Homework Problems

H5.1 Design context-free grammars for the following languages:

(i) $\{a^m b^n \mid m > n\}$.

(ii) $\{a^m b^n \mid m \neq n\}$. Observe that $m \neq n$ if and only if $m < n$ or $m > n$.

(iii) $\{ucv \mid u, v \in \{a, b\}^* \text{ and } |u| = |v|\}$.

Additionally, give a derivation for the string *aaab* using your first grammar, a derivation for *abb* using your second grammar, and a derivation for *abcbb* using your third grammar.

Exercise H5.1

(i) $\{a^m b^n \mid m > n\}$

Grammar: $S \to aSb \mid aS \mid a$

For the string aaab: $S \to aSb \to a(aS)b \to a(a(a))b \to aaab$

(ii) $\{a^m b^n \mid m \neq n\}$

Grammar: $S \to AT \mid TB$

$A \to Aa \mid a$

$B \to Bb \mid b$

$T \to aTb \mid \varepsilon$

For the string abb

$S \to TB \to (aTb)(b) \to (a(\varepsilon)b)(b)$

$\to abb$

(iii) $\{ucv \mid u, v \in \{a, b\}^* \text{ and } |u| = |v|\}$

Grammar: $S \to aSa \mid bSb \mid aSb \mid bSa \mid c$

For the string abcbb: $S \to aSb \to a(bSb)b \to a(b(c)b)b$

$\to abcbb$

H5.2 Design a context-free grammar that generates the language

$$L = \{a^i b^j c^k \mid i, j, k \geq 0,\ i + j = k\}.$$

Prove also, by using the pumping lemma, that the language L is not regular.

Exercise H5.2

- $L = \{a^i b^J c^k \mid i, J, k \geqslant 0,\ i + J = k\}$

The context free grammar is: $S_0 \rightarrow a S_0 c \mid S_1$

$S_1 \rightarrow b S_1 c \mid \varepsilon$

Use pumping lemma to prove that H5.2 is not regular

- 1st, we assume that L is a regular language

Let $w = a^i b^J c^k \Rightarrow |w| = i + J + k = 2k \geqslant k = n$

By pumping lemma, let $w = xyz$ where $|xy| \leqslant n = k$

Let $x = a^i$, $y = b^J$ and $z = c^k$, where $i + J = k \leqslant n$ and $J \neq 0$

$\Rightarrow |y| = |b^J| \neq 0$

Let multiple of y be 2. Then $xy^2z = a^i (b^J)^2 c^k = a^i b^{2J} c^k$

L is only satisfy if $i + J = k$. Since $i + 2J \neq k$ as $J \neq 0$ required above

$\Rightarrow xy^2z$ is not in L. Hence L is not regular

H5.3 Design right-linear context-free grammars for the following languages:

(i) $\{w \in \{a,b\}^* \mid w$ does not contain aba as a substring$\}$;

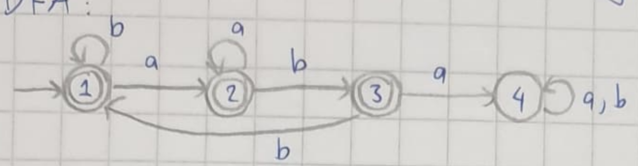
(ii) $\{w \in \{0,1\}^* \mid w$ contains an even number of 0's and an odd number of 1's$\}$.

Use the systematic construction presented at Lecture 5. That is, first design a finite automaton for the language in question and then translate the automaton into the corresponding right-linear context-free grammar. In addition to the final solutions, also show the intermediate phases, e.g., the automata that you designed.

Exercise 5.3

(i) $\{w \in \{a,b\}^* \mid w$ does not contain substring $aba\}$

The DFA:



The corresponding context-free grammar (right linear) is

$A_1 \to bA_1 \mid aA_2 \mid \varepsilon$

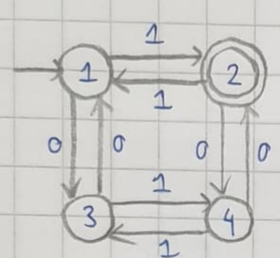
$A_2 \to aA_2 \mid bA_3 \mid \varepsilon$

$A_3 \to bA_1 \mid \varepsilon$

(There is no A_4 because it is a dead state, once entered it will not be accepted. CFG discards all failing state)

(ii) $\{w \in \{0,1\}^* \mid w$ contains even number of 0's and an odd number of 1's$\}$

The DFA:



A_1: even 0, even 1

A_2: even 0, odd 1

A_3: odd 0, even 1

A_4: odd 0, odd 1

and A_2 is accepting state

The corresponding right linear context-free grammar is

$A_1 \to 1A_2 \mid 0A_3$

$A_2 \to 1A_1 \mid 0A_4 \mid \varepsilon$

$A_3 \to 0A_1 \mid 1A_4$

$A_4 \to 0A_2 \mid 1A_3$