Homework Problems

H6.1 (a) Design a context-free grammar for the language

$$L = \{a^i b^j c^k \mid i = j \text{ or } j = k\}.$$

(b) Show that the grammar you gave in part (a) is ambiguous.

Exercise 1:

a) Design CFG for ~~grammar~~ language: $L = \{a^i b^j c^k \mid i=j \text{ or } j=k\}$

$S_0 \to S_1 \mid S_4$

$i=j$: $S_1 \to aS_2bS_3 \mid \varepsilon$; $S_2 \to aS_2b \mid \varepsilon$; $S_3 \to cS_3 \mid \varepsilon$

$j=k$: $S_4 \to S_5bS_6c \mid \varepsilon$; $S_5 \to aS_5 \mid \varepsilon$; $S_6 \to bS_6c \mid \varepsilon$

b) Show that the CFG is ambiguous

To prove that the CFG is ambiguous, if some word $x \in L(G)$ has 2 different parse trees

Let x = aabbcc and x has more than 1 parse trees

$S_0 \to S_1 \to aS_2bS_3 \to aaS_2bbS_3 \to aa(\varepsilon)bbcS_3 \to aabbccS_3 \to aabbcc$

$S_0 \to S_4 \to S_5bS_6c \to S_5bbS_6cc \to aS_5bbS_6cc \to aaS_5bbS_6cc \to aabbcc$

H6.2 (a) Design a context-free grammar for the language

$$L = \{w \in \{a, b\}^* \mid w \text{ contains equally many } a\text{'s and } b\text{'s}\}.$$

Draw the corresponding parse trees for sentences *aabb*, *abab* and *baab*.

(b) Is the grammar you designed in part (a) ambiguous or unambiguous? If it is ambiguous, then try to design also an unambiguous grammar for the language.

(c) Prove (precisely!) that the language in part (a) is not regular.

Exercise 2:

a) Design CFG for language: $L = \{w \in \{a,b\}^* \mid |a| = |b|\}$

The CFG is: $S \to SS \mid aSb \mid bSa \mid \varepsilon$

Parse tree for: aabb: $S \to aSb \to aaSbb \to aabb$

abab: $S \to aSb \to abSab \to abab$

baab: $S \to SS \to bSaaSb \to baab$

b) The grammar designed in part (a) is ambiguous

For example string "ab": $S \to SS \to (aSb)\varepsilon \to ab$

$S \to SS \to \varepsilon(aSb) \to ab$

There are two different parse tree for ab $\Rightarrow$ ambiguous

The unambiguous version would be:

$S \to aX \mid bY \mid \varepsilon$

$X \to bS \mid aXX$

$Y \to aS \mid bYY$

c) Prove (precisely!) that language in part (a) is not regular

1st, we assume that L is a regular language

Let $w = a^i b^j \Rightarrow |w| = i + j$, also $i = j \Rightarrow |w| = 2i \geq i = n$

By pumping lemma, let $w = xyz$ where $|xy| \leq n = i$

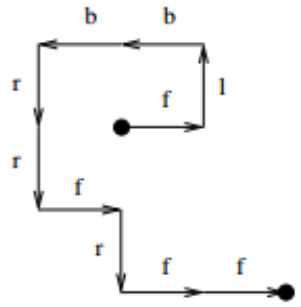
Let $x = a^i$, $y = b^j$, $j \neq 0$, $z = \varepsilon$

Let multiple of y be 2. Then $x y^2 z = a^i (b^j)^2 = a^i b^{2j}$

Since $j \neq 0$ and $i = j \Rightarrow i \neq 2j \Rightarrow xy^2z$ is not in L

$\Rightarrow$ L is not a regular language

H6.3 A *party walk* is a sequence of consequent steps, whose direction with respect to the starting point is either forward (abbr. f), backward (b), left (l) or right (r). For instance, the sequence *flbbrrfrff* describes the following walk, whose total result is to move the walker a distance of two steps forward (and concurrently two steps to the right):



Design a context-free grammar that generates all party walks whose total result is to move the walker at least one step forward from the starting point (ignoring any possible sideways movement).

Exercise 3:

By the problem formulation, it means that the step sequence can contain arbitrary number of left and right, while number of forwards > backward

⇒ The language: $L = \{ w \in \{l, r, f, b\}^* \mid |f| > |b| \}$

The CFG would be:

$S \to TfT$

$T \to fTbT \mid bTfT \mid lT \mid Tl \mid rT \mid Tr \mid fT \mid Tf \mid \varepsilon$