Instructions

- Classroom Problems C7.1–C7.3 will be discussed and solved onsite at the tutorial sessions in lecture week 7. No credit is given for these problems.
- Homework Problems H7.1–H7.3 you should solve on your own, and be available to present your solutions at one of the tutorial sessions in lecture week 8. In order to get course credit, you need to indicate your solved problems on the signup sheet circulated at the beginning of the session.
- Supplementary Problems S7.1–S7.3 provide further illustration and extension of the course material, but will usually not be covered at the tutorials. You are however invited to work on these problems too, and discuss them with the course staff. Sample solutions are provided on MyCourses.

Classroom Problems

C7.1 Convert the following grammar into Chomsky normal form:

$$\begin{aligned} S &\to AB \mid BA \mid \varepsilon \\ A &\to aS \\ B &\to bS \end{aligned}$$

C7.2 Determine, by using the CYK algorithm, whether strings aab and baa are generated by the grammar

$$\begin{aligned} S &\to AB \mid BA \\ A &\to BA \mid a \\ B &\to AB \mid b \end{aligned}$$

In the positive cases, give also the respective parse trees.

C7.3 Design a pushdown automaton that recognises the language

$$\mathrm{PAL} = \{ww^R \mid w \in \{a, b\}^*\}.$$

Homework Problems

H7.1 Convert the following grammar into Chomsky normal form:

$$\begin{aligned} S &\to AB \mid c \\ A &\to T \mid aA \\ B &\to TT \mid \varepsilon \\ T &\to bS \end{aligned}$$

H7.2 Determine, by using the CYK algorithm, whether strings *aba*, *abba* and *bbaa* are generated by the grammar

$$\begin{aligned} S &\to AB \mid BA \\ A &\to BA \mid a \\ B &\to AB \mid b \end{aligned}$$

In the positive cases, give also the respective parse trees.

H7.3 Design pushdown automata recognising the following languages.

(a) The language:

$$\{a^i b^j c^k \mid i = j \text{ or } j = k \text{ (or both)}\}.$$

(*Hint:* Have a look at the example automaton in Section 7.4 of the lecture slides.)

(b) The language generated by the grammar

$$S \to (S) \mid S, S \mid a$$

Supplementary Problems

S7.1 Show, using the pumping lemma for context-free languages, that the language

$$\{ww \mid w \in \{a, b\}^*\}$$

is not context-free. (*Hint:* Consider strings of the form $a^n b^n a^n b^n$.)

S7.2 Prove that the class of context-free languages is not closed under intersections and complements. (*Hint:* Represent the language $\{a^k b^k c^k \mid k \geq 0\}$ as the intersection of two context-free languages.)

S7.3 Design a pushdown automaton corresponding to the grammar

$$S \to (SS) \mid S^* \mid (S \cup S) \mid \emptyset \mid a \mid b$$

where the set of terminal symbols is $\Sigma = \{(,), \cup, *, \emptyset, a, b\}$.