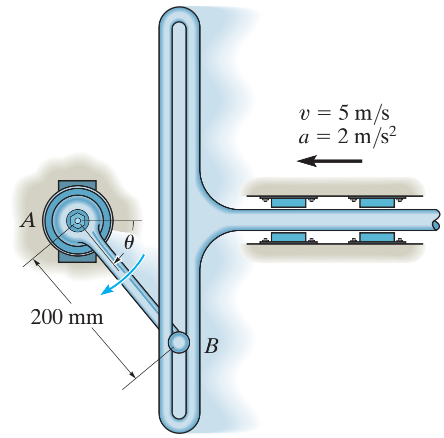
***16–40.**

At the instant $\theta = 60°$, the slotted guide rod is moving to the left with an acceleration of $2\ \text{m/s}^2$ and a velocity of $5\ \text{m/s}$. Determine the angular acceleration and angular velocity of link AB at this instant.



SOLUTION

Position Coordinate Equation. The rectilinear motion of the guide rod can be related to the angular motion of the crank by relating x and θ using the geometry shown in Fig. a, which is

$$x = 0.2\cos\theta\ \text{m}$$

Time Derivatives. Using the chain rule,

$$\dot{x} = -0.2(\sin\theta)\dot{\theta} \qquad \textbf{(1)}$$

$$\ddot{x} = -0.2[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}] \qquad \textbf{(2)}$$

Here $\dot{x} = v$, $\ddot{x} = a$, $\dot{\theta} = \omega$ and $\ddot{\theta} = \alpha$ when $\theta = 60°$. Realizing that the velocity and acceleration of the guide rod are directed toward the negative sense of x, $v = -5\ \text{m/s}$ and $a = -2\ \text{m/s}^2$. Then Eq (1) gives

$$-s = (-0.2(\sin 60°)\omega$$

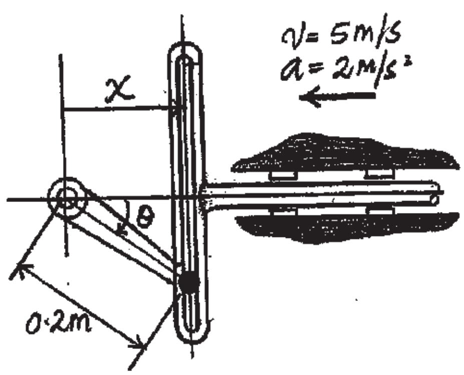
$$\omega = 28.87\ \text{rad/s} = 28.9\ \text{rad/s} \curvearrowright \qquad \textbf{Ans.}$$

Subsequently, Eq. (2) gives

$$-2 = -0.2[\cos 60°(28.87^2) + (\sin 60°)\alpha]$$

$$\alpha = -469.57\ \text{rad/s}^2 = 470\ \text{rad/s}^2 \curvearrowleft \qquad \textbf{Ans.}$$

The negative sign indicates that $\boldsymbol{\alpha}$ is directed in the negative sense of θ.

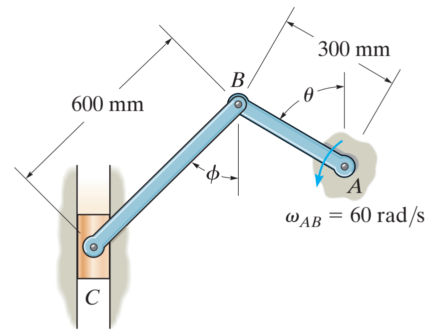


(a)

Ans:
$\omega = 28.9\ \text{rad/s} \curvearrowright$
$\alpha = 470\ \text{rad/s}^2 \curvearrowleft$

***16–68.**

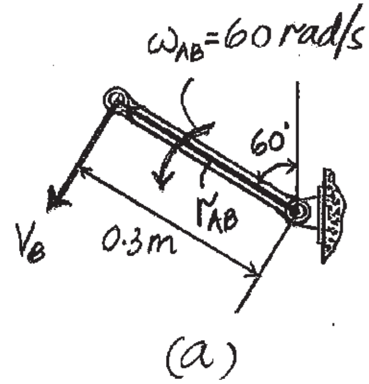
Rod AB is rotating with an angular velocity of $\omega_{AB} = 60$ rad/s. Determine the velocity of the slider C at the instant $\theta = 60°$ and $\phi = 45°$. Also, sketch the position of bar BC when $\theta = 30°$, $60°$ and $90°$ to show its general plane motion.



SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. *a*.

$$\mathbf{V}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$
$$= (60\mathbf{k}) \times (-0.3 \sin 60°\mathbf{i} + 0.3 \cos 60°\mathbf{j})$$
$$= \{-9\mathbf{i} - 9\sqrt{3}\mathbf{j}\} \text{ m/s}$$



General Plane Motion. For link BC, refer to Fig. *b*. Applying the relative velocity equation,

$$\mathbf{V}_C = \mathbf{V}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$
$$-v_C\mathbf{j} = (-9\mathbf{i} - 9\sqrt{3}\,\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (-0.6 \sin 45°\mathbf{i} - 0.6 \cos 45°\mathbf{j})$$
$$-v_C\mathbf{j} = \left(0.3\sqrt{2}\omega_{BC} - 9\right)\mathbf{i} + \left(-0.3\sqrt{2}\omega_{BC} - 9\sqrt{3}\right)\mathbf{j}$$

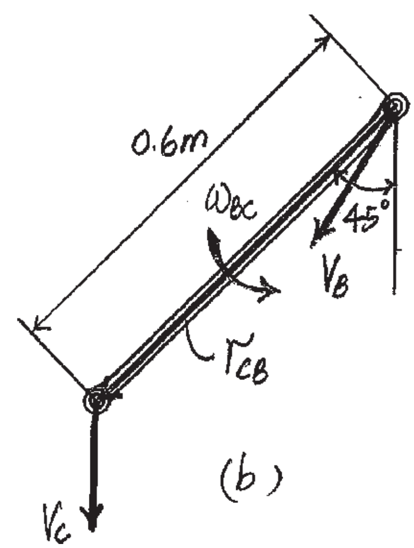
Equating **i** components,

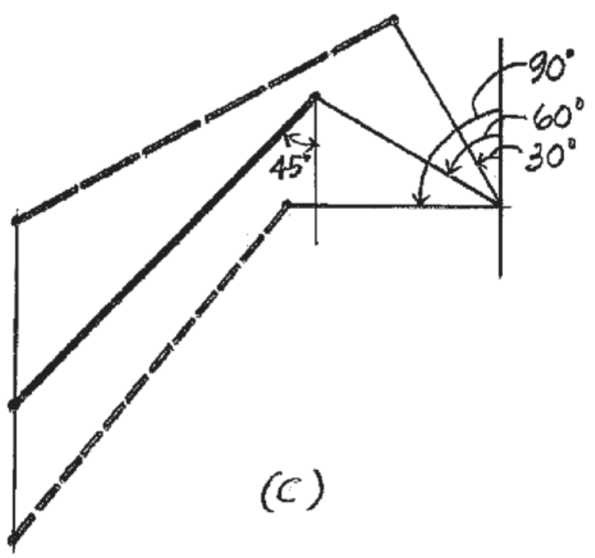
$$0 = 0.3\sqrt{2}\omega_{BC} - 9; \qquad \omega_{BC} = 15\sqrt{2} \text{ rad/s} = 21.2 \text{ rad/s} \circlearrowright$$

Then, equating **j** components,

$$-v_C = \left(-0.3\sqrt{2}\right)(15\sqrt{2}) - 9\sqrt{3}; \quad v_C = 24.59 \text{ m/s} = 24.6 \text{ m/s}\downarrow \quad \textbf{Ans.}$$

The general plane motion of link BC is described by its orientation when $\theta = 30°$, $60°$ and $90°$ shown in Fig. *c*.

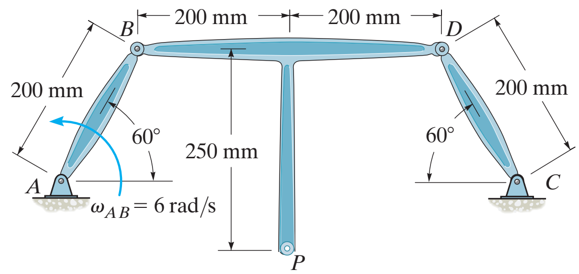




Ans:
$v_C = 24.6$ m/s$\downarrow$

***16–88.**

Member AB is rotating at $\omega_{AB} = 6$ rad/s. Determine the velocity of point P, and the angular velocity of member BPD.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of B and D are

$$v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20 \text{ m/s} \qquad v_D = \omega_{CD}(0.2)$$

And their direction are indicated in Fig. *a* and *b*

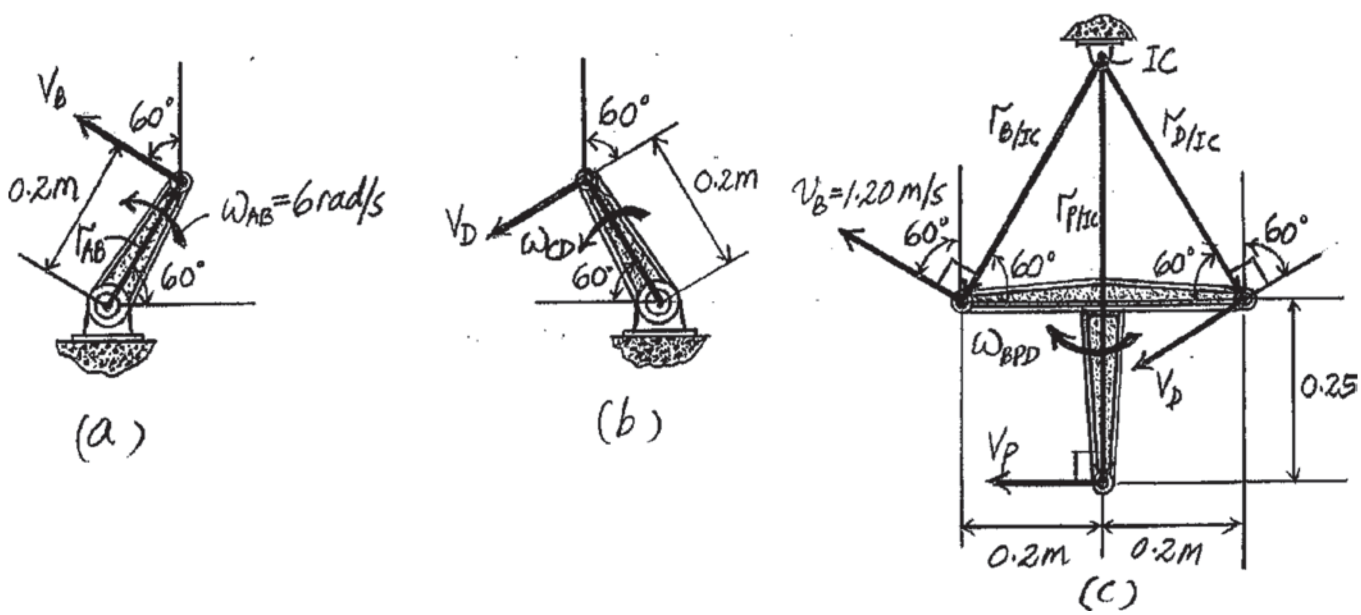
General Plane Motion. With the results of $\mathbf{v}_B$ and $\mathbf{v}_D$, the IC for member BPD can be located as shown in Fig. *c*. From the geometry of this figure

$$r_{B/IC} = 0.4 \text{ m} \qquad r_{P/IC} = 0.25 + 0.2 \tan 60° = 0.5964 \text{ m}$$

Then the kinematics give

$$\omega_{BPD} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.4} = 3.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

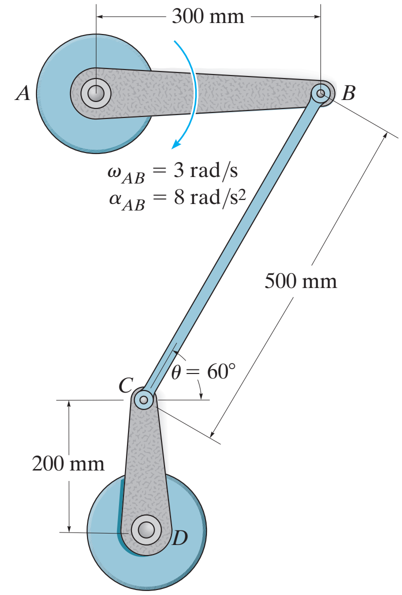
$$v_P = \omega_{BPD} r_{P/IC} = (3.00)(0.5964) = 1.7892 \text{ m/s} = 1.79 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$



Ans:
$\omega_{BPD} = 3.00$ rad/s $\circlearrowright$
$v_P = 1.79$ m/s $\leftarrow$

***16–120.**

If member AB has the angular motion shown, determine the velocity and acceleration of point C at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 3(0.3) = 0.9 \text{ m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-8\mathbf{k}) \times (0.3\mathbf{i}) - 3^2(0.3\mathbf{i})$$

$$= \{-2.70\mathbf{i} - 2.40\mathbf{j}\} \text{ m/s}^2$$

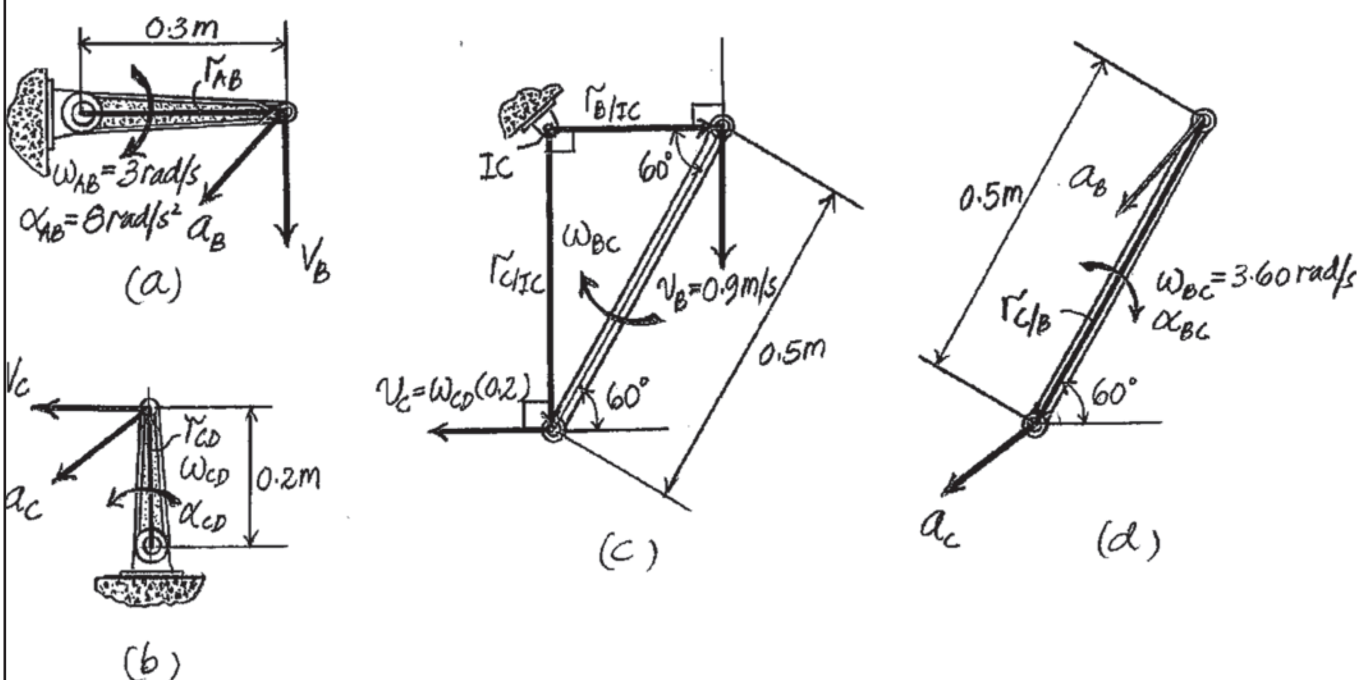
For link CD, refer to Fig. b.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.2) \leftarrow$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$\mathbf{a}_C = (\alpha_{CD}\mathbf{k}) \times (0.2\mathbf{j}) - \omega_{CD}^2(0.2\mathbf{j})$$

$$= -0.2\alpha_{CD}\mathbf{i} - 0.2\omega_{CD}^2\mathbf{j}$$



***16–120. Continued**

General Plane Motion. The *IC* of link *BC* can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. *c*. From the geometry of this figure,

$$r_{B/IC} = 0.5 \cos 60° = 0.25 \text{ m} \qquad r_{C/IC} = 0.5 \sin 60° = 0.25\sqrt{3} \text{ m}$$

Then kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \qquad 0.9 = \omega_{BC}(0.25) \qquad \omega_{BC} = 3.60 \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC} r_{C/IC}; \qquad \omega_{CD}(0.2) = (3.60)\left(0.25\sqrt{3}\right)$$

$$\omega_{CD} = 7.7942 \text{ rad/s} = 7.79 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. *d*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.2\alpha_{CD}\mathbf{i} - 0.2\left(7.7942^2\right)\mathbf{j} = (-2.70\mathbf{i} - 2.40\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$
$$- 3.60^2(-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$

$$-0.2\alpha_{CD}\mathbf{i} - 12.15\mathbf{j} = \left(0.54 - 0.25\sqrt{3}\alpha_{BC}\right)\mathbf{i} + (3.2118 + 0.25\alpha_{BC})\mathbf{j}$$

Equating the **j** components,

$$-12.15 = 3.2118 + 0.25\alpha_{BC}; \qquad \alpha_{BC} = -61.45 \text{ rad/s}^2 = 61.45 \text{ rad/s}^2 \circlearrowleft$$

Then the **i** component gives

$$-0.2\alpha_{CD} = 0.54 - 0.25\sqrt{3}(-61.4474); \quad \alpha_{CD} = -135.74 \text{ rad/s}^2 = 136 \text{ rad/s}^2 \quad \textbf{Ans.}$$

From the angular motion of *CD*,

$$v_C = w_{CD}(0.2) = (7.7942)(0.2) = 1.559 \text{ m/s} = 1.56 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = -0.2(-135.74)\mathbf{i} - 12.15\mathbf{j}$$
$$= \{27.15\mathbf{i} - 12.15\mathbf{j}\} \text{ m/s}$$

The magnitude of $\mathbf{a}_C$ is

$$a_C = \sqrt{27.15^2 + (-12.15)^2} = 29.74 \text{ m/s}^2 = 29.7 \text{ m/s}^2 \qquad \textbf{Ans.}$$

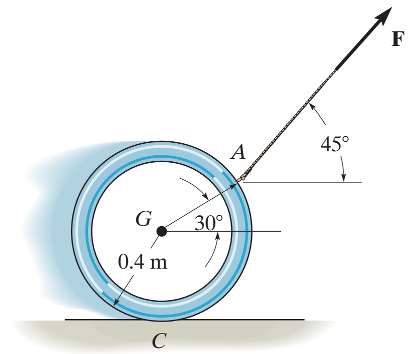
And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{12.15}{27.15}\right) = 24.11° = 24.1° \qquad \textbf{Ans.}$$

Ans:
$v_C = 1.56 \text{ m/s} \leftarrow$
$a_C = 29.7 \text{ m/s}^2$
$\theta = 24.1°$

17–98.

A force of $F = 10$ N is applied to the 10-kg ring as shown. If slipping does not occur, determine the ring's initial angular acceleration, and the acceleration of its mass center, G. Neglect the thickness of the ring.



SOLUTION

Equations of Motion. The mass moment of inertia of the ring about its center of gravity G is $I_G = mr^2 = 10\left(0.4^2\right) = 1.60 \text{ kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the ring, Fig. a,

$$\circlearrowleft + \Sigma M_C = (\mu_k)_C; \quad (10 \sin 45°)(0.4 \cos 30°) - (10 \cos 45°)[0.4(1 + \sin 30°)]$$

$$= -(10a_G)(0.4) - 1.60\alpha$$

$$4a_G + 1.60\alpha = 1.7932 \quad \textbf{(1)}$$

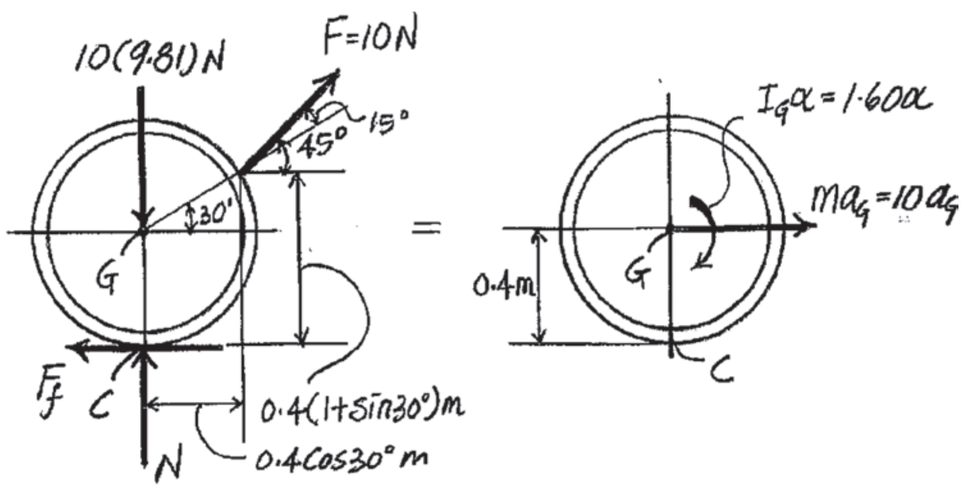
Kinematics. Since the ring rolls without slipping,

$$a_G = \alpha r = \alpha(0.4) \quad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$\alpha = 0.5604 \text{ rad/s}^2 = 0.560 \text{ rad/s}^2 \circlearrowright \quad \textbf{Ans.}$$

$$a_G = 0.2241 \text{ m/s}^2 = 0.224 \text{ m/s}^2 \rightarrow \quad \textbf{Ans.}$$



(a)

Ans:
$\alpha = 0.560 \text{ rad/s}^2 \circlearrowright$
$a_G = 0.224 \text{ m/s}^2 \rightarrow$