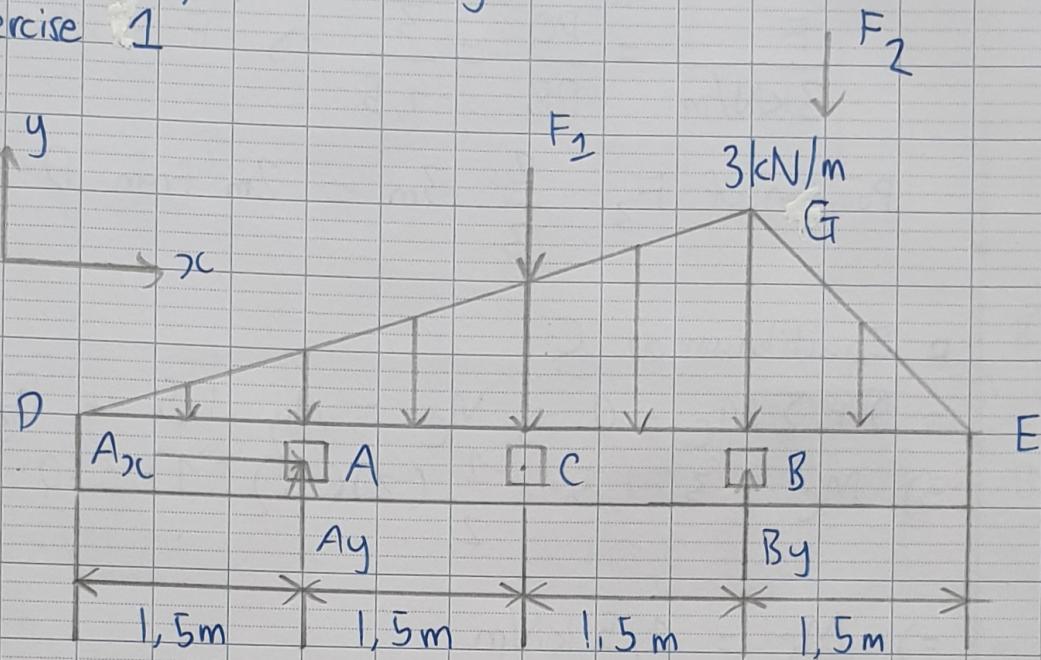


## Statics &amp; Dynamics (Assignment 3)

## Exercise 1



- Position of center of force for load DGB :  $\frac{2}{3} \cdot 4,5 = 3\text{m}$   
 from D and for load GBE :  $\frac{1}{3} \cdot 1,5 + 4,5 = 5\text{m}$  from D

- Moment equilibrium at B.

$$\sum M_B = 0$$

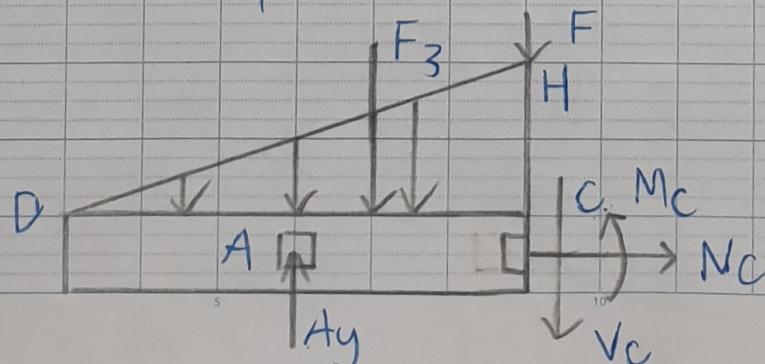
$$\Rightarrow -A_y \cdot 3 + F_1 \cdot (1,5) - F_2 \cdot (0,5) = 0$$

$$\Rightarrow -A_y \cdot 3 + \frac{1}{2} (4,5)(3)(1,5) - \frac{1}{2} (1,5)(3)(0,5) = 0$$

$$\Rightarrow -A_y \cdot 3 = -9$$

$$\Rightarrow A_y = 3 \text{ kN}$$

Cut the bar at point C and its FBD from left side



Force of F that tops the load DHC

$$\frac{F}{3 \text{ kN/m}} = \frac{DC}{DB} = \frac{3}{4,5} \Rightarrow F = 2 \text{ kN/m}$$

Position of  $F_3$ :  $\frac{2}{3} \cdot 3 \text{ m} = 2 \text{ m}$  from D

□ Equilibrium at C

$$\begin{aligned} - 5 + \sum M_C &= 0 \Rightarrow M_C - Ay \cdot 1,5 + F_3 \cdot 1 = 0 \\ \Rightarrow M_C - 3 \cdot 1,5 + \frac{1}{2} (2)(3) &= 0 \end{aligned}$$

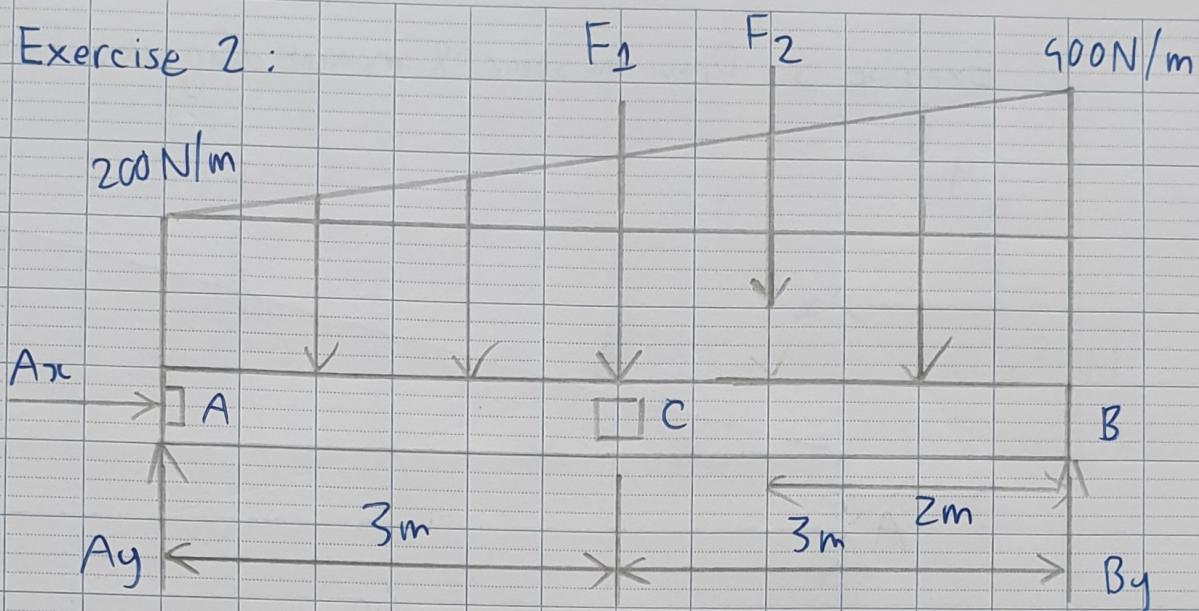
$$\Rightarrow M_C = 1,5 \text{ kNm}$$

$$\begin{aligned} - \uparrow + \sum F_y &= 0 \Rightarrow Ay - V_C - F_3 = 0 \\ \Rightarrow 3 - V_C - \frac{1}{2} (2)(3) &= 0 \\ \Rightarrow V_C &= 0 \end{aligned}$$

$$- \rightarrow + \sum F_{xC} = 0 \Rightarrow N_C = 0$$

Answer:  $N_C = 0$     $V_C = 0$     $M_C = 1,5 \text{ kNm}$

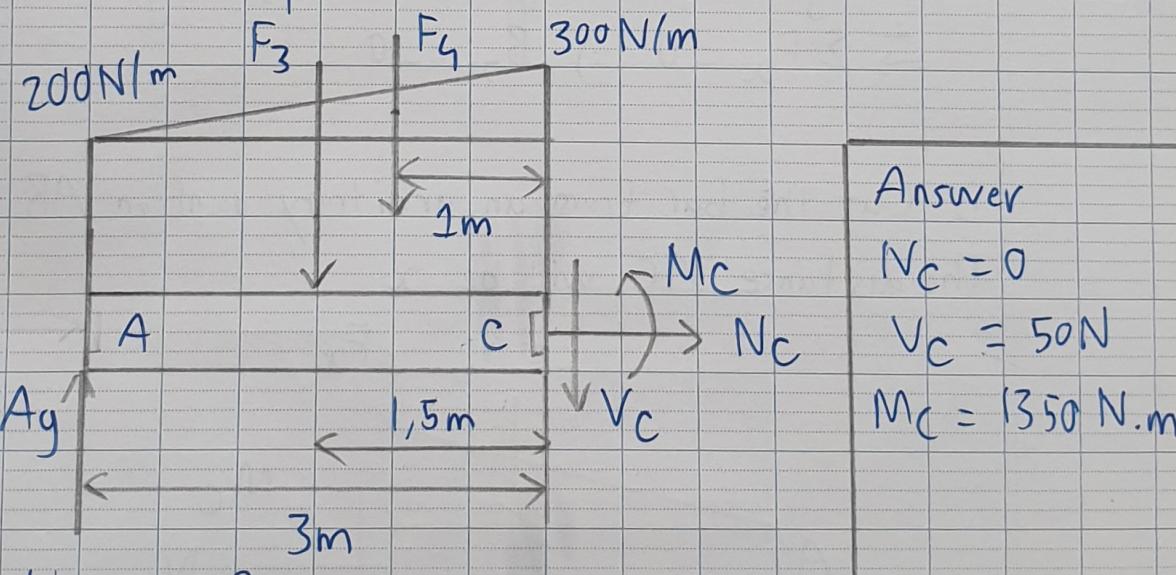
## Exercise 2:



□ Moment equilibrium at point B

$$\begin{aligned} \zeta + \sum M_B &= 0 \Rightarrow -Ay \cdot 6m + F_1 \cdot 3m + F_2 \cdot 2m = 0 \\ \Rightarrow -Ay \cdot 6m + 200N/m \cdot 6m \cdot 3m + \frac{1}{2} \cdot 2.00N/m \cdot 6m \cdot 2m &= 0 \\ \Rightarrow Ay &= 800N \end{aligned}$$

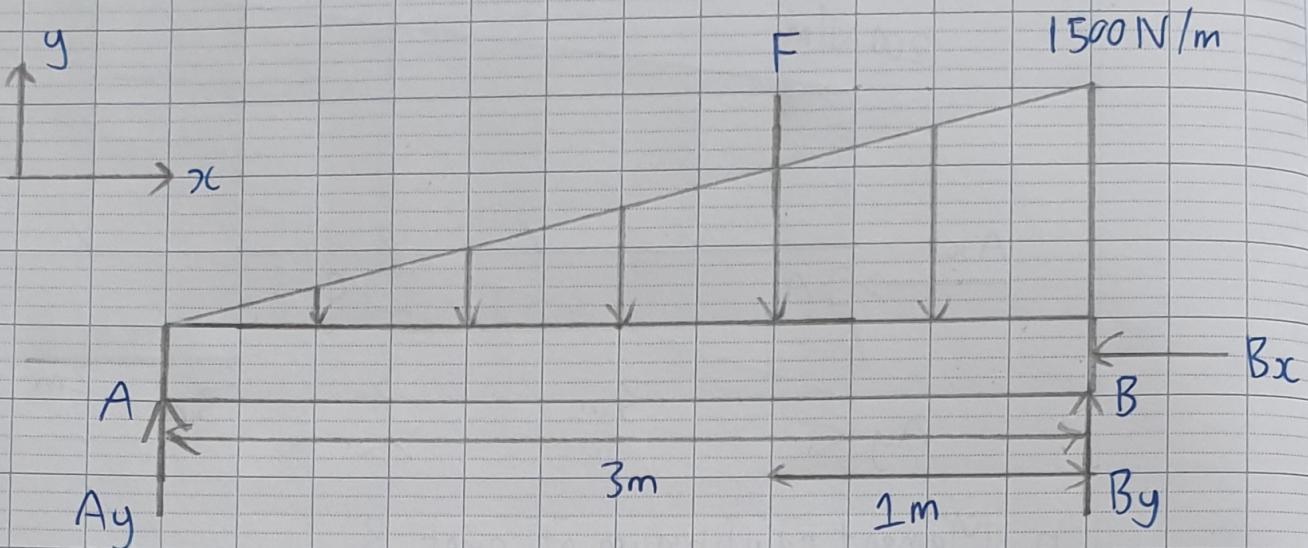
p Cut the bar at point C from the left



## ☐ Equilibrium at C

$$\begin{aligned}
 & \rightarrow \sum F_x = 0 \Rightarrow N_C = 0 \\
 & + \uparrow \bar{F}_y = 0 \Rightarrow A_y - F_3 - F_4 - V_C = 0 \\
 \Rightarrow & 800N - 200N/m \cdot 3m - \frac{1}{2}(100N/m) \cdot 3m - V_C = 0 \\
 \Rightarrow & V_C = 50N \\
 (\hookrightarrow + \sum M_C = 0 \Rightarrow M_C - A_y(3) + F_3(3) + F_4 \cdot 1 & = 0 \\
 \Rightarrow & M_C = 800 \cdot 3 - 600 \cdot (1,5) - 150 \cdot 1 \\
 M_C & = 1350 \text{ Nm}
 \end{aligned}$$

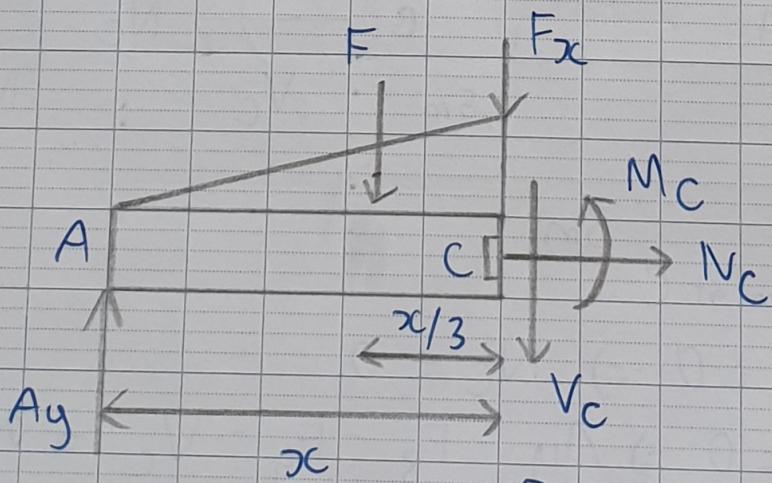
Exercise 3: Draw the shear & moment diagrams for the beam



□ Equilibrium at B

$$\begin{aligned}
 G + \sum M_B = 0 &\Rightarrow -Ay(3m) + F(1m) = 0 \\
 \Rightarrow -Ay(3m) + \frac{1}{2} \cdot 1500 \text{ N/m} (3m)(1m) &= 0 \\
 \Rightarrow -Ay(3m) = -2250 \text{ N} &\Rightarrow Ay = 750 \text{ N} \\
 \rightarrow \sum F_x = 0 &\Rightarrow B_x = 0
 \end{aligned}$$

□ Cut the bar from an arbitrary point on AB which is C. The distance AC will be  $x$



First we have:

$$\frac{AC}{AB} = \frac{Fx}{1500 \text{ N/m}} \Rightarrow \frac{x}{3m} = \frac{Fx}{1500 \text{ N/m}}$$

$$\Rightarrow F_x = 500x \text{ N/m}$$

Equilibrium at C:

$$+ \uparrow + \sum F_y = 0 \Rightarrow A_y - F - V_C = 0$$

$$\Rightarrow 750N - \frac{1}{2} AC \cdot F_x - V_C = 0$$

$$\Rightarrow V_C = 750 - \frac{1}{2} \times 500x = (750 - 250x^2)N$$

$$= (0,75 - 0,25x^2)kN$$

$$+ \rightarrow \sum F_x = 0 \Rightarrow N_C = 0$$

$$+ \text{G} + \sum M_C = 0 \Rightarrow M_C - A_y \cdot AC + F \cdot \frac{AC}{3} = 0$$

$$\Rightarrow M_C = 750x - 250x^2 \cdot \frac{x}{3}$$

$$\Rightarrow M_C = 750x - \frac{250}{3}x^3 (N)$$

$$= 0,75x - \frac{0,25}{3}x^3 (kN)$$

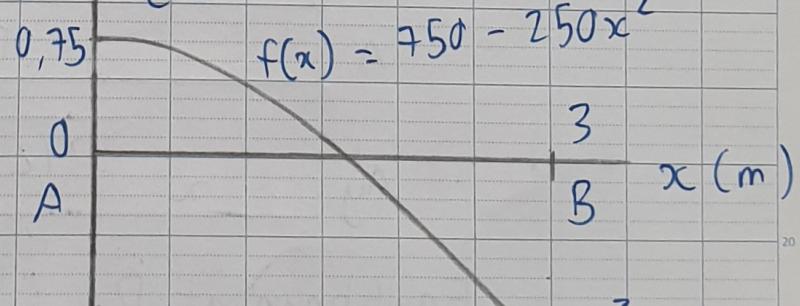
Answer

$$V_C = (0,75 - 0,25x^2) kN$$

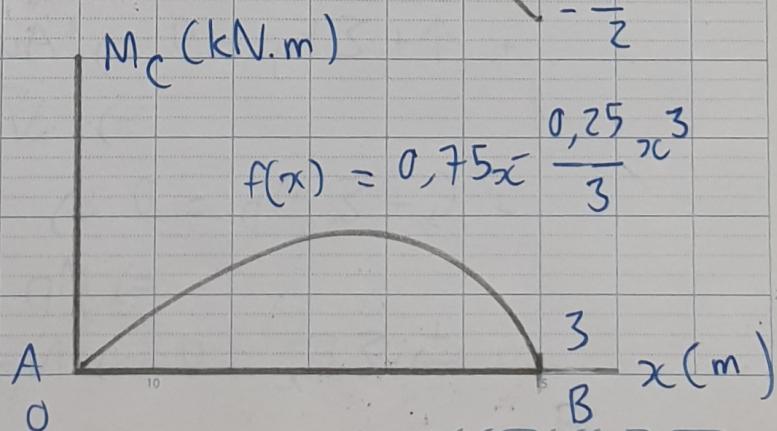
$$M_C = (0,75x - \frac{0,25}{3}x^3) kN$$

$$V_C (kN)$$

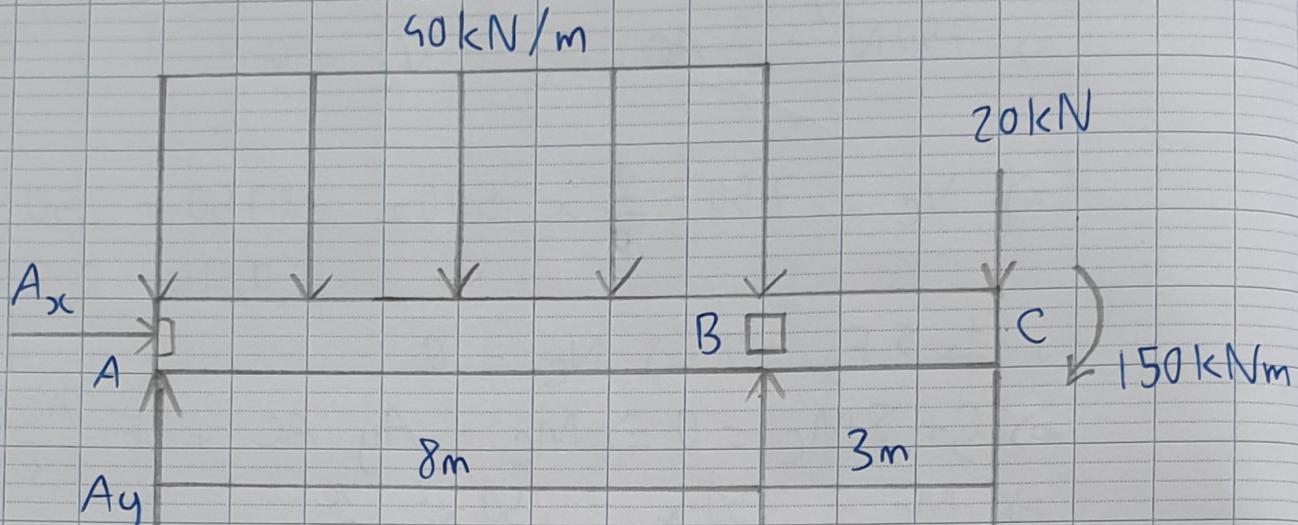
The diagram for  $V_C$



The diagram for  $M_C$



Exercise 4 : Draw the shear and moment diagrams for the beam



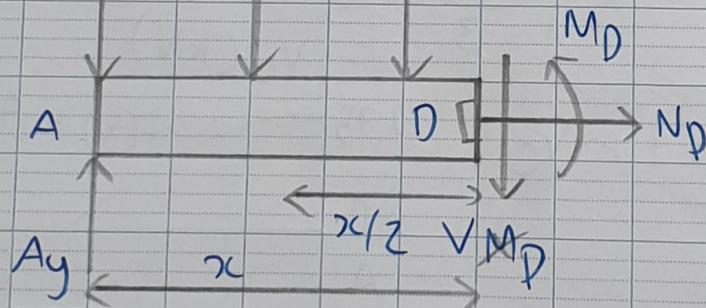
□ Equilibrium at B

$$-\sum M_B = 0 \Rightarrow -Ay(8) - 150 + 50(8)(4) = 0$$

$$\Rightarrow Ay = 133,75 \text{ N}$$

□ Cut the bar at D, an arbitrary point on AB

$$50 \text{ kN/m}$$



Equilibrium at D

$$+\uparrow + \sum F_y = 0 \Rightarrow Ay - 50x - V_D = 0$$

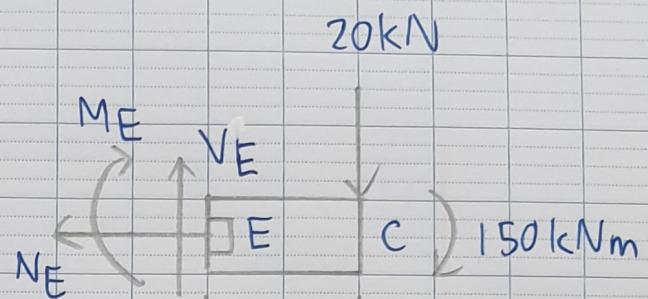
$$\Rightarrow V_D = (133,75 - 50x) \text{ N}$$

$$+\sum M_D = 0 \Rightarrow -Ay \cdot x + 50x \left(\frac{x}{2}\right) + M_D = 0$$

$$\Rightarrow M_D = (133,75x - 25x^2) \text{ Nm}$$

$$+\rightarrow \sum F_x = 0 \Rightarrow N_D = 0$$

□ Cut the bar at E, an arbitrary point on BC.



□ Equilibrium at E

$$+ \uparrow + \sum F_y = 0 \Rightarrow V_E - 20 \text{ kN} = 0 \Rightarrow V_E = 20 \text{ kN}$$

$$+ \zeta + \sum M_E = 0 \Rightarrow -M_E - 150 - 20(11 - x) = 0$$

$$\Rightarrow -M_E = 150 + 20(11 - x) \text{ kNm}$$

Since  $150 + 20(11 - x)$  is positive  $\Rightarrow M_E$  direction is opposite the diagram

Answer: at  $E \in AB$ :  $V_D = (133, 75 - 40x) \text{ kN}$

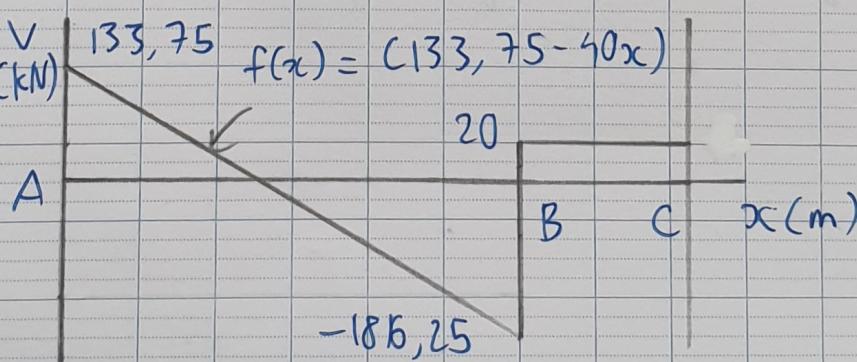
$$M_D = (133, 75x - 20x^2) \text{ kNm}$$

at  $E \in BC$ :  $V_E = 20 \text{ kN}$

$$M_E = (-370 + 20x) \text{ kNm}$$

Diagrams of shear and moment force

+ Shear:  $V$  (kN)  $f(x) = (133, 75 - 40x)$



+ Moment M (kNm)  $f(x) = 133, 75x - 20x^2$

$$f(x) = 20x - 370$$

