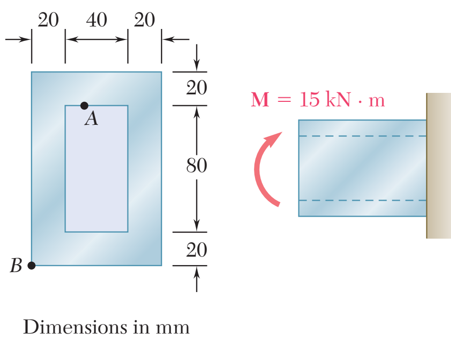
Chapter 4 - Solution Manual-Beer Johnston - Mechanics of Materials 7th c2015

Mechanics of Solids (Ghulam Ishaq Khan Institute of Engineering Sciences and Technology)

CHAPTER 4



PROBLEM 4.1

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

For rectangle: $I = \frac{1}{12}bh^3$

Outside rectangle: $I_1 = \frac{1}{12}(80)(120)^3$

$I_1 = 11.52 \times 10^6 \text{ mm}^4 = 11.52 \times 10^{-6} \text{ m}^4$

Cutout: $I_2 = \frac{1}{12}(40)(80)^3$

$I_2 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$

Section: $I = I_1 - I_2 = 9.81333 \times 10^{-6} \text{ m}^4$

(*a*) $y_A = 40 \text{ mm} = 0.040 \text{ m}$

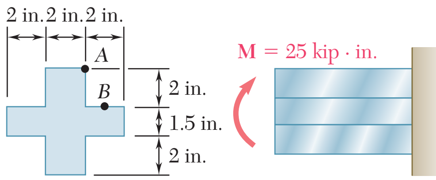
$$\sigma_A = -\frac{My_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$$

$\sigma_A = -61.6 \text{ MPa}$ ◀

(*b*) $y_B = -60 \text{ mm} = -0.060 \text{ m}$

$$\sigma_B = -\frac{My_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$$

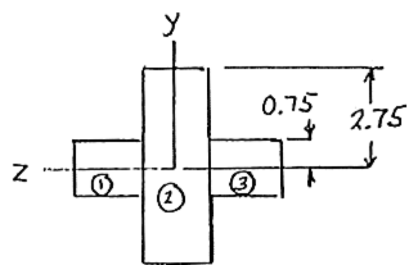
$\sigma_B = 91.7 \text{ MPa}$ ◀



PROBLEM 4.2

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



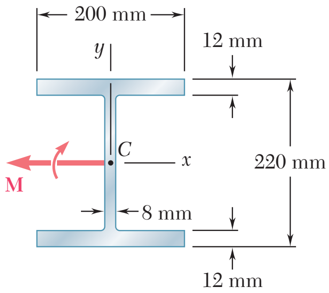
For rectangle: $I = \frac{1}{12}bh^3$

For cross sectional area:

$$I = I_1 + I_2 + I_3 = \frac{1}{12}(2)(1.5)^3 + \frac{1}{12}(2)(5.5)^3 + \frac{1}{12}(2)(1.5)^3 = 28.854 \text{ in}^4$$

(*a*) $y_A = 2.75$ in. $\qquad \sigma_A = -\frac{My_A}{I} = -\frac{(25)(2.75)}{28.854}$ $\qquad \sigma_A = -2.38$ ksi ◀

(*b*) $y_B = 0.75$ in. $\qquad \sigma_B = -\frac{My_B}{I} = -\frac{(25)(0.75)}{28.854}$ $\qquad \sigma_B = -0.650$ ksi ◀

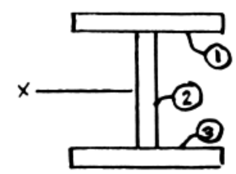


PROBLEM 4.3

Using an allowable stress of 155 MPa, determine the largest bending moment **M** that can be applied to the wide-flange beam shown. Neglect the effect of fillets.

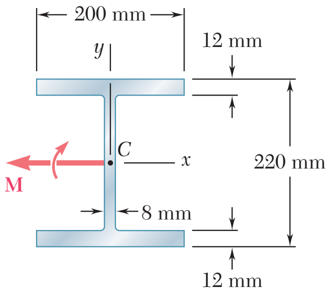
SOLUTION

Moment of inertia about x-axis:



$$I_1 = \frac{1}{12}(200)(12)^3 + (200)(12)(104)^2$$
$$= 25.9872 \times 10^6 \text{ mm}^4$$
$$I_2 = \frac{1}{12}(8)(196)^3 = 5.0197 \times 10^6 \text{ mm}^4$$
$$I_3 = I_1 = 25.9872 \times 10^6 \text{ mm}^4$$
$$I = I_1 + I_2 + I_3 = 56.944 \times 10^6 \text{ mm}^4 = 56.944 \times 10^{-6} \text{ m}^4$$
$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(220) = 110 \text{ mm} = 0.110 \text{ m}$$
$$M = \frac{I\sigma}{c} \quad \text{with} \quad \sigma = 155 \times 10^6 \text{ Pa}$$
$$M_x = \frac{(56.944 \times 10^{-6})(155 \times 10^6)}{0.110} = 80.2 \times 10^3 \text{ N} \cdot \text{m}$$

$M_x = 80.2 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 4.4

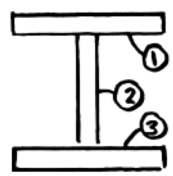
Solve Prob. 4.3, assuming that the wide-flange beam is bent about the y axis by a couple of moment M_y.

PROBLEM 4.3. Using an allowable stress of 155 MPa, determine the largest bending moment **M** that can be applied to the wide-flange beam shown. Neglect the effect of fillets.

SOLUTION

Moment of inertia about y axis:

$$I_1 = \frac{1}{12}(12)(200)^3 = 8 \times 10^6 \text{ mm}^4$$



$$I_2 = \frac{1}{12}(196)(8)^3 = 8.3627 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 8 \times 10^6 \text{ mm}^4$$

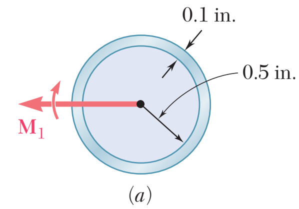
$$I = I_1 + I_2 + I_3 = 16.0084 \times 10^6 \text{ mm}^4 = 16.0084 \times 10^{-6} \text{ m}^4$$

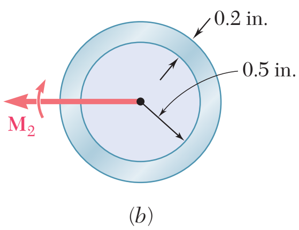
$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(200) = 100 \text{ mm} = 0.100 \text{ m}$$

$$M_y = \frac{I\sigma}{c} \quad \text{with} \quad \sigma = 155 \times 10^6 \text{ Pa}$$

$$M_y = \frac{(16.0084 \times 10^{-6})(155 \times 10^6)}{0.100} = 24.8 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_y = 24.8 \text{ kN} \cdot \text{m} \blacktriangleleft$$





PROBLEM 4.5

Using an allowable stress of 16 ksi, determine the largest couple that can be applied to each pipe.

SOLUTION

(*a*) $I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = \frac{\pi}{4}(0.6^4 - 0.5^4) = 52.7 \times 10^{-3} \text{ in}^4$

$c = 0.6 \text{ in.}$

$$\sigma = \frac{Mc}{I}: \qquad M = \frac{\sigma I}{c} = \frac{(16)(52.7 \times 10^{-3})}{0.6}$$

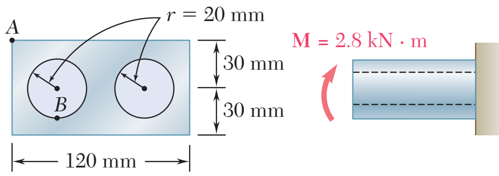
$M = 1.405 \text{ kip} \cdot \text{in.}$ ◀

(*b*) $I = \frac{\pi}{4}(0.7^4 - 0.5^4) = 139.49 \times 10^{-3} \text{ in}^4$

$c = 0.7 \text{ in.}$

$$\sigma = \frac{Mc}{I}: \qquad M = \frac{\sigma I}{c} = \frac{(16)(139.49 \times 10^{-3})}{0.7}$$

$M = 3.19 \text{ kip} \cdot \text{in.}$ ◀



PROBLEM 4.6

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$I = \frac{1}{12}(0.120\text{ m})(0.06\text{ m})^3 - 2\left[\frac{1}{12}\cdot\frac{\pi}{4}(0.02\text{ m})^4\right]$$

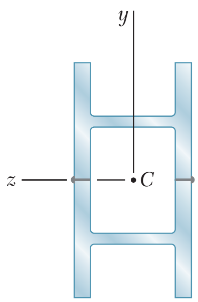
$$= 2.1391\times 10^{-6}\text{ mm}^4$$

(*a*) $$\sigma_A = -\frac{My_A}{I} = -\frac{(2.8\times 10^3\text{ N}\cdot\text{m})(0.03\text{ m})}{2.1391\times 10^{-6}\text{ mm}^4}$$

$\sigma_A = -39.3\text{ MPa}$ ◀

(*b*) $$\sigma_B = \frac{My_B}{I} = \frac{(2.8\times 10^3\text{ N}\cdot\text{m})(0.02\text{ m})}{2.1391\times 10^{-6}\text{ m}^4}$$

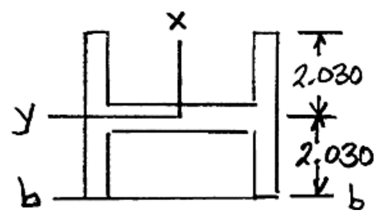
$\sigma_B = 26.2\text{ MPa}$ ◀



PROBLEM 4.7

Two $W4 \times 13$ rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the *z* axis.

SOLUTION



Properties of W4 × 13 rolled section.

(See Appendix C.)

$$\text{Area} = 3.83 \text{ in}^2$$
$$\text{Width} = 4.060 \text{ in.}$$
$$I_y = 3.86 \text{ in}^4$$

For one rolled section, moment of inertia about axis *b*-*b* is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

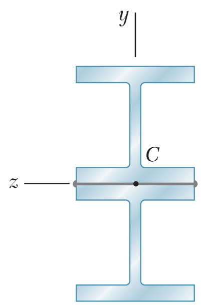
For both sections, $I_z = 2I_b = 39.286 \text{ in}^4$

$$c = \text{width} = 4.060 \text{ in.}$$

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{\text{all}} = \frac{\sigma_{\text{all}} I}{c} = \frac{(19.333)(39.286)}{4.060}$$

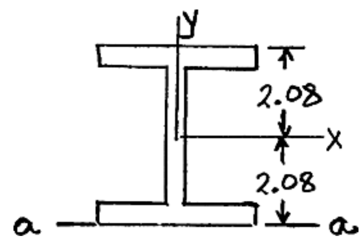
$M_{\text{all}} = 187.1 \text{ kip} \cdot \text{in.}$ ◀



PROBLEM 4.8

Two W4×13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the *z* axis.

SOLUTION



Properties of W4 × 13 rolled section.

(See Appendix C.)

$\text{Area} = 3.83 \text{ in}^2$

$\text{Depth} = 4.16 \text{ in.}$

$I_x = 11.3 \text{ in}^4$

For one rolled section, moment of inertia about axis *a*-*a* is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

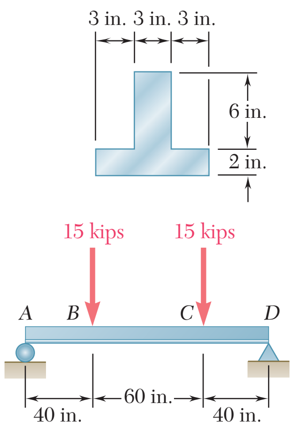
For both sections, $I_z = 2I_a = 55.74 \text{ in}^4$

$$c = \text{depth} = 4.16 \text{ in.}$$

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{\text{all}} = \frac{\sigma_{\text{all}} I}{c} = \frac{(19.333)(55.74)}{4.16}$$

$M_{\text{all}} = 259 \text{ kip} \cdot \text{in.}$ ◀

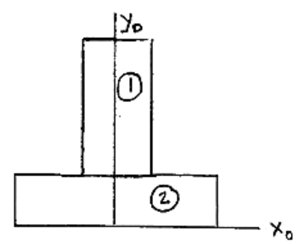


PROBLEM 4.9

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.

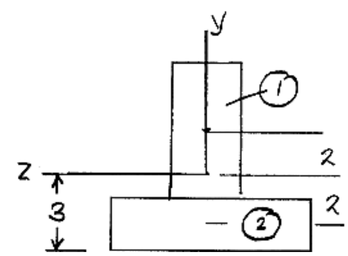
SOLUTION

	A	$\bar{y}_0$	$A\bar{y}_0$
①	18	5	90
②	18	1	18
Σ	36		108



$$\bar{Y}_0 = \frac{108}{36} = 3 \text{ in.}$$

Neutral axis lies 3 in. above the base.

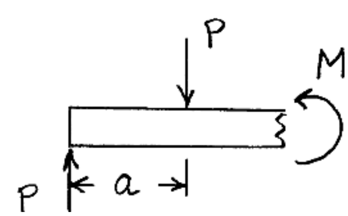


$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$y_{top} = 5$ in. $\quad y_{bot} = -3$ in.



$$M - Pa = 0$$

$$M = Pa = (15)(40) = 600 \text{ kip} \cdot \text{in.}$$

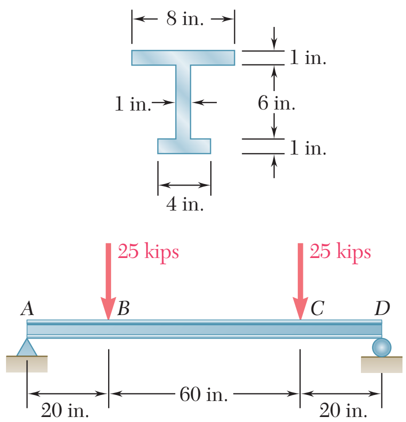
$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(600)(5)}{204}$$

$\sigma_{top} = -14.71$ ksi (compression) ◀

$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(600)(-3)}{204}$$

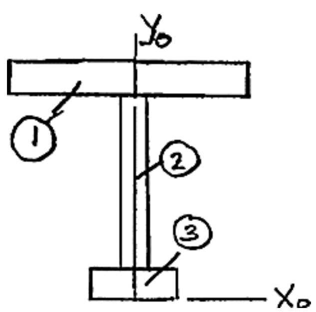
$\sigma_{bot} = 8.82$ ksi (tension) ◀

PROBLEM 4.10



Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

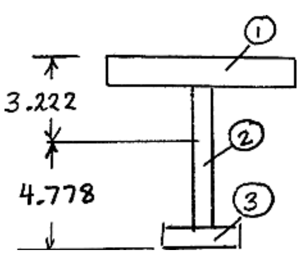
SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86

$$\bar{Y}_o = \frac{86}{18} = 4.778 \text{ in.}$$

Neutral axis lies 4.778 in. above the base.



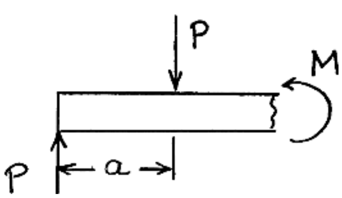
$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.772)^2 = 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2 = 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.57 = 155.16 \text{ in}^4$$

$$y_{\text{top}} = 3.222 \text{ in.} \quad y_{\text{bot}} = -4.778 \text{ in.}$$

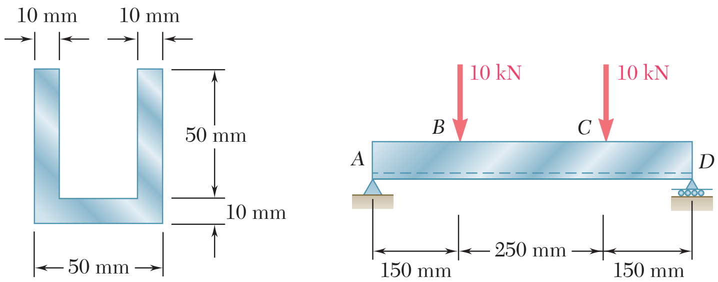


$$M - Pa = 0$$

$$M = Pa = (25)(20) = 500 \text{ kip} \cdot \text{in.}$$

$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(500)(3.222)}{155.16} \qquad \sigma_{\text{top}} = -10.38 \text{ ksi (compression)} \blacktriangleleft$$

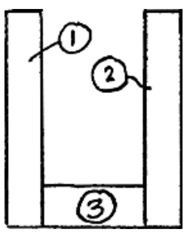
$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(500)(-4.778)}{155.16} \qquad \sigma_{\text{bot}} = 15.40 \text{ ksi (tension)} \blacktriangleleft$$



PROBLEM 4.11

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

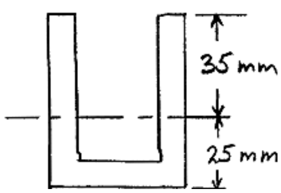
SOLUTION



	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3

$$\bar{Y}_0 = \frac{37.5\times10^3}{1500} = 25\,\text{mm}$$

Neutral axis lies 25 mm above the base.



$$I_1 = \frac{1}{12}(10)(60)^3 + (600)(5)^2 = 195\times10^3\,\text{mm}^4 \qquad I_2 = I_1 = 195\ \text{mm}^4$$

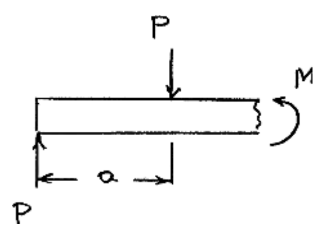
$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5\times10^3\,\text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 512.5\times10^3\,\text{mm}^4 = 512.5\times10^{-9}\,\text{m}^4$$

$$y_{\text{top}} = 35\ \text{mm} = 0.035\ \text{m} \qquad y_{\text{bot}} = -25\ \text{mm} = -0.025\ \text{m}$$

$$a = 150\ \text{mm} = 0.150\ \text{m} \qquad P = 10\times10^3\,\text{N}$$

$$M = Pa = (10\times10^3)(0.150) = 1.5\times10^3\,\text{N}\cdot\text{m}$$

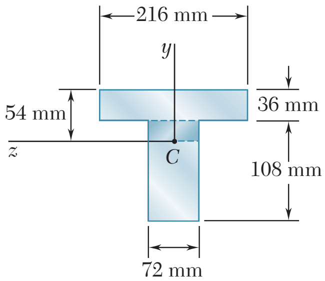


$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(1.5\times10^3)(0.035)}{512.5\times10^{-9}} = -102.4\times10^6\,\text{Pa}$$

$\sigma_{\text{top}} = -102.4$ MPa (compression) ◀

$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(1.5\times10^3)(-0.025)}{512.5\times10^{-9}} = 73.2\times10^6\,\text{Pa}$$

$\sigma_{\text{bot}} = 73.2$ MPa (tension) ◀



PROBLEM 4.12

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN · m, determine the total force acting on the shaded portion of the web.

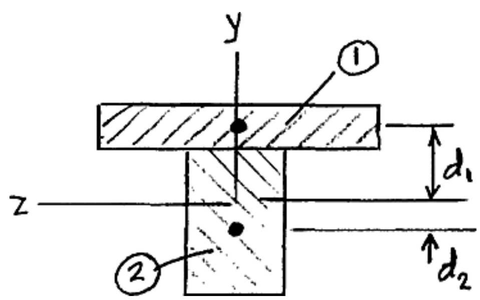
SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula:

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA, the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$



The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$d_1 = 54 - 18 = 36 \text{ mm}$$
$$d_2 = 54 + 36 - 54 = 36 \text{ mm}$$

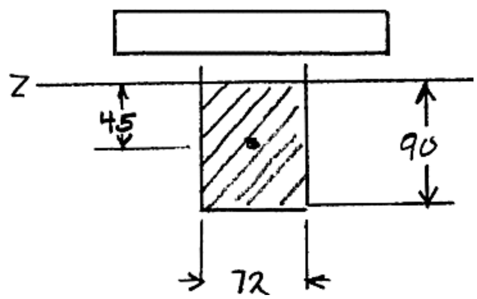
PROBLEM 4.12 *(Continued)*

Moment of inertia of entire cross section:

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(216)(36)^3 + (216)(36)(36)^2 = 10.9175\times 10^6\,\text{mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(72)(108)^3 + (72)(108)(36)^2 = 17.6360\times 10^6\,\text{mm}^4$$

$$I = I_1 + I_2 = 28.5535\times 10^6\,\text{mm}^4 = 28.5535\times 10^{-6}\,\text{m}^4$$



For the shaded area,

$$A^* = (72)(90) = 6480\ \text{mm}^2$$

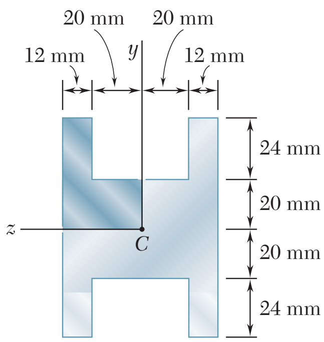
$$\bar{y}^* = 45\ \text{mm}$$

$$A^*\bar{y}^* = 291.6\times 10^3\,\text{mm}^3 = 291.6\times 10^{-6}\,\text{m}$$

$$F = \left|\frac{MA^*\bar{y}^*}{I}\right| = \frac{(6\times 10^3)(291.6\times 10^{-6})}{28.5535\times 10^{-6}}$$

$$= 61.3\times 10^3\,\text{N}$$

$F = 61.3$ kN ◀



PROBLEM 4.13

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is $4\ \text{kN}\cdot\text{m}$, determine the total force acting on the shaded portion of the beam.

SOLUTION

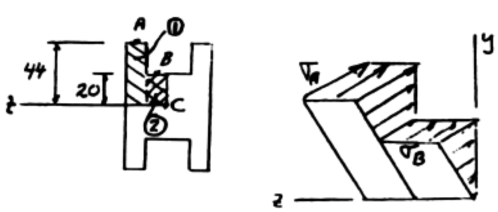
Dimensions in mm:

$$I_z = \frac{1}{12}(12+12)(88)^3 + \frac{1}{12}(40)(40)^3$$
$$= 1.3629\times 10^6 + 0.213\times 10^6$$
$$= 1.5763\times 10^6\,\text{mm}^4 = 1.5763\times 10^{-6}\,\text{m}^4$$

For use in Prob. 4.14,

$$I_y = \frac{1}{12}(88)(64)^3 - \frac{1}{12}(24+24)(40)^3$$
$$= 1.9224\times 10^6 - 0.256\times 10^6$$
$$= 1.6664\times 10^6\,\text{mm}^4 = 1.6664\times 10^{-6}\,\text{m}^4$$

Bending about horizontal axis. $\quad M_z = 4\ \text{kN}\cdot\text{m}$



$$\sigma_A = \frac{M_z c}{I_z} = \frac{(4\ \text{kN}\cdot\text{m})(0.044\ \text{m})}{1.5763\times 10^{-6}\,\text{m}^4} = 111.654\ \text{MPa}$$

$$\sigma_B = \frac{M_z c}{I_z} = \frac{(4\ \text{kN}\cdot\text{m})(0.020\ \text{m})}{1.5763\times 10^{-6}\ \text{m}^4} = 50.752\ \text{MPa}$$

Portion (1): $A = (44)(12) = 528\ \text{mm}^2 = 528 \times 10^{-6}\ \text{m}^2$

$$\sigma_{\text{avg}} = \frac{1}{2}\sigma_A = \frac{1}{2}(111.654) = 55.83\ \text{MPa}$$

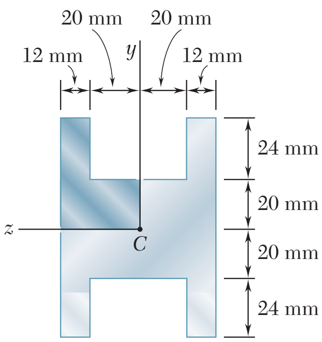
$$\text{Force}_1 = \sigma_{\text{avg}} A = (55.83\ \text{MPa})(528 \times 10^{-6}\ \text{m}^2) = 29.477\ \text{kN}$$

Portion (2): $A = (20)(20) = 400\ \text{mm}^2 = 400 \times 10^{-6}\ \text{m}^2$

$$\sigma_{\text{avg}} = \frac{1}{2}\sigma_B = \frac{1}{2}(50.752) = 25.376\ \text{MPa}$$

$$\text{Force}_2 = \sigma_{\text{avg}} A = (25.376\ \text{MPa})(400 \times 10^{-6}\ \text{m}^2) = 10.150\ \text{kN}$$

Total force on shaded area $= 29.477 + 10.150 = 39.6\ \text{kN}$ ◀



PROBLEM 4.14

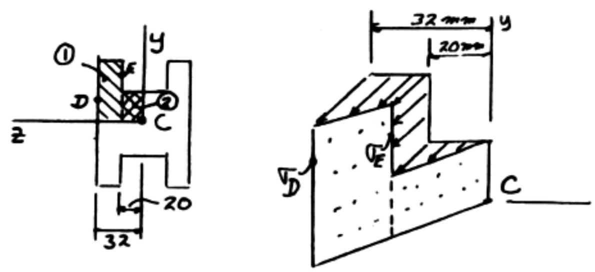
Solve Prob. 4.13, assuming that the beam is bent about a vertical axis by a couple of moment $4\ \text{kN}\cdot\text{m}$.

PROBLEM 4.13. Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is $4\ \text{kN}\cdot\text{m}$, determine the total force acting on the shaded portion of the beam.

SOLUTION

Bending about vertical axis. $\quad M_y = 4\ \text{kN}\cdot\text{m}$

See Prob. 4.13 for sketch and $\quad I_y = 1.6664\times10^{-6}\ \text{m}^4$



$$\sigma_D = \frac{Mc}{I_y} = \frac{(4\ \text{kN}\cdot\text{m})(0.032\ \text{m})}{1.6664\times10^{-6}\,\text{m}^4} = 76.81\ \text{MPa}$$

$$\sigma_E = \frac{Mc}{I_y} = \frac{(4\ \text{kN}\cdot\text{m})(0.020\ \text{m})}{1.6664\times10^{-6}\ \text{m}^4} = 48.01\ \text{MPa}$$

Portion (1): $\quad A = (44)(12) = 528\ \text{mm}^2 = 528\times10^{-6}\ \text{m}^2$

$$\sigma_{\text{avg}} = \frac{1}{2}(\sigma_D + \sigma_E) = \frac{1}{2}(76.81 + 48.01) = 62.41\ \text{MPa}$$

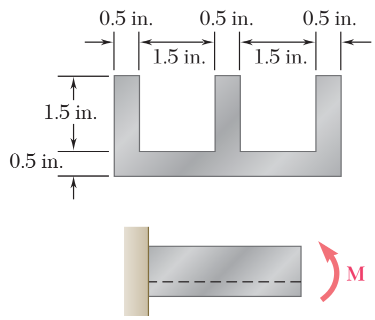
$$\text{Force}_1 = \sigma_{\text{avg}}A = (62.41\ \text{MPa})(528\times10^{-6}\ \text{m}^2) = 32.952\ \text{kN}$$

Portion (2): $\quad A = (20)(20) = 400\ \text{mm}^2 = 400\times10^{-6}\ \text{m}^2$

$$\sigma_{\text{avg}} = \frac{1}{2}\sigma_E = \frac{1}{2}(48.01) = 24.005\ \text{MPa}$$

$$\text{Force}_2 = \sigma_{\text{avg}}A = (24.005\ \text{MPa})(400\times10^{-6}\ \text{m}^2) = 9.602\ \text{kN}$$

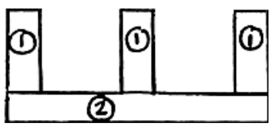
Total force on shaded area $= 32.952 + 9.602 = 42.6\ \text{kN}$ ◀



PROBLEM 4.15

Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in.}$$

The neutral axis lies 0.75 in. above bottom.

$$y_{\text{top}} = 2.0 - 0.75 = 1.25 \text{ in.}, \qquad y_{\text{bot}} = -0.75 \text{ in.}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^2 + A_2 d_2^2 = \frac{1}{12}(4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

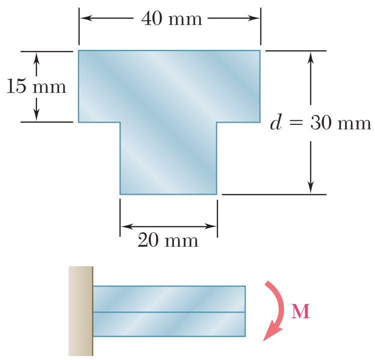
$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

Top: (compression) $\qquad M = \dfrac{(16)(1.59375)}{1.25} = 20.4 \text{ kip} \cdot \text{in.}$

Bottom: (tension) $\qquad M = \dfrac{(12)(1.59375)}{0.75} = 25.5 \text{ kip} \cdot \text{in.}$

Choose the smaller as M_{all}. $\qquad M_{\text{all}} = 20.4 \text{ kip} \cdot \text{in.}$ ◀

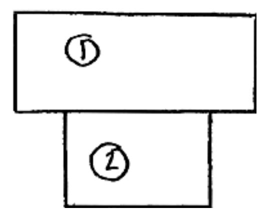


PROBLEM 4.16

The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3



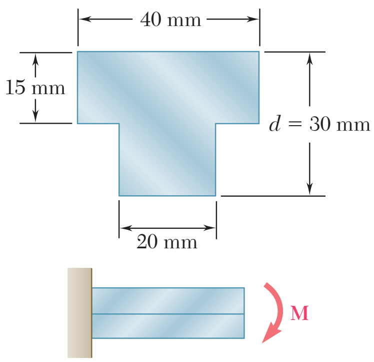
$$\bar{Y}_0 = \frac{15.5\times10^3}{900} = 17.5 \text{ mm}$$ The neutral axis lies 17.5 mm above the bottom.

$$y_{\text{top}} = 30 - 17.5 = 12.5 \text{ mm} = 0.0125 \text{ m}$$
$$y_{\text{bot}} = -17.5 \text{ mm} = -0.0175 \text{ m}$$
$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(5)^2 = 26.25\times10^3 \text{ mm}^4$$
$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(20)(15)^3 + (300)(10)^2 = 35.625\times10^3 \text{ mm}^4$$
$$I = I_1 + I_2 = 61.875\times10^3 \text{ mm}^4 = 61.875\times10^{-9} \text{ m}^4$$
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(24\times10^6)(61.875\times10^{-9})}{0.0125} = 118.8 \text{ N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(30\times10^6)(61.875\times10^{-9})}{0.0175} = 106.1 \text{ N}\cdot\text{m}$$

Choose smaller value. $M = 106.1 \text{ N}\cdot\text{m}$ ◀



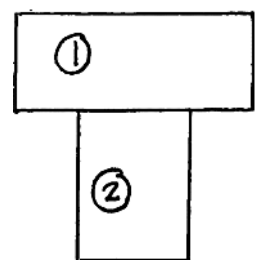
PROBLEM 4.17

Solve Prob. 4.16, assuming that $d = 40$ mm.

PROBLEM 4.16 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	32.5	19.5×10^3
②	500	12.5	6.25×10^3
Σ	1100		25.75×10^3



$$\bar{Y}_0 = \frac{25.75\times10^3}{1100} = 23.41 \text{ mm}$$ The neutral axis lies 23.41 mm above the bottom.

$$y_{top} = 40 - 23.41 = 16.59 \text{ mm} = 0.01659 \text{ m}$$

$$y_{bot} = -23.41 \text{ mm} = -0.02341 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(9.09)^2 = 60.827\times10^3\,\text{mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^2 + A_2d_2^2 = \frac{1}{12}(20)(25)^3 + (500)(10.91)^2 = 85.556\times10^3\,\text{mm}^4$$

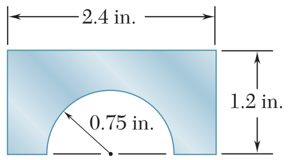
$$I = I_1 + I_2 = 146.383\times10^3\,\text{mm}^4 = 146.383\times10^{-9}\,\text{m}^4$$

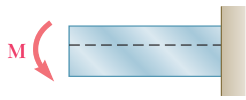
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(24\times10^6)(146.383\times10^{-9})}{0.01659} = 212 \text{ N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(30\times10^6)(146.383\times10^{-9})}{0.02341} = 187.6 \text{ N}\cdot\text{m}$$

Choose smaller value. $M = 187.6 \text{ N}\cdot\text{m}$ ◀





PROBLEM 4.18

Knowing that for the beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION

① = rectangle   ② = semi-circular cutout

$$A_1 = (2.4)(1.2) = 2.88 \text{ in}^2$$

$$A_2 = \frac{\pi}{2}(0.75)^2 = 0.8836 \text{ in}^2$$

$$A = 2.88 - 0.8836 = 1.9964 \text{ in}^2$$

$$\bar{y}_1 = 0.6 \text{ in.}$$

$$\bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(0.75)}{3\pi} = 0.3183 \text{ in.}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(2.88)(0.6) - (0.8836)(0.3183)}{1.9964} = 0.7247 \text{ in.}$$

Neutral axis lies 0.7247 in. above the bottom.

Moment of inertia about the base:

$$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(2.4)(1.2)^3 - \frac{\pi}{8}(0.75)^4 = 1.25815 \text{ in}^4$$

Centroidal moment of inertia:

$$\bar{I} = I_b - A\bar{Y}^2 = 1.25815 - (1.9964)(0.7247)^2 = 0.2097 \text{ in}^4$$

$$y_{\text{top}} = 1.2 - 0.7247 = 0.4753 \text{ in.},$$

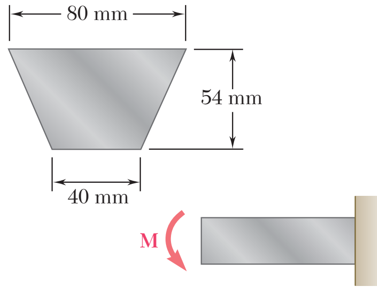
$$y_{\text{bot}} = -0.7247 \text{ in.}$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $M = \dfrac{(12)(0.2097)}{0.4753} = 5.29 \text{ kip} \cdot \text{in.}$

Bottom: (compression) $M = \dfrac{(16)(0.2097)}{0.7247} = 4.63 \text{ kip} \cdot \text{in.}$

Choose the smaller value. $M = 4.63 \text{ kip} \cdot \text{in.}$ ◀

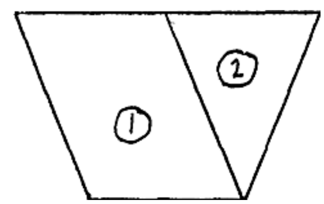


PROBLEM 4.19

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3	d, mm
①	2160	27	58,320	3
②	1080	36	38,880	3
Σ	3240		97,200	



$$\bar{Y} = \frac{97,200}{3240} = 30 \text{ mm}$$ The neutral axis lies 30 mm above the bottom.

$$y_{\text{top}} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m} \qquad y_{\text{bot}} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(54)^3 + (40)(54)(3)^2 = 544.32\times10^3\,\text{mm}^4$$

$$I_2 = \frac{1}{36}b_2h_2^2 + A_2d_2^2 = \frac{1}{36}(40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.84\times10^3\,\text{mm}^4$$

$$I = I_1 + I_2 = 758.16\times10^3\,\text{mm}^4 = 758.16\times10^{-9}\,\text{m}^4$$

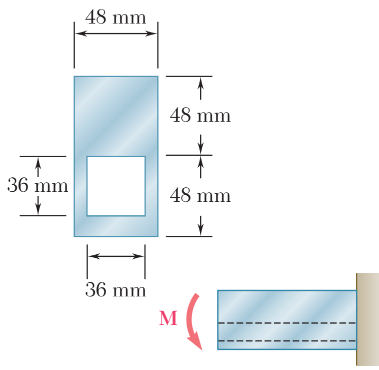
$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(120\times10^6)(758.16\times10^{-9})}{0.024} = 3.7908\times10^3\,\text{N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(150\times10^6)(758.16\times10^{-9})}{0.030} = 3.7908\times10^3\,\text{N}\cdot\text{m}$$

Choose the smaller as M_{all}. $M_{\text{all}} = 3.7908\times10^3\,\text{N}\cdot\text{m}$

$M_{\text{all}} = 3.79 \text{ kN}\cdot\text{m}$ ◀



PROBLEM 4.20

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
① Solid rectangle	4608	48	221,184
② Square cutout	–1296	30	–38,880
Σ	3312		182,304

$$\bar{Y} = \frac{182,304}{3312} = 55.04 \text{ mm} \qquad \text{Neutral axis lies 55.04 mm above bottom.}$$

$$y_{\text{top}} = 96 - 55.04 = 40.96 \text{ mm} = 0.04096 \text{ m}$$

$$y_{\text{bot}} = -55.04 \text{ mm} = -0.05504 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(48)(96)^3 + (48)(96)(7.04)^2 = 3.7673 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(36)(36)^3 + (36)(36)(25.04)^2 = 0.9526 \times 10^6 \text{ mm}^4$$

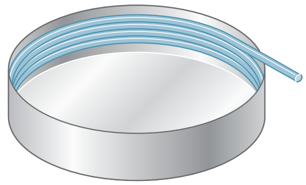
$$I = I_1 - I_2 = 2.8147 \times 10^6 \text{ mm}^4 = 2.8147 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad \therefore \quad M = +\left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $\qquad M = \dfrac{(120 \times 10^6)(2.8147 \times 10^{-6})}{0.04096} = 8.25 \times 10^3 \text{ N} \cdot \text{m}$

Bottom: (compression) $\qquad M = \dfrac{(150 \times 10^6)(2.8147 \times 10^6)}{0.05504} = 7.67 \times 10^3 \text{ N} \cdot \text{m}$

M_{all} is the smaller value. $\qquad M = 7.67 \times 10^3 \text{ N} \cdot \text{m} \qquad$ 7.67 kN · m ◀



PROBLEM 4.21

Straight rods of 6-mm diameter and 30-m length are stored by coiling the rods inside a drum of 1.25-m inside diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a coiled rod, (*b*) the corresponding bending moment in the rod. Use $E = 200$ GPa.

SOLUTION

Let $D =$ inside diameter of the drum,

$d =$ diameter of rod, $c = \frac{1}{2}d$,

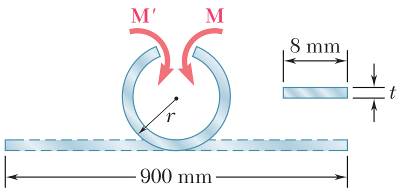
$\rho =$ radius of curvature of center line of rods when bent.

$$\rho = \frac{1}{2}D - \frac{1}{2}d = \frac{1}{2}(1.25) - \frac{1}{2}(6 \times 10^{-3}) = 0.622 \text{ m}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.003)^4 = 63.617 \times 10^{-12} \text{m}^4$$

(*a*) $\sigma_{\max} = \dfrac{Ec}{\rho} = \dfrac{(200 \times 10^9)(0.003)}{0.622} = 965 \times 10^6 \text{ Pa}$ $\quad\sigma = 965 \text{ MPa}$ ◀

(*b*) $M = \dfrac{EI}{\rho} = \dfrac{(200 \times 10^9)(63.617 \times 10^{-12})}{0.622} = 20.5 \text{ N} \cdot \text{m}$ $\quad M = 20.5 \text{ N} \cdot \text{m}$ ◀



PROBLEM 4.22

A 900-mm strip of steel is bent into a full circle by two couples applied as shown. Determine (*a*) the maximum thickness t of the strip if the allowable stress of the steel is 420 MPa, (*b*) the corresponding moment M of the couples. Use $E = 200$ GPa.

SOLUTION

When the rod is bent into a full circle, the circumference is 900 mm. Since the circumference is equal to 2π times ρ, the radius of curvature, we get

$$\rho = \frac{900 \text{ mm}}{2\pi} = 143.24 \text{ mm} = 0.14324 \text{ m}$$

Stress: $\sigma = E\varepsilon = \dfrac{Ec}{\rho}$ or $c = \dfrac{\rho\sigma}{E}$

For $\sigma = 420$ MPa and $E = 200$ GPa,

$$c = \frac{(0.14324)(420 \times 10^6)}{200 \times 10^9} = 0.3008 \times 10^{-3} \text{ m}$$

(*a*) Maximum thickness: $t = 2c = 0.6016 \times 10^{-3}$ m

$t = 0.602$ mm ◀

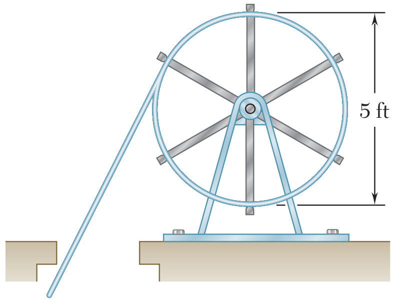
<u>Moment of inertia for a rectangular section.</u>

$$I = \frac{bt^3}{12} = \frac{(8 \times 10^{-3})(0.6016 \times 10^{-3})^3}{12} = 145.16 \times 10^{-15} \text{ m}^4$$

(*b*) Bending moment: $M = \dfrac{EI}{\rho}$

$$M = \frac{(200 \times 10^9)(145.16 \times 10^{-15})}{0.14324} = 0.203 \text{ N} \cdot \text{m}$$

$M = 0.203$ N · m ◀



PROBLEM 4.23

Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a rod, when the rod, which is initially straight, is wrapped on a spool, (*b*) the corresponding bending moment in the rod. Use $E = 29 \times 10^6$ psi .

SOLUTION

Radius of cross section: $r = \frac{1}{2}d = \frac{1}{2}(0.30) = 0.15 \text{ in.}$

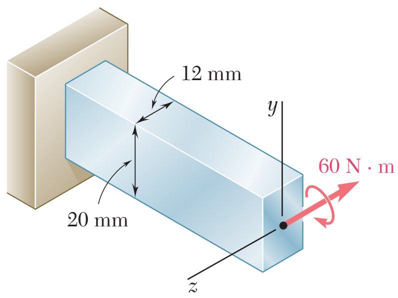
Moment of inertia: $I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-6} \text{ in}^4$

$D = 5 \text{ ft} = 60 \text{ in.} \qquad \rho = \frac{1}{2}D = 30 \text{ in.}$

$c = r = 0.15 \text{ in.}$

(*a*) $\sigma_{max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.15)}{30} = 145.0 \times 10^3 \text{ psi}$ $\qquad \sigma_{max} = 145.0 \text{ ksi}$ ◀

(*b*) $M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(397.61 \times 10^{-6})}{30}$ $\qquad M = 384 \text{ lb} \cdot \text{in.}$ ◀



PROBLEM 4.24

A 60-N · m couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.

SOLUTION

(*a*) Bending about *z*-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(20)^3 = 8\times10^3\text{ mm}^4 = 8\times10^{-9}\text{ m}^4$$

$$c = \frac{20}{2} = 10\text{ mm} = 0.010\text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.010)}{8\times10^{-9}} = 75.0\times10^6\text{ Pa}$$

$\sigma = 75.0$ MPa ◀

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times10^9)(8\times10^{-9})} = 37.5\times10^{-3}\text{ m}^{-1}$$

$\rho = 26.7$ m ◀

(*b*) Bending about *y*-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(20)(12)^3 = 2.88\times10^3\text{ mm}^4 = 2.88\times10^{-9}\text{ m}^4$$

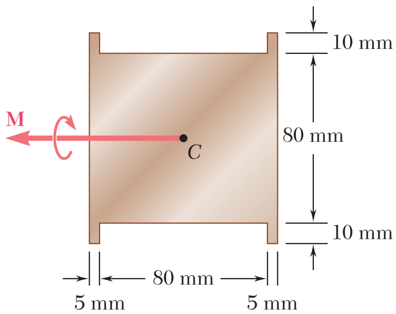
$$c = \frac{12}{2} = 6\text{ mm} = 0.006\text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.006)}{2.88\times10^{-9}} = 125.0\times10^6\text{ Pa}$$

$\sigma = 125.0$ MPa ◀

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times10^9)(2.88\times10^{-9})} = 104.17\times10^{-3}\text{ m}^{-1}$$

$\rho = 9.60$ m ◀



PROBLEM 4.25

(*a*) Using an allowable stress of 120 MPa, determine the largest couple **M** that can be applied to a beam of the cross section shown. (*b*) Solve part *a*, assuming that the cross section of the beam is an 80-mm square.

SOLUTION

(*a*) $I = I_1 + 4I_2$, where I_1 is the moment of inertia of an 80-mm square and I_2 is the moment of inertia of one of the four protruding ears.

$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}(80)(80)^3 = 3.4133 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(5)(10)^3 + (5)(10)(45)^2 = 101.667 \times 10^3 \text{ mm}^4$$

$$I = I_1 + 4I_2 = 3.82 \times 10^6 \text{ mm}^4 = 3.82 \times 10^{-6} \text{ mm}^4, \qquad c = 50 \text{ mm} = 0.050 \text{ m}$$

$$\sigma = \frac{Mc}{I} \qquad \therefore\ M = \frac{\sigma I}{c} = \frac{(120 \times 10^6)(3.82 \times 10^{-6})}{0.050}$$

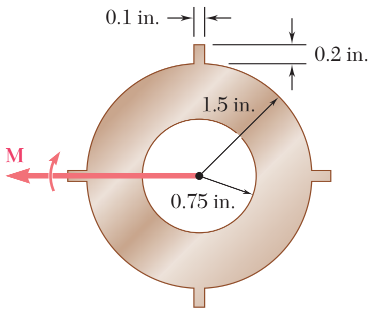
$$= 9.168 \times 10^3 \text{ N} \cdot \text{m}$$

$$= 9.17 \text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) Without the ears:

$$I = I_1 = 3.4133 \times 10^{-6} \text{ m}^2, \qquad c = 40 \text{ mm} = 0.040 \text{ m}$$

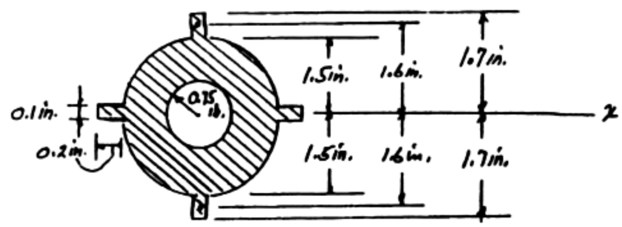
$$M = \frac{\sigma I}{c} = \frac{(120 \times 10^6)(3.4133 \times 10^{-6})}{0.040} = 10.24 \times 10^3 \text{ N} \cdot \text{m} \qquad = 10.24 \text{ kN} \cdot \text{m} \blacktriangleleft$$



PROBLEM 4.26

A thick-walled pipe is bent about a horizontal axis by a couple **M**. The pipe may be designed with or without four fins. (*a*) Using an allowable stress of 20 ksi, determine the largest couple that may be applied if the pipe is designed with four fins as shown. (*b*) Solve part *a*, assuming that the pipe is designed with no fins.

SOLUTION



I_x of hollow pipe: $\quad I_x = \frac{\pi}{4}\left[(1.5 \text{ in.})^4 - (0.75 \text{ in.})^4\right] = 3.7276 \text{ in}^4$

I_x of fins: $\quad I_x = 2\left[\frac{1}{12}(0.1)(0.2)^3 + (0.1 \times 0.2)(1.6)^2\right] + 2\left[\frac{1}{12}(0.2)(0.1)^3\right]$

$$= 0.1026 \text{ in}^4$$

(*a*) Pipe as designed, with fins:

$$I_x = 3.8302 \text{ in}^4, \qquad c = 1.7 \text{ in.}$$

$$\sigma_{\text{all}} = 20 \text{ ksi}, \qquad M = \sigma_{\text{all}} \frac{I_x}{c} = (20 \text{ ksi})\frac{3.8302 \text{ in}^4}{1.7 \text{ in.}}$$

$M = 45.1 \text{ kip} \cdot \text{in.}$ ◀

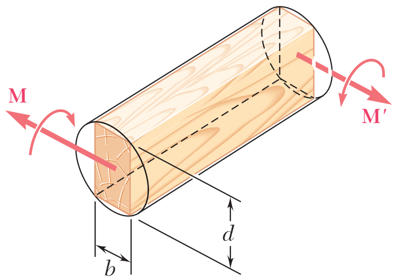
(*b*) Pipe with no fins:

$$\sigma_{\text{all}} = 20 \text{ ksi}, \qquad I_x = 3.7276 \text{ in}^4, \qquad c = 1.5 \text{ in.}$$

$$M = \sigma_{\text{all}} \frac{I_x}{c} = (20 \text{ ksi})\frac{3.7276 \text{ in}^4}{1.5 \text{ in.}}$$

$M = 49.7 \text{ kip} \cdot \text{in.}$ ◀

PROBLEM 4.27



A couple **M** will be applied to a beam of rectangular cross section that is to be sawed from a log of circular cross section. Determine the ratio d/b for which (*a*) the maximum stress σ_m will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

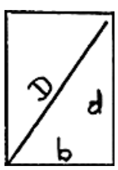
SOLUTION

Let D be the diameter of the log.

$$D^2 = b^2 + d^2 \qquad d^2 = D^2 - b^2$$

$$I = \frac{1}{12}bd^3 \qquad c = \frac{1}{2}d \qquad \frac{I}{c} = \frac{1}{6}bd^2$$

(*a*) σ_m is the minimum when $\dfrac{I}{c}$ is maximum.



$$\frac{I}{c} = \frac{1}{6}b(D^2 - b^2) = \frac{1}{6}D^2 b - \frac{1}{6}b^3$$

$$\frac{d}{db}\left(\frac{I}{c}\right) = \frac{1}{6}D^2 - \frac{3}{6}b^2 = 0 \qquad b = \frac{1}{\sqrt{3}}D$$

$$d = \sqrt{D^2 - \frac{1}{3}D^2} = \sqrt{\frac{2}{3}}D \qquad \frac{d}{b} = \sqrt{2} \blacktriangleleft$$

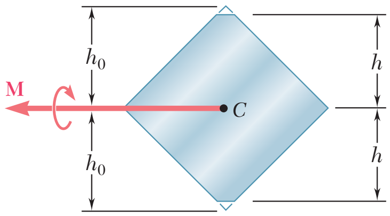
(*b*) $\rho = \dfrac{EI}{M}$

ρ is maximum when I is maximum, $\dfrac{1}{12}bd^3$ is maximum, or b^2d^6 is maximum.

$(D^2 - d^2)d^6$ is maximum.

$$6D^2d^5 - 8d^7 = 0 \qquad d = \frac{\sqrt{3}}{2}D$$

$$b = \sqrt{D^2 - \frac{3}{4}D^2} = \frac{1}{2}D \qquad \frac{d}{b} = \sqrt{3} \blacktriangleleft$$



PROBLEM 4.28

A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

SOLUTION

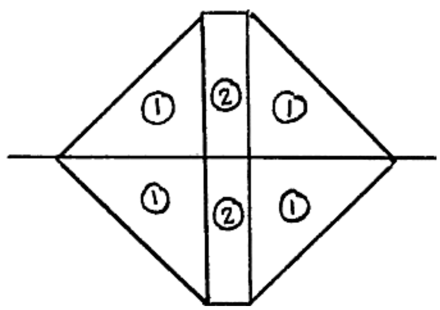
$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)h\,h^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)(h^3)$$

$$= \frac{1}{3}h^4 + \frac{4}{3}h_0h^3 - \frac{4}{3}h\,h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h$$

$$\sigma = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0h^3 - h^4} = \frac{3M}{(4h_0 - 3h)\,h^2}$$



For the original square, $h = h_0$, $c = h_0$.

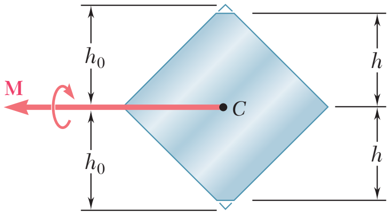
$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9\,h_0^2)} = 0.950$$

$$\sigma = 0.950\,\sigma_0$$

$k = 0.950$ ◀

PROBLEM 4.29



In Prob. 4.28, determine (*a*) the value of h for which the maximum stress σ_m is as small as possible, (*b*) the corresponding value of k.

PROBLEM 4.28 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

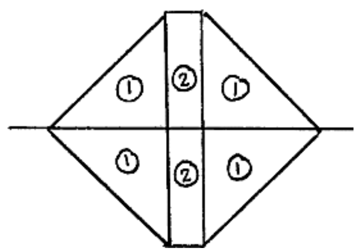
SOLUTION

$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)hh^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)h^3$$

$$= \frac{1}{3}h^4 - \frac{4}{3}h_0h^3 - \frac{4}{3}h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h \qquad \frac{I}{c} = \frac{4}{3}h_0h^2 - h^3$$



$\frac{I}{c}$ is maximum at $\frac{d}{dh}\left[\frac{4}{3}h_0h^2 - h^3\right] = 0.$

$$\frac{8}{3}h_0h - 3h^2 = 0 \qquad h = \frac{8}{9}h_0 \blacktriangleleft$$

$$\frac{I}{c} = \frac{4}{3}h_0\left(\frac{8}{9}h_0\right)^2 - \left(\frac{8}{9}h_0\right)^3 = \frac{256}{729}h_0^3 \qquad \sigma = \frac{Mc}{I} = \frac{729M}{256h_0^3}$$

For the original square, $h = h_0 \quad c = h_0 \quad \frac{I_0}{c_0} = \frac{1}{3}h_0^3$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^2}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256}\cdot\frac{1}{3} = \frac{729}{768} = 0.949 \qquad k = 0.949 \blacktriangleleft$$

PROBLEM 4.30

For the bar and loading of Concept Application 4.1, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section, (*c*) the angle between the sides of the bar that were originally vertical. Use $E = 29\times10^6\,\text{psi}$ and $v = 0.29.$

SOLUTION

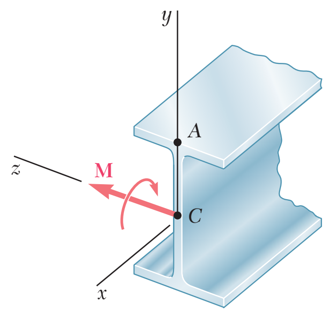
From Example 4.01, $M = 30\ \text{kip}\cdot\text{in.}\quad I = 1.042\ \text{in}^4$

(*a*) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{(30\times10^3)}{(29\times10^6)(1.042)} = 993\times10^{-6}\ \text{in.}^{-1}$$ $\rho = 1007\ \text{in.}$ ◀

(*b*) $$\varepsilon^1 = v\varepsilon = \frac{vc}{\rho} = v\frac{c}{\rho'}$$

$$\frac{1}{\rho'} = v\frac{1}{\rho} = (0.29)(993\times10^{-6})\,\text{in.}^{-1} = 288\times10^{-6}\ \text{in.}^{-1}$$ $\rho' = 3470\ \text{in.}$ ◀

(*c*) $$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.8}{3470} = 230\times10^{-6}\ \text{rad}$$ $\theta = 0.01320°$ ◀



PROBLEM 4.31

A $W200 \times 31.3$ rolled-steel beam is subjected to a couple **M** of moment 45 kN · m. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section.

SOLUTION

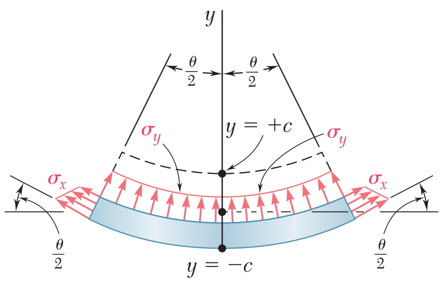
For $W\,200 \times 31.3$ rolled steel section,

$$I = 31.3 \times 10^6 \text{ mm}^4$$
$$= 31.3 \times 10^{-6} \text{ m}^4$$

(*a*) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{45 \times 10^3}{(200 \times 10^9)(31.3 \times 10^{-6})} = 7.1885 \times 10^{-3} \text{ m}^{-1}$$ $\rho = 139.1$ m ◀

(*b*) $$\frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(7.1885 \times 10^{-3}) = 2.0847 \times 10^{-3} \text{ m}^{-1}$$ $\rho' = 480$ m ◀

PROBLEM 4.32



It was assumed in Sec. 4.1B that the normal stresses σ_y in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (*a*) derive an approximate expression for σ_y as a function of *y*, (*b*) show that $(\sigma_y)_{\max} = -(c/2\rho)(\sigma_x)_{\max}$ and, thus, that σ_y can be neglected in all practical situations. (*Hint:* Consider the free-body diagram of the portion of beam located below the surface of ordinate *y* and assume that the distribution of the stress σ_x is still linear.)

SOLUTION

Denote the width of the beam by *b* and the length by *L*.

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos\frac{\theta}{2} \approx 1$

$$\Sigma F_y = 0: \quad \sigma_y bL + 2\int_{-c}^{y} \sigma_x b\,dy\,\sin\frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L}\sin\frac{\theta}{2}\int_{-c}^{y}\sigma_x\,dy \approx -\frac{\theta}{L}\int_{-c}^{y}\sigma_x\,dy = -\frac{1}{\rho}\int_{-c}^{y}\sigma_x\,dy$$

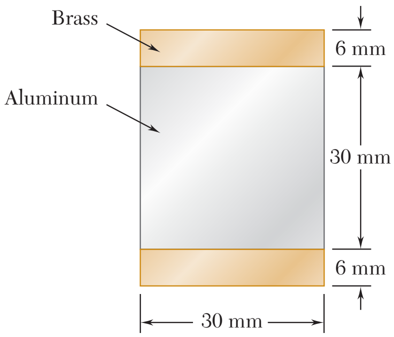
But, $$\sigma_x = -(\sigma_x)_{\max}\frac{y}{c}$$

(*a*) $$\sigma_y = \frac{(\sigma_x)_{\max}}{\rho c}\int_{-c}^{y} y\,dy = \frac{(\sigma_x)_{\max}}{\rho c}\left.\frac{y^2}{2}\right|_{-c}^{y}$$

$$\sigma_y = \frac{(\sigma_x)_{\max}}{2\rho c}(y^2 - c^2) \blacktriangleleft$$

The maximum value σ_y occurs at $y = 0$.

(*b*) $$(\sigma_y)_{\max} = -\frac{(\sigma_x)_{\max}c^2}{2\rho c} = -\frac{(\sigma_x)_{\max}c}{2\rho} \blacktriangleleft$$

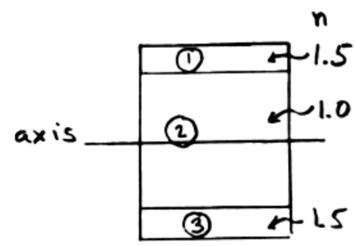


PROBLEM 4.33

A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

$$n = 1.0 \text{ in aluminum}$$

$$n = E_b/E_a = 105/70 = 1.5 \text{ in brass}$$

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.5}{12}(30)(6)^3 + (1.5)(30)(6)(18)^3 = 88.29 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.0}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 88.29 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 244.08 \times 10^3 \text{ mm}^4$$

$$= 244.08 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \frac{\sigma I}{ny}$$

Aluminum: $n = 1.0, \quad y = 15 \text{ mm} = 0.015 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

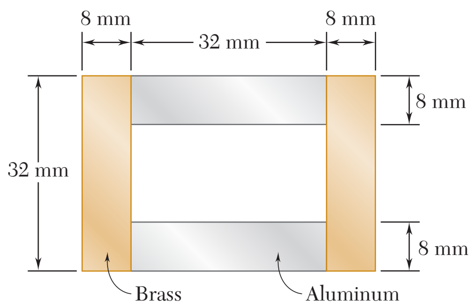
$$M = \frac{(100 \times 10^6)(244.08 \times 10^{-9})}{(1.0)(0.015)} = 1.627 \times 10^3 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \quad y = 21 \text{ mm} = 0.021 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(244.08 \times 10^{-9})}{(1.5)(0.021)} = 1.240 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value $M = 1.240 \times 10^3 \text{ N} \cdot \text{m}$ 1.240 kN · m ◀

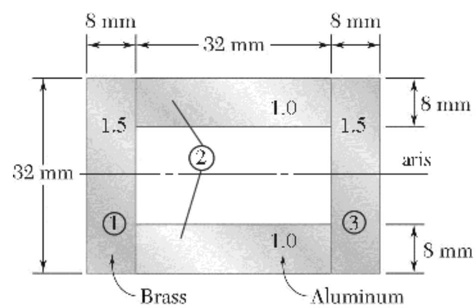
PROBLEM 4.34



A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12}(8)(32)^3 = 32.768 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 \left(H_2^3 - h_2^3 \right) = \frac{1.0}{12}(32)(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \qquad M = \left| \frac{\sigma I}{ny} \right|$$

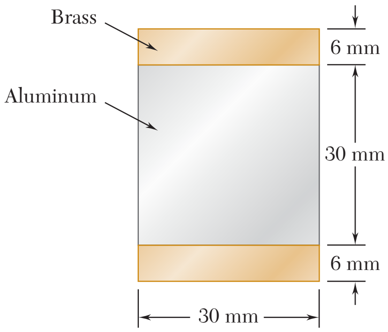
Aluminum: $n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 887 \text{ N} \cdot \text{m}$ ◀



PROBLEM 4.35

For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

PROBLEM 4.35. Bar of Prob. 4.33.

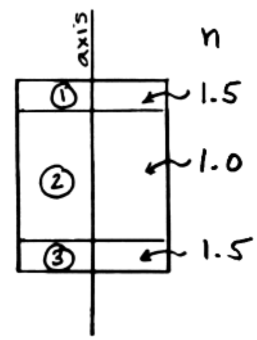
	Aluminum	**Brass**
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION

Use aluminum as reference material.

$$n = 1.0 \text{ in aluminum}$$

$$n = E_b/E_a = 105/70 = 1.5 \text{ in brass}$$



For transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3$$

$$= \frac{1.5}{12}(6)(30)^3 = 20.25 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3$$

$$= \frac{1.0}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 20.25 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 108 \times 10^3 \text{ mm}^4 = 108 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

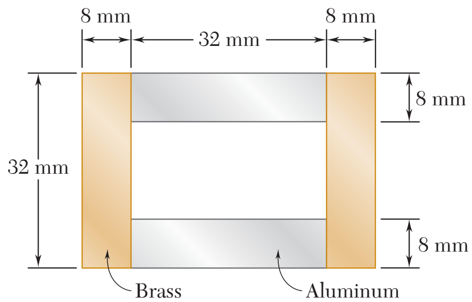
Aluminum: $n = 1.0, \quad y = 15 \text{ mm} = 0.015 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(108 \times 10^{-9})}{(1.0)(0.015)} = 720 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \quad y = 15 \text{ mm} = 0.015 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(108 \times 10^{-9})}{(1.5)(0.015)} = 768 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 720 \text{ N} \cdot \text{m}$ ◀



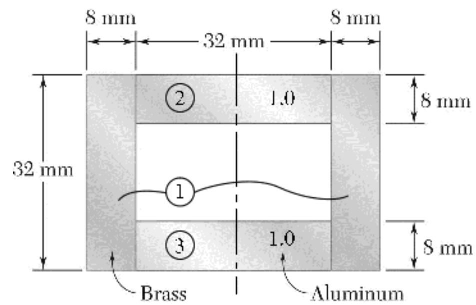
PROBLEM 4.36

For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

PROBLEM 4.36 Bar of Prob. 4.34.

	Aluminum	**Brass**
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} h_1 \left(B_1^3 - b_1^3\right) = \frac{1.5}{12}(32)(48^3 - 32^3) = 311.296 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 354.99 \times 10^3 \text{ mm}^4 = 354.99 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(354.99 \times 10^{-9})}{(1.0)(0.016)} = 2.2187 \times 10^3 \text{ N}\cdot\text{m}$$

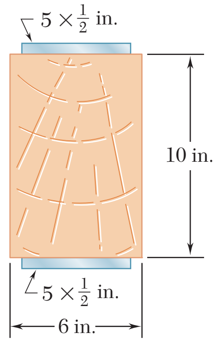
Brass: $n = 1.5 \quad |y| = 24 \text{ mm} = 0.024 \text{ m} \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(354.99 \times 10^{-9})}{(1.5)(0.024)} = 1.57773 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value.

$M = 1.57773 \times 10^3 \text{ N}\cdot\text{m}$ $\qquad$ $M = 1.578 \text{ kN}\cdot\text{m}$ ◀

PROBLEM 4.37



Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

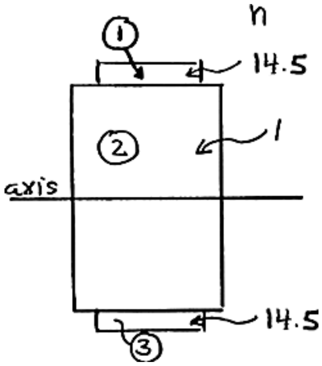
	Wood	Steel
Modulus of elasticity:	2×10^6 psi	29×10^6 psi
Allowable stress:	2000 psi	22 ksi

SOLUTION

Use wood as the reference material.

$$n = 1.0 \text{ in wood}$$

$$n = E_s/E_w = 29/2 = 14.5 \text{ in steel}$$



For the transformed section,

$$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2$$

$$= \frac{14.5}{12}(5)\left(\frac{1}{2}\right)^3 + (14.5)(5)\left(\frac{1}{2}\right)(5.25)^2 = 999.36 \text{ in}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^2 = \frac{1.0}{12}(6)(10)^3 = 500 \text{ in}^4$$

$$I_3 = I_1 = 999.36 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 2498.7 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

Wood: $n = 1.0, \quad y = 5 \text{ in.}, \quad \sigma = 2000 \text{ psi}$

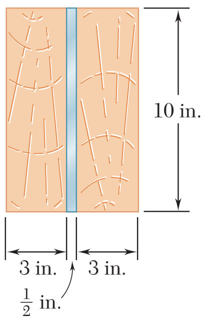
$$M = \frac{(2000)(2499)}{(1.0)(5)} = 999.5\times10^3 \text{ lb}\cdot\text{in.}$$

Steel: $n = 14.5, \quad y = 5.5 \text{ in.}, \quad \sigma = 22 \text{ ksi} = 22\times10^3 \text{ psi}$

$$M = \frac{(22\times10^3)(2499)}{(14.5)(5.5)} = 689.3\times10^3 \text{ lb}\cdot\text{in.}$$

Choose the smaller value. $M = 689\times10^3$ lb · in. $M = 689$ kip · in. ◀

PROBLEM 4.38



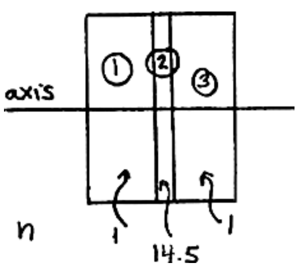
Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity:	2×10^6 psi	29×10^6 psi
Allowable stress:	2000 psi	22 ksi

SOLUTION

Use wood as the reference material.

$$n = 1.0 \text{ in wood}$$
$$n = E_s/E_w = 29/2 = 14.5 \text{ in steel}$$



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(3)(10)^3 = 250 \text{ in}^4$$
$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{14.5}{12}\left(\frac{1}{2}\right)(10)^3 = 604.17 \text{ in}^4$$
$$I_3 = I_1 = 250 \text{ in}^4$$
$$I = I_1 + I_2 + I_3 = 1104.2 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

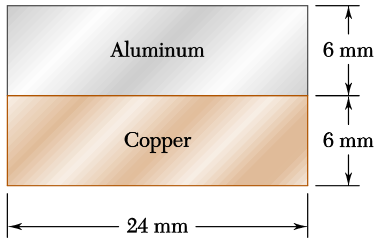
Wood: $\quad n = 1.0, \quad y = 5 \text{ in.}, \quad \sigma = 2000 \text{ psi}$

$$M = \frac{(2000)(1104.2)}{(1.0)(5)} = 441.7 \times 10^3 \text{ lb} \cdot \text{in.}$$

Steel: $\quad n = 14.5, \quad y = 5 \text{ in.}, \quad \sigma = 22 \text{ ksi} = 22 \times 10^3 \text{ psi}$

$$M = \frac{(22 \times 10^3)(1104.2)}{(14.5)(5)} = 335.1 \times 10^3 \text{ lb} \cdot \text{in.}$$

Choose the smaller value. $\quad M = 335 \times 10^3 \text{ lb} \cdot \text{in.} \qquad M = 335 \text{ kip} \cdot \text{in.}$ ◀



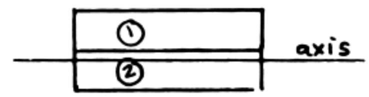
PROBLEM 4.39

A copper strip (E_c = 105 GPa) and an aluminum strip (E_a = 75 GPa) are bonded together to form the composite beam shown. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 35$ N · m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

SOLUTION

Use aluminum as the reference material.

$$n = 1.0 \text{ in aluminum}$$
$$n = E_c/E_a = 105/75 = 1.4 \text{ in copper}$$



Transformed section:

	A, mm^2	nA, mm^2	$\bar{y}_0$, mm	$nA\bar{y}_0$, mm^3
①	144	144	9	1296
②	144	201.6	3	604.8
Σ		345.6		1900.8

$$\bar{Y}_0 = \frac{1900.8}{345.6} = 5.50 \text{ mm}$$

The neutral axis lies 5.50 mm above the bottom.

$$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2 = \frac{1.0}{12}(24)(6)^3 + (1.0)(24)(6)(3.5)^2 = 2196 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{1.4}{12}(24)(6)^3 + (1.4)(24)(6)(2.5)^2 = 1864.8 \text{ mm}^4$$

$$I = I_1 + I_2 = 4060.8 \text{ mm}^4 = 4.0608 \times 10^{-9} \text{ m}^4$$

(*a*) Aluminum: $n = 1.0, \quad y = 12 - 5.5 = 6.5 \text{ mm} = 0.0065 \text{ m}$

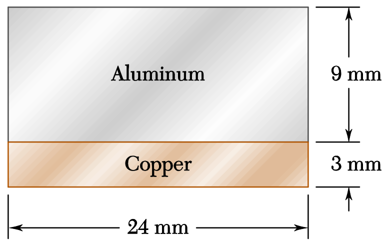
$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0065)}{4.0608 \times 10^{-9}} = -56.0 \times 10^6 \text{ Pa} = -56.0 \text{ MPa}$$

$\sigma = -56.0$ MPa ◀

(*b*) Copper: $n = 1.4, \quad y = -5.5 \text{ mm} = -0.0055 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055)}{4.0608 \times 10^{-9}} = 66.4 \times 10^6 \text{ Pa} = 66.4 \text{ MPa}$$

$\sigma = 66.4$ MPa ◀



PROBLEM 4.40

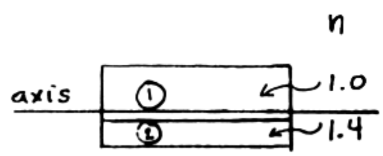
A copper strip (E_c = 105 GPa) and an aluminum strip (E_a = 75 GPa) are bonded together to form the composite beam shown. Knowing that the beam is bent about a horizontal axis by a couple of moment M = 35 N · m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

SOLUTION

Use aluminum as the reference material.

$$n = 1.0 \text{ in aluminum}$$

$$n = E_c/E_a = 105/75 = 1.4 \text{ in copper}$$



Transformed section:

	A, mm^2	nA, mm^2	$A\bar{y}_0$, mm	$nA\bar{y}_0$, mm^3
①	216	216	7.5	1620
②	72	100.8	1.5	151.8
Σ		316.8		1771.2

$$\bar{Y}_0 = \frac{1771.2}{316.8} = 5.5909 \text{ mm}$$

The neutral axis lies 5.5909 mm above the bottom.

$$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2 = \frac{1.0}{12}(24)(9)^3 + (1.0)(24)(9)(1.9091)^2 = 2245.2 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{1.4}{12}(24)(3)^3 + (1.4)(24)(3)(4.0909)^2 = 1762.5 \text{ mm}^4$$

$$I = I_1 + I_2 = 4839 \text{ mm}^4 = 4.008\times 10^{-9} \text{ m}^4$$

(*a*) Aluminum: $n = 1.0,\quad y = 12 - 5.5909 = 6.4091 \text{ mm} = 0.0064091$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0064091)}{4.008\times 10^{-9}} = -56.0\times 10^{-6} \text{ Pa}$$

$$= -56.0 \text{ MPa}$$

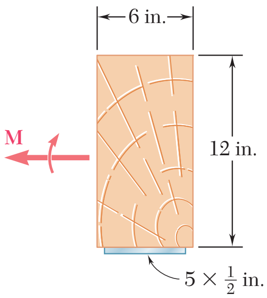
$\sigma = -56.0$ MPa ◀

(*b*) Copper: $n = 1.4,\quad y = -5.5909 \text{ mm} = -0.0055909 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055909)}{4.008\times 10^{-9}} = 68.4\times 10^{6} \text{ Pa}$$

$$= 68.4 \text{ MPa}$$

$\sigma = 68.4$ MPa ◀



PROBLEM 4.41

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel, 29×10^6 psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip·in., determine the maximum stress in (*a*) the wood, (*b*) the steel.

SOLUTION

Use wood as the reference material.

For wood, $n = 1$

For steel, $n = E_s / E_w = 29/1.8 = 16.1111$

Transformed section: ① = wood ② = steel

$$\bar{Y}_o = \frac{421.931}{112.278} = 3.758 \text{ in.}$$

	A, in^2	nA, in^2	$\bar{y}_0$	$nA\bar{y}_0$, in^3
①	72	72	6	432
②	2.5	40.278	−0.25	−10.069
		112.278		421.931

The neutral axis lies 3.758 in. above the wood-steel interface.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(6)(12)^3 + (72)(6 - 3.758)^2 = 1225.91 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{16.1111}{12}(5)(0.5)^3 + (40.278)(3.578 + 0.25)^2 = 647.87 \text{ in}^4$$

$$I = I_1 + I_2 = 1873.77 \text{ in}^4$$

$$M = 450 \text{ kip} \cdot \text{in.} \qquad \sigma = -\frac{nMy}{I}$$

(*a*) Wood: $n = 1, \quad y = 12 - 3.758 = 8.242$ in.

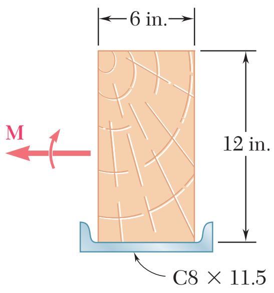
$$\sigma_w = -\frac{(1)(450)(8.242)}{1873.77} = -1.979 \text{ ksi}$$

$\sigma_w = -1.979$ ksi ◀

(*b*) Steel: $n = 16.1111, \quad y = -3.758 - 0.5 = -4.258$ in.

$$\sigma_s = -\frac{(16.1111)(450)(-4.258)}{1873.77} = 16.48 \text{ ksi}$$

$\sigma_s = 16.48$ ksi ◀

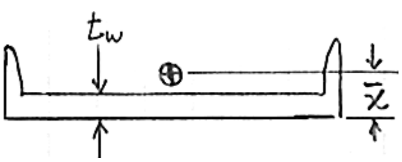


PROBLEM 4.42

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel, 29×10^6 psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip · in., determine the maximum stress in (*a*) the wood, (*b*) the steel.

SOLUTION

Use wood as the reference material.



For wood, $n = 1$

For steel, $n = \dfrac{E_s}{E_w} = \dfrac{29 \times 10^6}{1.8 \times 10^6} = 16.1111$

For C8 × 11.5 channel section,

$A = 3.38 \text{ in}^2, \quad t_w = 0.220 \text{ in.}, \quad \bar{x} = 0.571 \text{ in.}, \quad I_y = 1.32 \text{ in}^4$

For the composite section, the centroid of the channel (part 1) lies 0.571 in. above the bottom of the section. The centroid of the wood (part 2) lies 0.220 + 6.00 = 6.22 in. above the bottom.

Transformed section:

Part	A, in^2	nA, in^2	$\bar{y}$, in.	$nA\bar{y}$, in^3	d, in.
1	3.38	54.456	0.571	31.091	3.216
2	72	72	6.22	447.84	2.433
Σ		126.456		478.93	

$$\bar{Y}_0 = \frac{478.93 \text{ in}^3}{126.456 \text{ in}^2} = 3.787 \text{ in.} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

The neutral axis lies 3.787 in. above the bottom of the section.

$$I_1 = n_1 \bar{I}_1 + n_1 A_1 d_1^2 = (16.1111)(1.32) + (54.456)(3.216)^2 = 584.49 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(6)(12)^3 + (72)(2.433)^2 = 1290.20 \text{ in}^4$$

$$I = I_1 + I_2 = 1874.69 \text{ in}^4$$

$$M = 450 \text{ kip} \cdot \text{in} \qquad \sigma = -\frac{nMy}{I}$$

(*a*) Wood: $n = 1, \quad y = 12 + 0.220 - 3.787 = 8.433$ in.

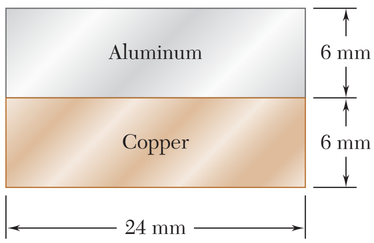
$$\sigma_w = -\frac{(1)(450)(8.433)}{1874.69} = -2.02 \text{ ksi}$$

$\sigma_w = -2.02$ ksi ◀

(*b*) Steel: $n = 16.1111, \quad y = -3.787$ in.

$$\sigma_s = -\frac{(16.1111)(450)(-3.787)}{1874.67} = 14.65 \text{ ksi}$$

$\sigma_s = 14.65$ ksi ◀

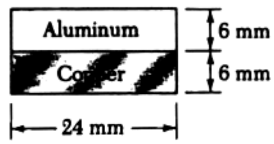


PROBLEM 4.43

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 35 N · m.

Beam of Prob. 4.39.

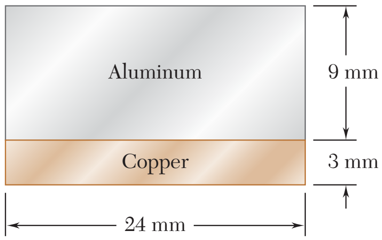
SOLUTION



See solution to Prob. 4.39 for the calculation of *I*.

$$\frac{1}{\rho}=\frac{M}{E_a I}=\frac{35}{(75\times10^9)(4.0608\times10^{-9})}=0.1149\ \text{m}^{-1}$$

$\rho = 8.70$ m ◀



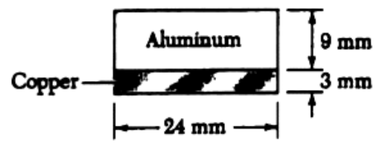
PROBLEM 4.44

For the composite beam indicated, determine the radius of curvature caused by the couple of moment $35\ \text{N}\cdot\text{m}$.

Beam of Prob. 4.40.

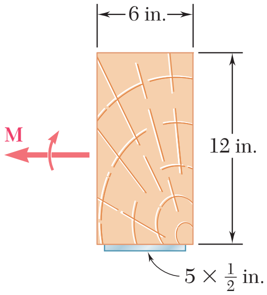
SOLUTION

See solution to Prob. 4.40 for the calculation of *I*.



$$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{35}{(75\times 10^9)(4.008\times 10^{-9})} = 0.1164\ \text{m}^{-1}$$

$\rho = 8.59\ \text{m}$ ◀



PROBLEM 4.45

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

Beam of Prob. 4.41.

SOLUTION

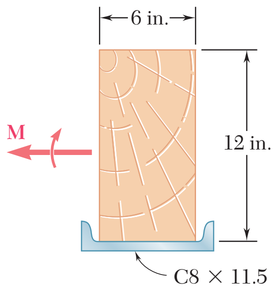
See solution to Prob. 4.41 for calculation of I.

$$I = 1873.77 \text{ in}^4 \qquad E_w = 1.8 \times 10^6 \text{ psi}$$

$$M = 450 \text{ kip} \cdot \text{in} = 450 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1873.77)} = 133.421 \times 10^{-6} \text{ in.}^{-1}$$

$$\rho = 7495 \text{ in.} = 625 \text{ ft} \blacktriangleleft$$



PROBLEM 4.46

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

Beam of Prob. 4.42.

SOLUTION

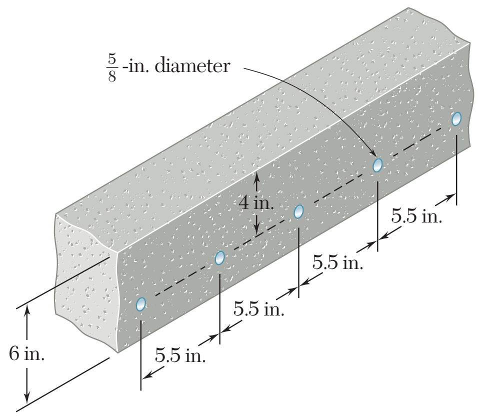
See solution to Prob. 4.42 for calculation of I.

$$I = 1874.69 \text{ in}^4 \qquad E_w = 1.8 \times 10^6 \text{ psi}$$

$$M = 450 \text{ kip} \cdot \text{in.} = 450 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1874.69)} = 133.355 \times 10^{-6} \text{ in.}^{-1}$$

$$\rho = 7499 \text{ in.} = 625 \text{ ft} \blacktriangleleft$$



PROBLEM 4.47

A concrete slab is reinforced by $\frac{5}{8}$-in.-diameter steel rods placed on 5.5-in. centers as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1400 psi for the concrete and 20 ksi for the steel, determine the largest bending moment in a portion of slab 1 ft wide.

SOLUTION

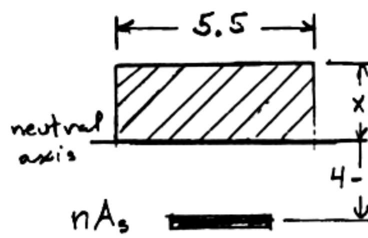
$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3 \times 10^6} = 9.6667$$

Consider a section 5.5 in. wide.

$$A_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.3068 \text{ in}^2$$

$$nA_s = 2.9657 \text{ in}^2$$

Locate the natural axis.



$$5.5x\frac{x}{2} - (4 - x)(2.9657) = 0$$

$$2.75x^2 + 2.9657x - 11.8628 = 0$$

Solve for x.

$$x = 1.6066 \text{ in.} \qquad 4 - x = 2.3934 \text{ in.}$$

$$I = \frac{1}{3}(5.5)x^3 + (2.9657)(4 - x)^2$$

$$= \frac{1}{3}(5.5)(1.6066)^3 + (2.9657)(2.3934)^2 = 24.591 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{I\sigma}{ny}\right|$$

Concrete: $n = 1$, $y = 1.6066$ in., $\sigma = 1400$ psi

$$M = \frac{(24.591)(1400)}{(1.0)(1.6066)} = 21.429 \times 10^3 \text{ lb} \cdot \text{in.}$$

Steel: $n = 9.6667, \quad y = 2.3934 \text{ in.}, \quad \sigma = 20 \text{ ksi} = 20 \times 10^3 \text{ psi}$

$$M = \frac{(24.591)(20 \times 10^3)}{(9.6667)(2.3934)} = 21.258 \times 10^3 \text{ lb} \cdot \text{in.}$$

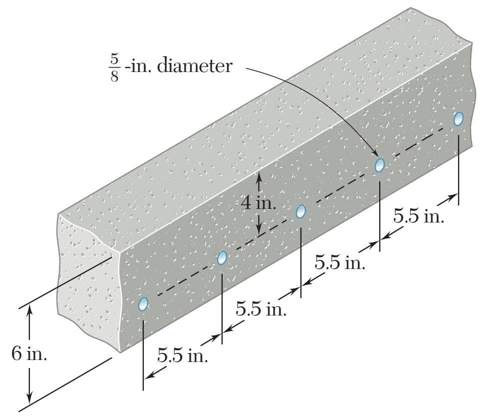
Choose the smaller value as the allowable moment for a 5.5 in. width.

$$M = 21.258 \times 10^3 \text{ lb} \cdot \text{in.}$$

For a 1 ft = 12 in. width,

$$M = \frac{12}{5.5}(21.258 \times 10^3) = 46.38 \times 10^3 \text{ lb} \cdot \text{in.}$$

$M = 46.38 \text{ kip} \cdot \text{in.}$ $\qquad$ $3.87 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 4.48

Solve Prob. 4.47, assuming that the spacing of the $\frac{5}{8}$-in.-diameter steel rods is increased to 7.5 in.

PROBLEM 4.47 A concrete slab is reinforced by $\frac{5}{8}$-in.-diameter steel rods placed on 5.5-in. centers as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1400 psi for the concrete and 20 ksi for the steel, determine the largest bending moment in a portion of slab 1 ft wide.

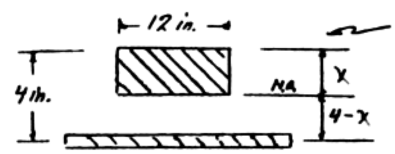
SOLUTION

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6 \text{ psi}}{3 \times 10^6 \text{ psi}} = 9.667$$

Number of rails per foot:

$$= \frac{12 \text{ in.}}{7.5 \text{ in.}} = 1.6$$

Area of $\frac{5}{8}$-in.-diameter bars per foot: $1.6\frac{\pi}{4}\left(\frac{5}{8}\right)^2$ $\qquad A_s = 0.4909 \text{ in}^2$



<u>Transformed section, all concrete</u>.

First moment of area:

$$12x\left(\frac{x}{2}\right) - 4.745(4 - x) = 0$$

$$x = 1.4266 \text{ in.}$$

$$nA_s = 9.667(0.4909) = 4.745 \text{ in}^2$$

$$I_{NA} = \frac{1}{3}(12)(1.4266)^3 + 4.745(4 - 1.4266)^2 = 43.037 \text{ in}^4$$

<u>For concrete</u>: $\sigma_{\text{all}} = 1400 \text{ psi} \quad c = x = 1.4266 \text{ in.}$

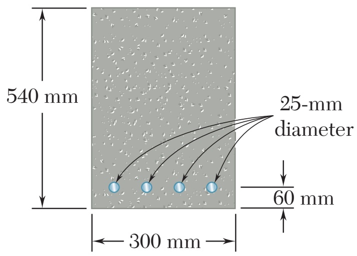
$$M = \sigma\frac{I}{c} = (1400 \text{ psi})\ \frac{43.037 \text{ in}^4}{1.4266 \text{ in.}} \qquad M = 42.24 \text{ kip.in.} ◀$$

<u>For steel</u>: $\sigma_{\text{all}} = 20 \text{ ksi} \quad c = 4 - x = 4 - 1.4266 = 2.5734 \text{ in.}$

$$M = \frac{\sigma_{\text{steel}}}{n}\cdot\frac{I}{c} = \frac{20 \text{ ksi}}{9.667}\cdot\frac{43.042 \text{ in}^4}{2.5734 \text{ in.}} \qquad M = 34.60 \text{ kip}\cdot\text{in.} ◀$$

We choose the smaller M. $\qquad M = 34.60 \text{ kip}\cdot\text{in.}$

Steel controls. $\qquad M = 2.88 \text{ kip}\cdot\text{ft}$ ◀



PROBLEM 4.49

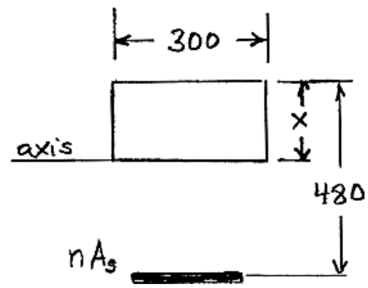
The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$nA_s = 15.708 \times 10^3 \text{ mm}^2$$



Locate the neutral axis.

$$300\, x \frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$150x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x.

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(150)(7.5398 \times 10^6)}}{(2)(150)}$$

$$x = 177.87 \text{ mm}, \quad 480 - x = 302.13 \text{ mm}$$

$$I = \frac{1}{3}(300)x^3 + (15.708 \times 10^3)(480 - x)^2$$

$$= \frac{1}{3}(300)(177.87)^3 + (15.708 \times 10^3)(302.13)^2$$

$$= 1.9966 \times 10^9 \text{ mm}^4 = 1.9966 \times 10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

(*a*) Steel: $y = -302.45 \text{ mm} = -0.30245 \text{ m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.30245)}{1.9966 \times 10^{-3}} = 212 \times 10^6 \text{ Pa}$$

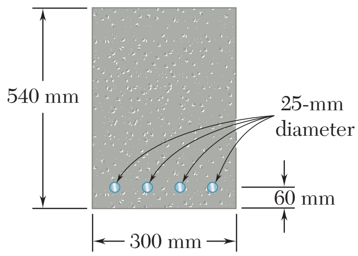
$\sigma = 212 \text{ MPa}$ ◀

(*b*) Concrete: $y = 177.87 \text{ mm} = 0.17787 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.17787)}{1.9966 \times 10^{-3}} = -15.59 \times 10^6 \text{ Pa}$$

$\sigma = -15.59 \text{ MPa}$ ◀

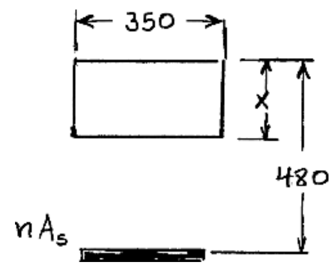
PROBLEM 4.50



Solve Prob. 4.49, assuming that the 300-mm width is increased to 350 mm.

PROBLEM 4.49 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{200\text{ GPa}}{25\text{ GPa}} = 8.0$$

$$A_s = 4\frac{\pi}{4}d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2$$

$$= 1.9635\times 10^3\text{ mm}^2$$

$$nA_s = 15.708\times 10^3\text{ mm}^2$$

Locate the neutral axis.

$$350x\frac{x}{2} - (15.708\times 10^3)(480 - x) = 0$$

$$175x^2 + 15.708\times 10^3 x - 7.5398\times 10^6 = 0$$

Solve for x.

$$x = \frac{-15.708\times 10^3 + \sqrt{(15.708\times 10^3)^2 + (4)(175)(7.5398\times 10^6)}}{(2)(175)}$$

$$x = 167.48\text{ mm}, \quad 480 - x = 312.52\text{ mm}$$

$$I = \frac{1}{3}(350)x^3 + (15.708\times 10^3)(480 - x)^2$$

$$= \frac{1}{3}(350)(167.48)^3 + (15.708\times 10^3)(312.52)^2$$

$$= 2.0823\times 10^9\text{ mm}^4 = 2.0823\times 10^{-3}\text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

(*a*) Steel: $y = -312.52\text{ mm} = -0.31252\text{ m}$

$$\sigma = -\frac{(8.0)(175\times 10^3)(-0.31252)}{2.0823\times 10^{-3}} = 210\times 10^6\text{ Pa}$$

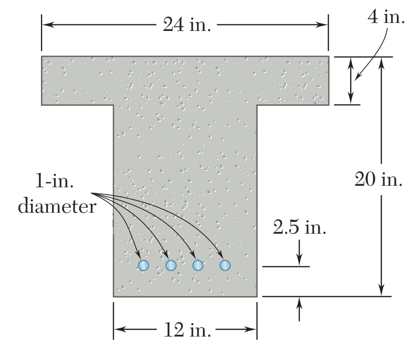
$\sigma = 210$ MPa ◀

(*b*) Concrete: $y = 167.48\text{ mm} = 0.16748\text{ m}$

$$\sigma = -\frac{(1.0)(175\times 10^3)(0.16748)}{2.0823\times 10^{-3}} = -14.08\times 10^6\text{ Pa}$$

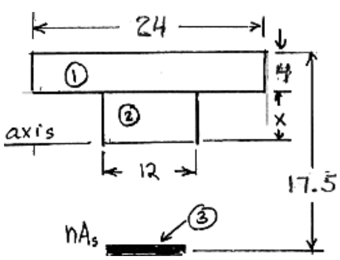
$\sigma = -14.08$ MPa ◀

PROBLEM 4.51



Knowing that the bending moment in the reinforced concrete beam is +100 kip · ft and that the modulus of elasticity is 3.625×10^6 psi for the concrete and 29×10^6 psi for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.625 \times 10^6} = 8.0$$

$$A_s = (4)\left(\frac{\pi}{4}\right)(1)^2 = 3.1416 \text{ in}^2 \qquad nA_s = 25.133 \text{ in}^2$$

Locate the neutral axis.

$$(24)(4)(x + 2) + (12x)\left(\frac{x}{2}\right) - (25.133)(17.5 - 4 - x) = 0$$

$$96x + 192 + 6x^2 - 339.3 + 25.133x = 0 \qquad \text{or} \qquad 6x^2 + 121.133x - 147.3 = 0$$

Solve for x.
$$x = \frac{-121.133 + \sqrt{(121.133)^2 + (4)(6)(147.3)}}{(2)(6)} = 1.150 \text{ in.}$$

$$d_3 = 17.5 - 4 - x = 12.350 \text{ in.}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(24)(4)^3 + (24)(4)(3.150)^2 = 1080.6 \text{ in}^4$$

$$I_2 = \frac{1}{3}b_2x^3 = \frac{1}{3}(12)(1.150)^3 = 6.1 \text{ in}^4$$

$$I_3 = nA_3d_3^2 = (25.133)(12.350)^2 = 3833.3 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 4920 \text{ in}^4$$

$$\sigma = -\frac{nMy}{I} \qquad \text{where } M = 100 \text{ kip} \cdot \text{ft} = 1200 \text{ kip} \cdot \text{in.}$$

(*a*) Steel: $n = 8.0 \qquad y = -12.350$ in.

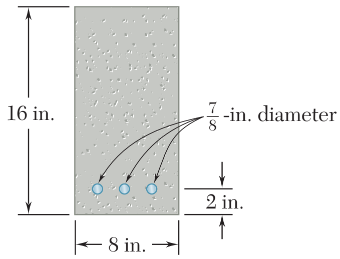
$$\sigma_s = -\frac{(8.0)(1200)(-12.350)}{4920}$$

$\sigma_s = 24.1$ ksi ◀

(*b*) Concrete: $n = 1.0, \quad y = 4 + 1.150 = 5.150$ in.

$$\sigma_c = -\frac{(1.0)(1200)(5.150)}{4920}$$

$\sigma_c = -1.256$ ksi ◀



PROBLEM 4.52

A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

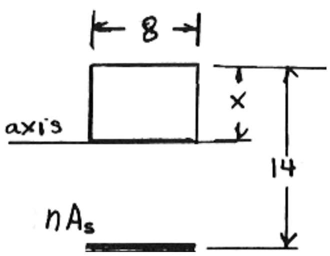
SOLUTION

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3 \times 10^6} = 9.67$$

$$A_s = 3\frac{\pi}{4}d^2 = (3)\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.8040 \text{ in}^2 \qquad nA_s = 17.438 \text{ in}^2$$

Locate the neutral axis:

$$8x\frac{x}{2} - (17.438)(14 - x) = 0$$



$$4x^2 + 17.438x - 244.14 = 0$$

Solve for x. $\quad x = \dfrac{-17.438 + \sqrt{17.438^2 + (4)(4)(244.14)}}{(2)(4)} = 5.6326 \text{ in.}$

$$14 - x = 8.3674 \text{ in.}$$

$$I = \frac{1}{3}8x^3 + nA_s(14 - x)^2 = \frac{1}{3}(8)(5.6326)^3 + (17.438)(8.3674)^2 = 1697.45 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $\quad n = 1.0, \quad |y| = 5.6326 \text{ in.}, \quad |\sigma| = 1350 \text{ psi}$

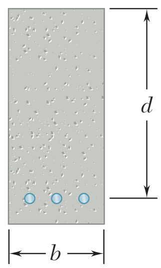
$$M = \frac{(1350)(1697.45)}{(1.0)(5.6326)} = 406.835 \times 10^3 \text{ lb} \cdot \text{in.} = 407 \text{ kip} \cdot \text{in.}$$

Steel: $\quad n = 9.67, \quad |y| = 8.3674 \text{ in.}, \quad \sigma = 20 \times 10^3 \text{ psi}$

$$M = \frac{(20 \times 10^3)(1697.45)}{(9.67)(8.3674)} = 419.72 \text{ lb} \cdot \text{in.} = 420 \text{ kip} \cdot \text{in.}$$

Choose the smaller value. $\quad M = 407 \text{ kip} \cdot \text{in.} \qquad M = 33.9 \text{ kip} \cdot \text{ft}$ ◀

PROBLEM 4.53



The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

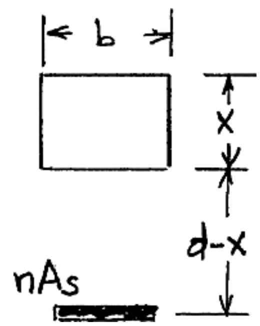
SOLUTION

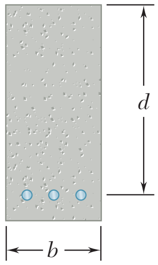
$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

$$x = \frac{d}{1 + \dfrac{E_c \sigma_s}{E_s \sigma_c}}$$





PROBLEM 4.54

For the concrete beam shown, the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel. Knowing that b = 200 mm and d = 450 mm, and using an allowable stress of 12.5 MPa for the concrete and 140 MPa for the steel, determine (*a*) the required area A_s of the steel reinforcement if the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.53 for definition of a balanced beam.)

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200\times 10^9}{25\times 10^9} = 8.0$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.0}\cdot\frac{140\times 10^6}{12.5\times 10^6} = 2.40$$

$$x = 0.41667d = (0.41667)(450) = 187.5 \text{ mm}$$

Locate neutral axis. $\quad bx\frac{x}{2} - nA_s(d-x)$

(*a*) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(200)(187.5)^2}{(2)(8.0)(262.5)} = 1674 \text{ mm}^2$$

$A_s = 1674 \text{ mm}^2$ ◀

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(200)(187.5)^3 + (8.0)(1674)(262.5)^2$$

$$= 1.3623\times 10^9 \text{ mm}^4 = 1.3623\times 10^{-3} \text{ m}^4$$

$$\sigma = \frac{nMy}{I} \qquad M = \frac{I\sigma}{ny}$$

(b) Concrete: $\quad n = 1.0 \quad y = 187.5 \text{ mm} = 0.1875 \text{ m} \quad \sigma = 12.5\times 10^6 \text{ Pa}$

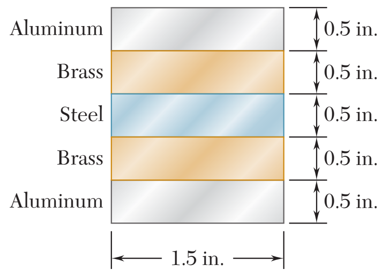
$$M = \frac{(1.3623\times 10^{-3})(12.5\times 10^6)}{(1.0)(0.1875)} = 90.8\times 10^3 \text{ N}\cdot\text{m}$$

Steel: $\quad n = 8.0 \quad y = 262.5 \text{ mm} = 0.2625 \text{ m} \quad \sigma = 140\times 10^6 \text{ Pa}$

$$M = \frac{(1.3623\times 10^{-3})(140\times 10^6)}{(8.0)(0.2625)} = 90.8\times 10^3 \text{ N}\cdot\text{m}$$

Note that both values are the same for balanced design.

$M = 90.8 \text{ kN}\cdot\text{m}$ ◀



PROBLEM 4.55

Five metal strips, each a 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 12 kip·in., determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

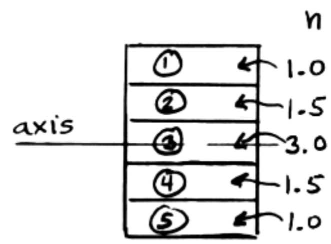
SOLUTION

Use aluminum as the reference material.

$$n = \frac{E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum}$$



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(1.5)(0.5)^3 + (0.75)(1.0)^2$$
$$= 0.7656 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(1.5)(0.5)^3 + (1.5)(0.75)(0.5)^2 = 0.3047 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3.0}{12}(1.5)(0.5)^3 = 0.0469 \text{ in}^4$$

$$I_4 = I_2 = 0.3047 \text{ in}^4 \qquad I_5 = I_1 = 0.7656 \text{ in}^4$$

$$I = \sum_1^5 I_i = 2.1875 \text{ in}^4$$

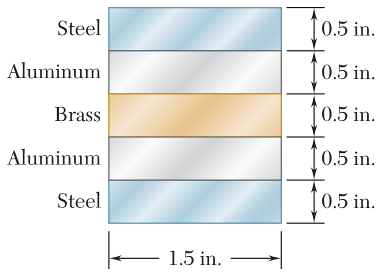
(*a*) Aluminum: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.0)(12)(1.25)}{2.1875} = 6.86 \text{ ksi}$ ◀

Brass: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.5)(12)(0.75)}{2.1875} = 6.17 \text{ ksi}$ ◀

Steel: $\sigma = \dfrac{nMy}{I} = \dfrac{(3.0)(12)(0.25)}{2.1875} = 4.11 \text{ ksi}$ ◀

(*b*) $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{12 \times 10^3}{(10 \times 10^6)(2.1875)} = 548.57 \times 10^{-6} \text{ in.}^{-1}$

$\rho = 1823 \text{ in.} = 151.9 \text{ ft}$ ◀



PROBLEM 4.56

Five metal strips, each a 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 12 kip · in., determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

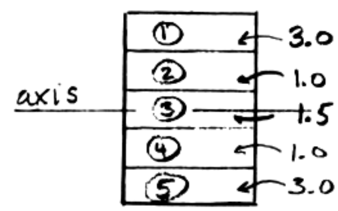
SOLUTION

Use aluminum as the reference material.

$$n = \frac{E_s}{E_a} = \frac{30\times 10^6}{10\times 10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15\times 10^6}{10\times 10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum}$$



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{3.0}{12}(1.5)(0.5)^3 + (3.0)(0.75)(1.0)^2$$

$$= 2.2969 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12}(1.5)(0.5)^3 + (1.0)(0.75)(0.5)^2 = 0.2031 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12}(1.5)(0.5)^3 = 0.0234 \text{ in}^4$$

$$I_4 = I_2 = 0.2031 \text{ in}^4 \qquad I_5 = I_1 = 2.2969 \text{ in}^4$$

$$I = \sum_1^5 I_i = 5.0234 \text{ in}^4$$

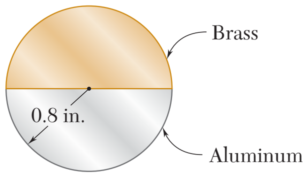
(*a*) Steel: $\sigma = \dfrac{nMy}{I} = \dfrac{(3.0)(12)(1.25)}{5.0234} = 8.96 \text{ ksi}$ ◀

Aluminum: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.0)(12)(0.75)}{5.0234} = 1.792 \text{ ksi}$ ◀

Brass: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.5)(12)(0.25)}{5.0234} = 0.896 \text{ ksi}$ ◀

(*b*) $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{12\times 10^3}{(10\times 10^6)(5.0234)} = 238.89\times 10^{-6} \text{ in.}^{-1}$

$\rho = 4186 \text{ in.} = 349 \text{ ft}$ ◀



PROBLEM 4.57

The composite beam shown is formed by bonding together a brass rod and an aluminum rod of semicircular cross sections. The modulus of elasticity is 15×10^6 psi for the brass and 10×10^6 psi for the aluminum. Knowing that the composite beam is bent about a horizontal axis by couples of moment $8 \text{ kip} \cdot \text{in.}$, determine the maximum stress (*a*) in the brass, (*b*) in the aluminum.

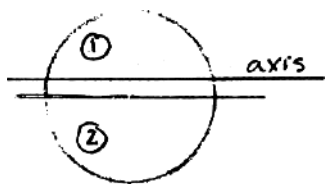
SOLUTION

For each semicircle, $r = 0.8 \text{ in.}$ $A = \dfrac{\pi}{2} r^2 = 1.00531 \text{ in}^2,$

$$\bar{y}_0 = \frac{4r}{3\pi} = \frac{(4)(0.8)}{3\pi} = 0.33953 \text{ in.} \qquad I_{\text{base}} = \frac{\pi}{8} r^4 = 0.160850 \text{ in}^4$$

$$\bar{I} = I_{\text{base}} - A\bar{y}_0^2 = 0.160850 - (1.00531)(0.33953)^2 = 0.044953 \text{ in}^4$$

Use aluminum as the reference material.



$n = 1.0$ in aluminum

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

Locate the neutral axis.

	A, in^2	nA, in^2	$\bar{y}_0$, in.	$nA\bar{y}_0$, in^3
①	1.00531	1.50796	0.33953	0.51200
②	1.00531	1.00531	−0.33953	−0.34133
Σ		2.51327		0.17067

$$\bar{Y}_0 = \frac{0.17067}{2.51327} = 0.06791 \text{ in.}$$

The neutral axis lies 0.06791 in. above the material interface.

$$d_1 = 0.33953 - 0.06791 = 0.27162 \text{ in.}, \quad d_2 = 0.33953 + 0.06791 = 0.40744 \text{ in.}$$

$$I_1 = n_1 \bar{I} + n_1 A d_1^2 = (1.5)(0.044957) + (1.5)(1.00531)(0.27162)^2 = 0.17869 \text{ in}^4$$

$$I_2 = n_2 \bar{I} + n_2 A d_2^2 = (1.0)(0.044957) + (1.0)(1.00531)(0.40744)^2 = 0.21185 \text{ in}^4$$

$$I = I_1 + I_2 = 0.39054 \text{ in}^4$$

(*a*) Brass: $n = 1.5, \quad y = 0.8 - 0.06791 = 0.73209 \text{ in.}$

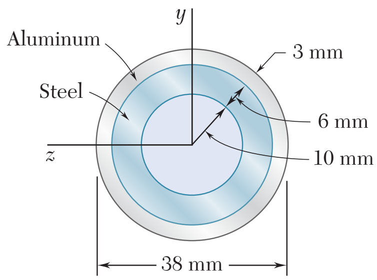
$$\sigma = -\frac{nMy}{I} = -\frac{(1.5)(8)(0.73209)}{0.39054}$$

$\sigma = -22.5 \text{ ksi}$ ◀

(*b*) Aluminium: $n = 1.0, \quad y = -0.8 - 0.06791 = -0.86791 \text{ in.}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(8)(-0.86791)}{0.39054}$$

$\sigma = 17.78 \text{ ksi}$ ◀



PROBLEM 4.58

A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by a couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

SOLUTION

Use aluminum as the reference material.

$$n = 1.0 \quad \text{in aluminum}$$

$$n = E_s / E_a = 200/70 = 2.857 \quad \text{in steel}$$

For the transformed section,

Steel: $$I_s = n_s \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = (2.857)\left(\frac{\pi}{4}\right)(16^4 - 10^4) = 124.62 \times 10^3 \text{ mm}^4$$

Aluminium: $$I_a = n_a \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = (1.0)\left(\frac{\pi}{4}\right)(19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$$

$$I = I_s + I_a = 175.50 \times 10^3 \text{ mm}^4 = 175.5 \times 10^{-9} \text{ m}^4$$

(*a*) Aluminum: $c = 19 \text{ mm} = 0.019 \text{ m}$

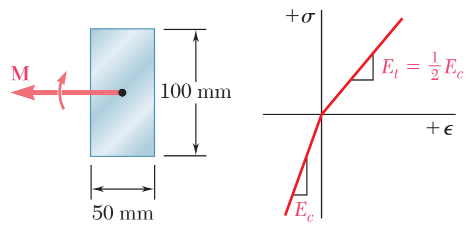
$$\sigma_a = \frac{n_a Mc}{I} = \frac{(1.0)(500)(0.019)}{175.5 \times 10^{-9}} = 54.1 \times 10^6 \text{ Pa}$$

$\sigma_a = 54.1$ MPa ◀

(*b*) Steel: $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\sigma_s = \frac{n_s Mc}{I} = \frac{(2.857)(500)(0.016)}{175.5 \times 10^{-9}} = 130.2 \times 10^6 \text{ Pa}$$

$\sigma_s = 130.2$ MPa ◀



PROBLEM 4.59

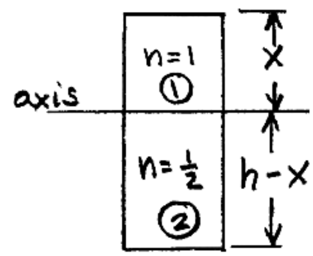
The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600\ \text{N}\cdot\text{m}$, determine the maximum (*a*) tensile stress, (*b*) compressive stress.

SOLUTION

$$n = \frac{1}{2} \text{ on the tension side of neutral axis}$$

$$n = 1 \text{ on the compression side}$$

Locate neutral axis.



$$n_1 bx\frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \qquad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421h = 41.421\ \text{mm}$$

$$h - x = 58.579\ \text{mm}$$

$$I_1 = n_1\frac{1}{3}bx^3 = (1)\left(\frac{1}{3}\right)(50)(41.421)^3 = 1.1844\times10^6\ \text{mm}^4$$

$$I_2 = n_2\frac{1}{3}b(h-x)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(50)(58.579)^3 = 1.6751\times10^6\ \text{mm}^4$$

$$I = I_1 + I_2 = 2.8595\times10^6\ \text{mm}^4 = 2.8595\times10^{-6}\ \text{m}^4$$

(*a*) Tensile stress: $n = \frac{1}{2}, \quad y = -58.579\ \text{mm} = -0.058579\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times10^{-6}} = 6.15\times10^6\ \text{Pa} \qquad \sigma_t = 6.15\ \text{MPa} \blacktriangleleft$$

(*b*) Compressive stress: $n = 1, \quad y = 41.421\ \text{mm} = 0.041421\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times10^{-6}} = -8.69\times10^6\ \text{Pa} \qquad \sigma_c = -8.69\ \text{MPa} \blacktriangleleft$$

PROBLEM 4.60*

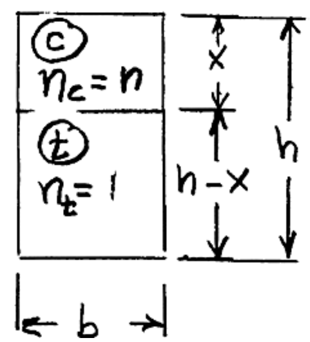
A rectangular beam is made of material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$

SOLUTION



Use E_t as the reference modulus.

Then $E_c = nE_t$.

Locate neutral axis.

$$nbx\frac{x}{2} - b(h-x)\frac{h-x}{2} = 0$$

$$nx^2 - (h-x)^2 = 0 \quad \sqrt{n}x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \quad h - x = \frac{\sqrt{n}h}{\sqrt{n}+1}$$

$$I_{\text{trans}} = \frac{n}{3}bx^3 + \frac{1}{3}b(h-x)^3 = \left[\frac{h}{3}\left(\frac{1}{\sqrt{n}+1}\right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^3\right]bh^3$$

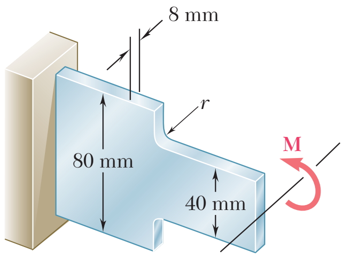
$$= \frac{1}{3}\frac{n+n^{3/2}}{\left(\sqrt{n}+1\right)^3}bh^3 = \frac{1}{3}\frac{n\left(1+\sqrt{n}\right)}{\left(\sqrt{n}+1\right)^3}bh^3 = \frac{1}{3}\times\frac{n}{\left(\sqrt{n}+1\right)^2}bh^3$$

$$\frac{1}{\rho} = \frac{M}{E_t I_{\text{trans}}} = \frac{M}{E_r I} \quad \text{where} \quad I = \frac{1}{12}bh^3$$

$$E_r I = E_t I_{\text{trans}}$$

$$E_r = \frac{E_t I_{\text{trans}}}{I} = \frac{12}{bh^3}\times E_t \times \frac{n}{3\left(\sqrt{n}+1\right)^2}bh^3$$

$$= \frac{4E_t E_c/E_t}{\left(\sqrt{E_c/E_t}+1\right)^2} = \frac{4E_t E_c}{\left(\sqrt{E_c}+\sqrt{E_t}\right)^2}$$



PROBLEM 4.61

Knowing that $M = 250\ \text{N}\cdot\text{m}$, determine the maximum stress in the beam shown when the radius r of the fillets is (*a*) 4 mm, (*b*) 8 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667\times 10^3\ \text{mm}^4 = 42.667\times 10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

(*a*) $\dfrac{r}{d} = \dfrac{4\ \text{mm}}{40\ \text{mm}} = 0.10$ From Fig. 4.27, $K = 1.87$

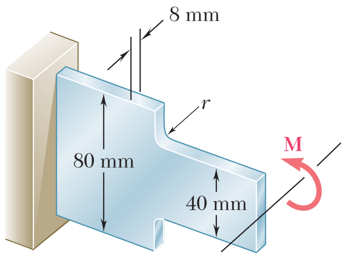
$$\sigma_{\max} = K\frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667\times 10^{-9}} = 219\times 10^6\ \text{Pa}$$

$\sigma_{\max} = 219\ \text{MPa}$ ◀

(*b*) $\dfrac{r}{d} = \dfrac{8\ \text{mm}}{40\ \text{mm}} = 0.20$ From Fig. 4.27, $K = 1.50$

$$\sigma_{\max} = K\frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667\times 10^{-9}} = 176.0\times 10^6\ \text{Pa}$$

$\sigma_{\max} = 176.0\ \text{MPa}$ ◀



PROBLEM 4.62

Knowing that the allowable stress for the beam shown is 90 MPa, determine the allowable bending moment M when the radius r of the fillets is (*a*) 8 mm, (*b*) 12 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

(*a*) $\frac{r}{d} = \frac{8 \text{ mm}}{40 \text{ mm}} = 0.2$ From Fig. 4.27, $K = 1.50$

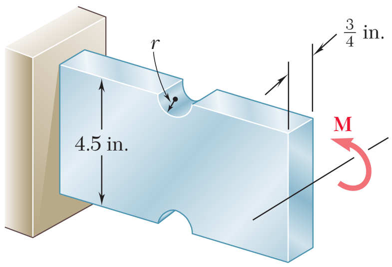
$$\sigma_{\max} = K\frac{Mc}{I} \qquad M = \frac{\sigma_{\max} I}{Kc} = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.50)(0.020)}$$

$M = 128.0 \text{ N} \cdot \text{m}$ ◀

(*b*) $\frac{r}{d} = \frac{12 \text{ mm}}{40 \text{ mm}} = 0.3$ From Fig. 4.27, $K = 1.35$

$$M = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.35)(0.020)}$$

$M = 142.0 \text{ N} \cdot \text{m}$ ◀



PROBLEM 4.63

Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 8 ksi, determine the largest bending moment that can be applied to the member when (*a*) $r = \frac{3}{8}$ in., (*b*) $r = \frac{3}{4}$ in.

SOLUTION

(*a*) $$d = D - 2r = 4.5 - (2)\left(\frac{3}{8}\right) = 3.75 \text{ in.}$$

$$\frac{D}{d} = \frac{4.5}{3.75} = 1.20 \qquad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$$

From Fig. 4.28, $K = 2.07$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)(3.75)^3 = 3.296 \text{ in}^4 \qquad c = \frac{1}{2} = 1.875 \text{ in.}$$

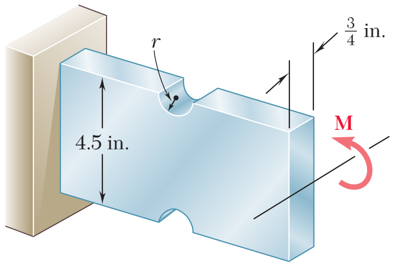
$$\sigma = K\frac{Mc}{I} \qquad \therefore \quad M = \frac{\sigma I}{Kc} = \frac{(8)(3.296)}{(2.07)(1.875)} = 6.79 \text{ kip} \cdot \text{in.}$$

$$\sigma = 6.79 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

(*b*) $$d = D - 2r = 4.5 - (2)\left(\frac{3}{4}\right) = 3.0 \qquad \frac{D}{d} = \frac{4.5}{3.0} = 1.5 \qquad \frac{r}{d} = \frac{0.75}{3.0} = 0.25$$

From Fig. 4.28, $K = 1.61$ $$I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)(3.0)^3 = 1.6875 \text{ in}^4$$

$$c = \frac{1}{2}d = 1.5 \text{ in.} \qquad M = \frac{\sigma I}{Kc} = \frac{(8)(1.6875)}{(1.61)(1.5)} = 5.59 \text{ kip} \cdot \text{in.} \blacktriangleleft$$



PROBLEM 4.64

Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 4$ kip · in., determine the maximum stress in the member when the radius r of the semicircular grooves is (*a*) $r = \frac{3}{8}$ in., (*b*) $r = \frac{3}{4}$ in.

SOLUTION

(*a*)

$$d = D - 2r = 4.5 - (2)\left(\frac{3}{8}\right) = 3.75 \text{ in.}$$

$$\frac{D}{d} = \frac{4.5}{3.75} = 1.20 \qquad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$$

From Fig. 4.28, $\quad K = 2.07$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)(3.75)^3 = 3.296 \text{ in}^4 \qquad c = \frac{1}{2} = 1.875 \text{ in.}$$

$$\sigma = K\frac{Mc}{I} = \frac{(2.07)(4)(1.875)}{3.296} = 4.71 \text{ ksi}$$

$\sigma = 4.71$ ksi ◀

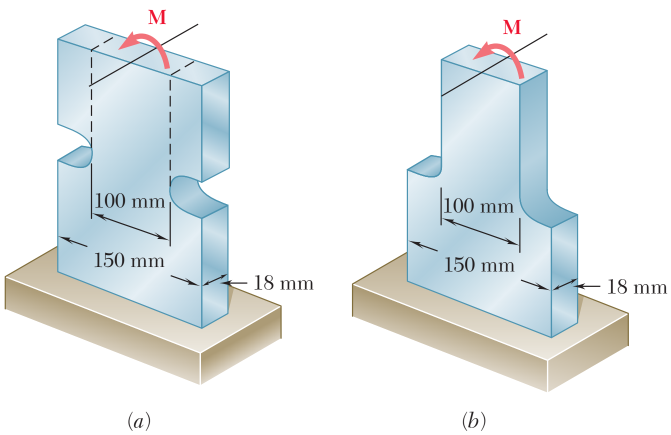
(*b*) $\quad d = D - 2r = 4.5 - (2)\left(\frac{3}{4}\right) = 3.00$ in. $\quad \frac{D}{d} = \frac{4.5}{3.00} = 1.50 \quad \frac{r}{d} = \frac{0.75}{3.0} = 0.25$

From Fig. 4.28,

$K = 1.61 \qquad I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)(3.00)^3 = 1.6875 \text{ in}^4 \qquad c = \frac{1}{2}d = 1.5$ in.

$$\sigma = K\frac{Mc}{I} = \frac{(1.61)(4)(1.5)}{1.6875} = 5.72 \text{ ksi}$$

$\sigma = 5.72$ ksi ◀



PROBLEM 4.65

A couple of moment $M = 2$ kN · m is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 10$ mm, as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material to the left and right of the dashed lines as shown in Fig. *b*.

SOLUTION

For both configurations,

$$D = 150 \text{ mm}$$
$$d = 100 \text{ mm}$$
$$r = 10 \text{ mm}$$
$$\frac{D}{d} = \frac{150}{100} = 1.50$$
$$\frac{r}{d} = \frac{10}{100} = 0.10$$

For configuration (*a*),

Fig. 4.28 gives $K_a = 2.21.$

For configuration (*b*), Fig. 4.27 gives $K_b = 1.79.$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

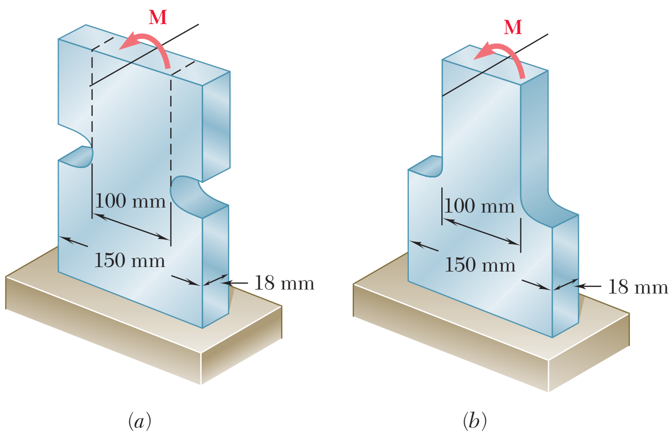
$$c = \frac{1}{2}d = 50 \text{ mm} = 0.05 \text{ m}$$

(*a*) $$\sigma = \frac{KMc}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147.0 \times 10^6 \text{ Pa} = 147.0 \text{ MPa}$$

$\sigma = 147.0$ MPa ◀

(*b*) $$\sigma = \frac{KMc}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119.0 \times 10^6 \text{ Pa} = 119.0 \text{ MPa}$$

$\sigma = 119.0$ MPa ◀



PROBLEM 4.66

The allowable stress used in the design of a steel bar is 80 MPa. Determine the largest couple **M** that can be applied to the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 15$ mm, as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material to the left and right of the dashed lines as shown in Fig. *b*.

SOLUTION

For both configurations,

$$D = 150 \text{ mm} \qquad d = 100 \text{ mm}$$
$$r = 15 \text{ mm}$$
$$\frac{D}{d} = \frac{150}{100} = 1.50$$
$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (*a*), Fig. 4.28 gives $K_a = 1.92$.

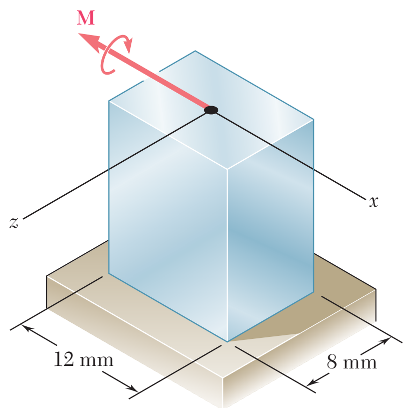
For configuration (*b*), Fig. 4.27 gives $K_b = 1.57$.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$
$$c = \frac{1}{2}d = 50 \text{ mm} = 0.050 \text{ m}$$

(*a*) $\quad \sigma = \dfrac{KMc}{I} \quad \therefore \quad M = \dfrac{\sigma I}{Kc} = \dfrac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.92) \times (0.05)} = 1.250 \times 10^3 \text{ N} \cdot \text{m}$

$$= 1.250 \text{ kN} \cdot \text{m} \qquad ◀$$

(*a*) $\quad M = \dfrac{\sigma I}{Kc} = \dfrac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.57)(0.050)} = 1.530 \times 10^3 \text{ N} \cdot \text{m} = 1.530 \text{ kN} \cdot \text{m} \qquad ◀$



PROBLEM 4.67

The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the *x* axis. Determine the moment *M* of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

SOLUTION

(*a*) $I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(8)^3 = 512 \text{ mm}^4$

$$= 512 \times 10^{-12} \text{ m}^4$$

$$c = \frac{1}{2}h = 4 \text{ mm} = 0.004 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(512 \times 10^{-12})}{0.004}$$

$$= 38.4 \text{ N} \cdot \text{m}$$

$M_Y = 38.4 \text{ N} \cdot \text{m}$ ◀

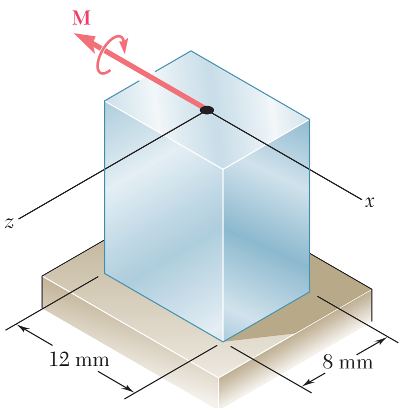
(*b*) $y_Y = \frac{1}{2}(4) = 2 \text{ mm} \qquad \frac{y_Y}{c} = \frac{2}{4} = 0.5$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(38.4)\left[1 - \frac{1}{3}(0.5)^2\right]$$

$$= 52.8 \text{ N} \cdot \text{m}$$

$M = 52.8 \text{ N} \cdot \text{m}$ ◀



PROBLEM 4.68

Solve Prob. 4.67, assuming that the couple **M** is parallel to the z axis.

PROBLEM 4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the x axis. Determine the moment M of the couple for which (a) yield first occurs, (b) the elastic core of the bar is 4 mm thick.

SOLUTION

(a) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(12)^3 = 1.152 \times 10^3 \text{mm}^4$$

$$= 1.152 \times 10^{-9}\ \text{m}^4$$

$$c = \frac{1}{2}h = 6\ \text{mm} = 0.006\ \text{m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(1.152 \times 10^{-9})}{0.006}$$

$$= 57.6\ \text{N} \cdot \text{m}$$

$M_Y = 57.6\ \text{N} \cdot \text{m}$ ◀

(b) $$y_Y = \frac{1}{2}(4) = 2\ \text{mm} \qquad \frac{y_Y}{c} = \frac{2}{6} = \frac{1}{3}$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(57.6)\left[1 - \frac{1}{3}\left(\frac{1}{3}\right)^2\right]$$

$$= 83.2\ \text{N} \cdot \text{m}$$

$M = 83.2\ \text{N} \cdot \text{m}$ ◀

PROBLEM 4.69

A solid square rod of side 0.6 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 48$ ksi. Knowing that a couple **M** is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is 6 ft.

SOLUTION

$$I = \frac{1}{12}(0.6)(0.6)^3 = 10.8 \times 10^{-3}\ \text{in}^4 \qquad c = \frac{1}{2}h = 0.3\ \text{in.}$$

$$M_Y = \frac{I\sigma_Y}{c} = \frac{(10.8 \times 10^{-3})(48 \times 10^3)}{0.3} = 1728\ \text{lb} \cdot \text{in.} \qquad \rho = 6\ \text{ft} = 72\ \text{in.}$$

$$\varepsilon_{\max} = \frac{c}{\rho} = \frac{0.3}{72} = 4.16667 \times 10^{-3} \qquad \varepsilon_Y = \frac{\sigma_Y}{E} = \frac{48 \times 10^3}{29 \times 10^6} = 1.65517 \times 10^{-3}$$

$$\frac{y_Y}{c} = \frac{\varepsilon_Y}{\varepsilon_M} = \frac{1.65517 \times 10^{-3}}{4.16667 \times 10^{-3}} = 0.39724$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{Y_Y}{c}\right)^2\right] = \frac{3}{2}(1728)\left[1 - \frac{1}{3}(0.39724)^2\right]$$

$M = 2460\ \text{lb} \cdot \text{in.}$ ◀

PROBLEM 4.70

For the solid square rod of Prob. 4.69, determine the moment M for which the radius of curvature is 3 ft.

PROBLEM 4.69. A solid square rod of side 0.6 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 48$ ksi. Knowing that a couple **M** is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is 6 ft.

SOLUTION

$$I = \frac{1}{12}(0.6)(0.6)^3 = 10.8 \times 10^{-3} \text{ in}^4 \qquad c = \frac{1}{2}h = 0.3 \text{ in.}$$

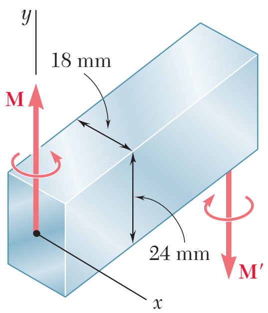
$$M_Y = \frac{I\sigma_Y}{c} = \frac{(10.8 \times 10^{-3})(48 \times 10^3)}{0.3} = 1728 \text{ lb} \cdot \text{in.} \qquad \rho = 3 \text{ ft} = 36 \text{ in.}$$

$$\varepsilon_{\max} = \frac{c}{\rho} = \frac{0.3}{36} = 8.3333 \times 10^{-3} \qquad \varepsilon_Y = \frac{\sigma_Y}{E} = \frac{48 \times 10^3}{29 \times 10^6} = 1.65517 \times 10^{-3}$$

$$\frac{y_Y}{c} = \frac{\varepsilon_Y}{\varepsilon_{\max}} = \frac{1.65517 \times 10^{-3}}{8.3333 \times 10^{-3}} = 0.19862$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{Y_Y}{c}\right)^2\right] = \frac{3}{2}(1728)\left[1 - \frac{1}{3}(0.19862)^2\right]$$

$M = 2560 \text{ lb} \cdot \text{in.}$ ◀



PROBLEM 4.71

The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 280$ MPa. Knowing that couples **M** and **M′** of moment 525 N · m are applied and maintained about axes parallel to the y axis, determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(24)(18)^3 = 11.664 \times 10^3 \text{ mm}^4 = 11.664 \times 10^{-3} \text{ m}^4$$

$$c = \frac{1}{2}h = 9 \text{ mm} = 0.009 \text{ m}$$

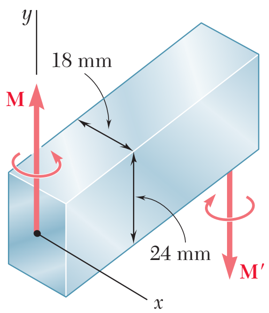
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(280 \times 10^6)(11.664 \times 10^{-9})}{0.009} = 362.88 \text{ N} \cdot \text{m}$$

$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y{}^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

$$\frac{y_Y}{c} = \sqrt{3 - \frac{(2)(525)}{362.88}} = 0.32632 \qquad y_Y = 0.32632c = 2.9368 \text{ mm}$$

(a) $\qquad t_{\text{core}} = 2y_Y = 5.87 \text{ mm}$ ◀

(b) $\qquad \varepsilon_Y = \dfrac{y_Y}{\rho} = \dfrac{\sigma_Y}{E} \qquad \therefore \quad \rho = \dfrac{Ey_Y}{\sigma_Y} = \dfrac{(200 \times 10^9)(2.9368 \times 10^{-3})}{280 \times 10^6} = 2.09 \text{ m}$ ◀



PROBLEM 4.72

Solve Prob. 4.71, assuming that the couples **M** and **M′** are applied and maintained about axes parallel to the x axis.

PROBLEM 4.71 The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 280$ MPa. Knowing that couples **M** and **M′** of moment 525 N · m are applied and maintained about axes parallel to the y axis, determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(24)^3 = 20.736\times 10^3 \text{ mm}^4 = 20.736\times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}h = 12 \text{ mm} = 0.012 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(280\times 10^6)(20.736\times 10^{-9})}{0.012} = 483.84 \text{ N}\cdot\text{m}$$

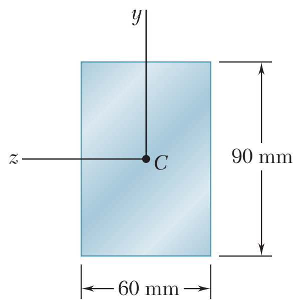
$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y{}^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

$$\frac{y_Y}{c} = \sqrt{3 - \frac{(2)(525)}{483.84}} = 0.91097 \qquad y_Y = 0.91097 \qquad c = 10.932 \text{ mm}$$

(a) $\qquad t_{\text{core}} = 2y_Y = 21.9 \text{ mm}$ ◀

(b) $\qquad \varepsilon_Y = \frac{y_Y}{\rho} = \frac{\sigma_Y}{E} \qquad \therefore \quad \rho = \frac{Ey_Y}{\sigma_Y} = \frac{(200\times 10^9)(10.932\times 10^{-3})}{280\times 10^6} = 7.81 \text{ m}$ ◀

PROBLEM 4.73



A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30 mm thick.

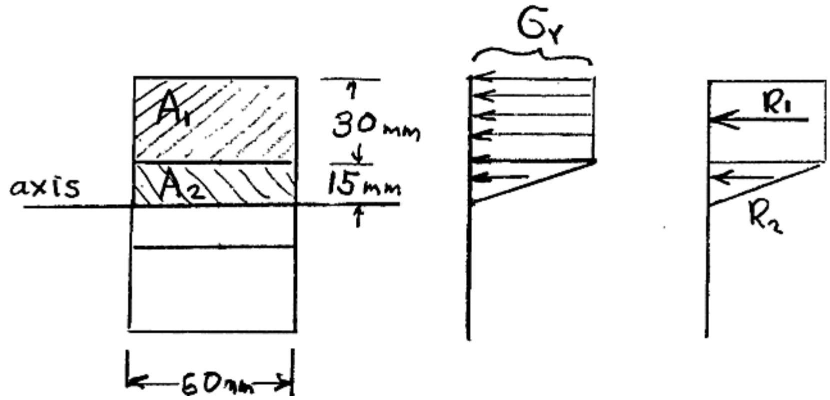
SOLUTION

(*a*) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10 \text{ N} \cdot \text{m}$$

$M_Y = 19.44 \text{ kN} \cdot \text{m}$ ◀

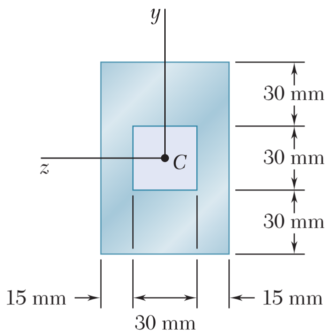


$$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.060)(0.030)$$
$$= 432 \times 10^3 \text{ N}$$
$$y_1 = 15 \text{ mm} + 15 \text{ mm} = 0.030 \text{ m}$$
$$R_2 = \frac{1}{2}\sigma_Y A_2 = \left(\frac{1}{2}\right)(240 \times 10^6)(0.060)(0.015)$$
$$= 108 \times 10^3 \text{ N}$$
$$y_2 = \frac{2}{3}(15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$$

(*b*) $$M = 2(R_1 y_1 + R_2 y_2) = 2[(432 \times 10^3)(0.030) + (108 \times 10^3)(0.010)]$$

$$= 28.08 \times 10^3 \text{ N} \cdot \text{m}$$

$M = 28.1 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 4.74

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 30 mm thick.

SOLUTION

(a) $I_{\text{rect}} = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4$

$I_{\text{cutout}} = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$

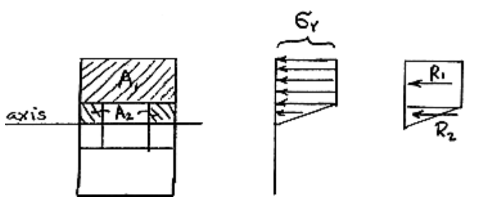
$I = 3.645 \times 10^6 - 67.5 \times 10^3 = 3.5775 \times 10^6 \text{ mm}^4$

$= 3.5775 \times 10^{-6} \text{ mm}^4$

$c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.5775 \times 10^{-6})}{0.045}$

$= 19.08 \times 10^3 \text{ N} \cdot \text{m}$ $\qquad M_Y = 19.08 \text{ kN} \cdot \text{m}$ ◀



$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.060)(0.030) = 432 \times 10^3 \text{ N}$

$y_1 = 15 \text{ mm} + 15 \text{ mm} = 30 \text{ mm} = 0.030 \text{ m}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(240 \times 10^6)(0.030)(0.015) = 54 \times 10^3 \text{ N}$

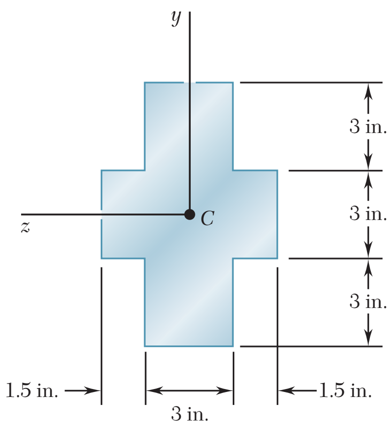
$y_2 = \frac{2}{3}(15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$

(b) $M = 2(R_1 y_1 + R_2 y_2)$

$= 2[(432 \times 10^3)(0.030) + (54 \times 10^3)(0.010)]$

$= 27.00 \times 10^3 \text{ N} \cdot \text{m}$ $\qquad M = 27.0 \text{ kN} \cdot \text{m}$ ◀

PROBLEM 4.75



A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 3 in. thick.

SOLUTION

(a) $$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$$

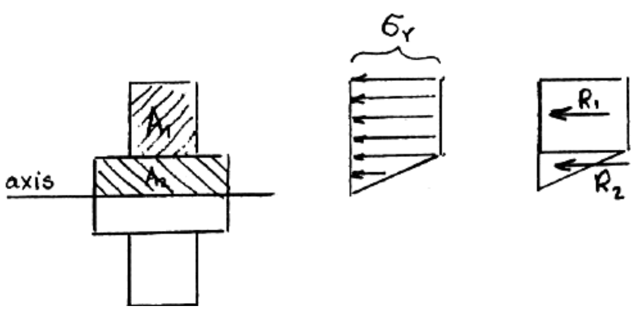
$$I_3 = I_1 = 87.75 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$$

$$c = 4.5 \text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5}$$

$M_Y = 1759$ kip · in. ◀



$$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378 \text{ kip}$$

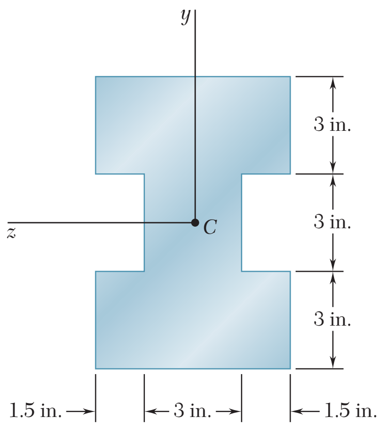
$$y_1 = 1.5 + 1.5 = 3.0 \text{ in.}$$

$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(6)(1.5) = 189 \text{ kip}$$

$$y_2 = \frac{2}{3}(1.5) = 1.0 \text{ in.}$$

(b) $$M = 2(R_1y_1 + R_2y_2) = 2[(378)(3.0) + (189)(1.0)] = 2646 \text{ kip}\cdot\text{in.}$$

$M = 2650$ kip · in. ◀



PROBLEM 4.76

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 3 in. thick.

SOLUTION

(a) $$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75 \text{ in}^4$$

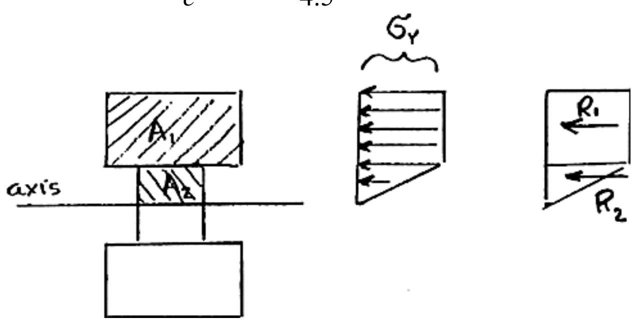
$$I_3 = I_1 = 175.5 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$$

$$c = 4.5 \text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339 \text{ kip} \cdot \text{in.}$$

$M_Y = 3340 \text{ kip} \cdot \text{in.}$ ◀



$$R_1 = \sigma_Y A_1 = (42)(6)(3) = 756 \text{ kip}$$

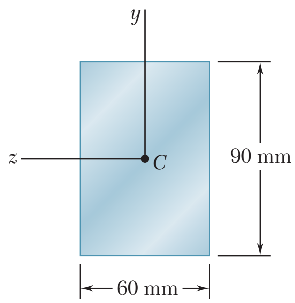
$$y_1 = 1.5 + 1.5 = 3 \text{ in.}$$

$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(3)(1.5) = 94.5 \text{ kip}$$

$$y_2 = \frac{2}{3}(1.5) = 1.0 \text{ in.}$$

(b) $$M = 2(R_1y_1 + R_2y_2) = 2[(756)(3) + (94.5)(1.0)] = 4725 \text{ kip} \cdot \text{in.}$$

$M = 4730 \text{ kip} \cdot \text{in.}$ ◀



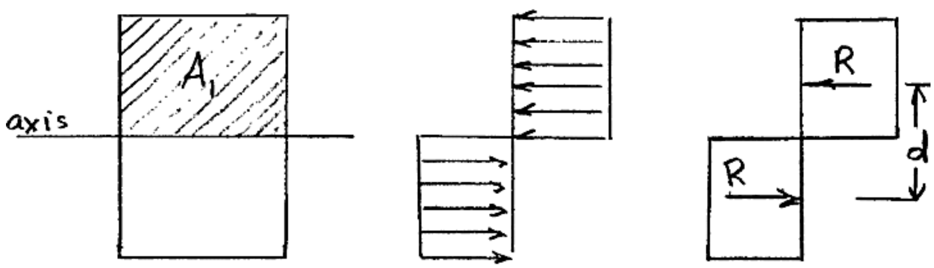
PROBLEM 4.77

For the beam indicated (of Prob. 4.73), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

From Problem 4.73, $E = 200 \text{ GPa}$ and $\sigma_Y = 240 \text{ MPa}$

$$A_1 = (60)(45) = 2700 \text{ mm}^2$$
$$= 2700 \times 10^{-6} \text{ m}^2$$
$$R = \sigma_Y A_1$$
$$= (240 \times 10^6)(2700 \times 10^{-6})$$
$$= 648 \times 10^3 \text{ N}$$
$$d = 45 \text{ mm} = 0.045 \text{ m}$$



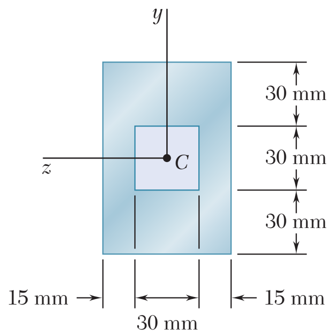
(*a*) $M_p = Rd = (648 \times 10^3)(0.045) = 29.16 \times 10^3 \text{ N} \cdot \text{m}$ $\qquad M_p = 29.2 \text{ kN} \cdot \text{m}$ ◀

(*b*) $I = \frac{1}{12} bh^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$

$c = 45 \text{ mm} = 0.045 \text{ m}$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10^3 \text{ N} \cdot \text{m}$$

$$k = \frac{M_p}{M_Y} = \frac{29.16}{19.44}$$ $\qquad k = 1.500$ ◀



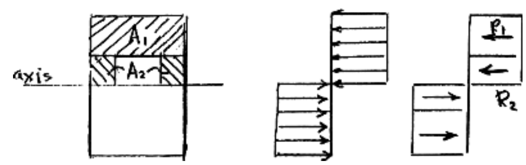
PROBLEM 4.78

For the beam indicated (of Prob. 4.74), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

From Problem 4.74, $E = 200\text{ GPa}$ and $\sigma_Y = 240\text{ MPa}$

(*a*)
$$R_1 = \sigma_Y A_1 = (240\times10^6)(0.060)(0.030) = 432\times10^3\text{ N}$$
$$y_1 = 15\text{ mm} + 15\text{ mm} = 30\text{ mm} = 0.030\text{ m}$$



$$R_2 = \sigma_Y A_2 = (240\times10^6)(0.030)(0.015) = 108\times10^3\text{ N}$$
$$y_2 = \frac{1}{2}(15) = 7.5\text{ mm} = 0.0075\text{ m}$$

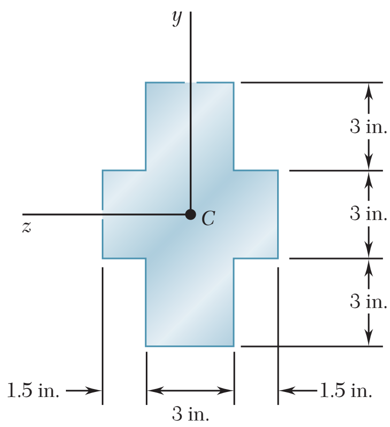
$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(432\times10^3)(0.030) + (108\times10^3)(0.0075)]$$
$$= 27.54\times10^3\text{ N}\cdot\text{m}$$

$M_p = 27.5\text{ kN}\cdot\text{m}$ ◀

(*b*)
$$I_{\text{rect}} = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6\text{ mm}^4$$
$$I_{\text{cutout}} = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67.5\times10^3\text{ mm}^4$$
$$I = I_{\text{rect}} - I_{\text{cutout}} = 3.645\times10^6 - 67.5\times10^3 = 3.5775\times10^3\text{ mm}^4$$
$$= 3.5775\times10^{-9}\text{ m}^4$$
$$c = \frac{1}{2}h = 45\text{ mm} = 0.045\text{ m}$$
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.5775\times10^{-9})}{0.045} = 19.08\times10^3\text{ N}\cdot\text{m}$$
$$k = \frac{M_p}{M_Y} = \frac{27.54}{19.08}$$

$k = 1.443$ ◀

PROBLEM 4.79



For the beam indicated (of Prob. 4.75), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

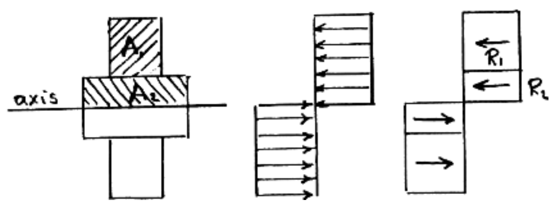
From Problem 4.75, $E = 29\times 10^6$ psi and $\sigma_Y = 42$ ksi.

(*a*) $R_1 = \sigma_Y A_1 = (42)(3)(3) = 378$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in.

$R_2 = \sigma_Y A_2 = (42)(6)(1.5) = 378$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.



$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (378)(0.75)]$ $\qquad M_p = 2840$ kip·in. ◀

(*b*) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$

$I_3 = I_1 = 87.75 \text{ in}^4$

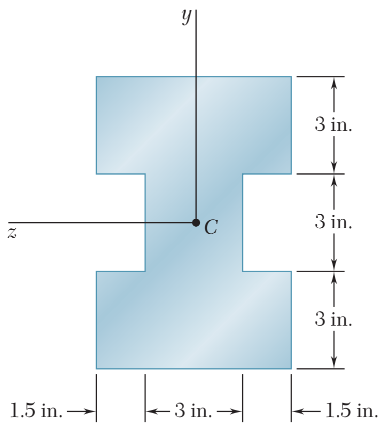
$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$

$c = 4.5$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759.3$ kip·in

$k = \frac{M_p}{M_Y} = \frac{2835}{1759.3}$ $\qquad k = 1.611$ ◀

PROBLEM 4.80



For the beam indicated (of Prob. 4.76), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

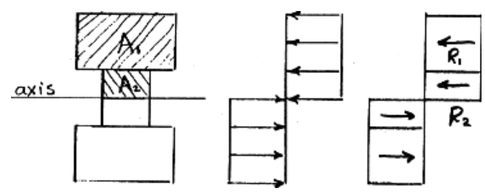
From Problem 4.76, $E = 29 \times 10^6$ and $\sigma_Y = 42$ ksi

(*a*) $R_1 = \sigma_Y A_1 = (42)(6)(3) = 756$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in.

$R_2 = \sigma_Y A_2 = (42)(3)(1.5) = 189$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.



$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3.0) + (189)(0.75)]$ $\qquad M_p = 4820$ kip · in. ◀

(*b*) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75 \text{ in}^4$

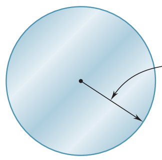
$I_3 = I_1 = 175.5 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$

$c = 4.5$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339$ kip · in.

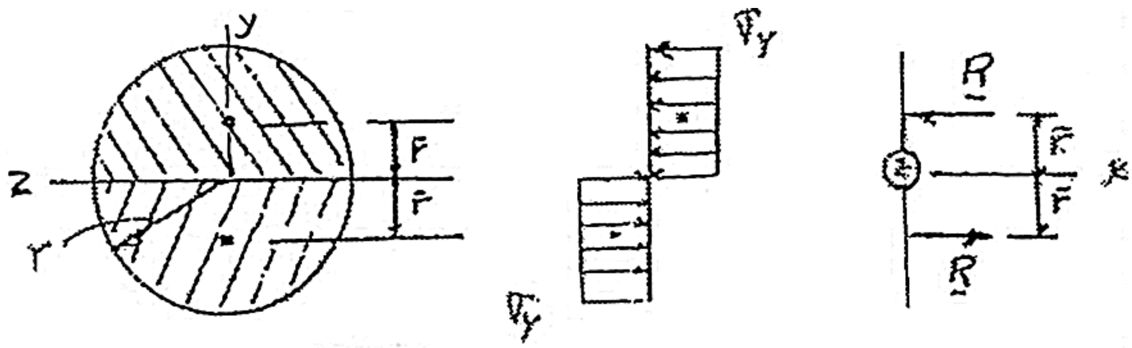
$k = \frac{M_p}{M_Y} = \frac{4819.5}{3339}$ $\qquad k = 1.443$ ◀



PROBLEM 4.81

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION



For a semicircle, $A = \dfrac{\pi}{2} r^2; \qquad \bar{r} = \dfrac{4r}{3\pi}$

Resultant force on semicircular section: $R = \sigma_Y A$

Resultant moment on entire cross section:

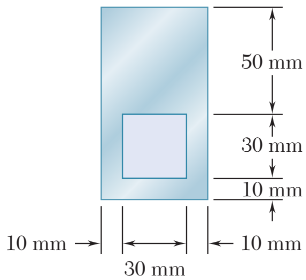
$$M_p = 2R\bar{r} = \frac{4}{3}\sigma_Y r^3$$

Data: $\sigma_Y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}, \quad r = 18 \text{ mm} = 0.018 \text{ m}$

$$M_p = \frac{4}{3}(240 \times 10^6)(0.018)^3 = 1866 \text{ N} \cdot \text{m}$$

$M_p = 1.866 \text{ kN} \cdot \text{m}$ ◀

PROBLEM 4.82



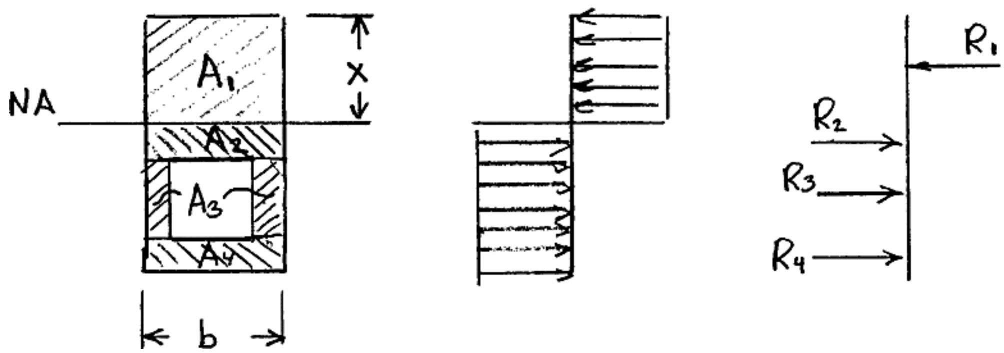
Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area: $$A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$$

$$\frac{1}{2}A = 1800 \text{ mm}^2$$

$$x = \frac{\frac{1}{2}A}{b} = \frac{1800}{50} = 36 \text{ mm}$$



$$A_1 = (50)(36) = 1800 \text{ mm}^2, \quad \bar{y}_1 = 18 \text{ mm}, \quad A_1\bar{y}_1 = 32.4 \times 10^3 \text{ mm}^3$$

$$A_2 = (50)(14) = 700 \text{ mm}^2, \quad \bar{y}_2 = 7 \text{ mm}, \quad A_2\bar{y}_2 = 4.9 \times 10^3 \text{ mm}^3$$

$$A_3 = (20)(30) = 600 \text{ mm}^2, \quad \bar{y}_3 = 29 \text{ mm}, \quad A_3\bar{y}_3 = 17.4 \times 10^3 \text{ mm}^3$$

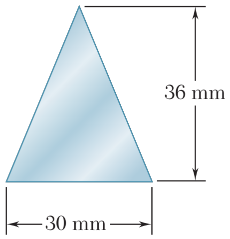
$$A_4 = (50)(10) = 500 \text{ mm}^2, \quad \bar{y}_4 = 49 \text{ mm}, \quad A_4\bar{y}_4 = 24.5 \times 10^3 \text{ mm}^3$$

$$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 = 79.2 \times 10^3 \text{ mm}^3 = 79.2 \times 10^{-6} \text{ m}^3$$

$$M_p = \sigma_Y \Sigma A_i \bar{y}_i = (240 \times 10^6)(79.2 \times 10^{-6}) = 19.008 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_p = 19.01 \text{ kN} \cdot \text{m} \blacktriangleleft$$

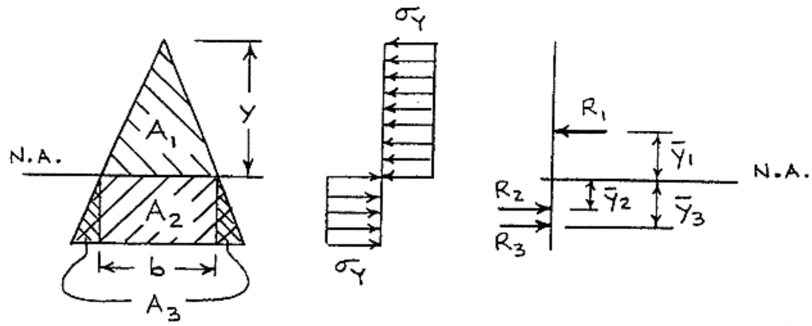
PROBLEM 4.83



Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area: $A = \frac{1}{2}(30)(36) = 540\ \text{mm}^2$ Half area: $\frac{1}{2}A = 270\ \text{mm}^2 = A_1$



By similar triangles, $\frac{b}{y} = \frac{30}{36} \qquad b = \frac{5}{6}y$

Since $A_1 = \frac{1}{2}by = \frac{5}{12}y^2, \qquad y^2 = \frac{12}{5}A_1$

$$y = \sqrt{\frac{12}{5}(270)} = 25.4558\ \text{mm}$$

$$b = 21.2132\ \text{mm}$$

$$A_1 = \frac{1}{2}(21.2132)(25.4558) = 270\ \text{mm}^2 = 270 \times 10^{-6}\ \text{m}^2$$

$$A_2 = (21.2132)(36 - 25.4558) = 223.676\ \text{mm}^2 = 223.676 \times 10^{-6}\ \text{m}^2$$

$$A_3 = A - A_1 - A_2 = 46.324\ \text{mm}^2 = 46.324 \times 10^{-6}\ \text{m}^2$$

$$R_i = \sigma_Y A_i = 240 \times 10^6 A_i$$

$$R_1 = 64.8 \times 10^3\ \text{N}, \quad R_2 = 53.6822 \times 10^3\ \text{N}, \quad R_3 = 11.1178 \times 10^3\ \text{N}$$

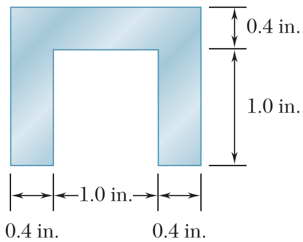
$$\bar{y}_1 = \frac{1}{3}y = 8.4853\ \text{mm} = 8.4853 \times 10^{-3}\ \text{m}$$

$$\bar{y}_2 = \frac{1}{2}(36 - 25.4558) = 5.2721\ \text{mm} = 5.2721 \times 10^{-3}\ \text{m}$$

$$\bar{y}_3 = \frac{2}{3}(36 - 25.4558) = 7.0295\ \text{mm} = 7.0295 \times 10^{-3}\ \text{m}$$

$$M_p = R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3 = 911\ \text{N} \cdot \text{m}$$

$M_p = 911\ \text{N} \cdot \text{m}$ ◀



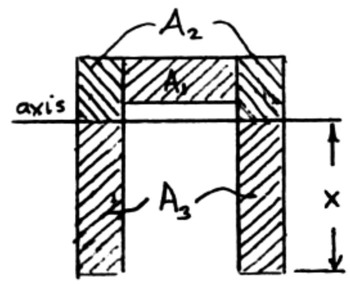
PROBLEM 4.84

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 42 ksi.

SOLUTION

$$A = (1.0\text{ in.})(0.4\text{ in.}) + 2(1.4\text{ in.})(0.4\text{ in.}) = 1.52\text{ in}^2$$

$$x = \frac{\frac{1}{2}(1.52)}{2(0.4)} = 0.95\text{ in.}$$



$$R_1 = (1.0)(0.4) = 0.4\text{ in}^2$$

$$\bar{y}_1 = 1.2 - 0.95 = 0.25\text{ in.}$$

$$R_2 = 2(0.4)(1.4 - 0.95) = 0.36\text{ in}^2$$

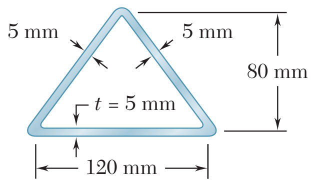
$$\bar{y}_2 = \frac{1}{2}(1.4 - 0.95) = 0.225\text{ in.}$$

$$R_3 = 2(0.4)(0.95) = 0.760\text{ in}^2$$

$$\bar{y}_3 = \frac{1}{2}(0.95) = 0.475\text{ in.}$$

$$M_p = (R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3)(\sigma_y)$$
$$= [(0.4)(0.25) + (0.36)(0.225) + (0.760)(0.475)](42)$$

$$M_p = 22.8\text{ kip}\cdot\text{in.} \blacktriangleleft$$

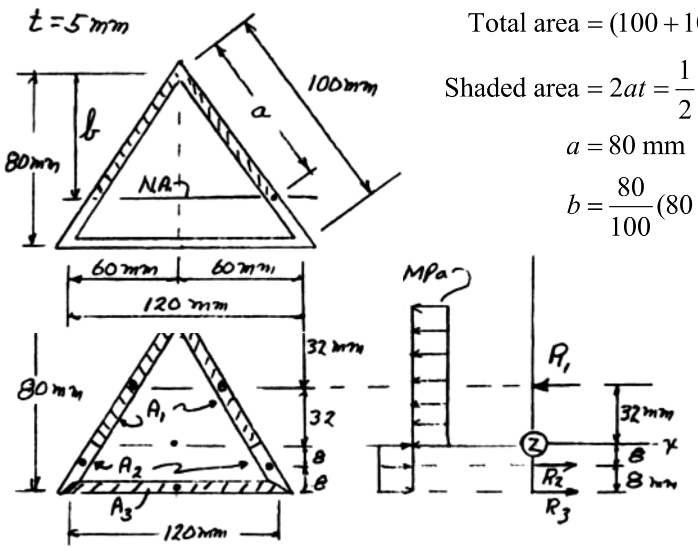


PROBLEM 4.85

Determine the plastic moment M_p of the cross section shown when the beam is bent about a horizontal axis. Assume the material to be elastoplastic with a yield strength of 175 MPa.

SOLUTION

For M_p, the neutral axis divides the area into two equal parts.



$$\text{Total area} = (100+100+120)t = 320t$$

$$\text{Shaded area} = 2at = \frac{1}{2}(320)t$$

$$a = 80 \text{ mm}$$

$$b = \frac{80}{100}(80 \text{ mm}) = 64 \text{ mm}$$

$$A_1 = 2at = 2(0.08 \text{ m})(0.005 \text{ m}) = 800\times10^{-6} \text{ m}^2$$

$$A_2 = 2(100-a)t = 2(0.02 \text{ m})(0.005 \text{ m}) = 200\times10^{-6} \text{ m}^2$$

$$A_3 = (120 \text{ mm})t = (0.120 \text{ m})(0.005 \text{ m}) = 600\times10^{-6} \text{ m}^2$$

$$R_1 = \sigma_Y A_1 = (175 \text{ MPa})(800\times10^{-6} \text{ m}^2) = 140 \text{ kN}$$

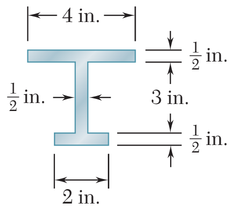
$$R_2 = \sigma_Y A_2 = (175 \text{ MPa})(200\times10^{-6} \text{ m}^2) = 35 \text{ kN}$$

$$R_3 = \sigma_Y A_3 = (175 \text{ MPa})(600\times10^{-6} \text{ m}^2) = 105 \text{ kN}$$

$$\sum M_z : M_p = R_1(32 \text{ mm}) + R_2(8 \text{ mm}) + R_3(16 \text{ mm})$$

$$= (140 \text{ kN})(0.032 \text{ m}) + (35 \text{ kN})(0.008 \text{ m}) + (105 \text{ kN})(0.016 \text{ m})$$

$$M_p = 6.44 \text{ kN}\cdot\text{m} \blacktriangleleft$$



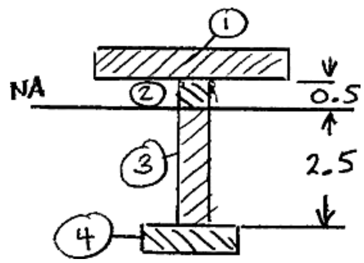
PROBLEM 4.86

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.

SOLUTION

Total area: $$A = (4)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)(3) + (2)\left(\frac{1}{2}\right) = 4.5 \text{ in}^2$$

$$\frac{1}{2}A = 2.25 \text{ in}^2$$

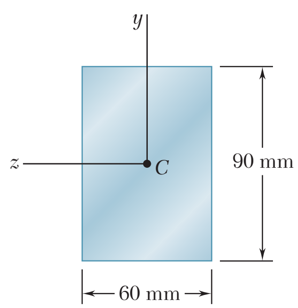


$$A_1 = 2.00 \text{ in}^2, \quad \bar{y}_1 = 0.75, \quad A_1 y_1 = 1.50 \text{ in}^3$$
$$A_2 = 0.25 \text{ in}^2, \quad \bar{y}_2 = 0.25, \quad A_2 \bar{y}_2 = 0.0625 \text{ in}^3$$
$$A_3 = 1.25 \text{ in}^2, \quad \bar{y}_3 = 1.25, \quad A_3 \bar{y}_3 = 1.5625 \text{ in}^3$$
$$A_4 = 1.00 \text{ in}^2, \quad \bar{y}_4 = 2.75, \quad A_4 \bar{y}_4 = 2.75 \text{ in}^3$$

$$M_p = \sigma_Y (A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4)$$
$$= (36)(1.50 + 0.0625 + 1.5625 + 2.75)$$

$M_p = 212 \text{ kip} \cdot \text{in.}$ ◀

PROBLEM 4.87



For the beam indicated (of Prob. 4.73), a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45\,\text{mm}$.

SOLUTION

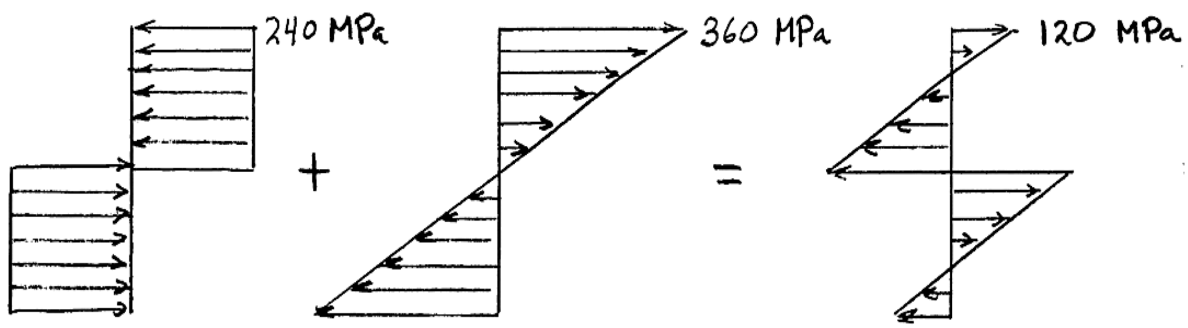
$$M_p = 29.16 \times 10^3\ \text{N} \cdot \text{m}$$

See solutions to Problems 4.73 and 4.77.

$$I = 3.645 \times 10^{-6}\ \text{m}^4$$

$$c = 0.045\ \text{m}$$

$$\sigma' = \frac{M_{\max} y}{I} = \frac{M_p c}{I} \qquad \text{at } y = c = 45\ \text{mm}$$

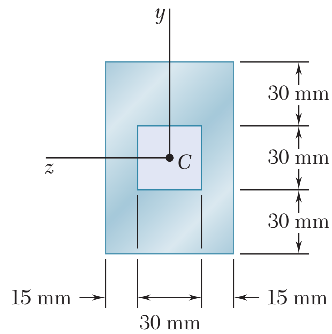


$$\sigma' = \frac{(29.16 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 360 \times 10^6\ \text{Pa}$$

$$\sigma_{\text{res}} = \sigma' - \sigma_Y = 360 \times 10^6 - 240 \times 10^6$$

$$= 120 \times 10^6\ \text{Pa} \qquad \sigma_{\text{res}} = 120.0\ \text{MPa} \blacktriangleleft$$

PROBLEM 4.88



For the beam indicated (of Prob. 4.74), a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45\,\text{mm}$.

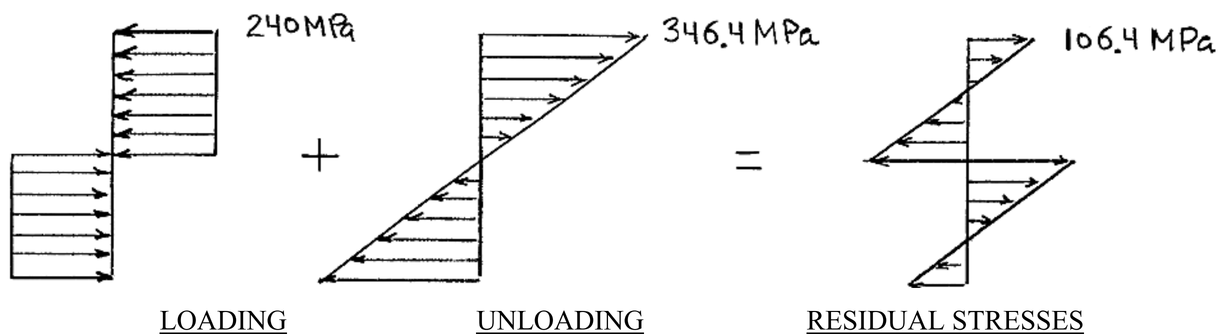
SOLUTION

$$M_p = 27.54 \times 10^3\ \text{N} \cdot \text{m} \ \ \text{(See solutions to Problems 4.74 and 4.78.)}$$

$$I = 3.5775 \times 10^{-6}\ \text{m}^4, \qquad c = 0.045\ \text{m}$$

$$\sigma' = \frac{M_{\max} y}{I} = \frac{M_p c}{I} \quad \text{at} \quad y = c$$

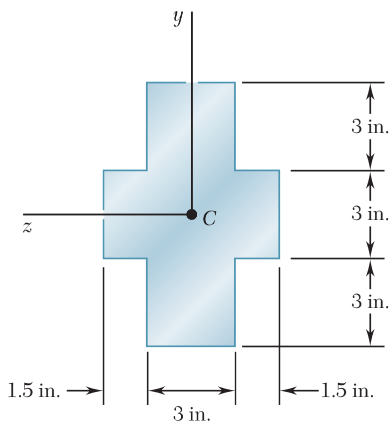
$$\sigma' = \frac{(27.54 \times 10^3)(0.045)}{3.5775 \times 10^{-6}} = 346.4 \times 10^6\ \text{Pa}$$



$$\sigma_{\text{res}} = \sigma' - \sigma_Y = 346.4 \times 10^6 - 240 \times 10^6 = 106.4 \times 10^6\ \text{Pa}$$

$$\sigma_{\text{res}} = 106.4\ \text{MPa} \ \blacktriangleleft$$

PROBLEM 4.89



A bending couple is applied to the bar indicated, causing plastic zones 3 in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

Beam of Prob. 4.75.

SOLUTION

See solution to Problem 4.75 for bending couple and stress distribution during loading.

$$M = 2646 \text{ kip} \cdot \text{in.} \quad y_Y = 1.5 \text{ in.} \quad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

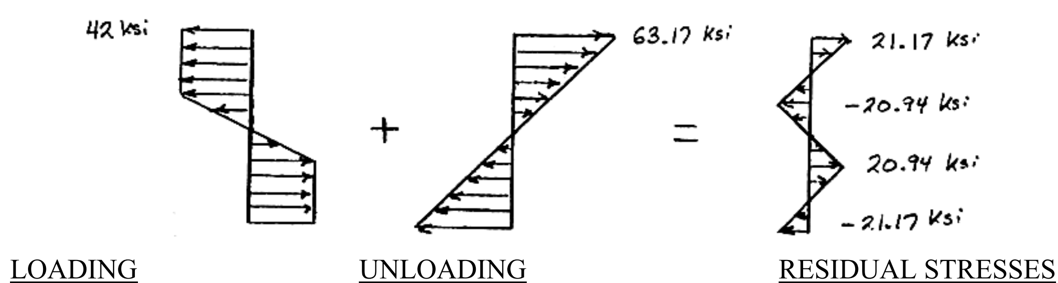
$$\sigma_Y = 42 \text{ ksi} \quad I = 188.5 \text{ in}^4 \quad c = 4.5 \text{ in.}$$

(*a*) $\sigma' = \dfrac{Mc}{I} = \dfrac{(2646)(4.5)}{188.5} = 63.167 \text{ ksi}$

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(2646)(1.5)}{188.5} = 21.056 \text{ ksi}$

At $y = c, \quad \sigma_{res} = \sigma' - \sigma_Y = 63.167 - 42 = 21.167 \text{ ksi}$

At $y = y_Y, \quad \sigma_{res} = \sigma'' - \sigma_Y = 21.056 - 42 = -20.944 \text{ ksi}$ $\qquad \sigma_{res} = -20.9 \text{ ksi}$ ◀

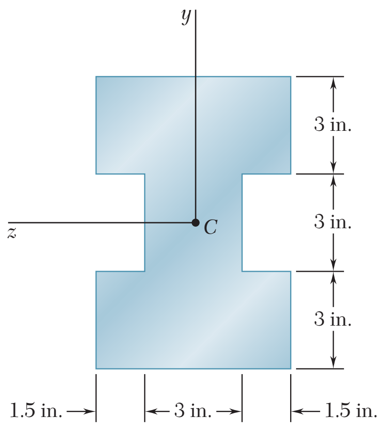


(*b*) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} = \sigma_Y$

$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(188.5)(42)}{2646} = 2.99 \text{ in.}$ $\qquad$ **Answer:** $y_0 = -2.99$ in., 0, 2.99 in. ◀

(*c*) At $y = y_Y, \quad \sigma_{res} = -20.944 \text{ ksi}$

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(29 \times 10^3)(1.5)}{20.944} = 2077 \text{ in.}$ $\qquad \rho = 173.1 \text{ ft}$ ◀



PROBLEM 4.90

A bending couple is applied to the bar indicated, causing plastic zones 3 in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

Beam of Prob. 4.76.

SOLUTION

See solution to Problem 4.76 for bending couple and stress distribution.

$$M = 4725 \text{ kip}\cdot\text{in.} \qquad y_Y = 1.5 \text{ in.} \qquad E = 29\times 10^6 \text{ psi} = 29\times 10^3 \text{ ksi}$$

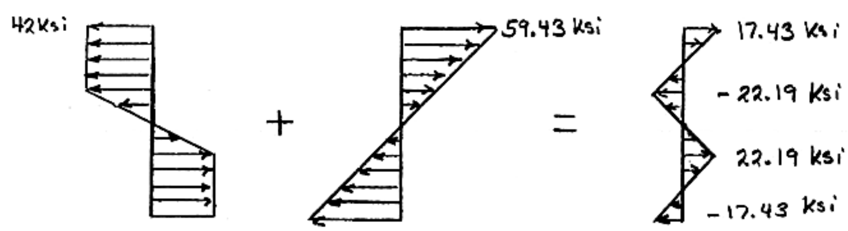
$$\sigma_Y = 42 \text{ ksi} \qquad I = 357.75 \text{ in}^4 \qquad c = 4.5 \text{ in.}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(4725)(4.5)}{357.75} = 59.434 \text{ ksi} \qquad \sigma' = 59.4 \text{ ksi} \blacktriangleleft$$

$$\sigma'' = \frac{My_Y}{I} = \frac{(4725)(1.5)}{357.75} = 19.8113 \text{ ksi}$$

At $y = c$, $\sigma_{res} = \sigma' - \sigma_Y = 59.434 - 42 = 17.4340$ ksi

At $y = y_Y$, $\sigma_{res} = \sigma'' - \sigma_Y = 19.8113 - 42 = -22.189$ ksi



LOADING UNLOADING RESIDUAL STRESSES

(*b*) $$\sigma_{res} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_Y = 0$$

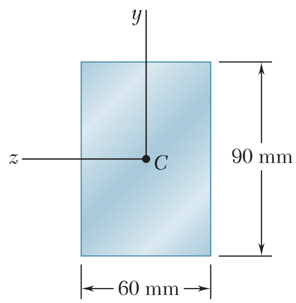
$$y_0 = \frac{I\sigma_Y}{M} = \frac{(357.75)(42)}{4725} = 3.18 \text{ in.}$$

Answer: $y_0 = -3.18$ in., 0, 3.18 in. ◀

(*c*) At $y = y_Y$, $\sigma_{res} = -22.189$ ksi

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29\times 10^3)(1.5)}{22.189} = 1960.43 \text{ in.} \qquad \rho = 163.4 \text{ ft.} \blacktriangleleft$$

PROBLEM 4.91



A bending couple is applied to the beam of Prob. 4.73, causing plastic zones 30 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 45$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

SOLUTION

See solution to Problem 4.73 for bending couple and stress distribution during loading.

$$M = 28.08 \times 10^3 \text{ N} \cdot \text{m} \quad y_Y = 15 \text{ mm} = 0.015 \text{ m} \quad E = 200 \text{ GPa}$$

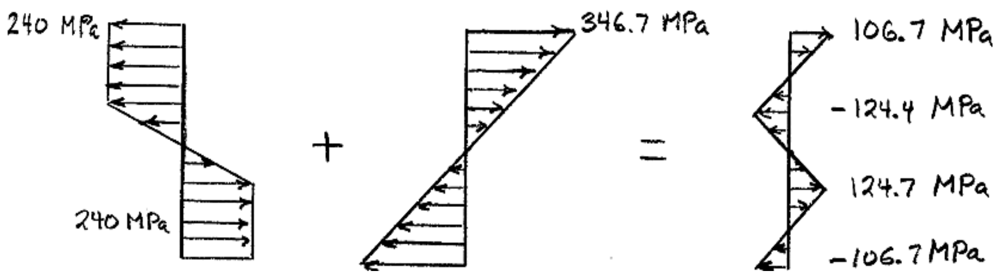
$$\sigma_Y = 240 \text{ MPa} \quad I = 3.645 \times 10^{-6} \text{ m}^4 \quad c = 0.045 \text{ m}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(28.08 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 346.67 \times 10^6 \text{ Pa} = 346.67 \text{ MPa}$$

$$\sigma'' = \frac{M y_Y}{I} = \frac{(28.08 \times 10^3)(0.015)}{3.645 \times 10^{-6}} = 115.556 \times 10^6 \text{ Pa} = 115.556 \text{ MPa}$$

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 346.67 - 240 = 106.670$ MPa $\qquad \sigma_{\text{res}} = 106.7$ MPa ◀

At $y = y_Y$, $\sigma_{\text{res}} = \sigma'' - \sigma_Y = 115.556 - 240 = -124.444$ MPa $\qquad \sigma_{\text{res}} = -124.4$ MPa



(*b*) $$\sigma_{\text{res}} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_Y = 0$$

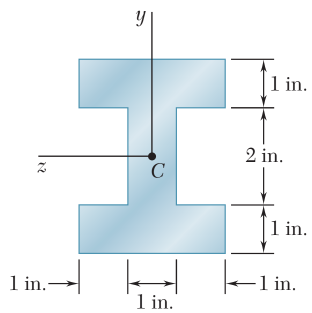
$$y_0 = \frac{I\sigma_Y}{M} = \frac{(3.645 \times 10^{-6})(240 \times 10^6)}{28.08 \times 10^3} = 31.15 \times 10^{-3} \text{ m} = 31.15 \text{ mm}$$

Answer: $y_0 = -31.2$ mm, 0, 31.2 mm ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -124.444 \times 10^6$ Pa

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(200 \times 10^9)(0.015)}{-124.444 \times 10^6}$$

$\rho = 24.1$ m ◀



PROBLEM 4.92

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. A bending couple is applied to the beam about the z axis, causing plastic zones 2 in. thick to develop at the top and bottom of the beam. After the couple has been removed, determine (a) the residual stress at $y = 2$ in., (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the beam.

SOLUTION

Flange: $$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(1)^3 + (3)(1)(1.5)^2 = 7.0 \text{ in}^4$$

Web: $$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(1)(2)^3 = 0.6667 \text{ in}^4$$

$$I_3 = I_1 = 7.0 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 14.6667 \text{ in}^4$$

$$c = 2 \text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(14.6667)}{2}$$

$M_Y = 308 \text{ kip} \cdot \text{in.}$ ◀

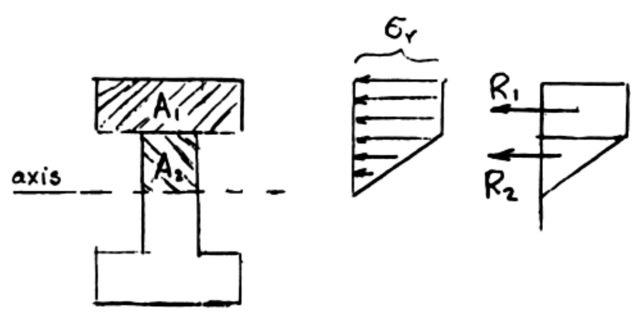
$$R_1 = \sigma_Y A_1 = (42)(3)(1) = 126 \text{ kips}$$

$$\bar{y}_1 = 1.0 + 0.5 = 1.5 \text{ in.}$$

$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(1)(1)$$

$$= 21 \text{ kips}$$

$$\bar{y}_2 = \frac{2}{3}(1.0) = 0.6667 \text{ in.}$$



$$M = 2(R_1\bar{y}_1 + R_2\bar{y}_2) = 2[(126)(1.5) + (21)(0.6667)]$$

$M = 406 \text{ kip} \cdot \text{in.}$ ◀

$M = 406$ kip · in. $\quad y_Y = 1.0$ in. $\quad E = 29 \times 10^6 = 29 \times 10^3$ ksi

$\sigma_Y = 42$ ksi $\quad I = 14.6667 \text{ in}^4 \quad c = 2$ in.

(a) $$\sigma' = \frac{Mc}{I}\frac{(406)(2)}{14.6667} = 55.364$$

$$\sigma'' = \frac{My_Y}{I}\frac{(406)(1.0)}{14.662} = 27.682$$

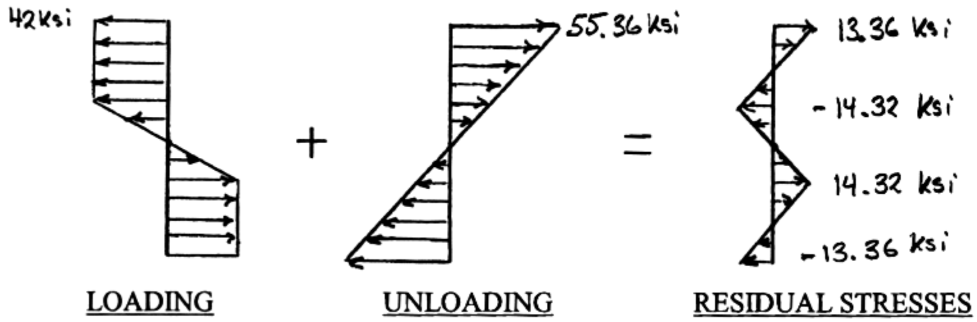
PROBLEM 4.92 *(Continued)*

At $y = c$, $\sigma_{res} = \sigma' - \sigma_Y = 55.364 - 42 = 13.3640$ ksi

$\sigma_{res} = 13.36$ ksi ◀

At $y = y_Y$, $\sigma_{res} = \sigma' - \sigma_Y = 27.682 - 42 = -14.3180$ ksi

$\sigma_{res} = -14.32$ ksi ◀



(*b*) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} - \sigma_Y = 0$

$$y_0 = \frac{I\sigma_Y}{M} = \frac{(14.667)(42)}{406} = 1.517 \text{ in}$$

Answer : $y_0 = -1.517$ in., 0, 1.517 in. ◀

(*c*) At $y = y_Y$, $\sigma_{res} = -14.3180$ ksi

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29\times 10^3)(1.0)}{14.3180} = 2025.42 \text{ in.}$$

$\rho = 168.8$ ft ◀

PROBLEM 4.93*

A rectangular bar that is straight and unstressed is bent into an arc of circle of radius by two couples of moment M. After the couples are removed, it is observed that the radius of curvature of the bar is ρ_R. Denoting by ρ_Y the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation:

$$\frac{1}{\rho_R} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left[1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]\right\}$$

SOLUTION

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right),$$

Let m denote $\frac{M}{M_Y}$.

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{\rho^2}{\rho_Y^2}\right) \quad \therefore \quad \frac{\rho^2}{\rho_Y^2} = 3 - 2\,m$$

$$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y}$$

$$= \frac{1}{\rho}\left\{1 - \frac{\rho}{\rho_Y}m\right\} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)\right\} \quad ◀$$

PROBLEM 4.94

A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (*a*) the radius of curvature when a couple of moment $M = 1.25 M_Y$ is applied to the bar, (*b*) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.93.

SOLUTION

(*a*) $$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2} M_Y \left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m = \frac{M}{M_Y} = 1.25$$

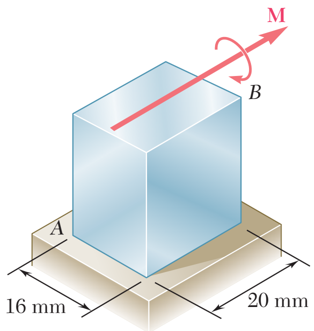
$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.70711$$

$$\rho = 0.707\rho_Y \blacktriangleleft$$

(*b*) $$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.70711\rho_Y} - \frac{1.25}{\rho_Y}$$

$$= \frac{0.16421}{\rho_Y}$$

$$\rho_R = 6.09\rho_Y \blacktriangleleft$$



PROBLEM 4.95

The prismatic bar AB is made of a steel that is assumed to be elastoplastic and for which $E = 200\text{ GPa}$. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 350\text{ N}\cdot\text{m}$ is applied as shown, determine (*a*) the yield strength of the steel, (*b*) the thickness of the elastic core of the bar.

SOLUTION

$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)$$

$$= \frac{3}{2}\frac{\sigma_Y I}{c}\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

$$= \frac{3}{2}\frac{\sigma_Y b(2c)^3}{12c}\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

$$= \sigma_Y bc^2\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

(*a*) $bc^2\sigma_Y\left(1 - \dfrac{\rho^2\sigma_Y^2}{3E^2c^2}\right) = M$ Cubic equation for σ_Y

Data:

$$E = 200 \times 10^9\text{ Pa}$$

$$M = 420\text{ N}\cdot\text{m}$$

$$\rho = 2.4\text{ m}$$

$$b = 20\text{ mm} = 0.020\text{ m}$$

$$c = \frac{1}{2}h = 8\text{ mm} = 0.008\text{ m}$$

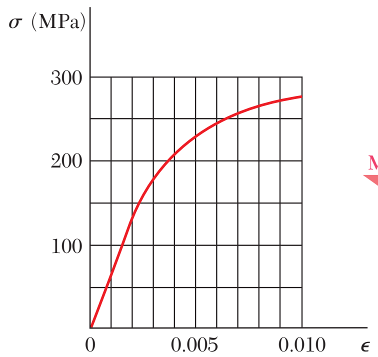
$$(1.28 \times 10^{-6})\,\sigma_Y\left[1 - 750 \times 10^{-21}\sigma_Y^2\right] = 350$$

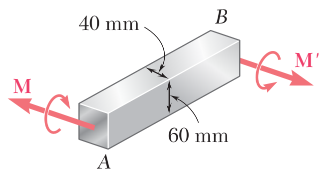
$$\sigma_Y\left[1 - 750 \times 10^{-21}\sigma_Y^2\right] = 273.44 \times 10^6$$

Solving by trial, $\sigma_Y = 292 \times 10^6\text{ Pa}$ $\sigma_Y = 292\text{ MPa}$ ◀

(*b*) $$y_Y = \frac{\sigma_Y\rho}{E} = \frac{(292 \times 10^6)(2.4)}{200 \times 10^9} = 3.504 \times 10^{-3}\text{ m} = 3.504\text{ mm}$$

thickness of elastic core $= 2y_Y = 7.01\text{ mm}$ ◀





PROBLEM 4.96

The prismatic bar AB is made of an aluminum alloy for which the tensile stress-strain diagram is shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine (*a*) the radius of curvature of the bar when the maximum stress is 250 MPa, (*b*) the corresponding value of the bending moment. (*Hint:* For part *b*, plot σ versus y and use an approximate method of integration.)

SOLUTION

(*a*) $\sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$

$\varepsilon_m = 0.0064$ from curve

$$c = \frac{1}{2}\text{h} = 30 \text{ mm} = 0.030 \text{ m}$$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1} \qquad \rho = 4.69 \text{ m} \blacktriangleleft$$

(*b*) Strain distribution: $\quad \varepsilon = -\varepsilon_m \dfrac{y}{c} = -\varepsilon_m u \quad$ where $\quad u = \dfrac{y}{\varepsilon}$

Bending couple: $\quad M = -\int_{-c}^{c} y\sigma b\, dy = 2b\int_0^c y\,|\sigma|\, dy = 2bc^2\int_0^1 u|\sigma|\, du = 2bc^2 J$

where the integral J is given by $\int_0^1 u|\sigma|\, du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\Sigma wu|\sigma|$$

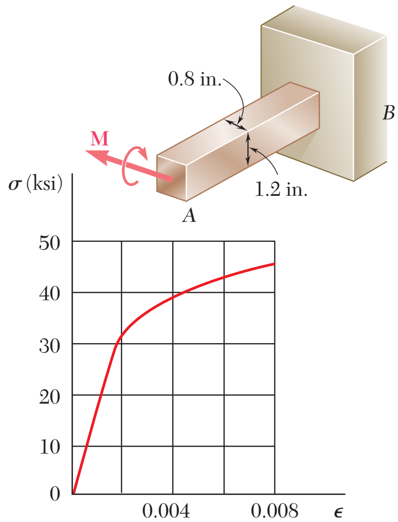
where w is a weighting factor.

Using $\Delta u = 0.25,$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, (MPa)	$u\,\lvert\sigma\rvert$, (MPa)	w	$wu\,\lvert\sigma\rvert$, (MPa)
0	0	0	0	1	0
0.25	0.0016	110	27.5	4	110
0.5	0.0032	180	90	2	180
0.75	0.0048	225	168.75	4	675
1.00	0.0064	250	250	1	250
					1215 ← $\Sigma wu\,\lvert\sigma\rvert$

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$$

$$M = (2)(0.040)(0.030)^2(101.25 \times 10^6) = 7.29 \times 10^3 \text{ N}\cdot\text{m} \qquad M = 7.29 \text{ kN}\cdot\text{m} \blacktriangleleft$$



PROBLEM 4.97

The prismatic bar AB is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine (*a*) the maximum stress in the bar when the radius of curvature of the bar is 100 in., (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.96.)

SOLUTION

(*a*) $\rho = 100$ in., $b = 0.8$ in., $c = 0.6$ in.

$$\varepsilon_m = \frac{c}{\rho} = \frac{0.6}{100} = 0.006$$

From the curve, $\sigma_m = 43.0$ ksi ◀

(*b*) Strain distribution: $\varepsilon = -\varepsilon_m \dfrac{y}{c} = -\varepsilon_m u$ where $u = \dfrac{y}{\varepsilon}$

Bending couple: $M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y\,|\sigma|dy = 2bc^2\int_0^1 u\,|\sigma|\,du = 2bc^2 J$

where the integral J is given by $\int_0^1 u\,|\sigma|\,du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\Sigma wu|\sigma|$$

where w is a weighting factor.

Using $\Delta u = 0.25$, we get the values given the table below:

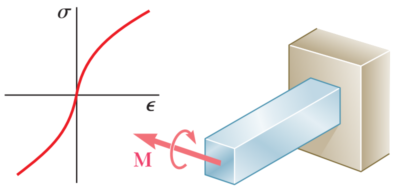
u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, ksi	$u\,\lvert\sigma\rvert$, ksi	w	$wu\,\lvert\sigma\rvert$, ksi
0	0	0	0	1	0
0.25	0.0015	25	6.25	4	25
0.5	0.003	36	18	2	36
0.75	0.0045	40	30	4	120
1.00	0.006	43	43	1	43
					224 ← $\Sigma wu\,\lvert\sigma\rvert$

$$J = \frac{(0.25)(224)}{3} = 18.67 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(18.67)$$

$M = 10.75$ kip · in. ◀

PROBLEM 4.98



A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\varepsilon = k\sigma^n$ for $\sigma > 0$ and $\varepsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple **M** is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{1+2n}{3n}\frac{Mc}{I}$$

SOLUTION

Strain distribution: $\varepsilon = -\varepsilon_m \dfrac{y}{c} = -\varepsilon_m u \quad \text{where} \quad u = \dfrac{y}{c}$

Bending couple:

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y\,|\sigma| dy = 2bc^2\int_0^c \frac{y}{c}|\sigma|\frac{dy}{c}$$

$$= 2bc^2\int_0^1 u\,|\sigma|\,du$$

For $\varepsilon = K\sigma^n, \quad \varepsilon_m = K\sigma_m$

$$\frac{\varepsilon}{\varepsilon_m} = u = \left(\frac{\sigma}{\sigma_m}\right)^n \qquad \therefore \qquad |\sigma| = \sigma_m u^{\frac{1}{n}}$$

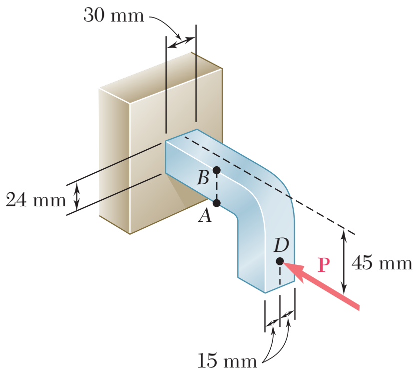
Then $M = 2bc^2\int_0^1 u\sigma_m u^{\frac{1}{n}}\,du = 2bc^2\sigma_m\int_0^1 u^{1+\frac{1}{n}}du$

$$= 2bc^2\sigma_m \frac{u^{2+\frac{1}{n}}}{2+\frac{1}{n}}\Big|_0^1 = \frac{2n}{2n+1}bc^2\sigma_m$$

$$\sigma_m = \frac{2n+1}{2}\frac{M}{bc^2}$$

Recall that $\dfrac{I}{c} = \dfrac{1}{12}\dfrac{b(2c)^3}{c} = \dfrac{2}{3}bc^2 \quad \therefore \quad \dfrac{1}{bc^2} = \dfrac{2}{3}\dfrac{c}{I}$

Then $\sigma_m = \dfrac{2n+1}{3n}\dfrac{Mc}{I}$



PROBLEM 4.99

Knowing that the magnitude of the horizontal force **P** is 8 kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = (30)(24) = 720 \text{ mm}^2 = 720 \times 10^{-6} \text{ m}^2$$

$$e = 45 - 12 = 33 \text{ mm} = 0.033 \text{ m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(30)(24)^3 = 34.56 \times 10^3 \text{ mm}^4 = 34.56 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}(24 \text{ mm}) = 12 \text{ mm} = 0.012 \text{ m} \qquad P = 8 \times 10^3 \text{ N}$$

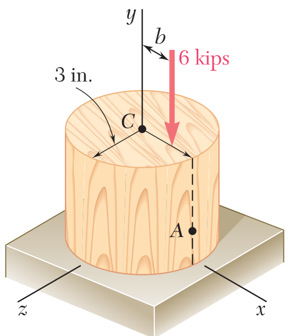
$$M = Pe = (8 \times 10^3)(0.033) = 264 \text{ N} \cdot \text{m}$$

(*a*) $$\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} - \frac{(264)(0.012)}{34.56 \times 10^{-9}} = -102.8 \times 10^6 \text{ Pa}$$

$$\sigma_A = -102.8 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} + \frac{(264)(0.012)}{34.56 \times 10^{-9}} = 80.6 \times 10^6 \text{ Pa}$$

$$\sigma_B = 80.6 \text{ MPa} \blacktriangleleft$$



PROBLEM 4.100

A short wooden post supports a 6-kip axial load as shown. Determine the stress at point A when (*a*) $b = 0$, (*b*) $b = 1.5$ in., (*c*) $b = 3$ in.

SOLUTION

$$A = \pi r^2 = \pi(3)^2 = 28.27 \text{ in}^2$$

$$I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(3)^4 = 63.62 \text{ in}^4$$

$$S = \frac{I}{c} = \frac{63.62}{3} = 21.206 \text{ in}^3$$

$$P = 6 \text{ kips} \qquad M = Pb$$

(*a*) $b = 0 \qquad M = 0$

$$\sigma = -\frac{P}{A} = -\frac{6}{28.27} = -0.212 \text{ ksi}$$

$\sigma = -212$ psi ◀

(*b*) $b = 1.5 \text{ in.} \quad M = (6)(1.5) = 9 \text{ kip} \cdot \text{in.}$

$$\sigma = -\frac{P}{A} - \frac{M}{S} = -\frac{6}{28.27} - \frac{9}{21.206} = -0.637 \text{ ksi}$$

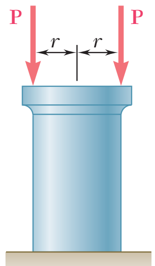
$\sigma = -637$ psi ◀

(*c*) $b = 3 \text{ in.} \quad M = (6)(3) = 18 \text{ kip} \cdot \text{in.}$

$$\sigma = -\frac{P}{A} - \frac{M}{S} = -\frac{6}{28.27} - \frac{18}{21.206} = -1.061 \text{ ksi}$$

$\sigma = -1061$ psi ◀

PROBLEM 4.101



Two forces **P** can be applied separately or at the same time to a plate that is welded to a solid circular bar of radius r. Determine the largest compressive stress in the circular bar, (a) when both forces are applied, (b) when only one of the forces is applied.

SOLUTION

For a solid section, $A = \pi r^2, \quad I = \frac{\pi}{4} r^4, \quad c = r$

Compressive stress
$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{F}{\pi r^2} - \frac{4M}{\pi r^3}$$

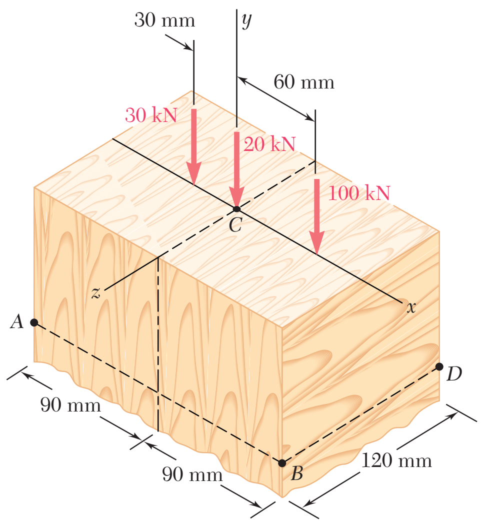
(a) Both forces applied. $F = 2P, \quad M = 0$

$$\sigma = \frac{-2P}{\pi r^2} \blacktriangleleft$$

(b) One force applied. $F = P, \quad M = Pr$

$$\sigma = -\frac{F}{\pi r^2} - \frac{4P_r}{\pi r^2}$$

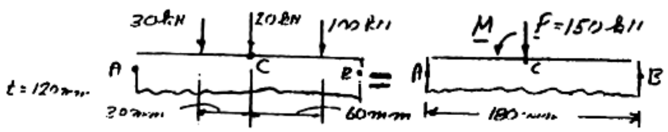
$$\sigma = \frac{-5P}{\pi r^2} \blacktriangleleft$$



PROBLEM 4.102

A short 120 × 180-mm column supports the three axial loads shown. Knowing that section *ABD* is sufficiently far from the loads to remain plane, determine the stress at (*a*) corner *A*, (*b*) corner *B*.

SOLUTION



$$A = (0.120 \text{ m})(0.180 \text{ m}) = 21.6 \times 10^{-3} \text{ m}^2$$

$$S = \frac{1}{6}(0.120 \text{ m})(0.180 \text{ m})^2 = 6.48 \times 10^{-4} \text{ m}^2$$

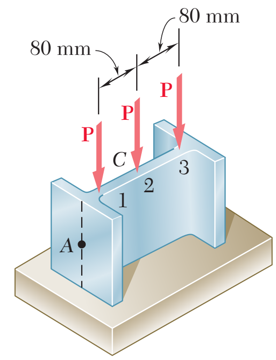
$$M = (30 \text{ kN})(0.03 \text{ m}) - (100 \text{ kN})(0.06 \text{ m}) = -5.10 \text{ kN} \cdot \text{m}$$

(*a*) $$\sigma_A = -\frac{P}{A} - \frac{M}{S} = \frac{-150 \times 10^3 \text{ N}}{21.6 \times 10^{-3} \text{ m}^2} - \frac{-5.10 \times 10^3 \text{ N} \cdot \text{m}}{6.48 \times 10^{-4} \text{ m}^3}$$

$$\sigma_A = 0.926 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} + \frac{M}{S} = \frac{-150 \times 10^3 \text{ N}}{21.6 \times 10^{-3} \text{ m}^2} + \frac{-5.10 \times 10^3 \text{ N} \cdot \text{m}}{6.48 \times 10^{-4} \text{ m}^3}$$

$$\sigma_B = -14.81 \text{ MPa} \blacktriangleleft$$



PROBLEM 4.103

As many as three axial loads, each of magnitude $P = 50$ kN, can be applied to the end of a W200 × 31.1 rolled-steel shape. Determine the stress at point A (a) for the loading shown, (b) if loads are applied at points 1 and 2 only.

SOLUTION

For W200 × 31.3 rolled-steel shape.

$$A = 3970 \text{ mm}^2 = 4.000 \times 10^{-3} \text{ m}^2$$

$$c = \frac{1}{2}d = \frac{1}{2}(210) = 105 \text{ mm} = 0.105 \text{ m}$$

$$I = 31.3 \times 10^6 \text{ mm}^4 = 31.3 \times 10^{-6} \text{ m}^4$$

(a) Centric load: $3P = 50 + 50 + 50 = 150 \text{ kN} = 150 \times 10^3 \text{ N}$

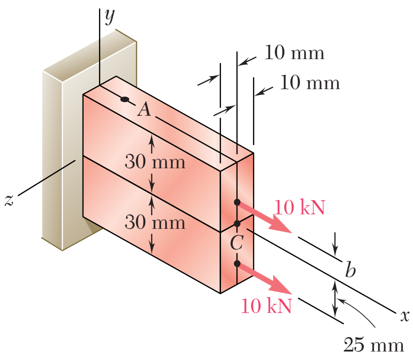
$$\sigma = -\frac{3P}{A} = -\frac{150 \times 10^3}{3.970 \times 10^{-3}} = -37.783 \times 10^6 \text{ Pa} = -37.8 \text{ MPa} \quad \blacktriangleleft$$

(b) Ececentric loading: $e = 80 \text{ mm} = 0.080 \text{ m}$

$$2P = 50 + 50 = 100 \text{ kN} = 100 \times 10^3 \text{ N}$$

$$M = Pe = (50 \times 10^3)(0.080) = 4.0 \times 10^3 \text{ N} \cdot \text{m}$$

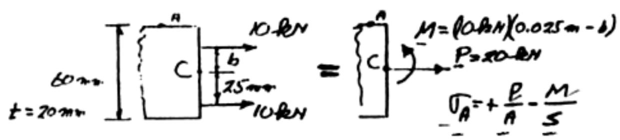
$$\sigma_A = -\frac{2P}{A} - \frac{Mc}{I} = -\frac{100 \times 10^3}{3.970 \times 10^{-3}} - \frac{(4.0 \times 10^3)(0.105)}{31.3 \times 10^{-6}} = -38.607 \times 10^6 \text{ Pa} = -38.6 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 4.104

Two 10-kN forces are applied to a 20 × 60-mm rectangular bar as shown. Determine the stress at point A when (*a*) $b = 0$, (*b*) $b = 15$ mm, (*c*) $b = 25$ mm.

SOLUTION



$$A = (0.060 \text{ m})(0.020 \text{ m}) = 1.2 \times 10^{-3} \text{ m}^2$$

$$S = \frac{1}{6}(0.020 \text{ m})(0.060 \text{ m})^2 = 12 \times 10^{-6} \text{ m}^3$$

(*a*) $b = 0, \ M = (10 \text{ kN})(0.025 \text{ m}) = 250 \text{ N} \cdot \text{m}$

$$\sigma_A = (20 \text{ kN})/(1.2 \times 10^{-3} \text{ m}^2) - (250 \text{ N} \cdot \text{m})/(12 \times 10^{-6} \text{ m}^3)$$
$$= 16.667 \text{ MPa} - 20.833 \text{ MPa}$$

$\sigma_A = -4.17$ MPa ◀

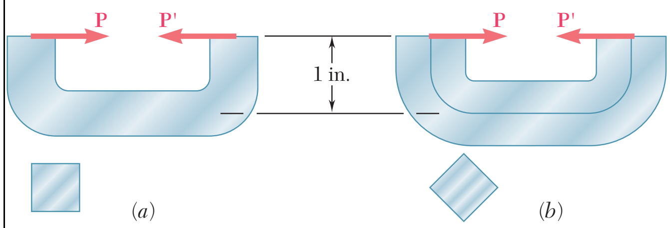
(*b*) $b = 15 \text{ mm}, \ M = (10 \text{ kN})(0.025 \text{ m} - 0.015 \text{ m}) = 100 \text{ N} \cdot \text{m}$

$$\sigma_A = (20 \text{ kN})/(1.2 \times 10^{-3} \text{ m}^2) - (100 \text{ N} \cdot \text{m})/(12 \times 10^{-6} \text{ m}^3)$$
$$= 16.667 \text{ MPa} - 8.333 \text{ MPa}$$

$\sigma_A = 8.33$ MPa ◀

(*c*) $b = 25 \text{ mm}, \ M = 0: \quad \sigma_A = (20 \text{ kN})/(1.2 \times 10^{-3} \text{ m}^2)$

$\sigma_A = 16.67$ MPa ◀

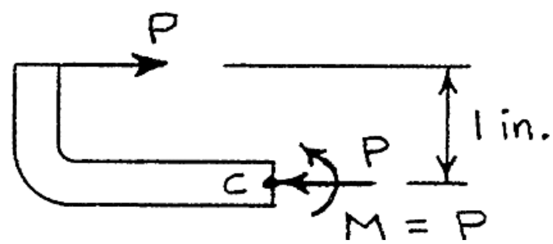


PROBLEM 4.105

Portions of a $\frac{1}{2} \times \frac{1}{2}$-in. square bar have been bent to form the two machine components shown. Knowing that the allowable stress is 15 ksi, determine the maximum load that can be applied to each component.

SOLUTION

The maximum stress occurs at point B.



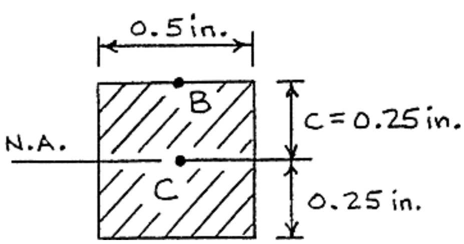
$$\sigma_B = -15 \text{ ksi} = -15 \times 10^3 \text{ psi}$$

$$\sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP$$

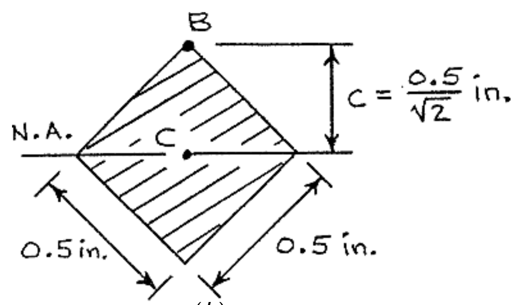
where $\quad K = \frac{1}{A} + \frac{ec}{I} \qquad e = 1.0 \text{ in.}$

$$A = (0.5)(0.5) = 0.25 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)(0.5)^3 = 5.2083 \times 10^{-3} \text{ in}^4 \text{ for all centroidal axes.}$$



(a)



(b)

(a) $\quad c = 0.25$ in.

$$K = \frac{1}{0.25} + \frac{(1.0)(0.25)}{5.2083 \times 10^{-3}} = 52 \text{ in}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-15 \times 10^3)}{52}$$

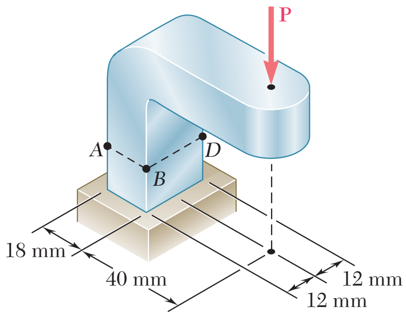
$P = 288$ lb ◀

(b) $\quad c = \frac{0.5}{\sqrt{2}} = 0.35355$ in.

$$K = \frac{1}{0.25} + \frac{(1.0)(0.35355)}{5.2083 \times 10^{-3}} = 71.882 \text{ in}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-15 \times 10^3)}{71.882}$$

$P = 209$ lb ◀



PROBLEM 4.106

Knowing that the allowable stress in section *ABD* is 80 MPa, determine the largest force **P** that can be applied to the bracket shown.

SOLUTION

$$A = (24)(18) = 432 \text{ mm}^2 = 432 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(24)(18)^3 - 11.664 \times 10^3 \text{ mm}^4 = 11.664 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}(18) = 9 \text{ mm} = 0.009 \text{ m}$$

$$e = \frac{1}{2}(18) + 40 = 49 \text{ mm} = 0.049 \text{ m}$$

On line *BD*, both axial and bending stresses are compressive.

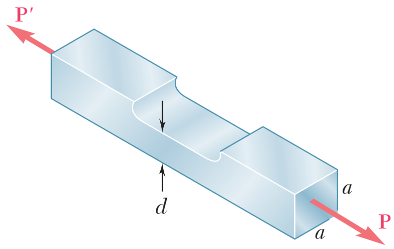
Therefore $$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I}$$

$$= \frac{P}{A} + \frac{Pec}{I}$$

Solving for *P* gives $$P = \frac{\sigma_{max}}{\left(\frac{1}{A} + \frac{ec}{I}\right)}$$

$$P = \frac{80 \times 10^6 \text{ Pa}}{\left(\frac{1}{432 \times 10^{-6} \text{ m}^2} + \frac{(0.049 \text{ m})(0.009 \text{ m})}{11.664 \times 10^{-9} \text{ m}^4}\right)}$$

$P = 1.994$ kN ◀



PROBLEM 4.107

A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 30$ mm, $d = 20$ mm, and $\sigma_{all} = 60$ MPa, determine the magnitude P of the largest forces that can be safely applied at the centers of the ends of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

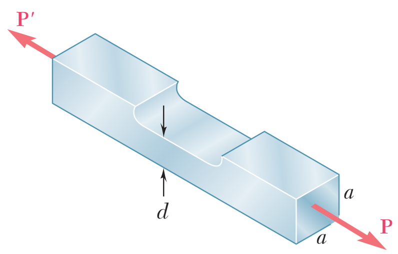
$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP \quad \text{where} \quad K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

Data: $a = 30 \text{ mm} = 0.030 \text{ m} \quad d = 20 \text{ mm} = 0.020 \text{ m}$

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{60 \times 10^6}{4.1667 \times 10^3} = 14.40 \times 10^3 \text{ N}$$

$P = 14.40$ kN ◀



PROBLEM 4.108

A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude $P = 18$ kN are applied at the centers of the ends of the bar. Knowing that $a = 30$ mm and $\sigma_{\text{all}} = 135$ MPa, determine the smallest allowable depth d of the milled portion of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P\frac{1}{2}(a-d)\frac{1}{2}d}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

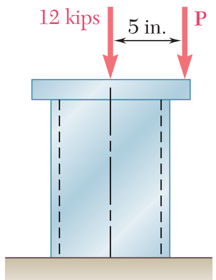
$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \qquad \text{or} \qquad \sigma d^2 + \frac{2P}{a}d - 3P = 0$$

Solving for d,
$$d = \frac{1}{2\sigma}\left\{\sqrt{\left(\frac{2P}{a}\right)^2 + 12P\sigma} - \frac{2P}{a}\right\}$$

Data: $a = 0.030$ m, $\quad P = 18 \times 10^3$ N, $\quad \sigma = 135 \times 10^6$ Pa

$$d = \frac{1}{(2)(135 \times 10^6)}\left\{\sqrt{\left[\frac{(2)(18 \times 10^3)}{0.030}\right]^2 + 12(18 \times 10^3)(135 \times 10^6)} - \frac{(2)(18 \times 10^3)}{0.030}\right\}$$

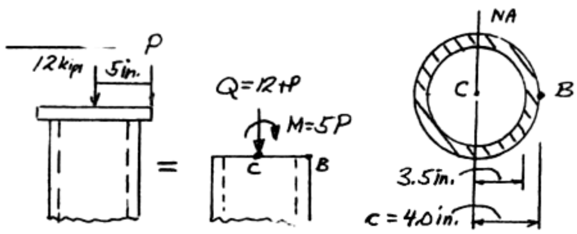
$= 16.04 \times 10^{-3}$ m $\qquad d = 16.04$ mm ◀



PROBLEM 4.109

The two forces shown are applied to a rigid plate supported by a steel pipe of 8-in. outer diameter and 7-in. inner diameter. Determine the value of **P** for which the maximum compressive stress in the pipe is 15 ksi.

SOLUTION



$$\sigma_{\text{all}} = -15 \text{ ksi} \qquad \text{I}_{NA} = \frac{\pi}{4}(4 \text{ in.})^4 - \frac{\pi}{4}(3.5 \text{ in.})^4 = 83.2 \text{ in}^4$$

$$A = \pi(4 \text{ in.})^2 - \pi(3.5 \text{ in.})^2 = 11.78 \text{ in}^2$$

Max. compressive stress is at point B.

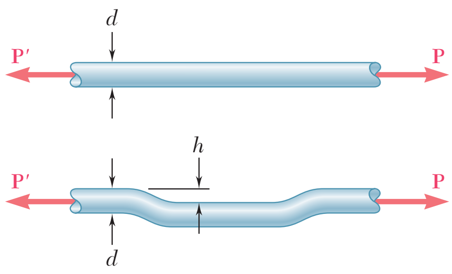
$$\sigma_B = -\frac{Q}{A} - \frac{Mc}{I} = -\frac{12+P}{11.78 \text{ in}^2} - \frac{(5P)(4.0 \text{ in.})}{83.2 \text{ in}^4}$$

$$-15 \text{ ksi} = -1.019 - 0.085P - 0.240P$$

$$-13.981 = -0.325P$$

$$P = 43.0 \text{ kips} \blacktriangleleft$$

PROBLEM 4.110



An offset h must be introduced into a solid circular rod of diameter d. Knowing that the maximum stress after the offset is introduced must not exceed 5 times the stress in the rod when it is straight, determine the largest offset that can be used.

SOLUTION

For centric loading, $\sigma_c = \dfrac{P}{A}$

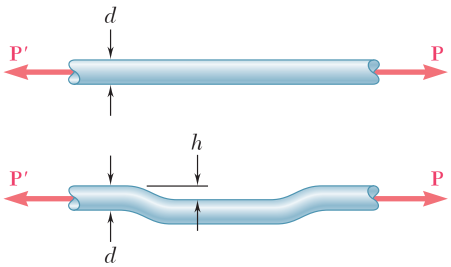
For eccentric loading, $\sigma_e = \dfrac{P}{A} + \dfrac{Phc}{I}$

Given $\sigma_e = 5\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 5\frac{P}{A}$$

$$\frac{Phc}{I} = 4\frac{P}{A} \qquad \therefore \quad h = \frac{4I}{cA} = \frac{(4)\left(\dfrac{\pi}{64}d^4\right)}{\left(\dfrac{d}{2}\right)\left(\dfrac{\pi}{4}d^2\right)} = \frac{1}{2}d$$

$h = 0.500\,d$ ◀



PROBLEM 4.111

An offset h must be introduced into a metal tube of 0.75-in. outer diameter and 0.08-in. wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed 4 times the stress in the tube when it is straight, determine the largest offset that can be used.

SOLUTION

$$c = \frac{1}{2}d = 0.375 \text{ in.}$$

$$c_1 = c - t = 0.375 - 0.08 = 0.295 \text{ in.}$$

$$A = \pi\left(c^2 - c_1^2\right) = \pi(0.375^2 - 0.295^2)$$

$$= 0.168389 \text{ in}^2$$

$$I = \frac{\pi}{4}\left(c^4 - c_1^4\right) = \frac{\pi}{4}(0.375^4 - 0.295^4)$$

$$= 9.5835 \times 10^{-3} \text{ in}^4$$

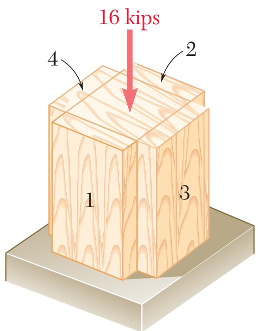
For centric loading, $$\sigma_{\text{cen}} = \frac{P}{A}$$

For eccentric loading, $$\sigma_{\text{ecc}} = \frac{P}{A} + \frac{Phc}{I}$$

$$\sigma_{\text{ecc}} = 4\sigma_{\text{cen}} \quad \text{or} \quad \frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$$

$$\frac{hc}{I} = \frac{3}{A} \qquad h = \frac{3I}{Ac} = \frac{(3)(9.5835 \times 10^{-3})}{(0.168389)(0.375)}$$

$h = 0.455$ in. ◀

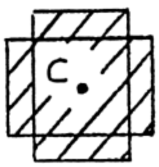


PROBLEM 4.112

A short column is made by nailing four 1 × 4-in. planks to a 4 × 4-in. timber. Using an allowable stress of 600 psi, determine the largest compressive load **P** that can be applied at the center of the top section of the timber column as shown if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) planks 1 and 2 are removed, (*d*) planks 1, 2, and 3 are removed, (*e*) all planks are removed.

SOLUTION

(*a*) Centric loading: $M = 0 \qquad \sigma = \dfrac{P}{A}$



$$A = (4 \times 4) + 4(1)(4) = 32 \text{ in}^2$$

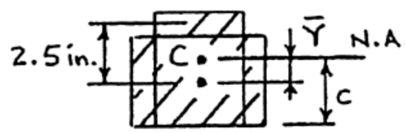
$$\sigma = \frac{P}{A} \qquad \therefore \qquad P = (0.600 \text{ ksi})(32 \text{ in}^2)$$

$$= 19.20 \text{ kips} \quad ◀$$

(*b*) Eccentric loading:

$$M = Pe \qquad \sigma = +\frac{P}{A} + \frac{Pec}{I}$$

$$A = (4)(4) + (3)(1)(3) = 28 \text{ in}^2 \qquad e = \bar{y}$$



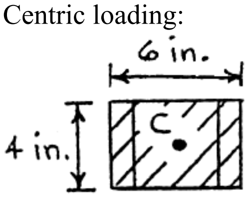
$$\bar{y} = \frac{\sum A\bar{y}}{A} = \frac{(1)(4)(2.5)}{28} = 0.35714 \text{ in.}$$

$$I = \sum(\bar{I} + Ad^2)$$

$$= \frac{1}{12}(6)(4)^3 + (6)(4)(0.35714)^2 + \frac{1}{12}(4)(1)^3 + (4)(1)(2.14286)^2 = 53.762 \text{ in}^4$$

$$\sigma = P\left(\frac{1}{A} + \frac{ec}{I}\right) \qquad \therefore \quad P = \frac{(0.600 \text{ ksi})}{\left[\dfrac{1}{28} + \dfrac{(0.35714)(2.35714)}{53.762}\right]} = 11.68 \text{ kips} \quad ◀$$

(*c*) Centric loading: $M = 0 \qquad \sigma = \dfrac{P}{A}$

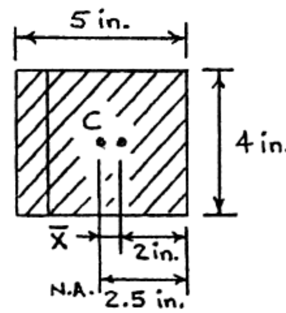


$$A = (6)(4) = 24 \text{ in}^2$$

$$P = (0.600 \text{ ksi})(24) = 14.40 \text{ kips} \quad ◀$$

PROBLEM 4.112 *(Continued)*

(*d*) Eccentric loading: $M = Pe \qquad \sigma = -\dfrac{P}{A} - \dfrac{Pec}{I}$



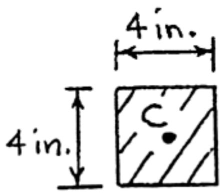
$A = (4)(4) + (1)(4)(1) = 20\text{ in}^2 \quad e = \overline{x}$

$\overline{x} = 2.5 - 2 = 0.5\text{ in.}$

$$I = \frac{1}{12}(4)(5)^3 = 41.667\text{ in}^4$$

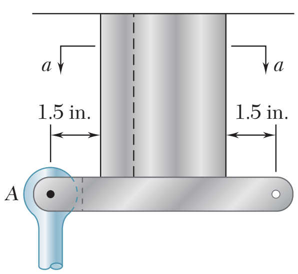
$$P = \frac{(0.600\text{ ksi})}{\left[\frac{1}{20} + \frac{(0.5)(2.5)}{41.667}\right]} = 7.50\text{ kips} \quad ◀$$

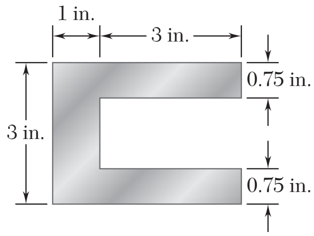
(*e*) Centric loading: $M = 0 \qquad \sigma = -\dfrac{P}{A}$



$A = (4)(4) = 16\text{ in}^2$

$P = (0.600\text{ ksi})(16.0\text{ in}^2) = 9.60\text{ kips} \quad ◀$



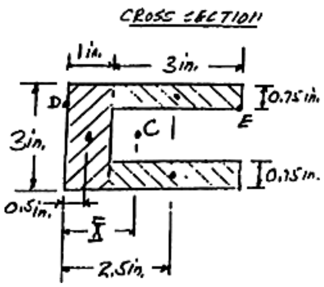


Section a–a

PROBLEM 4.113

A vertical rod is attached at point A to the cast iron hanger shown. Knowing that the allowable stresses in the hanger are $\sigma_{\text{all}} = +5$ ksi and $\sigma_{\text{all}} = -12$ ksi, determine the largest downward force and the largest upward force that can be exerted by the rod.

SOLUTION



$$\bar{X} = \frac{\sum A\bar{y}}{\sum A} = \frac{(1\times 3)(0.5) + 2(3\times 0.25)(2.5)}{(1\times 3) + 2(3\times 0.75)}$$

$$\bar{X} = \frac{12.75 \text{ in}^3}{7.5 \text{ in}^2} = 1.700 \text{ in.}$$

$$A = 7.5 \text{ in}^2$$

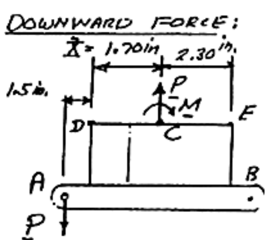
$$\sigma_{\text{all}} = +5 \text{ ksi} \qquad \sigma_{\text{all}} = -12 \text{ ksi}$$

$$I_c = \sum\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(3)(1)^3 + (3\times 1)(1.70 - 0.5)^2 + \frac{1}{12}(1.5)(3)^3 + (1.5\times 3)(2.5 - 1.70)^2$$

$$I_c = 10.825 \text{ in}^4$$

Downward force.



$$M = P(1.5 \text{ in.} + 1.70 \text{ in.}) = (3.20 \text{ in.})P$$

At D: $\sigma_D = +\dfrac{P}{A} + \dfrac{Mc}{I}$

$$+5 \text{ ksi} = \frac{P}{7.5} + \frac{(3.20)P(1.70)}{10.825}$$

$$+5 = P(+0.6359) \qquad \mathbf{P} = 7.86 \text{ kips} \downarrow$$

At E: $\sigma_E = +\dfrac{P}{A} - \dfrac{Mc}{I}$

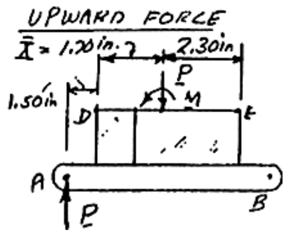
$$-12 \text{ ksi} = \frac{P}{7.5} - \frac{(3.20)P(2.30)}{10.825}$$

$$-12 = P(-0.5466) \qquad \mathbf{P} = 21.95 \text{ kips} \downarrow$$

We choose the smaller value. $\mathbf{P} = 7.86$ kips $\downarrow$ ◀

PROBLEM 4.113 *(Continued)*

Upward force.



$$M = P(1.5 \text{ in.} + 1.70 \text{ in.}) = (3.20 \text{ in.})P$$

At D: $\sigma_D = +\dfrac{P}{A} - \dfrac{Mc}{I}$

$$-12 \text{ ksi} = -\frac{P}{7.5} - \frac{(3.20)P(1.70)}{10.825}$$

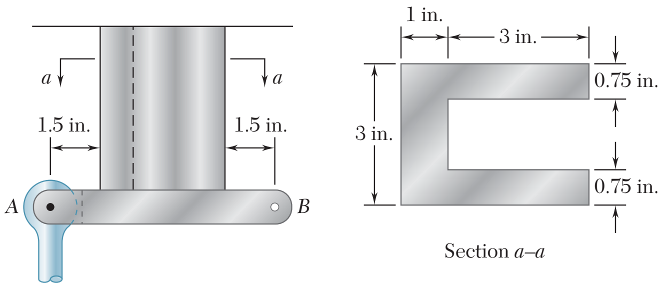
$$-12 = P(-0.6359) \qquad \mathbf{P} = 18.87 \text{ kips} \uparrow$$

At E: $\sigma_E = +\dfrac{P}{A} + \dfrac{Mc}{I}$

$$+5 \text{ ksi} = \frac{P}{7.5} + \frac{(3.20)P(2.30)}{10.825}$$

$$+5 = P(+0.5466) \qquad \mathbf{P} = 9.15 \text{ kips} \uparrow$$

We choose the smaller value. $\mathbf{P} = 9.15 \text{ kips} \uparrow$ ◀

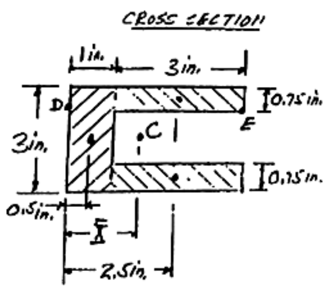


PROBLEM 4.114

Solve Prob. 4.113, assuming that the vertical rod is attached at point B instead of point A.

PROBLEM 4.113 A vertical rod is attached at point A to the cast iron hanger shown. Knowing that the allowable stresses in the hanger are $\sigma_{\text{all}} = +5$ ksi and $\sigma_{\text{all}} = -12$ ksi, determine the largest downward force and the largest upward force that can be exerted by the rod.

SOLUTION



$$\bar{X} = \frac{\sum A\bar{y}}{\sum A} = \frac{(1 \times 3)(0.5) + 2(3 \times 0.25)(2.5)}{(1 \times 3) + 2(3 \times 0.75)}$$

$$\bar{X} = \frac{12.75 \text{ in}^3}{7.5 \text{ in}^2} = 1.700 \text{ in.}$$

$$A = 7.5 \text{ in}^2$$

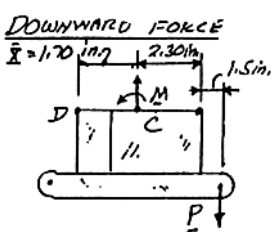
$$\sigma_{\text{all}} = +5 \text{ ksi} \qquad \sigma_{\text{all}} = -12 \text{ ksi}$$

$$I_c = \sum\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(3)(1)^3 + (3 \times 1)(1.70 - 0.5)^2 + \frac{1}{12}(1.5)(3)^3 + (1.5 \times 3)(2.5 - 1.70)^2$$

$$I_c = 10.825 \text{ in}^4$$

<u>Downward force</u>.



$$\sigma_{\text{all}} = +5 \text{ ksi} \qquad \sigma_{\text{all}} = -12 \text{ ksi}$$

$$M = (2.30 \text{ in.} + 1.5 \text{ in.}) = (3.80 \text{ in.})P$$

<u>At D:</u> $\sigma_D = +\frac{P}{A} - \frac{Mc}{I}$

$$-12 \text{ ksi} = +\frac{P}{7.5} - \frac{(3.80)P(1.70)}{10.825}$$

$$-12 = P(-0.4634) \qquad \mathbf{P} = 25.9 \text{ kips} \downarrow$$

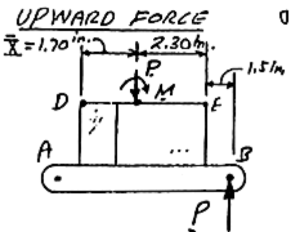
<u>At E:</u> $\sigma_E = +\frac{P}{A} + \frac{Mc}{I}$

$$+5 \text{ ksi} = +\frac{P}{7.5} + \frac{(3.80)P(2.30)}{10.825}$$

$$+5 = P(+0.9407) \qquad \mathbf{P} = 5.32 \text{ kips} \downarrow$$

We choose the smaller value. $\mathbf{P} = 5.32 \text{ kips} \downarrow$ ◀

Upward force.



$$\sigma_{\text{all}} = +5 \text{ ksi} \qquad \sigma_{\text{all}} = -12 \text{ ksi}$$

$$M = (2.30 \text{ in.} + 1.5 \text{ in.})P = (3.80 \text{ in.})P$$

<u>At D:</u> $\sigma_D = -\dfrac{P}{A} + \dfrac{Mc}{I}$

$$5 \text{ ksi} = -\frac{P}{7.5} + \frac{(3.80)P(1.70)}{10.825}$$

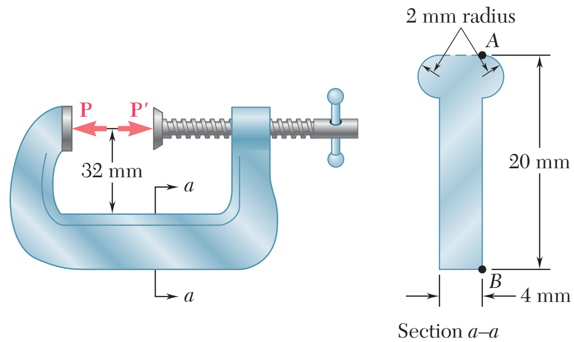
$$5 = P(+0.4634) \qquad \mathbf{P} = 10.79 \text{ kips} \uparrow$$

<u>At E:</u> $\sigma_E = -\dfrac{P}{A} - \dfrac{Mc}{I}$

$$-12 \text{ ksi} = -\frac{P}{7.5} - \frac{(3.80)P(2.30)}{10.825}$$

$$-12 = P(-0.9407) \qquad \mathbf{P} = 12.76 \text{ kips} \uparrow$$

We choose the smaller value. $\mathbf{P} = 10.79 \text{ kips} \uparrow$ ◀

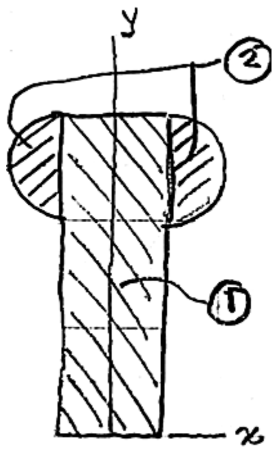


PROBLEM 4.115

Knowing that the clamp shown has been tightened until $P = 400$ N, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the location of the neutral axis of section $a - a$.

SOLUTION

Cross section: Rectangle ① + Circle ②



$$A_1 = (20 \text{ mm})(4 \text{ mm}) = 80 \text{ mm}^2$$

$$\bar{y}_1 = \frac{1}{2}(20 \text{ mm}) = 10 \text{ mm}$$

$$A_2 = \pi(2 \text{ mm})^2 = 4\pi \text{ mm}^2$$

$$\bar{y}_2 = 20 - 2 = 18 \text{ mm}$$

$$c_B = \bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{(80)(10) + (4\pi)(18)}{80 + 4\pi} = 11.086 \text{ mm}$$

$$c_A = 20 - \bar{y} = 8.914 \text{ mm}$$

$$d_1 = 11.086 - 10 = 1.086 \text{ mm}$$

$$d_2 = 18 - 11.086 = 6.914 \text{ mm}$$

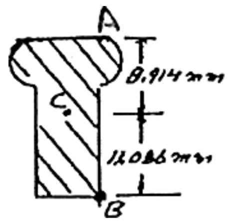
$$I_1 = \bar{I}_1 + A_1 d_1^2 = \frac{1}{12}(4)(20)^3 + (80)(1.086)^2 = 2.761 \times 10^3 \text{ mm}^4$$

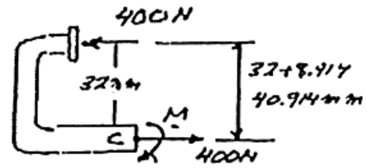
$$I_2 = \bar{I}_2 + A_2 d_2^2 = \frac{\pi}{4}(2)^4 + (4\pi)(6.914)^2 = 0.613 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 3.374 \times 10^3 \text{ mm}^4 = 3.374 \times 10^{-9} \text{ m}^4$$

$$A = A_1 + A_2 = 92.566 \text{ mm}^2 = 92.566 \times 10^{-6} \text{ m}^2$$

PROBLEM 4.115 *(Continued)*





$$e = 32 + 8.914 = 40.914 \text{ mm} = 0.040914 \text{ m}$$

$$M = Pe = (400 \text{ N})(0.040914 \text{ m}) = 16.3656 \text{ N} \cdot \text{m}$$

(*a*) Point *A*:

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{400}{92.566 \times 10^{-6}} + \frac{(16.3656)(8.914 \times 10^{-3})}{3.374 \times 10^{-9}}$$

$$= 4.321 \times 10^6 + 43.23 \times 10^6 = 47.55 \times 10^6 \text{ Pa}$$

$\sigma_A = 47.6$ MPa ◀

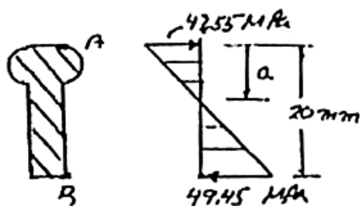
(*b*) Point *B*:

$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{92.566 \times 10^{-6}} - \frac{(16.3656)(11.086)}{3.374 \times 10^{-9}}$$

$$= 4.321 \times 10^6 - 53.72 \times 10^6 = -49.45 \times 10^6 \text{ Pa}$$

$\sigma_B = -49.4$ MPa ◀

(*c*) Neutral axis:

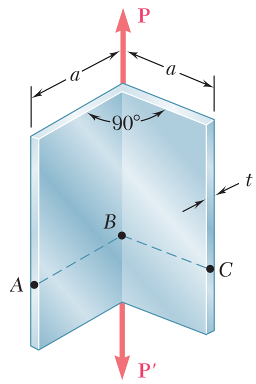


By proportions,

$$\frac{a}{47.55} = \frac{20}{47.55 + 49.45}$$

$$a = 9.80 \text{ mm}$$

9.80 mm below top of section ◀

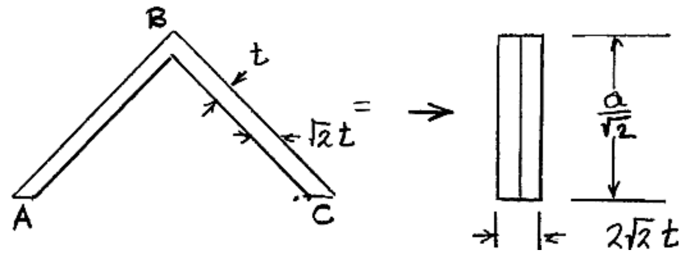


PROBLEM 4.116

The shape shown was formed by bending a thin steel plate. Assuming that the thickness t is small compared to the length a of a side of the shape, determine the stress (a) at A, (b) at B, (c) at C.

SOLUTION

Moment of inertia about centroid:



$$I = \frac{1}{12}\left(2\sqrt{2}t\right)\left(\frac{a}{\sqrt{2}}\right)^3$$

$$= \frac{1}{12}ta^3$$

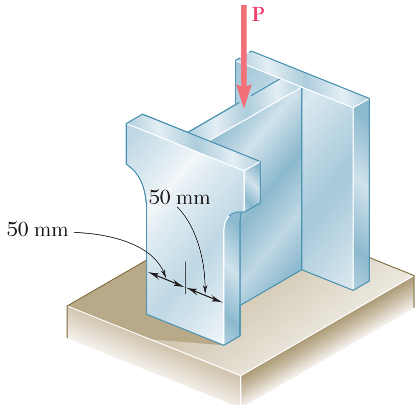
Area: $\quad A = \left(2\sqrt{2}t\right)\left(\frac{a}{\sqrt{2}}\right) = 2at, \quad c = \frac{a}{2\sqrt{2}}$

(a) $\quad \sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3}$ $\qquad \sigma_A = -\frac{P}{2at}$ ◀

(b) $\quad \sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}ta^3}$ $\qquad \sigma_B = -\frac{2P}{at}$ ◀

(c) $\quad \sigma_C = \sigma_A$ $\qquad \sigma_C = -\frac{P}{2at}$ ◀

PROBLEM 4.117



Three steel plates, each of 25×150-mm cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 25-mm strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 100 MPa, determine the largest force **P** (*a*) that could be applied to the original column, (*b*) that can be applied to the modified column.

SOLUTION

(*a*) Centric loading: $\sigma = -\dfrac{P}{A}$

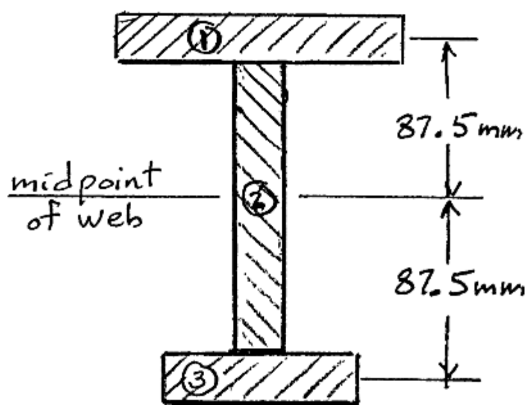
$$A = (3)(150)(25) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

$$P = -\sigma A = -(-100 \times 10^6)(11.25 \times 10^{-3})$$

$$= 1.125 \times 10^6 \text{ N}$$

$P = 1125$ kN ◀

(*b*) Eccentric loading (reduced cross section):



	A, 10^3 mm^2	$\bar{y}$, mm	$A\bar{y}$ (10^3 mm^3)	d, mm
①	3.75	87.5	328.125	76.5625
②	3.75	0	0	10.9375
③	2.50	−87.5	−218.75	98.4375
Σ	10.00		109.375	

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{109.375 \times 10^3}{10.00 \times 10^3} = 10.9375 \text{ mm}$$

PROBLEM 4.117 *(Continued)*

The centroid lies 10.9375 mm from the midpoint of the web.

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(150)(25)^3 + (3.75 \times 10^3)(76.5625)^2 = 22.177 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(25)(150)^3 + (3.75 \times 10^3)(10.9375)^2 = 7.480 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(100)(25)^3 + (2.50 \times 10^3)(98.4375)^2 = 24.355 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 54.012 \times 10^6 \text{ mm}^4 = 54.012 \times 10^{-6} \text{ m}^4$$

$$c = 10.9375 + 75 + 25 = 110.9375 \text{ mm} = 0.1109375 \text{ m}$$

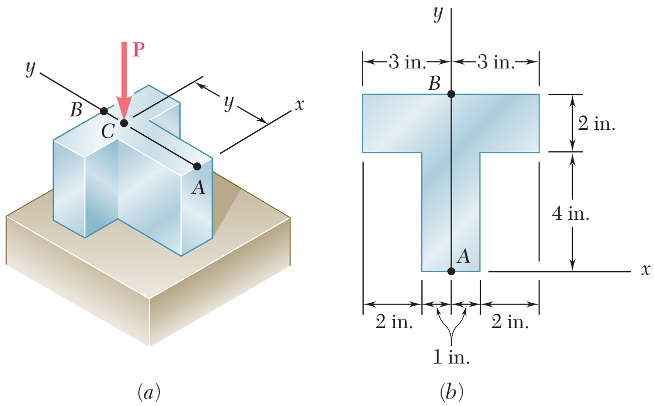
$$M = Pe \qquad \text{where} \qquad e = 10.4375 \text{ mm} = 10.4375 \times 10^{-3} \text{ m}$$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP \qquad A = 10.00 \times 10^{-3} \text{ m}^2$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{10.00 \times 10^{-3}} + \frac{(101.9375 \times 10^{-3})(0.1109375)}{54.012 \times 10^{-6}} = 122.465 \text{ m}^{-2}$$

$$P = -\frac{\sigma}{K} = -\frac{(-100 \times 10^6)}{122.465} = 817 \times 10^3 \text{ N}$$

$P = 817$ kN ◀



PROBLEM 4.118

A vertical force **P** of magnitude 20 kips is applied at point C located on the axis of symmetry of the cross section of a short column. Knowing that $y = 5$ in., determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the location of the neutral axis.

SOLUTION

Locate centroid.

Part	A, in^2	$\bar{y}$, in.	$A\bar{y}$, in^3
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{76}{20} = 3.8 \text{ in.}$$

Eccentricity of load: $e = 5 - 3.8 = 1.2$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4 \qquad I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(*a*) Stress at A: $c_A = 3.8$ in.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = -\frac{20}{20} + \frac{20(1.2)(3.8)}{57.867} \qquad \sigma_A = 0.576 \text{ ksi} ◀$$

(*b*) Stress at B: $c_B = 6 - 3.8 = 2.2$ in.

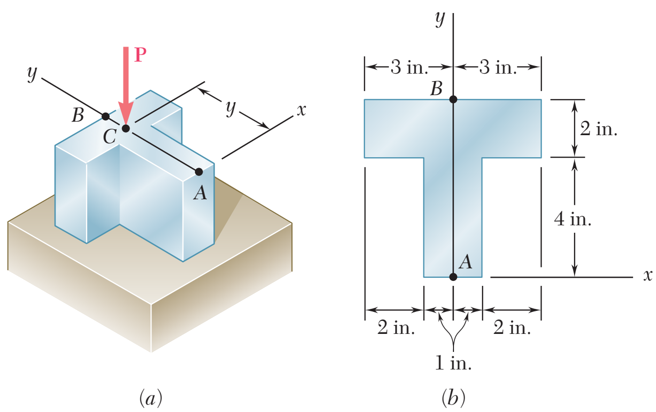
$$\sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = -\frac{20}{20} - \frac{20(1.2)(2.2)}{57.867} \qquad \sigma_B = -1.912 \text{ ksi} ◀$$

(*c*) Location of neutral axis: $\sigma = 0 \qquad \sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \quad \frac{ea}{I} = \frac{1}{A}$

$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in.}$$

Neutral axis lies 2.411 in. below centroid or $3.8 - 2.411 = 1.389$ in. above point A.

Answer: 1.389 in. from point A ◀



PROBLEM 4.119

A vertical force **P** is applied at point C located on the axis of symmetry of the cross section of a short column. Determine the range of values of y for which tensile stresses do not occur in the column.

SOLUTION

Locate centroid.

	A, in^2	$\bar{y}$, in.	$A\bar{y}$, in^3
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in.}$$

Eccentricity of load: $e = y - 3.8$ in. $\quad y = e + 3.8$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4 \qquad I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

If stress at A equals zero, $\quad c_A = 3.8$ in.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = 0 \qquad \therefore \qquad \frac{ec_A}{I} = \frac{1}{A}$$

$$e = \frac{I}{Ac_A} = \frac{57.867}{(20)(3.8)} = 0.761 \text{ in.} \qquad y = 0.761 + 3.8 = 4.561 \text{ in.}$$

If stress at B equals zero, $\quad c_B = 6 - 3.8 = 2.2$ in.

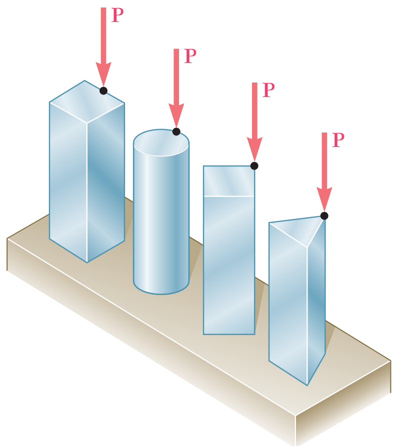
$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = 0 \qquad \therefore \qquad \frac{ec_B}{I} = -\frac{1}{A}$$

$$e = -\frac{I}{Ac_B} = -\frac{57.867}{(20)(2.2)} = -1.315 \text{ in.}$$

$$y = -1.315 + 3.8 = 2.485 \text{ in.}$$

Answer: $2.49 \text{ in.} < y < 4.56 \text{ in.}$ ◀

PROBLEM 4.120



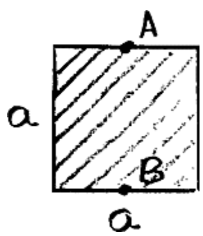
The four bars shown have the same cross-sectional area. For the given loadings, show that (*a*) the maximum compressive stresses are in the ratio 4:5:7:9, (*b*) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note:* the cross section of the triangular bar is an equilateral triangle.)

SOLUTION

Stresses:

At A,
$$\sigma_A = -\frac{P}{A} - \frac{Pec_A}{I} = -\frac{P}{A}\left(1 + \frac{Aec_A}{I}\right)$$

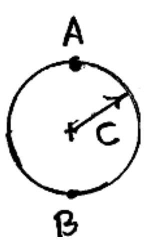
At B,
$$\sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = \frac{P}{A}\left(\frac{Aec_B}{I} - 1\right)$$



$$A_1 = a^2, \quad I_1 = \frac{1}{12}a^4, \quad c_A = c_B = \frac{1}{2}a, \quad e = \frac{1}{2}a$$

$$\sigma_A = -\frac{P}{A}\left(1 + \frac{(a^2)\left(\frac{1}{2}a\right)\left(\frac{1}{2}a\right)}{\frac{1}{12}a^2}\right) \qquad \sigma_A = -4\frac{P}{A_1} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A}\left(\frac{(a^2)\left(\frac{1}{2}a\right)\left(\frac{1}{2}a\right)}{\frac{1}{12}a^2} - 1\right) \qquad \sigma_B = 2\frac{P}{A_1} \blacktriangleleft$$

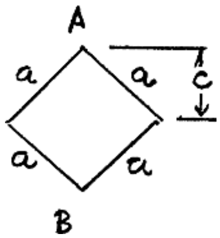


$$A_2 = \pi c^2 = a^2 \quad \therefore \quad c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4}c^4, \quad e = c$$

$$\sigma_A = -\frac{P}{A_2}\left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4}\right) \qquad \sigma_A = -5\frac{P}{A_2} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_2}\left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4} - 1\right) \qquad \sigma_B = 3\frac{P}{A_2} \blacktriangleleft$$

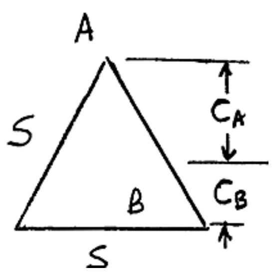
PROBLEM 4.120 *(Continued)*



$$\begin{cases} A_3 = a^2 \quad c = \dfrac{\sqrt{2}}{2}a \quad I_3 = \dfrac{1}{12}a^4 \quad e = c \\ \sigma_A = -\dfrac{P}{A_3}\left(1 + \dfrac{(a^2)\left(\frac{\sqrt{2}}{2}a\right)\left(\frac{\sqrt{2}}{2}a\right)}{\frac{1}{12}a^4}\right) \\ \sigma_B = \dfrac{P}{A_3}\left(\dfrac{(a^2)\left(\frac{\sqrt{2}}{2}a\right)\left(\frac{\sqrt{2}}{2}a\right)}{\frac{1}{12}a^4} - 1\right) \end{cases}$$

$$\sigma_A = -7\frac{P}{A_3} \blacktriangleleft$$

$$\sigma_B = 5\frac{P}{A_3} \blacktriangleleft$$



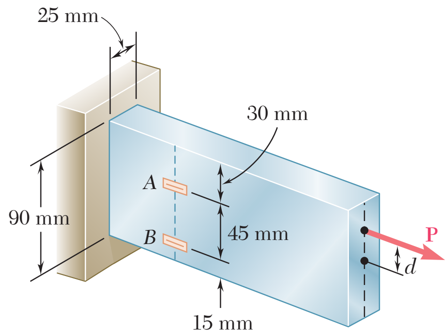
$$A_4 = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right) = \frac{\sqrt{3}}{4}s^2$$

$$I_4 = \frac{1}{36}s\left(\frac{\sqrt{3}}{2}s\right)^3 = \frac{\sqrt{3}}{96}s^4$$

$$c_A = \frac{2}{3}\frac{\sqrt{3}}{2}s = \frac{s}{\sqrt{3}} = e \qquad c_B = s$$

$$\sigma_A = -\frac{P}{A_4}\left(1 + \frac{\left(\frac{\sqrt{3}}{4}s^2\right)\left(\frac{s}{\sqrt{3}}\right)\left(\frac{s}{\sqrt{3}}\right)}{\frac{\sqrt{3}}{96}s^4}\right) \qquad \sigma_A = -9\frac{P}{A_4} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_4}\left(\frac{\left(\frac{\sqrt{3}}{4}s^2\right)\left(\frac{s}{\sqrt{3}}\right)\left(\frac{s}{2\sqrt{3}}\right)}{\frac{\sqrt{3}}{96}s^4} - 1\right) \qquad \sigma_B = 3\frac{P}{A_4} \blacktriangleleft$$



PROBLEM 4.121

An eccentric force **P** is applied as shown to a steel bar of 25×90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\,\mu \qquad \varepsilon_B = -70\,\mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm} \qquad b = 25 \text{ mm} \qquad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh - (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m} \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B: $\sigma_A = E\varepsilon_A = (200 \times 10^9)(350 \times 10^{-6}) = 70 \times 10^6 \text{ Pa}$

$$\sigma_B = E\varepsilon_B = (200 \times 10^9)(-70 \times 10^{-6}) = -14 \times 10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting, $\sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-6})(84 \times 10^6)}{0.045} = -2835 \text{ N} \cdot \text{m}$$

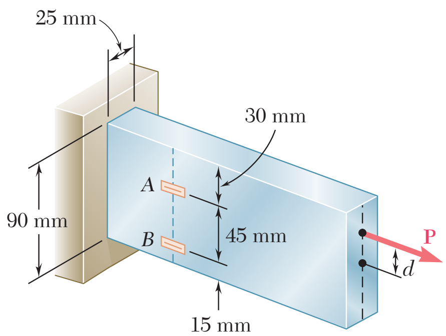
Multiplying (2) by y_A and (1) by y_B and subtracting, $y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\dfrac{P}{A}$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(-14 \times 10^6) - (-0.030)(70 \times 10^6)]}{0.045} = 94.5 \times 10^3 \text{ N}$$

(*a*) $M = -Pd \quad \therefore \quad d = -\dfrac{M}{P} = -\dfrac{-2835}{94.5 \times 10^3} = 0.030 \text{ m}$ $\qquad d = 30.0 \text{ mm}$ ◀

(*b*) $\qquad P = 94.5 \text{ kN}$ ◀

PROBLEM 4.122



Solve Prob. 4.121, assuming that the measured strains are

$$\varepsilon_A = +600\,\mu \qquad \varepsilon_B = +420\,\mu$$

PROBLEM 4.121 An eccentric force **P** is applied as shown to a steel bar of 25×90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\mu \qquad \varepsilon_B = -70\mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm} \qquad b = 25 \text{ mm} \qquad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m} \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B: $\sigma_A = E\varepsilon_A = (200 \times 10^9)(600 \times 10^{-6}) = 120 \times 10^6 \text{ Pa}$

$$\sigma_B = E\varepsilon_B = (200 \times 10^9)(420 \times 10^{-6}) = 84 \times 10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting, $\sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

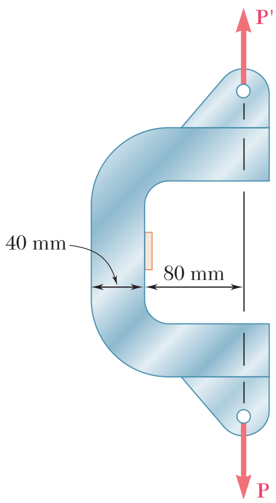
$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-6})(36 \times 10^6)}{0.045} = -1215 \text{ N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting, $y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\dfrac{P}{A}$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(84 \times 10^6) - (-0.030)(120 \times 10^6)]}{0.045} = 243 \times 10^3 \text{ N}$$

(*a*) $M = -Pd \quad \therefore \quad d = -\dfrac{M}{P} = -\dfrac{-1215}{243 \times 10^3} = 5 \times 10^{-3} \text{ m}$ $\qquad d = 5.00 \text{ mm}$ ◀

(*b*) $\qquad P = 243 \text{ kN}$ ◀



PROBLEM 4.123

The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and that strain on the inner edge was measured and found to be 450μ, determine the magnitude P of the forces. Use $E = 200$ GPa.

SOLUTION

At the strain gage location,

$$\sigma = E\varepsilon = (200\times10^9)(450\times10^{-6}) = 90\times10^6 \text{ Pa}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600\times10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(40)(40)^3 = 213.33\times10^3 \text{ mm}^4 = 213.33\times10^{-9} \text{ m}^4$$

$$e = 80 + 20 = 100 \text{ mm} = 0.100 \text{ m}$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

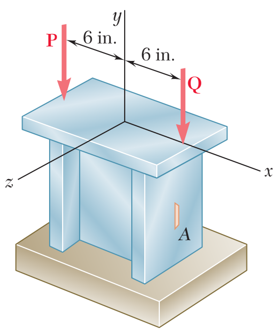
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

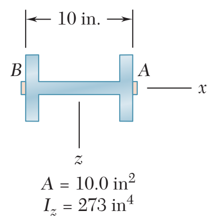
$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600\times10^{-6}} + \frac{(0.100)(0.020)}{213.33\times10^{-9}} = 10.00\times10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{90\times10^6}{10.00\times10^3} = 9.00\times10^3 \text{ N}$$

$P = 9.00$ kN ◀

PROBLEM 4.124



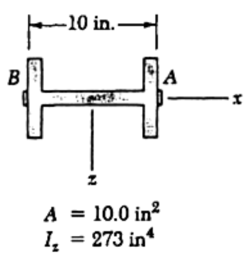


A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points A and B on the centerline of the outer faces of the flanges have been measured and found to be

$$\varepsilon_A = -400\times10^{-6}\text{ in./in.} \qquad \varepsilon_B = -300\times10^{-6}\text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

SOLUTION



Stresses at A and B from strain gages:

$$\sigma_A = E\varepsilon_A = (29\times10^6)(-400\times10^{-6}) = -11.6\times10^3\text{ psi}$$

$$\sigma_B = E\varepsilon_B = (29\times10^6)(-300\times10^{-6}) = -8.7\times10^3\text{ psi}$$

Centric force: $F = P + Q$

Bending couple: $M = 6P - 6Q$

$c = 5$ in.

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P-6Q)(5)}{273}$$

$$-11.6\times10^3 = +0.00989P - 0.20989Q \qquad (1)$$

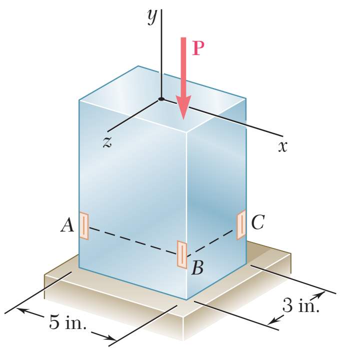
$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P-6Q)(5)}{273}$$

$$-8.7\times10^3 = -0.20989P - 0.00989Q \qquad (2)$$

Solving (1) and (2) simultaneously,

$P = 44.2\times10^3\text{ lb} = 44.2\text{ kips}$ ◀

$Q = 57.3\times10^3\text{ lb} = 57.3\text{ kips}$ ◀



PROBLEM 4.125

A single vertical force **P** is applied to a short steel post as shown. Gages located at *A, B,* and *C* indicate the following strains:

$$\varepsilon_A = -500\mu \qquad \varepsilon_B = -1000\mu \qquad \varepsilon_C = -200\mu$$

Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the magnitude of **P**, (*b*) the line of action of **P**, (*c*) the corresponding strain at the hidden edge of the post, where $x = -2.5$ in. and $z = -1.5$ in.

SOLUTION

$$I_x = \frac{1}{12}(5)(3)^3 = 11.25 \text{ in}^4 \qquad I_z = \frac{1}{12}(3)(5)^3 = 31.25 \text{ in}^4 \qquad A = (5)(3) = 15 \text{ in}^2$$

$$M_x = Pz \qquad M_z = -Px$$

$$x_A = -2.5 \text{ in.}, \quad x_B = 2.5 \text{ in.}, \quad x_C = 2.5 \text{ in.}, \quad x_D = -2.5 \text{ in.}$$

$$z_A = 1.5 \text{ in.}, \quad z_B = 1.5 \text{ in.}, \quad z_C = -1.5 \text{ in.}, \quad z_D = -1.5 \text{ in.}$$

$$\sigma_A = E\varepsilon_A = (29 \times 10^6)(-500 \times 10^{-6}) = -14{,}500 \text{ psi} = -14.5 \text{ ksi}$$

$$\sigma_B = E\varepsilon_B = (29 \times 10^6)(-1000 \times 10^{-6}) = -29{,}000 \text{ psi} = -29 \text{ ksi}$$

$$\sigma_C = E\varepsilon_C = (29 \times 10^6)(-200 \times 10^{-6}) = -5800 \text{ psi} = -5.8 \text{ ksi}$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -0.06667P - 0.13333M_x - 0.08M_z \qquad (1)$$

$$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -0.06667P - 0.13333M_x + 0.08M_z \qquad (2)$$

$$\sigma_C = -\frac{P}{A} - \frac{M_x z_C}{I_x} + \frac{M_z x_C}{I_z} = -0.06667P + 0.13333M_x + 0.08M_z \qquad (3)$$

Substituting the values for σ_A, σ_B, and σ_C into (1), (2), and (3) and solving the simultaneous equations gives

$$M_x = 87 \text{ kip} \cdot \text{in.} \qquad M_z = -90.625 \text{ kip} \cdot \text{in.}$$

(*a*) $P = 152.3$ kips ◀

$$x = -\frac{M_z}{P} = -\frac{-90.625}{152.25}$$

(*b*) $x = 0.595$ in. ◀

$$z = \frac{M_x}{P} = \frac{87}{152.25}$$

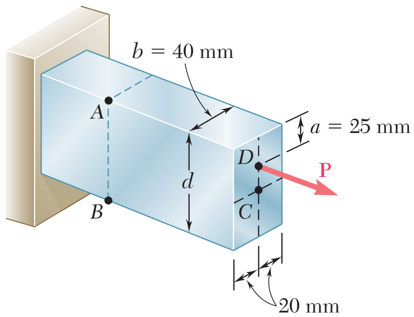
$z = 0.571$ in. ◀

$$\sigma_D = -\frac{P}{A} - \frac{M_x z_D}{I_x} + \frac{M_z x_D}{I_z} = -0.06667P + 0.13333M_x - 0.08M_z$$

$$= -(0.06667)(152.25) + (0.13333)(87) - (0.08)(-90.625) = 8.70 \text{ ksi}$$

(*c*) Strain at hidden edge: $\varepsilon = \dfrac{\sigma_D}{E} = \dfrac{8.70 \times 10^3}{29 \times 10^6}$

$\varepsilon = 300\mu$ ◀



PROBLEM 4.126

The eccentric axial force **P** acts at point D, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (a) the depth d of the bar for which the tensile stress at point A is maximum, (b) the corresponding stress at point A.

SOLUTION

$$A = bd \qquad I = \frac{1}{12}bd^3$$

$$c = \frac{1}{2}d \qquad e = \frac{1}{2}d - a$$

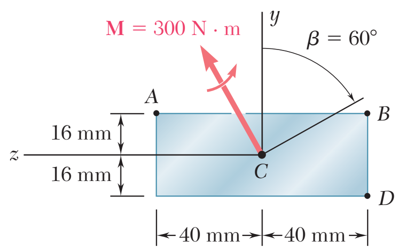
$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

$$\sigma_A = \frac{P}{b}\left\{\frac{1}{d} + \frac{12\left(\frac{1}{2}d - a\right)\left(\frac{1}{2}d\right)}{d^3}\right\} = \frac{P}{b}\left\{\frac{4}{d} - \frac{6a}{d^2}\right\}$$

(a) Depth d for maximum σ_A: Differentiate with respect to d.

$$\frac{d\sigma_A}{dd} = \frac{P}{b}\left\{-\frac{4}{d^2} + \frac{12a}{d^3}\right\} = 0 \qquad d = 3a \qquad d = 75 \text{ mm} \blacktriangleleft$$

(b) $$\sigma_A = \frac{60\times10^3}{40\times10^{-3}}\left\{\frac{4}{75\times10^{-3}} - \frac{(6)(25\times10^{-3})}{(75\times10^{-3})^2}\right\} = 40\times10^6 \text{ Pa} \qquad \sigma_A = 40 \text{ MPa} \blacktriangleleft$$



PROBLEM 4.127

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(80)(32)^3 = 218.45\times 10^3 \text{ mm}^4 = 218.45\times 10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12}(32)(80)^3 = 1.36533\times 10^6 \text{ mm}^4 = 1.36533\times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 16 \text{ mm}$$

$$z_A = -z_B = -z_D = 40 \text{ mm}$$

$$M_y = 300\cos 30° = 259.81 \text{ N}\cdot\text{m} \qquad M_z = 300\sin 30° = 150 \text{ N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(150)(16\times 10^{-3})}{218.45\times 10^{-9}} + \frac{(259.81)(40\times 10^{-3})}{1.36533\times 10^{-6}}$$

$$= -3.37\times 10^6 \text{ Pa}$$

$\sigma_A = -3.37$ MPa ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(150)(16\times 10^{-3})}{218.45\times 10^{-9}} + \frac{(259.81)(-40\times 10^{-3})}{1.36533\times 10^{-6}}$$

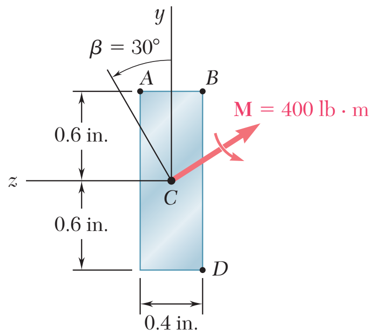
$$= -18.60\times 10^6 \text{ Pa}$$

$\sigma_B = -18.60$ MPa ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(150)(-16\times 10^{-3})}{218.45\times 10^{-9}} + \frac{(259.81)(-40\times 10^{-3})}{1.36533\times 10^{-6}}$$

$$= 3.37\times 10^6 \text{ Pa}$$

$\sigma_D = 3.37$ MPa ◀



PROBLEM 4.128

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(0.4)(1.2)^3 = 57.6\times10^{-3}\ \text{in}^4$$

$$I_y = \frac{1}{12}(1.2)(0.4)^3 = 6.40\times10^{-3}\ \text{in}^4$$

$$y_A = y_B = -y_D = 0.6\ \text{in.}$$

$$z_A = -z_B = -z_D = \left(\frac{1}{2}\right)(0.4) = 0.2\ \text{in.}$$

$$M_y = 400\cos 60° = 200\ \text{lb}\cdot\text{in.},\qquad M_z = -400\sin 60° = -346.41\ \text{lb}\cdot\text{in.}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-346.41)(0.6)}{57.6\times10^{-3}} + \frac{(200)(0.2)}{6.40\times10^{-3}}$$

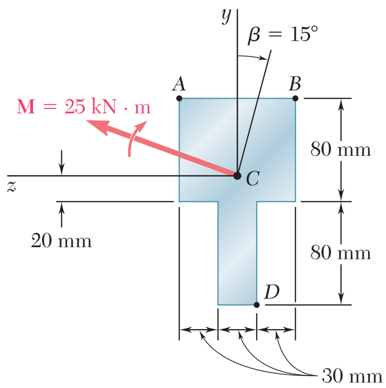
$$= 9.86\times10^3\ \text{psi} = 9.86\ \text{ksi} \quad ◀$$

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-346.41)(0.6)}{57.6\times10^{-6}} + \frac{(200)(-0.2)}{6.4\times10^{-3}}$$

$$= -2.64\times10^3\ \text{psi} = -2.64\ \text{ksi} \quad ◀$$

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-346.41)(-0.6)}{57.6\times10^{-3}} + \frac{(200)(-0.2)}{6.40\times10^{-3}}$$

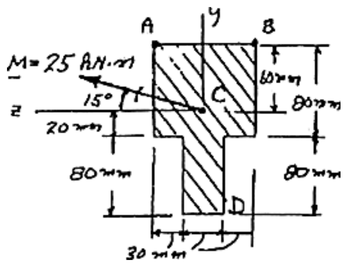
$$= -9.86\times10^3\ \text{psi} = -9.86\ \text{ksi} \quad ◀$$



PROBLEM 4.129

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION



$$M_y = 25\sin 15° = 6.4705 \text{ kN}\cdot\text{m}$$

$$M_z = 25\cos 15° = 24.148 \text{ kN}\cdot\text{m}$$

$$I_y = \frac{1}{12}(80)(90)^3 + \frac{1}{12}(80)(30)^3 = 5.04\times 10^6 \text{ mm}^4$$

$$I_y = 5.04\times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{3}(90)(60)^3 + \frac{1}{3}(60)(20)^3 + \frac{1}{3}(30)(100)^3 = 16.64\times 10^6 \text{mm}^4 = 16.64\times 10^{-6} \text{m}^4$$

Stress: $\sigma = \dfrac{M_y z}{I_y} - \dfrac{M_z y}{I_z}$

(*a*) $\sigma_A = \dfrac{(6.4705 \text{ kN}\cdot\text{m})(0.045 \text{ m})}{5.04\times 10^{-6} \text{ m}^4} - \dfrac{(24.148 \text{ kN}\cdot\text{m})(0.060 \text{ m})}{16.64\times 10^{-6} \text{ m}^4}$

$= 57.772 \text{ MPa} - 87.072 \text{ MPa}$ $\sigma_A = -29.3 \text{ MPa}$ ◀

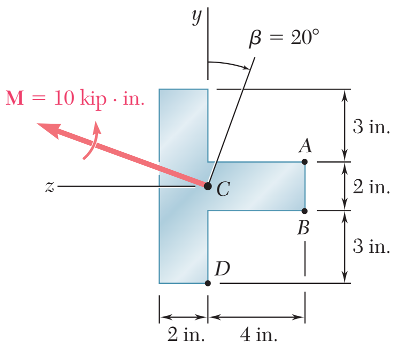
(*b*) $\sigma_B = \dfrac{(6.4705 \text{ kN}\cdot\text{m})(-0.045 \text{ m})}{5.04\times 10^{-6} \text{ m}^4} - \dfrac{(24.148 \text{ kN}\cdot\text{m})(0.060 \text{ m})}{16.64\times 10^{-6} \text{ m}^4}$

$= -57.772 \text{ MPa} - 87.072 \text{ MPa}$ $\sigma_B = -144.8 \text{ MPa}$ ◀

(*c*) $\sigma_D = \dfrac{(6.4705 \text{ kN}\cdot\text{m})(-0.015 \text{ m})}{5.04\times 10^{-6} \text{ m}^4} - \dfrac{(24.148 \text{ kN}\cdot\text{m})(-0.100 \text{ m})}{16.64\times 10^{-6} \text{ m}^4}$

$= -19.257 \text{ MPa} + 145.12 \text{ MPa}$ $\sigma_D = -125.9 \text{ MPa}$ ◀

PROBLEM 4.130

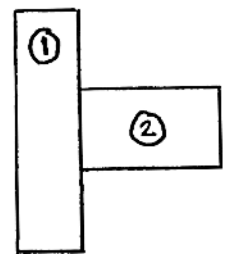


The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

Locate centroid.

	A, in^2	$\bar{z}$, in.	$A\bar{z}$, in^3
①	16	−1	−16
②	8	2	16
Σ	24		0



The centroid lies at point *C*.

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$$y_A = -y_B = 1 \text{ in.}, \qquad y_D = -4 \text{ in.}$$

$$z_A = z_B = -4 \text{ in.}, \qquad z_D = 0$$

$$M_z = 10 \cos 20° = 9.3969 \text{ kip} \cdot \text{in.}$$

$$M_y = 10 \sin 20° = 3.4202 \text{ kip} \cdot \text{in.}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(9.3969)(1)}{88} + \frac{(3.4202)(-4)}{64}$$

$\sigma_A = 0.321$ ksi ◀

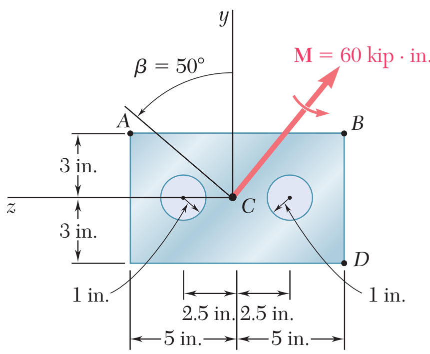
(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(9.3969)(-1)}{88} + \frac{(3.4202)(-4)}{64}$$

$\sigma_B = -0.107$ ksi ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(9.3969)(-4)}{88} + \frac{(3.4202)(0)}{64}$$

$\sigma_D = 0.427$ ksi ◀

PROBLEM 4.131



The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$M_z = -60 \sin 40° = -38.567 \text{ kip} \cdot \text{in.}$$

$$M_y = 60 \cos 40° = 45.963 \text{ kip} \cdot \text{in.}$$

$$y_A = y_B = -y_D = 3 \text{ in.}$$

$$z_A = -z_B = -z_D = 5 \text{ in.}$$

$$I_z = \frac{1}{12}(10)(6)^3 - 2\left[\frac{\pi}{4}(1)^2\right] = 178.429 \text{ in}^4$$

$$I_y = \frac{1}{12}(6)(10)^3 - 2\left[\frac{\pi}{4}(1)^4 + \pi(1)^2(2.5)^2\right] = 459.16 \text{ in}^4$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-38.567)(3)}{178.429} + \frac{(45.963)(5)}{459.16}$$

$$= 1.149 \text{ ksi}$$

$\sigma_A = 1.149 \text{ ksi}$ ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-38.567)(3)}{178.429} + \frac{(45.963)(-5)}{459.16}$$

$$= 0.1479$$

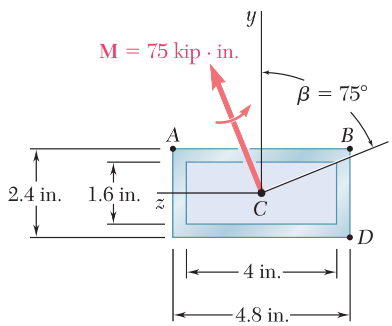
$\sigma_B = 0.1479 \text{ ksi}$ ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-38.567)(-3)}{178.429} + \frac{(45.963)(-5)}{459.16}$$

$$= -1.149 \text{ ksi}$$

$\sigma_D = -1.149 \text{ ksi}$ ◀

PROBLEM 4.132



The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(4.8)(2.4)^3 - \frac{1}{12}(4)(1.6)^3 = 4.1643 \text{ in}^4$$

$$I_y = \frac{1}{12}(2.4)(4.8)^3 - \frac{1}{12}(1.6)(4)^3 = 13.5851 \text{ in}^4$$

$$y_A = y_B = -y_D = 1.2 \text{ in.}$$

$$z_A = -z_B = -z_D = 2.4 \text{ in.}$$

$$M_z = 75\sin 15° = 19.4114 \text{ kip}\cdot\text{in.}$$

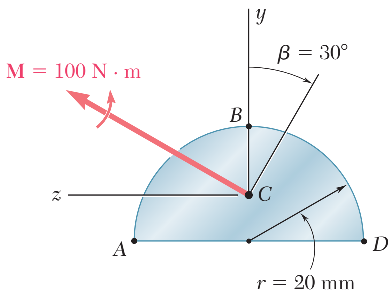
$$M_y = 75\cos 15° = 72.444 \text{ kip}\cdot\text{in.}$$

(*a*) $\sigma_A = -\dfrac{M_z y_A}{I_z} + \dfrac{M_y z_A}{I_y} = -\dfrac{(19.4114)(1.2)}{4.1643} + \dfrac{(72.444)(2.4)}{13.5851}$ $\qquad \sigma_A = 7.20 \text{ ksi}$ ◀

(*b*) $\sigma_B = -\dfrac{M_z y_B}{I_z} + \dfrac{M_y z_B}{I_y} = -\dfrac{(19.4114)(1.2)}{4.1643} + \dfrac{(72.444)(-2.4)}{13.5851}$ $\qquad \sigma_B = -18.39 \text{ ksi}$ ◀

(*c*) $\sigma_D = -\dfrac{M_z y_D}{I_z} + \dfrac{M_y z_D}{I_y} = -\dfrac{(19.4114)(-1.2)}{4.1643} + \dfrac{(72.444)(-2.4)}{13.5851}$ $\qquad \sigma_D = -7.20 \text{ ksi}$ ◀

PROBLEM 4.133



The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(20)^4 = 17.5611\times 10^{-3}\ \text{mm}^4 = 17.5611\times 10^{-9}\ \text{m}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi(20)^4}{8} = 62.832\times 10^{-3}\ \text{mm}^4 = 62.832\times 10^{-9}\ \text{m}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(20)}{3\pi} = -8.4883\ \text{mm}$$

$$y_B = 20 - 8.4883 = 11.5117\ \text{mm}$$

$$z_A = -z_D = 20\ \text{mm} \qquad z_B = 0$$

$$M_z = 100\cos 30° = 86.603\ \text{N}\cdot\text{m}$$

$$M_y = 100\sin 30° = 50\ \text{N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(86.603)(-8.4883\times 10^{-3})}{17.5611\times 10^{-9}} + \frac{(50)(20\times 10^{-3})}{62.832\times 10^{-9}}$$

$$= 57.8\times 10^6\ \text{Pa}$$

$\sigma_A = 57.8$ MPa ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(86.603)(11.5117\times 10^{-3})}{17.5611\times 10^{-9}} + \frac{(50)(0)}{62.832\times 10^{-9}}$$

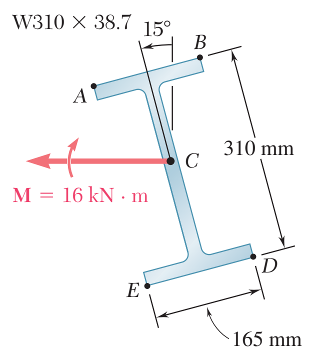
$$= -56.8\times 10^6\ \text{Pa}$$

$\sigma_B = -56.8$ MPa ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(86.603)(-8.4883\times 10^{-3})}{17.5611\times 10^{-9}} + \frac{(50)(-20\times 10^{3})}{62.832\times 10^{-9}}$$

$$= 25.9\times 10^6\ \text{Pa}$$

$\sigma_D = 25.9$ MPa ◀



PROBLEM 4.134

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

For $W310 \times 38.7$ rolled steel shape,

$$I_z = 84.9 \times 10^6 \text{ mm}^4 = 84.9 \times 10^{-6} \text{ m}^4$$

$$I_y = 7.20 \times 10^6 \text{ mm}^4 = 7.20 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = \frac{1}{2} y_D = -y_E = \left(\frac{1}{2}\right)(310) = 155 \text{ mm}$$

$$z_A = z_E = -z_B = -z_D = \left(\frac{1}{2}\right)(165) = 82.5 \text{ mm}$$

$$M_z = (16 \times 10^3) \cos 15° = 15.455 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_y = (16 \times 10^3) \sin 15° = 4.1411 \times 10^3 \text{ N} \cdot \text{m}$$

(*a*) $\tan \phi = \frac{I_z}{I} \tan \theta = \frac{84.9 \times 10^{-6}}{7.20 \times 10^{-6}} \tan 15° = 3.1596$

$$\phi = 72.4°$$

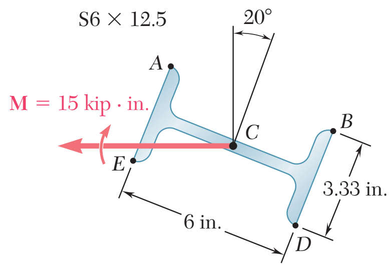
$$\alpha - 72.4 - 15 = 57.4° \quad \blacktriangleleft$$

(*b*) Maximum tensile stress occurs at point *E*.

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455 \times 10^3)(-155 \times 10^{-3})}{84.9 \times 10^{-6}} + \frac{(4.1411 \times 10^3)(82.5 \times 10^{-3})}{7.20 \times 10^{-6}}$$

$$= 75.7 \times 10^6 \text{ Pa} = 75.7 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 4.135



The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

For S6×12.5 rolled steel shape,

$$I_z = 22.0 \text{ in}^4$$

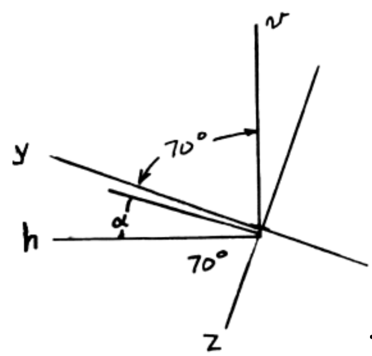
$$I_y = 1.80 \text{ in}^4$$

$$z_E = -z_A = -z_B = z_D = \frac{1}{2}(3.33) = 1.665 \text{ in.}$$

$$y_A = y_B = -y_D = -y_E = \frac{1}{2}(6) = 3 \text{ in.}$$

$$M_z = 15 \sin 20° = 5.1303 \text{ kip} \cdot \text{in.}$$

$$M_y = 15 \cos 20° = 14.095 \text{ kip} \cdot \text{in.}$$



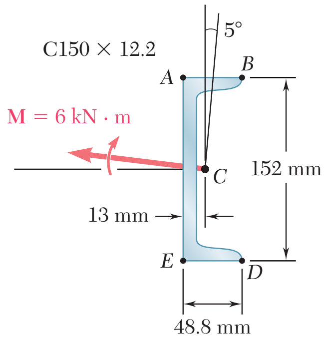
(*a*) $\tan \phi = \dfrac{I_z}{I_y} \tan \theta = \dfrac{22.0}{1.80} \tan (90° - 20°) = 33.58°$

$$\phi = 88.29°$$

$$\alpha = 88.29° - 70° = 18.29° \quad ◀$$

(*b*) Maximum tensile stress occurs at point *D*.

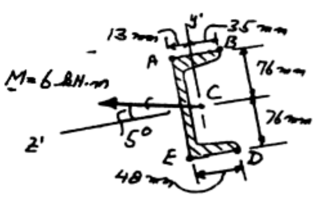
$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(5.1303)(-3)}{22.0} + \frac{(14.095)(1.665)}{1.80} = 13.74 \text{ ksi} \quad ◀$$



PROBLEM 4.136

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION



$$M_{y'} = 6 \sin 5° = 0.52293 \text{ kN} \cdot \text{m}$$

$$M_{z'} = 6 \cos 5° = 5.9772 \text{ kN} \cdot \text{m}$$

$$\underline{\text{C}150 \times 12.2}$$

$$I_{y'} = 0.286 \times 10^{-6} \text{ m}^4$$

$$I_{z'} = 5.45 \times 10^{-6} \text{ m}^4$$

(*a*) Neutral axis:

$$\tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{5.45 \times 10^{-6} \text{ m}^4}{0.286 \times 10^{-6} \text{ m}^4} \tan 5°$$

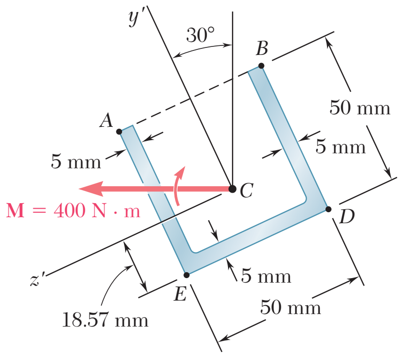
$$\tan \phi = 1.66718 \qquad \phi = 59.044°$$

$$\alpha = \phi - 5° \qquad \alpha = 54.0° \quad \blacktriangleleft$$

(*b*) Maximum tensile stress at E: $y'_E = -76$ mm, $z'_E = 13$ mm

$$\sigma_E = -\frac{M_{y'} z'_E}{I_{y'}} - \frac{M_{z'} y'_E}{I_{z'}} = \frac{(0.52293 \text{ kN} \cdot \text{m})(0.013 \text{ m})}{0.286 \times 10^{-6} \text{ m}^4} - \frac{(5.9772 \text{ kN} \cdot \text{m})(-0.076 \text{ m})}{5.45 \times 10^{-6} \text{ m}^4}$$

$$\sigma_E = 107.1 \text{ MPa} \quad \blacktriangleleft$$



$I_{y'} = 281 \times 10^3 \text{ mm}^4$
$I_{z'} = 176.9 \times 10^3 \text{ mm}^4$

PROBLEM 4.137

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 176.9 \times 10^3 \text{ mm}^4 = 176.9 \times 10^{-9} \text{ m}^4$$
$$I_{y'} = 281 \times 10^3 \text{ mm}^4 = 281 \times 10^{-9} \text{ m}^4$$
$$y'_E = -18.57 \text{ mm}, \quad z_E = 25 \text{ mm}$$
$$M_{z'} = 400 \cos 30° = 346.41 \text{ N} \cdot \text{m}$$
$$M_{y'} = 400 \sin 30° = 200 \text{ N} \cdot \text{m}$$

(*a*) $$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{176.9 \times 10^{-9}}{281 \times 10^{-9}} \cdot \tan 30° = 0.36346$$

$$\varphi = 19.97°$$

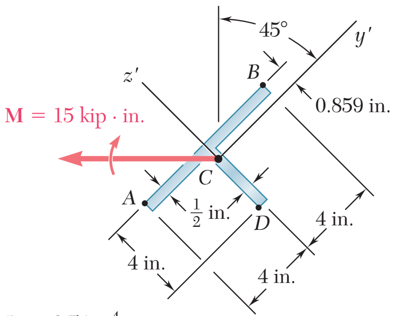
$$\alpha = 30° - 19.97° \qquad \alpha = 10.03° \blacktriangleleft$$

(*b*) Maximum tensile stress occurs at point *E*.

$$\sigma_E = -\frac{M_{z'} y'_E}{I_{z'}} + \frac{M_{y'} z'_E}{I_{y'}} = -\frac{(346.41)(-18.57 \times 10^{-3})}{176.9 \times 10^{-9}} + \frac{(200)(25 \times 10^{-3})}{281 \times 10^{-9}}$$

$$= 36.36 \times 10^6 + 17.79 \times 10^6 = 54.2 \times 10^6 \text{ Pa}$$

$$\sigma_E = 54.2 \text{ MPa} \blacktriangleleft$$



$I_{y'} = 6.74\ \text{in}^4$
$I_{z'} = 21.4\ \text{in}^4$

PROBLEM 4.138

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 21.4\ \text{in}^4 \qquad I_{y'} = 6.74\ \text{in}^4$$

$$z'_A = z'_B = 0.859\ \text{in.} \qquad z'_D = -4 + 0.859\ \text{in.} = -3.141\ \text{in.}$$

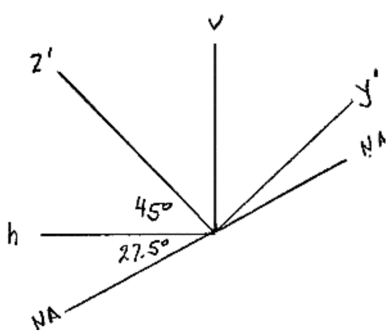
$$y_A = -4\ \text{in.} \qquad y_{B'} = 4\ \text{in.} \qquad y_{D'} = -0.25\ \text{in.}$$

$$M_{y'} = -15\sin 45° = -10.6066\ \text{kip}\cdot\text{in.}$$

$$M_{z'} = 15\cos 45° = 10.6066\ \text{kip}\cdot\text{in.}$$

(*a*) Angle of neutral axis:

$$\tan\varphi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{21.4}{6.74}\tan(-45°) = 3.1751$$



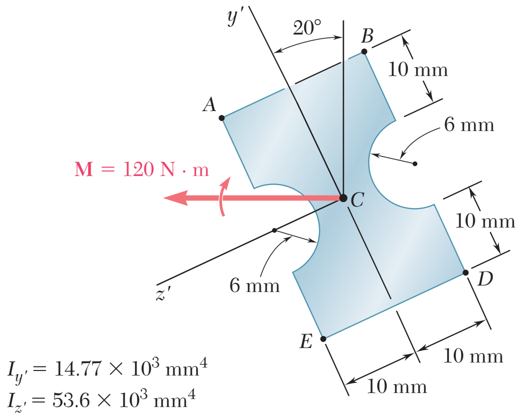
$$\varphi = -72.5°$$

$$\alpha = 72.5° - 45° \qquad \alpha = 27.5° \blacktriangleleft$$

(*b*) The maximum tensile stress occurs at point *D*.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(10.6066)(-0.25)}{21.4} + \frac{(-10.6066)(-3.141)}{6.74}$$

$$= 0.12391 + 4.9429 \qquad \sigma_D = 5.07\ \text{ksi} \blacktriangleleft$$



PROBLEM 4.139

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

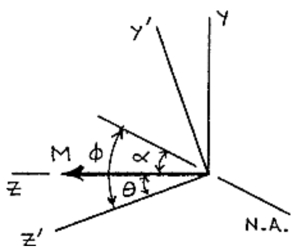
$$I_{z'} = 53.6 \times 10^3 \text{ mm}^4 = 53.6 \times 10^{-9} \text{ m}^4$$

$$I_{y'} = 14.77 \times 10^3 \text{ mm}^4 = 14.77 \times 10^{-9} \text{ m}^4$$

$$M_{z'} = 120 \sin 70° = 112.763 \text{ N} \cdot \text{m}$$

$$M_{y'} = 120 \cos 70° = 41.042 \text{ N} \cdot \text{m}$$

(*a*) Angle of neutral axis: $\theta = 20°$



$$\tan \varphi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{53.6 \times 10^{-9}}{14.77 \times 10^{-9}} \tan 20° = 1.32084$$

$$\varphi = 52.871°$$

$$\alpha = 52.871° - 20° \qquad \alpha = 32.9° \blacktriangleleft$$

(*b*) The maximum tensile stress occurs at point *E*.

$$y'_E = -16 \text{ mm} = -0.016 \text{ m}$$

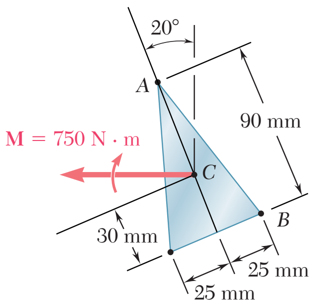
$$z_E = 10 \text{ mm} = 0.010 \text{ m}$$

$$\sigma_E = -\frac{M_{z'} y'_E}{I_{z'}} + \frac{M_{y'} z'_E}{I_{y'}}$$

$$= -\frac{(112.763)(-0.016)}{53.6 \times 10^{-9}} + \frac{(41.042)(0.010)}{14.77 \times 10^{-9}}$$

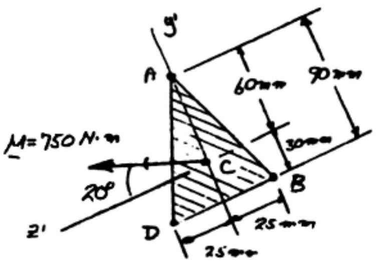
$$= 61.448 \times 10^6 \text{ Pa} \qquad \sigma_E = 61.4 \text{ MPa} \blacktriangleleft$$

PROBLEM 4.140



The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION



$$M_{y'} = 750 \sin 20°$$

$$M_{y'} = 256.5 \text{ N} \cdot \text{m}$$

$$M_{z'} = 750 \cos 20°$$

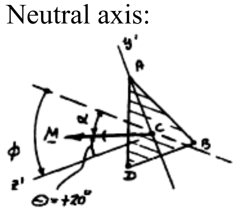
$$M_{z'} = 704.8 \text{ N} \cdot \text{m}$$

$$I_{y'} = 2\left[\frac{1}{12}(90)(25)^3\right] = 0.2344 \times 10^6 \text{ mm}^4$$

$$I_{y'} = 0.2344 \times 10^{-6} \text{ m}^4$$

$$I_{z'} = \frac{1}{36}(50)(90)^3 = 1.0125 \times 10^6 \text{ mm}^4 = 1.0125 \times 10^{-6} \text{ m}^4$$

(*a*) Neutral axis:



$$\tan\phi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1.0125 \times 10^{-6} \text{ m}^4}{0.2344 \times 10^6 \text{ m}^4} \tan 20°$$

$$\tan\phi = 1.5724 \qquad \phi = 57.5°$$

$$\alpha = \phi - 20° = 57.5° - 30° = 37.5°$$

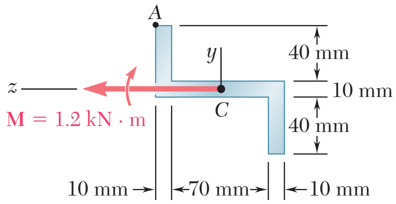
$$\alpha = 37.5° \blacktriangleleft$$

(*b*) Maximum tensile stress, at *D*: $y'_D = -30$ mm $\qquad z' = 25$ mm

$$\sigma_D = \frac{M_{y'}z'_D}{I_{y'}} - \frac{M_{z'}y'_D}{I_{z'}} = \frac{(256.5 \text{ N} \cdot \text{m})(-0.030 \text{ m})}{0.2344 \times 10^{-6} \text{ m}^4} - \frac{(704.8 \text{ N} \cdot \text{m})(0.025 \text{ m})}{1.0125 \times 10^{-6} \text{ m}^4}$$

$$= 32.83 \text{ MPa} + 17.40 \text{ MPa} = 50.23 \text{ MPa} \qquad \sigma_D = 50.2 \text{ MPa} \blacktriangleleft$$

PROBLEM 4.141



$I_y = 1.894 \times 10^6\ \text{mm}^4$
$I_z = 0.614 \times 10^6\ \text{mm}^4$
$I_{yz} = +0.800 \times 10^6\ \text{mm}^4$

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.

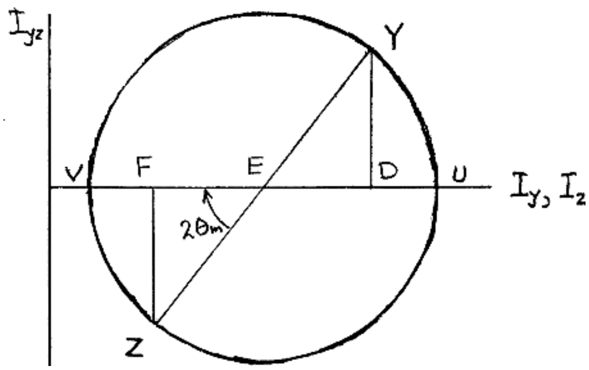
SOLUTION

Using Mohr's circle, determine the principal axes and principal moments of inertia.

$Y: (1.894, 0.800) \times 10^6\ \text{mm}^4$

$Z: (0.614, 0.800) \times 10^6\ \text{mm}^4$

$E: (1.254, 0) \times 10^6\ \text{mm}^4$



$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-6} = 1.0245 \times 10^6\ \text{mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6\ \text{mm}^4 = 0.2295 \times 10^6\ \text{mm}^4 = 0.2295 \times 10^{-6}\ \text{m}^4$$

$$I_u = (1.254 + 1.0245) \times 10^6\ \text{mm}^4 = 2.2785 \times 10^6\ \text{mm}^4 = 2.2785 \times 10^{-6}\ \text{m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \qquad \theta_m = 25.67^\circ$$

$$M_v = M\cos\theta_m = (1.2 \times 10^3)\cos 25.67^\circ = 1.0816 \times 10^3\ \text{N}\cdot\text{m}$$

$$M_u = -M\sin\theta_m = -(1.2 \times 10^3)\sin 25.67^\circ = -0.5198 \times 10^3\ \text{N}\cdot\text{m}$$

$$u_A = y_A\cos\theta_m - z_A\sin\theta_m = 45\cos 25.67^\circ - 45\sin 25.67^\circ = 21.07\ \text{mm}$$

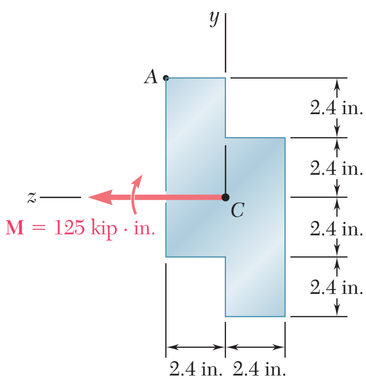
$$v_A = z_A\cos\theta_m + y_A\sin\theta_m = 45\cos 25.67^\circ + 45\sin 25.67^\circ = 60.05\ \text{mm}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6\ \text{Pa}$$

$\sigma_A = 113.0\ \text{MPa}$ ◀

PROBLEM 4.142



The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.

SOLUTION

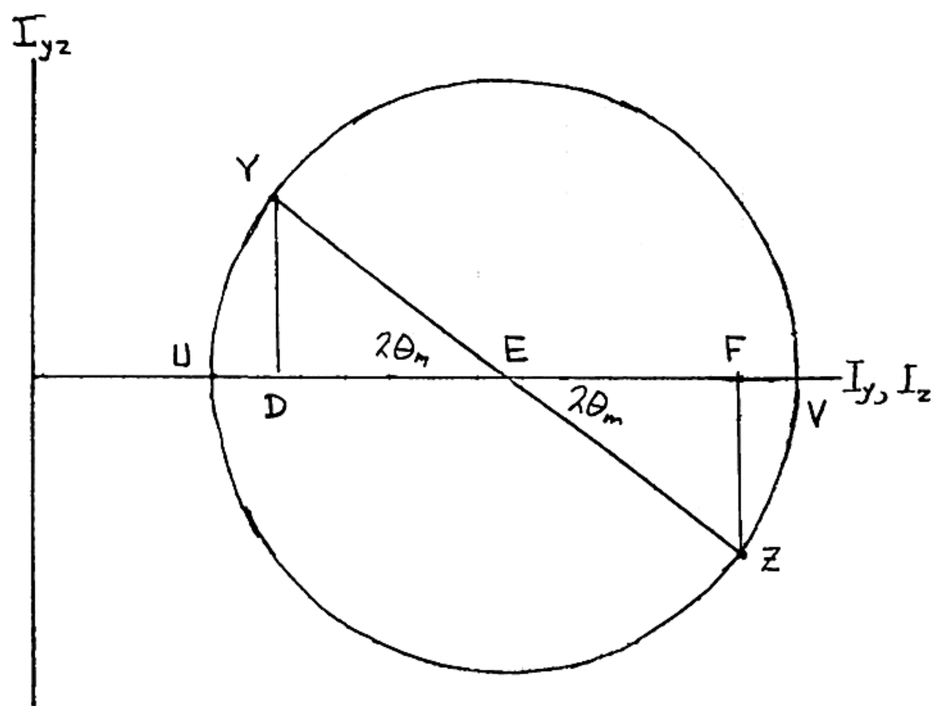
$$I_y = 2\left\{\frac{1}{3}(7.2)(2.4)^3\right\} = 66.355 \text{ in}^4$$

$$I_z = 2\left\{\frac{1}{12}(2.4)(7.2)^3 + (2.4)(7.2)(1.2)^2\right\} = 199.066 \text{ in}^4$$

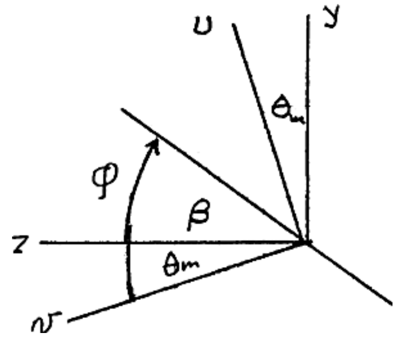
$$I_{yz} = 2\{(2.4)(7.2)(1.2)(1.2)\} = 49.766 \text{ in}^4$$

Using Mohr's circle, determine the principal axes and principal moments of inertia.

$$Y: (66.355 \text{ in}^4,\ 49.766 \text{ in}^4)$$
$$Z: (199.066 \text{ in}^4,\ -49.766 \text{ in}^4)$$
$$E: (132.710 \text{ in}^4,\ 0)$$



PROBLEM 4.142 *(Continued)*



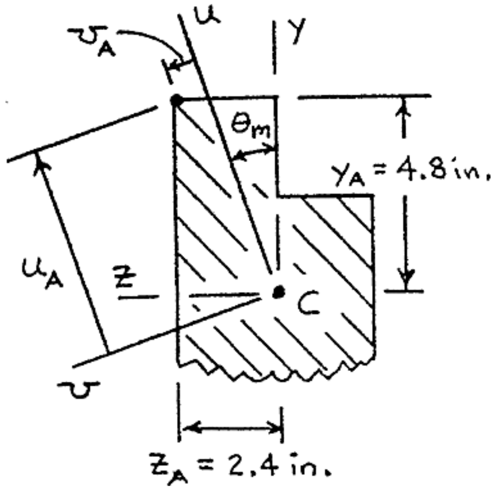
$$\tan 2\theta_m = \frac{DY}{DE} = \frac{49.766}{66.355}$$

$$2\theta_m = 36.87° \quad \theta_m = 18.435°$$

$$R = \sqrt{\overline{DE}^2 + \overline{DY}^2} = 82.944 \text{ in}^4$$

$$I_u = 132.710 - 82.944 = 49.766 \text{ in}^4$$

$$I_v = 132.710 + 82.944 = 215.654 \text{ in}^4$$



$$M_u = 125 \sin 18.435° = 39.529 \text{ kip} \cdot \text{in.}$$

$$M_v = 125 \cos 18.435° = 118.585 \text{ kip} \cdot \text{in.}$$

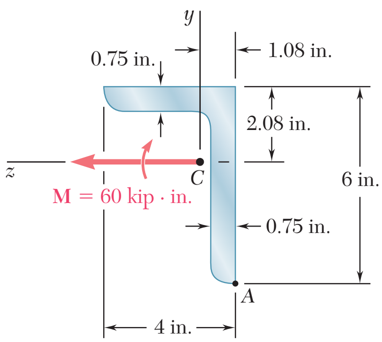
$$u_A = 4.8 \cos 18.435° + 2.4 \sin 18.435° = 5.3126 \text{ in.}$$

$$v_A = -4.8 \sin 18.435° + 2.4 \cos 18.435° = 0.7589 \text{ in.}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u}$$

$$= -\frac{(118.585)(5.3126)}{215.654} + \frac{(39.529)(0.7589)}{49.766}$$

$$= -2.32 \text{ ksi} \qquad \sigma_A = -2.32 \text{ ksi} \blacktriangleleft$$



$I_y = 8.7\ \text{in}^4$
$I_z = 24.5\ \text{in}^4$
$I_{yz} = +8.3\ \text{in}^4$

PROBLEM 4.143

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.

SOLUTION

Using Mohr's circle, determine the principal axes and principal moments of inertia.

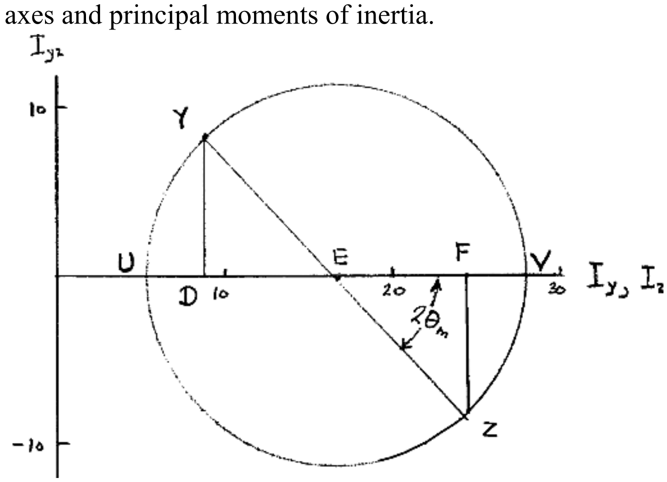
$Y: (8.7,\ 8.3)\ \text{in}^4$

$Z: (24.5,\ -8.3)\ \text{in}^4$

$E: (16.6,\ 0)\ \text{in}^4$

$EF = 7.9\ \text{in}^4$

$FZ = 8.3\ \text{in}^4$



$$R = \sqrt{7.9^2 + 8.3^2} = 11.46\ \text{in}^4 \qquad \tan 2\theta_m = \frac{FZ}{EF} = \frac{8.3}{7.9} = 1.0506$$

$$\theta_m = 23.2^\circ \quad I_u = 16.6 - 11.46 = 5.14\ \text{in}^4 \quad I_v = 16.6 + 11.46 = 28.06\ \text{in}^4$$

$$M_u = M \sin\theta_m = (60)\sin 23.2^\circ = 23.64\ \text{kip}\cdot\text{in.}$$

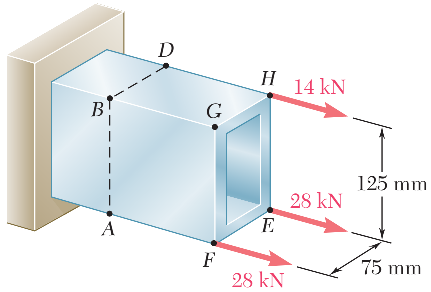
$$M_v = M \cos\theta_m = (60)\cos 23.2^\circ = 55.15\ \text{kip}\cdot\text{in.}$$

$$u_A = y_A \cos\theta_m + z_A \sin\theta_m = -3.92\cos 23.2^\circ - 1.08\sin 23.2^\circ = -4.03\ \text{in.}$$

$$v_A = z_A \cos\theta_m - y_A \sin\theta_m = -1.08\cos 23.2^\circ + 3.92\sin 23.2^\circ = 0.552\ \text{in.}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(55.15)(-4.03)}{28.06} + \frac{(23.64)(0.552)}{5.14}$$

$\sigma_A = 10.46\ \text{ksi}$ ◀

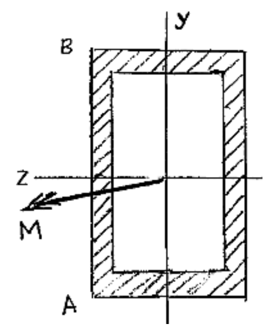


PROBLEM 4.144

The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

SOLUTION

Add *y*- and *z*-axes as shown. Cross section is a 75 mm × 125-mm rectangle with a 51 mm × 101-mm rectangular cutout.



$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283\times10^6\,\text{mm}^4 = 7.8283\times10^{-6}\,\text{m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781\times10^3\,\text{mm}^4 = 3.2781\times10^{-6}\,\text{m}^4$$

$$A = (75)(125) - (51)(101) = 4.224\times10^3\,\text{mm}^2 = 4.224\times10^{-3}\,\text{m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 + 28 = 70\text{ kN} = 70\times10^3\text{ N}$$

$$M_z = -(62.5\text{ mm})(14\text{ kN}) + (62.5\text{ mm})(28\text{kN}) + (62.5\text{ mm})(28\text{ kN}) = 2625\text{ N}\cdot\text{m}$$

$$M_y = -(37.5\text{ mm})(14\text{ kN}) + (37.5\text{ mm})(28\text{ kN}) + (37.5\text{ mm})(28\text{ kN}) = -525\text{ N}\cdot\text{m}$$

(*a*) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{70\times10^3}{4.224\times10^{-3}} - \frac{(2625)(-0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= 31.524\times10^6\,\text{Pa}$$

$\sigma_A = 31.5$ MPa ◀

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{70\times10^3}{4.224\times10^{-3}} - \frac{(2625)(0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= -10.39\times10^6\,\text{Pa}$$

$\sigma_B = -10.39$ MPa ◀

(*b*) Let point *H* be the point where the neutral axis intersects *AB*.

$$z_H = 0.0375\text{ m},\quad y_H = ?,\quad \sigma_H = 0$$

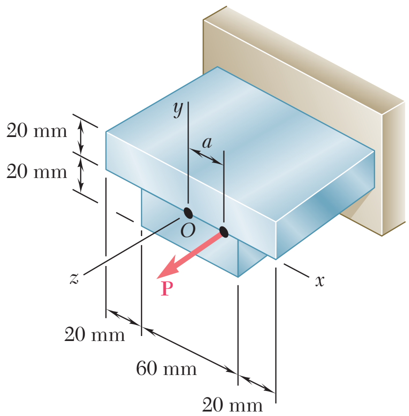
$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

$$y_H = \frac{I_z}{M_z}\left(\frac{P}{A} + \frac{M z_H}{I_y}\right) = \frac{7.8283\times10^{-6}}{2625}\left[\frac{70\times10^3}{4.224\times10^{-3}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}\right]$$

$$= 0.03151\text{ m} = 31.51\text{ mm}$$

$$31.51 + 62.5 = 94.0\text{ mm}$$

Answer: 94.0 mm above point *A*. ◀

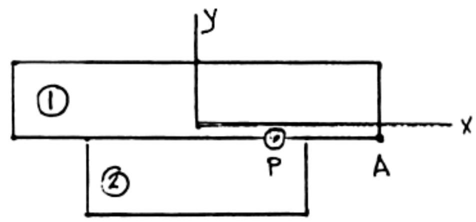


PROBLEM 4.145

A horizontal load **P** of magnitude 100 kN is applied to the beam shown. Determine the largest distance *a* for which the maximum tensile stress in the beam does not exceed 75 MPa.

SOLUTION

Locate the centroid.



	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, mm^3
①	2000	10	20×10^3
②	1200	–10	-12×10^3
Σ	3200		8×10^3

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{8 \times 10^3}{3200} = 2.5 \text{ mm}$$

Move coordinate origin to the centroid.

Coordinates of load point: $X_P = a, \quad y_P = -2.5 \text{ mm}$

Bending couples: $M_x = y_P P \quad M_y = -aP$

$$I_x = \frac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \frac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4$$

$$= 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(20)(100)^3 + \frac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} \qquad \sigma_A = 75 \times 10^6 \text{ Pa}, \quad P = 100 \times 10^3 \text{ N}$$

PROBLEM 4.145 *(Continued)*

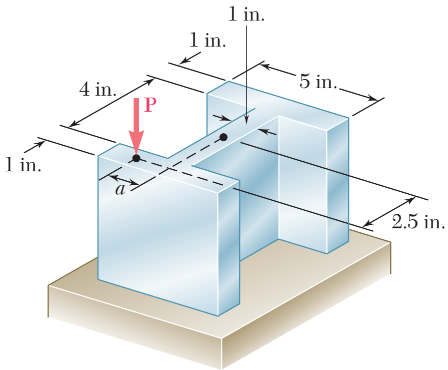
$$M_y = \frac{I_y}{x}\left\{\frac{P}{A} + \frac{M_x y}{I_x} - \sigma\right\} \qquad \text{For point } A, \quad x = 50 \text{ mm}, \; y = -2.5 \text{ mm}$$

$$M_y = \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}}\left\{\frac{100 \times 10^{3}}{3200 \times 10^{-6}} + \frac{(-2.5)(100 \times 10^{3})(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} - 75 \times 10^{6}\right\}$$

$$= \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}}\{31.25 + 1.537 - 75\} \times 10^{6} = -1.7111 \times 10^{3} \text{ N} \cdot \text{m}$$

$$a = -\frac{M_y}{P} = -\frac{(1.7111 \times 10^{3})}{100 \times 10^{3}} = 17.11 \times 10^{3} \text{ m}$$

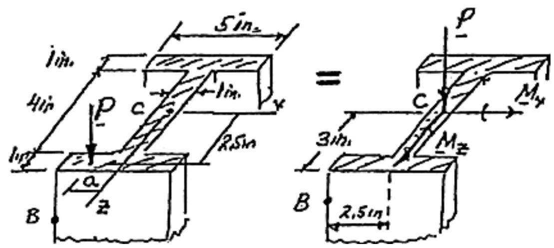
$a = 17.11$ mm ◀



PROBLEM 4.146

Knowing that $P = 90$ kips, determine the largest distance a for which the maximum compressive stress dose not exceed 18 ksi.

SOLUTION



$$A = (5\text{ in.})(6\text{ in.}) - 2(2\text{ in.})(4\text{ in.}) = 14\text{ in}^2$$

$$I_x = \frac{1}{12}(5\text{ in.})(6\text{ in.})^3 - 2\frac{1}{12}(2\text{ in.})(4\text{ in.})^3 = 68.67\text{ in}^4$$

$$I_z = 2\frac{1}{12}(1\text{ in.})(5\text{ in.})^3 + \frac{1}{12}(4\text{ in.})(1\text{ in.})^3 = 21.17\text{ in}^4$$

Force-couple system at C: $\quad P = P \quad M_x = P(2.5\text{ in.}) \quad M_z = Pa$

For $P = 90$ kips: $\quad P = 90\text{ kips} \quad M_x = (90\text{ kips})(2.5\text{ in.}) = 225\text{ kip}\cdot\text{in.} \quad M_z = (90\text{kips})\,a$

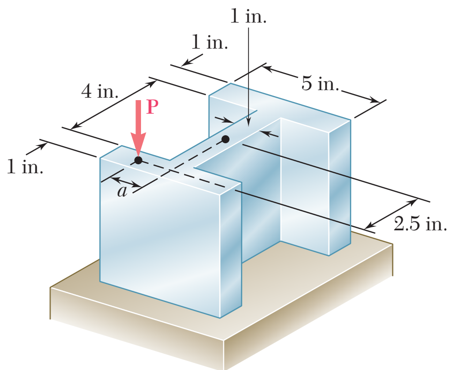
Maximum compressive stress at B: $\quad \sigma_B = -18\text{ ksi}$

$$\sigma_B = -\frac{P}{A} - \frac{M_x(3\text{ in.})}{I_x} - \frac{M_z(2.5\text{ in.})}{I_z}$$

$$-18\text{ ksi} = -\frac{90\text{ kips}}{14\text{ in}^2} - \frac{(225\text{ kip}\cdot\text{in.})(3\text{ in.})}{68.67\text{ in}^4} - \frac{(90\text{ kips})\,a\,(2.5\text{ in.})}{21.17\text{ in}^4}$$

$$-18 = -6.429 - 9.830 - 10.628a$$

$$-1.741 = -10.628\,a \qquad a = 0.1638\text{ in.} \blacktriangleleft$$



PROBLEM 4.147

Knowing that $a = 1.25$ in., determine the largest value of **P** that can be applied without exceeding either of the following allowable stresses:

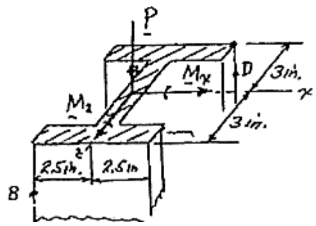
$$\sigma_{\text{ten}} = 10 \text{ ksi} \qquad \sigma_{\text{comp}} = 18 \text{ ksi}$$

SOLUTION

$$A = (5 \text{ in.})(6 \text{ in.}) - (2)(2 \text{ in.})(4 \text{ in.}) = 14 \text{ in}^2$$

$$I_x = \frac{1}{12}(5 \text{ in.})(6 \text{ in.})^3 - 2\frac{1}{12}(2 \text{ in.})(4 \text{ in.})^3 = 68.67 \text{ in}^4$$

$$I_z = 2\frac{1}{12}(1 \text{ in.})(5 \text{ in.})^3 + \frac{1}{12}(4 \text{ in.})(1 \text{ in.})^3 = 21.17 \text{ in}^4$$



Force-couple system at C: For $a = 1.25$ in.,

$$P = P \quad M_x = P(2.5 \text{ in.})$$

$$M_y = Pa = (1.25 \text{ in.})$$

Maximum compressive stress at B: $\sigma_B = -18$ ksi

$$\sigma_B = -\frac{P}{A} - \frac{M_x(3 \text{ in.})}{I_x} - \frac{M_z(2.5 \text{ in.})}{I_z}$$

$$-18 \text{ ksi} = -\frac{P}{14 \text{ in}^2} - \frac{P(2.5 \text{ in.})(3 \text{ in.})}{68.67 \text{ in}^4} - \frac{P(1.25 \text{ in.})(2.5 \text{ in.})}{21.17 \text{ in}^4}$$

$$-18 = -0.0714P - 0.1092P - 0.1476P$$

$$-18 = 0.3282P \qquad P = 54.8 \text{ kips}$$

Maximum tensile stress at D: $\sigma_D = +10$ ksi

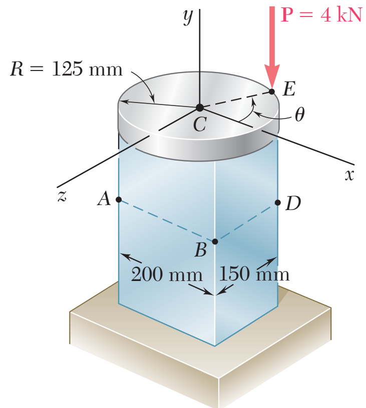
$$\sigma_D = -\frac{P}{A} + \frac{M_x(3 \text{ in.})}{I_x} + \frac{M_z(2.5 \text{ in.})}{I_z}$$

$$+10 \text{ ksi} = -0.0714P + 0.1092P + 0.1476P$$

$$10 = 0.1854P \qquad P = 53.9 \text{ kips}$$

The smaller value of **P** is the largest allowable value.

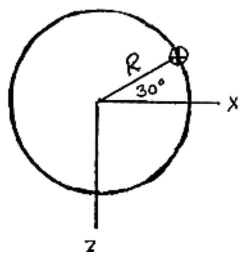
$P = 53.9$ kips ◀



PROBLEM 4.148

A rigid circular plate of 125-mm radius is attached to a solid 150×200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at E with $\theta = 30°$, determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the point where the neutral axis intersects line ABD.

SOLUTION

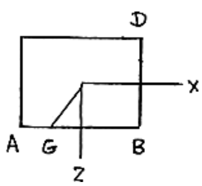


$$P = 4 \times 10^3 \text{ N (compression)}$$

$$M_x = -PR\sin 30° = -(4 \times 10^3)(125 \times 10^{-3})\sin 30° = -250 \text{ N} \cdot \text{m}$$

$$M_z = -PR\cos 30° = -(4 \times 10^3)(125 \times 10^{-3})\cos 30° = -433 \text{ N} \cdot \text{m}$$

$$I_x = \frac{1}{12}(200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4 = 56.25 \times 10^{-6} \text{ m}^4$$



$$I_z = \frac{1}{12}(150)(200)^3 = 100 \times 10^6 \text{ mm}^4 = 100 \times 10^{-6} \text{ m}^4$$

$$-x_A = x_B = 100 \text{ mm} \qquad z_A = z_B = 75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-3} \text{ m}^2$$

(*a*) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(-100 \times 10^{-3})}{100 \times 10^{-6}}$$

$$\sigma_A = 633 \times 10^3 \text{ Pa} = 633 \text{ kPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(100 \times 10^{-3})}{100 \times 10^{-6}}$$

$$\sigma_B = -233 \times 10^3 \text{ Pa} = -233 \text{ kPa} \blacktriangleleft$$

(*c*) Let G be the point on AB where the neutral axis intersects.

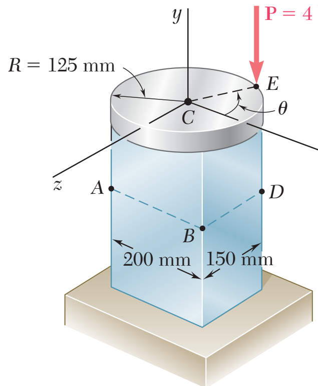
$$\sigma_G = 0 \qquad z_G = 75 \text{ mm} \qquad x_G = ?$$

$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z}\left\{\frac{P}{A} + \frac{M_x z_G}{I_x}\right\} = \frac{100 \times 10^{-6}}{-433}\left\{\frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}}\right\}$$

$$= 46.2 \times 10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point G lies 146.2 mm from point A. ◀



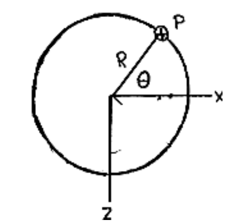
PROBLEM 4.149

In Prob. 4.148, determine (*a*) the value of θ for which the stress at D reaches it largest value, (*b*) the corresponding values of the stress at A, B, C, and D.

PROBLEM 4.148 A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at E with $\theta = 30°$, determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the point where the neutral axis intersects line ABD.

SOLUTION

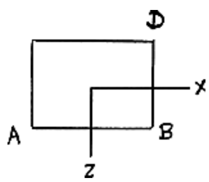
(*a*)



$$P = 4 \times 10^3\ \text{N} \qquad PR = (4 \times 10^3)(125 \times 10^{-3}) = 500\ \text{N} \cdot \text{m}$$

$$M_x = -PR\sin\theta = -500\sin\theta \qquad M_x = -PR\cos\theta = -500\cos\theta$$

$$I_x = \frac{1}{2}(200)(150)^3 = 56.25 \times 10^6\ \text{mm}^4 = 56.25 \times 10^{-6}\ \text{m}^4$$



$$I_z = \frac{1}{2}(150)(200)^3 = 100 \times 10^6\ \text{mm}^4 = 100 \times 10^{-6}\ \text{m}^4$$

$$x_D = 100\ \text{mm} \qquad z_D = -75\ \text{mm}$$

$$A = (200)(150) = 30 \times 10^3\ \text{mm}^2 = 30 \times 10^{-3}\ \text{m}^2$$

$$\sigma = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -P\left\{\frac{1}{A} - \frac{Rz\sin\theta}{I_x} + \frac{Rx\cos\theta}{I_z}\right\}$$

For σ to be a maximum, $\dfrac{d\sigma}{d\theta} = 0$ with $z = z_D$, $x = x_D$

$$\frac{d\sigma_D}{d\theta} = -P\left\{0 + \frac{Rz_D\cos\theta}{I_x} + \frac{Rx_D\sin\theta}{I_Z}\right\} = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{I_z z_D}{I_x x_D} = -\frac{(100 \times 10^{-6})(-75 \times 10^{-3})}{(56.25 \times 10^{-6})(100 \times 10^{-3})} = \frac{4}{3}$$

$$\sin\theta = 0.8, \qquad \cos\theta = 0.6 \qquad\qquad \theta = 53.1° \blacktriangleleft$$

(*b*)

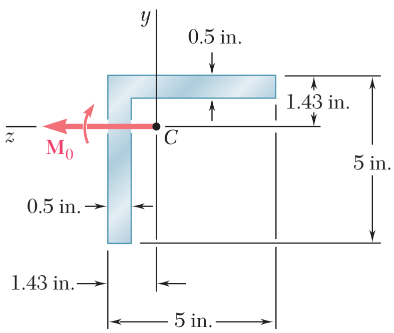
$$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(500)(0.8)(75 \times 10^{-3})}{56.25 \times 10^{-6}} - \frac{(500)(0.6)(-100 \times 10^{-3})}{100 \times 10^{-6}}$$

$$= (-0.13333 + 0.53333 + 0.300) \times 10^6\ \text{Pa} = 0.700 \times 10^6\ \text{Pa} \qquad \sigma_A = 700\ \text{kPa} \blacktriangleleft$$

$$\sigma_B = (-0.13333 + 0.53333 - 0.300) \times 10^6\ \text{Pa} = 0.100 \times 10^6\ \text{Pa} \qquad \sigma_B = 100\ \text{kPa} \blacktriangleleft$$

$$\sigma_C = (-0.13333 + 0 + 0) \times 10^6\ \text{Pa} \qquad \sigma_C = -133.3\ \text{kPa} \blacktriangleleft$$

$$\sigma_D = (-0.13333 - 0.53333 - 0.300) \times 10^6\ \text{Pa} = -0.967 \times 10^6\ \text{Pa} \qquad \sigma_D = -967\ \text{kPa} \blacktriangleleft$$



PROBLEM 4.150

A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3 \text{ in}^4$, $A = 4.75 \text{ in}^2$, $k_{\min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{\min} = Ak_{\min}^2$ and $I_{\min} + I_{\max} = I_y + I_z$.)

SOLUTION

$$M_u = M_0 \sin 45° = 0.70711 M_0$$

$$M_v = M_0 \cos 45° = 0.70711 M_0$$

$$I_{\min} = Ak_{\min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$u_B = y_B \cos 45° + z_B \sin 45° = -3.57 \cos 45° + 0.93 \sin 45° = -1.866 \text{ in.}$$

$$v_B = z_B \cos 45° - y_B \sin 45° = 0.93 \cos 45° - (-3.57) \sin 45° = 3.182 \text{ in.}$$

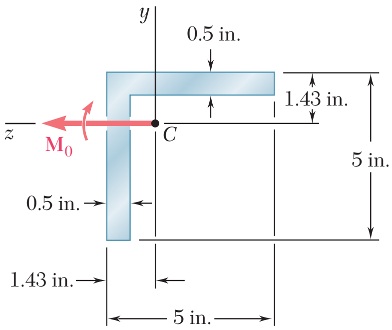
$$\sigma_B = -\frac{M_v u_B}{Iv} + \frac{M_u v_B}{Iu} = -0.70711 M_0 \left[-\frac{u_B}{I_{\min}} + \frac{v_B}{I_{\max}} \right]$$

$$= 0.70711 M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124 M_0$$

$$M_0 = \frac{\sigma_B}{0.4124} = \frac{12}{0.4124}$$

$M_0 = 29.1 \text{ kip} \cdot \text{in.}$ ◀

PROBLEM 4.151



Solve Prob. 4.150, assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

PROBLEM 4.150 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3 \text{ in}^4$, $A = 4.75 \text{ in}^2$, $k_{min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak_{min}^2$ and $I_{min} + I_{max} = I_y + I_z$.)

SOLUTION

$$M_u = M_0 \cos 45° = 0.70711 M_0$$

$$M_v = -M_0 \sin 45° = -0.70711 M_0$$

$$I_{min} = Ak_{min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{max} = I_y + I_z - I_{min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$u_D = y_D \cos 45° + z_D \sin 45° = -0.93 \cos 45° + (-3.57 \sin 45°) = -1.866 \text{ in.}$$

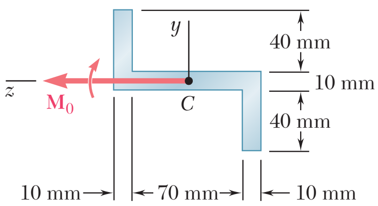
$$v_D = z_D \cos 45° - y_D \sin 45° = (-3.57) \cos 45° - (0.93) \sin 45° = 3.182 \text{ in.}$$

$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -0.70711 M_0 \left[-\frac{u_D}{I_{min}} + \frac{v_D}{I_{max}} \right]$$

$$= 0.70711 M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124 M_0$$

$$M_0 = \frac{\sigma_D}{0.4124} = \frac{12}{0.4124}$$

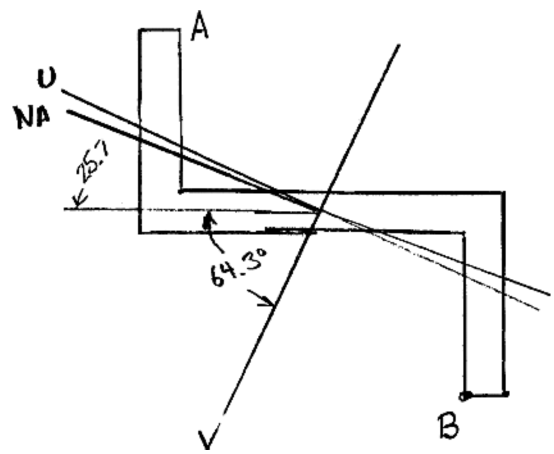
$M_0 = 29.1 \text{ kip} \cdot \text{in.}$ ◀



PROBLEM 4.152

The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{max} = 2.28 \times 10^{-6}$ mm^4, $I_{min} = 0.23 \times 10^{-6}$ mm^4, principal axes 25.7° and 64.3°.

SOLUTION



$$I_v = I_{max} = 2.28 \times 10^6 \text{ mm}^4 = 2.28 \times 10^{-6} \text{ m}^4$$

$$I_u = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^{-6} \text{ m}^4$$

$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan \varphi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3°$$

$$= 20.597$$

$$\varphi = 87.22°$$

Points A and B are farthest from the neutral axis.

$$u_B = y_B \cos 64.3° + z_B \sin 64.3° = (-45) \cos 64.3° + (-35) \sin 64.3°$$

$$= -51.05 \text{ mm}$$

$$v_B = z_B \cos 64.3° - y_B \sin 64.3° = (-35) \cos 64.3° - (-45) \sin 64.3°$$

$$= +25.37 \text{ mm}$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u}$$

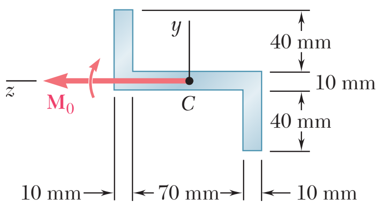
$$80 \times 10^6 = -\frac{(M_0 \cos 64.3°)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_0 \sin 64.3°)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$

$$= 109.1 \times 10^3 M_0$$

$$M_0 = \frac{80 \times 10^6}{109.1 \times 10^3}$$

$M_0 = 733 \text{ N} \cdot \text{m}$ ◀

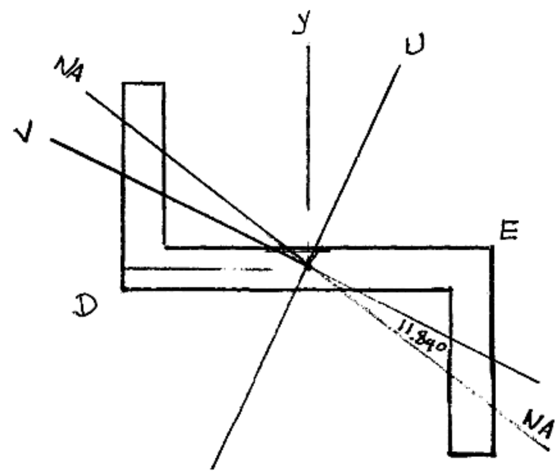
PROBLEM 4.153



Solve Prob. 4.152 assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

PROBLEM 4.152 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{max} = 2.28 \times 10^{-6}$ mm^4, $I_{min} = 0.23 \times 10^{-6}$ mm^4, principal axes 25.7° and 64.3°.

SOLUTION



$$I_v = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^6 \text{ m}^4$$

$$I_u = I_{max} = 2.28 \times 10^6 \text{ mm}^4 = 2.28 \times 10^6 \text{ m}^4$$

$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan \varphi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{0.23 \times 10^{-6}}{2.28 \times 10^{-6}} \tan 64.3°$$

$$= 0.20961$$

$$\varphi = 11.84°$$

Points D and E are farthest from the neutral axis.

$$u_D = y_D \cos 25.7° - z_D \sin 25.7° = (-5) \cos 25.7° - 45 \sin 25.7°$$

$$= -24.02 \text{ mm}$$

$$v_D = z_D \cos 25.7° + y_D \sin 25.7° = 45 \cos 25.7° + (-5) \sin 25.7°$$

$$= 38.38 \text{ mm}$$

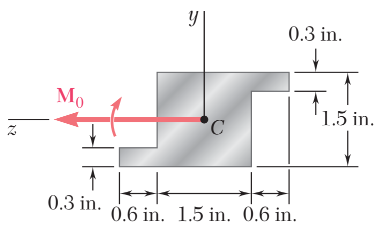
$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -\frac{(M_D \cos 64.3°)(-24.02 \times 10^{-3})}{0.23 \times 10^{-6}}$$

$$+ \frac{(M_0 \sin 64.3°)(38.38 \times 10^{-3})}{2.28 \times 10^{-6}}$$

$$80 \times 10^6 = 60.48 \times 10^3 M_0$$

$$M_0 = 1.323 \times 10^3 \text{ N} \cdot \text{m}$$

$M_0 = 1.323$ kN · m ◀



PROBLEM 4.154

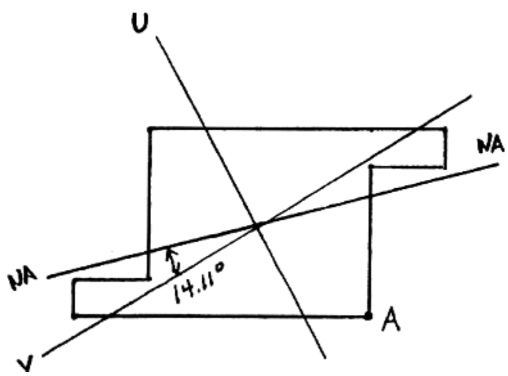
An extruded aluminum member having the cross section shown is subjected to a couple acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 12 ksi. *Given:* $I_{max} = 0.957\ \text{in}^4$, $I_{min} = 0.427\ \text{in}^4$, principal axes 29.4° and 60.6°.

SOLUTION

$$I_u = I_{max} = 0.957\ \text{in}^4$$

$$I_v = I_{min} = 0.427\ \text{in}^4$$

$$M_u = M_0 \sin 29.4°, \quad M_v = M_0 \cos 29.4°$$



$$\theta = 29.4°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.427}{0.957}\tan 29.4°$$

$$= 0.2514 \qquad \varphi = 14.11°$$

Point A is farthest from the neutral axis.

$$y_A = -0.75\ \text{in.}, \quad z_A = -0.75\ \text{in.}$$

$$u_A = y_A \cos 29.4° + z_A \sin 29.4° = -1.0216\ \text{in.}$$

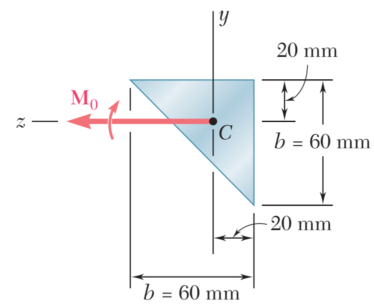
$$v_A = z_A \cos 29.4° - y_A \sin 29.4° = -0.2852\ \text{in.}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u V_A}{I_u} = -\frac{(M_0 \cos 29.4°)(-1.0216)}{0.427} + \frac{(M_0 \sin 29.4°)(-0.2852)}{0.957}$$

$$= 1.9381 M_0$$

$$M_0 = \frac{\sigma_A}{1.9381} = \frac{12}{1.9381}$$

$M_0 = 6.19\ \text{kip} \cdot \text{in.}$ ◀



PROBLEM 4.155

A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 100 MPa. *Given:* $I_y = I_z = b^4/36$ and $I_{yz} = b^4/72$.

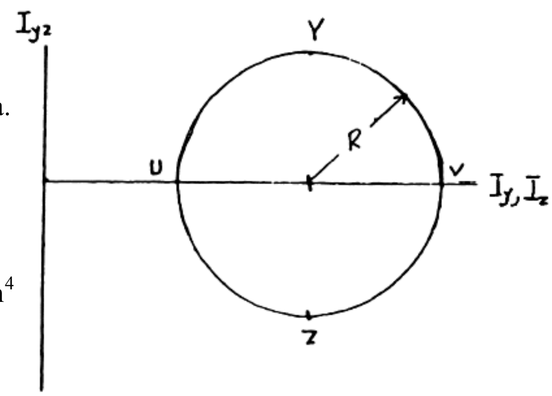
SOLUTION

$$I_y = I_z = \frac{b^4}{36} = \frac{60^4}{36} = 0.360 \times 10^6 \text{ mm}^4$$

$$I_{yz} = \frac{b^4}{72} = \frac{60^4}{72} = 0.180 \times 10^6 \text{ mm}^4$$

Principal axes are symmetry axes.

Using Mohr's circle, determine the principal moments of inertia.



$$R = |I_{yz}| = 0.180 \times 10^6 \text{ mm}^4$$

$$I_v = \frac{I_y + I_z}{2} + R$$

$$= 0.540 \times 10^6 \text{ mm}^4 = 0.540 \times 10^{-6} \text{ m}^4$$

$$I_u = \frac{I_y + I_z}{2} - R$$

$$= 0.180 \times 10^6 \text{ mm}^4 = 0.180 \times 10^{-6} \text{ m}^4$$

$$M_u = M_0 \sin 45° = 0.70711 M_0, \qquad M_v = M_0 \cos 45° = 0.70711 M_0$$

$$\theta = 45° \qquad \tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.540 \times 10^{-6}}{0.180 \times 10^{-6}}\tan 45° = 3$$

$$\varphi = 71.56°$$

<u>Point *A*</u>: $\quad u_A = 0 \qquad v_A = -20\sqrt{2}$ mm

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = 0 + \frac{(0.70711 M_0)(-20\sqrt{2} \times 10^3)}{0.180 \times 10^{-6}} = -11.11 \times 10^{-3} M_0$$

$$M_0 = -\frac{\sigma_A}{111.11 \times 10^3} = -\frac{-100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N} \cdot \text{m}$$

PROBLEM 4.155 *(Continued)*

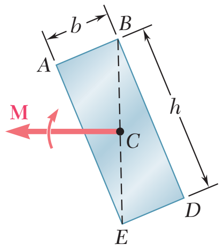
Point B: $u_B = -\dfrac{60}{\sqrt{2}}$ mm, $v_B = \dfrac{20}{\sqrt{2}}$ mm

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u} = -\frac{(0.70711 M_0)\left(-\frac{60}{\sqrt{2}} \times 10^{-3}\right)}{0.540 \times 10^{-6}} + \frac{(0.70711 M_0)\left(\frac{20}{\sqrt{2}} \times 10^{-3}\right)}{0.180 \times 10^{-6}}$$

$$= -111.11 \times 10^3 M_0$$

$$M_0 = \frac{\sigma_B}{111.11 \times 10^{-3}} = \frac{100 \times 10^6}{111.11 \times 10^{-3}} = 900 \text{ N} \cdot \text{m}$$

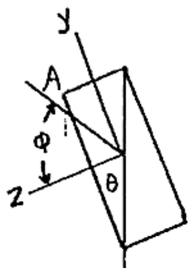
Choose the smaller value. $M_0 = 900 \text{ N} \cdot \text{m}$ ◀



PROBLEM 4.156

Show that, if a solid rectangular beam is bent by a couple applied in a plane containing one diagonal of a rectangular cross section, the neutral axis will lie along the other diagonal.

SOLUTION



$$\tan\theta = \frac{b}{h}$$

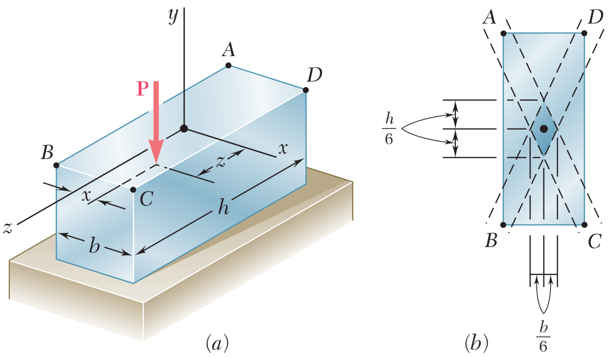
$$M_z = M\cos\theta, \quad M_z = M\sin\theta$$

$$I_z = \frac{1}{12}bh^3 \qquad I_y = \frac{1}{12}hb^3$$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{\frac{1}{12}bh^3}{\frac{1}{12}hb^3}\cdot\frac{b}{h} = \frac{h}{b}$$

The neutral axis passes through corner A of the diagonal AD. ◀

PROBLEM 4.157



(*a*) Show that the stress at corner *A* of the prismatic member shown in part *a* of the figure will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6} + \frac{z}{h/6} = 1$$

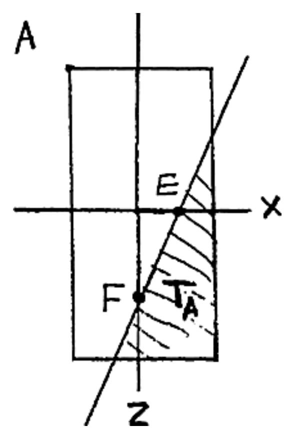
(*b*) Further show that, if no tensile stress is to occur in the member, the force **P** must be applied at a point located within the area bounded by the line found in part *a* and three similar lines corresponding to the condition of zero stress at *B*, *C*, and *D*, respectively. This area, shown in part *b* of the figure, is known as the *kern* of the cross section.

SOLUTION

$$I_z = \frac{1}{12}hb^3 \qquad I_x = \frac{1}{12}bh^3 \qquad A = bh$$

$$z_A = -\frac{h}{2} \qquad x_A = -\frac{b}{2}$$

Let *P* be the load point.



$$M_z = -Px_P \quad M_x = Pz_P$$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x}$$

$$= -\frac{P}{bh} + \frac{(-Px_P)\left(-\frac{b}{2}\right)}{\frac{1}{12}hb^3} - \frac{\left(Pz_P - \frac{h}{2}\right)}{\frac{1}{12}bh^3}$$

$$= -\frac{P}{bh}\left[1 - \frac{x_P}{b/6} - \frac{z_P}{h/6}\right]$$

(*a*) For $\sigma_A = 0, \quad 1 - \frac{x}{b/6} - \frac{z}{h/6} = 0 \qquad \frac{x}{b/6} + \frac{z}{h/6} = 1$

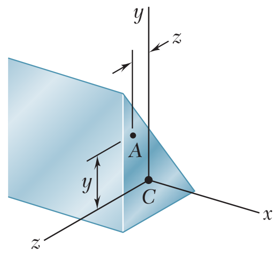
(*b*) At point E: $z = 0 \quad \therefore \quad x_E = b/6$

At point F: $x = 0 \quad \therefore \quad z_F = h/6$

If the line of action (x_P, z_P) lies within the portion marked T_A, a tensile stress will occur at corner *A*.

By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.

PROBLEM 4.158



A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the horizontal plane xz. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = \frac{zI_z - yI_{yz}}{I_y I_z - I_{yz}^2} M_y$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_y the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes:

$$\sigma_A = C_1 y + C_2 z$$

where C_1 and C_2 are constants.

$$M_z = -\int y\sigma_A dA = -C_1 \int y^2 dA - C_2 \int yz\, dA$$

$$= -I_z C_1 - I_{yz} C_2 = 0$$

$$C_1 = -\frac{I_{yz}}{I_z} C_2$$

$$M_y = \int z\sigma_A dA = C_1 \int yz\, dA + C_2 \int z^2 dA$$

$$= I_{yz} C_1 + I_y C_2$$

$$- I_{yz} \frac{I_{yz}}{I_z} C_2 + I_y C_2$$

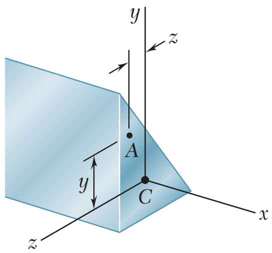
$$I_z M_y = \left(I_y I_z - I_{yz}^2\right) C_2$$

$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2}$$

$$C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{I_z z - I_{yz} y}{I_y I_z - I_{yz}^2} M_y \quad \blacktriangleleft$$

PROBLEM 4.159



A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the vertical plane xy. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = \frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_z$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_z the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes:

$$\sigma_A = C_1 y + C_2 z$$

where C_1 and C_2 are constants.

$$M_y = \int z\sigma_A dA = C_1 \int yz\, dA + C_2 \int z^2 dA$$

$$= I_{yz}C_1 + I_y C_2 = 0$$

$$C_2 = -\frac{I_{yz}}{I_y} C_1$$

$$M_z = -\int y\sigma_A dz = -C_1 \int y^2 dA + C_2 \int yz\, dA$$

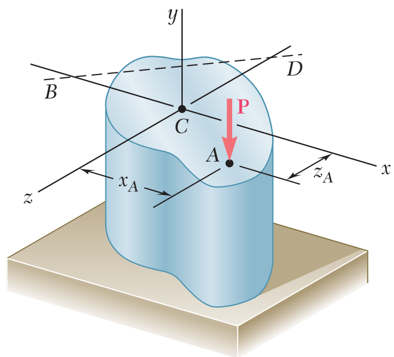
$$= -I_z C_1 - I_{yz}\frac{I_{yz}}{I_y} C_1$$

$$I_y M_z = -\left(I_y I_z - I_{yz}^2\right) C_1$$

$$C_1 = -\frac{I_y M_z}{I_y I_z - I_{yz}^2}$$

$$C_2 = +\frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = -\frac{I_y y - I_{yz^2}}{I_y I_z - I_{yz}^2} M_z \quad \blacktriangleleft$$



PROBLEM 4.160

(a) Show that, if a vertical force **P** is applied at point A of the section shown, the equation of the neutral axis BD is

$$\left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = -1$$

where r_z and r_x denote the radius of gyration of the cross section with respect to the z axis and the x axis, respectively. (b) Further show that, if a vertical force **Q** is applied at any point located on line BD, the stress at point A will be zero.

SOLUTION

Definitions:

$$r_x^2 = \frac{I_x}{A}, \quad r_z^2 = \frac{I_z}{A}$$

(a) $\quad M_x = Pz_A \qquad M_z = -Px_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{Px_A x_E}{Ar_z^2} - \frac{Pz_A z_E}{Ar_x^2}$$

$$= -\frac{P}{A}\left[1 + \left(\frac{x_A}{r_z^2}\right)x_E + \left(\frac{z_A}{r_x^2}\right)z_E\right] = 0$$

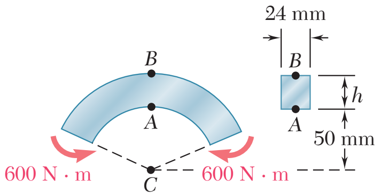
if E lies on neutral axis.

$$1 + \left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = 0, \qquad \left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = -1$$ ◀

(b) $\quad M_x = Pz_E \quad M_z = -Px_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_y} = -\frac{P}{A} - \frac{Px_E x_A}{Ar_z^2} - \frac{Pz_E z_A}{Ar_x^2}$$

$= 0$ by equation from part (a). ◀



PROBLEM 4.161

For the curved bar shown, determine the stress at point A when (*a*) $h = 50$ mm, (*b*) $h = 60$ mm.

SOLUTION

(*a*) $h = 50 \text{ mm}, \qquad r_1 = 50 \text{ mm}, \qquad r_2 = 100 \text{ mm}$

$$A = (24)(50) = 1.200 \times 10^3 \text{ mm}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{50}{\ln \frac{100}{50}} = 72.13475 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 75 \text{ mm}$$

$$e = \bar{r} - R = 2.8652 \text{ mm}$$

$$y_A = 72.13475 - 50 = 22.13475 \text{ mm} \qquad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(22.13475 \times 10^{-3})}{(1.200 \times 10^{-3})(2.8652 \times 10^{-3})(50 \times 10^{-3})} = -77.3 \times 10^6 \text{ Pa}$$

−77.3 MPa ◀

(*b*) $h = 60 \text{ mm}, \qquad r_1 = 50 \text{ mm}, \qquad r_2 = 110 \text{ mm}, \qquad A = (24)(60) = 1.440 \times 10^3 \text{ mm}^2$

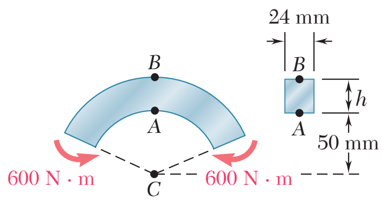
$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{60}{\ln \frac{110}{50}} = 76.09796 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 80 \text{ mm} \qquad e = \bar{r} - R = 3.90204 \text{ mm}$$

$$y_A = 76.09796 - 50 = 26.09796 \text{ mm} \qquad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(26.09796 \times 10^{-3})}{(1.440 \times 10^{-3})(3.90204 \times 10^{-3})(50 \times 10^{-3})} = -55.7 \times 10^6 \text{ Pa}$$

−55.7 MPa ◀



PROBLEM 4.162

For the curved bar shown, determine the stress at points A and B when $h = 55$ mm.

SOLUTION

$$h = 55 \text{ mm}, \qquad r_1 = 50 \text{ mm}, \qquad r_2 = 105 \text{ mm}$$

$$A = (24)(55) = 1.320 \times 10^3 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r1}} = \frac{50}{\ln \frac{105}{50}} = 74.13025 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 77.5 \text{ mm}$$

$$e = \bar{r} - R = 3.36975 \text{ mm}$$

$$y_A = 74.13025 - 50 = 24.13025 \text{ mm} \qquad r_A = 50 \text{ mm}$$

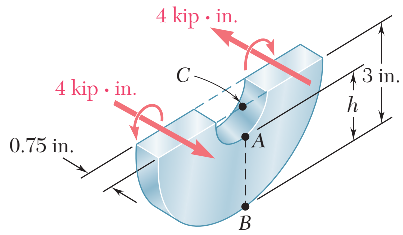
$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(600)(24.13025 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(50 \times 10^{-3})} = -65.1 \times 10^6 \text{ Pa}$$

-65.1 MPa ◀

$$y_B = 74.13025 - 105 = -30.86975 \text{ mm} \qquad r_B = 105 \text{ mm}$$

$$\sigma_B = \frac{M y_B}{A e r_B} = -\frac{(600)(-30.8697 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(105 \times 10^{-3})} = 39.7 \times 10^6 \text{ Pa}$$

39.7 MPa ◀



PROBLEM 4.163

For the machine component and loading shown, determine the stress at point A when (*a*) $h = 2$ in., (*b*) $h = 2.6$ in.

SOLUTION

$$M = -4 \text{ kip} \cdot \text{in.}$$

Rectangular cross section: $A = bh \quad r_2 = 3 \text{ in.} \quad r_1 = r_2 - h$

$$\bar{r} = \frac{1}{2}(r_1 + r_2), \quad R = \frac{h}{\ln \frac{r_2}{r_1}}, \quad e = \bar{r} - R$$

(*a*) $h = 2$ in. $\quad A = (0.75)(2) = 1.5 \text{ in}^2$

$$r_1 = 3 - 2 = 1 \text{ in.} \qquad \bar{r} = \frac{1}{2}(3 + 1) = 2 \text{ in.}$$

$$R = \frac{2}{\ln \frac{3}{1}} = 1.8205 \text{ in.} \qquad e = 2 - 1.8205 = 0.1795 \text{ in.}$$

At point A: $\quad r = r_1 = 1$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4)(1 - 1.8205)}{(1.5)(0.1795)(1)} = 12.19 \text{ ksi}$$

$\sigma_A = 12.19$ ksi ◀

(*b*) $h = 2.6$ in. $\quad A = (0.75)(2.6) = 1.95 \text{ in}^2$

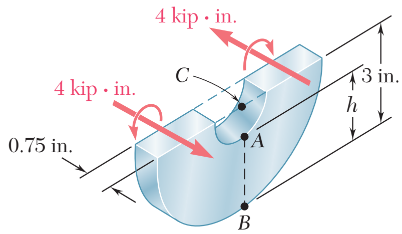
$$r_1 = 3 - 2.6 = 0.4 \text{ in.} \qquad \bar{r} = \frac{1}{2}(3 + 0.4) = 1.7 \text{ in.}$$

$$R = \frac{2.6}{\ln \frac{3}{0.4}} = 1.2904 \text{ in.} \qquad e = 1.7 - 1.2904 = 0.4906 \text{ in.}$$

At point A: $\quad r = r_1 = 0.4$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4)(0.4 - 1.2904)}{(1.95)(0.4096)(0.4)} = 11.15 \text{ ksi}$$

$\sigma_A = 11.15$ ksi ◀



PROBLEM 4.164

For the machine component and loading shown, determine the stress at points A and B when $h = 2.5$ in.

SOLUTION

$$M = -4 \text{ kip} \cdot \text{in.}$$

Rectangular cross section: $h = 2.5$ in. $b = 0.75$ in. $A = 1.875$ in.^{2}

$$r_2 = 3 \text{ in.} \quad r_1 = r_2 - h = 0.5 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = \frac{1}{2}(0.5 + 3.0) = 1.75 \text{ in.}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.5}{\ln \frac{3.0}{0.5}} = 1.3953 \text{ in.}$$

$$e = \bar{r} - R = 1.75 - 1.3953 = 0.3547 \text{ in.}$$

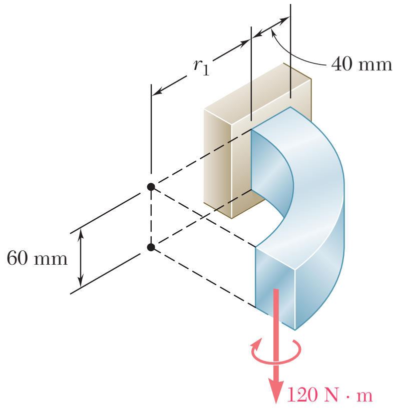
<u>At point A</u>: $r = r_1 = 0.5$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4 \text{ kip} \cdot \text{in.})(0.5 \text{ in.} - 1.3953 \text{ in.})}{(0.75 \text{ in.})(2.5 \text{ in.})(0.3547 \text{ in.})(0.5 \text{ in.})} \qquad \sigma_A = 10.77 \text{ ksi} \blacktriangleleft$$

<u>At point B</u>: $r = r_2 = 3$ in.

$$\sigma_B = \frac{M(r - R)}{Aer} = \frac{(-4 \text{ kip} \cdot \text{in.})(3 \text{ in.} - 1.3953 \text{ in.})}{(0.75 \text{ in.} \times 2.5 \text{ in.})(0.3547 \text{ in.})(3 \text{ in.})} \qquad \sigma_B = -3.22 \text{ ksi} \blacktriangleleft$$

PROBLEM 4.165



The curved bar shown has a cross section of 40×60 mm and an inner radius $r_1 = 15$ mm. For the loading shown, determine the largest tensile and compressive stresses.

SOLUTION

$$h = 40 \text{ mm}, \quad r_1 = 15 \text{ mm}, \quad r_2 = 55 \text{ mm}$$

$$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{m}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{55}{40}} = 30.786 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35 \text{ mm}$$

$$e = \bar{r} - R = 4.214 \text{ mm} \qquad \sigma = -\frac{My}{Aer}$$

At $r = 15$ mm, $\quad y = 30.786 - 15 = 15.756$ mm

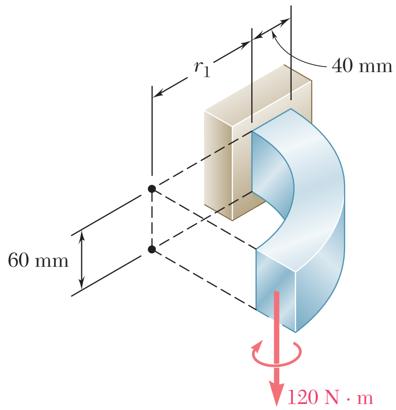
$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^{-6} \text{ Pa}$$

$\sigma = -12.49$ MPa ◀

At $r = 55$ mm, $\quad y = 30.786 - 55 = -24.214$ mm

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^{6} \text{ Pa}$$

$\sigma = 5.22$ MPa ◀



PROBLEM 4.166

For the curved bar and loading shown, determine the percent error introduced in the computation of the maximum stress by assuming that the bar is straight. Consider the case when (*a*) $r_1 = 20$ mm, (*b*) $r_1 = 200$ mm, (*c*) $r_1 = 2$ m.

SOLUTION

$$h = 40 \text{ mm}, \quad A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2, \quad M = 120 \text{ N} \cdot \text{m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(40)^3 = 0.32 \times 10^6 \text{ mm}^4 = 0.32 \times 10^{-6} \text{ mm}^4, \quad c = \frac{1}{2}h = 20 \text{ mm}$$

Assuming that the bar is straight,

$$\sigma_s = -\frac{Mc}{I} = -\frac{(120)(20 \times 10^{-8})}{(0.32 \times 10^{-6})} = 7.5 \times 10^6 \text{Pa} = 7.5 \text{ MPa}$$

(*a*) $\quad r_1 = 20 \text{ mm} \quad r_2 = 60 \text{ mm}$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{60}{20}} = 36.4096 \text{ mm} \qquad r_1 - R = -16.4096 \text{ mm}$$

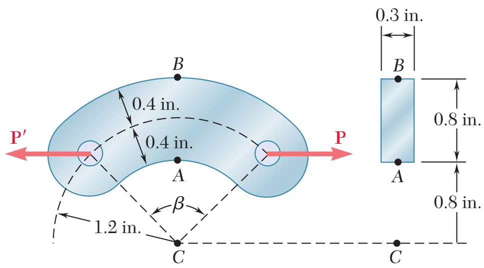
$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40 \text{ mm} \qquad e = \bar{r} - R = 3.5904 \text{ mm}$$

$$\sigma_a = \frac{M(r_1 - R)}{Aer} = \frac{(120)(-16.4096 \times 10^{-3})}{(2400 \times 10^{-6})(3.5904 \times 10^{-3})(20 \times 10^{-3})} = -11.426 \times 10^6 \text{ Pa} = -11.426 \text{ MPa}$$

$$\% \text{ error} = \frac{-11.426 - (-7.5)}{-11.426} \times 100\% = -34.4\% \quad ◀$$

For parts (*b*) and (*c*), we get the values in the table below:

	r_1, mm	r_2, mm	R, mm	$\bar{r}$, mm	e, mm	σ, MPa	% error
(*a*)	20	60	36.4096	40	3.5904	−11.426	−34.4 %
(*b*)	200	240	219.3926	220	0.6074	−7.982	6.0 % ◀
(*c*)	2000	2040	2019.9340	2020	0.0660	−7.546	0.6 % ◀



PROBLEM 4.167

Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 12 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

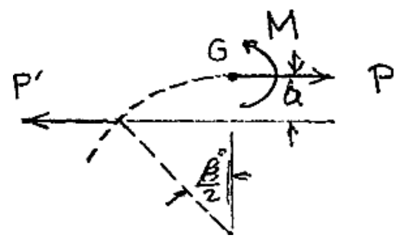
SOLUTION

Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$

The bending couple is $M = -Pa$.

For the rectangular section, the neutral axis for bending couple only lies at



$$R = \frac{h}{\ln\frac{r_2}{r_1}}.$$

Also, $e = \bar{r} - R$

At point A, the tensile stress is $\sigma_A = \dfrac{P}{A} - \dfrac{My_A}{Aer_1} = \dfrac{P}{A} + \dfrac{Pay_A}{Aer_1} = \dfrac{P}{A}\left(1 + \dfrac{ay_A}{er_1}\right) = K\dfrac{P}{A}$

where $K = 1 + \dfrac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 1.2$ in., $r_1 = 0.8$ in., $r_2 = 1.6$ in., $h = 0.8$ in., $b = 0.3$ in.

$A = (0.3)(0.8) = 0.24\ \text{in}^2$ $\quad R = \dfrac{0.8}{\ln\frac{1.6}{0.8}} = 1.154156$ in.

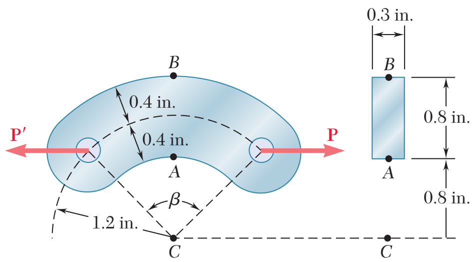
$e = 1.2 - 1.154156 = 0.045844$ in., $\quad y_A = 1.154156 - 0.8 = 0.35416$ in.

$a = 1.2(1 - \cos 45°) = 0.35147$ in.

$$K = 1 + \frac{(0.35147)(0.35416)}{(0.045844)(0.8)} = 4.3940$$

$$P = \frac{(0.24)(12)}{4.3940} = 0.65544\ \text{kips}$$

$P = 655$ lb ◀



PROBLEM 4.168

Solve Prob. 4.167, assuming that $\beta = 60°$.

PROBLEM 4.167 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 12 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

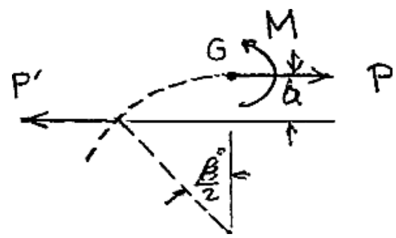
SOLUTION

Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$

The bending couple is $M = -Pa$.

For the rectangular section, the neutral axis for bending couple only lies at



$$R = \frac{h}{\ln\frac{r_2}{r_1}}.$$

Also, $e = \bar{r} - R$

At point A, the tensile stress is $\sigma_A = \dfrac{P}{A} - \dfrac{My_A}{Aer_1} = \dfrac{P}{A} + \dfrac{Pay_A}{Aer_1} = \dfrac{P}{A}\left(1 + \dfrac{ay_A}{er_1}\right) = K\dfrac{P}{A}$

where $K = 1 + \dfrac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 1.2$ in., $r_1 = 0.8$ in., $r_2 = 1.6$ in., $h = 0.8$ in., $b = 0.3$ in.

$A = (0.3)(0.8) = 0.24 \text{ in}^2$ $\quad R = \dfrac{0.8}{\ln\frac{1.6}{0.8}} = 1.154156$ in.

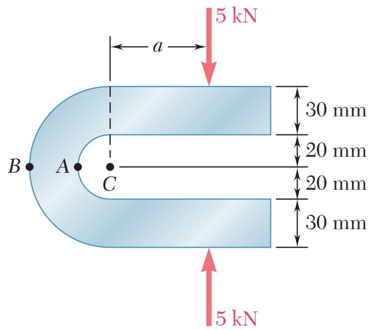
$e = 1.2 - 1.154156 = 0.045844$ in. $\quad y_A = 1.154156 - 0.8 = 0.35416$ in.

$a = (1.2)(1 - \cos 30°) = 0.160770$ in.

$$K = 1 + \frac{(0.160770)(0.35416)}{(0.045844)(0.8)} = 2.5525$$

$$P = \frac{(0.24)(12)}{2.5525} = 1.128 \text{ kips}$$

$P = 1128$ lb ◀



PROBLEM 4.169

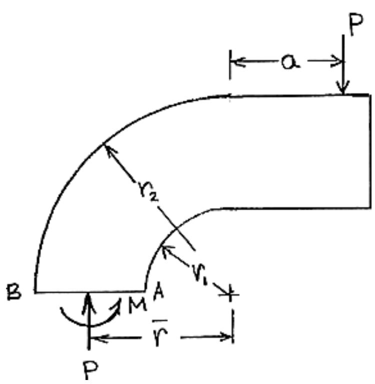
The curved bar shown has a cross section of 30×30 mm. Knowing that the allowable compressive stress is 175 MPa, determine the largest allowable distance a.

SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at $R = \dfrac{h}{\ln \dfrac{r_2}{r_1}}$.



Also, $e = \bar{r} - R$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \quad \text{with} \quad y_A = R - r_1$$

Thus, $$K = 1 + \frac{(a+\bar{r})(R-r_1)}{er_1} \qquad (1)$$

Data: $h = 30$ mm, $r_1 = 20$ mm, $r_2 = 50$ mm, $\bar{r} = 35$ mm, $R = \dfrac{30}{\ln \frac{50}{20}} = 32.7407$ mm

$e = 35 - 32.7407 = 2.2593$ mm, $b = 30$ mm, $R - r_1 = 12.7407$ mm, $a = ?$

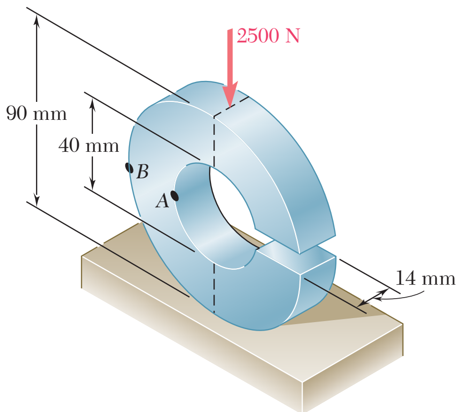
$\sigma_A = -175 \text{ MPa} = -175 \times 10^6$ Pa, $P = 5 \text{ kN} = 5 \times 10^3$ N

$$K = -\frac{A\sigma_A}{P} = -\frac{(900 \times 10^{-6})(-175 \times 10^6)}{5 \times 10^3} = 31.5$$

Solving (1) for $a + \bar{r}$, $\quad a + \bar{r} = \dfrac{(K-1)er_1}{R - r_1}$

$$a + \bar{r} = \frac{(30.5)(2.2593)(20)}{12.7407} = 108.17 \text{ mm}$$

$a = 108.17 \text{ mm} - 35 \text{ mm}$ $\qquad a = 73.2 \text{ mm}$ ◀



PROBLEM 4.170

For the split ring shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

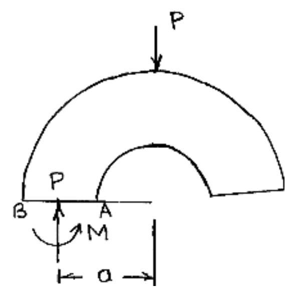
SOLUTION

$$r_1 = \frac{1}{2}40 = 20 \text{ mm}, \qquad r_2 = \frac{1}{2}(90) = 45 \text{ mm} \qquad h = r_2 - r_1 = 25 \text{ mm}$$

$$A = (14)(25) = 350 \text{ mm}^2 \qquad R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 32.5 \text{ mm} \qquad e = \bar{r} - R = 1.6712 \text{ mm}$$

Reduce the internal forces transmitted across section *AB* to a force-couple system at the centroid of the cross section. The bending couple is



$$M = Pa = P\bar{r} = (2500)(32.5 \times 10^{-3}) = 81.25 \text{ N} \cdot \text{m}$$

(*a*) Point *A*: $r_A = 20$ mm $\quad y_A = 30.8288 - 20 = 10.8288$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{AeR} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(10.8288 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(20 \times 10^{-3})}$$

$$= -82.4 \times 10^6 \text{ Pa}$$

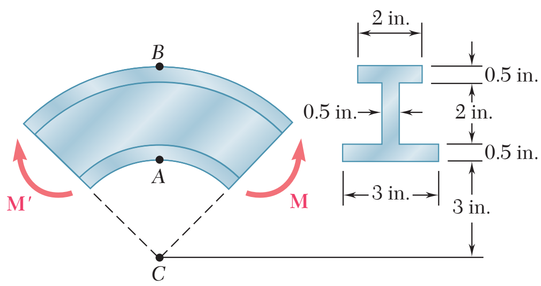
$\sigma_A = -82.4$ MPa ◀

(*b*) Point *B*: $r_B = 45$ mm $\quad y_B = 30.8288 - 45 = -14.1712$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_B} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(-14.1712 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(45 \times 10^{-3})}$$

$$= 36.6 \times 10^6 \text{ Pa}$$

$\sigma_B = 36.6$ MPa ◀



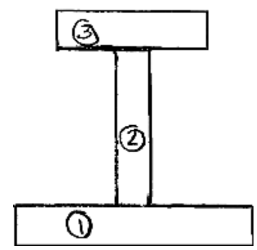
PROBLEM 4.171

Three plates are welded together to form the curved beam shown. For $M = 8$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) the centroid of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\,dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	Part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.,} \qquad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.} \qquad M = -8 \text{ kip} \cdot \text{in.}$$

(*a*) $y_A = R - r_1 = 4.05812 - 3 = 1.05812$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-8)(1.05812)}{(3.5)(0.26331)(3)}$$

$\sigma_A = 3.06$ ksi ◀

(*b*) $y_B = R - r_2 = 4.05812 - 6 = -1.94188$ in.

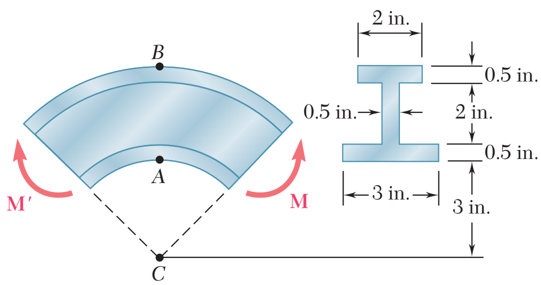
$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-8)(-1.94188)}{(3.5)(0.26331)(6)}$$

$\sigma_B = -2.81$ ksi ◀

(*c*) $y_C = R - \bar{r} = -e$

$$\sigma_C = -\frac{My_C}{Ae\bar{r}} = -\frac{Me}{Ae\bar{r}} = -\frac{M}{A\bar{r}} = -\frac{-8}{(3.5)(4.32143)}$$

$\sigma_C = 0.529$ ksi ◀



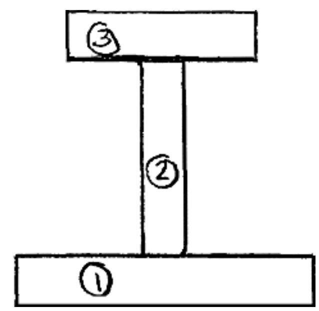
PROBLEM 4.172

Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

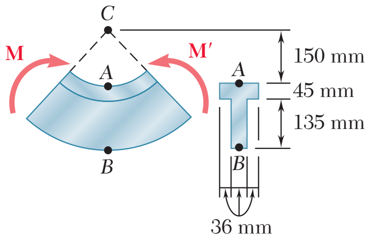
$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	Part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3							
3.5	①	3	0.5	1.5	0.462452	3.25	4.875
5.5	②	0.5	2	1.0	0.225993	4.5	4.5
6	③	2	0.5	1.0	0.174023	5.75	5.75
	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.} \qquad e = 0.263 \text{ in.} \blacktriangleleft$$



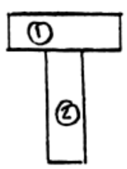
PROBLEM 4.173

Knowing that the maximum allowable stress is 45 MPa, determine the magnitude of the largest moment M that can be applied to the components shown.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i 2 \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm	Part	b_i, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150	①	108	45	4860	28.3353	172.5	838.35×10^3
195	②	36	135	4860	18.9394	262.5	1275.75×10^3
330	Σ			9720	47.2747		2114.1×10^3

$$R = \frac{9720}{47.2747} = 205.606 \text{ mm} \qquad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5 \text{ mm}$$

$$e = \bar{r} - R = 11.894 \text{ mm}$$

$$y_A = R - r_1 = 205.606 - 150 = 55.606 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_1}$$

$$M = \frac{\sigma_A A e r_1}{y_A}$$

$$= \frac{(45 \times 10^6)(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}{(55.606 \times 10^{-3})} = 14.03 \text{ kN} \cdot \text{m}$$

$$y_B = R - r_2 = 205.606 - 330 = -124.394 \text{ mm}$$

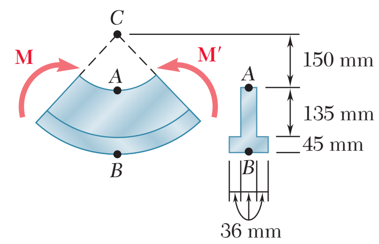
$$\sigma_B = -\frac{My_B}{Aer_2}$$

$$M = \frac{\sigma_B A e r_2}{y_B}$$

$$= \frac{(45 \times 10^6)(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}{(124.394 \times 10^{-3})}$$

$$= 13.80 \text{ kN} \cdot \text{m}$$

$M = 13.80$ kN · m ◀



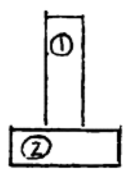
PROBLEM 4.174

Knowing that the maximum allowable stress is 45 MPa, determine the magnitude of the largest moment M that can be applied to the components shown.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\, dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \dfrac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \dfrac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm		b_i, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150	①	36	135	4860	23.1067	217.5	1.05705×10^6
285	②	108	45	4860	15.8332	307.5	1.49445×10^6
330	Σ			9720	38.9399		2.5515×10^6

$$R = \frac{9720}{38.9399} = 249.615 \text{ mm}, \qquad \bar{r} = \frac{2.5515 \times 10^6}{9720} = 262.5 \text{ mm}$$

$$e = \bar{r} - R = 12.885 \text{ mm}, \qquad M = 20 \times 10^3 \text{ N} \cdot \text{m}$$

$$y_A = R - r_1 = 249.615 - 150 = 99.615 \text{ mm}$$

$$\sigma_A = -\frac{M y_A}{A e r_1}$$

$$M = \frac{\sigma_A A e r_1}{y_A}$$

$$= \frac{(45 \times 10^6)(9720 \times 10^{-6})(12.885 \times 10^{-3})(150 \times 10^{-3})}{(99.615 \times 10^{-3})} = 8.49 \text{ kN} \cdot \text{m}$$

$$y_B = R - r_2 = 249.615 - 330 = -80.385 \text{ mm}$$

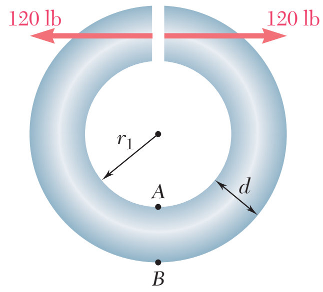
$$\sigma_B = -\frac{M y_B}{A e r_2}$$

$$M = \frac{\sigma_B A e r_2}{y_B}$$

$$= \frac{(45 \times 10^6)(9720 \times 10^{-6})(12.885 \times 10^{-3})(330 \times 10^{-3})}{(80.385 \times 10^{-3})}$$

$$= 23.1 \text{ kN} \cdot \text{m}$$

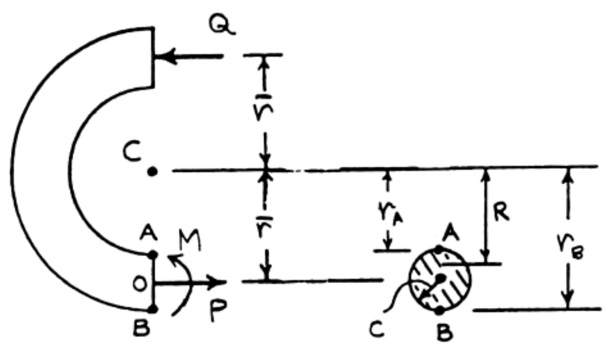
$$M = 8.49 \text{ kN} \cdot \text{m} \blacktriangleleft$$



PROBLEM 4.175

The split ring shown has an inner radius $r_1 = 0.8$ in. and a *circular* cross section of diameter $d = 0.6$ in. Knowing that each of the 120-lb forces is applied at the centroid of the cross section, determine the stress (*a*) at point *A*, (*b*) at point *B*.

SOLUTION



$$r_A = r_1 = 0.8 \text{ in.}$$

$$r_B = r_A + d = 0.8 + 0.6 = 1.4 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_A + r_B) = 1.1 \text{ in.} \qquad c = \frac{1}{2}d = 0.3 \text{ in.}$$

$$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2 \qquad \text{for solid circular section}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[1.1 + \sqrt{(1.1)^2 - (0.3)^2}\right] = 1.079150 \text{ in.}$$

$$e = \bar{r} - R = 1.1 - 1.0791503 = 0.020850 \text{ in.}$$

$$Q = 120 \text{ lb} \qquad \xrightarrow{+} \sum F_x = 0: \quad P - Q = 0 \qquad P = 120 \text{ lb}$$

$$+\circlearrowleft \sum M_0 = 0: \quad 2\bar{r}Q + M = 0 \qquad M = -2\bar{r}Q = -(2)(1.1)(120) = -264 \text{ lb} \cdot \text{in.}$$

(*a*) $r = r_A = 0.8$ in.

$$\sigma_A = \frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = \frac{120}{0.28274} + \frac{(-264)(0.8 - 1.079150)}{(0.28274)(0.020850)(0.8)} = 16.05 \times 10^3 \text{ psi}$$

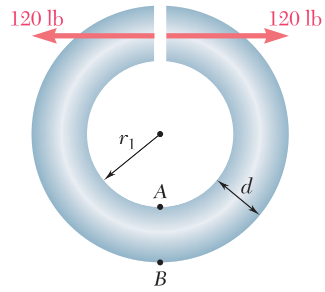
$\sigma_A = 16.05$ ksi ◀

(*b*) $r = r_B = 1.4$ in.

$$\sigma_B = \frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = \frac{120}{0.28274} + \frac{(-264)(1.4 - 1.079150)}{(0.28274)(0.020850)(1.4)}$$

$$= -9.84 \times 10^3 \text{ psi}$$

$\sigma_B = -9.84$ ksi ◀

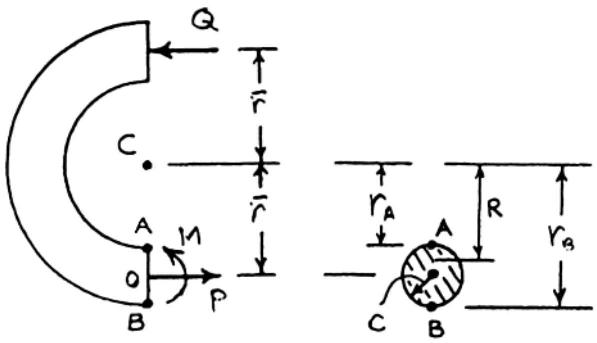


PROBLEM 4.176

Solve Prob. 4.175, assuming that the ring has an inner radius $r_1 = 0.6$ in. and a cross-sectional diameter $d = 0.8$ in.

PROBLEM 4.175 The split ring shown has an inner radius $r_1 = 0.8$ in. and a *circular* cross section of diameter $d = 0.6$ in. Knowing that each of the 120-lb forces is applied at the centroid of the cross section, determine the stress (*a*) at point *A*, (*b*) at point *B*.

SOLUTION



$$r_A = r_1 = 0.6 \text{ in.}$$

$$r_B = r_A + d = 0.6 + 0.8 = 1.4 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_A + r_B) = 1.0 \text{ in.} \qquad c = \frac{1}{2}d = 0.4 \text{ in.}$$

$$A = \pi c^2 = \pi(0.4)^2 = 0.50265 \text{ in}^2 \qquad \text{for solid circular section.}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[1.0 + \sqrt{(1.0)^2 - (0.4)^2}\right] = 0.958258 \text{ in.}$$

$$e = \bar{r} - R = 1.0 - 0.958258 = 0.041742 \text{ in.}$$

$$Q = 120 \text{ lb} \qquad \underrightarrow{+} \Sigma F_x = 0 \qquad P - Q = 0 \qquad P = 120 \text{ lb}$$

$$+\circlearrowleft \Sigma M_0 = 0: \quad 2\bar{r}Q + M = 0 \qquad M = -2\bar{r}Q = -(2)(1.0)(120) = -240 \text{ lb} \cdot \text{in.}$$

(*a*) $r = r_A = 0.6$ in.

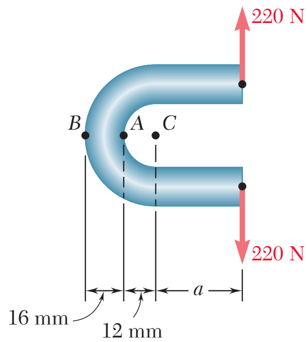
$$\sigma_A = \frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = \frac{120}{0.50265} + \frac{(-240)(0.6 - 0.958258)}{(0.50265)(0.041742)(0.6)}$$

$$= 7.069 \times 10^3 \text{ psi} \qquad \sigma_A = 7.07 \text{ ksi} \blacktriangleleft$$

(*b*) $r = r_B = 1.4$ in.

$$\sigma_B = \frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = \frac{120}{0.50265} + \frac{(-240)(1.4 - 0.958258)}{(0.50265)(0.041742)(1.4)}$$

$$= -3.37 \times 10^3 \text{ psi} \qquad \sigma_B = -3.37 \text{ ksi} \blacktriangleleft$$



PROBLEM 4.177

The bar shown has a circular cross section of 14-mm diameter. Knowing that $a = 32$ mm, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$c = \frac{1}{2}d = 8 \text{ mm} \qquad \bar{r} = 12 + 8 = 20 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[20 + \sqrt{20^2 - 8^2}\right] = 19.1652 \text{ mm}$$

$$e = \bar{r} - R = 20 - 19.1652 = 0.83485 \text{ mm}$$

$$A = \pi r^2 = \pi(8)^2 = 201.06 \text{ mm}^2$$

$$P = 220 \text{ N}$$

$$M = -P(a + \bar{r}) = 220(0.032 + 0.020) = 11.44 \text{ N}\cdot\text{m}$$

$$y_A = R - r_1 = 19.1652 - 12 = 7.1652 \text{ mm}$$

$$y_b = R - r_2 = 19.1652 - 28 = -8.8348 \text{ mm}$$

(*a*) $$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1}$$

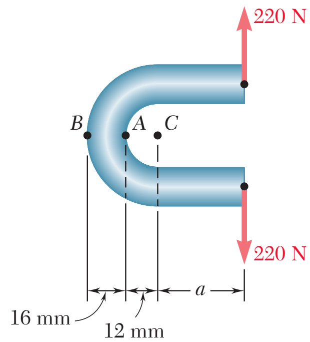
$$= \frac{220}{201.06 \times 10^{-6}} - \frac{(-11.44)(7.1652 \times 10^{-3})}{(201.06 \times 10^{-6})(0.83485 \times 10^{-3})(12 \times 10^{-3})}$$

$\sigma_A = 41.8 \text{ MPa}$ ◀

(*b*) $$\sigma_B = \frac{P}{A} - \frac{My_B}{Aer_2}$$

$$= \frac{220}{201.06 \times 10^{-6}} - \frac{(-11.44)(8.8348 \times 10^{-3})}{(201.06 \times 10^{-6})(0.83485 \times 10^{-3})(28 \times 10^{-3})}$$

$\sigma_B = -20.4 \text{ MPa}$ ◀



PROBLEM 4.178

The bar shown has a circular cross section of 14-mm diameter. Knowing that the allowable stress is 38 MPa, determine the largest permissible distance a from the line of action of the 220-N forces to the plane containing the center of curvature of the bar.

SOLUTION

$$c = \frac{1}{2}d = 8 \text{ mm} \qquad \bar{r} = 12 + 8 = 20 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[20 + \sqrt{20^2 - 8^2}\right] = 19.1652 \text{ mm}$$

$$e = \bar{r} - R = 20 - 19.1652 = 0.83485 \text{ mm}$$

$$A = \pi r^2 = \pi(8)^2 = 201.06 \text{ mm}^2$$

$$P = 220 \text{ N}$$

$$M = -P(a + \bar{r})$$

$$y_A = R - r_1 = 19.1652 - 12 = 7.1652 \text{ mm}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{P(a+\bar{r})y_A}{Aer_1} = \frac{P}{A}\left[1 + \frac{(a+\bar{r})y_A}{er_1}\right]$$

$$= \frac{KP}{A} \quad \text{where} \quad K = 1 + \frac{(a+\bar{r})y_A}{er_1}$$

$$K = \frac{\sigma_A A}{P} = \frac{(38 \times 10^6)(201.06 \times 10^{-6})}{220} = 34.729$$

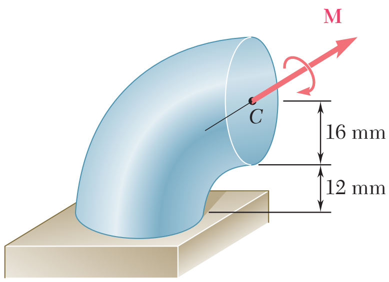
$$a + \bar{r} = \frac{(K-1)er_1}{y_A}$$

$$= \frac{(34.729 - 1)(0.83485 \times 10^{-3})(12 \times 10^{-3})}{(7.1652 \times 10^{-3})}$$

$$= 0.047158 \text{ m}$$

$$a = 0.047158 - 0.020 = 0.027158$$

$$a = 27.2 \text{ mm} \blacktriangleleft$$



PROBLEM 4.179

The curved bar shown has a circular cross section of 32-mm diameter. Determine the largest couple **M** that can be applied to the bar about a horizontal axis if the maximum stress is not to exceed 60 MPa.

SOLUTION

$$c = 16 \text{ mm} \qquad \bar{r} = 12 + 16 = 28 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right]$$

$$= \frac{1}{2}\left[28 + \sqrt{28^2 - 16^2}\right] = 25.4891 \text{ mm}$$

$$e = \bar{r} - R = 28 - 25.4891 = 2.5109 \text{ mm}$$

$\sigma_{\max}$ occurs at A, which lies at the inner radius.

It is given by $|\sigma_{\max}| = \left|\dfrac{My_A}{Aer_1}\right|$ from which $M = \dfrac{Aer_1|\sigma_{\max}|}{y_A}$.

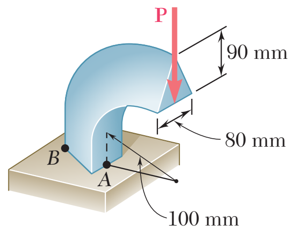
Also, $A = \pi c^2 = \pi(16)^2 = 804.25 \text{ mm}^2$

Data: $y_A = R - r_1 = 25.4891 - 12 = 13.4891 \text{ mm}$

$$M = \frac{(804.25 \times 10^{-6})(2.5109 \times 10^{-3})(12 \times 10^{-3})(60 \times 10^{6})}{13.4891 \times 10^{-3}}$$

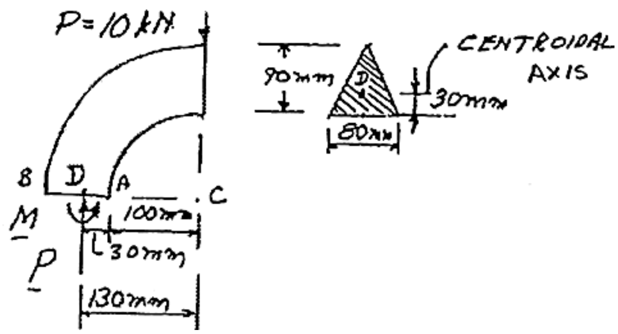
$M = 107.8 \text{ N} \cdot \text{m}$ ◀

PROBLEM 4.180



Knowing that $P = 10$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



Locate the centroid *D* of the cross section.

$$\bar{r} = 100 \text{ mm} + \frac{90 \text{ mm}}{3} = 130 \text{ mm}$$

Force-couple system at *D*.

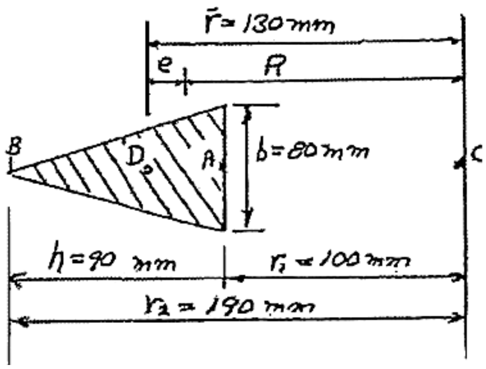
$$P = 10 \text{ kN}$$

$$M = P\bar{r} = (10 \text{ kN})(130 \text{ mm}) = 1300 \text{ N} \cdot \text{m}$$

Triangular cross section.

$$A = \frac{1}{2}bh = \frac{1}{2}(90 \text{ mm})(80 \text{ mm})$$

$$= 3600 \text{ mm}^2 = 3600 \times 10^{-6} \text{ m}^2$$



$$R = \frac{\frac{1}{2}h}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1} = \frac{\frac{1}{2}(90)}{\frac{190}{90}\ln\frac{190}{100} - 1} = \frac{45 \text{ mm}}{0.355025}$$

$$R = 126.752 \text{ mm}$$

$$e = \bar{r} - R = 130 \text{ mm} - 126.752 \text{ mm} = 3.248 \text{ mm}$$

(*a*) Point *A*: $r_A = 100 \text{ mm} = 0.100 \text{ m}$

$$\sigma_A = -\frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = -\frac{10 \text{ kN}}{3600 \times 10^{-6} \text{m}^2} + \frac{(1300 \text{ N} \cdot \text{m})(0.100 \text{ m} - 0.126752 \text{ m})}{(3600 \times 10^{-6} \text{ m}^2)(3248 \times 10^{-3} \text{ m})(0.100 \text{ m})}$$

$$= -2.778 \text{ MPa} - 29.743 \text{ MPa}$$

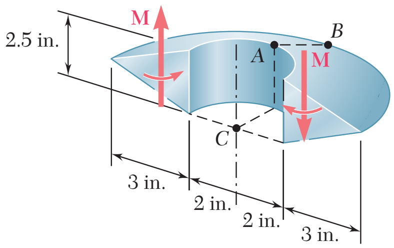
$\sigma_A = -32.5$ MPa ◀

(*b*) Point *B*: $r_B = 190 \text{ mm} = 0.190 \text{ m}$

$$\sigma_B = -\frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = -\frac{10 \text{ kN}}{3600 \times 10^{-6} \text{ m}^2} + \frac{(1300 \text{ N} \cdot \text{m})(0.190 \text{ m} - 0.126752 \text{ m})}{(3600 \times 10^{-6} \text{ m}^2)(3.248 \times 10^{-3} \text{ m})(0.190 \text{ m})}$$

$$= -2.778 \text{ MPa} + 37.01 \text{ MPa}$$

$\sigma_B = +34.2$ MPa ◀



PROBLEM 4.181

Knowing that $M = 5$ kip · in., determine the stress at (*a*) point A, (*b*) point B.

SOLUTION

$$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 1 = 3.00000 \text{ in.}$$

$$b_1 = 2.5 \text{ in.}, \quad r_1 = 2 \text{ in.}, \quad b_2 = 0, \quad r_2 = 5 \text{ in.}$$

Use formula for trapezoid with $b_2 = 0.$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(2.5 + 0)}{[(2.5)(5) - (0)(2)]\ln\frac{5}{2} - (3)(2.5 - 0)} = 2.84548 \text{ in.}$$

$$e = \bar{r} - R = 0.15452 \text{ in.} \qquad M = 5 \text{ kip} \cdot \text{in.}$$

(*a*) $y_A = R - r_1 = 0.84548$ in.

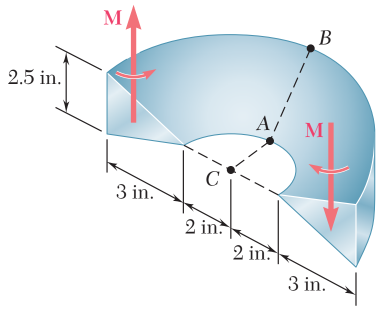
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(0.84548)}{(3.75)(0.15452)(2)}$$

$\sigma_A = -3.65$ ksi ◀

(*b*) $y_B = R - r_2 = -2.15452$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-2.15452)}{(3.75)(0.15452)(5)}$$

$\sigma_B = 3.72$ ksi ◀



PROBLEM 4.182

Knowing that $M = 5$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 2 = 4.00000 \text{ in.}$$

$$b_1 = 0, \quad r_1 = 2 \text{ in.}, \quad b_2 = 2.5 \text{ in.}, \quad r_2 = 5 \text{ in.}$$

Use formula for trapezoid with $b_1 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(0 + 2.5)}{[(0)(5) - (2.5)(2)]\ln\frac{5}{2} - (3)(0 - 2.5)} = 3.85466 \text{ in.}$$

$$e = \bar{r} - R = 0.14534 \text{ in.} \qquad M = 5 \text{ kip} \cdot \text{in.}$$

(*a*) $y_A = R - r_1 = 1.85466$ in.

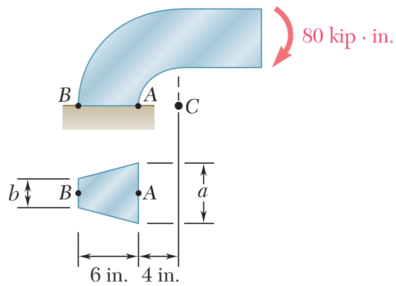
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(1.85466)}{(3.75)(0.14534)(2)}$$

$\sigma_A = -8.51$ ksi ◀

(*b*) $y_B = R - r_2 = -1.14534$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-1.14534)}{(3.75)(0.14534)(5)}$$

$\sigma_B = 2.10$ ksi ◀



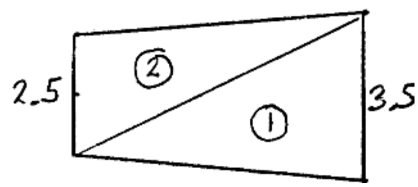
PROBLEM 4.183

Knowing that the machine component shown has a trapezoidal cross section with $a = 3.5$ in. and $b = 2.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.

	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	10.5	6	63
②	7.5	8	60
Σ	18		123



$$\bar{r} = \frac{123}{18} = 6.8333 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(3.5 + 2.5)}{[(3.5)(10) - (2.5)(4)]\ln\frac{10}{4} - (6)(3.5 - 2.5)} = 6.3878 \text{ in.}$$

$$e = \bar{r} - R = 0.4452 \text{ in.} \qquad M = 80 \text{ kip} \cdot \text{in.}$$

(*a*) $y_A = R - r_1 = 6.3878 - 4 = 2.3878$ in.

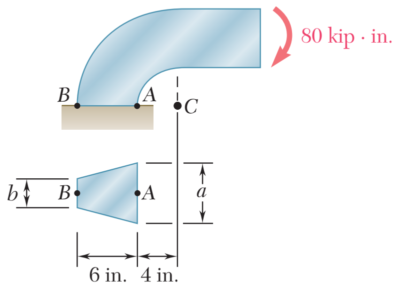
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.3878)}{(18)(0.4452)(4)}$$

$\sigma_A = -5.96$ ksi ◀

(*b*) $y_B = R - r_2 = 6.3878 - 10 = -3.6122$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.6122)}{(18)(0.4452)(10)}$$

$\sigma_B = 3.61$ ksi ◀



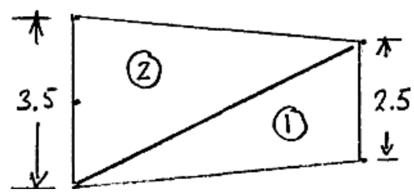
PROBLEM 4.184

Knowing that the machine component shown has a trapezoidal cross section with $a = 2.5$ in. and $b = 3.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.

	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	7.5	6	45
②	10.5	8	84
Σ	18		129



$$\bar{r} = \frac{129}{18} = 7.1667 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(2.5 + 3.5)}{[(2.5)(10) - (3.5)(4)]\ln\frac{10}{4} - (6)(2.5 - 3.5)} = 6.7168 \text{ in.}$$

$$e = \bar{r} - R = 0.4499 \text{ in.} \qquad M = 80 \text{ kip} \cdot \text{in.}$$

(*a*) $y_A = R - r_1 = 2.7168$ in.

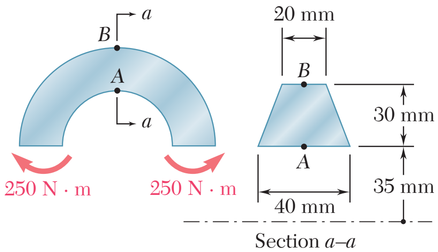
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.7168)}{(18)(0.4499)(4)}$$

$\sigma_A = -6.71$ ksi ◀

(*b*) $y_B = R - r_2 = -3.2832$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.2832)}{(18)(0.4499)(10)}$$

$\sigma_B = 3.24$ ksi ◀

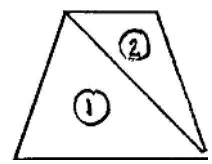


PROBLEM 4.185

For the curved beam and loading shown, determine the stress at (*a*) point A, (*b*) point B.

SOLUTION

Locate centroid.



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	600	45	27×10^3
②	300	55	16.5×10^3
Σ	900		43.5×10^3

$$\bar{r} = \frac{43.5 \times 10^3}{900} = 48.333 \text{ mm}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2(40 + 20)}{[(40)(65) - (20)(35)]\ln\frac{65}{35} - (30)(40 - 20)} = 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm} \qquad M = -250 \text{ N} \cdot \text{m}$$

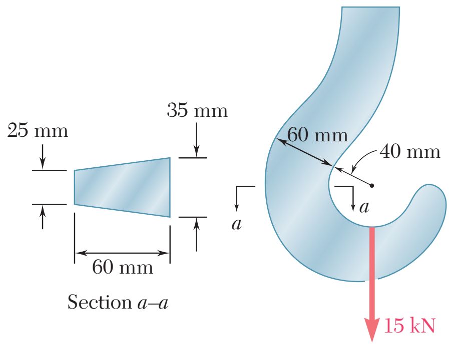
(*a*) $\quad y_A = R - r_1 = 11.8608$ mm

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(35 \times 10^{-3})} = 63.9 \times 10^6 \text{ Pa} \qquad \sigma_A = 63.9 \text{ MPa} \blacktriangleleft$$

(*b*) $\quad y_B = R - r_2 = -18.1392$ mm

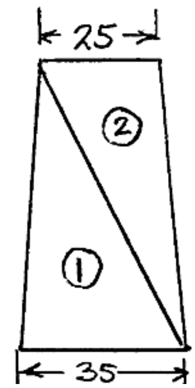
$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(65 \times 10^{-3})} = -52.6 \times 10^6 \text{ Pa} \qquad \sigma_B = -52.6 \text{ MPa} \blacktriangleleft$$

PROBLEM 4.186



For the crane hook shown, determine the largest tensile stress in section *a-a*.

SOLUTION



Locate centroid.

	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	1050	60	63×10^3
②	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r} = \frac{103\times10^3}{1800} = 63.333 \text{ mm}$$

Force-couple system at centroid: $P = 15\times10^3$ N

$$M = -P\bar{r} = -(15\times10^3)(68.333\times10^{-3}) = -1.025\times10^3 \text{ N}\cdot\text{m}$$

$$R = \frac{\frac{1}{2}h^2(b_1+b_2)}{(b_1r_2 - b_2r_1)\ln\frac{r_2}{r_1} - h(b_1-b_2)}$$

$$= \frac{(0.5)(60)^2(35+25)}{[(35)(100)-(25)(40)]\ln\frac{100}{40} - (60)(35+25)} = 63.878 \text{ mm}$$

$$e = \bar{r} - R = 4.452 \text{ mm}$$

Maximum tensile stress occurs at point A.

$$y_A = R - r_1 = 23.878 \text{ mm}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1}$$

$$= \frac{15\times10^3}{1800\times10^{-6}} - \frac{-(1.025\times10^3)(23.878\times10^{-3})}{(1800\times10^{-6})(4.452\times10^{-3})(40\times10^{-3})}$$

$$= 84.7\times10^6 \text{ Pa}$$

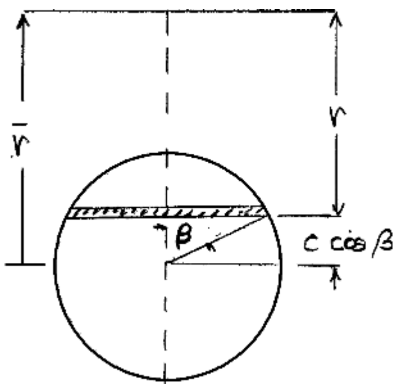
$\sigma_A = 84.7$ MPa ◀

PROBLEM 4.187

Using Eq. (4.66), derive the expression for R given in Fig. 4.61 for a circular cross section.

SOLUTION

Use polar coordinate β as shown. Let w be the width as a function of β



$$w = 2c\sin\beta$$

$$r = \bar{r} - c\cos\beta$$

$$dr = c\sin\beta\, d\beta$$

$$dA = w\,dr = 2c^2\sin^2\beta\, d\beta$$

$$\int\frac{dA}{r} = \int_0^\pi \frac{2c^2\sin\beta}{\bar{r}-c\cos\beta}\,d\beta$$

$$\int\frac{dA}{r} = \int_0^\pi \frac{c^2(1-\cos^2\beta)}{\bar{r}-c\cos\beta}\,d\beta$$

$$= 2\int_0^\pi \frac{\bar{r}^2 - c^2\cos^2\beta - (\bar{r}^2 - c^2)}{\bar{r}-c\cos\beta}\,d\beta$$

$$= 2\int_0^\pi (\bar{r} + c\cos\beta)\,d\beta - 2(\bar{r}^2 - c^2)\int_0^\pi \frac{dr}{\bar{r}-c\cos\beta}$$

$$= 2\bar{r}\,\beta\Big|_0^\pi + 2c\sin\beta\Big|_0^\pi$$

$$- 2(\bar{r}^2 - c^2)\frac{2}{\sqrt{\bar{r}^2-c^2}}\tan^{-1}\frac{\sqrt{\bar{r}^2-c^2}\,\tan\left(\frac{1}{2}\beta\right)}{\bar{r}+c}\Bigg|_0^\pi$$

$$= 2\bar{r}(\pi - 0) + 2c(0-0) - 4\sqrt{\bar{r}^2-c^2}\cdot\left(\frac{\pi}{2}-0\right)$$

$$= 2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2-c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int\frac{dA}{r}} = \frac{\pi c^2}{2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2-c^2}} = \frac{1}{2}\frac{c^2}{\bar{r}-\sqrt{r^2-c^2}} \times \frac{\bar{r}+\sqrt{\bar{r}^2-c^2}}{\bar{r}+\sqrt{\bar{r}^2-c^2}}$$

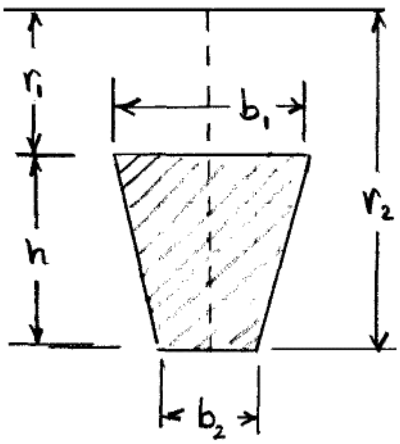
$$= \frac{1}{2}\frac{c^2\left(\bar{r}+\sqrt{\bar{r}^2-c^2}\right)}{\bar{r}^2 - (\bar{r}^2-c^2)} = \frac{1}{2}\frac{c^2\left(\bar{r}+\sqrt{\bar{r}^2-c^2}\right)}{c^2} = \frac{1}{2}\left(\bar{r}+\sqrt{\bar{r}^2-c^2}\right) \quad \blacktriangleleft$$

PROBLEM 4.188

Using Eq. (4.66), derive the expression for R given in Fig. 4.73 for a trapezoidal cross section.

SOLUTION

The section width w varies linearly with r.



$$w = c_0 + c_1 r$$

$$w = b_1 \text{ at } r = r_1 \text{ and } w = b_2 \quad \text{at} \quad r = r_2$$

$$b_1 = c_0 + c_1 r_1$$

$$b_2 = c_0 + c_1 r_2$$

$$b_1 - b_2 = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1)c_0 = hc_0$$

$$c_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$

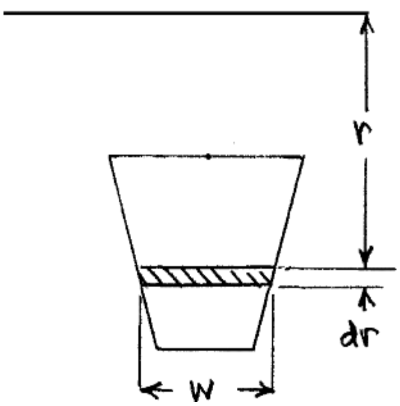
$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r} dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1(r_2 - r_1)$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2)$$



$$A = \frac{1}{2}(b_1 + b_2) h$$

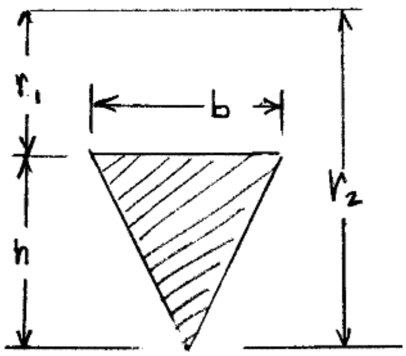
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln \frac{r_2}{r_1} - h(b_1 - b_2)} \quad \blacktriangleleft$$

PROBLEM 4.189

Using Equation (4.66), derive the expression for R given in Fig. 4.73 for a triangular cross section.

SOLUTION

The section width w varies linearly with r.



$$w = c_0 + c_1 r$$

$$w = b \text{ at } r = r_1 \quad \text{and} \quad w = 0 \text{ at } r = r_2$$

$$b = c_0 + c_1 r_1$$

$$0 = c_0 + c_1 r_2$$

$$b = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b}{h} \quad \text{and} \quad c_0 = -c_1 r_2 = \frac{b r_2}{h}$$

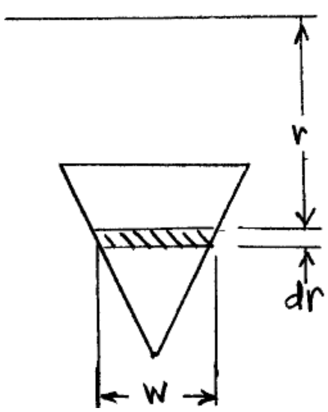
$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r} dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1(r_2 - r_1)$$

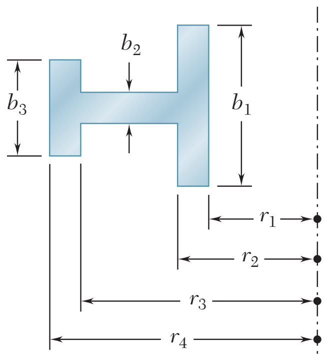
$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)$$



$$A = \frac{1}{2} bh$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} bh}{b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1} \quad \blacktriangleleft$$



PROBLEM 4.190

Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

where A is the total area of the cross section.

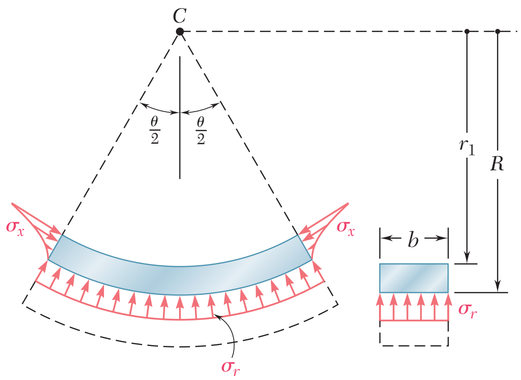
SOLUTION

$$R = \frac{\Sigma A}{\Sigma\int \frac{1}{r}\,dA} = \frac{A}{\Sigma b_i \ln\frac{r_{i+1}}{r_i}}$$

$$= \frac{A}{\Sigma \ln\left(\frac{r_{i+1}}{r_i}\right)^{b_i}} = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]} \blacktriangleleft$$

Note that for each rectangle, $\int \frac{1}{r}\,dA = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$

$$= b_i \int_{r_2}^{r_{i+1}} \frac{dr}{r} = b_i \ln\frac{r_{i+1}}{r_i}$$



PROBLEM 4.191

For a curved bar of rectangular cross section subjected to a bending couple **M**, show that the radial stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

and compute the value of σ_r for the curved bar of Concept Applications 4.10 and 4.11. (*Hint:* consider the free-body diagram of the portion of the beam located above the neutral surface.)

SOLUTION

At radial distance r, $\quad \sigma_r = \dfrac{M(r-R)}{Aer} = \dfrac{M}{Ae} - \dfrac{MR}{Aer}$

For portion above the neutral axis, the resultant force is

$$H = \int \sigma_r\,dA = \int_{r_1}^{R} \sigma_r b\,dr$$

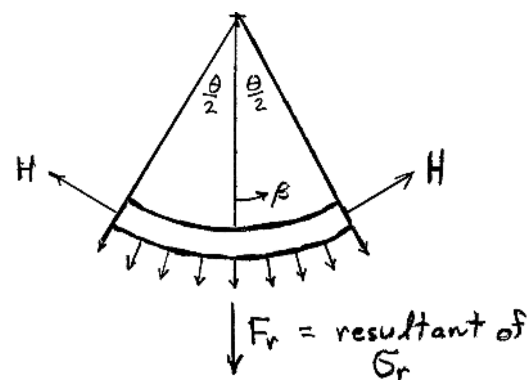
$$= \frac{Mb}{Ae}\int_{r_1}^{R} dr - \frac{MRb}{Ae}\int_{r_1}^{R}\frac{dr}{r}$$

$$= \frac{Mb}{Ae}(R - r_1) - \frac{MRb}{Ae}\ln\frac{R}{r_1} = \frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Resultant of σ_n: $\quad F_r = \int \sigma_r \cos\beta\; dA$

$$= \int_{-\theta/2}^{\theta/2} \sigma_r \cos\beta (bR\,d\beta) = \sigma_r bR \int_{-\theta/2}^{\theta/2} \cos\beta\,d\beta$$

$$= \sigma_r\,bR \;\; \sin\beta\Big|_{-\theta/2}^{\theta/2} = 2\sigma_r\,bR\,\sin\frac{\theta}{2}$$



For equilibrium: $\quad F_r - 2H\sin\dfrac{\theta}{2} = 0$

$$2\sigma_r bR\sin\frac{\theta}{2} - 2\frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)\sin\frac{\theta}{2} = 0$$

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right) \quad ◀$$

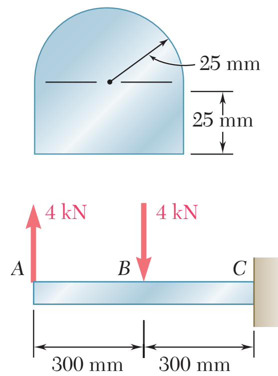
Using results of Examples 4.10 and 4.11 as data,

$M = 8$ kip · in., $\quad A = 3.75\text{ in}^2$, $\quad R = 5.9686$ in., $\quad e = 0.0314$ in., $\quad r_1 = 5.25$ in.

$$\sigma_r = \frac{8}{(3.75)(0.0314)}\left[1 - \frac{5.25}{5.9686} - \ln\frac{5.9686}{5.25}\right]$$

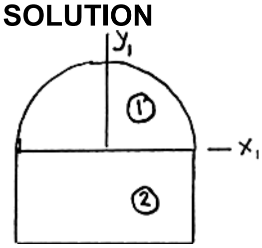
$\sigma_r = -0.536$ ksi ◀

PROBLEM 4.192



Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.

SOLUTION



$$A_1 = \frac{\pi}{2}r^2 = \frac{\pi}{2}(25)^2 = 981.7 \text{ mm}^2 \quad \bar{y}_1 = \frac{4r}{3\pi} = \frac{(4)(25)}{3\pi} = 10.610 \text{ mm}$$

$$A_2 = bh = (50)(25) = 1250 \text{ mm}^2 \quad \bar{y}_2 = -\frac{h}{2} = -\frac{25}{2} = -12.5 \text{ mm}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = \frac{(981.7)(10.610) + (1250)(-12.5)}{981.7 + 1250} = -2.334 \text{ mm}$$

$$\bar{I}_1 = I_{x_1} - A_1\bar{y}_1^2 = \frac{\pi}{8}r^4 - A_1\bar{y}_1^2 = \frac{\pi}{8}(25)^4 - (981.7)(10.610)^2 = 42.886\times10^6 \text{ mm}^4$$

$$d_1 = \bar{y}_1 - \bar{y} = 10.610 - (-2.334) = 12.944 \text{ mm}$$

$$I_1 = \bar{I}_1 + A_1 d_1^2 = 42.866\times10^3 + (981.7)(12.944)^2 = 207.35\times10^3 \text{ mm}^4$$

$$\bar{I}_2 = \frac{1}{12}bh^3 = \frac{1}{12}(50)(25)^3 = 65.104\times10^3 \text{ mm}^4$$

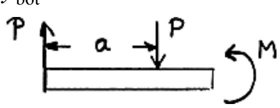
$$d_2 = |\bar{y}_2 - \bar{y}| = |-12.5 - (-2.334)| = 10.166 \text{ mm}$$

$$I_2 = \bar{I}_2 + A_2 d_2^2 = 65.104\times10^3 + (1250)(10.166)^2 = 194.288\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 401.16\times10^3 \text{ mm}^4 = 401.16\times10^{-9} \text{ m}^4$$

$$y_{\text{top}} = 25 + 2.334 = 27.334 \text{ mm} = 0.027334 \text{ m}$$

$$y_{\text{bot}} = -25 + 2.334 = -22.666 \text{ mm} = -0.022666 \text{ m}$$



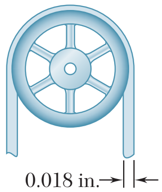
$$M - Pa = 0: \quad M = Pa = (4\times10^3)(300\times10^{-3}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{top}} = \frac{-My_{\text{top}}}{I} = -\frac{(1200)(0.027334)}{401.16\times10^{-9}} = -81.76\times10^6 \text{ Pa}$$

$\sigma_{\text{top}} = -81.8$ MPa ◀

$$\sigma_{\text{bot}} = \frac{-My_{\text{bot}}}{I} = -\frac{(1200)(-0.022666)}{401.16\times10^{-9}} = 67.80\times10^6 \text{ Pa}$$

$\sigma_{\text{bot}} = 67.8$ MPa ◀



PROBLEM 4.193

A steel band saw blade that was originally straight passes over 8-in.-diameter pulleys when mounted on a band saw. Determine the maximum stress in the blade, knowing that it is 0.018 in. thick and 0.625 in. wide. Use $E = 29 \times 10^6$ psi.

SOLUTION

Band blade thickness: $t = 0.018$ in.

Radius of pulley: $r = \frac{1}{2}d = 4.000$ in.

Radius of curvature of centerline of blade:

$$\rho = r + \frac{1}{2}t = 4.009 \text{ in.}$$

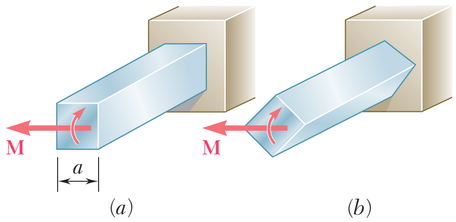
$$c = \frac{1}{2}t = 0.009 \text{ in.}$$

Maximum strain: $\varepsilon_m = \frac{c}{\rho} = \frac{0.009}{4.009} = 0.002245$

Maximum stress: $\sigma_m = E\varepsilon_m = (29 \times 10^6)(0.002245)$

$$\sigma_m = 65.1 \times 10^3 \text{ psi}$$

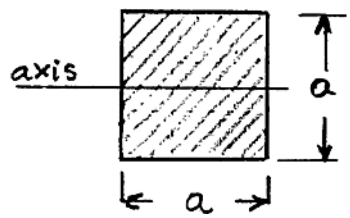
$\sigma_m = 65.1$ ksi ◀



PROBLEM 4.194

A couple of magnitude M is applied to a square bar of side a. For each of the orientations shown, determine the maximum stress and the curvature of the bar.

SOLUTION



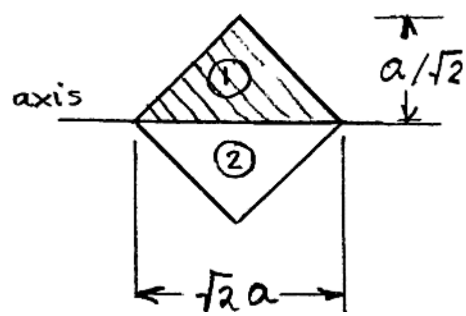
$$I = \frac{1}{12}bh^3 = \frac{1}{12}aa^3 = \frac{a^4}{12}$$

$$c = \frac{a}{2}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M\frac{a}{2}}{\frac{a^4}{12}} \qquad \sigma_{\max} = \frac{6M}{a^3} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4} \blacktriangleleft$$

For one triangle, the moment of inertia about its base is



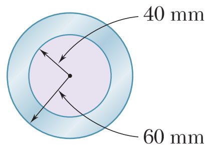
$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}\left(\sqrt{2}a\right)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

$$I_2 = I_1 = \frac{a^4}{24}$$

$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}} \qquad \sigma_{\max} = \frac{Mc}{I} = \frac{Ma/\sqrt{2}}{a^4/12} = \frac{6\sqrt{2}M}{a^3} \qquad \sigma_{\max} = \frac{8.49M}{a^3} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4} \blacktriangleleft$$

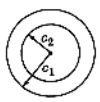


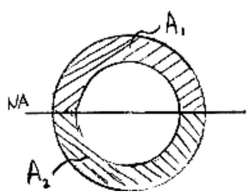
PROBLEM 4.195

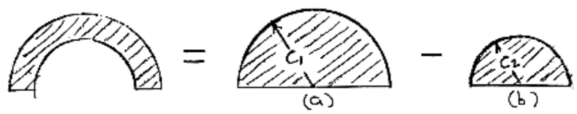
Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Let c_1 be the outer radius and c_2 the inner radius.







$$A_1\bar{y}_1 = A_a\bar{y}_a - A_b\bar{y}_b$$

$$= \left(\frac{\pi}{2}c_1^2\right)\left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2}c_2^2\right)\left(\frac{4c_2}{3\pi}\right)$$

$$= \frac{2}{3}\left(c_1^3 - c_2\right)^3$$

$$A_2\bar{y}_2 = A_1\bar{y}_1 = \frac{2}{3}\left(c_1^3 - c_2^2\right)$$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2) = \frac{4}{3}\sigma_Y\left(c_1^3 - c_2^3\right)$$

Data: $\sigma_Y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}$

$$c_1 = 60 \text{ mm} = 0.060 \text{ m}$$

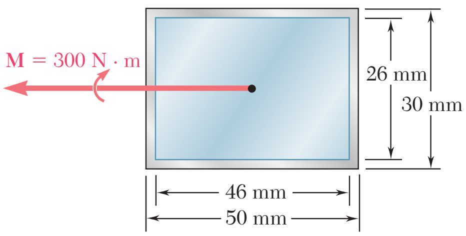
$$c_2 = 40 \text{ mm} = 0.040 \text{ m}$$

$$M_p = \frac{4}{3}(240 \times 10^6)(0.060^3 - 0.040^3)$$

$$= 48.64 \times 10^3 \text{ N} \cdot \text{m}$$

$M_p = 48.6 \text{ kN} \cdot \text{m}$ ◀

PROBLEM 4.196



In order to increase corrosion resistance, a 2-mm-thick cladding of aluminum has been added to a steel bar as shown. The modulus of elasticity is 200 GPa for steel and 70 GPa for aluminum. For a bending moment of 300 N · m, determine (*a*) the maximum stress in the steel, (*b*) the maximum stress in the aluminum, (*c*) the radius of curvature of the bar.

SOLUTION

Use aluminum as the reference material.

$$n = 1 \text{ in aluminum} \qquad n = \frac{E_s}{E_a} = \frac{200}{70} = 2.857 \text{ in steel}$$

Cross section geometry:

$$\text{Steel: } A_s = (46\,\text{mm})(26\,\text{mm}) = 1196\text{ mm}^2 \qquad I_s = \frac{1}{12}(46\text{ mm})(26\text{ mm})^3 = 67{,}375\text{ mm}^4$$

$$\text{Aluminum: } A_a = (50\text{ mm})(30\text{ mm}) - 1196\text{ mm}^2 = 304\text{ mm}^2$$

$$I_a = \frac{1}{12}(50\text{ mm})(30\text{ mm}^3) - 67{,}375\text{ mm}^4 = 45{,}125\text{ mm}^4$$

Transformed section.

$$I = n_a I_a + n_s I_s = (1)(45{,}125) + (2.857)(67{,}375) = 237{,}615\text{ mm}^4 = 237.615\times10^{-9}\text{ m}^4$$

Bending moment. $M = 300\text{ N}\cdot\text{m}$

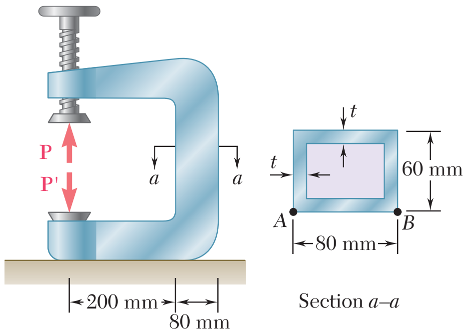
(*a*) Maximum stress in steel: $n_s = 2.857 \qquad y_s = 13\text{ mm} = 0.013\text{ m}$

$$\sigma_s = \frac{n_s M y_s}{I} = \frac{(2.857)(300)(0.013)}{237.615\times10^{-9}} = 46.9\times10^6\text{ Pa} \qquad \sigma_s = 46.9\text{ MPa} \blacktriangleleft$$

(*b*) Maximum stress in aluminum: $n_a = 1, \qquad y_a = 15\text{ mm} = 0.015\text{ m}$

$$\sigma_a = \frac{n_a M y_a}{I} = \frac{(1)(300)(0.015)}{237.615\times10^{-9}} = 18.94\times10^6\text{ Pa} \qquad \sigma_a = 18.94\text{ MPa} \blacktriangleleft$$

(*c*) Radius of curvative: $\rho = \dfrac{EI}{M} \qquad \rho = \dfrac{(70\times10^9)(237.615\times10^{-9})}{300} \qquad \rho = 55.4\text{ m} \blacktriangleleft$



PROBLEM 4.197

The vertical portion of the press shown consists of a rectangular tube of wall thickness t = 10 mm. Knowing that the press has been tightened on wooden planks being glued together until P = 20 kN, determine the stress at (a) point A, (b) point B.

SOLUTION

Rectangular cutout is $60 \text{ mm} \times 40 \text{ mm}$.

$$A = (80)(60) - (60)(40) = 2.4 \times 10^3 \text{ mm}^2 = 2.4 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84 \times 10^6 \text{ mm}^4$$

$$= 1.84 \times 10^{-6} \text{ m}^4$$

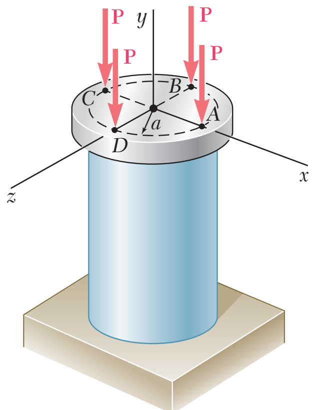
$$c = 40 \text{ mm} = 0.040 \text{ m} \quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$$

$$P = 20 \times 10^3 \text{ N}$$

$$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N} \cdot \text{m}$$

(a) $$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^3}{2.4 \times 10^{-3}} + \frac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = 112.7 \times 10^6 \text{ Pa}$$ $\sigma_A = 112.7$ MPa ◀

(b) $$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{2.4 \times 10^{-3}} - \frac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = -96.0 \times 10^6 \text{ Pa}$$ $\sigma_B = -96.0$ MPa ◀



PROBLEM 4.198

The four forces shown are applied to a rigid plate supported by a solid steel post of radius a. Knowing that $P = 24$ kips and $a = 1.6$ in., determine the maximum stress in the post when (a) the force at D is removed, (b) the forces at C and D are removed.

SOLUTION

For a solid circular section of radius a,

$$A = \pi a^2 \qquad I = \frac{\pi}{4}a^4$$

Centric force: $\quad F = 4P, \quad M_x = M_z = 0 \qquad \sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2}$

(a) Force at D is removed.

$$F = 3P, \qquad M_x = -Pa, \qquad M_z = 0$$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4}a^2} = -\frac{7P}{\pi a^2}$$

(b) Forces at C and D are removed.

$$F = 2P, \qquad M_x = -Pa, \qquad M_z = -Pa$$

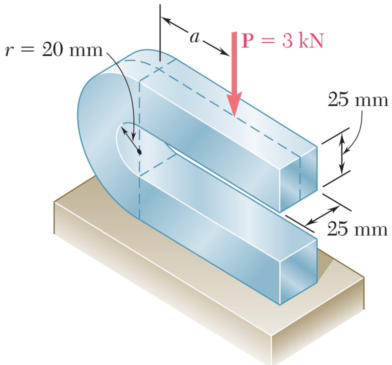
Resultant bending couple: $\quad M = \sqrt{M_x^2 + M_z^2} = \sqrt{2}Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2}Paa}{\frac{\pi}{4}a^2} = -\frac{2 + 4\sqrt{2}}{\pi}\frac{P}{a^2} = -2.437P/a^2$$

Numerical data: $\quad P = 24.0$ kips, $\quad a = 1.6$ in.

Answers: (a) $\quad \sigma = -\frac{(7)(24.0)}{\pi(1.6)^2}$ $\qquad \sigma = -20.9$ ksi ◀

(b) $\quad \sigma = -\frac{(2.437)(24.0)}{(1.6)^2}$ $\qquad \sigma = -22.8$ ksi ◀



PROBLEM 4.199

The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance *a* from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.

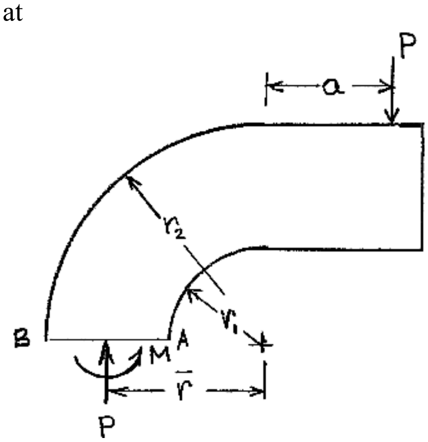
SOLUTION

Reduce the internal forces transmitted across section *AB* to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}$$



Also, $e = \bar{r} - R$

The maximum compressive stress occurs at point *A*. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1} = -K\frac{P}{A}$$

with $\qquad y_A = R - r_1$

Thus, $\qquad K = 1 + \frac{(a+\bar{r})(R - r_1)}{er_1}$

Data: $\qquad h = 25 \text{ mm}, \quad r_1 = 20 \text{ mm}, \quad r_2 = 45 \text{ mm}, \quad \bar{r} = 32.5 \text{ mm}$

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

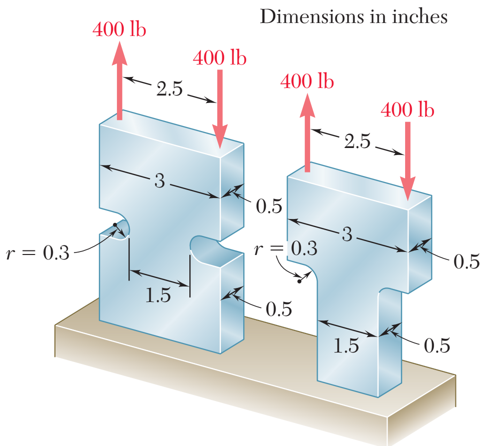
$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2 \quad R - r_1 = 10.8288 \text{ mm}$$

$$P = 3 \times 10^3 \text{ N} \cdot \text{m}, \quad \sigma_A = -150 \times 10^6 \text{ Pa}$$

$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-6})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)er_1}{R - r_1} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$$a = 93.37 - 32.5 \qquad a = 60.9 \text{ mm} \blacktriangleleft$$



PROBLEM 4.200

Determine the maximum stress in each of the two machine elements shown.

SOLUTION

For each case, $M = (400)(2.5) = 1000 \text{ lb} \cdot \text{in.}$

At the minimum section,

$$I = \frac{1}{12}(0.5)(1.5)^3 = 0.140625 \text{ in}^4$$

$$c = 0.75 \text{ in.}$$

(*a*) $D/d = 3/1.5 = 2$

$r/d = 0.3/1.5 = 0.2$

From Fig 4.32, $K = 1.75$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.75)(1000)(0.75)}{0.140625} = 9.33 \times 10^3 \text{ psi}$$

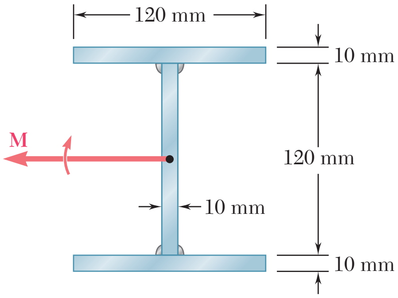
$\sigma_{\max} = 9.33 \text{ ksi}$ ◀

(*b*) $D/d = 3/1.5 = 2 \quad r/d = 0.3/1.5 = 0.2$

From Fig. 4.31, $K = 1.50$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.50)(1000)(0.75)}{0.140625} = 8.00 \times 10^3 \text{ psi}$$

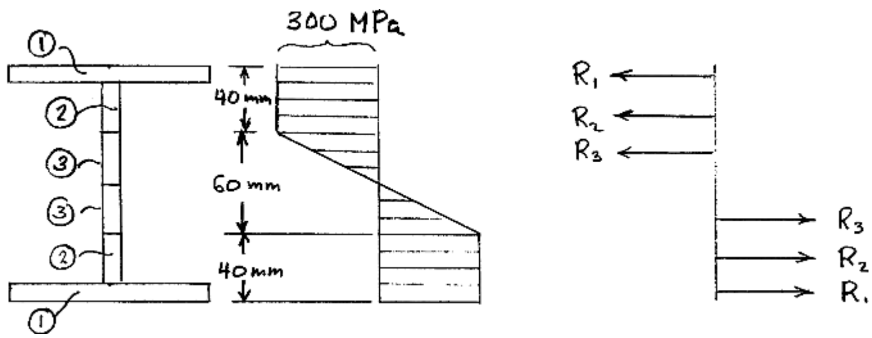
$\sigma_{\max} = 8.00 \text{ ksi}$ ◀



PROBLEM 4.201

Three 120 × 10-mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (*a*) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (*b*) the corresponding radius of curvature of the beam.

SOLUTION



$$A_1 = (120)(10) = 1200 \text{ mm}^2$$

$$R_1 = \sigma_Y A_1 = (300\times 10^6)(1200\times 10^{-6}) = 360\times 10^3 \text{ N}$$

$$A_2 = (30)(10) = 300 \text{ mm}^2$$

$$R_2 = \sigma_Y A_2 = (300\times 10^6)(300\times 10^{-6}) = 90\times 10^3 \text{ N}$$

$$A_3 = (30)(10) = 300 \text{ mm}^2$$

$$R_3 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times 10^6)(300\times 10^{-6}) = 45\times 10^3 \text{ N}$$

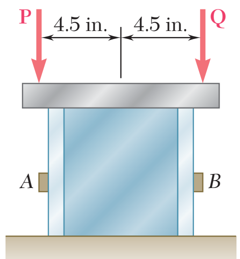
$$y_1 = 65 \text{ mm} = 65\times 10^{-3} \text{ m} \qquad y_2 = 45 \text{ mm} = 45\times 10^{-3} \text{ m} \qquad y_3 = 20 \text{ mm} = 20\times 10^{-3} \text{ m}$$

(*a*) $M = 2(R_1 y_1 + R_2 y_2 + R_3 y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$

$= 56.7\times 10^3 \text{ N}\cdot\text{m}$ $\qquad M = 56.7 \text{ kN}\cdot\text{m}$ ◀

(*b*) $\dfrac{y_Y}{\rho} = \dfrac{\sigma_Y}{E} \qquad \rho = \dfrac{E y_Y}{\sigma_Y} = \dfrac{(200\times 10^9)(30\times 10^{-3})}{300\times 10^6}$ $\qquad \rho = 20.0$ mm ◀

PROBLEM 4.202



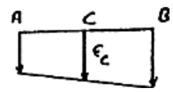
A short length of a W8 × 31 rolled-steel shape supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points A and B on the centerline of the outer faces of the flanges have been measured and found to be

$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.} \qquad \varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

SOLUTION

Strains:



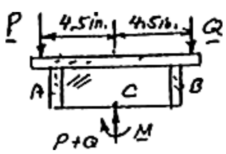
$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.} \qquad \varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

$$\varepsilon_C = \frac{1}{2}(\varepsilon_A + \varepsilon_B) = \frac{1}{2}(-550 - 680)10^{-6} = -615 \times 10^{-6} \text{ in./in.}$$

Stresses:

$$\sigma_A = E\varepsilon_A = (29 \times 10^6 \text{ psi})(-550 \times 10^{-6} \text{ in./in.}) = -15.95 \text{ ksi}$$

$$\sigma_C = E\varepsilon_C = (29 \times 10^6 \text{ psi})(-615 \times 10^{-6} \text{ in./in.}) = -17.835 \text{ ksi}$$



W8 × 31: $\quad A = 9.12 \text{ in}^2$

$S = 27.5 \text{ in}^3$

$$M = (4.5 \text{ in.})(P - Q)$$

At point C:

$$\sigma_C = -\frac{P+Q}{A}; \quad -17.835 \text{ ksi} = -\frac{P+Q}{9.12 \text{ in}^2}$$

$$P + Q = 162.655 \text{ kips} \quad (1)$$

At point A:

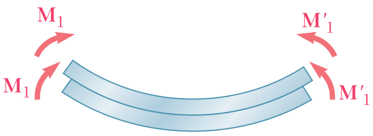
$$\sigma_A = -\frac{P+Q}{A} - \frac{M}{S}$$

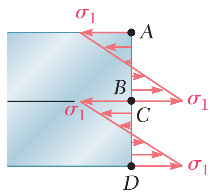
$$-15.95 \text{ ksi} = -17.835 \text{ ksi} - \frac{(4.5 \text{ in.})(P-Q)}{27.5 \text{ in}^3}; \qquad P - Q = -11.5194 \text{ kips} \quad (2)$$

Solve simultaneously. $\quad P = 75.6$ kips $\quad Q = 87.1$ kips

$\mathbf{P} = 75.6 \text{ kips} \downarrow$ ◀

$\mathbf{Q} = 87.1 \text{ kips} \downarrow$ ◀





PROBLEM 4.203

Two thin strips of the same material and same cross section are bent by couples of the same magnitude and glued together. After the two surfaces of contact have been securely bonded, the couples are removed. Denoting by σ_1 the maximum stress and by ρ_1 the radius of curvature of each strip while the couples were applied, determine (*a*) the final stresses at points *A, B, C,* and *D,* (*b*) the final radius of curvature.

SOLUTION

Let b = width and t = thickness of one strip.

Loading one strip, $M = M_1$

$$I_1 = \frac{1}{12}bt^3, \quad c = \frac{1}{2}t$$

$$\sigma_1 = \frac{M_1 c}{I} = \frac{\sigma M_1}{bt^2}$$

$$\frac{1}{\rho_1} = \frac{M_1}{EI_1} = \frac{12M_1}{Et^3}$$

After M_1 is applied to each of the strips, the stresses are those given in the sketch above. They are

$$\sigma_A = -\sigma_1, \quad \sigma_B = \sigma_1, \quad \sigma_C = -\sigma_1, \quad \sigma_D = \sigma_1$$

The total bending couple is $2M_1$.

After the strips are glued together, this couple is removed.

$$M' = 2M_1, \quad I' = \frac{1}{12}b(2t)^3 = \frac{2}{3}bt^3 \qquad c = t$$

The stresses removed are

$$\sigma' = -\frac{M'y}{I} = -\frac{2M_1 y}{\frac{2}{3}bt^3} = -\frac{3M_1 y}{bt^2}$$

$$\sigma'_A = -\frac{3M_1}{bt^2} = -\frac{1}{2}\sigma_1, \quad \sigma'_B = \sigma'_C = 0, \quad \sigma'_D = \frac{3M_1}{bt^2} = \frac{1}{2}\sigma_1$$

PROBLEM 4.203 *(Continued)*

(*a*) Final stresses: $\sigma_A = -\sigma_1 - (-\frac{1}{2}\sigma_1)$ $\qquad \sigma_A = -\frac{1}{2}\sigma_1$ ◀

$\sigma_B = \sigma_1$ ◀

$\sigma_C = -\sigma_1$ ◀

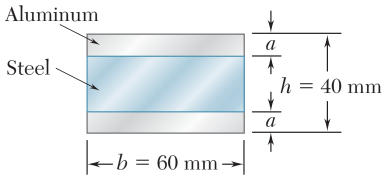
$\sigma_D = \sigma_1 - \frac{1}{2}\sigma_1$ $\qquad \sigma_D = \frac{1}{2}\sigma_1$ ◀

$$\frac{1}{\rho'} = \frac{M'}{EI'} = \frac{2M_1}{E\frac{2}{3}bt^3} = \frac{3M_1}{Et^3} = \frac{1}{4}\frac{1}{\rho'}$$

(*b*) Final radius: $\frac{1}{\rho} = \frac{1}{\rho_1} - \frac{1}{\rho'} = \frac{1}{\rho_1} - \frac{1}{4}\frac{1}{\rho_1} = \frac{3}{4}\frac{1}{\rho_1}$

$\rho = \frac{4}{3}\rho_1$ ◀

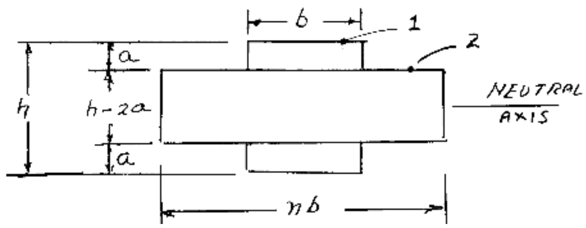
PROBLEM 4.C1



Two aluminum strips and a steel strip are to be bonded together to form a composite member of width $b = 60$ mm and depth $h = 40$ mm. The modulus of elasticity is 200 GPa for the steel and 75 GPa for the aluminum. Knowing that $M = 1500$ N · m, write a computer program to calculate the maximum stress in the aluminum and in the steel for values of a from 0 to 20 mm using 2-mm increments. Using appropriate smaller increments, determine (a) the largest stress that can occur in the steel, (b) the corresponding value of a.

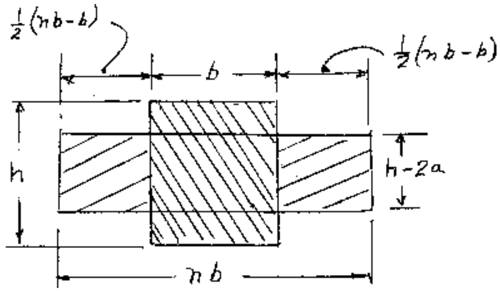
SOLUTION

Transformed section: (all steel) $n = \dfrac{E_{\text{steel}}}{E_{\text{alum}}}$



$$\bar{I} = \frac{1}{12}bh^3 + \frac{1}{12}\left[2\left(\frac{1}{2}\right)(nb-b)\right](h-2a)^3$$

At Point 1: $\sigma_{\text{alum}} = \dfrac{M\left(\frac{h}{2}\right)}{\bar{I}}$



At Point 2: $\sigma_{\text{steel}} = n\dfrac{M\left(\frac{h}{2} - a\right)}{\bar{I}}$

PROBLEM 4.C1 *(Continued)*

For $a = 0$ to 20 mm using 2-mm intervals: compute: n, $\bar{I}$, σ_{alum}, σ_{steel}.

$$b = 60 \text{ mm} \quad h = 40 \text{ mm} \quad M = 1500 \text{ N} \cdot \text{m}$$

Moduli of elasticity: Steel = 200 GPa Aluminum = 75 GPa

Program Output

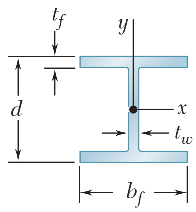
a mm	I $\text{m}^4/10^6$	Sigma Aluminum MPa	Sigma Steel MPa
0.000	0.8533	35.156	93.750
2.000	0.7088	42.325	101.580
4.000	0.5931	50.585	107.914
6.000	0.5029	59.650	111.347
8.000	0.4352	68.934	110.294
10.000	0.3867	77.586	103.448
12.000	0.3541	84.714	90.361
14.000	0.3344	89.713	71.770
16.000	0.3243	92.516	49.342
18.000	0.3205	93.594	24.958
20.000	0.3200	93.750	0.000

Find 'a' for max. steel stress and the corresponding aluminum stress.

6.600	0.4804	62.447	111.572083
6.610	0.4800	62.494	111.572159
6.620	0.4797	62.540	111.572113

Max. steel stress = 111.6 MPa occurs when $a = 6.61$ mm.

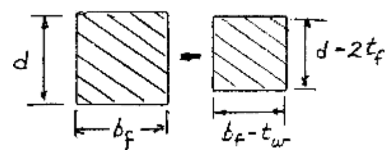
Corresponding aluminum stress $= 62.5$ MPa



PROBLEM 4.C2

A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength σ_Y and a modulus of elasticity E, is bent about the x axis. (a) Denoting by y_Y the half thickness of the elastic core, write a computer program to calculate the bending moment M and the radius of curvature ρ for values of y_Y from $\frac{1}{2}d$ to $\frac{1}{6}d$ using decrements equal to $\frac{1}{2}t_f$. Neglect the effect of fillets. (b) Use this program to solve Prob. 4.201.

SOLUTION

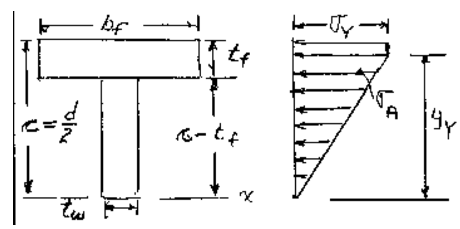


Compute moment of inertia I_x.

$$I_x = \frac{1}{12}b_f d^3 - \frac{1}{12}(b_f - t_w)(d - 2t_f)^3$$

Maximum elastic moment: $\quad M_Y = \sigma_Y \dfrac{I_x}{(d/2)}$

For yielding in the flanges,

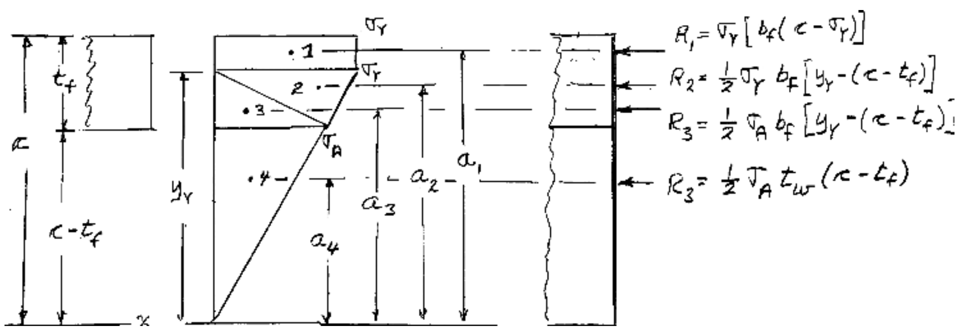


(Consider upper half of cross section.) $\quad c = \dfrac{d}{2}$

Stress at junction of web and flange:

$$\sigma_A = \frac{(d/2) - t_f}{y_Y}\sigma_Y$$

Detail of stress diagram: Resultant forces:



$$a_1 = \frac{1}{2}(c + y_Y)$$

$$a_2 = y_Y - \frac{1}{3}\left[y_Y - (c - t_f)\right]$$

$$a_3 = y_Y - \frac{2}{3}[y_Y - (c - t_f)]$$

$$a_4 = \frac{2}{3}(c - t_f)$$

PROBLEM 4.C2 *(Continued)*

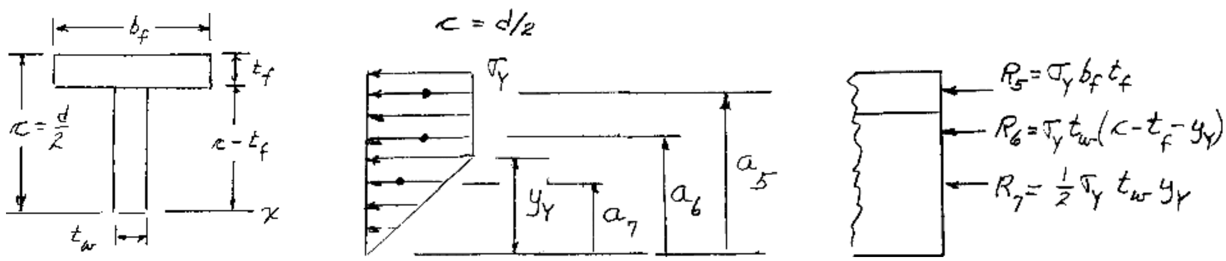
Bending moment.

$$M = 2\sum_{n=1}^{4} R_n a_n$$

Radius of curvature.

$$y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho; \quad \rho = \frac{y_Y E}{\sigma_Y}$$

For yielding in the web, (Consider upper half of cross section.)



$$a_5 = c - \frac{1}{2}t_f$$

$$a_6 = \frac{1}{2}[y_Y + (c - t_f)]$$

$$a_7 = \frac{2}{3}y_Y$$

Bending moment. $$M = 2\sum_{n=5}^{7} R_n a_n$$

Radius of curvature. $$y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho \qquad \rho = \frac{y_Y E}{\sigma_Y}$$

Program: Key in expressions for a_n and R_n for $n = 1$ to 7.

For $y_Y = c$ to $(c - t_f)$ at $-t_f/2$ decrements, compute $M = 2\Sigma R_n a_n$ for $n = 1$ to 4 and $\rho = \dfrac{y_Y E}{\sigma_Y}$, then print.

For $y_Y = (c - t_w)$ to $c/3$ at $-t_f/2$ decrements, compute $M = 2\Sigma R_n a_n$ for $n = 5$ to 7 and $\rho = \dfrac{y_Y E}{\sigma_Y}$, then print.

Input numerical values and run program.

PROBLEM 4.C2 *(Continued)*

Program Output

For a beam of Problem 4.201,

Depth $d = 140.00$ mm

Width of flange $b_f = 120.00$ mm

Thickness of flange $t_f = 10.00$ mm

Thickness of web $t_w = 10.00$ mm

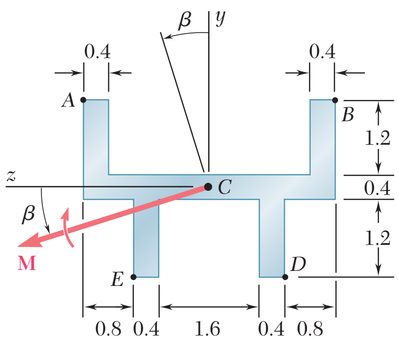
$I = 0.000011600$ m to the 4th

Yield strength of steel sigma$Y$ = 300 MPa

Yield moment $M_Y = 49.71$ kip $\cdot$ in.

y_Y (mm)	M (kN $\cdot$ m)	ρ (m)
For yielding still in the flange,		
70.000	49.71	46.67
65.000	52.59	43.33
60.000	54.00	40.00
For yielding in the web,		
60.000	54.00	40.00
55.000	54.58	36.67
50.000	55.10	33.33
45.000	55.58	30.00
40.000	56.00	26.67
35.000	56.38	23.33
30.000	56.70	20.00
25.000	56.97	16.67

PROBLEM 4.C3



An 8 kip · in. couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Noting that the centroid of the cross section is located at C and that the y and z axes are principal axes, write a computer program to calculate the stress at A, B, C, and D for values of β from 0 to 180° using 10° increments. (*Given:* $I_y = 6.23 \text{ in}^4$ and $I_z = 1.481 \text{ in}^4$.)

SOLUTION

Input coordinates of A, B, C, D.

$$z_A = z(1) = 2 \qquad y_A = y(1) = 1.4$$
$$z_B = z(2) = -2 \qquad y_B = y(2) = 1.4$$
$$z_C = z(3) = -1 \qquad y_C = y(3) = -1.4$$
$$z_D = z(4) = 1 \qquad y_D = y(4) = -1.4$$

Components of M.

$$M_y = -M \sin\beta$$
$$M_z = M \cos\beta$$

Equation 4.55, Page 305:
$$\sigma(n) = -\frac{M_z y(n)}{I_z} + \frac{M_y z(n)}{I_y}$$

Program: For $\beta = 0$ to 180° using 10° increments.

For $n = 1$ to 4 using unit increments.

Evaluate Equation 4.55 and print stresses.

Return

Return

Program Output

Moment of couple: $M = 8.00 \text{ kip} \cdot \text{in.}$

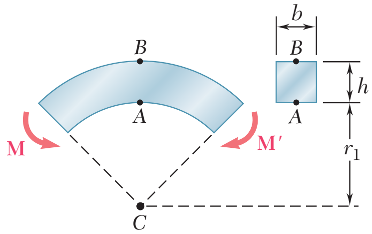
Moments of inertia: $I_y = 6.23 \text{ in}^4 \quad I_z = 1.481 \text{ in}^4$

Coordinates of Points A, B, D, and E:

Point A: $z(1) = 2$: $y(1) = 1.4$
Point B: $z(2) = -2$: $y(2) = 1.4$
Point D: $z(3) = -1$: $y(3) = -1.4$
Point E: $z(4) = 1$: $y(4) = -1.4$

---Stress at Points---

Beta °	A ksi	B ksi	D ksi	E ksi
0	–7.565	–7.565	7.565	7.565
10	–7.896	–7.004	7.673	7.227
20	–7.987	–6.230	7.548	6.669
30	–7.836	–5.267	7.193	5.909
40	–7.446	–4.144	6.621	4.970
50	–6.830	–2.895	5.846	3.879
60	–6.007	–1.558	4.895	2.670
70	–5.001	–0.174	3.794	1.381
80	–3.843	1.216	2.578	0.049
90	–2.569	2.569	1.284	–1.284
100	–1.216	3.843	–0.049	–2.578
110	0.174	5.001	–1.381	–3.794
120	1.558	6.007	–2.670	–4.895
130	2.895	6.830	–3.879	–5.846
140	4.144	7.446	–4.970	–6.621
150	5.267	7.836	–5.909	–7.193
160	6.230	7.987	–6.669	–7.548
170	7.004	7.896	–7.227	–7.673
180	7.565	7.565	–7.565	–7.565



PROBLEM 4.C4

Couples of moment $M = 2\ \text{kN}\cdot\text{m}$ are applied as shown to a curved bar having a rectangular cross section with $h = 100$ mm and $b = 25$ mm. Write a computer program and use it to calculate the stresses at points A and B for values of the ratio r_1/h from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio r_1/h for which the maximum stress in the curved bar is 50% larger than the maximum stress in a straight bar of the same cross section.

SOLUTION

<u>Input</u>:

$$h = 100\ \text{mm},$$
$$b = 25\ \text{mm},$$
$$M = 2\ \text{kN}\cdot\text{m}$$

<u>For straight bar,</u>

$$\sigma_{\text{straight}} = \frac{M}{S} = \frac{6M}{h^2 b} = 48\ \text{MPa}$$

<u>Following notation of Section 4.15</u>, key in the following:

$$r_2 = h + r_1;\quad R = h/\ln(r_2 - r_1);\quad \bar{r} = r_1 + r_2:\quad e = \bar{r} - R;\quad A = bh = 2500 \qquad \text{(I)}$$

<u>Stresses</u>:

$$\sigma_A = \sigma_1 = M(r_1 - R)(Aer_1) \qquad \sigma_B = \sigma_2 = M(r_2 - R)/(Aer_2) \qquad \text{(II)}$$

Since $h = 100$ mm, for $r_1/h = 10$, $r_1 = 1000$ mm. Also, $r_1/h = 10$, $r_1 = 100$

<u>Program</u>: <u>For $r_1 = 1000$ to 100</u> at -100 decrements,

using equations of Lines I and II, evaluate r_2, R, $\bar{r}$, e, σ_1, and σ_2

Also evaluate ratio $= \sigma_1/\sigma_{\text{straight}}$

Return and repeat for $r_1 = 100$ to 10 at -10 decrements.

PROBLEM 4.C4 *(Continued)*

Program Output

M = Bending moment = 2 kN·m $h = 100.000$ in. $A = 2500.00\ \text{mm}^2$

Stress in straight beam = 48.00 MPa

r_1 mm	rbar mm	R mm	e mm	σ_1 MPa	σ_2 MPa	r_1/h –	ratio –
1000	1050	1049	0.794	–49.57	46.51	10.000	–1.033
900	950	949	0.878	–49.74	46.36	9.000	–1.036
800	850	849	0.981	–49.95	46.18	8.000	–1.041
700	750	749	1.112	–50.22	45.95	7.000	–1.046
600	650	649	1.284	–50.59	45.64	6.000	–1.054
500	550	548	1.518	–51.08	45.24	5.000	–1.064
400	450	448	1.858	–51.82	44.66	4.000	–1.080
300	350	348	2.394	–53.03	43.77	3.000	–1.105
200	250	247	3.370	–55.35	42.24	2.000	–1.153
100	150	144	5.730	–61.80	38.90	1.000	–1.288
==========	==========	==========	==========	==========	==========		
100	150	144	5.730	–61.80	38.90	1.000	–1.288
90	140	134	6.170	–63.15	38.33	0.900	–1.316
80	130	123	6.685	–64.80	37.69	0.800	–1.350
70	120	113	7.299	–66.86	36.94	0.700	–1.393
60	110	102	8.045	–69.53	36.07	0.600	–1.449
50	100	91	8.976	–73.13	35.04	0.500	–1.523
40	90	80	10.176	–78.27	33.79	0.400	–1.631
30	80	68	11.803	–86.30	32.22	0.300	–1.798
20	70	56	14.189	–100.95	30.16	0.200	–2.103
10	60	42	18.297	–138.62	27.15	0.100	–2.888

==

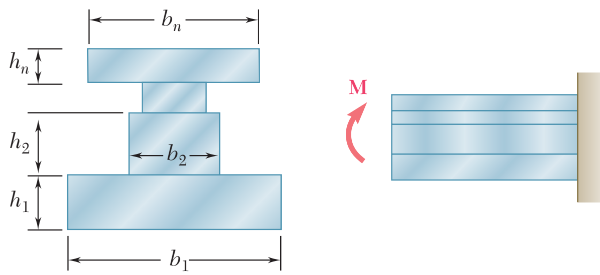
Find r_1/h for $(\sigma_{\text{max}})/(\sigma_{\text{straight}}) = 1.5$

52.70	103	94	8.703	–72.036	35.34	0.527	–1.501
52.80	103	94	8.693	–71.998	35.35	0.528	–1.500
52.90	103	94	8.683	–71.959	35.36	0.529	–1.499

Ratio of stresses is 1.5 for $r_1 = 52.8$ mm or $r_1/h = 0.529$.

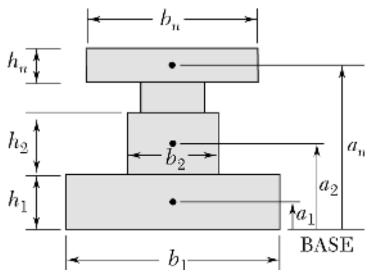
[*Note:* The desired ratio r_1/h is valid for any beam having a rectangular cross section.]

PROBLEM 4.C5



The couple **M** is applied to a beam of the cross section shown. (*a*) Write a computer program that, for loads expressed in either SI or U.S. customary units, can be used to calculate the maximum tensile and compressive stresses in the beam. (*b*) Use this program to solve Probs. 4.9, 4.10, and 4.11.

SOLUTION



Input: Bending moment M.

For $n = 1$ to n, Enter b_n and h_n

$$\Delta\text{Area} = b_n h_n \qquad \text{(Print)}$$

$$a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$$

[Moment of rectangle about base]

$$\Delta m = (\Delta\text{Area})a_n$$

[For whole cross section]

$$m = m + \Delta m; \quad \text{Area} = \text{Area} + \Delta\text{Area}$$

Location of centroid above base.

$$\bar{y} = m/\text{Area} \qquad \text{(Print)}$$

Moment of inertia about horizontal centroidal axis.

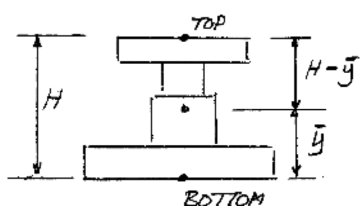
For $n = 1$ to n,

$$a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$$

$$\Delta I = b_n h_n^3/12 + (b_n h_n)(\bar{y} - a_n)^2$$

$$I = I + \Delta I \qquad \text{(Print)}$$

Computation of stresses.



Total height: For $n = 1$ to n,

$$H = H + h_n$$

Stress at top:

$$M_{\text{top}} = -M\frac{H - \bar{y}}{I} \qquad \text{(Print)}$$

Stress at bottom:

$$M_{\text{bottom}} = M\frac{\bar{y}}{I} \qquad \text{(Print)}$$

PROBLEM 4.C5 *(Continued)*

Problem 4.9

Summary of cross section dimensions:

Width (in.)	Height (in.)
9.00	2.00
3.00	6.00

Bending moment $= 600.000$ kip $\cdot$ in.

Centroid is 3.000 mm above lower edge.

Centroidal moment of inertia is 204.000 in^4.

Stress at top of beam $= -14.706$ ksi

Stress at bottom of beam $= 8.824$ ksi

Problem 4.10

Summary of cross section dimensions:

Width (in.)	Height (in.)
4.00	1.00
1.00	6.00
8.00	1.00

Bending moment $= 500.000$ kip $\cdot$ in.

Centroid is 4.778 in. above lower edge.

Centroidal moment of inertia is 155.111 in^4.

Stress at top of beam $= -10.387$ ksi

Stress at bottom of beam $= 15.401$ ksi

Problem 4.11

Summary of cross section dimensions:

Width (mm)	Height (mm)
50	10
20	50

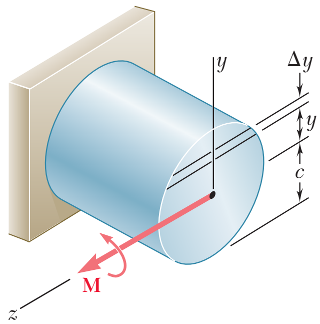
Bending moment $= 1500.0000$ N $\cdot$ m

Centroid is 25.000 mm above lower edge.

Centroidal moment of inertia is 512,500 mm^4.

Stress at top of beam $= -102.439$ MPa

Stress at bottom of beam $= 72.171$ MPa



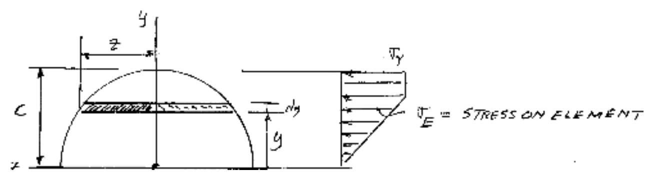
PROBLEM 4.C6

A solid rod of radius $c = 1.2$ in. is made of a steel that is assumed to be elastoplastic with $E = 29{,}000$ ksi and $\sigma_Y = 42$ ksi. The rod is subjected to a couple of moment M that increases from zero to the maximum elastic moment M_Y and then to the plastic moment M_p. Denoting by y_Y the half thickness of the elastic core, write a computer program and use it to calculate the bending moment M and the radius of curvature ρ for values of y_Y from 1.2 in. to 0 using 0.2-in. decrements. (*Hint:* Divide the cross section into 80 horizontal elements of 0.03-in. height.)

SOLUTION

$$M_Y = \sigma_Y \frac{\pi}{4} c^3 = (42 \text{ ksi}) \frac{\pi}{4} (1.2 \text{ in.})^3 = 57 \text{ kip} \cdot \text{in.}$$

$$M_p = \sigma_Y \frac{4}{3} c^3 = (42 \text{ ksi}) \frac{4}{3} (1.2 \text{ in.})^3 = 96.8 \text{ kip} \cdot \text{in.}$$



Consider top half of rod.

Let i = Number of elements in top half.

Δh = Height of each element: $\Delta h = \dfrac{c}{L}$

For $n = 0$ to $i-1$, Step 1:

$y = n(\Delta h)$

$z = \left[c^2 - \{(n+0.5)\Delta h\}^2\right]$ ← z at midheight of element

If $y \geq y_Y$ go to 100

$\sigma_E = \sigma_Y \dfrac{(n+0.5)\Delta h}{\sigma_y}$ ← Stress in elastic core

go to 200

100 $\sigma_E = \sigma_Y$ ← Stress in plastic zone

200 $\Delta\text{Area} = 2z(\Delta h)$

$\Delta\text{Force} = \sigma_E(\Delta\text{Area})$

$\Delta\text{Moment} = \Delta\text{Force } (n+0.5)\Delta h$

$M = M + \Delta\text{Moment}$

$P = y_Y E/\sigma_Y$

Print y_Y, M, and ρ.

Next

Repeat for $y_Y = 1.2$ in.
to $y_Y = 0$
At –0.2-in. decrements

PROBLEM 4.C6 *(Continued)*

Program Output

Radius of rod $= 1.2$ in.

Yield point of steel $= 42$ ksi

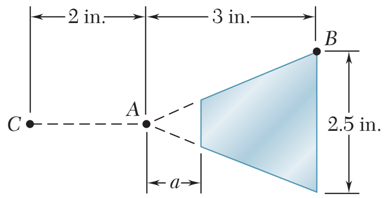
Yield moment $= 57.0$ kip · in.

Plastic moment $= 96.8$ kip · in.

Number of elements in half of the rod $= 40$

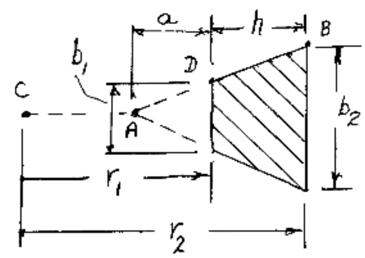
For $y_Y = 1.20$ in.,	$M = 57.1$ kip · in.	Radius of curvature $= 828.57$ in.
For $y_Y = 1.00$ in.,	$M = 67.2$ kip · in.	Radius of curvature $= 690.48$ in.
For $y_Y = 0.80$ in.,	$M = 76.9$ kip · in.	Radius of curvature $= 552.38$ in.
For $y_Y = 0.60$ in.,	$M = 85.2$ kip · in.	Radius of curvature $= 414.29$ in.
For $y_Y = 0.40$ in.,	$M = 91.6$ kip · in.	Radius of curvature $= 276.19$ in.
For $y_Y = 0.20$ in.,	$M = 95.5$ kip · in.	Radius of curvature $= 138.10$ in.
For $y_Y = 0.00$ in.,	$M =$ infinite	Radius of curvature $=$ zero

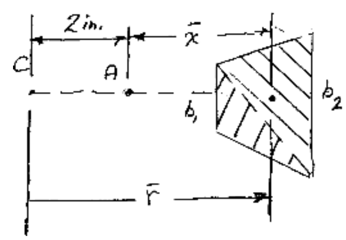
PROBLEM 4.C7



The machine element of Prob 4.178 is to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width a will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of a from 0 to 1 in. using 0.1-in. increments. Using appropriate smaller increments, determine the distance a for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

SOLUTION





See Figure 4.79, Page 289.

$$M = 5 \text{ kip} \cdot \text{in.} \quad r_2 = 5 \text{ in.} \quad b_2 = 2.5 \text{ in.}$$

For $a = 0$ to 1.0 at 0.1 intervals,

$$h = 3 - a$$

$$r_1 = 2 + a$$

$$b_1 = b_2(a/(h+a))$$

$$\text{Area} = (b_1 + b_2)(h/2)$$

$$\bar{x} = a + \left[\frac{1}{2} b_1 h(h/3) + \frac{1}{2} b_2 h\left(2h/3\right)\right] \Big/ \text{Area}$$

$$\bar{r} = r_2 - (h - \bar{x})$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$e = \bar{r} - R$$

$$\sigma_D = M(r_1 - R)/[\text{Area } (e)(r_1)]$$

$$\sigma_B = M(r_2 - R)/[\text{Area } (e)(r_2)]$$

Print and Return

Program Output

a in.	R in.	σ_D ksi	σ_B ksi	b_1	$\bar{r}$	e
0.00	3.855	−8.5071	2.1014	0.00	4.00	0.145
0.10	3.858	−7.7736	2.1197	0.08	4.00	0.144
0.20	3.869	−7.2700	2.1689	0.17	4.01	0.140
0.30	3.884	−6.9260	2.2438	0.25	4.02	0.134
0.40	3.904	−6.7004	2.3423	0.33	4.03	0.127
0.50	3.928	−6.5683	2.4641	0.42	4.05	0.119
0.60	3.956	−6.5143	2.6102	0.50	4.07	0.111
0.70	3.985	−6.5296	2.7828	0.58	4.09	0.103
0.80	4.018	−6.6098	2.9852	0.67	4.11	0.094
0.90	4.052	−6.7541	3.2220	0.75	4.14	0.086
1.00	4.089	−6.9647	3.4992	0.83	4.17	0.078

Determination of the maximum compressive stress that is as small as possible.

a in.	R in.	σ_D ksi	σ_B ksi	b_1	$\bar{r}$	e
0.620	3.961	−6.51198	2.6425	0.52	4.07	0.109
0.625	3.963	−6.51185	2.6507	0.52	4.07	0.109
0.630	3.964	−6.51188	2.6591	0.52	4.07	0.109

Answer: When a = 625 in., the compressive stress is 6.51 ksi.