# Recursion equation for Random Walk expected passage time (Section 4.3)

Consider the expected passage time for the Random Walk on {0,1,...,M} to the absorbing states {0,M}. We can solve for it using Theorem 4.1. What is the behavior when M grows?

## Assume first that q is not 1/2. The Recursion Equation is inhomogeneous, and its solution has the form:

In[12]:=
```
RSolveValue[{q * f[x + 1] - f[x] + (1 - q) * f[x - 1] == -1, f[0] == 0, f[M] == 0}, f[x], x]
```

Out[12]=

$$-\frac{1}{\left(-1+\left(\frac{1-q}{q}\right)^M\right)(-1+2q)^2}\left(-1+\frac{1}{q}\right)^{-M-x}\Bigg(-M\left(-1+\frac{1}{q}\right)^{M+x}+M\left(-1+\frac{1}{q}\right)^{M+x}\left(\frac{1-q}{q}\right)^x+2M\left(-1+\frac{1}{q}\right)^{M+x}q-\left(-1+\frac{1}{q}\right)^x\left(\frac{1-q}{q}\right)^M q+\left(-1+\frac{1}{q}\right)^{M+x}\left(\frac{1-q}{q}\right)^M q+\left(-1+\frac{1}{q}\right)^M\left(\frac{1-q}{q}\right)^x q-\left(-1+\frac{1}{q}\right)^{M+x}\left(\frac{1-q}{q}\right)^x q-2M\left(-1+\frac{1}{q}\right)^{M+x}\left(\frac{1-q}{q}\right)^x q-\left(-1+\frac{1}{q}\right)^M\left(\frac{1-q}{q}\right)^{M+x}q+\left(-1+\frac{1}{q}\right)^x\left(\frac{1-q}{q}\right)^{M+x}q+\left(-1+\frac{1}{q}\right)^{M+x}x-\left(-1+\frac{1}{q}\right)^{M+x}\left(\frac{1-q}{q}\right)^M x-2\left(-1+\frac{1}{q}\right)^{M+x}q\,x+2\left(-1+\frac{1}{q}\right)^{M+x}\left(\frac{1-q}{q}\right)^M q\,x\Bigg)$$

In[13]:=
```
FullSimplify[%]
```

Out[13]=

$$\frac{M\left(-1+\left(-1+\frac{1}{q}\right)^x\right)+x-\left(-1+\frac{1}{q}\right)^M x}{\left(-1+\left(-1+\frac{1}{q}\right)^M\right)(-1+2q)}$$

## Let' s plot with the value x = 1:

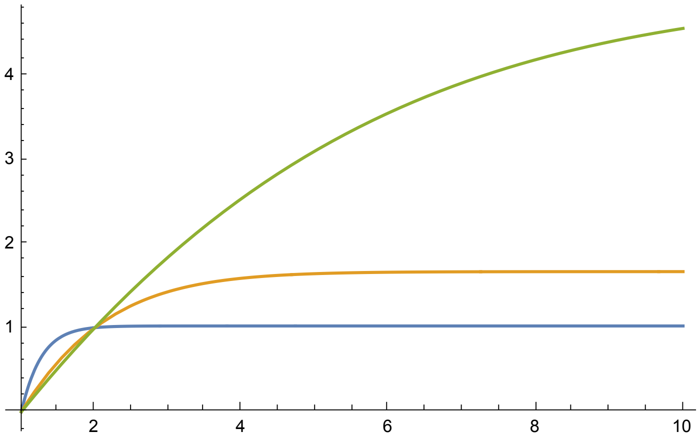
In[14]:=

$$\text{F[M\_, q\_] :=}\left(\frac{M\left(-1+\left(-1+\frac{1}{q}\right)^x\right)+x-\left(-1+\frac{1}{q}\right)^M x}{\left(-1+\left(-1+\frac{1}{q}\right)^M\right)(-1+2q)}\right)\text{ /. \{x → 1\};}$$

Plotting for some q' s less than 1/2:

In[15]:=
```
Plot[{F[M, .01], F[M, .2], F[M, .4]}, {M, 1, 10}]
```

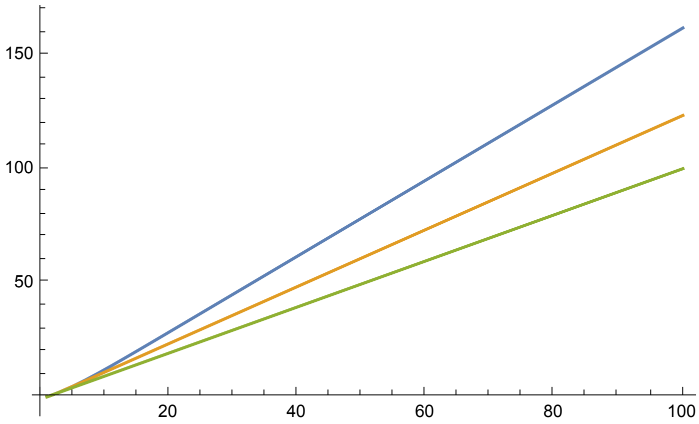
Out[15]=




Plotting for some q' s greater than 1/2:

In[16]:=
```
Plot[{F[M, .6], F[M, .8], F[M, .99]}, {M, 1, 100}]
```

Out[16]=




Example limits:

In[17]:=
```
Limit[F[M, .1], M → Infinity]
Limit[F[M, .45], M → Infinity]
Limit[F[M, .55], M → Infinity]
Limit[F[M, .9], M → Infinity]
```

Out[17]= 1.25

Out[18]= 10.

Out[19]= ∞

Out[20]= ∞

We see that the limit (as M grows) exists if q < 1/2 and is infinite if q > 1/2.

Assume then that q = 1/2. The Recursion Equation is inhomogeneous, and its solution has the form:

In[21]:=
```
RSolveValue[
 {(1 / 2) * f[x + 1] - f[x] + (1 - (1 / 2)) * f[x - 1] == -1, f[0] == 0, f[M] == 0}, f[x], x]
```

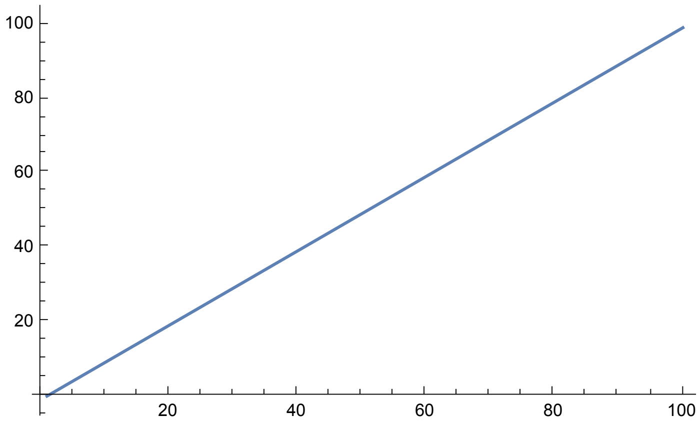
Out[21]= $M x - x^2$

Let' s plot with the value x = 1:

In[22]:=
```
G[M_] := (M x - x^2) /. {x → 1};
```

In[23]:=
```
Plot[G[M], {M, 1, 100}]
```

Out[23]=




We see that the limit (as M grows) is infinite also if q = 1/2.