Let $x_1, x_2, \ldots, x_n$ be an i.i.d. sample from the normal distribution $\mathcal{N}(\mu, \sigma^2)$ where both $\mu \in \mathbb{R}$ and $\sigma^2 > 0$ are unknown.

A level $100(1-\alpha)\%$ confidence interval for μ is obtained as,

$$\left(\bar{x} - t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}, \bar{x} + t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}\right),$$

where $t_{n-1,\alpha/2}$ is the $(1-\alpha/2)$-quantile of the Student's t-distribution with $n-1$ degrees of freedom.

1. Consider the confidence interval for the expected value of the normal distribution on page 2.9 of the lecture notes. Describe what will (most likely) happen to the width of the confidence interval (does it get smaller, larger or stay the same?) if we,

- a. Increase the sample size .

In the formula, n increases => s/sqrt(n) decreases => The confidence interval decreases

- b. Decrease the confidence level

Decreasing the confidence level will make the error bound narrower => The confidence interval decreases

- c. Increase the variance

Increased variance => s^2 increases => s increases. In the formula, s increases => s/sqrt(n) increases => The confidence interval increases

- d. Decrease the expected value

Expected value of the random variable doesnt affect the confidence interval. It just shifts the confidence interval. In fact the formula above doesnt have expected value => decrease expected value, the confidence interval sat the same

2. Consider the following four hypothesis testing scenarios. For each scenario, describe what the Type I error and Type II error mean in that particular context. Comment also on the possible consequences of the two errors in each case (which one of the errors is more "dangerous"?). For part d, come up with a typical hypothesis testing scenario from your own field of science.

Definition: Type I error is rejected true H0 and Type II error is accepted wrong H0

- a. A suspect is brought before a judge.

H0: The suspect is innocent.
H1: The suspect is guilty.
Type I error: H0 is true (The suspect is indeed innocent) but was rejected => H1 is accepted: An innocent suspect is found guilty and put to jail
Type II error: H0 is false (The suspect is indeed guilty from H1) and was accepted => A guilty suspect is found innocent and is let free

Now in this case type II is more dangerous, if the suspect is a murderer for example. Type I is less dangerous because if it happens, it only affects the suspect.

- b. A new experimental cancer treatment is compared to placebo.

H0: The new treatment is no better than placebo.
H1: The new treatment is better than placebo.
Type I error: H0 is true (The new treatment is indeed no better than placebo) but was rejected => H1 is accepted: The new treatment, although no better than placebo, was wrongly thought to be better than placebo
Type II error: H0 is false (The new treatment is indeed better than placebo from H1) and was accepted => The new treatment, although better than placebo, was wrongly thought to be no better than placebo
In this case, Type I error is probably a little bit more dangerous. Because the treatment has no effect at all (because it is no better than placebo), but still administered to the patients, the cancer will not be treated which is unknown to the doctors and patients. Thus it will lead to death. In type II error, at least if that treatment is not administered to the patient, other treatments will be worked on. At least a future treatment which is more effective than placebo will be given to cancer patients.

- c. An automated security screening scans passengers at the airport.

H0: The passenger is not carrying dangerous items.
H1: The passenger is carrying dangerous items.
Type I error: H0 is true (The passenger is indeed not carrying dangerous items) but was rejected => H1 is accepted: The passenger, although not carrying dangerous items, is thought to be carrying dangerous items
Type II error: H0 is false (The passenger is indeed carrying dangerous items from H1) and was accepted => The passenger, although carrying dangerous items, is thought to be not carrying dangerous items
In this case type II error is obviously more dangerous. If that passenger is allowed onboard, the dangerous items will put the whole plane at perils. Meanwhile in Type I error, if it is a mistake done by the customs, only that passenger is affected and theres nothing bad happening, except an apology. It is better safe than sorry

- d. Your own scenario here!: High grade steel is considered to be used in highrising structures. It should be non-flammable to stand against the weather

H0: The high grade steel is non flammable in highrise buildings
H1: The high grade steel is flammable in highrise building
Type I error: H0 is true (The high grade steel is indeed non flammable in highrise buildings) but was rejected => H1 is accepted: The high grade steel, although non flammable in highrise buildings, is thought to be flammable otherwise
Type II error: H0 is false (The high grade steel is indeed flammable in highrise buildings from H1) and was accepted => The high grade steel, although flammable in highrise buildings, is thought to be non flammable otherwise
=> Type II error is much more dangerous, because it may lead to fire accidents that put people currently staying in the building at jeopardy. Meanwhile if type I error occurs, building planners simply find another steel type and there is nothing dangerous happening.