Homework Problems

H1.1 List all the equivalence relations (partitions) on the set $\{a, b, c\} = A$

Definition of an equivalence relation R on set S

- Reflexive : $\forall s \in S$, $(s,s) \in R$
- Symmetry : if $s R t \Rightarrow t R s$ holds for all $s, t \in S$
- Transitive : if $s R t \wedge t R u \Rightarrow s R u$ for all $s, t, u \in S$

Draw a matrix table of relation R

	a	b	c
a	1	0	0
b	0	1	0
c	0	0	1

Reflexive : (s,s) holds $\Rightarrow$ diagonal is non zero

Symmetry : $R = R^T$

Transitive : $R^2 = R$ for non zero entries

$\Rightarrow$ The only matrix that satisfies the condition is the identity matrix

$\Rightarrow$ Equivalence relations on the set $\{a, b, c\}$ is $R = \{(a,a), (b,b), (c,c)\}$

H1.2 Prove by induction that $n! > 2^n$ for all $n \geq 4$

Proof:

- Induction basis : $n = 4 \Rightarrow 4! > 2^4 \Rightarrow 24 > 16$ (True)
- Induction hypothesis : assume for some $k \geq 4$, $P(k)$: $k! > 2^k$ holds

$\Rightarrow$ Induction step : prove $(k+1)! > 2^{k+1}$ when $n = k+1$ $(k \geq 4)$

We have : $(k+1)! = (k+1)k! > (k+1)2^k$ $(k! > 2^k \ \forall k \geq 4)$

$\Rightarrow (k+1)k! > 2 \cdot 2^k$ (since $k+1 > 2 \ \forall k \geq 4$)

$\Rightarrow (k+1)k! > 2^{k+1}$

Therefore, $(k+1)! > 2^{k+1}$ (proven)

$\Rightarrow n! > 2^n$ for all $n \geq 4$

H1.3 Let $\Sigma = \{a, b\}$. Give examples of strings from each of the following languages (min 3 strings / language)

(i) $\{w \in \Sigma^* \mid$ a's in w is odd and b's in w is prime $\geq 2\}$

"abbaa", "abbb", "ababababab"

(ii) $\{w \in \Sigma^* \mid w$ has exactly two occurences of substrings ab and/or ba $\}$

"baaba", "abbba", "abbabaab"

(iii) $\{(ab)^n(ba)^n \mid n \geq 1\}$

"abba", "ababbaba", "abababbababa"

(iv) $\{w \in \Sigma^* \mid \exists u, v \in \Sigma^*$ s.e. $w = uvuv\}$

"abab", "baba", "aaaa"