



Aalto University
School of Science

CS-C2160 Theory of Computation

Appetizer: Computational Problems, Models of Computation

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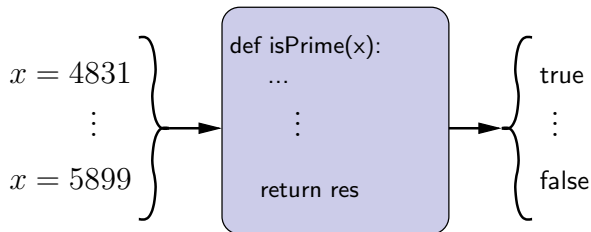
Department of Computer Science

Spring 2022

Computational Problems

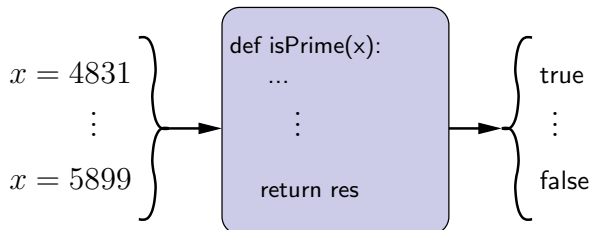
Computational Problems

Can one make a program that detects whether a given number x is a prime number? (Very large primes are important in cryptography.)



Computational Problems

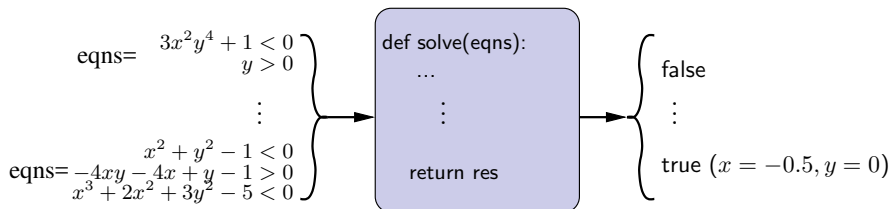
Can one make a program that detects whether a given number x is a prime number? (Very large primes are important in cryptography.)



Answer: Yes, a very inefficient way is to just go through all the pairs of integers in $\{2, \dots, x-1\}$ and check that their product does not equal x .

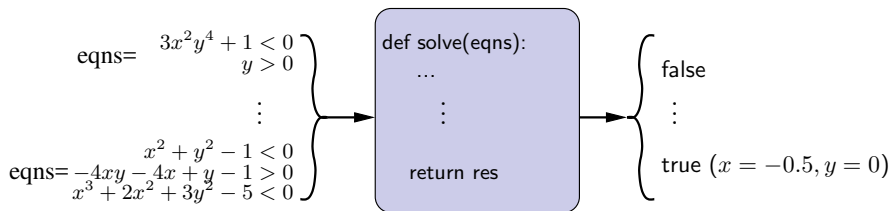
Computational Problems

Can one make a program that tells whether a multivariate polynomial equation system has a *real-valued* solution?



Computational Problems

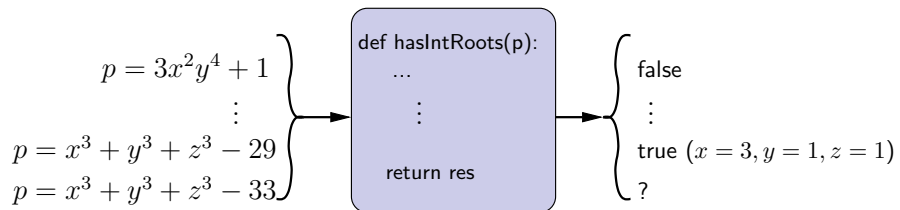
Can one make a program that tells whether a multivariate polynomial equation system has a *real-valued* solution?



Answer: Yes, but this is not easy, see e.g. [this report](#).

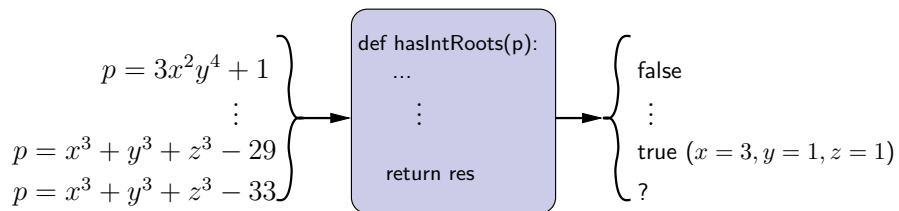
Computational Problems

Can one make a program that tells whether a multivariate polynomial has *integer-valued* roots?



Computational Problems

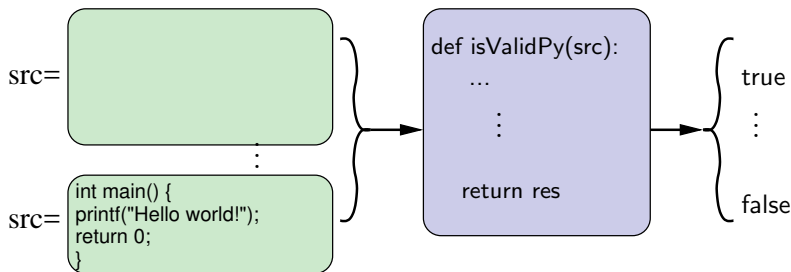
Can one make a program that tells whether a multivariate polynomial has *integer-valued* roots?



Answer: No, this is called **Hilbert's tenth problem** and it cannot be solved by a computer.

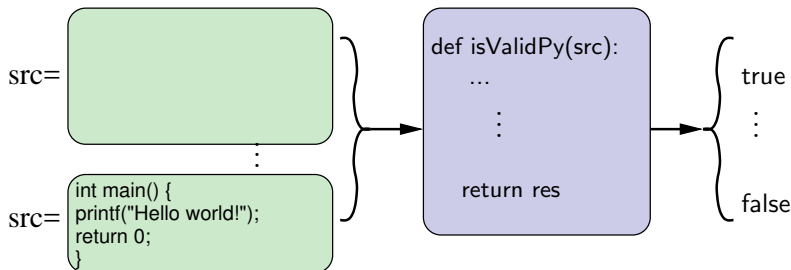
Computational Problems

Can one make a program that tells if a text file is a valid Python program?



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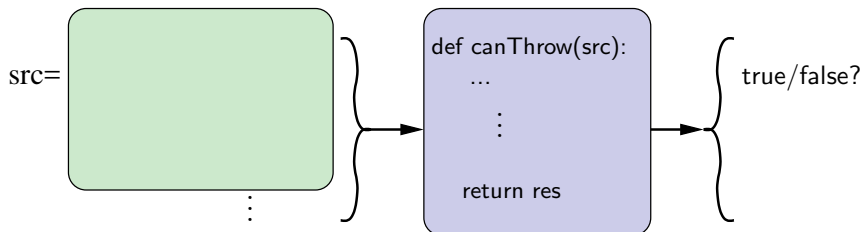


Answer: Yes, the Python interpreter/compiler does this as the first step.

Computational Problems

Can one make a program that tells if there exists an input that causes a given Python program to throw an exception, e.g.

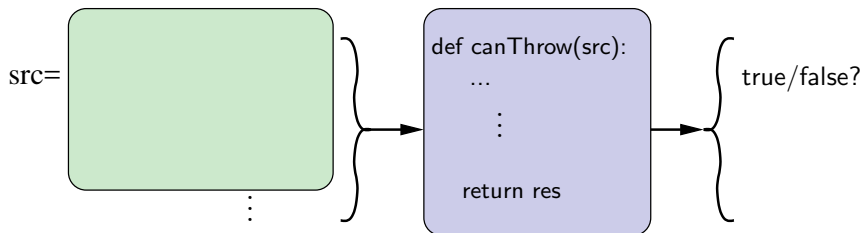
`ZeroDivisionError`? (Such a program would be a very convenient “perfect compiler”, able to detect all possibilities for run-time errors in advance.)



Computational Problems

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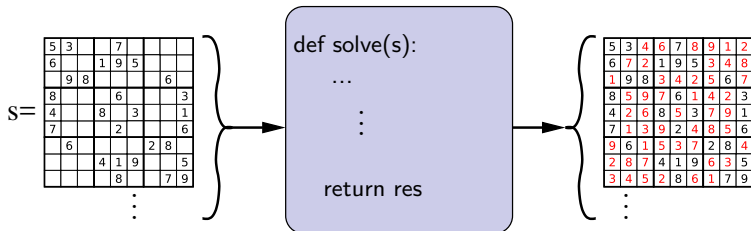
`ZeroDivisionError`? (Such a program would be a very convenient “perfect compiler”, able to detect all possibilities for run-time errors in advance.)



Answer: Assuming that the source Python program can access unlimited memory resources, this is not possible.

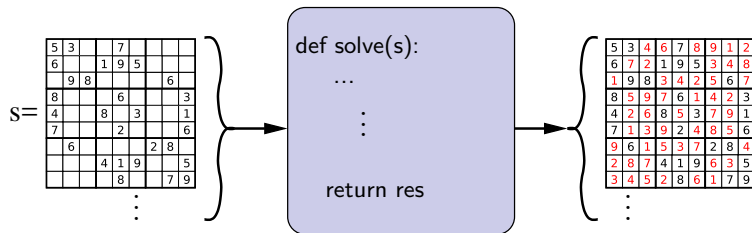
Computational Problems

Can one make a program that solves sudokus?



Computational Problems

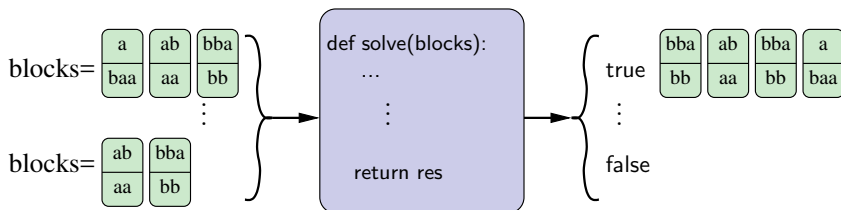
Can one make a program that solves sudokus?



Answer: Yes, an inefficient way is to just enumerate through all the solution candidates.

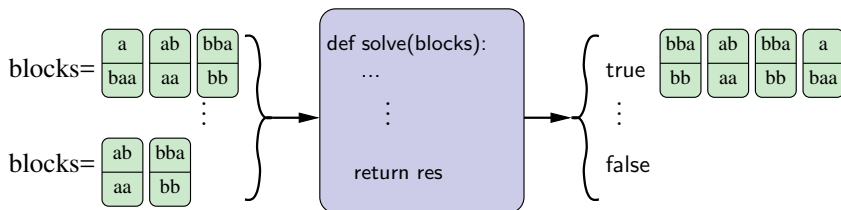
Computational Problems

Can one make a program that decides whether blocks from a given set (multiple copies are allowed) can be organised so that the top and bottom row have the same string?



Computational Problems

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Answer: No, this is called **Post's Correspondence Problem** and it cannot be solved by a computer.

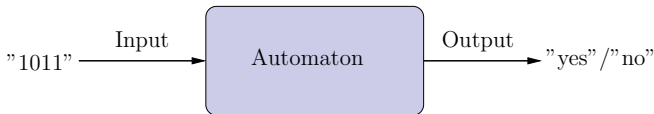
Models of Computation

Problem: To answer these kinds of questions, we need a model of a computer/computer program.

Question: Mathematical model of a computer/computer program?

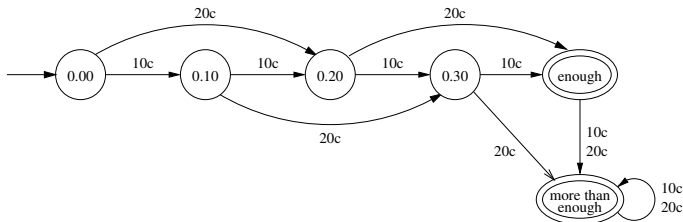
This course:

- Different classes of automata that process input.
- The inputs “accepted” by an automaton form its language.
- Also: grammars (duals of automata) generating the same languages.



E.g. the language of a primality testing automaton would be $\{2, 3, 5, 7, 11, 13, 17, 23, \dots\}$

Model I: Finite state automata



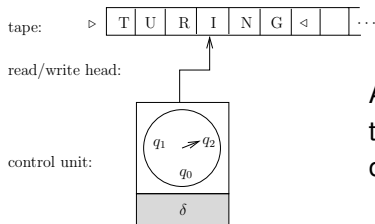
- Finite amount of memory
- Can only read input, one step at time

Application areas of variants:

- Communication protocols
- Simple embedded controllers
- Lexers in compilers (e.g. recognising keywords etc. when compiling Java to bytecode)
- ...

Model II: Turing machines

Turing machines (Alan Turing 1935–36)

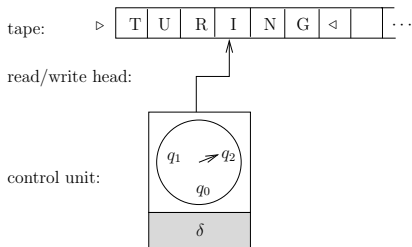


A finite state automaton with an unbounded tape that it can read, write, and move left/right one step at time.

Church-Turing thesis:

Any mechanically (= algorithmically) solvable problem can be solved with a Turing machine.

Model II: Turing machines



A Turing machine is a 7-tuple

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}}),$$

where:

- Q is a finite set of *states*;
- Σ is the *input alphabet*;
- $\Gamma \supseteq \Sigma$ is the *tape alphabet* (s.t. $\triangleright, \triangleleft \notin \Gamma$);
- $\delta: (Q - \{q_{\text{acc}}, q_{\text{rej}}\}) \times (\Gamma \cup \{\triangleright, \triangleleft\}) \rightarrow Q \times (\Gamma \cup \{\triangleright, \triangleleft\}) \times \{L, R\}$ is the *transition function*;
- $q_0 \in Q$ is the *initial state*;
- $q_{\text{acc}} \in Q$ is the *accepting state*; and
- $q_{\text{rej}} \in Q$ is the *rejecting state*.

FAQ: Why do we need all these symbols and formal definitions?

Answers:

- To **precisely communicate** complex constructions (concepts, problems, algorithms, etc) to other people.
- To make **solid, mathematical arguments** about properties of constructions.
 - ▶ In the general case, this problem cannot always be solved because ...
 - ▶ This problem is computationally difficult because ...
 - ▶ My algorithm outputs “yes” if and only if the input ...

Some interesting courses to continue after this course

- CS-E3190 Principles of Algorithmic Techniques
- CS-E4530 Computational Complexity Theory
- CS-E4340 Cryptography
- Many other CS courses