Course Log

What has been going on? Here will appear Lecture topics & Additional materials:

Lecture 1 (24.10.): Chapter 1 in Lecture notes

Markov chains (MC, X) & Markov property

Transition matrix (P), transition diagram

Examples (weather; more examples in exercises!)

Distribution (\mu_t), computing it using powers of transition matrix P

Transition probabilities for many steps / given path

Occupancy (M): expected time spent in one state

Computing occupancy matrix M using transition matrix P

One can simulate MC using Unif(0,1) random variables; see more details in Lecture notes

Additional material: Some example Mathematica codes (NB: the syntax is very specific to Wolfram Mathematica -- don't try to use as such elsewhere...):

Predicting weather (nb)

Path probabilities (nb)

Occupancy matrix (nb)

Simple weather simulation (nb)

Lecture 2 (26.10.): Chapter 2 in Lecture notes

Limit distribution (\mu_\infty)

Invariant distribution (\pi), balance equations, law of total probability

Limit distribution is also invariant distribution

When does limit distribution exist?

Irreducibility and aperiodicity

Irreducible MC has unique invariant distribution

Irreducible and aperiodic MC has unique limit distribution that is also its unique invariant distribution

(We assume MC has finite state space)

Additional material: Some example Mathematica codes:

Brand loyalty model (nb)

Ehrenfest model (nb)

Getting stuck (nb)

General example (nb)

One more example: reducible MC (nb)

Lecture 3 (31.10.): Chapter 3 in Lecture notes

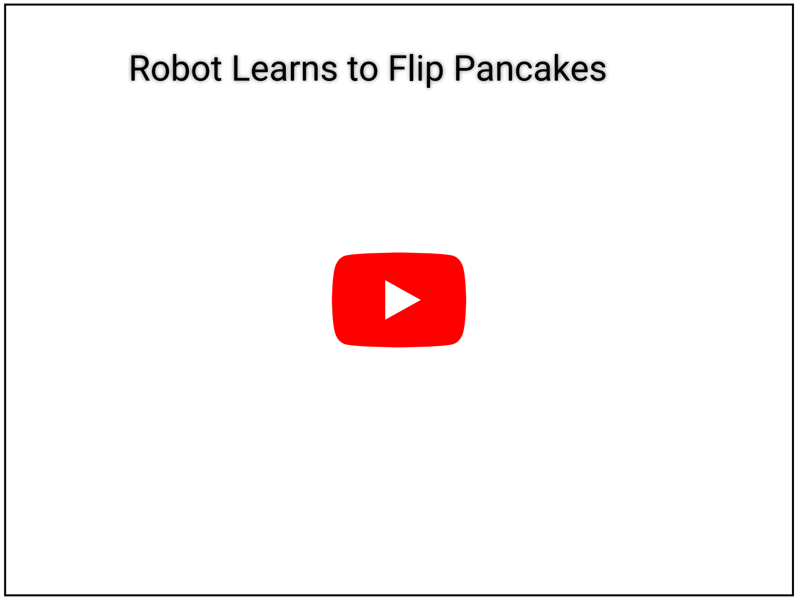
Markov additive processes and their time behavior

Ergodicity of long-term time averages

Relative frequencies and their infinite-time limit

Expected growth rate

Video:



Additional material: Some example Mathematica codes

Inventory model (nb)

Lecture 4 (02.11.): Chapter 4 in Lecture notes

Passage times

Calculation of expected passage time

Calculation of hitting probabilities

Gambler's ruin

(We assume MC has finite state space)

Additional material: Some example Mathematica codes:

Gambler's ruin (nb)

Hitting probability - general recursion (nb)

Expected passage time - general recursion (nb)

Lecture 5 (07.11.): Chapter 5 in Lecture notes

Countable state spaces: What is different?

Markov chain limit theorem

Reversibility and detailed balance equations

Random Walk on non-negative integers

Existence/non-existence of invariant (and limit) distribution

Lecture 6 (09.11.): Chapter 6 in Lecture notes

Branching processes (Galton-Watson processes, X)

Definition using offspring distribution (Y \sim p), tree structure

Very useful tool: probability generating functions (p.g.f.) -- they determine distributions uniquely!

Key properties of p.g.f.

Computing p.g.f. of X using p.g.f. of Y (composition property)

Expected population size in terms of E[Y]

Finding extinction probability using properties of the p.g.f. of X (fixed point equation)

Lecture 7 (14.11.): Chapter 7 in Lecture notes

Random point patterns and their counting measure and counting process

Limit of Binomial distribution is the Poisson distribution (law of small numbers)

For any independently scattered and homogeneous random point pattern, its counting process is a Poisson Point Process

Waiting times of Poisson Point Processes are exponentially distributed

Lecture 8 (16.11.): Chapter 8 in Lecture notes

Superposed Poisson Process (sum of independent PP)

Compound Poisson Process (PP with random jump sizes)

Thinned Poisson Process

Lecture 9 (21.11.): Parts of Chapters 9-10 in Lecture notes

Markov property and MC in continuous time

(Note: We always assume time-homogeneity.)

Semigroup of transition matrices (P_t)_t: suffices to know P_s for some small time s>0

Idea: build continuous-time MC from underlying discrete-time MC and independent jump times give by Poisson process

Generator matrix of MC

Behavior in the long run, statistical equilibrium

Examples

Lecture 10 (23.11.): Perts of Chapters 9-10 in Lecture notes

Continuing to study continuous-time MC more precisely

What is the generator matrix (Q) exactly?

Jump probabilities and jump rates

Computing Q from jump probabilities and jump rates

Overclocking procedure

Example: busy taxis; similar to Example 10.4 in **Lecture notes**

Lecture 11 (28.11.): Parts of Chapters 11-12 in Lecture notes

Conditional expected value with respect to information --> random variable!

Useful properties of conditional expected value: unbiasedness, pulling out known factors, independence, ...

What is a martingale? Key property: best prediction for tomorrow is its current value.

When is random walk a martingale?

Martingales remain constant over time *in expectation*, but can fluctuate a lot

Optional times with respect to information (idea: can determine whether they happened using history up to present)

Lecture 12 (30.11.): Parts of Chapters 11-12 in Lecture notes

Optional stopping and properties of martingales

Gambling strategies

Integral process and cumulative wealth

Doob's Optional Stopping Theorem(s)

Long-term behavior of martingales

◄ Quizzes

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