Homework Problems

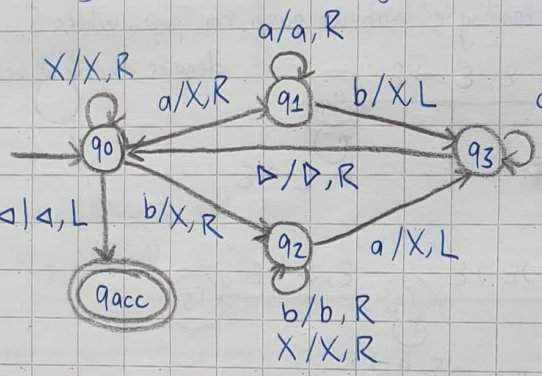
H8.1 Design a deterministic single-tape Turing machine that decides (i.e. recognises and halts on every input) the language

$$\{w \in \{a, b\}^* \mid w \text{ contains equally many } a\text{'s and } b\text{'s}\}.$$

Show the accepting computation sequence ("run") of your machine on input *aabb* and the rejecting sequence on input *baa*.

We have to scan the input from left to right.
Convert first 'a' and first 'b' in the scanning to 'X', then in the second turn convert second 'a' and second 'b' to 'X' and so on. We have to repeat the process until we convert all a's and b's to 'X'. Characters scanned in between 'a' and 'b' will not be changed.
=> a and b are converted to X in pairs. The input is rejected when it reads end of input symbol at state q1 or q2

Exercise 1:
Design deterministic single-tape Turing Machine that recognizes the language
$\{ w \in \{a, b\}^* \mid w$ contains equal a and b $\}$
The Turing Machine for this language would be:



▷: start of tape
◁: end of input
q_3: rewinding back to start of tape at q_0

- Accepting computation sequence on input aabb (start rewinding)
  $(q_0, \underline{a}abb) \vdash (q_1, X\underline{a}bb) \vdash (q_1, Xa\underline{b}b) \vdash (q_3, X\underline{a}Xb)$
  $\vdash (q_3, \underline{X}aXb) \vdash (q_3, \underline{\triangleright}XaXb) \vdash (q_0, \underline{X}aXb)$ (end rewinding)
  $\vdash (q_0, X\underline{a}Xb) \vdash (q_1, XX\underline{X}b) \vdash (q_1, XXX\underline{b}) \vdash (q_3, XX\underline{X}X)$
  $\vdash$ rewinding $\vdash (q_0, \underline{X}XXX)$ $\vdash$ just move left for scanning X
  $\vdash (q_0, XXXX\underline{\triangleleft}) \vdash q_{acc}$
- Rejecting computation sequence on input bba
  $(q_0, \underline{b}aa) \vdash (q_2, X\underline{a}a) \vdash (q_3, \underline{X}Xa) \vdash (q_3, \triangleright XXa)$
  $\vdash (q_0, \underline{X}Xa) \vdash (q_0, X\underline{X}a) \vdash (q_0, XX\underline{a}) \vdash (q_1, XXX\underline{\triangleleft})$
  $\vdash q_{rej}$

H8.2 Design a three-tape Turing machine ADD that functions as follows. The machine gets as input on the tapes 1 and 2 two binary numbers written in reverse, i.e. with their least significant bits first. It then computes on the tape 3 the sum of the two given numbers in the same notation. For simplicity, you may assume that the input numbers are of the same length, i.e. that the possibly shorter one is padded with leading zeros. Thus, for instance, the calculation $7 + 11 = 18$ is represented as:

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1110
1101
01001
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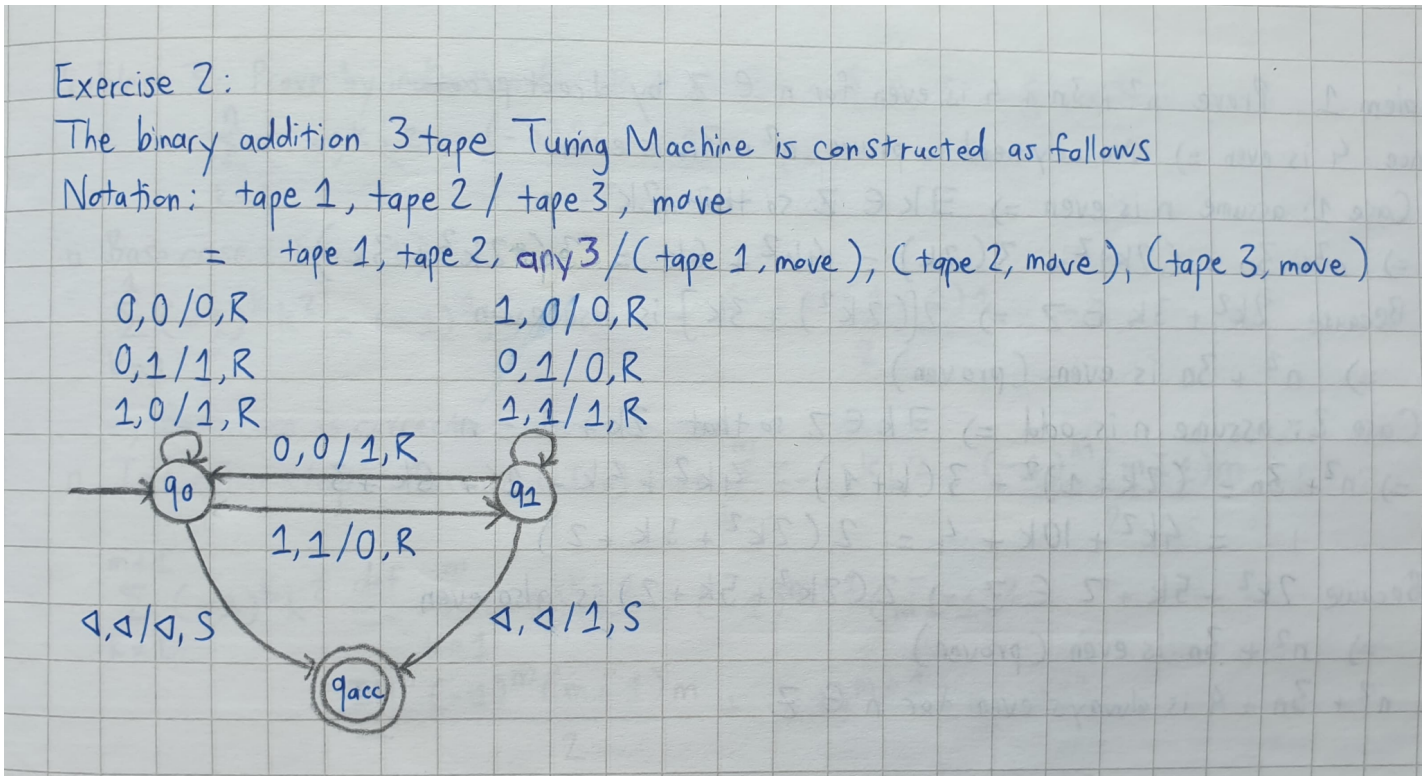
You may assume that a tape head of the machine can also stay stationary in a transition (the move direction "S").



H8.3 Design a two-tape nondeterministic Turing machine that recognises the language $\{ww \mid w \in \{a,b\}^*\}$. You may assume that a tape head of the machine can also stay stationary in a transition (the move direction "S").

Exercise 3:

First we need to find the middle position of the word if we use deterministic Turing machine. However, non-deterministic machine is allowed, so we just need to scan the input one character more each time and store it in tape 2, then duplicate it to see if it matches the input

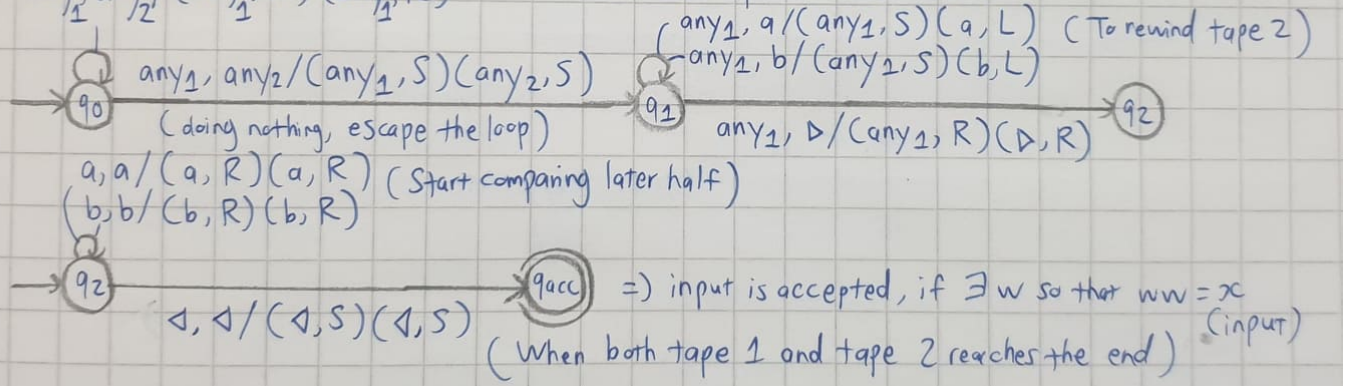
For ex: abbabb: $w = a \Rightarrow ww = aa \neq$ input

$w = ab \Rightarrow ww = abab \neq$ input

$w = abb \Rightarrow ww = abbabb =$ input $\Rightarrow$ abbabb is accepted

The nondeterministic Turing machine for this model would be:

$any_1, any_2 / (any_1, R)(any_1, R)$ ← Non-deterministic part that chooses w



q_0 q_1 q_2 q_{acc}

$\Rightarrow$ input is accepted, if $\exists w$ so that $ww = x$ (input)

Deterministic version:
https://www.geeksforgeeks.org/turing-machine-in-toc/