Example 2.4: MC with many limit distributions (Section 2.1)

Transition matrix

In[56]:=
```
P := {{0.5, 0.5, 0, 0}, {0.5, 0.5, 0, 0}, {0, 0.1, 0.8, 0.1}, {0, 0, 0, 1}};
P // MatrixForm
```

Out[57]//MatrixForm=

$$\begin{pmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Start from one possible initial distribution: we are at state 1

In[58]:=
```
mu0 := {{1, 0, 0, 0}};
mu0 // MatrixForm
```

Out[59]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

One step

In[60]:=
```
mu1 := mu0 . P;
mu1 // MatrixForm
```

Out[61]//MatrixForm=

$$\begin{pmatrix} 0.5 & 0.5 & 0. & 0. \end{pmatrix}$$

Two steps

In[62]:=
```
mu2 := mu0 . P.P;
mu2 // MatrixForm
```

Out[63]//MatrixForm=

$$\begin{pmatrix} 0.5 & 0.5 & 0. & 0. \end{pmatrix}$$

The MC converges to limiting distribution very fast!

Hundred steps

In[64]:=
```
mu100 := mu0 . MatrixPower[P, 100];
mu100 // MatrixForm
```

Out[65]//MatrixForm=

$$\begin{pmatrix} 0.5 & 0.5 & 0. & 0. \end{pmatrix}$$

Start from another possible initial distribution: we are at state 4

In[66]:=
```
nu0 := {{0, 0, 0, 1}};
nu0 // MatrixForm
```

Out[67]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$$

One step

In[68]:=
```
nu1 := nu0 . P;
nu1 // MatrixForm
```

Out[69]//MatrixForm=

$$\begin{pmatrix} 0. & 0. & 0. & 1. \end{pmatrix}$$

Two steps

In[70]:=
```
nu2 := nu0 . P.P;
nu2 // MatrixForm
```

Out[71]//MatrixForm=

$$\begin{pmatrix} 0. & 0. & 0. & 1. \end{pmatrix}$$

Hundred steps

In[72]:=
```
nu100 :=  nu0 . MatrixPower[P, 100];
nu100 // MatrixForm
```

Out[73]//MatrixForm=

$$\begin{pmatrix} 0. & 0. & 0. & 1. \end{pmatrix}$$

Again, the MC converges to limiting distribution very fast!

But the two limiting distributions are different!

In[74]:=
```
mu100 :=  mu0 . MatrixPower[P, 100];
mu100 // MatrixForm
```

Out[75]//MatrixForm=

$$\begin{pmatrix} 0.5 & 0.5 & 0. & 0. \end{pmatrix}$$

In[76]:=
```
nu100 :=  nu0 . MatrixPower[P, 100];
nu100 // MatrixForm
```

Out[77]//MatrixForm=

$$\begin{pmatrix} 0. & 0. & 0. & 1. \end{pmatrix}$$

One more possibility: start from another possible initial distribution: we are at state 3

In[78]:=
```
rho0 := {{0, 0, 1, 0}};
rho0 // MatrixForm
```

Out[79]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$$

One step

In[80]:=
```
rho1 := rho0 . P;
rho1 // MatrixForm
```

Out[81]//MatrixForm=

$$\begin{pmatrix} 0. & 0.1 & 0.8 & 0.1 \end{pmatrix}$$

Two steps

In[82]:=
```
rho2 := rho0 . P.P;
rho2 // MatrixForm
```

Out[83]//MatrixForm=

$$\begin{pmatrix} 0.05 & 0.13 & 0.64 & 0.18 \end{pmatrix}$$

Hundred steps

In[84]:=
```
rho100 := rho0 . MatrixPower[P, 100];
rho100 // MatrixForm
```

Out[85]//MatrixForm=

$$\begin{pmatrix} 0.25 & 0.25 & 2.03704 \times 10^{-10} & 0.5 \end{pmatrix}$$

Thousand steps

In[86]:=
```
rho1000 := rho0 . MatrixPower[P, 1000];
rho1000 // MatrixForm
```

Out[87]//MatrixForm=

$$\begin{pmatrix} 0.25 & 0.25 & 1.23023 \times 10^{-97} & 0.5 \end{pmatrix}$$

It seems that we found yet another limiting distribution, but now the convergence is slower!

Million steps

In[88]:=
```
nu100000 := nu0 . MatrixPower[P, 100 000];
nu100000 // MatrixForm
```

Out[89]//MatrixForm=

$$\begin{pmatrix} 0. & 0. & 0. & 1. \end{pmatrix}$$