

Exercise 12

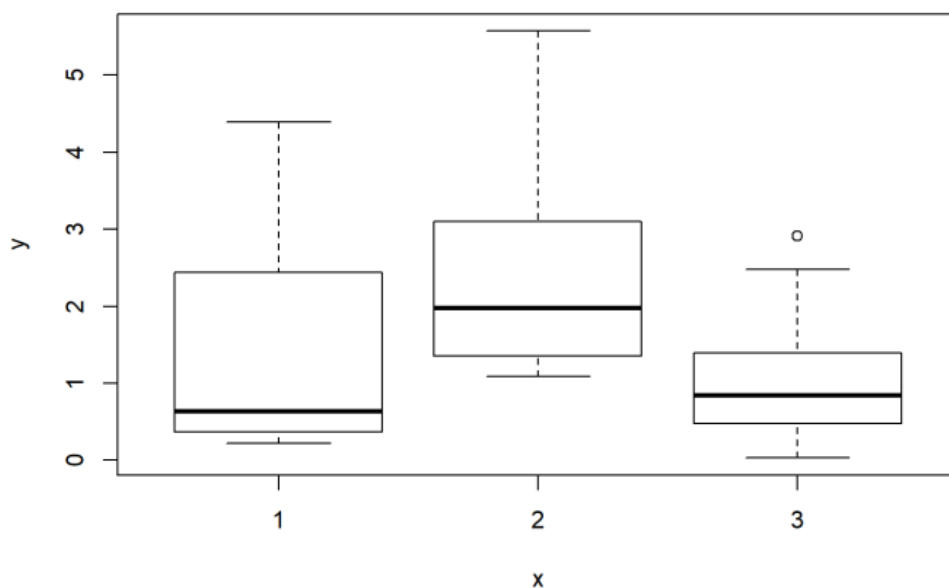
Homework exercise

To be solved at home before the exercise session.

1. Consider a data set with measurements of the variable y for three groups (x). Each group has sample size 15. Below are shown boxplots of the groups, along with outputs given by ANOVA and the Kruskal-Wallis test for the data.

- What are the conclusions of the two tests?
- Which test (if either) would you trust and why?
- How would you continue the analysis?

```
boxplot(y ~ x, data = my_data)
```



```
summary(aov(y ~ x, data = my_data))
```

```
##           Df Sum Sq Mean Sq F value Pr(>F)
## x           1   1.13   1.129    0.586  0.448
## Residuals  43  82.89   1.928
```

```
kruskal.test(y ~ x, data = my_data)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data:  y by x
## Kruskal-Wallis chi-squared = 10.185, df = 2, p-value = 0.006142
```

a. What are the conclusions of the two tests?

- Null hypothesis for ANOVA test: $H_0 : \mu_1 = \mu_2 = \dots = \mu_k$ (no difference in mean between the sample groups)

Conclusion of ANOVA test: F-value is quite low. The expected value of the F-value is $(n-k)/(n-k-2)$ which is $(15 \cdot 3 - 3)/(15 \cdot 3 - 3 - 2) = 1.05$. Since the F-value is close to the expected value, ANOVA test accepts the null hypothesis

- Null hypothesis for Kruskal-Wallis test: $H_0 : m_1 = m_2 = \dots = m_k$ (no difference in median between the sample groups)

- Conclusion of Kruskal-Wallis test: p-value of this test is $0.006142 < \alpha = 0.05$, which indicates that the null hypothesis is false

b. Which test (if either) would you trust and why?

Normal / gaussian distribution should be analyzed with ANOVA while a non-normal / non-gaussian distribution should be analyzed with the Kruskal-Wallis. Moreover, Kruskal-Wallis test assumes that the distribution have the same "shape" but possibly different medians/ locations. Looking at the bar plots above, it seems that the samples do not have the same distribution => ANOVA can be trusted.

c. How would you continue the analysis?

The analysis is continued to find out which pairs are statistically significant different, using pairwise t-test. However, we should take into account of multiple test problem, where we may commit type I error => Bonferroni correction should also be applied to find out the statistically significant pairs.