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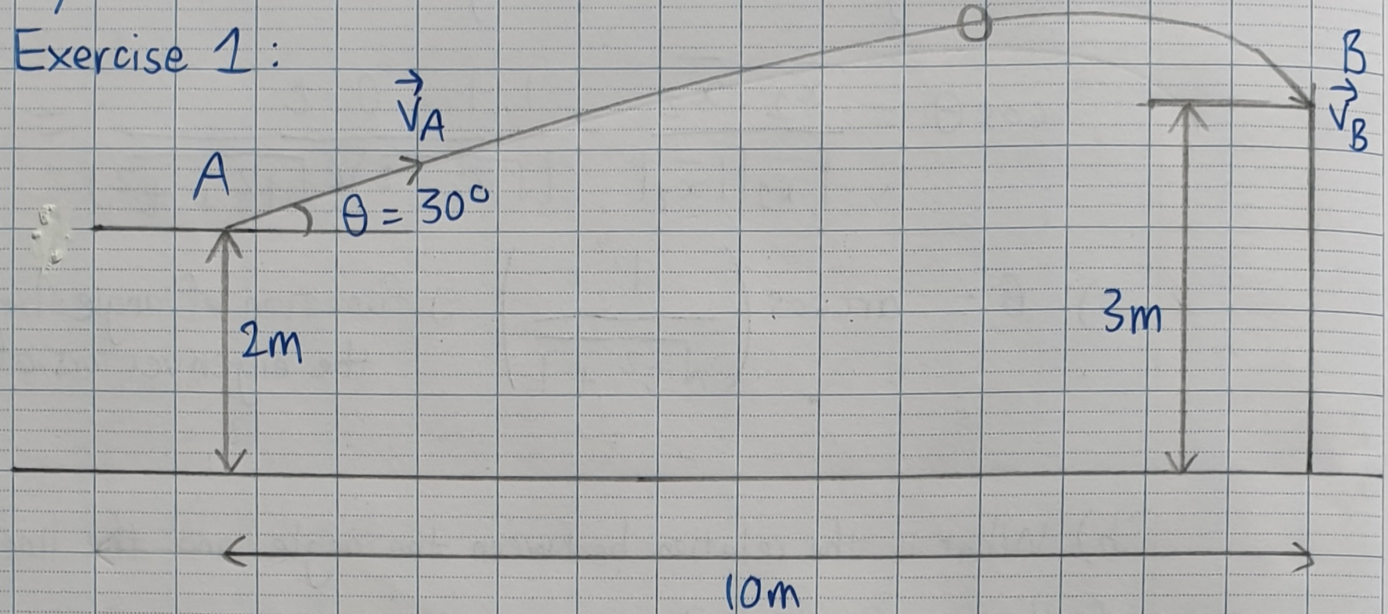
Thứ

Ngày . . No.

Dynamics Final Exam

Exercise 1:



□ Since initial velocity is unknown, we may use system of linear equation to find v_A

Motion of a projectile : $a_c = g = 9,81\ m/s^2$ in y direction

* Horizontal motion ($a_c = 0$)

$\xrightarrow{+} (v_B)_x = (v_A)_x + a_c t = (v_A)_x \quad (3)$

$\xrightarrow{+} (s_B)_x = (s_A)_x + (v_A)_x t + \frac{1}{2} a_c t^2$

$\Rightarrow 10 = 0 + (v_A)_x t$

$\Rightarrow (v_A)_x t = 10 \quad (1)$

* Vertical motion ($a_c = -g$)

$+\uparrow (v_B)_y = (v_A)_y + a_c t = (v_A)_y - 9,81t \quad (4)$

$+\uparrow (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_c t^2$

$\Rightarrow 3 = 2 + (v_A)_y t - 4,905 t^2$

$\Rightarrow (v_A)_y t - 4,905 t^2 = 1 \quad (2)$

□ From (1) & (2)

$$\Rightarrow \begin{cases} (v_A)_x t = 10 \\ (v_A)_y t - 4,905 t^2 = 1 \end{cases} \Rightarrow \begin{cases} v_A \cos 30° t = 10 \\ v_A \sin 30° t - 4,905 t^2 = 1 \end{cases}$$

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$$\Rightarrow \begin{cases} \frac{\sqrt{3}}{2} v_A t = 10 \\ \frac{1}{2} v_A t - 4,905 t^2 = 1 \end{cases} \Rightarrow \begin{cases} \frac{\sqrt{3}}{2} v_A t = 10 \\ \frac{1}{2} \cdot \frac{20\sqrt{3}}{3} - 4,905 t^2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} v_A = \frac{20\sqrt{3}}{3t} = 11,7 \text{ m/s} \\ t = 0,9865 \text{ s} \end{cases}$$

From (3) and (4)

$$\begin{cases} (v_B)_x = (v_A)_x \\ (v_B)_y = (v_A)_y - 9,81t \end{cases} \Rightarrow \begin{cases} (v_B)_x = v_A \cos 30^\circ \\ (v_B)_y = v_A \sin 30^\circ - 9,81t \end{cases}$$

$$\Rightarrow \begin{cases} (v_B)_x = 11,7 \cdot \cos 30^\circ \\ (v_B)_y = 11,7 \cdot \sin 30^\circ - 9,81 \cdot 0,9865 \end{cases}$$

$$\Rightarrow \begin{cases} (v_B)_x = 10,132 \text{ m/s} \\ (v_B)_y = -3,827 \text{ m/s} \end{cases}$$

Magnitude of the velocity when passing through B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{(10,132)^2 + (-3,827)^2}$$

$$\Rightarrow v_B = 10,83 \text{ m/s}$$

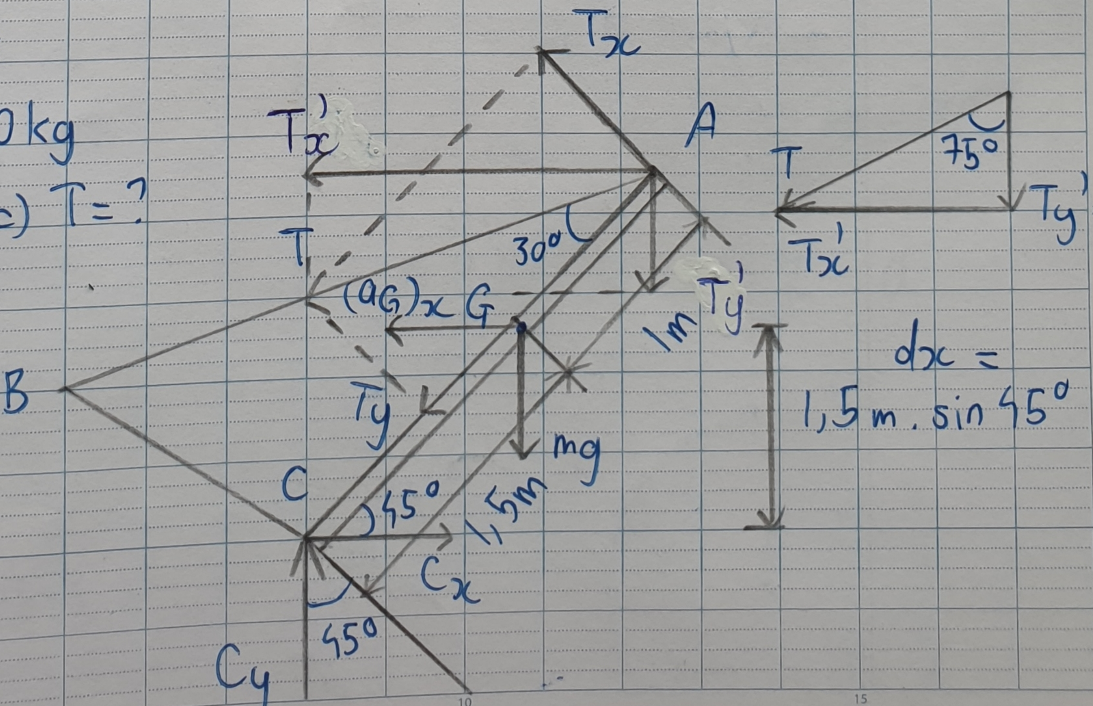
Answer: $v_A = 11,7$ m/s $\quad v_B = 10,83$ m/s

Exercise 2:

$m_{AC} = 1250$ kg

$a = 5 \text{ m/s}^2 \Rightarrow T = ?$

$C_y, C_x = ?$



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Since acceleration will determine the force in the drop gate, we'll find kinetic moment of the drop gate around C

$\circlearrowleft + \Sigma M_C = \Sigma (M_k)_C$

$\Rightarrow -T_x . 2,5 + mg . 1,5 \cos 45^\circ = -m(a_G)_x . dx$

$\Rightarrow -T . \sin 30^\circ . 2,5 + 1250 . 9,81 . 1,5 . \cos 45^\circ$

$= -1250 . 5 . 1,5 . \sin 45^\circ$

$\Rightarrow T \sin 30^\circ . 2,5 \approx 19635,47 N$

$\Rightarrow T \approx 15708,37 (N)$

□ The truck moves to the left $\Rightarrow \Sigma F_x = m(a_G)_x , \Sigma F_y = 0$

We have: $\uparrow + \Sigma F_y = 0$

$\Rightarrow C_y - mg - T_y' = 0$

$\Rightarrow C_y - mg - T . \cos 75^\circ = 0$

$\Rightarrow C_y - 1250 . 9,81 - 15708,37 . \cos 75^\circ = 0$

$\Rightarrow C_y = 16328,12 N$

We have: $\xrightarrow{+} \Sigma F_x = m(a_G)_x$

$\Rightarrow C_x - T_x' = -m(a_G)_x$

$\Rightarrow C_x - T \sin 75^\circ = -m(a_G)_x$

$\Rightarrow C_x - 15708,37 \sin 75^\circ = -1250 . 5$

$\Rightarrow C_x = 8923,12 N$

Answer : $T = 15708,37 N$

$C_x = 8923,12 N$

$C_y = 16328,12 N$

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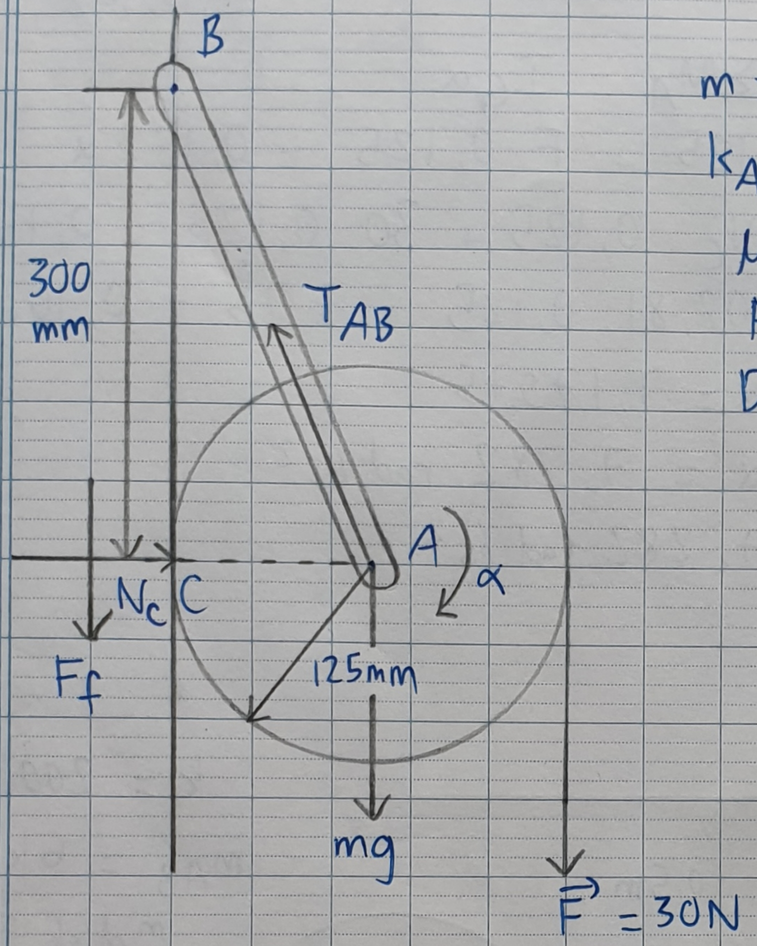
Exercise 3:



$m = 20\ kg$

$k_A = 90\ mm = 0,09$

$\mu_k = 0,2$

$F = 30N$

Determine α

$\Delta ABC : AB = \sqrt{0,3^2 + 0,125^2}$

$\Rightarrow AB = 0,325\ m$

▫ Since the roll of paper only rotates but doesn't translate

$\Rightarrow \Sigma F_x = \Sigma F_y = 0$

We have:

* $+\uparrow \Sigma F_y = 0 \Rightarrow T_{AB,y} - F_f - mg - F = 0$

$\Rightarrow T_{AB} \cdot \frac{0,3}{0,325} - \mu_k N_C - 20 . 9,81 - 30 = 0$

$\Rightarrow \frac{12}{13} T_{AB} - 0,2 N_C - 226,2 = 0$

* $\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow -T_{AB,x} + N_C = 0$

$\Rightarrow -\frac{5}{13} T_{AB} + N_C = 0$

$\Rightarrow \begin{cases} T_{AB} = 267,32N \\ N_C = 102,81\ N \end{cases}$

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Since $k_A = 0{,}09$ m $\Rightarrow I_A = m k_A^2 = 20 \cdot (0{,}09)^2$

$= 0{,}162$

- We have: $\circlearrowright + \Sigma M_A = I_G \alpha$

$\Rightarrow -F_f \cdot 0{,}125 + F \cdot 0{,}125 = 0{,}162\alpha$

$\Rightarrow -0{,}2 \cdot N_C \cdot 0{,}125 + 30 \cdot 0{,}125 = 0{,}162\alpha$

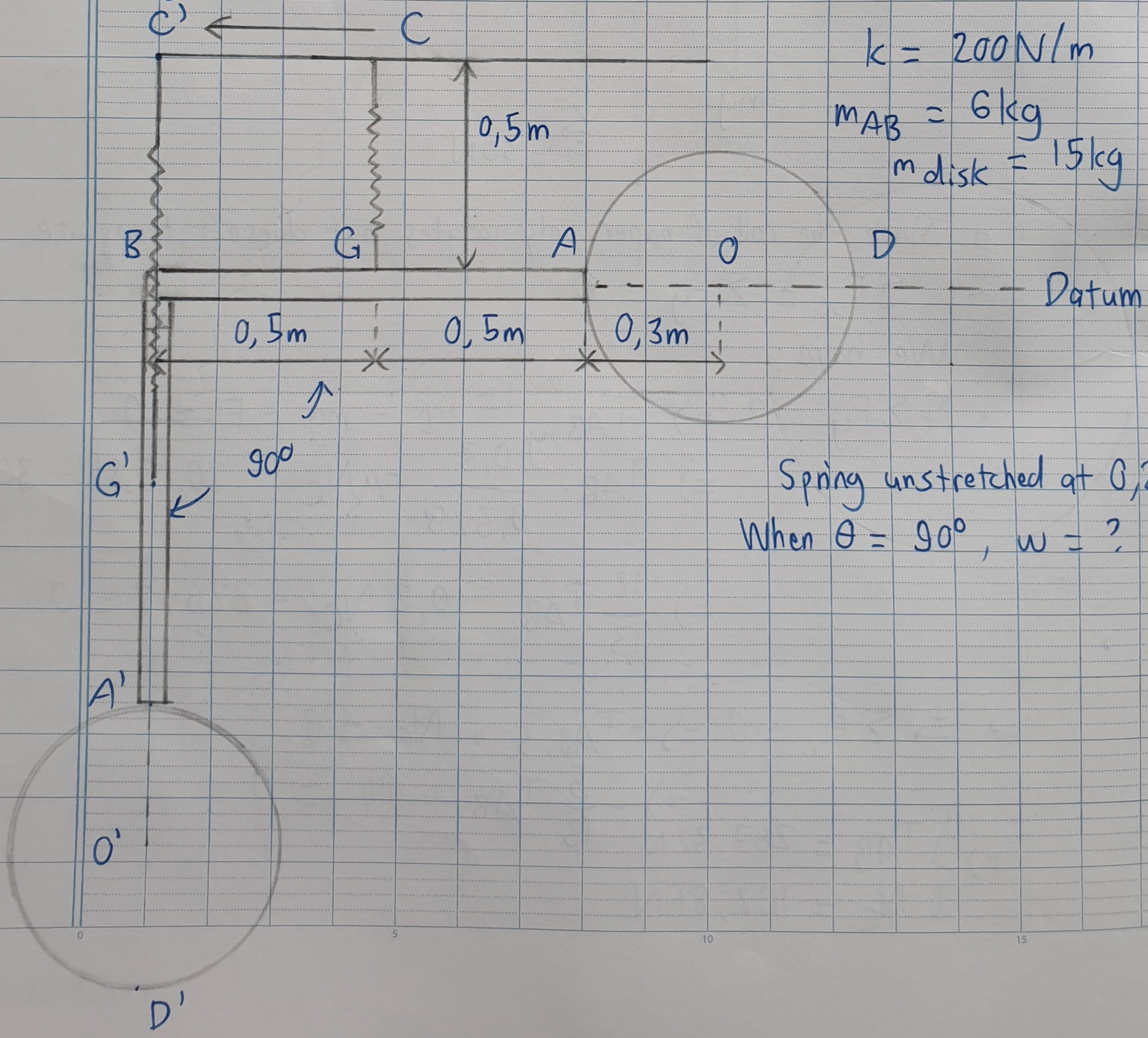
$\Rightarrow -0{,}2 \cdot 102{,}81 \cdot 0{,}125 + 3{,}75 = 0{,}162\alpha$

$\Rightarrow 0{,}162\alpha = 1{,}17975$

$\Rightarrow \alpha = 7{,}282 \text{ rad/s}^2$

Answer: $\alpha = 7{,}282 \text{ rad/s}^2$

Exercise 4:



$k = 200$ N/m

$m_{AB} = 6$ kg

$m_{disk} = 15$ kg

Spring unstretched at 0,2m

When $\theta = 90°$, $\omega = ?$

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Let state 1 be when $\theta = 0$ and state 2 when $\theta = 90^\circ$.

The datum pass through the rod at $\theta = 0$

- Potential energy: Gravitational

$(V_g)_1 = W(y_G)_1 = 0$

$(V_g)_2 = -W(y_G)_2$

$= -6 \cdot 9{,}81 \cdot 0{,}5 - 15 \cdot 9{,}81 \cdot 1{,}3$

$= -220{,}725\ J$

- Potential energy: elastic

We have: $s_1 = 0{,}5 - 0{,}2 = 0{,}3 m$

$s_2 = 0{,}5 \cdot 2 - 0{,}2 = 0{,}8 m$

$\Rightarrow (V_e)_1 = \frac{1}{2} k s_1^2 = \frac{1}{2} \cdot 200 \cdot 0{,}3^2 = 9 J$

$(V_e)_2 = \frac{1}{2} k s_2^2 = \frac{1}{2} \cdot 200 \cdot 0{,}8^2 = 64 J$

- Kinetic energy

Since the bar is at rest when $\theta = 0 \Rightarrow T_1 = 0$

Kinetic energy of the rod

$T_{2, rod} = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$

$= \frac{1}{2} m (\omega r_G)^2 + \frac{1}{2} \cdot \frac{1}{12} m l^2 \omega^2$

$= \frac{1}{12} \cdot 6 (\omega \cdot 0{,}5)^2 + \frac{1}{2} \cdot \frac{1}{12} \cdot 6 \cdot 1^2 \cdot \omega^2$

$= 0{,}75 \omega^2 + 0{,}25 \omega^2 = \omega^2$

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$$T_{2,\,ball} = \frac{1}{2} m v_0^2 + \frac{1}{2} I_0 \omega^2$$

$$= \frac{1}{2} m (\omega r_0)^2 + \frac{1}{2} \cdot \frac{1}{2} m r^2 \omega^2$$

$$= \frac{1}{2} \cdot 15 (\omega \cdot 1,3)^2 + \frac{1}{2} \cdot \frac{1}{2} \cdot 15 \cdot 0,3^2 \omega^2$$

$$= 12,675\, \omega^2 + 0,3375\, \omega^2 = 13,0125\, \omega^2$$

$$\Rightarrow T_2 = T_{2,\,rod} + T_{2,\,ball} = 1 + 13,0125 = 14,0125\, \omega^2$$

Conservation of energy

$$T_1 + V_1 = T_2 + V_2$$

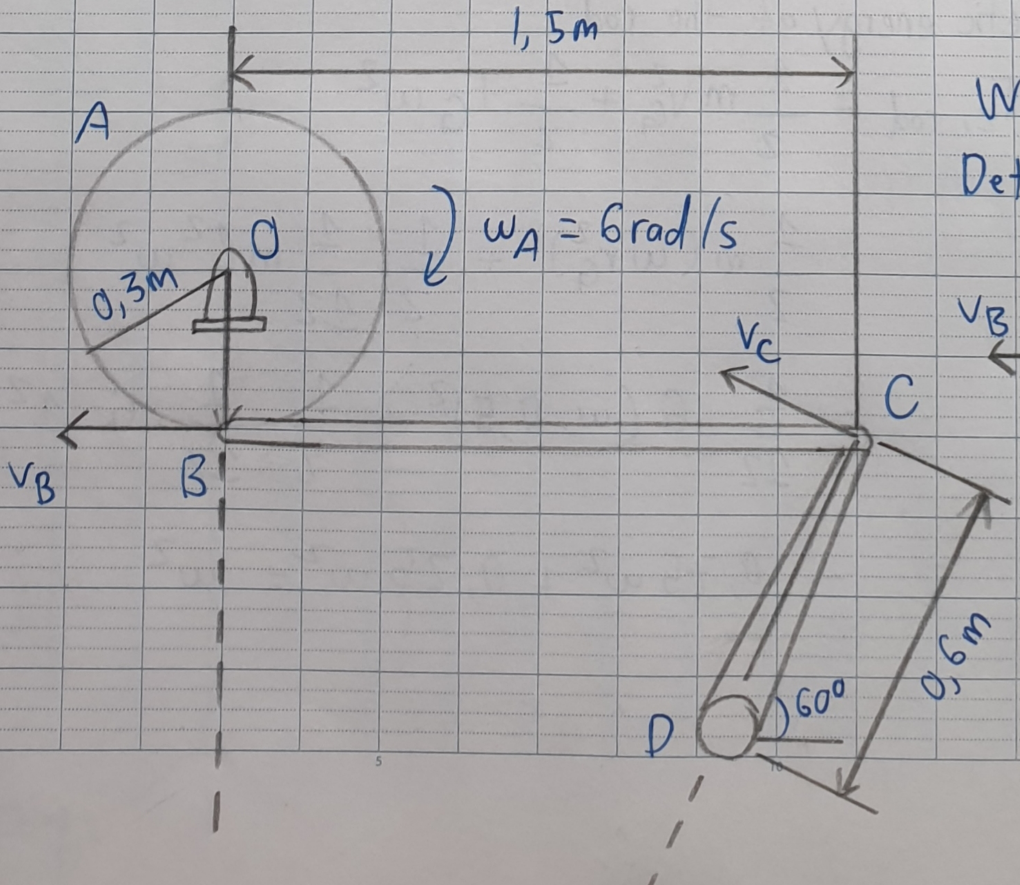
$$\Rightarrow 0 + 0 + 9 = 14,0125\, \omega^2 + 64 - 220,725$$

$$\Rightarrow 14,0125\, \omega^2 = 165,725$$

$$\Rightarrow \quad \omega \approx 3,439 \text{ rad/s}$$

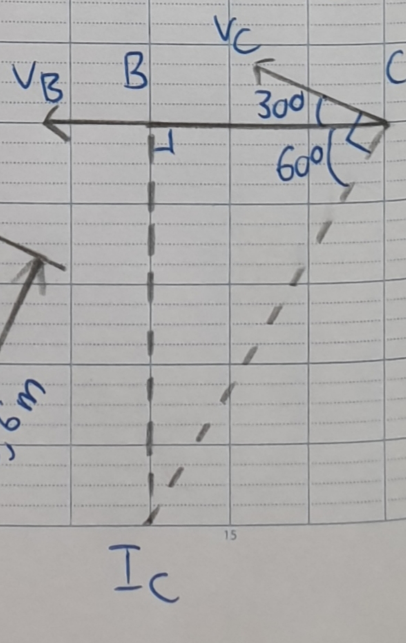
Answer: $\omega \approx 3,439$ rad/s

Exercise 5:



When $\omega_A = 6$ rad/s

Determine ω_{BC}



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- We have : $v_B = \omega_A . r$

$= 6 . 0,3 = 1,8 \ m/s$

- I_C is situated like illustrated above

$\Rightarrow r_{B/IC} = BC . \tan 60^0$

$= 1,5 . \sqrt{3}$

- Angular velocity of rod BC at the instant shown

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1,8}{1,5\sqrt{3}} = \frac{2\sqrt{3}}{5} \approx 0,6928 \ rad/s$$

Answer : $\omega_{BC} = 0,6928 \ rad/s$