
Example: Reducible MC (Section 2.1)

Transition matrix

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In[106]:= P := {{1, 0, 0, 0, 0}, {1/3, 0, 1/3, 1/3, 0},  
               {0, 2/3, 0, 0, 1/3}, {0, 0, 0, 2/3, 1/3}, {0, 0, 0, 2/3, 1/3}};  
P // MatrixForm
```

Out[107]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Powers of transition matrix P

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In[108]:= MatrixPower[P, 2] // MatrixForm  
MatrixPower[P, 3] // MatrixForm  
MatrixPower[P, 5] // MatrixForm  
MatrixPower[P, 10] // MatrixForm  
MatrixPower[P, 20] // MatrixForm  
MatrixPower[P, 50] // MatrixForm
```

Out[108]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{9} & 0 & \frac{2}{9} & \frac{2}{9} \\ \frac{2}{9} & 0 & \frac{2}{9} & \frac{4}{9} & \frac{1}{9} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Out[109]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{11}{27} & 0 & \frac{2}{27} & \frac{10}{27} & \frac{4}{27} \\ \frac{2}{9} & \frac{4}{27} & 0 & \frac{10}{27} & \frac{7}{27} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Out[110]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{103}{243} & 0 & \frac{4}{243} & \frac{92}{243} & \frac{44}{243} \\ \frac{22}{81} & \frac{8}{243} & 0 & \frac{110}{243} & \frac{59}{243} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Out[111]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{8431}{19683} & \frac{32}{59049} & 0 & \frac{22472}{59049} & \frac{11252}{59049} \\ \frac{16862}{59049} & 0 & \frac{32}{59049} & \frac{28114}{59049} & \frac{14041}{59049} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Out[112]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{498111911}{1162261467} & \frac{1024}{3486784401} & 0 & \frac{1328298088}{3486784401} & \frac{664149556}{3486784401} \\ \frac{996223822}{3486784401} & 0 & \frac{1024}{3486784401} & \frac{1660373378}{3486784401} & \frac{830186177}{3486784401} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Out[113]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{102556855384550365030831}{239299329230617529590083} & \frac{33554432}{717897987691852588770249} & 0 & \frac{2734849476921342955640}{717897987691852588770249} \\ \frac{205113710769100730061662}{717897987691852588770249} & 0 & \frac{33554432}{717897987691852588770249} & \frac{3418561846151678946209}{717897987691852588770249} \\ 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} \end{pmatrix}$$

In[114]:= N[MatrixPower[P, 50] // MatrixForm]

Out[114]//MatrixForm=

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. \\ 0.428571 & 4.67398 \times 10^{-17} & 0. & 0.380952 & 0.190476 \\ 0.285714 & 0. & 4.67398 \times 10^{-17} & 0.47619 & 0.238095 \\ 0. & 0. & 0. & 0.666667 & 0.333333 \\ 0. & 0. & 0. & 0.666667 & 0.333333 \end{pmatrix}$$

Can you split the transition matrix into components that correspond to irreducible MCs?