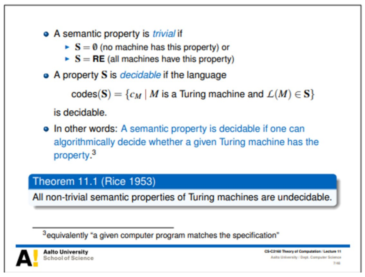
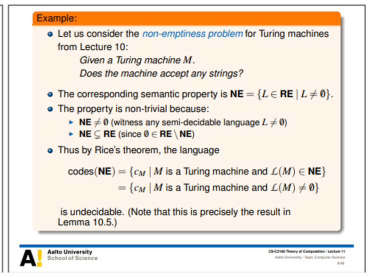
Wednesday, 30 March 2022 10.21

Apply Rice's theorem for all parts





(i) Denote the class of all regular languages by REG.

$RL = \{L \in RE \mid L \in REG\}$

a) Non-triviality:

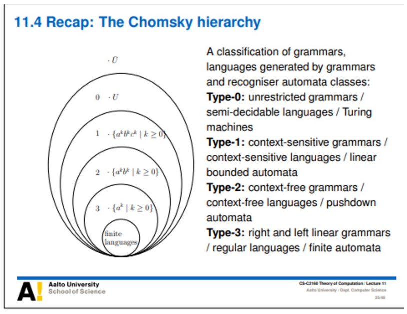
By CH, there exists a language $L \notin REG$

$\Rightarrow RL \neq RE$

b) On the other hand, there exists at least one regular language $L \in REG$

$\Rightarrow RL \neq \emptyset$

$RL \neq \emptyset \wedge RL \neq RE \Rightarrow RL$ is non-trivial



C11.1 Prove that the following problems are undecidable:

(i) Given a Turing machine M, does M recognise a regular language?

(ii) Given a Turing machine M, does M accept exactly one input string?

(iii) Given Turing machines M_1 and M_2, is $\mathcal{L}(M_1) \cap \mathcal{L}(M_2) = \emptyset$?

(ii) $OS = \{L \in RE \mid |L| = 1\}$

a) For example, construct TM over $\Sigma = \{a, b\}$ so that it only accepts a

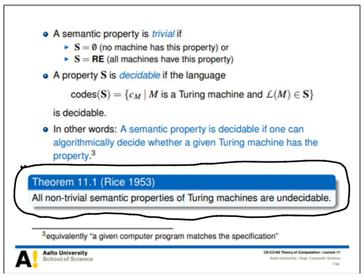
$\Rightarrow \exists TM$, so that $|L(TM)| = 1$

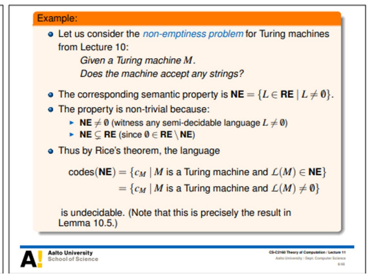
$\Rightarrow OS \neq \emptyset$

b) Construct a TM s.t. it accepts the strings a and b

$\Rightarrow \exists TM$, so that $|L(M)| = 2 \neq 1$

$\Rightarrow OS \neq RE$





(iii) $ES = \{L \in RE \mid L = L(M_1) \cap L(M_2)\}$

a) For example, $L(M_1) = \{a\}$, $L(M_2) = \{b\}$

b) $L(M_1) = \{a, b\}$, $L(M_2) = \{a\}$

Non-trivial semantic property