Example 2.2: Brand loyalty has unique limit distribution (Section 2.1)

Transition matrix

In[1]:=
```
P := {{0.8, 0.1, 0.1}, {0.2, 0.6, 0.2}, {0.3, 0.3, 0.4}};
P // MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}$$

Powers of transition matrix P

In[3]:=
```
MatrixPower[P, 2] // MatrixForm
MatrixPower[P, 3] // MatrixForm
MatrixPower[P, 5] // MatrixForm
MatrixPower[P, 10] // MatrixForm
MatrixPower[P, 20] // MatrixForm
MatrixPower[P, 50] // MatrixForm
```

Out[3]//MatrixForm=

$$\begin{pmatrix} 0.69 & 0.17 & 0.14 \\ 0.34 & 0.44 & 0.22 \\ 0.42 & 0.33 & 0.25 \end{pmatrix}$$

Out[4]//MatrixForm=

$$\begin{pmatrix} 0.628 & 0.213 & 0.159 \\ 0.426 & 0.364 & 0.21 \\ 0.477 & 0.315 & 0.208 \end{pmatrix}$$

Out[5]//MatrixForm=

$$\begin{pmatrix} 0.57252 & 0.25295 & 0.17453 \\ 0.5059 & 0.30188 & 0.19222 \\ 0.52359 & 0.28833 & 0.18808 \end{pmatrix}$$

Out[6]//MatrixForm=

$$\begin{pmatrix} 0.547129 & 0.271502 & 0.18137 \\ 0.543003 & 0.274522 & 0.182475 \\ 0.544109 & 0.273712 & 0.182179 \end{pmatrix}$$

Out[7]//MatrixForm=

$$\begin{pmatrix} 0.545461 & 0.272723 & 0.181816 \\ 0.545445 & 0.272734 & 0.181821 \\ 0.545449 & 0.272731 & 0.18182 \end{pmatrix}$$

Out[8]//MatrixForm=

$$\begin{pmatrix} 0.545455 & 0.272727 & 0.181818 \\ 0.545455 & 0.272727 & 0.181818 \\ 0.545455 & 0.272727 & 0.181818 \end{pmatrix}$$

It seems that the entries stabilize when time grows!

Balance equations

In[9]:=
```
pii := {{p1, p2, p3}};
pii // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} p1 & p2 & p3 \end{pmatrix}$$

In[11]:=
```
pii.P
```

Out[11]=
```
{{0.8 p1 + 0.2 p2 + 0.3 p3, 0.1 p1 + 0.6 p2 + 0.3 p3, 0.1 p1 + 0.2 p2 + 0.4 p3}}
```

Eigenvector equations (pii is a left eigenvector of P with eigenvalue 1) :

In[12]:=
```
0.8` p1 + 0.2` p2 + 0.3` p3 == p1;
0.1` p1 + 0.6` p2 + 0.3` p3 == p2;
0.1` p1 + 0.2` p2 + 0.4` p3 == p3;
```

Normalization (sum of elements of the vector pii is 1, since they are probabilities (law of total probability)) :

In[15]:=
```
p1 + p2 + p3 == 1;
```

Solving the equations :

In[16]:=
```
Solve[{0.8` p1 + 0.2` p2 + 0.3` p3 == p1, 0.1` p1 + 0.6` p2 + 0.3` p3 == p2,
  0.1` p1 + 0.2` p2 + 0.4` p3 == p3, p1 + p2 + p3 == 1}, {p1, p2, p3}]
```

Out[16]=
```
{{p1 → 0.545455, p2 → 0.272727, p3 → 0.181818}}
```

This is a row in the limiting transition matrix!

In[17]:=
```
MatrixPower[P, 50] // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0.545455 & 0.272727 & 0.181818 \\ 0.545455 & 0.272727 & 0.181818 \\ 0.545455 & 0.272727 & 0.181818 \end{pmatrix}$$

(All rows are the same because the limiting distribution does not depend on the initial state.)