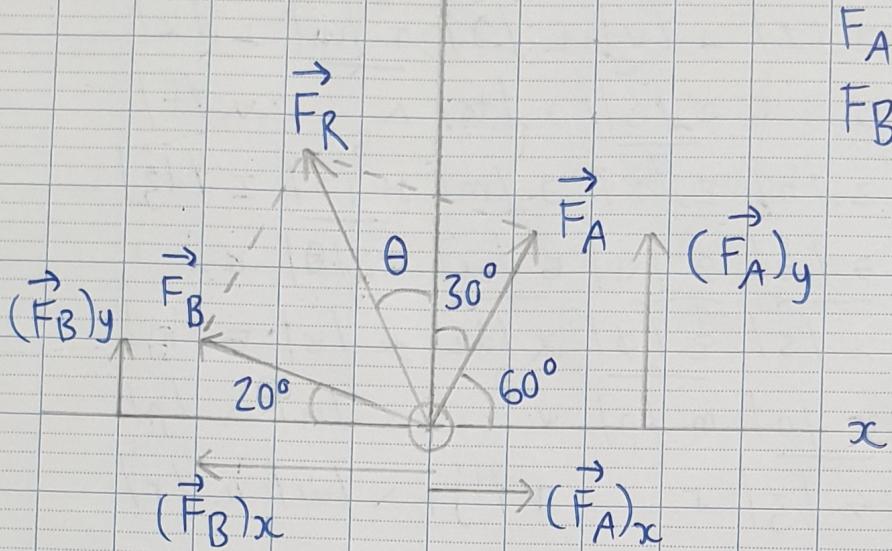


Statics & Dynamics (Assignment 1)

Exercise 1:



$$F_A = 700 \text{ N}$$

$$F_B = 600 \text{ N}$$

* The components of the resultant force are

$$\square (\vec{F}_R)_x = (\vec{F}_B)_x - (\vec{F}_A)_x$$

$$\Rightarrow (\vec{F}_R)_x = 600 \cos 20^\circ - 700 \cos 60^\circ \approx 213,82 \text{ N}$$

$$\square (\vec{F}_R)_y = (\vec{F}_B)_y + (\vec{F}_A)_y$$

$$= 600 \sin 20^\circ + 700 \sin 60^\circ \approx 811,43 \text{ N}$$

* The magnitude of \vec{F}_R

$$\vec{F}_R = \sqrt{(\vec{F}_R)_x^2 + (\vec{F}_R)_y^2} = \sqrt{213,82^2 + 811,43^2}$$

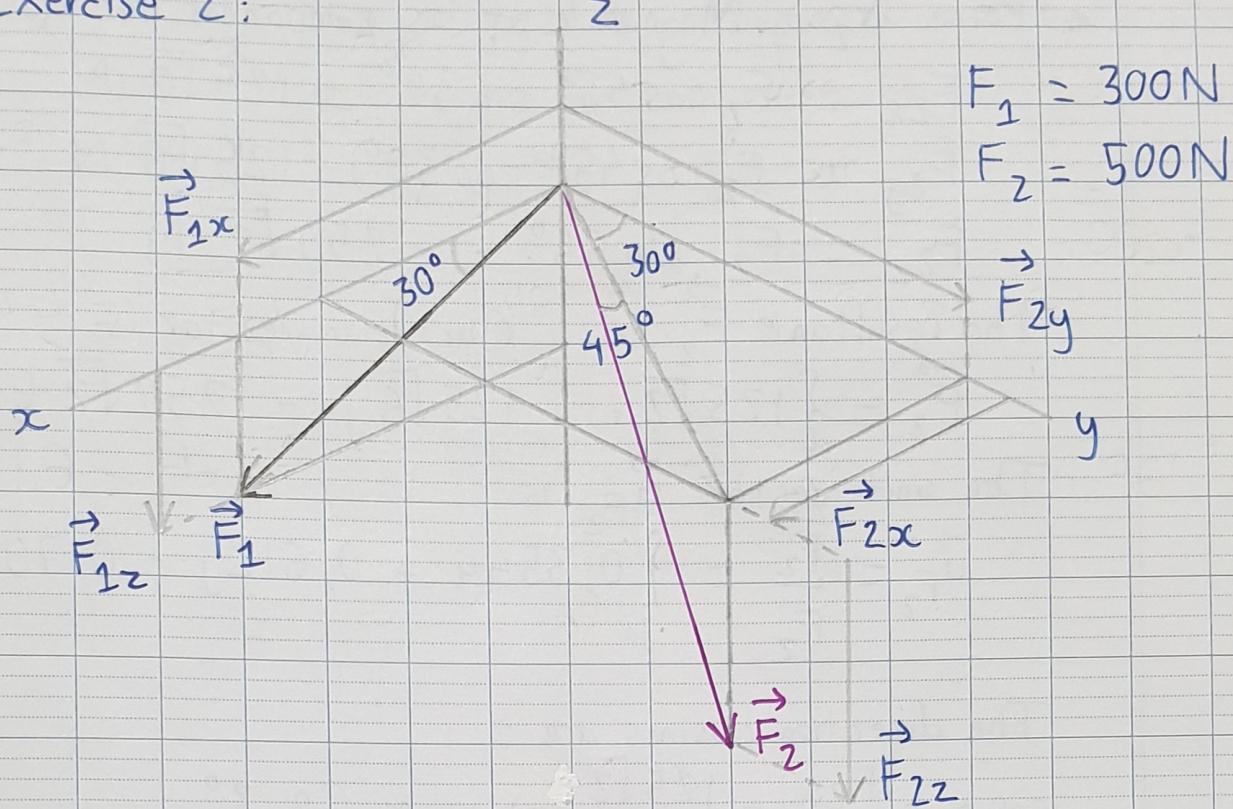
$$\Rightarrow F_R \approx 839,13 \text{ N}$$

* Orientation θ of \vec{F}_R

$$\theta = \tan^{-1} \left(\frac{(\vec{F}_R)_x}{(\vec{F}_R)_y} \right) = \tan^{-1} \left(\frac{213,82}{811,43} \right) \approx 14,762^\circ$$

The resultant force \vec{F}_R has the magnitude of $839,13 \text{ N}$ and orientation of $\theta = 14,762^\circ$ measured counter-clockwise from the positive y -axis

Exercise 2:



* Force \vec{F}_1 : Magnitude of its component vectors are

$$F_{1x} = F_1 \cdot \cos 30^\circ = 300 \cos 30^\circ = 259,8 \text{ N}$$

$$F_{1z} = -F_1 \cdot \sin 30^\circ = -300 \cdot \sin 30^\circ = -150 \text{ N}$$

$$F_{1y} = 0$$

The resultant \vec{F}_1 as a cartesian vector is

$$\vec{F}_1 = \{ 259,8i - 150k \} \text{ N}$$

The unit vector \vec{u}_{F_1} is

$$\vec{u}_{F_1} = \frac{\vec{F}_1}{F_1} = 0,866i - 0,5k$$

The direction cosines are

$$\cos \alpha = \frac{F_{1x}}{F_1} \Rightarrow \alpha = 30^\circ$$

$$\cos \beta = \frac{F_{1y}}{F_1} = 0 \Rightarrow \beta = 90^\circ$$

$$\cos \gamma = \frac{F_{1z}}{F_1} \Rightarrow \gamma = 120^\circ$$

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* Force \vec{F}_2 : Magnitude of its component vectors are:

$$F_{2x} = F_2 \cos 45^\circ \cdot \sin 30^\circ = 125\sqrt{2} \text{ N}$$

$$F_{2y} = F_2 \cos 45^\circ \cdot \cos 30^\circ = 125\sqrt{6} \text{ N}$$

$$F_{2z} = -F_2 \cdot \sin 45^\circ = -250\sqrt{2} \text{ N}$$

The resultant \vec{F}_2 as a Cartesian vector is

$$\vec{F}_2 = \{125\sqrt{2} i + 125\sqrt{6} j - 250\sqrt{2} k\} \text{ N}$$

The unit vector \vec{u}_{F_2} is

$$\vec{u}_{F_2} = \frac{\vec{F}_2}{F_2} = 0,354i + 0,612j - 0,707k$$

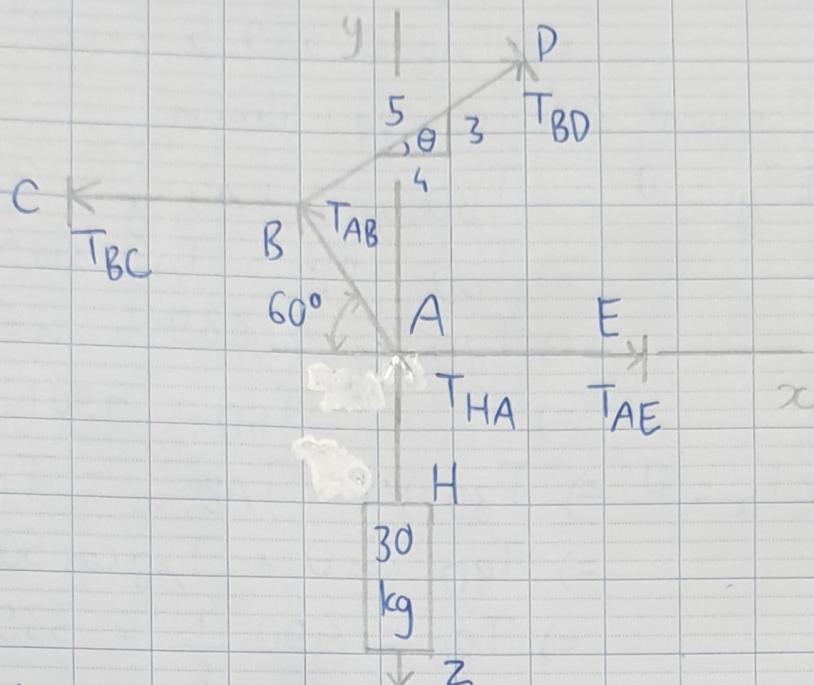
The direction cosines are:

$$\cos \alpha = \frac{F_{2x}}{F_2} = \frac{125\sqrt{2}}{500} \Rightarrow \alpha \approx 69,3^\circ$$

$$\cos \beta = \frac{F_{2y}}{F_2} = \frac{125\sqrt{6}}{500} \Rightarrow \beta \approx 52,2^\circ$$

$$\cos \gamma = \frac{F_{2z}}{F_2} = \frac{-250\sqrt{2}}{500} \Rightarrow \gamma = 135^\circ$$

Exercise 3: The 30 kg pipe is supported at A by a system of five cords. Determine the force in each cord for equilibrium



The condition for equilibrium is: $\sum F = 0$

$$\Rightarrow \sum F_x i + \sum F_y j = 0 \Rightarrow \sum F_x = 0, \sum F_y = 0$$

$\sum F_x = 0$

$$\Rightarrow T_{BD,x} + T_{AE,x} - \cancel{T_{AB,x}} = 0$$

$\sum F_y = 0$

$$\Rightarrow \cancel{T_{AB,y}} - T_{AH,y} + T_{BD,y} = 0$$

We have: 30 kg creates force of $30 \times \text{gravity} = 30 \cdot 9,81$

$$= 294 \text{ N} = T_{HZ}$$

* For point H to be in equilibrium

$$\Rightarrow \sum F_y = 0 \Rightarrow T_{HA} = T_{HZ} = 294 \text{ N}$$

* For point A to be in equilibrium

$$\sum F_y = 0 \Rightarrow T_{AB,y} - T_{HZ} = 0$$

$$\Rightarrow T_{AB} \sin 60^\circ - T_{HZ} = 0$$

$$\Rightarrow T_{AB} = \frac{T_{HZ}}{\sin 60^\circ} = \frac{294}{\sin 60^\circ} \approx 339,5 \text{ N}$$

$$\begin{aligned}\sum F_x = 0 &\Rightarrow T_{AE} - T_{AB,x} = 0 \\ &\Rightarrow T_{AE} - T_{AB} \cos 60^\circ = 0 \\ &\Rightarrow T_{AE} = 339,5 \cdot \cos 60^\circ \approx 169,7 \text{ N}\end{aligned}$$

* For point B to be in equilibrium

$$\begin{aligned}\sum F_y = 0 &\Rightarrow T_{BD,y} - T_{AB,y} = 0 \\ &\Rightarrow T_{BD} \cdot \sin \theta - T_{AB} \cdot \sin 60^\circ = 0 \\ &\Rightarrow T_{BD} \frac{3}{5} - 339,5 \sin 60^\circ = 0 \\ &\Rightarrow T_{BD} = 490 \text{ N}\end{aligned}$$

$$\begin{aligned}\sum F_x = 0 &\Rightarrow T_{BD,x} + T_{AB,x} - T_{BC} = 0 \\ &\Rightarrow T_{BD} \cdot \cos \theta + T_{AB} \cos 60^\circ - T_{BC} = 0 \\ &\Rightarrow T_{BC} = 490 \cdot \frac{4}{5} + 339,5 \cdot \cos 60^\circ \\ &\Rightarrow T_{BC} \approx 561,7 \text{ N}\end{aligned}$$

The force in each cord are

$$T_{HA} = 294 \text{ N} \quad T_{AB} = 339,5 \text{ N} \approx 340 \text{ N}$$

$$T_{AE} = 169,7 \text{ N} \approx 170 \text{ N} \quad T_{BD} = 490 \text{ N}$$

$$T_{BC} \approx 561,7 \text{ N} \approx 562 \text{ N}$$

Verify:

$$\square \sum F_x = 0$$

$$\Rightarrow T_{BD,x} + T_{AE} - T_{BC} = 0$$

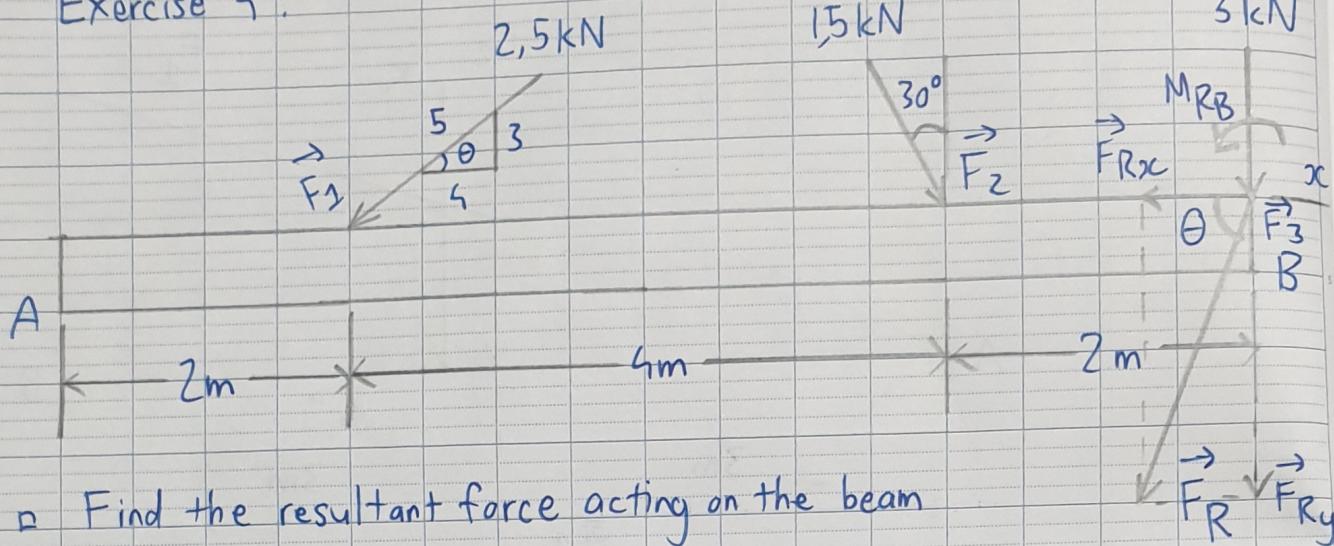
$$\Rightarrow 490 \frac{4}{5} + 169,7 - 561,7 \approx 0$$

$$\square \sum F_y = 0$$

$$\Rightarrow T_{BD,y} - T_{HZ} = 0$$

$$\Rightarrow 490 \cdot \frac{3}{5} - 294 = 0$$

Exercise 5:



□ Find the resultant force acting on the beam

- Its components

$$\begin{aligned} F_{Rx} &= \sum F_x = -2,5 \text{ kN} \cos \theta + 1,5 \text{ kN} \cdot \sin 30^\circ \\ &= -2,5 \text{ kN} \cdot \frac{4}{5} + 1,5 \text{ kN} \cdot \sin 30^\circ \\ &= -1,25 \text{ kN} \end{aligned}$$

$$\begin{aligned} F_{Ry} &= \sum F_y = -2,5 \text{ kN} \sin \theta - 1,5 \text{ kN} \cos 30^\circ - 3 \text{ kN} \\ &= -2,5 \text{ kN} \cdot \frac{3}{5} - 1,5 \text{ kN} \cos 30^\circ - 3 \text{ kN} \\ &\approx -5,8 \text{ kN} \end{aligned}$$

- Its magnitude

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-1,25)^2 + (-5,8)^2} = 5,93 \text{ kN}$$

- Its direction from x negative axis

$$F_{R\theta} = \tan^{-1} \left(\frac{F_{Ry}}{F_{Rx}} \right) = \tan^{-1} \left(\frac{-5,8}{-1,25} \right) \approx 77,83^\circ$$

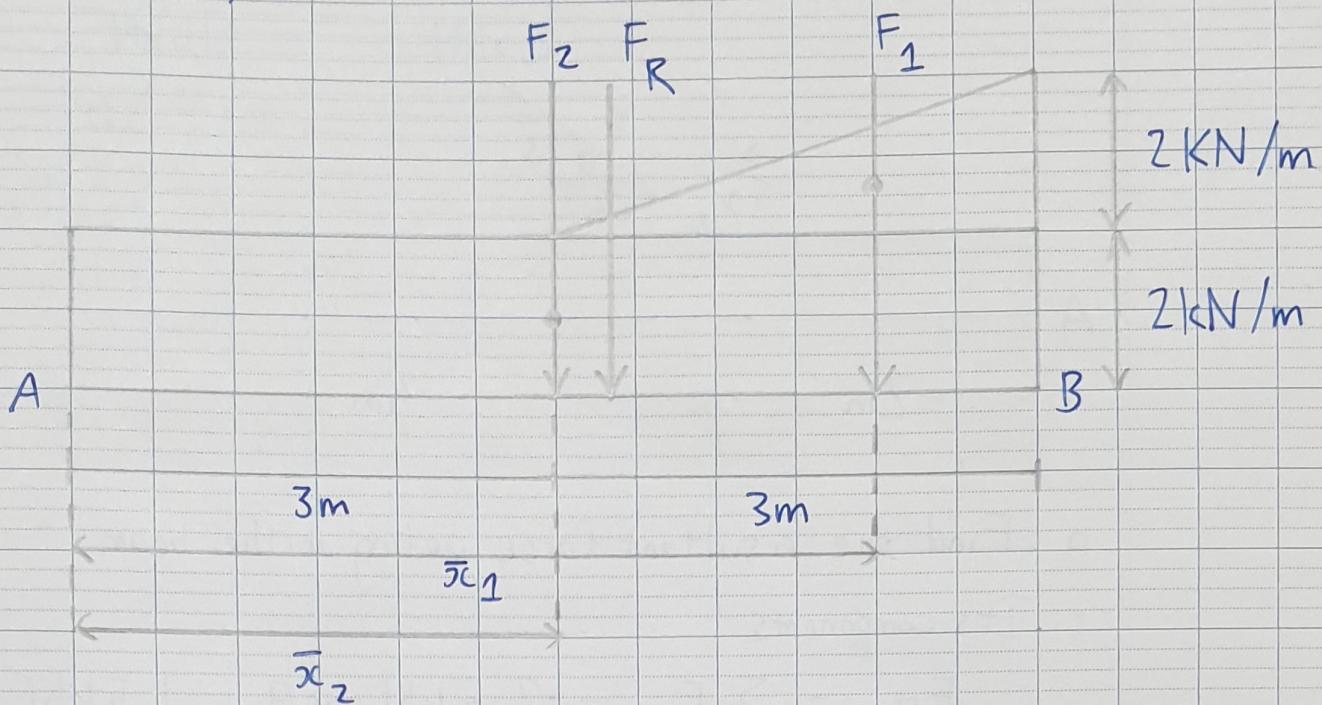
□ Let rotation as counter-clockwise positive

Couple moment at point B is

$$\begin{aligned} G + M_{RB} &= \sum M_B = \vec{F}_{1y} \cdot d_1 + \vec{F}_{2y} \cdot d_2 + \vec{F}_{3y} \cdot d_3 \\ &= 2,5 \text{ kN} \cdot \frac{3}{5} \text{ m} + 1,5 \text{ kN} \cdot (2 \text{ m}) + 3 \text{ kN} \cdot 0 \text{ m} \\ &= 9 + 2,598 = 11,598 \text{ kN.m} \end{aligned}$$

This couple moment is counter clockwise

Exercise 5:



□ Divide the distribution in 2 areas

$$\text{Triangle: } +\downarrow F_1 = \frac{1}{2} (2 \text{ kN/m}) \cdot 3 \text{ m} = 3 \text{ kN}$$

$$\text{Rectangle: } +\downarrow F_2 = (2 \text{ kN/m}) \cdot 3 \text{ m} = 12 \text{ kN}$$

□ The centroid for each distribution is located at

$$\bar{x}_1 = 3 \text{ m} + \frac{2}{3} \cdot 3 \text{ m} = 5 \text{ m}$$

$$\bar{x}_2 = \frac{1}{2} (3 \text{ m} + 3 \text{ m}) = 3 \text{ m}$$

□ The magnitude of the resultant force is

$$+\downarrow F_R = F_1 + F_2 = 3 + 12 = 15 \text{ kN}$$

□ Location of the resultant force on the beam, measured from pin A

$$+\text{C}(\text{M}_R)_A = \sum M_A$$

$$\Rightarrow -F_R \bar{x}_R = -F_1 \bar{x}_1 - F_2 \bar{x}_2$$

$$\Rightarrow -15 \text{ kN} \cdot \bar{x}_R = -3 \text{ kN} \cdot 5 \text{ m} - 12 \text{ kN} \cdot 3 \text{ m}$$

$$\Rightarrow \bar{x}_R = \frac{51}{15} \text{ m} = 3,4 \text{ m}$$