Nguyen Xuan Binh 887799

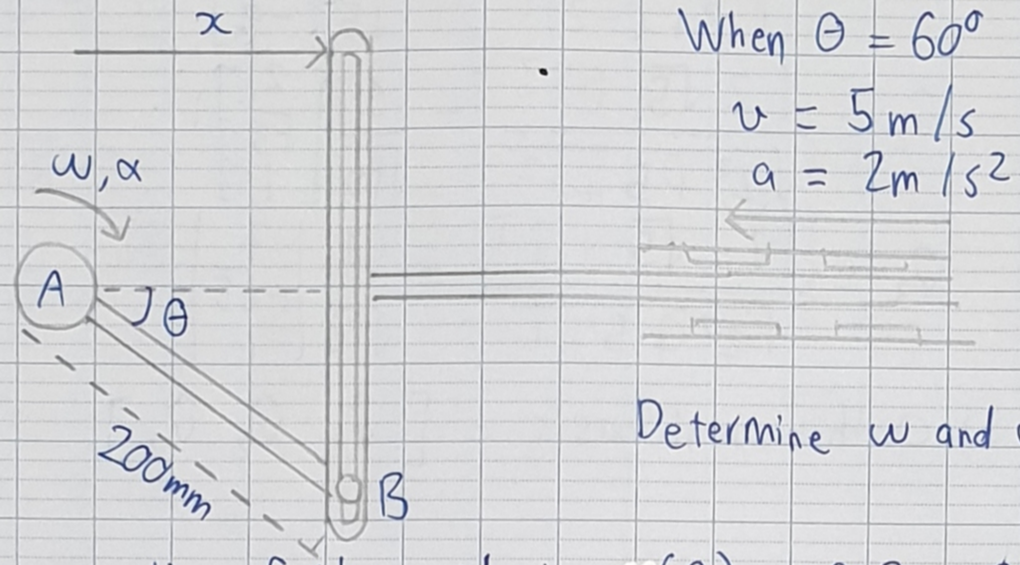
Thứ

Ngày . .

No.

Statics & Dynamics Problem Sheet 6

Exercise 1:



When $\theta = 60^\circ$

$v = 5 m/s$

$a = 2 m/s^2$

Determine ω and α of AB

- The position of the guide is $x(\theta) = 0,2\cos\theta$
- Derivation of $x(\theta)$

$$v = \frac{dx}{dt} = 0,2\frac{d}{dt}(\cos\theta) = -0,2\sin\theta\,\omega$$

=> Right at $\theta = 60^\circ$, the angular velocity is

$$v = -0,2\sin\theta . \omega$$

$$\Rightarrow \omega = \frac{v}{-0,2\sin\theta} = \frac{-5}{-0,2\sin 60} = 28,86 \text{ rad/s}$$

- Derivate $x(\theta)$ twice

$$a = \frac{dv}{dt} = (-0,2\sin\theta\,\omega)\frac{d}{dt} =$$

$$= -0,2\left[\cos\theta\,\omega^2 + \sin\theta\,\alpha\right]$$

$$\Rightarrow \alpha = \frac{a + 0,2\cos\theta\,\omega^2}{-0,2\sin\theta} = \frac{-2 + 0,2\cos 60^\circ . 28,86^2}{-0,2\sin 60}$$

$$\Rightarrow \alpha = -469,33 \ (\text{rad}/s^2)$$

Since α is negative, its direction will be counterclockwise

Answer: $\omega = 28,86$ rad/s , $\alpha = -469,33$ rad/s^2

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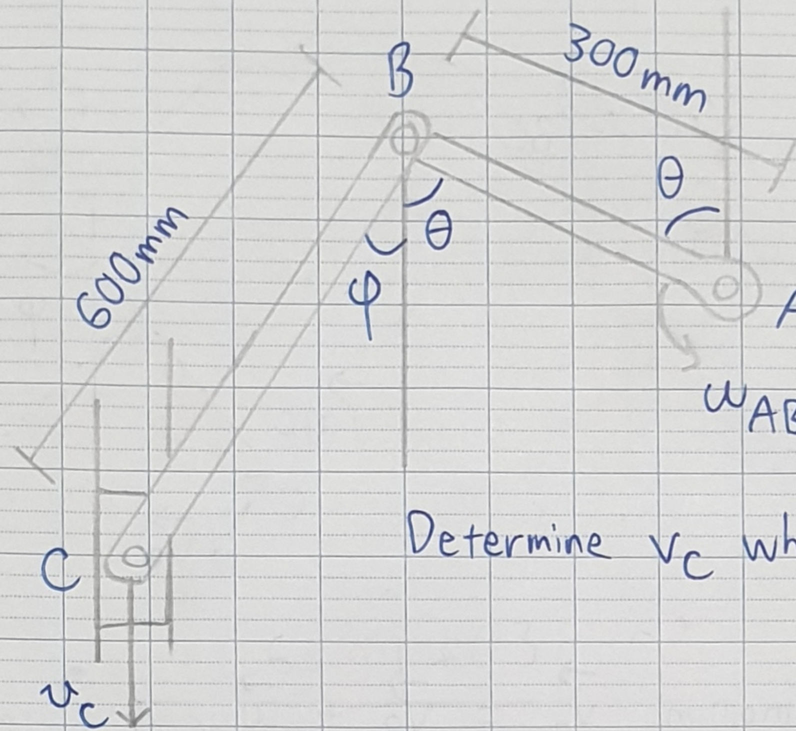
Exercise 2:



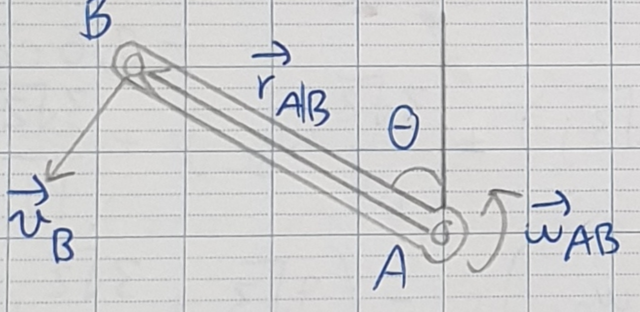
$\omega_{AB} = 60$ rad/s

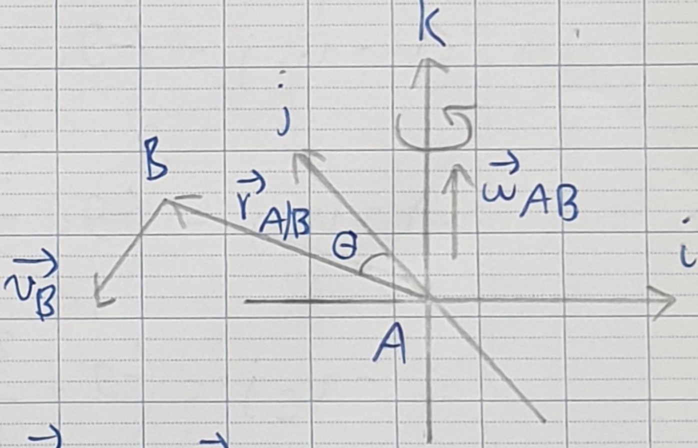
Determine v_C when $\theta = 60^\circ$ and $\varphi = 45^\circ$

We will apply relative motion analysis

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + \vec{\omega}_A \times \vec{r}_{B/A}$

- Analysis of rod AB





Relative motion: $\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{A/B}$

Since A is fixed $\Rightarrow \vec{v}_A = 0$

$\Rightarrow \vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{A/B}$

$= \omega_{AB} \cdot k \times (r_{AB} \sin\theta (-i) + r_{AB} \cos\theta j)$

$= 60k \times (-0{,}3 \sin 60 i + 0{,}3 \cos 60 j)$

$= -9\sqrt{3} j - 9i$ (m/s)

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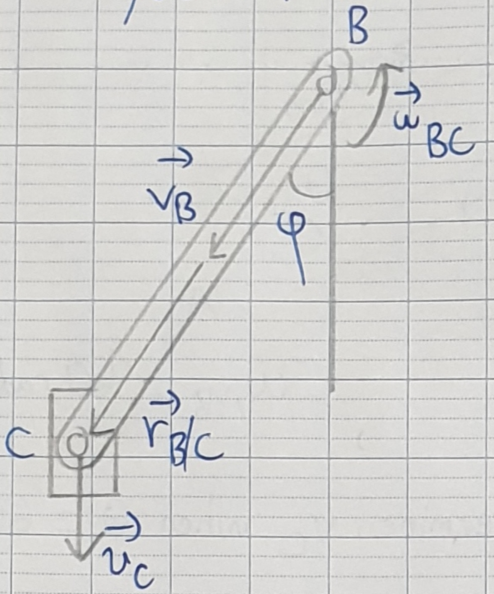
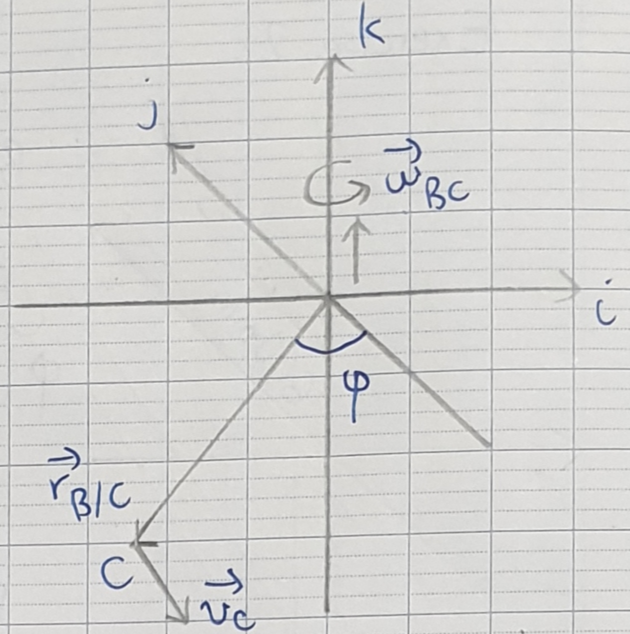
- Analysis of rod BC





Relative motion: $\vec{v}_C = \vec{v}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$

Since C only translates along y axis $\Rightarrow \vec{v}_C = -v_C j$

$$-v_C j = (-9\sqrt{3} j - 9i) + \omega_{BC} k \times (r_{CB} \sin\varphi (-i) + r_{CB} \cos\varphi (-j))$$

$$\Rightarrow -v_C j = (-9i - 9\sqrt{3} j) + \omega_{BC} k \times (-0{,}6 \sin 45 i - 0{,}6 \cos 45 j)$$

$$\Rightarrow -v_C j = (-9i - 9\sqrt{3} j) - \frac{3\sqrt{2}}{10}\omega_{BC} j + \frac{3\sqrt{2}}{10}\omega_{BC} i$$

$$\Rightarrow -v_C j = \left(-9 + \frac{3\sqrt{2}}{10}\omega_{BC}\right) i - \left(9\sqrt{3} + \frac{3\sqrt{2}}{10}\omega_{BC}\right) j$$

$$\Rightarrow \begin{cases} -9 + \frac{3\sqrt{2}}{10}\omega_{BC} = 0 \\ v_C - 9\sqrt{3} - \frac{3\sqrt{2}}{10}\omega_{BC} = 0 \end{cases} \Rightarrow \begin{cases} \omega_{BC} = 15\sqrt{2} \text{ rad/s} \\ v_C - 9\sqrt{3} - \frac{3\sqrt{2}}{10} \cdot 15\sqrt{2} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \omega_{BC} = 15\sqrt{2} \text{ rad/s} \\ v_C = 9\sqrt{3} + 9 \text{ m/s} \approx 24{,}58 \text{ m/s} \end{cases}$$

At the moment $\theta = 60^\circ$ and $\varphi = 45^\circ$, v_C is $24{,}58$ m/s

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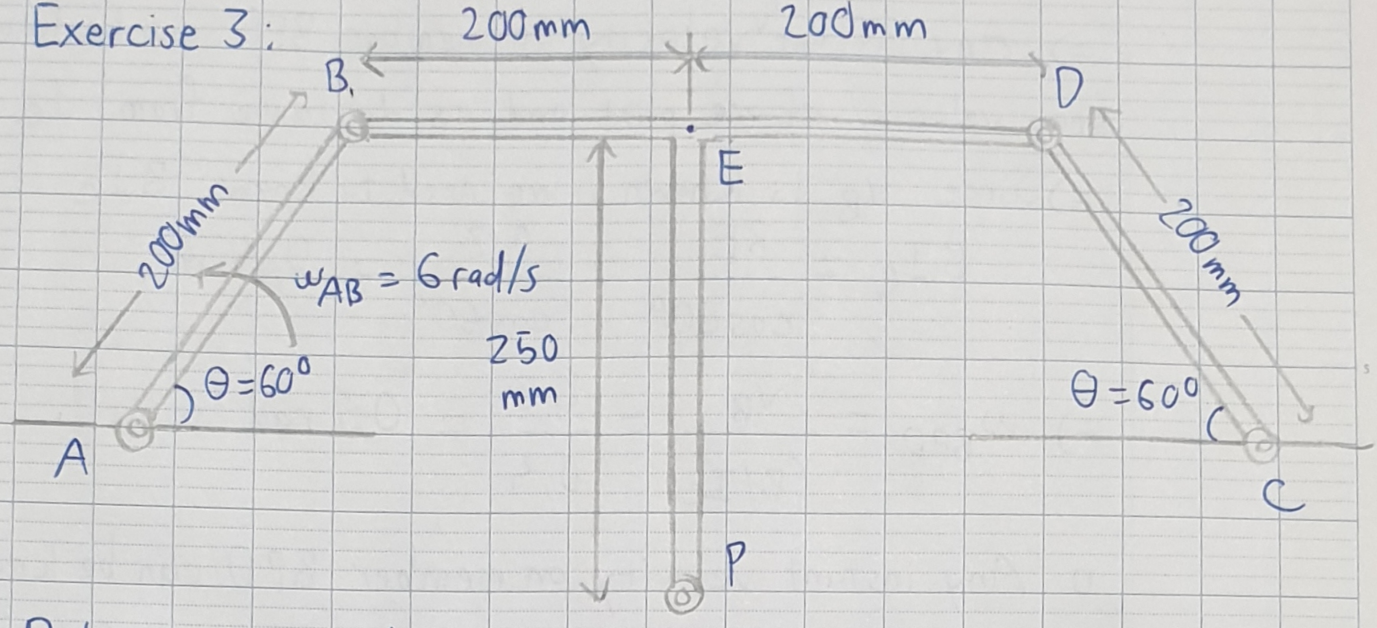
Exercise 3:



Determine v_P and ω_{BPD}

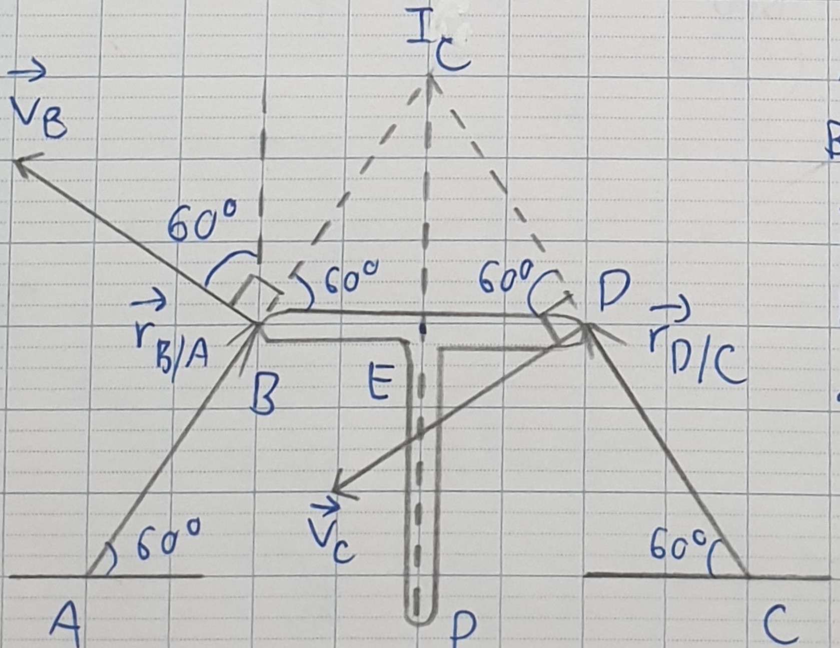
□ Relative motion: $\vec{v}_B = \vec{v}_A + \vec{\omega}_{BA} \times \vec{r}_{B/A}$

$\Rightarrow \vec{v}_B = 0 + 6k \times (0,2\sin60^\circ j + 0,2\cos60^\circ i)$

$\vec{v}_B = 6k \times \left(\frac{\sqrt{3}}{10} j + \frac{1}{10} i\right) = -\frac{3\sqrt{5}}{5} i + \frac{3}{5} j$

Magnitude of v_B: $v_B = \sqrt{\left(-\frac{3\sqrt{5}}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = 1,2$ m/s

□ Since v_P and ω_{BPD} need to be known $\Rightarrow$ an instantaneous center of zero velocity is needed for the whole member BPD



At the instant member BPD is upright, AB and BI_C are on the same line because they are both orthogonal to $\vec{v}_B$

It is the same for I_CD and DC

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$\Rightarrow$ Any instant angular velocity on member BPD can be known if we know velocity of one point and its distance from I_C

Since v_B is known, we need to know BI_C

$$BI_C = \frac{BE}{\cos 60^\circ} = \frac{0,2}{\cos 60^\circ} = 0,4 \text{ m}$$

$$\Rightarrow \omega_{BPD} = \frac{v_B}{r_{B/I_C}} = \frac{1,2}{0,4} = 3 \text{ rad/s}$$

▫ Any instant velocity on member BPD can be known if we know angular velocity of BPD and distance of the point to I_C.

To find v_P, find I_CP

We have: $I_CP = I_CE + EP = 0,2\tan 60^\circ + 0,25$

$$= \frac{5 + 4\sqrt{3}}{20} \text{ m}$$

$$\Rightarrow v_P = \omega_{BPD} \cdot r_{P/I_C} = 3 \cdot \frac{5 + 4\sqrt{3}}{20} = \frac{15 + 12\sqrt{3}}{20}$$

$\Rightarrow v_P \approx 1,789$ m/s

Answer: $\omega_{BPD} = 3$ rad/s , $v_P = 1,789$ m/s

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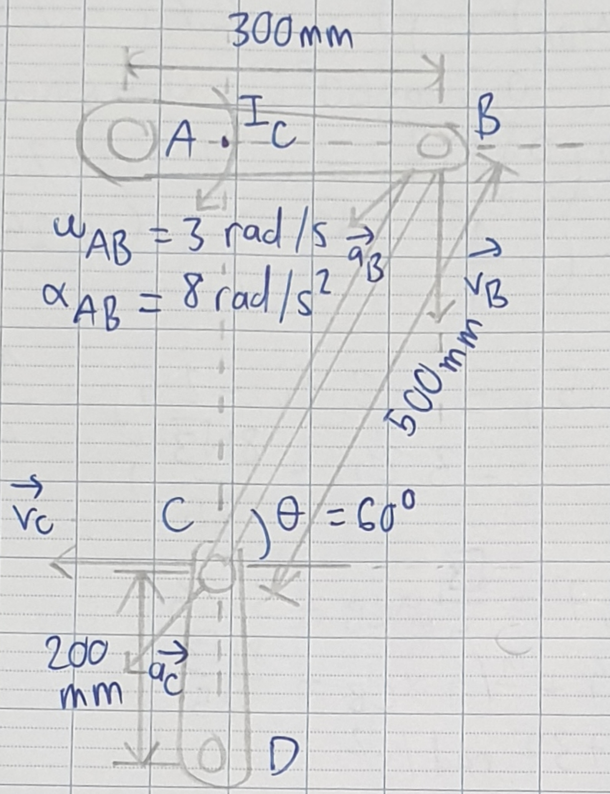
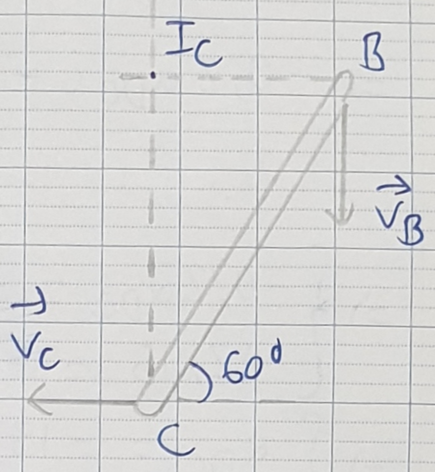
Exercise 4 :



Find v_C and a_C



- Since A is fix $\Rightarrow \vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$$\vec{v}_B = -3k \times 0,3i = -0,9j$$

$\Rightarrow$ Magnitude of $\vec{v}_B$ is $0,9$ m/s

- Since A is fix $\Rightarrow \vec{a}_A = 0$

$$\vec{a}_B = \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$
$$= -8k \times 0,3i - 3^2 . 0,3i$$
$$= -2,4j - 2,7i \ (m/s^2)$$

- We know direction of $\vec{v}_B$ and $\vec{v}_C$ $\Rightarrow$ instant center of zero velocity will be intersection of lines passing through B and C and perpendicular to $\vec{v}_B$ and $\vec{v}_C$

$$\Rightarrow \omega_{BC} = \frac{v_B}{r_{B/I_C}} = \frac{0,9}{0,5\cos 60^\circ} = 3,6 \text{ rad/s}$$

$$\Rightarrow v_C = \omega_{BC} . r_{C/I_C} = 3,6 . 0,5 \sin 60^\circ = 0,9\sqrt{3} \text{ m/s}$$

$$\Rightarrow \omega_{DC} = \frac{v_C}{r_{D/C}} = \frac{0,9\sqrt{3}}{0,2} = \frac{9\sqrt{3}}{2} \text{ rad/s}$$

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□ We have: $\vec{a}_C = \vec{a}_B + \vec{\alpha}_{CB} \times \vec{r}_{C/B} - \omega^2_{BC} \vec{r}_{C/B}$

$\Rightarrow r_{CD}\alpha_{DC}(-i) + r_{CD}(\omega_{DC})^2(-j) = (-2,7i - 2,4j)$
$+ (-\alpha_{CB}k) \times (-0,5\cos 60^\circ i - 0,5\sin 60^\circ j) - (3,6)^2 \vec{r}_{C/B}$

$\Rightarrow -0,2\alpha_{DC} i - 0,2\left(\frac{9\sqrt{3}}{2}\right)^2 j = (-2,7i - 2,4j)$
$+ (-\alpha_{CB}k) \times \left(-\frac{1}{4}i - \frac{\sqrt{3}}{4}j\right) + \left(3,24i + \frac{81\sqrt{3}}{25}j\right)$

$\Rightarrow -0,2\alpha_{DC} i - 12,15 j = -2,7i - 2,4j + \frac{1}{4}\alpha_{CB} j - \frac{\sqrt{3}}{4}\alpha_{CB} i$
$+ 3,24i + \frac{81\sqrt{3}}{25} j$

$\Rightarrow \begin{cases} -0,2\alpha_{DC} = -2,7 - \frac{\sqrt{3}}{4}\alpha_{CB} + 3,24 \\ -12,15 = -2,4 + 1/4\,\alpha_{CB} + \frac{81\sqrt{3}}{25} \end{cases}$

$\Rightarrow \begin{cases} \alpha_{DC} = -135,737 \text{ rad/s}^2 \\ \alpha_{CB} = -61,447 \text{ rad/s}^2 \end{cases}$

□ We have: $\vec{a}_C = r_{CD}\alpha_{DC}(-i) + r_{CD}(\omega_{DC})^2(-j)$
$= -0,2.(-135,737)i - 0,2\left(\frac{9\sqrt{3}}{2}\right)^2 j$
$= 27,1474i - 12,15j \ (m/s^2)$

$\Rightarrow$ Magnitude of a_C is $\sqrt{27,1474^2 + (-12,15)^2} = 29,74 \text{ m/s}^2$

$\Rightarrow$ Answer: $v_C = 0,9\sqrt{3}$ m/s $= 1,5588$ m/s
$a_C = 29,74 \text{ m/s}^2$

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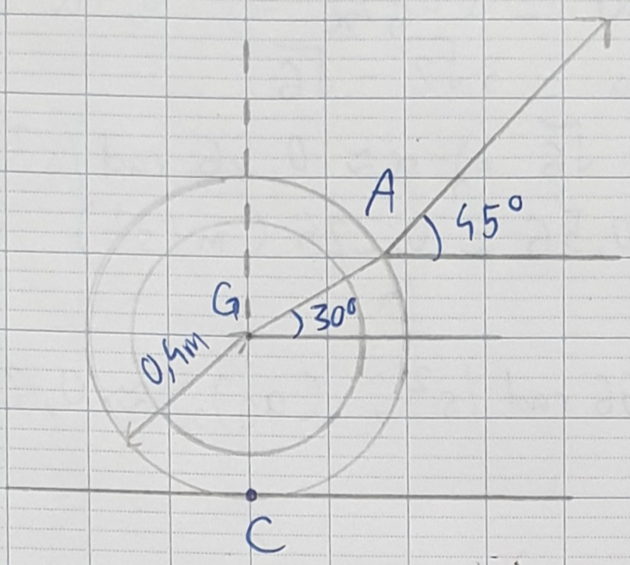
Exercise 5 :

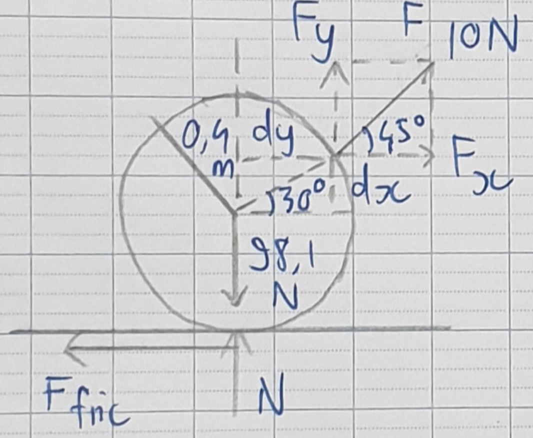


$\vec{F} = 10N$

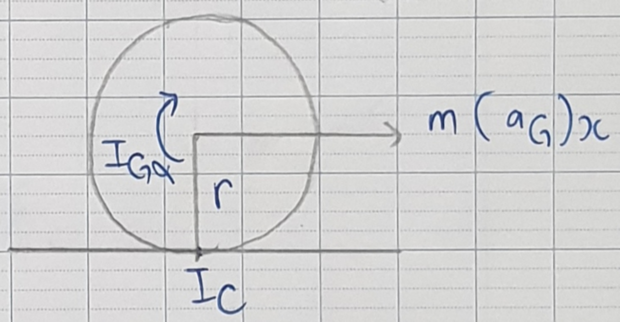
$m = 10kg$

$\alpha = ?$ $\quad (a_G)_x = ?$

Free-body diagram



Kinetic Diagram



We have: $\curvearrowright + \Sigma M_C = -F_y \cdot d_y + F_x(d_x + r)$

$$\Rightarrow \curvearrowright + \Sigma M_C = -(F \cdot \sin 45^\circ)(r \cdot \cos 30^\circ) + (F \cos 45^\circ)(r \cdot \sin 30^\circ + r)$$

$$= -(10 \sin 45^\circ)(0,4 \cos 30^\circ) + (10 \cos 45^\circ)(0,4 \cancel{\cos} \sin 30^\circ + 0,4)$$

$$= -\sqrt{6} + 3\sqrt{2} \quad (1)$$

We also have: $\curvearrowright + \Sigma M_C = m(a_G)_x r + \overset{mr^2}{\bar{I}_G}\alpha$

$$= 10(a_G)_x \cdot 0,4 + 10 \cdot 0,4^2 \alpha$$

$$= 4(a_G)_x + 1,6\alpha \quad (2)$$

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From (1) & (2) $\Rightarrow 4(a_G)_x + 1{,}6\alpha = 3\sqrt{2} - \sqrt{6}$

We also have : $\alpha r = (a_G)_x \Rightarrow 0{,}4\alpha = (a_G)_x$

$\Rightarrow 4 . 0{,}4\alpha + 1{,}6\alpha = 3\sqrt{2} - \sqrt{6}$

$3{,}2\alpha = 3\sqrt{2} - \sqrt{6} \Rightarrow \alpha \approx 0{,}56 \text{ rad}/s^2$

$\Rightarrow (a_G)_x = 0{,}4 . 0{,}56 \approx 0{,}224 \ (m/s^2)$

Answer: $\alpha \approx 0{,}56 \text{ rad}/s^2$, $(a_G)_x \approx 0{,}224 \ m/s^2$