Aalto-yliopisto
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COE-C1001: Statics

4. Structural analysis

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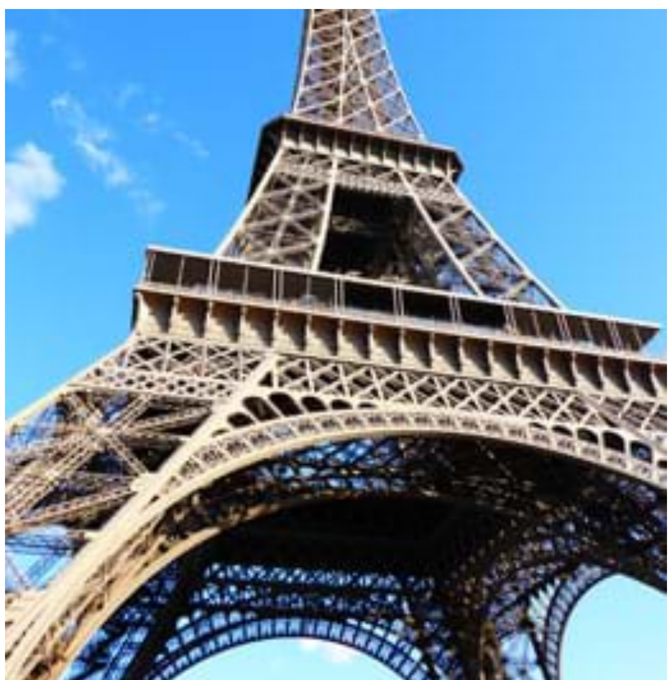
Learning outcomes

After this lecture, you should be able to:

- Define what is a simple truss structure,
- Determine the internal forces in the members of a simple truss structure using:
  - The method of joints, and
  - The method of sections.

Truss structures

Truss structures are used in a wide range of applications because they are an extremely efficient use of material.



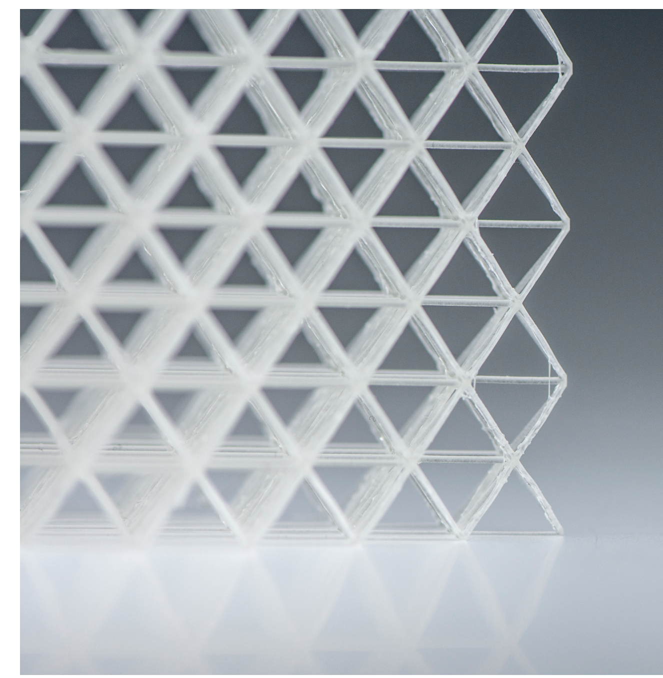


Photo by Valeria Azovskaya

I-35W Mississippi River bridge, before 1.8.2007

I-35W Mississippi River bridge, 1.8.2007

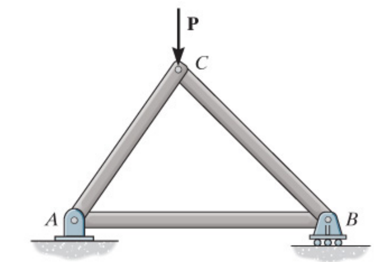
Missisippi River Bridge I-35W

The accident was due to:

- Undersized joint plates.
- Large weight of construction equipment, machines and material (that were on the bridge when the accident occured).
- Addition of concrete on the road surface (an increase in weight of 20%).

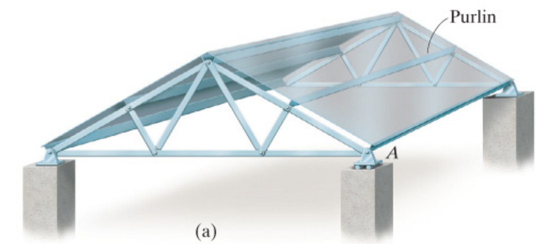
https://en.wikipedia.org/wiki/I-35W_Mississippi_River_bridge

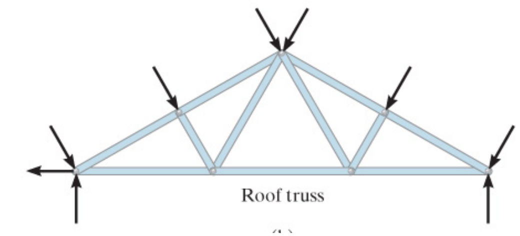
Assumptions of simple truss

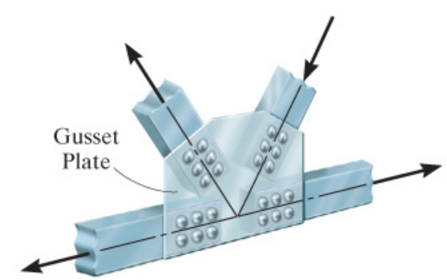


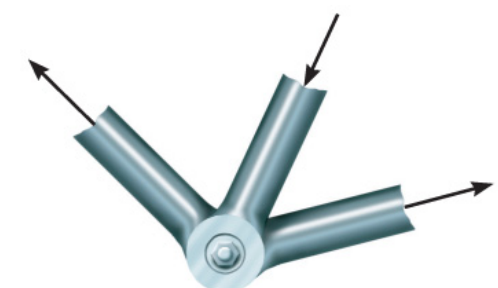
The analysis of simple truss structures is based on the following assumptions:

- All loadings are applied at the joints. (If their weight has to be considered, apply it as a force on the joints).
- The members are connected by pin joints.
- All members are straight.









Simple truss

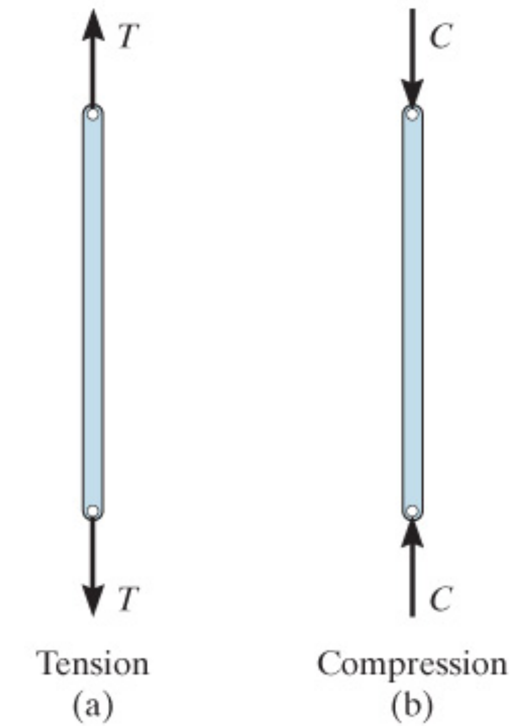
Because of these assumptions, each truss member:

- Is a two-force member,
- where the forces are directed along the member's length (i.e. axial forces only).

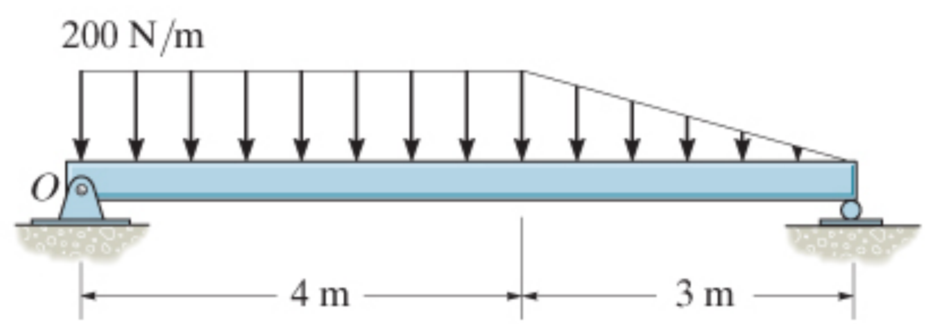
An important distinction:

- Truss: carries axial forces only.
- Beam: carries axial and transverse forces, and bending moments.

Truss



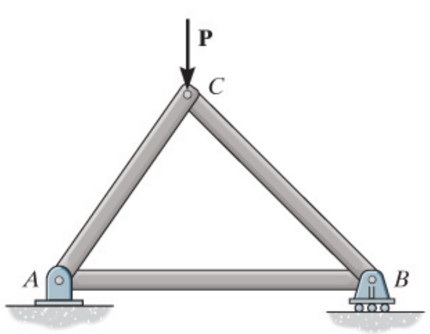
Beam



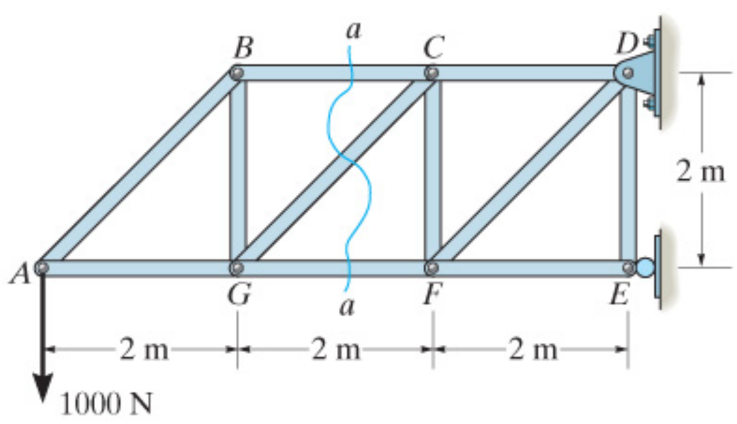
Is it a simple truss structure?

- If three bars are connected at their ends they form a triangular truss.
- We can add two bars to create a larger truss.
- Repeat this procedure multiple times to expand the truss structure.

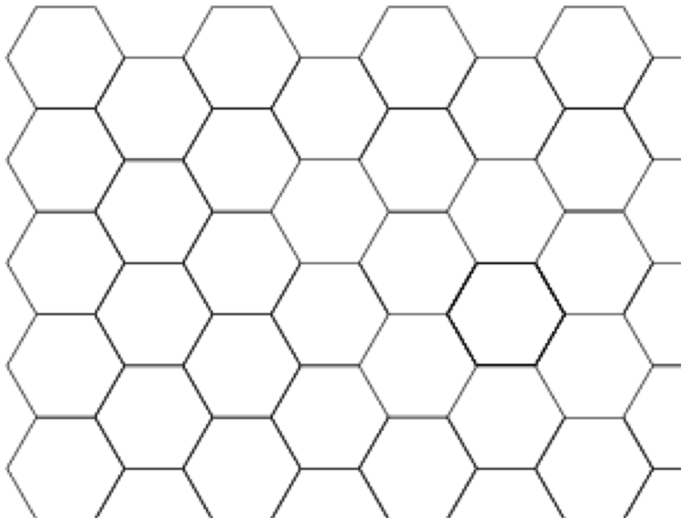
Simple truss



Simple truss



Not a simple truss!



Important points

- Simple trusses are made of triangular elements. The bars are connected at their ends by pins.
- Loads are applied at the joints only.

- If a truss is in equilibrium, then each joint and each bar has to be in equilibrium.
- We will see two different methods to find the internal forces in the members.
  - Method of joints, and
  - Method of sections.

Method of joints

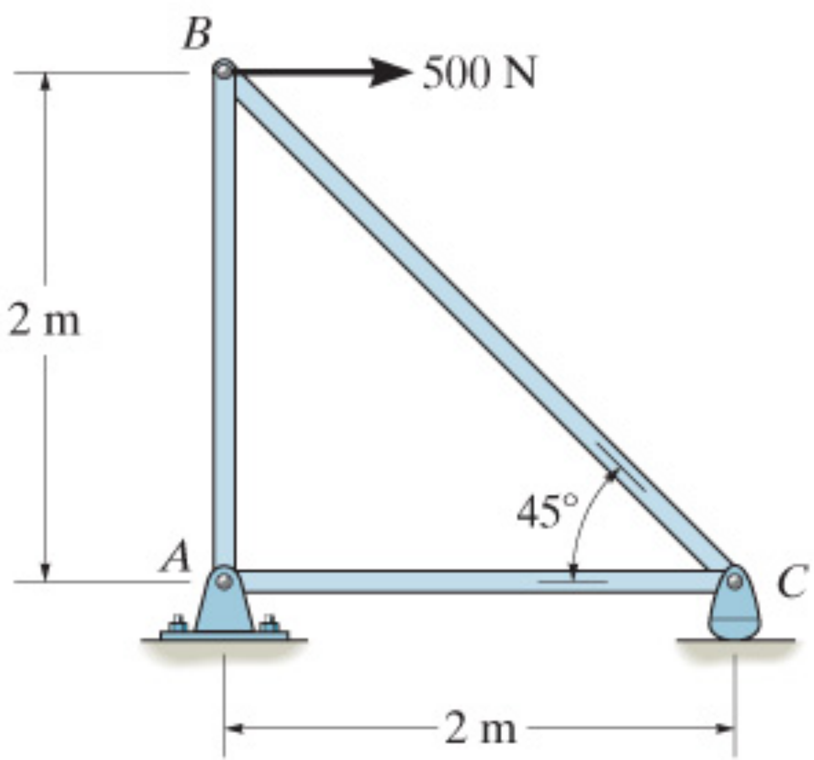
The method of joints follows this procedure:

1. Draw a free-body diagram of the structure, and find the support reactions.
2. Create a free-body diagram of each joint. You can (i) guess if each member is in tension or compression or (ii) assume that each member is in tension.
3. Use equilibrium equations ($\Sigma F_x = 0$ and $\Sigma F_y = 0$) to find the internal force in each bar. Begin with a joint that has at most two unknown forces.

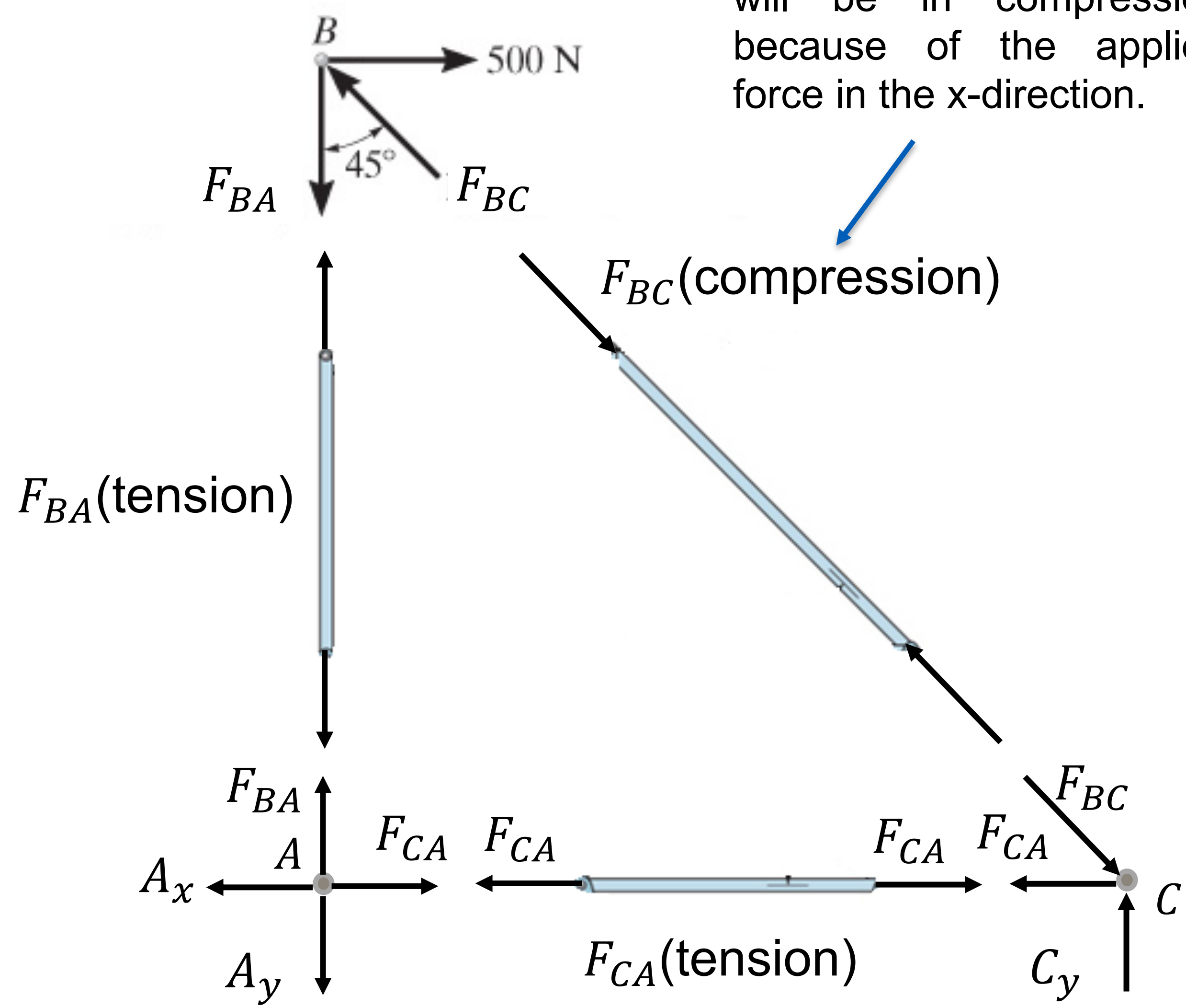
If you find that an internal force is negative it means that the loading direction should be reversed (tension/compression).

Method of joints

Create free-body diagram for each bar and joint.

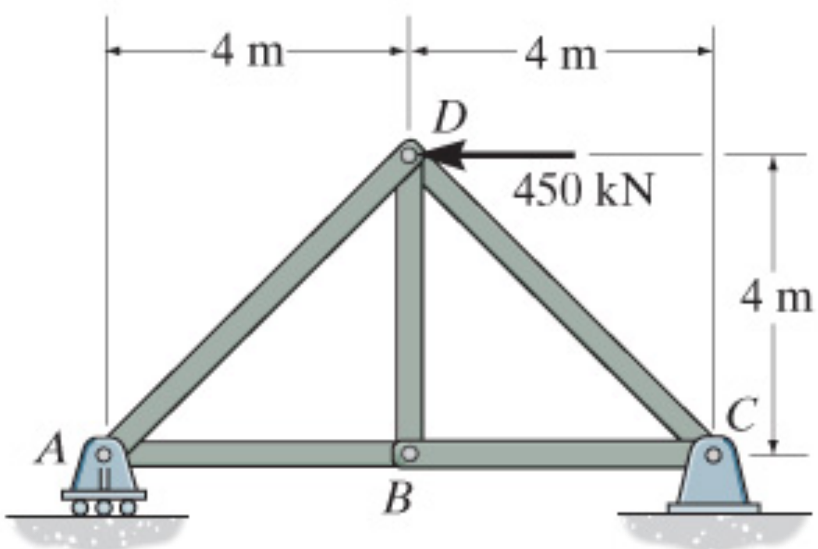


We can guess that bar BC will be in compression because of the applied force in the x-direction.

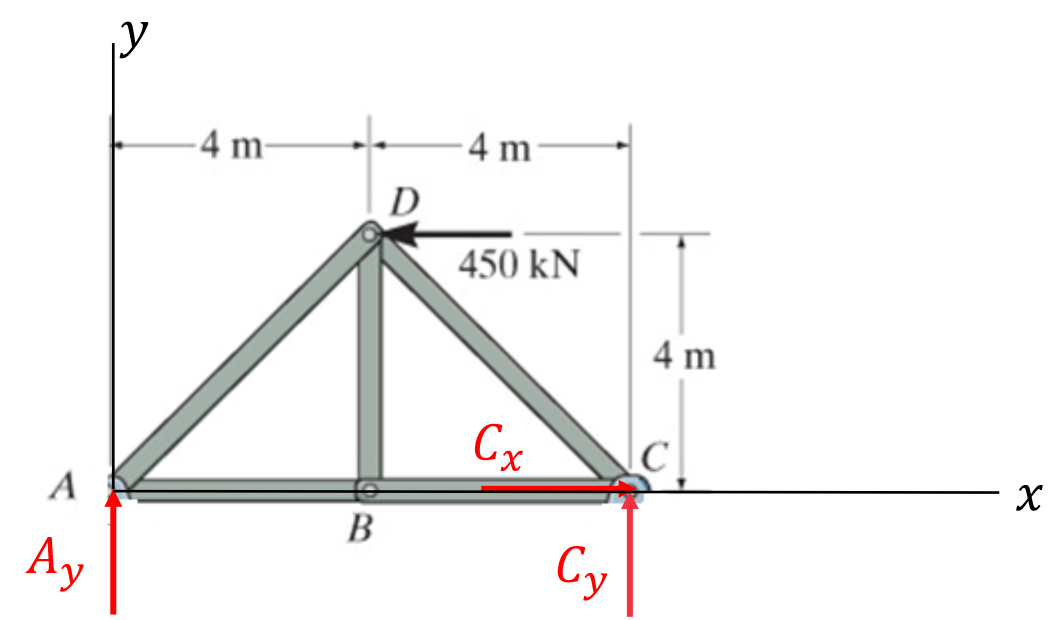


Example

Determine the force in each truss, and if they are loaded in tension or compression.



1. Determine the support reactions.
2. Find the forces in each truss member.



$\rightarrow + \quad \Sigma F_x = 0 \qquad -450\text{ kN} + C_x = 0 \qquad C_x = 450\text{ kN}$

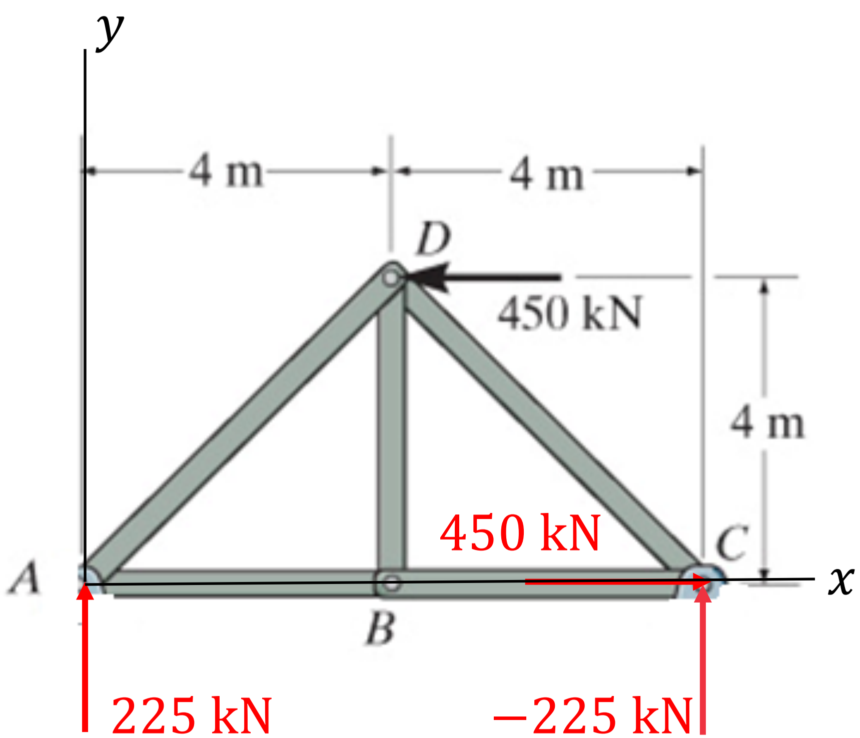
$\circlearrowleft + \quad \Sigma M_A = 0 \qquad 450\text{ kN}(4\text{m}) + C_y(8\text{m}) = 0 \qquad C_y = -225\text{ kN}$

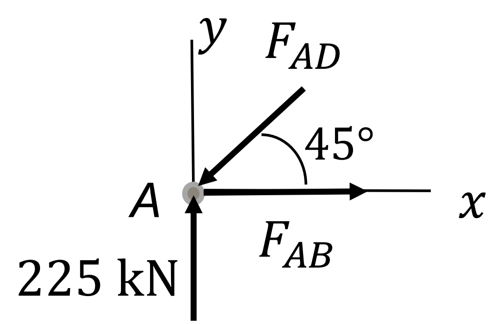
$\uparrow + \quad \Sigma F_y = 0 \qquad -225\text{ kN} + A_y = 0 \qquad A_y = 225\text{ kN}$

Example

Joints *A* and *C* have two unknowns, whereas joints *B* and *D* have three. Let's start with the equilibirum equations for joints *A* and *C*.

Determine the force in each truss, and if they are loaded in tension or compression.





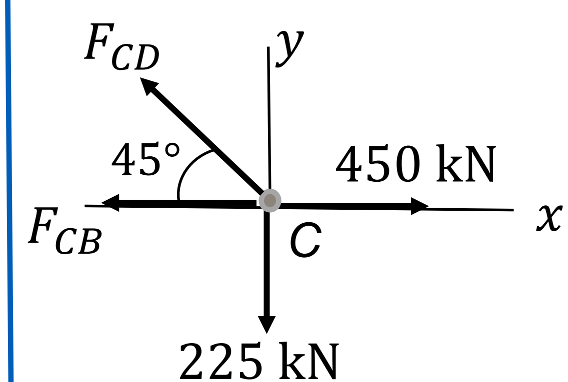
$$\uparrow + \quad \Sigma F_y = 0 \qquad 225\ \text{kN} - F_{AD} \sin 45° = 0$$

$$F_{AD} = 318\ \text{kN}$$

$$\rightarrow + \quad \Sigma F_x = 0 \quad -318\ \text{kN} \cos 45° + F_{AB} = 0$$

$$F_{AB} = 225\ \text{kN}$$

Next, we look at joint *C*.



$$\uparrow + \quad \Sigma F_y = 0 \quad -225\ \text{kN} + F_{CD} \sin 45° = 0$$

$$F_{CD} = 318\ \text{kN}$$

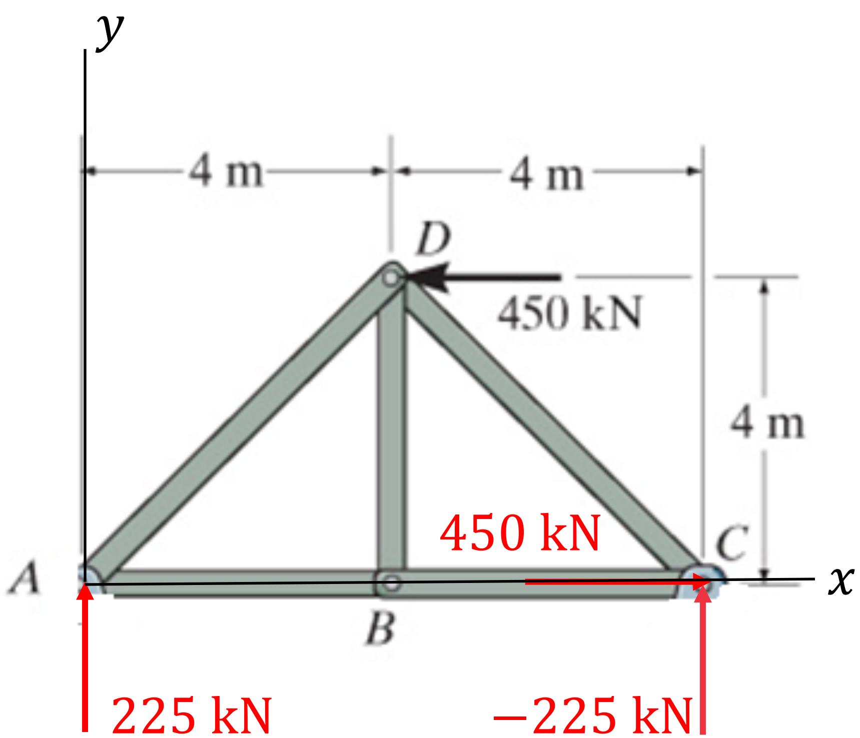
$$\rightarrow + \quad \Sigma F_x = 0$$

$$-318\ \text{kN} \cos 45° - F_{CB} + 450\ \text{kN} = 0$$

$$F_{CB} = 225\ \text{kN}$$

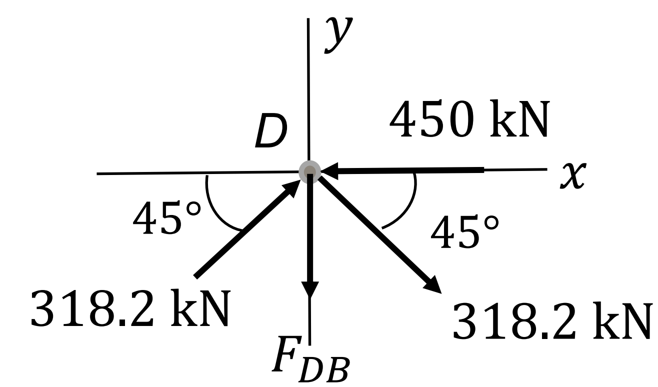
Example

Determine the force in each truss, and if they are loaded in tension or compression.



Force in bar *DB* can be obtained from joint *D* or *B*.

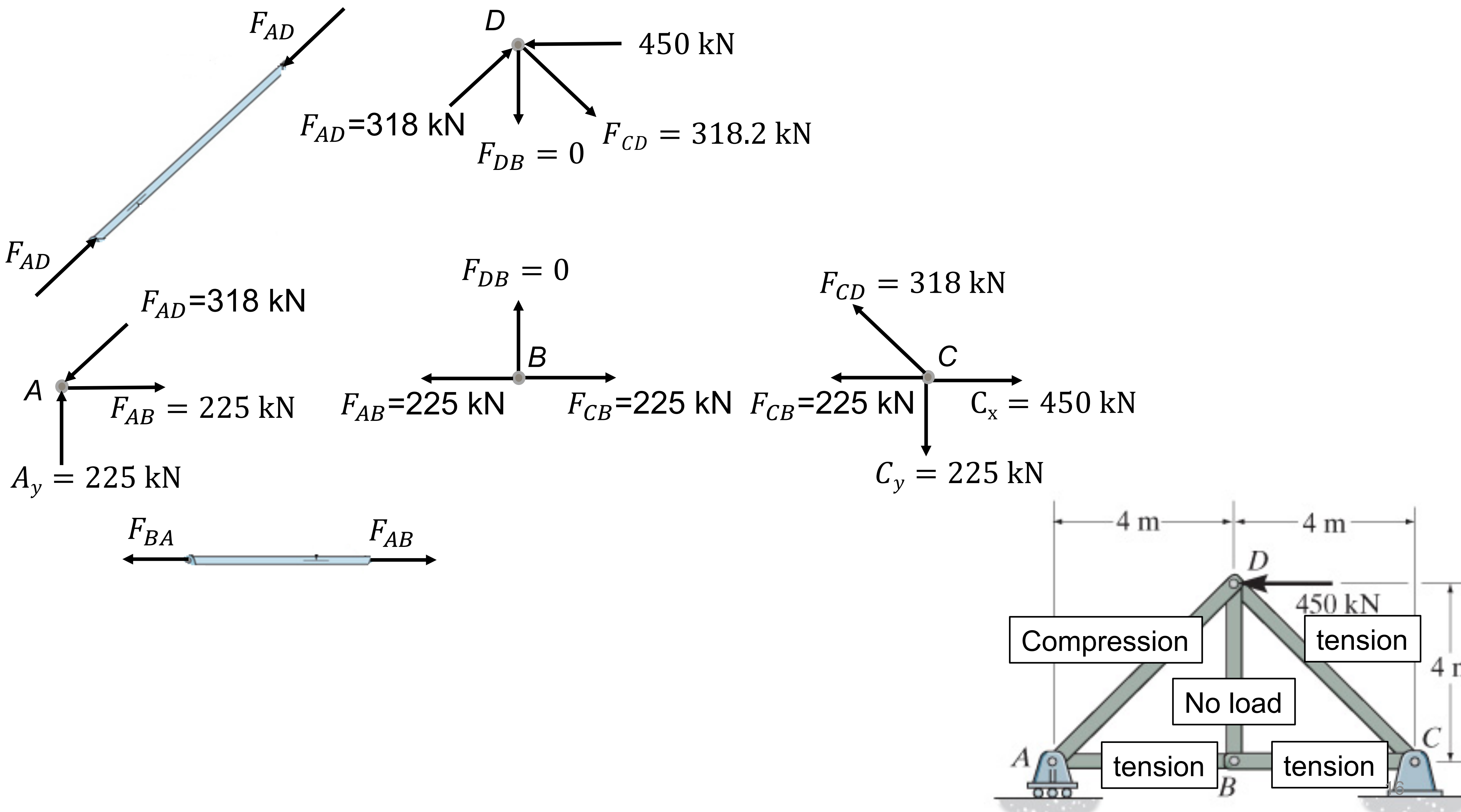
Considering joint *D*:



$$\uparrow + \quad \Sigma F_y = 0 \qquad 318.2\ \text{kN} \sin 45° - 318.2\ \text{kN} \sin 45° - F_{DB} = 0$$

$$F_{DB} = 0\ \text{kN}$$

Truss DB carries no load.



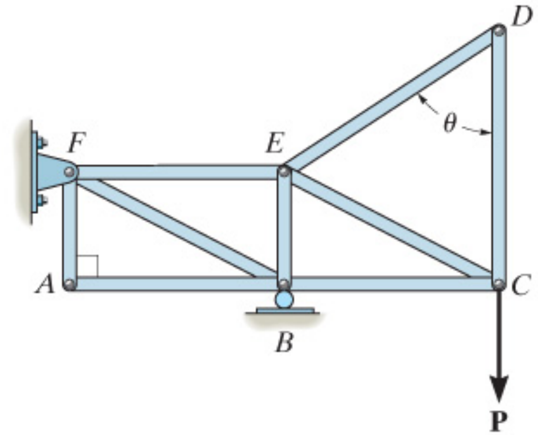
F_{AD}
F_{AD}
D
450 kN
F_{AD}=318 kN
$F_{DB} = 0$
$F_{CD} = 318.2$ kN
F_{AD}=318 kN
A
$F_{AB} = 225$ kN
$A_y = 225$ kN
$F_{DB} = 0$
B
F_{AB}=225 kN
F_{CB}=225 kN
$F_{CD} = 318$ kN
C
F_{CB}=225 kN
$C_x = 450$ kN
$C_y = 225$ kN
F_{BA}
F_{AB}
4 m
4 m
D
450 kN
4 m
Compression
tension
No load
tension
tension
A
B
C

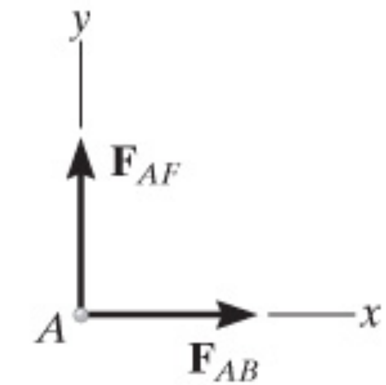
Zero-force member

Truss analysis is simplified if we can identify **zero-force members**, i.e. bars carrying no load.

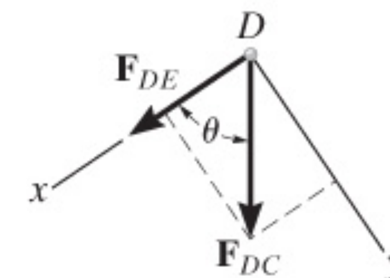
From joint A, we conclude that bars AF and AB are zero-force members.

Likewise, from joint D, bars DE and CD are also zero-force members.





$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AB} = 0$$
$$+\uparrow \Sigma F_y = 0; \quad F_{AF} = 0$$



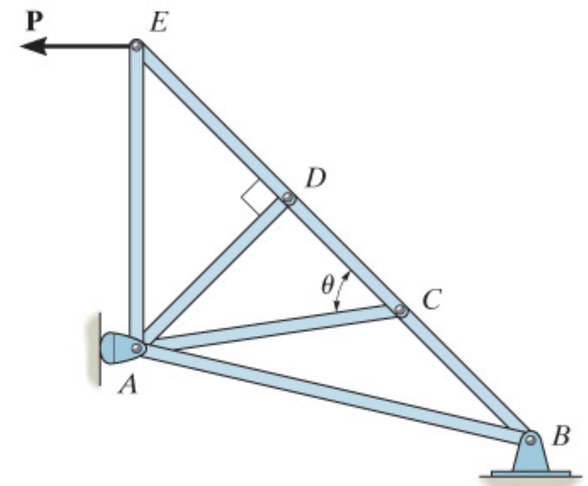
$$+\searrow \Sigma F_y = 0; F_{DC} \sin\theta = 0; \quad F_{DC} = 0 \text{ since } \sin\theta \neq 0$$
$$+\swarrow \Sigma F_x = 0; F_{DE} + 0 = 0; \quad F_{DE} = 0$$

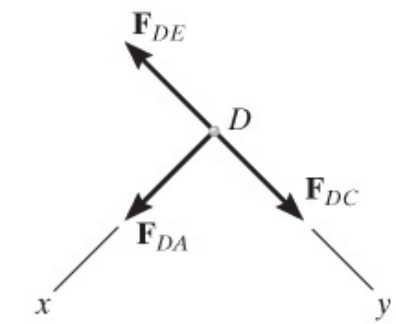
Zero-force member

Consider the truss shown here.

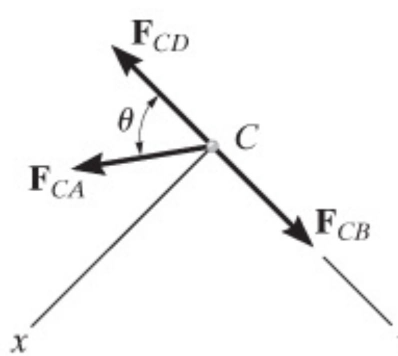
From joint D, we conclude that bar DA is a zero-force member.

From joint C, it follows that bar CA is also a zero-force member.





$$+\swarrow \Sigma F_x = 0; \quad F_{DA} = 0$$
$$+\searrow \Sigma F_y = 0; \quad F_{DC} = F_{DE}$$



$$+\swarrow \Sigma F_x = 0; \quad F_{CA} \sin\theta = 0; \quad F_{CA} = 0 \text{ since } \sin\theta \neq 0;$$
$$+\searrow \Sigma F_y = 0; \quad F_{CB} = F_{CD}$$

Zero-force member

Zero-force members can be identified following these two rules:

- If a joint has only two non-collinear members, and no external load or support reaction is applied to the joint, then these two bars are zero-force members.

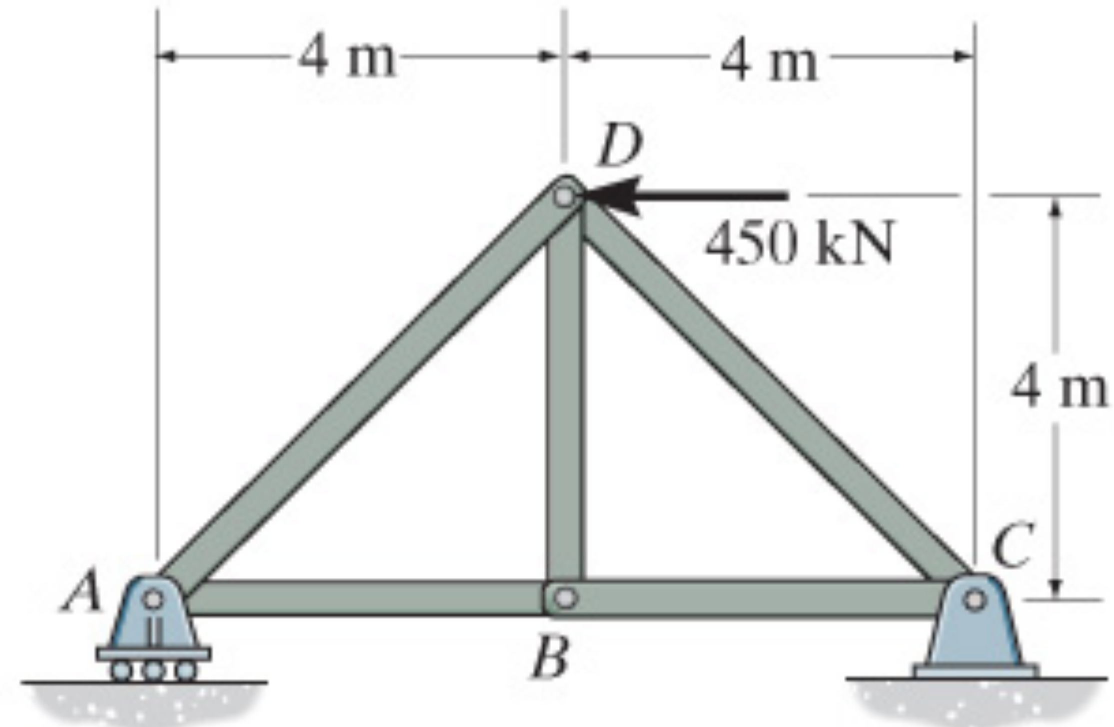
- If three members form a joint and two members are collinear, then the third one is a zero-force member provided that no external loading or support reaction is applied to this joint.

Be careful before assuming a bar is a zero-force member!

Zero-force member

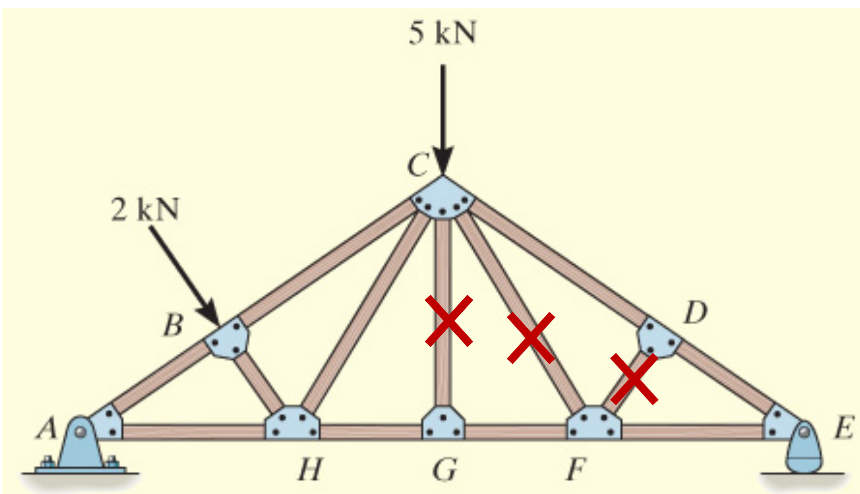
This is the example we solved earlier.

Based on the rules mentioned previously, we could guess that bar BD is a zero-force member.

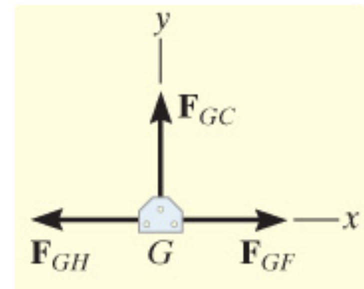


Example

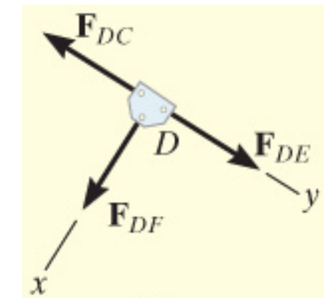
How many zero-force members are there in this example?



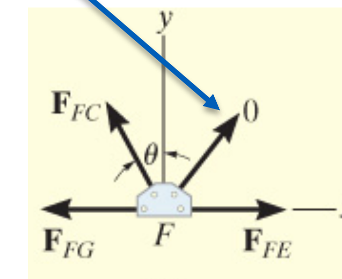
Because bar FD is a zero-force member



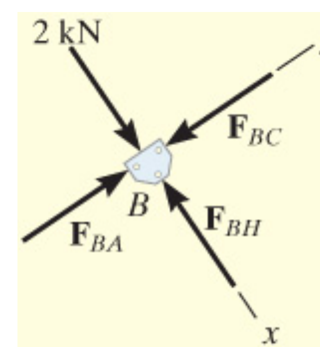
Bar GC is a zero-force member.



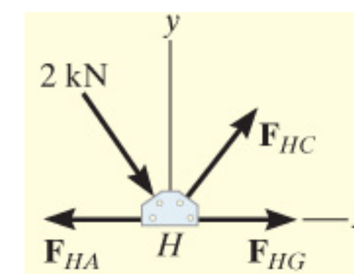
Bar DF is a zero-force member.



Bar FC is a zero-force member.



Bar BH carries load.

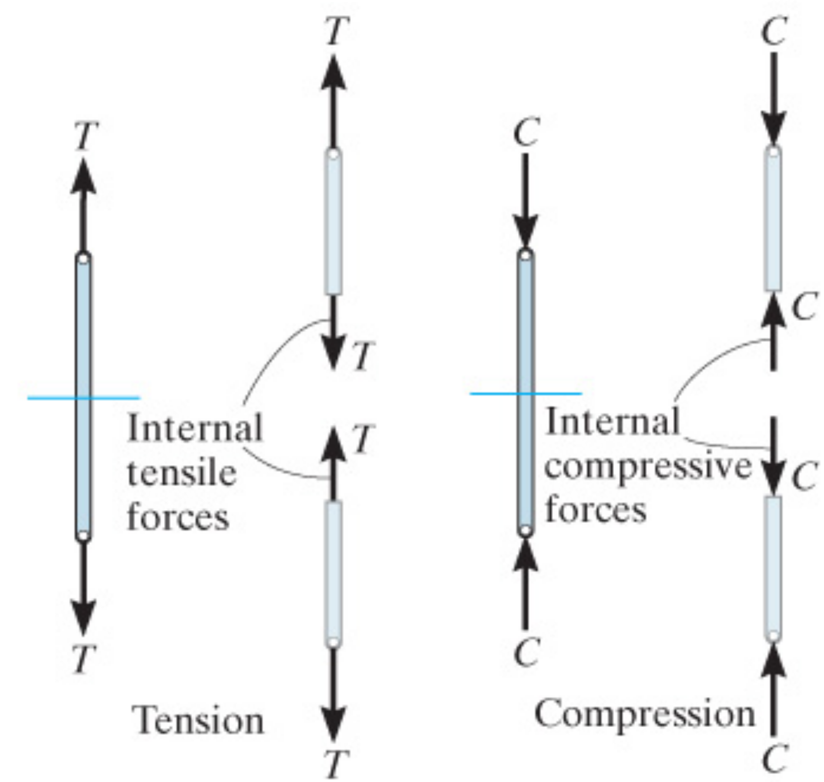


Bar HC carries load.

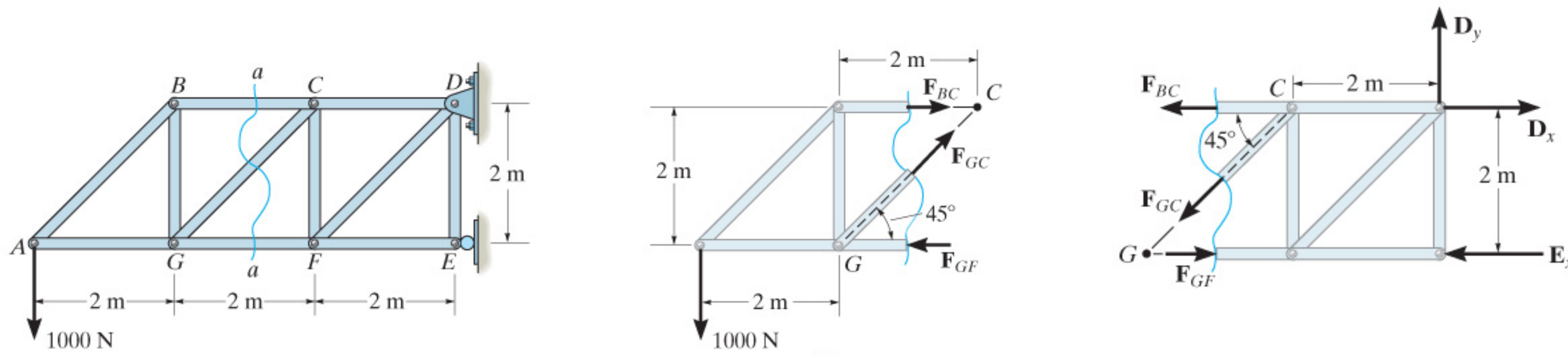
Method of sections

The method of sections is based on the principle that if a member is in equilibrium then any segment of the truss is also in equilibrium.

Therefore, we can cut a member in two parts and expose its internal force as shown on the right.



Method of sections

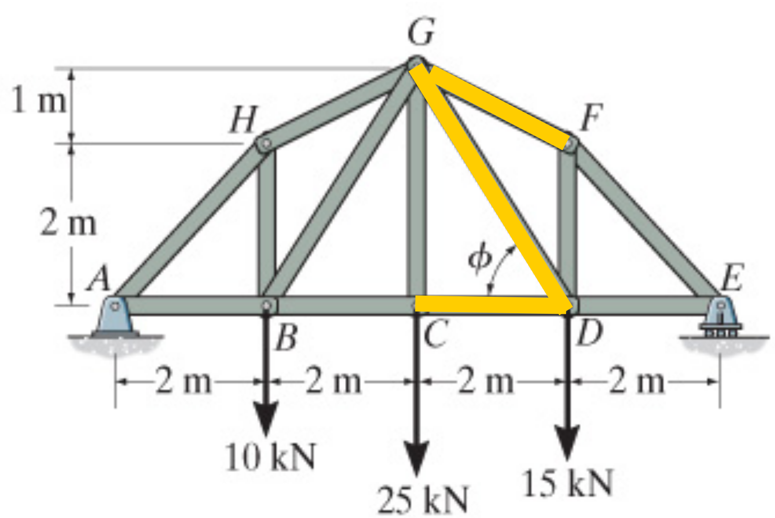


The method of sections is particularly useful if you want to find the force in a few members only. Follow this procedure:

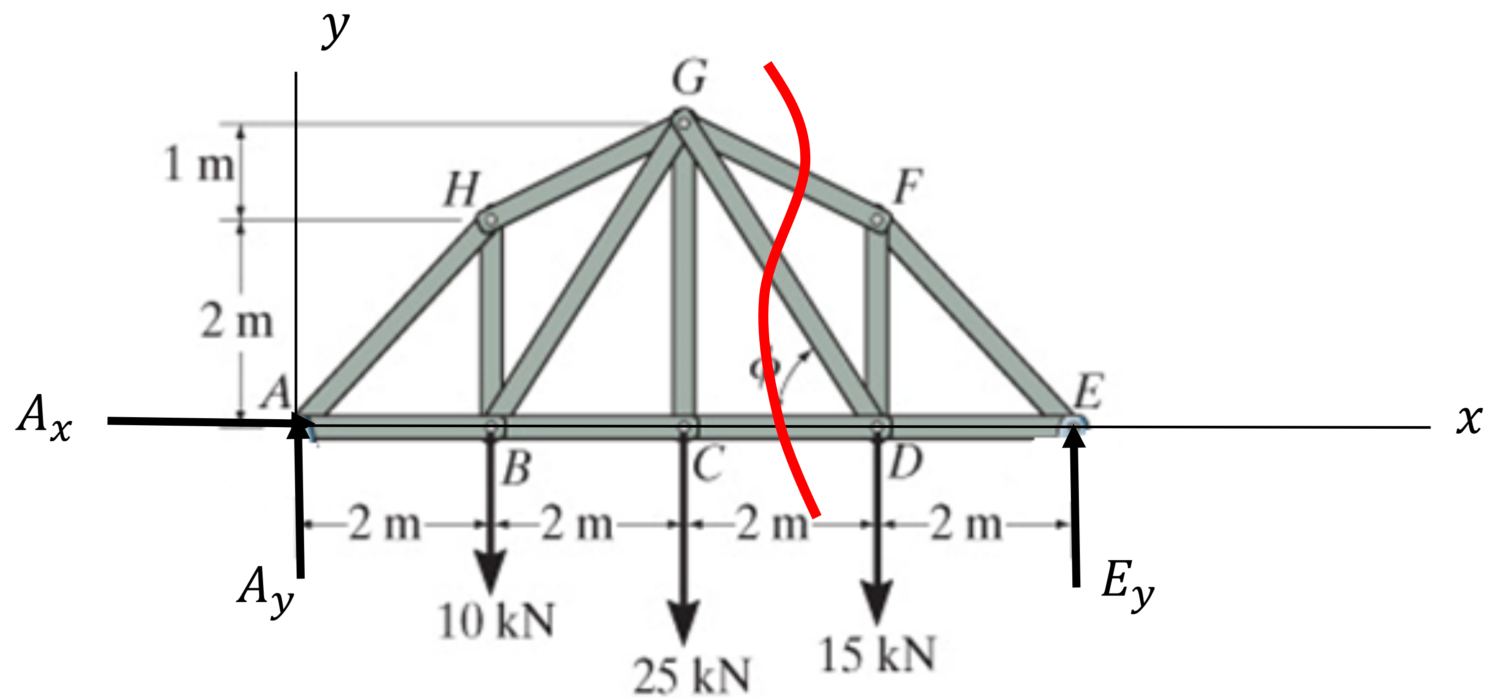
1. Cut the truss through the members where the forces are to be determined.
2. Draw the free-body of the cut segment, including internal forces.
3. Use equilibrium equations to find the unknown internal forces.

Example

Find the forces in bars *GF, GD* and *CD*. Specify if the bars are in tension or compression.



1. Find the support reactions.
2. Find the force in bars *GF, GD* and *CD* using the method of sections cutting along the red line.



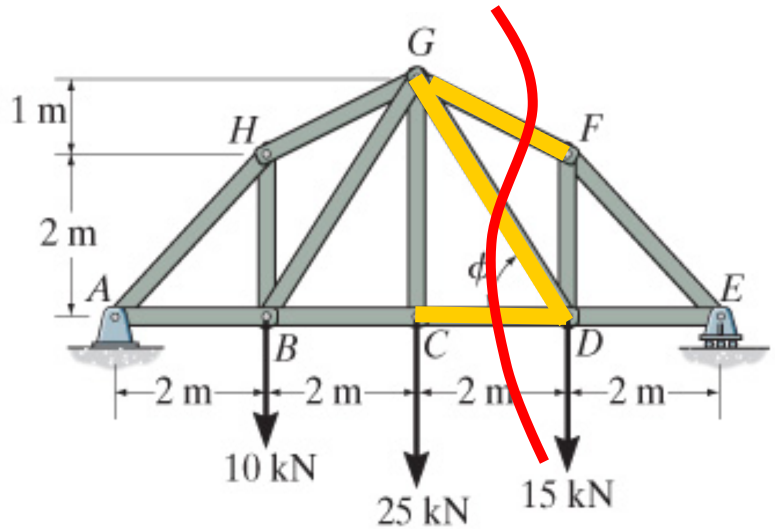
In step 2, we will consider the right part of the truss structure; hence, we only need the reaction force at point *E*.

$$\circlearrowleft + \quad \Sigma M_A = 0 \quad -10\ \text{kN}(2\text{m}) - 25\ \text{kN}(4\text{m}) - 15\ \text{kN}(6\text{m}) + E_y(8\text{m}) = 0$$

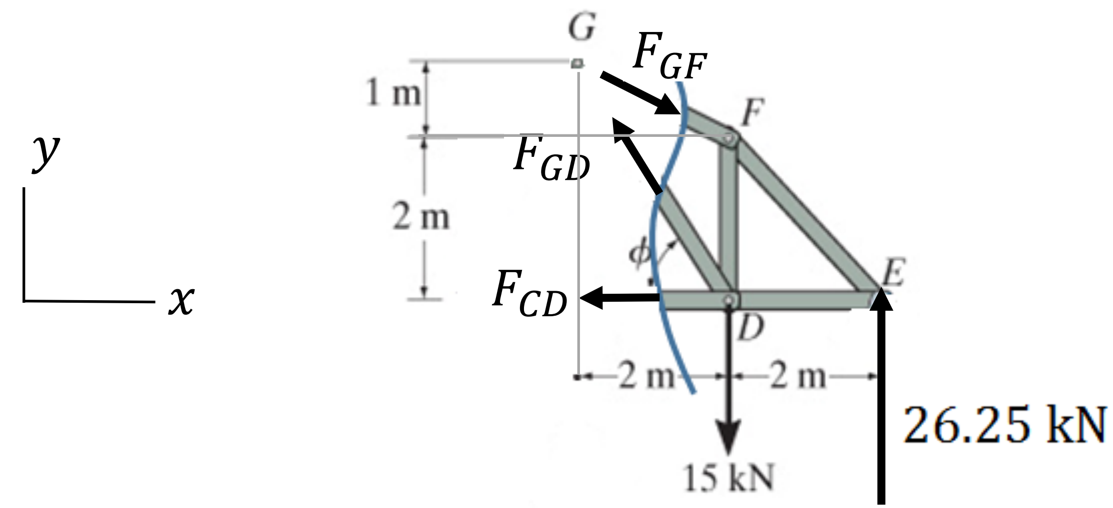
$$E_y = 26.25\ \text{kN}$$

Example

Find the forces in bars *GF, GD* and *CD*. Specify if the bars are in tension or compression.



In step 2, we analyse the right side of the truss structure:



We can solve the three unknowns using equilibrium equations.

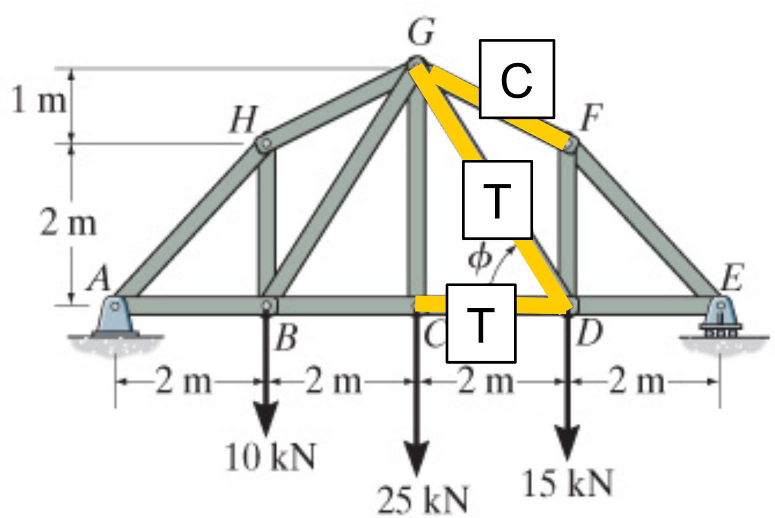
We begin with sum of moments about point *G*.

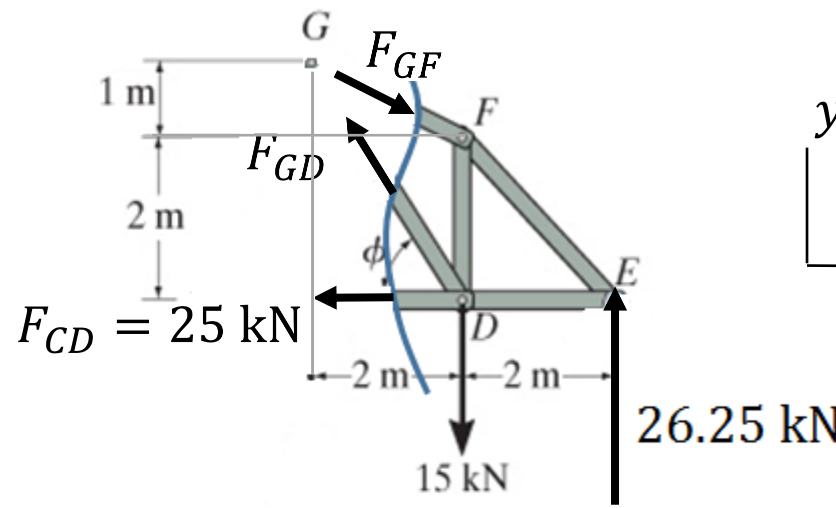
$$\circlearrowleft + \quad \Sigma M_G = 0 \qquad\qquad 26.25\ \text{kN}(4\text{m}) - 15\text{kN}(2\text{m}) - F_{CD}(3\text{m}) = 0$$

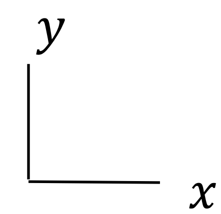
$$F_{CD} = 25\ \text{kN}$$

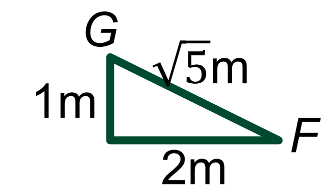
Example

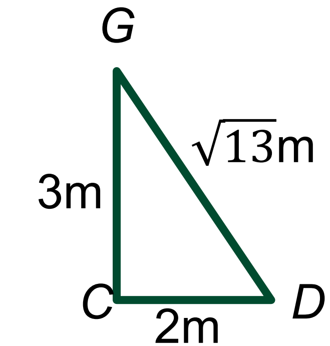
Find the forces in bars *GF, GD* and *CD*. Specify if the bars are in tension or compression.











Calculating the remaining two forces:

$$\xrightarrow{+} \quad \Sigma F_x = 0 \qquad -25\text{ kN} + F_{GF}\left(\frac{2}{\sqrt{5}}\right) - F_{GD}\left(\frac{2}{\sqrt{13}}\right) = 0$$

$$\uparrow + \quad \Sigma F_y = 0 \qquad 26.25\text{ kN} - 15\text{kN} - F_{GF}\left(\frac{1}{\sqrt{5}}\right) + F_{GD}\left(\frac{3}{\sqrt{13}}\right) = 0$$

Solving this system of two equations and two unknowns gives:

$$F_{GF} = 29.3\text{ kN} \qquad F_{GD} = 2.25\text{ kN}$$

The loading direction: tension (T) or compression (C) is labelled on the figure.

Summary

A simple truss structure is:

- Made of straight bars connected by pin joints,
- Loaded at the joints only.

Each bar carries axial forces only (tension or compression), and their magnitude can be found with the:

- method of joints, or
- Method of sections.

Need more explanations?

This lecture covered sections 6.1 to 6.4 (inclusively) of the textbook.