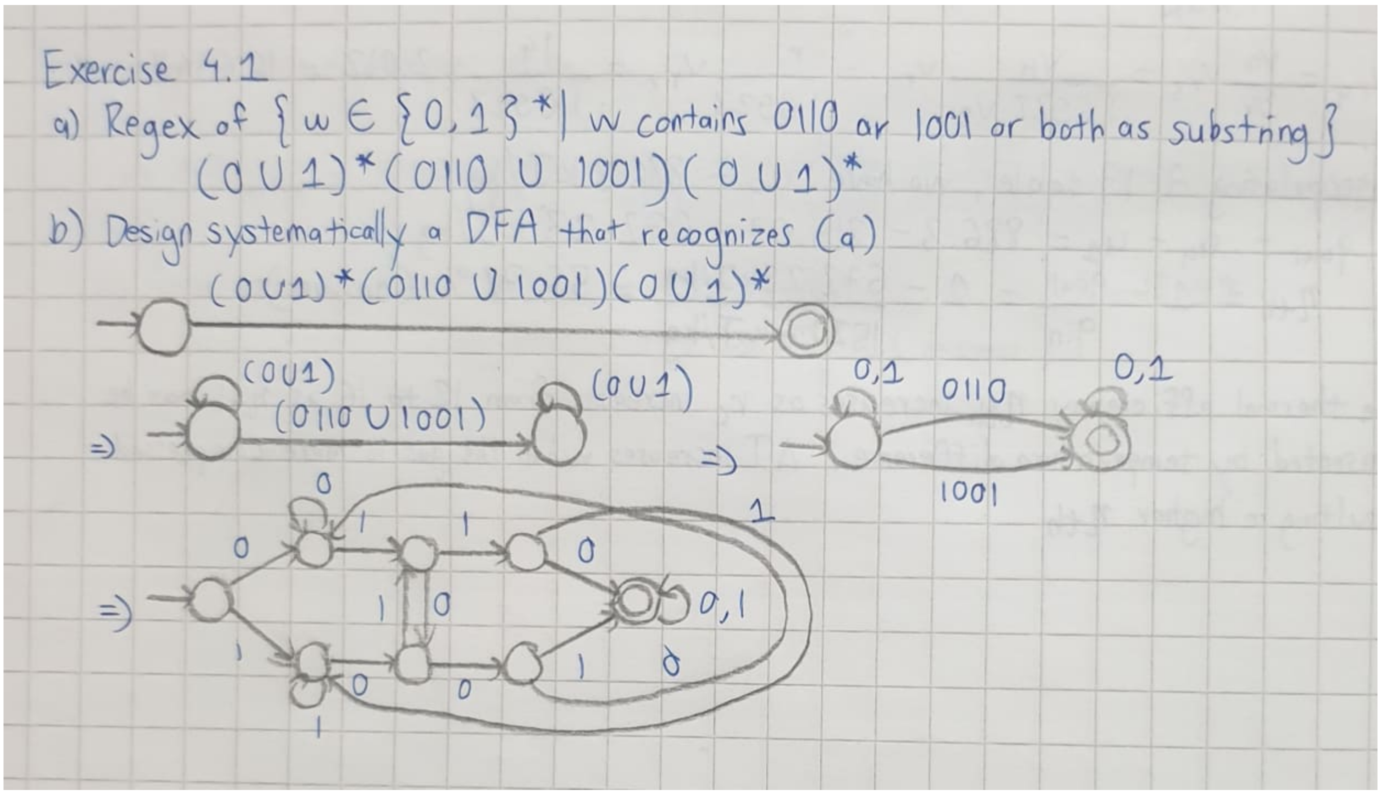
H4.1

(a) Give a regular expression that describes the language

$$\{w \in \{0,1\}^* \mid w \text{ contains 0110 or 1001 as a substring (possibly both)}\}.$$

(b) Following the guidelines presented in the lectures, design in a systematic way a deterministic finite automaton that recognises the language in part (a).



H4.2 Consider the following languages:

(a) $\{w \in \{a,b\}^* \mid w \text{ does not contain } aba \text{ as a substring}\}$;

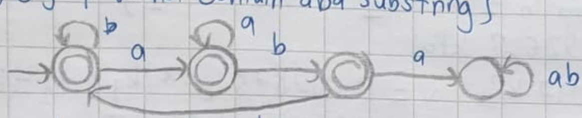
(b) $\{w \in \{0,1\}^* \mid w \text{ contains an even number of both 0's and 1's}\}$.

In both cases, design a regular expression describing the language, by first constructing a finite automaton recognising it, and then converting the automaton in a systematic manner into the corresponding expression.

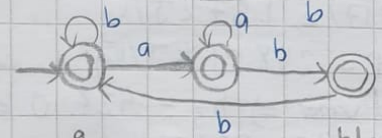
Exercise 4.2

a) $\{ w \in \{a, b\}^* \mid w$ not contain aba substring $\}$

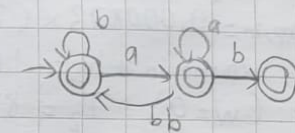
The DFA:



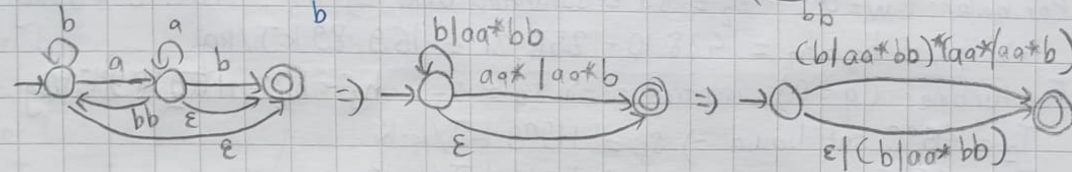
=) NFA:



=) GNFA:



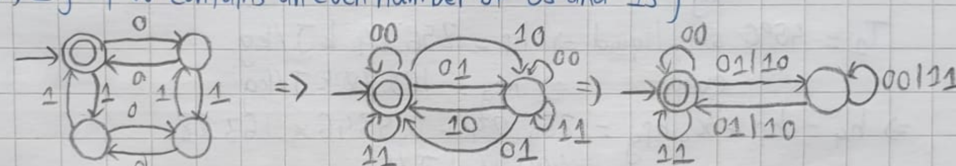
=)



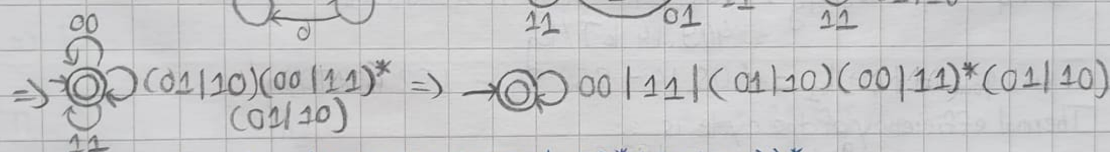
=) Regex: ε|(b|aa*bb)|(b|aa*bb)*(aa*|aa*b)

b) $\{ w \in \{0, 1\}^* \mid w$ contains an even number of 0s and 1s $\}$

The DFA



=)



=) Regex: (00|11|(01|10)(00|11)*(01|10))*

H4.3 Design (in outline) algorithms for determining whether the language described by a regular expression r over the alphabet $\{0, 1\}$ is (a) empty, i.e. $L(r) = \emptyset$, (b) contains all possible binary strings, i.e. $L(r) = \{0, 1\}^*$.

Exercise 4.3

Theorem: for all regex r, there always exist a DFA for the regular language

a) The language of a DFA is empty if and only if there are no final states reachable from the starting state.

Algorithm: first convert regex "r" into DFA. Then do depth-first search traversing from starting node. If there are no final states encountered after search =) $L(r) = \emptyset$

Example: r = ε →◯ ◎ ← non reachable state =) $L(r) = \emptyset$

b) The language of a DFA contains all possible binary strings if the DFA has only one state, which is both begin and final state and has a loop into its own over the alphabet =) $L(r) = \{0, 1\}^*$

Example: r = (0|1)* =) →◎ 0|1 =) $L(r) = \{0, 1\}^*$