Instructions

- Classroom Problems C5.1–C5.2 will be discussed and solved onsite at the tutorial sessions in lecture week 5. No credit is given for these problems.
- Homework Problems H5.1–H5.3 you should solve on your own, and be available to present your solutions at one of the tutorial sessions in lecture week 6. In order to get course credit, you need to indicate your solved problems on the signup sheet circulated at the beginning of the session.
- Supplementary Problems S5.1–S5.3 provide further illustration and extension of the course material, but will usually not be covered at the tutorials. You are however invited to work on these problems too, and discuss them with the course staff. Sample solutions are provided on MyCourses.

Classroom Problems

C5.1 Consider the following context-free grammars:

(i)
$$\begin{aligned} A &\to aAcc \mid B \\ B &\to bbBc \mid \varepsilon \end{aligned}$$

(ii)
$$S \to +S- \mid SS \mid \varepsilon$$

(a) Give a derivation for the string *abbccc* according to grammar (i), and a derivation for the string $+-++-+--$ according to grammar (ii).

(b) Describe the language generated by each grammar mathematically or verbally as simply as you can.

C5.2 A *palindrome* is a string s such that for the string w obtained by stripping away punctuation (blanks, commas etc.) from s it holds that $w = w^R$. (E.g. "MADAM, I'M ADAM", "ABLE WAS I, ERE I SAW ELBA" are palindromes, cf. `http://www.derf.net/palindromes/`.)

Consider the set of "stripped" palindromes over the alphabet $\{a, b\}$:

$$\text{PAL} = \{w \in \{a, b\}^* \mid w = w^R\}.$$

(a) Design a context-free grammar generating the language PAL. (Remember that the number of characters in a palindrome can be even or odd.)

(b) Show, by using the pumping lemma for regular languages, that the language PAL is not regular.

Homework Problems

H5.1 Design context-free grammars for the following languages:

(i) $\{a^m b^n \mid m > n\}$.

(ii) $\{a^m b^n \mid m \neq n\}$. Observe that $m \neq n$ if and only if $m < n$ or $m > n$.

(iii) $\{ucv \mid u, v \in \{a, b\}^* \text{ and } |u| = |v|\}$.

Additionally, give a derivation for the string *aaab* using your first grammar, a derivation for *abb* using your second grammar, and a derivation for *abcbb* using your third grammar.

H5.2 Design a context-free grammar that generates the language

$$L = \{a^i b^j c^k \mid i, j, k \geq 0,\ i + j = k\}.$$

Prove also, by using the pumping lemma, that the language L is not regular.

H5.3 Design right-linear context-free grammars for the following languages:

(i) $\{w \in \{a, b\}^* \mid w$ does not contain *aba* as a substring$\}$;

(ii) $\{w \in \{0, 1\}^* \mid w$ contains an even number of 0's and an odd number of 1's$\}$.

Use the systematic construction presented at Lecture 5. That is, first design a finite automaton for the language in question and then translate the automaton into the corresponding right-linear context-free grammar. In addition to the final solutions, also show the intermediate phases, e.g., the automata that you designed.

Supplementary Problems

S5.1 Prove that the language $L = \{w \mid w$ contains equally many a's as b's$\}$ is not regular, and design a context-free grammar generating it.

S5.2 Design a context-free grammar that describes simple programs consisting of nested `for` loops, compound statements enclosed by `begin-end` pairs and elementary operations `a`. An example "program" in this language is:

```
a;
for 3 times do
begin
   for 5 times do a;
   a; a
end.
```

For simplicity, you may assume that the loop counters are always integer constants in the range 0,. . . , 9.

S5.3 In the data description language XML, designers can construct their own "data type definitions" (abbr. DTD), which are essentially context free grammars describing the structure of the data contained in a file. Acquaint yourself with the notation used in this XML/DTD description language (from e.g. `http://www.rpbourret.com/xml/xmldtd.htm`), and give a context-free grammar corresponding to the following XML/DTD description:

```
<!DOCTYPE Book [
 <!ELEMENT Book (Title, Chapter+)>
 <!ATTLIST Book Author CDATA #REQUIRED>
 <!ELEMENT Title (#PCDATA)>
 <!ELEMENT Chapter (#PCDATA)>
 <!ATTLIST Chapter id ID #REQUIRED>
]>
```