# Instructions

- Classroom Problems C8.1–C8.2 will be discussed and solved onsite at the tutorial sessions in lecture week 8. No credit is given for these problems.
- Homework Problems H8.1–H8.3 you should solve on your own, and be available to present your solutions at one of the tutorial sessions in lecture week 9. In order to get course credit, you need to indicate your solved problems on the signup sheet circulated at the beginning of the session.
- Supplementary Problems S8.1–S8.3 provide further illustration and extension of the course material, but will usually not be covered at the tutorials. You are however invited to work on these problems too, and discuss them with the course staff. Sample solutions are provided on MyCourses.

# Classroom Problems

**C8.1** Design a deterministic single-tape Turing machine that decides (i.e. recognises and halts on every input) the language

$$\{1^n 0 1^n \mid n \geq 0\}.$$

**C8.2** Design a deterministic single-tape Turing machine that decides (i.e. recognises and halts on every input) the language

$$\{ww^R \mid w \in \{a, b\}^*\}.$$

# Homework Problems

**H8.1** Design a deterministic single-tape Turing machine that decides (i.e. recognises and halts on every input) the language

$$\{w \in \{a, b\}^* \mid w \text{ contains equally many } a\text{'s and } b\text{'s}\}.$$

Show the accepting computation sequence ("run") of your machine on input *aabb* and the rejecting sequence on input *baa*.

**H8.2** Design a three-tape Turing machine ADD that functions as follows. The machine gets as input on the tapes 1 and 2 two binary numbers written in reverse, i.e. with their least significant bits first. It then computes on the tape 3 the sum of the two given numbers in the same notation. For

simplicity, you may assume that the input numbers are of the same length, i.e. that the possibly shorter one is padded with leading zeros. Thus, for instance, the calculation 7 + 11 = 18 is represented as:

```
1110
1101
01001
```

You may assume that a tape head of the machine can also stay stationary in a transition (the move direction "S").

**H8.3** Design a two-tape nondeterministic Turing machine that recognises the language $\{ww \mid w \in \{a, b\}^*\}$. You may assume that a tape head of the machine can also stay stationary in a transition (the move direction "S").

## Supplementary Problems

**S8.1** Extend the notion of a Turing machine by providing the possibility of a *two-way infinite tape*. Show that nevertheless such machines recognise exactly the same languages as the standard machines whose tape is only one-way infinite.

**S8.2** Design deterministic Turing machines NEXT and DUP that perform the following tasks:

1. NEXT replaces the string on the tape with the string that is next in the shortlex order[1] (lexicographical order except that shorter strings precede longer ones).
2. DUP copies the tape contents to the end of the tape (e.g., the tape contents *abb* is replaced with *abbabb*).

**S8.3** Show that pushdown automata with *two* stacks (rather than just one as permitted by the standard definition) would be capable of recognising exactly the same languages as Turing machines.

[1] `http://en.wikipedia.org/wiki/Shortlex_order`