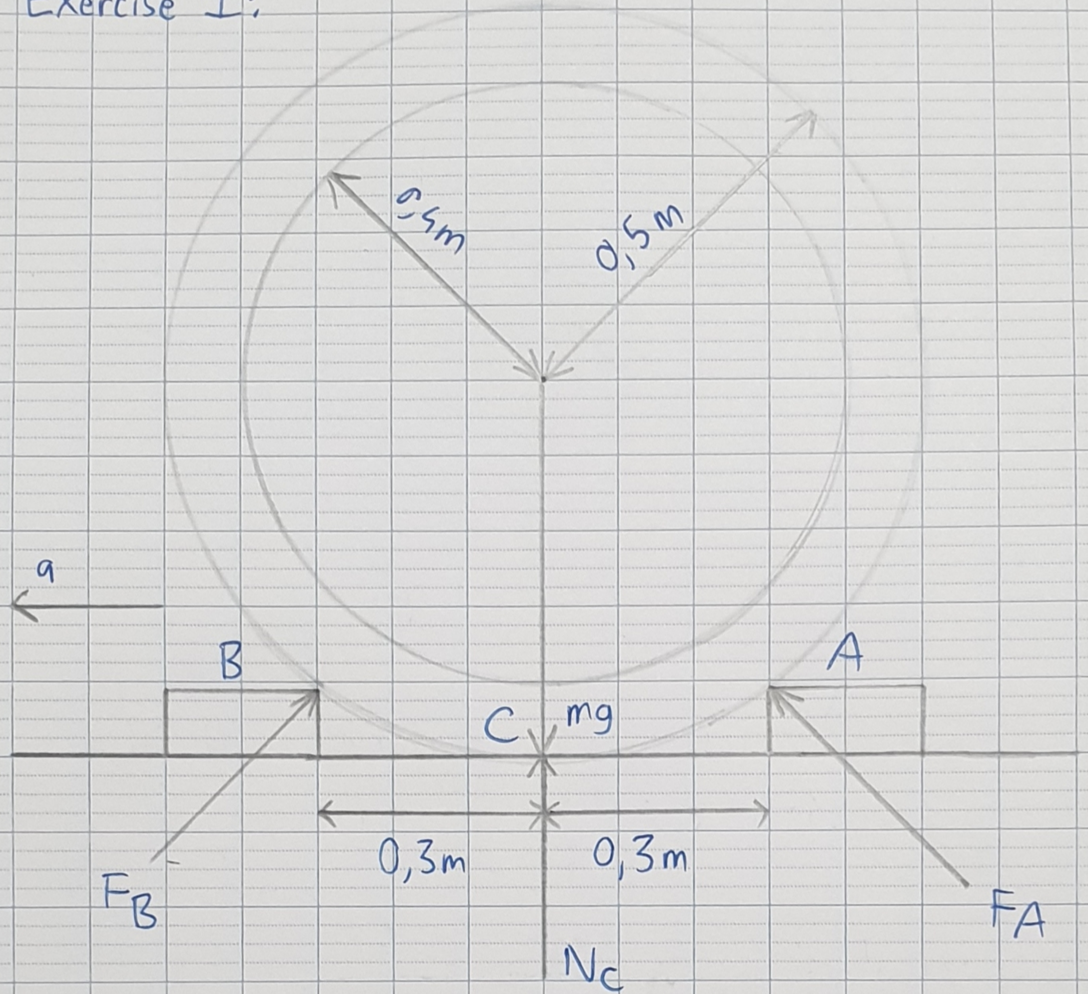
Nguyen Xuan Binh 887799

Thứ

Ngày . . No.

Statics & Dynamics Problem Sheet 5

Exercise 1:



When the pipe hasn't started to pivot around B, it will stay in equilibrium

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow (F_B)_x - (F_A)_x = 0$

$\uparrow + \Sigma F_y = 0 \Rightarrow (F_B)_y + (F_A)_y + N_C - mg = 0$

$\circlearrowleft + \Sigma G = 0 \Rightarrow (F_B)_y \cdot 0{,}3 - (F_B)_x \cdot 0{,}4$

$- (F_A)_y\, 0.3 + (F_A)_x\, 0{,}4 = 0$

When the pipe start to pivot around B, it will lose contact with A and the ground C. Moreover, it will start to translate horizontally to the left $\Rightarrow F_A = 0,\ N_C = 0$

$\xrightarrow{+} \Sigma F_x = ma_x \Rightarrow (F_B)_x = ma_x$

$\uparrow + \Sigma F_y = 0 \Rightarrow (F_B)_y - mg = 0$

$\circlearrowleft + \Sigma G = 0 \Rightarrow (F_B)_y \cdot 0{,}3 - (F_B)_x \cdot 0{,}4 = 0$

KOKUYO

We have: $(F_B)_y - mg = 0$

$\Rightarrow (F_B)_y = 460 \cdot 9{,}81 = 4512{,}6\ N$

$+ (F_B)_y\, 0{,}3 - (F_B)_x \cdot 0{,}4 = 0$

$\Rightarrow (F_B)_x = 3384{,}45N$

$+ (F_B)_x = ma_x$

$$\Rightarrow a_x = \frac{3384{,}45}{460} \approx 7{,}3575\ m/s^2$$

The total force acting on ~~B~~ the pile at B

$$F_B = \sqrt{(F_B)_y^2 + (F_B)_x^2} = 5640{,}75\ N$$

Answer: right at the moment the pipe starts to pivot around B, acceleration is $7{,}3575\ m/s^2$ and the force acting on the pipe at B is $5640{,}75N$. Acceleration's direction is opposite the running direction of the truck

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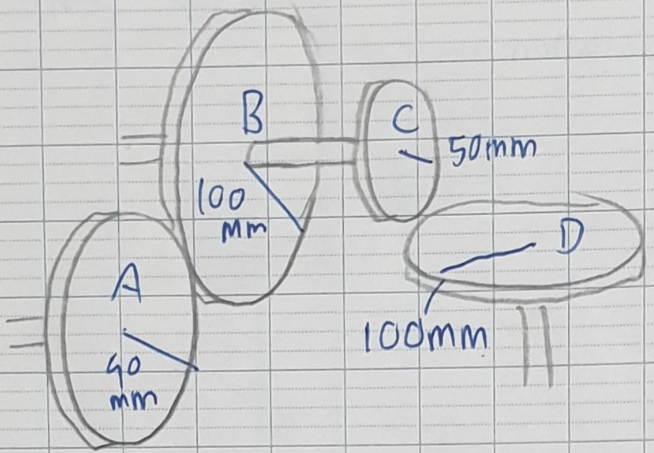
Exercise 2:



$\alpha_A = 3 \text{ rad}/s^2$

$\omega_A = 60 \text{ rad}/s$

- We have: $v_A = v_B$

$\Rightarrow \omega_A r_A = \omega_B r_B$

$\Rightarrow \omega_B = \dfrac{60 \text{ rad}/s \cdot 40 \text{ mm}}{100 \text{ mm}} = 24 \text{ rad}/s$

Since B and C are concentric $\Rightarrow \omega_B = \omega_C = 24 \text{ rad}/s$

We have: $v_C = v_D$

$\Rightarrow \omega_C r_C = \omega_D r_D$

$\Rightarrow \omega_D = \dfrac{24 \text{ rad}/s \cdot 50 \text{ mm}}{100 \text{ mm}} = 12 \text{ rad}/s$

Angular velocity of gear D is $\omega_D = 12 \text{ rad}/s$

- We have: $(a_A)_t = (a_B)_t$

$\Rightarrow \alpha_A r_A = \alpha_B r_B$

$\Rightarrow \alpha_B = \dfrac{3 \text{ rad}/s^2 \cdot 40 \text{ mm}}{100 \text{ mm}} = 1,2 \text{ rad}/s^2$

Since B and C are concentric $\Rightarrow \alpha_B = \alpha_C = 1,2 \text{ rad}/s^2$

We have: $(\cancel{a_A}\, a_C)_t = (a_D)_t$

$\Rightarrow \alpha_C \cdot r_C = \alpha_D \cdot r_D$

$\Rightarrow \alpha_D = \dfrac{1,2 \text{ rad}/s^2 \cdot 50 \text{ mm}}{100 \text{ mm}} = 0,6 \text{ rad}/s^2$

Angular acceleration of gear D is $\alpha_D = 0,6 \text{ rad}/s^2$

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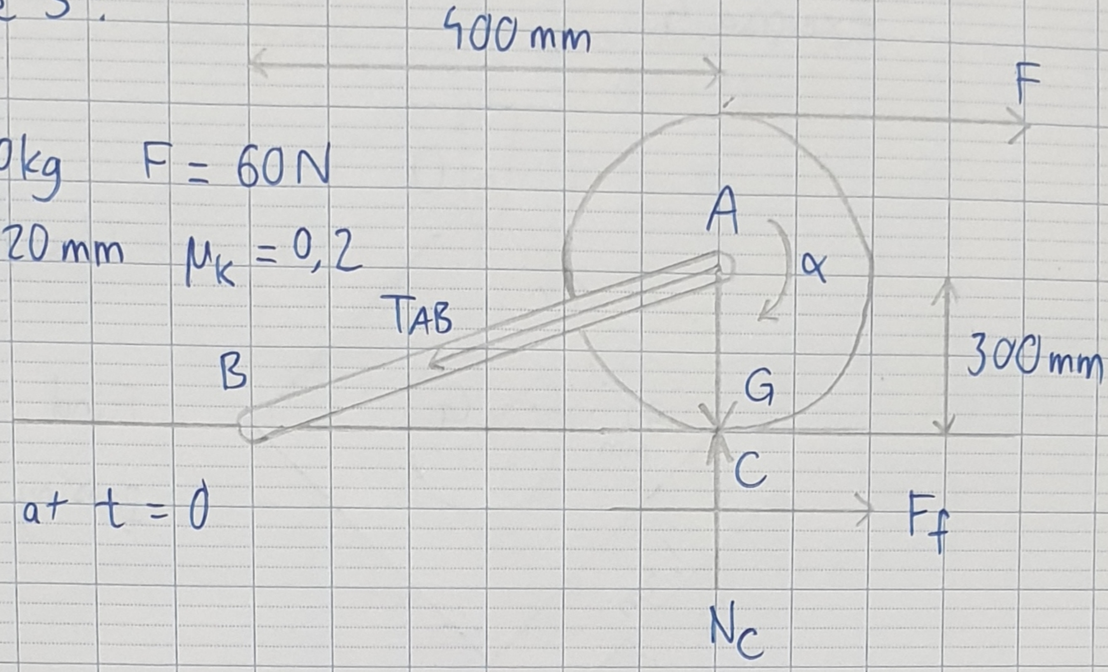
Exercise 3:

$m = 20kg \quad F = 60N$

$k_A = 120\,mm \quad \mu_k = 0,2$



$\alpha = ?$ at $t = 0$

We have $AB^2 = AC^2 + BC^2 \Rightarrow AB = 500\,mm$

□ Since the roll of paper is held firm by bar AB ⇒ it is static in translation, but dynamic in rotation

$\Rightarrow \xrightarrow{+} \Sigma F_x = 0 \Rightarrow F + F_f - T_{AB}\frac{4}{5} = 0$

$\Rightarrow N_C . 0,2 - T_{AB}\frac{4}{5} = -60 \quad (1)$

$\uparrow + \Sigma F_y = 0 \Rightarrow N_C - 20.(9,81) - T_{AB}\frac{3}{5} = 0$

$\Rightarrow N_C - T_{AB}\frac{3}{5} = 196,2 \quad (2)$

Solve (1) & (2)

$\Rightarrow N_C = 283,76N \quad , T_{AB} = 155,94N$

□ Since the roll of paper rotates around A

$\Rightarrow \circlearrowright + \Sigma M_A = I_A\alpha$

$\Rightarrow F.r - F_f.r = mk_A^2\alpha$

$\Rightarrow 60.0,3 - (283,76).0,2.0,3 = 20.(0,12)^2\alpha$

$\Rightarrow 0,288\alpha = 0,9744$

$\Rightarrow \alpha = 3,383\ (rad/s^2)$

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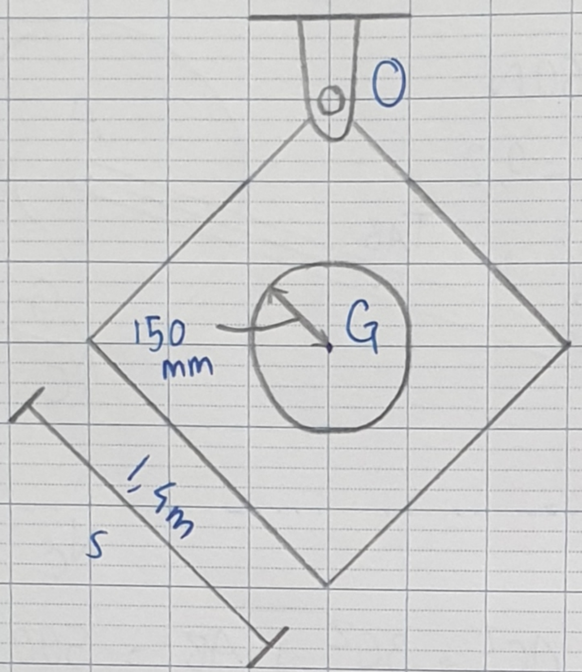
Exercise 4:



Thickness = 50 mm

$\rho = 50\ kg/m^3$

Find I_O of axis $\perp$ paper passing through O

- Volume of empty hole : $(0,15)^2 . \pi . 0,05 = 0,00113\pi\ (m^3)$

Volume of the whole sheet ~~with~~ without empty hole : $(1,4)^2 . 0,05$

$= 0,098\ (m^3)$

=) Volume of the sheet with the hole: $0,098 - 0,00113\pi\ (m^3)$

=) Mass of the sheet: $50(0,098 - 0,00113\pi) \approx 4,723$ kg

Mass of the empty hole if it was solid

$50 . 0,00113\pi = 0,177$ kg

- The distance OG : $OG = \left(\frac{1,4^2}{2}\right)^{\frac{1}{2}} = 0,989\ (m)$

$I_{G,\text{ sheet with full hole}} = \frac{1}{12} m (s^2 + s^2)$

$= \frac{1}{12} . 50 . 0,098 (2 . 1,4^2)$

$= 1,6\ kg . m^2$

$I_{G,\text{ empty hole}} = \frac{1}{2} m r^2 = \frac{1}{2} . 50 . 0,00113\pi . 0,15^2$

$= 0,00198\ kg . m^2$

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$\Rightarrow I_{G,\text{sheet with empty hole}} = I_{G,\text{sheet with full hole}} - I_{G,\text{empty hole}}$

$= 1,6 - 0,00198$

$= 1,598 \text{ kg.m}^2$

Parallel axis theorem:

$I_{O,\text{sheet with empty hole}} = I_{G,\text{sheet with empty hole}} + md^2$

$= 1,598 + 4,723 . (\sqrt{0,98})^2$

$= 6,2265 \approx 6,23 \text{ kg.m}^2$

Moment of inertia around O is $I_O = 6,23 \text{ kg.m}^2$

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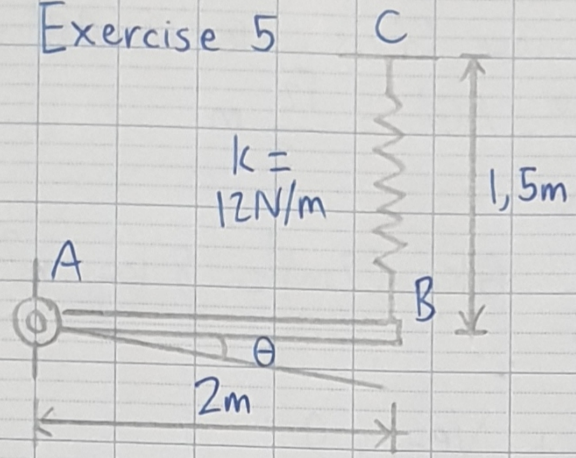
Exercise 5

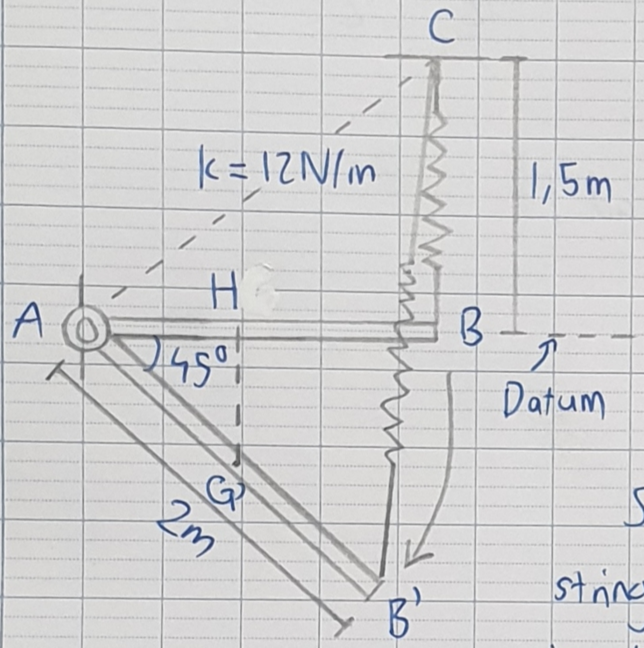


$m_{AB} = 6kg$

$\theta = 0, s = 0$

$\theta = 45° \Rightarrow \omega = ?$

We'll use conservation of energy to find ω when $\theta = 45°$



Let datum passing through A when $\theta = 0°$

$\Rightarrow$ Let state 1 when $\theta = 0°$ and state 2 when $\theta = 45°$

$\Rightarrow T_1 + V_1 = T_2 + V_2$

Since the datum is at $\theta = 0$, the string is unstretched at $\theta = 0$ and the bar is at rest $\Rightarrow T_1 = V_1 = 0$

$\Rightarrow T_2 + V_2 = 0 \Rightarrow T_2 + V_{g_2} + V_{e_2} = 0$

$\Rightarrow \frac{1}{2} I_A \omega^2 - mgy_2 + \frac{1}{2} ks_2^2 = 0$

When stretched, the length of the spring is CB'

We have: $AC = \sqrt{AB^2 + BC^2} = 2,5m$

$\widehat{CAB} = \tan^{-1}\left(\frac{1,5}{2}\right) = 36,87° \Rightarrow \widehat{CAB'} = \widehat{CAB} + 45°$
$= 81,87°$

$\Rightarrow CB' = \sqrt{AC^2 + AB'^2 - 2AC.AB'\cos 81,87}$
$= 2,97 m$

$\Rightarrow s_2 = CB' - CB = 2,97 - 1,5 = 1,47m$

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- We have $y_2 = HG'$. Since G' is center of AB' $\Rightarrow AG' = 1m$

$\Rightarrow y_2 = HG' = \sin 45^0 . AG' = \frac{\sqrt{2}}{2} m$

- The moment of inertia is

$$I_A = \frac{1}{3} ml^2 = \frac{1}{3} . 6kg.(2m)^2 = 8 \, kg.m^2$$

- We have $T_2 + Vg_2 + Ve_2 = 0$

$\Rightarrow \frac{1}{2} . 8 . \omega^2 - 6 . 9,81 . \frac{\sqrt{2}}{2} + \frac{1}{2} . 12 . 1,47^2 = 0$

$\Rightarrow 4\omega^2 = 28,655$

$\Rightarrow \omega \approx 2,676$ rad/s