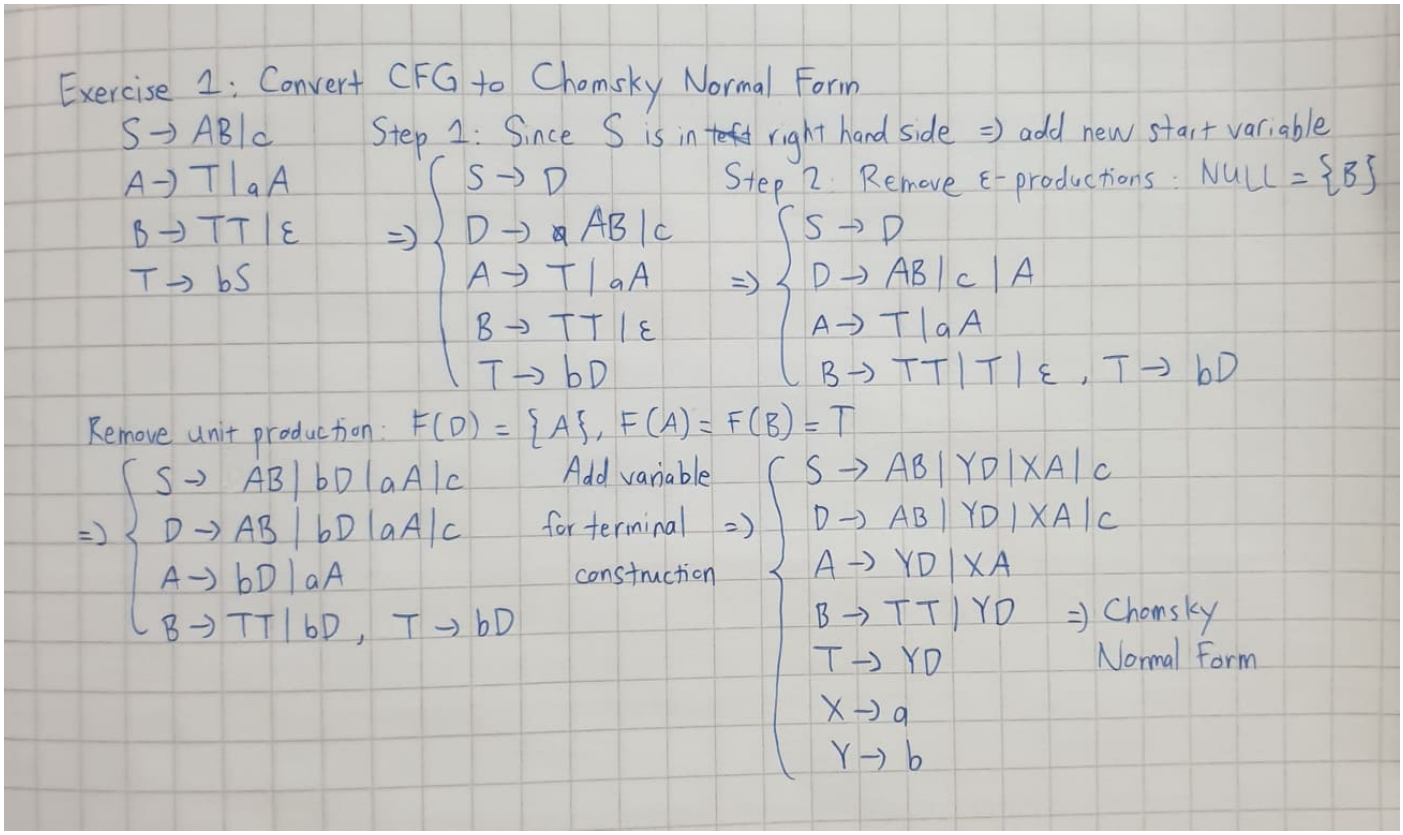
Homework Problems

H7.1 Convert the following grammar into Chomsky normal form:

$$
\begin{aligned}
S &\rightarrow AB \mid c \\
A &\rightarrow T \mid aA \\
B &\rightarrow TT \mid \varepsilon \\
T &\rightarrow bS
\end{aligned}
$$



H7.2 Determine, by using the CYK algorithm, whether strings *aba*, *abba* and *bbaa* are generated by the grammar

$$\begin{aligned} S &\rightarrow AB \mid BA \\ A &\rightarrow BA \mid a \\ B &\rightarrow AB \mid b \end{aligned}$$

In the positive cases, give also the respective parse trees.

Exercise 2:

- For string aba, the CYK table is

$N_{i,k}$	1: a	2: b	3: a	→ i
k↓ 1	A	B	A	
2	S, B	S, A		
3	S, A			

$S \rightarrow AB \mid BA$
$A \rightarrow BA \mid a$
$B \rightarrow AB \mid b$

$N_{1,2} = N_{1,1} \times N_{2,1} = \{AB\}$
$N_{2,2} = N_{2,1} \times N_{3,1} = \{BA\}$
$N_{1,3} = (N_{1,1} \times N_{2,2}) \cup (N_{1,2} \times N_{3,1})$
$= \{AS, AA, SA, BA\}$

Start variable is in set $N_{1,3}$ ⇒ aba belongs to the language
$S \in N_{1,3}$ as $S \rightarrow BA$ and $B \in N_{1,2}$, $A \in N_{3,1}$
⇒ $S \rightarrow (AB)A \rightarrow aba$

- For the string abba, the CYK table is

$N_{i,k}$	1: a	2: b	3: b	4: a
k↓ 1	A	B	B	A
2	S, B	∅	S, A	
3	∅	S, A		
4	S, A			

$N_{1,2} = N_{1,1} \times N_{2,1} = \{AB\}$
$N_{2,2} = N_{2,1} \times N_{3,1} = \{BB\}$
$N_{3,2} = N_{3,1} \times N_{4,1} = \{BA\}$
$N_{1,3} = \{SB, BB\}$
$N_{2,3} = \{BS, BA\}$
$N_{1,4} = \{AS, AA, SS, SA, BS, BA\}$

Start variable is in set $N_{1,4}$ ⇒ abba belongs to the language
$S \rightarrow BA \rightarrow (AB)(BA) \rightarrow abba$

- For the string bbaa, the CYK table is

$N_{i,k}$	1: b	2: b	3: a	4: a
k↓ 1	B	B	A	A
2	∅	S, A	∅	
3	S, A	∅		
4	∅			

Start variable is not in $N_{1,4}$
⇒ bbaa doesn't belong to the language

H7.3 Design pushdown automata recognising the following languages.

(a) The language:

$$\{a^i b^j c^k \mid i = j \text{ or } j = k \text{ (or both)}\}.$$

(*Hint:* Have a look at the example automaton in Section 7.4 of the lecture slides.)

(b) The language generated by the grammar

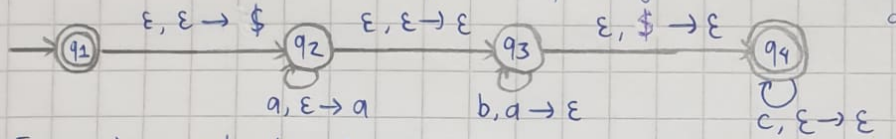
$$S \to (S) \mid S, S \mid a$$

Exercise 3 : Design PDA recognizing the language

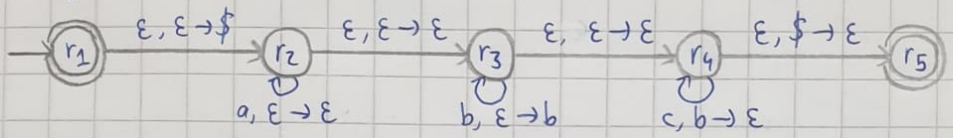
a) The language

$\{a^i b^j c^k \mid i = j$ or $j = k$ or both $\}$

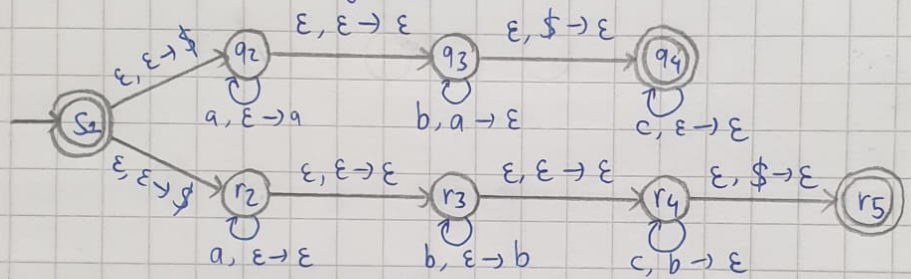
- For $i = j$: as PDA reads a, it adds a to the stack. Then when it reads b, it pops a from the stack until stack is empty. Finally reading c without push nor pop, where \$ denotes empty stack



- For $j = k$: similar like above



=) Combine both PDA, we got final result



b) The CFG : $S \to (S) \mid S, S \mid a$