PLEASE REMEMBER TO REGISTER FOR THE COURSE!

Instructions

- Classroom Problems C1.1–C1.5 will be discussed and solved onsite at the tutorial sessions in lecture week 1. No credit is given for these problems.
- Homework Problems H1.1–H1.3 you should solve on your own, and be available to present your solutions at one of the tutorial sessions in lecture week 2. In order to get course credit, you need to indicate your solved problems on the signup sheet circulated at the beginning of the session.
- Supplementary Problems S1.1–S1.2 provide further illustration and extension of the course material, but will usually not be covered at the tutorials. You are however invited to work on these problems too, and discuss them with the course staff. Sample solutions are provided on MyCourses.

Classroom Problems

C1.1 Let $A = \{a, b, c\}$, $B = \{b, d\}$, and $C = \{a, c, d, e\}$. List the elements of the following sets:

(i) $A \cup (C - B)$;
(ii) $B \times (A \cap C)$;
(iii) $\mathcal{P}(A - B)$;
(iv) $\mathcal{P}(\{\emptyset\}) - \mathcal{P}(\emptyset)$.

Here $\mathcal{P}(A)$ denotes the power set of a set A, i.e. the set of all subsets of A.

C1.2 Let $A = \{a, b, c, d\}$, and define a relation $R \subseteq A \times A$ as follows:

$$R = \{(a, b), (b, c), (b, d), (c, a), (d, d)\}.$$

Draw the graphs corresponding to the following relations:

(i) R, (ii) R^{-1}, (iii) $R \circ R$, (iv) $(R \circ R) - R^{-1}$.

Are some of these relations actually functions?

C1.3 Prove by induction the following equality:

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2 (n+1)^2}{4}$$

C1.4 Let $\Sigma = \{a, b\}$. Give some examples of strings from each of the following languages (at least three strings per language):

(i) $\{w \in \Sigma^* \mid \text{the number of } a\text{'s in } w \text{ is even and the number of } b\text{'s is divisible by three}\}$;
(ii) $\{a^{2n}b^{3m} \mid n > m \geq 0\}$;
(iii) $\{uvuv \mid u, v \in \Sigma^*\}$;
(iv) $\{w \in \Sigma^* \mid \text{ there exist } u, v \in \Sigma^* \text{ s.t. } w = uu \text{ and } w = vvv\}$.

C1.5 The *reversal* of a string $w \in \Sigma^*$, denoted w^R, is defined inductively by the rules:

(i) $\varepsilon^R = \varepsilon$;
(ii) if $w = ua$, where $u \in \Sigma^*$ and $a \in \Sigma$, then $w^R = au^R$.

It was proved in Lecture 1 that for any strings $u, v \in \Sigma^*$ it is the case that $(uv)^R = v^R u^R$. Prove in a similar manner, by induction based on the above definition of reversal, the following facts:

(i) $(w^R)^R = w$;
(ii) $(w^k)^R = (w^R)^k$, for any $k \geq 0$.

Homework Problems

H1.1 List all the equivalence relations (partitions) on the set $\{a, b, c\}$.

H1.2 Prove by induction that $n! > 2^n$ for all $n \geq 4$.

H1.3 Let $\Sigma = \{a, b\}$. Give some examples of strings from each of the following languages (at least three strings per language):

(i) $\{w \in \Sigma^* \mid \text{the number of } a\text{'s in } w \text{ is odd and the number of } b\text{'s is a prime} \geq 2\}$;
(ii) $\{w \in \Sigma^* \mid w \text{ contains exactly two occurrences of the substrings } ab \text{ and/or } ba\}$;
(iii) $\{(ab)^n(ba)^n \mid n \geq 1\}$
(iv) $\{w \in \Sigma^* \mid \exists u, v \in \Sigma^* \text{ s.e. } w = uvuv\}$;

Supplementary Problems

S1.1 Define a relation $\sim$ on the set $\mathbb{N} \times \mathbb{N}$ by the rule:

$$(m, n) \sim (p, q) \quad \Leftrightarrow \quad m + n = p + q.$$

Prove that this is an equivalence relation, and describe intuitively ("geometrically") the equivalence classes it determines.

S1.2 Prove by induction that if X is a finite set of cardinality $n = |X|$, then its power set $\mathcal{P}(X)$ is of cardinality $|\mathcal{P}(X)| = 2^n$.