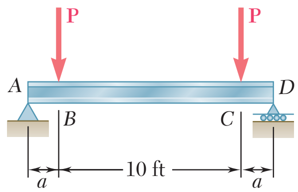
Chapter 8 - Solution Manual-Beer Johnston - Mechanics of Materials 7th c2015

Mechanics of Solids (Ghulam Ishaq Khan Institute of Engineering Sciences and Technology)

CHAPTER 8

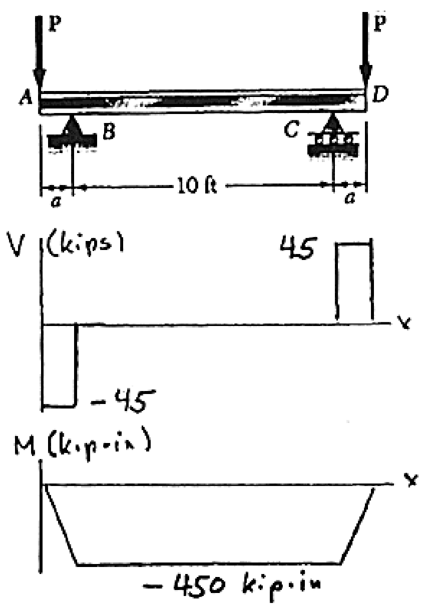
PROBLEM 8.1



A W10×39 rolled-steel beam supports a load **P** as shown. Knowing that $P = 45$ kips, $a = 10$ in., and $\sigma_{all} = 18$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION

$$|V|_{max} = 90 \text{ kips}$$
$$|M|_{max} = (45)(10) = 450 \text{ kip}\cdot\text{in.}$$



For W10 × 39 rolled steel section,

$$d = 9.92 \text{ in.}, \quad b_f = 7.99 \text{ in.}, \quad t_f = 0.530 \text{ in.},$$
$$t_w = 0.315 \text{ in.}, \quad I_x = 209 \text{ in}^4 \quad S_x = 42.1 \text{ in}^3$$
$$c = \frac{1}{2}d = 4.96 \text{ in.} \qquad y_b = c - t_f = 4.43 \text{ in.}$$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450}{42.1} \qquad \sigma_m = 10.69 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.43}{4.96}\right)(10.69) = 9.55 \text{ ksi}$$
$$A_f = b_f t_f = 4.2347 \text{ in}^2$$
$$\bar{y}_f = \frac{1}{2}(c + y_b) = 4.695 \text{ in.}$$
$$Q_b = A_f \bar{y}_f = 19.8819 \text{ in}^3$$
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(45)(19.8819)}{(209)(0.315)} = 13.5898 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 14.4043 \text{ ksi}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 19.18 \text{ ksi} \qquad \sigma_{max} = 19.18 \text{ ksi} \blacktriangleleft$$

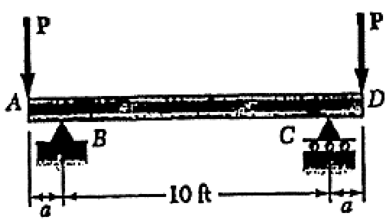
(*c*) Since $\sigma_{max} > \sigma_{all}$ (= 18 ksi), W10 × 39 is <u>not acceptable</u>. ◀

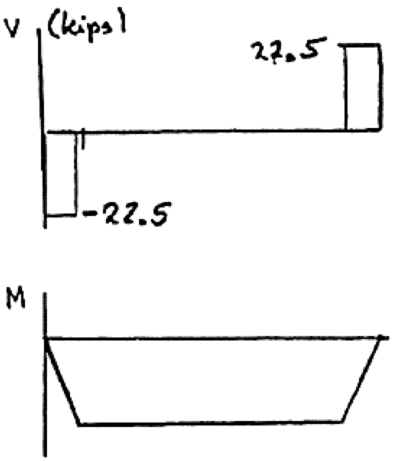
PROBLEM 8.2

Solve Prob. 8.1, assuming that $P = 22.5$ kips and $a = 20$ in.

PROBLEM 8.1 A $W10 \times 39$ rolled-steel beam supports a load **P** as shown. Knowing that $P = 45$ kips, $a = 10$ in., and $\sigma_{all} = 18$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION





$$|V|_{max} = 22.5 \text{ kips}$$

$$|M|_{max} = (22.5)(20) = 450 \text{ kip} \cdot \text{in.}$$

For $W10 \times 39$ rolled steel section,

$$d = 9.92 \text{ in.,} \qquad b_f = 7.99 \text{ in.,} \qquad t_f = 0.530 \text{ in.,}$$

$$t_w = 0.315 \text{ in.,} \qquad I_x = 209 \text{ in}^4, \qquad S_x = 42.1 \text{ in}^3$$

$$c = \frac{1}{2}d = 4.96 \text{ in.} \qquad y_b = c - t_f = 4.43 \text{ in.}$$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450}{42.1}$$ $\sigma_m = 10.69$ ksi ◀

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.43}{4.96}\right)(10.69) = 9.55 \text{ ksi}$$

$$A_f = b_f t_f = 4.2347 \text{ in}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 4.695 \text{ in.}$$

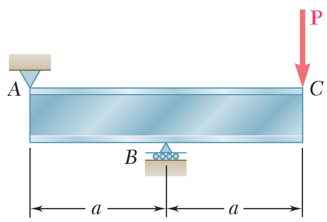
$$Q_b = A_f \bar{y}_f = 19.8819 \text{ in}^3$$

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(22.5)(19.8819)}{(209)(0.315)} = 6.7949 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 8.3049 \text{ ksi}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 13.08 \text{ ksi}$$ $\sigma_{max} = 13.08$ ksi ◀

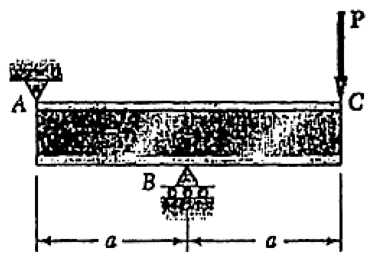
(*c*) Since $\sigma_{max} < \sigma_{all}$ $(= 18 \text{ ksi})$, $W10 \times 39$ is acceptable. ◀



PROBLEM 8.3

An overhanging $W920 \times 449$ rolled-steel beam supports a load **P** as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{all} = 100$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION



$$|V|_{max} = 700 \text{ kN} = 700 \times 10^3 \text{ N}$$

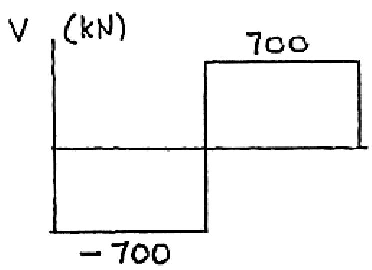
$$|M|_{max} = (700 \times 10^3)(2.5) = 1.75 \times 10^6 \text{ N} \cdot \text{m}$$

For $W920 \times 449$ rolled steel beam,

$$d = 947 \text{ mm}, \quad b_f = 424 \text{ mm}, \quad t_f = 42.7 \text{ mm},$$

$$t_w = 24.0 \text{ mm}, \quad I_x = 8780 \times 10^6 \text{ mm}^4, \quad S_x = 18,500 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 473.5 \text{ mm}, \qquad y_b = c - t_f = 430.8 \text{ mm}$$



(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.75 \times 10^6}{18,500 \times 10^{-6}}$$

$\sigma_m = 94.595$ MPa

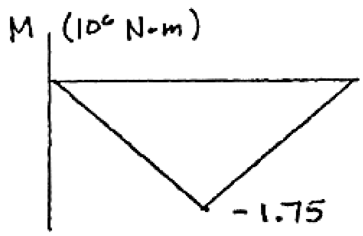
$\sigma_m = 94.6$ MPa ◀

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{430.8}{473.5}(94.595) = 86.064 \text{ MPa}$$

$$A_f = b_f t_t = 18.1048 \times 10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 452.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8186.1 \times 10^3 \text{ mm}^3 = 8186.1 \times 10^{-6} \text{ m}^3$$



$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(700 \times 10^3)(8186.1 \times 10^{-6})}{(8780 \times 10^{-6})(24.0 \times 10^{-3})} = 27.194 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{86.064}{2}\right)^2 + 27.194^2} = 50.904 \text{ MPa}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 93.9 \text{ MPa}$$

$\sigma_{max} = 93.9$ MPa ◀

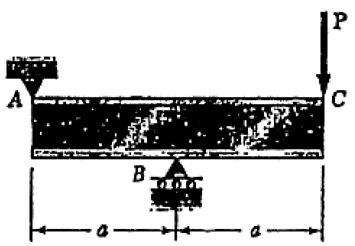
(*c*) Since 94.6 MPa $< \sigma_{all}$ ($= 100$ MPa), $W920 \times 449$ is acceptable. ◀

PROBLEM 8.4

Solve Prob. 8.3, assuming that $P = 850$ kN and $a = 2.0$ m.

PROBLEM 8.3 An overhanging $W920 \times 449$ rolled-steel beam supports a load **P** as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{all} = 100$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION



$$|V|_{max} = 850 \text{ kN} = 850 \times 10^3 \text{ N}$$

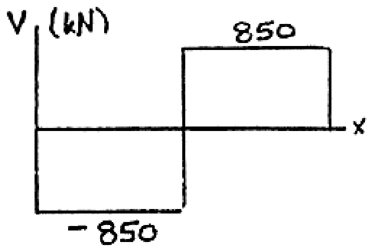
$$|M|_{max} = (850 \times 10^3)(2.0) = 1.70 \times 10^6 \text{ N} \cdot \text{m}$$

For $W920 \times 449$ rolled steel section,

$$d = 947 \text{ mm}, \quad b_f = 424 \text{ mm}, \quad t_f = 42.7 \text{ mm},$$

$$t_w = 24.0 \text{ mm}, \quad I_x = 8780 \times 10^6 \text{ mm}^4, \quad S_x = 18{,}500 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 473.5 \text{ mm} \qquad y_b = c - t_f = 430.8 \text{ mm}$$



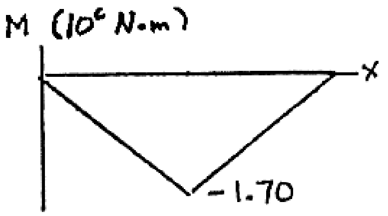
(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.70 \times 10^6}{18{,}500 \times 10^{-6}}$$

$\sigma_m = 91.892$ MPa

$\sigma_m = 91.9$ MPa ◀

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{430.8}{473.5}(91.892) = 83.605 \text{ MPa}$$

$$A_f = b_f t_f = 18.1048 \times 10^3 \text{ mm}^2$$



$$\bar{y}_f = \frac{1}{2}(c + y_b) = 452.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8186.1 \times 10^3 \text{ mm}^3 = 8186.1 \times 10^{-6} \text{ m}^3$$

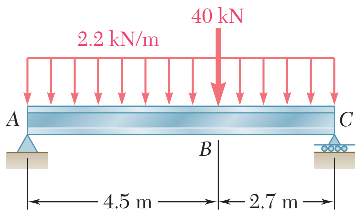
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(850 \times 10^3)(8186.1 \times 10^{-6})}{(8780 \times 10^{-6})(24.0 \times 10^{-3})} = 33.021 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{83.605}{2}\right)^2 + 33.021^2} = 53.271 \text{ MPa}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 95.1 \text{ MPa}$$

$\sigma_{max} = 95.1$ MPa ◀

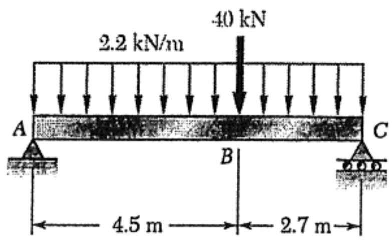
(*c*) Since 95.1 MPa $< \sigma_{all}$ $(= 100$ MPa$)$, $W920 \times 449$ is <u>acceptable</u>. ◀

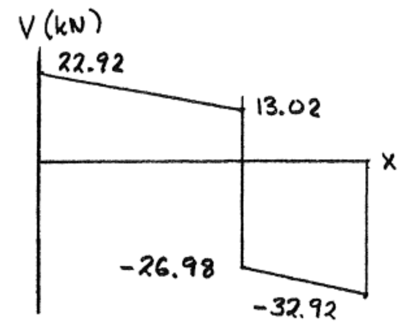


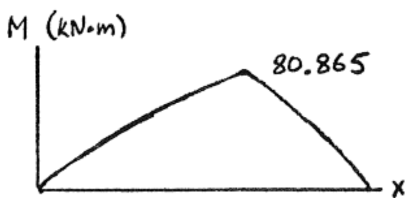
PROBLEM 8.5

(a) Knowing that $\sigma_{\text{all}} = 160$ MPa and $\tau_{\text{all}} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m, τ_m, and the principal stress $\sigma_{\max}$ at the junction of a flange and the web of the selected beam.

SOLUTION







$$+\sum M_C = 0: \quad -7.2R_A + (2.2)(7.2)(3.6) + (40)(2.7) = 0$$

$$R_A = 22.92 \text{ kN}$$

$$V_A = R_A = 22.92 \text{ kN}$$

$$V_B^- = 22.92 - (2.2)(4.5) = 13.02 \text{ kN}$$

$$V_B^+ = 13.02 - 40 = -26.98 \text{ kN}$$

$$V_C = -26.98 - (2.2)(2.7) = -32.92 \text{ kN}$$

$$M_A = 0$$

$$M_B = 0 + \frac{1}{2}(22.92 + 13.02)(4.5) = 80.865 \text{ kN} \cdot \text{m}$$

$$M_C = 0$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{80.865 \times 10^3}{165 \times 10^6} = 490 \times 10^{-6} \text{ m}^3$$

$$= 490 \times 10^3 \text{ mm}^3$$

Shape	S (10^3 mm^3)
W360 × 39	578
W310 × 38.7	547 ←
W250 × 44.8	531
W200 × 52	511

(a) Use W310 × 38.7. ◀

$d = 310$ mm $\qquad t_f = 9.65$ mm

$t_w = 5.84$ mm

PROBLEM 8.5 *(Continued)*

(*b*) $$\sigma_m = \frac{M_B}{S} = \frac{80.865 \times 10^3}{547 \times 10^{-6}} = 147.834 \times 10^6 \text{ Pa}$$

$\sigma_m = 147.8 \text{ MPa}$ ◀

$$\tau_m = \frac{|V|_{\max}}{dt_w} = \frac{32.92 \times 10^3}{(310 \times 10^{-3})(5.84 \times 10^{-3})} = 18.1838 \times 10^6 \text{ Pa}$$

$\tau_m = 18.18 \text{ MPa}$ ◀

$$c = \frac{1}{2}d = 155 \text{ mm} \qquad y_b = c - t_f = 155 - 9.65 = 145.35 \text{ mm}$$

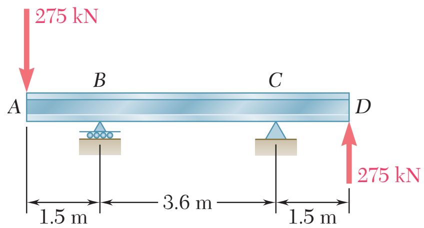
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{145.35}{155}\right)(147.834) = 138.630 \text{ MPa}$$

At point *B*, $$\tau_w = \frac{V}{dt_w} = \frac{(26.98 \times 10^3)}{(310 \times 10^{-3})(5.84 \times 10^{-3})} = 14.9028 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(69.315)^2 + (14.9028)^2} = 70.899 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R = 69.315 + 70.899$$

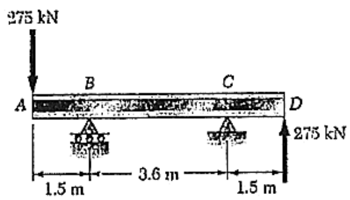
$\sigma_{\max} = 140.2 \text{ MPa}$ ◀

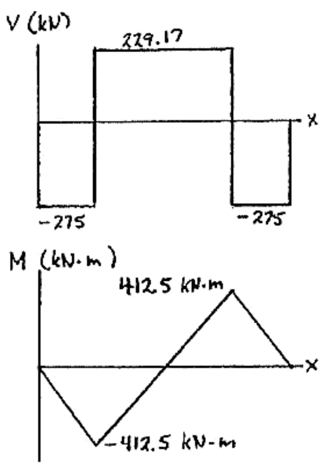


PROBLEM 8.6

(*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION





$$R_B = 504.17 \text{ kN} \uparrow \qquad R_C = 504.17 \downarrow$$

$$|V|_{max} = 275 \text{ kN} \qquad |M|_{max} = 412.5 \text{ kN} \cdot \text{m}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{412.5 \times 10^2}{160 \times 10^6} = 2578 \times 10^{-6} \text{ m}^3$$

$$= 2578 \times 10^3 \text{ mm}^3$$

	Shape	$S_x(10^3 \text{ mm}^3)$
	W760 × 147	4410
→	W690 × 125	3490
	W530 × 150	3720
	W460 × 158	3340
	W360 × 216	3800

(*a*) Use W690 × 125. ◀

$$d = 678 \text{ mm} \quad t_f = 16.3 \text{ mm} \quad t_w = 11.7 \text{ mm}$$

(*b*)

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{412.5 \times 10^3}{3490 \times 10^{-6}} = 118.195 \times 10^6 \text{ Pa} \qquad \sigma_m = 118.2 \text{ MPa} ◀$$

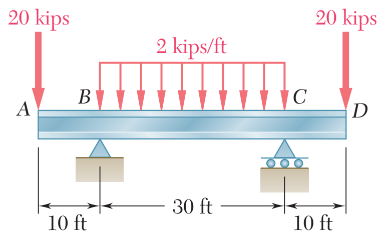
$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{dt_w} = \frac{275 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 34.667 \times 10^6 \text{ Pa} \qquad \tau_m = 34.7 \text{ MPa} ◀$$

$$c = \frac{1}{2}d = \frac{67.8}{2} = 339 \text{ mm}, \quad t_f = 16.3 \text{ mm}, \quad y_b = c - t_f = 339 - 16.3 = 322.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(118.195) = 112.512 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(56.256)^2 + (34.667)^2} = 66.080 \text{ MPa}$$

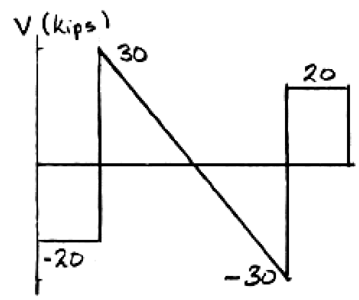
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 56.256 + 66.080 \qquad \sigma_{max} = 122.3 \text{ MPa} ◀$$

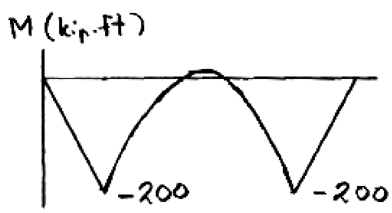


PROBLEM 8.7

(a) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION





$$R_A = 50 \text{ kips}\uparrow \qquad R_D = 50 \text{ kips}\uparrow$$

$$|V|_{max} = 30 \text{ kips} \qquad |M|_{max} = 200 \text{ kip} \cdot \text{ft} = 2400 \text{ kip} \cdot \text{in.}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2400}{24} = 100 \text{ in}^3$$

	Shape	S (in^3)
	W24 × 68	154
→	W21 × 62	127
	W18 × 76	146
	W16 × 77	134
	W12 × 96	131
	W10 × 112	126

(a) Use W21 × 62. ◀

$$d = 21.0 \text{ in.} \quad t_f = 0.615 \text{ in.} \quad t_w = 0.400 \text{ in.}$$

(b)

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{2400}{127} = 18.8976 \text{ ksi} \qquad \sigma_m = 18.9 \text{ ksi} ◀$$

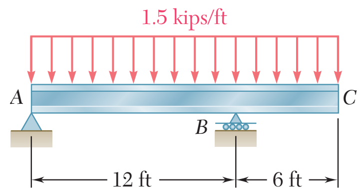
$$\tau_m = \frac{|V|_{max}}{dt_w} = \frac{30}{(21.0)(0.400)} = 3.5714 \text{ ksi} \qquad \tau_m = 3.57 \text{ ksi} ◀$$

$$c = \frac{1}{2}d = \frac{21.0}{2} = 10.50 \text{ in.} \qquad y_b = c - t_f = 10.50 - 0.615 = 9.8850 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{9.8850}{10.50}\right)(18.8976) = 17.7907 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(8.8954)^2 + (3.5714)^2} = 9.5856 \text{ ksi}$$

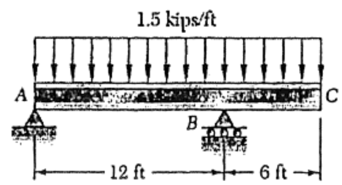
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.8954 + 9.5856 = 18.4810 \text{ ksi} \qquad \sigma_{max} = 18.48 \text{ ksi} ◀$$

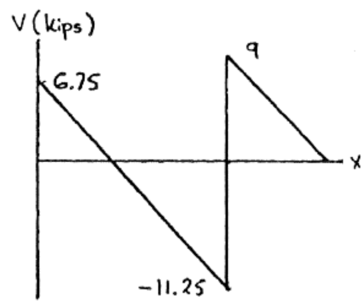


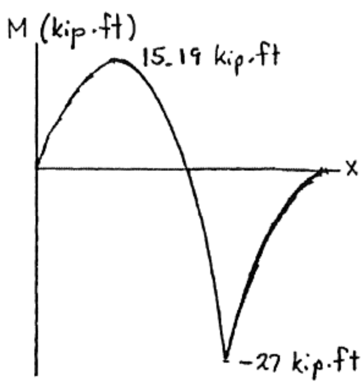
PROBLEM 8.8

(*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION







$$+\circlearrowleft \Sigma M_B = 0: \quad -12R_A + (1.5)(18)(3) = 0 \qquad R_A = 6.75 \text{ kips}\uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 12R_B + (1.5)(18)(9) = 0 \qquad R_B = 20.25 \text{ kips}\uparrow$$

$$|V|_{max} = 11.25 \text{ kips}$$

$$|M|_{max} = 27 \text{ kip}\cdot\text{ft} = 324 \text{ kip}\cdot\text{in.}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{324}{24} = 13.5 \text{ in}^3$$

	Shape	$S(\text{in}^3)$
	W12 × 16	17.1
→	W10 × 15	13.8
	W8 × 18	15.2
	W6 × 20	13.4

(*a*) Use W10 × 15. ◀

$$d = 10.0 \text{ in.}$$

$$t_f = 0.270 \text{ in.}$$

$$t_w = 0.230 \text{ in.}$$

(*b*)

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{324}{13.8} = 23.478 \text{ ksi}$$

$$\sigma_m = 23.5 \text{ ksi} \blacktriangleleft$$

$$\tau_m = \frac{|V|_{max}}{dt_w} = \frac{11.25}{(10.0)(0.230)} = 4.8913 \text{ ksi}$$

$$\tau_m = 4.89 \text{ ksi} \blacktriangleleft$$

$$c = \frac{1}{2}d = \frac{10.0}{2} = 5.00 \text{ in.}$$

PROBLEM 8.8 *(Continued)*

$$y_b = c - t_f = 5.00 - 0.270 = 4.73 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.73}{5.00}\right)(23.478) = 22.210 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{22.210}{2}\right)^2 + (4.8913)^2} = 12.1345 \text{ ksi}$$

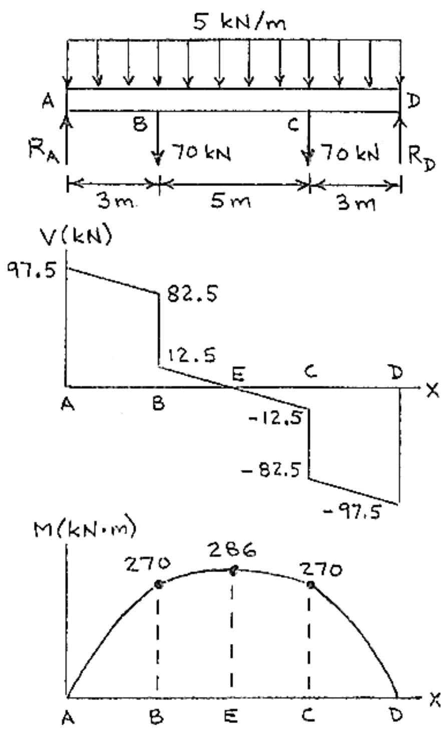
$$\sigma_{\max} = \frac{\sigma_b}{2} + R = \frac{22.210}{2} + 12.1345$$

$\sigma_{\max} = 23.2 \text{ ksi}$ ◀

PROBLEM 8.9

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.73 and selected W530 × 92 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION



Reactions: $R_A = 97.5 \text{ kN} \uparrow \quad R_D = 97.5 \text{ kN} \uparrow$

$|V|_{max} = 97.5 \text{ kN}$

$|M|_{max} = 286 \text{ kN} \cdot \text{m}$

For W530 × 92 rolled-steel section,

$$d = 533 \text{ mm}, \quad b_f = 209 \text{ mm}, \quad t_f = 15.6 \text{ mm},$$

$$t_w = 10.2 \text{ mm}, \quad c = \frac{1}{2}d = 266.5 \text{ mm}$$

$$I = 554 \times 10^6 \text{ mm}^4 \quad S = 2080 \times 10^3 \text{ mm}^3$$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{286 \times 10^3}{2080 \times 10^{-6}} = 137.5 \times 10^6 \text{ Pa}$$

$$\sigma_m = 137.5 \text{ MPa} \blacktriangleleft$$

$$y_b = c - t_f = 250.9 \text{ mm}$$

$$A_f = b_f t_f = 3260.4 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 258.7 \text{ mm}$$

$$Q = A_f \bar{y} = 843.47 \times 10^3 \text{mm}^3$$

At midspan: $V = 0 \quad \tau_b = 0$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{250.9}{266.5}(137.5) = 129.5 \text{ MPa}$$

$$\sigma_{max} = 129.5 \text{ MPa} \blacktriangleleft$$

PROBLEM 8.9 *(Continued)*

At sections B and C:

$$\sigma_m = \frac{M}{S} = \frac{270 \times 10^3}{2080 \times 10^{-6}} = 129.808 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{250.9}{266.5}(129.808) = 122.209 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\overline{y}}{It_w} = \frac{(82.5 \times 10^3)(3260.4 \times 10^{-6})(258.7 \times 10^{-3})}{(554 \times 10^{-6})(10.2 \times 10^{-3})}$$

$$= 12.3143 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 62.333 \text{ MPa}$$

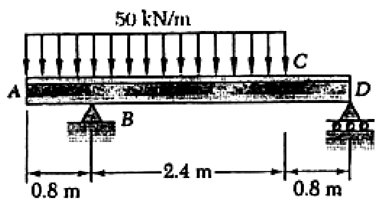
$$\sigma_{\max} = \frac{\sigma_b}{2} + R$$

$\sigma_{\max} = 123.4 \text{ MPa}$ ◀

PROBLEM 8.10

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design (use the loading of Prob. 5.74 and selected W250 × 28.4 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION



From Problem 5.74, $\sigma_{all} = 160$ MPa

$|M|_{max} = 48$ kN · m at section E, which lies 1.6 m to the right of B.

$|V| = 0$

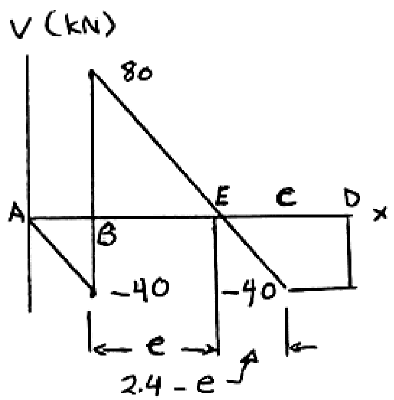
For W250 × 28.4 rolled-steel section,

$d = 259$ mm $\quad b_f = 102$ mm $\quad t_f = 10.0$ mm

$t_w = 6.35$ mm $\quad I = 40.1 \times 10^6$ mm^4

$S = 308 \times 10^3$ mm^3

$$c = \frac{1}{2}d = 129.5 \text{ mm}$$



(*a*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{48 \times 10^3}{308 \times 10^{-6}}$$

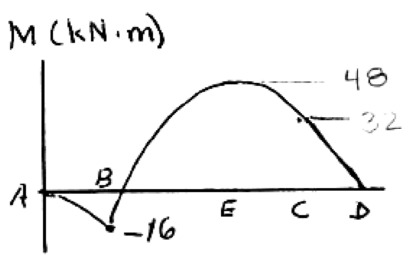
$\sigma_m = 155.8$ MPa ◀

$$y_b = c - t_f = 119.5 \text{ mm}$$

$$A_f = \frac{1}{2}b_f t_f = 1020 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 124.5 \text{ mm}$$

$$Q = A_f\bar{y} = 126.99 \times 10^3 \text{ mm}^3 = 126.99 \times 10^{-6} \text{ m}^3$$



At section E, $\quad V = 0 \quad M = M_{max}$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 143.8 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It_w} = 0$$

(*b*) $\sigma_{max} = \sigma_b$ $\qquad$ $\sigma_{max} = 143.8$ MPa ◀

PROBLEM 8.10 *(Continued)*

At section C,

$$|V| = 40 \text{ kN} \qquad M = 32 \text{ kN} \cdot \text{m}$$

$$\sigma_b = \frac{My_b}{I} = \frac{(32 \times 10^3)(119.5 \times 10^{-3})}{40.0 \times 10^{-6}} = 95.6 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It_w} = \frac{(40 \times 10^3)(126.99 \times 10^{-6})}{(40.1 \times 10^{-6})(6.35 \times 10^{-3})} = 19.95 \text{ MPa}$$

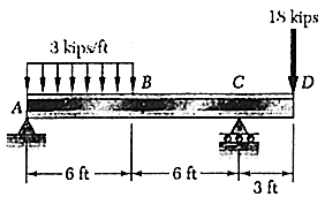
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{47.8^2 + 19.95^2} = 51.08 \text{ MPa}$$

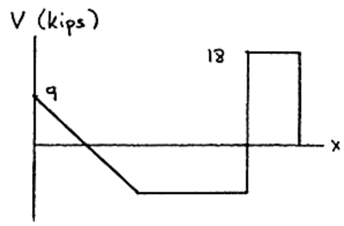
$$\sigma_{\text{max}} = \frac{\sigma_b}{2} + R = 99.6 \text{ MPa} \quad \text{(less than value at section } E\text{)}$$

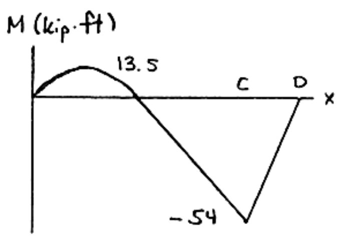
PROBLEM 8.11

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.75 and selected S12 × 31.8 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION







From Prob. 5.75, $\sigma_{all} = 24$ ksi

$|M|_{max} = 54 \text{ kip}\cdot\text{ft} = 648 \text{ kip}\cdot\text{in. at } C.$

At C, $|V| = 18$ kips

For S12×31.8 rolled-steel shape,

$d = 12.0$ in., $b_f = 5.00$ in., $t_f = 0.544$ in.,

$t_w = 0.350$ in. $I_z = 217 \text{ in}^4$, $S_z = 36.2 \text{ in}^3$

$$c = \frac{1}{2}d = 6.00 \text{ in.}$$

$$\sigma_m = \frac{|M|}{S_z} = \frac{648}{36.2} = 17.9006 \text{ ksi}$$

(*a*) $\sigma_m = 17.90$ ksi ◀

$$y_b = c - t_f = 5.456 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 16.2776 \text{ ksi} \qquad \frac{\sigma_b}{2} = 8.1388 \text{ ksi}$$

$$A_f = b_f t_f = 2.72 \text{ in}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 5.728 \text{ in.}$$

$$Q = A_f \bar{y} = 15.5802 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(18)(15.5802)}{(217)(0.350)} = 3.6925 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.1388^2 + 3.6925^2} = 8.9373 \text{ ksi}$$

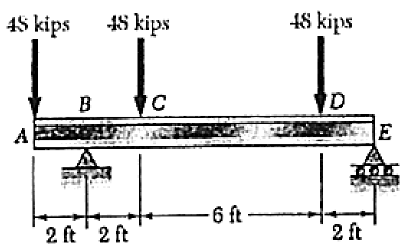
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.1388 + 8.9373$$

(*b*) $\sigma_{max} = 17.08$ ksi ◀

PROBLEM 8.12

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.76 and selected S15 × 42.9 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

SOLUTION



From Prob. 5.76, $\sigma_{\text{all}} = 24$ ksi

$$|M|_{\max} = 96 \text{ kip}\cdot\text{ft} = 1152 \text{ kip}\cdot\text{in. at } D.$$

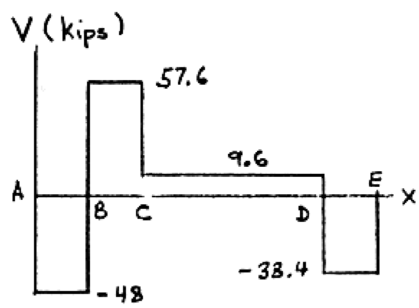
At D, $|V| = 38.4$ kips

For S15×42.9 shape,

$$d = 15.0 \text{ in.}, \qquad b_f = 5.50 \text{ in.} \qquad t_f = 0.622 \text{ in.},$$

$$t_w = 0.411 \text{ in.}, \qquad I_z = 446 \text{ in}^4, \qquad S_z = 59.4 \text{ in}^3$$

$$c = \frac{1}{2}d = 7.5 \text{ in.}$$

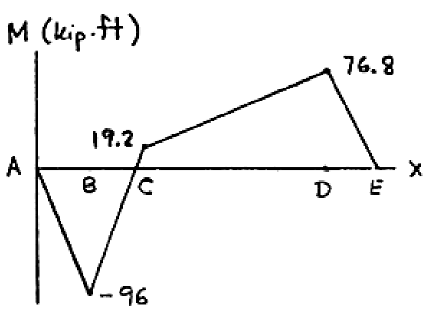


$$\sigma_m = \frac{|M|}{S} = \frac{1152}{59.4} = 19.3939 \text{ ksi} \qquad (a) \qquad \sigma_m = 19.39 \text{ ksi} \blacktriangleleft$$

$$y_b = c - t_f = 6.878 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 17.7855 \text{ ksi} \qquad \frac{\sigma_b}{2} = 8.8928 \text{ ksi}$$

$$A_f = b_f t_f = 3.421 \text{ in}^2$$



$$\bar{y} = \frac{1}{2}(c + y_b) = 7.189 \text{ in.}$$

$$Q = A_f \bar{y} = 24.594 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(57.6)(24.594)}{(446)(0.411)} = 7.7281 \text{ ksi}$$

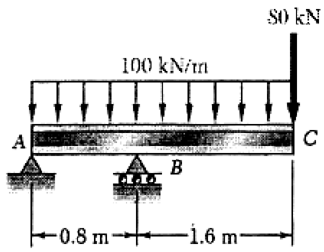
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.8928^2 + 7.7281^2} = 11.7816 \text{ ksi}$$

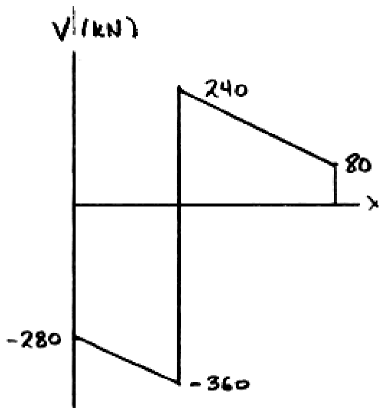
$$\sigma_{\max} = \frac{\sigma_b}{2} + R = 8.8928 + 11.7816 \qquad (b) \qquad \sigma_{\max} = 20.7 \text{ ksi} \blacktriangleleft$$

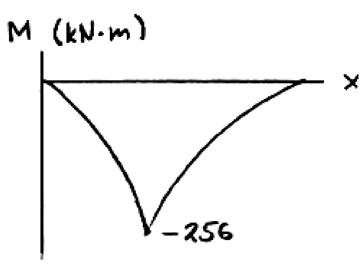
PROBLEM 8.13

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.77 and selected S510 × 98.2 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

SOLUTION







From Problem 5.77, $\sigma_{\text{all}} = 160 \text{ MPa}$

$$|M|_{\max} = 256 \text{ kN} \cdot \text{m at point } B$$

$$|V| = 360 \text{ kN at } B$$

For S510 × 98.2 rolled-steel section,

$$d = 508 \text{ mm}, \quad b_f = 159 \text{ mm}, \quad t_f = 20.2 \text{ mm}$$

$$t_w = 12.8 \text{ mm}, \quad I_x = 495 \times 10^6 \text{ mm}^4, \quad S_x = 1950 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 254 \text{ mm}$$

(*a*) $$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{256 \times 10^3}{1950 \times 10^{-6}} = 131.3 \text{ MPa} \quad ◀$$

$$y_b = c - t_f = 233.8$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 120.9 \text{ MPa} \qquad \frac{\sigma_b}{2} = 60.45 \text{ MPa}$$

$$A_f = b_f t_f = 3212 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 243.9 \text{ mm}$$

$$Q = A_f \bar{y} = 783.4 \times 10^3 \text{ mm}^3$$

$$\tau_b = \frac{VQ}{It_w} = \frac{(360 \times 10^3)(783.4 \times 10^{-6})}{(495 \times 10^{-6})(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$$

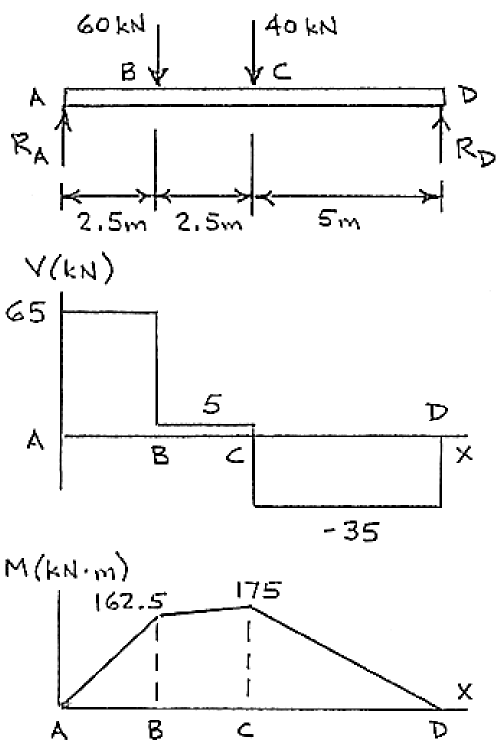
(*b*) $$\sigma_{\max} = \frac{\sigma_b}{2} + R = 60.45 + 75.06 = 135.5 \text{ MPa}$$

$$\sigma_{\max} = 135.5 \text{ MPa} \quad ◀$$

PROBLEM 8.14

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.78 and selected S460 × 81.4 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

SOLUTION



Reactions: $R_A = 65 \text{ kN} \uparrow \qquad R_D = 35 \text{ kN} \uparrow$

$$|V|_{\max} = 65 \text{ kN}$$
$$|M|_{\max} = 175 \text{ kN} \cdot \text{m}$$

For $S460 \times 81.4$ rolled-steel section,

$$d = 457 \text{ mm}, \qquad b_f = 152 \text{ mm}, \qquad t_f = 17.6 \text{ mm},$$
$$t_w = 11.7 \text{ mm}, \qquad c = \frac{1}{2}d = 228.5 \text{ mm}$$
$$I = 333 \times 10^6 \text{ mm}^4 \quad S = 1460 \times 10^3 \text{ mm}^3$$

(*a*) $$\sigma_m = \frac{|M|_{\max}}{S} = \frac{175 \times 10^3}{1460 \times 10^{-6}} = 119.863 \times 10^6 \text{ Pa}$$

$$\sigma_m = 119.9 \text{ MPa} \blacktriangleleft$$

(*b*) $$y_b = c - t_f = 210.9 \text{ mm}$$
$$A_f = b_f t_f = 2675.2 \text{ mm}^2$$
$$\overline{y} = \frac{1}{2}(c + y_b) = 219.7 \text{ mm}$$
$$Q = A_f \overline{y} = 587.74 \times 10^3 \text{ mm}^3$$

At section *C*, $$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{210.9}{228.5}(119.863) = 110.631 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\overline{y}}{It_w} = \frac{(35 \times 10^3)(2675.2 \times 10^{-6})(219.7 \times 10^{-3})}{(333 \times 10^{-6})(11.7 \times 10^{-3})} = 5.2799 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 55.567 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R \qquad \sigma_{\max} = 110.9 \text{ MPa} \blacktriangleleft$$

PROBLEM 8.14 *(Continued)*

At section B,

$$\sigma_m = \frac{M}{S} = \frac{162.5 \times 10^3}{1460 \times 10^{-6}} = 111.301 \text{ MPa}$$

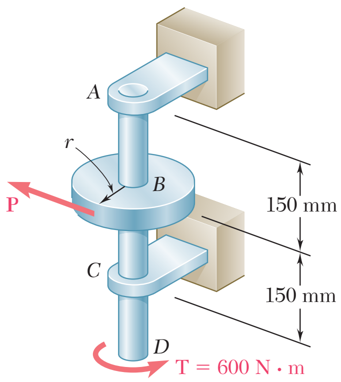
$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{210.9}{228.5}(111.301) = 102.728 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f \bar{y}}{It_w} = \frac{(65 \times 10^3)(2675.2 \times 10^{-6})(219.7 \times 10^{-3})}{(333 \times 10^{-6})(11.7 \times 10^{-3})} = 9.8055 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 52.292 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R$$

$\sigma_{\max} = 103.7 \text{ MPa}$ ◀



PROBLEM 8.15

Determine the smallest allowable diameter of the solid shaft $ABCD$, knowing that $\tau_{all} = 60$ MPa and that the radius of disk B is $r = 80$ mm.

SOLUTION

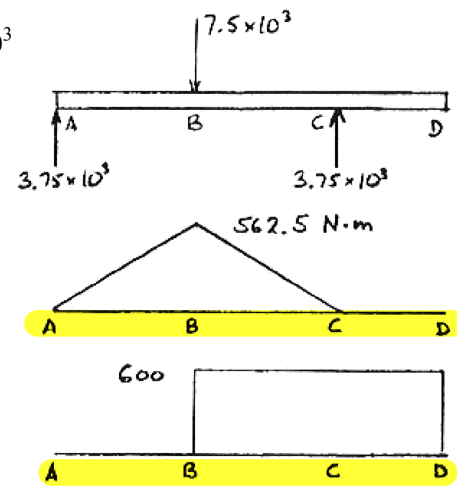
$$\Sigma M_{axis} = 0: \quad T - Pr = 0 \quad P = \frac{T}{r} = \frac{600}{80 \times 10^{-3}} = 7.5 \times 10^3$$

$$R_A = R_C = \frac{1}{2}P$$

$$= 3.75 \times 10^3 \text{N}$$

$$M_B = (3.75 \times 10^3)(150 \times 10^{-3})$$

$$= 562.5 \text{ N} \cdot \text{m}$$



Bending moment: (See sketch).

Torque: (See sketch).

Critical section lies at point B.

$$M = 562.5 \text{ N} \cdot \text{m}, \qquad T = 600 \text{ N} \cdot \text{m}$$

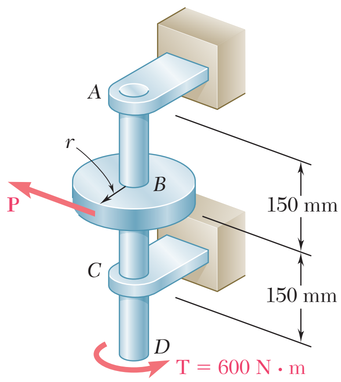
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2 + T^2}\right)_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{(562.5)^2 + (600)^2}}{60 \times 10^6}$$

$$= 8.726 \times 10^{-6} \text{m}^3$$

$$c = 20.58 \times 10^{-3} \text{m} \qquad d = 2c = 41.2 \times 10^{-3} \text{m}$$

$$d = 41.2 \text{ mm} \blacktriangleleft$$



PROBLEM 8.16

Determine the smallest allowable diameter of the solid shaft *ABCD*, knowing that $\tau_{\text{all}} = 60$ MPa and that the radius of disk *B* is $r = 120$ mm.

SOLUTION

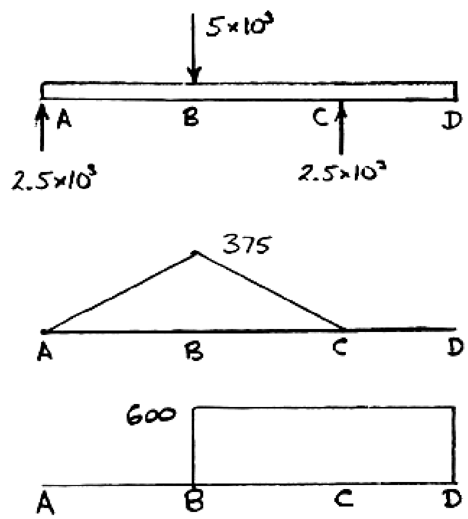
$$\Sigma M_{AD} = 0, \qquad T - Pr = 0 \qquad P = \frac{T}{r} = \frac{600}{120 \times 10^{-3}} = 5 \times 10^3 \text{ N}$$

$$R_A = R_C = \frac{1}{2}P$$

$$= 2.5 \times 10^3 \text{ N}$$

$$M_B = (2.5 \times 10^3)(0.150 \times 10^{-3})$$

$$= 375 \text{ N} \cdot \text{m}$$



Bending moment: (See sketch).

Torque: (See sketch).

Critical section lies at point *B*.

$$M = 375 \text{ N} \cdot \text{m}, \qquad T = 600 \text{ N} \cdot \text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2 + T^2}\right)_{\text{max}}}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{375^2 + 600^2}}{60 \times 10^6} = 7.5073 \times 10^{-6} \text{ m}^3$$

$$c = 19.5807 \times 10^{-3} \text{ m} \qquad d = 2c = 39.2 \times 10^{-3} \text{ m} = 39.2 \text{ mm}$$ ◀

PROBLEM 8.17

Using the notation of Sec. 8.2 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a circular shaft can be expressed as follows:

$$\sigma_{\max} = \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{1/2} + \left(M_y^2 + M_z^2 + T^2\right)^{1/2}\right]_{\max}$$

SOLUTION

Maximum bending stress:

$$\sigma_m = \frac{|M|c}{I} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{I}$$

Maximum torsional stress:

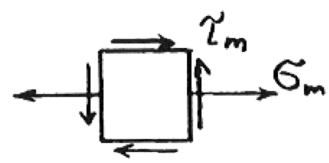
$$\tau_m = \frac{Tc}{J}$$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{2I} = \frac{c}{J}\sqrt{M_y^2 + M_z^2}$$

Using Mohr's circle,

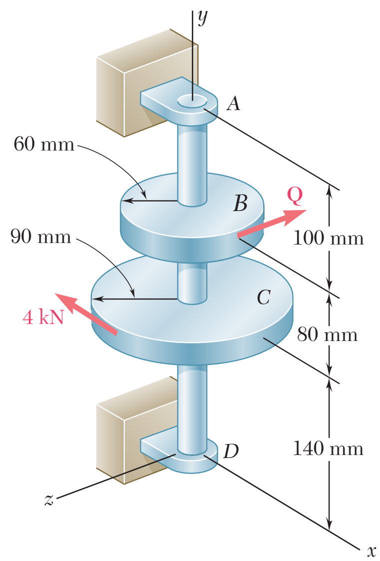
$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2}\left(M_y^2 + M_z^2\right) + \frac{T^2c^2}{J^2}}$$

$$= \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$



$$\sigma_{\max} = \frac{\sigma_m}{2} + R = \frac{c}{J}\sqrt{M_y^2 + M_z^2} + \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$= \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{1/2} + \left(M_y^2 + M_z^2 + T^2\right)^{1/2}\right] \blacktriangleleft$$



PROBLEM 8.18

The 4-kN force is parallel to the x axis, and the force **Q** is parallel to the z axis. The shaft AD is hollow. Knowing that the inner diameter is half the outer diameter and that $\tau_{all} = 60$ MPa, determine the smallest permissible outer diameter of the shaft.

SOLUTION

$$\Sigma M_y = 0: \quad 60\times10^{-3}Q - (90\times10^{-3})(4\times10^3) = 0$$

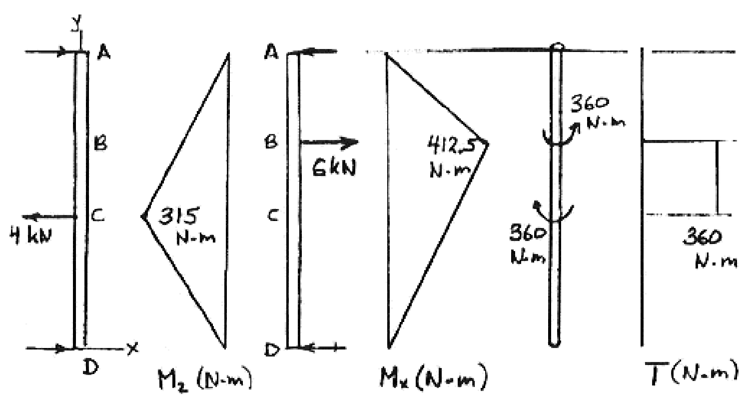
$$Q = 6\times10^3\ \text{N} = 6\ \text{kN}$$

Bending moment and torque diagrams.

In xy plane: $(M_z)_{max} = 315\ \text{N}\cdot\text{m}$ at C.

In yz plane: $(M_x)_{max} = 412.5\ \text{N}\cdot\text{m}$ at B.

About z axis: $T_{max} = 360\ \text{N}\cdot\text{m}$ between B and C.



PROBLEM 8.18 *(Continued)*

At B,

$$M_z = \left(\frac{100}{180}\right)(315) = 175 \text{ N} \cdot \text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{175^2 + 412.5^2 + 360^2} = 574.79 \text{ N} \cdot \text{m}$$

At C,

$$M_x = \left(\frac{140}{220}\right)(412.5) = 262.5 \text{ N} \cdot \text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{315^2 + 262.5^2 + 360^2} = 545.65 \text{ N} \cdot \text{m}$$

Largest value is $574.79 \text{ N} \cdot \text{m}$.

$$\tau_{\max} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}\, c}{J}$$

$$\frac{J}{c} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}}{\tau_{\max}} = \frac{574.79}{60 \times 10^6}$$

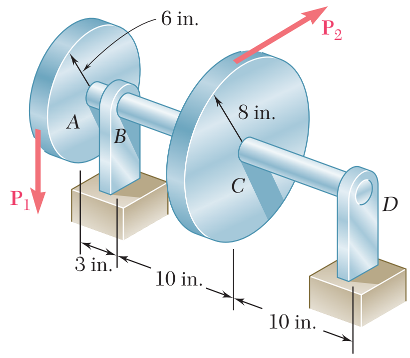
$$= 9.5798 \times 10^{-6} \text{ m}^3 = 9.5798 \times 10^3 \text{ mm}^3$$

$$\frac{J}{c} = \frac{\frac{\pi}{2}\left(c_o^4 - c_i^4\right)}{c_o} = \frac{\pi}{2} c_o^3 \left(1 - \frac{c_i^4}{c_o^4}\right) = \frac{\pi}{2} c_o^3 \left[1 - \left(\frac{1}{2}\right)^4\right]$$

$$= 1.47262 c_o^3$$

$$1.47262 c_o^3 = 9.5798 \times 10^3 \qquad c_o = 18.67 \text{ mm}$$

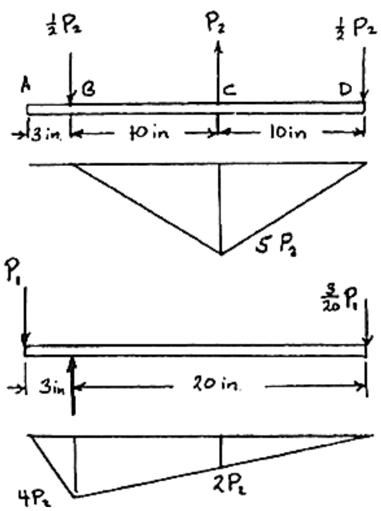
$$d_o = 2c_o \qquad\qquad d_o = 37.3 \text{ mm} \blacktriangleleft$$



PROBLEM 8.19

The vertical force $\mathbf{P_1}$ and the horizontal force $\mathbf{P_2}$ are applied as shown to disks welded to the solid shaft *AD*. Knowing that the diameter of the shaft is 1.75 in. and that $\tau_{all} = 8$ ksi, determine the largest permissible magnitude of the force $\mathbf{P_2}$.

SOLUTION



Let P_2 be in kips.

$$\sum M_{shaft} = 0: \quad 6P_1 - 8P_2 = 0 \qquad P_1 = \frac{4}{3}P_2$$

Torque over portion *ABC*: $\quad T = 8P_2$

Bending in horizontal plane: $\quad M_{Cy} = 10 \cdot \frac{1}{2}P_2 = 5P_2$

Bending in vertical plane: $\quad M_{Bz} = 3P_1$

$$= 3 \cdot \frac{4}{3}P_2$$

$$= 4P_1$$

Critical point is just to the left of point *C*.

$$T = 8P_2 \qquad M_y = 5P_2 \qquad M_z = 2P_2$$

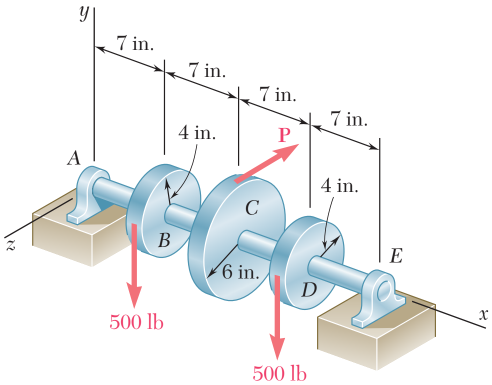
$$d = 1.75 \text{ in.} \qquad c = \frac{1}{2}d = 0.875 \text{ in.}$$

$$J = \frac{\pi}{2}(0.875)^4 = 0.92077 \text{ in}^4$$

$$\tau_{all} = \frac{c}{J}\sqrt{T^2 + M_y^2 + M_z^2}$$

$$8 = \frac{0.875}{0.92077}\sqrt{(8P_2)^2 + (5P_2)^2 + (2P_2)^2} = 9.164\,P_2$$

$P_2 = 0.873$ kips $\qquad\qquad P_2 = 873$ lb ◀

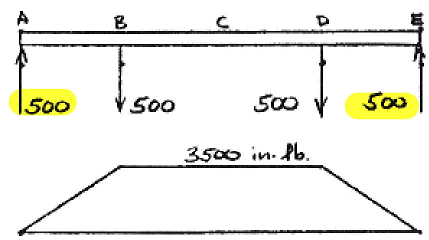


PROBLEM 8.20

The two 500-lb forces are vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{\text{all}} = 8$ ksi, determine the smallest permissible diameter of the solid shaft AE.

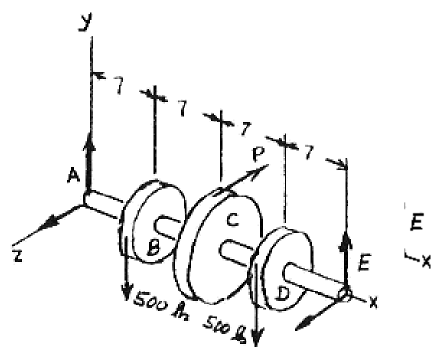
SOLUTION

Forces in vertical plane:



$$\Sigma M_x = 0: \quad (4)(500) - 6P + (4)(500) = 0$$

$$P = 666.67 \text{ lb}$$



Torques:

$$AB: \quad T = 0$$

$$BC: \quad T = -(4)(500) = -2000 \text{ lb}\cdot\text{in.}$$

$$CD: \quad T = 4(500) = 2000 \text{ lb}\cdot\text{in.}$$

$$DE: \quad T = 0$$

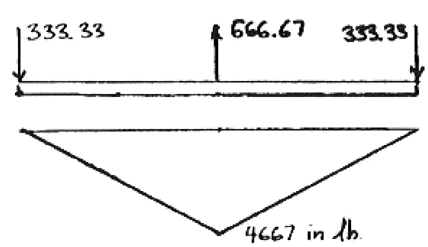
Critical sections are on either side of disk C.

$$T = 2000 \text{ lb}\cdot\text{in.}$$

$$M_z = 3500 \text{ lb}\cdot\text{in.}$$

$$M_y = 4667 \text{ lb}\cdot\text{in.}$$

Forces in horizontal plane:



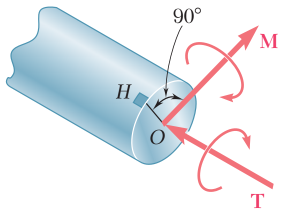
$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{\text{all}}} = \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8 \times 10^3} = 0.77083 \text{ in}^3$$

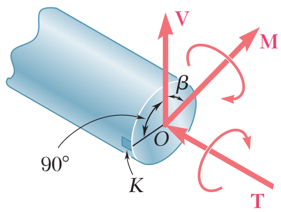
$$c = 0.7888 \text{ in.} \qquad d = 2c$$

$$d = 1.578 \text{ in.} \blacktriangleleft$$

PROBLEM 8.21



(a)



(b)

It was stated in Sec. 8.2 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems, their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point H (Fig. P8.21a) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point K (Fig. P8.21b), where the effect of the shear V is greatest, can be expressed as

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\frac{2}{3}cV + T\right)^2}$$

where β is the angle between the vectors **V** and **M**. It is clear that the effect of the shear V cannot be ignored when $\tau_K \geq \tau_H$. (*Hint:* Only the component of **M** along **V** contributes to the shearing stress at K.)

SOLUTION

Shearing stress at point K.

Due to V: For a semicircle, $\quad Q = \frac{2}{3}c^3$

For a circle cut across its diameter, $\quad t = d = 2c$

For a circular section, $\quad I = \frac{1}{2}J$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)\left(\frac{2}{3}c^3\right)}{\left(\frac{1}{2}J\right)(2c)} = \frac{2}{3}\frac{Vc^2}{J}$$

Due to T: $\quad \tau_{xy} = \frac{Tc}{J}$

Since these shearing stresses have the same orientation,

$$\tau_{xy} = \frac{c}{J}\left(\frac{2}{3}Vc + T\right)$$

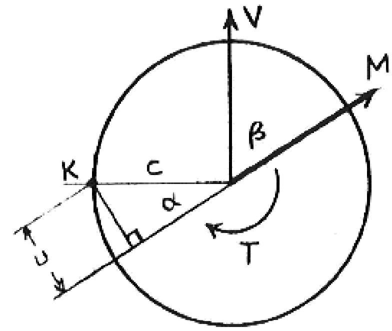
PROBLEM 8.21 *(Continued)*

Bending stress at point K: $\sigma_x = \frac{Mu}{I} = \frac{2Mu}{J}$

Where u is the distance between point K and the neutral axis,

$$u = c\ \sin\,\alpha = c\sin\left(\frac{\pi}{2} - \beta\right) = c\cos\beta$$

$$\sigma_x = \frac{2Mc\cos\beta}{J}$$

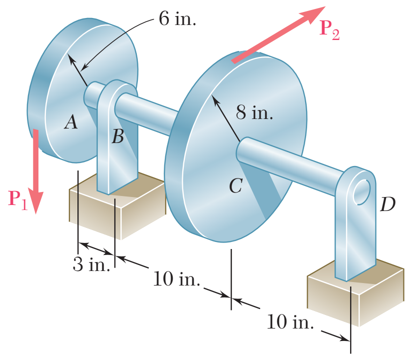


Cross section

Using Mohr's circle,

$$\tau_K = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\frac{2}{3}Vc + T\right)^2}$$



PROBLEM 8.22

Assuming that the magnitudes of the forces applied to disks A and C of Prob. 8.19 are, respectively, $P_1 = 1080$ lb and $P_2 = 810$ lb, and using the expressions given in Prob. 8.21, determine the values τ_H and τ_K in a section (*a*) just to the left of B, (*b*) just to the left of C.

SOLUTION

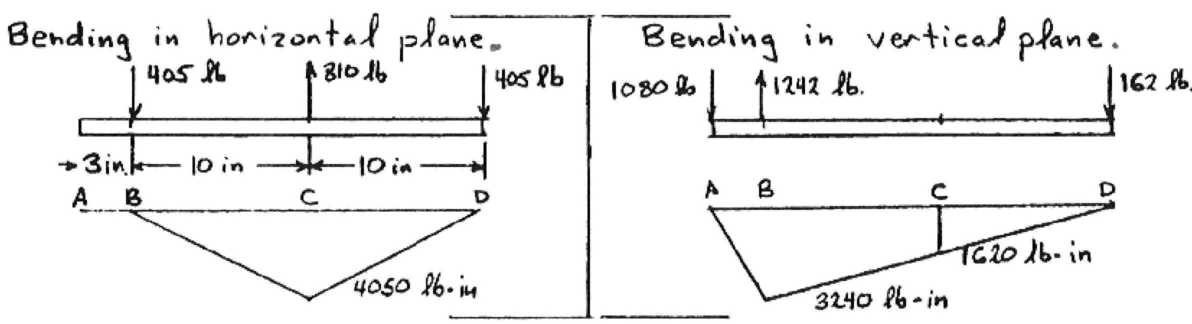
From Prob. 8.19, shaft diameter = 1.75 in.

$$c = \frac{1}{2}d = 0.875 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = 0.92077$$

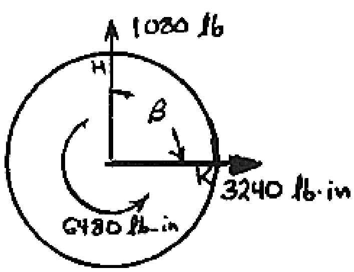
Torque over portion ABC:

$$T = (6)(1080) = (8)(810) = 6480 \text{ lb} \cdot \text{in.}$$



(*a*) Just to the left of point B: $V = 1080$ lb $\quad M = 3240$ lb · in.

$\beta = 90°$ $\quad T = 6480$ lb · in.



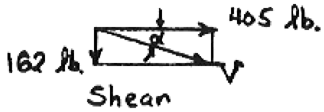
$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2} = \frac{0.875}{0.92077}\sqrt{(3240)^2 + (6480)^2}$$

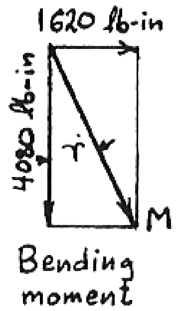
$$\tau_H = 6880 \text{ psi} \blacktriangleleft$$

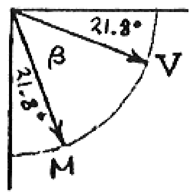
$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\tfrac{2}{3}V_C + T\right)^2} = \frac{c}{J}\left[\frac{2}{3}V_C + T\right]$$

$$= \frac{0.875}{0.92077}\left[\left(\frac{2}{3}\right)(1080)(0.875) + 6480\right] \qquad \tau_K = 6760 \text{ psi} \blacktriangleleft$$

(*b*) Just to the left of point *C*:







$$V = \sqrt{(162)^2 + (405)^2} = 436.2 \text{ lb}$$

$$\alpha = \tan^{-1}\frac{162}{405} = 21.80°$$

$$M = \sqrt{(1620)^2 + (4050)^2} = 4362 \text{ lb} \cdot \text{in.}$$

$$\gamma = \tan^{-1}\frac{1620}{4050} = 21.80°$$

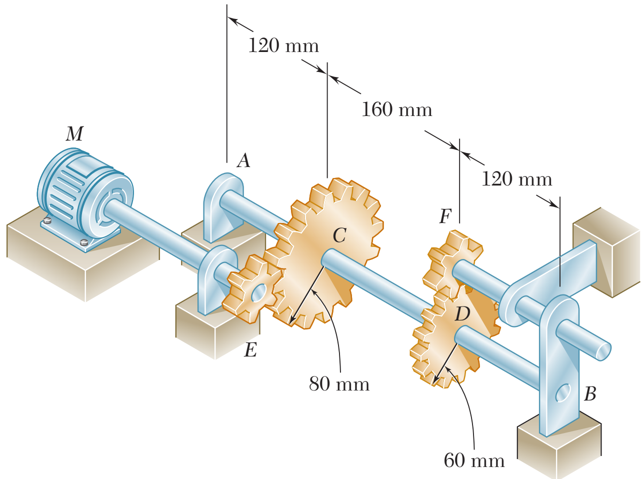
$$\beta = 90° - 21.8° - 21.8° = 46.4°$$

$$\tau_H = \frac{0.875}{0.92077}\sqrt{(6480)^2 + (4362)^2} \qquad \tau_H = 7420 \text{ psi} \blacktriangleleft$$

$$\frac{2}{3}V_C + T = \left(\frac{2}{3}\right)(436.2)(0.875) + 6480 = 6734 \text{ lb} \cdot \text{in.}$$

$$M\cos\beta = 4362\cos 46.4° = 3008 \text{ lb} \cdot \text{in.}$$

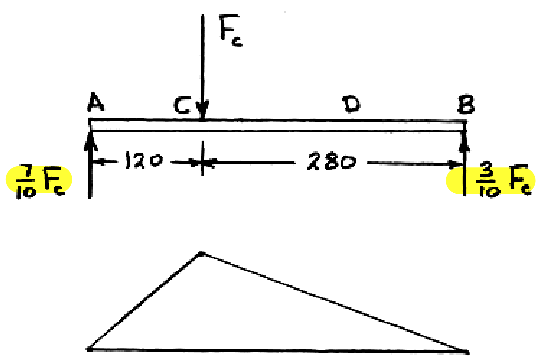
$$\tau_K = \frac{0.875}{0.92077}\sqrt{(3008)^2 + (6734)^2} \qquad \tau_K = 7010 \text{ psi} \blacktriangleleft$$

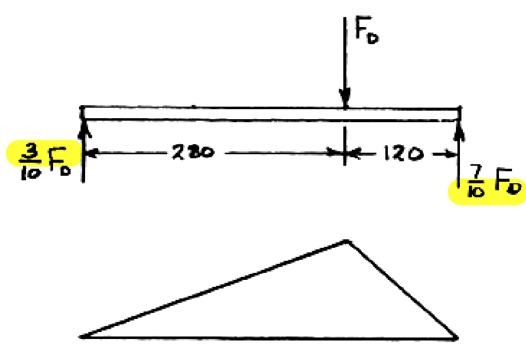


PROBLEM 8.23

The solid shaft AB rotates at 600 rpm and transmits 80 kW from the motor M to a machine tool connected to gear F. Knowing that $\tau_{\text{all}} = 60$ MPa, determine the smallest permissible diameter of shaft AB.

SOLUTION





$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{80 \times 10^3}{(2\pi)(10)} = 1273.24 \text{ N}\cdot\text{m}$$

Gear C: $$F_C = \frac{T}{r_C}$$

$$F_C = \frac{1273.24}{80 \times 10^{-3}} = 15.9155 \times 10^3 \text{N}$$

Gear D: $$F_D = \frac{T}{r_D}$$

$$F_D = \frac{1273.24}{60 \times 10^{-3}} = 21.221 \times 10^3 \text{ N}$$

Forces in vertical plane:

$$M_{Cz} = (120 \times 10^{-3})\left(\frac{7}{10}F_c\right) = 1336.90 \text{ N}\cdot\text{m}$$

$$M_{Dz} = \frac{120}{280}M_{Cz} = 572.96 \text{ N}\cdot\text{m}$$

Forces in horizontal plane:

$$M_{Dy} = (120 \times 10^{-3})\left(\frac{7}{10}F_D\right) = 1782.56 \text{ N.m}$$

$$M_{Cy} = \frac{120}{280}M_{Dy} = 763.95 \text{ N}\cdot\text{m}$$

PROBLEM 8.23 *(Continued)*

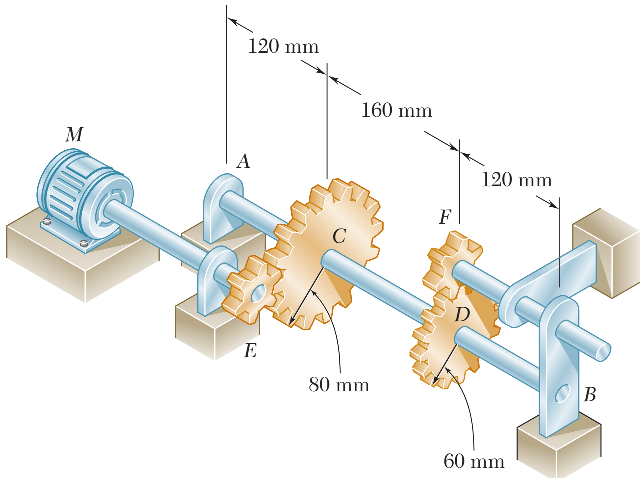
At C, $\sqrt{M_y^2 + M_z^2 + T^2} = 1998.01\ \text{N}\cdot\text{m}$

At D, $\sqrt{M_y^2 + M_z^2 + T^2} = 2264.3\ \text{N}\cdot\text{m}$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}}{\tau_{\text{all}}} = \frac{2264.3}{60 \times 10^6} = 37.738 \times 10^{-6}\ \text{m}^3$$

$c = 28.855 \times 10^{-3}\ \text{m} \qquad d = 2c = 57.7 \times 10^{-3}\ \text{m} \qquad d = 57.7\ \text{mm}$ ◄

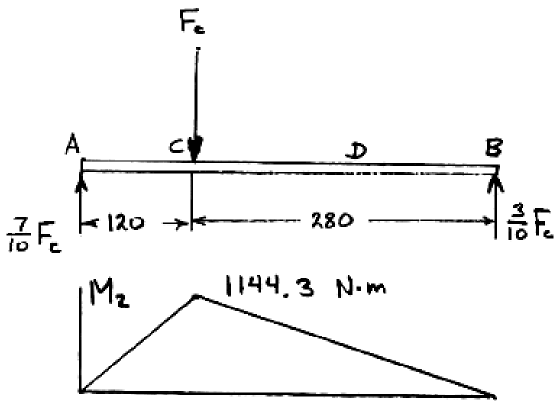


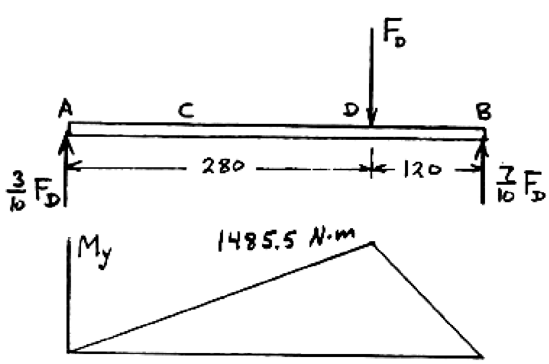
PROBLEM 8.24

Solve Prob. 8.23, assuming that shaft AB rotates at 720 rpm.

PROBLEM 8.23 The solid shaft AB rotates at 600 rpm and transmits 80 kW from the motor M to a machine tool connected to gear F. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft AB.

SOLUTION





$$f = \frac{720 \text{ rpm}}{60 \text{ sec/min}} = 12 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{80 \times 10^3}{(2\pi)(12)} = 1061.03 \text{ N} \cdot \text{m}$$

Gear C: $\quad F_C = \dfrac{I}{r_C}$

$$F_C = \frac{1061.03}{80 \times 10^{-3}} = 13.2629 \times 10^3 \text{ N}$$

Gear D: $\quad F_D = \dfrac{T}{r_D}$

$$F_D = \frac{1061.03}{60 \times 10^{-3}} = 17.6838 \times 10^3 \text{ N}$$

Forces in vertical plane:

$$M_{Cz} = (120 \times 10^{-3})\left(\frac{7}{10}F_C\right) = 1114.08 \text{ N} \cdot \text{m}$$

$$M_{Dz} = \frac{120}{280} M_{Cz} = 477.46 \text{ N.m}$$

Forces in horizontal plane:

$$M_{Dy} = (120 \times 10^{-3})\left(\frac{7}{10}F_D\right) = 1485.44 \text{ N} \cdot \text{m}$$

$$M_{Cy} = \frac{120}{280} M_{Dy} = 636.62 \text{ N} \cdot \text{m}$$

PROBLEM 8.24 *(Continued)*

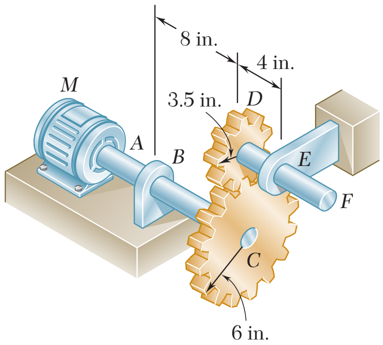
At C, $\sqrt{M_y^2 + M_z^2 + T^2} = 1665.01\ \text{N}\cdot\text{m}$

At D, $\sqrt{M_y^2 + M_z^2 + T^2} = 1886.87\ \text{N}\cdot\text{m}$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\text{max}}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\text{max}}}{\tau_{\text{all}}} = \frac{1886.87}{60 \times 10^6} = 31.448 \times 10^{-6}\ \text{m}^3$$

$c = 27.153 \times 10^{-3}\ \text{m}$ $\quad d = 2c = 54.3 \times 10^{-3}$ $\quad d = 54.3\ \text{mm}$ ◄



PROBLEM 8.25

The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in.} \cdot \text{lb/s}$$

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

(*a*) Shaft *ABC*: $T = \dfrac{P}{2\pi f} = \dfrac{132 \times 10^3}{(2\pi)(4)} = 5252 \text{ lb} \cdot \text{in.}$

Gear *C*: $F_{CD} = \dfrac{T}{r_C} = \dfrac{5252}{6} = 875.4 \text{ lb}$

Bending moment at *B*: $M_B = (8)(875.4) = 7003 \text{ lb} \cdot \text{in.}$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} = \frac{\sqrt{(5252)^2 + (7003)^2}}{7500} = 1.1671 \text{ in}^3$$

$c = 0.9057$ in. $\quad d = 2c$ $\qquad$ $d = 1.811$ in. ◀

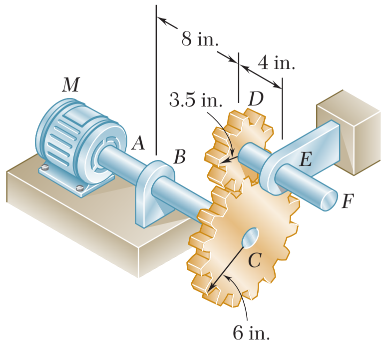
(*b*) Shaft *DEF*: $T = r_D F_{CD} = (3.5)(875.4) = 3064 \text{ lb} \cdot \text{in.}$

Bending moment at *E*: $M_E = (4)(875.4) = 3502 \text{ lb} \cdot \text{in.}$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} = \frac{\sqrt{(3502)^2 + (3064)^2}}{7500} = 0.6204 \text{ in}^3$$

$c = 0.7337$ in. $\quad d = 2c$ $\qquad$ $d = 1.467$ in. ◀



PROBLEM 8.26

Solve Prob. 8.25, assuming that the motor rotates at 360 rpm.

PROBLEM 8.25 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

SOLUTION

$$20 \text{ hp} = (20)(6600) = 132\times 10^3 \text{ in.}\cdot\text{lb/s}$$

$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

(*a*) Shaft *ABC*: $\quad T = \dfrac{P}{2\pi f} = \dfrac{132\times 10^3}{(2\pi)(6)} = 3501 \text{ lb}\cdot\text{in.}$

Gear *C*: $\quad F_{CD} = \dfrac{T}{r_C} = \dfrac{3501}{6} = 583.6 \text{ lb}$

Bending moment at *B*: $\quad M_B = (8)(583.6) = 4669 \text{ lb}\cdot\text{in.}$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{\text{all}}} = \frac{\sqrt{4669^2 + 3501^2}}{7500} = 0.77806 \text{ in}^3$$

$c = 0.791$ in. $\quad d = 2c \qquad\qquad d = 1.582$ in. ◀

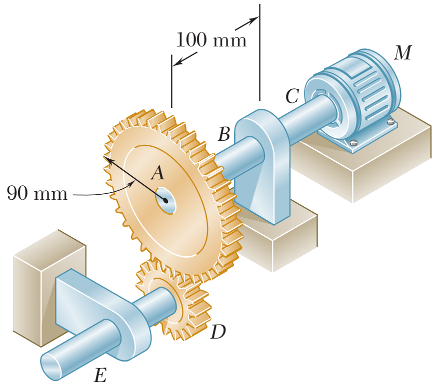
(*b*) Shaft *DEF*: $\quad T = r_D F_{CD} = (3.5)(583.6) = 2043 \text{ lb}\cdot\text{in.}$

Bending moment at *E*: $\quad M_E = (4)(583.6) = 2334 \text{ lb}\cdot\text{in.}$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{\text{all}}} = \frac{\sqrt{2334^2 + 2043^3}}{7500} = 0.41362 \text{ in}^3$$

$c = 0.6410$ in. $\quad d = 2c \qquad\qquad d = 1.282$ in. ◀



PROBLEM 8.27

The solid shaft ABC and the gears shown are used to transmit 10 kW from the motor M to a machine tool connected to gear D. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 60$ MPa, determine the smallest permissible diameter of shaft ABC.

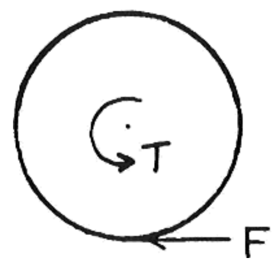
SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/ min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N} \cdot \text{m}$$

Gear A: $\quad Fr_A - T = 0$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$



Bending moment at B:

$$M_B = L_{AB}F = (100 \times 10^{-3})(4421) = 442.1 \text{ N} \cdot \text{m}$$

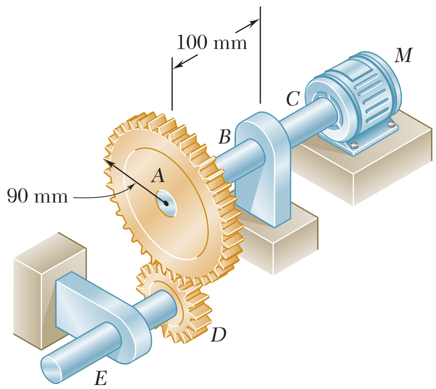
$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} = \frac{(2)\sqrt{442.1^2 + 397.89^2}}{\pi(60 \times 10^6)} = 6.3108 \times 10^{-6} \text{m}^3$$

$$c = 18.479 \times 10^{-3} \text{m} \qquad d = 2c = 37.0 \times 10^{-3} \text{m}$$

$d = 37.0$ mm ◀



PROBLEM 8.28

Assuming that shaft ABC of Prob. 8.27 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.

PROBLEM 8.27 The solid shaft ABC and the gears shown are used to transmit 10 kW from the motor M to a machine tool connected to gear D. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 60$ MPa, determine the smallest permissible diameter of shaft ABC.

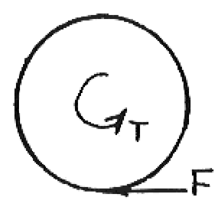
SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N} \cdot \text{m}$$

Gear A: $\quad Fr_A - T = 0$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$



Bending moment at B:

$$M_B = L_{AB}F = (100 \times 10^{-3})(4421) = 442.1 \text{ N} \cdot \text{m}$$

$$\tau_{\text{all}} = \frac{c_o}{J}\sqrt{M^2 + T^2} \qquad c_o = \frac{1}{2}d_o = 25 \times 10^{-3} \text{ m}$$

$$\frac{J}{c_o} = \frac{\pi}{2}\frac{(c_o^4 - c_i^4)}{c_o} = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}}$$

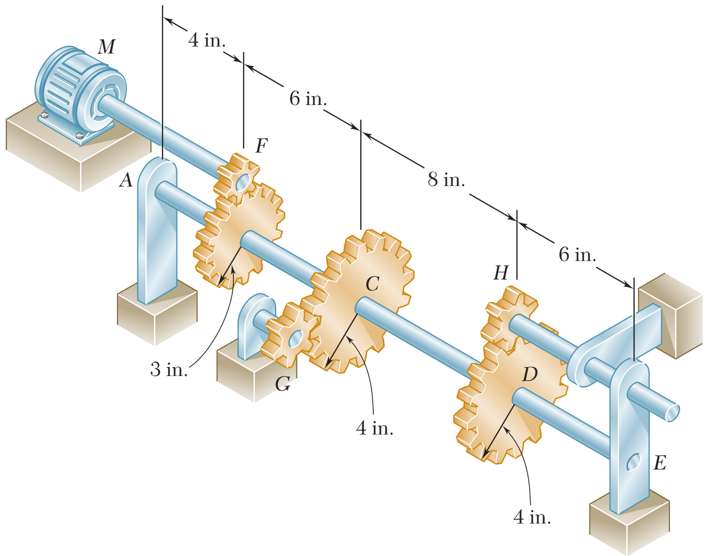
$$c_i^4 = c_o^4 - \frac{2c_o\sqrt{M^2 + T^2}}{\pi\tau_{\text{all}}}$$

$$= (25 \times 10^{-3})^4 - \frac{(2)(25 \times 10^{-3})\sqrt{442.1^2 + 397.89^2}}{\pi(60 \times 10^6)}$$

$$= 390.625 \times 10^{-9} - 157.772 \times 10^{-9} = 232.85 \times 10^{-9}$$

$$c_i = 21.967 \times 10^{-3} \text{ m} \qquad d_i = 2c_i = 43.93 \times 10^{-3} \text{ m}$$

$d = 43.9$ mm ◀



PROBLEM 8.29

The solid shaft *AE* rotates at 600 rpm and transmits 60 hp from the motor *M* to machine tools connected to gears *G* and *H*. Knowing that $\tau_{\text{all}} = 8\text{ ksi}$ and that 40 hp is taken off at gear *G* and 20 hp is taken off at gear *H*, determine the smallest permissible diameter of shaft *AE*.

SOLUTION

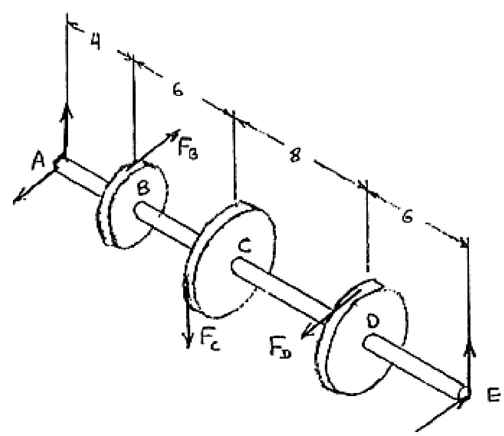
$$60\text{ hp} = (60)(6600) = 396 \times 10^3\text{ in.}\cdot\text{lb/sec}$$

$$f = \frac{600\text{ rpm}}{60\text{ sec/min}} = 10\text{ Hz}$$

Torque on gear *B*:

$$T_B = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(10)} = 6302.5\text{ lb}\cdot\text{in.}$$

Torques on gears *C* and *D*:



$$T_C = \frac{40}{60}T_B = 4201.7\text{ lb}\cdot\text{in.}$$

$$T_D = \frac{20}{60}T_B = 2100.8\text{ lb}\cdot\text{in.}$$

Shaft torques:

$$AB:\quad T_{AB} = 0$$
$$BC:\quad T_{BC} = 6302.5\text{ lb}\cdot\text{in.}$$
$$CD:\quad T_{CD} = 2100.8\text{ lb}\cdot\text{in.}$$
$$DE:\quad T_{DE} = 0$$

Gear forces:

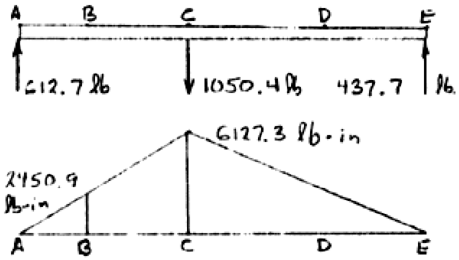
$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8\text{ lb}$$

$$F_C = \frac{T_C}{r_C} = \frac{4201.7}{4} = 1050.4\text{ lb}$$

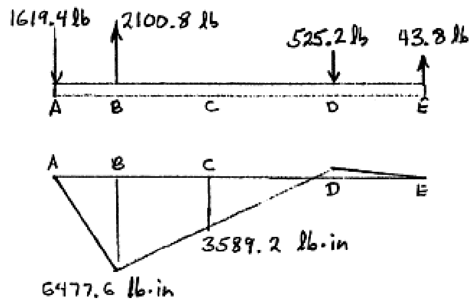
$$F_D = \frac{T_D}{r_D} = \frac{2100.8}{4} = 525.2\text{ lb}$$

PROBLEM 8.29 *(Continued)*

Forces in vertical plane:



Forces in horizontal plane:



At B^+, $\sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{2450.9^2 + 6477.6^2 + 6302.5^2}$

$$= 9364 \text{ lb} \cdot \text{in.}$$

At C^-, $\sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{6127.3^2 + 3589.2^2 + 6302.5^2}$

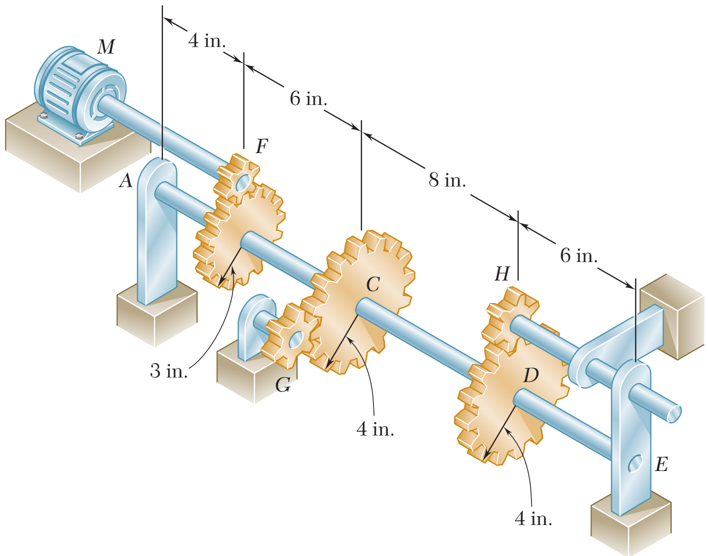
$$= 9495 \text{ lb} \cdot \text{in.} \quad \text{(maximum)}$$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{\text{all}}}$$

$$= \frac{9495}{8 \times 10^3} = 1.1868 \text{ in}^3$$

$c = 0.911$ in. $\quad d = 2c$ $\qquad d = 1.822$ in. ◀



PROBLEM 8.30

Solve Prob. 8.29, assuming that 30 hp is taken off at gear *G* and 30 hp is taken off at gear *H*.

PROBLEM 8.29 The solid shaft *AE* rotates at 600 rpm and transmits 60 hp from the motor *M* to machine tools connected to gears *G* and *H*. Knowing that $\tau_{all} = 8$ ksi and that 40 hp is taken off at gear *G* and 20 hp is taken off at gear *H*, determine the smallest permissible diameter of shaft *AE*.

SOLUTION

$$60 \text{ hp} = (60)(6600)$$

$$= 396 \times 10^3 \text{ in.} \cdot \text{lb/sec}$$

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear *B*:

$$T_B = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(10)} = 6302.5 \text{ lb} \cdot \text{in.}$$

Torques on gears *C* and *D*:

$$T_C = \frac{30}{60} T_B = 3151.3 \text{ lb} \cdot \text{in.}$$

$$T_D = \frac{30}{60} T_B = 3151.3 \text{ lb} \cdot \text{in.}$$

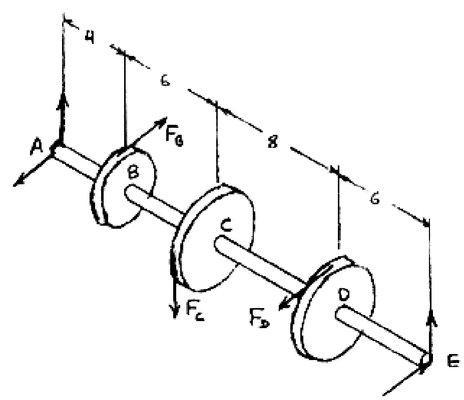
Shaft torques:

$$AB: \quad T_{AB} = 0$$

$$BC: \quad T_{BC} = 6302.5 \text{ lb} \cdot \text{in.}$$

$$CD: \quad T_{CD} = 3151.3 \text{ lb} \cdot \text{in.}$$

$$DE: \quad T_{DE} = 0$$



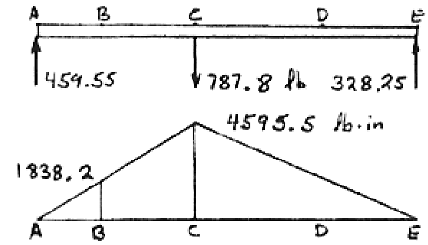
Gear forces:

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb}$$

$$F_C = \frac{T_C}{r_C} = \frac{3151.3}{4} = 787.8 \text{ lb}$$

$$F_D = \frac{T_D}{r_D} = \frac{3151.3}{4} = 787.8 \text{ lb}$$

Forces in vertical plane:



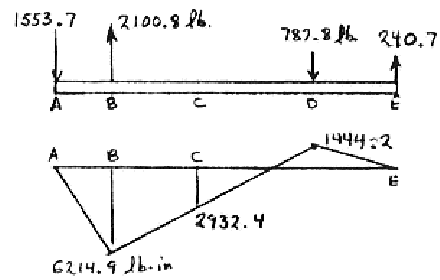
At B^+, $\sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{1838.2^2 + 6214.9^2 + 6302.5^2}$

$= 9040.2 \text{ lb} \cdot \text{in.}$ (maximum)

At C^-, $\sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{4595.5^2 + 2932.4^2 + 6302.5^2}$

$= 8333.0 \text{ lb} \cdot \text{in.}$

Forces in horizontal plane:

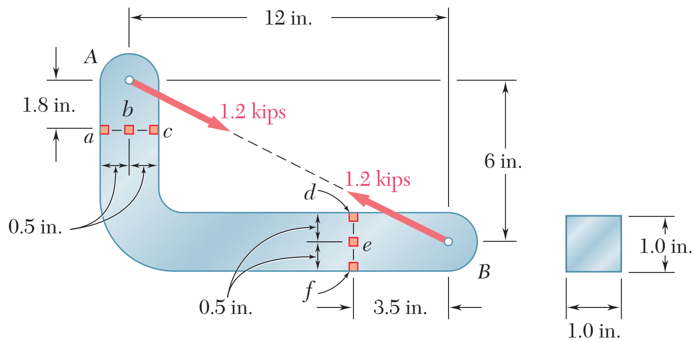


$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{\text{all}}}$$

$$= \frac{9040.3}{8 \times 10^3} = 1.1300 \text{ in}^3$$

$c = 0.8960$ in. $\quad d = 2c \qquad\qquad d = 1.792$ in. ◀



PROBLEM 8.31

Two 1.2-kip forces are applied to an L-shaped machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Let β be the slope angle of line *AB*.

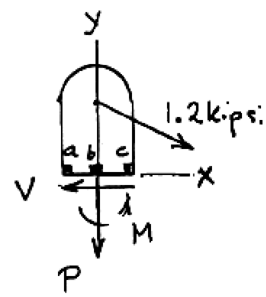
$$\tan\beta = \frac{6}{12} \qquad \beta = 26.565°$$

Draw a free body sketch of the portion of the machine element lying above section <u>*abc*</u>.

$$P = -(1.2)\sin\beta = -0.53666 \text{ kips}$$

$$V = 1.2\cos\beta = 1.07331 \text{ kips}$$

$$M = (1.8)(1.2\cos\beta) = 1.93196 \text{ kip}\cdot\text{in.}$$



Section properties:

$$A = (1.0)^2 = 1.0 \text{ in}^2$$

$$I = \frac{1}{12}(1.0)(1.0)^3 = 0.083333 \text{ in}^4$$

$$c = 0.5 \text{ in.}$$

(*a*) Point *a*:

$$\sigma = \frac{P}{A} - \frac{Mx}{I} = \frac{-0.53666}{1.0} - \frac{(1.93196)(-0.5)}{0.083333}$$

$$\sigma = 11.06 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

(*b*) Point *b*:

$$\sigma = \frac{P}{A} = \frac{-0.53666}{1.0} \qquad \sigma = -0.537 \text{ ksi} \blacktriangleleft$$

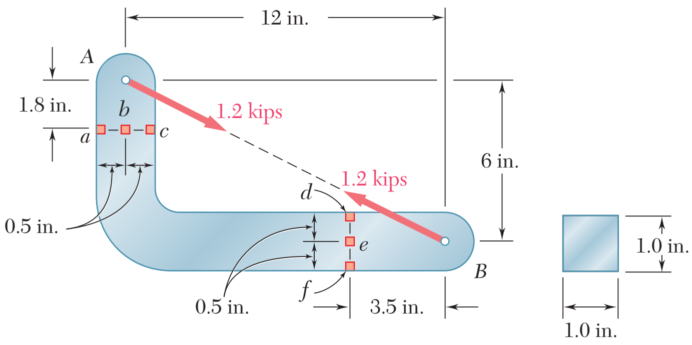
$$Q = (0.5)(1.0)(0.25) = 0.125 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(1.07331)(0.125)}{(0.083333)(1.0)} \qquad \tau = 1.610 \text{ ksi} \blacktriangleleft$$

(*c*) Point *c*:

$$\sigma = \frac{P}{A} - \frac{Mx}{I} = \frac{-0.53660}{1.0} - \frac{(1.93196)(0.5)}{0.083333} \qquad \sigma = -12.13 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$



PROBLEM 8.32

Two 1.2-kip forces are applied to an L-shaped machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *d*, (*b*) point *e*, (*c*) point *f*.

SOLUTION

Let β be the slope angle of line *AB*.

$$\tan\beta = \frac{6}{12} \qquad \beta = 26.535°$$

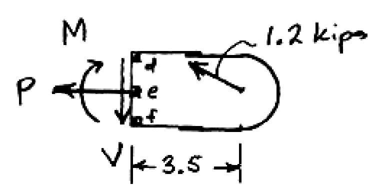
Draw a free body sketch of the portion of the machine element lying to the right of section <u>def</u>.

$$P = -(1.2)\cos\beta$$

$$= -1.07331 \text{ kips}$$

$$V = 1.2\sin\beta = 0.53666 \text{ kips}$$

$$M = (3.5)(1.2\sin\beta) = 1.87831 \text{ kip}\cdot\text{in.}$$



Section properties: $A = (1.0)^2 = 1.0 \text{ in}^2$

$$I = \frac{1}{12}(1.0)(1.0)^3 = 0.083333 \text{ in}^4$$

$$c = 0.5 \text{ in.}$$

(*a*) Point *d*: $\sigma = \dfrac{P}{A} - \dfrac{My}{I} = \dfrac{-1.07331}{1.0} - \dfrac{(1.87831)(0.5)}{0.083333}$

$\sigma = -12.34 \text{ ksi}$ ◀

$\tau = 0$ ◀

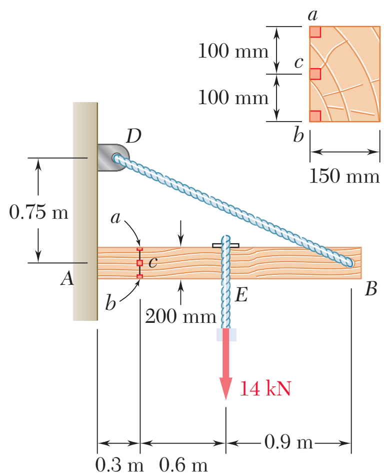
(*b*) Point *e*: $\sigma = \dfrac{P}{A} = \dfrac{-1.07331}{1.0}$ $\sigma = -1.073 \text{ ksi}$ ◀

$$Q = (0.5)(1.0)(0.25) = 0.125 \text{ in}^3$$

$\tau = \dfrac{VQ}{It} = \dfrac{(0.53666)(0.125)}{(0.083333)(1.0)}$ $\tau = 0.805 \text{ ksi}$ ◀

(*c*) Point *f*: $\sigma = \dfrac{P}{A} - \dfrac{My}{I} = \dfrac{-1.07331}{1.0} - \dfrac{(1.87831)(-0.5)}{0.083333}$ $\sigma = 10.20 \text{ ksi}$ ◀

$\tau = 0$ ◀

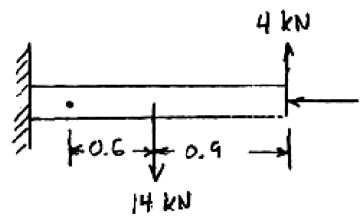


PROBLEM 8.33

The cantilever beam AB has a rectangular cross section of 150×200 mm. Knowing that the tension in the cable BD is 10.4 kN and neglecting the weight of the beam, determine the normal and shearing stresses at the three points indicated.

SOLUTION

$$\overline{DB} = \sqrt{0.75^2 + 1.8^2}$$
$$= 1.95 \text{ m}$$



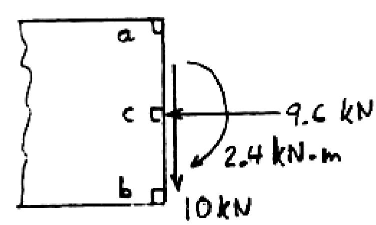
Vertical component of T_{DB}: $\left(\dfrac{0.75}{1.95}\right)(10.4) = 4 \text{ kN}$

Horizontal components of T_{DB}: $\left(\dfrac{1.8}{1.95}\right)(10.4) = 9.6 \text{ kN}$

At section containing points a, b, and c,

$$P = -9.6 \text{ kN} \qquad V = 14 - 4 = 10 \text{ kN}$$

$$M = (1.5)(4) - (0.6)(14) = -2.4 \text{ kN} \cdot \text{m}$$



Section properties:

$$A = (0.150)(0.200) = 0.030 \text{ m}^2$$

$$I = \frac{1}{12}(0.150)(0.200)^3 = 100 \times 10^{-6} \text{ m}^2$$

$$c = 0.100 \text{ m}$$

At point a, $\sigma_x = -\dfrac{P}{A} - \dfrac{Mc}{I} = -\dfrac{9.6 \times 10^3}{0.030} + \dfrac{(2.4 \times 10^3)(0.100)}{100 \times 10^{-6}} = 2.08 \text{ MPa}$ $\qquad \sigma_x = 2.08 \text{ MPa}$ ◀

$\tau_{xy} = 0$ ◀

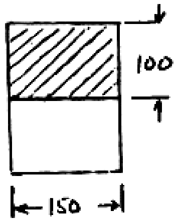
At point b, $\sigma_x = -\dfrac{P}{A} + \dfrac{Mc}{I} = -\dfrac{9.6 \times 10^3}{0.030} - \dfrac{(2.4 \times 10^3)(0.100)}{100 \times 10^{-6}} = -2.72 \text{ MPa}$ $\qquad \sigma_x = -2.72 \text{ MPa}$ ◀

$\tau_{xy} = 0$ ◀

PROBLEM 8.33 *(Continued)*

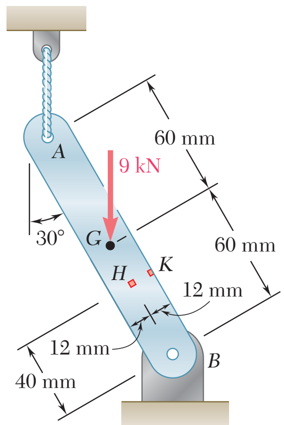
At point c,

$$\sigma_x = -\frac{P}{A} = -\frac{9.6 \times 10^3}{0.030} = -0.320\ \text{MPa} \quad ◄$$



$$Q = (150)(100)(50) = 750 \times 10^3\ \text{mm}^3 = 750 \times 10^{-6}\ \text{m}^3$$

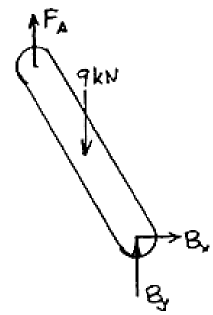
$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(10 \times 10^3)(750 \times 10^{-6})}{(100 \times 10^{-6})(0.150)} = -0.500\ \text{MPa} \quad ◄$$



PROBLEM 8.34

Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (a) point H, (b) point K.

SOLUTION

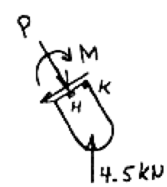


$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(120 \sin 30°) - 9(60 \sin 30°) = 0$$

$$B_y = 4.5 \text{ kN}$$

At the section containing points H and K,



$$P = 4.5 \cos 30° = 3.897 \text{ kN}$$

$$V = 4.5 \sin 30° = 2.25 \text{ kN}$$

$$M = (4.5 \times 10^3)(40 \times 10^{-3} \sin 30°)$$

$$= 90 \text{ N} \cdot \text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{mm}^4$$

$$= 11.52 \times 10^{-9} \text{m}^4$$

(a) <u>At point H,</u>

$$\sigma_x = -\frac{P}{A} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}}$$

$\sigma = -16.24$ MPa ◀

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}}$$

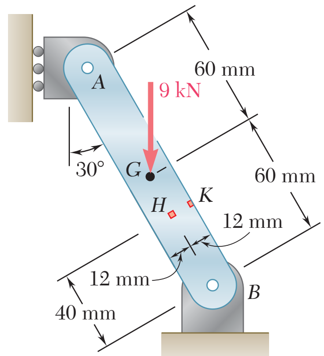
$\tau = 14.06$ MPa ◀

(b) <u>At point K,</u>

$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$$

$\sigma = 110.0$ MPa ◀

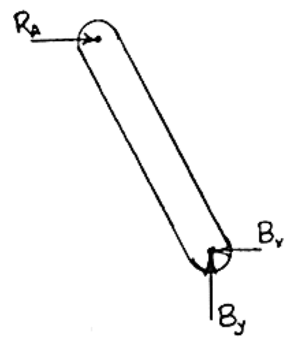
$\tau = 0$ ◀



PROBLEM 8.35

Member *AB* has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (120\cos 30°)R_A - (60\sin 30°)(9) = 0$$

$$R_A = 2.598 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 9 = 0 \qquad B_y = 9 \text{ kN} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2.598 - B_x = 0 \quad B_x = 2.598 \text{ kN} \leftarrow$$

At the section containing points *H* and *K*,

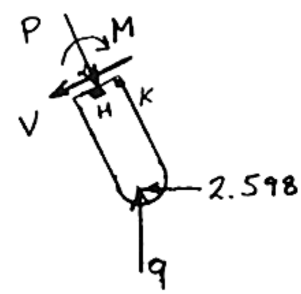
$$P = 9 \cos 30° + 2.598 \sin 30° = 9.093 \text{ kN}$$

$$V = 9 \sin 30° - 2.598 \cos 30° = 2.25 \text{ kN}$$

$$M = (9 \times 10^3)(40 \times 10^{-3} \sin 30°) - (2.598 \times 10^3)(40 \times 10^{-3} \cos 30°) = 90 \text{ N} \cdot \text{m}$$

$$A = 10 \times 240 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{mm}^4 = 11.52 \times 10^{-9} \text{m}^4$$



(*a*) At point *H*,

$$\sigma_x = -\frac{P}{A} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}}$$

$\sigma = -37.9$ MPa ◀

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}}$$

$\tau = 14.06$ MPa ◀

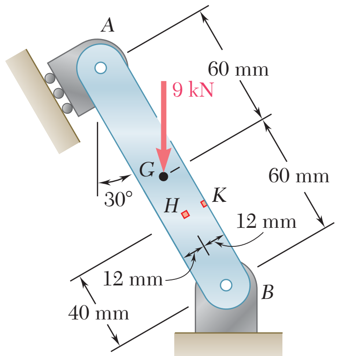
(*b*) At point *K*,

$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I}$$

$$= -\frac{9.093 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$$

$\sigma = -131.6$ MPa ◀

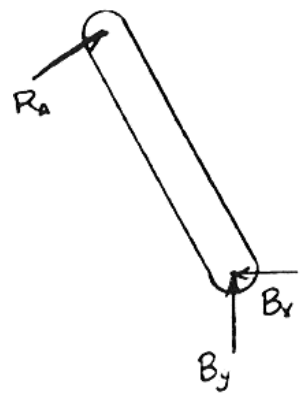
$\tau = 0$ ◀



PROBLEM 8.36

Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point H, (*b*) point K.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (9)(60\sin 30°) - 120R_A = 0$$

$$R_A = 2.25 \text{ kN}$$

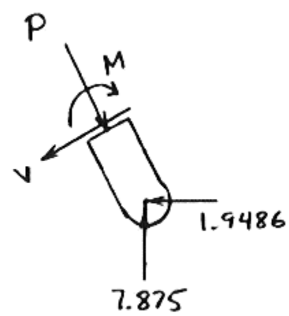
$$\xrightarrow{+} \Sigma F_x = 0: \quad 2.25\cos 30° - B_x = 0$$

$$B_x = 1.9486 \text{ kN} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 2.25\sin 30° - 9 + B_y = 0$$

$$B_y = 7.875 \text{ kN} \uparrow$$

At the section containing points H and K,



$$P = 7.875\cos 30° + 1.9486\sin 30° = 7.794 \text{ kN}$$

$$V = 7.875\sin 30° - 1.9486\cos 30° = 2.25 \text{ kN}$$

$$M = (7.875 \times 10^3)(40 \times 10^{-3}\sin 30°) - (1.9486 \times 10^3)(40 \times 10^{-3}\cos 30°) = 90 \text{ N} \cdot \text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{mm}^4 = 11.52 \times 10^{-9} \text{m}^4$$

(*a*) <u>At point H,</u>

$$\sigma_x = -\frac{P}{A} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} \qquad \sigma_x = -32.5 \text{ MPa} \blacktriangleleft$$

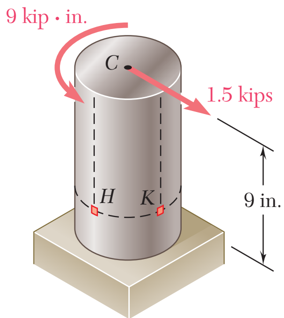
$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}} \qquad \tau_{xy} = 14.06 \text{ MPa} \blacktriangleleft$$

(*b*) <u>At point K,</u>

$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$$

$$\sigma_x = -126.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = 0 \blacktriangleleft$$



PROBLEM 8.37

A 1.5-kip force and a 9-kip · in. couple are applied at the top of the 2.5-in.-diameter cast-iron post shown. Determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION

Diameter $= 2.5$ in.

At the section containing points *H* and *K*,

$$P = 0 \qquad V = 1.5 \text{ kips}$$

$$T = 9 \text{ kip} \cdot \text{in.} \qquad M = (1.5)(9) = 13.5 \text{ kip} \cdot \text{in.}$$

$$d = 2.5 \text{ in.} \qquad c = \frac{1}{2}d = 1.25 \text{ in.}$$

$$A = \pi c^2 = 4.909 \text{ in}^2 \qquad I = \frac{\pi}{4}c^4 = 1.9175 \text{ in}^4 \qquad J = 2I = 3.835 \text{ in}^4$$

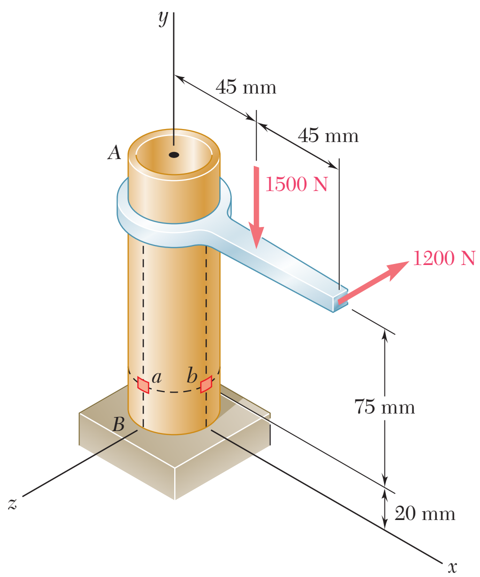
For a semicircle, $$Q = \frac{2}{3}c^3 = 1.3021 \text{ in}^3$$

(*a*) At point *H*, $\sigma_H = 0$ ◀

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(9)(1.25)}{3.835} + \frac{(1.5)(1.3021)}{(1.9175)(2.5)} = 2.934 + 0.407 = 3.34 \text{ ksi} \qquad \tau_H = 3.34 \text{ ksi} ◀$$

(*b*) At point *K*, $$\sigma_K = -\frac{Mc}{I} = -\frac{(13.5)(1.25)}{1.9175} = -8.80 \text{ ksi} \qquad \sigma_K = -8.80 \text{ ksi} ◀$$

$$\tau_K = \frac{Tc}{J} = \frac{(9)(1.25)}{3.835} = 2.93 \text{ ksi} \qquad \tau_K = 2.93 \text{ ksi} ◀$$



PROBLEM 8.38

Two forces are applied to the pipe AB as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

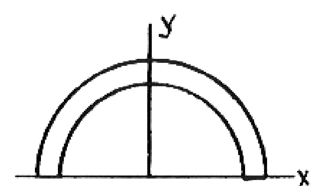
SOLUTION

$$c_o = \frac{d_o}{2} = 21\text{ mm}, \quad c_i = \frac{d_i}{2} = 17.5\text{ mm} \quad A = \pi\left(c_o^2 - c_i^2\right) = 423.33\text{ mm}^2$$

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = 158.166 \times 10^3\text{mm}^4 \quad I = \frac{1}{2}J = 79.083 \times 10^3\text{mm}^4$$

For semicircle with semicircular cutout,

$$Q = \frac{2}{3}\left(c_o^3 - c_i^3\right) = 2.6011 \times 10^3\text{mm}^3$$



At the section containing points *a* and *b*,

$$P = -1500\text{ N} \qquad V_z = -1200\text{ N} \qquad V_x = 0$$

$$M_z = -(45 \times 10^{-3})(1500) = -67.5\text{ N}\cdot\text{m}$$

$$M_x = -(75 \times 10^{-3})(1200) = -90\text{ N}\cdot\text{m}$$

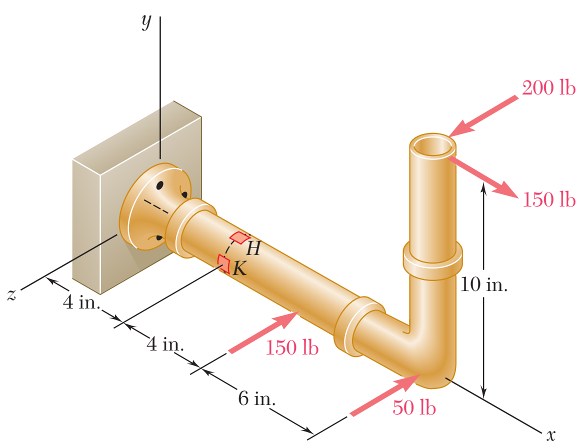
$$T = (90 \times 10^{-3})(1200) = 108\text{ N}\cdot\text{m}$$

(*a*) $$\sigma = \frac{P}{A} - \frac{M_x c}{I} = \frac{-1500}{423.33 \times 10^{-6}} - \frac{(-90)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$ $\sigma = 20.4\text{ MPa}$ ◀

$$\tau = \frac{Tc}{J} + \frac{V_x Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + 0$$ $\tau = 14.34\text{ MPa}$ ◀

(*b*) $$\sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} + \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$ $\sigma = -21.5\text{ MPa}$ ◀

$$\tau = \frac{Tc}{J} + \frac{|V_z| Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + \frac{(1200)(2.6011 \times 10^{-6})}{(79.083 \times 10^{-9})(7 \times 10^{-3})}$$ $\tau = 19.98\text{ MPa}$ ◀

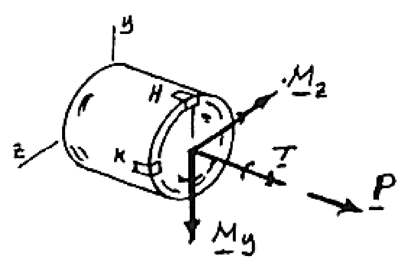


PROBLEM 8.39

Several forces are applied to the pipe assembly shown. Knowing that the pipe has inner and outer diameters equal to 1.61 in. and 1.90 in., respectively, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION

Section properties:



$$P = 150 \text{ lb}$$

$$T = (200 \text{ lb})(10 \text{ in.}) = 2000 \text{ lb}\cdot\text{in.}$$

$$M_z = (150 \text{ lb})(10 \text{ in.}) = 1500 \text{ lb}\cdot\text{in.}$$

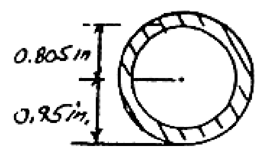
$$M_y = (200 \text{ lb} - 50 \text{ lb})(10 \text{ in.}) - (150 \text{ lb})(4 \text{ in.})$$

$$= 900 \text{ lb}\cdot\text{in.}$$

$$V_z = 200 - 150 - 50 = 0$$

$$V_y = 0$$

$$A = \pi(0.95^2 - 0.805^2) = 0.79946 \text{ in}^2$$



$$I = \frac{\pi}{4}(0.95^4 - 0.805^4) = 0.30989 \text{ in}^4$$

$$J = 2I = 0.61979 \text{ in}^4$$

(*a*) Point *H*:

$$\sigma_H = \frac{P}{A} + \frac{M_z c}{I} = \frac{150 \text{ lb}}{0.79946 \text{ in}^2} + \frac{(1500 \text{ lb}\cdot\text{in.})(0.95 \text{ in.})}{0.30989 \text{ in}^4}$$

$$= 187.6 \text{ psi} + 4593 \text{ psi}$$

$\sigma_H = 4.79$ ksi ◀

$$\tau_H = \frac{Tc}{J} = \frac{(2000 \text{ lb}\cdot\text{in.})(0.95 \text{ in.})}{0.61979 \text{ in}^4} = 3065.6 \text{ psi}$$

$\tau_H = 3.07$ ksi ◀

(*b*) Point *K*:

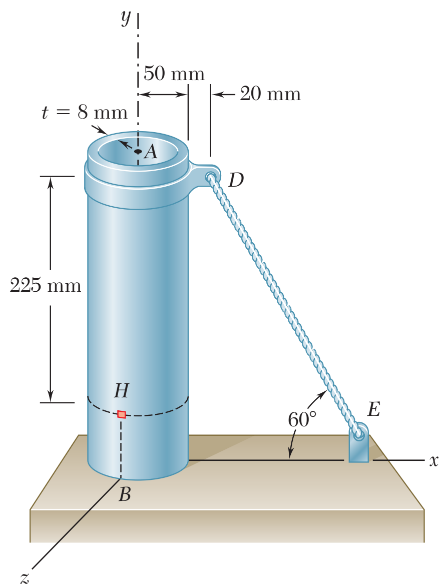
$$\sigma_K = \frac{P}{A} + \frac{M_y c}{I} = \frac{150 \text{ lb}}{0.79946 \text{ in}^2} - \frac{(900 \text{ lb}\cdot\text{in.})(0.95 \text{ in.})}{0.30989 \text{ in}^4}$$

$$= 187.6 \text{ psi} - 2759 \text{ psi}$$

$\sigma_K = -2.57$ ksi ◀

$$\tau_K = \frac{Tc}{J} = \text{same as for } \tau_H$$

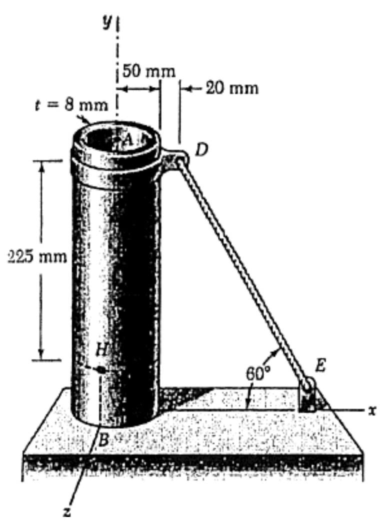
$\tau_K = 3.07$ ksi ◀

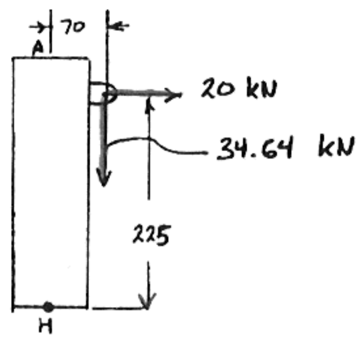


PROBLEM 8.40

The steel pipe AB has a 100-mm outer diameter and an 8-mm wall thickness. Knowing that the tension in the cable is 40 kN, determine the normal and shearing stresses at point H.

SOLUTION





Vertical force:

$40\cos 30° = 34.64$ kN

Horizontal force:

$40\sin 30° = 20$ kN

Point H lies on neutral axis of bending.

Section properties:

$$d_o = 100 \text{ mm}, \qquad c_o = \frac{1}{2}d_o = 50 \text{ mm}, \qquad c_i = c_o - t = 42 \text{ mm},$$

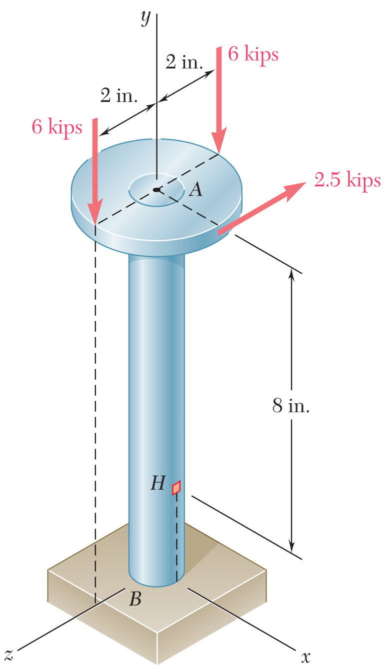
$$A = \pi(c_o^2 - c_i^2) = 2.312 \times 10^3 \text{ mm}^2 = 2.312 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{34.64 \times 10^3}{2.312 \times 10^{-6}}$$

$\sigma = -14.98$ MPa ◀

For thin pipe, $$\tau = 2\frac{V}{A} = \frac{(2)(20 \times 10^3)}{2.314 \times 10^{-3}}$$

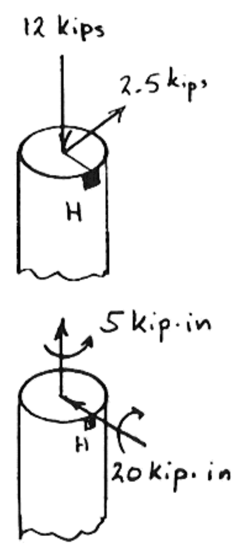
$\tau = 17.29$ MPa ◀



PROBLEM 8.41

Three forces are applied to a 4-in.-diameter plate that is attached to the solid 1.8-in. diameter shaft *AB*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

SOLUTION



At the section containing point *H*,

$$P = 12 \text{ kips} \quad \text{(compression)}$$

$$V = 2.5 \text{ kips}$$

$$T = (2)(2.5) = 5 \text{ kip} \cdot \text{in.}$$

$$M = (8)(2.5) = 20 \text{ kip} \cdot \text{in.}$$

$$d = 1.8 \text{ in.} \quad c = \frac{1}{2}d = 0.9 \text{ in.}$$

$$A = \pi c^2 = 2.545 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 0.5153 \text{ in}^4$$

$$J = 2I = 1.0306 \text{ in}^4$$

For a semicircle,

$$Q = \frac{2}{3}c^3 = 0.486 \text{ in}^3$$

Point *H* lies on neutral axis of bending.

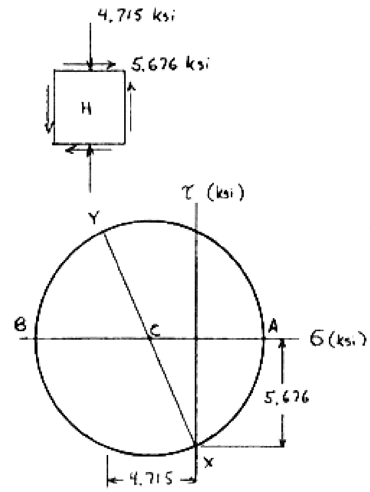
$$\sigma_H = \frac{P}{A} = -\frac{12}{2.545} = -4.715 \text{ ksi}$$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5)(0.9)}{1.0306} + \frac{(2.5)(0.486)}{(0.5153)(1.8)}$$

$$= 5.676 \text{ ksi}$$

PROBLEM 8.41 *(Continued)*

Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}(-4.715)$$

$$= -2.3575 \text{ ksi}$$

$$R = \sqrt{\left(\frac{4.715}{2}\right)^2 + 5.676^2}$$

$$= 6.1461 \text{ ksi}$$

(*a*) $\sigma_a = \sigma_{\text{ave}} + R$ $\qquad \sigma_a = 3.79 \text{ ksi} \blacktriangleleft$

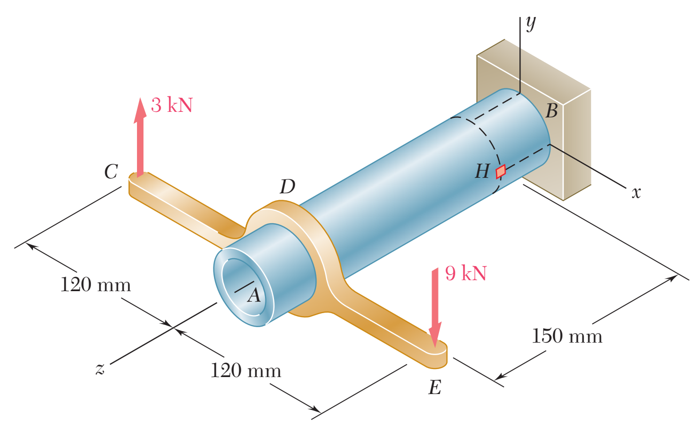
$\sigma_b = \sigma_{\text{ave}} - R$ $\qquad \sigma_b = -8.50 \text{ ksi} \blacktriangleleft$

$$\tan 2\theta_p = \frac{(2)(5.676)}{4.715} = 2.408$$

$\theta_a = 33.7° \blacktriangleleft$

$\theta_b = 123.7° \blacktriangleleft$

(*b*) $\tau_{\max} = R$ $\qquad \tau_{\max} = 6.15 \text{ ksi} \blacktriangleleft$



PROBLEM 8.42

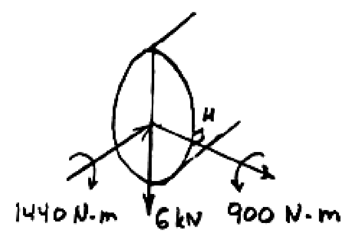
The steel pipe AB has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that the arm CDE is rigidly attached to the pipe, determine the principal stresses, principal planes, and the maximum shearing stress at point H.

SOLUTION

Replace the forces at C and E by an equivalent force-couple system at D.

$$F_D = 9 - 3 = 6 \text{ kN}\downarrow$$

$$T_D = (9\times10^3)(120\times10^{-3}) + (3\times10^3)(120\times10^{-3}) = 1440 \text{ N}\cdot\text{m}$$



At the section containing point H,

$$P = 0 \qquad V = 6 \text{ kN} \qquad T = 1440 \text{ N}\cdot\text{m}$$

$$M = (6\times10^3)(150\times10^{-3}) = 900 \text{ N}\cdot\text{m}$$

Section properties: $\quad d_o = 72 \text{ mm} \quad c_o = \frac{1}{2}d_o = 36 \text{ mm} \quad c_i = c_o - t = 31 \text{ mm}$

$$A = \pi(c_o^2 - c_i^2) = 1.0524\times10^3 \text{ mm}^2 = 1.0524\times10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84\times10^{-3} \text{ mm}^4 = 593.84\times10^{-9} \text{ m}^4$$

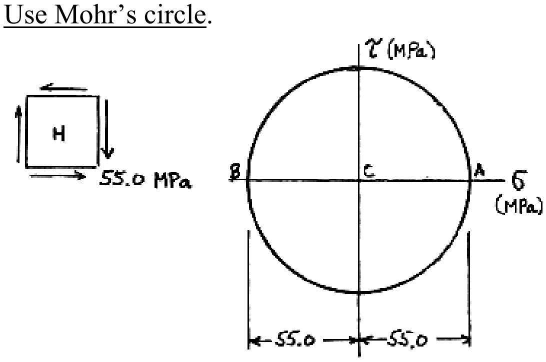
$$J = 2I = 1.1877\times10^{-6} \text{ m}^4$$

For half-pipe, $\quad Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243\times10^3 \text{ mm}^3 = 11.243\times10^{-6} \text{ m}^3$

At point H, point H lies on the neutral axis of bending. $\quad \sigma_H = 0.$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} + \frac{(6\times10^3)(11.243\times10^{-6})}{(593.84\times10^{-9})(10\times10^3)} = 55.0 \text{ MPa}$$

Use Mohr's circle.



$$\sigma_c = 0$$

$$R = 55.0 \text{ MPa}$$

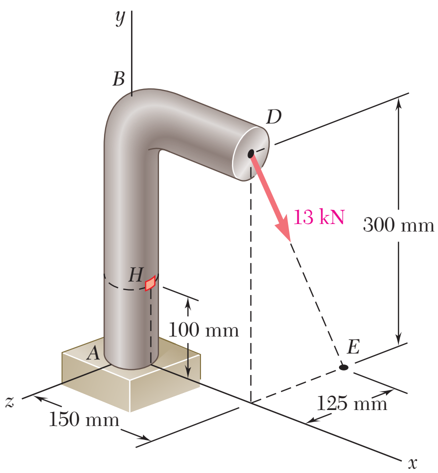
$\sigma_a = \sigma_c + R \qquad \sigma_a = 55.0 \text{ MPa}$ ◀

$\sigma_b = \sigma_c - R \qquad \sigma_b = -55.0 \text{ MPa}$ ◀

$\theta_a = -45°, \theta_b = +45°$ ◀

$\tau_{max} = R \qquad \tau_{max} = 55.0 \text{ MPa}$ ◀

PROBLEM 8.43



A 13-kN force is applied as shown to the 60-mm-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

SOLUTION

$$DE = \sqrt{125^2 + 300^2} = 325 \text{ mm}$$

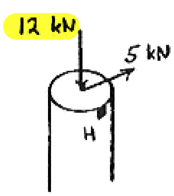
At point *D*, $F_x = 0$

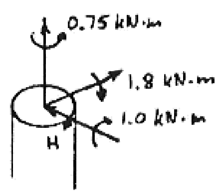
$$F_y = -\left(\frac{300}{325}\right)(13) = -12 \text{ kN}$$

$$F_z = -\left(\frac{125}{300}\right)(13) = -5 \text{ kN}$$

Moment of equivalent force-couple system at *C*, the centroid of the section containing point *H*:

$$\vec{\boldsymbol{M}} = \begin{vmatrix} \vec{\boldsymbol{i}} & \vec{\boldsymbol{j}} & \vec{\boldsymbol{k}} \\ 0.150 & 0.200 & 0 \\ 0 & -12 & -5 \end{vmatrix} = -1.00\vec{\boldsymbol{i}} + 0.75\vec{\boldsymbol{j}} - 1.8\vec{\boldsymbol{k}} \text{ kN} \cdot \text{m}$$





Section properties:

$$d = 60 \text{ mm} \quad c = \frac{1}{2}d = 30 \text{ mm}$$

$$A = \pi c^2 = 2.8274 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 636.17 \times 10^3 \text{ mm}^4$$

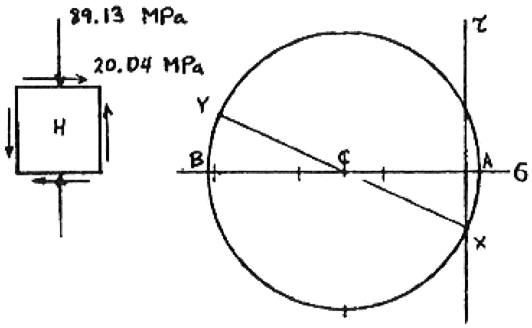
$$J = 2I = 1.2723 \times 10^6 \text{ mm}^4$$

For a semicircle, $Q = \frac{2}{3}c^3 = 18.00 \times 10^3 \text{ mm}^3$

PROBLEM 8.43 *(Continued)*

At point H, $$\sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{12 \times 10^3}{2.8274 \times 10^{-3}} - \frac{(1.8 \times 10^3)(30 \times 10^{-3})}{636.17 \times 10^{-9}} = -89.13 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.75 \times 10^3)(30 \times 10^{-3})}{1.2723 \times 10^{-6}} + \frac{(5 \times 10^3)(18.00 \times 10^{-6})}{(636.17 \times 10^{-9})(60 \times 10^{-3})} = 20.04 \text{ MPa}$$



(*a*) $$\sigma_{\text{ave}} = \frac{\sigma_H}{2} = -44.565 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 48.863 \text{ MPa}$$

$\sigma_a = \sigma_{\text{ave}} + R$ $\qquad \sigma_a = 4.30 \text{ MPa}$ ◀

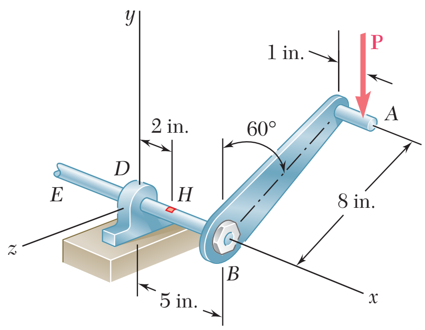
$\sigma_b = \sigma_{\text{ave}} - R$ $\qquad \sigma_b = -93.4 \text{ MPa}$ ◀

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4497$$

$\theta_a = 12.1°, \quad \theta_b = 102.1°$ ◀

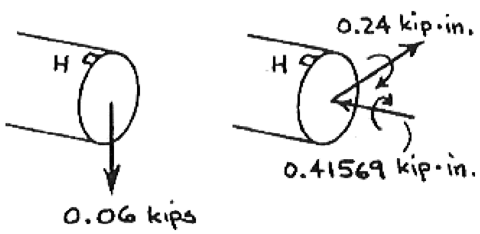
(*b*) $\tau_{\max} = R = 48.9 \text{ MPa}$ ◀

PROBLEM 8.44



A vertical force **P** of magnitude 60 lb is applied to the crank at point A. Knowing that the shaft BDE has a diameter of 0.75 in., determine the principal stresses and the maximum shearing stress at point H located at the top of the shaft, 2 in. to the right of support D.

SOLUTION



Force-couple system at the centroid of the section containing point H:

$$F_x = 0, \quad V_y = -0.06 \text{ kips}, \quad V_z = 0$$

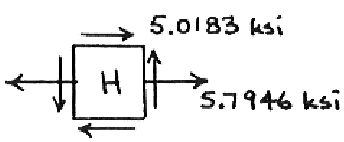
$$M_z = -(5-2+1)(0.06) = -0.24 \text{ kip} \cdot \text{in.}$$

$$M_x = -(8\sin 60°)(0.06) = -0.41569 \text{ kip} \cdot \text{in.}$$

$$d = 0.75 \text{ in.} \qquad c = \frac{1}{2}d = 0.375 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.375)^4 = 15.5316 \times 10^{-3} \text{ in}^4$$

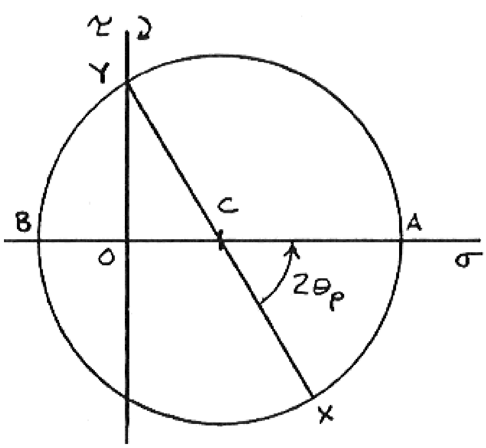
$$J = 2I = 31.063 \times 10^{-3} \text{ in}^4$$



At point H,

$$\sigma_H = -\frac{M_z y}{I_z} = -\frac{(-0.24)(0.375)}{15.5316 \times 10^{-3}} = 5.7946 \text{ ksi}$$

$$\tau_H = \frac{Tc}{J} = \frac{(0.41569)(0.375)}{31.063 \times 10^{-3}} = 5.0183 \text{ ksi}$$



Use Mohr's circle. $\quad \sigma_{ave} = \frac{1}{2}\sigma_H = 2.8973 \text{ ksi}$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 5.7946 \text{ ksi}$$

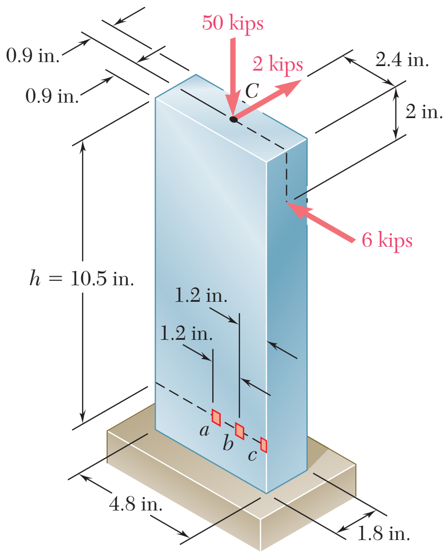
$\sigma_a = \sigma_{ave} + R$ $\qquad\qquad$ $\sigma_{max} = 8.69 \text{ ksi}$ ◀

$\sigma_b = \sigma_{ave} - R$ $\qquad\qquad$ $\sigma_{min} = -2.90 \text{ ksi}$ ◀

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{2(5.0183)}{5.7946} = 1.7321$$

$$\theta_a = 30.0° \qquad \theta_b = 120.0°$$

$\tau_{max} = R$ $\qquad\qquad$ $\tau_{max} = 5.79 \text{ ksi}$ ◀

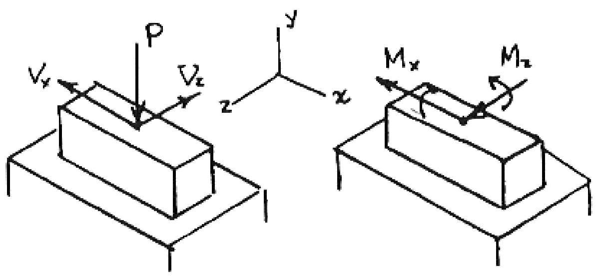


PROBLEM 8.45

Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Calculate forces and couples at section containing points *a*, *b*, and *c*.



Forces   Couples

$$h = 10.5 \text{ in.}$$

$$P = 50 \text{ kips} \quad V_x = 6 \text{ kips} \quad V_z = 2 \text{ kips}$$

$$M_z = (10.5 - 2)(6) = 51 \text{ kip} \cdot \text{in.}$$

$$M_x = (10.5)(2) = 21 \text{ kip} \cdot \text{in.}$$

Section properties.

$$A = (1.8)(4.8) = 8.64 \text{ in}^2$$

$$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4$$

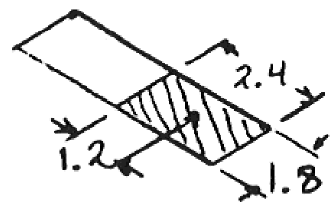
$$I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4$$

Stresses.

$$\sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$$

PROBLEM 8.45 *(Continued)*

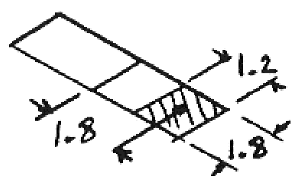
(*a*) Point *a*: $x = 0, \quad z = 0.9$ in., $\quad Q = (1.8)(2.4)(1.2) = 5.184 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(21)(0.9)}{2.3328} \qquad \sigma = 2.31 \text{ ksi} \blacktriangleleft$$

$$\tau = \frac{(6)(5.184)}{(16.5888)(1.8)} \qquad \tau = 1.042 \text{ ksi} \blacktriangleleft$$

(*b*) Point *b*: $x = 1.2$ in., $\quad z = 0.9$ in., $\quad Q = (1.8)(1.2)(1.8) = 3.888 \text{ in}^3$



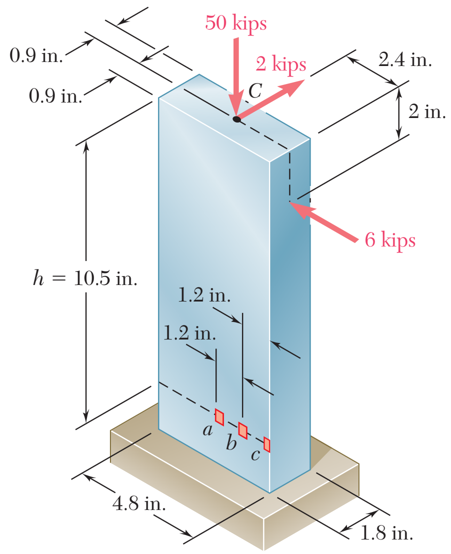
$$\sigma = -\frac{50}{8.64} + \frac{(51)(1.2)}{16.5888} + \frac{(21)(0.9)}{2.3328} \qquad \sigma = 6.00 \text{ ksi} \blacktriangleleft$$

$$\tau = \frac{(6)(3.888)}{(16.5888)(1.8)} = 0.781 \text{ ksi} \qquad \tau = 0.781 \text{ ksi} \blacktriangleleft$$

(*c*) Point *c*: $x = 2.4$ in., $\quad z = 0.9$ in., $\quad Q = 0$

$$\sigma = -\frac{50}{8.64} + \frac{(51)(2.4)}{16.5888} + \frac{(21)(0.9)}{2.3328} \qquad \sigma = 9.69 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$



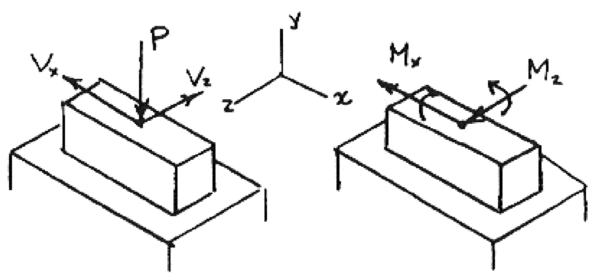
PROBLEM 8.46

Solve Prob. 8.45, assuming that $h = 12$ in.

PROBLEM 8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Calculate forces and couples at section containing points *a*, *b*, and *c*.



$$h = 12 \text{ in.}$$

$$P = 50 \text{ kips} \quad V_x = 6 \text{ kips} \quad V_z = 2 \text{ kips}$$

$$M_z = (12 - 2)(6) = 60 \text{ kip} \cdot \text{in.}$$

$$M_x = (12)(2) = 24 \text{ kip} \cdot \text{in.}$$

Section properties.

$$A = (1.8)(4.8) = 8.64 \text{ in}^2$$

$$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4$$

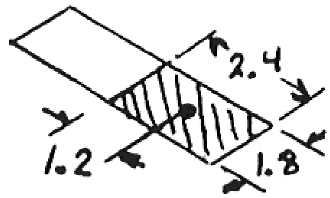
$$I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4$$

Stresses.

$$\sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$$

PROBLEM 8.46 *(Continued)*

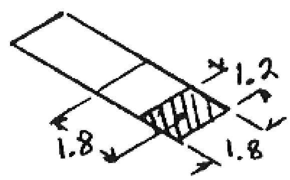
(*a*) Point *a*: $x = 0, \quad z = 0.9 \text{ in.}, \quad Q = (1.8)(2.4)(1.2) = 5.184 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(24)(0.9)}{2.3328} \qquad \sigma = 3.47 \text{ ksi} \blacktriangleleft$$

$$\tau = \frac{(6)(5.184)}{(16.5888)(1.8)} \qquad \tau = 1.042 \text{ ksi} \blacktriangleleft$$

(*b*) Point *b*: $x = 1.2 \text{ in.}, \quad z = 0.9 \text{ in.}, \quad Q = (1.8)(1.2)(1.8) = 3.888 \text{ in}^3$



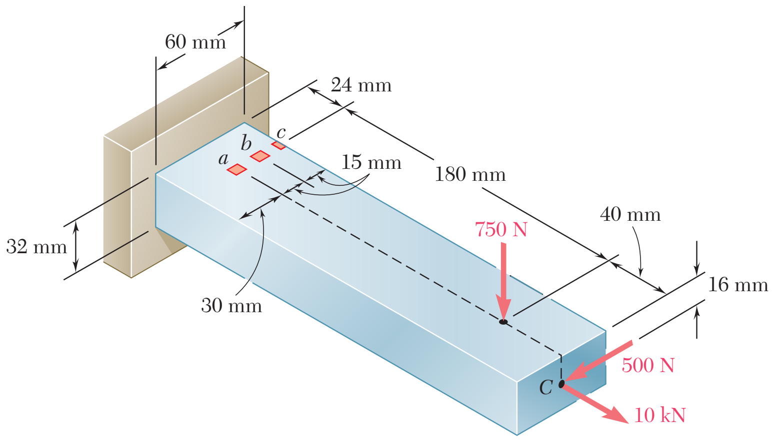
$$\sigma = -\frac{50}{8.64} + \frac{(60)(1.2)}{16.5888} + \frac{(24)(0.9)}{2.3328} \qquad \sigma = 7.81 \text{ ksi} \blacktriangleleft$$

$$\tau = \frac{(6)(3.888)}{(16.5888)(1.8)} \qquad \tau = 0.781 \text{ ksi} \blacktriangleleft$$

(*c*) Point *c*: $x = 2.4 \text{ in.}, \quad z = 0.9 \text{ in.}, \quad Q = 0$

$$\sigma = -\frac{50}{8.64} + \frac{(60)(2.4)}{16.5888} + \frac{(24)(0.9)}{2.3328} \qquad \sigma = 12.15 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$



PROBLEM 8.47

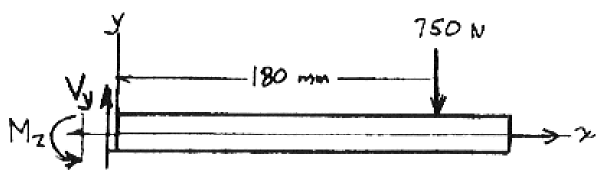
Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

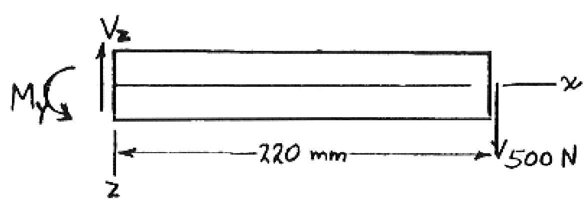
$$A = (60)(32) = 1920 \text{ mm}^2$$
$$= 1920 \times 10^{-6} \text{ m}^2$$
$$I_z = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3 \text{ mm}^4$$
$$= 163.84 \times 10^{-9} \text{ m}^4$$
$$I_y = \frac{1}{12}(32)(60)^3$$
$$= 579 \times 10^3 \text{ mm}^4$$
$$= 576 \times 10^{-9} \text{ m}^4$$

At the section containing points *a*, *b*, and *c*,

$$P = 10 \text{ kN}$$
$$V_y = 750 \text{ N},$$
$$V_z = 500 \text{ N}$$
$$M_z = (180 \times 10^{-3})(750)$$
$$= 135 \text{ N} \cdot \text{m}$$
$$M_y = (220 \times 10^{-3})(500)$$
$$= 110 \text{ N} \cdot \text{m}$$
$$T = 0$$
$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$
$$\tau = \frac{V_z Q}{I_y t}$$



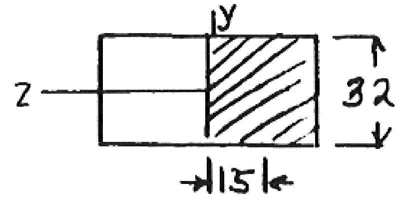
Side View



Top View

PROBLEM 8.47 *(Continued)*

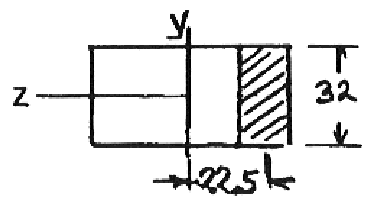
(*a*) Point *a*: $y = 16\text{ mm}, \quad z = 0, \quad Q = A\bar{z} = (32)(30)(15) = 14.4\times 10^3\text{ mm}^3$



$$\sigma = \frac{10\times 10^3}{1920\times 10^{-6}} + \frac{(135)(16\times 10^{-3})}{163.84\times 10^{-9}} - 0$$ $\sigma = 18.39\text{ MPa}$ ◀

$$\tau = \frac{(500)(14.4\times 10^{-6})}{(576\times 10^{-9})(32\times 10^{-3})}$$ $\tau = 0.391\text{ MPa}$ ◀

(*b*) Point *b*: $y = 16\text{ mm}, \quad z = -15\text{ mm}, \quad Q = A\bar{z} = (32)(15)(22.5) = 10.8\times 10^3\text{ mm}^3$



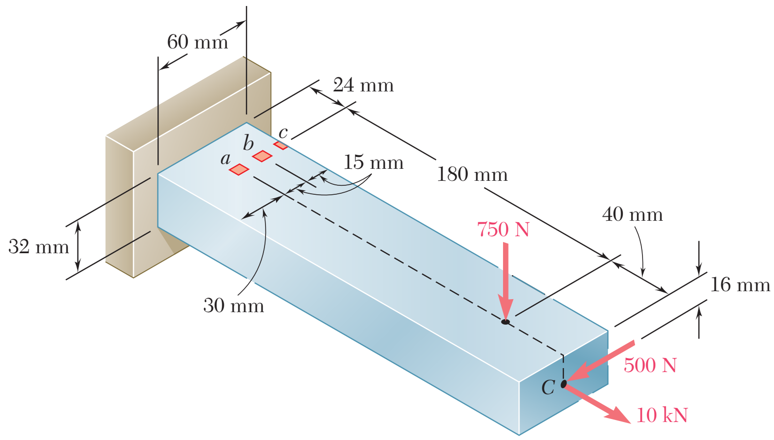
$$\sigma = \frac{10\times 10^3}{1920\times 10^{-6}} + \frac{(135)(16\times 10^{-3})}{163.84\times 10^{-9}} - \frac{(110)(-15\times 10^{-3})}{576\times 10^{-9}}$$ $\sigma = 21.3\text{ MPa}$ ◀

$$\tau = \frac{(500)(10.8\times 10^{-6})}{(576\times 10^{-9})(32\times 10^{-3})}$$ $\tau = 0.293\text{ MPa}$ ◀

(*c*) Point *c*: $y = 16\text{ mm}, \quad z = -30\text{ mm}, \quad Q = 0$

$$\sigma = \frac{10\times 10^3}{1920\times 10^{-6}} + \frac{(135)(16\times 10^{-3})}{163.84\times 10^{-9}} - \frac{(110)(-30\times 10^{-6})}{576\times 10^{-9}}$$ $\sigma = 24.1\text{ MPa}$ ◀

$\tau = 0$ ◀



PROBLEM 8.48

Solve Prob. 8.47, assuming that the 750-N force is directed vertically upward.

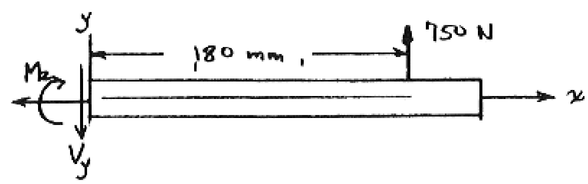
PROBLEM 8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

$$A = (60)(32) = 1920 \text{ mm}^2$$
$$= 1920\times 10^{-6}\,\text{m}^2$$
$$I_z = \frac{1}{12}(60)(32)^3$$
$$= 163.84\times 10^3\,\text{mm}^4$$
$$= 163.84\times 10^{-9}\,\text{m}^4$$
$$I_y = \frac{1}{12}(32)(60)^3$$
$$= 576\times 10^3\,\text{mm}^4$$
$$= 576\times 10^{-9}\,\text{m}^4$$

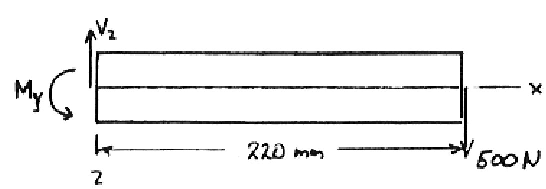
At the section containing points *a*, *b*, and *c*,

$$P = 10\text{ kN} \qquad T = 0$$
$$V_y = 750\text{ N}$$
$$V_z = 500\text{ N}$$
$$M_z = (180\times 10^{-3})(750)$$
$$= 135\text{ N}\cdot\text{m}$$



Side View

$$M_y = (220\times 10^{-3})(500)$$
$$= 110\text{ N}\cdot\text{m}$$



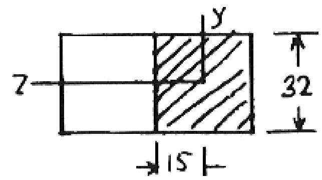
Top View

$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$

$$\tau = \frac{V_z Q}{I_y t}$$

PROBLEM 8.48 *(Continued)*

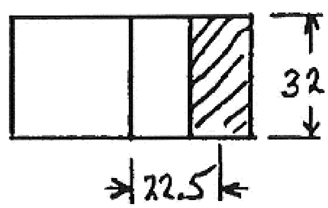
(*a*) Point *a*: $y = 16\text{ mm}, \quad z = 0, \quad Q = A\bar{z} = (32)(30)(15) = 14.4\times10^3\text{ mm}^3$



$$\sigma = \frac{10\times10^3}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} = 0 \qquad \sigma = -7.98\text{ MPa} ◀$$

$$\tau = \frac{(500)(14.4\times10^{-6})}{(163.84\times10^{-9})(32\times10^{-3})} \qquad \tau = 0.391\text{ MPa} ◀$$

(*b*) Point *b*: $y = 16\text{ mm}, \quad z = -15\text{ mm}, \quad Q = A\bar{z} = (32)(15)(22.5) = 10.8\times10^3\text{ mm}^3$



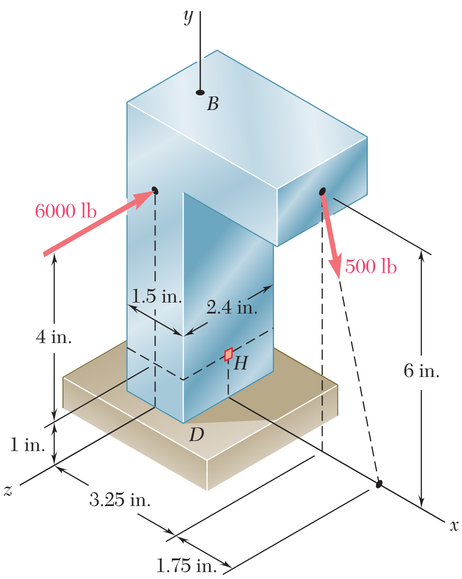
$$\sigma = \frac{10\times10^3}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-15\times10^{-3})}{576\times10^{-9}} \qquad \sigma = -5.11\text{ MPa} ◀$$

$$\tau = \frac{(500)(10.8\times10^{-6})}{(163.84\times10^{-9})(32\times10^{-9})} \qquad \tau = 0.293\text{ MPa} ◀$$

(*c*) Point *c*: $y = 16\text{ mm}, \quad z = -30\text{ mm}, \quad Q = 0$

$$\sigma = \frac{10\times10^3}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-30\times10^{-3})}{576\times10^{-9}} \qquad \sigma = -2.25\text{ MPa} ◀$$

$$\tau = 0 ◀$$



PROBLEM 8.49

Two forces are applied to the small post BD as shown. Knowing that the vertical portion of the post has a cross section of 1.5×2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point H.

SOLUTION

Components of 500-lb force:

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb}$$

Moment arm of 500-lb force:

$$\vec{r} = 3.25\vec{i} + (6-1)\vec{j}$$

Moment of 500-lb force:

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\vec{k} \text{ lb} \cdot \text{in.}$$

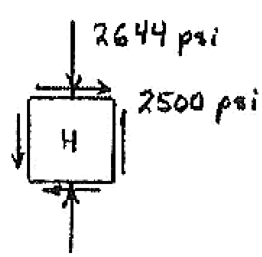
At the section containing point H, $\quad P = -480$ lb $\quad V_x = 140$ lb

$$V_z = -6000 \text{ lb}, \quad M_z = -2260 \text{ lb} \cdot \text{in.}, \quad M_x = -(4)(6000) = -24{,}000 \text{ lb} \cdot \text{in.}$$

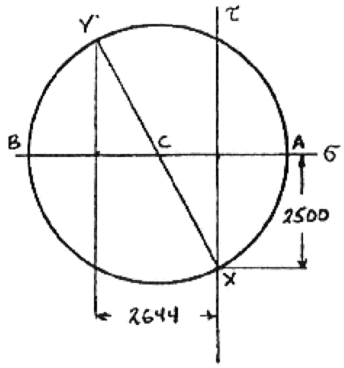
$$A = (1.5)(2.4) = 3.6 \text{ in}^2 \qquad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{6000}{3.6} = 2500 \text{ psi}$$



PROBLEM 8.49 *(Continued)*



Use Mohr's circle.

$$\sigma_{\text{ave}} = -\frac{2644}{2} = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (2500)^2} = 2828 \text{ psi}$$

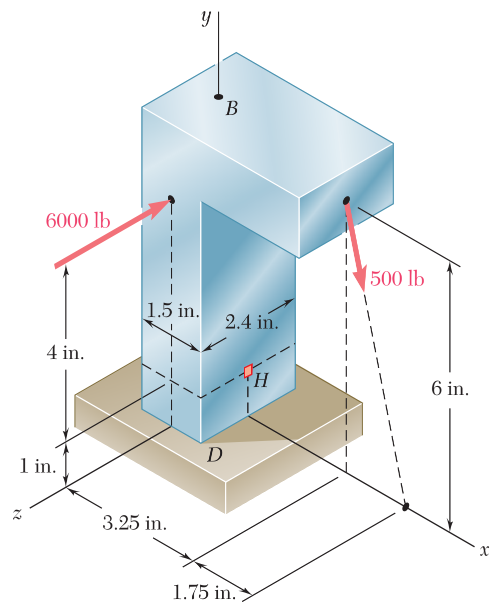
$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 1506 \text{ psi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -4150 \text{ psi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(2500)}{2644} = 1.891$$

$$\theta_a = 31.1°, \qquad \theta_b = 121.1° \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 2830 \text{ psi} \blacktriangleleft$$



PROBLEM 8.50

Solve Prob. 8.49, assuming that the magnitude of the 6000-lb force is reduced to 1500 lb.

PROBLEM 8.49 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5×2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

SOLUTION

Components of 500-lb force:

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb}$$

Moment arm of 500-lb force:

$$\vec{r} = 3.25\vec{i} + (6-1)\vec{j}$$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\vec{k} \text{ lb} \cdot \text{in.}$$

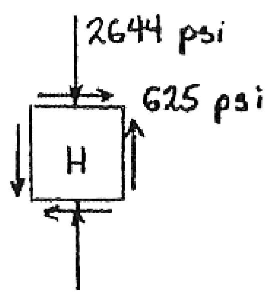
At the section containing point *H*, $P = -480$ lb $\quad V_x = 140$ lb

$$V_z = -1500 \text{ lb}, \quad M_z = -2260 \text{ lb} \cdot \text{in.}, \quad M_x = -(4)(1500) = -6000 \text{ lb} \cdot \text{in.}$$

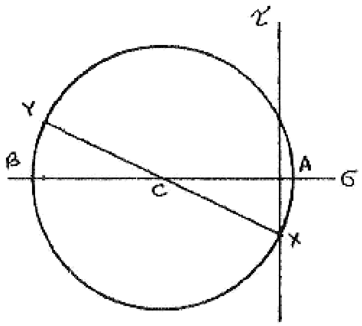
$$A = (1.5)(2.4) = 3.6 \text{ in}^2 \quad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{1500}{3.6} = 625 \text{ psi}$$



PROBLEM 8.50 *(Continued)*



Use Mohr's circle.

$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (625)^2} = 1462.30 \text{ psi}$$

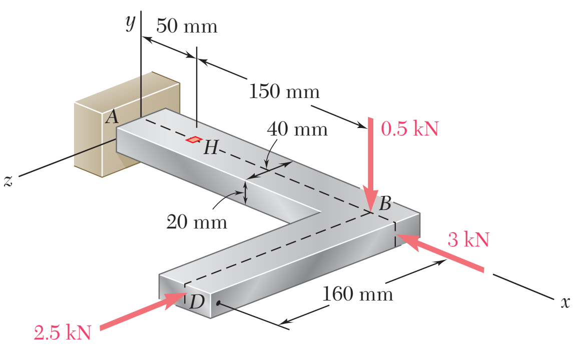
$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 140.3 \text{ psi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -2780 \text{ psi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(625)}{2644} = 0.4728$$

$$\theta_a = 12.6^\circ \qquad \theta_b = 102.6^\circ \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 1462 \text{ psi} \blacktriangleleft$$



PROBLEM 8.51

Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20×40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

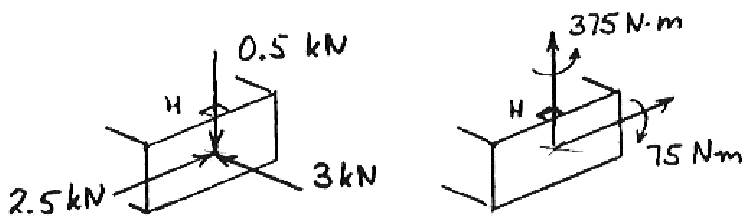
SOLUTION

Equivalent force-couple system at section containing point *H*:

$$F_x = -3\text{ kN}, \quad F_y = -0.5\text{ kN}, \quad F_z = -2.5\text{ kN}$$

$$M_x = 0, \quad M_y = (0.150)(2500) = 375\text{ N}\cdot\text{m}$$

$$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$$



$$A = (20)(40) = 800\text{ mm}^2$$

$$= 800 \times 10^{-6}\text{m}^2$$

$$I_z = \frac{1}{12}(40)(20)^3$$

$$= 26.667 \times 10^3\text{mm}^4$$

$$= 26.667 \times 10^{-9}\text{m}^4$$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z}$$

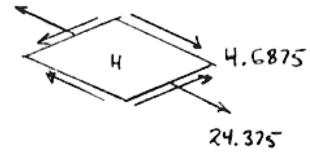
$$= \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}}$$

$$= 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2}\cdot\frac{2500}{800 \times 10^{-6}}$$

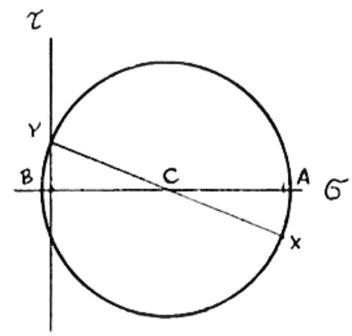
$$= 4.6875\text{ MPa}$$

PROBLEM 8.51 *(Continued)*



Use Mohr's circle.

$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H$$
$$= 12.1875 \text{ MPa}$$
$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2}$$
$$= 13.0579 \text{ MPa}$$



$$\sigma_a = \sigma_{\text{ave}} + R$$

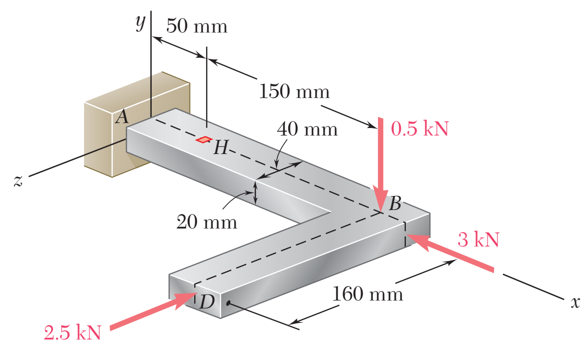
$\sigma_a = 25.2$ MPa ◀

$$\sigma_b = \sigma_{\text{ave}} - R$$

$\sigma_b = -0.87$ MPa ◀

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$$
$$\theta_a = 10.5°, \quad \theta_b = 100.5°$$
$$\tau_{\max} = R$$

$\tau_{\max} = 13.06$ MPa ◀



PROBLEM 8.52

Solve Prob. 8.51, assuming that the magnitude of the 2.5-kN force is increased to 10 kN.

PROBLEM 8.51 Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20×40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

SOLUTION

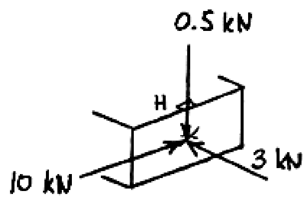
Equivalent force-couple system at section containing point *H*:

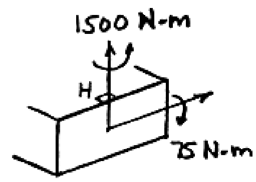
$$F_x = -3\text{ kN},\quad F_y = -0.5\text{ kN},\quad F_z = -10\text{ kN}$$

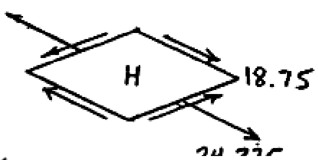
$$M_x = 0, M_y = (0.150)(10{,}000) = 1500\text{ N}\cdot\text{m}$$

$$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$$

$$A = (20)(40) = 800\text{ mm}^2 = 800\times 10^{-6}\text{ m}^2$$





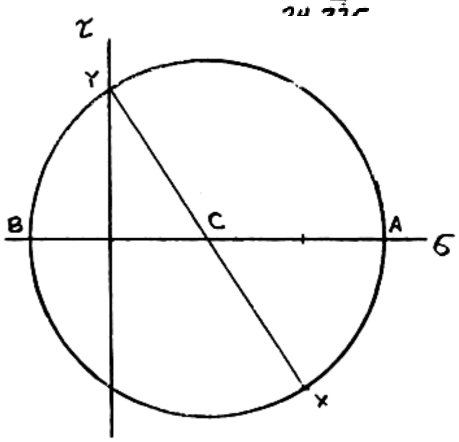


$$I_z = \frac{1}{12}(40)(20)^3 = 26.667\times 10^3\text{ mm}^4 = 26.667\times 10^{-9}\text{ m}^4$$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800\times 10^{-6}} - \frac{(-75)(10\times 10^{-3})}{26.667\times 10^{-9}} = 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2}\cdot\frac{10{,}000}{800\times 10^{-6}} = 18.75\text{ MPa}$$

$$\sigma_c = \frac{1}{2}\sigma_H = 12.1875\text{ MPa}$$



$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (18.75)^2} = 22.363\text{ MPa}$$

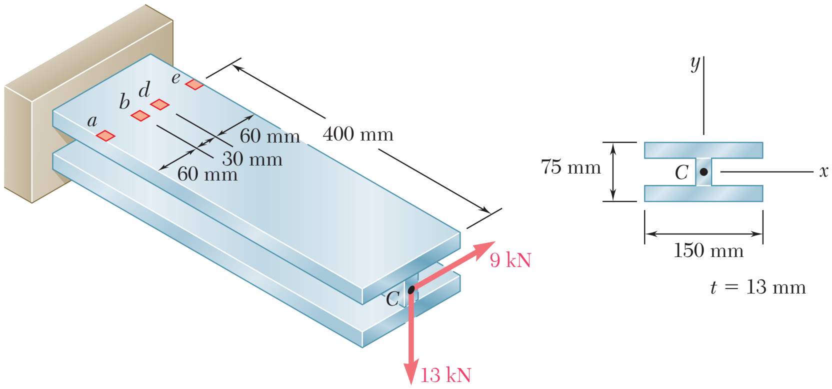
$$\sigma_a = \sigma_c + R \qquad \sigma_a = 34.6\text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_c - R \qquad \sigma_b = -10.18\text{ MPa} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(18.75)}{24.375} = 1.5385$$

$$\theta_a = 28.5°,\quad \theta_b = 118.5°$$

$$\tau_{max} = R \qquad \tau_{max} = 22.4\text{ MPa} \blacktriangleleft$$



PROBLEM 8.53

Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.

SOLUTION

Equivalent force-couple system at section containing points *a* and *b*:

$$F_x = 9 \text{ kN}, \quad F_y = -13 \text{ kN}, \quad F_z = 0$$

$$M_x = (0.400)(13 \times 10^3) = 5200 \text{ N}\cdot\text{m}$$

$$M_y = (0.400)(9 \times 10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75 - 26)$$

$$= 4537 \text{ mm}^2$$

$$= 4537 \times 10^{-6} \text{m}^2$$

$$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5 - 6.5)^2\right] + \frac{1}{12}(13)(75 - 26)^3$$

$$= 3.9303 \times 10^6 \text{mm}^4$$

$$= 3.9303 \times 10^{-6} \text{m}^4$$

$$I_y = 2 \cdot \frac{1}{12}(13)(150)^3 + \frac{1}{12}(75 - 26)(13)^3$$

$$= 7.3215 \times 10^6 \text{mm}^4 = 7.3215 \times 10^{-6} \text{mm}^4$$

For point *a*,

$$Q_x = 0, \quad Q_y = 0$$

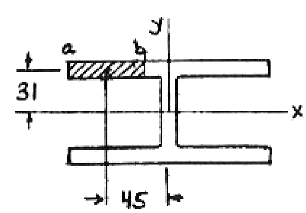
For point *b*,

$$A^* = (60)(13) = 780 \text{ mm}^2$$

$$\bar{x} = -45 \text{ mm} \quad \bar{y} = 31 \text{ mm}$$

$$Q_x = A^* \bar{y} = 24.18 \times 10^3 \text{ mm}^3 = 24.18 \times 10^{-6} \text{ m}^3$$

$$Q_y = A^* \bar{x} = -35.1 \times 10^{-3} \text{ mm}^3 = -35.1 \times 10^{-6} \text{ m}^3$$



PROBLEM 8.53 *(Continued)*

Direction of shearing stress for horizontal and for vertical components of shear:

At point *a*,

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-75\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 86.5 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

At point *b*,

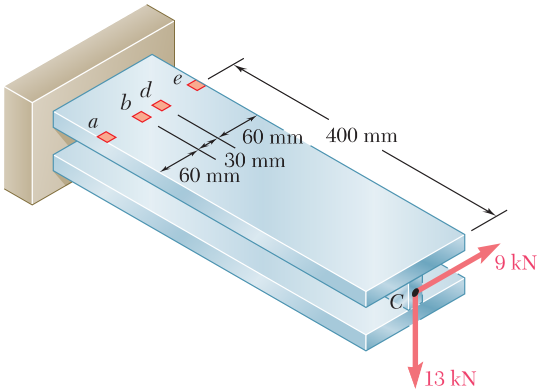
$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

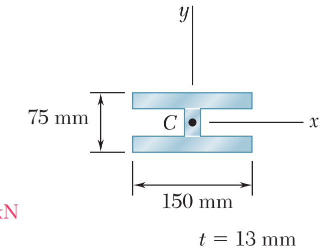
$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-15\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 57.0 \text{ MPa} \blacktriangleleft$$

$$\tau = \frac{|V_x||Q_y|}{I_y t} + \frac{|V_y||Q_x|}{I_x t}$$

$$= \frac{(9\times10^3)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} + \frac{(13\times10^3)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})}$$

$$= 3.32 \text{ MPa} + 6.15 \text{ MPa} \qquad \tau = 9.47 \text{ MPa} \blacktriangleleft$$





PROBLEM 8.54

Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.

SOLUTION

Equivalent force-couple system at section containing points *d* and *e*.

$$F_x = 9 \text{ kN}, \quad F_y = -13 \text{ kN}, \quad F_z = 0$$

$$M_x = (0.400)(13 \times 10^3) = 5200 \text{ N} \cdot \text{m}$$

$$M_y = (0.400)(9 \times 10^3) = 3600 \text{ N} \cdot \text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75 - 26)$$

$$= 4537 \text{ mm}^2$$

$$= 4537 \times 10^{-6} \text{m}^2$$

$$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5 - 6.5)^2\right] + \frac{1}{12}(13)(75 - 26)^3$$

$$= 3.9303 \times 10^6 \text{mm}^4$$

$$= 3.9303 \times 10^{-6} \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(13)(150)^3\right] + \frac{1}{12}(75 - 26)(13)^3$$

$$= 7.3215 \times 10^6 \text{mm}^4 = 7.3215 \times 10^{-6} \text{m}^4$$

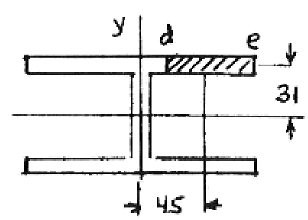
For point *d*,

$$A^* = (60)(13) = 780 \text{ mm}^2$$

$$\bar{x} = 45 \text{ mm} \quad \bar{y} = 31 \text{ mm}$$

$$Q_x = A^* \bar{y} = 24.18 \times 10^3 \text{ mm}^3 = 24.18 \times 10^{-6} \text{ m}^3$$

$$Q_y = A^* \bar{x} = 35.1 \times 10^3 \text{ mm}^3 = 35.1 \times 10^{-6} \text{ m}^3$$



For point *e*, $Q_x = 0, \quad Q_y = 0$

PROBLEM 8.54 *(Continued)*

At point d,

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

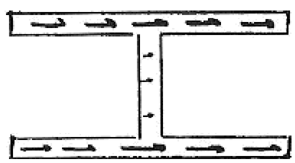
$$= \frac{(5200)(37.5\times 10^{-3})}{3.9303\times 10^{-6}} - \frac{(3600)(15\times 10^{-3})}{7.3215\times 10^{-6}}$$

$\sigma = 42.2\text{ MPa}$ ◀

Due to V_x:

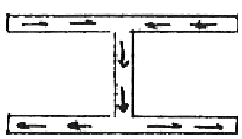
$$\tau = \frac{|V_x||Q_y|}{I_y t} = \frac{(9000)(35.1\times 10^{-6})}{(7.3215\times 10^{-6})(13\times 10^{-3})}$$

$$= 3.32\text{ MPa} \rightarrow$$



Due to V_y:

$$\tau = \frac{|V_y||Q_x|}{I_x t} = \frac{(13{,}000)(24.18\times 10^{-6})}{(3.9303\times 10^{-6})(13\times 10^{-3})} = 6.15\text{ MPa} \leftarrow$$



By superposition, the net value is $\tau = 2.83\text{ MPa}$ ◀

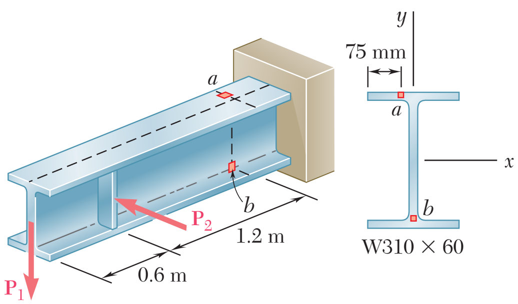
At point e,

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(5200)(37.5\times 10^{-3})}{3.9303\times 10^{-6}} - \frac{(3600)(75\times 10^{-3})}{7.3215\times 10^{-6}}$$

$\sigma = 12.74\text{ MPa}$ ◀

$\tau = 0$ ◀

PROBLEM 8.55



Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $P_1 = 25$ kN and $P_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point a.

SOLUTION

At the section containing points a and b,

$$M_x = (1.8)(25) = 45 \text{ kN}\cdot\text{m}$$

$$M_y = -(1.2)(24) = -28.8 \text{ kN}\cdot\text{m}$$

$$V_x = -24 \text{ kN} \qquad V_y = -25 \text{ kN}$$

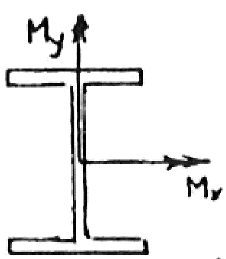
For W310 × 60 rolled-steel section,

$$d = 302 \text{ mm}, \quad b_f = 203 \text{ mm}, \quad t_f = 13.1 \text{ mm}, \quad t_w = 7.49 \text{ mm}$$

$$I_x = 128\times 10^6 \text{ mm}^4 = 128\times 10^{-6} \text{ m}^4, \quad I_y = 18.4\times 10^6 \text{ mm}^4 = 18.4\times 10^{-6} \text{ m}^4$$

Normal stress at point a: $\quad x = -\dfrac{b_f}{2} + 75 = -26.5 \text{ mm}$

$$y = \frac{1}{2}d = 151 \text{ mm}$$

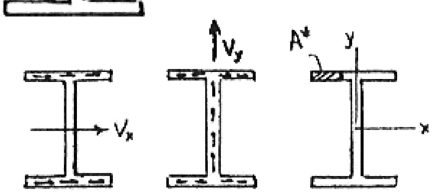


$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45\times 10^3)(151\times 10^{-3})}{128\times 10^{-6}} - \frac{(-28.8\times 10^3)(-26.5\times 10^{-3})}{18.4\times 10^{-6}}$$

$$= 53.086 \text{ MPa} - 41.478 \text{ MPa} = 11.608 \text{ MPa}$$

Shearing stress at point a:

$$\tau_{xz} = -\frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$



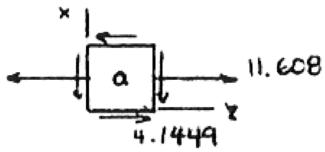
$$A^* = (75\times 10^{-3})(13.1\times 10^{-3}) = 982.5\times 10^{-6} \text{ m}^2$$

$$\bar{x} = -\frac{b_f}{2} + \frac{75}{2} = -64 \text{ mm}$$

$$\bar{y} = \frac{d}{2} - \frac{t_f}{2} = 144.45 \text{ mm}$$

$$\tau_{xz} = -\frac{(-24\times 10^3)(982.5\times 10^{-6})(-64\times 10^{-3})}{(18.4\times 10^{-6})(13.1\times 10^{-3})} - \frac{(-25\times 10^3)(982.5\times 10^{-6})(144.45\times 10^{-3})}{(128\times 10^{-6})(13.1\times 10^{-3})}$$

$$= -6.2609 \text{ MPa} + 2.1160 \text{ MPa} = -4.1449 \text{ MPa}$$



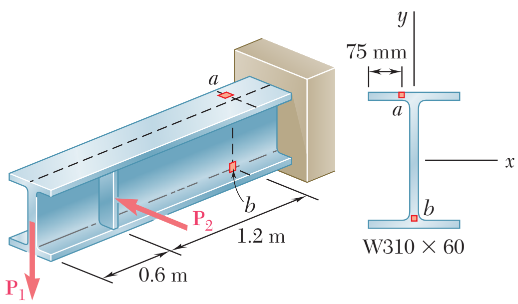
$$\sigma_{ave} = \frac{11.608}{2} = 5.804 \text{ MPa}$$

$$R = \sqrt{\left(\frac{11.608}{2}\right)^2 + (4.1449)^2} = 7.1321 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 12.94 \text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -1.328 \text{ MPa} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 7.13 \text{ MPa} \blacktriangleleft$$



PROBLEM 8.56

Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $P_1 = 25$ kN and $P_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point *b*.

SOLUTION

At the section containing points *a* and *b*,

$$M_x = (1.8)(25) = 45 \text{ kN} \cdot \text{m}$$

$$M_y = -(1.2)(24) = -28.8 \text{ kN} \cdot \text{m}$$

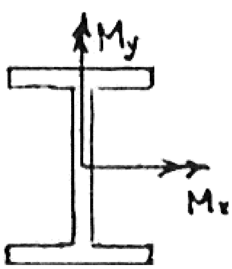
$$V_x = -24 \text{ kN}, \qquad V_y = -25 \text{ kN}$$

For W310 × 60 rolled-steel section,

$$d = 302 \text{ mm}, \quad b_f = 203 \text{ mm}, \quad t_f = 13.1 \text{ mm}, \quad t_w = 7.49 \text{ mm}$$

$$I_x = 128 \times 10^6 \text{ mm}^4 = 128 \times 10^{-6} \text{ m}^4, \quad I_y = 18.4 \times 10^6 \text{ mm}^4 = 18.4 \times 10^{-6} \text{ m}^4$$

Normal stress at point *b*: $\quad x \approx 0, \quad y = -\frac{1}{2}d + t_f = -137.9$ mm

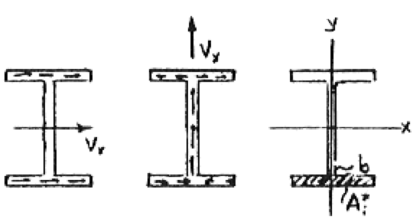


$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45 \times 10^3)(-137.9 \times 10^{-3})}{128 \times 10^{-6}} - 0$$

$$= -48.480 \text{ MPa}$$

Shearing stress at point *b*:

$$\tau_{yz} = -\frac{V_y A^* \overline{y}}{I_x t_w}$$



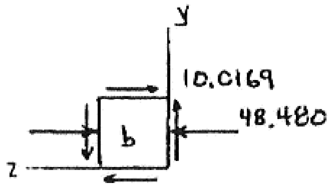
$$A^* = A_f = b_f t_f = 2659.3 \text{ mm}^2$$

$$= 2659.3 \times 10^{-6} \text{ m}^2$$

$$\overline{x} = 0, \qquad \overline{y} = -\frac{1}{2}d + \frac{1}{2}t_f = -144.45 \text{ mm}$$

$$\tau_{yz} = -\frac{(-25 \times 10^3)(2659.3 \times 10^{-6})(-144.45 \times 10^{-3})}{(128 \times 10^{-6})(7.49 \times 10^{-3})} = -10.0169 \text{ MPa}$$

PROBLEM 8.56 *(Continued)*



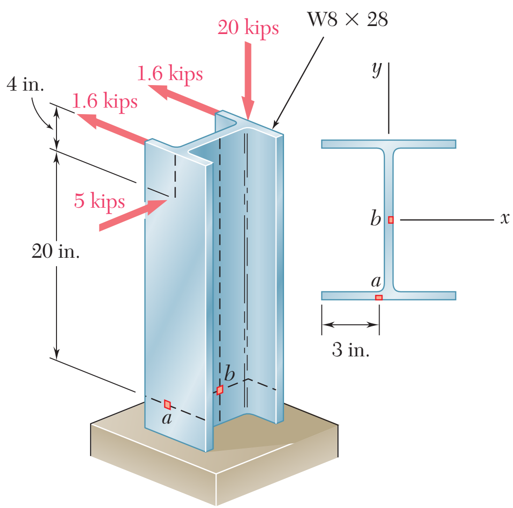
$$\sigma_{\text{ave}} = -\frac{48.480}{2} = -24.240 \text{ MPa}$$

$$R = \sqrt{\left(\frac{48.48}{2}\right)^2 + (10.0169)^2} = 26.228 \text{ MPa}$$

$\sigma_{\max} = \sigma_{\text{ave}} + R$ $\qquad$ $\sigma_{\max} = 1.988 \text{ MPa}$ ◀

$\sigma_{\min} = \sigma_{\text{ave}} - R$ $\qquad$ $\sigma_{\min} = -50.5 \text{ MPa}$ ◀

$\tau_{\max} = R$ $\qquad$ $\tau_{\max} = 26.2 \text{ MPa}$ ◀

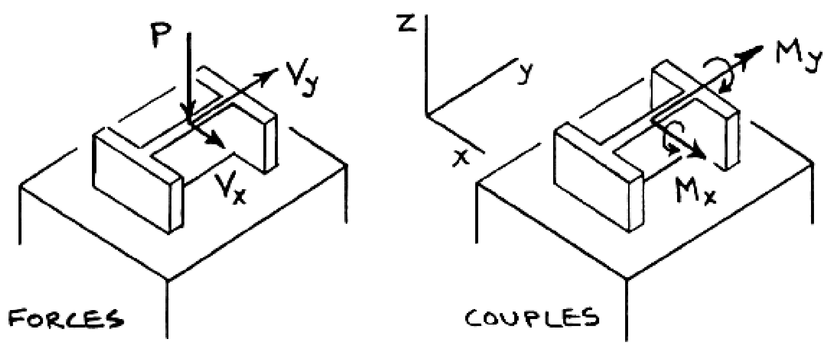


PROBLEM 8.57

Four forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses and maximum shearing stress at point *a*.

SOLUTION

Calculate forces and couples at section containing point of interest.



$$P = 20 \text{ kips} \qquad V_x = -3.2 \text{ kips} \qquad V_y = 5 \text{ kips}$$

$$M_x = -(20)(5) - (4.03)(20) = -180.6 \text{ kip} \cdot \text{in.}$$

$$M_y = -(20+4)(3.2) = -76.8 \text{ kip} \cdot \text{in.}$$

Section properties:

$$A = 8.24 \text{ in}^4 \qquad d = 8.06 \text{ in.} \qquad b_f = 6.54 \text{ in.} \qquad t_f = 0.465 \text{ in.}$$

$$t_w = 0.285 \text{ in.} \qquad I_x = 98.0 \text{ in}^4 \qquad I_y = 21.7 \text{ in}^4$$

Point *a*:

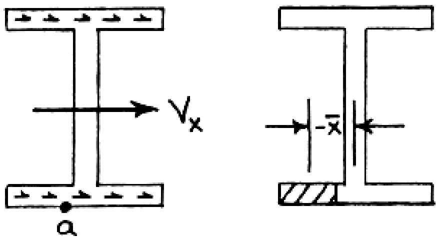
$$x_a = -\frac{6.54}{2} + 3 = -0.27 \text{ in.} \qquad y_a = -\frac{8.06}{2} = -4.03 \text{ in.}$$

$$\sigma_a = -\frac{P}{A} + \frac{M_x y_a}{I_x} - \frac{M_y x_a}{I_y}$$

$$= -\frac{20}{8.24} + \frac{(-180.6)(-4.03)}{98.0} - \frac{(-76.8)(-0.27)}{21.7}$$

$$= -2.4272 + 7.4267 - 0.95558 = 4.0439 \text{ ksi}$$

Shearing stress at point a due to V_x:



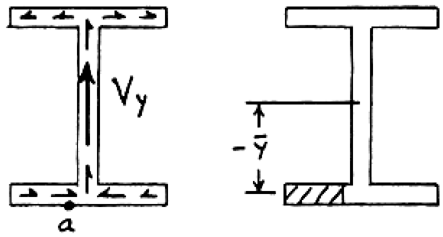
$$A = (3)(0.465) = 1.395 \text{ in}^2$$

$$\bar{x} = -\frac{6.54}{2} + \frac{3}{2} = -1.77 \text{ in.}$$

$$Q = -A\bar{x} = 2.4692 \text{ in}^3$$

$$\tau_{xz} = \frac{V_x Q}{I_y t} = \frac{(-3.2)(2.4692)}{(21.7)(0.465)} = -0.78306 \text{ ksi}$$

Shearing stress at point a due to V_y:



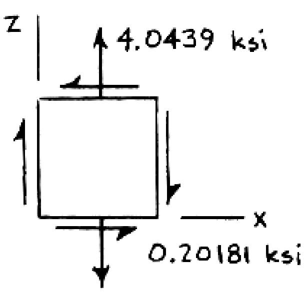
$$A = (3)(0.465) = 1.395 \text{ in}^2$$

$$\bar{y} = -\frac{8.06}{2} + \frac{0.465}{2} = -3.7975 \text{ in.}$$

$$Q = -A\bar{y} = 5.2975 \text{ in}^3$$

$$\tau_{xz} = \frac{V_y Q}{I_x t} = \frac{(5)(5.2975)}{(98.0)(0.465)} = 0.58125 \text{ ksi}$$

Combined shearing stress: $\tau_a = -0.78306 + 0.58125 = 0.20181 \text{ ksi}$



$$\sigma_{\text{ave}} = \frac{4.0439 + 0}{2} = 2.0220 \text{ ksi}$$

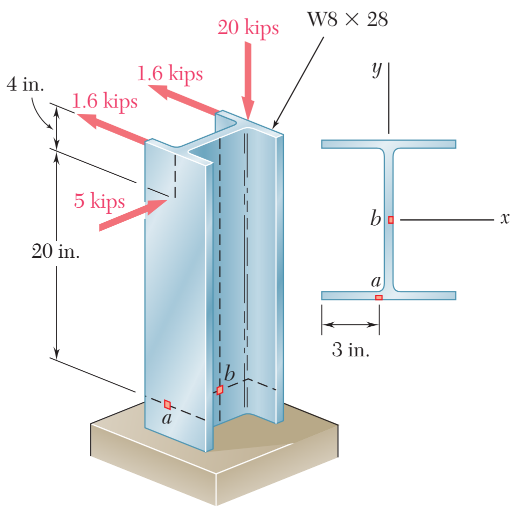
$$R = \sqrt{\left(\frac{4.0439 - 0}{2}\right)^2 + (0.20181)^2} = 2.0320 \text{ ksi}$$

$\sigma_{\max} = \sigma_{\text{ave}} + R$ $\qquad$ $\sigma_{\max} = 4.05 \text{ ksi}$ ◄

$\sigma_{\min} = \sigma_{\text{ave}} - R$ $\qquad$ $\sigma_{\min} = -0.010 \text{ ksi}$ ◄

$\tau_{\max} = R$ $\qquad$ $\tau_{\max} = 2.03 \text{ ksi}$ ◄

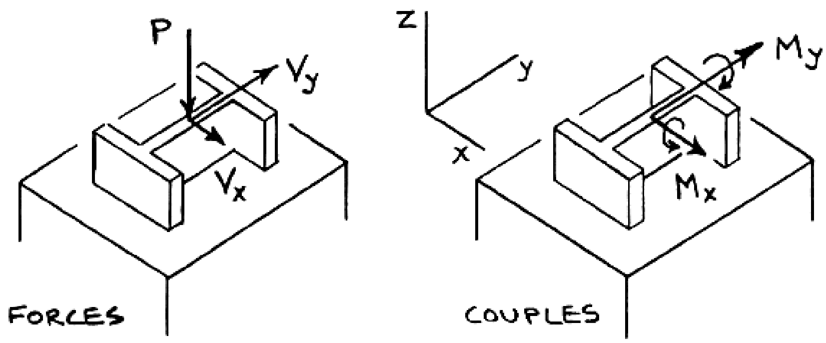
PROBLEM 8.58



Four forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses and maximum shearing stress at point b.

SOLUTION

Calculate forces and couples at section containing point of interest.



$$P = 20 \text{ kips} \qquad V_x = -3.2 \text{ kips} \qquad V_y = 5 \text{ kips}$$

$$M_x = -(20)(5) - (4.03)(20) = -180.6 \text{ kip} \cdot \text{in.}$$

$$M_y = -(20+4)(3.2) = -76.8 \text{ kip} \cdot \text{in.}$$

Section properties:

$$A = 8.24 \text{ in}^4 \qquad d = 8.06 \text{ in.} \qquad b_f = 6.54 \text{ in.} \qquad t_f = 0.465 \text{ in.}$$

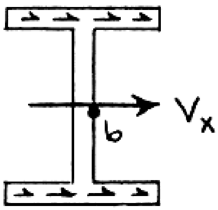
$$t_w = 0.285 \text{ in.} \qquad I_x = 98.0 \text{ in}^4 \qquad I_y = 21.7 \text{ in}^4$$

Point b: $\quad x_b = 0 \qquad y_b = 0$

$$\sigma_b = -\frac{P}{A} + \frac{M_x y_b}{I_x} + \frac{M_y x_b}{I_y}$$

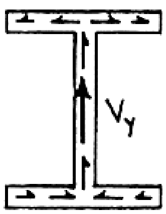
$$= -\frac{20}{8.24} + 0 + 0 = -2.4272 \text{ ksi}$$

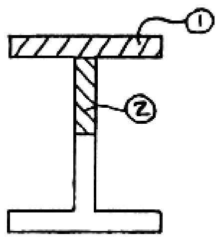
Shearing stress at point b due to V_x:



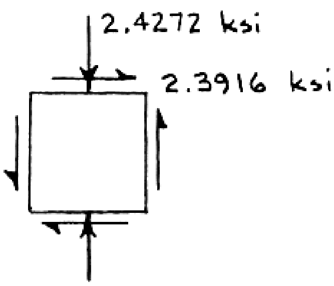
$$\tau_{xz} = 0$$

Shearing stress at point b due to V_y:





	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$
①	3.0411	3.7975	11.5486
②	1.01603	1.7825	1.81107
Σ			13.3597



$$Q = \Sigma A\bar{y} = 13.3597 \text{ in}^3$$

$$\tau_b = \frac{V_y Q}{I_x t_w} = \frac{(5)(13.3597)}{(98.0)(0.285)} = 2.3916 \text{ ksi}$$

$$\sigma_{\text{ave}} = \frac{-2.4272 + 0}{2} = -1.2136 \text{ ksi}$$

$$R = \sqrt{\left(\frac{-2.4272 - 0}{2}\right)^2 + (2.3916)^2}$$

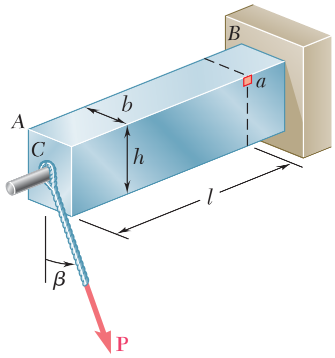
$$= 2.6819 \text{ ksi}$$

$\sigma_{\max} = \sigma_{\text{ave}} + R$ $\qquad\qquad$ $\sigma_{\max} = 1.468 \text{ ksi}$ ◀

$\sigma_{\min} = \sigma_{\text{ave}} - R$ $\qquad\qquad$ $\sigma_{\min} = -3.90 \text{ ksi}$ ◀

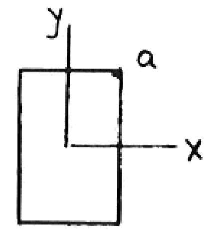
$\tau_{\max} = R$ $\qquad\qquad$ $\tau_{\max} = 2.68 \text{ ksi}$ ◀

PROBLEM 8.59



A force **P** is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that **P** acts in a direction perpendicular to the longitudinal axis of the beam, determine (*a*) the normal stress at point *a* in terms of *P*, *b*, *h*, *l*, and β, (*b*) the values of β for which the normal stress at *a* is zero.

SOLUTION



$$I_x = \frac{1}{12}bh^3 \quad I_y = \frac{1}{12}hb^3$$

$$\sigma = \frac{M_x(h/2)}{I_x} - \frac{M_y(b/2)}{I_y}$$

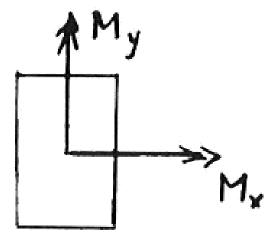
$$= \frac{6M_x}{bh^2} - \frac{6M_y}{hb^2}$$

$$\boldsymbol{P} = P\sin\beta\,\boldsymbol{i} - P\cos\beta\,\boldsymbol{j} \quad \boldsymbol{r} = l\boldsymbol{k}$$

$$\boldsymbol{M} = \boldsymbol{r}\times\boldsymbol{P} = l\boldsymbol{k}\times(P\sin\beta\boldsymbol{i} - P\cos\beta\,\boldsymbol{j})$$

$$= Pl\cos\beta\,\boldsymbol{i} + Pl\sin\beta\,\boldsymbol{j}$$

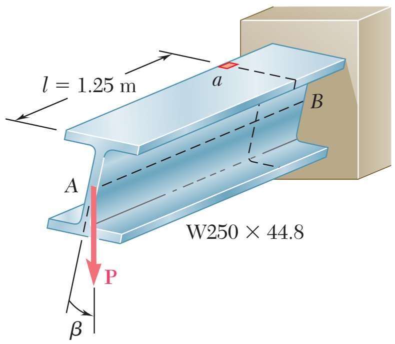
$$M_x = Pl\cos\beta \qquad M_y = Pl\sin\beta$$



(*a*) $\sigma = \frac{6\,Pl\cos\beta}{bh^2} - \frac{6\,Pl\sin\beta}{hb^2} = \frac{6\,Pl}{bh}\left[\frac{\cos\beta}{h} - \frac{\sin\beta}{b}\right]$ ◀

(*b*) $\sigma = 0 \quad \frac{\cos\beta}{h} - \frac{\sin\beta}{b} = 0 \quad \tan\beta = \frac{b}{h}$

$$\beta = \tan^{-1}\left(\frac{b}{h}\right)$$ ◀



PROBLEM 8.60

A vertical force **P** is applied at the center of the free end of cantilever beam AB. (a) If the beam is installed with the web vertical ($\beta = 0$) and with its longitudinal axis AB horizontal, determine the magnitude of the force **P** for which the normal stress at point a is +120 MPa. (b) Solve part a, assuming that the beam is installed with $\beta = 3°$.

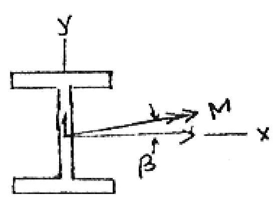
SOLUTION

For W250 × 44.8 rolled-steel section,

$$S_x = 531 \times 10^3 \text{ mm}^3 = 531 \times 10^{-6} \text{ m}^3$$

$$S_y = 94.2 \times 10^3 \text{ mm}^3 = 94.2 \times 10^{-6} \text{ m}^3$$

At the section containing point a,



$$M_x = Pl\cos\beta, \qquad M_y = Pl\sin\beta$$

Stress at a:

$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl\cos\beta}{S_x} + \frac{Pl\sin\beta}{S_y}$$

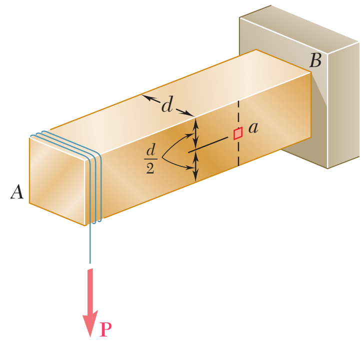
<u>Allowable load</u>. $$P_{\text{all}} = \frac{\sigma_{\text{all}}}{l}\left[\frac{\cos\beta}{S_x} + \frac{\sin\beta}{S_y}\right]^{-1}$$

(a) $\underline{\beta = 0}$: $$P_{\text{all}} = \frac{120 \times 10^6}{1.25}\left[\frac{1}{531 \times 10^{-6}} + 0\right]^{-1} = 51.0 \times 10^3 \text{ N}$$

$P_{\text{all}} = 51.0$ kN ◀

(b) $\underline{\beta = 3°}$: $$P_{\text{all}} = \frac{120 \times 10^6}{1.25}\left[\frac{\cos 3°}{531 \times 10^{-6}} + \frac{\sin 3°}{94.2 \times 10^{-6}}\right]^{-1} = 39.4 \times 10^3 \text{ N}$$

$P_{\text{all}} = 39.4$ kN ◀



PROBLEM 8.61*

A 5-kN force **P** is applied to a wire that is wrapped around bar AB as shown. Knowing that the cross section of the bar is a square of side $d = 40$ mm, determine the principal stresses and the maximum shearing stress at point a.

SOLUTION

Bending: Point a lies on the neutral axis.

$$\sigma = 0$$

Torsion:

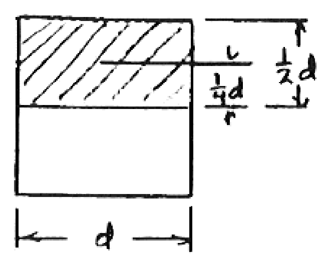
$$\tau = \frac{T}{c_1 ab^2} \quad \text{where} \quad a = b = d$$

and $c_1 = 0.208$ for a square section.

Since

$$T = \frac{Pd}{2}, \quad \tau_T = \frac{P}{0.416d^2} = 2.404\frac{P}{d^2}.$$

Transverse shear:



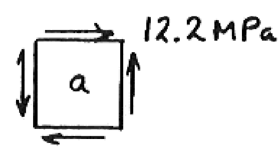
$$V = P \qquad I = \frac{1}{12}d^4$$

$$A = \frac{1}{2}d^2 \qquad \bar{y} = \frac{1}{4}d \qquad Q = A\bar{y} = \frac{1}{8}d^3$$

$$t = d$$

$$\tau_V = \frac{VQ}{It} = 1.5\frac{P}{d^2}$$

By superposition,



$$\tau = \tau_T + \tau_V = 3.904\frac{P}{d^2}$$

$$\tau = \frac{(3.904)(5 \times 10^3)}{(40 \times 10^{-3})^2}$$

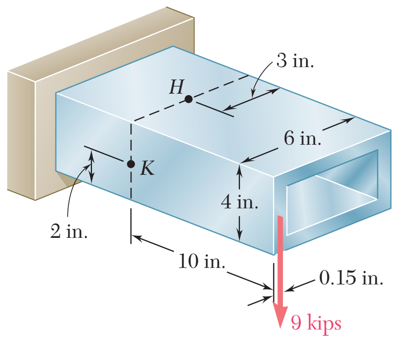
$$= 12.2 \times 10^6 \text{ Pa} = 12.2 \text{ MPa}$$

By Mohr's circle,

$$\sigma_{max} = 12.2 \text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = -12.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{max} = 12.2 \text{ MPa} \blacktriangleleft$$



PROBLEM 8.62*

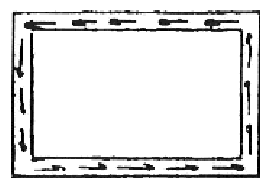
Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the principal stresses, principal planes, and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION

At the section containing points *H* and *K*,

$$V = 9 \text{ kips} \quad M = (9)(10) = 90 \text{ kip} \cdot \text{in.}$$
$$T = (9)(3 - 0.15) = 25.65 \text{ kip} \cdot \text{in.}$$

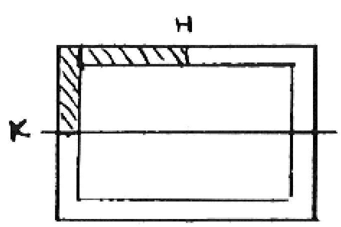
Torsion:

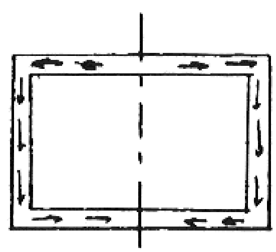


$$\mathscr{A} = (5.7)(3.7) = 21.09 \text{ in}^2$$
$$\tau = \frac{T}{2t\mathscr{A}} = \frac{25.65}{(2)(0.3)(21.09)} = 2.027 \text{ ksi}$$

Transverse shear:

$$Q_H = 0$$
$$Q_K = (3)(2)(1) - (2.7)(1.7)(0.85) = 2.0985 \text{ in}^3$$
$$I = \frac{1}{12}(6)(4)^3 - \frac{1}{12}(5.4)(3.4)^3 = 14.3132 \text{ in}^4$$
$$\tau_H = 0 \qquad \tau_K = \frac{VQ_K}{It} = \frac{(9)(2.0985)}{(14.3132)(0.3)} = 4.398 \text{ ksi}$$





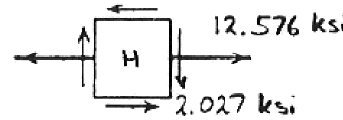
Bending:

$$\sigma_H = \frac{Mc}{I} = \frac{(90)(2)}{14.3132} = 12.576 \text{ ksi}, \qquad \sigma_K = 0$$

PROBLEM 8.62* *(Continued)*

(*a*) Point *H*:

$$\sigma_{ave} = \frac{12.576}{2} = 6.288 \text{ ksi}$$



$$R = \sqrt{\left(\frac{12.576}{2}\right)^2 + (2.027)^2} = 6.607 \text{ ksi}$$

$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 12.90 \text{ ksi} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -0.32 \text{ ksi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau}{\sigma} = -0.3224 \qquad \theta_p = -8.9°,\ 81.1° \blacktriangleleft$$

$$\tau_{max} = R = 6.61 \text{ ksi} \qquad \tau_{max} = 6.61 \text{ ksi} \blacktriangleleft$$

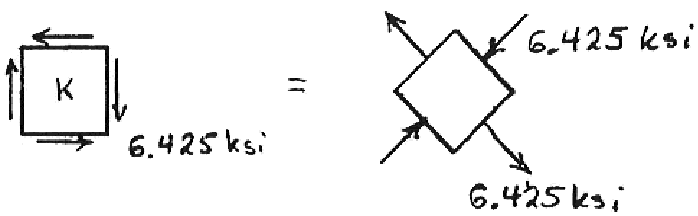
(*b*) Point *K*:

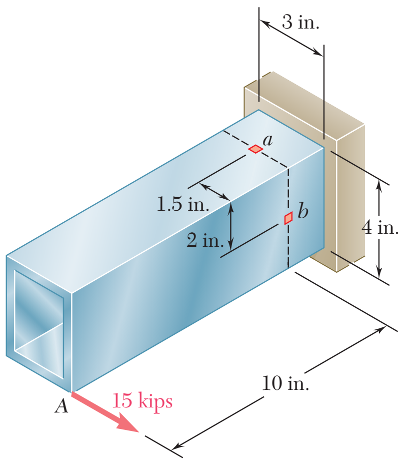
$$\sigma = 0 \qquad \tau = 2.027 + 4.398 = 6.425 \text{ ksi} \qquad \sigma_{max} = 6.43 \text{ ksi} \blacktriangleleft$$

$$\sigma_{min} = -6.43 \text{ ksi} \blacktriangleleft$$

$$\theta = \pm 45° \blacktriangleleft$$

$$\tau_{max} = 6.43 \text{ ksi} \blacktriangleleft$$





PROBLEM 8.63*

The structural tube shown has a uniform wall thickness of 0.3 in. Knowing that the 15-kip load is applied 0.15 in. above the base of the tube, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

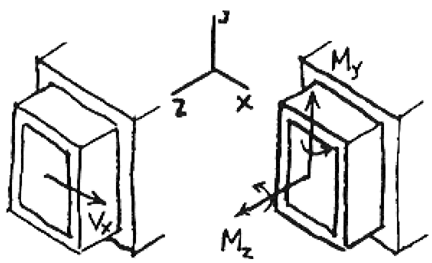
SOLUTION

Calculate forces and couples at section containing points *a* and *b*.

$$V_x = 15 \text{ kips}$$

$$M_z = (2 - 0.15)(15) = 27.75 \text{ kip} \cdot \text{in.}$$

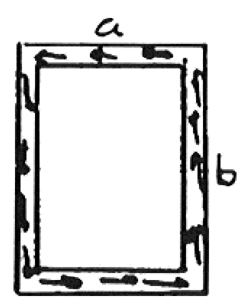
$$M_y = (10)(15) = 150 \text{ kip} \cdot \text{in.}$$



Shearing stresses due to torque $T = M_z$:

$$\mathscr{A} = [3 - (2)(0.15)][4 - (2)(0.15)] = 9.99 \text{ in}^2$$

$$q = \frac{M_z}{2\mathscr{A}} = \frac{27.75}{(2)(9.99)} = 1.3889 \text{ kip/in.}$$



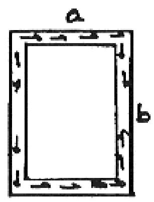
At point *a*, $\quad t = 0.3 \text{ in.} \quad \tau_a = \frac{q}{t} = \frac{1.3889}{0.3} = 4.630 \text{ ksi} \leftarrow$

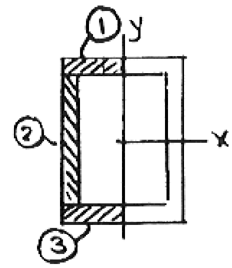
At point *b*, $\quad t = 0.3 \text{ in.} \quad \tau_b = \frac{q}{t} = \frac{1.3889}{0.3} = 4.630 \text{ ksi} \uparrow$

PROBLEM 8.63* *(Continued)*

Shearing stresses due to V_x:

At point *a*,





Part	$A(\text{in}^2)$	$\bar{x}(\text{in.})$	$A\bar{x}(\text{in}^3)$
①	0.45	−0.75	−0.3375
②	1.02	−1.35	−1.377
③	0.45	−0.75	−0.3375
Σ			−2.052

$$Q = \left|\Sigma A\bar{x}\right| = 2.052 \text{ in}^3$$

$$t = (2)(0.3) = 0.6 \text{ in.}$$

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832$$

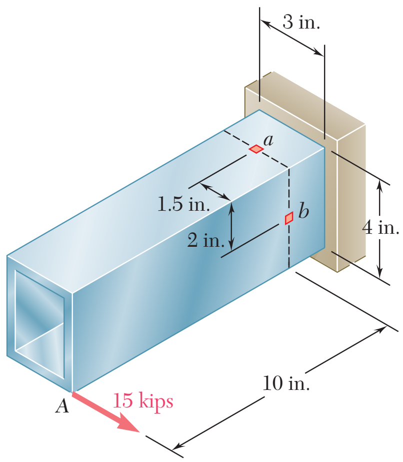
$$\tau = \frac{V_x Q}{I_y t} = \frac{(15)(2.052)}{(5.0832)(0.6)} = 10.092 \text{ ksi}$$

At point *b*, $\tau_b = 0$

Combined shearing stresses.

(*a*) At point *a*, $\tau_a = 4.630 \leftarrow +10.092 \rightarrow = 5.46 \text{ ksi} \rightarrow$ $\tau_a = 5.46$ ksi ◀

(*b*) At point *b*, $\tau_b = 4.630 \uparrow + 0 = 4.63 \text{ ksi} \uparrow$ $\tau_b = 4.63$ ksi ◀



PROBLEM 8.64*

For the tube and loading of Prob. 8.63, determine the principal stresses and the maximum shearing stress at point b.

PROBLEM 8.63* The structural tube shown has a uniform wall thickness of 0.3 in. Knowing that the 15-kip load is applied 0.15 in. above the base of the tube, determine the shearing stress at (a) point a, (b) point b.

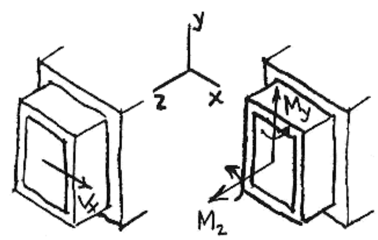
SOLUTION

Calculate forces and couples at section containing point b.

$$V_x = 15 \text{ kips}$$

$$M_z = (2 - 0.15)(15) = 27.75 \text{ kip} \cdot \text{in.}$$

$$M_y = (10)(15) = 150 \text{ kip} \cdot \text{in.}$$



Forces Couples

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832 \text{ in}^4$$

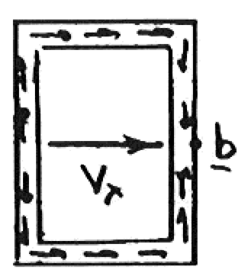
$$\sigma_b = -\frac{M_y x_b}{I_y} = -\frac{(150)(1.5)}{5.0832} = -44.26 \text{ ksi}$$

Shearing stress at point b due to torque M_z:

$$\mathscr{A} = [3 - (2)(0.15)][4 - (2)(0.15)] = 9.99 \text{ in}^2$$

$$q = \frac{M_z}{2\mathscr{A}} = \frac{27.75}{(2)(9.99)} = 1.38889 \text{ kip/in.}$$

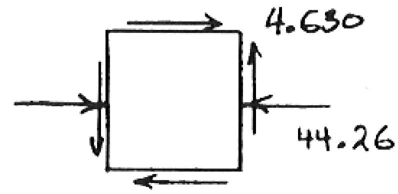
$$\tau = \frac{q}{t} = \frac{1.38889}{0.3} = 4.630 \text{ ksi}$$



Shearing at point b due to V_x:

$$\tau = 0$$

PROBLEM 8.64* *(Continued)*



Calculation of principal stresses and maximum shearing stress.

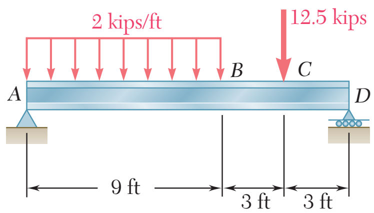
$$\sigma_{ave} = \frac{-44.26 + 0}{2} = -22.13 \text{ ksi}$$

$$R = \sqrt{\left(-\frac{44.26 - 0}{2}\right)^2 + (4.630)^2} = 22.61 \text{ ksi}$$

$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 0.48 \text{ ksi} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -44.7 \text{ ksi} \blacktriangleleft$$

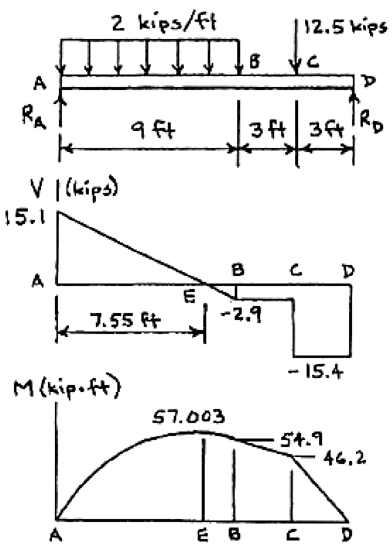
$$\tau_{max} = R \qquad \tau_{max} = 22.6 \text{ ksi} \blacktriangleleft$$



PROBLEM 8.65

(*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION



$$+\circlearrowleft \Sigma M_D = 0: \quad -15R_A + (10.5)(9)(2) + (3)(12.5) = 0$$

$$R_A = 15.1 \text{ kips} \uparrow \qquad R_D = 15.4 \text{ kips} \uparrow$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{(57.003 \times 12 \text{ kip} \cdot \text{in.})}{24 \text{ ksi}} = 28.502 \text{ in}^3$$

	Shape	S (in^3)
	W16 × 26	38.4
→	W14 × 22	29.0
	W12 × 26	33.4
	W10 × 30	32.4
	W8 × 35	31.2

(*a*) Use W14 × 22. ◀

For W14 × 22, $\quad A_{web} = dt_w = (13.7)(0.230) = 3.151 \text{ in}^2$

(*b*) Point *E*:

$$\sigma_m = \frac{M_E}{S} = \frac{(57.003)(12)}{29.0} = 23.587 \text{ ksi} \qquad \sigma_m = 23.6 \text{ ksi} \blacktriangleleft$$

$$c = \frac{1}{2}d = 6.85 \text{ in.} \qquad y_b = c - t_f = 6.515 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{6.515}{6.85}\right)(23.587) = 22.433 \text{ ksi}$$

Since $\tau_m = 0$, $\qquad \sigma_{max} = \sigma_b = 22.4$ ksi ◀

Point *C*:

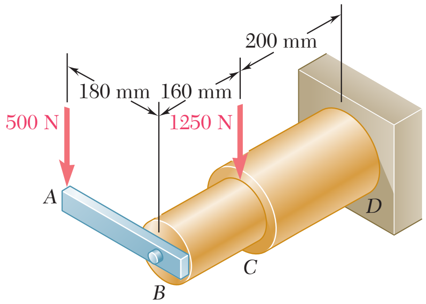
$$\sigma_m = \frac{M_C}{S} = \frac{(46.2)(12)}{29.0} = 19.1172 \text{ ksi} \qquad \sigma_m = 19.12 \text{ ksi}$$

$$\tau_m = \frac{|V|}{A_{web}} = \frac{15.4}{3.151} = 4.8873 \text{ ksi} \qquad \tau_m = 4.89 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{6.515}{6.85}\right)(19.1172) = 18.1823 \text{ ksi} \qquad R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = 10.3216 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R \qquad \sigma_{max} = 19.41 \text{ ksi} \blacktriangleleft$$

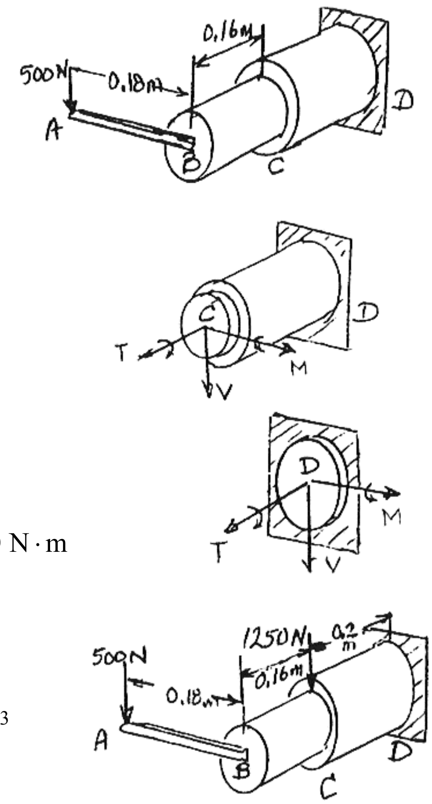
For σ_{max}, point *B* controls; $\qquad \sigma_{max} = 22.4$ ksi ◀



PROBLEM 8.66

Neglecting the effect of fillets and of stress concentrations, determine the smallest permissible diameters of the solid rods BC and CD. Use $\tau_{\text{all}} = 60$ MPa.

SOLUTION



$$\tau_{\text{all}} = 60 \times 10^6 \text{ Pa}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} \qquad d = 2c$$

<u>Bending moments and torques</u>.

Just to the left of C:

$$M = (500)(0.16) = 80 \text{ N} \cdot \text{m}$$

$$T = (500)(0.18) = 90 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 120.416 \text{ N} \cdot \text{m}$$

Just to the left of D:

$$T = 90 \text{ N} \cdot \text{m}$$

$$M = (500)(0.36) + (1250)(0.2) = 430 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 439.32 \text{ N} \cdot \text{m}$$

<u>Smallest permissible diameter d_{BC}</u>.

$$c^3 = \frac{(2)(120.416)}{\pi(60 \times 10^6)} = 1.27765 \times 10^{-6} \text{ m}^3$$

$$c = 0.01085 \text{ m} = 10.85 \text{ mm}$$

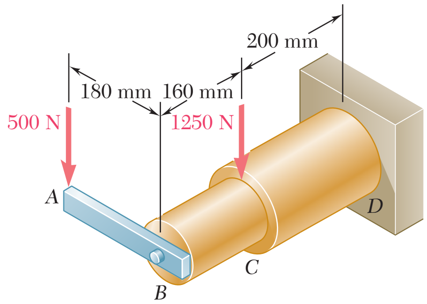
$$d_{BC} = 21.7 \text{ mm} \blacktriangleleft$$

<u>Smallest permissible diameter d_{CD}</u>.

$$c^3 = \frac{(2)(439.32)}{\pi(60 \times 10^6)} = 4.6613 \times 10^{-6} \text{ m}^3$$

$$c = 0.01670 \text{ m} = 16.7 \text{ mm}$$

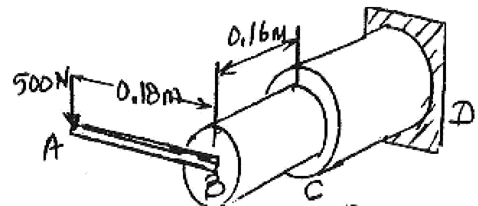
$$d_{CD} = 33.4 \text{ mm} \blacktriangleleft$$



PROBLEM 8.67

Knowing that rods BC and CD are of diameter 24 mm and 36 mm, respectively, determine the maximum shearing stress in each rod. Neglect the effect of fillets and of stress concentrations.

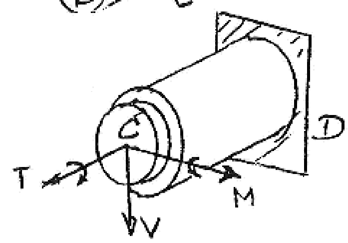
SOLUTION



Over BC: $$c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$$

Over CD: $$c = \frac{1}{2}d = 18 \text{ mm} = 0.018 \text{ m}$$

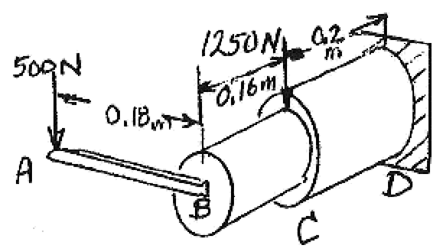
$$\tau = \frac{\sqrt{M^2+T^2}\,c}{J} \quad \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{c^3}$$



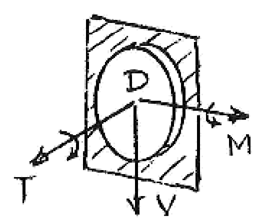
Bending moments and torques.

Just to the left of C:

$$M = (500)(0.16) = 80 \text{ N}\cdot\text{m}$$
$$T = (500)(0.18) = 90 \text{ N}\cdot\text{m}$$
$$\sqrt{M^2+T^2} = 120.416 \text{ N}\cdot\text{m}$$



Just to the left of D:



$$T = 90 \text{ N}\cdot\text{m}$$
$$M = (500)(0.36) + (1250)(0.2) = 430 \text{ N}\cdot\text{m}$$
$$\sqrt{M^2+T^2} = 439.32 \text{ N}\cdot\text{m}$$

Maximum shearing stress in portion BC.

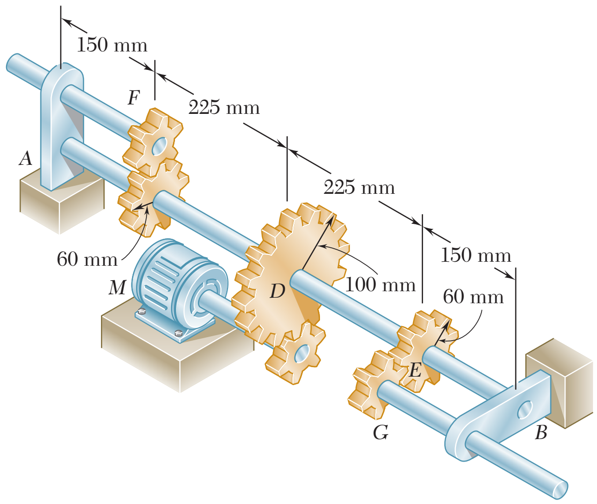
$$\tau_{\max} = \frac{(2)(120.416)}{\pi(0.012)^3} = 44.36\times 10^6 \text{ Pa}$$

$\tau_{\max} = 44.4$ MPa ◀

Maximum shearing stress in portion CD.

$$\tau_{\max} = \frac{(2)(439.32)}{\pi(0.018)^3} = 47.96\times 10^6 \text{ Pa}$$

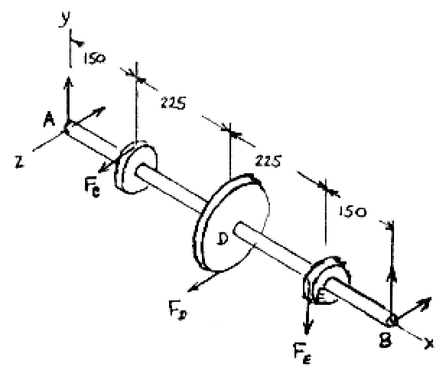
$\tau_{\max} = 48.0$ MPa ◀



PROBLEM 8.68

The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G. Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G, determine the smallest permissible diameter of shaft AB.

SOLUTION



$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D:

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)}$$
$$= 424.41 \text{ N} \cdot \text{m}$$

Torques on gears C and E:

$$T_C = \frac{8}{20} T_D = 169.76 \text{ N} \cdot \text{m}$$

$$T_E = \frac{12}{20} T_D = 254.65 \text{ N} \cdot \text{m}$$

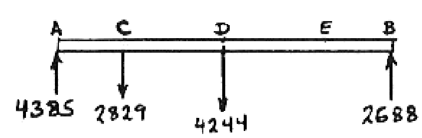
<u>Forces on gears</u>:

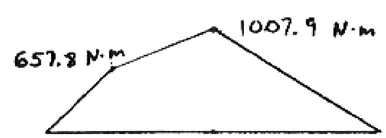
$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60 \times 10^{-3}} = 2829 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60 \times 10^{-3}} = 4244 \text{ N}$$

<u>Forces in horizontal plane:</u>





<u>Torques in various parts</u>:

AC: $T = 0$

CD: $T = 169.76 \text{ N} \cdot \text{m}$

DE: $T = 254.65 \text{ N} \cdot \text{m}$

EB: $T = 0$

PROBLEM 8.68 *(Continued)*

Critical point lies just to the right of D.

$$T = 254.65\ \text{N}\cdot\text{m}$$

$$M_y = 1007.9\ \text{N}\cdot\text{m}$$

$$M_z = 318.3\ \text{N}\cdot\text{m}$$

$$\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max} = 1087.2\ \text{N}\cdot\text{m}$$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}}{\tau_{\text{all}}}$$

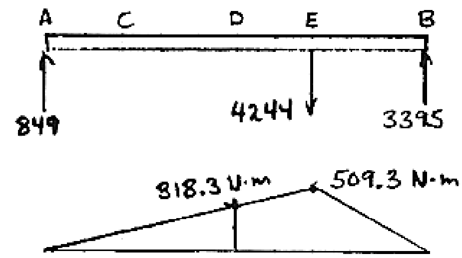
$$= \frac{1087.2}{55 \times 10^6}$$

$$= 19.767 \times 10^{-3}\ \text{m}^3$$

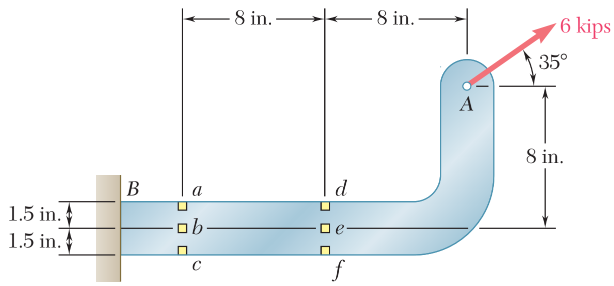
$$c = 23.26 \times 10^{-3}\ \text{m}$$

$$d = 2c = 46.5 \times 10^{-3}\ \text{m}$$

Forces in vertical plane:



$d = 46.5\ \text{mm}$ ◀



PROBLEM 8.69

A 6-kip force is applied to the machine element *AB* as shown. Knowing that the uniform thickness of the element is 0.8 in., determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Thickness = 0.8 in.

At the section containing points *a*, *b*, and *c*,

$$P = 6\cos 35° = 4.9149 \text{ kips} \qquad V = 6\sin 35° = 3.4415 \text{ kips}$$

$$M = (6\sin 35°)(16) - (6\cos 35°)(8) = 15.744 \text{ kip}\cdot\text{in.}$$

$$A = (0.8)(3.0) = 2.4 \text{ in}^2$$

$$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80 \text{ in}^4$$

(*a*) At point *a*, $\sigma_x = \dfrac{P}{A} - \dfrac{Mc}{I} = \dfrac{4.9149}{2.4} - \dfrac{(15.744)(1.5)}{1.80}$ $\qquad \sigma_x = -11.07 \text{ ksi}$ ◀

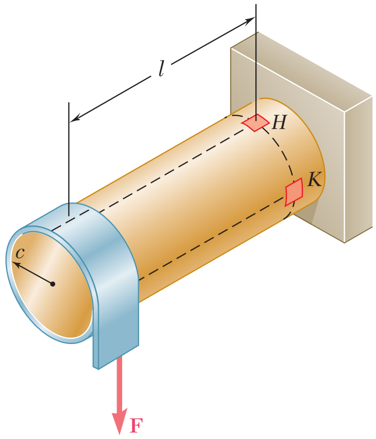
$\tau_{xy} = 0$ ◀

(*b*) At point *b*, $\sigma_x = \dfrac{P}{A} = \dfrac{4.9149}{2.4}$ $\qquad \sigma_x = 2.05 \text{ ksi}$ ◀

$\tau_{xy} = \dfrac{3}{2}\dfrac{V}{A} = \dfrac{3}{2}\cdot\dfrac{3.4415}{2.4}$ $\qquad \tau_{xy} = 2.15 \text{ ksi}$ ◀

(*c*) At point *c*, $\sigma_x = \dfrac{P}{A} + \dfrac{Mc}{I} = \dfrac{4.9149}{2.4} + \dfrac{(15.744)(1.5)}{1.80}$ $\qquad \sigma_x = 15.17 \text{ ksi}$ ◀

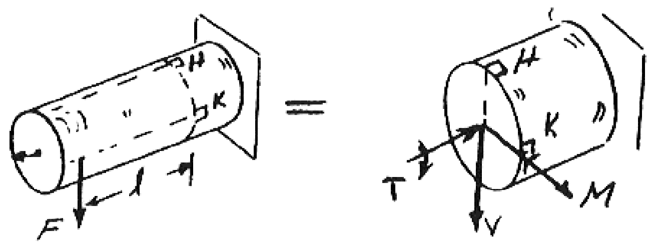
$\tau_{xy} = 0$ ◀



PROBLEM 8.70

A thin strap is wrapped around a solid rod of radius $c = 20$ mm as shown. Knowing that $l = 100$ mm and $F = 5$ kN, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION



At the section containing points *H* and *K*,

$$T = Fc \qquad M = Fl \qquad V = F$$

$$J = \frac{\pi}{2}c^4 \qquad I = \frac{\pi}{4}c^4$$

Point *H*:

$$\sigma = \frac{Mc}{I} = \frac{Flc}{\frac{\pi}{4}c^4} \qquad \sigma = \frac{4Fl}{\pi c^2}$$

$$\tau = \frac{Tc}{J} = \frac{Fc^2}{\frac{\pi}{4}c^4} \qquad \tau = \frac{2F}{\pi c^2}$$

Point *K*: Point *K* lies on the neutral axis. $\sigma = 0$

Due to torque: $\tau = \dfrac{Tc}{J} = \dfrac{2F}{\pi c^2}$

Due to shear: For a semicircle, $Q = \dfrac{2}{3}c^3, \quad t = d = 2c$

$$\tau = \frac{VQ}{It} = \frac{F\frac{2}{3}c^3}{\frac{\pi}{4}c^4(2c)} = \frac{4F}{3\pi c^2}$$

Combined: $\tau = \dfrac{2F}{\pi c^2} + \dfrac{4F}{3\pi c^2} \qquad \tau = \dfrac{10F}{3\pi c^2}$

PROBLEM 8.70 *(Continued)*

Data: $F = 5\text{ kN} = 5\times10^3\text{ N},\quad l = 100\text{ mm} = 0.100\text{ m}$

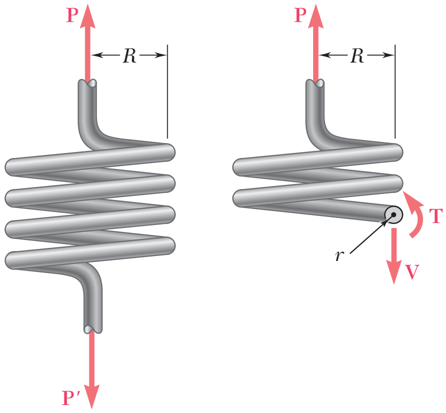
$c = 20\text{ mm} = 0.020\text{ m}$

(*a*) Point *H*: $\sigma = \dfrac{(4)(5\times10^3)(0.100)}{\pi(0.020)^3}$ $\sigma = 79.6\text{ MPa}$ ◀

$\tau = \dfrac{(2)(5\times10^3)}{\pi(0.020)^2}$ $\tau = 7.96\text{ MPa}$ ◀

(*b*) Point *K*: $\sigma = 0$ ◀

$\tau = \dfrac{(10)(5\times10^3)}{3\pi(0.020)^2}$ $\tau = 13.26\text{ MPa}$ ◀



PROBLEM 8.71

A close-coiled spring is made of a circular wire of radius r that is formed into a helix of radius R. Determine the maximum shearing stress produced by the two equal and opposite forces **P** and **P′**. (*Hint:* First determine the shear **V** and the torque **T** in a transverse cross section.)

SOLUTION

$$+\uparrow \Sigma F_y = 0: \qquad P - V = 0 \qquad V = P$$

$$+\circlearrowleft \Sigma M_C = 0: \qquad T - PR = 0 \qquad T = PR$$

<u>Shearing stress due to T.</u>

$$\tau_T = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2PR}{\pi r^3}$$

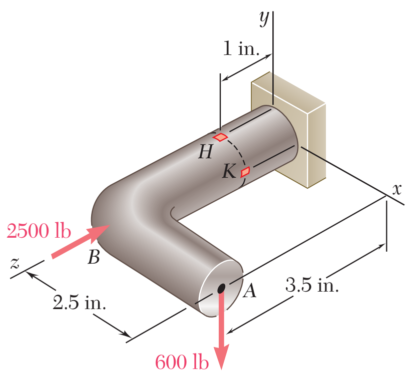
<u>Shearing stress due to V.</u>

For semicircle, $$Q = \frac{2}{3}r^3, \quad t = d = 2r$$

For solid circular section, $$I = \frac{1}{2}J = \frac{\pi}{4}r^3$$

$$\tau_V = \frac{VQ}{It} = \frac{V\left(\frac{2}{3}r^3\right)}{\frac{\pi}{4}r^4(2r)} = \frac{4V}{3\pi r^2} = \frac{4P}{3\pi r^2}$$

By superposition, $$\tau_{\max} = \tau_T + \tau_V \qquad \tau_{\max} = P(2R + 4r/3)/\pi r^3 \blacktriangleleft$$



PROBLEM 8.72

Forces are applied at points A and B of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 0.8 in., determine the principal stresses and the maximum shearing stress at (a) point H, (b) point K.

SOLUTION

At the section containing points H and K,

$$P = 2500 \text{ lb} \quad \text{(compression)}$$

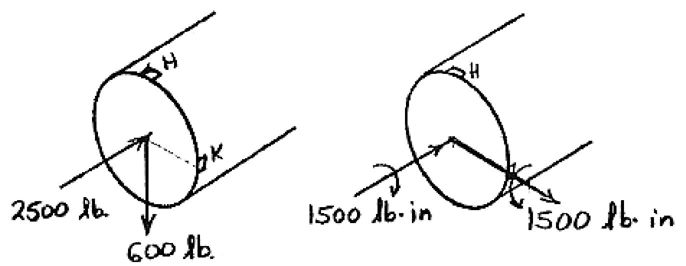
$$V_y = -600 \text{ lb}$$

$$V_x = 0$$

$$M_x = (3.5-1)(600) = 1500 \text{ lb}\cdot\text{in.}$$

$$M_y = 0$$

$$M_z = -(2.5)(600) = -1500 \text{ lb}\cdot\text{in.}$$



Forces Couples

$$c = \frac{1}{2}d = 0.4 \text{ in.}$$

$$A = \pi c^2 = 0.50265 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 20.106\times10^{-3} \text{ in}^4$$

$$J = 2I = 40.212\times10^{-3} \text{ in}^4$$

For semicircle,

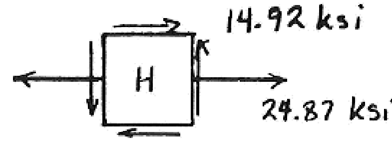
$$Q = \frac{2}{3}c^3$$

$$= 42.667\times10^{-3} \text{ in}^3$$

(*a*) At point H,

$$\sigma_H = \frac{P}{A} + \frac{Mc}{I} = -\frac{2500}{0.50265} + \frac{(1500)(0.4)}{20.106 \times 10^{-3}}$$

$$= 24.87 \times 10^3 \text{ psi}$$



$$\tau_H = \frac{Tc}{J} = \frac{(1500)(0.4)}{40.212 \times 10^{-3}} = 14.92 \times 10^3 \text{ psi}$$

$$\sigma_{\text{ave}} = \frac{24.87}{2} = 12.435 \text{ ksi}$$

$$R = \sqrt{\left(\frac{24.87}{2}\right)^2 + (14.92)^2} = 19.423 \text{ ksi}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 31.9 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -6.99 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 19.42 \text{ ksi} \blacktriangleleft$$

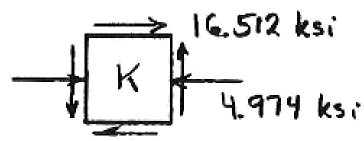
(*b*) At point K,

$$\sigma_K = \frac{P}{A} = -\frac{2500}{0.50265} = -4.974 \times 10^3 \text{ psi}$$

$$\tau_K = \frac{Tc}{J} + \frac{VQ}{It}$$

$$= \frac{(1500)(0.4)}{40.212 \times 10^{-3}} + \frac{(600)(42.667 \times 10^{-3})}{(20.106 \times 10^{-3})(0.8)}$$

$$= 16.512 \times 10^3 \text{ psi}$$



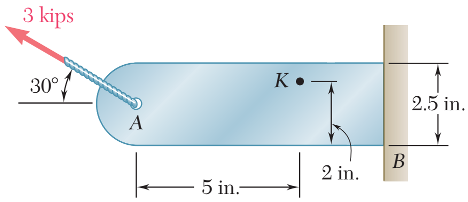
$$\sigma_{\text{ave}} = -\frac{4.974}{2} = -2.487 \text{ ksi}$$

$$R = \sqrt{\left(-\frac{4.974}{2}\right)^2 + (16.512)^2} = 16.698 \text{ ksi}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 14.21 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -19.18 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 16.70 \text{ ksi} \blacktriangleleft$$



PROBLEM 8.73

Knowing that the bracket AB has a uniform thickness of $\frac{5}{8}$ in., determine (*a*) the principal planes and principal stresses at point K, (*b*) the maximum shearing stress at point K.

SOLUTION

Resolve the 3-kip force **F** at point A into x and y components.

$$F_x = -F\cos 30° = -(3)\cos 30° = -2.598 \text{ kips}$$

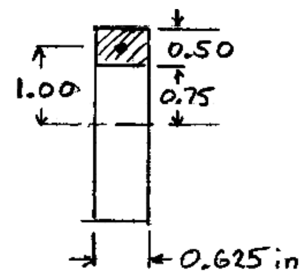
$$F_y = F\sin 30° = (3)\sin 30° = 1.5 \text{ kips}$$

At the section containing points H and K,

$$P = -F_x = 2.598 \text{ kips}, \qquad V = F_y = 1.5 \text{ kips}$$

$$M = (5)(1.5) = 7.5 \text{ kip} \cdot \text{in.}$$

Section properties. $\quad t = \frac{5}{8} \text{ in.} = 0.625 \text{ in.}, \qquad A = \left(\frac{5}{8}\right)(2.5) = 1.5625 \text{ in}^2,$

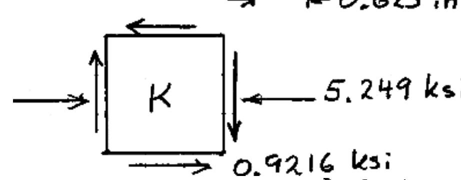


$$I = \frac{1}{12}(0.625)(2.5)^3 = 0.8138 \text{ in}^4, \qquad c = 1.25 \text{ in.}$$

At point k, $\quad y = 0.75$ in.

$$Q = (0.625)(0.50)(1.00) = 0.3125 \text{ in}^3$$

Stresses at point K.

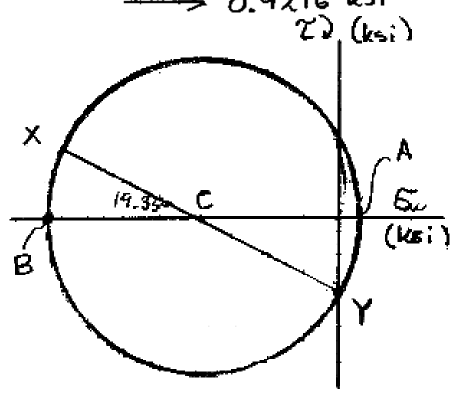


$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{2.598}{1.5625} - \frac{(7.5)(0.75)}{0.8138} = -5.249 \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(1.5)(0.3125)}{(0.8138)(0.625)} = 0.9216 \text{ ksi}$$

$$\sigma_x = -5.249 \text{ ksi}, \qquad \sigma_y = 0, \qquad \tau_{xy} = -0.9216 \text{ ksi}$$

Mohr's circle.



$$X: (\sigma_x, -\tau_{xy}) = (-5.249 \text{ ksi}, 0.9216 \text{ ksi})$$

$$Y: (\sigma_y, \tau_{xy}) = (0, -0.9216 \text{ ksi})$$

$$C: (\sigma_{ave}, 0) = (-2.6245 \text{ ksi}, 0)$$

$$\frac{\sigma_x - \sigma_y}{2} = -2.6245 \text{ ksi}$$

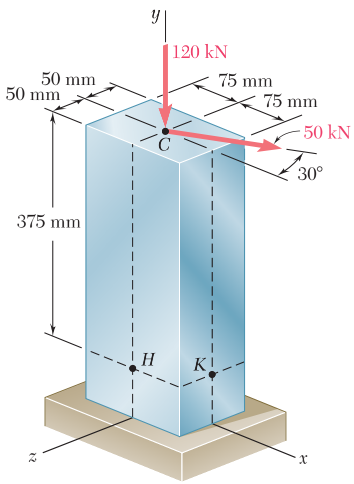
$$R = \sqrt{(2.6245)^2 + (0.9216)^2} = 2.7816 \text{ ksi}$$

$$\tan 2\theta_p = \frac{0.9216}{2.6245} = 0.35112 \quad 2\theta_p = 19.35°$$

(*a*) $\quad \sigma_{max} = \sigma_{ave} + R = -2.6245 + 2.7816 \qquad \sigma_{max} = 0.157 \text{ ksi at } 80.3°$ ◀

(*b*) $\quad \sigma_{min} = \sigma_{ave} - R = -2.6245 - 2.7816 \qquad \sigma_{min} = -5.41 \text{ ksi at } 9.7°$ ◀

$\quad \tau_{max} = R = 2.7816 \text{ ksi} \qquad \tau_{max} = 2.78 \text{ ksi}$ ◀



PROBLEM 8.74

For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point H.

SOLUTION

Components of force at point C:

$$F_x = 50 \cos 30° = 43.301 \text{ kN}$$

$$F_z = -50 \sin 30° = -25 \text{ kN}$$

$$F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points H and K:

$$P = 120 \text{ kN} \qquad \text{(compression)}$$

$$V_x = 43.301 \text{ kN}$$

$$V_z = -25 \text{ kN}$$

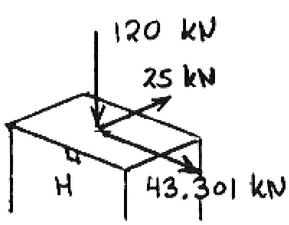
$$M_x = -(25)(0.375) = -9.375 \text{ kN} \cdot \text{m}$$

$$M_y = 0$$

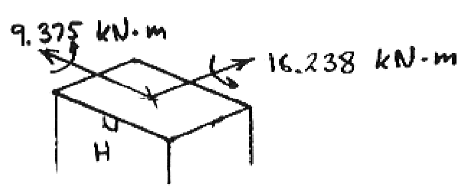
$$M_z = -(43.301)(0.375) = -16.238 \text{ kN} \cdot \text{m}$$

$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2 = 15 \times 10^{-3} \text{ m}^2$$

$$I_x = \frac{1}{12}(150)(100)^3 = 12.5 \times 10^6 \text{ mm}^4 = 12.5 \times 10^{-6} \text{ m}^4$$



Forces



Couples

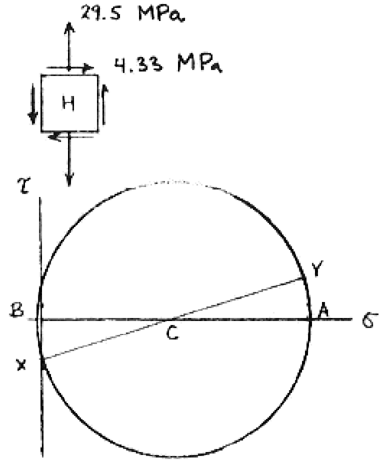
PROBLEM 8.74 *(Continued)*

Stresses at point H:

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120\times10^3)}{15\times10^{-3}} - \frac{(-9.375\times10^3)(50\times10^{-3})}{12.5\times10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_x}{A} = \frac{3}{2}\frac{43.301\times10^3}{15\times10^{-3}} = 4.33 \text{ MPa}$$

Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H = 14.75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37 \text{ MPa}$$

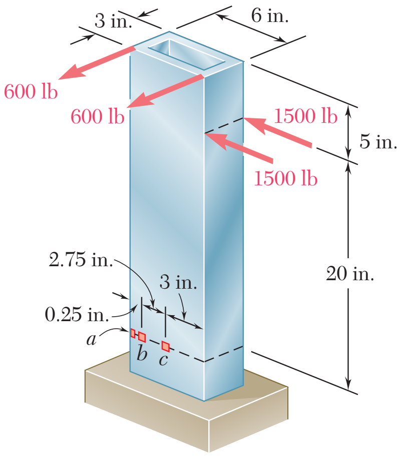
$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 30.1 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.62 \text{ MPa} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$$

$$\theta_a = -8.2^\circ \quad \theta_b = 81.8^\circ \blacktriangleleft$$

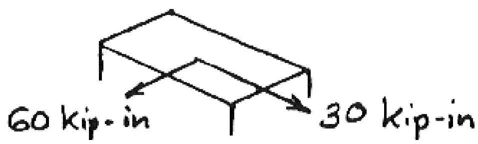
$$\tau_{\max} = R \qquad \tau_{\max} = 15.37 \text{ MPa} \blacktriangleleft$$



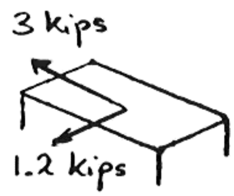
PROBLEM 8.75

Knowing that the structural tube shown has a uniform wall thickness of 0.25 in., determine the normal and shearing stresses at the three points indicated.

SOLUTION



Bending moment



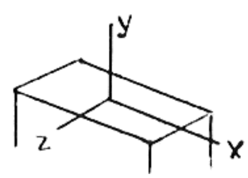
Shear

$$b_o = 6 \text{ in.} \quad b_i = b_o - 2t = 5.5 \text{ in.}$$

$$h_o = 3 \text{ in.} \quad h_i = h_o - 2t = 2.5 \text{ in.}$$

$$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 6.3385 \text{ in}^4$$

$$I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 19.3385 \text{ in}^4$$



Normal stresses.

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x} \begin{cases} \underline{\text{At } a}: \dfrac{(60)(-3)}{19.3385} - \dfrac{(30)(1.5)}{6.3385} \\ \underline{\text{At } b}: \dfrac{(60)(-2.75)}{19.3385} - \dfrac{(30)(1.5)}{6.3385} \\ \underline{\text{At } c}: \dfrac{(60)(0)}{19.3385} - \dfrac{(30)(1.5)}{6.3385} \end{cases}$$

$\sigma_a = -16.41 \text{ ksi}$ ◀

$\sigma_b = -15.63 \text{ ksi}$ ◀

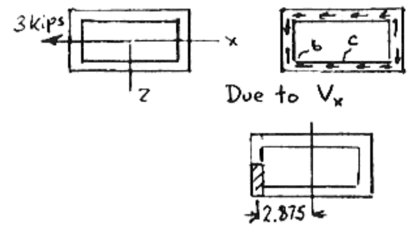
$\sigma_c = -7.10 \text{ ksi}$ ◀

PROBLEM 8.75 *(Continued)*

<u>Shearing stresses</u>.

Point a is an outside corner. $\tau_a = 0$ ◀

At point b,



$$Q_{zb} = (1.5)(0.25)(2.875) = 1.0781 \text{ in}^3$$

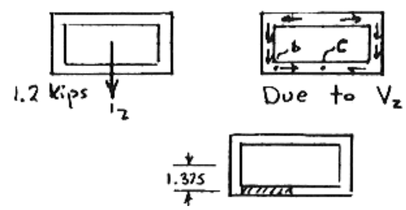
$$\tau_{b,v_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(1.0781)}{(19.3385)(0.25)} = 0.66899 \text{ ksi}$$

At point c,

$$Q_{zc} = Q_{zb} + (2.75)(0.25)\left(\frac{2.75}{2}\right) = 2.0234 \text{ in}^3$$

$$\tau_{c,v_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(2.0234)}{(19.3385)(0.25)} = 1.256 \text{ ksi}$$

At point b,



$$Q_{xb} = (2.75)(0.25)(1.375) = 0.9453 \text{ in}^3$$

$$\tau_{b,v_z} = \frac{V_z Q_y}{I_x t} = \frac{(1.2)(0.9453)}{(6.3385)(0.25)} = 0.71585 \text{ ksi}$$

At point c, (symmetry axis)

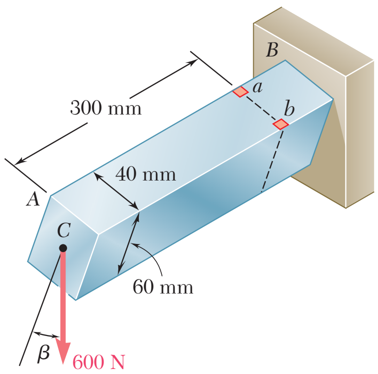
$$\tau_{c,v_z} = 0$$

Net shearing stress at points b and c:

$$\tau_b = 0.71585 - 0.66899$$ $\tau_b = 0.0469$ ksi ◀

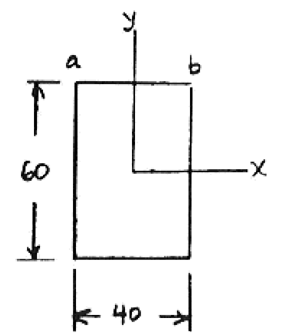
$$\tau_c = 1.256$$ $\tau_c = 1.256$ ksi ◀

PROBLEM 8.76



The cantilever beam AB will be installed so that the 60-mm side forms an angle β between 0 and 90° with the vertical. Knowing that the 600-N vertical force is applied at the center of the free end of the beam, determine the normal stress at point a when (*a*) $\beta = 0$, (*b*) $\beta = 90°$. (*c*) Also, determine the value of β for which the normal stress at point a is a maximum and the corresponding value of that stress.

SOLUTION



$$S_x = \frac{1}{6}(40)(60)^2 = 24 \times 10^3 \text{ mm}^3$$

$$= 24 \times 10^{-6} \text{ m}^3$$

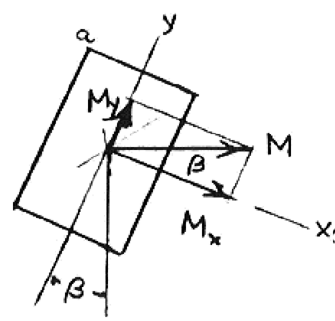
$$S_y = \frac{1}{6}(60)(40)^2 = 16 \times 10^3 \text{ mm}^3$$

$$= 16 \times 10^{-3} \text{ m}^3$$

$$M = Pl = (600)(300 \times 10^{-3}) = 180 \text{ N} \cdot \text{m}$$

$$M_x = M \cos\beta = 180 \cos\beta$$

$$M_y = M \sin\beta = 180 \sin\beta$$



$$\sigma_a = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{180 \cos\beta}{24 \times 10^{-6}} + \frac{180 \sin\beta}{16 \times 10^{-6}}$$

$$= (7.5 \times 10^6)\left(\cos\beta + \frac{3}{2}\sin\beta\right) \text{ Pa}$$

$$= 7.5\left(\cos\beta + \frac{3}{2}\sin\beta\right) \text{ MPa}$$

(*a*) $\underline{\beta = 0.}$ $\qquad \sigma_a = 7.50$ MPa $\qquad \sigma = 7.50$ MPa ◀

(*b*) $\underline{\beta = 90°.}$ $\qquad \sigma_a = 11.25$ MPa $\qquad \sigma = 11.25$ MPa ◀

(*c*) <u>Maximum</u>.

$$\frac{d\sigma_a}{d\beta} = 7.5\left(-\sin\beta + \frac{3}{2}\cos\beta\right) = 0$$

$$\sin\beta = \frac{3}{2}\cos\beta \qquad \tan\beta = \frac{3}{2}$$

$\beta = 56.3°$ ◀

$$\sigma_a = 7.5\left(\cos 56.3° + \frac{3}{2}\sin 56.3°\right)$$

$\sigma = 13.52$ MPa ◀

PROBLEM 8.C1

Let us assume that the shear V and the bending moment M have been determined in a given section of a rolled-steel beam. Write a computer program to calculate in that section, from the data available in Appendix C, (*a*) the maximum normal stress σ_m, (*b*) the principal stress $\sigma_{\max}$ at the junction of a flange and the web. Use this program to solve parts *a* and *b* of the following problems:

(1) Prob. 8.1 (Use $V = 45$ kips and $M = 450$ kip · in.)

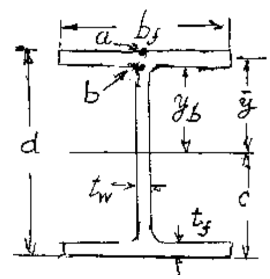
(2) Prob. 8.2 (Use $V = 22.5$ kips and $M = 450$ kip · in.)

(3) Prob. 8.3 (Use $V = 700$ kN and $M = 1750$ kN · m.)

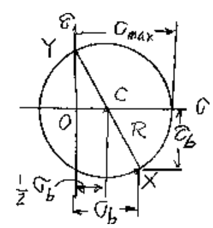
(4) Prob. 8.4 (Use $V = 850$ kN and $M = 1700$ kN · m.)

SOLUTION

We enter the given values of V and M and obtain from Appendix C the values of d, b_f, t_f, t_w, I, and S for the given WF shape.



We compute
$$c = \frac{d}{2}, \quad y_b = c - t_f$$
$$\bar{y} = c - \frac{1}{2}t_f, \quad \sigma_a = \frac{M}{S}, \quad \sigma_b = \sigma_a\left(\frac{y_b}{c}\right)$$
$$Q = b_f t_f \bar{y}, \qquad \tau_b = \frac{VQ}{It_w}$$



From Mohr's circle, $$\sigma_{\max} = \frac{1}{2}\sigma_b + R$$
$$\sigma_{\max} = \frac{1}{2}\sigma_b + \sqrt{\left(\frac{1}{2}\sigma_b\right)^2 + \tau_b^2}$$

<u>Program Outputs</u>

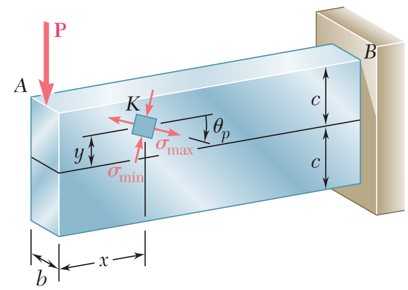
<u>Problem 8.1</u> Given Data: $V = 45$ kips, $M = 450$ kip · in. $d = 9.92$ in., $b_f = 7.985$ in. $t_f = 0.530$ in., $t_w = 0.315$ in. $I = 209.00$ in.^{4}, $S = 42.10$ in.^{4} Answers: (a) σ_A = 10,688.84 psi (b) σ_m = 19,169.08 psi	<u>Problem 8.2</u> Given Data: $V = 22.5$ kips, $M = 450$ kip · in. $d = 9.92$ in., $b_f = 7.985$ in. $t_f = 0.530$ in., $t_w = 0.315$ in. $I = 209.00$ in.^{4}, $S = 42.10$ in.^{4} Answers: (a) σ_A = 10,688.84 psi (b) σ_m = 13,073.82 psi

PROBLEM 8.C1 *(Continued)*

Program Outputs (***Continued***)

Problem 8.3	Problem 8.4
Given Data:	Given Data:
$V = 700$ kN, $M = 1750$ kN $\cdot$ m	$V = 850$ kN, $M = 1700$ kN $\cdot$ m
$d = 9.30$ mm, $b_f = 423$ mm	$d = 930$ mm, $b_f = 423$ mm
$t_f = 43$ mm, $t_w = 24$ mm	$t_f = 43$ mm, $t_w = 24$ mm
$I = 8470\ (10^6\ \text{mm}^4)$	$I = 8470\ (10^6\ \text{mm}^4)$
$S = 18{,}200 (10^3\ \text{mm}^3)$	$S = 18{,}200\ (10^3\ \text{mm}^3)$
Answers:	Answers:
(a) $\sigma_A = 96.15$ MPa	(a) $\sigma_A = 93.40$ MPa
(b) $\sigma_m = 95.39$ MPa	(b) $\sigma_m = 96.56$ MPa

PROBLEM 8.C2



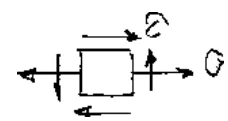
A cantilever beam AB with a rectangular cross section of width b and depth $2c$ supports a single concentrated load **P** at its end A. Write a computer program to calculate, for any values of x/c and y/c, (a) the ratios σ_{max}/σ_m and σ_{min}/σ_m, where σ_{max} and σ_{min} are the principal stresses at Point $K(x, y)$ and σ_m the maximum normal stress in the same transverse section, (b) the angle θ_p that the principal planes at K form with a transverse and a horizontal plane through K. Use this program to check the values shown in Fig. 8.8 and to verify that σ_{max} exceeds σ_m if $x \leqslant 0.544c$, as indicated in the second footnote on Page 560.

SOLUTION

Since the distribution of the normal stresses is linear, we have $\sigma = \sigma_m \left(\dfrac{y}{c}\right)$ (1)

where $$\sigma_m = \frac{MC}{I} = \frac{Pxc}{I} \qquad (2)$$

We use Equation (8.4), Page 498: $$\tau = \frac{3}{2}\frac{P}{A}\left(1 - \frac{y^2}{c^2}\right) \qquad (3)$$



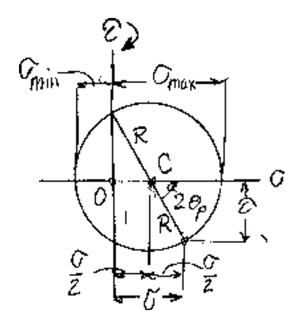
Dividing (3) by (2), $$\frac{\tau}{\sigma_m} = \frac{3}{2}\frac{I}{A}\frac{1-\left(\frac{y}{c}\right)^2}{xc}$$

or, since $$\frac{I}{A} = \frac{\frac{1}{12}b(2c)^3}{b(2c)} = \frac{1}{3}c^2, \quad \frac{\tau}{\sigma_m} = \frac{1}{2}\frac{1-\left(\frac{y}{c}\right)^2}{\frac{x}{c}} \qquad (4)$$

Letting $X = \dfrac{x}{c}$ and $Y = \dfrac{y}{c}$, Equations (1) and (4) yield

$$\sigma = \sigma_m Y$$

$$\tau = \sigma_m \frac{1 - Y^2}{2X}$$



Using Mohr's circle, we calculate

$$R = \sqrt{\left(\frac{1}{2}\sigma\right)^2 + \tau^2} = \frac{1}{2}\sigma_m \sqrt{Y^2 + \left(\frac{1-Y^2}{X}\right)^2}$$

$$\frac{\sigma_{max}}{\sigma_m} = \frac{1}{2}Y + R$$

$$\frac{\sigma_{min}}{\sigma_m} = \frac{1}{2}Y - R \quad ◀$$

$$\tan 2\theta_P = \frac{\tau}{\frac{\sigma}{2}} = \frac{1-Y^2}{2X(\frac{y}{2})} = \frac{1-Y^2}{XY}$$

$$\theta_P = \frac{1}{2}\tan^{-1}\left(\frac{1-Y^2}{XY}\right) \circlearrowleft \quad \blacktriangleleft$$

<u>Note</u>:

For $y > 0$, the angle θ_P is $\circlearrowleft$, which is opposite to what was arbitrarily assumed in Figure P8.C2.

<u>Program Outputs</u>

For $\frac{x}{c} = 2$,

$\frac{x}{c}$	$\frac{\sigma_{min}}{\sigma_m}$	$\frac{\sigma_{max}}{\sigma_m}$	$\theta \circlearrowleft$
1.0	0.000	1.000	0.00
0.8	–0.010	0.810	6.34
0.6	–0.040	0.640	14.04
0.4	–0.090	0.490	23.20
0.2	–0.160	0.360	33.69
0.0	–0.250	0.250	45.00
–0.2	–0.360	0.160	–33.69
–0.4	–0.490	0.090	–23.20
–0.6	–0.640	0.040	–14.04
–0.8	–0.810	0.010	–6.34
–1.0	–1.000	0.000	–0.00

For $\frac{x}{c} = 8$,

$\frac{y}{c}$	$\frac{\sigma_{min}}{\sigma_m}$	$\frac{\sigma_{max}}{\sigma_m}$	$\theta \circlearrowleft$
1.0	0.000	1.000	0.00
0.8	–0.001	0.801	1.61
0.6	–0.003	0.603	3.80
0.4	–0.007	0.407	7.35
0.2	–0.017	0.217	15.48
0.0	–0.062	0.063	45.00
–0.2	–0.217	0.017	–15.48
–0.4	–0.407	0.007	–7.35
–0.6	–0.603	0.003	–3.80
–0.8	–0.801	0.001	–1.61
–1.0	–1.000	0.000	–0.00

To check that $\sigma_{max} > \sigma_m$ if $x \leqslant 0.544c$, we run the program for $\frac{x}{c} = 0.544$ and for $\frac{x}{c} = 0.545$ and observe that $\frac{\sigma_{max}}{\sigma_m}$ exceeds 1 for several values of $\frac{y}{c}$ in the first case, but does not exceed 1 in the second case.

For $\frac{x}{c} = 0.544$,

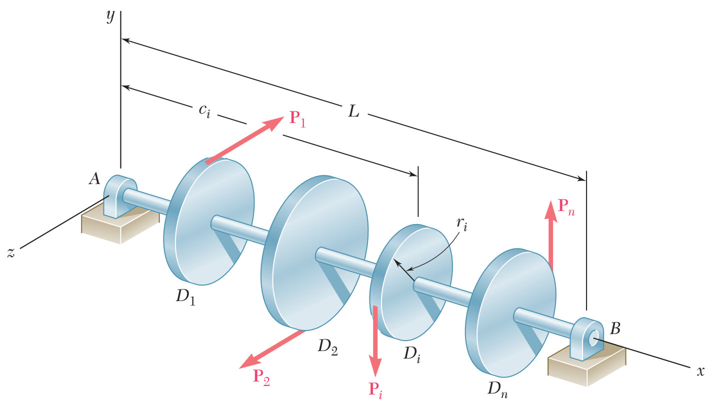
$\frac{y}{c}$	$\frac{\sigma_{min}}{\sigma_m}$	$\frac{\sigma_{max}}{\sigma_m}$	$\theta \circlearrowleft$
0.30	–0.700	0.9997	39.92
0.31	–0.690	1.0001	39.72
0.32	–0.680	1.0004	39.51
0.33	–0.670	1.0005	39.30
0.34	–0.660	1.0005	39.09
0.35	–0.650	1.0003	38.88
0.36	–0.640	1.0000	38.66
0.37	–0.630	0.9996	38.44
0.38	–0.619	0.9990	38.21
0.39	–0.608	0.9983	37.98
0.40	–0.598	0.9975	37.74

<u>Program Outputs</u> (***Continued***)

For $\frac{x}{c} = 0.545$,

$\frac{y}{c}$	$\frac{\sigma_{\min}}{\sigma_m}$	$\frac{\sigma_{\max}}{\sigma_m}$	θ ↺
0.30	–0.698	0.9982	39.91
0.31	–0.689	0.9986	39.71
0.32	–0.679	0.9989	39.50
0.33	–0.669	0.9990	39.29
0.34	–0.659	0.9990	39.08
0.35	–0.649	0.9988	38.87
0.36	–0.639	0.9986	38.65
0.37	–0.628	0.9982	38.42
0.38	–0.618	0.9976	38.20
0.39	–0.607	0.9970	37.96
0.40	–0.596	0.9962	37.73

PROBLEM 8.C3



Disks $D_1, D_2, \ldots, D_n$ are attached as shown in Fig. 8.C3 to the solid shaft AB of length L, uniform diameter d, and allowable shearing stress τ_{all}. Forces $\mathbf{P}_1$, $\mathbf{P}_2$, . . . , $\mathbf{P}_n$ of known magnitude (except for one of them) are applied to the disks, either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. Denoting by r_i the radius of disk D_i and by c_i its distance from the support at A, write a computer program to calculate (*a*) the magnitude of the unknown force $\mathbf{P}_i$, (*b*) the smallest permissible value of the diameter d of shaft AB. Use this program to solve Prob. 8.18.

SOLUTION

1. Determine the unknown force P_i by equating to zero the sum of their torques T_i about the x axis.

2. Determine the components $(F_y)_i$ and $(F_z)_i$ of all forces.

3. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' // z$ and $By' // y$:

$$\Sigma M_{z'} = 0: \quad -A_y L - \Sigma(F_y)_i (L - c_i) = 0, \qquad A_y = -\frac{1}{L}\Sigma(F_y)_i (L - c_i)$$

$$\Sigma M_{y'} = 0: \quad A_z L + \Sigma(F_z)_i (L - c_i) = 0, \qquad A_z = -\frac{1}{L}\Sigma(F_z)_i (L - c_i)$$

4. Determine $(M_y)_i$, $(M_z)_i$ and torque T_i just to the left of disk D_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle^1$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

where < > indicates a singularity function.

5. The minimum diameter d required to the left of D_i is obtained by first computing $(J/c)_i$ from Equation (8.7).

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{\text{all}}}$$

PROBLEM 8.C3 *(Continued)*

6. Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$, we have $c_i = \frac{2}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ ◀

   This is the required diameter just to the left of disk D_i.

7. The required diameter just to the right of disk D_i is obtained by replacing T_i with T_{i+1} in the above computation.

8. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for D_i.

Program Output

Problem 8.19

Length of shaft = 28 in.

τ(ksi) = 8

For Disk 1,

Force = 0.500 kips

Radius of disk = 4.0 in.

Distance from A = 7.0 in.

For Disk 2,

Force = 0.000 kips

Radius of disk = 6.0 in.

Distance from A = 14.0 in.

For Disk 3,

Force = 0.500 kips

Radius of disk = 4.0 in.

Distance from A = 21.0 in.

Unknown force = –0.667 kips

$A_Y = 0.500$ kips, $A_Z = 0.333$ kips

$B_Y = 0.500$ kips, $B_Z = 0.333$ kips

Just to the left of Disk 1:

$M_Y = 2.3333$ kip · in.

$M_Z = -3.5000$ kip · in.

$T = 0.0000$ kip · in.

Diameter must be at least 1.389 in.

Just to the right of Disk 1:

$T = 2.00$ kip · in.

Diameter must be at least 1.437 in.

PROBLEM 8.C3 *(Continued)*

Program Output (***Continued***)

Just to the left of Disk 2:

$$M_Y = 4.6667 \text{ kip} \cdot \text{in.}$$
$$M_Z = -3.5000 \text{ kip} \cdot \text{in.}$$
$$T = 2.0000 \text{ kip} \cdot \text{in.}$$

Diameter must be at least 1.578 in. ◀

Just to the right of Disk 2:

$$T = -2.00 \text{ kip} \cdot \text{in.}$$

Diameter must be at least 1.578 in.

Just to the left of Disk 3:

$$M_Y = 2.3333 \text{ kip} \cdot \text{in.}$$
$$M_Z = -3.5000 \text{ kip} \cdot \text{in.}$$
$$T = -2.0000 \text{ kip} \cdot \text{in.}$$

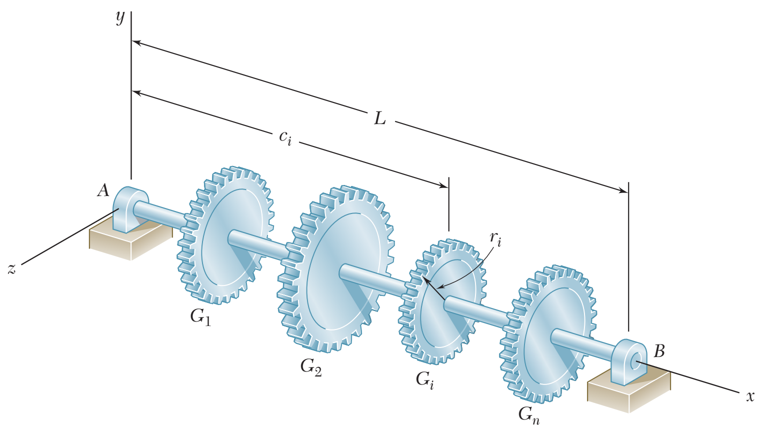
Diameter must be at least 1.437 in.

Just to the right of Disk 3:

$$T = 0.00 \text{ kip} \cdot \text{in.}$$

Diameter must be at least 1.389 in.

PROBLEM 8.C4



The solid shaft AB of length L, uniform diameter d, and allowable shearing stress τ_{all} rotates at a given speed expressed in rpm (Fig. 8.C4). Gears G_1, G_2, . . . , G_n are attached to the shaft and each of these gears meshes with another gear (not shown), either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. One of these gears is connected to a motor and the rest of them to various machine tools. Denoting by r_i the radius of disk G_i, by c_i its distance from the support at A, and by P_i the power transmitted to that gear (+sign) or taken of that gear (–sign), write a computer program to calculate the smallest permissible value of the diameter d of shaft AB. Use this program to solve Probs 8.27 and 8.68.

SOLUTION

1. Enter w in rpm and determine frequency $f = w/60$.
2. For each gear, determine the torque $T_i = P_i/2\pi f$, where P_i is the power input (+) or output (–) at the gear.
3. For each gear, determine the force $F_i = T_i/r_i$ exerted on the gear and its components $(F_y)_i$ and $(F_z)_i$.
4. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' // z$ and $By' // y$:

$$\Sigma M_{z'} = 0: \quad -A_y L - \Sigma(F_y)_i(L - c_i) = 0, \quad A_y = -\frac{1}{L}\Sigma(F_y)_i(L - c_i)$$

$$\Sigma M_{y'} = 0: \quad A_z L + \Sigma(F_z)_i(L - c_i) = 0, \quad A_z = -\frac{1}{L}\Sigma(F_z)_i(L - c_i)$$

5. Determine $(M_y)_i, (M_z)_i$, and torque T_i just to the left of gear G_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle^1$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

where $\langle\,\rangle$ indicates a singularity function.

PROBLEM 8.C4 *(Continued)*

6. The minimum diameter d required to the left of G_i is obtained by first computing $(J/c)_i$ from Equation (8.7).

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{\left(M_y\right)_i^2 + \left(M_z\right)_i^2 + T_i^2}}{\tau_{\text{all}}}$$

7. Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$,

   we have $c_i = \frac{2}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ ◀

   This is the required diameter just to the left of gear G_i.

8. The required diameter just to the right of gear G_i is obtained by replacing T_i with $T_{\text{i+1}}$ in the above computation.
9. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for d_i.

Program Outputs

Problem 8.25

$\omega = 450$ rpm
Number of Gears: 3
Length of shaft = 750 mm
$\tau = 55$ MPa

For Gear 1,
Power input = –8.00 kW
Radius of gear = 60 mm
Distance from A in mm = 150

For Gear 2,
Power input = 20.00 kW
Radius of gear = 100 mm
Distance from A in mm = 375

For Gear 3,
Power input = –12.00 kW
Radius of gear = 60 mm
Distance from A in mm = 600
$A_Y = -0.849$ kN, $A_Z = 4.386$
$B_Y = -3.395$ kN, $B_Z = 2.688$

Just to the left of Gear 1:
$M_Y = 657.84$ Nm
$M_Z = 127.32$ Nm
$T = 0.00$ Nm

Problem 8.27

$\omega = 600$ rpm
Number of Gears: 3
Length of shaft = 24 in.
$\tau = 8$ ksi

For Gear 1,
Power input = 60.00 hp
Radius of gear = 3.00 in.
Distance from A in inches = 4.0
$F_Y = 0$
$F_z = 2.100845$

For Gear 2,
Power input = –40.00 hp
Radius of gear = 4.00 in.
Distance from A in inches = 10.0
$F_Y = 1.050423$
$F_Z = 0$

For Gear 3,
Power input = –20.00 hp
Radius of gear = 4.00 in.
Distance from A in inches = 18.0
$F_Y = 0$
$F_Z = -0.5252113$

Diameter must be at least 39.59 mm.

Just to the right of Gear 1:

$T = -169.77 \text{ N} \cdot \text{m}$

Diameter must be at least 40.00 mm.

Just to the left of Gear 2:

$M_Y = 1007.98 \text{ N} \cdot \text{m}$

$M_Z = 318.31 \text{ N} \cdot \text{m}$

$T = -169.77 \text{ N} \cdot \text{m}$

Diameter must be at least 46.28 mm.

Just to the right of Gear 2:

$T = 254.65 \text{ N} \cdot \text{m}$

Diameter must be at least 46.52 mm. ◀

Just to the left of Gear 3:

$M_Y = 403.19 \text{ N} \cdot \text{m}$

$M_Z = 509.30 \text{ N} \cdot \text{m}$

$T = 254.65 \text{ N} \cdot \text{m}$

Diameter must be at least 40.13 mm.

Just to the right of Gear 3:

$T = 0.00 \text{ N} \cdot \text{m}$

Diameter must be at least 39.18 mm.

$A_Y = -0.6127$ kips, $A_Z = -1.6194$ kips

$B_Y = -0.4377$ kips, $B_Z = 0.438$ kips

Just to the left of Gear 1:

$M_Y = -6.478 \text{ kip} \cdot \text{in.}$

$M_Z = 2.451 \text{ kip} \cdot \text{in.}$

$T = 0.000 \text{ kip} \cdot \text{in.}$

Diameter must be at least 1.640 in.

Just to the right of Gear 1:

$T = 6.3025 \text{ kip} \cdot \text{in.}$

Diameter must be at least 1.813 in.

Just to the left of Gear 2:

$M_Y = -3.589 \text{ kip} \cdot \text{in.}$

$M_Z = 6.127 \text{ kip} \cdot \text{in.}$

$T = 6.303 \text{ kip} \cdot \text{in.}$

Diameter must be at least 1.822 in. ◀

Just to the right of Gear 2:

$T = 2.1008 \text{ kip} \cdot \text{in.}$

Diameter must be at least 1.677 in.

Just to the left of Gear 3:

$M_Y = 0.263 \text{ kip} \cdot \text{in.}$

$M_Z = 2.626 \text{ kip} \cdot \text{in.}$

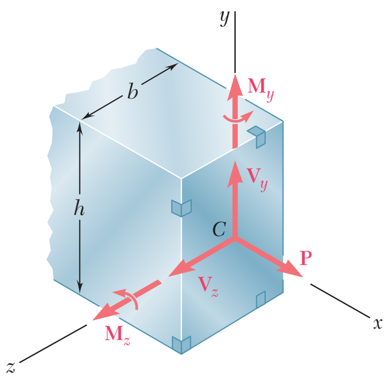
$T = 2.101 \text{ kip} \cdot \text{in.}$

Diameter must be at least 1.290 in.

Just to the right of Gear 3:

$T = 0.000 \text{ kip} \cdot \text{in.}$

Diameter must be at least 1.189 in.



PROBLEM 8.C5

Write a computer program that can be used to calculate the normal and shearing stresses at points with given coordinates y and z located on the surface of a machine part having a rectangular cross section. The internal forces are known to be equivalent to the force-couple system shown. Write the program so that the loads and dimensions can be expressed in either SI or U.S. customary units. Use this program to solve (*a*) Prob. 8.45*b*, (*b*) Prob. 8.47*a*.

SOLUTION

Enter: b and h

Program: $A = bh \quad I_y = \frac{b^3h}{12} \quad I_z = \frac{bh^3}{12}$

For point on surface, enter y and z.

Note: y and z must satisfy one of following:

$$y^2 = \frac{h^2}{4} \quad \text{and} \quad z^2 \le \frac{b^2}{4} \qquad (1)$$

or

$$z^2 = \frac{b^2}{4} \quad \text{and} \quad y^2 \le \frac{h^2}{4} \qquad (2)$$

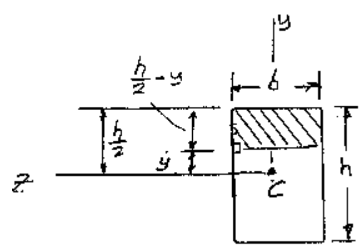
If either (1) and (2) are satisfied, compute

$$\sigma = \frac{P}{A} + \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

If $z^2 = b^2/4$, then point is on vertical surface and

$$Q_z = b\left(\frac{h}{2} - y\right)\left(\frac{h}{2} + y\right)\frac{1}{2} = b\left(\frac{h^2}{8} - \frac{y^2}{2}\right)$$

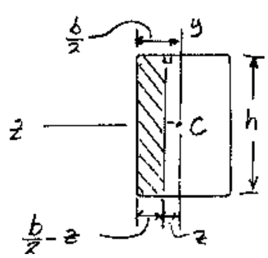
$$\gamma = \frac{v_y Q_z}{I_z b}$$



If $y^2 = h^2/4$, the point is on horizontal surface, and

$$Q_y = h\left(\frac{b}{2} - z\right)\left(\frac{b}{2} + z\right)\frac{1}{2} = h\left(\frac{b^2}{8} - \frac{z^2}{2}\right)$$

$$\gamma = \frac{V_z Q_y}{I_y h}$$

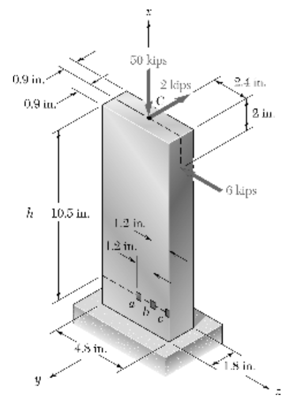


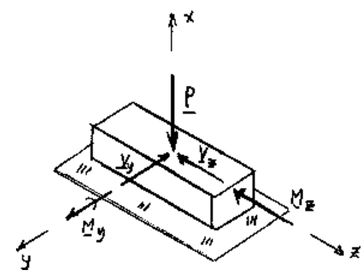
Force-couple system: $P = -50 \text{ kips} \quad V_z = -6 \text{ kips} \quad V_y = -2 \text{ kips}$

$$M_y = (6 \text{ kips})(8.5 \text{ in.}) = 51 \text{ kip} \cdot \text{in.}$$

$$M_z = (2 \text{ kips})(10.5 \text{ in.}) = 21 \text{ kip} \cdot \text{in.}$$

PROBLEM 8.C5 *(Continued)*





<u>Problem 8.45*b*</u> <u>Force-Couple at Centroid</u>

$P = -50.00$ kips

$M_Y = 51.00$ kip $\cdot$ in. $\quad M_Z = -21.00$ kip $\cdot$ in.

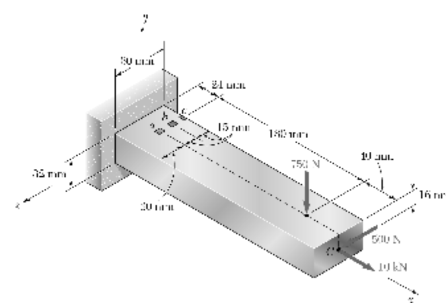
$V_Y = -2.00$ kips $\quad V_Z = -6.00$ kips

\+ +

At point of coordinates: $y = 0.90$ in. $\quad z = 1.20$ in.

$\sigma = \ 6.004$ ksi

$\tau = -0.781$ ksi



<u>Force-Couple System</u>

$P = 10$ kN $\quad V_y = -750$ N $\quad V_z = 500$ N

$M_y = (500 \text{ N})(220 \text{ mm}) = 110 \text{ N} \cdot \text{m}$

$M_z = (750 \text{ N})(180 \text{ mm}) = 135 \text{ N} \cdot \text{m}$

<u>Problem 8.47*a*</u> <u>Force-Couple at Centroid</u>

$P = 10000.00$ N

$M_Y = 110.00 \text{ N} \cdot \text{m} \quad M_Z = -135.00 \text{ N} \cdot \text{m}$

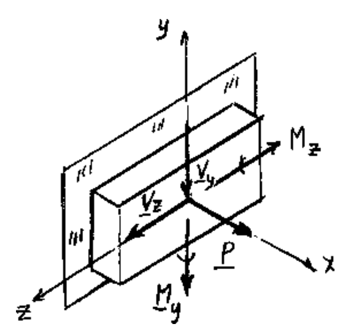
$V_Y = 750.00$ N $\quad V_Z = \ 500.00$ N

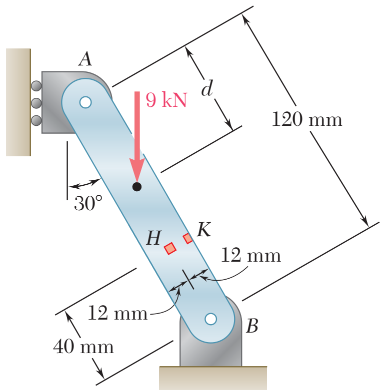
\+ +

At point of coordinates: $y = 16.00$ mm $\quad x = 0.00$ mm

$\sigma = 18.392$ MPa

$\tau = \ 0.391$ MPa





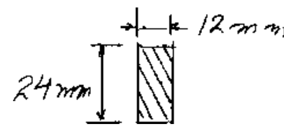
PROBLEM 8.C6

Member AB has a rectangular cross section of 10×24 mm. For the loading shown, write a computer program that can be used to determine the normal and shearing stresses at Points H and K for values of d from 0 to 120 mm, using 15-mm increments. Use this program to solve Prob. 8.35.

SOLUTION

Cross section:

Enter



$$A = (0.010 \text{ m})(0.024 \text{ m}) = 240 \times 10^{-6} \text{ m}^2$$

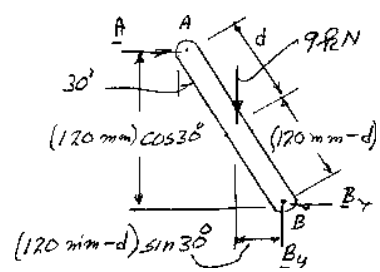
$$I = (0.010 \text{ m})(0.024 \text{ m})^3/12 = 138.24 \times 10^{-9} \text{ m}^4$$

$$R = 0.5(0.029 \text{ m}) = 12 \times 10^{-3} \text{ m}$$

Compute reaction at A.

$$+\circlearrowleft \Sigma M_B = 0: \quad (9 \text{ kN})(120 - d) \sin 30° - A\,(120) \cos 30° = 0$$

$$A = (9 \text{ kN}) \frac{(120 \text{ mm} - d)}{120 \text{ mm}} \tan 30°$$



Free Body: From A to section containing Points H and K,

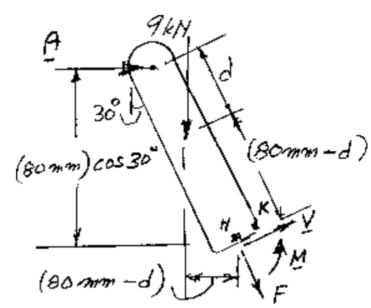
Define: If $d < 80$ mm, Then STP = 1 Else STP = 0

Program force-couple system.

$$F = -A \sin 30° - (9 \text{ kN}) \cos 30° \,(\text{Step})$$

$$V = -A \cos 30° + (9 \text{ kN}) \sin 30° \,(\text{Step})$$

$$M = A(80 \text{ mm}) \cos 30° - (9 \text{ kN})(80 \text{ mm} - d) \sin 30° \,(\text{STP})$$



At Point H,

$$\sigma_H = +F/A$$

$$\tau_H = \frac{3}{2} V/A$$

At Point K,

$$\sigma_K = +F/A - Mc/I$$

$$\tau_K = 0$$

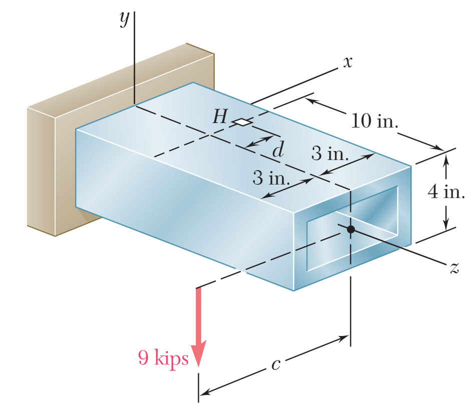
PROBLEM 8.C6 *(Continued)*

Program Output

Problem 8.35

		Stresses in MPa		
d (mm)	σ_H	τ_H	σ_K	τ_K
0.0	–43.30	0.00	–43.30	0.00
15.0	–41.95	3.52	–65.39	0.00
30.0	–40.59	7.03	–87.47	0.00
45.0	–39.24	10.55	–109.55	0.00
60.0	–37.89	14.06	–131.64	0.00
75.0	–36.54	17.58	–153.72	0.00
90.0	–2.71	–7.03	–96.46	0.00
105.0	–1.35	–3.52	–48.23	0.00
120.0	0.00	0.00	0.00	0.00

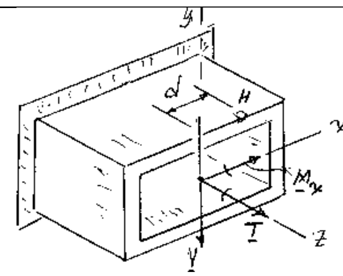
◀



PROBLEM 8.C7*

The structural tube shown has a uniform wall thickness of 0.3 in. A 9-kip force is applied at a bar (not shown) that is welded to the end of the tube. Write a computer program that can be used to determine, for any given value of c, the principal stresses, principal planes, and maximum shearing stress at Point H for values of d from -3 in. to 3 in., using one-inch increments. Use this program to solve Prob. 8.62a.

SOLUTION

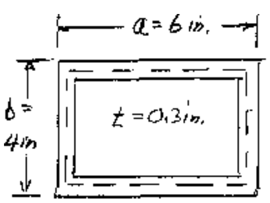


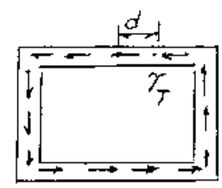
Force-couple system.

Enter:

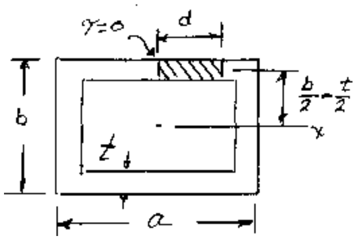
$$V = 9 \text{ kips} \downarrow$$
$$M_x = (9 \text{ kips})(10 \text{ in.})$$
$$= 90 \text{ kips} \cdot \text{in.}$$
$$T = (9 \text{ kips})c$$

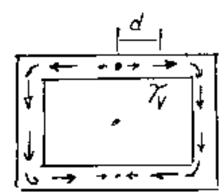
Area enclosed.



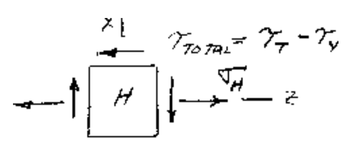


$$a = (a-t)(b-t)$$
$$\tau_T = \frac{T}{2ta} = \frac{9c}{2ta}$$
$$\tau_T = \text{Shearing stress due to torsion}$$
$$Q = dt\left(\frac{b}{2} - \frac{t}{2}\right)$$





$$\bar{I} = ab^3/12 - (a-2t)(b-2t)^3/12$$
$$\tau_V = \frac{VQ}{It}$$
$$\tau_V = \text{Shearing stress due to } V$$
$$\tau_{\text{total}} = \tau_T - \tau_Y$$



Bending:

$$\sigma_H = \frac{M_x\left(\frac{b}{2}\right)}{I}$$

Principal stresses:

$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H; \quad R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{\text{total}}^2}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R; \quad \sigma_{\min} = \sigma_{\text{ave}} - R;$$

$$\theta_P = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{\text{total}}}{\sigma_{\text{ave}}}\right); \quad \tau_{\max} = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{\text{total}}^2}$$

PROBLEM 8.C7* *(Continued)*

Rectangular tube of uniform thickness $t = 0.3$ in.

<u>Outside dimensions</u>.

Horizontal width: $a = 6$ in.

Vertical depth: $b = 4$ in.

Vertical load: $P = 9$ kips;

Line of action at $x = -c$

Find normal and shearing stresses at point H. $(x = d,\ y = b/2)$

<u>Problem 8.62a</u>

Program output for value of $c = 2.85$ in.

d in.	σ ksi	τ_V ksi	τ_T ksi	τ_{Total} ksi	$\sigma_{\max}$ ksi	$\sigma_{\min}$ ksi	$\tau_{\max}$ ksi	θ_P Degrees
–3.00	12.58	–3.49	–2.03	–5.52	14.65	–2.08	8.36	–18.49
–2.00	12.58	–2.33	–2.03	–4.35	13.94	–1.36	7.65	–16.00
–1.00	12.58	–1.16	–2.03	–3.19	13.34	–0.76	7.05	–12.78
$\rightarrow$ 0.00	12.58	0.00	–2.03	–2.03	12.89	–0.32	6.61	–8.73 $\leftarrow$
1.00	12.58	1.16	–2.03	–0.86	12.63	–0.06	6.35	–3.89
2.00	12.58	2.33	–2.03	0.30	12.58	–0.01	6.30	1.36
3.00	12.58	3.49	–2.03	1.46	12.74	–0.17	6.46	6.46