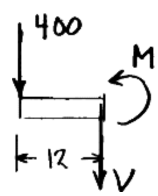


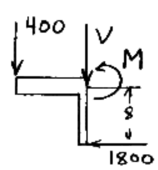
A to E: $\quad V = -400$ lb

E to F: $\quad V = -2000$ lb

F to B: $\quad V = 400$ lb

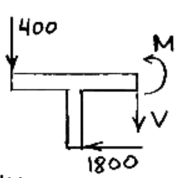
At A and B, $\quad M = 0$





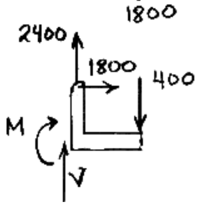
At D^-, $\quad +\circlearrowleft \Sigma M_D = 0: \; (12)(400) + M = 0 \qquad M = -4800 \text{ lb} \cdot \text{in.}$

At D^+, $\quad +\circlearrowleft \Sigma M_D = 0: \; (12)(400) - (8)(1800) + M = 0 \qquad M = 9600 \text{ lb} \cdot \text{in.}$

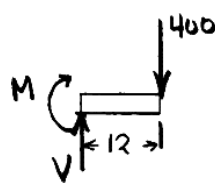


At E, $\quad +\circlearrowleft \Sigma M_E = 0: \; (24)(400) - (8)(1800) + M = 0 \qquad M = 4800 \text{ lb} \cdot \text{in.}$

At F^-, $\quad +\circlearrowleft \Sigma M_F = 0: \; -M - (8)(1800) - (12)(400) = 0 \qquad M = -19{,}200 \text{ lb} \cdot \text{in.}$

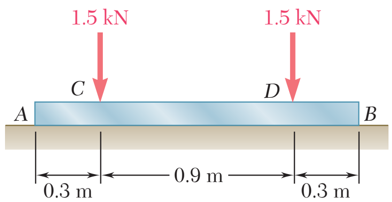


At F^+, $\quad +\circlearrowleft \Sigma M_F = 0: \; -M - (12)(400) = 0 \qquad M = -4800 \text{ lb} \cdot \text{in.}$



(*a*) Maximum $|V| = 2000$ lb ◀

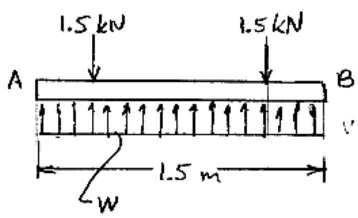
(*b*) Maximum $|M| = 19{,}200$ lb · in. ◀



PROBLEM 5.13

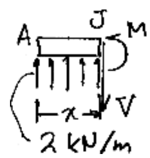
Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

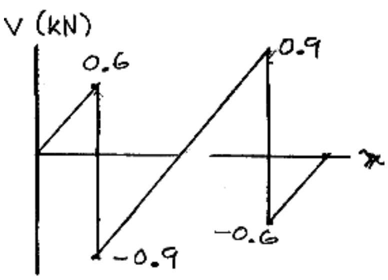
SOLUTION



Over the whole beam,

$$+\uparrow \Sigma F_y = 0: \quad 1.5w - 1.5 - 1.5 = 0 \qquad\qquad w = 2 \text{ kN/m}$$



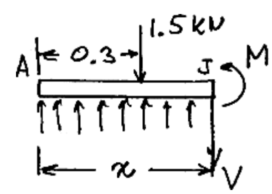


A to C: $\quad 0 \le x < 0.3$ m

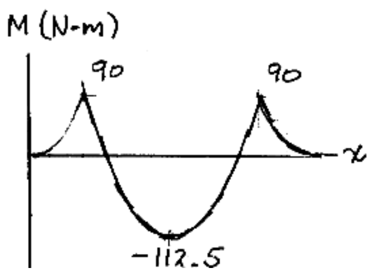
$$+\uparrow \Sigma F_y = 0: \quad 2x - V = 0 \qquad\qquad V = (2x) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -(2x)\left(\frac{x}{2}\right) + M = 0 \qquad\qquad M = (x^2) \text{ kN} \cdot \text{m}$$

At C^-, $\qquad x = 0.3$ m



$$V = 0.6 \text{ kN}, \quad M = 0.090 \text{ kN} \cdot \text{m}$$
$$= 90 \text{ N} \cdot \text{m}$$



C to D: $\quad 0.3 \text{ m} < x < 1.2$ m

$$+\uparrow \Sigma F_y = 0: \quad 2x - 1.5 - V = 0 \qquad\qquad V = (2x - 1.5) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -(2x)\left(\frac{x}{2}\right) + (1.5)(x - 0.3) + M = 0$$

$$M = (x^2 - 1.5x + 0.45) \text{ kN} \cdot \text{m}$$

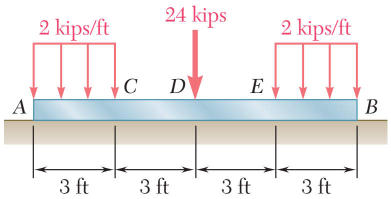
At the center of the beam, $x = 0.75$ m

$$V = 0 \qquad M = -0.1125 \text{ kN} \cdot \text{m}$$
$$= -112.5 \text{ N} \cdot \text{m}$$

At C^+, $\qquad x = 0.3$ m, $\qquad V = -0.9$ kN

(*a*) Maximum $|V| = 0.9 \text{ kN} = 900 \text{ N}$ ◀

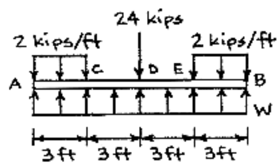
(*b*) Maximum $|M| = 112.5 \text{ N} \cdot \text{m}$ ◀

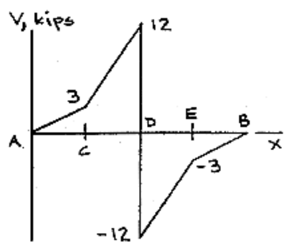


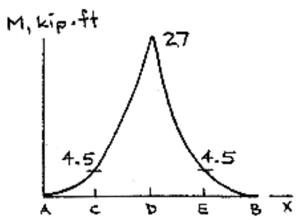
PROBLEM 5.14

Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam *AB* and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



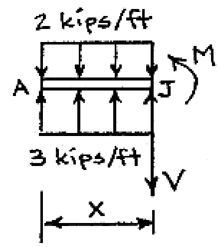




Over the whole beam,

$$+\uparrow\Sigma F_y = 0: \quad 12w - (3)(2) - 24 - (3)(2) = 0 \qquad w = 3 \text{ kips/ft}$$

A to *C*: $(0 \le x < 3 \text{ ft})$



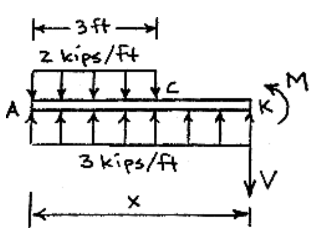
$$+\uparrow\Sigma F_y = 0: \quad 3x - 2x - V = 0 \qquad V = (x) \text{ kips}$$

$$+\Sigma M_J = 0: \quad -(3x)\frac{x}{2} + (2x)\frac{x}{2} + M = 0 \qquad M = (0.5x^2) \text{ kip}\cdot\text{ft}$$

At *C*, $x = 3$ ft

$V = 3$ kips, $M = 4.5$ kip · ft

C to *D*: $(3 \text{ ft} \le x < 6 \text{ ft})$



$$+\uparrow\Sigma F_y = 0: \quad 3x - (2)(3) - V = 0 \qquad V = (3x - 6) \text{ kips}$$

$$\circlearrowleft \Sigma M_K = 0: \quad -(3x)\left(\frac{x}{2}\right) + (2)(3)\left(x - \frac{3}{2}\right) + M = 0$$

$$M = (1.5x^2 - 6x + 9) \text{ kip}\cdot\text{ft}$$

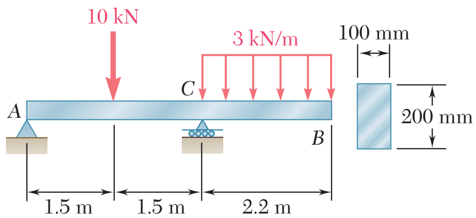
At D^-, $x = 6$ ft

$V = 12$ kips, $M = 27$ kip · ft

D to *B*: Use symmetry to evaluate.

(*a*) $|V|_{max} = 12.00$ kips ◀

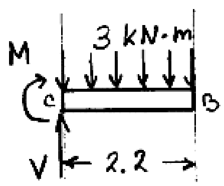
(*b*) $|M|_{max} = 27.0$ kip · ft ◀



PROBLEM 5.15

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION



Using *CB* as a free body,

$$\circlearrowleft \Sigma M_C = 0: \quad -M + (2.2)(3 \times 10^3)(1.1) = 0$$

$$M = 7.26 \times 10^3 \text{ N} \cdot \text{m}$$

Section modulus for rectangle:

$$S = \frac{1}{6} b h^2$$

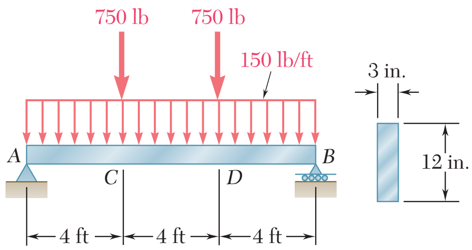
$$= \frac{1}{6}(100)(200)^2 = 666.7 \times 10^3 \text{ mm}^3$$

$$= 666.7 \times 10^{-6} \text{ m}^3$$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{7.26 \times 10^3}{666.7 \times 10^{-6}} = 10.8895 \times 10^6 \text{ Pa}$$

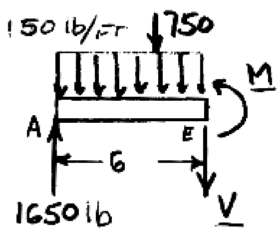
$$\sigma = 10.89 \text{ MPa} \blacktriangleleft$$



PROBLEM 5.16

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION



Reactions: $C = A$ by symmetry.

$+\uparrow \Sigma F_y = 0: \quad A + C - (2)(750) - (12)(150) = 0$

$A = C = 1650 \text{ lb}$

Use left half of beam as free body.

$+\circlearrowleft \Sigma M_E = 0:$

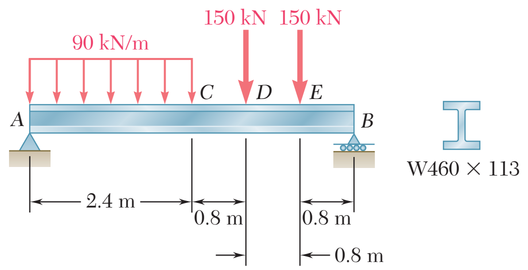
$$-(1650)(6) + (750)(2) + (150)(6)(3) + M = 0$$

$$M = 5700 \text{ lb}\cdot\text{ft} = 68.4 \times 10^3 \text{ lb}\cdot\text{in.}$$

Section modulus: $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(3)(12)^2 = 72 \text{ in}^3$

Normal stress: $\sigma = \frac{M}{S} = \frac{68.4 \times 10^3}{72} = 950 \text{ psi}$

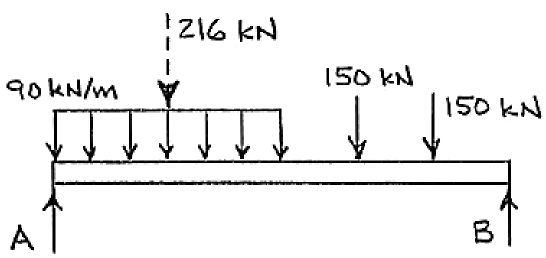
$\sigma = 950 \text{ psi}$ ◀



PROBLEM 5.17

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION

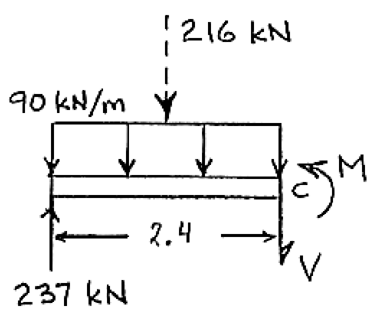


Use entire beam as free body.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$-4.8A + (3.6)(216) + (1.6)(150) + (0.8)(150) = 0$$

$$\mathbf{A} = 237 \text{ kN}\uparrow$$



Use portion AC as free body.

$$+\circlearrowleft \Sigma M_C = 0:$$

$$M - (2.4)(237) + (1.2)(216) = 0$$

$$M = 309.6 \text{ kN} \cdot \text{m}$$

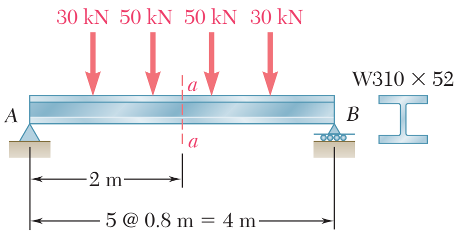
For W460 × 113, $S = 2390 \times 10^6 \text{mm}^3$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{309.6 \times 10^3 \text{ N} \cdot \text{m}}{2390 \times 10^{-6} \text{ m}^3}$$

$$= 129.5 \times 10^6 \text{ Pa}$$

$$\sigma = 129.5 \text{ MPa} \blacktriangleleft$$



PROBLEM 5.18

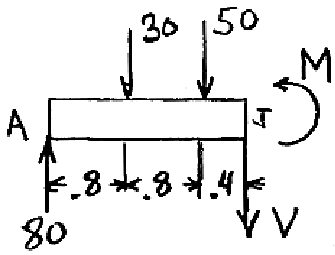
For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.

SOLUTION

Reactions: By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad \mathbf{A} = \mathbf{B} = 80 \text{ kN} \uparrow$$

Using left half of beam as free body,



$$+\circlearrowleft \Sigma M_J = 0:$$

$$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$$

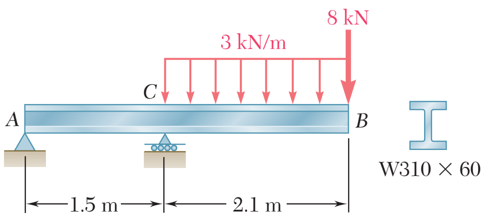
$$M = 104 \text{ kN} \cdot \text{m} = 104 \times 10^3 \text{ N} \cdot \text{m}$$

For W310 × 52, $S = 747 \times 10^3 \text{ mm}^3$

$$= 747 \times 10^{-6} \text{ m}^3$$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{104 \times 10^3}{747 \times 10^{-6}} = 139.2 \times 10^6 \text{ Pa}$

$\sigma = 139.2 \text{ MPa}$ ◀



PROBLEM 5.19

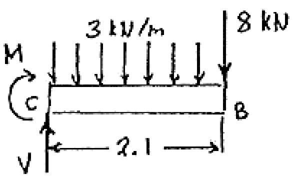
For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION

Use portion CB as free body.

$$+\circlearrowleft \Sigma M_C = 0: \quad -M + (3)(2.1)(1.05) + (8)(2.1) = 0$$

$$M = 23.415 \text{ kN} \cdot \text{m} = 23.415 \times 10^3 \text{ N} \cdot \text{m}$$



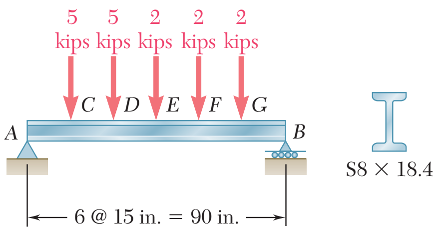
For W310 × 60, $S = 844 \times 10^3 \text{ mm}^3$

$$= 844 \times 10^{-6} \text{ m}^3$$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{23.415 \times 10^3}{844 \times 10^{-6}} = 27.7 \times 10^6 \text{ Pa}$

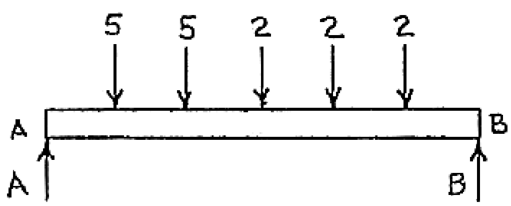
$\sigma = 27.7 \text{ MPa}$ ◀

PROBLEM 5.20



For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION



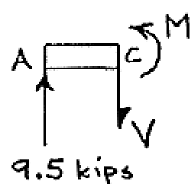
Use entire beam as free body.

$+\circlearrowleft \Sigma M_B = 0$:

$$-90A + (75)(5) + (60)(5) + (45)(2) + (30)(2) + (15)(2) = 0$$

$$A = 9.5 \text{ kips} \uparrow$$

Use portion AC as free body.



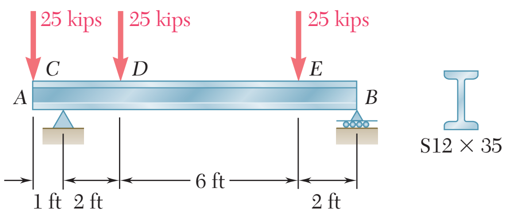
$+\circlearrowleft \Sigma M_C = 0: \quad M - (15)(9.5) = 0$

$$M = 142.5 \text{ kip} \cdot \text{in.}$$

For $S8 \times 18.4$, $\quad S = 14.4 \text{ in}^3$

Normal stress: $\quad \sigma = \dfrac{M}{S} = \dfrac{142.5}{14.4}$

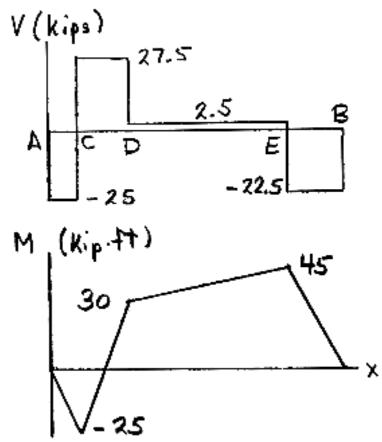
$$\sigma = 9.90 \text{ ksi} \blacktriangleleft$$

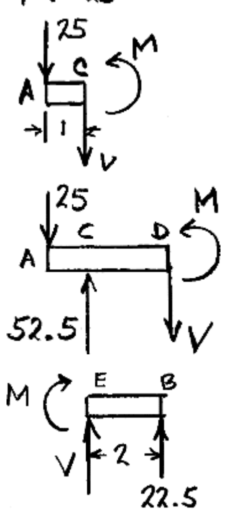


PROBLEM 5.21

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION





$+\circlearrowleft \Sigma M_B = 0$:

$(11)(25) - 10C + (8)(25) + (2)(25) = 0 \qquad \boldsymbol{C} = 52.5 \text{ kips} \uparrow$

$+\circlearrowleft \Sigma M_C = 0$:

$(1)(25) - (2)(25) - (8)(25) + 10B = 0 \qquad \boldsymbol{B} = 22.5 \text{ kips}$

Shear:

A to C^-: $\quad V = -25$ kips

C^+ to D^-: $\quad V = 27.5$ kips

D^+ to E^-: $\quad V = 2.5$ kips

E^+ to B: $\quad V = -22.5$ kips

Bending moments:

At C, $\quad +\circlearrowleft \Sigma M_C = 0: \quad (1)(25) + M = 0$

$$M = -25 \text{ kip} \cdot \text{ft}$$

At D, $\quad +\circlearrowleft \Sigma M_D = 0: \quad (3)(25) - (2)(52.5) + M = 0$

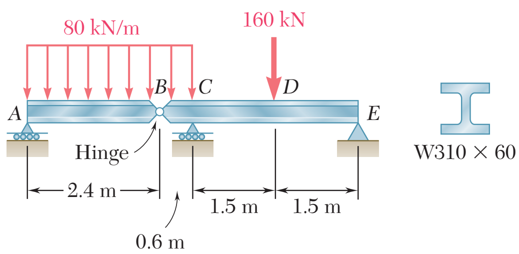
$$M = 30 \text{ kip} \cdot \text{ft}$$

At E, $\quad +\circlearrowleft \Sigma M_E = 0: \quad -M + (2)(22.5) = 0 \qquad M = 45 \text{ kip} \cdot \text{ft}$

$\max |M| = 45 \text{ kip} \cdot \text{ft} = 540 \text{ kip} \cdot \text{in.}$

For $S12 \times 35$ rolled steel section, $\quad S = 38.1 \text{ in}^3$

Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{540}{38.1} = 14.17 \text{ ksi} \qquad \sigma = 14.17 \text{ ksi} \blacktriangleleft$

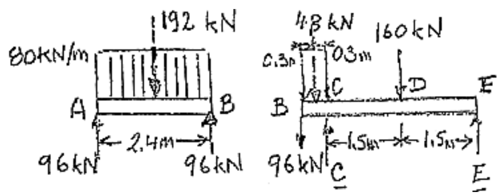


PROBLEM 5.22

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

Statics: Consider portion AB and BE separately.



Portion BE:

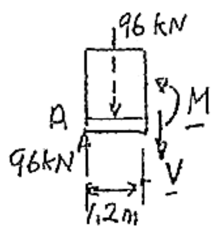
$+\circlearrowleft \sum M_E = 0:$

$(96)(3.6) + (48)(3.3) - C(3) + (160)(1.5) = 0$

$\mathbf{C} = 248\,\text{kN} \uparrow$

$\mathbf{E} = 56\,\text{kN} \uparrow$

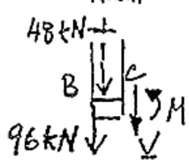
$$M_A = M_B = M_E = 0$$



At midpoint of AB:

$\sum F_y = 0: \quad V = 0$

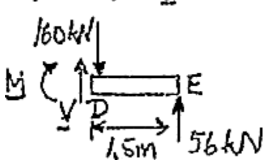
$\sum M = 0: \quad M = (96)(1.2) - (96)(0.6) = 57.6\text{ kN}\cdot\text{m}$



Just to the left of C:

$\sum F_y = 0: \quad V = -96 - 48 = -144\text{ kN}$

$\sum M_C = 0: \quad M = -(96)(0.6) - (48)(0.3) = -72\text{ kN}$

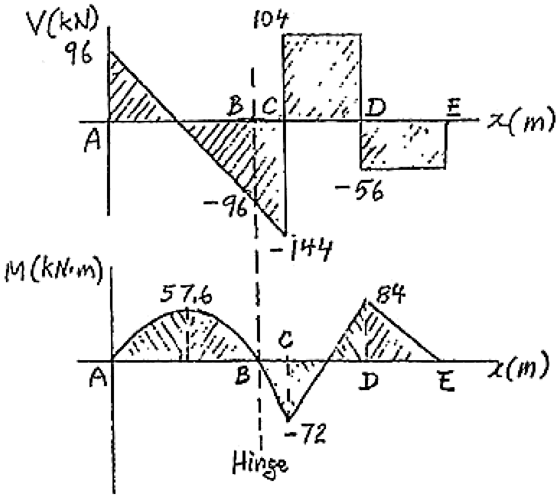


Just to the left of D:

$\sum F_y = 0: \quad V = 160 - 56 = +104\text{ kN}$

$\sum M_D = 0: \quad M = (56)(1.5) = +84\text{ kN}\cdot\text{m}$

PROBLEM 5.22 *(Continued)*



From the diagram,

$$|M|_{\max} = 84 \text{ kN} \cdot \text{m} = 84 \times 10^3 \text{ N} \cdot \text{m}$$

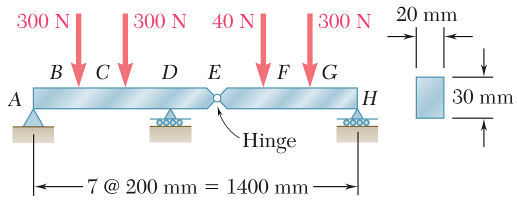
For $W310 \times 60$ rolled-steel shape,

$$S_x = 844 \times 10^3 \text{mm}^3$$
$$= 844 \times 10^{-6} \text{m}^3$$

Stress: $\sigma_m = \dfrac{|M|_{\max}}{S}$

$$\sigma_m = \frac{84 \times 10^3}{844 \times 10^{-6}} = 99.5 \times 10^6 \text{ Pa}$$

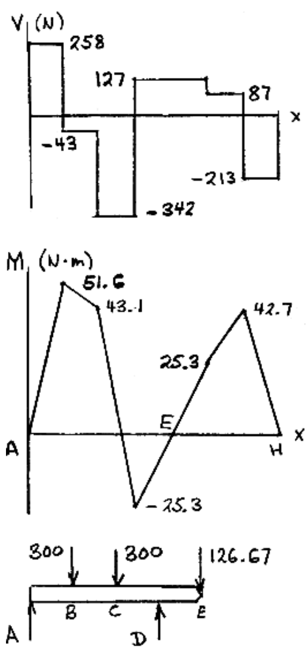
$\sigma_m = 99.5$ MPa ◀



PROBLEM 5.23

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

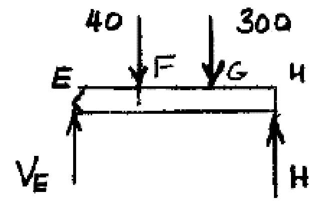
SOLUTION



Free body $EFGH$. Note that $M_E = 0$ due to hinge.

$$+\circlearrowleft \Sigma M_E = 0: \quad 0.6\,H - (0.2)(40) - (0.40)(300) = 0$$

$$H = 213.33 \text{ N}$$



$$+\uparrow \Sigma F_y = 0: \quad V_E - 40 - 300 + 213.33 = 0$$

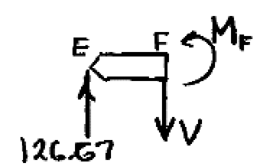
$$V_E = 126.67 \text{ N}$$

Shear:

E to F: $V = 126.67 \text{ N} \cdot \text{m}$

F to G: $V = 86.67 \text{ N} \cdot \text{m}$

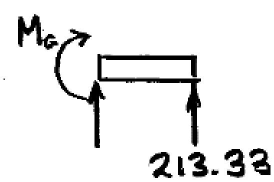
G to H: $V = -213.33 \text{ N} \cdot \text{m}$



Bending moment at F:

$$+\circlearrowleft \Sigma M_F = 0: \quad M_F - (0.2)(126.67) = 0$$

$$M_F = 25.33 \text{ N} \cdot \text{m}$$



Bending moment at G:

$$+\circlearrowleft \Sigma M_G = 0: \quad -M_G + (0.2)(213.33) = 0$$

$$M_G = 42.67 \text{ N} \cdot \text{m}$$

Free body $ABCDE$.

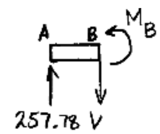
$$+\circlearrowleft \Sigma M_B = 0: \quad 0.6A + (0.4)(300) + (0.2)(300) - (0.2)(126.63) = 0$$

$$A = 257.78 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6D = 0$$

$$D = 468.89 \text{ N}$$

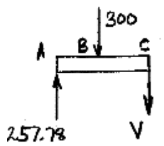
PROBLEM 5.23 *(Continued)*



Bending moment at B.

$$+\circlearrowleft \Sigma M_B = 0: \ -(0.2)(257.78) + M_B = 0$$

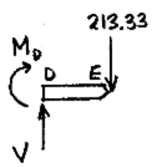
$$M_B = 51.56 \text{ N} \cdot \text{m}$$



Bending moment at C.

$$+\circlearrowleft \Sigma M_C = 0: \ -(0.4)(257.78) + (0.2)(300) + M_C = 0$$

$$M_C = 43.11 \text{ N} \cdot \text{m}$$



Bending moment at D.

$$+\circlearrowleft \Sigma M_D = 0: \ -M_D - (0.2)(213.33) = 0$$

$$M_D = -25.33 \text{ N} \cdot \text{m}$$

$$\max|M| = 51.56 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

$$S = \frac{1}{6}bh^2 = \frac{1}{6}(20)(30)^2$$

$$= 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$$

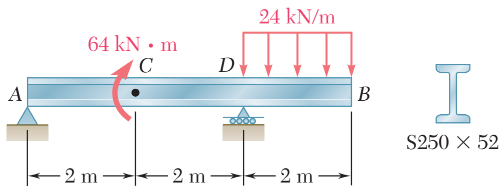
Normal stress:

$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa}$$

$$\sigma = 17.19 \text{ MPa} \quad \blacktriangleleft$$

$$|V|_{\max} = 342 \text{ N} \quad \blacktriangleleft$$

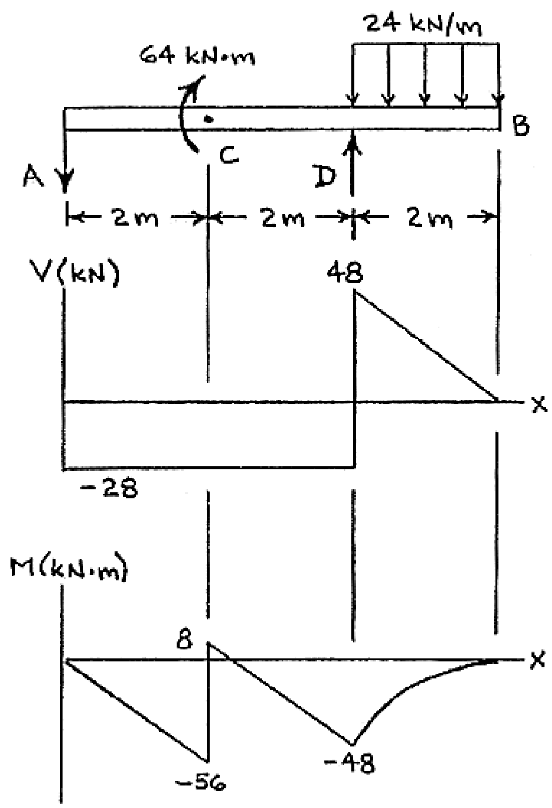
$$|M|_{\max} = 516 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 5.24

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

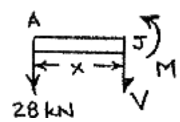


Reactions:

$+\circlearrowleft \Sigma M_D = 0: \quad 4A - 64 - (24)(2)(1) = 0 \quad \boldsymbol{A} = 28 \text{ kN}\downarrow$

$+\uparrow \Sigma F_y = 0: \quad -28 + D - (24)(2) = 0 \quad \boldsymbol{D} = 76 \text{ kN}\uparrow$

A to *C*: $\quad 0 < x < 2\text{m}$



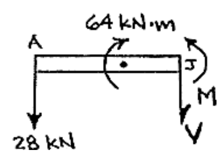
$+\uparrow \Sigma F_y = 0: \quad -V - 28 = 0$

$V = -28 \text{ kN}$

$+\circlearrowleft \Sigma M_J = 0: \quad M + 28x = 0$

$M = (-28x) \text{ kN} \cdot \text{m}$

C to *D*: $\quad 2\text{m} < x < 4\text{m}$



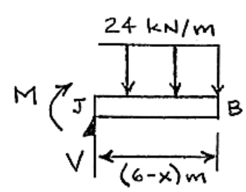
$+\uparrow \Sigma F_y = 0: \quad -V - 28 = 0$

$V = -28 \text{ kN}$

$+\circlearrowleft \Sigma M_J = 0: \quad M + 28x - 64 = 0$

$M = (-28x + 64) \text{ kN} \cdot \text{m}$

D to *B*: $\quad 4\text{m} < x < 6\text{m}$



$+\uparrow \Sigma F_y = 0:$

$V - 24(6 - x) = 0$

$V = (-24x + 144) \text{ kN}$

$+\circlearrowleft \Sigma M_J = 0:$

$$-M - 24(6 - x)\left(\frac{6 - x}{2}\right) = 0$$

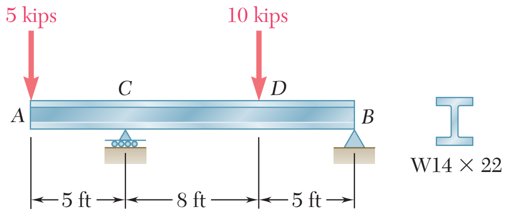
$M = -12(6 - x)^2 \text{ kN} \cdot \text{m}$

$\max|M| = 56 \text{ kN} \cdot \text{m} = 56 \times 10^3 \text{ N} \cdot \text{m}$

For S250 × 52 section, $\quad S = 482 \times 10^3 \text{ mm}^3$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{56 \times 10^3 \text{N} \cdot \text{m}}{482 \times 10^{-6} \text{m}^3} = 116.2 \times 10^6 \text{ Pa}$

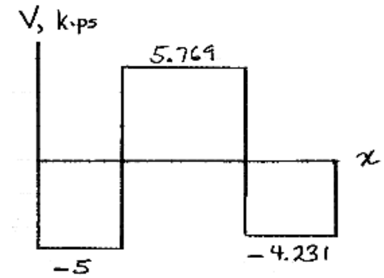
$\sigma = 116.2 \text{ MPa}$ ◀



PROBLEM 5.25

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION



Reaction at C: $\quad +\circlearrowleft \Sigma M_B = 0$: $(18)(5) - 13C + (5)(10) = 0$

$$C = 10.769 \text{ kips}$$

Reaction at B: $\quad +\circlearrowleft M_C = 0$: $(5)(5) - (8)(10) + 13B = 0$

$$B = 4.231 \text{ kips}$$

Shear diagram:

A to C^-: $\quad V = -5$ kips

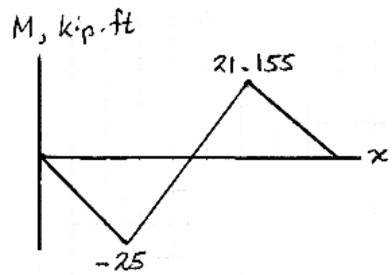
C^+ to D^-: $\quad V = -5 + 10.769 = 5.769$ kips

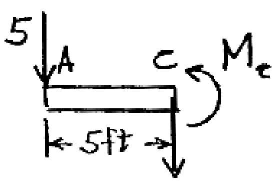
D^+ to B: $\quad V = 5.769 - 10 = -4.231$ kips

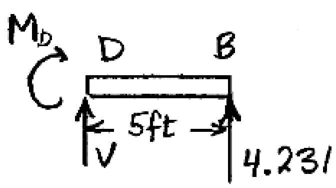
At A and B, $\quad M = 0$

At C, $\quad +\circlearrowleft M_C = 0$: $(5)(5) + M_C = 0$

$$M_C = -25 \text{ kip} \cdot \text{ft}$$







At D, $\quad +\circlearrowleft \Sigma M_D = 0$: $-M_D + (5)(4.231)$

$$M_D = 21.155 \text{ kip} \cdot \text{ft}$$

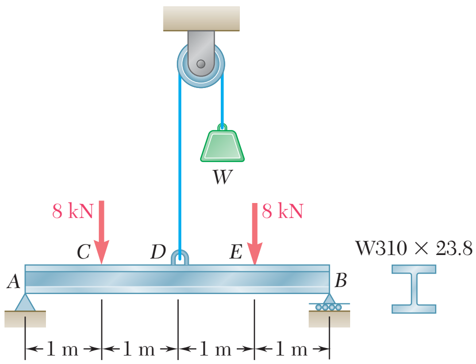
$$|V|_{\max} = 5.77 \text{ kips} \blacktriangleleft$$

$|M|_{\max}$ occurs at C. $\quad |M|_{\max} = 25 \text{ kip} \cdot \text{ft} = 300 \text{ kip} \cdot \text{in.} \blacktriangleleft$

For W14 × 22 rolled-steel section, $\quad S = 29.0 \text{ in}^3$

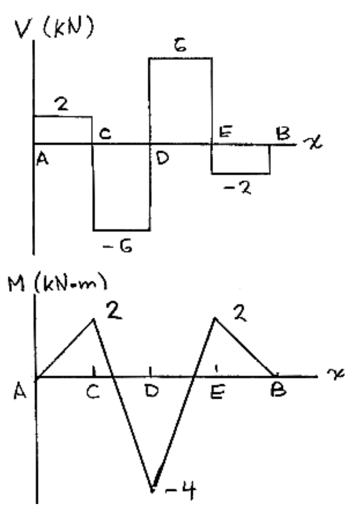
Normal stress: $\quad \sigma = \dfrac{M}{S} = \dfrac{300}{29.0} \qquad \sigma = 10.34 \text{ ksi} \blacktriangleleft$

PROBLEM 5.26



Knowing that $W = 12$ kN, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.

SOLUTION



By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad A - 8 + 12 - 8 + B = 0$$

$$A = B = 2 \text{ kN}$$

Shear: A to C^-: $V = 2$ kN

C^+ to D^-: $V = -6$ kN

D^+ to E^-: $V = 6$ kN

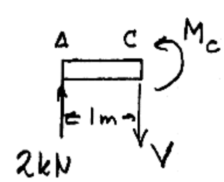
E^+ to B: $V = -2$ kN

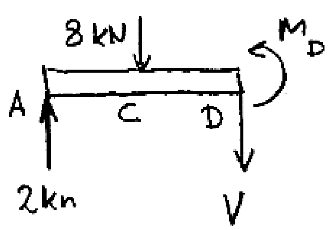
$$|V|_{\max} = 6.00 \text{ kN} \blacktriangleleft$$

Bending moment:

At C, $\quad +\circlearrowleft \Sigma M_C = 0: \quad M_C - (1)(2) = 0$

$$M_C = 2 \text{ kN} \cdot \text{m}$$





At D, $\quad +\Sigma M_D = 0: \quad M_D - (2)(2) + (8)(1) = 0$

$$M_D = -4 \text{ kN} \cdot \text{m}$$

By symmetry, $\quad M = 2 \text{ kN} \cdot \text{m}$ at D. $\qquad M_E = 2 \text{ kN} \cdot \text{m}$

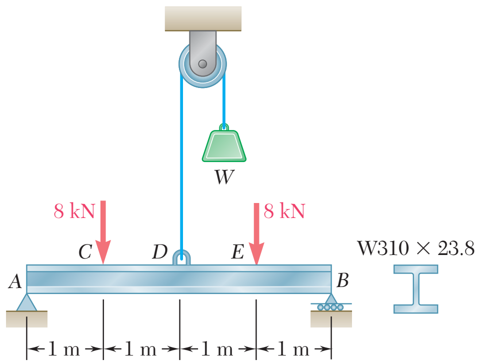
$$\max|M| = 4.00 \text{ kN} \cdot \text{m} \text{ occurs at } E. \blacktriangleleft$$

For W310 × 23.8, $\quad S_x = 280 \times 10^3 \text{mm}^3 = 280 \times 10^{-6} \text{m}^3$

Normal stress: $\quad \sigma_{\max} = \dfrac{|M|_{\max}}{S_x} = \dfrac{4 \times 10^3}{280 \times 10^{-6}}$

$$= 14.29 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 14.29 \text{ MPa} \blacktriangleleft$$

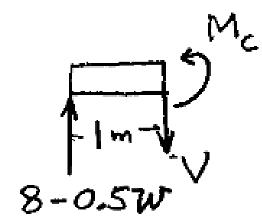
PROBLEM 5.27



Determine (*a*) the magnitude of the counterweight W for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (*Hint:* Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION

By symmetry, $A = B$



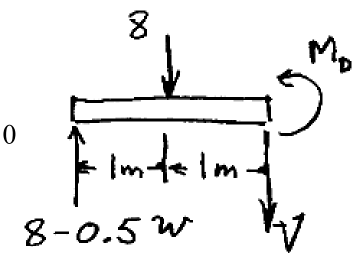
$$+\uparrow \Sigma F_y = 0: \quad A - 8 + W - 8 + B = 0$$

$$A = B = 8 - 0.5W$$

Bending moment at C:

$$+\circlearrowleft \Sigma M_C = 0: \quad -(8 - 0.5W)(1) + M_C = 0$$

$$M_C = (8 - 0.5W) \text{ kN} \cdot \text{m}$$



Bending moment at D:

$$+\circlearrowleft \Sigma M_D = 0: \quad -(8 - 0.5W)(2) + (8)(1) + M_D = 0$$

$$M_D = (8 - W) \text{ kN} \cdot \text{m}$$

Equate: $\quad -M_D = M_C \qquad W - 8 = 8 - 0.5W$

$W = 10.67$ kN ◀

(*a*) $\quad W = 10.6667$ kN

$M_C = -2.6667 \text{ kN} \cdot \text{m}$

$M_D = 2.6667 \text{ kN} \cdot \text{m} = 2.6667.10^3 \text{ N} \cdot \text{m}$

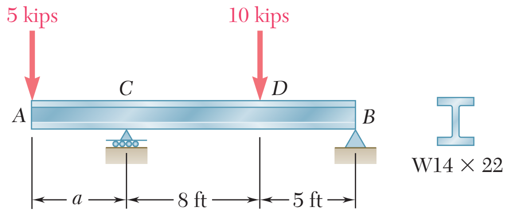
$|M|_{\max} = 2.6667 \text{ kN} \cdot \text{m}$

For W310 × 23.8 rolled-steel shape,

$$S_x = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{m}^3$$

(*b*) $\quad \sigma_{\max} = \dfrac{|M|_{\max}}{S_x} = \dfrac{2.6667 \times 10^3}{280 \times 10^{-6}} = 9.52 \times 10^6 \text{ Pa}$

$\sigma_{\max} = 9.52$ MPa ◀



PROBLEM 5.28

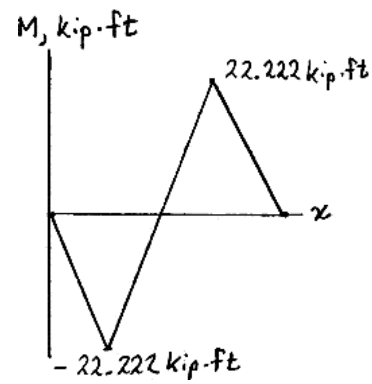
Determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

Reaction at B:

$$+\circlearrowleft \Sigma M_C = 0: \quad 5a - (8)(10) + 13R_B = 0$$

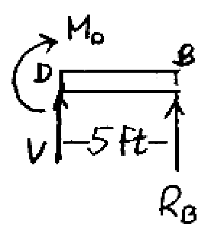
$$R_B = \frac{1}{18}(80 - 5a)$$



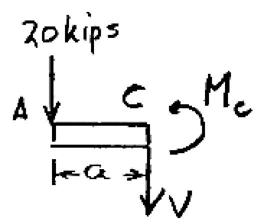
Bending moment at D:

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D + 5R_B = 0$$

$$M_D = 5R_B = \frac{5}{13}(80 - 5a)$$



Bending moment at C:



$$+\circlearrowleft M_C = 0 \quad 5a + M_C = 0$$

$$M_C = -5a$$

Equate:

$$-M_C = M_D$$

$$5a = \frac{5}{13}(80 - 5a)$$

$$a = 4.4444 \text{ ft}$$

(*a*) $a = 4.44$ ft ◀

Then

$$-M_C = M_D = (5)(4.4444) = 22.222 \text{ kip} \cdot \text{ft}$$

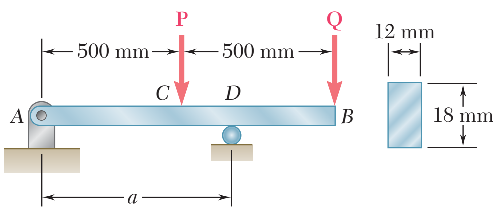
$$|M|_{max} = 22.222 \text{ kip} \cdot \text{ft} = 266.67 \text{ kip} \cdot \text{in.}$$

For W14 × 22 rolled-steel section, $S = 29.0 \text{ in}^3$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{266.67}{29.0} = 9.20 \text{ ksi}$$

(*b*) 9.20 ksi ◀

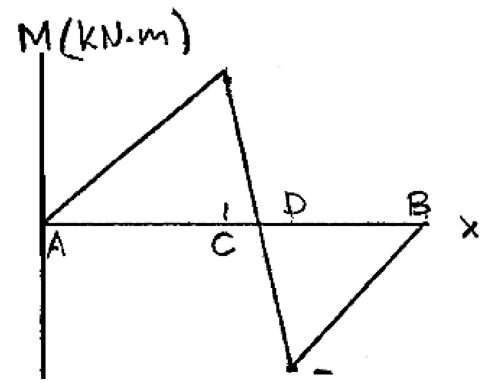


PROBLEM 5.29

Knowing that $P = Q = 480$ N, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$$P = 480 \text{ N} \qquad Q = 480 \text{ N}$$

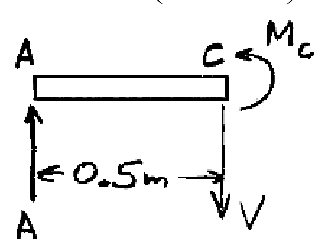


Reaction at A: $\quad +\circlearrowleft \Sigma M_D = 0: \quad -Aa + 480(a - 0.5) - 480(1 - a) = 0$

$$A = \left(960 - \frac{720}{a}\right) \text{ N}$$

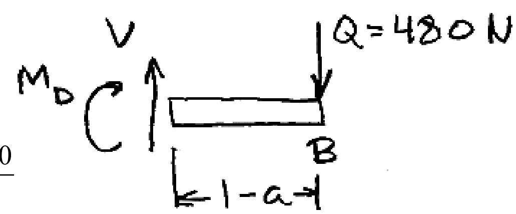
Bending moment at C: $\quad +\circlearrowleft \Sigma M_C = 0: \quad -0.5A + M_C = 0$

$$M_C = 0.5A = \left(480 - \frac{360}{a}\right) \text{ N} \cdot \text{m}$$



Bending moment at D: $\quad +\circlearrowleft \Sigma M_D = 0: \quad -M_D - 480(1 - a) = 0$

$$M_D = -480(1 - a) \text{ N} \cdot \text{m}$$



(*a*) Equate: $\quad -M_D = M_C \qquad 480(1 - a) = 480 - \dfrac{360}{a}$

$$a = 0.86603 \text{ m}$$

$a = 866$ mm ◀

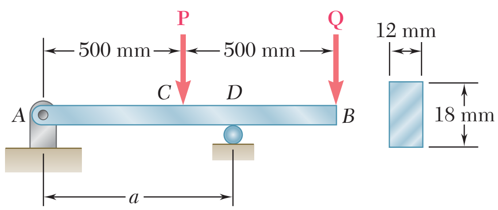
$$A = 128.62 \text{ N} \qquad M_C = 64.31 \text{ N} \cdot \text{m} \qquad M_D = -64.31 \text{ N} \cdot \text{m}$$

(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(13)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{m}^3$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{64.31}{6.48 \times 10^{-9}} = 99.2 \times 10^6 \text{ Pa}$$

$\sigma_{\max} = 99.2$ MPa ◀



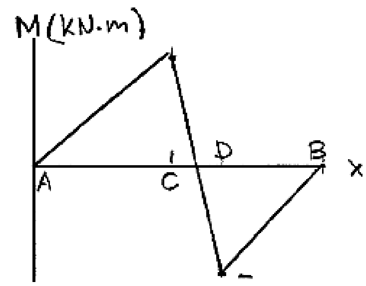
PROBLEM 5.30

Solve Prob. 5.29, assuming that $P = 480\ \text{N}$ and $Q = 320\ \text{N}$.

PROBLEM 5.29 Knowing that $P = Q = 480\ \text{N}$, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$$P = 480\ \text{N} \qquad Q = 320\ \text{N}$$

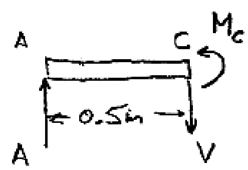


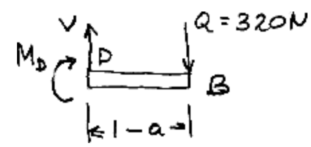
Reaction at A: $\circlearrowleft + \Sigma M_D = 0\text{:}\quad Aa + 480(a - 0.5) - 320(1 - a) = 0$

$$A = \left(800 - \frac{560}{a}\right)\ \text{N}$$

Bending moment at C: $\circlearrowleft + \Sigma M_C = 0\text{:}\ -0.5A + M_C = 0$

$$M_C = 0.5A = \left(400 - \frac{280}{a}\right)\ \text{N} \cdot \text{m}$$





Bending moment at D: $\circlearrowleft + \Sigma M_D = 0\text{:}\quad -M_D - 320(1 - a) = 0$

$$M_D = (-320 + 320a)\ \text{N} \cdot \text{m}$$

(*a*) Equate: $\qquad -M_D = M_C \qquad 320 - 320a = 400 - \dfrac{280}{a}$

$$320a^2 + 80a - 280 = 0 \qquad a = 0.81873\ \text{m},\ -1.06873\ \text{m}$$

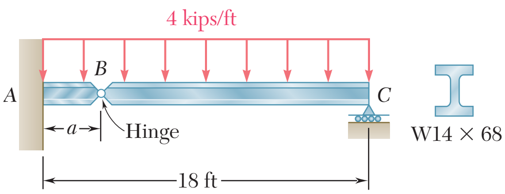
Reject negative root. $\qquad a = 819\ \text{mm}$ ◀

$A = 116.014\ \text{N} \qquad M_C = 58.007\ \text{N} \cdot \text{m} \qquad M_D = -58.006\ \text{N} \cdot \text{m}$

(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(18)^2 = 648\ \text{mm}^3 = 648 \times 10^{-9}\ \text{m}^3$$

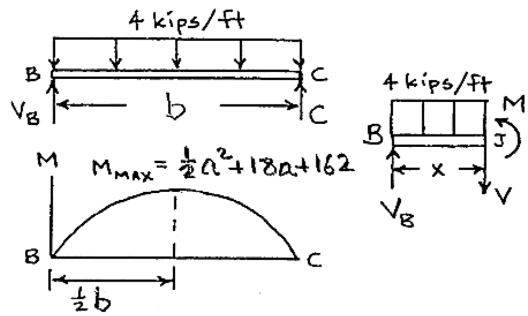
$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{58.0065}{648 \times 10^{-9}} = 89.5 \times 10^6\ \text{Pa} \qquad \sigma_{\max} = 89.5\ \text{MPa}\ ◀$$



PROBLEM 5.31

Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION



For W14 × 68, $\quad S_x = 103 \text{ in}^3$

Let $b = (18 - a)$ ft

Segment *BC*:

By symmetry, $\quad V_B = C$

$$+\uparrow \Sigma F_y = 0: \quad V_B + C - 4b = 0$$

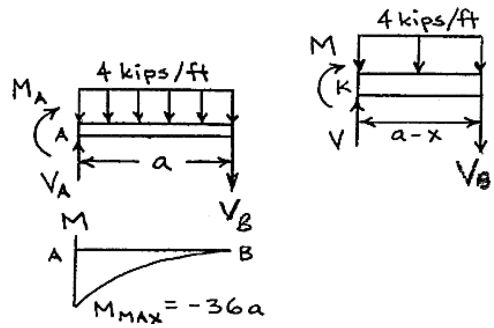
$$V_B = 2b$$

$$+\circlearrowleft \Sigma M_J = 0: \; -V_B x + (4x)\left(\frac{x}{2}\right) - M = 0$$

$$M = V_B x - 2x^2 = 2bx - 2x^2 \quad \text{lb} \cdot \text{ft}$$

$$\frac{dM}{dx} = 2b - x_m = 0 \qquad x_m = \frac{1}{2}b$$

$$M_{\text{max}} = b^2 - \frac{1}{2}b^2 = \frac{1}{2}b^2$$



Segment *AB*:

$$+\circlearrowleft \Sigma M_K = 0: \; -4(a - x)\frac{(a - x)}{2}$$

$$-V_B(a - x) - M = 0$$

$$M = -2(a - x)^2 + 2b(a - x)$$

$|M_{\text{max}}|$ occurs at $x = 0$.

$$|M_{\text{max}}| = -2a^2 - 2ab = -2a^2 - 2a(18 - a) = 36a$$

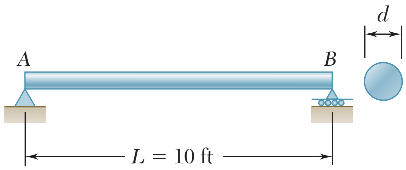
(*a*) Equate the two values of $|M_{\text{max}}|$: $\quad 36a = \frac{1}{2}b^2 = \frac{1}{2}(18 - a)^2 = 162 - 18a + \frac{1}{2}a^2$

$$\frac{1}{2}a^2 - 54a + 162 = 0 \qquad a = 54 \pm \sqrt{(54)^2 - (4)\left(\tfrac{1}{2}\right)(162)}$$

$$a = 54 \pm 50.9118 = 3.0883 \text{ ft} \qquad\qquad a = 3.09 \text{ ft} \blacktriangleleft$$

(*b*) $\quad |M|_{\text{max}} = 36a = 111.179 \text{ kip} \cdot \text{ft} = 1334.15 \text{ kip} \cdot \text{in.}$

$$\sigma = \frac{|M|_{\text{max}}}{S_x} = \frac{1334.15}{103} = 12.95 \text{ kips/in}^2 \qquad\qquad \sigma_m = 12.95 \text{ ksi} \blacktriangleleft$$



PROBLEM 5.32

A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490 \text{ lb/ft}^3$, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 4 ksi.

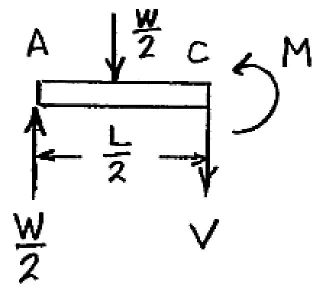
SOLUTION

Let W = total weight.

$$W = AL\gamma = \frac{\pi}{4} d^2 L\gamma$$

Reaction at A:

$$A = \frac{1}{2}W$$



Bending moment at center of beam:

$$+\circlearrowleft \Sigma M_C = 0: \quad -\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32} d^2 L^2 \gamma$$

For circular cross section, $\left(c = \frac{1}{2}d\right)$

$$I = \frac{\pi}{4}c^4, \qquad S = \frac{I}{c} = \frac{\pi}{4}c^3 = \frac{\pi}{32}d^3$$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32} d^2 L^2 \gamma}{\frac{\pi}{32} d^3} = \frac{L^2 \gamma}{d}$$

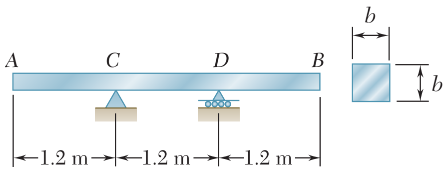
Solving for d, $\qquad d = \dfrac{L^2 \gamma}{\sigma}$

Data: $\qquad L = 10 \text{ ft} = (12)(10) = 120 \text{ in.}$

$$\gamma = 490 \text{ lb/ft}^3 = \frac{490}{12^3} = 0.28356 \text{ lb/in}^3$$

$$\sigma = 4 \text{ ksi} = 4000 \text{ lb/in}^2$$

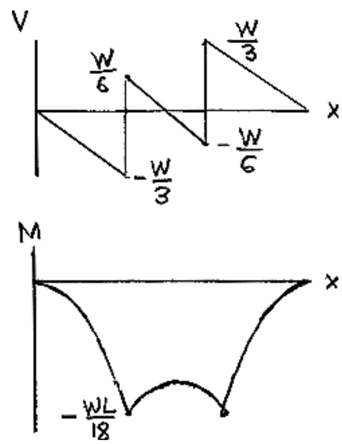
$$d = \frac{(120)^2 (0.28356)}{4000} \qquad d = 1.021 \text{ in.} \blacktriangleleft$$



PROBLEM 5.33

A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860 \text{ kg/m}^3$, determine the dimension b for which the maximum normal stress due to bending is (*a*) 10 MPa, (*b*) 50 MPa.

SOLUTION

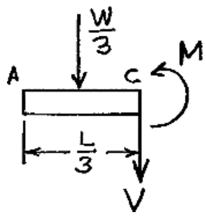


Weight density: $\gamma = \rho g$

Let L = total length of beam.

$$W = AL\rho g = b^2 L\rho g$$

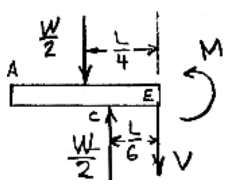
Reactions at C and D: $\quad C = D = \dfrac{W}{2}$



Bending moment at C:

$$+\circlearrowleft \Sigma M_C = 0: \left(\frac{L}{6}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$



Bending moment at center of beam:

$$+\circlearrowleft \Sigma M_E = 0: \left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0 \qquad M = -\frac{WL}{24}$$

$$\max|M| = \frac{WL}{18} = \frac{b^2L^2\rho g}{18}$$

For a square section, $\quad S = \dfrac{1}{6}b^3$

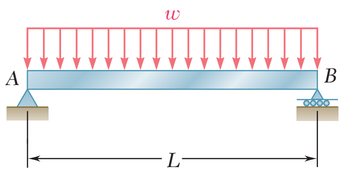
Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{b^2L^2\rho g/18}{b^3/6} = \dfrac{L^2\rho g}{3b}$

Solve for b: $\quad b = \dfrac{L^2\rho g}{3\sigma}$

Data: $L = 3.6 \text{ m} \quad \rho = 7860 \text{ kg/m}^3 \quad g = 9.81 \text{ m/s}^2 \quad (a)\ \sigma = 10\times10^6 \text{ Pa} \quad (b)\ \sigma = 50\times10^6 \text{ Pa}$

(*a*) $\quad b = \dfrac{(3.6)^2(7860)(9.81)}{(3)(10\times10^6)} = 33.3\times10^{-3} \text{ m}$ $\qquad b = 33.3 \text{ mm}$ ◀

(*b*) $\quad b = \dfrac{(3.6)^2(7860)(9.81)}{(3)(50\times10^6)} = 6.66\times10^{-3} \text{ m}$ $\qquad b = 6.66 \text{ mm}$ ◀



PROBLEM 5.34

Using the method of Sec. 5.2, solve Prob. 5.1*a*.

PROBLEM 5.1 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

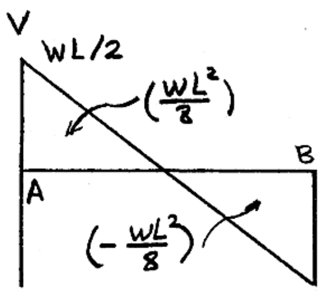
$$+\circlearrowleft \Sigma M_B = 0: \quad -AL + wL \cdot \frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad BL - wL \cdot \frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = A - wx \qquad V = \frac{wL}{2} - wx \blacktriangleleft$$

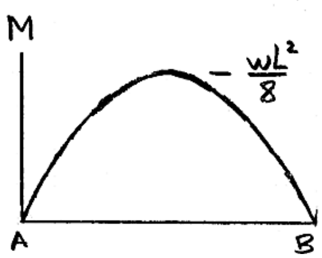


$$\frac{dM}{dx} = V$$

$$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{wL}{2} - wx\right) dx$$

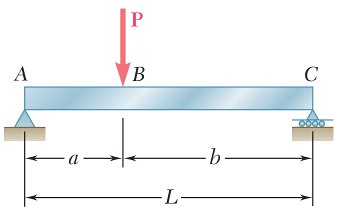
$$= \frac{wLx}{2} - \frac{wx^2}{2}$$

$$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} \qquad M = \frac{w}{2}(Lx - x^2) \blacktriangleleft$$



Maximum M occurs at $x = \frac{1}{2}$, where

$$V = \frac{dM}{dx} = 0 \qquad |M|_{max} = \frac{wL^2}{8} \blacktriangleleft$$

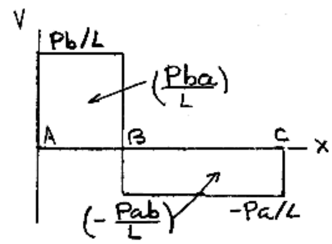


PROBLEM 5.35

Using the method of Sec. 5.2, solve Prob. 5.2*a*.

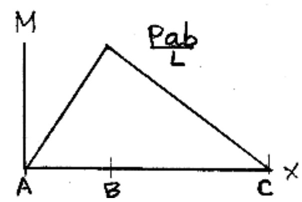
PROBLEM 5.2 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$$



At A^+; $\quad V = A = \dfrac{Pb}{L} \qquad M = 0$

A to B^-: $\quad 0 < x < a$

$$w = 0 \qquad \int_0^x w\,dx = 0$$

$$V - V_A = 0 \qquad\qquad V = \frac{Pb}{L} \blacktriangleleft$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{Pb}{L}\,dx = \frac{Pba}{L} \qquad\qquad M_B = \frac{Pba}{L} \blacktriangleleft$$

At B^+; $\quad V = A - P = \dfrac{Pb}{L} - P = -\dfrac{Pa}{L}$

B^+ to C: $\quad a < x < L$

$$w = 0 \qquad \int_a^x w\,dx = 0$$

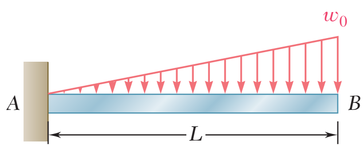
$$V_C - V_B = 0 \qquad\qquad V = -\frac{Pa}{L} \blacktriangleleft$$

$$M_C - M_B = \int_a^L V\,dx = -\frac{Pa}{L}(L-a) = -\frac{Pab}{L}$$

$$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$$

$$|M|_{\max} = \frac{Pab}{L} \blacktriangleleft$$

PROBLEM 5.36

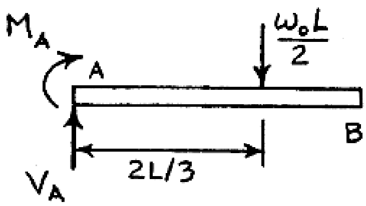


Using the method of Sec. 5.2, solve Prob. 5.3*a*.

PROBLEM 5.3 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Free body diagram for determining reactions.

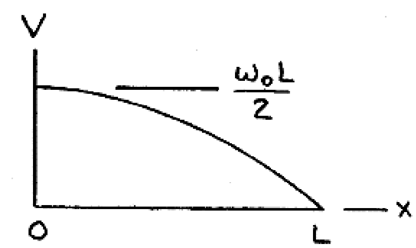


Reactions:

$$+\uparrow \Sigma F_y = 0: \quad V_A - \frac{w_0 L}{2} = 0 \qquad V_A = \frac{w_0 L}{2}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \left(\frac{w_0 L}{2}\right)\left(\frac{2L}{3}\right) = 0$$

$$M_A = -\frac{w_0 L^2}{3}$$

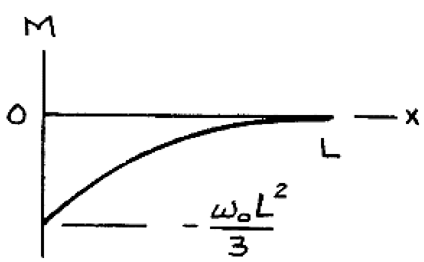


$$w = w_0 \frac{x}{L}, \quad V_A = \frac{w_0 L}{2}, \quad M_A = -\frac{w_0 L^2}{3}$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V - V_A = -\int_0^x \frac{w_0 x}{L}\,dx = -\frac{w_0 x^2}{2L}$$

$$V = \frac{w_0 L}{2} - \frac{w_0 x^2}{2L} \blacktriangleleft$$



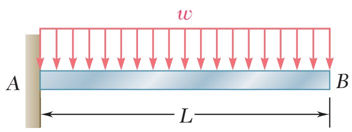
$$\frac{dM}{dx} = V = \frac{w_0 L}{2} - \frac{w_0 x^2}{2L}$$

$$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{w_0 L}{2} - \frac{w_0 x^2}{2L}\right) dx$$

$$= \frac{w_0 L}{2} x - \frac{w_0 x^3}{6L}$$

$$M = -\frac{w_0 L^2}{3} + \frac{w_0 L}{2} x - \frac{w_0 x^3}{6L} \blacktriangleleft$$

PROBLEM 5.37



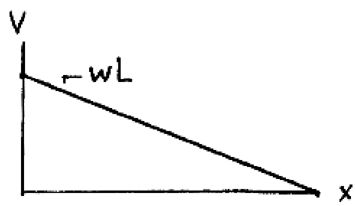
Using the method of Sec. 5.2, solve Prob. 5.4*a*.

PROBLEM 5.4 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad V_A - wL = 0 \qquad V_A = wL$$

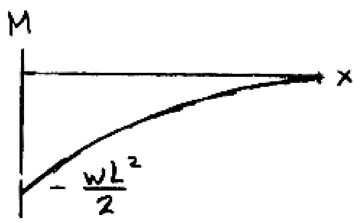
$$+\circlearrowleft \Sigma M_A = 0: \quad -M - (wL)\left(\frac{L}{2}\right) = 0 \qquad M_A = -\frac{wL^2}{2}$$



$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = wL - wx \blacktriangleleft$$



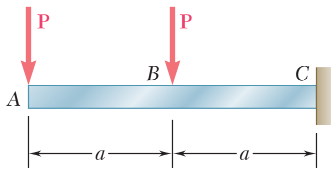
$$\frac{dM}{dx} = V = wL - wx$$

$$M - M_A = -\int_0^x (wL - wx)dx = wLx - \frac{wx^2}{2}$$

$$M = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2} \blacktriangleleft$$

$$|V|_{\max} = wL$$

$$|M|_{\max} = \frac{wL^2}{2}$$

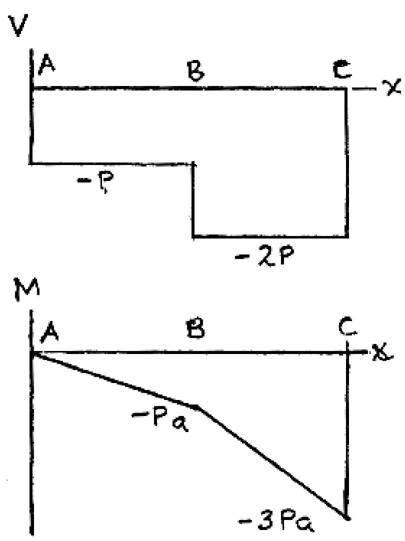


PROBLEM 5.38

Using the method of Sec. 5.2, solve Prob. 5.5*a*.

PROBLEM 5.5 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



At A^+: $V_A = -P$

Over AB: $\frac{dV}{dx} = -w = 0$

$$\frac{dM}{dx} = V = V_A = -P \blacktriangleleft$$

$M = -Px + C$

$M = 0$ at $x = 0$ $C_1 = 0$

$$M = -Px \blacktriangleleft$$

At point B: $x = a$ $M = -Pa$

At point B^+: $V = -P - P = -2P$

Over BC: $\frac{dV}{dx} = -w = 0$

$$\frac{dM}{dx} = V = -2P \blacktriangleleft$$

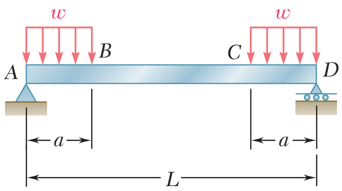
$M = -2Px + C_2$

At B: $x = a$ $M = -Pa$

$-Pa = -2Pa + C_2$ $C_2 = Pa$

$$M = -2Px + Pa \blacktriangleleft$$

At C: $x = 2a$ $M = -3Pa$



PROBLEM 5.39

Using the method of Sec. 5.2, solve Prob. 5.6*a*.

PROBLEM 5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

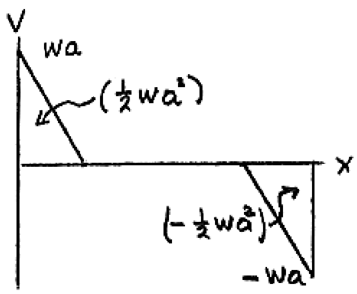
Reactions: $A = D = wa$

A to B: $0 < x < a \qquad w = w$

$$V_A = A = wa, \qquad M_A = 0$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

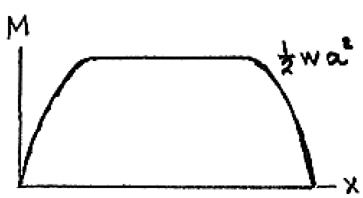
$$V = w(a - x) \blacktriangleleft$$



$$\frac{dM}{dx} = V = wa - wx$$

$$M - M_A = \int_0^x V\,dx = \int_0^x (wa - wx)dx$$

$$M = wax - \frac{1}{2}wx^2 \blacktriangleleft$$



$$V_B = 0 \qquad M_B = \frac{1}{2}wa^2$$

B to C: $a < x < L - a$ $\qquad V = 0 \blacktriangleleft$

$$\frac{dM}{dx} = V = 0$$

$$M - M_B = \int_a^x V\,dx = 0$$

$$M = M_B \qquad M = \frac{1}{2}wa^2 \blacktriangleleft$$

PROBLEM 5.39 *(Continued)*

$$L - a < x < L$$

C to D: $V - V_C = -\int_{L-a}^{x} w\,dx = -w[x - (L - a)]$

$$V = w[L - x - a)] \blacktriangleleft$$

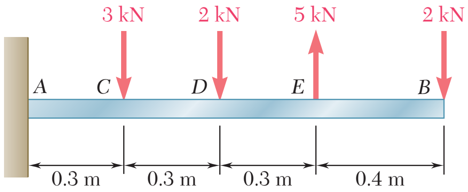
$$M - M_C = \int_{L-a}^{x} V\,dx = \int_{L-a}^{x} -w[x - (L - a)]dx$$

$$= -w\left[\frac{x^2}{2} - (L - a)x\right]\Bigg|_{L-a}^{x}$$

$$= -w\left[\frac{x^2}{2} - (L - a)x - \frac{(L - a)^2}{2} + (L - a)^2\right]$$

$$= -w\left[\frac{x^2}{2} - (L - a)x + \frac{(L - a)^2}{2}\right]$$

$$M = \frac{1}{2}wa^2 - w\left[\frac{x^2}{2} - (L - a)x + \frac{(L - a)^2}{2}\right] \blacktriangleleft$$



PROBLEM 5.40

Using the method of Sec. 5.2, solve Prob. 5.7.

PROBLEM 5.7 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

<u>Free body diagram for determining reactions</u>.

Reactions:

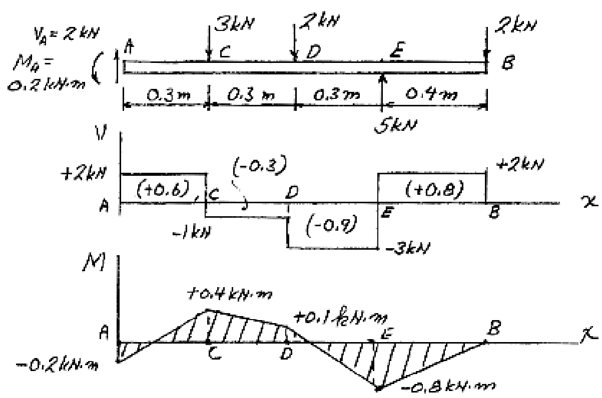
$$+\uparrow \Sigma F_y = 0: \quad V_A - 3\text{ kN} - 2\text{ kN} + 5\text{ kN} + 2\text{ kN} = 0$$

$$V_A = 2\text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (3\text{ kN})(0.3\text{ m}) - (2\text{ kN})(0.6\text{ m}) + (5\text{ kN})(0.9\text{ m}) - (2\text{ kN})(1.3\text{ m}) = 0$$

$$M_A = 0.2\text{ kN}\cdot\text{m}$$

Between concentrated loads and the vertical reaction, the scope of the shear diagram is ϕ, i.e., the shear is constant. Thus, the area under the shear diagram is equal to the change in bending moment.



A to *C*:

$V = 2\text{ kN} \quad M_C - M_A = +0.6 \quad M_C = +0.4\text{ kN}$

C to *D*:

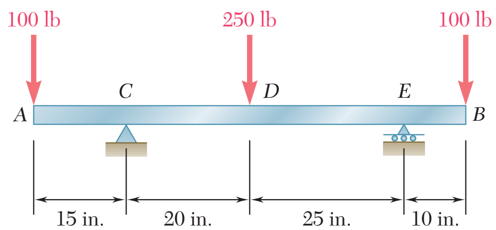
$V = -1\text{ kN} \quad M_D - M_C = -0.3 \quad M_D = +0.1\text{ kN}\cdot\text{m}$

D to *E*:

$V = -3\text{ kN} \quad M_E - M_D = -0.9 \quad M_E = -0.8\text{ kN}\cdot\text{m}$

E to *B*:

$V = +2\text{ kN} \quad M_B - M_E = +0.8 \quad M_B = 0 \text{ (Checks)}$



PROBLEM 5.41

Using the method of Sec. 5.2, solve Prob. 5.8.

PROBLEM 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

<u>Free body diagram for determining reactions</u>.

Reactions:

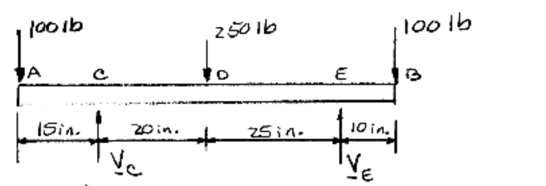
$$+\uparrow \Sigma F_Y = 0: \quad V_C + V_E - 100\text{ lb} - 250\text{ lb} - 100\text{ lb} = 0$$

$$V_C = V_E = 450\text{ lb}$$

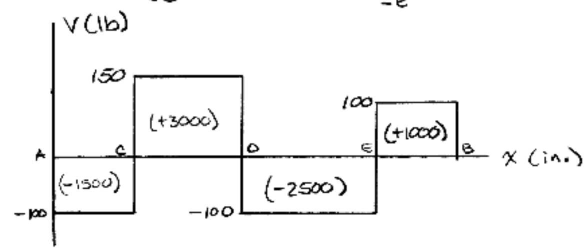
$$+\circlearrowleft \Sigma M_C = 0: \quad V_E(45\text{ in.}) + (100\text{ lb})(15\text{ in.}) - (250\text{ lb})(20\text{ in.}) - (100\text{ lb})(55\text{ in.}) = 0$$

$$V_E = 200\text{ lb}$$

$$\therefore \quad V_C = 250\text{ lb}$$



Between concentrated loads and the vertical reaction, the scope of the shear diagram is ϕ, i.e., the shear is constant. Thus, the area under the shear diagram is equal to he change in bending moment.

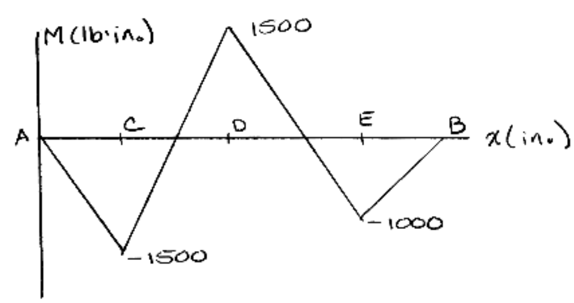


A to *C*:

$V = -100\text{ lb}, \quad M_C - M_A = -1500, \quad M_C = -1500\text{ lb}\cdot\text{in.}$

C to *D*:

$V = 150\text{ lb} \quad M_D - M_C = +3000, \quad M_D = +1500\text{ lb}\cdot\text{in.}$

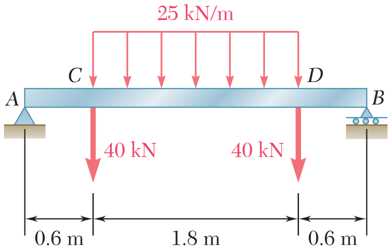


D to *E*:

$V = -100\text{ lb}, \quad M_E - M_D = -2500, \quad M_E = -1000\text{ lb}\cdot\text{in.}$

E to *B*:

$V = 100\text{ lb}, \quad M_B - M_E = +1000, \quad M_B = 0 \quad \text{(Checks)}$



PROBLEM 5.42

Using the method of Sec. 5.2, solve Prob. 5.9.

PROBLEM 5.9 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Free body diagram to determine reactions:

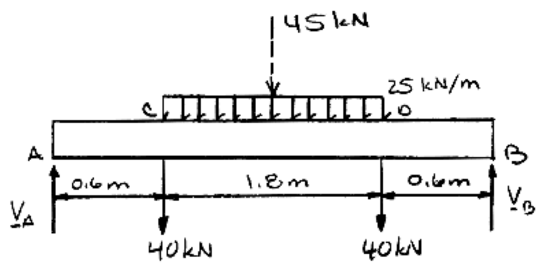
$+\circlearrowleft \Sigma M_A = 0:$

$$V_B(3.0\text{ m}) - 45\text{ kN}(1.5\text{ m}) - (40\text{ kN})(0.6\text{ m}) - (40\text{ kN})(2.4\text{ m}) = 0$$

$$V_B = 62.5\text{ kN}$$

$+\uparrow \Sigma F_y = 0: \quad V_A - 40\text{ kN} - 45\text{ kN} - 40\text{ kN} + 62.5\text{ kN} = 0$

$$V_A = 62.5\text{ kN}$$



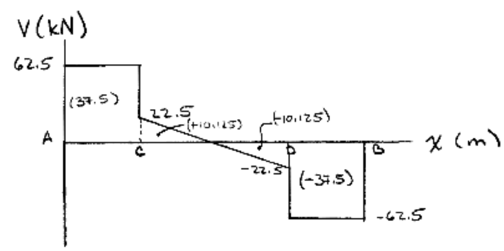
Change in bending moment is equal to area under shear diagram.

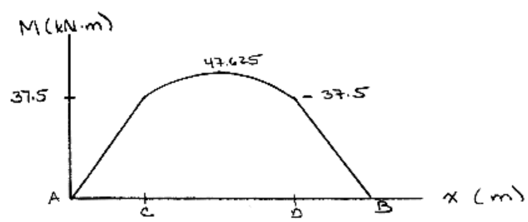
A to *C*: $(62.5\text{ kN})(0.6\text{ m}) = 37.5\text{ kN}\cdot\text{m}$

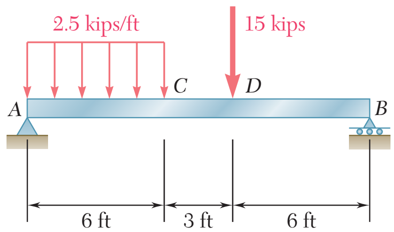
C to *E*: $\frac{1}{2}(0.9\text{ m})(22.5\text{ kN}) = 10.125\text{ kN}\cdot\text{m}$

E to *D*: $\frac{1}{2}(0.9\text{ m})(-22.5\text{ kN}) = -10.125\text{ kN}\cdot\text{m}$

D to *B*: $(-62.5\text{ kN})(0.6\text{ m}) = -37.5\text{ kN}\cdot\text{m}$





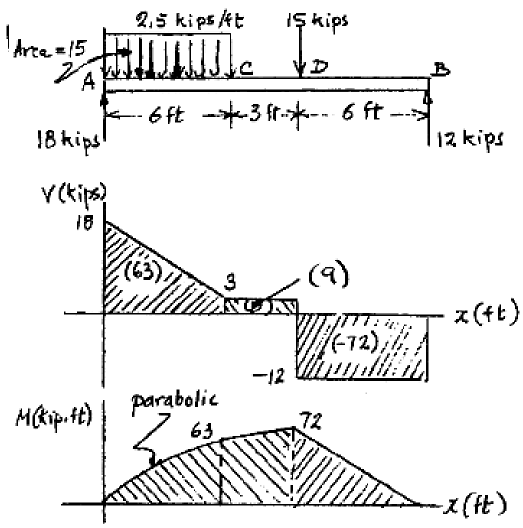


PROBLEM 5.43

Using the method of Sec. 5.2, solve Prob. 5.10.

PROBLEM 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



Reactions at supports A and B:

$$+\circlearrowleft \Sigma M_B = 0: \quad -15(R_A) + (12)(6)(2.5) + (6)(15) = 0$$

$$R_A = 18 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 15R_B - (3)(6)(2.5) - (9)(15) = 0$$

$$R_B = 12 \text{ kips}$$

Areas under shear diagram:

A to C: $\quad (6)(3) + \frac{1}{2}(6)(15) = 63 \text{ kip}\cdot\text{ft}$

C to D: $\quad (3)(3) = 9 \text{ kip}\cdot\text{ft}$

D to B: $\quad (6)(-12) = -72 \text{ kip}\cdot\text{ft}$

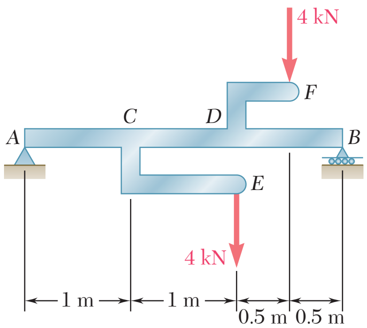
Bending moments:

$$M_A = 0$$

$$M_C = 0 + 63 = 63 \text{ kip}\cdot\text{ft}$$

$$M_D = 63 + 9 = 72 \text{ kip}\cdot\text{ft}$$

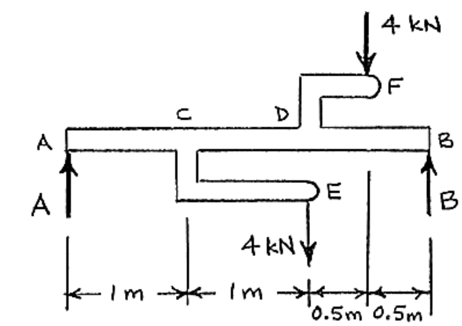
$$M_B = 72 - 72 = 0$$

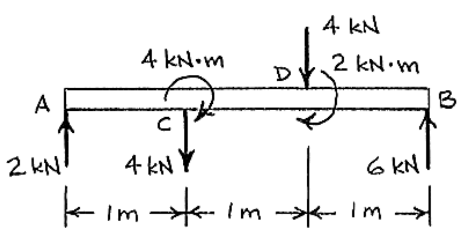


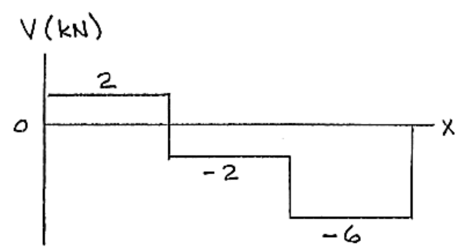
PROBLEM 5.44

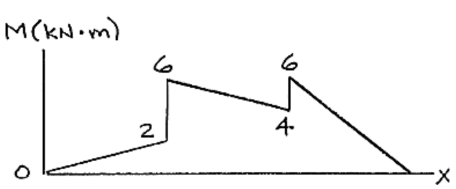
Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION









$+\circlearrowleft \Sigma M_B = 0:$

$$-3A + (1)(4) + (0.5)(4) = 0$$

$$\boldsymbol{A} = 2 \text{ kN} \uparrow$$

$+\circlearrowleft \Sigma M_A = 0: 3B - (2)(4) - (2.5)(4) = 0$

$$\boldsymbol{B} = 6 \text{ kN} \uparrow$$

Shear diagram:

A to *C*: $V = 2$ kN

C to *D*: $V = 2 - 4 = -2$ kN

D to *B*: $V = -2 - 4 = -6$ kN

Areas of shear diagram:

A to *C*: $\int V\,dx = (1)(2) = 2 \text{ kN} \cdot \text{m}$

C to *D*: $\int V\,dx = (1)(-2) = -2 \text{ kN} \cdot \text{m}$

D to *E*: $\int V\,dx = (1)(-6) = -6 \text{ kN} \cdot \text{m}$

Bending moments:

$M_A = 0$

$M_{C^-} = 0 + 2 = 2 \text{ kN} \cdot \text{m}$

$M_{C^+} = 2 + 4 = 6 \text{ kN} \cdot \text{m}$

$M_{D^-} = 6 - 2 = 4 \text{ kN} \cdot \text{m}$

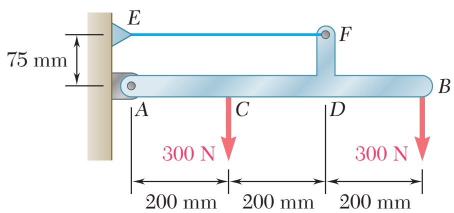
$M_{D^+} = 4 + 2 = 6 \text{kN} \cdot \text{m}$

$M_B = 6 - 6 = 0$

(*a*) $|V|_{\max} = 6.00 \text{ kN}$ ◀

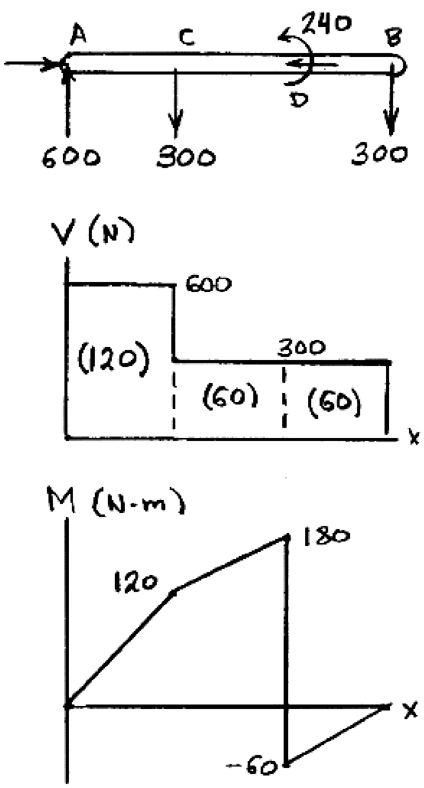
(*b*) $|M|_{\max} = 6.00 \text{ kN} \cdot \text{m}$ ◀

PROBLEM 5.45



Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



$+\circlearrowleft \Sigma M_A = 0:$

$$0.075\, F_{EF} - (0.2)(300) - (0.6)(300) = 0$$

$$F_{EF} = 3.2 \times 10^3 \text{ N}$$

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - F_{EF} = 0 \quad A_x = 3.2 \times 10^3 \text{ N}$

$+\uparrow \Sigma F_y = 0: \quad A_y - 300 - 300 = 0$

$$A_y = 600 \text{ N}$$

Couple at D: $\quad M_D = (0.075)(3.2 \times 10^3)$

$$= 240 \text{ N} \cdot \text{m}$$

Shear:

A to C: $\quad V = 600 \text{ N}$

C to B: $\quad V = 600 - 300 = 300 \text{ N}$

Areas under shear diagram:

A to C: $\quad \int V\,dx = (0.2)(600) = 120 \text{ N} \cdot \text{m}$

C to D: $\quad \int V\,dx = (0.2)(300) = 60 \text{ N} \cdot \text{m}$

D to B: $\quad \int V\,dx = (0.2)(300) = 60 \text{ N} \cdot \text{m}$

Bending moments:

$M_A = 0$

$M_C = 0 + 120 = 120 \text{ N} \cdot \text{m}$

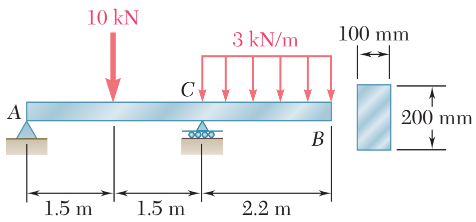
$M_{D^-} = 120 + 60 = 180 \text{ N} \cdot \text{m}$

$M_{D^+} = 180 - 240 = -60 \text{ N} \cdot \text{m}$

$M_B = -60 + 60 = 0$

Maximum $|V| = 600 \text{ N}$ ◀

Maximum $|M| = 180.0 \text{ N} \cdot \text{m}$ ◀



PROBLEM 5.46

Using the method of Sec. 5.2, solve Prob. 5.15.

PROBLEM 5.15 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0:$

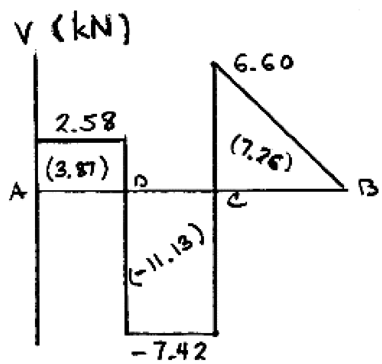
$$-3A + (1.5)(10) - (1.1)(2.2)(3) = 0$$

$$A = 2.58 \text{ kN}$$

$+\circlearrowleft \Sigma M_A = 0:$

$$-(1.5)(10) + 3C - (4.1)(2.2)(3) = 0$$

$$C = 14.02 \text{ kN}$$



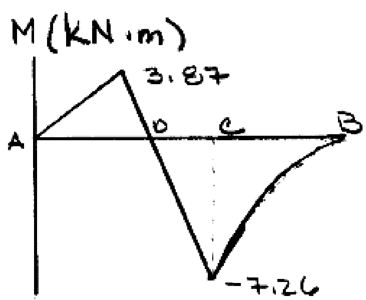
Shear:

A to D^-: $\quad V = 2.58$ kN

D^+ to C^-: $\quad V = 2.58 - 10 = -7.42$ kN

C^+: $\quad V = -7.42 + 14.02 = 6.60$ kN

B: $\quad V = 6.60 - (2.2)(3) = 0$



Areas under shear diagram:

A to D: $\quad \int V\,dx = (1.5)(2.58) = 3.87 \text{ kN}\cdot\text{m}$

D to C: $\quad \int V\,dx = (1.5)(-7.42) = -11.13 \text{ kN}\cdot\text{m}$

C to B: $\quad \int V\,dx = \left(\frac{1}{2}\right)(2.2)(6.60) = 7.26 \text{ kN}\cdot\text{m}$

Bending moments:

$M_A = 0$

$M_D = 0 + 3.87 = 3.87 \text{ kN}\cdot\text{m}$

$M_C = 3.87 - 11.13 = -7.26 \text{ kN}\cdot\text{m}$

$M_B = 7.26 - 7.26 = 0$

$$|M_C| = 7.26 \text{ kN}\cdot\text{m} = 7.26 \times 10^3 \text{ N}\cdot\text{m}$$

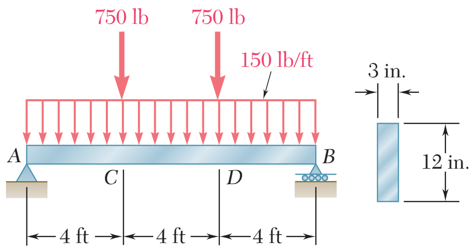
PROBLEM 5.46 *(Continued)*

For rectangular cross section, $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(100)(200)^2$

$$= 666.67 \times 10^3 \text{ mm}^3 = 666.67 \times 10^{-6} \text{ m}^2$$

Normal stress: $\sigma = \frac{|M_C|}{S} = \frac{7.26 \times 10^3}{666.67 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa}$

10.89 MPa ◀



PROBLEM 5.47

Using the method of Sec. 5.2, solve Prob. 5.16.

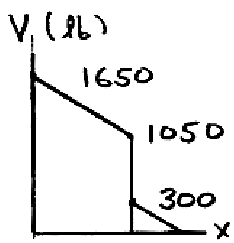
PROBLEM 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION

Reactions: $\quad C = A$ by symmetry

$$+\uparrow \Sigma F_y = 0: \quad A + C - (2)(750) - (12)(150) = 0$$

$$A = C = 1650 \text{ lb}$$



Shear:

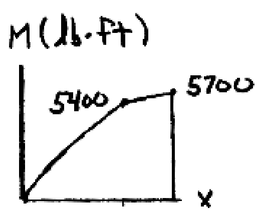
$$V_A = 1650 \text{ lb}$$

$$V_{C^-} = 1650 - (4)(150) = 1050 \text{ lb}$$

$$V_{C^+} = 1050 - 750 = 300 \text{ lb}$$

$$V_E = 300 - (2)(150) = 0$$

Areas under shear diagram:



A to C: $\quad \int V\,dx = \left(\frac{1}{2}\right)(1650 + 1050)(4)$

$$= 5400 \text{ lb} \cdot \text{ft}$$

C to E: $\quad \int V\,dx = \left(\frac{1}{2}\right)(300)(2) = 300 \text{ lb} \cdot \text{ft}$

Bending moments:

$$M_A = 0$$

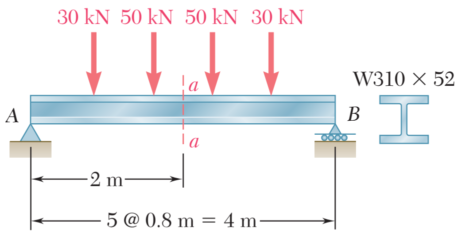
$$M_C = 0 + 5400 = 5400 \text{ lb} \cdot \text{ft}$$

$$M_E = 5400 + 300 = 5700 \text{ lb} \cdot \text{ft}$$

$$M_E = 5700 \text{ lb} \cdot \text{ft} = 68.4 \times 10^3 \text{ lb} \cdot \text{in.}$$

For rectangular cross section, $\quad S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(3)(12)^2 = 72 \text{ in}^3$

Normal stress: $\quad \sigma = \frac{|M|}{S} = \frac{68.4 \times 10^3}{72} = 950 \text{ psi}$ ◀

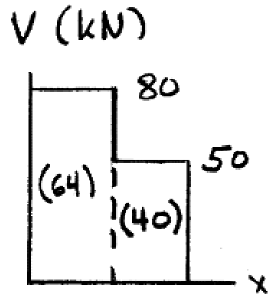


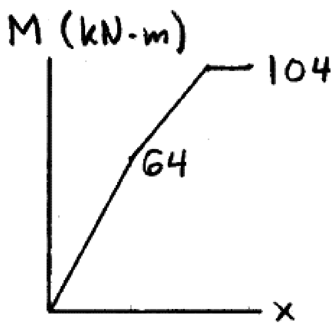
PROBLEM 5.48

Using the method of Sec. 5.2, solve Prob. 5.18.

PROBLEM 5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.

SOLUTION





Reactions: By symmetry, $A = B$.

$+\uparrow \Sigma F_y = 0$: $\boldsymbol{A} = \boldsymbol{B} = 80 \text{ kN} \uparrow$

Shear diagram:

A to C: $V = 80 \text{ kN}$

C to D: $V = 80 - 30 = 50 \text{ kN}$

D to E: $V = 50 - 50 = 0$

Areas of shear diagram:

A to C: $\int V\,dx = (80)(0.8) = 64 \text{ kN} \cdot \text{m}$

C to D: $\int V\,dx = (50)(0.8) = 40 \text{ kN} \cdot \text{m}$

D to E: $\int V\,dx = 0$

Bending moments:

$M_A = 0$

$M_C = 0 + 64 = 64 \text{ kN} \cdot \text{m}$

$M_D = 64 + 40 = 104 \text{ kN} \cdot \text{m}$

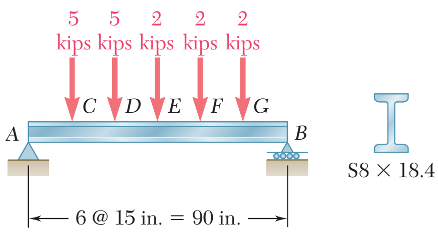
$M_E = 104 + 0 = 104 \text{ kN} \cdot \text{m}$

$$|M|_{\max} = 104 \text{ kN} \cdot \text{m} = 104 \times 10^3 \text{ N} \cdot \text{m}$$

For W310 × 52, $S = 747 \times 10^3 \text{ mm}^3 = 747 \times 10^{-6} \text{ m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{104 \times 10^3}{747 \times 10^{-6}} = 139.2 \times 10^6 \text{ Pa}$$

$\sigma = 139.2 \text{ MPa}$ ◀

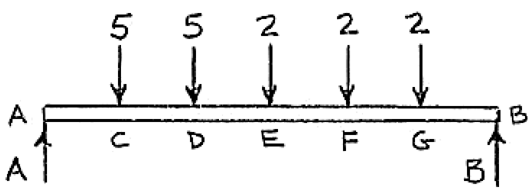


PROBLEM 5.49

Using the method of Sec. 5.2, solve Prob. 5.20.

PROBLEM 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION



Use entire beam as free body.

$+\circlearrowleft \Sigma M_B = 0$:

$-90A + (75)(5) + (60)(5) + (45)(2) + (30)(2) + (15)(2) = 0$

$A = 9.5 \text{ kips} \uparrow$

Shear A to C: $V = 9.5$ kips

Area under shear curve A to C: $\int V\,dx = (15)(9.5)$

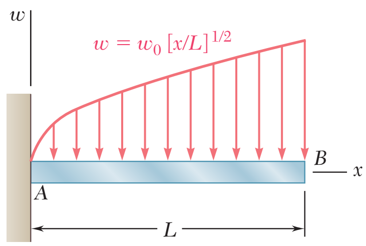
$= 142.5 \text{ kip} \cdot \text{in.}$

$M_A = 0$

$M_C = 0 + 142.5 = 142.5 \text{ kip} \cdot \text{in.}$

For $S8 \times 18.4$, $S = 14.4 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{142.5}{14.4}$ $\sigma = 9.90$ ksi ◀



PROBLEM 5.50

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0\left(\frac{x}{L}\right)^{\frac{1}{2}} = -\frac{w_0 x^{1/2}}{L^{1/2}}$$

$$V = -\frac{2}{3}\frac{w_o x^{3/2}}{L^{1/2}} + C_1$$

$$V = 0 \quad \text{at} \quad x = L$$

$$0 = -\frac{2}{3}w_0 L + C_1 \qquad C_1 = \frac{2}{3}w_0 L$$

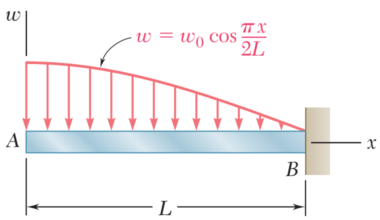
$$V = \frac{2}{3}w_0 L - \left(\frac{2}{3}\right)\frac{w_0 x^{3/2}}{L^{1/2}} \blacktriangleleft$$

$$\frac{dM}{dx} = V \qquad M = C_2 + \frac{2}{3}w_0 Lx - \frac{2}{3}\cdot\frac{2}{5}\frac{w_0 x^{5/2}}{L^{1/2}}$$

$$M = 0 \quad \text{at} \quad x = L \qquad 0 = C + \frac{2}{3}w_0 L^2 - \frac{4}{15}w_0 L^2 \qquad C_2 = -\frac{2}{5}w_0 L^2$$

$$M = \frac{2}{3}w_0 Lx - \frac{4}{15}\frac{w_0 x^{5/2}}{L^{1/2}} - \frac{2}{5}w_0 L^2 \blacktriangleleft$$

$|M|_{\text{max}}$ occurs at $x = 0$ $\qquad |M|_{\text{max}} = \frac{2}{5}w_0 L^2 \blacktriangleleft$



PROBLEM 5.51

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \cos \frac{\pi x}{2L}$$

$$V = -\frac{2Lw_0}{\pi} \sin \frac{\pi x}{2L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{4L^2 w_0}{\pi^2} \cos \frac{\pi x}{2L} + C_1 x + C_2$$

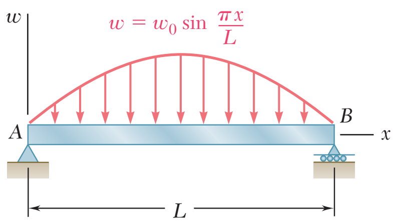
$$V = 0 \quad \text{at} \quad x = 0. \quad \text{Hence,} \quad C_1 = 0.$$

$$M = 0 \quad \text{at} \quad x = 0. \quad \text{Hence,} \quad C_2 = -\frac{4L^2 w_0}{\pi^2}.$$

$$(a) \quad V = -(2Lw_0/\pi)\sin(\pi x/2L) \blacktriangleleft$$

$$M = -(4L^2 w_0/\pi^2)[1 - \cos(\pi x/2L)] \blacktriangleleft$$

$$(b) \quad |M|_{\max} = 4w_0 L^2/\pi^2 \blacktriangleleft$$



PROBLEM 5.52

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0 \qquad C_2 = 0$$

$$M = 0 \text{ at } x = L \qquad 0 = 0 + C_1 L + 0$$

$$C_1 = 0$$

(*a*)

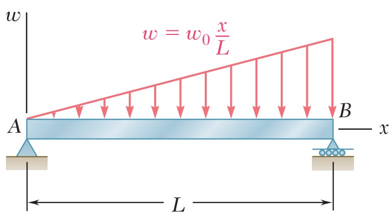
$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} \blacktriangleleft$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} \blacktriangleleft$$

$$\frac{dM}{dx} = V = 0 \quad \text{at} \quad x = \frac{L}{2}$$

(*b*) $M_{\max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2}$

$$M_{\max} = \frac{w_0 L^2}{\pi^2} \blacktriangleleft$$



PROBLEM 5.53

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \frac{x}{L}$$

$$V = -\frac{1}{2} w_0 \frac{x^2}{L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + C_1 x + C_2$$

$$M = 0 \quad \text{at} \quad x = 0 \qquad C_2 = 0$$

$$M = 0 \quad \text{at} \quad x = L \qquad 0 = -\frac{1}{6} w_0 L^2 + C_1 L \qquad C_1 = \frac{1}{6} w_0 L$$

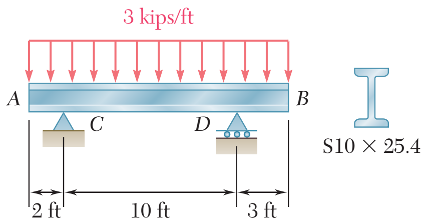
(*a*) $V = -\frac{1}{2} w_0 \frac{x^2}{L} + \frac{1}{6} w_0 L^2$ $\qquad V = \frac{1}{6} w_0 (L^2 - 3x^2)/L$ ◀

$M = -\frac{1}{6} w_0 \frac{x^3}{L} + \frac{1}{6} w_0 L x$ $\qquad M = \frac{1}{6} w_0 (Lx - x^3/L)$ ◀

(*b*) $M_{\max}$ occurs when $\frac{dM}{dx} = V = 0. \quad L^2 - 3x_m^2 = 0$

$$x_m = \frac{L}{\sqrt{3}} \qquad M_{\max} = \frac{1}{6} w_0 \left(\frac{L^2}{\sqrt{3}} - \frac{L^2}{3\sqrt{3}} \right)$$

$M_{\max} = 0.0642 w_0 L^2$ ◀



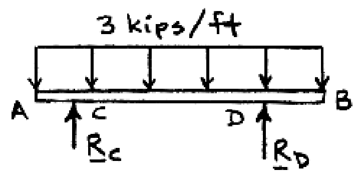
PROBLEM 5.54

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad R_D(10) - (5.5)(15)(3) = 0 \qquad R_D = 24.75 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (4.5)(15)(3) - R_C(10) = 0 \qquad R_C = 20.25 \text{ kips} \uparrow$$



Shear:

$$V_A = 0$$

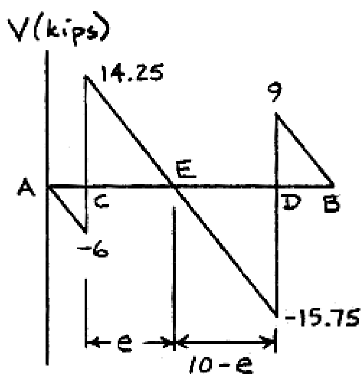
$$V_C = 0 - (2)(3) = -6 \text{ kips}$$

$$V_{C^+} = -6 + 20.50 = 14.25 \text{ kips}$$

$$V_{D^-} = 14.25 - (10)(3) = -15.75 \text{ kips}$$

$$V_{D^+} = -15.75 + 24.75 = 9 \text{ kips}$$

$$V_B = 9 - (3)(3) = 0$$



Locate point E where $V = 0$:

$$\frac{e}{14.25} = \frac{10-e}{15.75} \qquad e = 4.75 \text{ ft} \qquad 10 - e = 5.25 \text{ ft}$$

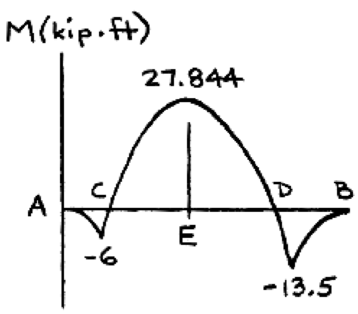
Areas of shear diagram:

A to C: $\quad \int V\,dx = \left(\frac{1}{2}\right)(2)(-6) = -6 \text{ kip} \cdot \text{ft}$

C to E: $\quad \int V\,dx = \left(\frac{1}{2}\right)(4.75)(14.25) = 33.84375 \text{ kip} \cdot \text{ft}$

E to D: $\quad \int V\,dx = \left(\frac{1}{2}\right)(5.25)(-15.75) = -41.34375 \text{ kip} \cdot \text{ft}$

D to B: $\quad \int V\,dx = \left(\frac{1}{2}\right)(3)(9) = 13.5 \text{ kip} \cdot \text{ft}$



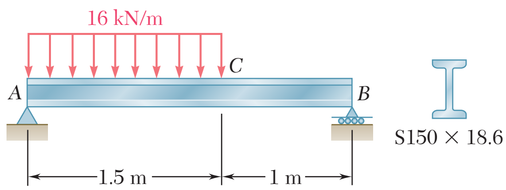
PROBLEM 5.54 *(Continued)*

Bending moments:

$$M_A = 0$$
$$M_C = 0 - 6 = -6 \text{ kip} \cdot \text{ft}$$
$$M_E = -6 + 33.84375 = 27.84375 \text{ kip} \cdot \text{ft}$$
$$M_D = 27.84375 - 41.34375 = -13.5 \text{ kip} \cdot \text{ft}$$
$$M_B = -13.5 + 13.5 = 0$$

Maximum $|M| = 27.84375 \text{ kip} \cdot \text{ft} = 334.125 \text{ kip} \cdot \text{in.}$

For $\text{S}10 \times 25.4$, $S = 24.6 \text{ in}^3$ Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{334.125}{24.6} = 13.58 \text{ ksi}$ ◀



PROBLEM 5.55

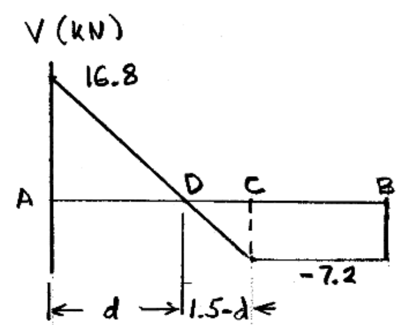
Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -2.5A + (1.75)(1.5)(16) = 0$$
$$A = 16.8 \text{ kN}$$
$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.75) + (1.5)(16) + 2.5B = 0$$
$$B = 7.2 \text{ kN}$$

Shear diagram:

$$V_A = 16.8 \text{ kN}$$
$$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$$
$$V_B = -7.2 \text{ kN}$$



Locate point D where $V = 0$.

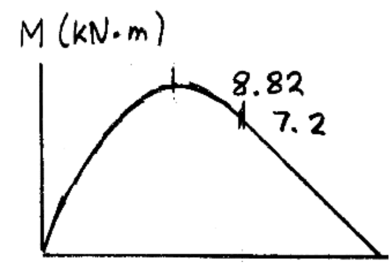
$$\frac{d}{16.8} = \frac{1.5-d}{7.2} \qquad 24d = 25.2$$
$$d = 1.05 \text{ m} \quad 1.5 - d = 0.45 \text{ m}$$

Areas of the shear diagram:

A to D: $\displaystyle\int V\,dx = \left(\frac{1}{2}\right)(1.05)(16.8) = 8.82 \text{ kN}\cdot\text{m}$

D to C: $\displaystyle\int V\,dx = \left(\frac{1}{2}\right)(0.45)(-7.2) = -1.62 \text{ kN}\cdot\text{m}$

C to B: $\displaystyle\int V\,dx = (1)(-7.2) = -7.2 \text{ kN}\cdot\text{m}$



Bending moments:

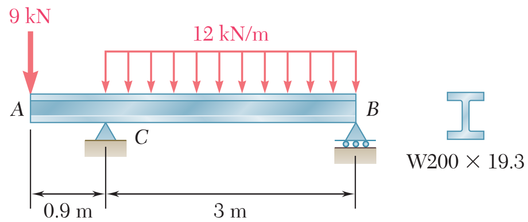
$$M_A = 0$$
$$M_D = 0 + 8.82 = 8.82 \text{ kN}\cdot\text{m}$$
$$M_C = 8.82 - 1.62 = 7.2 \text{ kN}\cdot\text{m}$$
$$M_B = 7.2 - 7.2 = 0$$

Maximum $|M| = 8.82 \text{ kN}\cdot\text{m} = 8.82\times 10^3 \text{ N}\cdot\text{m}$

For S150×18.6 rolled-steel section, $S = 120\times 10^3 \text{ mm}^3 = 120\times 10^{-6} \text{ m}^3$

Normal stress: $\displaystyle\sigma = \frac{|M|}{S} = \frac{8.82\times 10^3}{120\times 10^{-6}} = 73.5\times 10^6 \text{ Pa}$

$\sigma = 73.5$ MPa ◀



PROBLEM 5.56

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

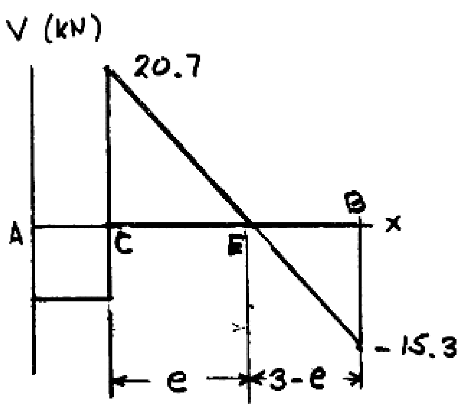
SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad (0.9)(9) - (1.5)(3)(12) + 3B = 0$$

$$B = 15.3 \text{ kN}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (3.9)(9) - 3C + (1.5)(3)(12) = 0$$

$$C = 29.7 \text{ kN}$$



Shear:

A to C: $\quad V = -9$ kN

C^+: $\quad V = -9 + 29.7 = 20.7$ kN

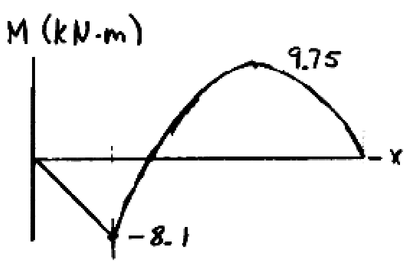
B: $\quad V = 20.7 - (3)(12) = -15.3$ kN

$$|V|_{max} = 20.7 \text{ kN} \blacktriangleleft$$

Locate point E where $V = 0$.

$$\frac{e}{20.7} = \frac{3-e}{15.3} \qquad 36e = (20.7)(3)$$

$$e = 1.725 \text{ ft} \qquad 3 - e = 1.275 \text{ ft}$$



Areas under shear diagram:

A to C: $\quad \int V\,dx = (0.9)(9) = 8.1 \text{ kN} \cdot \text{m}$

C to E: $\quad \int V\,dx = \left(\frac{1}{2}\right)(1.725)(20.7) = 17.8538 \text{ kN} \cdot \text{m}$

E to B: $\quad \int V\,dx = \left(\frac{1}{2}\right)(-1.275)(15.3) = -9.7538 \text{ kN} \cdot \text{m}$

PROBLEM 5.56 *(Continued)*

Bending moments:

$$M_A = 0$$

$$M_C = 0 - 8.1 = -8.1 \text{ kN} \cdot \text{m}$$

$$M_E = -8.1 + 17.8538 = 9.7538 \text{ kN} \cdot \text{m}$$

$$M_B = 9.7538 - 9.7538 = 0$$

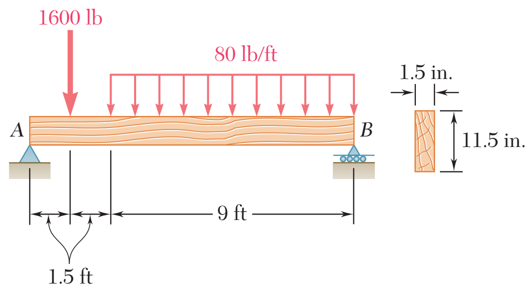
$$|M|_{max} = 9.7538 \times 10^3 \text{ N} \cdot \text{m at point } E \blacktriangleleft$$

For W200 × 19.3 rolled-steel section, $S = 162 \times 10^3 \text{ mm}^3 = 162 \times 10^{-6} \text{ m}^3$

Normal stress:

$$\sigma = \frac{|M|}{S} = \frac{9.7538 \times 10^3}{162 \times 10^{-6}} = 60.2 \times 10^6 \text{ Pa} = 60.2 \text{ MPa}$$

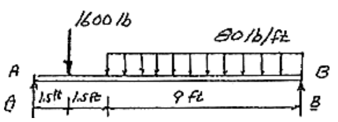
$$\sigma = 60.2 \text{ MPa} \blacktriangleleft$$



PROBLEM 5.57

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

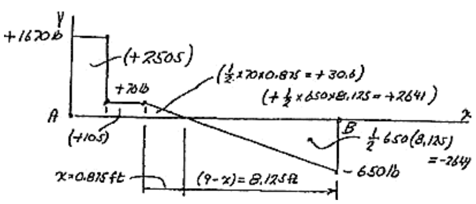


$$\circlearrowleft \Sigma M_A = 0: \quad (1600 \text{ lb})(1.5 \text{ ft}) + [(80 \text{ lb/ft})(9 \text{ ft})](7.5 \text{ ft}) - 12B = 0$$

$$B = +650 \text{ lb} \qquad \mathbf{B} = 650 \text{ lb} \uparrow$$

$$+\!\uparrow \Sigma F_y = 0: \quad A - 1600 \text{ lb} - [(80 \text{ lb/ft})(9 \text{ ft})] + 650 \text{ lb} = 0$$

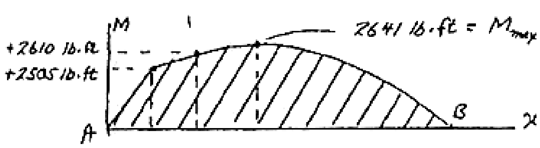
$$A = +1670 \text{ lb} \qquad \mathbf{A} = 1670 \text{ lb} \uparrow$$



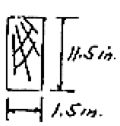
$$\frac{x}{70 \text{ lb}} = \frac{9-x}{650 \text{ lb}} \qquad x = 0.875 \text{ ft} \qquad 2641 \text{ lb} \cdot \text{ft} = M_{\max}$$

$$c = \frac{1}{2}(11.5 \text{ in.}) = 5.75 \text{ in.}$$

$$I = \frac{1}{12}(1.5 \text{ in.})(11.5 \text{ in.})^3 = 190.1 \text{ in}^4$$

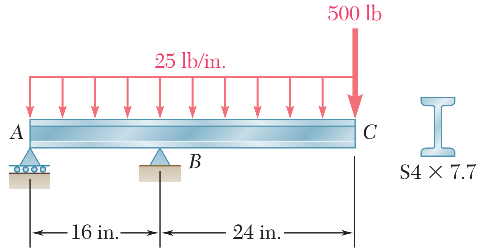


◀



$$M_{\max} = 2641 \text{ lb} \cdot \text{ft} = 31{,}690 \text{ lb} \cdot \text{in.}$$

$$\sigma_m = \frac{M_{\max} c}{I} = \frac{(31{,}690 \text{ lb} \cdot \text{in.})(5.75 \text{ in.})}{190.1 \text{ in}^4} \qquad \sigma_m = 959 \text{ psi} \blacktriangleleft$$



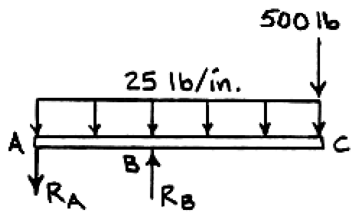
PROBLEM 5.58

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad R_A(16) - (4)(40)(25) - (24)(500) = 0 \quad R_A = 1000 \text{ lb} \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B(16) - (20)(40)(25) - (40)(500) = 0 \quad R_B = 2500 \text{ lb} \uparrow$$



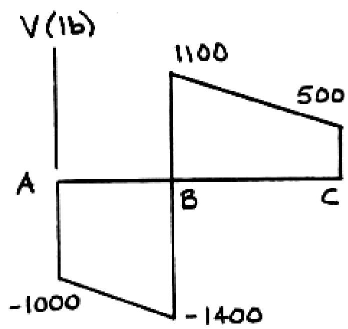
Shear:

$$V_A = -1000 \text{ lb}$$

$$V_B = -1000 - (16)(25) = -1400 \text{ lb}$$

$$V_{B^+} = -1400 + 2500 = 1100 \text{ lb}$$

$$V_C = 1100 - (24)(25) = 500 \text{ lb}$$



Areas of shear diagram:

A to B: $$\int V\,dx = \frac{1}{2}(-1000 - 1400)(16) = -19{,}200 \text{ lb} \cdot \text{in.}$$

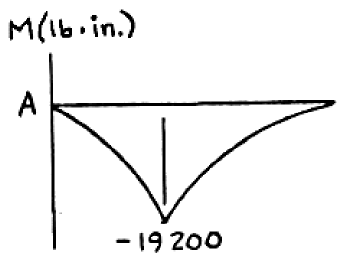
B to C: $$\int V\,dx = \left(\frac{1}{2}\right)(1100 + 500)(24) = 19{,}200 \text{ lb} \cdot \text{in.}$$

Bending moments:

$$M_A = 0$$

$$M_B = 0 - 19{,}200 = -19{,}200 \text{ lb} \cdot \text{in.}$$

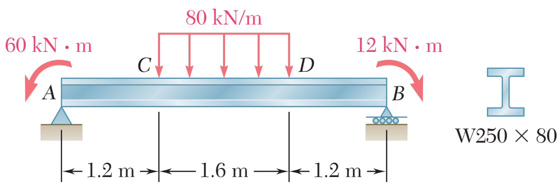
$$M_C = -19{,}200 + 19{,}200 = 0$$



Maximum $|M| = 19.2 \text{ kip} \cdot \text{in.}$

For S4 × 7.7 rolled-steel section, $S = 3.03 \text{ in}^3$

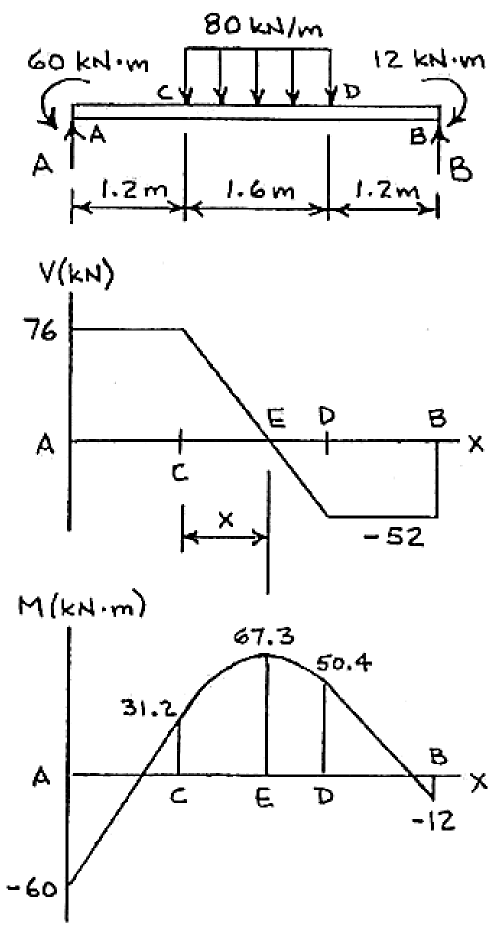
Normal stress: $$\sigma = \frac{|M|}{S} = \frac{19.2}{3.03} = 6.34 \text{ ksi} \quad \blacktriangleleft$$



PROBLEM 5.59

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION



Reaction:

$$+\circlearrowleft \Sigma M_B = 0: \quad -4A + 60 + (80)(1.6)(2) - 12 = 0$$

$$\mathbf{A} = 76 \text{ kN} \uparrow$$

Shear: $V_A = 76$ kN

A to C: $\quad V = 76.0$ kN ◀

$$V_D = 76 - (80)(1.6) = -52 \text{ kN}$$

D to C: $\quad V = -52$ kN

Locate point where $V = 0$.

$V(x) = -80x + 76 = 0 \qquad x = 0.95$ m

Areas of shear diagram:

A to C: $\int V\,dx = (1.2)(76) = 91.2 \text{ kN} \cdot \text{m}$

C to E: $\int V\,dx = \frac{1}{2}(0.95)(76) = 36.1 \text{ kN} \cdot \text{m}$

E to D: $\int V\,dx = \frac{1}{2}(0.65)(-52) = -16.9 \text{ kN} \cdot \text{m}$

D to B: $\int V\,dx = (1.2)(-52) = -62.4 \text{ kN} \cdot \text{m}$

Bending moments: $\quad M_A = -60 \text{ kN} \cdot \text{m}$

$M_C = -60 + 91.2 = 31.2 \text{ kN} \cdot \text{m}$

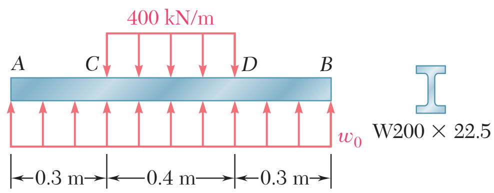
$M_E = 31.2 + 36.1 = 67.3 \text{ kN} \cdot \text{m}$ ◀

$M_D = 67.3 - 16.9 = 50.4 \text{ kN} \cdot \text{m}$

$M_B = 50.4 - 62.4 = -12 \text{ kN} \cdot \text{m}$

For W250 × 80, $S = 983 \times 10^3 \text{mm}^3$

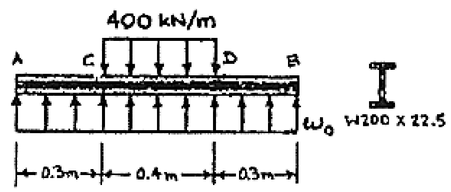
Normal stress: $$\sigma_{\max} = \frac{|M|}{S} = \frac{67.3 \times 10^3 \text{ N} \cdot \text{m}}{983 \times 10^{-6} \text{ m}^3} = 68.5 \times 10^6 \text{ Pa} \qquad \sigma_m = 68.5 \text{ MPa}$$ ◀



PROBLEM 5.60

Knowing that beam AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION



$$+\uparrow \sum F_y = 0: (1)(w_0) - (0.4)(400) = 0$$

$$w_0 = 160 \text{ kN/m}$$

Shear diagram: $V_A = 0$

$$V_C = 0 + (0.3)(160) = 48 \text{ kN}$$

$$V_D = 48 - (0.3)(400) + (0.3)(160) = -48 \text{ kN}$$

$$V_B = -48 + (0.3)(160) = 0$$

Locate point E where $V = 0$.

By symmetry, E is the midpoint of CD.

Areas of shear diagram:

A to C: $\frac{1}{2}(0.3)(48) = 7.2 \text{ kN} \cdot \text{m}$

C to E: $\frac{1}{2}(0.2)(48) = 4.8 \text{ kN} \cdot \text{m}$

E to D: $\frac{1}{2}(0.2)(-48) = -4.8 \text{ kN} \cdot \text{m}$

D to B: $\frac{1}{2}(0.3)(-48) = -7.2 \text{ kN} \cdot \text{m}$

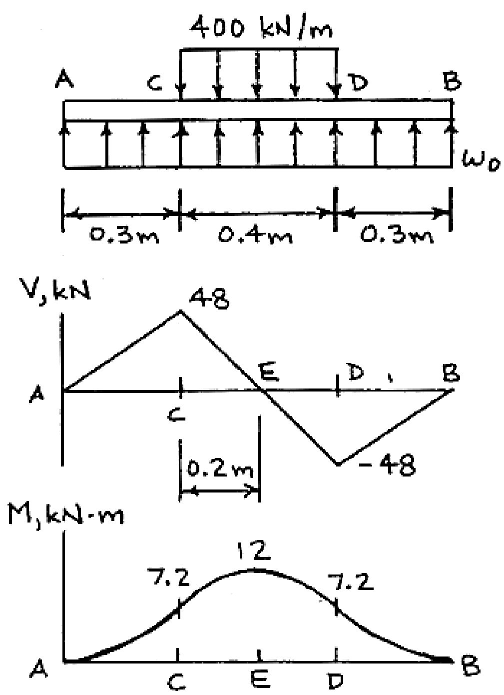
Bending moments: $M_A = 0$

$$M_C = 0 + 7.2 = 7.2 \text{ kN}$$

$$M_E = 7.2 + 4.8 = 12.00 \text{ kN}$$

$$M_D = 12.0 - 4.8 = 7.2 \text{ kN}$$

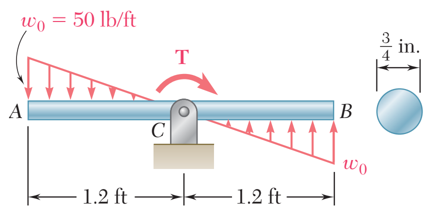
$$M_B = 7.2 - 7.2 = 0$$



$$|M|_{\max} = 12.00 \text{ kN} \cdot \text{m} = 12.00 \times 10^3 \text{ N} \cdot \text{m}$$

For W200 × 22.5 rolled-steel shape, $S_x = 193 \times 10^3 \text{ mm}^3 = 193 \times 10^{-6} \text{ m}^3$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{12.00 \times 10^3}{193 \times 10^{-6}} = 62.2 \times 10^6 \text{ Pa}$ $\qquad \sigma = 62.2 \text{ MPa}$ ◀



PROBLEM 5.61

Knowing that beam AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION

A to C: $\quad 0 < x < 1.2$ ft

$$w = 50\left(1 - \frac{x}{1.2}\right) = 50 - 41.667x$$

$$\frac{dV}{dx} = -w = 41.667x - 50$$

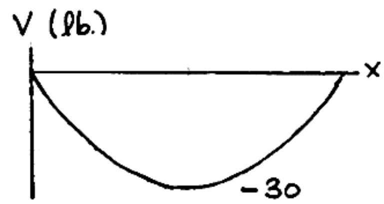
$$V = V_A + \int_0^x (41.667x - 50)dx$$

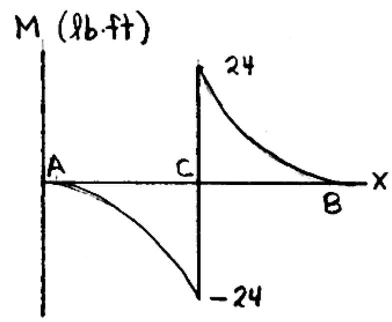
$$= 0 + 20.833x^2 - 50x = \frac{dM}{dx}$$

$$M = M_A + \int_0^x V\,dx$$

$$= 0 + \int_0^x (20.833x^2 - 50x)dx$$

$$= 6.944x^3 - 25x^2$$





At C to B, use symmetry conditions. $\quad x = 1.2$ ft, $\quad V = -30.0$ lb

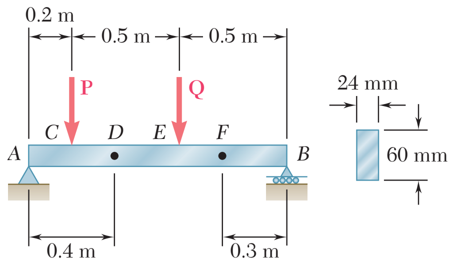
$M = -24.0$ lb $\cdot$ in.

Maximum $|M| = 24.0$ lb $\cdot$ ft $= 288$ lb $\cdot$ in.

Cross section: $\quad c = \dfrac{d}{2} = \left(\dfrac{1}{2}\right)(0.75) = 0.375$ in.

$$I = \frac{\pi}{4}c^4 = \left(\frac{\pi}{4}\right)(0.375) = 15.532 \times 10^{-3}\,\text{in}^4$$

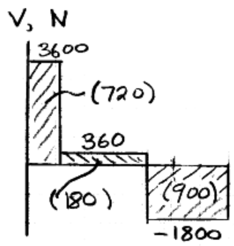
Normal stress: $\quad \sigma = \dfrac{|M|c}{I} = \dfrac{(2.88)(0.375)}{15.532 \times 10^{-3}} = 6.95 \times 10^3$ psi $\qquad \sigma = 6.95$ ksi ◀

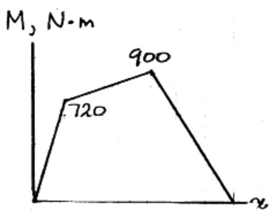


PROBLEM 5.62*

The beam AB supports two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the beam is +55 MPa at D and +37.5 MPa at F. (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending that occurs in the beam.

SOLUTION



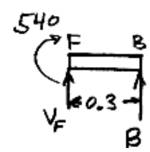


(a) $$I = \frac{1}{12}(24)(60)^3 = 432\times 10^3\ \text{mm}^4 \qquad c = 30\ \text{mm}$$

$$S = \frac{I}{c} = 14.4\times 10^3\ \text{mm}^3 = 14.4\times 10^{-6}\ \text{m}^3 \qquad M = S\sigma$$

At D, $M_D = (14.4\times 10^{-6})(55\times 10^6) = 792\ \text{N}\cdot\text{m}$

At F, $M_F = (14.4\times 10^{-6})(37.5\times 10^6) = 540\ \text{N}\cdot\text{m}$

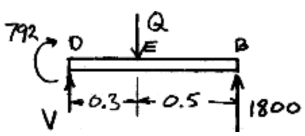


Using free body FB, $\quad +\circlearrowleft \Sigma M_F = 0:\quad -540 + 0.3B = 0$

$$B = \frac{540}{0.3} = 1800\ \text{N}$$

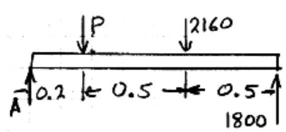
Using free body $DEFB$, $\quad +\circlearrowleft \Sigma M_D = 0:\quad -792 - 3Q + (0.8)(1800) = 0$

$$Q = 2160\ \text{N}$$



Using entire beam, $\quad +\circlearrowleft \Sigma M_A = 0:\quad -0.2P - (0.7)(2160) + (1.2)(1800) = 0$

$$P = 3240\ \text{N}$$



$+\uparrow \Sigma F_y = 0:\quad A - 3240 - 2160 + 1800 = 0$

$$A = 3600\ \text{N}$$

Shear diagram and its areas:

A to C^-: $\quad V = 3600$ N $\quad A_{AC} = (0.2)(3600) = 720\ \text{N}\cdot\text{m}$

C^+ to E^-: $\quad V = 3600 - 3240 = 360$ N $\quad A_{CE} = (0.5)(360) = 180\ \text{N}\cdot\text{m}$

E^+ to B: $\quad V = 360 - 2160 = -1800$ N $\quad A_{EB} = (0.5)(-1800) = -900\ \text{N}\cdot\text{m}$

Bending moments:

$M_A = 0$

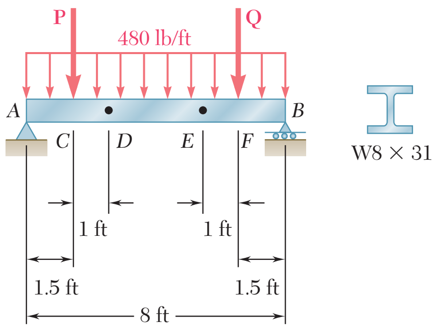
$M_C = 0 + 720 = 720\ \text{N}\cdot\text{m}$

$M_E = 720 + 180 = 900\ \text{N}\cdot\text{m}$ $\qquad |M|_{\max} = 900\ \text{N}\cdot\text{m}$

$M_B = 900 - 900 = 0$

(b) Normal stress: $$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{900}{14.4\times 10^{-6}} = 62.5\times 10^6\ \text{Pa}$$

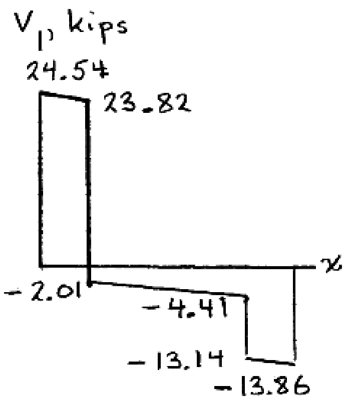
$$\sigma_{\max} = 62.5\ \text{MPa} \blacktriangleleft$$

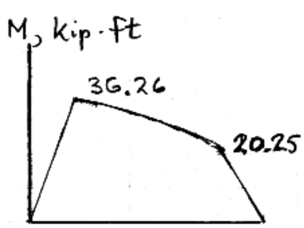


PROBLEM 5.63*

The beam *AB* supports a uniformly distributed load of 480 lb/ft and two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the lower flange is +14.85 ksi at *D* and +10.65 ksi at *E*. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending that occurs in the beam.

SOLUTION





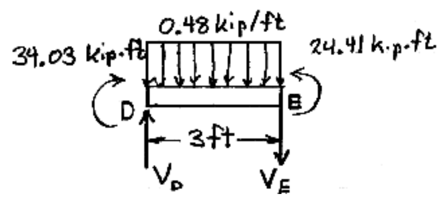
(*a*) For W8 × 31 rolled-steel section, $S = 27.5\text{ in}^3$

$$M = S\sigma$$

At *D*, $M_D = (27.5)(14.85) = 408.375\text{ kip}\cdot\text{in.}$

At *E*, $M_E = (27.5)(10.65) = 292.875\text{ kip}\cdot\text{in.}$

$$M_D = 34.03\text{ kip}\cdot\text{ft} \quad M_E = 24.41\text{ kip}\cdot\text{ft}$$



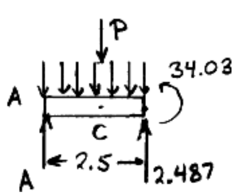
Use free body *DE*.

$$+\circlearrowleft \Sigma M_E = 0: \quad -34.03 + 24.41 + (1.5)(3)(0.48) - 3V_D = 0$$

$$V_D = -2.487\text{ kips}$$

$$+\circlearrowleft M_D = 0: \quad -34.03 + 24.41 - (1.5)(3)(0.48) - 3V_E = 0$$

$$V_E = -3.927\text{ kips}$$



Use free body *ACD*.

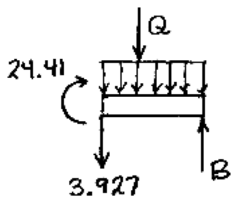
$$+\circlearrowleft \Sigma M_A = 0: \quad -1.5P - (1.25)(2.5)(0.48) + (2.5)(2.487) + 34.03 = 0$$

$$\mathbf{P} = 25.83\text{ kips} \downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A - (2.5)(0.48) + 2.487 - 25.83 = 0$$

$$\mathbf{A} = 24.54\text{ kips} \uparrow$$

PROBLEM 5.63* *(Continued)*



Use free body *EFB*.

$$+\circlearrowleft \Sigma M_B = 0: \quad 1.5Q + (1.25)(2.5)(0.48) + (2.5)(3.927) - 24.41 = 0$$

$$Q = 8.728 \text{ kips}$$

$$+\uparrow \Sigma F_y = 0: \quad B - 3.927 - (2.5)(0.48) - 8.7 = 0$$

$$B = 13.855 \text{ kips}$$

Areas of load diagram:

A to C: $(1.5)(0.48) = 0.72 \text{ kip} \cdot \text{ft}$

C to F: $(5)(0.48) = 2.4 \text{ kip} \cdot \text{ft}$

F to B: $(1.5)(0.48) = 0.72 \text{ kip} \cdot \text{ft}$

Shear diagram: $V_A = 24.54 \text{ kips}$

$$V_C^- = 24.54 - 0.72 = 23.82 \text{ kips}$$

$$V_C^+ = 23.82 - 25.83 = -2.01 \text{ kips}$$

$$V_F^- = -2.01 - 2.4 = 4.41 \text{ kips}$$

$$V_F^+ = -4.41 - 8.728 = -13.14 \text{ kips}$$

$$V_B = -13.14 - 0.72 = -13.86 \text{ kips}$$

Areas of shear diagram:

A to C: $\frac{1}{2}(1.5)(24.52 + 23.82) = 36.23 \text{ kip} \cdot \text{ft}$

C to F: $\frac{1}{2}(5)(-2.01 - 4.41) = -16.05 \text{ kip} \cdot \text{ft}$

F to B: $\frac{1}{2}(1.5)(-13.14 - 13.86) = 20.25 \text{ kip} \cdot \text{ft}$

Bending moments: $M_A = 0$

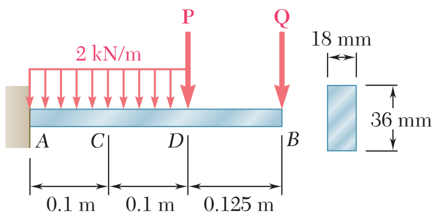
$$M_C = 0 + 36.26 = 36.26 \text{ kip} \cdot \text{ft}$$

$$M_F = 36.26 - 16.05 = 20.21 \text{ kip} \cdot \text{ft}$$

$$M_B = 20.21 - 20.25 \approx 0$$

Maximum $|M|$ occurs at C: $|M|_{\text{max}} = 36.26 \text{ kip} \cdot \text{ft} = 435.1 \text{ kip} \cdot \text{in.}$

(*b*) Maximum stress: $\sigma = \dfrac{|M|_{\text{max}}}{S} = \dfrac{435.1}{27.5}$ $\qquad \sigma = 15.82 \text{ ksi}$ ◀



PROBLEM 5.64*

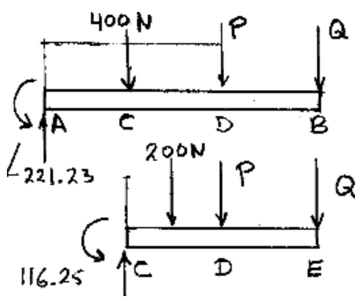
Beam AB supports a uniformly distributed load of $2\,\text{kN/m}$ and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending in the bottom edge of the beam is −56.9 MPa at A and −29.9 MPa at C. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads **P** and **Q**.

SOLUTION

$$I = \frac{1}{12}(18)(36)^3 = 69.984\times 10^3\ \text{mm}^4$$

$$c = \frac{1}{2}d = 18\ \text{mm}$$

$$S = \frac{I}{c} = 3.888\times 10^3\ \text{mm}^3 = 3.888\times 10^{-6}\ \text{m}^3$$



At A, $\quad M_A = S\sigma_A = (3.888\times 10^{-6})(-56.9) = -221.25\ \text{N}\cdot\text{m}$

At C, $\quad M_C = S\sigma_C = (3.888\times 10^{-6})(-29.9) = -116.25\ \text{N}\cdot\text{m}$

$$+\circlearrowright \Sigma M_A = 0:\quad 221.23 - (0.1)(400) - 0.2P - 0.325Q = 0$$

$$0.2P + 0.325Q = 181.25 \qquad (1)$$

$$+\circlearrowright \Sigma M_C = 0:\quad 116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$$

$$0.1P + 0.225Q = 106.25 \qquad (2)$$

Solving (1) and (2) simultaneously, $\quad P = 500\ \text{N}$ ◀

$Q = 250\ \text{N}$ ◀

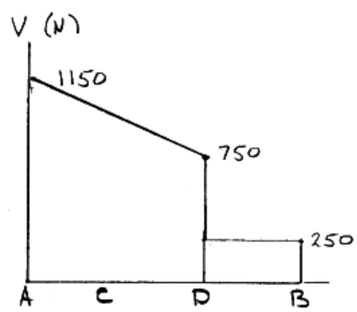
Reaction force at A: $\quad R_A - 400 - 500 - 250 = 0 \quad R_A = 1150\ \text{N}\cdot\text{m}$

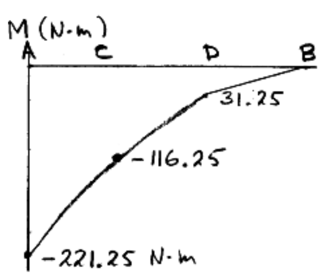
$V_A = 1150\ \text{N} \quad V_D = 250$

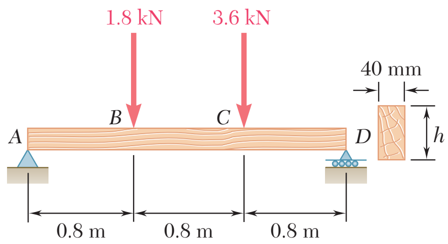
$M_A = -221.25\ \text{N}\cdot\text{m} \quad M_C = -116.25\ \text{N}\cdot\text{m} \quad M_D = -31.25\ \text{N}\cdot\text{m}$

$|V|_{\max} = 1150\ \text{N}$ ◀

$|M|_{\max} = 221\ \text{N}\cdot\text{m}$ ◀







PROBLEM 5.65

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

Reactions:

$$+\circlearrowleft \Sigma M_D = 0: \quad -2.4A + (1.6)(1.8) + (0.8)(3.6) = 0 \qquad A = 2.4 \text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0 \qquad D = 3 \text{ kN}$$

Construct shear and bending moment diagrams:

$$|M|_{\max} = 2.4 \text{ kN}\cdot\text{m} = 2.4\times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa}$$

$$= 12\times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{2.4\times 10^3}{12\times 10^6}$$

$$= 200\times 10^{-6} \text{ m}^3$$

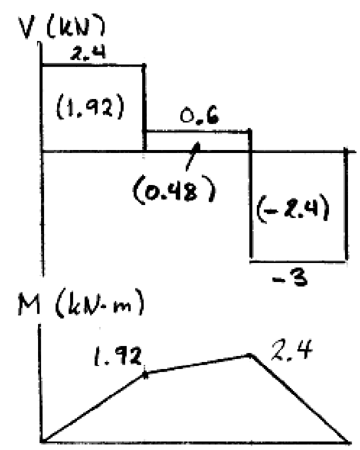
$$= 200\times 10^3 \text{ mm}^3$$

$$S = \frac{1}{6}bh^2 = \frac{1}{6}(40)h^2$$

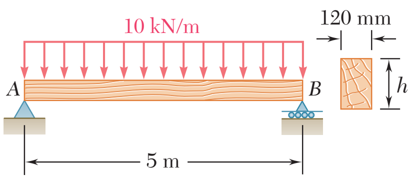
$$= 200\times 10^3$$

$$h^2 = \frac{(6)(200\times 10^3)}{40}$$

$$= 30\times 10^3 \text{ mm}^2$$



$h = 173.2 \text{ mm}$ ◀



PROBLEM 5.66

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

Reactions:

$A = B$ by symmetry

$$+\uparrow \Sigma F_y = 0: \quad A + B - (5)(10) = 0$$

$$A = B = 25 \text{ kN}$$

From bending moment diagram,

$$|M|_{\max} = 31.25 \text{ kN}\cdot\text{m} = 31.25\times10^3 \text{ N}\cdot\text{m}$$

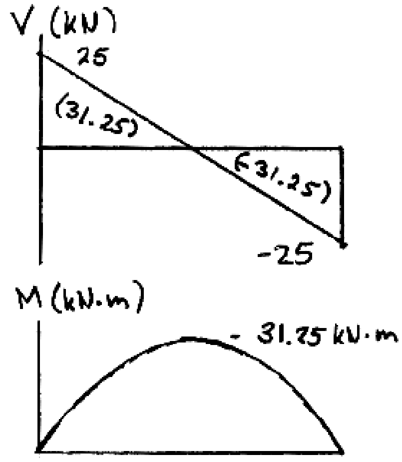
$$\sigma_{\text{all}} = 12 \text{ MPa} = 12\times10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{31.25\times10^3}{12\times10^6} = 2.604\times10^{-3} \text{ m}^3$$

$$= 2.604\times10^6 \text{ mm}^3$$

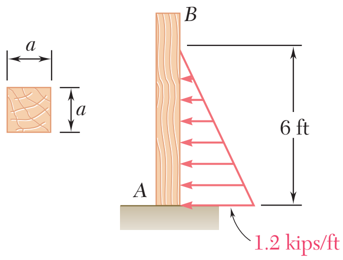
$$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(120)h^2 = 2.604\times10^6$$

$$h^2 = \frac{(6)(2.064\times10^6)}{120} = 130.21\times10^3 \text{ mm}^2$$



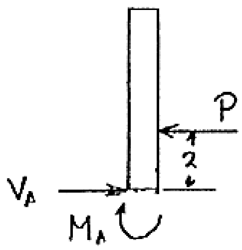
$h = 361$ mm ◀

PROBLEM 5.67



For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION



Equivalent concentrated load:

$$P = \left(\frac{1}{2}\right)(6)(1.2) = 3.6 \text{ kips}$$

Bending moment at A:

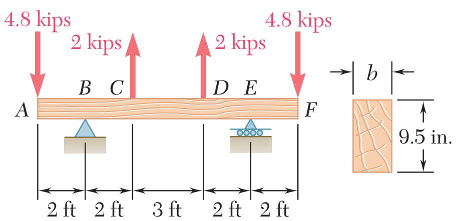
$$M_A = (2)(3.6) = 7.2 \text{ kip} \cdot \text{ft} = 86.4 \text{ kip} \cdot \text{in.}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{86.4}{1.75} = 49.37 \text{ in}^3$$

For a square section, $\quad S = \dfrac{1}{6}a^3$

$$a = \sqrt[3]{6S}$$

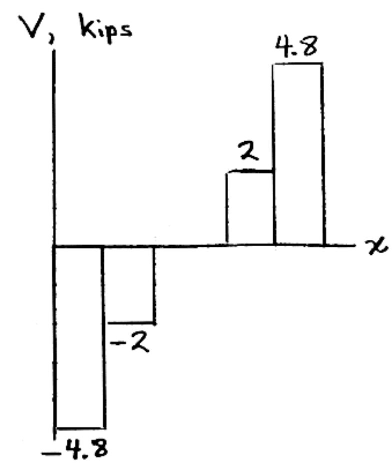
$$a_{\min} = \sqrt[3]{(6)(49.37)} \qquad a_{\min} = 6.67 \text{ in.} \blacktriangleleft$$

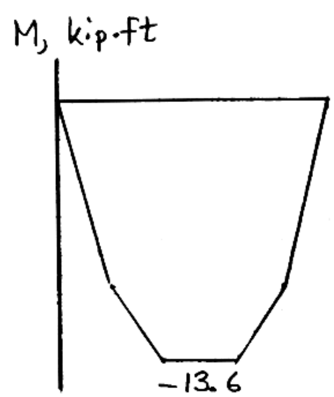


PROBLEM 5.68

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION





For equilibrium, $B = E = 2.8$ kips

Shear diagram:

A to B^-: $V = -4.8$ kips

B^+ to C^-: $V = -4.8 + 2.8 = -2$ kips

C^+ to D^-: $V = -2 + 2 = 0$

D^+ to E^-: $V = 0 + 2 = 2$ kips

E^+ to F: $V = 2 + 2.8 = 4.8$ kips

Areas of shear diagram:

A to B: $(2)(-4.8) = -9.6$ kip·ft

B to C: $(2)(-2) = -4$ kip·ft

C to D: $(3)(0) = 0$

D to E: $(2)(2) = 4$ kip·ft

E to F: $(2)(4.8) = 9.6$ kip·ft

Bending moments: $M_A = 0$

$M_B = 0 - 9.6 = -9.6$ kip·ft

$M_C = -9.6 - 4 = -13.6$ kip·ft

$M_D = -13.6 + 0 = -13.6$ kip·ft

$M_E = -13.6 + 4 = -9.6$ kip·ft

$M_F = -9.6 + 9.6 = 0$

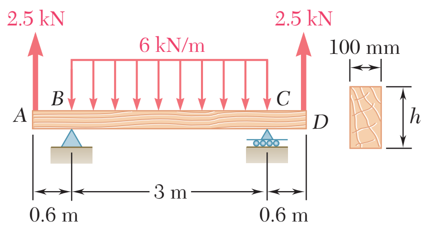
$$|M|_{\max} = 13.6 \text{ kip}\cdot\text{ft} = 162.3 \text{ kip}\cdot\text{in.} = 162.3\times10^3 \text{ lb}\cdot\text{in.}$$

Required value for S:

$$S = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{162.3\times10^3}{1750} = 93.257 \text{ in}^3$$

For a rectangular section, $I = \frac{1}{12}bh^3$, $c = \frac{1}{2}h$ $S = \frac{I}{c} = \frac{bh^2}{6} = \frac{(b)(9.5)^2}{6} = 15.0417b$

Equating the two expressions for S, $15.0417b = 93.257$ $b = 6.20$ in. ◀



PROBLEM 5.69

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

By symmetry, $B = C$

$$+\uparrow \Sigma F_y = 0: \quad B + C + 2.5 + 2.5 - (3)(6) = 0 \qquad B = C = 6.5 \text{ kN}$$

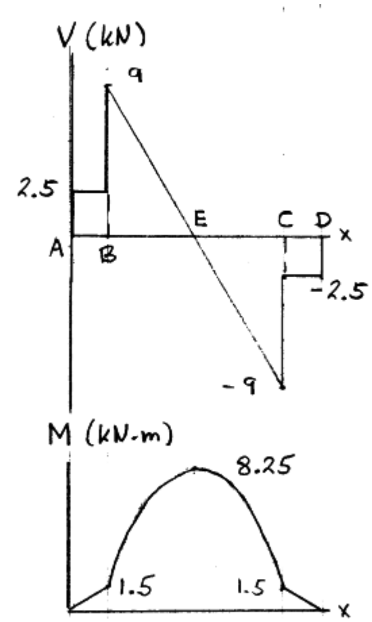
Shear:

A to B: $V = 2.5 \text{ kN}$

$$V_{B^+} = 2.5 + 6.5 = 9 \text{ kN}$$

$$V_{C^-} = 9 - (3)(6) = -9 \text{ kN}$$

C to D: $V = -9 + 6.5 = -2.5 \text{ kN}$



Areas of the shear diagram:

A to B: $\int V\,dx = (0.6)(2.5) = 1.5 \text{ kN}\cdot\text{m}$

B to E: $\int V\,dx = \left(\frac{1}{2}\right)(1.5)(9) = 6.75 \text{ kN}\cdot\text{m}$

E to C: $\int V\,dx = -6.75 \text{ kN}\cdot\text{m}$

C to D: $\int V\,dx = -1.5 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$

$$M_B = 0 + 1.5 = 1.5 \text{ kN}\cdot\text{m}$$

$$M_E = 1.5 + 6.75 = 8.25 \text{ kN}\cdot\text{m}$$

$$M_C = 8.25 - 6.75 = 1.5 \text{ kN}\cdot\text{m}$$

$$M_D = 1.5 - 1.5 = 0$$

Maximum $|M| = 8.25 \text{ kN}\cdot\text{m} = 8.25 \times 10^3 \text{ N}\cdot\text{m}$

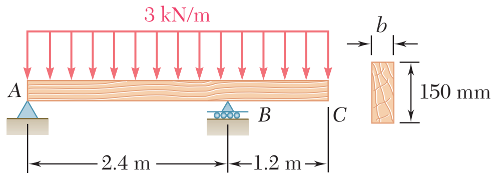
$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$$

For a rectangular section, $S = \frac{1}{6} b h^2$

$$687.5 \times 10^3 = \left(\frac{1}{6}\right)(100) h^2$$

$$h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2 \qquad h = 203 \text{ mm} \blacktriangleleft$$



PROBLEM 5.70

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

$$+\circlearrowleft M_B = 0: \quad -2.4A + (0.6)(3.6)(3) = 0 \qquad A = 2.7 \text{ kN}$$

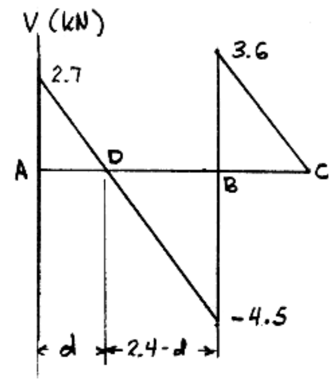
$$+\circlearrowleft M_A = 0: \quad -(1.8)(3.6)(3) + 2.4B = 0 \qquad B = 8.1 \text{ kN}$$

Shear: $V_A = 2.7$ kN

$V_{B^-} = 2.7 - (2.4)(3) = -4.5$ kN

$V_{B^+} = -4.5 + 8.1 = 3.6$ kN

$V_C = 3.6 - (1.2)(3) = 0$



Locate point D where $V = 0$. $\dfrac{d}{2.7} = \dfrac{2.4 - d}{4.5} \qquad 7.2d = 6.48$

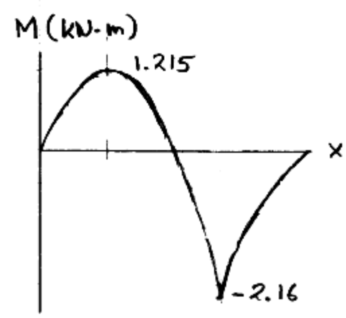
$d = 0.9$ m $\qquad 2.4 - d = 1.5$ m

Areas of the shear diagram:

A to D: $\int V\,dx = \left(\frac{1}{2}\right)(0.9)(2.7) = 1.215 \text{ kN}\cdot\text{m}$

D to B: $\int V\,dx = \left(\frac{1}{2}\right)(1.5)(-4.5) = -3.375 \text{ kN}\cdot\text{m}$

B to C: $\int V\,dx = \left(\frac{1}{2}\right)(1.2)(3.6) = 2.16 \text{ kN}\cdot\text{m}$



Bending moments: $M_A = 0$

$M_D = 0 + 1.215 = 1.215 \text{ kN}\cdot\text{m}$

$M_B = 1.215 - 3.375 = -2.16 \text{ kN}\cdot\text{m}$

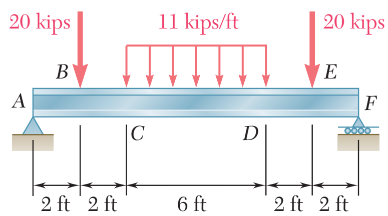
$M_C = -2.16 + 2.16 = 0$

Maximum $|M| = 2.16 \text{ kN}\cdot\text{m} = 2.16\times10^3 \text{ N}\cdot\text{m} \qquad \sigma_{\text{all}} = 12 \text{ MPa} = 12\times10^6 \text{ Pa}$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{2.16\times10^3}{12\times10^6} = 180\times10^{-6} \text{ m}^3 = 180\times10^3 \text{ mm}^3$$

For rectangular section, $S = \dfrac{1}{6}bh^2 = \dfrac{1}{6}b(150)^2 = 180\times10^3$

$$b = \frac{(6)(180\times10^3)}{150^2} \qquad b = 48.0 \text{ mm} \blacktriangleleft$$



PROBLEM 5.71

Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

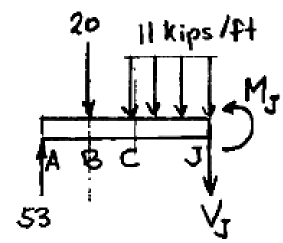
SOLUTION

By symmetry, $R_A = R_F$

$$+\uparrow \Sigma F_y = 0: \quad R_A - 20 - (6)(11) - 20 + R_F = 0$$

$$R_A = R_F = 50 \text{ kips}$$

Maximum bending moment occurs at center of beam.



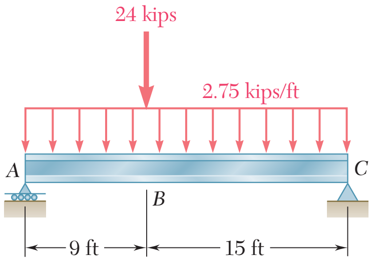
$$+\circlearrowleft \Sigma M_J = 0: \quad -(7)(53) + (5)(20) + (1.5)(3)(11) + M_J = 0$$

$$M_J = 221.5 \text{ kip.ft} = 2658 \text{ kip} \cdot \text{in.}$$

$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{2658}{24} = 110.75 \text{ in}^3$$

Shape	S (in^2)
W24 × 68	154
W21 × 62	127
W18 × 76	146
W16 × 77	134
W14 × 82	123
W12 × 96	131

Use W21 × 62. ◀



PROBLEM 5.72

Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -24A + (12)(24)(2.75) + (15)(24) = 0 \qquad A = 48 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 24C - (12)(24)(2.75) - (9)(24) = 0 \qquad C = 42 \text{ kips}$$

Shear:

$$V_A = 48$$
$$V_{B^-} = 48 - (9)(2.75) = 23.25 \text{ kips}$$
$$V_{B^+} = 23.25 - 24 = -0.75 \text{ kips}$$
$$V_C = -0.75 - (15)(2.75) = -42 \text{ kips}$$

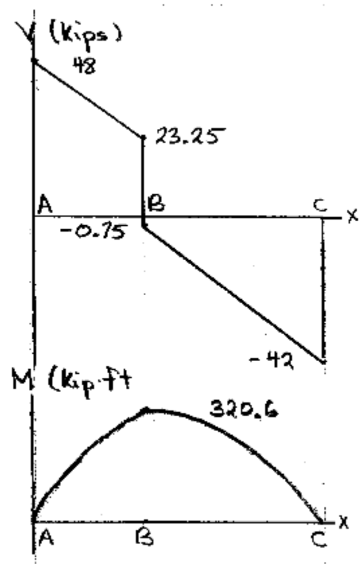
Areas of the shear diagram:

A to *B*: $$\int V\,dx = \left(\frac{1}{2}\right)(9)(48 + 23.25) = 320.6 \text{ kip}\cdot\text{ft}$$

B to *C*: $$\int V\,dx = \left(\frac{1}{2}\right)(15)(-0.75 - 42) = -320.6 \text{ kip}\cdot\text{ft}$$

Bending moments:

$$M_A = 0$$
$$M_B = 0 + 320.6 = 320.6 \text{ kip}\cdot\text{ft}$$
$$M_C = 320.6 - 320.6 = 0$$



Maximum $|M| = 320.6 \text{ kip}\cdot\text{ft} = 3848 \text{ kip}\cdot\text{in.}$

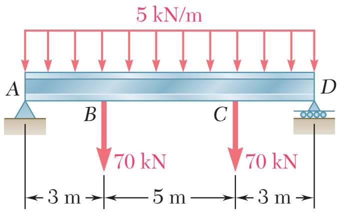
$$\sigma_{\text{all}} = 24 \text{ ksi}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3848}{24} = 160.3 \text{ in}^3$$

Shape	S, (in^3)
W30×99	269
W27×84	213 ←
W24×104	258
W21×101	227
W18×106	204

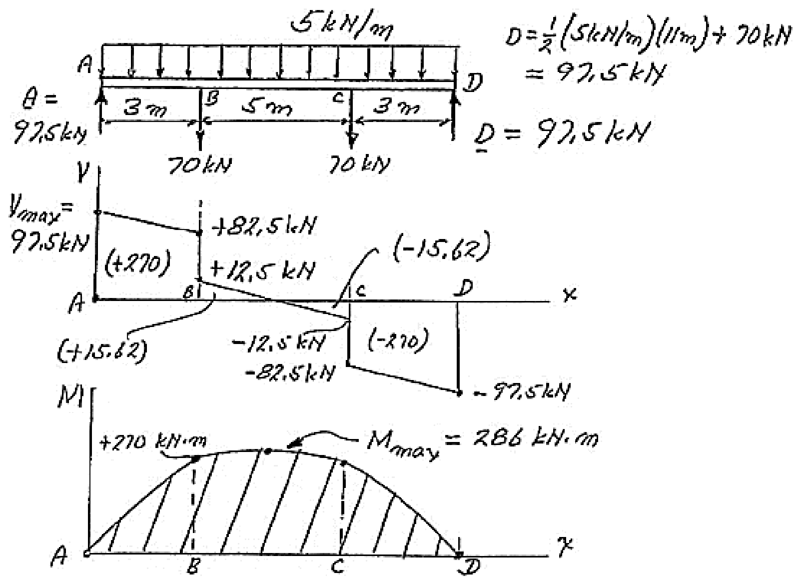
Lightest wide flange beam: W27×84 @ 84 lb/ft ◀

PROBLEM 5.73



Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION



Section modulus

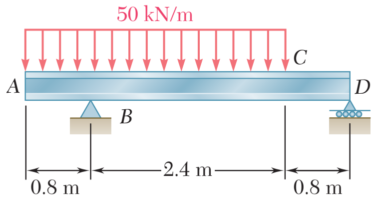
$\sigma_{all} = 160$ Mpa

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{286 \text{ kN} \cdot \text{m}}{160 \text{ MPa}} = 1787 \times 10^{-6} \text{ m}^3$$

$$= 1787 \times 10^3 \text{ mm}^3$$

Shape	S, (10^3 mm^3)
W610 × 101	2520
W530 × 92	2080 ← ◀
W460 × 113	2390
W410 × 114	2200
W360 × 122	2020
W310 × 143	2150

Use W530 × 92. ◀



PROBLEM 5.74

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -3.2B + (24)(3.2)(50) = 0 \qquad B = 120 \text{ kN}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 3.2D - (0.8)(3.2)(50) = 0 \qquad D = 40 \text{ kN}$$

Shear:

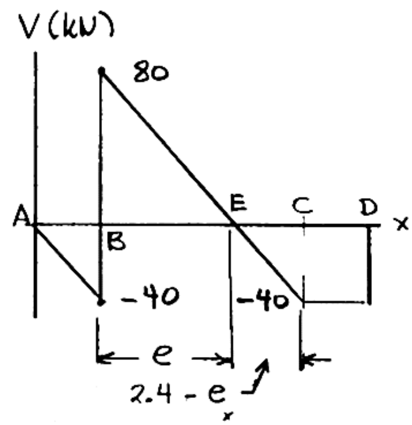
$$V_A = 0$$

$$V_{B^-} = 0 - (0.8)(50) = -40 \text{ kN}$$

$$V_{B^+} = -40 + 120 = 80 \text{ kN}$$

$$V_C = 80 - (2.4)(50) = -40 \text{ kN}$$

$$V_D = -40 + 0 = -40 \text{ kN}$$



Locate point E where $V = 0$.

$$\frac{e}{80} = \frac{2.4 - e}{40} \qquad 120e = 192$$

$$e = 1.6 \text{ m} \qquad 2.4 - e = 0.8 \text{ m}$$

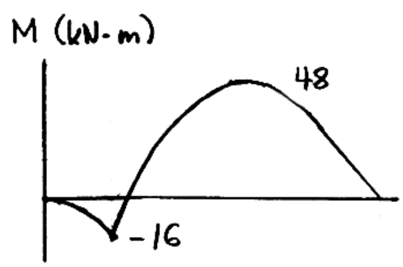
Areas:

A to B: $\int V\,dx = \left(\frac{1}{2}\right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m}$

B to E: $\int V\,dx = \left(\frac{1}{2}\right)(1.6)(80) = 64 \text{ kN} \cdot \text{m}$

E to C: $\int V\,dx = \left(\frac{1}{2}\right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m}$

C to D: $\int V\,dx = (0.8)(-40) = -32 \text{ kN} \cdot \text{m}$



Bending moments:

$$M_A = 0$$

$$M_B = 0 - 16 = -16 \text{ kN} \cdot \text{m}$$

$$M_E = -16 + 64 = 48 \text{ kN} \cdot \text{m}$$

$$M_C = 48 - 16 = 32 \text{ kN} \cdot \text{m}$$

$$M_D = 32 - 32 = 0$$

PROBLEM 5.74 *(Continued)*

Maximum $|M| = 48\ \text{kN}\cdot\text{m} = 48\times 10^3\ \text{N}\cdot\text{m}$

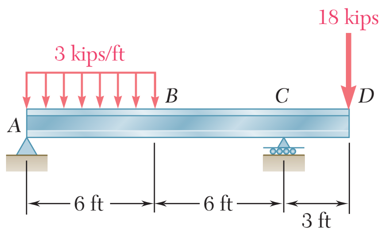
$$\sigma_{\text{all}} = 160\ \text{MPa} = 160\times 10^6\ \text{Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{48\times 10^3}{160\times 10^6} = 300\times 10^{-6}\ \text{m}^3 = 300\times 10^3\ \text{mm}^3$$

Shape	$S(10^3\ \text{mm}^3)$
W310×32.7	415
W250×28.4	308 ←
W200×35.9	342

Lightest wide flange beam: W250×28.4 @ 28.4 kg/m ◀

PROBLEM 5.75

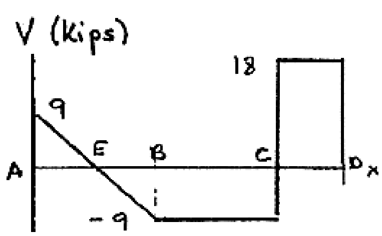


Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -12A + (9)(6)(3) - (3)(18) = 0 \qquad A = 9 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 12C - (3)(6)(3) - (15)(18) = 0 \qquad C = 27 \text{ kips}$$



Shear: $V_A = 9$ kips

B to C: $V = 9 - (6)(3) = -9$ kips

C to D: $V = -9 + 27 = 18$ kips

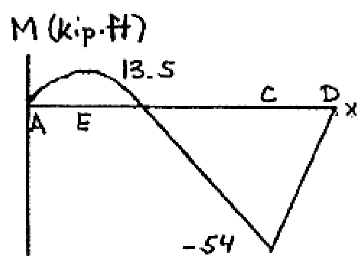
Areas:

A to E: $(0.5)(3)(9) = 13.5 \text{ kip} \cdot \text{ft}$

E to B: $(0.5)(3)(-9) = -13.5 \text{ kip} \cdot \text{ft}$

B to C: $(6)(-9) = -54 \text{ kip} \cdot \text{ft}$

C to D: $(3)(18) = 54 \text{ kip} \cdot \text{ft}$



Bending moments: $M_A = 0$

$M_E = 0 + 13.5 = 13.5 \text{ kip} \cdot \text{ft}$

$M_B = 13.5 - 13.5 = 0$

$M_C = 0 + 54 = 54 \text{ kip} \cdot \text{ft}$

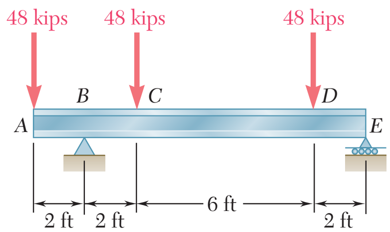
$M_D = 54 - 54 = 0$

Maximum $|M| = 54 \text{ kip} \cdot \text{ft} = 648 \text{ kip} \cdot \text{in.} \quad \sigma_{\text{all}} = 24 \text{ ksi}$

$$S_{\min} = \frac{648}{24} = 27 \text{ in}^3$$

Shape	$S(\text{in}^3)$
S12 × 31.8	36.2
S10 × 35	29.4

Lightest S-shaped beam: S12 × 31.8 ◀



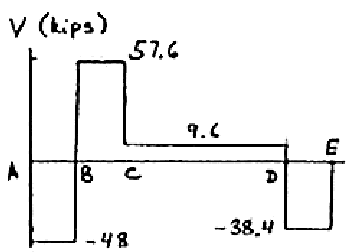
PROBLEM 5.76

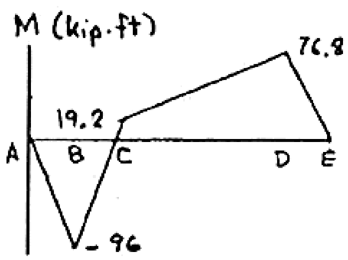
Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0: \quad (12)(48) - 10B + (8)(48) + (2)(48) = 0 \qquad \boldsymbol{B} = 105.6 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (2)(48) - (2)(48) - (8)(48) + 10E = 0 \qquad \boldsymbol{E} = 38.4 \text{ kips} \uparrow$$





Shear: A to B: $V = -48$ kips

B to C: $V = -48 + 105.6 = 57.6$ kips

C to D: $V = 57.6 - 48 = 9.6$ kips

D to E: $V = 9.6 - 48 = -38.4$ kips

Areas: A to B: $(2)(-48) = -96 \text{ kip} \cdot \text{ft}$

B to C: $(2)(57.6) = 115.2 \text{ kip} \cdot \text{ft}$

C to D: $(6)(9.6) = 57.6 \text{ kip} \cdot \text{ft}$

D to E: $(2)(-38.4) = 76.8 \text{ kip} \cdot \text{ft}$

Bending moments: $M_A = 0$

$$M_B = 0 - 96 = -96 \text{ kip} \cdot \text{ft}$$

$$M_C = -96 + 115.2 = 19.2 \text{ kip} \cdot \text{ft}$$

$$M_D = 19.2 + 57.2 = 76.8 \text{ kip} \cdot \text{ft}$$

$$M_E = 76.8 - 76.8 = 0$$

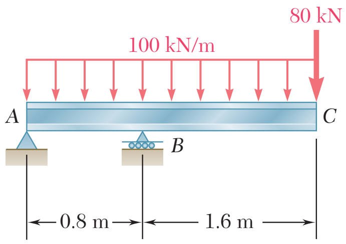
Maximum $|M| = 96 \text{ kip} \cdot \text{ft} = 1152 \text{ kip} \cdot \text{in.}$

$$\sigma_{\text{all}} = 24 \text{ ksi}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1152}{24} = 48 \text{ in}^3$$

Shape	S (in^3)
$S15 \times 42.9$	59.4
$S12 \times 50$	50.6

Lightest S-shaped beam: $S15 \times 42.9$ ◀



PROBLEM 5.77

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

SOLUTION

$$\circlearrowleft \Sigma M_B = 0: \quad 0.8A - (0.4)(2.4)(100) - (1.6)(80) = 0$$

$$A = 280 \text{ kN} \downarrow$$

$$\circlearrowleft \Sigma M_A = 0: \quad 0.8B - (1.2)(2.4)(100) - (2.4)(80) = 0$$

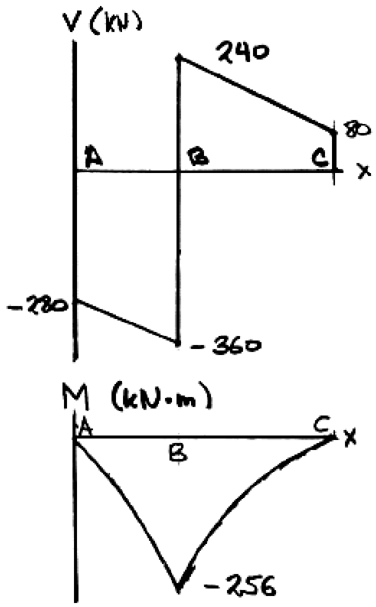
$$B = 600 \text{ kN} \uparrow$$

Shear: $\quad V_A = -280 \text{ kN}$

$$V_{B^-} = -280 - (0.8)(100) = -360 \text{ kN}$$

$$V_{B^+} = -360 + 600 = 240 \text{ kN}$$

$$V_C = 240 - (1.6)(100) = 80 \text{ kN}$$



Areas under shear diagram:

A to B: $\quad \frac{1}{2}(0.8)(-280 - 360) = -256 \text{ kN} \cdot \text{m}$

B to C: $\quad \frac{1}{2}(1.6)(240 + 80) = 256 \text{ kN} \cdot \text{m}$

Bending moments: $\quad M_A = 0$

$$M_B = 0 - 256 = -256 \text{ kN} \cdot \text{m}$$

$$M_C = -256 + 256 = 0$$

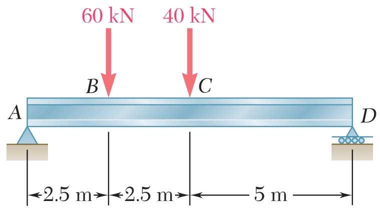
Maximum $|M| = 256 \text{ kN} \cdot \text{m} = 256 \times 10^3 \text{ N} \cdot \text{m}$

$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3} \text{ m}^3 = 1600 \times 10^3 \text{ mm}^3$$

Shape	S (10^3 mm^3)
S510×98.2	1950
S460×104	1685

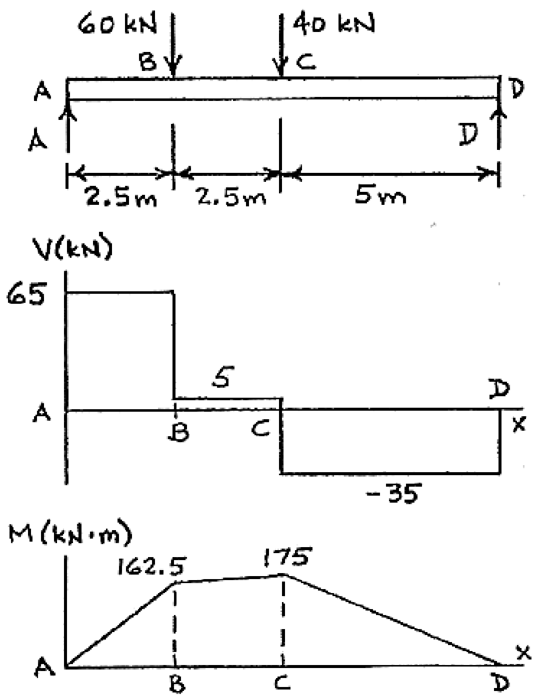
Lightest S-section: $\quad$ S510×98.2 ◀



PROBLEM 5.78

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

SOLUTION



Reaction: $\circlearrowleft \Sigma M_D = 0: \quad -10A + (7.5)(60) + (5)(40) = 0$

$$A = 65 \text{ kN} \uparrow$$

Shear diagram:

A to B: $\quad V = 65 \text{ kN}$

B to C: $\quad V = 65 - 60 = 5 \text{ kN}$

C to D: $\quad V = 5 - 40 = -35 \text{ kN}$

Areas of shear diagram:

A to B: $(2.5)(65) = 162.5 \text{ kN}\cdot\text{m}$

B to C: $(2.5)(5) = 12.5 \text{ kN}\cdot\text{m}$

C to D: $(5)(-35) = -175 \text{ kN}\cdot\text{m}$

Bending moments: $\quad M_A = 0$

$$M_B = 0 + 162.5 = 162.5 \text{ kN}\cdot\text{m}$$

$$M_C = 162.5 + 12.5 = 175 \text{ kN}\cdot\text{m}$$

$$M_D = 175 - 175 = 0$$

$$|M|_{max} = 175 \text{ kN}\cdot\text{m} = 175 \times 10^3 \text{ N}\cdot\text{m}$$

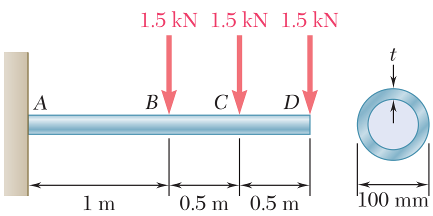
$$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

Shape	S_x, (10^3 mm^3)
S610 × 119	2870
S510 × 98.2	1950
S460 × 81.4	1460 ←

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{175 \times 10^3}{160 \times 10^6} = 1093.75 \times 10^{-6} \text{ m}^3$$

$$= 1093.75 \times 10^3 \text{ mm}^3$$

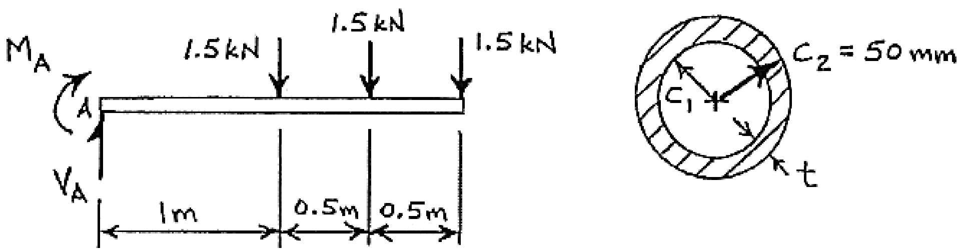
Lightest S-section: $\quad$ S460 × 81.4 ◀



PROBLEM 5.79

A steel pipe of 100-mm diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from 6 mm to 24 mm in 3-mm increments, and that the allowable normal stress for the steel used is 150 MPa, determine the minimum wall thickness t that can be used.

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (1)(1.5) - (1.5)(1.5) - (2)(1.5) = 0 \qquad M_A = -6.75 \text{ kN} \cdot \text{m}$$

$$|M|_{\max} = |M_A| = 6.75 \text{ kN} \cdot \text{m}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{6.75 \times 10^3 \text{ N} \cdot \text{m}}{150 \times 10^6 \text{ Pa}} = 45 \times 10^{-6} \text{ m}^3 = 45 \times 10^3 \text{ mm}^3$$

$$S_{\min} = \frac{I_{\min}}{c_2} \qquad I_{\min} = c_2 S_{\min} = (50)(45 \times 10^3) = 2.25 \times 10^6 \text{ mm}^4$$

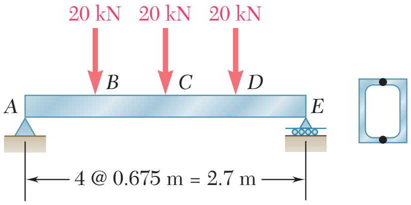
$$I_{\min} = \frac{\pi}{4}\left(c_2^4 - c_{1\max}^4\right)$$

$$c_{1\max}^4 = c_2^4 - \frac{4}{\pi} I_{\min} = (50)^4 - \frac{4}{\pi}(2.25 \times 10^6) = 3.3852 \times 10^6 \text{ mm}^4$$

$$c_{1\max} = 42.894 \text{ mm}$$

$$t_{\min} = c_2 - c_{1\max} = 50 - 42.894 = 7.106 \text{ mm}$$

$t = 9$ mm ◀



PROBLEM 5.80

Two metric rolled-steel channels are to be welded along their edges and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 150 MPa, determine the most economical channels that can be used.

SOLUTION

By symmetry, $A = E$

$$+\uparrow \Sigma F_y = 0: \quad A + E - 20 - 20 - 20 = 0$$

$$A = E = 30 \text{ kN}$$

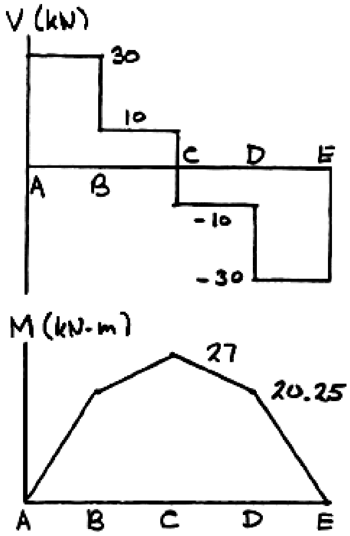
Shear:

A to B: $V = 30$ kN

B to C: $V = 30 - 20 = 10$ kN

C to D: $V = 10 - 20 = -10$ kN

D to E: $V = -10 - 20 = -30$ kN



Areas:

A to B: $(0.675)(30) = 20.25$ kN · m

B to C: $(0.675)(10) = 6.75$ kN · m

C to D: $(0.675)(-10) = -6.75$ kN · m

D to E: $(0.675)(-30) = -20.25$ kN · m

Bending moments: $M_A = 0$

$$M_B = 0 + 20.25 = 20.25 \text{ kN} \cdot \text{m}$$

$$M_C = 20.25 + 6.75 = 27 \text{ kN} \cdot \text{m}$$

$$M_D = 27 - 6.75 = 20.25 \text{ kN} \cdot \text{m}$$

$$M_E = 20.25 - 20.25 = 0$$

$$\text{Maximum } |M| = 27 \text{ kN} \cdot \text{m} = 27 \times 10^3 \text{ N} \cdot \text{m}$$

$$\sigma_{\text{all}} = 150 \text{ MPa} = 150 \times 10^6 \text{ Pa}$$

For a section consisting of two channels,

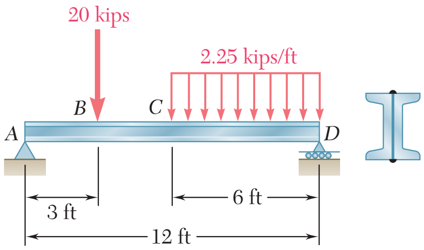
$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{27 \times 10^3}{150 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

For each channels,

$$S_{\min} = \left(\frac{1}{2}\right)(180 \times 10^3) = 90 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{ mm}^3)$
C180 × 14.6	100
C150 × 19.3	94.7 ←

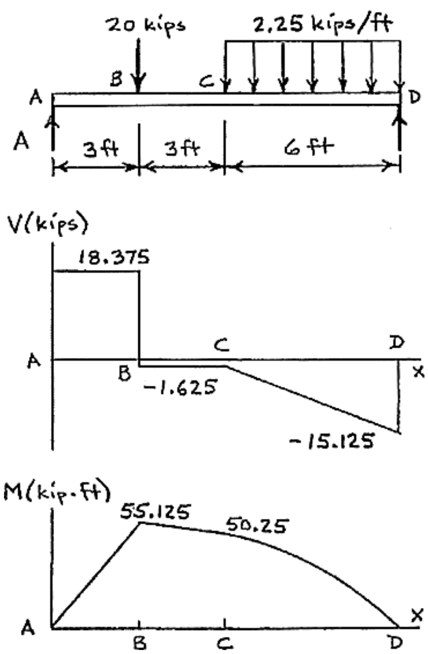
Lightest channel section: C180 × 14.6 ◀



PROBLEM 5.81

Two rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 30 ksi, determine the most economical channels that can be used.

SOLUTION



Reaction: $\overset{+}{\circlearrowleft}\Sigma M_D = 0: \quad -12A + 9(20) + (6)(2.25)(3) = 0$

$$\boldsymbol{A} = 18.375 \text{ kips} \uparrow$$

Shear diagram:

A to B: $\quad V = 18.375 \text{ kips}$

B to C: $\quad V = 18.375 - 20 = -1.625 \text{ kips}$

$$V_D = -1.625 - (6)(2.25) = -15.125 \text{ kips}$$

Areas of shear diagram:

A to B: $\quad (3)(18.375) = 55.125 \text{ kip} \cdot \text{ft}$

B to C: $\quad (3)(-1.625) = -4.875 \text{ kip} \cdot \text{ft}$

C to D: $\quad 0.5(6)(-1.625 - 15.125) = -50.25 \text{ kip} \cdot \text{ft}$

Bending moments: $\quad M_A = 0$

$$M_B = 0 + 55.125 = 55.125 \text{ kip} \cdot \text{ft}$$

$$M_C = 55.125 - 4.875 = 50.25 \text{ kip} \cdot \text{ft}$$

$$M_D = 50.25 - 50.25 = 0$$

$$|M|_{max} = 55.125 \text{ kip} \cdot \text{ft} = 661.5 \text{ kip} \cdot \text{in.}$$

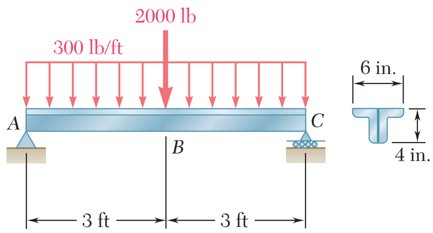
$$\sigma_{all} = 30 \text{ ksi}$$

Shape	$S(\text{in})^3$
$C10 \times 15.3$	13.5
$C9 \times 15$	11.3 ←
$C8 \times 18.7$	11.0

For double channel, $\quad S_{min} = \dfrac{|M|}{\sigma_{all}} = \dfrac{661.5}{30} = 22.05 \text{ in}^3$

For single channel, $\quad S_{min} = 0.5(22.05) = 11.025 \text{ in}^3$

Lightest channel section: $C9 \times 15$ ◀



PROBLEM 5.82

Two L4 × 3 rolled-steel angles are bolted together and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine the minimum angle thickness that can be used.

SOLUTION

By symmetry, $A = C$

$$+\uparrow \Sigma F_y = 0: \quad A + C - 2000 - (6)(300) = 0$$

$$A = C = 1900 \text{ lb}$$

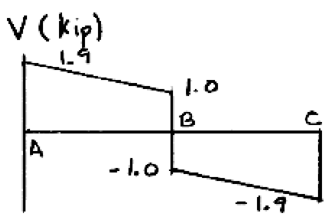
Shear:

$$V_A = 1900 \text{ lb} = 1.9 \text{ kips}$$

$$V_{B^-} = 1900 - (3)(300) = 1000 \text{ lb} = 1 \text{ kip}$$

$$V_{B^+} = 1000 - 2000 = -1000 \text{ lb} = -1 \text{ kip}$$

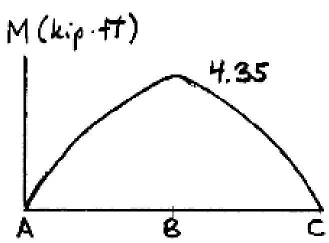
$$V_C = -1000 - (3)(300) = -1900 \text{ lb} = -1.9 \text{ kip}$$



Areas:

A to B: $$\left(\frac{1}{2}\right)(3)(1.9 + 1) = 4.35 \text{ kip} \cdot \text{ft}$$

B to C: $$\left(\frac{1}{2}\right)(3)(-1 - 1.9) = -4.35 \text{ kip} \cdot \text{ft}$$



Bending moments: $M_A = 0$

$$M_B = 0 + 4.35 = 4.35 \text{ kip} \cdot \text{ft}$$

$$M_C = 4.35 - 4.35 = 0$$

Maximum $|M| = 4.35 \text{ kip} \cdot \text{ft} = 52.2 \text{ kip} \cdot \text{in.}$

$$\sigma_{\text{all}} = 24 \text{ ksi}$$

For section consisting of two angles,

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{52.2}{24} = 2.175 \text{ in}^3$$

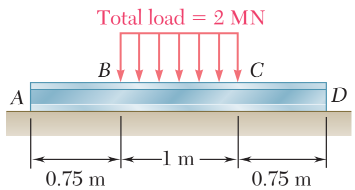
For each angle,

$$S_{\min} = \left(\frac{1}{2}\right)(2.175) = 1.0875 \text{ in}^3$$

PROBLEM 5.82 *(Continued)*

Angle section	$S(\text{in}^3)$
$\text{L}4 \times 3 \times \frac{1}{2}$	1.87
$\text{L}4 \times 3 \times \frac{3}{8}$	1.44
$\text{L}4 \times 3 \times \frac{1}{4}$	0.988

Smallest allowable thickness: $t = \frac{3}{8}$ in. ◀



PROBLEM 5.83

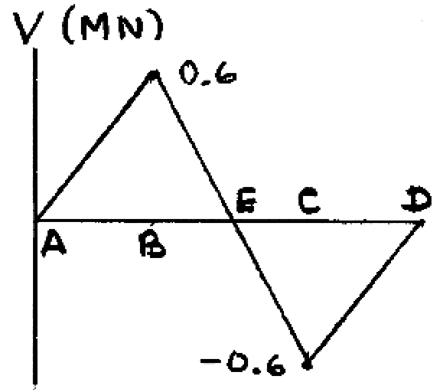
Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION

Downward distributed load: $w = \dfrac{2}{1.0} = 2 \text{ MN/m}$

Upward distributed reaction: $q = \dfrac{2}{2.5} = 0.8 \text{ MN/m}$

Net distributed load over BC: 1.2 MN/m



Shear:

$$V_A = 0$$
$$V_B = 0 + (0.75)(0.8) = 0.6 \text{ MN}$$
$$V_C = 0.6 - (1.0)(1.2) = -0.6 \text{ MN}$$
$$V_D = -0.6 + (0.75)(0.8) = 0$$

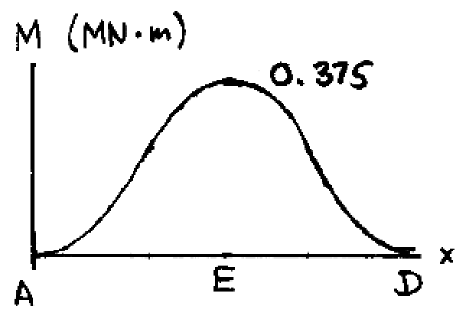
Areas:

A to B: $\left(\dfrac{1}{2}\right)(0.75)(0.6) = 0.225 \text{ MN} \cdot \text{m}$

B to E: $\left(\dfrac{1}{2}\right)(0.5)(0.6) = 0.150 \text{ MN} \cdot \text{m}$

E to C: $\left(\dfrac{1}{2}\right)(0.5)(-0.6) = -0.150 \text{ MN} \cdot \text{m}$

C to D: $\left(\dfrac{1}{2}\right)(0.75)(-0.6) = -0.225 \text{ MN} \cdot \text{m}$



Bending moments:

$$M_A = 0$$
$$M_B = 0 + 0.225 = 0.225 \text{ MN} \cdot \text{m}$$
$$M_E = 0.225 + 0.150 = 0.375 \text{ MN} \cdot \text{m}$$
$$M_C = 0.375 - 0.150 = 0.225 \text{ MN} \cdot \text{m}$$
$$M_D = 0.225 - 0.225 = 0$$

Maximum $|M| = 0.375 \text{ MN} \cdot \text{m} = 375 \times 10^3 \text{ N} \cdot \text{m}$

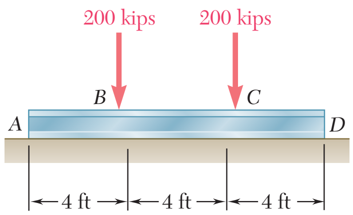
$$\sigma_{\text{all}} = 170 \text{ MPa} = 170 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3} \text{ m}^3 = 2206 \times 10^3 \text{ mm}^3$$

PROBLEM 5.83 *(Continued)*

Shape	$S\,(10^3\ \text{mm}^3)$
W690 × 125	3490
W610 × 101	2520 ←
W530 × 150	3720
W460 × 113	2390

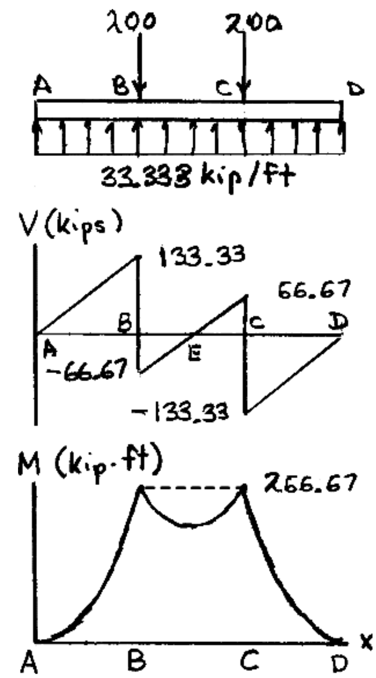
Lightest wide flange section: W610 × 101 ◀



PROBLEM 5.84

Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION



Distributed reaction: $q = \dfrac{400}{12} = 33.333$ kip/ft

Shear:

$$V_A = 0$$
$$V_{B^-} = 0 + (4)(33.333) = 133.33 \text{ kips}$$
$$V_{B^+} = 133.33 - 200 = -66.67 \text{ kips}$$
$$V_{C^-} = -66.67 + 4(33.333) = 66.67 \text{ kips}$$
$$V_{C^+} = 66.67 - 200 = -133.33 \text{ kips}$$
$$V_D = -133.33 + (4)(33.333) = 0 \text{ kips}$$

Areas:

A to B: $\left(\dfrac{1}{2}\right)(4)(133.33) = 266.67$ kip · ft

B to E: $\left(\dfrac{1}{2}\right)(2)(-66.67) = -66.67$ kip · ft

E to C: $\left(\dfrac{1}{2}\right)(2)(66.67) = 66.67$ kip · ft

C to D: $\left(\dfrac{1}{2}\right)(4)(-133.33) = -266.67$ kip · ft

Bending moments:

$$M_A = 0$$
$$M_B = 0 + 266.67 = 266.67 \text{ kip} \cdot \text{ft}$$
$$M_E = 266.67 - 66.67 = 200 \text{ kip} \cdot \text{ft}$$
$$M_C = 200 + 66.67 = 266.67 \text{ kip} \cdot \text{ft}$$
$$M_D = 266.67 - 266.67 = 0$$

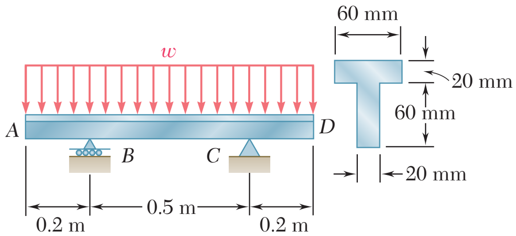
Maximum $|M| = 266.67$ kip · ft $= 3200$ kip · in.

$$\sigma_{\text{all}} = 24 \text{ ksi}$$
$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3200}{24} = 133.3 \text{ in}^3$$

PROBLEM 5.84 *(Continued)*

Shape	$S(\text{in}^3)$	
$W27 \times 84$	213	
$W24 \times 68$	154	$\leftarrow$
$W21 \times 101$	227	
$W18 \times 76$	146	
$W16 \times 77$	134	
$W14 \times 145$	232	

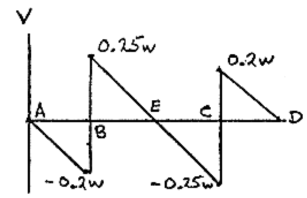
Lightest W-shaped section: $W24 \times 68$ ◀

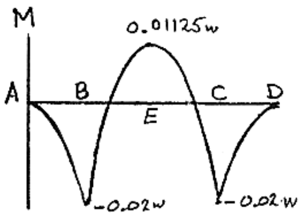


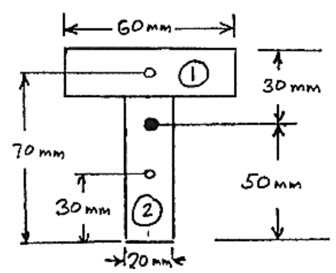
PROBLEM 5.85

Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and −130 MPa in compression.

SOLUTION







Reactions: By symmetry, $B = C$

$$+\uparrow \sum F_y = 0: \quad B + C - 0.9w = 0$$

$$B = C = 0.45w \uparrow$$

Shear:

$$V_A = 0$$

$$V_{B^-} = 0 - 0.2w = -0.2w$$

$$V_{B^+} = -0.2w + 0.45w = 0.25w$$

$$V_{C^-} = 0.25w - 0.5w = -0.25w$$

$$V_{C^+} = -0.25w + 0.45w = 0.2w$$

$$V_D = 0.2w - 0.2w = 0$$

Areas:

A to B: $$\frac{1}{2}(0.2)(-0.2w) = -0.02w$$

B to E: $$\frac{1}{2}(0.25)(0.25w) = 0.03125w$$

Bending moments:

$$M_A = 0$$

$$M_B = 0 - 0.02w = -0.02w$$

$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia:

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}(10^3\ \text{mm}^3)$	d, mm	$Ad^2(10^3\ \text{mm}^4)$	$\bar{I}(10^3\ \text{mm}^4)$
①	1200	70	84	20	480	40
②	1200	30	36	20	480	360
Σ	2400		120		960	400

$$\bar{Y} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 1360 \times 10^3 \text{ mm}^4$$

PROBLEM 5.85 *(Continued)*

Top: $I/y = (1360 \times 10^3)/30 = 45.333 \times 10^3\ \text{mm}^3 = 45.333 \times 10^{-6}\ \text{m}^3$

Bottom: $I/y = (1360 \times 10^3)/(-50) = -27.2 \times 10^3\ \text{mm}^3 = -27.2 \times 10^{-6}\ \text{m}^3$

Bending moment limits $(M = -\sigma I/y)$ and load limits w.

Tension at B and C: $-0.02w = -(80 \times 10^6)(45.333 \times 10^{-6})$ $w = 181.3 \times 10^3$ N/m

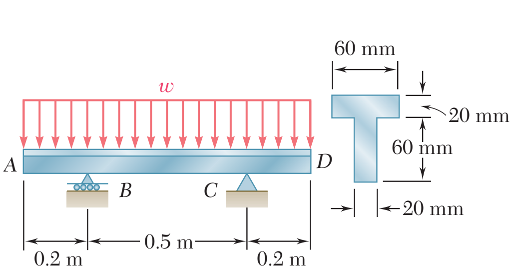
Compression at B and C: $-0.02w = -(-130 \times 10^6)(27.2 \times 10^{-6})$ $w = 176.8 \times 10^3$ N/m

Tension at E: $0.01125w = -(80 \times 10^6)(27.2 \times 10^{-6})$ $w = 193.4 \times 10^3$ N/m

Compression at E: $0.01125w = -(-130 \times 10)(45.333 \times 10^{-6})$ $w = 523.8 \times 10^3$ N/m

The smallest allowable load controls. $w = 176.8 \times 10^3$ N/m

$w = 176.8$ kN/m ◀

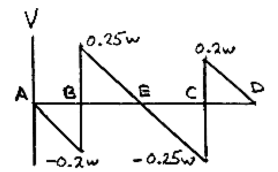


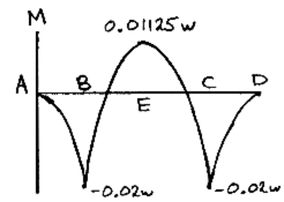
PROBLEM 5.86

Solve Prob. 5.85, assuming that the cross section of the beam is inverted, with the flange of the beam resting on the supports at B and C.

PROBLEM 5.85 Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and −130 MPa in compression.

SOLUTION





Reactions: By symmetry, $B = C$

$$+\uparrow \Sigma F_y = 0:\ B + C - 0.9w = 0$$

$$\boldsymbol{B} = \boldsymbol{C} = 0.45w\uparrow$$

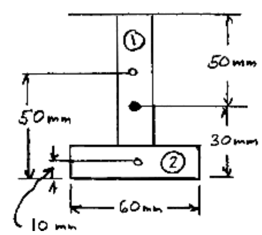
Shear:

$$V_A = 0$$
$$V_{B^-} = 0 - 0.2w = -0.2w$$
$$V_{B^+} = -0.2w + 0.45w = 0.25w$$
$$V_{C^-} = 0.25w - 0.5w = -0.25w$$
$$V_{C^+} = -0.25w + 0.45w = 0.2w$$
$$V_D = 0.2w - 0.2w = 0$$

Areas:

A to B: $\quad \frac{1}{2}(0.2)(-0.2w) = -0.02w$

B to E: $\quad \frac{1}{2}(0.25)(0.25w) = 0.03125w$



Bending moments:

$$M_A = 0$$
$$M_B = 0 - 0.02w = -0.02w$$
$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia:

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, $(10^3\ \text{mm}^3)$	d, mm	$Ad^2 (10^3\ \text{mm}^4)$	$\bar{I}$, $(10^3\ \text{mm}^4)$
①	1200	50	60	20	480	360
②	1200	10	12	20	480	40
Σ	2400		72		960	400

$$\bar{Y} = \frac{72 \times 10^3}{2400} = 30\ \text{mm} \qquad I = \Sigma Ad^2 + \Sigma\bar{I} = 1360 \times 10^3\ \text{mm}^3$$

PROBLEM 5.86 *(Continued)*

Top: $I/y = (1360\times10^3)/(50) = 27.2\times10^3 \text{ mm}^3 = 27.2\times10^{-6} \text{ m}^3$

Bottom: $I/y = (1360\times10^3)/(-30) = -45.333\times10^8 \text{ mm}^3 = -45.333\times10^{-6} \text{ m}^3$

Bending moment limits $(M = -\sigma I/y)$ and load limits w.

Tension at B and C: $-0.02w = -(80\times10^6)(27.2\times10^{-6})$ $w = 108.8\times10^3 \text{ N/m}$

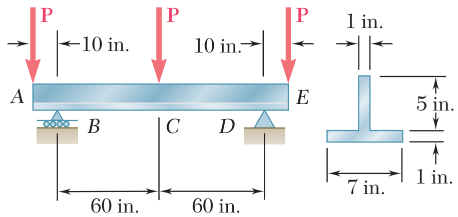
Compression at B and C: $-0.02w = -(-130\times10^6)(-45.333\times10^{-6})$ $w = 294.7\times10^3 \text{ N/m}$

Tension at E: $0.01125w = -(80\times10^6)(-45.333\times10^{-6})$ $w = 322.4\times10^3 \text{ N/m}$

Compression at E: $0.01125w = -(-130\times10^6)(27.2\times10^{-6})$ $w = 314.3\times10^3 \text{ N/m}$

The smallest allowable load controls. $w = 108.8\times10^3 \text{ N/m}$

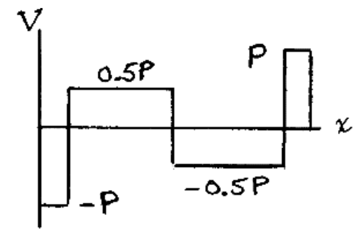
$w = 108.8 \text{ kN/m}$ ◀



PROBLEM 5.87

Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +8 ksi in tension and −18 ksi in compression.

SOLUTION



Reactions: $\boldsymbol{B} = \boldsymbol{D} = 1.5P \uparrow$

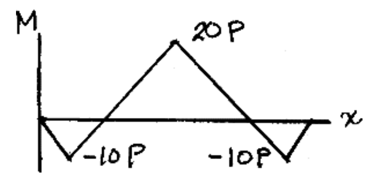
Shear diagram:

A to B^-: $V = -P$

B^+ to C^-: $V = -P + 1.5P = 0.5P$

C^+ to D^-: $V = 0.5P - P = -0.5P$

D^+ to E: $V = -0.5P + 1.5P = P$



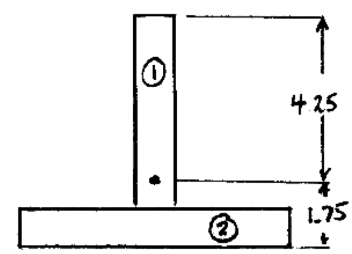
Areas:

A to B: $(10)(-P) = -10P$

B to C: $(60)(0.5P) = 30P$

C to D: $(60)(-0.5P) = -30P$

D to E: $(10)(P) = 10P$



Bending moments:

$$M_A = 0$$
$$M_B = 0 - 10P = -10P$$
$$M_C = -10P + 30P = 20P$$
$$M_D = 20P - 30P = -10P$$
$$M_E = -10P + 10P = 0$$

Largest positive bending moment: $20P$

Largest negative bending moment: $-10P$

Centroid and moment of inertia:

Part	A, in^2	$\bar{y}_0$, in.	$A\bar{y}_0$, in^3	d, in.	Ad^2, in^4	$\bar{I}$, in^4
①	5	3.5	17.5	1.75	15.3125	10.417
②	7	0.5	3.5	1.25	10.9375	0.583
Σ	12		21		26.25	11.000

$$\bar{Y} = \frac{21}{12} = 1.75 \text{ in.} \qquad I = \Sigma Ad^2 + \Sigma I = 37.25 \text{ in}^4$$

Top: $y = 4.25$ in. Bottom: $y = -1.75$ in. $\sigma = -\dfrac{My}{I}$

PROBLEM 5.87 *(Continued)*

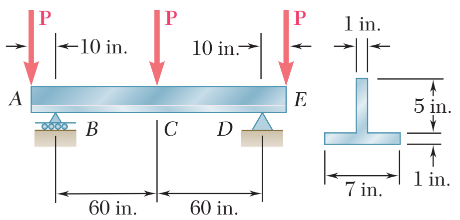
Top, tension: $\quad 8 = -\dfrac{(-10P)(4.25)}{37.25} \qquad P = 7.01 \text{ kips}$

Top, comp.: $\quad -18 = -\dfrac{(20P)(4.25)}{37.25} \qquad P = 7.89 \text{ kips}$

Bottom. tension: $\quad 8 = -\dfrac{(20P)(-1.75)}{37.25} \qquad P = 8.51 \text{ kips}$

Bottom. comp.: $\quad -18 = -\dfrac{(-10P)(-1.75)}{37.25} \qquad P = 38.3 \text{ kips}$

Smallest value of P is the allowable value. $\quad P = 7.01 \text{ kips}$ ◀



PROBLEM 5.88

Solve Prob. 5.87, assuming that the T-shaped beam is inverted.

PROBLEM 5.87 Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +8 ksi in tension and −18 ksi in compression.

SOLUTION

Reactions: $B = D = 1.5P\uparrow$

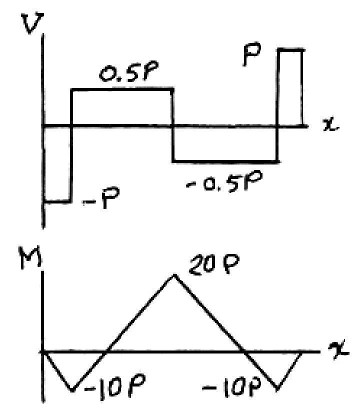
Shear diagram:

A to B^-: $V = -P$

B^+ to C^-: $V = -P + 1.5P = 0.5P$

C^+ to D^-: $V = 0.5P - P = -0.5P$

D^+ to E: $V = -0.5P + 1.5P = P$



Areas:

A to B: $(10)(-P) = -10P$

B to C: $(60)(0.5P) = 30P$

C to D: $(60)(-0.5P) = -30P$

D to E: $(10)(P) = 10P$

Bending moments:

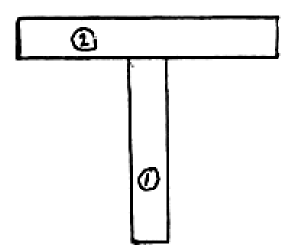
$M_A = 0$

$M_B = 0 - 10P = -10P$

$M_C = -10P + 30P = 20P$

$M_D = 20P - 30P = -10P$

$M_E = -10P + 10P = 0$



Largest positive bending moment $= 20P$

Largest negative bending moment $= -10P$

Centroid and moment of inertia:

Part	A, in^2	$\bar{y}_0$, in.	$A\bar{y}_0$, in^3	d, in.	Ad^2, in^4	$\bar{I}$, in^4
①	5	2.5	12.5	1.75	15.3125	10.417
②	7	5.5	38.5	1.25	10.9375	0.583
Σ	12		51		26.25	11.000

Top: $y = 1.75$ in. $\qquad \bar{Y} = \dfrac{51}{12} = 4.25$ in.

Bottom: $y = -4.25$ in. $\qquad I = \sum Ad^2 + \sum \bar{I} = 37.25\ \text{in}^4$

$$\sigma = -\frac{My}{I}$$

PROBLEM 5.88 *(Continued)*

Top, tension: $8=-\frac{(-10P)(1.75)}{37.25}$ $P=17.03$ kips

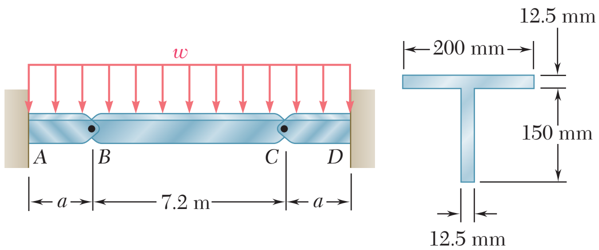
Top, compression: $-18=-\frac{(20P)(1.75)}{37.25}$ $P=19.16$ kips

Bottom, tension: $8=-\frac{(20P)(-4.25)}{37.25}$ $P=3.51$ kips

Bottom, compression: $-18=-\frac{(-10P)(-4.25)}{37.25}$ $P=15.78$ kips

Smallest value of P is the allowable value. $P=3.51$ kips ◀

PROBLEM 5.89



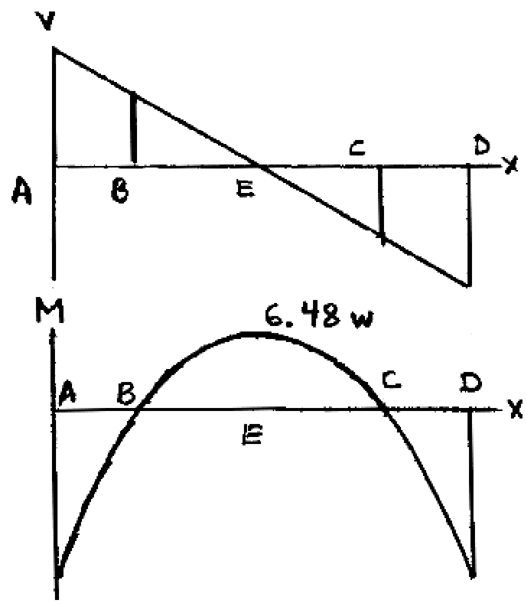
Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of w if beam BC is not to be overstressed, (*b*) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.

SOLUTION

$M_B = M_C = 0$

$V_B = -V_C = \left(\frac{1}{2}\right)(7.2)w = 3.6w$

Area B to E of shear diagram:



$$\left(\frac{1}{2}\right)(3.6)(3.6w) = 6.48w$$

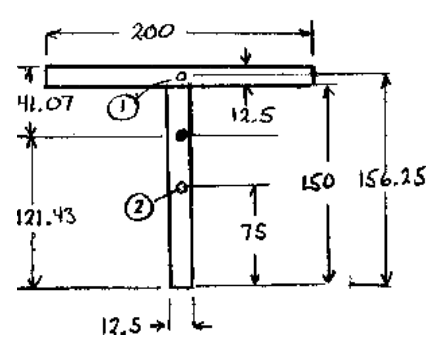
$$M_E = 0 + 6.48w = 6.48w$$

Centroid and moment of inertia:

Part	$A\,(\text{mm}^2)$	$\bar{y}\,(\text{mm})$	$A\bar{y}\,(\text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(\text{mm}^4)$	$\bar{I}\,(\text{mm}^4)$
①	2500	156.25	390,625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140,625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531,250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531,250}{4375} = 121.43 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 10.621 \times 10^6 \text{ mm}^4$$



Location	$y\,(\text{mm})$	$I/y\,(10^3\ \text{mm}^3)$
Top	41.07	258.6
Bottom	−121.43	−87.47

← also $(10^{-6}\ \text{m}^3)$

PROBLEM 5.89 *(Continued)*

Bending moment limits: $M = -\sigma I/y$

Tension at E: $-(110\times 10^6)(-87.47\times 10^{-6}) = 9.622\times 10^3\ \text{N}\cdot\text{m}$

Compression at E: $-(-150\times 10^{-6})(258.6\times 10^{-6}) = 38.8\times 10^3\ \text{N}\cdot\text{m}$

Tension at A and D: $-(110\times 10^6)(258.6\times 10^{-6}) = -28.45\times 10^3\ \text{N}\cdot\text{m}$

Compression at A and D: $-(-150\times 10^6)(-87.47\times 10^{-6}) = -13.121\times 10^3\ \text{N}\cdot\text{m}$

(a) Allowable load w: $6.48w = 9.622\times 10^3$ $\quad w = 1.485\times 10^3\ \text{N/m}$ $\quad w = 1.485\ \text{kN/m}$ ◀

Shear at A: $V_A = (a+3.6)\,w$

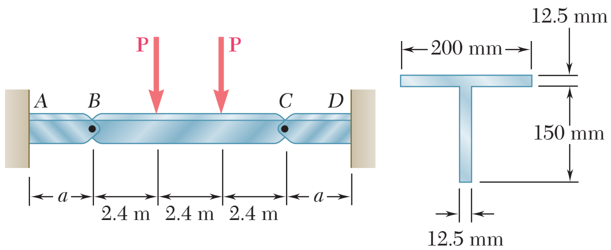
Area A to B of shear diagram: $\frac{1}{2}a(V_A + V_B) = \frac{1}{2}a(a+7.2)\,w$

Bending moment at A (also D): $M_A = -\frac{1}{2}a(a+7.2)\,w$

$$-\frac{1}{2}a(a+7.2)(4.485\times 10^3) = -13.121\times 10^3$$

(b) Distance a: $\frac{1}{2}a^2 + 3.6a - 8.837 = 0$ $\quad a = 1.935\ \text{m}$ ◀

PROBLEM 5.90



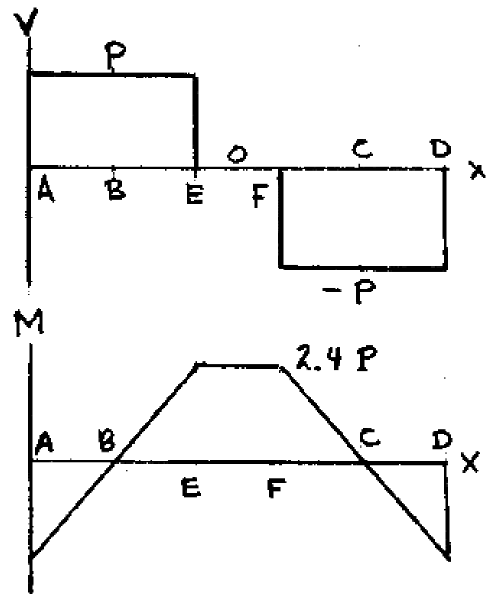
Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of **P** if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.

SOLUTION

$$M_B = M_C = 0$$
$$V_B = -V_C = P$$

Area *B* to *E* of shear diagram: 2.4*P*

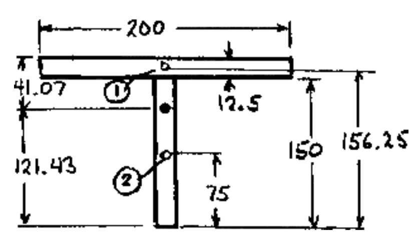
$$M_E = 0 + 2.4P = 2.4P = M_F$$



Centroid and moment of inertia:

Part	$A\,(\text{mm}^2)$	$\bar{y}\,(\text{mm})$	$A\bar{y}\,(\text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(\text{mm}^4)$	$\bar{I}\,(\text{mm}^4)$
①	2500	156.25	390,625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140,625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531,250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531,250}{4375} = 121.43 \text{ mm}$$
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6 \text{ mm}^4$$



Location	$y\,(\text{mm})$	$I/y\,(10^3\text{ mm}^3)$
Top	41.07	258.6
Bottom	−121.43	−87.47

← also (10^{-6} m^3)

PROBLEM 5.90 *(Continued)*

Bending moment limits: $M = -\sigma I/y$

Tension at E and F: $-(110\times 10^6)(-87.47\times 10^{-6}) = 9.622\times 10^3\ \text{N}\cdot\text{m}$

Compression at E and F: $-(-150\times 10^6)(258.6\times 10^{-6}) = 38.8\times 10^3\ \text{N}\cdot\text{m}$

Tension at A and D: $-(110\times 10^6)(258.6\times 10^{-6}) = -28.45\times 10^3\ \text{N}\cdot\text{m}$

Compression at A and D: $-(-150\times 10^6)(-87.47\times 10^{-6}) = -13.121\times 10^3\ \text{N}\cdot\text{m}$

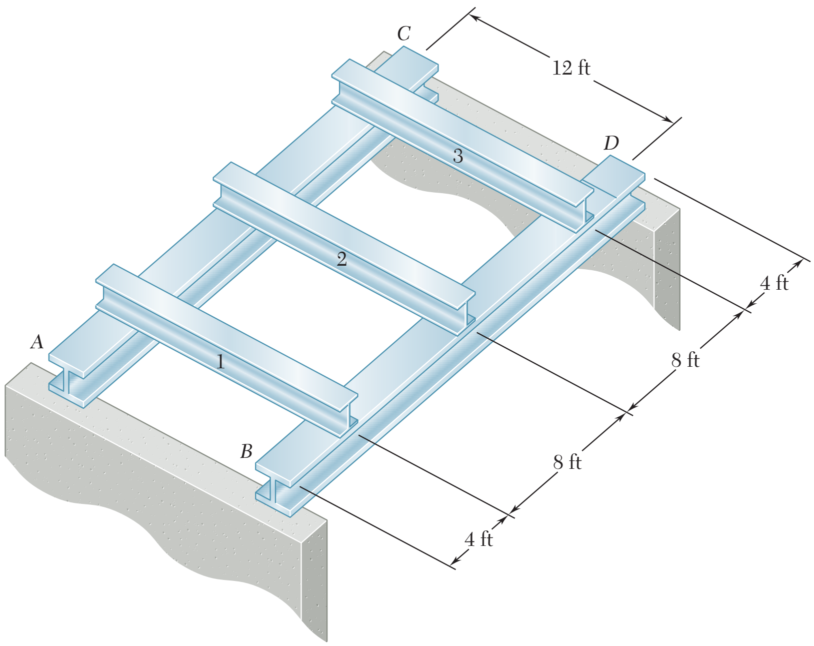
(a) Allowable load P: $2.4P = 9.622\times 10^3$ $P = 4.01\times 10^3$ N $P = 4.01$ kN ◀

Shear at A: $V_A = P$

Area A to B of shear diagram: $aV_A = aP$

Bending moment at A: $M_A = -aP = -4.01\times 10^3 a$

(b) Distance a: $-4.01\times 10^3 a = -13.121\times 10^3$ $a = 3.27$ m ◀



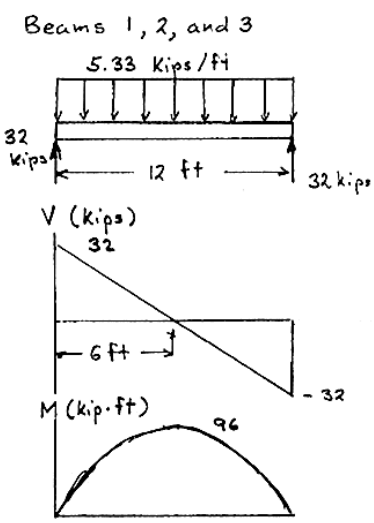
PROBLEM 5.91

Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders *AC* and *BD*. Knowing that the allowable normal stress for the steel used is 24 ksi, select (*a*) the most economical S shape for the three beams, (*b*) the most economical W shape for the two girders.

SOLUTION

For beams 1, 2, and 3,

$$\text{Maximum } M = \left(\frac{1}{2}\right)(6)(32) = 96 \text{ kip}\cdot\text{ft} = 1152 \text{ kip}\cdot\text{in.}$$

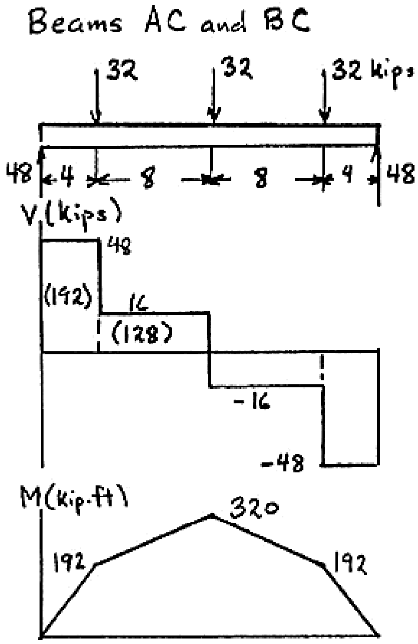


$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1152}{24} = 48 \text{ in}^3$$

Shape	$S(\text{in}^3)$
$S15 \times 42.9$	59.4
$S12 \times 50$	50.6

(*a*) Use $S15 \times 42.9$. ◀

PROBLEM 5.91 *(Continued)*



For beams AC and BC,
areas under shear digram:

$$(4)(48) = 192 \text{ kip} \cdot \text{ft}$$

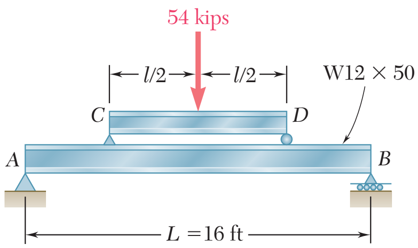
$$(8)(16) = 128 \text{ kip} \cdot \text{ft}$$

Maximum $M = 192 + 128 = 320 \text{ kip} \cdot \text{ft} = 3840 \text{ kip} \cdot \text{in.}$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3840}{24} = 160 \text{ in}^3$$

Shape	$S(\text{in}^3)$
W30 × 99	269
W27 × 84	213
W24 × 104	258
W21 × 101	227
W18 × 106	204

(b) Use W27 × 84. ◀

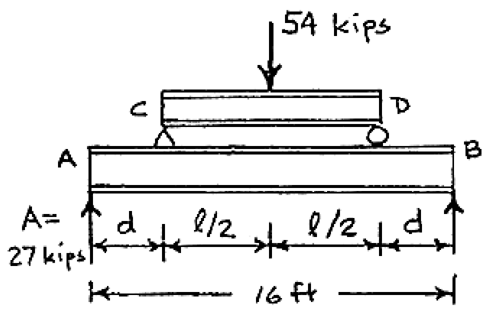


PROBLEM 5.92

A 54-kip is load is to be supported at the center of the 16-ft span shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine (*a*) the smallest allowable length l of beam *CD* if the W12 × 50 beam *AB* is not to be overstressed, (*b*) the most economical W shape that can be used for beam *CD*. Neglect the weight of both beams.

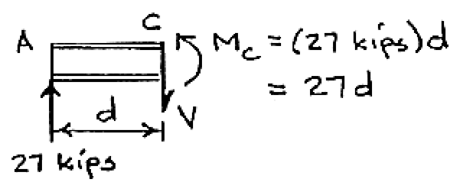
SOLUTION

(*a*)



$$d = 8\,\text{ft} - \frac{l}{2} \qquad l = 16\,\text{ft} - 2d \qquad (1)$$

<u>Beam *AB* (Portion *AC*)</u>:



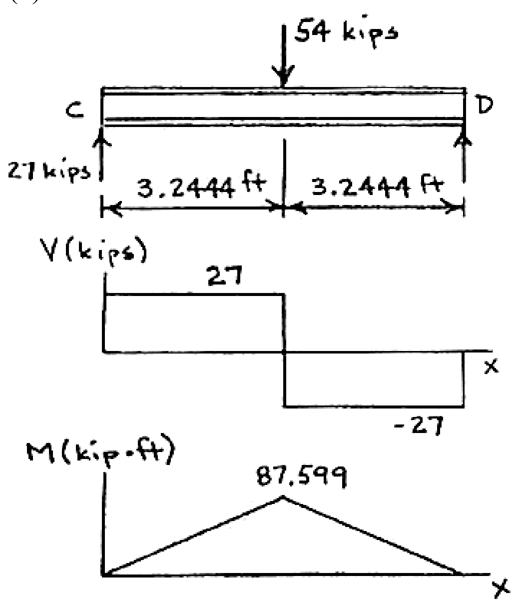
For W12 × 50, $S_x = 64.2\ \text{in}^3 \quad \sigma_{\text{all}} = 24\ \text{ksi}$

$$M_{\text{all}} = \sigma_{\text{all}} S_x = (24)(64.2) = 1540.8\ \text{kip}\cdot\text{in.} = 128.4\ \text{kip}\cdot\text{ft}$$

$$M_C = 27d = 128.4\ \text{kip}\cdot\text{ft} \qquad d = 4.7556\ \text{ft}$$

Using (1), $\quad l = 16 - 2d = 16 - 2(4.7556) = 6.4888\ \text{ft}$ $\qquad l = 6.49\ \text{ft}$ ◀

(*b*)



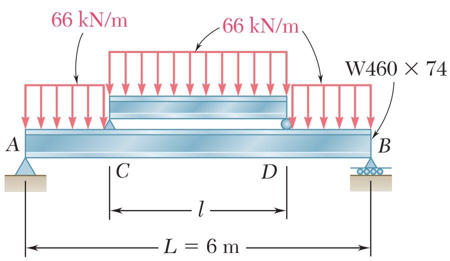
<u>Beam *CD*</u>: $\quad l = 6.4888\ \text{ft} \quad \sigma_{\text{all}} = 24\ \text{ksi}$

$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{(87.599 \times 12)\ \text{kip}\cdot\text{in.}}{24\ \text{ksi}}$$

$$= 43.800\ \text{in}^3$$

Shape	$S(\text{in}^3)$	
W18 × 35	57.6	
W16 × 31	47.2	←
W14 × 38	54.6	
W12 × 35	45.6	
W10 × 45	49.1	

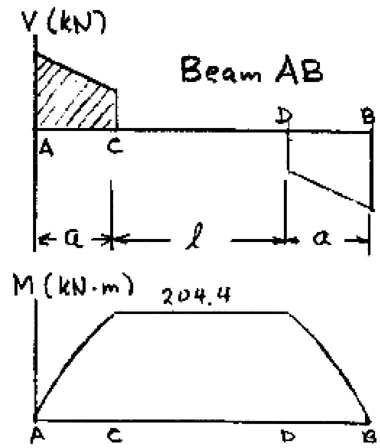
W16 × 31 ◀



PROBLEM 5.93

A uniformly distributed load of 66 kN/m is to be supported over the 6-m span shown. Knowing that the allowable normal stress for the steel used is 140 MPa, determine (*a*) the smallest allowable length *l* of beam *CD* if the $W460 \times 74$ beam *AB* is not to be overstressed, (*b*) the most economical W shape that can be used for beam *CD*. Neglect the weight of both beams.

SOLUTION



For $W460 \times 74$,

$$S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$$

$$\sigma_{\text{all}} = 140 \text{ MPa} = 140 \times 10^6 \text{ Pa}$$

$$M_{\text{all}} = S\sigma_{\text{all}} = (1460 \times 10^{-6})(140 \times 10^6)$$

$$= 204.4 \times 10^3 \text{ N} \cdot \text{m} = 204.4 \text{ kN} \cdot \text{m}$$

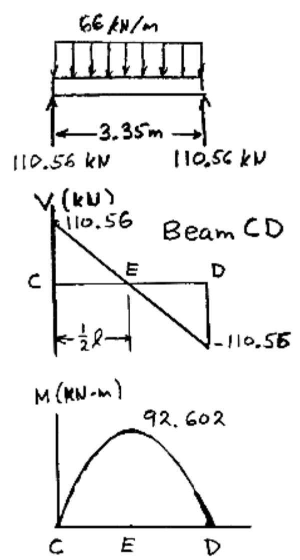
Reactions: By symmetry, $A = B, \quad C = D$

$$+\uparrow \Sigma F_y = 0: \; A + B - (6)(66) = 0$$

$$A = B = 198 \text{ kN} = 198 \times 10^3 \text{ N}$$

$$+\Sigma F_y = 0: \; C + D - 66l = 0$$

$$C = D = (33l) \text{ kN} \qquad (1)$$



Shear and bending moment in beam *AB*:

$$0 < x < a, \qquad V = 198 - 66x \text{ kN}$$

$$M = 198x - 33x^2 \text{ kN} \cdot \text{m}$$

At C, $x = a$. $\qquad M = M_{\max}$

$$M = 198a - 33a^2 \text{ kN} \cdot \text{m}$$

Set $M = M_{\text{all}}$. $\qquad 198a - 33a^2 = 204.4$

$$33a^2 - 198a + 204.4 = 0$$

$$a = \cancel{4.6751 \text{ m}}, \qquad 1.32487 \text{ m}$$

(*a*) By geometry, $\quad l = 6 - 2a = 3.35 \text{ m}$ $\qquad l = 3.35 \text{ m}$ ◀

From (1), $\quad C = D = 110.56 \text{ kN}$

Draw shear and bending moment diagrams for beam *CD*. $V = 0$ at point *E*, the midpoint of *CD*.

PROBLEM 5.93 *(Continued)*

Area from A to E: $$\int V\,dx = \frac{1}{2}(110.560)\left(\frac{1}{2}l\right) = 92.602 \text{ kN}\cdot\text{m}$$

$$M_E = 92.602 \text{ kN}\cdot\text{m} = 92.602\times 10^3 \text{ N}\cdot\text{m}$$

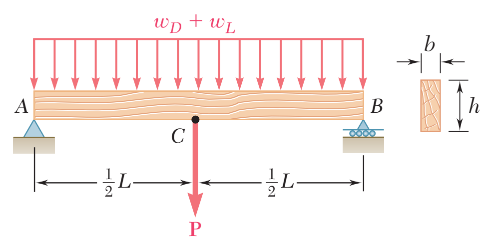
$$S_{\min} = \frac{M_E}{\sigma_{\text{all}}} = \frac{92.602\times 10^3}{140\times 10^6} = 661.44\times 10^{-6} \text{ m}^3$$

$$= 661.44\times 10^3 \text{ mm}^3$$

Shape	$S(10^3\text{ mm}^3)$	
$W410\times 46.1$	773	
$W360\times 44$	688	←
$W310\times 52$	747	
$W250\times 58$	690	
$W200\times 71$	708	

(*b*) Use $W360\times 44$. ◀

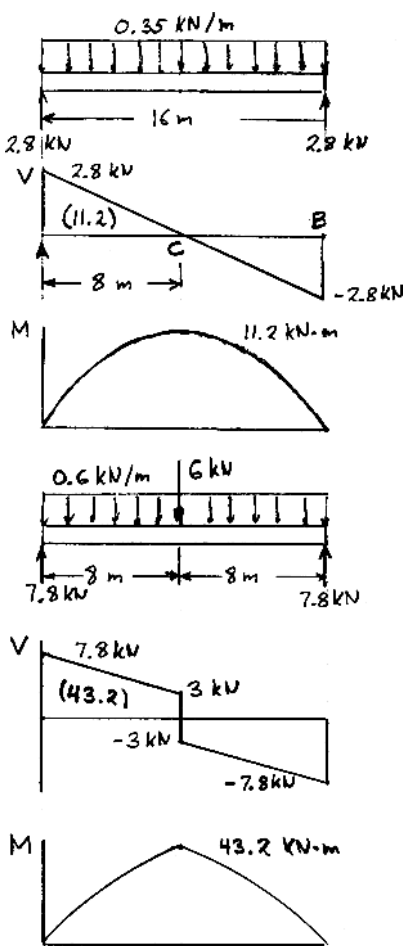
PROBLEM 5.94



A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

SOLUTION

$$L = 16 \text{ m}, \quad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m}$$
$$w_L = 600 \text{ N/m} = 0.6 \text{ kN/m}, \quad P = 6 \text{ kN}$$



Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8$ kN

Area A to C of shear diagram: $\left(\frac{1}{2}\right)(8)(2.8) = 11.2$ kN $\cdot$ m

Bending moment at C: $11.2 \text{ kN} \cdot \text{m} = 11.2 \times 10^3 \text{ N} \cdot \text{m}$

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8$ kN

Shear at C^-: $V = 7.8 - (8)(0.6) = 3$ kN

Area A to C of shear diagram: $\left(\frac{1}{2}\right)(8)(7.8 + 3) = 43.2$ kN $\cdot$ m

Bending moment at C: $43.2 \text{ kN} \cdot \text{m} = 43.2 \times 10^3 \text{ N} \cdot \text{m}$

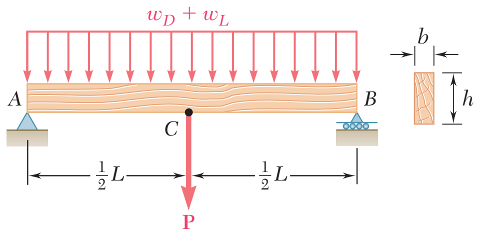
Design: $\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(43.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$
$$= 1.8347 \times 10^{-3} \text{ m}^3 = 1.8347 \times 10^6 \text{ mm}^3$$

For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347 \times 10^6)}{75}} \qquad h = 383 \text{ mm} \blacktriangleleft$$

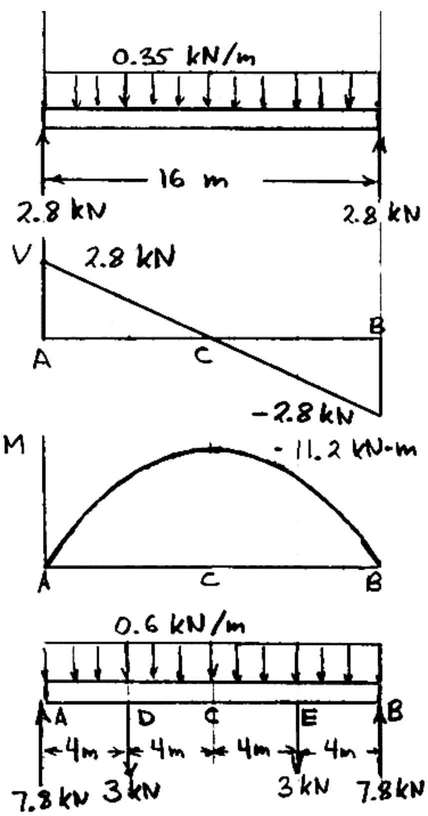
PROBLEM 5.95*



Solve Prob. 5.94, assuming that the 6-kN concentrated load **P** applied to each beam is replaced by 3-kN concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ applied at a distance of 4 m from each end of the beams.

PROBLEM 5.94* A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

SOLUTION



$$L = 16 \text{ m}, \qquad a = 4 \text{ m}, \qquad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m}$$

$$w_L = 600 \text{ N/m} = 0.6 \text{ kN/m}, \quad P = 3 \text{ kN}$$

Dead load: $\quad R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8 \text{ kN}$

Area A to C of shear diagram: $\quad \left(\frac{1}{2}\right)(8)(2.8) = 11.2 \text{ kN} \cdot \text{m}$

Bending moment at C: $\quad 11.2 \text{ kN} \cdot \text{m} = 11.2 \times 10^3 \text{ N} \cdot \text{m}$

Live load: $\quad R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8 \text{ kN}$

Shear at D^-: $\quad 7.8 - (4)(0.6) = 5.4 \text{ kN}$

Shear at D^+: $\quad 5.4 - 3 = 2.4 \text{ kN}$

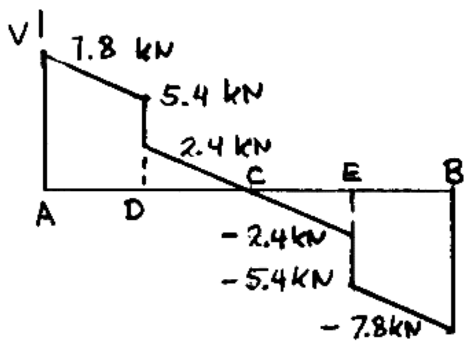
Area A to D: $\quad \left(\frac{1}{2}\right)(4)(7.8 + 5.4) = 26.4 \text{ kN} \cdot \text{m}$

Area D to C: $\quad \left(\frac{1}{2}\right)(4)(2.4) = 4.8 \text{ kN} \cdot \text{m}$

Bending moment at C: $\quad 26.4 + 4.8 = 31.2 \text{ kN} \cdot \text{m}$

$$= 31.2 \times 10^3 \text{ N} \cdot \text{m}$$

PROBLEM 5.95* *(Continued)*



Design: $$\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S$$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U} = \frac{(1.2)(11.2\times 10^3) + (1.6)(31.2\times 10^3)}{(0.9)(50\times 10^6)}$$

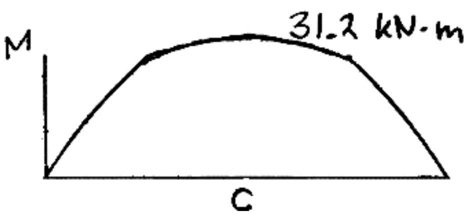
$$= 1.408\times 10^{-3}\,\text{m}^3 = 1.408\times 10^6\,\text{mm}^3$$

For a rectangular section,

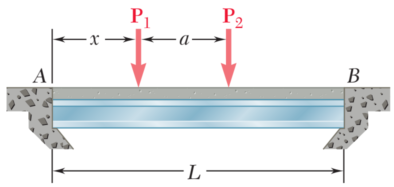
$$S = \frac{1}{6}bh^2$$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408\times 10^6)}{75}}$$

$h = 336$ mm ◀



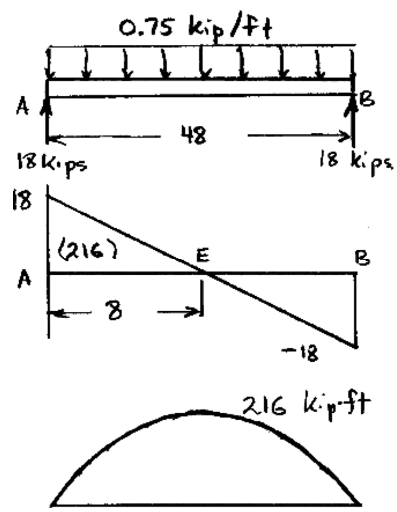
PROBLEM 5.96*



A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength of $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

SOLUTION

$$L = 48 \text{ ft} \quad a = 14 \text{ ft} \quad P_1 = 24 \text{ kips}$$
$$P_2 = 6 \text{ kips} \quad W = 0.75 \text{ kip/ft}$$



Dead load: $$R_A = R_B = \left(\frac{1}{2}\right)(48)(0.75) = 18 \text{ kips}$$

Area A to E of shear diagram:

$$\left(\frac{1}{2}\right)(8)(18) = 216 \text{ kip} \cdot \text{ft}$$

$$M_{\max} = 216 \text{ kip} \cdot \text{ft} = 2592 \text{ kip} \cdot \text{in. at point } E.$$

Live load: $$u = \frac{aP_2}{2(P_1 + P_2)} = \frac{(14)(6)}{(2)(30)} = 1.4 \text{ ft}$$

$$x = \frac{L}{2} - u = 24 - 1.4 = 22.6 \text{ ft}$$

$$x + a = 22.6 + 14 = 36.6 \text{ ft}$$

$$L - x - a = 48 - 36.6 = 11.4 \text{ ft}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -48R_A + (25.4)(24) + (11.4)(6) = 0$$

$$R_A = 14.125 \text{ kips}$$

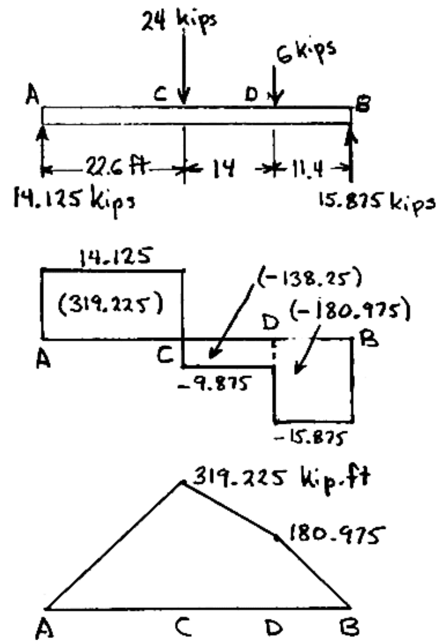
PROBLEM 5.96* *(Continued)*

Shear:

A to C: $V = 14.125$ kips

C to D: $V = 14.125 - 24 = -9.875$ kips

D to B: $V = -15.875$ kips



Area:

A to C: $(22.6)(14.125) = 319.225 \text{ kip} \cdot \text{ft}$

Bending moment: $M_C = 319.225 \text{ kip} \cdot \text{ft} = 3831 \text{ kip} \cdot \text{in.}$

Design:

$$\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S_{\min}$$

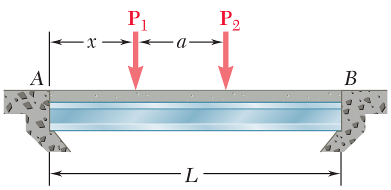
$$S_{\min} = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U} = \frac{(1.25)(2592) + (1.75)(3831)}{(0.9)(60)} = 184.2 \text{ in}^3$$

Shape	$S(\text{in}^3)$	
$W30 \times 99$	269	
$W27 \times 84$	213	←
$W24 \times 104$	258	
$W21 \times 101$	227	
$W18 \times 106$	204	

Use $W27 \times 84$. ◀

PROBLEM 5.97*

Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.96, determine how much heavier a truck could safely cross the bridge designed in that problem.



PROBLEM 5.96* A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength of $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

SOLUTION

$$L = 48 \text{ ft} \qquad a = 14 \text{ ft} \quad P_1 = 24 \text{ kips}$$
$$P_2 = 6 \text{ kips} \quad W = 0.75 \text{ kip/ft}$$

See solution to Prob. 5.96 for calculation of the following:

$$M_D = 2592 \text{ kip}\cdot\text{in.} \qquad M_L = 3831 \text{ kip}\cdot\text{in.}$$

For rolled-steel section W27 × 84, $S = 213 \text{ in}^3$

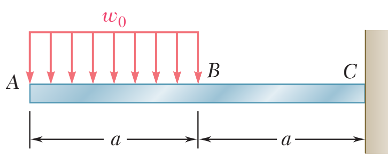
Allowable live load moment M_L^*:

$$\gamma_D M_D + \gamma_L M_L^* = \varphi M_U = \varphi \sigma_U S$$
$$M_L^* = \frac{\varphi \sigma_U S - \gamma_D M_D}{\gamma_L}$$
$$= \frac{(0.9)(60)(213) - (1.25)(2592)}{1.75}$$
$$= 4721 \text{ kip}\cdot\text{in.}$$

Ratio:
$$\frac{M_L^*}{M_L} = \frac{4721}{3831} = 1.232 = 1 + 0.232$$

Increase: 23.2% ◀

PROBLEM 5.98



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = w_0 - w_0 \langle x - a \rangle^0$$

$$= -\frac{dV}{dx}$$

(*a*) $V = -w_0 x + w_0 \langle x - a \rangle^1 = \frac{dM}{dx}$ ◀

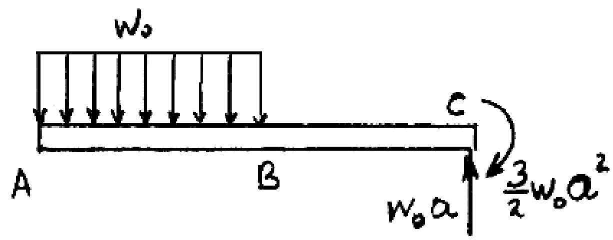
$$M = -\frac{1}{2} w_0 x^2 + \frac{1}{2} w_0 \langle x - a \rangle^2$$ ◀

At point *C*, $x = 2a$

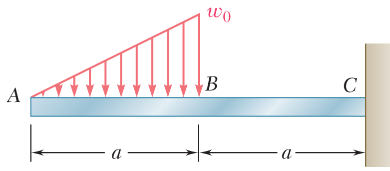
(*b*) $M_C = -\frac{1}{2} w_0 (2a)^2 + \frac{1}{2} w_0 a^2$ $\quad M_C = -\frac{3}{2} w_0 a^2$ ◀

Check: $+\circlearrowleft \Sigma M_C = 0: \left(\frac{3a}{2}\right)(w_0 a) + M_C = 0$

$$M_C = -\frac{3}{2} w_0 a^2$$



PROBLEM 5.99



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = \frac{w_0 x}{a} - w_0 \langle x-a\rangle^0 - \frac{w_0}{a}\langle x-a\rangle^1$$

$$= -\frac{dV}{dx}$$

(*a*) $$V = -\frac{w_0 x^2}{2a} + w_0\langle x-a\rangle^1 + \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$ ◀

$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2}\langle x-a\rangle^2 + \frac{w_0}{6a}\langle x-a\rangle^3$$ ◀

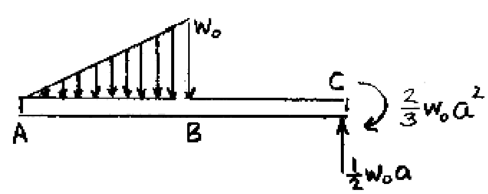
At point C, $x = 2a$

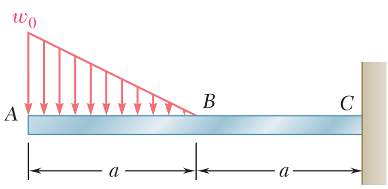
(*b*) $$M_C = -\frac{w_0(2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a}$$

$$M_C = -\frac{2}{3} w_0 a^2$$ ◀

Check: $+\circlearrowleft \Sigma M_C = 0$: $\left(\frac{4a}{3}\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$

$$M_C = -\frac{2}{3} w_0 a^2$$





PROBLEM 5.100

(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a}\langle x-a\rangle^1$$

$$= -\frac{dV}{dx}$$

(*a*) $$V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$ ◀

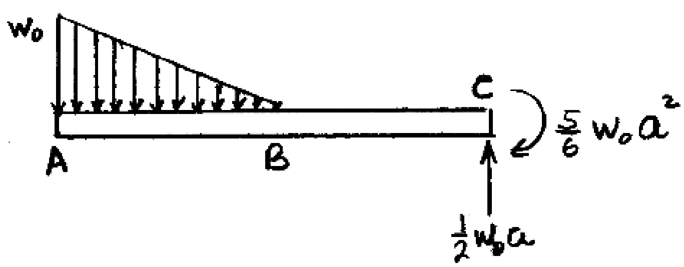
$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a}\langle x-a\rangle^3$$ ◀

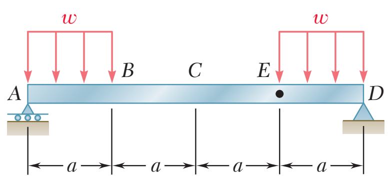
At point C, $x = 2a$

(*b*) $$M_C = -\frac{w_0(2a)^2}{2} + \frac{w_0(2a)^3}{6a} - \frac{w_0 a^3}{6a}$$ $$M_C = -\frac{5}{6}w_0 a^2$$ ◀

Check: $+\circlearrowleft \Sigma M_C = 0{:}\quad \left(\frac{5}{3}a\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$

$$M_C = -\frac{5}{6}w_0 a^2$$

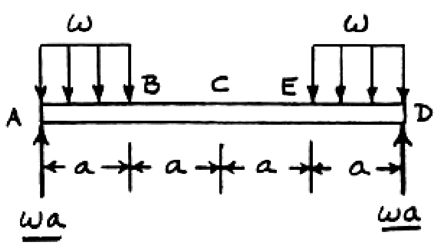




PROBLEM 5.101

(a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point E, and check your answer by drawing the free-body diagram of the portion of the beam to the right of E.

SOLUTION



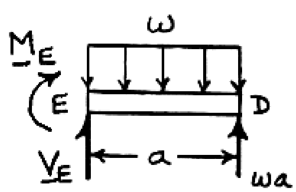
$$w = w - w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

(a) $$V = wa - \int w\,dx$$

$$= wa - wx + w\langle x-a\rangle^1 - w\langle x-3a\rangle^1$$

$$M = \int V\,dx$$

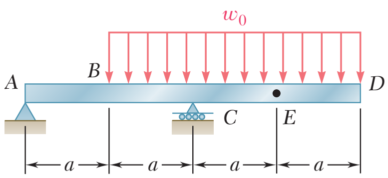
$$= wax - wx^2/2 + (w/2)\langle x-a\rangle^2 - (w/2)\langle x-3a\rangle^2$$



(b) At point E, $x = 3a$

$$M_E = wa(3a) - w(3a)^2/2 + (w/2)(2a)^2 \qquad = wa^2/2$$

$$+\circlearrowleft\ \Sigma M_E = 0: \quad wa(a) - (wa)\left(\frac{a}{2}\right) - M_E = 0 \qquad M_E = wa^2/2 \qquad \text{(Checks)}$$



PROBLEM 5.102

(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point E, and check your answer by drawing the free-body diagram of the portion of the beam to the right of E.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -2aA - \left(\frac{a}{2}\right) + (3aw_0) = 0 \qquad A = -\frac{3}{4} w_0 a$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 2aC - \left(\frac{5a}{2}\right) + (3aw_0) = 0 \qquad C = \frac{15}{4} w_0 a$$

$$w = w_0 \langle x - a \rangle^0 = -\frac{dV}{dx}$$

(*a*) $$V = -w_0 \langle x - a \rangle^1 - \frac{3}{4} w_0 a + \frac{15}{4} w_0 a \langle x - 2a \rangle^0 = \frac{dM}{dx} \quad \blacktriangleleft$$

$$M = -\frac{1}{2} w_0 \langle x - a \rangle^2 - \frac{3}{4} w_0 a x + \frac{15}{4} w_0 a \langle x - 2a \rangle^1 + 0 \quad \blacktriangleleft$$

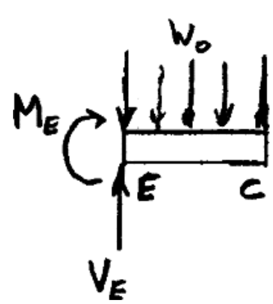
At point E, $\qquad x = 3a$

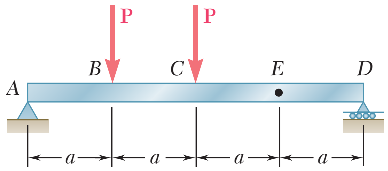
(*b*) $$M_E = -\frac{1}{2} w_0 (2a)^2 - \frac{3}{4} w_0 a(3a) + \frac{15}{4} w_0 a(a)$$

$$M_E = -\frac{1}{2} w_0 a^2 \quad \blacktriangleleft$$

Check: $$+\circlearrowleft \Sigma M_E = 0: \quad -M_E - \frac{a}{2}(w_0 a) = 0$$

$$M_E = -\frac{1}{2} w_0 a^2$$





PROBLEM 5.103

(a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point E, and check your answer by drawing the free-body diagram of the portion of the beam to the right of E.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -4aA + 3aP + 2aP = 0 \qquad A = 1.25P$$

(a) $\quad V = 1.25P - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$ ◀

$$M = 1.25Px - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1$$ ◀

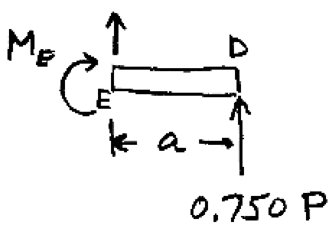
(b) $\quad$ At point E, $\quad x = 3a$

$$M_E = 1.25P(3a) - P(2a) - P(a) = 0.750Pa$$ ◀

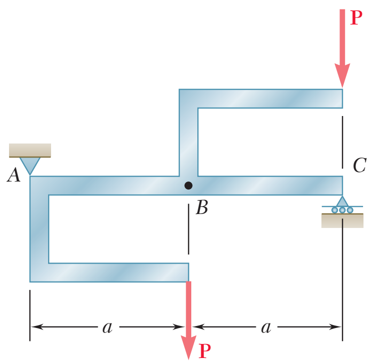
Reaction: $\quad +\uparrow \Sigma F_y = 0: \quad A - P - P + D = 0 \qquad \boldsymbol{D} = 0.750P \uparrow$

$$+\circlearrowleft \Sigma M_E = 0: \quad -M_E + 0.750Pa = 0$$

$$M_E = 0.750Pa$$

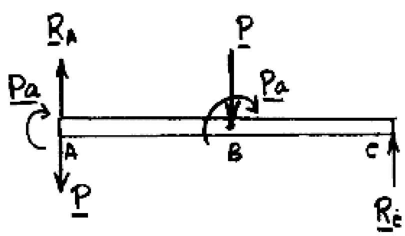


PROBLEM 5.104



(a) Using singularity functions, write the equations for the shear and bending moment for beam ABC under the loading shown. (b) Use the equation obtained for M to determine the bending moment just to the right of point B.

SOLUTION



(a) $\sum M_C = 0$: $(2a)P + aP - 2(Pa) + 2aR_A = 0$

$$R_A = \frac{1}{2}P$$

$$V = (R_A - P) - P\langle x-a\rangle^0$$

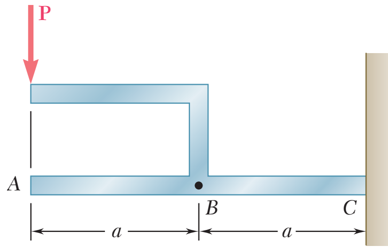
$$= -\frac{1}{2}P - P\langle x-a\rangle^0 \quad \blacktriangleleft$$

$$\frac{dM}{dx} = -\frac{1}{2}P - P\langle x-a\rangle^0$$

$$M = -\frac{1}{2}Px - P\langle x-a\rangle^1 + Pa + Pa\langle x-a\rangle^0$$

(b) Just to the right of point B, $x = a^+$

$$M = -\frac{1}{2}Pa - 0 + Pa + Pa = \frac{3}{2}Pa \quad \blacktriangleleft$$



PROBLEM 5.105

(*a*) Using singularity functions, write the equations for the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.

SOLUTION

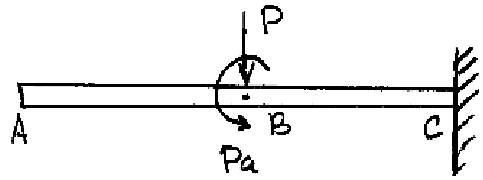
(*a*) $V = -P\langle x-a\rangle^0$ ◀

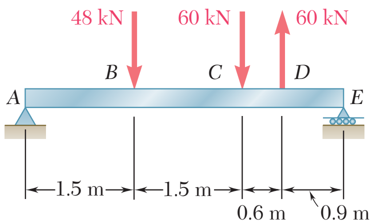
$$\frac{dM}{dx} = -P\langle x-a\rangle^0$$

$$M = -P\langle x-a\rangle^1 - Pa\langle x-a\rangle^0 \quad ◀$$

Just to the right of B, $x = a^1$.

(*b*) $M = -\ 0 - Pa$ $\qquad M = -Pa$ ◀





PROBLEM 5.106

(a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0: \quad -4.5R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$$

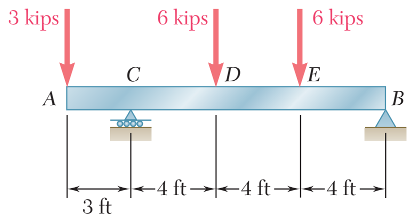
$$R_A = 40 \text{ kN}$$

(a) $V = 40 - 48\langle x - 1.5\rangle^0 - 60\langle x - 3.0\rangle^0 + 60\langle x - 3.6\rangle^0 \text{ kN}$ ◀

$M = 40x - 48\langle x - 1.5\rangle^1 - 60\langle x - 3.0\rangle^1 + 60\langle x - 3.6\rangle^1 \text{ kN}\cdot\text{m}$ ◀

Pt.	x(m)	$M\,(\text{kN}\cdot\text{m})$
A	0	0
B	1.5	$(40)(1.5) = 60 \text{ kN}\cdot\text{m}$
C	3.0	$(40)(3.0) - (48)(1.5) = 48 \text{ kN}\cdot\text{m}$
D	3.6	$(40)(3.6) - (48)(2.1) - (60)(0.6) = 7.2 \text{ kN}\cdot\text{m}$
E	4.5	$(40)(4.5) - (48)(3.0) - (60)(1.5) + (60)(0.9) = 0$

(b) $M_{\max} = 60 \text{ kN}\cdot\text{m}$ ◀



PROBLEM 5.107

(a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0:\ (15)(3) - 12C + (8)(6)C + (4)(6) = 0$$

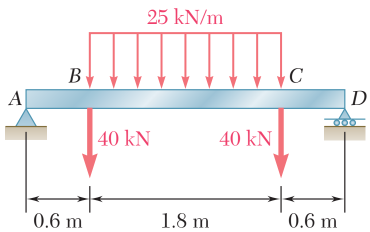
$$\mathbf{C} = 9.75 \text{ kips} \uparrow$$

(a) $V = -3 + 9.75\langle x-3\rangle^0 - 6\langle x-7\rangle^0 - 6\langle x-11\rangle^0$ kips ◀

$M = -3x + 9.75\langle x-3\rangle^1 - 6\langle x-7\rangle^1 - 6\langle x-11\rangle^1$ kip · ft ◀

Pt.	x(ft)	M(kip · ft)	
A	0	0	
C	3	$-(3)(3) = -9$	
D	7	$-(3)(7) + (9.75)(4) = 18$	
E	11	$-(3)(11) + (9.75)(8) - (6)(4) = 21$	← maximum
B	15	$-(3)(15) + (9.75)(12) - (6)(8) - (6)(4) = 0$	

(b) $|M|_{max} = 21.0$ kip · ft ◀



PROBLEM 5.108

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

(*a*) By symmetry, $R_A = R_D$

$$+\uparrow \sum F_y = 0: \quad R_A + R_D - 40 - (1.8)(25) - 40 = 0$$

$$R_A = R_D = 62.5 \text{ kN}$$

$$w = 25\langle x-0.6\rangle^0 - 25\langle x-2.4\rangle^0 = -\frac{dV}{dx}$$

$$V = 62.5 - 25\langle x-0.6\rangle^1 + 25\langle x-2.4\rangle^1 - 40\langle x-0.6\rangle^0 - 40\langle x-2.4\rangle^0 \text{ kN} \quad \blacktriangleleft$$

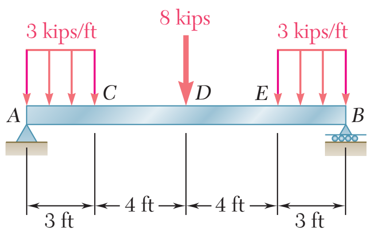
$$M = 62.5x - 12.5\langle x-0.6\rangle^2 + 12.5\langle x-2.4\rangle^2 - 40\langle x-0.6\rangle^1 - 40\langle x-2.4\rangle^1 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

(*b*) Locate point where $V = 0$. Assume $0.6 < x^* < 1.8$

$$0 = 62.5 - 25(x^* - 0.6) + 0 - 40 - 0 \qquad x^* = 1.5 \text{ m}$$

$$M = (62.5)(1.5) - (12.5)(0.9)^2 + 0 - (40)(0.9) - 0 = 47.6 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

PROBLEM 5.109



(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -14A + (12.5)(3)(3) + (7)(8) + (1.5)(3)(3) = 0$$

$$\mathbf{A} = 13 \text{ kips} \uparrow$$

$$w = 3 - 3\langle x-3\rangle^0 + 3\langle x-11\rangle^0 = -\frac{dV}{dx}$$

(*a*) $V = 13 - 3x + 3\langle x-3\rangle^1 - 8\langle x-7\rangle^0 - 3\langle x-11\rangle^1 \text{ kips}$ ◀

$M = 13x - 1.5x^2 + 1.5\langle x-3\rangle^2 - 8\langle x-7\rangle^1 - 1.5\langle x-11\rangle^2 \text{ kip}\cdot\text{ft}$ ◀

$$V_C = 13 - (3)(3) = 4 \text{ kips}$$

$$V_{D^-} = 13 - (3)(7) + (3)(4) = 4 \text{ kips}$$

$$V_{D^+} = 13 - (3)(7) + (3)(4) - 8 = -4 \text{ kips}$$

$$V_E = 13 - (3)(11) + (3)(8) - 8 = -4 \text{ kips}$$

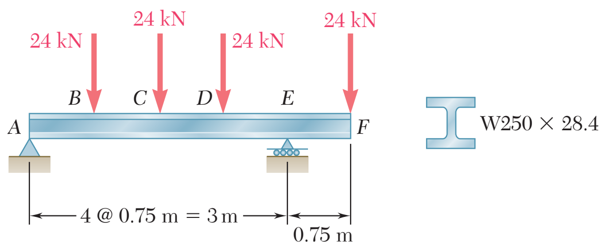
$$V_B = 13 - (3)(14) + (3)(11) - 8 - (3)(3) = -13 \text{ kips}$$

(*b*) Note that V changes sign at D.

$$|M|_{\max} = M_D = (13)(7) - (1.5)(7)^2 + (1.5)(4)^2 - 0 - 0$$

$|M|_{\max} = 41.5 \text{ kip}\cdot\text{ft}$ ◀

PROBLEM 5.110



(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0: \quad -3R_A + (2.25)(24) - (1.5)(24) - (0.75)(24) + (0.75)(24) = 0$$

$$R_A = 30 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$$

$$R_E = 66 \text{ kips}$$

(*a*) $V = 30 - 24\langle x - 0.75\rangle^0 - 24\langle x - 1.5\rangle^0 - 24\langle x - 2.25\rangle^0 + 66\langle x - 3\rangle^0 \text{ kN}$ ◀

$M = 30x - 24\langle x - 0.75\rangle^1 - 24\langle x - 1.5\rangle^1 - 24\langle x - 2.25\rangle^1 + 66\langle x - 3\rangle^1 \text{ kN}\cdot\text{m}$ ◀

Point	x(m)	M (kN · m)
B	0.75	$(30)(0.75) = 22.5 \text{ kN}\cdot\text{m}$
C	1.5	$(30)(1.5) - (24)(0.75) = 27 \text{ kN}\cdot\text{m}$
D	2.25	$(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5 \text{ kN}\cdot\text{m}$
E	3.0	$(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18 \text{ kN}\cdot\text{m}$
F	3.75	$(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$ ✓

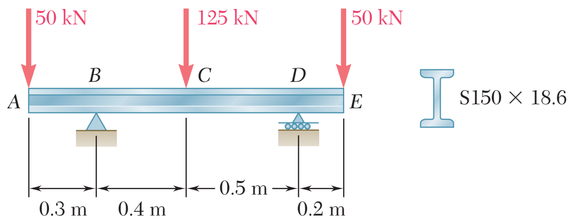
Maximum $|M| = 27 \text{ kN}\cdot\text{m} = 27\times\ 30^3 \text{ N}\cdot\text{m}$

For rolled-steel section W250 × 28.4, $S = 308\times 10^3 \text{ mm}^3 = 308\times 10^{-6} \text{ m}^3$

(*b*) Normal stress:

$$\sigma = \frac{|M|}{S} = \frac{27\times 10^3}{308\times 10^{-6}} = 87.7\times 10^6 \text{ Pa}$$

$\sigma = 87.7 \text{ MPa}$ ◀



PROBLEM 5.111

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad (1.2)(50) - 0.9B + (0.5)(125) - (0.2)(50) = 0$$

$$\boldsymbol{B} = 125 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (0.3)(50) - (0.4)(125) + 0.9D - (1.1)(50) = 0$$

$$\boldsymbol{D} = 100 \text{ kN} \uparrow$$

(*a*) $V = -50 + 125\langle x-0.3\rangle^0 - 125\langle x-0.7\rangle^0 + 100\langle x-1.2\rangle^0 \text{ kN}$ ◀

$M = -50x + 125\langle x-0.3\rangle^1 - 125\langle x-0.7\rangle^1 + 100\langle x-1.2\rangle^1 \text{ kN}\cdot\text{m}$ ◀

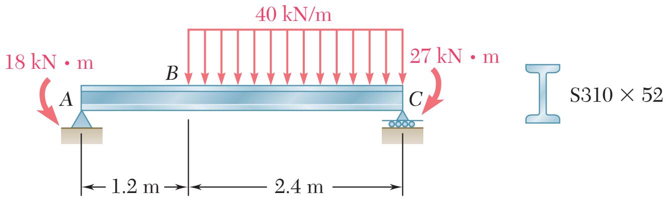
Point	x(m)	M(kN · m)
B	0.3	$-(50)(0.3)+0-0+0=-15 \text{ kN}\cdot\text{m}$
C	0.7	$-(50)(0.7)+(125)(0.4)-0+0=15 \text{ kN}\cdot\text{m}$
D	1.2	$-(50)(1.2)+(125)(0.9)-(125)(0.5)+0=-10 \text{ kN}\cdot\text{m}$
E	1.4	$-(50)(1.4)+(125)(1.1)-(125)(0.7)+(100)(0.2)=0$ (checks)

Maximum $|M| = 15 \text{ kN}\cdot\text{m} = 15\times10^3 \text{ N}\cdot\text{m}$

For S150×18.6 rolled-steel section, $S = 120\times10^3 \text{ mm}^3 = 120\times10^{-6} \text{ m}^3$

(*b*) Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{15\times10^3}{120\times10^{-6}} = 125\times10^6 \text{ Pa}$ $\qquad \sigma = 125.0 \text{ MPa}$ ◀

PROBLEM 5.112



(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft M_c = 0: \; 18 - 3.6A + (1.2)(2.4)(40) - 27 = 0$$

$$\boldsymbol{A} = 29.5 \text{ kN} \uparrow$$

$$V = 29.5 - 40\langle x - 1.2\rangle^1 \text{ kN}$$

Point D. $V = 0$ $\qquad 29.5 - 40(x_D - 1.2) = 0$

$$x_D = 1.9375 \text{ m}$$

$$M = -18 + 29.5x - 20\langle x - 1.2\rangle^2 \text{ kN} \cdot \text{m}$$

$$M_A = -18 \text{ kN} \cdot \text{m}$$

$$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN} \cdot \text{m}$$

$$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN} \cdot \text{m}$$

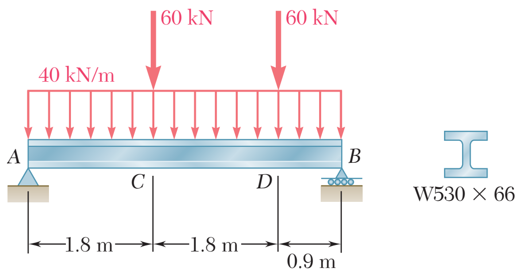
(a) Maximum $|M| = 28.3 \text{ kN} \cdot \text{m}$ at $x = 1.938 \text{ m}$ ◀

For S310 × 52 rolled-steel section, $S = 624 \times 10^3 \text{ mm}^3$

$$= 624 \times 10^{-6} \text{ m}^3$$

(*b*) Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{28.278 \times 10^3}{624 \times 10^{-6}} = 45.3 \times 10^6 \text{ Pa}$

$\sigma = 45.3 \text{ MPa}$ ◀



PROBLEM 5.113

(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -4.5A + (2.25)(4.5)(40) + (2.7)(60) + (0.9)(60) = 0$$

$$\mathbf{A} = 138 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(2.25)(4.5)(40) - (1.8)(60) - (3.6)(60) + 4.5B = 0$$

$$\mathbf{B} = 162 \text{ kN} \uparrow$$

$$w = 40 \text{ kN/m} = \frac{dV}{dx}$$

$$V = -40x + 138 - 60\langle x - 1.8\rangle^0 - 60\langle x - 3.6\rangle^0 = \frac{dM}{dx}$$

$$M = -20x^2 - 138x - 60\langle x - 1.8\rangle^1 - 60\langle x - 3.6\rangle^1$$

$$V_C^+ = -(40)(1.8) + 138 - 60 = 6 \text{ kN}$$

$$V_D^- = -(40)(3.6) + 138 - 60 = -66 \text{ kN}$$

Locate point E where $V = 0$. It lies between C and D.

$$V_E = -40x_E + 138 - 60 + 0 = 0 \qquad x_E = 1.95 \text{ m}$$

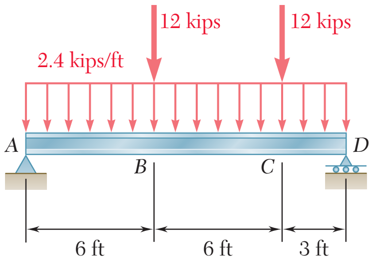
$$M_E = -(20)(1.95)^2 + (138)(1.95) - (60)(1.95 - 1.8) = 184 \text{ kN}\cdot\text{m}$$

(*a*) $$|M|_{\max} = 184 \text{ kN}\cdot\text{m} = 184 \times 10^3 \text{ N}\cdot\text{m} \quad \text{at} \quad x = 1.950 \text{ m} \blacktriangleleft$$

For W530 × 66 rolled-steel section, $S = 1340 \times 10^3 \text{ mm}^3 = 1340 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $$\sigma = \frac{|M|_{\max}}{S} = \frac{184 \times 10^3}{1340 \times 10^{-6}} = 137.3 \times 10^6 \text{ Pa}$$

$$\sigma = 137.3 \text{ MPa} \blacktriangleleft$$



PROBLEM 5.114

A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -15R_A + (7.5)(15)(2.4) - (9)(12) - (3)(12) = 0$$

$$R_A = 27.6 \text{ kips}$$

$$w = 2.4 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 27.6 - 2.4x - 12\langle x-6\rangle^0 - 12\langle x-12\rangle^0 \text{ kips}$$

$$V_{B^-} = 27.6 - (2.4)(6) = 13.2 \text{ kips}$$

$$\left.\begin{aligned} V_{B^+} &= 27.6 - (2.4)(6) - 12 = 1.2 \text{ kips} \\ V_{C^-} &= 27.6 - (2.4)(12) - 12 = -13.2 \text{ kips} \end{aligned}\right\} \text{Point where } V = 0 \text{ lies between } B \text{ and } C.$$

Locate point E where $V = 0$.

$$0 = 27.6 - 2.4x_E - 12 - 0 \qquad x_E = 6.50 \text{ ft}$$

$$M = 27.6x - 1.2x^2 - 12\langle x-6\rangle^1 - 12\langle x-12\rangle^1 \text{ kip}\cdot\text{ft}$$

At point E, $\quad (x = 6.5 \text{ ft}) \qquad M = (27.6)(6.5) - (1.2)(6.5)^2 - (12)(0.5) - 0$

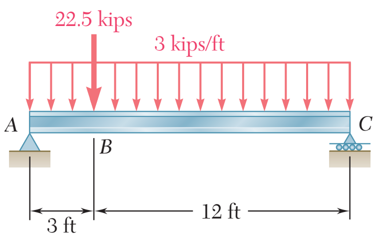
$$= 122.70 \text{ kip}\cdot\text{ft} = 1472.40 \text{ kip}\cdot\text{in.}$$

Maximum $|M|$ $\quad 122.7 \text{ kip}\cdot\text{ft}$ at $x = 6.50$ ft ◀

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1472.40}{24} = 61.35 \text{ in}^3$$

Shape	$S(\text{in}^3)$
W21 × 44	81.6
W18 × 50	88.9
W16 × 40	64.7 ←
W14 × 43	62.6
W12 × 50	64.2
W10 × 68	75.7

Answer: W16 × 40 ◀



PROBLEM 5.115

A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -15R_A + (7.5)(15)(3) + (12)(22.5) = 0$$

$$\mathbf{R}_A = 40.5 \text{ kips} \uparrow$$

$$w = 3 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 40.5 - 3x - 22.5\langle x-3\rangle^0 \text{ kips}$$

$$M = 40.5x - 1.5x^2 - 22.5\langle x-3\rangle^1 \text{ kip}\cdot\text{ft}$$

(*a*) Location of point D where $V = 0.$ Assume $3 \text{ ft} < x_D < 12 \text{ ft}.$

$$0 = 40.5 - 3x_D - 22.5 \qquad x_D = 6 \text{ ft}$$

At point D, $(x = 6 \text{ ft})$.

$$M = (40.5)(6) - (1.5)(6)^2 - (22.5)(3)$$
$$= 121.5 \text{ kip}\cdot\text{ft} = 1458 \text{ kip}\cdot\text{in.}$$

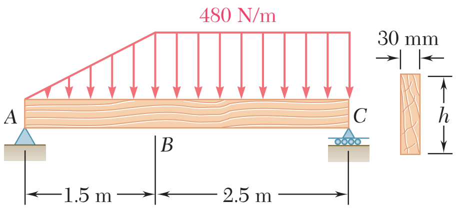
Maximum $|M|$: $|M|_{max} = 121.5 \text{ kip}\cdot\text{ft}$ at $x = 6.00$ ft ◀

$$S_{min} = \frac{M}{\sigma_{all}} = \frac{1458}{24} = 60.75 \text{ in}^3$$

(*b*)

Shape	$S\,(\text{in}^3)$	
W21×44	81.6	
W18×50	88.9	
W16×40	64.7	←
W14×43	62.6	
W12×50	64.2	
W10×68	75.7	

Wide-flange shape: W16×40 ◀



PROBLEM 5.116

A timber beam is being designed with supports and loads as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable normal stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$$480 \text{ N/m} = 0.48 \text{ kN/m}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -4R_A + (3)\left(\frac{1}{2}\right)(1.5)(0.48) + (1.25)(2.5)(0.48) = 0$$

$$\mathbf{R}_A = 0.645 \text{ kN} \uparrow$$

$$w = \frac{0.48}{1.5}x - \frac{0.48}{1.5}\langle x - 1.5\rangle^1 = 0.32x - 0.32\langle x-1.5\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.645 - 0.16x^2 + 0.16\langle x - 1.5\rangle^2 \text{ kN}$$

$$M = 0.645x - 0.053333x^3 + 0.053333\langle x-1.5\rangle^3 \text{ kN}\cdot\text{m}$$

(*a*) Locate point D where $V = 0$.

Assume $1.5 \text{ m} < x_D < 4$ m.

$$0 = 0.645 - 0.16x_D^2 + 0.16(x_D - 1.5)^2$$

$$= 0.645 - \cancel{0.16x_D^2} + \cancel{0.16x_D^2} - 0.48x_D + 0.36$$

$$x_D = 2.0938 \text{ m} \qquad x_D = 2.09 \text{ m} \blacktriangleleft$$

At point D, $\quad M_D = (0.645)(2.0938) - (0.053333)(2.0938)^3 + (0.053333)(0.59375)^3$

$$M_D = 0.87211$$

$$M_D = 0.872 \text{ kN}\cdot\text{m} \blacktriangleleft$$

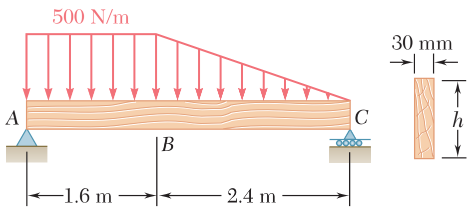
$$S_{\min} = \frac{M_D}{\sigma_{\text{all}}} = \frac{0.87211\times10^3}{12\times10^6} = 72.676\times10^{-6} \text{ m}^3 = 72.676\times10^3 \text{ mm}^3$$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}}$

$$h_{\min} = \sqrt{\frac{(6)(72.676\times10^3)}{30}} = 120.562 \text{ mm}$$

(*b*) At next larger 10-mm increment, $\qquad h = 130 \text{ mm} \blacktriangleleft$

PROBLEM 5.117



A timber beam is being designed with supports and loads as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth *h* varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$$500\text{ N/m} = 0.5\text{ kN/m}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -4R_A + (3.2)(1.6)(0.5) + (1.6)\left(\frac{1}{2}\right)(2.4)(0.5) = 0 \qquad \mathbf{R}_A = 0.880\text{ kN}\uparrow$$

$$w = 0.5 - \frac{0.5}{2.4}\langle x-1.6\rangle^1 = 0.5 - 0.20833\langle x-1.6\rangle^1\text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.880 - 0.5x + 0.104167\langle x-1.6\rangle^2\text{ kN}$$

$$V_A = 0.880\text{ kN}$$

$$\left.\begin{aligned} V_B &= 0.880 - (0.5)(1.6) = 0.080\text{ kN} \\ V_C &= 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520\text{ kN}\end{aligned}\right\}\ \text{Sign change}$$

Locate point *D* (between *B* and *C*) where $V = 0$. $\qquad 0 = 0.880 - 0.5x_D + 0.104167(x_D - 1.6)^2$

$$0.104167x_D^2 - 0.83333x_D + 1.14667 = 0$$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)} = \cancel{6.2342},\quad 1.7658\text{ m} \blacktriangleleft$$

$$M = 0.880x - 0.25x^2 + 0.347222\langle x-1.6\rangle^3\text{ kN}\cdot\text{m}$$

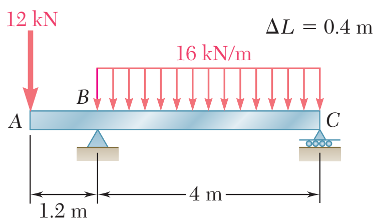
$$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.34722)(0.1658)^3 = 0.77597\text{ kN}\cdot\text{m}$$

(*a*) $\qquad M_{max} = 0.776\text{ kN}\cdot\text{m} \quad \text{at} \quad x = 1.766\text{ m} \blacktriangleleft$

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{0.77597\times 10^3}{12\times 10^6} = 64.664\times 10^{-6}\text{ m}^3 = 64.664\times 10^3\text{ mm}^3$$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}} \qquad h_{min} = \sqrt{\frac{(6)(64.664\times 10^3)}{30}} = 113.7\text{ mm}$

(*b*) At next higher 10-mm increment, $\qquad h = 120\text{ mm} \blacktriangleleft$



PROBLEM 5.118

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad (5.2)(12) - 4B + (2)(4)(16) = 0$$

$$\mathbf{B} = 47.6 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (1.2)(12) - (2)(4)(16) + 4C = 0$$

$$\mathbf{C} = 28.4 \text{ kN} \uparrow$$

$$w = 16\langle x - 1.2\rangle^0 = -\frac{dV}{dx}$$

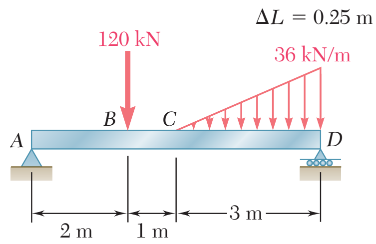
$$V = -16\langle x - 1.2\rangle^1 - 12 + 47.6\langle x - 1.2\rangle^0 \blacktriangleleft$$

$$M = -8\langle x - 1.2\rangle^2 - 12x + 47.6\langle x - 1.2\rangle^1 \blacktriangleleft$$

x	V	M
m	kN	kN · m
0.0	–12.0	0.00
0.4	–12.0	–4.80
0.8	–12.0	–9.60
1.2	35.6	–14.40
1.6	29.2	–1.44
2.0	22.8	8.96
2.4	16.4	16.80
2.8	10.0	22.08
3.2	3.6	24.80
3.6	–2.8	24.96
4.0	–9.2	22.56
4.4	–15.6	17.60
4.8	–22.0	10.08
5.2	–28.4	–0.00

$$|V|_{\max} = 35.6 \text{ kN} \blacktriangleleft$$

$$|M|_{\max} = 25.0 \text{ kN} \cdot \text{m} \blacktriangleleft$$



PROBLEM 5.119

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -6R_A + (4)(120) + (1)\left(\frac{1}{2}\right)(3)(36) = 0$$

$$R_A = 89 \text{ kN}$$

$$w = \frac{36}{3}\langle x-3\rangle^1 = 12\langle x-3\rangle^1$$

$$V = 89 - 120\langle x-2\rangle^0 - 6\langle x-3\rangle^2 \text{ kN} \quad \blacktriangleleft$$

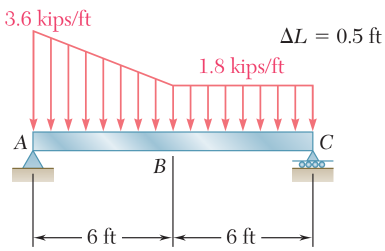
$$M = 89x - 120\langle x-2\rangle^1 - 2\langle x-3\rangle^3 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

x	V	M
m	kN	kN · m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	–31.0	178.0
2.3	–31.0	170.3
2.5	–31.0	162.5
2.8	–31.0	154.8
3.0	–31.0	147.0
3.3	–31.4	139.2
3.5	–32.5	131.3
3.8	–34.4	122.9
4.0	–37.0	114.0
4.3	–40.4	104.3
4.5	–44.5	93.8
4.8	–49.4	82.0

x	V	M
m	kN	kN · m
5.0	–55.0	69.0
5.3	–61.4	54.5
5.5	–68.5	38.3
5.8	–76.4	20.2
6.0	–85.0	–0.0

$$|V|_{max} = 89.0 \text{ kN} \quad \blacktriangleleft$$

$$|M|_{max} = 178.0 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$



PROBLEM 5.120

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -12R_A + (6)(12)(1.8) + (10)\left(\frac{1}{2}\right)(6)(1.8) = 0$$

$$R_A = 15.3 \text{ kips}$$

$$w = 3.6 - \frac{1.8}{6}x + \frac{1.8}{6}\langle x-6\rangle^1$$

$$= 3.6 - 0.3x + 0.3\langle x-6\rangle^1$$

$$V = 15.3 - 3.6x + 0.15x^2 - 0.15\langle x-6\rangle^2 \text{ kips} \blacktriangleleft$$

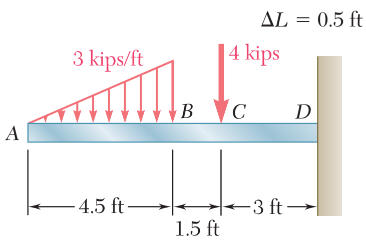
$$M = 15.3x - 1.8x^2 + 0.05x^3 - 0.05\langle x-6\rangle^3 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

x	V	M
ft	kips	kip · ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	–0.90	37.8
6.5	–1.80	37.1
7.0	–2.70	36.0
7.5	–3.60	34.4
8.0	–4.50	32.4
8.5	–5.40	29.9
9.0	–6.30	27.0

x	V	M
ft	kips	kip · ft
9.5	–7.20	23.6
10.0	–8.10	19.8
10.5	–9.00	15.5
11.0	–9.90	10.8
11.5	–10.80	5.6
12.0	–11.70	0.0

$$|V|_{max} = 15.30 \text{ kips} \blacktriangleleft$$

$$|M|_{max} = 38.0 \text{ kip}\cdot\text{ft} \blacktriangleleft$$



PROBLEM 5.121

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$w = \frac{3}{4.5}x - 3\langle x-4.5\rangle^0 - \frac{3}{4.5}\langle x-4.5\rangle^1$$

$$= \frac{2}{3}x - 3\langle x-4.5\rangle^0 - \frac{2}{3}\langle x-4.5\rangle^1 = -\frac{dV}{dx}$$

$$V = -\frac{1}{3}x^2 + 3\langle x-4.5\rangle^1 + \frac{1}{3}\langle x-4.5\rangle^2 - 4\langle x-6\rangle^0$$

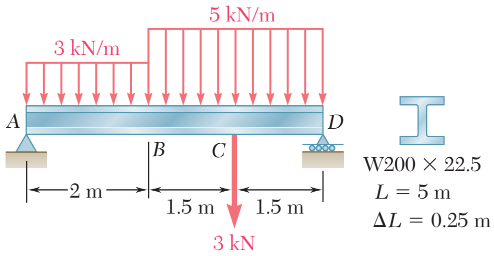
$$M = -\frac{1}{9}x^3 + \frac{3}{2}\langle x-4.5\rangle^2 + \frac{1}{9}\langle x-4.5\rangle^3 - 4\langle x-6\rangle^1$$

x	V	M
ft	kips	kip · ft
0.0	0.00	0.00
0.5	−0.08	−0.01
1.0	−0.33	−0.11
1.5	−0.75	−0.38
2.0	−1.33	−0.89
2.5	−2.08	−1.74
3.0	−3.00	−3.00
3.5	−4.08	−4.76
4.0	−5.33	−7.11
4.5	−6.75	−10.13
5.0	−6.75	−13.50
5.5	−6.75	−16.88
6.0	−10.75	−20.25
6.5	−10.75	−25.63
7.0	−10.75	−31.00
7.5	−10.75	−36.38
8.0	−10.75	−41.75
8.5	−10.75	−47.13
9.0	−10.75	−52.50

$|V|_{max} = 10.75$ kips ◀

$|M|_{max} = 52.5$ kip · ft ◀

PROBLEM 5.122



For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0:$$

$$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$$

$$R_A = 10.2 \text{ kN}$$

$$w = 3 + 2\langle x - 2\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 10.2 - 3x - 2\langle x - 2\rangle^1 - 3\langle x - 3.5\rangle^0 \text{ kN} \blacktriangleleft$$

$$M = 10.2x - 1.5x^2 - \langle x - 2\rangle^2 - 3\langle x - 3.5\rangle^1 \text{ kN}\cdot\text{m} \blacktriangleleft$$

(*b*) For rolled-steel section W200 × 22.5,

$$S = 193 \times 10^3 \text{ mm}^3 = 193 \times 10^{-6} \text{ m}^3$$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{16.164 \times 10^3}{193 \times 10^{-6}} = 83.8 \times 10^6 \text{ Pa}$$

$$\sigma = 83.8 \text{ MPa} \blacktriangleleft$$

x	V	M	σ
m	kN	kN · m	MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.73	24.5
0.75	7.95	6.81	35.3
1.00	7.20	8.70	45.1
1.25	6.45	10.41	53.9
1.50	5.70	11.93	61.8
1.75	4.95	13.26	68.7
2.00	4.20	14.40	74.6
2.25	2.95	15.29	79.2
2.50	1.70	15.88	82.3
2.75	0.45	16.14	83.6

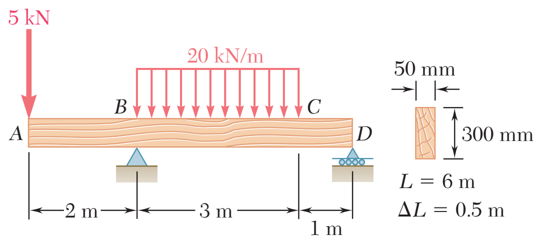
PROBLEM 5.122 *(Continued)*

x	V	M	σ	
m	kN	kN · m	MPa	
3.00	−0.80	16.10	83.4	
3.25	−2.05	15.74	81.6	
3.50	−6.30	15.08	78.1	
3.75	−7.55	13.34	69.1	
4.00	−8.80	11.30	58.5	
4.25	−10.05	8.94	46.3	
4.50	−11.30	6.28	32.5	
4.75	−12.55	3.29	17.1	
5.00	−13.80	0.00	0.0	
2.83	0.05	16.164	83.8	
2.84	0.00	16.164	83.8	←
2.85	−0.05	16.164	83.8	

(*a*)

$$|V|_{max} = 13.80 \text{ kN} \blacktriangleleft$$

$$|M|_{max} = 16.16 \text{ kN} \cdot \text{m} \blacktriangleleft$$



PROBLEM 5.123

For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (b) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -4R_B + (6)(5) + (2.5)(3)(20) = 0 \qquad R_B = 45 \text{ kN}$$

$$w = 20\langle x-2\rangle^0 - 20\langle x-5\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = -5 + 45\langle x-2\rangle^0 - 20\langle x-2\rangle^1 + 20\langle x-5\rangle^1 \text{ kN} \quad \blacktriangleleft$$

$$M = -5x + 45\langle x-2\rangle^1 - 10\langle x-2\rangle^2 + 10\langle x-5\rangle^2 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

(a)

x	V	M	stress	
m	kN	kN · m	MPa	
0.00	−5	0.00	0.0	
0.50	−5	−2.50	−3.3	
1.00	−5	−5.00	−6.7	
1.50	−5	−7.50	−10.0	
2.00	40	−10.00	−13.3	
2.50	30	7.50	10.0	
3.00	20	20.00	26.7	
3.50	10	27.50	36.7	
4.00	0	30.00	40.0	←
4.50	−10	27.50	36.7	
5.00	−20	20.00	26.7	
5.50	−20	10.00	13.3	
6.00	−20	0.00	0.0	

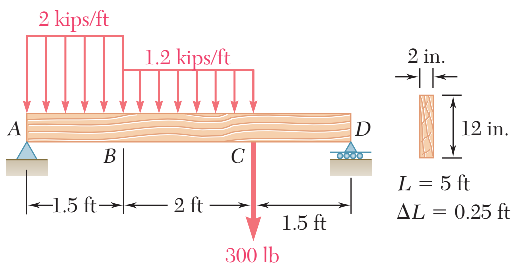
(b) Maximum $|M| = 30.0 \text{ kN}\cdot\text{m}$ at $x = 4.0$ m ◀

Maximum $|V| = 40.0$ kN ◀

For rectangular cross section, $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(50)(300)^2 = 750\times10^3 \text{ mm}^3 = 750\times10^{-6} \text{ m}^3$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{30\times10^3}{750\times10^{-6}} = 40\times10^6 \text{ Pa} \qquad \sigma_{\max} = 40.0 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 5.124



For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$300 \text{ lb} = 0.3 \text{ kips}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -5R_A + (4.25)(1.5)(2) + (2.5)(2)(1.2) + (1.5)(0.3) = 0$$

$$R_A = 3.84 \text{ kips}$$

$$w = 2 - 0.8\langle x - 1.5\rangle^0 - 1.2\langle x - 3.5\rangle^0 \text{ kip/ft}$$

$$V = 3.84 - 2x + 0.8\langle x - 1.5\rangle^1 + 1.2\langle x - 3.5\rangle^1 - 0.3\langle x - 3.5\rangle^0 \text{ kips} \quad \blacktriangleleft$$

$$M = 3.84x - x^2 + 0.4\langle x - 1.5\rangle^2 + 0.6\langle x - 3.5\rangle^2 - 0.3\langle x - 3.5\rangle^1 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

x	V	M	stress
ft	kips	kip · ft	ksi
0.00	3.84	0.00	0.000
0.25	3.34	0.90	0.224
0.50	2.84	1.67	0.417
0.75	2.34	2.32	0.579
1.00	1.84	2.84	0.710
1.25	1.34	3.24	0.809
1.50	0.84	3.51	0.877
1.75	0.54	3.68	0.921
2.00	0.24	3.78	0.945
2.25	−0.06	3.80	0.951
2.50	−0.36	3.75	0.937
2.75	−0.66	3.62	0.906
3.00	−0.96	3.42	0.855
3.25	−1.26	3.14	0.786
3.50	−1.86	2.79	0.697
3.75	−1.86	2.32	0.581

PROBLEM 5.124 *(Continued)*

x	V	M	stress
ft	kips	kip · ft	ksi
4.00	–1.86	1.86	0.465
4.25	–1.86	1.39	0.349
4.50	–1.86	0.93	0.232
4.75	–1.86	0.46	0.116
5.00	–1.86	–0.00	–0.000
2.10	0.12	3.80	0.949
2.20	0.00	3.80	0.951 ←
2.30	–0.12	3.80	0.949

Maximum $|M| = 3.804 \text{ kip} \cdot \text{ft} = 45.648 \text{ kip} \cdot \text{in.}$ at $x = 2.20$ ft ◀

Maximum $|V| = 3.84$ kip ◀

Rectangular section:

$$2 \text{ in.} \times 12 \text{ in.}$$

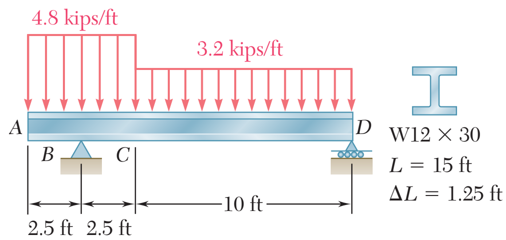
$$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(2)(12)^2$$

$$= 48 \text{ in}^3$$

$$\sigma = \frac{M}{S} = \frac{45.648}{48}$$

$\sigma = 0.951$ ksi ◀

PROBLEM 5.125



For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the *x* axis at end *A* of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -12.5 R_B + (12.5)(5.0)(4.8) + (5)(10)(3.2) = 0$$

$$R_B = 36.8 \text{ kips}$$

$$w = 4.8 - 1.6\langle x-5\rangle^0 \text{ kips/ft}$$

$$V = -4.8x + 36.8\langle x-2.5\rangle^0 + 1.6\langle x-5\rangle^1 \text{ kips} \quad ◀$$

$$M = -2.4x^2 + 36.8\langle x-2.5\rangle^1 + 0.8\langle x-5\rangle^2 \text{ kip} \cdot \text{ft} \quad ◀$$

x	V	M	stress
ft	kips	kip · ft	ksi
0.00	0.00	0.00	0.00
1.25	−6.0	−3.75	−1.17
2.50	24.8	−15.00	−4.66
3.75	18.8	12.25	3.81
5.00	12.8	32.00	9.95
6.25	8.8	45.50	14.15
7.50	4.8	54.00	16.79
8.75	0.8	57.50	17.88
10.00	−3.2	56.00	17.41
11.25	−7.2	49.50	15.39
12.50	−11.2	38.00	11.81
13.75	−15.2	21.50	6.68
15.00	−19.2	0.00	0.00
8.90	0.32	57.58	17.90
9.00	−0.00	57.60	17.91 ←
9.10	−0.32	57.58	17.90

PROBLEM 5.125 *(Continued)*

Maximum $|V| = 24.8$ kips ◀

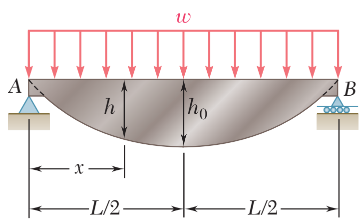
Maximum $|M| = 57.6 \text{ kip} \cdot \text{ft} = 691.2 \text{ kip} \cdot \text{in.}$ at $x = 9.0$ ft ◀

For rolled-steel section $W12 \times 30$,

$$S = 38.6 \text{ in}^3$$

Maximum normal stress:

$$\sigma = \frac{M}{S} = \frac{691.2}{38.6} \qquad \sigma = 17.91 \text{ ksi} \blacktriangleleft$$



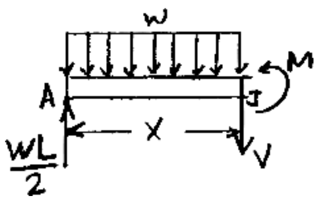
PROBLEM 5.126

The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wL = 0 \qquad\qquad R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{wL}{2}x - wx\frac{x}{2} + M = 0$$



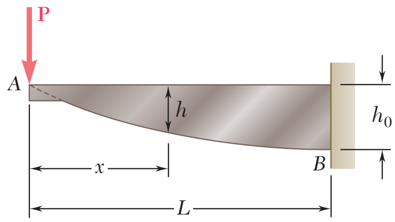
$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}} \qquad\qquad M = \frac{w}{2}x(L-x)$$

For a rectangular cross section, $\qquad S = \frac{1}{6}bh^2$

Equating, $\qquad \frac{1}{6}bh^2 = \frac{wx(L-x)}{2\sigma_{all}} \qquad\qquad h = \left\{\frac{3wx(L-x)}{\sigma_{all}b}\right\}^{1/2}$

(a) At $x = \frac{L}{2}$, $\qquad h = h_0 = \left\{\frac{3wL^2}{4\sigma_{all}b}\right\}^{1/2} \qquad\qquad h = h_0\left[\frac{x}{L}\left(1 - \frac{x}{L}\right)\right]^{1/2}$ ◀

(b) Solving for w, $\qquad w = \frac{4\sigma_{all}bh_0^2}{3L^2} = \frac{(4)(24)(1.25)(12)^2}{(3)(36)^2} \qquad\qquad w = 4.44$ kip/in. ◀



PROBLEM 5.127

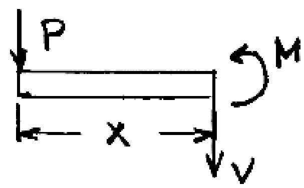
The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.

SOLUTION

$$V = -P$$

$$M = -Px \qquad |M| = Px$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P}{\sigma_{all}}x$$



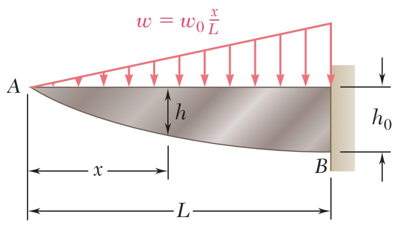
For a rectangular cross section, $$S = \frac{1}{6}bh^2$$

Equating, $$\frac{1}{6}bh^2 = \frac{Px}{\sigma_{all}} \qquad h = \left(\frac{6Px}{\sigma_{all}b}\right)^{1/2} \qquad (1)$$

At $x = L$, $$h = h_0 = \left\{\frac{6PL}{\sigma_{all}b}\right\}^{1/2} \qquad (2)$$

(*a*) Divide Eq. (1) by Eq. (2) and solve for h. $h = h_0(x/L)^{1/2}$ ◀

(*b*) Solving for P, $$P = \frac{\sigma_{all}bh_0^2}{6L} = \frac{(24)(1.25)(12)^2}{(6)(36)}$$ $P = 20.0$ kips ◀



PROBLEM 5.128

The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \qquad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w_0 x^3}{6L\sigma_{all}}$$

For a rectangular cross section, $\quad S = \frac{1}{6} bh^2$

Equating, $\quad \frac{1}{6} bh^2 = \frac{w_0 x^3}{6L\sigma_{all}} \qquad h = \sqrt{\frac{w_0 x^3}{\sigma_{all} bL}}$

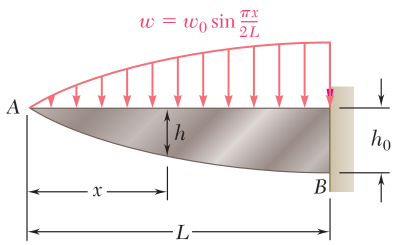
At $x = L$, $\quad h = h_0 = \sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$

(*a*) $\quad h = h_0 \left(\frac{x}{L}\right)^{3/2}$ ◀

Data: $\quad L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$

$w_0 = 300 \text{ kN/m} = 300 \times 10^3 \text{ N/m}, \quad \sigma_{all} = 200 \text{ MPa} = 200 \times 10^6 \text{ Pa}$

(*b*) $\quad h_0 = \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 167.7 \times 10^{-3} \text{ m} \qquad h_0 = 167.7 \text{ mm}$ ◀



PROBLEM 5.129

The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{\text{all}} = 200$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin\frac{\pi x}{2L}$$

$$V = \frac{2w_0 L}{\pi}\cos\frac{\pi x}{2L} + C_1$$

$$V = 0 \quad \text{at} \quad x = 0 \;\rightarrow\; C_1 = \frac{2w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi}\left(1 - \cos\frac{\pi x}{2L}\right)$$

$$M = -\frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right) \qquad |M| = \frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{L}\right)$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{2w_0 L}{\pi\sigma_{\text{all}}}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)$$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2$

Equating, $\quad \frac{1}{6}bh^2 = \frac{2w_0 L}{\pi\sigma_{\text{all}}}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)$

$$h = \left\{\frac{12 w_0 L}{\pi\sigma_{\text{all}} b}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)\right\}^{1/2}$$

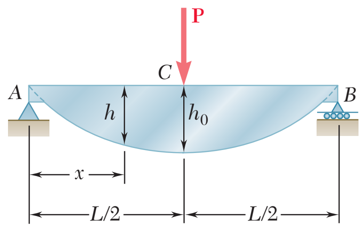
At $x = L$, $\quad h = h_0 = \left\{\frac{12 w_0 L^2}{\pi\sigma_{\text{all}} b}\left(1 - \frac{2}{\pi}\right)\right\}^{1/2} = 1.178\sqrt{\frac{w_0 L^2}{\sigma_{\text{all}} b}}$

(*a*) $\quad h = h_0\left[\left(\frac{x}{L} - \frac{2}{\pi}\sin\frac{\pi x}{2L}\right)\Big/\left(1 - \frac{2}{\pi}\right)\right]^{1/2} \qquad h = 1.659\, h_0\left[\frac{x}{L} - \frac{2}{\pi}\sin\frac{\pi x}{2L}\right]^{1/2}$ ◀

Data: $\quad L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$

$w_0 = 300 \text{ kN/m} = 300\times 10^3 \text{ N/m}, \quad \sigma_{\text{all}} = 200 \text{ MPa} = 200\times 10^6 \text{ Pa}$

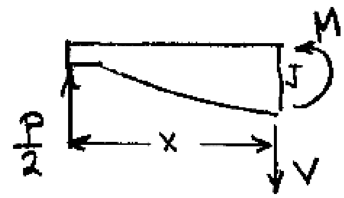
(*b*) $\quad h_0 = 1.178\sqrt{\frac{(300\times 10^3)(0.75)^2}{(200\times 10^6)(0.030)}} = 197.6\times 10^{-3} \text{ m} \qquad h_0 = 197.6 \text{ mm}$ ◀



PROBLEM 5.130

The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{\text{all}} = 72$ MPa.

SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{P}{2}\uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \qquad \left(0 < x < \frac{L}{2}\right)$$

$$S = \frac{M}{\sigma_{\text{all}}} = \frac{Px}{2\sigma_{\text{all}}}$$

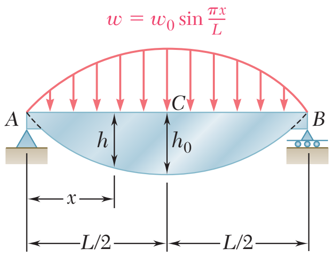
For a rectangular cross section, $\quad S = \dfrac{1}{6}bh^2$

Equating, $\quad \dfrac{1}{6}bh^2 = \dfrac{Px}{2\sigma_{\text{all}}} \qquad h = \sqrt{\dfrac{3Px}{\sigma_{\text{all}}b}}$

(a) At $x = \dfrac{L}{2}$, $\quad h = h_0 = \sqrt{\dfrac{3PL}{2\sigma_{\text{all}}b}} \qquad h = h_0\sqrt{\dfrac{2x}{L}}, \quad 0 < x < \dfrac{L}{2}$ ◀

For $x > \dfrac{L}{2}$, replace x by $L - x$.

(b) Solving for P, $\quad P = \dfrac{2\sigma_{\text{all}}bh_0^2}{3L} = \dfrac{(2)(72\times10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60.0\times10^3$ N $\qquad P = 60.0$ kN ◀



PROBLEM 5.131

The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin\frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = \frac{w_0 L}{\pi}\cos\frac{\pi x}{L} + C_1$$

$$M = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L} + C_1 x + C_2$$

At A, $x = 0$ $M = 0$ $\qquad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

At B, $x = L$ $M = 0$ $\qquad 0 = \dfrac{w_0 L^2}{\pi^2}\sin\pi + C_1 L \qquad C_1 = 0$

$$M = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L}$$

For constant strength, $\qquad S = \dfrac{|M|}{\sigma_{all}} = \dfrac{w_0 L^2}{\pi^2 \sigma_{all}}\sin\dfrac{\pi x}{L}$

For a rectangular section, $\qquad I = \dfrac{1}{12}bh^3, \quad c = \dfrac{h}{2}, \quad S = \dfrac{I}{C} = \dfrac{1}{6}bh^2$

Equating the two expressions for S, $\qquad \dfrac{1}{6}bh^2 = \dfrac{w_0 L^2}{\pi^2 \sigma_{all}}\sin\dfrac{\pi x}{L} \qquad (1)$

At $x = \dfrac{L}{2}$, $\quad h = h_0 \qquad \dfrac{1}{6}bh_0^2 = \dfrac{w_0 L^2}{\pi^2 \sigma_{all}} \qquad (2)$

(*a*) Dividing Eq. (1) by Eq. 2, $\qquad \dfrac{h^2}{h_0^2} = \sin\dfrac{\pi x}{L} \qquad h = h_0\left(\sin\dfrac{\pi x}{L}\right)^{\frac{1}{2}}$ ◀

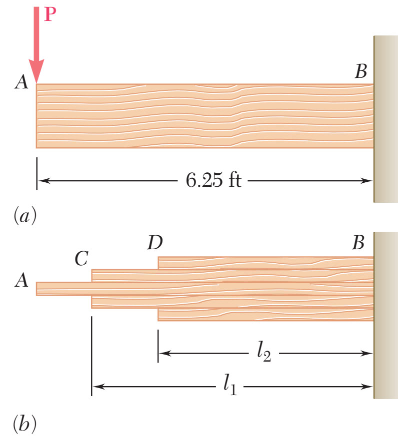
(*b*) Solving Eq. (1) for w_0, $\qquad w_0 = \dfrac{\pi^2 \sigma_{all} b h_0^2}{6L^2}$

Data: $\qquad \sigma_{all} = 72 \times 10^6$ Pa, $\quad L = 800 \text{ mm} = 0.800$ m, $\quad h_0 = 200 \text{ mm} = 0.200$ m,

$b = 25 \text{ mm} = 0.025$ m

$$w_0 = \frac{\pi^2 (72\times 10^6)(0.025)(0.200)^2}{(6)(0.800)^2} = 185.1\times 10^3 \text{ N/m}$$

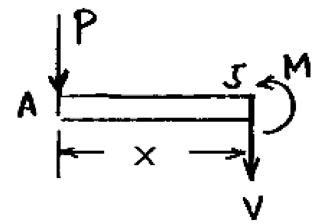
185.1 kN/m ◀



PROBLEM 5.132

A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2×2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION



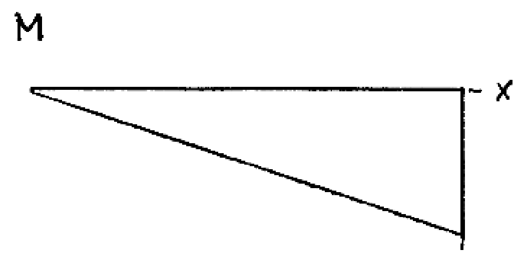
$$+\circlearrowleft \Sigma M_J = 0: \quad Px + M = 0 \qquad M = -Px$$

$$|M| = Px$$

At B, $\quad |M|_B = M_{\max}$

At C, $\quad |M|_C = M_{\max} x_C/6.25$

At D, $\quad |M|_D = M_{\max} x_D/6.25$



$$S_B = \frac{1}{6}bh^2 = \frac{1}{6}\cdot b(5b)^2 = \frac{25}{6}b^3$$

A to C: $\quad S_C = \frac{1}{6}\cdot b(b)^2 = \frac{1}{6}b^3$

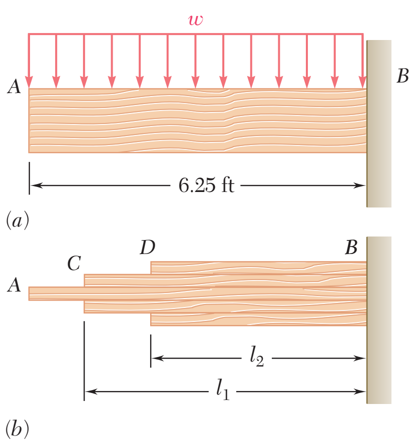
C to D: $\quad S_D = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

$$\frac{|M|_C}{|M|_B} = \frac{x_C}{6.25} = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{(1)(6.25)}{25} = 0.25 \text{ ft}$$

$$l_1 = 6.25 - 0.25 \qquad l_1 = 6.00 \text{ ft} \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \frac{x_D}{6.25} = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{(9)(6.25)}{25} = 2.25 \text{ ft}$$

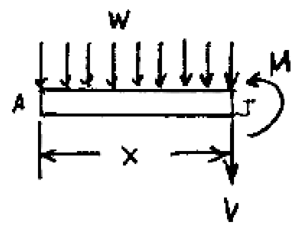
$$l_2 = 6.25 - 2.25 \qquad l_2 = 4.00 \text{ ft} \blacktriangleleft$$



PROBLEM 5.133

A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2×2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION



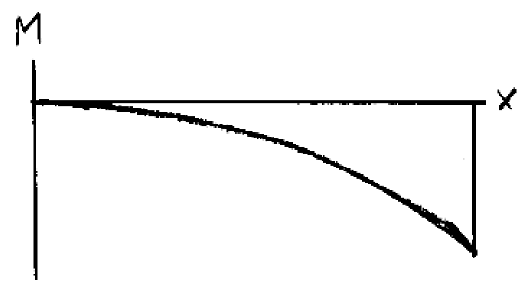
$$+\circlearrowleft \Sigma M_J = 0: \quad wx\frac{x}{2} + M = 0$$

$$M = -\frac{wx^2}{2} \qquad |M| = \frac{wx^2}{2}$$

At B, $\quad |M|_B = |M|_{\max}$

At C, $\quad |M|_C = |M|_{\max}\,(x_C/6.25)^2$

At D, $\quad |M|_D = |M|_{\max}\,(x_D/6.25)^2$



At B, $\quad S_B = \frac{1}{6}bh^2 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$

A to C: $\quad S_C = \frac{1}{6}bh^2 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$

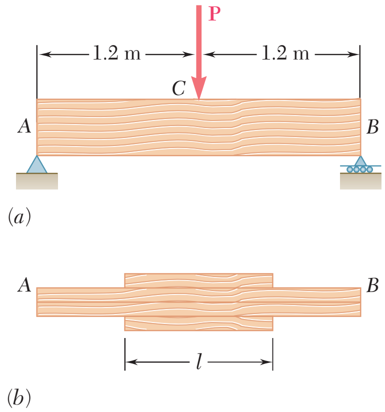
C to D: $\quad S_D = \frac{1}{6}bh^2 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

$$\frac{|M|_C}{|M|_B} = \left(\frac{x_C}{6.25}\right)^2 = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{6.25}{\sqrt{25}} = 1.25 \text{ ft}$$

$$l_1 = 6.25 - 1.25 \text{ ft} \qquad l_1 = 5.00 \text{ ft} \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{6.25}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{6.25\sqrt{9}}{\sqrt{25}} = 3.75 \text{ ft}$$

$$l_2 = 6.25 - 3.75 \text{ ft} \qquad l_2 = 2.50 \text{ ft} \blacktriangleleft$$



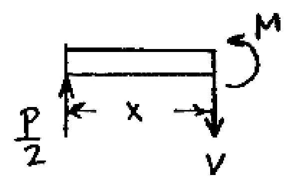
PROBLEM 5.134

A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50×50-mm cross section. Determine the length l of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{P}{2}$$

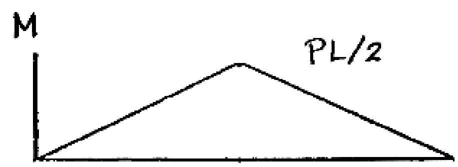
$$0 < x < \frac{1}{2}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \quad \text{or} \quad M = \frac{M_{\max} x}{1.2}$$

Bending moment diagram is two straight lines.



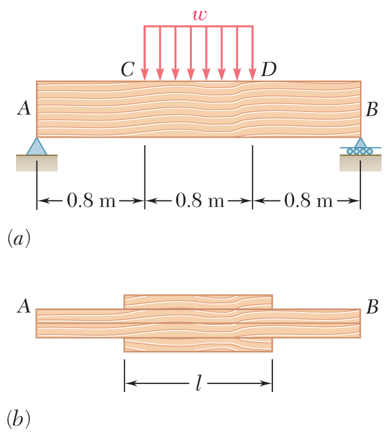
At C, $\quad S_C = \frac{1}{6} b h_C^2 \qquad M_C = M_{\max}$

Let D be the point where the thickness changes.

At D, $\quad S_D = \frac{1}{6} b h_D^2 \qquad M_D = \frac{M_{\max} x_D}{1.2}$

$$\frac{S_D}{S_C} = \frac{h_D^2}{h_C^2} = \left(\frac{100 \text{ mm}}{200 \text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_D}{M_C} = \frac{x_D}{1.2} \qquad x_D = 0.3 \text{ m}$$

$$\frac{l}{2} = 1.2 - x_D = 0.9 \qquad l = 1.800 \text{ m} \blacktriangleleft$$



PROBLEM 5.135

A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50×50-mm cross section. Determine the length l of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{0.8\ \text{N}}{2} = 0.4w$$

Shear:

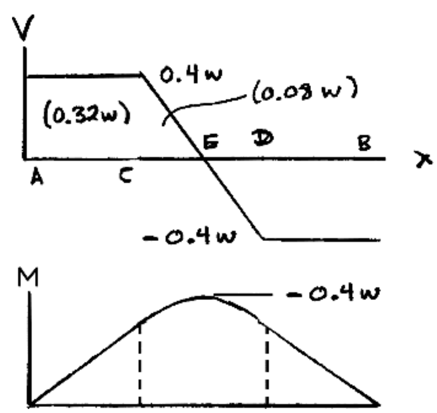
A to C: $\quad V = 0.4w$

D to B: $\quad V = -0.4w$

Areas:

A to C: $\quad (0.8)(0.4)w = 0.32w$

C to E: $\quad \left(\frac{1}{2}\right)(0.4)(0.4)w = 0.08w$



Bending moments:

At C, $\quad M_C = 0.40w$

A to C: $\quad M = 0.40wx$

At C, $\quad S_C = \frac{1}{6}bh_C^2 \qquad M_C = M_{\max} = 0.40w$

Let F be the point where the thickness changes.

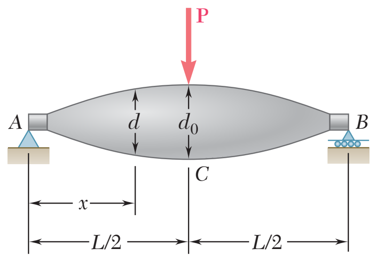
At F, $\quad S_F = \frac{1}{6}bh_F^2 \qquad M_F = 0.40wx_F$

$$\frac{S_F}{S_C} = \frac{h_F^2}{h_C^2} = \left(\frac{100\ \text{mm}}{200\ \text{mm}}\right)^2 = \frac{1}{4} = \frac{M_F}{M_C} = \frac{0.40wx_F}{0.40w}$$

$$x_F = 0.25\ \text{m} \qquad \frac{l}{2} = 1.2 - x_F = 0.95\ \text{m}$$

$l = 1.900$ m ◀

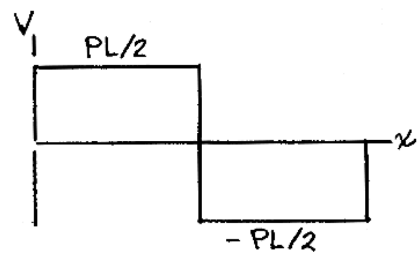
PROBLEM 5.136

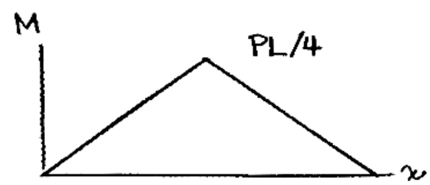


A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

SOLUTION

Draw shear and bending moment diagrams.





$$0 \le x \le \frac{L}{2}, \qquad M = \frac{Px}{2}$$

$$\frac{L}{2} \le x \le L, \qquad M = \frac{P(L-x)}{2}$$

For a solid circular section, $c = \frac{1}{2}d$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^4 \qquad S = \frac{I}{c} = \frac{\pi}{32}d^3$$

For constant strength design, $\sigma = \text{constant}$. $\qquad S = \dfrac{M}{\sigma}$

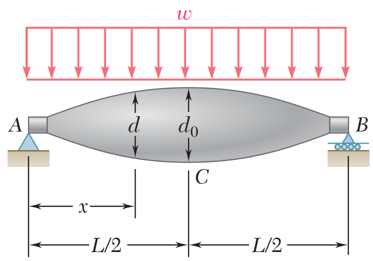
For $\qquad 0 \le x \le \frac{L}{2}, \qquad \frac{\pi}{32}d^3 = \frac{Px}{2} \qquad$ (1a)

For $\qquad \frac{L}{2} \le x \le L, \qquad \frac{\pi}{32}d^3 = \frac{P(L-x)}{2} \qquad$ (1b)

At point C, $\qquad \frac{\pi}{32}d_0^3 = \frac{PL}{4} \qquad$ (2)

Dividing Eq. (1a) by Eq. (2), $\qquad 0 \le x \le \frac{L}{2}, \qquad \frac{d^3}{d_0^3} = \frac{2x}{L} \qquad d = d_0(2x/L)^{1/3}$ ◀

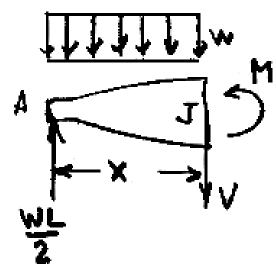
Dividing Eq. (1b) by Eq. (2), $\qquad \frac{L}{2} \le x \le L, \qquad \frac{d^3}{d_0^3} = \frac{2(L-x)}{L} \qquad d = d_0[2(L-x)/L]^{1/3}$ ◀



PROBLEM 5.137

A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

SOLUTION



$$R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

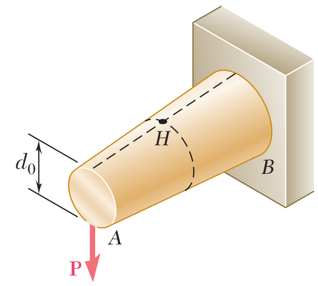
$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{wx(L-x)}{2\sigma_{\text{all}}}$$

For a solid circular cross section, $\quad c = \dfrac{d}{2} \qquad I = \dfrac{\pi}{4}c^3 \qquad S = \dfrac{I}{c} = \dfrac{\pi d^3}{32}$

Equating, $\quad \dfrac{\pi d^3}{32} = \dfrac{wx(L-x)}{2\sigma_{\text{all}}} \qquad d = \left\{\dfrac{16\,wx(L-x)}{\pi\sigma_{\text{all}}}\right\}^{1/3}$

At $x = \dfrac{L}{2}$, $\quad d = d_0 = \left\{\dfrac{4wL^2}{\pi\sigma_{\text{all}}}\right\}^{1/3}$

$$d = d_0\left\{4\frac{x}{L}\left(1 - \frac{x}{L}\right)\right\}^{1/3} \blacktriangleleft$$



PROBLEM 5.138

A transverse force **P** is applied as shown at end A of the conical taper AB. Denoting by d_0 the diameter of the taper at A, show that the maximum normal stress occurs at point H, which is contained in a transverse section of diameter $d = 1.5d_0$.

SOLUTION

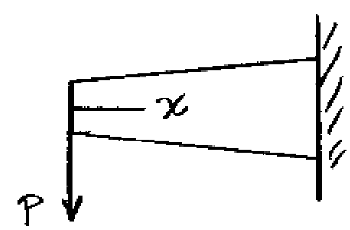
$$V = -P = \frac{dM}{dx} \qquad M = -Px$$

Let $$d = d_0 + kx$$

For a solid circular section, $$I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^3$$

$$c = \frac{d}{2} \qquad S = \frac{I}{c} = \frac{\pi}{32}d^3 = \frac{\pi}{32}(d_0 + kx)^3$$

$$\frac{dS}{dx} = \frac{3\pi}{32}(d_0 + kx)^2 k = \frac{3\pi}{32}d^2 k$$



Stress: $$\sigma = \frac{|M|}{S} = \frac{Px}{S}$$

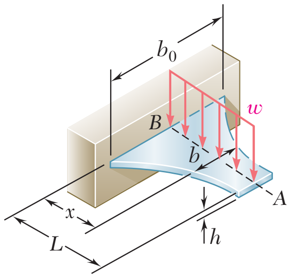
At H, $$\frac{d\sigma}{dx} = \frac{1}{S^2}\left(PS - Px_H\frac{dS}{dx}\right) = 0$$

$$S - x_H\frac{dS}{dx} = \frac{\pi}{32}d^3 - x_H\frac{3\pi}{32}d^2 k$$

$$kx_H = \frac{1}{3}d = \frac{1}{3}(d_0 + k_H x_H) \qquad kx_H = \frac{1}{2}d_0$$

$$d = d_0 + \frac{1}{2}d_0 = \frac{3}{2}d_0$$

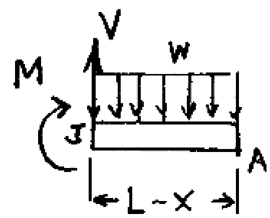
$d = 1.5d_0$ ◀



PROBLEM 5.139

A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support the distributed load w along its centerline AB. (a) Knowing that the beam is to be of constant strength, express b in terms of x, L, and b_0. (b) Determine the maximum allowable value of w if $L = 15$ in., $b_0 = 8$ in., $h = 0.75$ in., and $\sigma_{\text{all}} = 24$ ksi.

SOLUTION



$$+\circlearrowleft \Sigma M_J = 0: \quad -M - w(L-x)\frac{L-x}{2} = 0$$

$$M = -\frac{w(L-x)^2}{2} \qquad |M| = \frac{w(L-x)^2}{2}$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{w(L-x)^2}{2\sigma_{\text{all}}}$$

For a rectangular cross section, $\quad S = \dfrac{1}{6}bh^2$

$$\frac{1}{6}bh^2 = \frac{w(L-x)^2}{2\sigma_{\text{all}}} \qquad b = \frac{3w(L-x)^2}{\sigma_{\text{all}}h^2}$$

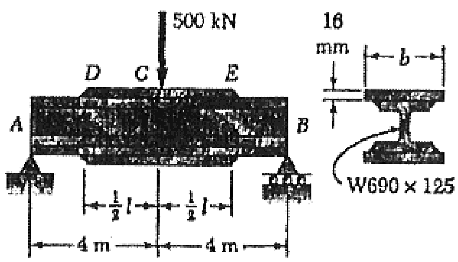
(a) At $\quad x = 0, \quad b = b_0 = \dfrac{3wL^2}{\sigma_{\text{all}}h^2}$ $\qquad b = b_0\left(1 - \dfrac{x}{L}\right)^2$ ◀

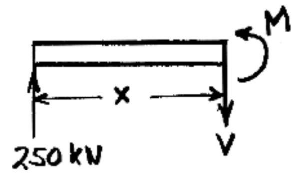
(b) Solving for w, $\quad w = \dfrac{\sigma_{\text{all}}b_0h^2}{3L^2} = \dfrac{(24)(8)(0.75)^2}{(3)(15)^2} = 0.160$ kip/in. $\qquad w = 160.0$ lb/in. ◀

PROBLEM 5.140

Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively, $l = 4$ m and $b = 285$ mm, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION





$A = B = 250$ kN ↑

$$+\circlearrowleft \Sigma M_J = 0: \quad -250x + M = 0$$

$$M = 250x \text{ kN} \cdot \text{m}$$

At center of beam, $\quad x = 4 \text{ m} \quad M_C = (250)(4) = 1000 \text{ kN} \cdot \text{m}$

At *D*, $\quad x = \frac{1}{2}(8 - l) = \frac{1}{2}(8 - 4) = 2 \text{ m} \qquad M_0 = 500 \text{ kN} \cdot \text{m}$

At center of beam, $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 1190 \times 10^6 + 2\left\{(285)(16)\left(\frac{678}{2} + \frac{16}{2}\right)^2 + \frac{1}{12}(285)(16)^3\right\}$$

$$= 2288 \times 10^6 \text{ mm}^4$$

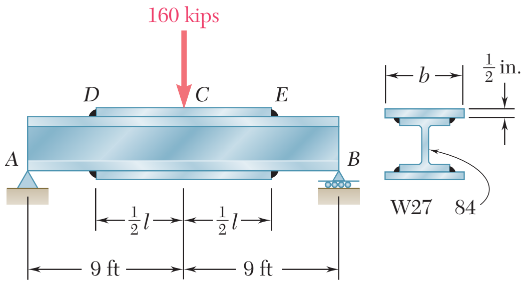
$$c = \frac{678}{2} + 16 = 355 \text{ mm} \qquad S = \frac{I}{c} = 6445 \times 10^3 \text{mm}^3$$

$$= 6445 \times 10^{-6} \text{m}^3$$

(*a*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{1000 \times 10^3}{6445 \times 10^{-6}} = 155.2 \times 10^6 \text{ Pa} \qquad \sigma = 155.2 \text{ MPa}$ ◀

At *D*, $\quad S = 3490 \times 10^3 \text{mm}^3 = 3510 \times 10^{-6} \text{m}^3$

(*b*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{500 \times 10^3}{3490 \times 10^{-6}} = 143.3 \times 10^6 \text{ Pa} \qquad \sigma = 143.3 \text{ MPa}$ ◀

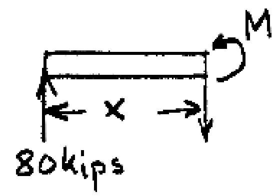


PROBLEM 5.141

Two cover plates, each $\frac{1}{2}$ in. thick, are welded to a W27 × 84 beam as shown. Knowing that $l = 10$ ft and $b = 10.5$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION

$$\mathbf{R}_A = \mathbf{R}_B = 80 \text{ kips} \uparrow$$



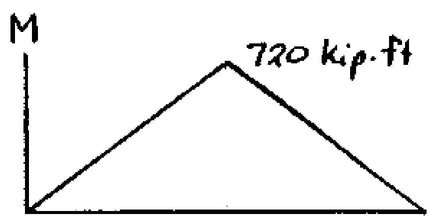
$$+\circlearrowleft \Sigma M_J = 0: \quad -80x + M = 0$$

$$M = 80x \text{ kip} \cdot \text{ft}$$

At *C*, $\quad x = 9 \text{ ft} \quad M_C = 720 \text{ kip} \cdot \text{ft} = 8640 \text{ kip} \cdot \text{in.}$

At *D*, $\quad x = 9 - 5 = 4 \text{ ft}$

$$M_D = (80)(4) = 320 \text{ kip} \cdot \text{ft} = 3840 \text{ kip} \cdot \text{in.}$$



At center of beam, $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$I = 2850 + 2\left\{(10.5)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(10.5)(0.500)^3\right\}$$

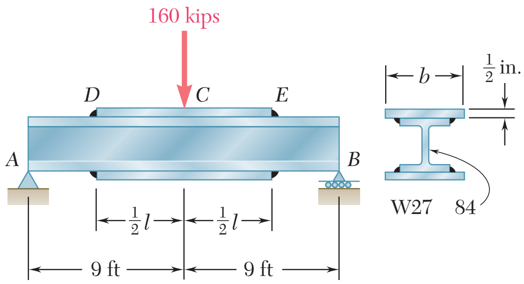
$$= 4794 \text{ in}^3$$

$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in.}$$

(*a*) Normal stress: $\quad \sigma = \dfrac{Mc}{I} = \dfrac{(8640)(13.855)}{4794} \qquad \sigma = 25.0 \text{ ksi} \blacktriangleleft$

At point *D*, $\quad S = 213 \text{ in}^3$

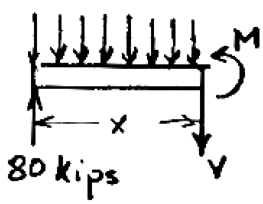
(*b*) Normal stress: $\quad \sigma = \dfrac{M}{S} = \dfrac{3840}{213} \qquad \sigma = 18.03 \text{ ksi} \blacktriangleleft$



PROBLEM 5.142

Two cover plates, each $\frac{1}{2}$ in. thick, are welded to a W27 × 84 beam as shown. Knowing that $\sigma_{\text{all}} = 24$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

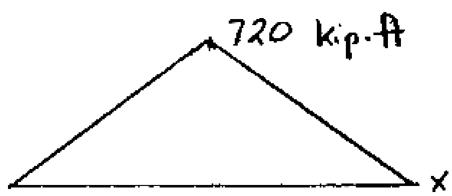
SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = 80 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -80x + M = 0$$

$$M = 80x \text{ kip} \cdot \text{ft}$$



At *D*, $\qquad S = 213 \text{ in}^3$

Allowable bending moment:

$$M_{\text{all}} = \sigma_{\text{all}} S = (24)(213) = 5112 \text{ kip} \cdot \text{in.}$$

$$= 426 \text{ kip} \cdot \text{ft}$$

Set $M_D = M_{\text{all}}$. $\qquad 80x_D = 426 \qquad x_D = 5.325$ ft

(*a*) $\qquad l = 18 - 2x_D \qquad\qquad l = 7.35$ ft ◀

At center of beam, $\qquad M = (80)(9) = 720 \text{ kip} \cdot \text{ft} = 8640 \text{ kip} \cdot \text{in.}$

$$S = \frac{M}{\sigma_{\text{all}}} = \frac{8640}{24} = 360 \text{ in}^3$$

$$c = \frac{26.7}{2} + 0.500 = 13.85 \text{ in.}$$

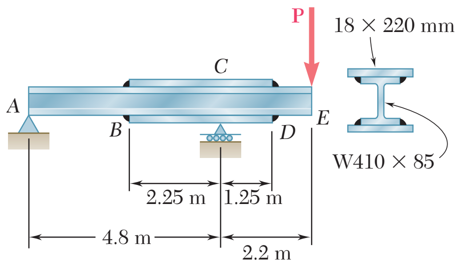
Required moment of inertia: $\qquad I = Sc = 4986 \text{ in}^4$

But $\qquad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$4986 = 2850 + 2\left\{(b)(0.500)\left(\frac{26.7}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(b)(0.500)^3\right\}$$

$$= 2850 + 184.981b$$

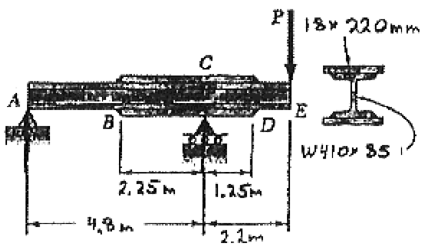
(*b*) $\qquad\qquad b = 11.55$ in. ◀



PROBLEM 5.143

Knowing that $\sigma_{\text{all}} = 150$ MPa, determine the largest concentrated load **P** that can be applied at end E of the beam shown.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad -4.8A - 2.2P = 0$$

$$A = -0.45833P \qquad \mathbf{A} = 0.45833P \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 4.8D - 7.0P = 0$$

$$\mathbf{D} = 1.45833P \uparrow$$

Shear: A to C: $V = -0.45833P$

C to E: $V = P$

Bending moments: $M_A = 0$

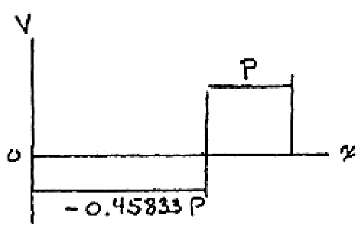
$$M_C = 0 + (4.8)(-0.45833P) = -2.2P$$

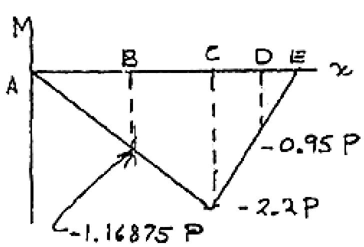
$$M_E = -2.2P + 2.2P = 0$$

$$M_B = \left(\frac{4.8 - 2.25}{48}\right)(-2.2P) = -1.16875P$$

$$M_D = \left(\frac{2.2 - 1.25}{2.2}\right)(-2.2P) = -0.95P$$

$$|M_D| < |M_B|$$





For W410 × 85, $S = 1510 \times 10^3\ \text{mm}^3 = 1510 \times 10^{-6}\ \text{m}^3$

Allowable value of P based on strength at B. $\sigma = \dfrac{|M_B|}{S}$

$$150 \times 10^6 = \frac{1.16875P}{1510 \times 10^{-6}} \qquad P = 193.8 \times 10^3\ \text{N}$$

Section properties over portion *BCD*:

$$\text{W}410\times 85: \quad d = 417 \text{ mm}, \quad \frac{1}{2}d = 208.5 \text{ mm}, \quad I_x = 316\times 10^6 \text{mm}^4$$

Plate:
$$A = (18)(220) = 3960 \text{ mm}^2 \qquad d = 208.5 + \left(\frac{1}{2}\right)(18) = 217.5 \text{ mm}$$

$$\bar{I} = \frac{1}{12}(220)(18)^3 = 106.92\times 10^3 \text{ mm}^4 \quad Ad^2 = 187.333\times 10^6 \text{ mm}^4$$

$$I_x = \bar{I} + Ad^2 = 187.440\times 10^6 \text{ mm}^4$$

For section,
$$I = 316\times 10^6 + (2)(187.440\times 10^6) = 690.88\times 10^6 \text{ mm}^4$$

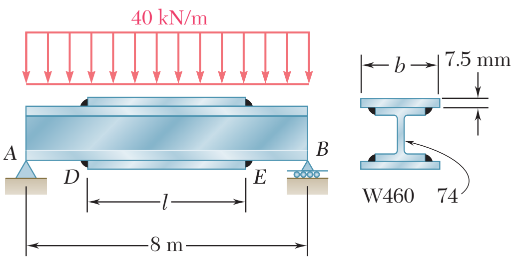
$$c = 208.5 + 18 = 226.5 \text{ mm}$$

$$S = \frac{I}{c} = \frac{690.88\times 10^6}{226.5} = 3050.2\times 10^3 \text{mm}^3 = 3050.2\times 10^{-6} \text{ m}^3$$

Allowable load based on strength at *C*: $\sigma = \dfrac{|M_C|}{S}$

$$150\times 10^6 = \frac{2.2P}{3050.2\times 10^{-6}} \qquad P = 208.0\times 10^3 \text{ N}$$

The smaller allowable load controls. $P = 193.8\times 10^3 \text{N}$ $\qquad P = 193.8 \text{ kN}$ ◀

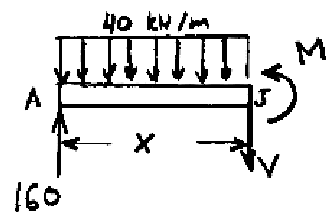


PROBLEM 5.144

Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $l = 5$ m and $b = 200$ mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 160 \text{ kN} \uparrow$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -160x + (40x)\frac{x}{2} + M = 0$$

$$M = 160x - 20x^2 \text{ kN} \cdot \text{m}$$

At center of beam, $\quad x = 4 \text{ m} \qquad M_C = 320 \text{ kN} \cdot \text{m}$

At *D*, $\quad x = \frac{1}{2}(8 - l) = 1.5 \text{ m} \qquad M_D = 195 \text{ kN} \cdot \text{m}$

At center of beam, $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 333 \times 10^6 + 2\left\{(200)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(200)(7.5)^3\right\}$$

$$= 494.8 \times 10^6 \text{ mm}^4$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm}$$

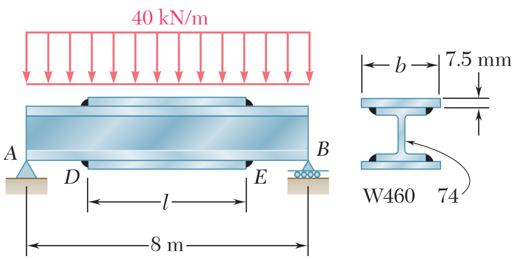
$$S = \frac{I}{c} = 2097 \times 10^3 \text{ mm}^3 = 2097 \times 10^{-6} \text{ m}^3$$

(*a*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{320 \times 10^3}{2097 \times 10^{-6}} = 152.6 \times 10^6 \text{ Pa}$

$\sigma = 152.6 \text{ MPa}$ ◀

At *D*, $\quad S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{195 \times 10^3}{1460 \times 10^{-6}} = 133.6 \times 10^6 \text{ Pa} \qquad \sigma = 133.6 \text{ MPa}$ ◀

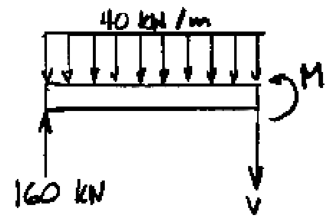


PROBLEM 5.145

Two cover plates, each 7.5 mm thick, are welded to a $W460 \times 74$ beam as shown. Knowing that $\sigma_{all} = 150$ MPa for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 160 \text{ kN} \uparrow$$

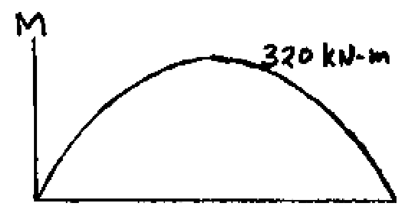


$$+\circlearrowleft \Sigma M_J = 0: \quad -160x + (40x)\left(\frac{x}{2}\right) + M = 0$$

$$M = 160x - 20x^2 \text{ kN} \cdot \text{m}$$

For $W460 \times 74$ rolled-steel beam,

$$S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$$



Allowable bending moment:

$$M_{all} = \sigma_{all} S = (150 \times 10^6)(1460 \times 10^{-6})$$
$$= 219 \times 10^3 \text{ N} \cdot \text{m} = 219 \text{ kN} \cdot \text{m}$$

To locate points D and E, set $\quad M = M_{all}$

$$160x - 20x^2 = 219 \qquad 20x^2 - 160x + 219 = 0$$

$$x = \frac{160 \pm \sqrt{160^2 - (4)(20)(219)}}{(2)(20)} \qquad x = 1.753 \text{ m and } x = 6.247 \text{ m}$$

(*a*) $\quad x_D = 1.753$ ft $\quad x_E = 6.247$ ft $\qquad l = x_E - x_D = 4.49 \text{ m}$ ◀

At center of beam, $\quad M = 320 \text{ kN} \cdot \text{m} = 320 \times 10^3 \text{N} \cdot \text{m} \quad c = \frac{457}{2} + 7.5 = 236 \text{ mm}^4$

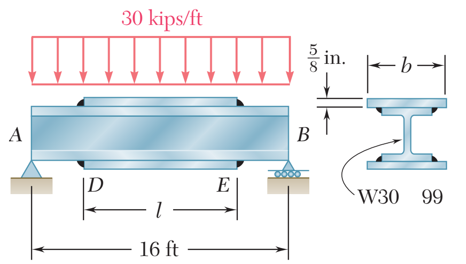
$$S = \frac{M}{\sigma_{all}} = \frac{320 \times 10^3}{150 \times 10^6} = 2133 \times 10^{-6} \text{m}^3 = 2133 \times 10^3 \text{mm}^3$$

Required moment of inertia: $\quad I = Sc = 503.4 \times 10^6 \text{ mm}^4$

But $\quad I = I_{beam} + 2I_{plate}$

$$503.4 \times 10^6 = 333 \times 10^6 + 2\left\{(b)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(b)(7.5)^3\right\}$$

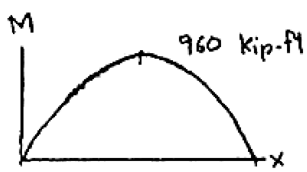
(*b*) $\qquad = 333 \times 10^6 + 809.2 \times 10^3 b \qquad b = 211 \text{ mm}$ ◀



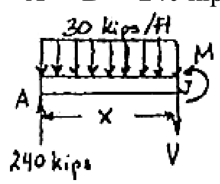
PROBLEM 5.146

Two cover plates, each $\frac{5}{8}$ in. thick, are welded to a W30 × 99 beam as shown. Knowing that $l = 9$ ft and $b = 12$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION



$\boldsymbol{A} = \boldsymbol{B} = 240 \text{ kips} \uparrow$



$$+\circlearrowleft \Sigma M_J = 0: \quad -240x + 30x\frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip} \cdot \text{ft}$$

At center of beam, $\quad x = 8 \text{ ft}$

$$M_C = 960 \text{ kip} \cdot \text{ft} = 11{,}520 \text{ kip} \cdot \text{in.}$$

At point D, $\quad x = \frac{1}{2}(16 - 9) = 3.5 \text{ ft}$

$$M_D = 656.25 \text{ kip} \cdot \text{ft} = 7875 \text{ kip} \cdot \text{in.}$$

At center of beam, $\quad I = I_{\text{beam}} + 2I_{\text{plate}}$

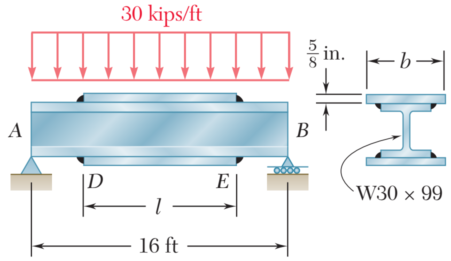
$$I = 3990 + 2\left\{(12)(0.625)\left(\frac{29.7}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(12)(0.625)^3\right\} = 7439 \text{ in}^4$$

$$c = \frac{29.7}{2} + 0.625 = 15.475 \text{ in.}$$

(*a*) Normal stress: $\quad \sigma = \frac{Mc}{I} = \frac{(11{,}520)(15.475)}{7439} \qquad \sigma = 24.0 \text{ ksi}$ ◀

At point D, $\quad S = 269 \text{ in}^3$

(*b*) Normal stress: $\quad \sigma = \frac{M}{S} = \frac{7875}{269} \qquad \sigma = 29.3 \text{ ksi}$ ◀

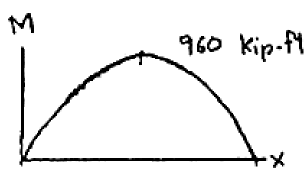


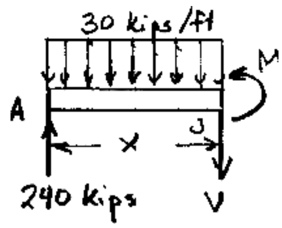
PROBLEM 5.147

Two cover plates, each $\frac{5}{8}$ in. thick, are welded to a W30 × 99 beam as shown. Knowing that $\sigma_{all} = 22$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 240 \text{ kips} \uparrow$$





$$+\circlearrowleft \Sigma M_J = 0: \quad -240x + 30x\frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip} \cdot \text{ft}$$

For W30 × 99 rolled-steel section, $S = 269 \text{ in}^3$

Allowable bending moment:

$$M_{all} = \sigma_{all} S = (22)(269) = 5918 \text{ kip} \cdot \text{in.} = 493.167 \text{ kip} \cdot \text{ft}$$

To locate points D and E, set $M = M_{all}$.

$$240x - 15x^2 = 493.167 \qquad 15x^2 - 240x + 493.167 = 0$$

$$x = \frac{240 \pm \sqrt{(240)^2 - (4)(15)(493.167)}}{(2)(15)} = 2.42 \text{ ft}, \quad 13.58 \text{ ft}$$

(*a*) $\quad l = x_E - x_D = 13.58 - 2.42$ $\qquad\qquad l = 11.16 \text{ ft} \blacktriangleleft$

Center of beam: $\quad M = 960 \text{ kip} \cdot \text{ft} = 11{,}520 \text{ kip} \cdot \text{in.}$

$$S = \frac{M}{\sigma_{all}} = \frac{11{,}520}{22} = 523.64 \text{ in}^3 \qquad c = \frac{29.7}{2} + 0.625 = 15.475 \text{ in.}$$

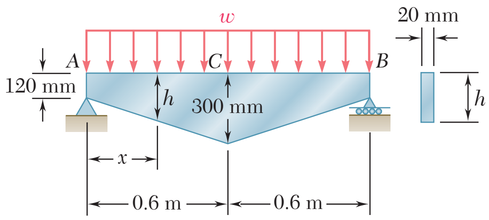
Required moment of inertia: $\quad I = Sc = 8103.3 \text{ in}^4$

But $\quad I = I_{beam} + 2I_{plate}$

$$8103.3 = 3990 + 2\left\{(b)(0.625)\left(\frac{29.7}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(b)(0.625)^3\right\}$$

$$= 3990 + 287.42b$$

(*b*) $\qquad\qquad b = 14.31 \text{ in.} \blacktriangleleft$

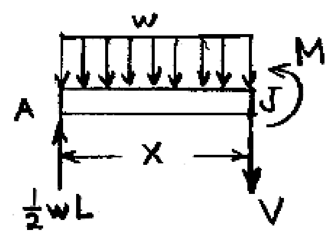


PROBLEM 5.148

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{\text{all}} = 140$ MPa.

SOLUTION

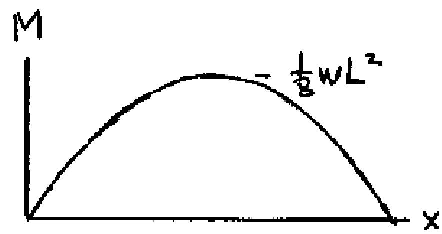
$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow \qquad L = 1.2 \text{ m}$$



$$\Sigma M_J = 0: \quad -\frac{1}{2}wL + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L-x)$$



For the tapered beam, $\quad h = a + kx$

$$a = 120 \text{ mm}$$

$$k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For rectangular cross section, $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\quad \sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0.$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a+kL)x}{(a+kx)^3}\right\} = 0$$

PROBLEM 5.148 *(Continued)*

(*a*) $x_m = \dfrac{aL}{2a+kL} = \dfrac{(120)(1.2)}{(2)(120)+(300)(1.2)} = 0.240 \text{ m}$ $x_m = 240 \text{ mm}$ ◀

$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$

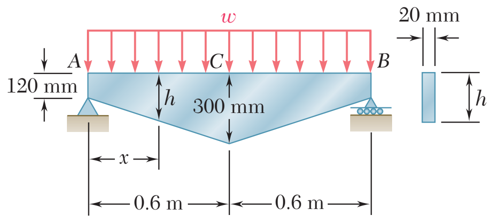
$S_m = \dfrac{1}{6}bh_m^2 = \dfrac{1}{6}(20)(192)^2 = 122.88\times 10^3 \text{ mm}^3 = 122.88\times 10^{-6} \text{ m}^3$

Allowable value of M_m: $M_m = S_m\sigma_{\text{all}} = (122.88\times 10^{-6})(140\times 10^6)$

$= 17.2032\times 10^3 \text{ N}\cdot\text{m}$

(*b*) Allowable value of w: $w = \dfrac{2M_m}{x_m(L-x_m)} = \dfrac{(2)(17.2032\times 10^3)}{(0.24)(0.96)}$

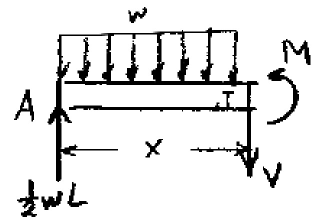
$= 149.3\times 10^3 \text{ N/m}$ $w = 149.3 \text{ kN/m}$ ◀



PROBLEM 5.149

For the tapered beam shown, knowing that $w = 160$ kN/m, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the corresponding value of the normal stress.

SOLUTION

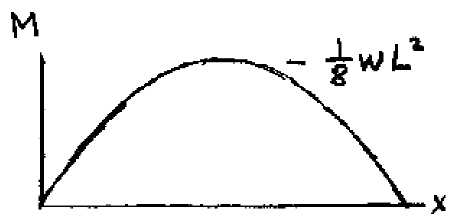


$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



where $w = 160$ kN/m and $L = 1.2$ m.

For the tapered beam, $h = a + kx$

$$a = 120 \text{ mm}$$

$$k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - 2ax + kLx}{(a + kx)^3}\right\} = 0$$

PROBLEM 5.149 *(Continued)*

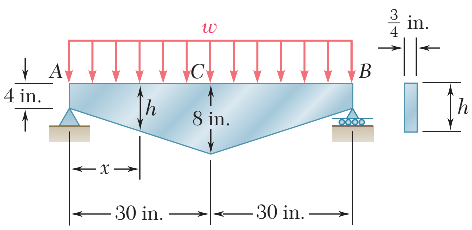
(*a*) $$x_m = \frac{aL}{2a+kL} = \frac{(120)(1.2)}{(2)(120)+(300)(1.2)} = 0.240 \text{ m}$$ $x_m = 240 \text{ mm}$ ◀

$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

$$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88\times 10^3 \text{ mm}^3 = 122.88\times 10^{-6} \text{ m}^3$$

$$M_m = \frac{w}{2}x_m(L-x_m) = \frac{160\times 10^3}{2}(0.24)(0.96) = 18.432\times 10^3 \text{ N}\cdot\text{m}$$

(*b*) Maximum bending stress: $$\sigma_m = \frac{M_m}{S_m} = \frac{18.432\times 10^3}{122.88\times 10^{-6}} = 150\times 10^6 \text{ Pa}$$ $\sigma_m = 150.0 \text{ MPa}$ ◀

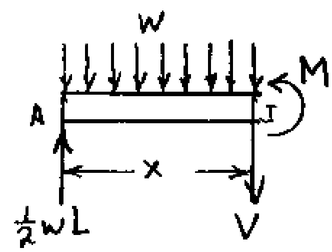


PROBLEM 5.150

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{\text{all}} = 24$ ksi.

SOLUTION

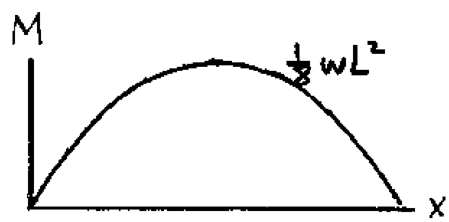
$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL\uparrow \qquad L = 60 \text{ in.}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



For the tapered beam, $\quad h = a + kx$

$$a = 4 \text{ in.} \quad k = \frac{8-4}{30} = \frac{2}{15} \text{ in./in.}$$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress: $\quad \sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0.$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a + kL)x}{(a + kx)^3}\right\} = 0$$

PROBLEM 5.150 *(Continued)*

(*a*)

$$x_m = \frac{aL}{2a+kL} = \frac{(4)(60)}{(2)(4)+\left(\frac{2}{15}\right)(6.0)}$$

$x_m = 15.00$ in. ◀

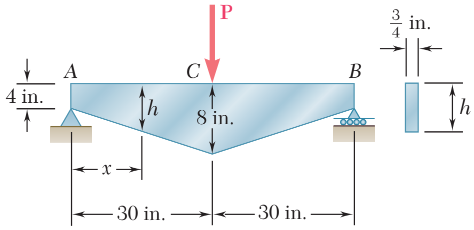
$$h_m = a + kx_m = 4 + \left(\frac{2}{15}\right)(15) = 6.00 \text{ in.}$$

$$S_m = \frac{1}{6}bh_m^3 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(6.00)^2 = 4.50 \text{ in}^3$$

Allowable value of M_m: $M_m = S_m\sigma_{\text{all}} = (4.50)(24) = 180.0 \text{ kip} \cdot \text{in.}$

(*b*) Allowable value of *w*: $w = \dfrac{2M_m}{x_m(L-x_m)} = \dfrac{(2)(108.0)}{(15)(45)} = 0.320 \text{ kip/in.}$

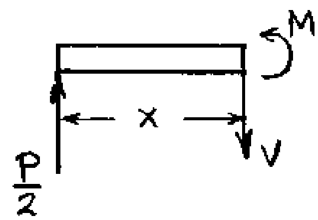
$w = 320$ lb/in. ◀



PROBLEM 5.151

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest concentrated load **P** that can be applied, knowing that $\sigma_{\text{all}} = 24$ ksi.

SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{P}{2}\uparrow$$

$$\Sigma M_J = 0: \quad -\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \qquad \left(0 < x < \frac{L}{2}\right)$$

For a tapered beam, $\qquad h = a + kx$

For a rectangular cross section, $\qquad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\qquad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress, set $\dfrac{d\sigma}{dx} = 0.$

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\,\frac{(a+kx)^2 - x - 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\frac{a-kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Data: $\qquad a = 4$ in., $\quad k = \dfrac{8-4}{30} = 0.13333$ in./in.

(*a*) $\qquad x_m = \dfrac{4}{0.13333} = 30$ in. $\qquad x_m = 30.0$ in. ◀

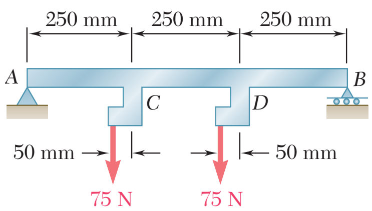
$$h_m = a + kx_m = 8 \text{ in.}$$

$$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(8)^2 = 8 \text{ in}^3$$

$$M_m = \sigma_{\text{all}}S_m = (24)(8) = 192 \text{ kip}\cdot\text{in.}$$

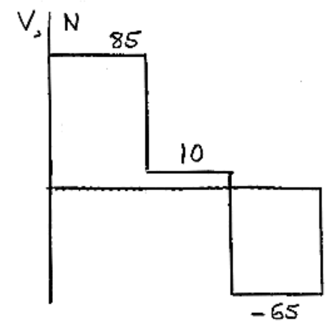
(*b*) $\qquad P = \dfrac{2M_m}{x_m} = \dfrac{(2)(192)}{30} = 12.8$ kips $\qquad P = 12.80$ kips ◀

PROBLEM 5.152



Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



Reaction at A: $\curvearrowleft M_B = 0: \; -0.750R_A + (0.550)(75) + (0.300)(75) = 0$

$$\boldsymbol{R}_A = 85\,\text{N}\uparrow$$

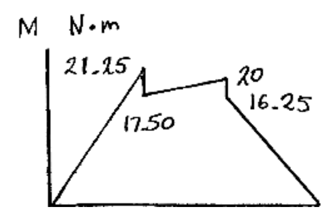
Also, $\boldsymbol{R}_B = 65\,\text{N}\uparrow$

A to C: $V = 85$ N

C to D: $V = 10$ N

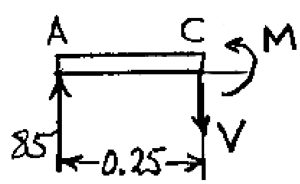
D to B: $V = -65$ N

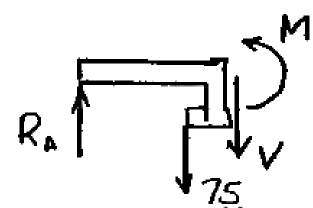
At A and B, $M = 0$



Just to the left of C,

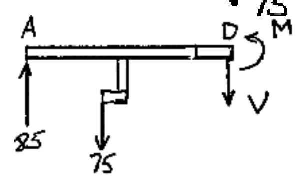
$$\curvearrowleft \Sigma M_C = 0: \; -(0.25)(85) + M = 0$$
$$M = 21.25\ \text{N}\cdot\text{m}$$





Just to the right of C,

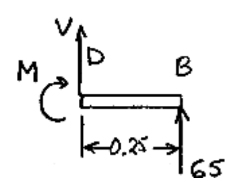
$$\curvearrowleft \Sigma M_C = 0: \quad -(0.25)(85) + (0.050)(75) + M = 0$$
$$M = 17.50\ \text{N}\cdot\text{m}$$



Just to the left of D,

$$\curvearrowleft \Sigma M_D = 0: \quad -(0.50)(85) + (0.300)(75) + M = 0$$
$$M = 20\ \text{N}\cdot\text{m}$$

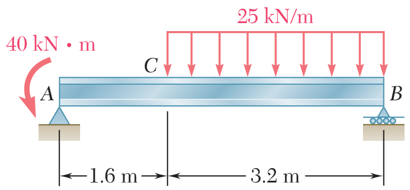
Just to the right of D,



$$\curvearrowleft \Sigma M_D = 0: \quad -M + (0.25)(65) = 0$$
$$M = 16.25\ \text{kN}$$

(*a*) $|V|_{max} = 85.0$ N ◀

(*b*) $|M|_{max} = 21.3\ \text{N}\cdot\text{m}$ ◀

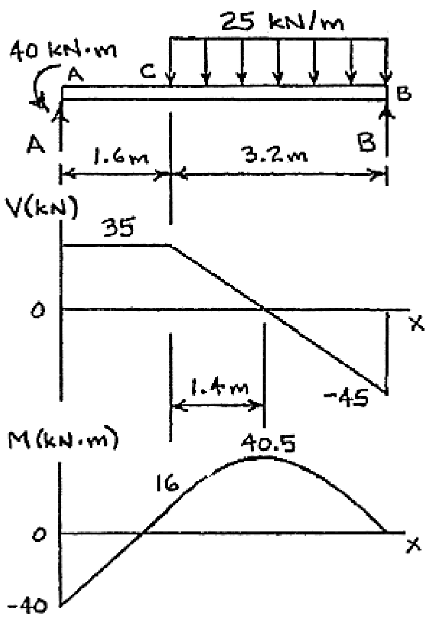


W200 × 31.3

PROBLEM 5.153

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

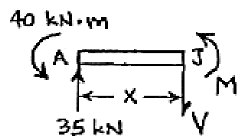


Reaction at A:

$$+\circlearrowleft \Sigma M_B = 0: \quad -4.8A + 40 + (25)(3.2)(1.6) = 0$$

$$\mathbf{A} = 35\text{ kN}\uparrow$$

A to *C*: $0 < x < 1.6\text{ m}$



$$+\uparrow \Sigma F_y = 0: \quad 35 - V = 0 \quad V = 35\text{ kN}$$

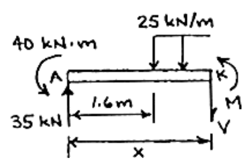
$$+\circlearrowleft \Sigma M_J = 0: \quad M + 40 - 35x = 0$$

$$M = (30x - 40)\text{ kN}\cdot\text{m}$$

C to *B*: $1.6\text{ m} < x < 4.8\text{ m}$

$$+\uparrow \Sigma F_y = 0: \; 35 - 25(x - 1.6) - V = 0$$

$$V = (-25x + 75)\text{ kN}$$



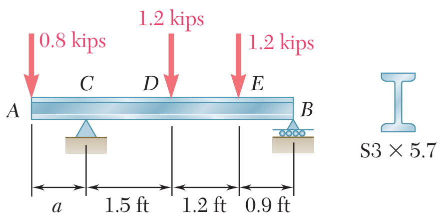
$$+\circlearrowleft \Sigma M_K = 0: \quad M + 40 - 35x + (25)(x - 1.6)\left(\frac{x - 1.6}{2}\right) = 0$$

$$M = (-12.5x^2 + 75x - 72)\text{ kN}\cdot\text{m}$$

Normal stress: For W200 × 31.3, $S = 298 \times 10^3\text{ mm}^3$

$$\sigma = \frac{|M|}{S} = \frac{40.5 \times 10^3\text{N}\cdot\text{m}}{298 \times 10^{-6}\text{ m}^3} = 135.9 \times 10^6\text{ Pa} \qquad \sigma = 135.9\text{ MPa} \blacktriangleleft$$

PROBLEM 5.154

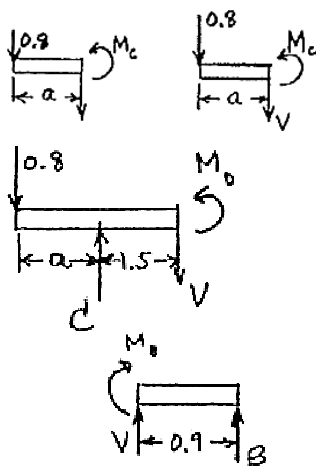


Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$+\circlearrowleft \sum M_C = 0: \quad 0.8a - (1.5)(1.2) - (2.7)(1.2) + (3.6)B = 0 \qquad \mathbf{B} = 1.4 - 0.22222a \uparrow$

$+\circlearrowleft \sum M_B = 0: \quad (0.8)(3.6 + a) - 3.6C + (2.1)(1.2) + (0.9)(1.2) = 0 \qquad \mathbf{C} = 1.8 + 0.22222a \uparrow$



Bending moment at *C*: $+\circlearrowleft \sum M_C = 0: \quad M_C + (0.8)(a) = 0$

$$M_C = -0.8a$$

Bending moment at *D*: $\sum M_D = 0:$

$$M_D + (0.8)(a + 1.5) - 1.5C = 0$$

$$M_D = 1.5 - 0.46667a$$

Bending moment at *E*: $+\circlearrowleft \sum M_E = 0: \quad -M_E + 0.9B = 0$

$$M_E = 1.26 - 0.2a$$

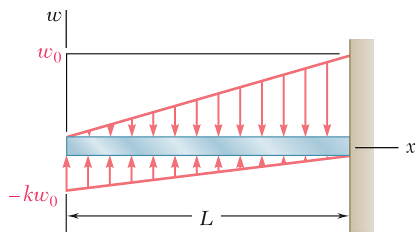
Assume $-M_C = M_E$: $\quad 0.8a = 1.26 - 0.2a \qquad a = 1.260 \text{ ft}$ ◀

$M_C = -1.008 \text{ kip} \cdot \text{ft} \qquad M_E = 1.008 \text{ kip} \cdot \text{ft} \qquad M_D = 0.912 \text{ kip} \cdot \text{ft}$

Note that $|M_D| < 1.008 \text{ kip} \cdot \text{ft} \qquad \max|M| = 1.008 \text{ kip} \cdot \text{ft} = 12.096 \text{ kip} \cdot \text{in.}$

For rolled-steel section S3 × 5.7: $\quad S = 1.67 \text{ in}^3$

Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{12.096}{1.67} \qquad \sigma = 7.24 \text{ ksi}$ ◀



PROBLEM 5.155

For the beam and loading shown, determine the equations of the shear and bending-moment curves, and the maximum absolute value of the bending moment in the beam, knowing that (*a*) $k = 1$, (*b*) $k = 0.5$.

SOLUTION

$$w = \frac{w_0 x}{L} - \frac{k w_0 (L-x)}{L} = (1+k)\frac{w_0 x}{L} - kw$$

$$\frac{dV}{dx} = -w = k w_0 - (1+k)\frac{w_0 x}{L}$$

$$V = k w_0 x - (1+k)\frac{w_0 x^2}{2L} + C_1$$

$$V = 0 \quad \text{at} \quad x = 0 \quad C_1 = 0$$

$$\frac{dM}{dx} = V = k w_0 x - (1+k)\frac{w_0 x^2}{2L}$$

$$M = \frac{k w_0 x^2}{2} - (1+k)\frac{w_0 x^3}{6L} + C_2$$

$$M = 0 \quad \text{at} \quad x = 0 \quad C_2 = 0$$

$$M = \frac{k w_0 x^2}{2} - \frac{(1+k) w_0 x^3}{6L}$$

(*a*) $\underline{k = 1}$.

$$V = w_0 x - \frac{w_0 x^2}{L} \blacktriangleleft$$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L} \blacktriangleleft$$

Maximum M occurs at $x = L$.

$$|M|_{\max} = \frac{w_0 L^2}{6} \blacktriangleleft$$

(*b*) $\underline{k = \frac{1}{2}}$.

$$V = \frac{w_0 x}{2} - \frac{3 w_0 x^2}{4L} \blacktriangleleft$$

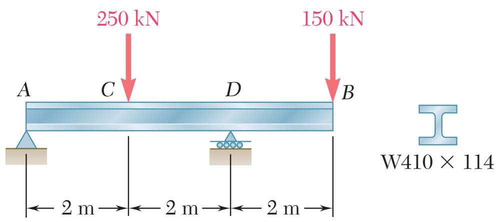
$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L} \blacktriangleleft$$

$$V = 0 \quad \text{at} \quad x = \frac{2}{3}L$$

At $x = \frac{2}{3}L$, $M = \frac{w_0\left(\frac{2}{3}L\right)^2}{4} - \frac{w_0\left(\frac{2}{3}L\right)^3}{4L} = \frac{w_0 L^2}{27} = 0.03704 w_0 L^2$

At $x = L$, $M = 0$

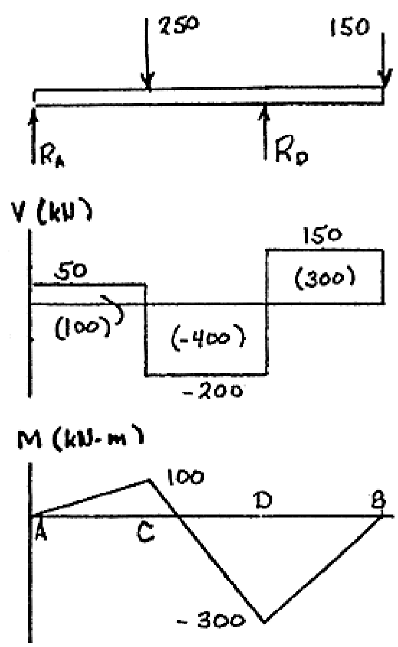
$$|M|_{\max} = \frac{w_0 L^2}{27} \blacktriangleleft$$



PROBLEM 5.156

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION



$$w = 0$$

$$+\circlearrowleft \Sigma M_D = 0:$$

$$-4R_A + (2)(250) - (2)(150) = 0$$

$$\mathbf{R}_A = 50 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0:$$

$$4R_D - (2)(250) - (6)(150) = 0$$

$$\mathbf{R}_D = 350 \text{ kN} \uparrow$$

Shear: $V_A = 50$ kN

A to *C*: $V = 50$ kN

C to *D*: $V = 50 - 250 = -200$ kN

D to *B*: $V = -200 + 350 = 150$ kN

Areas of shear diagram:

A to *C*: $\int V\,dx = (50)(2) = 100 \text{ kN} \cdot \text{m}$

C to *D*: $\int V\,dx = (-200)(2) = -400 \text{ kN} \cdot \text{m}$

D to *B*: $\int V\,dx = (150)(2) = 300 \text{ kN} \cdot \text{m}$

Bending moments: $M_A = 0$

$$M_C = M_A + \int V\,dx = 0 + 100 = 100 \text{ kN} \cdot \text{m}$$

$$M_D = M_C + \int V\,dx = 100 - 400 = -300 \text{ kN} \cdot \text{m}$$

$$M_B = M_D + \int V\,dx = -300 + 300 = 0$$

Maximum $|V| = 200$ kN

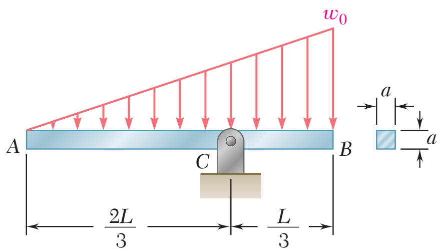
Maximum $|M| = 300 \text{ kN} \cdot \text{m} = 300 \times 10^3 \text{ N} \cdot \text{m}$

For W410 × 114 rolled-steel section, $S_x = 2200 \times 10^3 \text{ mm}^3 = 2200 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{300 \times 10^3}{2200 \times 10^{-6}} = 136.4 \times 10^6 \text{ Pa}$$

$$\sigma_m = 136.4 \text{ MPa} \blacktriangleleft$$

PROBLEM 5.157



Beam AB, of length L and square cross section of side a, is supported by a pivot at C and loaded as shown. (a) Check that the beam is in equilibrium. (b) Show that the maximum normal stress due to bending occurs at C and is equal to $w_0 L^2/(1.5a)^3$.

SOLUTION

(a) Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

For the triangular distribution, the centroid lies at $x = \dfrac{2L}{3}$. $\quad W = \dfrac{1}{2} w_0 L$

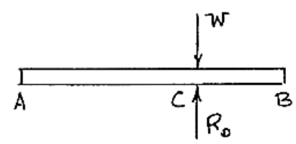
(a) $+\uparrow \sum F_y = 0: \quad R_D - W = 0 \qquad R_D = \dfrac{1}{2} w_0 L \qquad +\circlearrowleft \sum M_C = 0: \quad 0 = 0$ equilibrium ◀

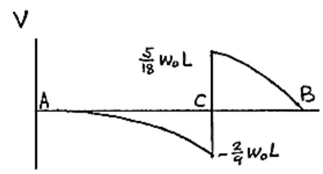
$$V = 0, \; M = 0, \text{ at } x = 0$$

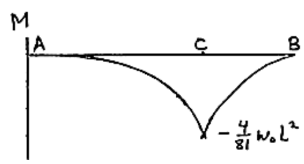
$$0 < x < \frac{2L}{3}, \quad \frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L} + C_1 = -\frac{w_0 x^2}{2L}$$

$$M = -\frac{w_0 x^3}{6L} + C_2 = -\frac{w_0 x^3}{6L}$$







Just to the left of C, $\quad V = -\dfrac{w_0 (2L/3)^2}{2L} = -\dfrac{2}{9} w_0 L$

Just to the right of C, $\quad V = -\dfrac{2}{9} w_0 L + R_D = \dfrac{5}{18} w_0 L$

Note sign change. Maximum $|M|$ occurs at C. $\quad M_C = -\dfrac{w_0 (2L/3)^3}{6L} = -\dfrac{4}{81} w_0 L^2$

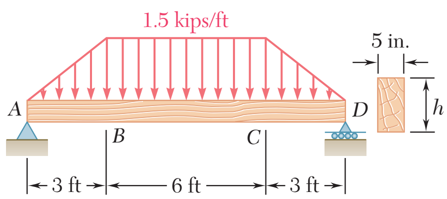
Maximum $|M| = \dfrac{4}{81} w_0 L^2$

For square cross section, $\quad I = \dfrac{1}{12} a^4 \qquad c = \dfrac{1}{2} a$

(b) $\quad \sigma_m = \dfrac{|M|_{\max} c}{I} = \dfrac{4}{81} \dfrac{w_0 L^2 6}{a^3} = \dfrac{8}{27} \dfrac{w_0 L^2}{a^3} = \left(\dfrac{2}{3}\right)^3 \dfrac{w_0 L^2}{a^3}$

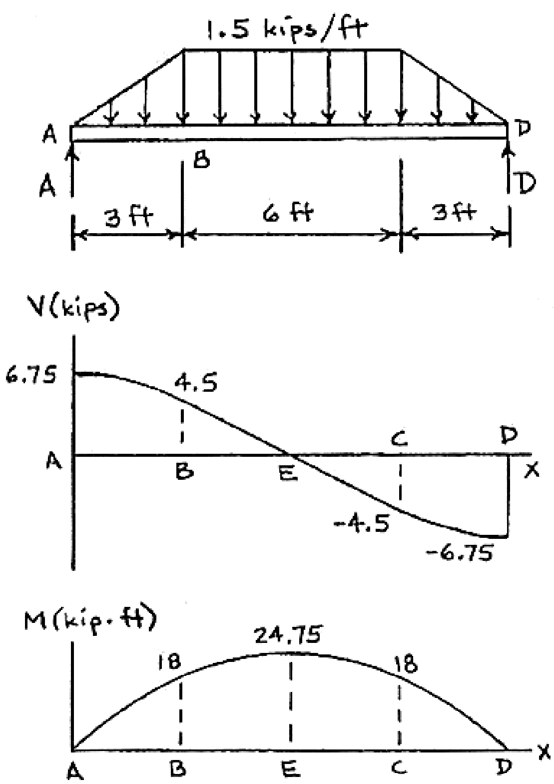
$$\sigma_m = \frac{w_0 L^2}{(1.5a)^3} \; ◀$$

PROBLEM 5.158



For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION



Reactions: By symmetry, $A = D$.

$$+\uparrow \Sigma F_y = 0: \quad A - \frac{1}{2}(3)(1.5) - (6)(1.5) - \frac{1}{2}(3)(1.5) - D = 0$$

$$A = D = 6.75 \text{ kips} \uparrow$$

Shear diagram: $V_A = 6.75$ kips

$$V_B = 6.75 - \frac{1}{2}(3)(1.5) = 4.5 \text{ kips}$$

$$V_C = 4.5 - (6)(1.5) = -4.5 \text{ kips}$$

$$V_D = -4.5 - \frac{1}{2}(3)(1.5) = -6.75 \text{ kips}$$

Locate point E where $V = 0$.

By symmetry, E is the midpoint of BC.

Areas of the shear diagram:

A to B: $(3)(4.5) + \frac{2}{3}(3)(2.25) = 18 \text{ kip} \cdot \text{ft}$

B to E: $\frac{1}{2}(3)(4.5) = 6.75 \text{ kip} \cdot \text{ft}$

E to C: $\frac{1}{2}(3)(-4.5) = -6.75 \text{ kip} \cdot \text{ft}$

C to D: By antisymmetry, $-18 \text{ kip} \cdot \text{ft}$

PROBLEM 5.158 *(Continued)*

Bending moments: $M_A = 0$

$$M_B = 0 + 18 = 18 \text{ kip} \cdot \text{ft}$$

$$M_E = 18 + 6.75 = 24.75 \text{ kip} \cdot \text{ft}$$

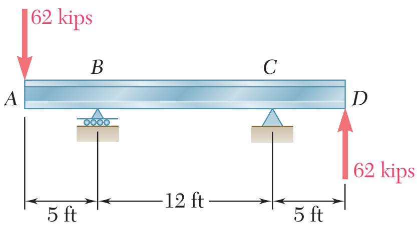
$$M_C = 24.75 - 6.75 = 18 \text{ kip} \cdot \text{ft}$$

$$M_D = 18 - 18 = 0$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} \qquad S = \frac{|M|_{\max}}{\sigma_{\max}} = \frac{(24.75 \text{ kip} \cdot \text{ft})(12 \text{ in./ft})}{1.750 \text{ ksi}} = 169.714 \text{ in}^3$$

For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(169.714)}{5}} = 14.27 \text{ in.} \qquad h = 14.27 \text{ in.} \blacktriangleleft$$



PROBLEM 5.159

Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0$: $(17)(62) - 12B + (5)(62) = 0$ $\qquad$ $\mathbf{B} = 113.667 \text{ kips} \uparrow$

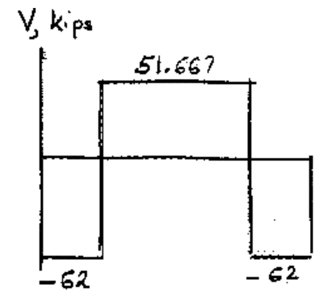
$+\circlearrowleft \Sigma M_B = 0$: $(5)(62) + 12C + (17)(62) = 0$ $\qquad$ $C = -113.667$ kips or $\mathbf{C} = 113.667 \text{ kips} \downarrow$

Shear diagram:

A to B^-: $\quad V = -62$ kips

B^+ to C^-: $\quad V = -62 + 113.667 = 51.667$ kips

C^+ to D: $\quad V = 51.667 - 113.667 = -62$ kips



Areas of shear diagram:

A to B: $\quad (5)(-62) = -310 \text{ kip}\cdot\text{ft}$

B to C: $\quad (12)(51.667) = 620 \text{ kip}\cdot\text{ft}$

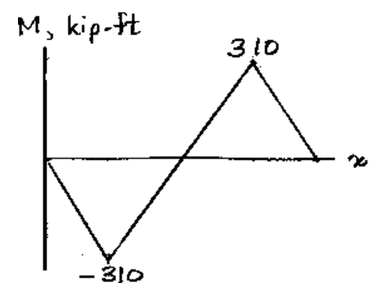
C to D: $\quad (5)(-62) = -310 \text{ kip}\cdot\text{ft}$

Bending moments: $\quad M_A = 0$

$M_B = 0 - 310 = -310 \text{ kip}\cdot\text{ft}$

$M_C = -310 + 620 = 310 \text{ kip}\cdot\text{ft}$

$M_D = 310 - 310 = 0$

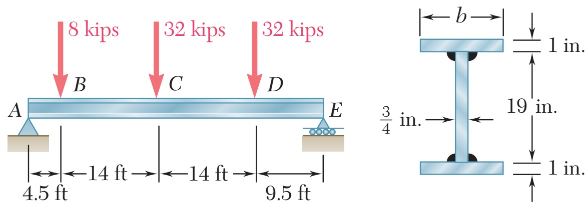


$$|M|_{\max} = 310 \text{ kip}\cdot\text{ft} = 3.72\times10^3 \text{ kip}\cdot\text{in.}$$

Required $S_{\min}$: $\quad S_{\min} = \dfrac{|M|_{\max}}{\sigma_{\text{all}}} = \dfrac{3.72\times10^3}{24} = 155 \text{ in}^3$

Shape	$S(\text{in}^3)$
W27 × 84	213
W21 × 101	227
W18 × 106	204
W14 × 145	232

Use W27 × 84. ◀



PROBLEM 5.160

Three steel plates are welded together to form the beam shown. Knowing that the allowable normal stress for the steel used is 22 ksi, determine the minimum flange width b that can be used.

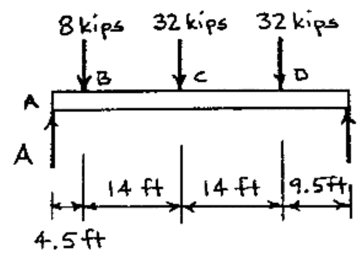
SOLUTION

Reactions: $+\circlearrowleft \Sigma M_E = 0: \quad -42A + (37.5)(8) + (23.5)(32) - (9.5)(32) = 0$

$$\boldsymbol{A} = 32.2857 \text{ kips} \uparrow$$

$+\circlearrowleft \Sigma M_A = 0: \quad 42E - (4.5)(8) - (18.5)(32) - (32.5)(32) = 0$

$$\boldsymbol{E} = 39.7143 \text{ kips} \uparrow$$



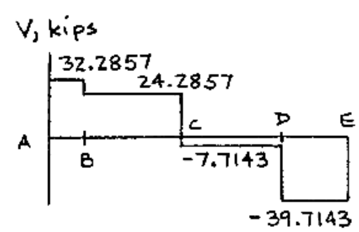
Shear:

A to B: 32.2857 kips

B to C: $32.2857 - 8 = 24.2857$ kips

C to D: $24.2857 - 32 = -7.7143$ kips

D to E: $-7.7143 - 32 = -39.7143$ kips



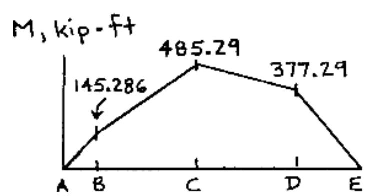
Areas:

A to B: $(4.5)(32.2857) = 145.286 \text{ kip} \cdot \text{ft}$

B to C: $(14)(24.2857) = 340 \text{ kip} \cdot \text{ft}$

C to D: $(14)(-7.7143) = -108 \text{ kip} \cdot \text{ft}$

D to E: $(9.5)(-39.7143) = -377.286 \text{ kip} \cdot \text{ft}$



Bending moments:

$$M_A = 0$$
$$M_B = 0 + 145.286 = 145.286 \text{ kip} \cdot \text{ft}$$
$$M_C = 145.286 + 340 = 485.29 \text{ kip} \cdot \text{ft}$$
$$M_D = 485.29 - 108 = 377.29 \text{ kip} \cdot \text{ft}$$
$$M_E = 377.29 - 377.286 = 0$$

Maximum $|M| = 485.29 \text{ kip} \cdot \text{ft} = 5.2834 \times 10^3 \text{ kip} \cdot \text{in.} \qquad \sigma_{\text{all}} = 22 \text{ ksi}$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{5.2834 \times 10^3}{22} = 264.70 \text{ in}^3$$

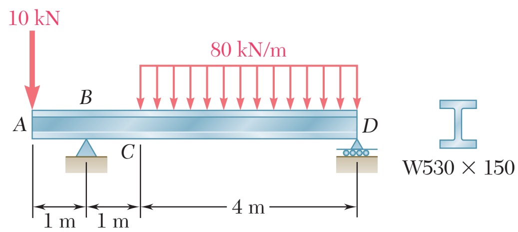
$$I = \frac{1}{12}\left(\frac{3}{4}\right)(19)^3 + 2\left[\frac{1}{12}(b)(1)^3 + (b)(1)(10)^2\right] = 428.69 + 200.17b$$

$$c = 9.5 + 1 = 10.5 \text{ in.}$$

$$S_{\min} = \frac{I}{c} = 40.828 + 19.063b = 264.70$$

$b = 11.74$ in. ◀

PROBLEM 5.161



(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft M_D = 0: \quad (6)(10) - 5R_B + (2)(4)(80) = 0$$

$$R_B = 140 \text{ kN}$$

$$w = 80\langle x-2\rangle^0 \text{ kN/m} = -dV/dx$$

$$V = -10 + 140\langle x-1\rangle^0 - 80\langle x-2\rangle^1 \text{ kN}$$

A to *B*: $\quad V = -10 \text{ kN}$

B to *C*: $\quad V = -10 + 140 = 130 \text{ kN}$

D: $\quad (x = 6) \quad V = -10 + 140 - 80(4) = -190 \text{ kN}$

V changes sign at B and at point E $(x = x_E)$ between C and D.

$$V = 0 = -10 + 140\langle x_E - 1\rangle^0 - 80\langle x_E - 2\rangle^1$$

$$= -10 + 140 - 80(x_E - 2) \qquad x_E = 3.625 \text{ m}$$

$$M = -10x + 140\langle x-1\rangle^1 - 40\langle x-2\rangle^2 \text{ kN}\cdot\text{m}$$

At pt. *B*, $\quad x = 1 \quad M_B = -(10)(1) = -10 \text{ kN}\cdot\text{m}$

At pt. *E*, $\quad x = 3.625$

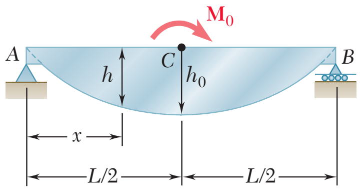
$$M_E = -(10)(3.625) + (140)(2.625) - (40)(1.625)^2 = 225.6 \text{ kN}\cdot\text{m}$$

(*a*) $\quad |M|_{\max} = 225.6 \text{ kN}\cdot\text{m} \text{ at } x = 3.63 \text{ m} \blacktriangleleft$

For W530 × 150, $\quad S = 3720 \times 10^3 \text{ mm}^3 = 3720 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{225.6 \times 10^3}{3720 \times 10^{-6}} = 60.6 \times 10^6 \text{ Pa}$

$$\sigma = 60.6 \text{ MPa} \blacktriangleleft$$

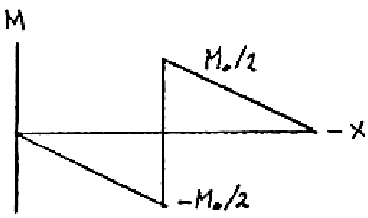


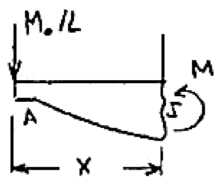
PROBLEM 5.162

The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

SOLUTION

$$\mathbf{A} = M_0/L \downarrow \qquad \mathbf{B} = M_0/L \uparrow$$





$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{M_0}{L}x + M = 0$$

$$M = -\frac{M_0 x}{L} \quad \left(0 < x < \frac{L}{2}\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{M_0 x}{\sigma_{all} L} \qquad \text{for} \qquad \left(0 < x \frac{L}{2}\right)$$

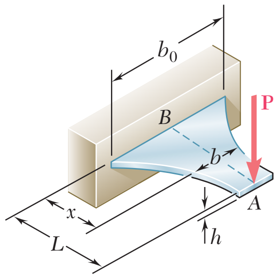
For a rectangular cross section, $\quad S = \dfrac{1}{6}bh^2$

Equating, $\quad \dfrac{1}{6}bh^2 = \dfrac{M_0 x}{\sigma_{all} L} \qquad h = \sqrt{\dfrac{6M_0 x}{\sigma_{all} bL}}$

(a) At $x = \dfrac{L}{2}$, $\qquad h = h_0 = \sqrt{\dfrac{3M_0}{\sigma_{all} b}} \qquad\qquad h = h_0\sqrt{2x/L}$ ◀

(b) Solving for M_0, $\quad M_0 = \dfrac{\sigma_{all} b h_0^2}{3} = \dfrac{(72\times10^6)(0.025)(0.200)^2}{3} = 24\times10^3\ \text{N}\cdot\text{m}$

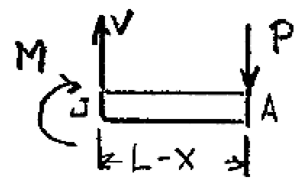
$$M_0 = 24.0\ \text{kN}\cdot\text{m}$$ ◀



PROBLEM 5.163

A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support the concentrated load **P** at point A. (a) Knowing that the beam is to be of constant strength, express b in terms of x, L, and b_0. (b) Determine the smallest allowable value of h if $L = 300$ mm, $b_0 = 375$ mm, $P = 14.4$ kN, and $\sigma_{\text{all}} = 160$ MPa.

SOLUTION



$$+\circlearrowleft \Sigma M_J = 0: \quad -M - P(L-x) = 0$$

$$M = -P(L-x)$$

$$|M| = P(L-x)$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{P(L-x)}{\sigma_{\text{all}}}$$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2$

Equating, $\quad \frac{1}{6}bh^2 = \frac{P(L-x)}{\sigma_{\text{all}}} \qquad b = \frac{6P(L-x)}{\sigma_{\text{all}}h^2}$

(a) At $x = 0$, $\quad b = b_0 = \frac{6PL}{\sigma_{\text{all}}h^2}$ $\qquad b = b_0\left(1 - \frac{x}{L}\right)$ ◀

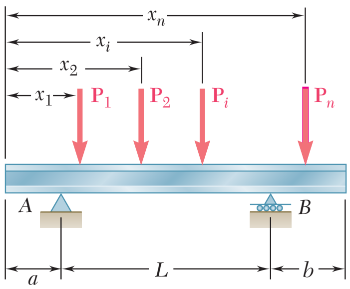
Solving for h, $\quad h = \sqrt{\frac{6PL}{\sigma_{\text{all}}b_0}}$

Data: $\quad L = 300 \text{ mm} = 0.300 \text{ m}, \quad b_0 = 375 \text{ mm} = 0.375 \text{ m}$

$P = 14.4 \text{ kN} = 14.4 \times 10^3 \text{ N} \cdot \text{m}, \quad \sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

(b) $\quad h = \sqrt{\frac{(6)(14.4 \times 10^3)(0.300)}{(160 \times 10^6)(0.375)}} = 20.8 \times 10^{-3} \text{ m}$ $\qquad h = 20.8$ mm ◀

PROBLEM 5.C1



Several concentrated loads $P_i(i=1, 2, \ldots, n)$ can be applied to a beam as shown. Write a computer program that can be used to calculate the shear, bending moment, and normal stress at any point of the beam for a given loading of the beam and a given value of its section modulus. Use this program to solve Probs. 5.18, 5.21, and 5.25. (*Hint:* Maximum values will occur at a support or under a load.)

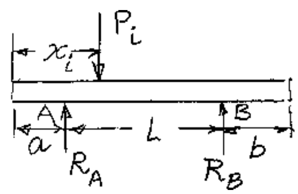
SOLUTION

Reactions at A and B

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - \sum_i P_i(x_i - a)$$

$$R_B = \left(\frac{1}{L}\right)\sum_i P_i(x_i - a)$$

$$R_A = \sum_i P_i - R_B$$



We use step functions (See bottom of Page 348 of text.)

We define:

If $x \geq a$	Then STP $A = 1$	Else STP $A = 0$
If $x \geq a + L$	Then STP $B = 1$	Else STP $B = 0$
If $x \geq x_i$	Then STP $(I) = 1$	Else STP (I) $= 0$

$$V = R_A \text{ STP } A + R_B \text{ STP } B - \sum_i P_i \text{ STP (I)}$$

$$M = R_A(x-a) \text{ STP } A + R_B(x-a-L) \text{ STP } B - \sum_i P_i(x-x_i) \text{ STP (I)}$$

$\sigma = M/S$, where S is obtained from Appendix C.

Program Outputs

Problem 5.18

$R_A = 80.0$ kN $\quad R_B = 80.0$ kN

X m	V kN	M kN · m	σ MPa
2.00	0.00	104.00	139.0

◀

PROBLEM 5.C1 *(Continued)*

Program Outputs ***(Continued)***

Problem 5.21

$R_1 = 52.5$ kips $R_2 = 22.5$ kips

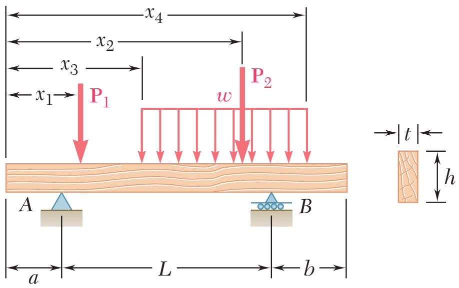
X ft	V kips	M kip · ft	σ ksi
0.00	–25.00	0.00	0.00
1.00	27.50	–25.00	–7.85
3.00	2.50	30.00	9.42
9.00	–22.50	45.00	14.14 ◀
11.00	0.00	0.00	0.00

Problem 5.25

$R_1 = 10.77$ kips $R_2 = 4.23$ kips

X ft	V kips	M kip · in.	σ ksi
0	–5.00	0	0
5	5.69	–300.00	–10.34
13	–4.23	253.8	8.75
18	–4.23	0	0

PROBLEM 5.C2



A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress in the beam will not exceed a given allowable value σ_{all}. Write a computer program that can be used to calculate at given intervals ΔL the shear, the bending moment, and the smallest acceptable value of the unknown dimension. Apply this program to solve the following problems, using the intervals ΔL indicated: (*a*) Prob. 5.65 ($\Delta L = 0.1$ m), (*b*) Prob. 5.69 ($\Delta L = 0.3$ m), (*c*) Prob. 5.70 ($\Delta L = 0.2$ m).

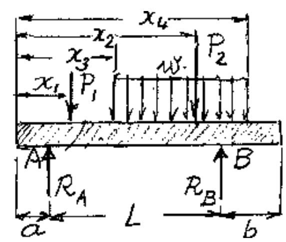
SOLUTION

Reactions at A and B:

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - P_1(x_1 - a) - P_2(x_2 - a) - w(x_4 - x_3)\left(\frac{x_4 + x_3}{2} - a\right) = 0$$

$$R_B = \frac{1}{L}\left[P_1(x_1 - a) + P_2(x_2 - a) + \frac{1}{2}w(x_4 - x_3)(x_4 + x_3 - 2a)\right]$$

$$R_A = P_1 + P_2 + w(x_4 - x_3) - R_B$$



We use step functions. (See bottom of Page 348 of text.)

Set $\quad n = (a + b + L)/\Delta L$

For $\quad i = 0$ to n: $\quad x = (\Delta L)i$

We define:

If $x \geq a$	Then STP $A = 1$	Else STP $A = 0$
If $x \geq a + L$	Then STP $B = 1$	Else STP $B = 0$
If $x \geq x_1$	Then STP $1 = 1$	Else STP $1 = 0$
If $x \geq x_2$	Then STP $2 = 1$	Else STP $2 = 0$
If $x \geq x_3$	Then STP $3 = 1$	Else STP $3 = 0$
If $x \geq x_4$	Then STP $4 = 1$	Else STP $4 = 0$

$$V = R_A \text{ Step } A + R_B \text{ STP } B - P_1 \text{ STP1} - P_2 \text{ STP2} - w(x - x_3)\text{STP3} + w(x - x_4)\text{STP4}$$

$$M = R_A(x - a)\text{STP } A + R_B(x - a - L)\text{STP } B - P_1(x - x_1)\text{STP1}$$
$$-P_2(x - x_2)\text{STP2} - \frac{1}{2}w(x - x_3)^2\text{ STP3} + \frac{1}{2}w(x - x_4)^4\text{ STP4}$$

$$S_{min} = \frac{|M|}{\sigma_{all}}$$

PROBLEM 5.C2 *(Continued)*

If unknown dimension is h:

From $\qquad S = \frac{1}{6}th^2, \quad$ we have $\quad h = \sqrt{6S/t}$

If unknown dimension is t:

From $\qquad S = \frac{1}{6}th^2, \quad$ we have $\quad t = 6S/h^2$

Program Outputs

Problem 5.65 $\qquad R_A = 2.40 \text{ kN} \quad R_B = 3.00 \text{ kN}$

X m	V kN	M kN · m	H mm
0.00	2.40	0.000	0.00
0.10	2.40	0.240	54.77
0.20	2.40	0.480	77.46
0.30	2.40	0.720	94.87
0.40	2.40	0.960	109.54
0.50	2.40	1.200	122.47
0.60	2.40	1.440	134.16
0.70	2.40	1.680	144.91
0.80	0.60	1.920	154.92
0.90	0.60	1.980	157.32
1.00	0.60	2.040	159.69
1.10	0.60	2.100	162.02
1.20	0.60	2.160	164.32
1.30	0.60	2.220	166.58
1.40	0.60	2.280	168.82
1.50	0.60	2.340	171.03
1.60	–3.00	2.400	173.21 ◀
1.70	–3.00	2.100	162.02
1.80	–3.00	1.800	150.00
1.90	–3.00	1.500	136.93
2.00	–3.00	1.200	122.47
2.10	–3.00	0.900	106.07
2.20	–3.00	0.600	86.60
2.30	–3.00	0.300	61.24
2.40	0.00	0.000	0.05

Program Outputs ***(Continued)***

Problem 5.69

$R_A = 6.50$ kN $\quad R_B = 6.50$ kN

X m	V kN	M kN·m	H mm
0.00	2.50	0.000	0.00
0.30	2.50	0.750	61.24
0.60	9.00	1.500	86.60
0.90	7.20	3.930	140.18
1.20	5.40	5.820	170.59
1.50	3.60	7.170	189.34
1.80	1.80	7.980	199.75
2.10	–0.00	8.250	203.10 ◀
2.40	–1.80	7.980	199.75
2.70	–3.60	7.170	189.34
3.00	–5.40	5.820	170.59
3.30	–7.20	3.930	140.18
3.60	–2.50	1.500	86.60
3.90	–2.50	0.750	61.24
4.20	0.00	0.000	0.06

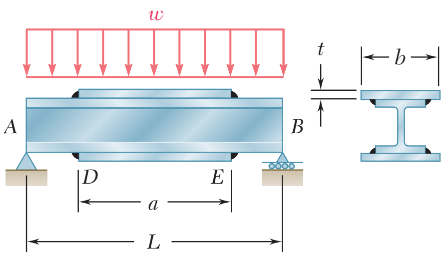
PROBLEM 5.C2 *(Continued)*

Program Outputs *(Continued)*

Problem 5.70

$R_A = 2.70$ kN $R_B = 8.10$ kN

X m	V kN	M $\text{kN} \cdot \text{m}$	T mm
0.00	2.70	0.000	0.00
0.20	2.10	0.480	10.67
0.40	1.50	0.840	18.67
0.60	0.90	1.080	24.00
0.80	0.30	1.200	26.67
1.00	–0.30	1.200	26.67
1.20	–0.90	1.080	24.00
1.40	–1.50	0.840	18.67
1.60	–2.10	0.480	10.67
1.80	–2.70	0.000	0.00
2.00	–3.30	–0.600	13.33
2.20	–3.90	–1.320	29.33
2.40	3.60	–2.160	48.00 ◀
2.60	3.00	–1.500	33.33
2.80	2.40	–0.960	21.33
3.00	1.80	–0.540	12.00
3.20	1.20	–0.240	5.33
3.40	0.60	–0.060	1.33
3.60	0.00	–0.000	0.00

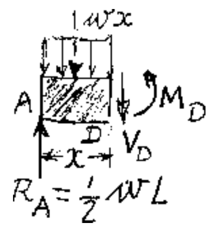


PROBLEM 5.C3

Two cover plates, each of thickness t, are to be welded to a wide-flange beam of length L that is to support a uniformly distributed load w. Denoting by σ_{all} the allowable normal stress in the beam and in the plates, by d the depth of the beam, and by I_b and S_b, respectively, the moment of inertia and the section modulus of the cross section of the unreinforced beam about a horizontal centroidal axis, write a computer program that can be used to calculate the required value of (*a*) the length a of the plates, (*b*) the width b of the plates. Use this program to solve Prob. 5.145.

SOLUTION

(*a*) <u>Required length of plates</u>:



$$FB = AD:$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D + wx\left(\frac{x}{2}\right) - R_A x = 0$$

But

$$R_A = \frac{1}{2}wL \quad \text{and} \quad M_D = S\sigma_{\text{all}}.$$

Divide by $\frac{1}{2}w$:

$$x^2 - Lx + (2S\sigma_{\text{all}}/w) = 0$$

Set $k = \dfrac{2S\sigma_{\text{all}}}{w}$:

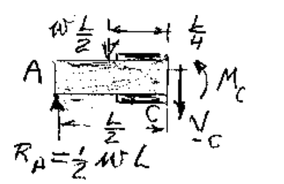
$$x^2 - Lx + k = 0$$

Solving the quadratic, $\quad x = \dfrac{L - \sqrt{L^2 - 4k}}{2}$

Compute x and $\quad a = L - 2x$ ◀

(*b*) <u>Required width of plates</u>:

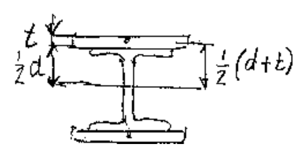
At midpoint C of beam:



$$FB = AC:$$

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + \frac{wL}{2}\frac{L}{4} - \frac{wL}{2}\frac{L}{2} = 0$$

Compute $\quad M_C = \dfrac{1}{8}wL^2$



PROBLEM 5.C3 *(Continued)*

Compute $C = t + \frac{1}{2}d$

From $\sigma_{all} = \frac{M_C C}{I}$ compute $I = \frac{M_C C}{\sigma_{all}}$

But $I = I_{beam} + I_{plates} = I_b + 2\left[\frac{1}{12}bt^3 + bt\left(\frac{d+t}{2}\right)^2\right]$

Solving for b, $b = \frac{6(I - I_b)}{t[t^2 + 3(d+t)^2]}$ ◀

Program Outputs

Problems 5.155:

$W460 \times 74, \quad \sigma_{all} = 150 \text{ MPa}$

$w = 40 \text{ kN/m}, \quad L = 8 \text{ m}, \quad t = 7.5 \text{ mm}$

$d = 457 \text{ mm}, \quad I_b = 333 \times 10^6 \text{ mm}^4, \quad S = 416 \times 10^3 \text{ mm}^3$

Problem 5.145

$a = 4.49 \text{ m}$

$b = 211 \text{ mm}$

Problem 5.157:

$W30 \times 99, \quad \sigma_{all} = 22 \text{ ksi}$

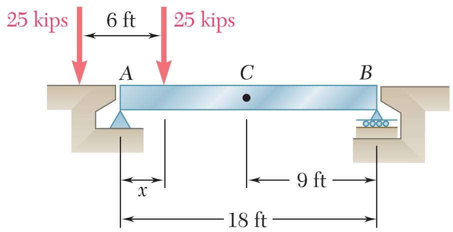
$w = 30 \text{ kips/ft}, \quad L = 16 \text{ ft}, \quad t = 5/8 \text{ in.}$

$d = 29.65 \text{ in.}, \quad I_b = 3990 \text{ in.}^4, \quad S = 269 \text{ in.}^3$

Problem 5.143

$a = 11.16 \text{ ft}$

$b = 14.31 \text{ in.}$



PROBLEM 5.C4

Two 25-kip loads are maintained 6 ft apart as they are moved slowly across the 18-ft beam AB. Write a computer program and use it to calculate the bending moment under each load and at the midpoint C of the beam for values of x from 0 to 24 ft at intervals $\Delta x = 1.5$ ft.

SOLUTION

<u>Notation</u>: Length of beam $= L = 18$ ft

Loads: $P_1 = P_2 = P = 25$ kips

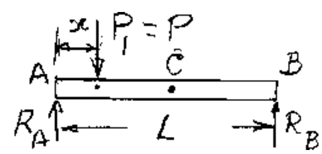
Distance between loads $= d = 6$ ft

We note that $d < L/2$.

(1) From $x = 0$ to $x = d$:

$$+\circlearrowleft \Sigma M_B = 0: \quad P(L-x) - R_A L = 0$$

$$R_A = P(L-x)/L$$



Under P_1: $M_1 = R_A x$

At C: $M_C = R_A\left(\dfrac{L}{2}\right) - P\left(\dfrac{L}{2} - x\right)$

(2) From $x = d$ to $x = L$:

$$+\circlearrowleft \Sigma M_B = 0: \quad P(L-x) + P(L-x+d) - R_A L = 0$$

$$R_A = P(2L - 2x + d)/L$$

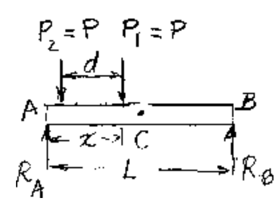
Under P_1: $M_1 = R_A x - Pd$

Under P_2: $M_2 = R_A(x-d)$

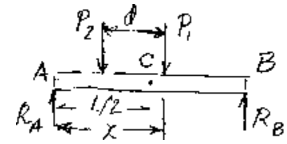
(2A) From $x = d$ to $x = L/2$:

$$M_C = R_A\left(\frac{L}{2}\right) - P\left(\frac{L}{2} - x\right) - P\left(\frac{L}{2} - x + d\right)$$

$$= R_A(L/2) - P(L - 2x + d)$$



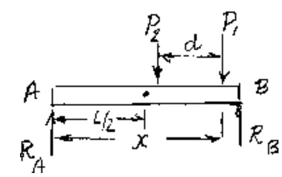
PROBLEM 5.C4 *(Continued)*



(2B) From

$$x = L/2 \quad \text{to} \quad x = L/2 + d:$$

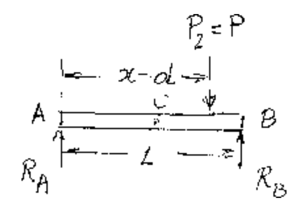
$$M_C = R_A(L/2) - P\left(\frac{L}{2} - x + d\right)$$



(2C) From

$$x = L/2 + d \quad \text{to} \quad x = L:$$

$$M_C = R_A L/2$$



(3) From

$$x = L \quad \text{to} \quad x = L + d:$$

$$+\circlearrowleft \Sigma M_B = 0: \quad P(L - x + d) - R_A L = 0$$

$$R_A = P(L - x + d)/L$$

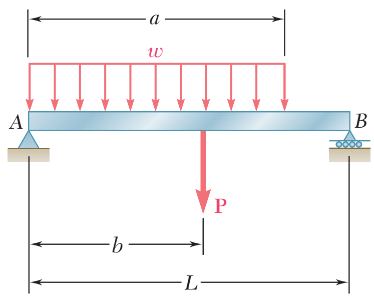
Under P_2: $\quad M_z = R_A(x - d)$

At C: $\quad M_C = R_A(L/2)$

Program Output

$$P = 25 \text{ kips}, \quad L = 18 \text{ ft}, \quad D = 6 \text{ ft}$$

X ft	M_C kip·ft	M_1 kip·ft	M_2 kip·ft
0.0	0.00	0.00	0.00
1.5	18.75	34.38	0.00
3.0	37.50	62.50	0.00
4.5	56.25	84.38	0.00
6.0	75.00	100.00	0.00
7.5	112.50	131.25	56.25
9.0	150.00	150.00	100.00
10.5	150.00	156.25	131.25
12.0	150.0	150.00	150.00
13.5	150.00	131.25	156.25
15.0	150.00	100.00	150.00
16.5	112.50	56.25	131.25
18.0	75.00	0.00	100.00
19.5	56.25	0.00	84.38
21.0	37.50	0.00	62.50
22.5	18.75	0.00	34.38
24.0	0.00	0.00	0.00



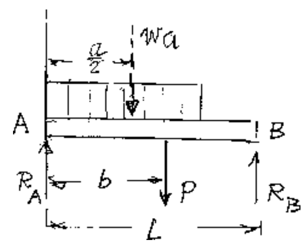
PROBLEM 5.C5

Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.2$ ft to the beam and loading of (*a*) Prob. 5.72, (*b*) Prob. 5.115.

SOLUTION

Reactions at *A* and *B*:

Using *FB* diagram of beam,



$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - Pb - wa(a/2) = 0$$

$$R_B = (1/L)\left(Pb + \frac{1}{2}wa^2\right)$$

$$R_A = P + wa - R_B$$

We use step functions (see bottom of Page 348 of text).

$$\text{set } n = L/\Delta L. \quad \text{For} \quad i = 0 \quad \text{to} \quad n: \quad x = (\Delta L)i$$

We define:

If $x \geqslant a$ Then STP $A = 1$ Else STP $A = 0$

If $x \geqslant b$ Then STP $B = 1$ Else STP $B = 0$

$$V = R_A - wx + w(x-a)\text{STP } A - P\text{STP } B$$

$$M = R_A x - \frac{1}{2}wx^2 + \frac{1}{2}w(x-a)^2\text{ STP } A - P(x-b)\text{STP } B$$

Locate and Print (x, v) and (x, M)

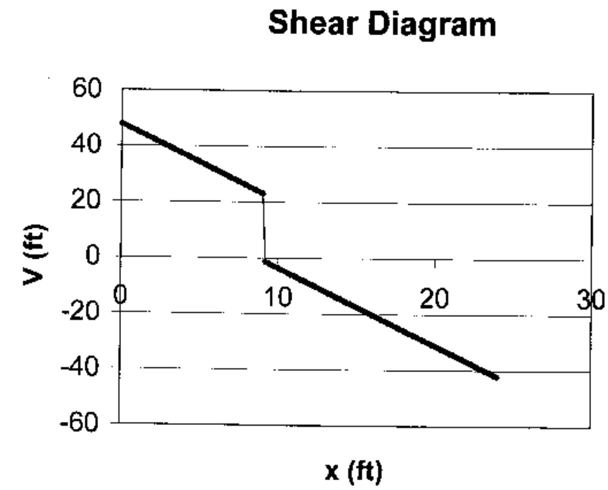
See next pages for program outputs

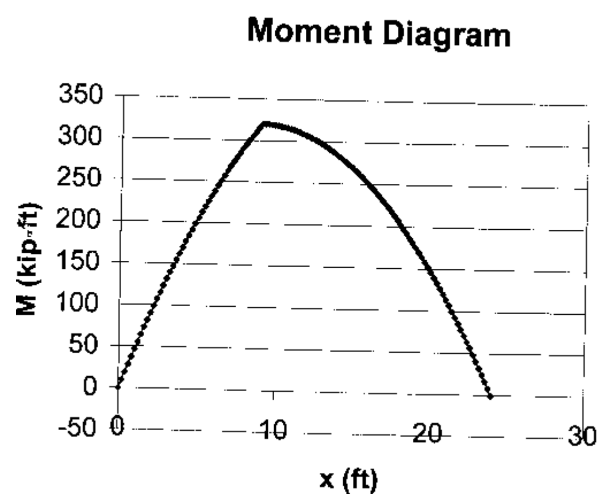
PROBLEM 5.C5 *(Continued)*

Program Outputs

Problem 5.72

$$R_A = 48.00 \text{ kips} \quad R_B = 42.00 \text{ kips}$$

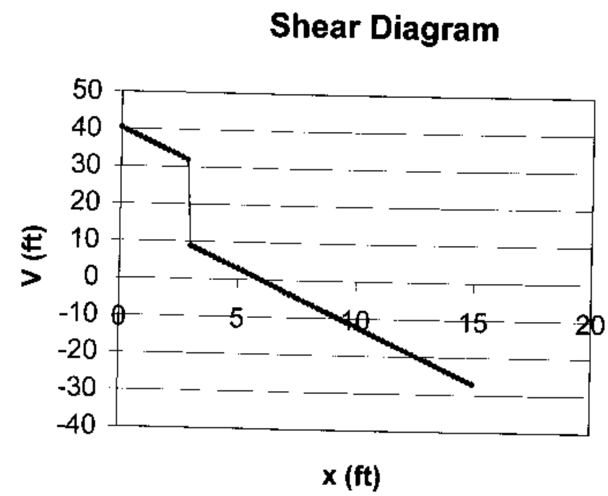


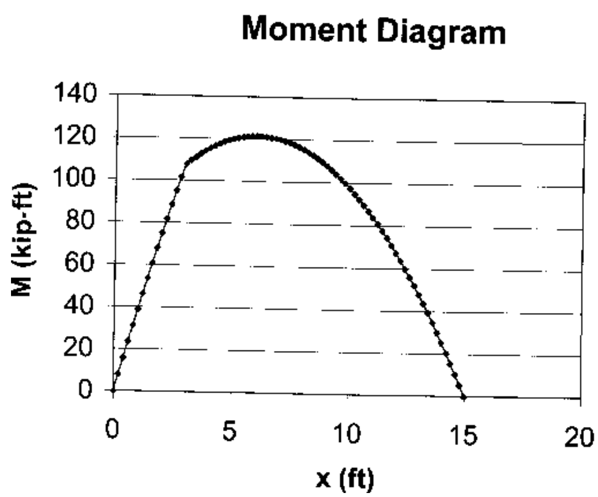


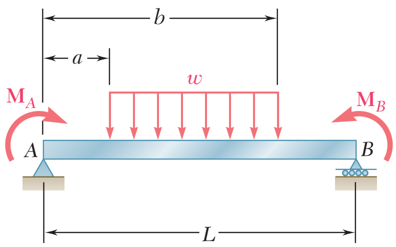
Program Outputs ***(Continued)***

Problem 5.115

$R_A = 40.50$ kips $R_B = 27.00$ kips





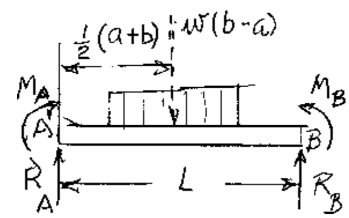


PROBLEM 5.C6

Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.025$ m to the beam and loading of Prob. 5.112.

SOLUTION

Reactions at A and B:



$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L + M_B - M_A = w(b-a)\frac{1}{2}(a+b) = 0$$

$$R_B = (1/L)\left[M_A - M_B + \frac{1}{2}w(b^2 - a^2)\right]$$

$$R_A = w(b-a) - R_B$$

We use step functions (see bottom of Page 348 of text).

Set $\quad n = \dfrac{L}{\Delta L}$

For $\quad i = 0 \text{ to } n: \quad x = (\Delta L)i$

We define:

If $x \geqslant a$ Then STP$A = 1$ Else STP$A = 0$

If $x \geqslant b$ Then STP$B = 1$ Else STP$B = 0$

$$V = R_A - w(x-a)\,\text{STP}A + w(x-b)\,\text{STP}B$$

$$M = M_A + R_A x - \frac{1}{2}w(x-a)^2\,\text{STP}A + \frac{1}{2}w(x-b)^2\,\text{STP}B$$

Locate and print (x, V) and (x, M).

Program output on next page

PROBLEM 5.C6 *(Continued)*

Program Output

Problem 5.112

$R_A = 29.50$ kips $\quad R_B = 66.50$ kips