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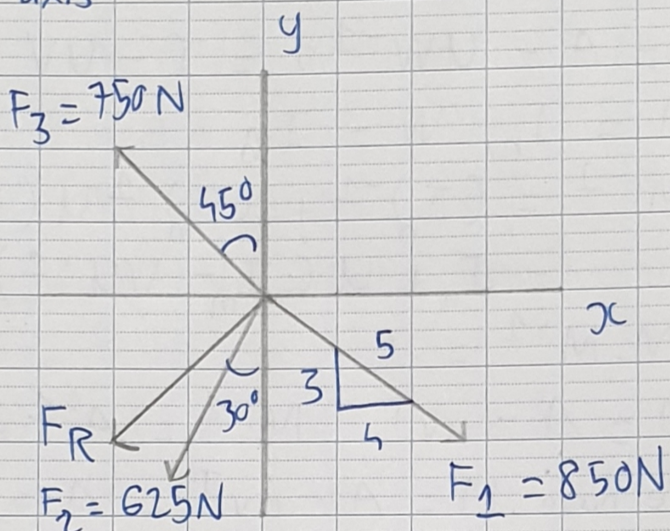
Statics & Dynamics Mid-term Exam

Exercise 1: The resultant force is measured counterclockwise from positive x-axis



We have:

$$\vec{F_1} = (F_1)_x i - (F_1)_y j = 850\left(\frac{4}{5}\right)i - 850\left(\frac{3}{5}\right)j$$

$$= \{680 i - 510 j\} N$$

$$\vec{F_2} = -(F_2)_x i - (F_2)_y j = -625(\sin 30^0) i - 625(\cos 30^0) j$$

$$= \{-312,5 i - 541,27 j\} N$$

$$\vec{F_3} = -(F_3)_x i + (F_3)_y j = -750(\sin 45^0) i + 750(\cos 45^0) j$$

$$= \{-530,33 i + 530,33 j\} N$$

Thus, the resultant force is

$$\vec{F}_R = \Sigma\vec{F}: \vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\Rightarrow \vec{F}_R = \{680 i - 312,5 i - 530,33 i$$
$$- 510 j - 541,72 j + 530,33 j\} N$$

$$\vec{F}_R = \{-162,83 i - 520,9 j\} N$$

The magnitude of $\vec{F}_R$ is

$$F_R = \sqrt{(-162,83)^2 + (-520,9)^2}$$

$$\Rightarrow F_R = 546,22 N$$

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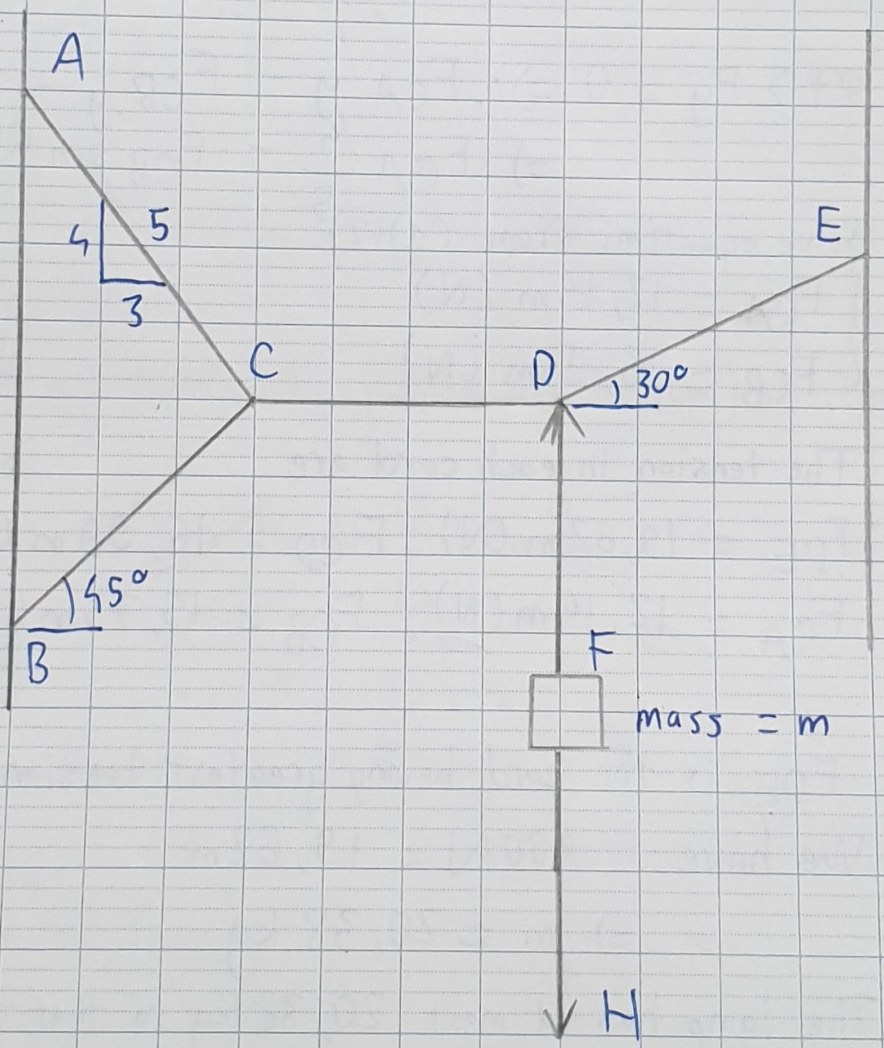
And its directional angle θ measured counterclockwise from the positive x - axis is

$$\theta = 180^\circ + \tan^{-1}\left(\frac{520,9}{162,8}\right) = 180^\circ + 72,64^\circ$$

$$\Rightarrow \theta = 252,64^\circ$$

Exercise 2 :



We have : $F_{FH} = 9,81m$ (N)

- Point F in equilibrium : $-F_{FH} + F_{DF} = 0 \Rightarrow F_{DF} = 9,81m$ (N)
- Point D in equilibrium :

$- \uparrow + \sum F_y = 0 \Rightarrow -F_{DF} + F_{DE,y} = 0$

$\Rightarrow -9,81m + F_{DE}\sin 30^\circ = 0$

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$\Rightarrow F_{DE} = 19,62m$ (N)

- $\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -F_{DC} + F_{DE,x} = 0$

$\Rightarrow -F_{DC} + F_{DE}\cos 30^\circ = 0$

$\Rightarrow F_{DC} = 16,99m$ (N)

□ Point C in equilibrium

- $\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -F_{CA,x} - F_{CB,x} + F_{CD} = 0$

$\Rightarrow F_{CA}\frac{3}{5} + F_{CB}\cos 45^\circ = 16,99m$ (N) (1)

- $\uparrow + \sum F_y = 0 \Rightarrow F_{CA,y} - F_{CB,y} = 0$

$\Rightarrow F_{CA}\frac{4}{5} - F_{CB}\sin 45^\circ = 0$ (2)

Solve equations from (1)(2)

$\Rightarrow \begin{cases} F_{CA} = 12,14m \text{ (N)} \\ F_{CB} = 13,73m \text{ (N)} \end{cases}$

The tension in each cord are

$F_{DE} = 19,62m$ (N) $F_{CD} = 16,99m$ (N)

$F_{CA} = 12,14m$ (N) $F_{CB} = 13,73m$ (N)

$\Rightarrow$ F_{DE} is the cord having greatest tension

We have: $400N = 19,62m$

$\Rightarrow m \approx 20,38$ kg

The lamp has at most 20,38 kg so that no tension in all cords will exceed 400 N

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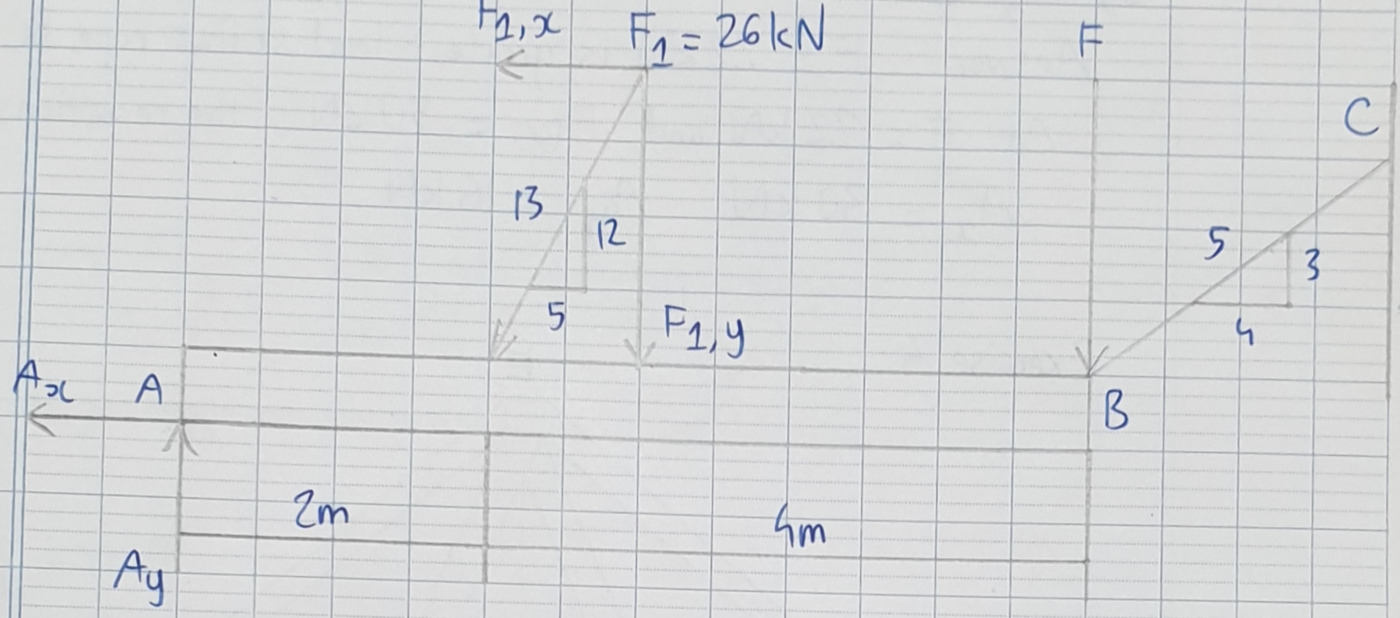
Exercise 3 :



Let the tension in rope BC equal to 50 kN

- Equilibrium at A :

- $\overset{+}{\circlearrowleft} \Sigma M_A = 0 \Rightarrow F_{1,y}(2m) + F(6m) - F_{BC,y}(6m) = 0$

$$\Rightarrow 26kN \cdot \frac{12}{13}(2m) + F(6m) - 50kN \cdot \frac{3}{5}(6m) = 0$$

$$\Rightarrow F = 22kN$$

From the equation, if F increases $\Rightarrow$ F_{BC} also increases

If $F_{BC} = 50kN \Rightarrow F = 22kN$ is the highest tension for F_{BC} to not fail

- $+\uparrow \Sigma F_y = 0 \Rightarrow A_y - F_{1,y} - F + F_{BC,y} = 0$

$$\Rightarrow A_y - 26kN\left(\frac{12}{13}\right) - 22kN + 50kN \cdot \frac{3}{5} = 0$$

$$\Rightarrow A_y = 16kN$$

- $\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow -A_x + F_{BC,x} - F_{1,x} = 0$

$$\Rightarrow -A_x + 50kN \cdot \frac{4}{5} - 26kN \cdot \frac{5}{13} = 0$$

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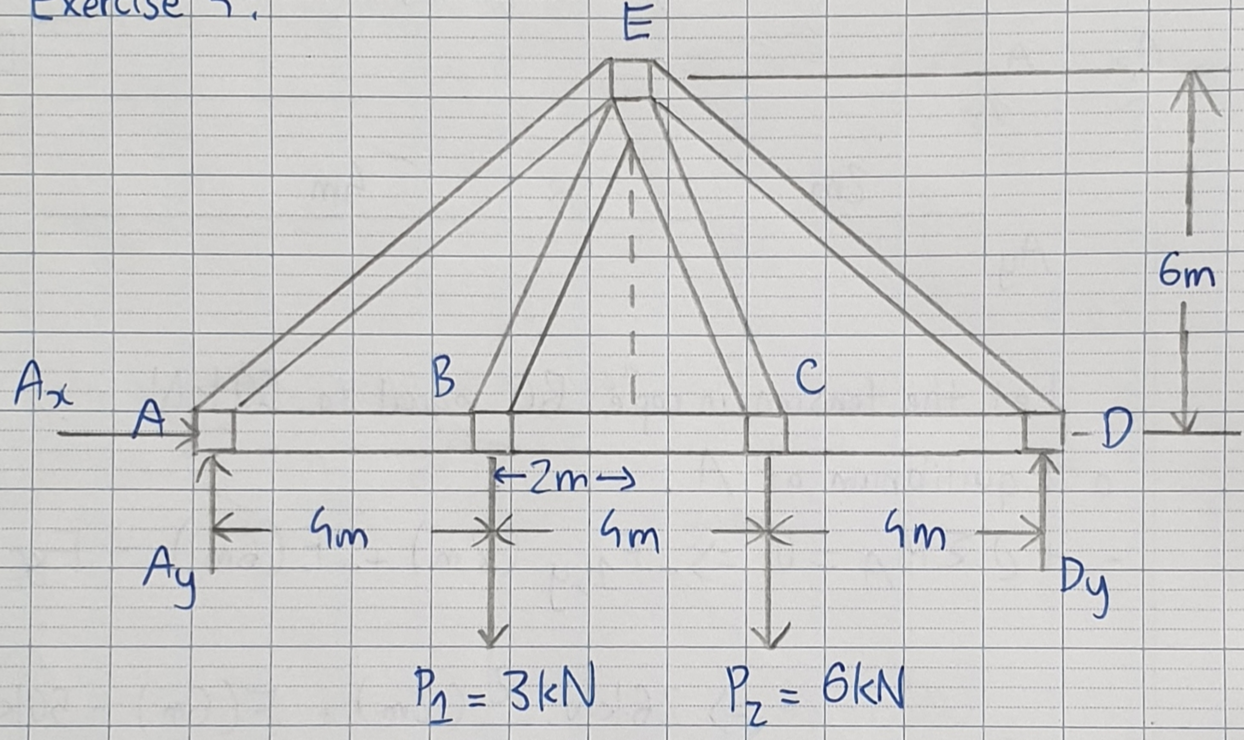
$\Rightarrow A_x = 30\,kN$

Answer : $F = 22\,kN$ is greatest load so that rope BC won't fail

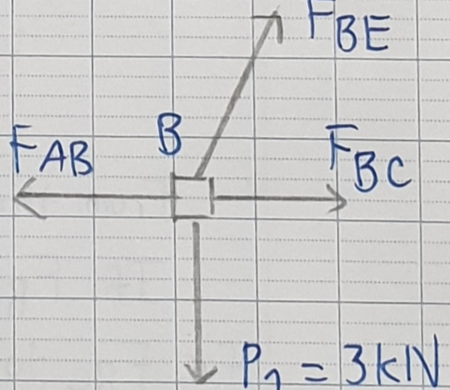
At $F = 22\,kN$ and $F_{BC} = 50\,kN$, the reactions at A are

$A_x = 30\,kN \quad A_y = 16\,kN$

Exercise 4:



- Equilibrium at B ($BE = 2\sqrt{10}$ m)



- $\uparrow + \sum F_y = 0$

$\Rightarrow F_{BE} \cdot \dfrac{6}{2\sqrt{10}} - P_1 = 0$

$\Rightarrow F_{BE} = \sqrt{10}\,kN$. F_{BE} is tension

- $\xrightarrow{+} \sum F_x = 0$

$\Rightarrow F_{BC} - F_{AB} + F_{BE}\left(\dfrac{2}{2\sqrt{10}}\right) = 0$

$\Rightarrow F_{BC} - F_{AB} + 1 = 0 \quad (1)$

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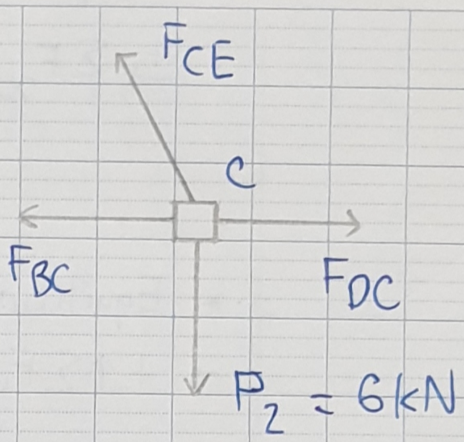
□ Equilibrium at C ($CE = 2\sqrt{10}\,m$)



- $+\uparrow \Sigma F_y = 0$

$\Rightarrow F_{CE} \cdot \dfrac{6}{2\sqrt{10}} - P_2 = 0$

$\Rightarrow F_{CE} = 2\sqrt{10}$ kN. F_{CE} is tension

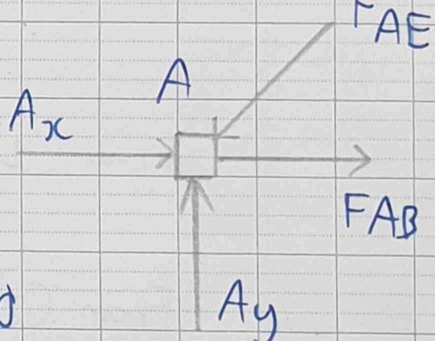
- $\xrightarrow{+} \Sigma F_x = 0$

$\Rightarrow F_{DC} - F_{BC} - F_{CE}\left(\dfrac{2}{2\sqrt{10}}\right) = 0$

$\Rightarrow F_{DC} - F_{BC} - 2 = 0$ (2)

□ Equilibrium at A ($AE = 6\sqrt{2}m$) & D ($DE = 6\sqrt{2}m$)

Considering the whole structure, if we take moment equilibrium at A, we will know D_y and vice versa



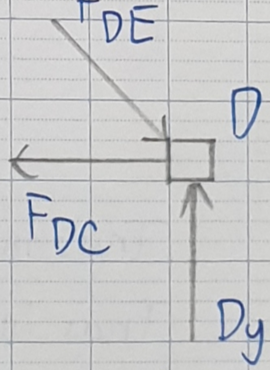
We have: $+\circlearrowleft \Sigma M_A = 0$

$\Rightarrow P_1 (4m) + P_2 (8m) - D_y (12m) = 0$

$\Rightarrow D_y (12m) = 3kN (4m) + 6kN (8m)$

$\Rightarrow D_y = 5$ kN

We have: $+\circlearrowright \Sigma M_D = 0$



$\Rightarrow P_2 (4m) + P_1 (8m) - A_y (12m) = 0$

$\Rightarrow A_y (12m) = 6kN.4m + 3kN.8m$

$\Rightarrow A_y = 4kN$

At A, we have $\xrightarrow{+} \Sigma F_x = 0 \Rightarrow A_x = 0$

(All forces in x direction are all balanced out in the truss)

* Equilibrium at A

$+\uparrow \Sigma F_y = 0 \Rightarrow -F_{AE}\left(\dfrac{1}{\sqrt{2}}\right) + A_y = 0$

$\Rightarrow F_{AE} = 4\sqrt{2}$ kN (Compression)

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$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow F_{AB} - F_{AE}\left(\frac{1}{\sqrt{2}}\right) = 0$

$\Rightarrow F_{AB} = 4\,kN$. F_{AB} is in tension

* Equilibrium at D:

- $+\uparrow \Sigma F_y = 0 \Rightarrow D_y - F_{DE}\frac{1}{\sqrt{2}} = 0 \Rightarrow 5 - F_{DE}\frac{1}{\sqrt{2}} = 0$

$\Rightarrow F_{DE} = 5\sqrt{2}\,kN$. F_{DE} is in compression

- $\xrightarrow{+} \Sigma F_x = 0 \Rightarrow -F_{DC} + F_{DE}\cdot\frac{1}{\sqrt{2}} = 0$

$\Rightarrow -F_{DC} + 5\sqrt{2}\cdot\frac{1}{\sqrt{2}} = 0$

$\Rightarrow F_{DC} = 5\,kN$ (F_{DC} is in tension)

From (1): $F_{BC} - F_{AB} + 1 = 0 \Rightarrow F_{BC} - 4 + 1 = 0$

$\Rightarrow F_{BC} = 3\,kN$. ($F_{BC}$ is in tension)

From (2): $F_{DC} - F_{BC} - 2 = 0 \Rightarrow F_{DC} - 3 - 2 = 0$

$\Rightarrow F_{DC} = 5\,kN$ (F_{DC} is in tension)

Answer: $F_{BE} = \sqrt{10}\,kN$ $\quad F_{BC} = 3\,kN$

$F_{CE} = 2\sqrt{10}\,kN$ $\quad F_{DC} = 5\,kN$

$F_{AE} = 4\sqrt{2}\,kN$ $\quad F_{AB} = 4\,kN$

$F_{DE} = 5\sqrt{2}\,kN$

Truss in tension: AB, CD, BC, BE, CE

Truss in compression: AE, DE

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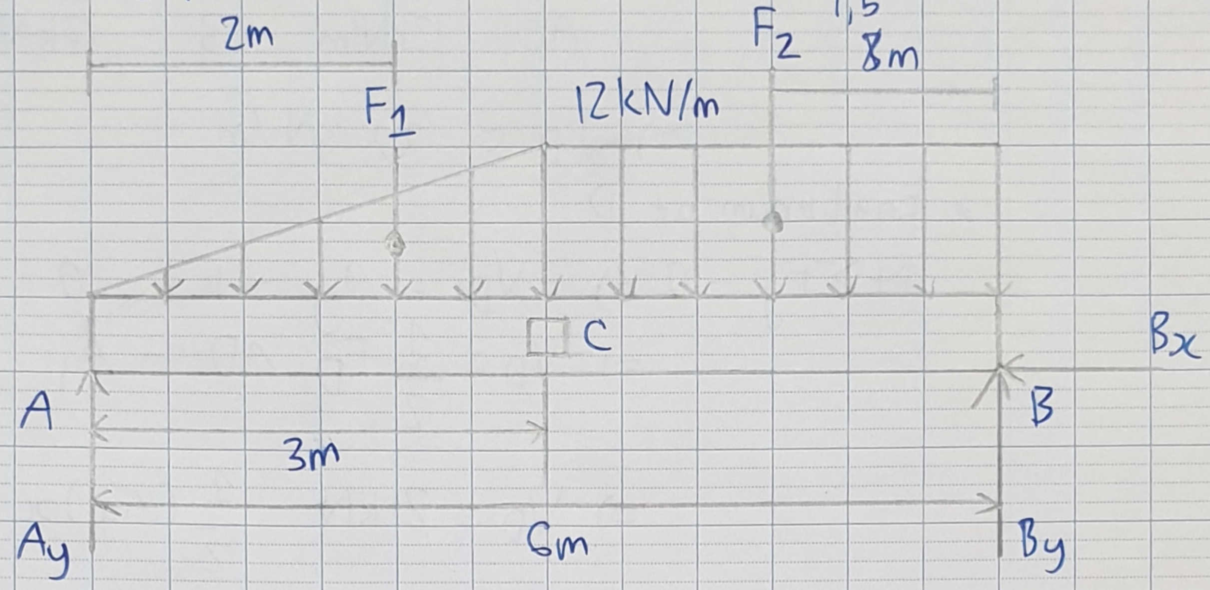
Exercise 5 :



- Equilibrium at A

$\circlearrowleft + \sum M_A = 0 \Rightarrow -F_1 (2m) - F_2 (4,5m) + B_y 6m = 0$

$\Rightarrow -\frac{1}{2} \cdot 12kN/m \cdot 3m \cdot 2m - 12kN/m \cdot 3m \cdot 4,5m$

$+ B_y \cdot 6m = 0$

$\Rightarrow B_y = 33\ kN$

- Equilibrium at B

$\circlearrowleft + \sum M_B = 0 \Rightarrow F_1 (4m) + F_2 (1,5m) - A_y (6m) = 0$

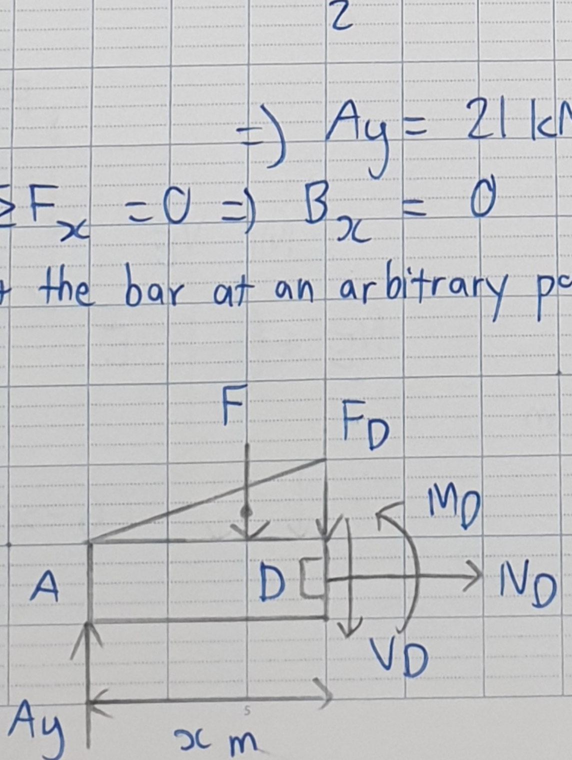
$\frac{1}{2} \cdot 12kN/m \cdot 3m \cdot 4m + 12kN/m \cdot 3m \cdot 1,5m$

$- A_y (6m) = 0$

$\Rightarrow A_y = 21\ kN$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow B_x = 0$

- Cut the bar at an arbitrary point D on AC



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First we have : $\frac{AD}{AC} = \frac{F_D}{12kN/m} \Rightarrow \frac{x}{3} = \frac{F_D}{12kN/m}$

$\Rightarrow F_D = 4x \; kN/m$

* Equilibrium at D

$\uparrow + \Sigma F_y = 0 \Rightarrow -V_D + A_y - F = 0$

$\Rightarrow -V_D = \frac{1}{2} F_D . AD - A_y$

$\Rightarrow V_D = 21kN - \frac{1}{2}(4x)x$

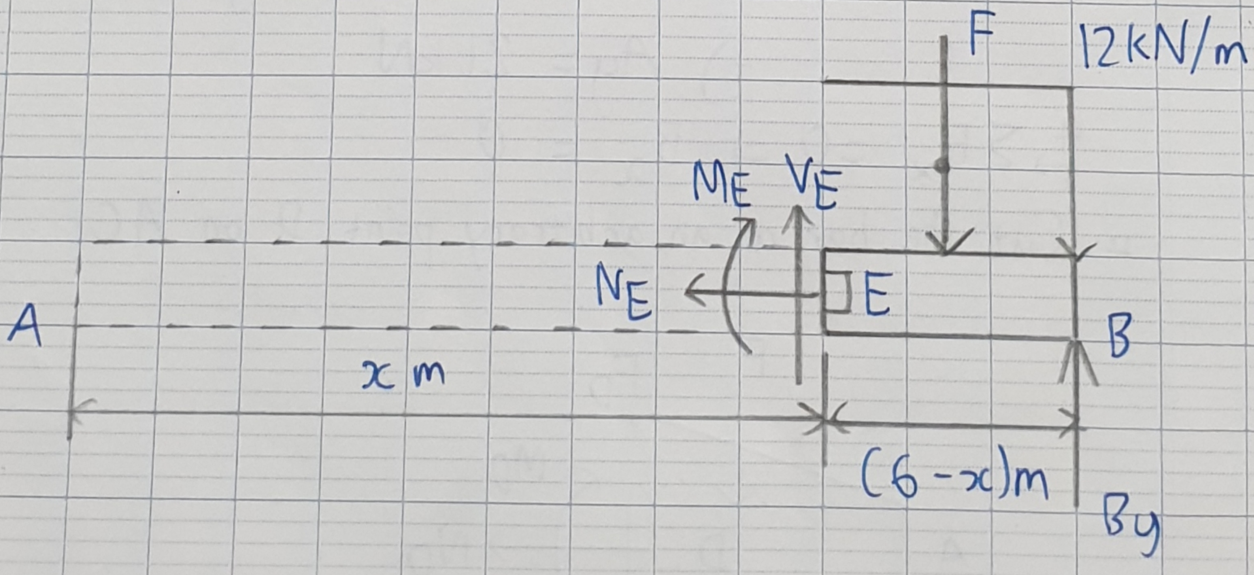
$\Rightarrow V_D = (21 - 2x^2) \; kN$

$\circlearrowleft + \Sigma M_D = 0 \Rightarrow M_D + F\left(\frac{AD}{3}\right) - A_y . AD = 0$

$\Rightarrow M_D + \frac{1}{2}(4x)x . \frac{x}{3} - 21x = 0$

$\Rightarrow M_D = \left(-\frac{2}{3}x^3 + 21x\right) kNm$

□ Cut the bar at an arbitrary point E on CB



* Equilibrium at E

$\uparrow + \sum F_y = 0 \Rightarrow V_E + B_y - F = 0$

$\Rightarrow V_E + 33\,kN - 12.(6-x) = 0$

$\Rightarrow V_E = (-12x + 39)\,kN$

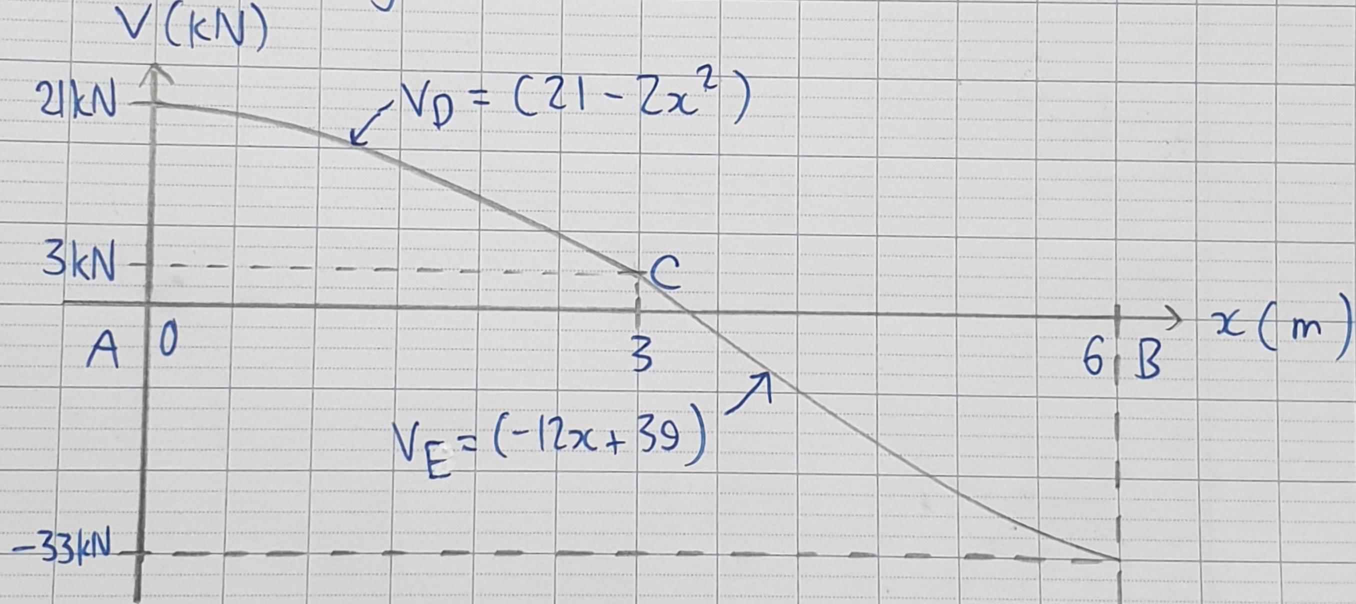
$\circlearrowleft + \sum M_E = 0 \Rightarrow -M_E + B_y\,EB - F.\frac{EB}{2} = 0$

$\Rightarrow M_E = 33\,kN\,(6-x)m - 12(6-x)kN.\frac{6-x}{2}m$

$\Rightarrow M_E = (-6x^2 + 39x - 18)\,kNm$

The shear and moment diagrams are

- The shear diagram



- The moment diagram