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C11.3 Which of the following claims are true and which are false?

(i) The computation of a deterministic Turing machine halts on every input.

(ii) The complement of any decidable language is semidecidable.

(iii) The intersection of any two semidecidable languages is decidable.

(iv) The problem of determining if a Turing machine accepts at least 7 strings is undecidable.

(v) The problem of determining if a Turing machine has at least 7 states is undecidable.

(vi) The problem of determining if a Turing machine runs for at least 7 steps on all inputs of length $|x| \leq 7$ is undecidable.

Not necessarily distinct languages →

Lemma 9.3

A language $A \subseteq \Sigma^*$ is decidable if and only if both languages A and $\bar{A}$ are semi-decidable.

- The "acceptance" decision problem for Turing machines is *Given a Turing machine M and a string w. Does M accept w?*
- The formal language representing this is the universal language
  $U = \{c_M w \mid M \text{ is a TM and } M \text{ accepts } w\}$.
- The language U is semi-decidable but not decidable.

= undecidable

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(i) For example, a deterministic TM which loops forever, i.e. never goes to a halting state. False

(ii) By Lemma 9.3. A is decidable $\Rightarrow \bar{A}$ semidecidable. True.

(iii) For example, $U \cap U = U$, which is semi-decidable and undecidable. False.

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(iv) Semantic property?

$AS = \{L \in RE \mid |L| \geq 7\}$

Non-trivial?

a) We can construct a TM s.t. $L(M) = \{a, aa, aaa \ldots, a^7\}$

$|L| = 7 \geq 7$

$AS \neq \emptyset$

b) On the other hand, if we construct TM s.t. $L(M) = \{a\}$

$|L| = 1 < 7$

$AS \neq RE$

And by Rice's theorem this problem is undecidable.

$\Rightarrow$ True.

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(v) Is this a property of the TM itself or $L(TM)$?

$|Q_{TM}| \geq 7$? So this is a property of the TM $\Rightarrow$ not a semantic property. We cannot apply Rice's theorem.

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So consider the encoding C_{TM} and input it to another Universal TM which reads the encoding and accepts if (and only if) C_{TM} contains seven distinct states. Otherwise it rejects.

$\longrightarrow$ False.

(vi) We have a way to determine when our TM has ran for 7 steps.

By definition of a TM, $|\Sigma| = n$ is finite. Therefore

$$\Sigma_{TM}^{*} = \{x \in \Sigma^{*} \mid |x| \leq 7\}$$ is also finite.

$$n, n^2 \ldots \Rightarrow \sum_{k=0}^{n} n^k = |\Sigma_{TM}^{*}| < \infty.$$

Enlist all strings in Σ_{TM}^{*} and feed them to our TM. If any string halts with less than 7 steps, output false. Otherwise output true.

$\Rightarrow$ False.