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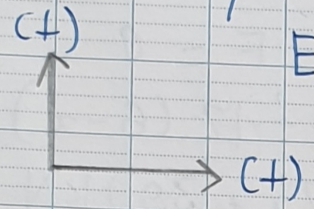
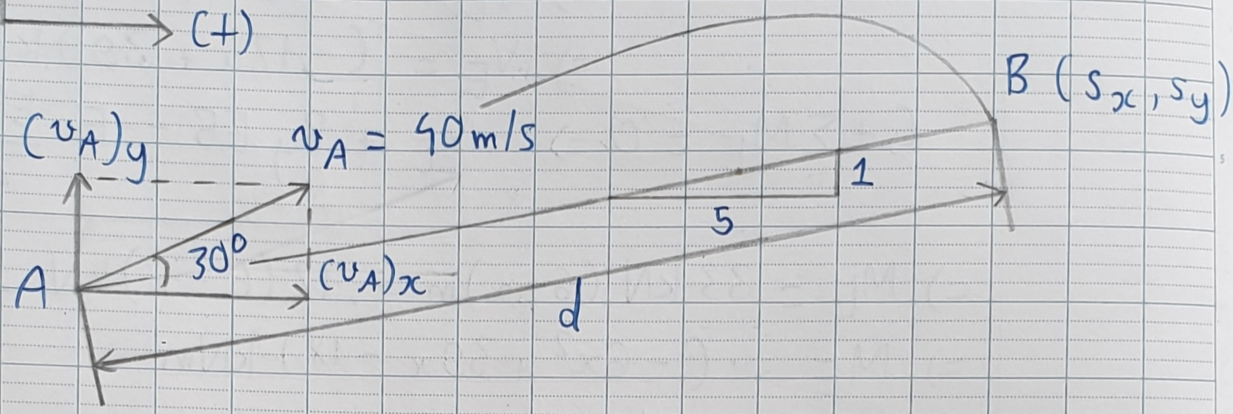
Statics & Dynamics Assignment 4

Exercise 1:





▫ Since $v_A = 40\,m/s \Rightarrow a_A = 0$ (Constant acceleration)

$\Rightarrow$ We can apply kinematics with constant acceleration

▫ For x-axis, the displacement of the ball is

$$s_x = s_{0x} + (v_A)_x t + \frac{1}{2} a_c t^2$$

$$\Rightarrow d \cdot \frac{5}{\sqrt{26}} = 0 + 40 \cdot \cos 30^0 t + \frac{1}{2} \cdot 0 \cdot t^2$$

$$\Rightarrow \frac{5}{\sqrt{26}} d = 20\sqrt{3}\, t \quad (1)$$

▫ For y-axis, the gravity is the constant acceleration. Since it accelerates towards y-negative $\Rightarrow a_c = -9,81$

$$s_y = s_{0y} + (v_A)_y t + \frac{1}{2} a_c t^2$$

$$\Rightarrow d \cdot \frac{1}{\sqrt{26}} = 0 + 40 \cdot \sin 30^0 t - \frac{1}{2} \cdot (9,81) t^2$$

$$\Rightarrow \frac{d}{\sqrt{26}} = 20t - 4,905 t^2 \quad (2)$$

At any time t, the distance d in both (1) (2) must yield

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the same result

$\Rightarrow$ Solve the equation $\begin{cases} \dfrac{5}{\sqrt{26}} d = 20\sqrt{3}\, t \\ \dfrac{d}{\sqrt{26}} = 20t - 4,905t^2 \end{cases}$

$\Rightarrow \cancel{\dfrac{6}{\sqrt{26}} d = -4,905t^2 + (20 + 20\sqrt{3})t}$

$\Rightarrow 20\sqrt{3}\, t = 5(20t - 4,905t^2)$

$\Rightarrow 20\sqrt{3}\, t = 100t - 24,525t^2$

$\Rightarrow \left[\begin{array}{l} t = 2,6649\ (s) \Rightarrow d = 94,1463\ (m) \\ t = 0\ (\text{omit}) \end{array}\right.$

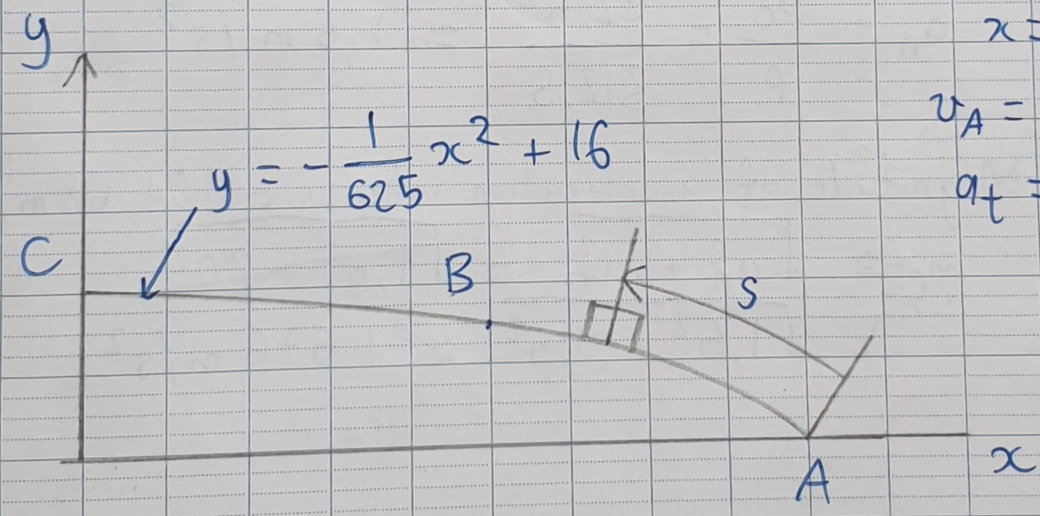
Answer: $d = 94,1463$ m at $t = 2,665$ s

Exercise 2: Determine car's acceleration when $s = 101,68$ m

$x = 0$

$v_A = 20$ m/s

$a_t = 0,5$ m/s^2



Initial position of the car : $y = 0 \Rightarrow s_A = 0$

Since a_t is constant

$\Rightarrow s = s_0 + v_A t + \frac{1}{2} a_t . t^2$

$\Rightarrow 101,68 = 0 + 20t + \frac{1}{2} . 0,5 t^2$

$\Rightarrow 20t + 0,25t^2 - 101,68 = 0$

$\Rightarrow t \approx 4,796\ (s)$

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We have : $v_C = v_A + a_t . t$

$= 20 + 0,5 . 4,796 = 22,4 \, (m/s)$

▫ Finding radius of curvature ρ

We have $y = -\dfrac{x^2}{625} + 16$

$\Rightarrow y' = -\dfrac{2x}{625} \Rightarrow y'' = -\dfrac{2}{625}$

$$\rho = \frac{(1 + y'^2)^{\frac{3}{2}}}{|y''|} = \frac{\left(1 + \left(\frac{2x}{625}\right)^2\right)^{\frac{3}{2}}}{\left|-\frac{2}{625}\right|}$$

As $x = 0 \Rightarrow \rho = \dfrac{1}{\left|-\frac{2}{625}\right|} = 312,5 \, (m)$

▫ The centripetal acceleration is :

$$a_n = \frac{v_C^2}{\rho} = \frac{22,4^2}{312,5} \approx 1,6 \, m/s^2$$

▫ Magnitude of acceleration when $s = 101,68 m$

$a_C = \sqrt{a_n^2 + a_t^2} = \cancel{\sqrt{1,6^2 + 20^2}}$

$= \sqrt{1,6^2 + 0,5^2} \approx 1,676 \, m/s^2$

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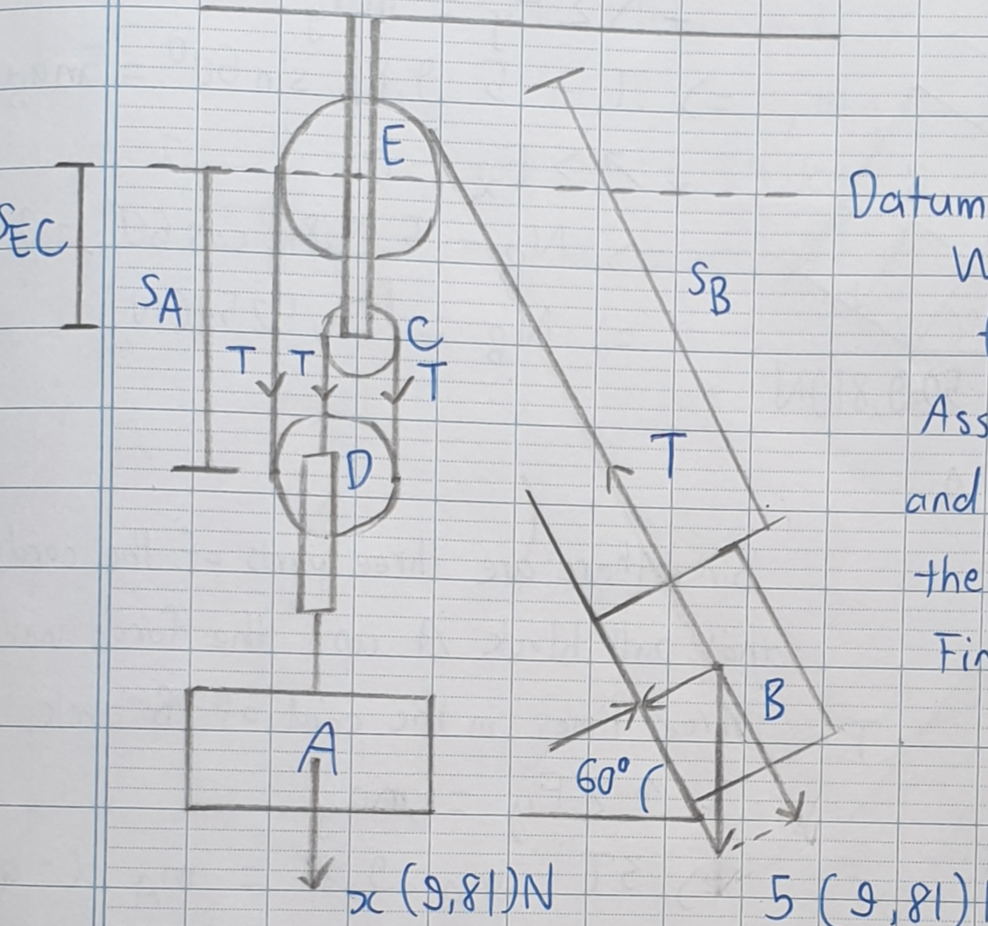
Exercise 3:



When blocked A is released.

$t = 2s \Rightarrow s_B = 0{,}75\,m$

Assume that there is no friction and the mass of pulleys and the cord are neglected.

Find mass of block A

Let ℓ be the total length of the cord (not counting parts of the rope in touch with the pulleys), then the position coordinates is the equation

$$s_B + s_A + 2(s_A - s_{EC}) = \ell$$

$$\Rightarrow s_B + 3s_A - 2s_{EC} = \ell$$

$$\Rightarrow s_B + 3s_A = 2s_{EC} + \ell$$

Since $2s_{EC}$ and ℓ are constant during the motion, the one and two time derivatives is

$$v_B + 3v_A = 0 \Rightarrow v_B = -3v_A$$

$$a_B + 3a_A = 0 \Rightarrow a_B = -3a_A$$

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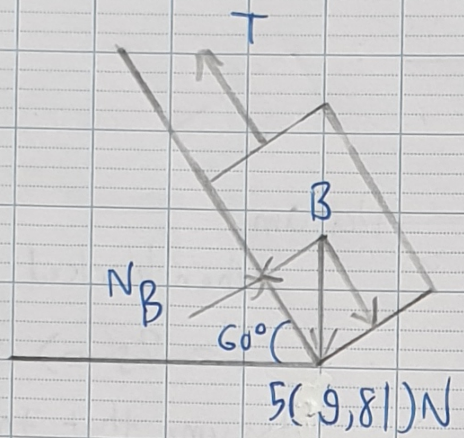
▫ At block B



$+\nwarrow \sum F_y = ma_y$

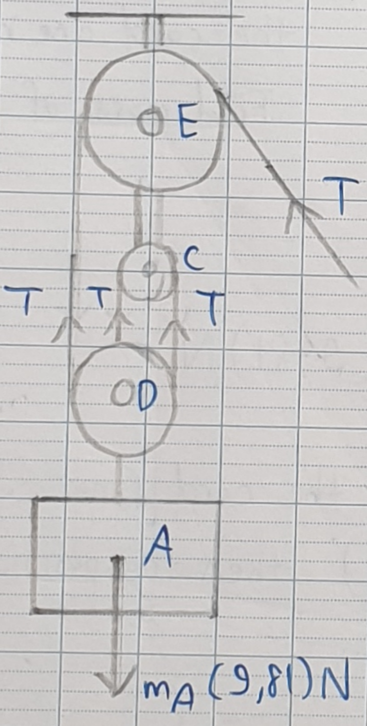
$\Rightarrow T - 5.9,81.\sin 60^0 = 5\,a_B$ (1)

$+\nearrow \sum F_x = 0$

$\Rightarrow N_B - 5.9,81.\cos 60^\circ = 0$

$\Rightarrow N_B = 24,525$ (N)

▫ At block A



Since there are three parts of the cords that pull block A up, the force will be three times in the cord of B block

$+\uparrow \sum F_y = ma_y$

$\Rightarrow 3T - m_A.9,81 = m_A.(-a_A)$ (2)

Since acceleration of block B is constant

$$\Rightarrow s_B = s_{B_0} + v_{B_0}t + \frac{1}{2}a_B t^2$$

$$\Rightarrow 0,75 = 0 + 0.t + \frac{1}{2}a_B(2)^2$$

$\Rightarrow a_B = 0,375\ m/s^2$

We have: $T - 5,9,81\ \sin 60^\circ = 5a_B$ (1)

$\Rightarrow T = 44,353$ (N)

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Since $|a_B| = |3a_A| \Rightarrow a_A = 0,125\ m/s^2$

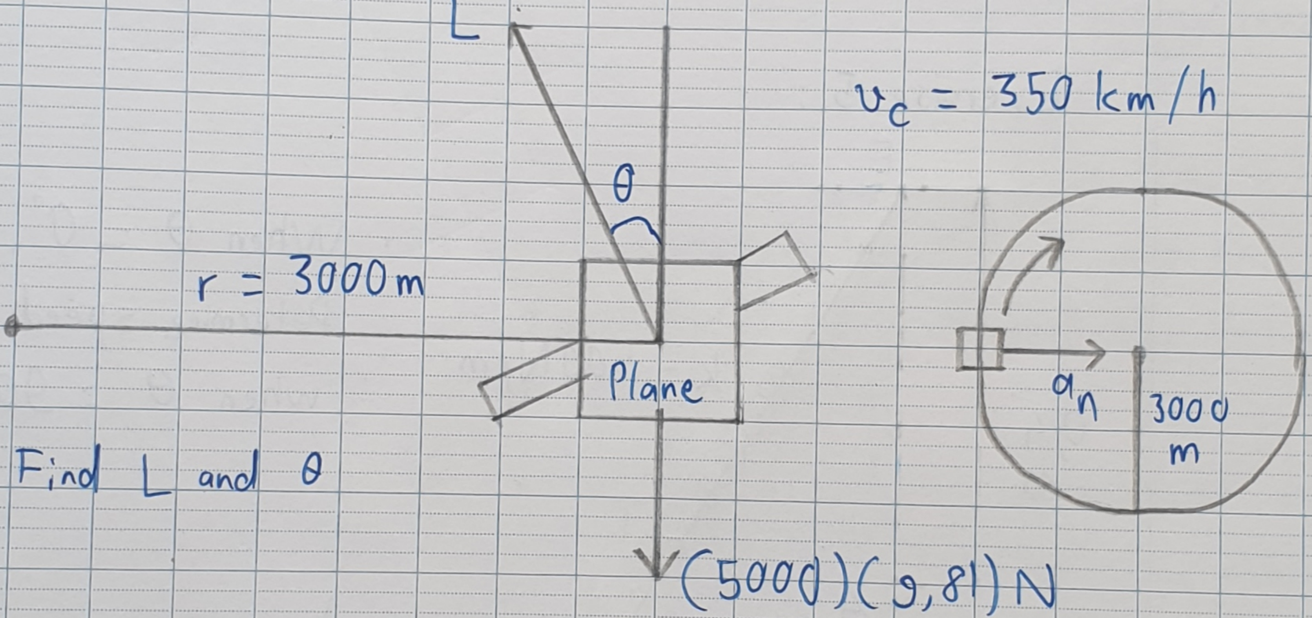
We have: $3T - m_A \cdot 9,81 = m_A \cdot (-a_A)$

$\Rightarrow 3 \cdot 44,353 - m_A \cdot 9,81 = m_A\,(-0,125)$

$\Rightarrow m_A \approx 13,74\ kg$

Answer: mass of block A is 13,74 kg

Exercise 4:



Find L and θ

Convert v_c to m/s: $350\ km/h = 97,2\ m/s$

- Since the plane is moving in the circle parallel to the horizon without flying higher or lower

$\Rightarrow +\uparrow \Sigma F_y = 0 \Rightarrow L \cdot \cos\theta - (5000)(9,81) = 0$

$\Rightarrow L \cdot \cos\theta = 49050\ (N)\ (1)$

- The centripetal acceleration of the plane is given by

$$a_n = \frac{v_c^2}{r} = \frac{97,2^2}{3000} = 3,15\ \cancel{(N)}\ (m/s^2)$$

- Since the plane is moving towards the center

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$\Rightarrow \overset{+}{\leftarrow} \Sigma F_x = m \cdot a_n$

$L \cdot \sin\theta = 5000 a_n = 15746,2 \text{ (N)} \quad (2)$

From (1) and (2)

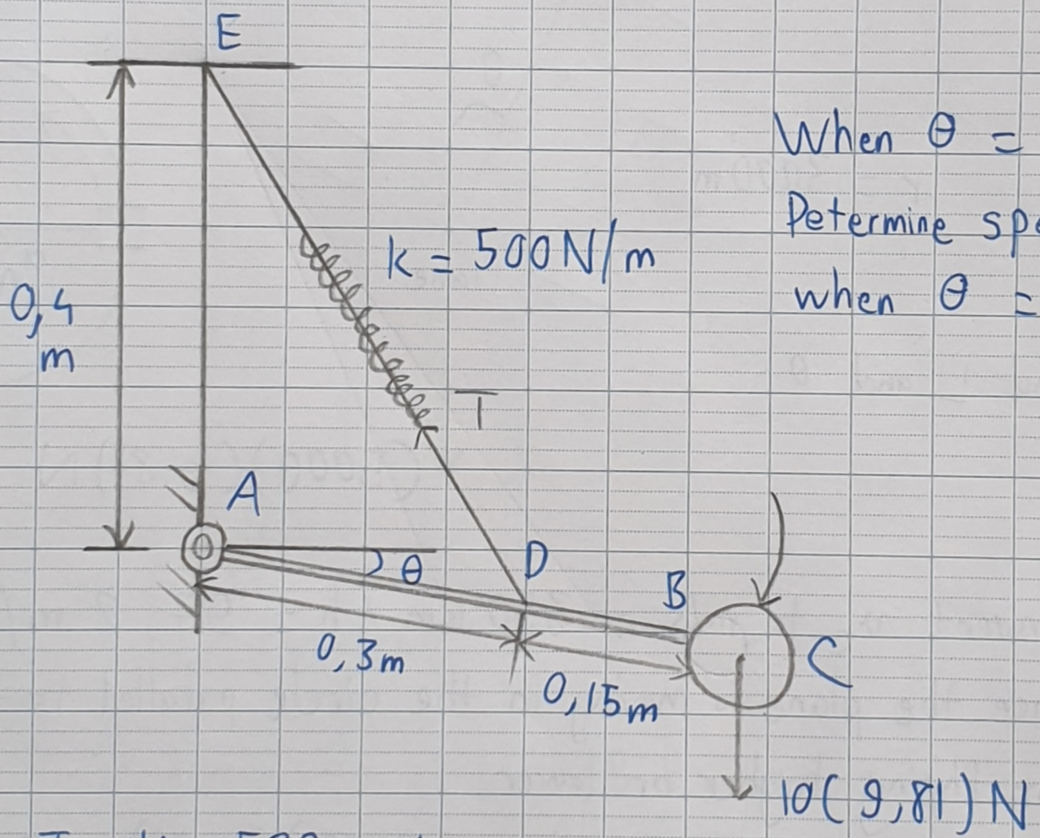
$\Rightarrow \dfrac{L \cdot \sin\theta}{L \cdot \cos\theta} = \dfrac{15746,2}{49050} \Rightarrow \tan\theta = 0,321$

$\Rightarrow \tan^{-1}(0,321) = 17,8^\circ$

- We have $L \cdot \cos\theta = 49050 \Rightarrow L = 49050 / \cos 17,8^\circ$

$\Rightarrow L = 51516,1 \text{ (N)}$

Exercise 5:



When $\theta = 0^\circ$, $T = 100$N

Determine speed of sphere C when $\theta = 90^\circ$

In this FBD, there are only conservative forces which are spring force and gravity

When $\theta = 0^\circ$, $T = 100$ N. Since $T > 0$

$\Rightarrow$ The string is stretched when $\theta = 0^\circ$

$\cancel{V_e = \frac{1}{2} ks^2 \Rightarrow 100 = \frac{1}{2} 500 s^2 \Rightarrow s =}$

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We have : $\Delta_{stretch} = \dfrac{T}{k} = \dfrac{100}{500} = 0,2\ m = s_1$

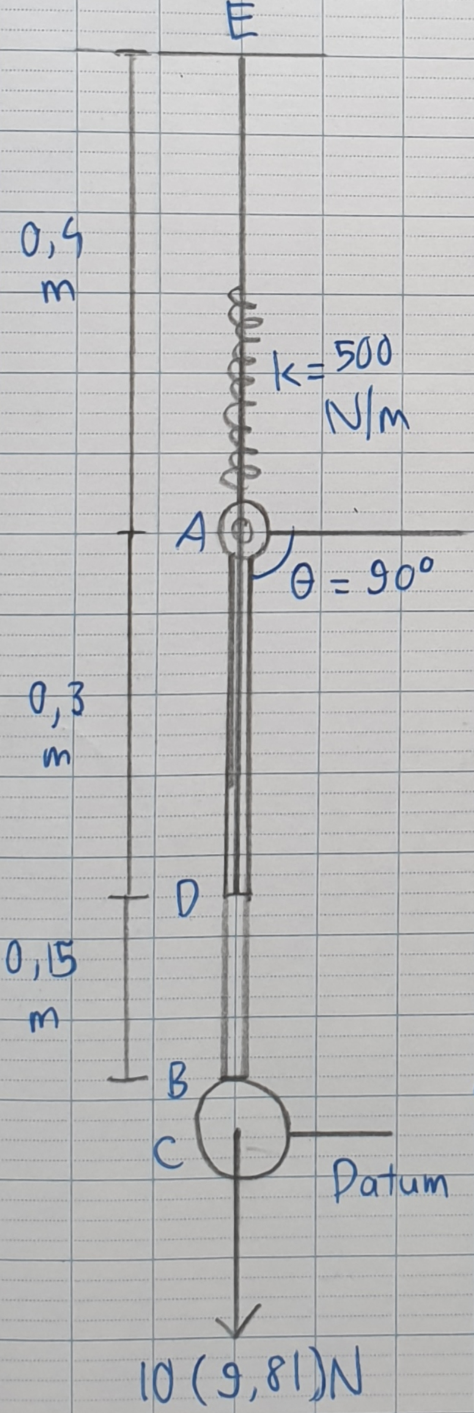
- When $\theta = 0°$, the length of the string is

$$ED = \sqrt{EA^2 + AD^2} = \sqrt{0,4^2 + 0,3^2} = 0,5\,m$$

We know that at $\theta = 0°$, the string is stretched $0,2\,m$

$\Rightarrow$ when unstretched, length of the string is $0,5 - 0,2 = 0,3\,m$

- Consider when $\theta = 90°$



According to conservation of energy law

$$T_1 + V_1 = T_2 + V_2$$

$$\Rightarrow T_1 + V_{e_1} + V_{g_1} = T_2 + V_{e_2} + V_{g_2}$$

Let the first state as $\theta = 0$ and the second state as $\theta = 0$ and the datum at the ball C

$\Rightarrow$ Since at $\theta = 0$, the sphere is at rest so $T_1 = 0$. At $\theta = 90°$, the sphere is at the datum $\Rightarrow V_{g_2} = 0$

$$\Rightarrow V_{e_1} + V_{g_1} = T_2 + V_{e_2}$$

$$\Rightarrow \frac{1}{2} k s_1^2 + W.AB = \frac{1}{2} m v_2^2 + \frac{1}{2} k s_2^2$$

Since the string is unstretched at $0,3\,m$, at $\theta = 90°$, the stretch is

$$s_2 = ED - 0,3 = 0,4 + 0,3 - 0,3$$
$$= 0,4\,m$$

We have : $s_1 = 0,2\,m$

$AB = 0,3 + 0,15 = 0,45\,m$

$W = 10 . 9,81 = 98,1\ (N)$

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$$\Rightarrow \frac{1}{2} . 500 . 0,2^2 + 98,1 . 0,45 = \frac{1}{2} . 10 . v_2^2 + \frac{1}{2} . 500 . 0,4^2$$

$$\Rightarrow 54,145 = 5v_2^2 + 40$$

$$\Rightarrow 5v_2^2 = 14,145 \Rightarrow v_2 \approx 1,682 \text{ m/s}$$

Answer: at $\theta = 90°$, the speed of the sphere is 1,682 m/s