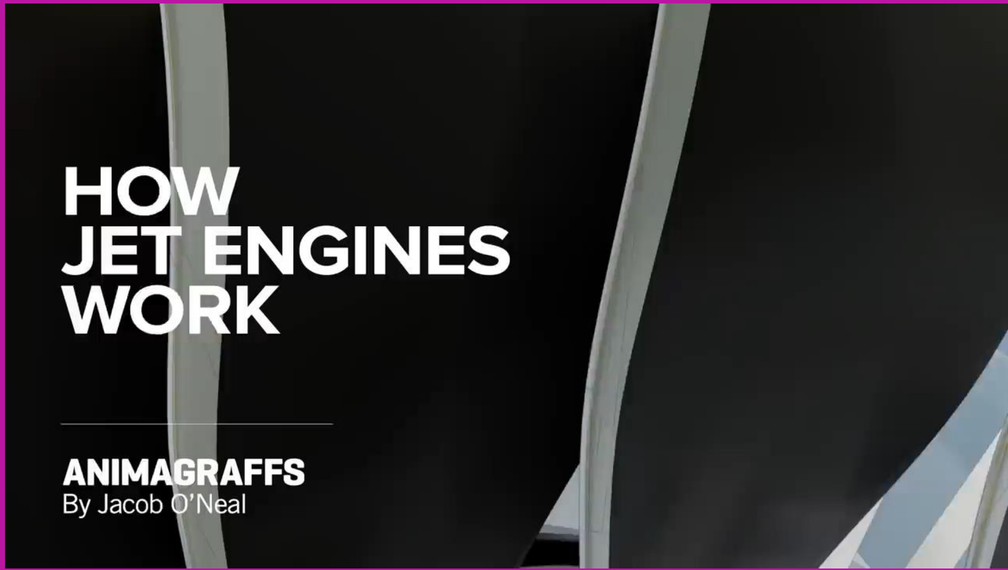
Thermodynamics

An Engineering Approach

Lecture 4: Mass and Energy Analysis of Control Volumes

Aalto University
School of Engineering

Qiang Cheng (Jonny)



Learning Outcomes

When you complete your study of this chapter, you will be able to…

describe key concepts related to control volume analysis, including distinguishing between steady-state and transient analysis, distinguishing between mass flow rate and volumetric flow rate, and explaining the meanings of one-dimensional flow and flow work.

apply mass and energy balances to control volumes.

develop appropriate engineering models for control volumes, with particular attention to analyzing components commonly encountered in engineering practice such as nozzles, diffusers, turbines, compressors, heat exchangers, throttling devices, and integrated systems that incorporate two or more components.

obtain and apply appropriate property data for control volume analyses.

Conservation of Mass for a Control Volume

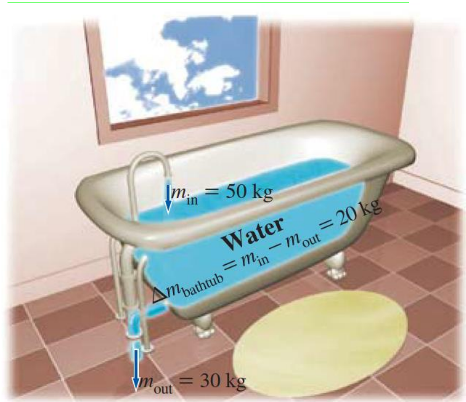


FIGURE 5–5
Conservation of mass principle for an ordinary bathtub.

When applied to such a control volume, the conservation of mass principle states

$$\begin{bmatrix}\text{time } \textit{rate of change} \text{ of} \\ \text{mass contained within the} \\ \text{control volume } \textit{at time t}\end{bmatrix} = \begin{bmatrix}\text{time } \textit{rate} \text{ of flow of} \\ \text{mass } \textit{in} \text{ across} \\ \text{inlet } i \textit{ at time t}\end{bmatrix} - \begin{bmatrix}\text{time } \textit{rate} \text{ of flow} \\ \text{of mass } \textit{out} \text{ across} \\ \text{exit } e \textit{ at time t}\end{bmatrix}$$

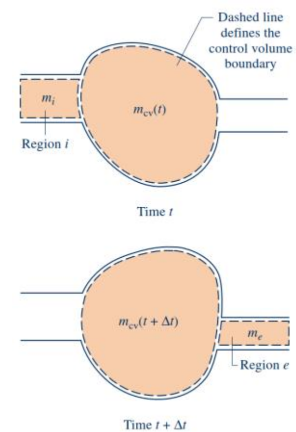
$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e \qquad (4.1)$$

In general, there may be several locations on the boundary through which mass enters or exits. This can be accounted for by summing, as follows

$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e \qquad (4.2)$$

Evaluating the Mass Flow Rate

$$\begin{bmatrix}\text{amount of mass} \\ \text{crossing } dA \text{ during} \\ \text{the time interval } \Delta t\end{bmatrix} = \rho(V_n \, \Delta t) \, dA \qquad \begin{bmatrix}\text{instantaneous rate} \\ \text{of mass flow} \\ \text{across } dA\end{bmatrix} = \rho V_n \, dA$$



$$\dot{m} = \int_A \rho V_n \, dA$$

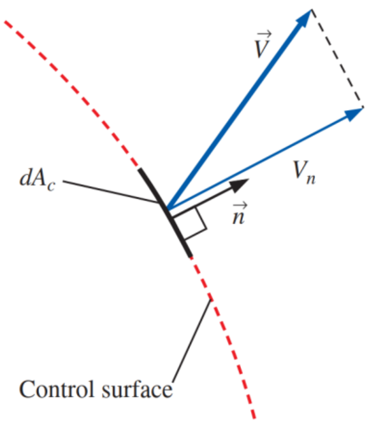
Forms of the Mass Rate Balance

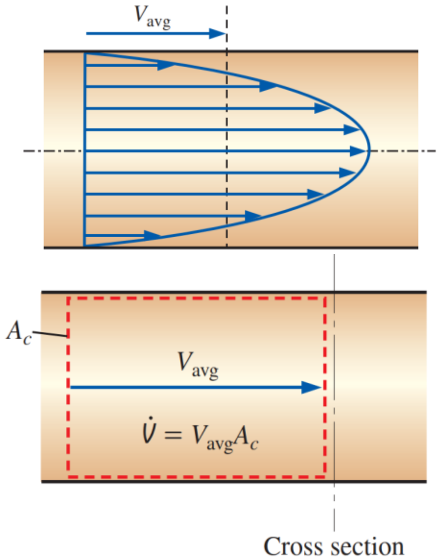
One-Dimensional Flow Form of the Mass Rate Balance

When a flowing stream of matter entering or exiting a control volume adheres to the following idealizations, the flow is said to be one-dimensional:

- The flow is normal to the boundary at locations where mass enters or exits the control volume.
- All intensive properties, including velocity and density, are uniform with position (bulk average values) over each inlet or exit area through which matter flows.

$$\frac{dm_{cv}}{dt} = \sum_i \frac{A_i V_i}{v_i} - \sum_e \frac{A_e V_e}{v_e} \quad \text{(one-dimensional flow)} \qquad (4.5)$$





When the flow is one-dimensional, Eq. 4.3 for the mass flow rate becomes

$$\dot{m} = \rho A V \quad \text{(one-dimensional flow)} \qquad (4.4a)$$

or in terms of specific volume

$$\dot{m} = \frac{AV}{v} \quad \text{(one-dimensional flow)} \qquad (4.4b)$$

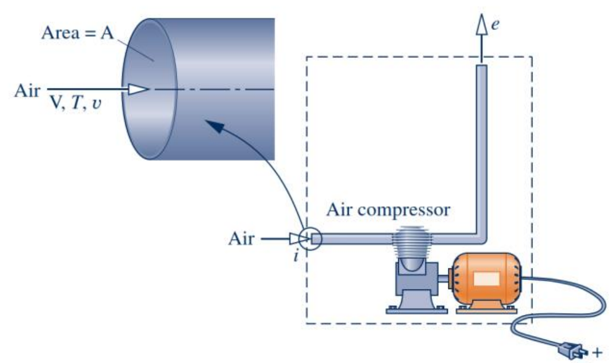


Fig. 4.3 Figure illustrating the one-dimensional flow model.

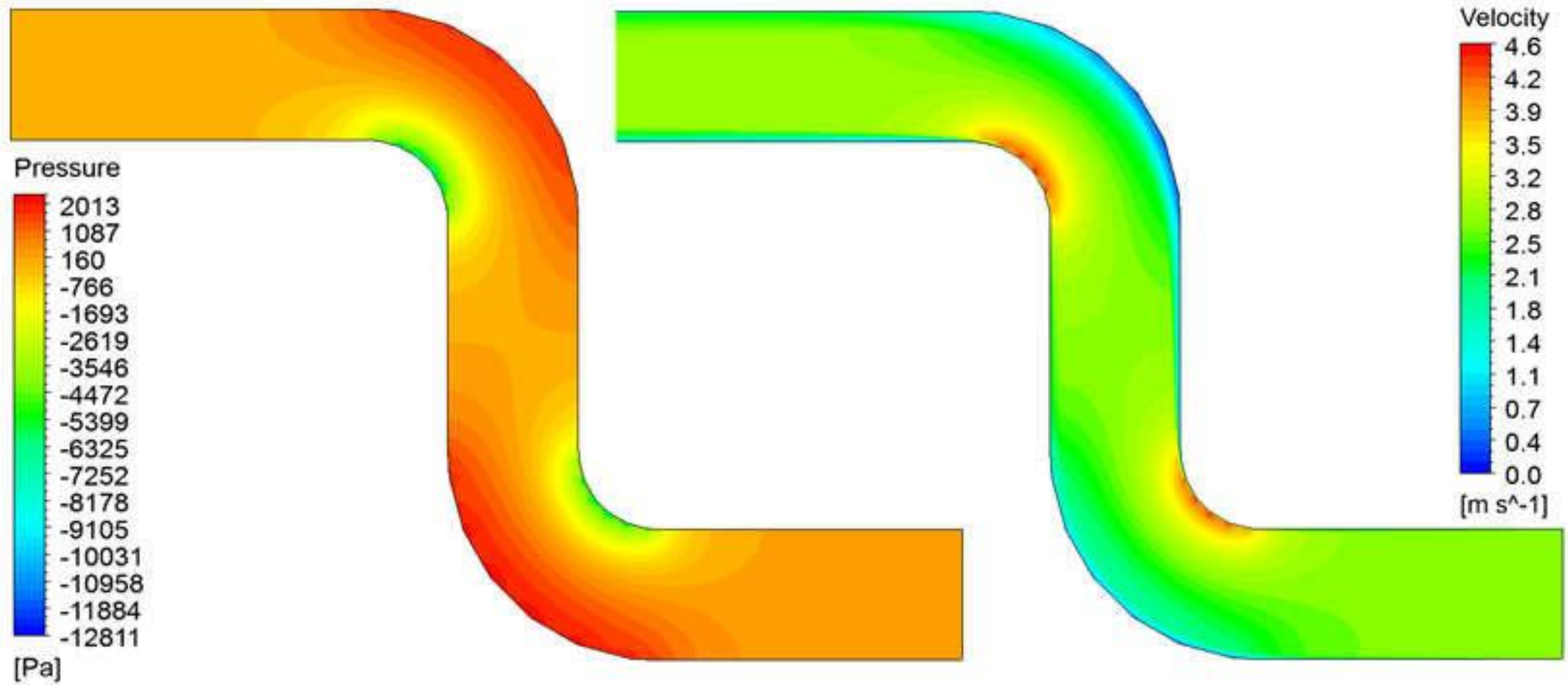
Aalto University
School of Engineering

Forms of the Mass Rate Balance

One-Dimensional Flow Form of the Mass Rate Balance

When a flowing stream of matter entering or exiting a control volume adheres to the following idealizations, the flow is said to be one-dimensional:

- The flow is normal to the boundary at locations where mass enters or exits the control volume.



Forms of the Mass Rate Balance

Steady-State Form of the Mass Rate Balance

all properties are unchanging in time.

$$\underset{\text{(mass rate in)}}{\sum_i \dot{m}_i} = \underset{\text{(mass rate out)}}{\sum_e \dot{m}_e} \qquad (4.6)$$

Integral Form of the Mass Rate Balance

The product ρV_n appearing in this equation, known as the mass flux, gives the time rate of mass flow per unit of area.

$$m_{cv}(t) = \int_V \rho \, dV \qquad (4.7)$$

$$\frac{d}{dt}\int_V \rho \, dV = \sum_i \left(\int_A \rho \mathrm{V}_n \, dA \right)_i - \sum_e \left(\int_A \rho \mathrm{V}_n \, dA \right)_e \qquad (4.8)$$

Applications of the Mass Rate Balance

Steady-State Application

EXAMPLE 4.1

Applying the Mass Rate Balance to a Feedwater Heater at Steady State

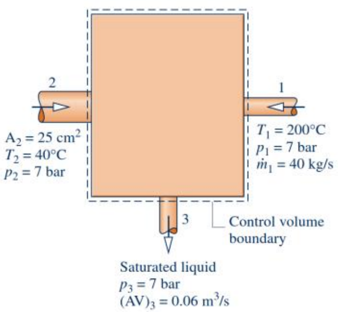
A feedwater heater operating at steady state has two inlets and one exit. At inlet 1, water vapor enters at $p_1 = 7$ bar, $T_1 = 200°C$ with a mass flow rate of 40 kg/s. At inlet 2, liquid water at $p_2 = 7$ bar, $T_2 = 40°C$ enters through an area $A_2 = 25\ cm^2$. Saturated liquid at 7 bar exits at 3 with a volumetric flow rate of 0.06 m^3/s. Determine the mass flow rates at inlet 2 and at the exit, in kg/s, and the velocity at inlet 2, in m/s.

SOLUTION

Known: A stream of water vapor mixes with a liquid water stream to produce a saturated liquid stream at the exit. The states at the inlets and exit are specified. Mass flow rate and volumetric flow rate data are given at one inlet and at the exit, respectively.

Find: Determine the mass flow rates at inlet 2 and at the exit, and the velocity V_2.

Schematic and Given Data:



Engineering Model: The control volume shown on the accompanying figure is at steady state.

Fig. E4.1

Analysis: The principal relations to be employed are the mass rate balance (Eq. 4.2) and the expression $\dot{m} = AV/v$ (Eq. 4.4b). At steady state the mass rate balance becomes

❶
$$\frac{dm_{cv}^{\ 0}}{dt} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3$$

Solving for $\dot{m}_2$,

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1$$

The mass flow rate $\dot{m}_1$ is given. The mass flow rate at the exit can be evaluated from the given volumetric flow rate

$$\dot{m}_3 = \frac{(AV)_3}{v_3}$$

where v_3 is the specific volume at the exit. In writing this expression, one-dimensional flow is assumed. From Table A-3, $v_3 = 1.108 \times 10^{-3}\ m^3/kg$. Hence,

$$\dot{m}_3 = \frac{0.06\ m^3/s}{(1.108 \times 10^{-3}\ m^3/kg)} = 54.15\ kg/s$$

The mass flow rate at inlet 2 is then

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 54.15 - 40 = 14.15\ kg/s$$

For one-dimensional flow at 2, $\dot{m}_2 = A_2V_2/v_2$, so

$$V_2 = \dot{m}_2 v_2 / A_2$$

State 2 is a compressed liquid. The specific volume at this state can be approximated by $v_2 \approx v_f(T_2)$ (Eq. 3.11). From Table A-2 at 40°C, $v_2 = 1.0078 \times 10^{-3}\ m^3/kg$. So,

$$V_2 = \frac{(14.15\ kg/s)(1.0078 \times 10^{-3}\ m^3/kg)}{25\ cm^2}\left|\frac{10^4\ cm^2}{1\ m^2}\right| = 5.7\ m/s$$

❶ In accord with Eq. 4.6, the mass flow rate at the exit equals the sum of the mass flow rates at the inlets. It is left as an exercise to show that the volumetric flow rate at the exit *does not equal* the sum of the volumetric flow rates at the inlets.

Skills Developed

Ability to...

- ❑ apply the steady-state mass rate balance.
- ❑ apply the mass flow rate expression, Eq. 4.4b.
- ❑ retrieve property data for water.

Quick Quiz Evaluate the volumetric flow rate, in m^3/s, at each inlet. **Ans.** $(AV)_1 = 12\ m^3/s$, $(AV)_2 = 0.01\ m^3/s$

Applications of the Mass Rate Balance

Time-Dependent (Transient) Application

EXAMPLE 4.2

Applying the Mass Rate Balance to a Barrel Filling with Water

Water flows into the top of an open barrel at a constant mass flow rate of 30 lb/s. Water exits through a pipe near the base with a mass flow rate proportional to the height of liquid inside: $\dot{m}_e = 9\,L$, where L is the instantaneous liquid height, in ft. The area of the base is 3 ft^2, and the density of water is 62.4 lb/ft^3. If the barrel is initially empty, plot the variation of liquid height with time and comment on the result.

SOLUTION

Known: Water enters and exits an initially empty barrel. The mass flow rate at the inlet is constant. At the exit, the mass flow rate is proportional to the height of the liquid in the barrel.

Find: Plot the variation of liquid height with time and comment.

Schematic and Given Data:

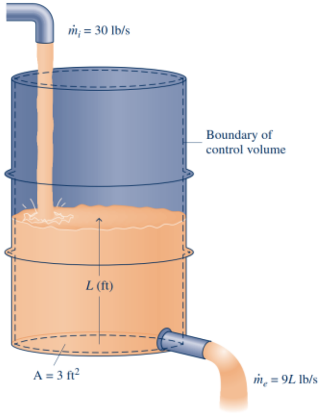


Fig. E4.2*a*

Engineering Model:

1. The control volume is defined by the dashed line on the accompanying diagram.
2. The water density is constant.

Analysis: For the one-inlet, one-exit control volume, Eq. 4.2 reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

The mass of water contained within the barrel at time t is given by

$$m_{cv}(t) = \rho A L(t)$$

where ρ is density, A is the area of the base, and $L(t)$ is the instantaneous liquid height. Substituting this into the mass rate balance together with the given mass flow rates

$$\frac{d(\rho A L)}{dt} = 30 - 9L$$

Since density and area are constant, this equation can be written as

$$\frac{dL}{dt} + \left(\frac{9}{\rho A}\right)L = \frac{30}{\rho A}$$

which is a first-order, ordinary differential equation with constant coefficients. The solution is

1 $$L = 3.33 + C\exp\left(-\frac{9t}{\rho A}\right)$$

where C is a constant of integration. The solution can be verified by substitution into the differential equation.

To evaluate C, use the initial condition: at $t = 0$, $L = 0$. Thus, $C = -3.33$, and the solution can be written as

$$L = 3.33[1 - \exp(-9t/\rho A)]$$

Substituting $\rho = 62.4$ lb/ft^3 and $A = 3$ ft^2 results in

$$L = 3.33[1 - \exp(-0.048t)]$$

This relation can be plotted by hand or using appropriate software. The result is

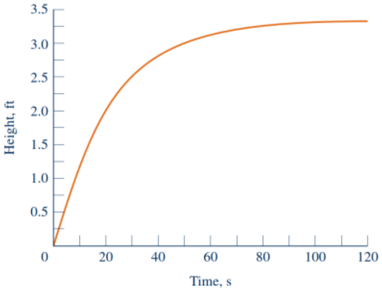


Fig. E4.2*b*

From the graph, we see that initially the liquid height increases rapidly and then levels out as steady-state operation is approached. After about 100 s, the height stays constant with time. At this point, the rate of water flow into the barrel equals the rate of flow out of the barrel. From the graph, the limiting value of L is 3.33 ft, which also can be verified by taking the limit of the analytical solution as $t \rightarrow \infty$.

Applications of the Mass Rate Balance

BIOCONNECTIONS

The human heart provides a good example of how biological systems can be modeled as control volumes. Figure 4.4 shows the cross section of a human heart. The flow is controlled by valves that intermittently allow blood to enter from veins and exit through arteries as the heart muscles pump. Work is done to increase the pressure of the blood leaving the heart to a level that will propel it through the cardiovascular system of the body. Observe that the boundary of the control volume enclosing the heart is not fixed but moves with time as the heart pulses.

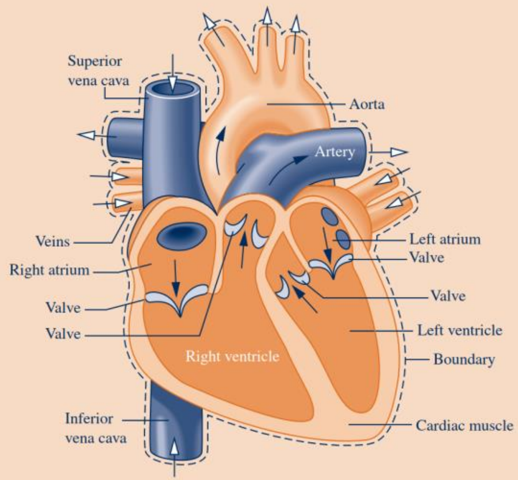


Fig. 4.4 Control volume enclosing the heart.

Understanding the medical condition known as *arrhythmia* requires consideration of the time-dependent behavior of the heart. An arrhythmia is a change in the regular beat of the heart. This can take several forms. The heart may beat irregularly, skip a beat, or beat very fast or slowly. An arrhythmia may be detectable by listening to the heart with a stethoscope, but an electrocardiogram offers a more precise approach. Although arrhythmia does occur in people without underlying heart disease, patients having serious symptoms may require treatment to keep their heartbeats regular. Many patients with arrhythmia may require no medical intervention at all.

Conservation of Energy for a Control Volume

conservation of energy

$$\begin{bmatrix}\text{time } \textit{rate of change} \\ \text{of the energy} \\ \text{contained within} \\ \text{the control volume} \\ \textit{at time } t\end{bmatrix} = \begin{bmatrix}\textit{net rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred in} \\ \text{by heat } \textit{at} \\ \textit{time } t\end{bmatrix} - \begin{bmatrix}\textit{net rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred out} \\ \text{by work } \textit{at} \\ \textit{time } t\end{bmatrix} + \begin{bmatrix}\textit{net rate} \text{ of energy} \\ \text{transfer } \textit{into} \text{ the} \\ \text{control volume} \\ \text{accompanying} \\ \text{mass flow}\end{bmatrix}$$

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \underline{\dot{m}_i\left(u_i + \frac{V_i^2}{2} + gz_i\right)} - \underline{\dot{m}_e\left(u_e + \frac{V_e^2}{2} + gz_e\right)} \quad \text{(4.9)}$$

Because work is always done on or by a control volume where matter flows across the boundary, it is convenient to separate the work term $\dot{W}$ of Eq. 4.9 into two contributions:

- One contribution is the work associated with the fluid pressure as mass is introduced at inlets and removed at exits.
- The other contribution, denoted by $\dot{W}_{CV}$, includes all other work effects, such as those associated with rotating shafts, displacement of the boundary, and electrical effects.

flow work

$$\begin{bmatrix}\text{time rate of energy transfer} \\ \text{by work } \textit{from} \text{ the control} \\ \text{volume at exit } e\end{bmatrix} = (p_e A_e) V_e \quad \text{(4.10)}$$

$$\dot{W} = \dot{W}_{cv} + (p_e A_e)V_e - (p_i A_i)V_i \quad \text{(4.11)}$$

$$\dot{W} = \dot{W}_{cv} + \dot{m}_e(p_e v_e) - \dot{m}_i(p_i v_i) \quad \text{(4.12)}$$

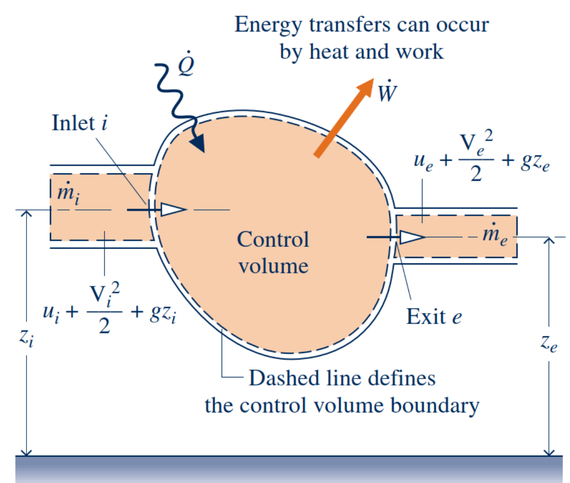


Fig. 4.5 Figure used to develop Eq. 4.9.

One-Dimensional Flow Form of the Control Volume Energy Rate Balance

energy rate balance

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right) \quad (4.15)$$

Integral Form of the Control Volume Energy Rate Balance

$$E_{cv}(t) = \int_V \rho e\, dV = \int_V \rho\left(u + \frac{V^2}{2} + gz\right) dV \quad (4.16)$$

$$\frac{d}{dt}\int_V \rho e\, dV = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \left[\int_A \left(h + \frac{V^2}{2} + gz\right)\rho V_n\, dA\right]_i - \sum_e \left[\int_A \left(h + \frac{V^2}{2} + gz\right)\rho V_n\, dA\right]_e \quad (4.17)$$

Steady-State Forms of the Mass and Energy Rate Balances

Steady-State Forms of the Mass and Energy Rate Balances

$$\underset{\text{(mass rate in)}}{\sum_i \dot{m}_i} = \underset{\text{(mass rate out)}}{\sum_e \dot{m}_e} \qquad \textbf{(4.6)}$$

saturation temperature and pressure

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right) \qquad \textbf{(4.18)}$$

$$\underbrace{\dot{Q}_{\text{cv}} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right)}_{\text{(energy rate in)}} = \underbrace{\dot{W}_{\text{cv}} + \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)}_{\text{(energy rate out)}} \qquad \textbf{(4.19)}$$

one-inlet, one-exit control volumes at steady state.

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right] \qquad \textbf{(4.20a)}$$

Or, dividing by the mass flow rate,

$$0 = \frac{\dot{Q}_{\text{cv}}}{\dot{m}} - \frac{\dot{W}_{\text{cv}}}{\dot{m}} + (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \qquad \textbf{(4.20b)}$$

Nozzles and Diffusers

Nozzle

A nozzle is a flow passage of varying cross-sectional area in which the velocity of a gas or liquid increases in the direction of flow.

Diffuser

In a diffuser, the gas or liquid decelerates in the direction of flow.

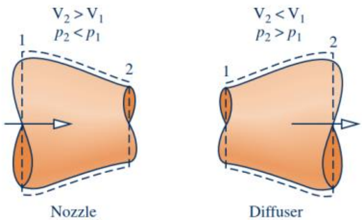


Fig. 4.7 Illustration of a nozzle and a diffuser.

For a control volume enclosing a nozzle or diffuser, the only work is flow work at locations where mass enters and exits the control volume, so the term $\dot{W}_{cv}$ drops out of the energy rate balance. The change in potential energy from inlet to exit is negligible under most conditions.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + \underline{g(z_1 - z_2)}\right]$$

$$0 = \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2}\right] \qquad \text{(a)}$$

Application to a Steam Nozzle

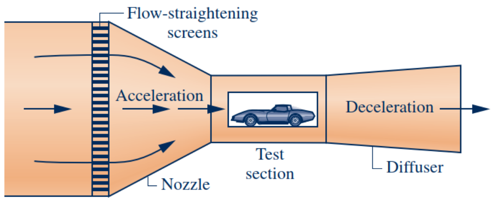


Fig. 4.8 Wind-tunnel test facility.

Aalto University
School of Engineering

EXAMPLE 4.3

Calculating Exit Area of a Steam Nozzle

Steam enters a converging–diverging nozzle operating at steady state with $p_1 = 40$ bar, $T_1 = 400°C$, and a velocity of 10 m/s. The steam flows through the nozzle with negligible heat transfer and no significant change in potential energy. At the exit, $p_2 = 15$ bar, and the velocity is 665 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle, in m^2.

SOLUTION

Known: Steam flows through a nozzle at steady state with known properties at the inlet and exit, a known mass flow rate, and negligible effects of heat transfer and potential energy.

Find: Determine the exit area.

Schematic and Given Data:

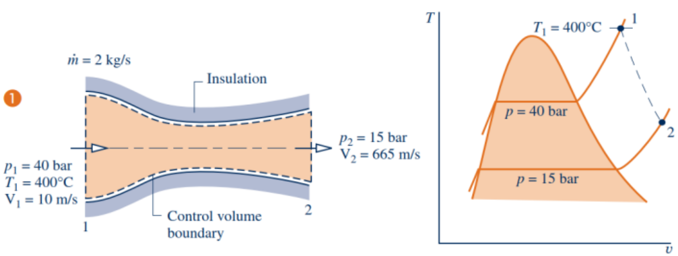


Fig. E4.3

Engineering Model:

1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer is negligible and $\dot{W}_{cv} = 0$.
3. The change in potential energy from inlet to exit can be neglected.

Analysis: The exit area can be determined from the mass flow rate $\dot{m}$ and Eq. 4.4b, which can be arranged to read

$$A_2 = \frac{\dot{m} v_2}{V_2}$$

To evaluate A_2 from this equation requires the specific volume v_2 at the exit, and this requires that the exit state be fixed.

The state at the exit is fixed by the values of two independent intensive properties. One is the pressure p_2, which is known. The other is the specific enthalpy h_2, determined from the steady-state energy rate balance Eq. 4.20a, as follows

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

The terms $\dot{Q}_{cv}$ and $\dot{W}_{cv}$ are deleted by assumption 2. The change in specific potential energy drops out in accordance with assumption 3 and $\dot{m}$ cancels, leaving

$$0 = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

Solving for h_2

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

From Table A-4, $h_1 = 3213.6$ kJ/kg. The velocities V_1 and V_2 are given. Inserting values and converting the units of the kinetic energy terms to kJ/kg results in

2

$$h_2 = 3213.6 \text{ kJ/kg} + \left[\frac{(10)^2 - (665)^2}{2}\right]\left(\frac{m^2}{s^2}\right)\left|\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}}\right|$$
$$= 3213.6 - 221.1 = 2992.5 \text{ kJ/kg}$$

Finally, referring to Table A-4 at $p_2 = 15$ bar with $h_2 = 2992.5$ kJ/kg, the specific volume at the exit is $v_2 = 0.1627 \text{ m}^3/\text{kg}$. The exit area is then

$$A_2 = \frac{(2 \text{ kg/s})(0.1627 \text{ m}^3/\text{kg})}{665 \text{ m/s}} = 4.89 \times 10^{-4} \text{ m}^2$$

1 Although equilibrium property relations apply at the inlet and exit of the control volume, the intervening states of the steam are not necessarily equilibrium states. Accordingly, the expansion through the nozzle is represented on the T–v diagram as a dashed line.

2 Care must be taken in converting the units for specific kinetic energy to kJ/kg.

Skills Developed

Ability to...

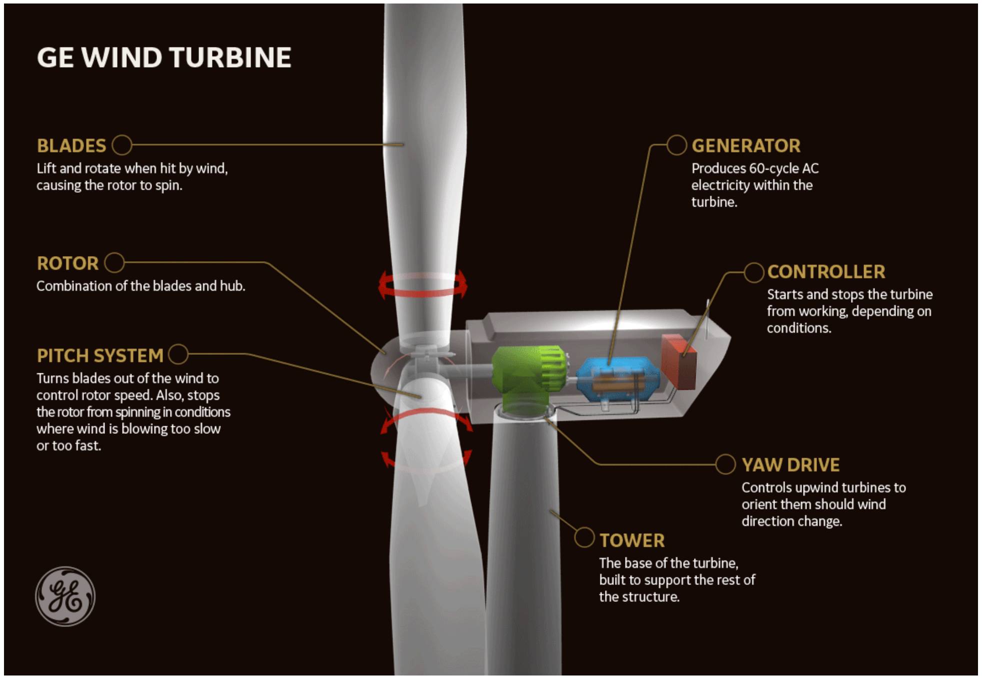
- apply the steady-state energy rate balance to a control volume.
- apply the mass flow rate expression, Eq. 4.4b.
- develop an engineering model.
- retrieve property data for water.

Quick Quiz Evaluate the nozzle inlet area, in m^2. **Ans.** 1.47×10^{-2} m^2.

Turbines

Turbine

A turbine is a device in which power is developed as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate.



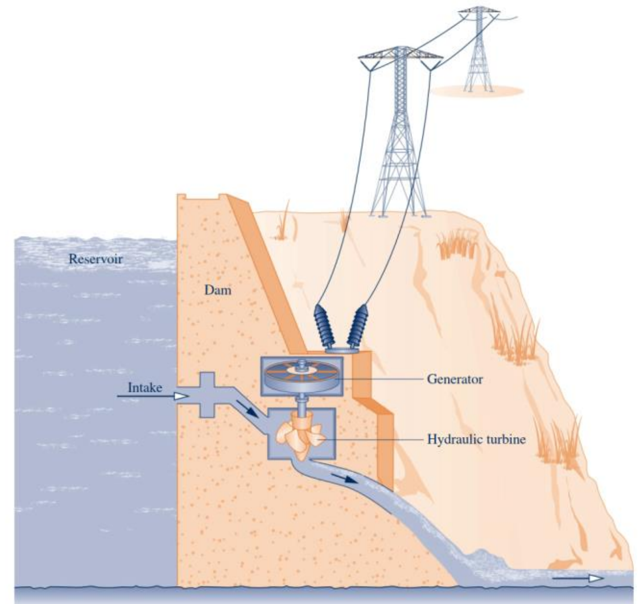


Fig. 4.10 Hydraulic turbine installed in a dam.

Aalto University
School of Engineering

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \underline{\frac{(V_1^2 - V_2^2)}{2}} + \underline{g(z_1 - z_2)}\right]$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \quad \text{(a)}$$

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) \quad \text{(b)}$$

EXAMPLE 4.4

Calculating Heat Transfer from a Steam Turbine

Steam enters a turbine operating at steady state with a mass flow rate of 4600 kg/h. The turbine develops a power output of 1000 kW. At the inlet, the pressure is 60 bar, the temperature is 400°C, and the velocity is 10 m/s. At the exit, the pressure is 0.1 bar, the quality is 0.9 (90%), and the velocity is 30 m/s. Calculate the rate of heat transfer between the turbine and surroundings, in kW.

SOLUTION

Known: A steam turbine operates at steady state. The mass flow rate, power output, and states of the steam at the inlet and exit are known.

Find: Calculate the rate of heat transfer.

Schematic and Given Data:

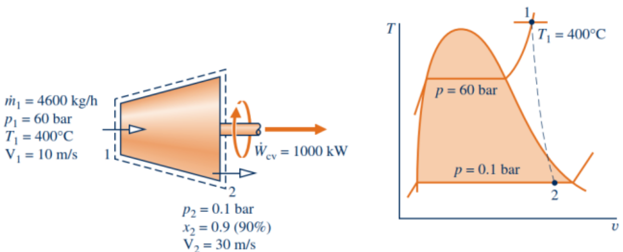


Fig. E4.4

Engineering Model:

1. The control volume shown on the accompanying figure is at steady state.
2. The change in potential energy from inlet to exit can be neglected.

Analysis: To calculate the heat transfer rate, begin with the one-inlet, one-exit form of the energy rate balance for a control volume at steady state, Eq. 4.20a. That is,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

where $\dot{m}$ is the mass flow rate. Solving for $\dot{Q}_{cv}$ and dropping the potential energy change from inlet to exit

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}\left[(h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)\right] \qquad \text{(a)}$$

To compare the magnitudes of the enthalpy and kinetic energy terms, and stress the unit conversions needed, each of these terms is evaluated separately.

First, the specific *enthalpy difference* $h_2 - h_1$ is found. Using Table A-4, $h_1 = 3177.2$ kJ/kg. State 2 is a two-phase liquid–vapor mixture, so with data from Table A-3 and the given quality

$$\begin{aligned} h_2 &= h_{f2} + x_2(h_{g2} - h_{f2}) \\ &= 191.83 + (0.9)(2392.8) = 2345.4 \text{ kJ/kg} \end{aligned}$$

Hence,

$$h_2 - h_1 = 2345.4 - 3177.2 = -831.8 \text{ kJ/kg}$$

Consider next the specific *kinetic energy difference*. Using the given values for the velocities,

❶
$$\begin{aligned} \left(\frac{V_2^2 - V_1^2}{2}\right) &= \left[\frac{(30)^2 - (10)^2}{2}\right]\left(\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}}\right| \\ &= 0.4 \text{ kJ/kg} \end{aligned}$$

Calculating $\dot{Q}_{cv}$ from Eq. (a),

❷
$$\begin{aligned} \dot{Q}_{cv} &= (1000 \text{ kW}) + \left(4600 \frac{\text{kg}}{\text{h}}\right)(-831.8 + 0.4)\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1 \text{ h}}{3600 \text{ s}}\right|\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| \\ &= -62.3 \text{ kW} \end{aligned}$$

❶ The magnitude of the change in specific kinetic energy from inlet to exit is much smaller than the specific enthalpy change. Note the use of unit conversion factors here and in the calculation of $\dot{Q}_{cv}$ to follow.

❷ The negative value of $\dot{Q}_{cv}$ means that there is heat transfer from the turbine to its surroundings, as would be expected. The magnitude of $\dot{Q}_{cv}$ is small relative to the power developed.

Skills Developed

Ability to...

- apply the steady-state energy rate balance to a control volume.
- develop an engineering model.
- retrieve property data for water.

Quick Quiz If the change in kinetic energy from inlet to exit were neglected, evaluate the heat transfer rate, in kW, keeping all other data unchanged. Comment. **Ans.** −62.9 kW.

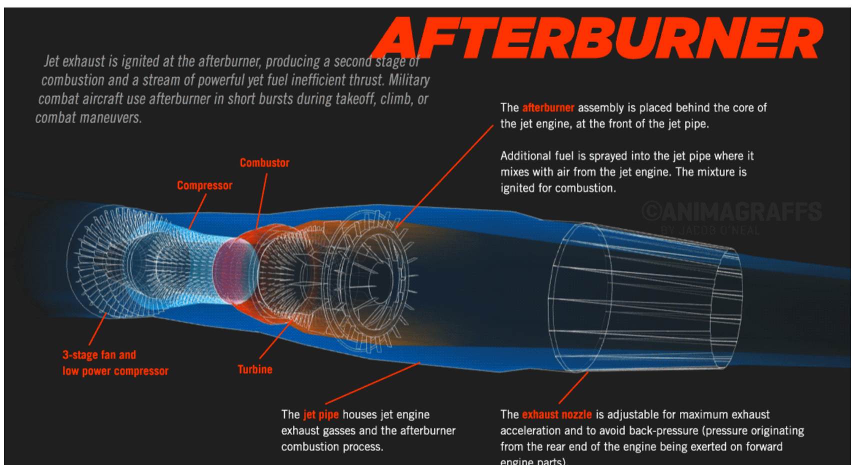
Compressors and Pumps

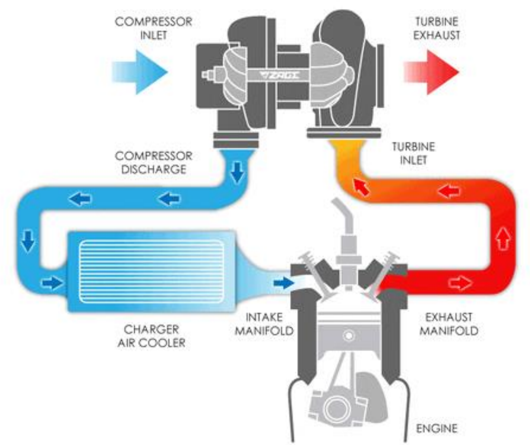
Compressors and pumps

Compressors and pumps are devices in which work is done on the substance flowing through them in order to change the state of the substance, typically to increase the pressure and/or elevation. The term compressor is used when the substance is a gas (vapor) and the term pump is used when the substance is a liquid.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \qquad \text{(a)}$$

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) \qquad \text{(b)}$$





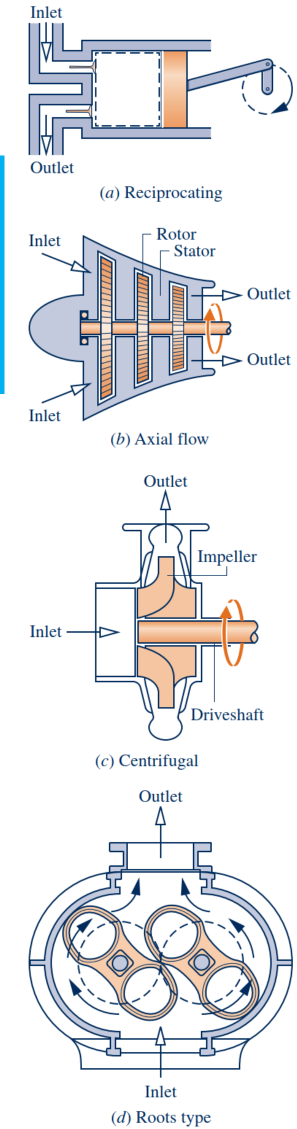


Fig. 4.11 Compressor types.

EXAMPLE 4.5

Calculating Compressor Power

Air enters a compressor operating at steady state at a pressure of 1 bar, a temperature of 290 K, and a velocity of 6 m/s through an inlet with an area of 0.1 m^2. At the exit, the pressure is 7 bar, the temperature is 450 K, and the velocity is 2 m/s. Heat transfer from the compressor to its surroundings occurs at a rate of 180 kJ/min. Employing the ideal gas model, calculate the power input to the compressor, in kW.

SOLUTION

Known: An air compressor operates at steady state with known inlet and exit states and a known heat transfer rate.

Find: Calculate the power required by the compressor.

Schematic and Given Data:

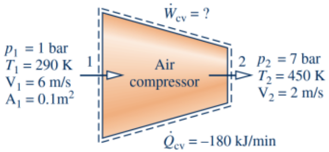


Fig. E4.5

Engineering Model:

1. The control volume shown on the accompanying figure is at steady state.
2. The change in potential energy from inlet to exit can be neglected.
3. ❶ The ideal gas model applies for the air.

Analysis: To calculate the power input to the compressor, begin with the one-inlet, one-exit form of the energy rate balance for a control volume at steady state, Eq. 4.20a. That is,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

Solving

$$\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

The change in potential energy from inlet to exit drops out by assumption 2.

The mass flow rate $\dot{m}$ can be evaluated with given data at the inlet and the ideal gas equation of state.

$$\dot{m} = \frac{A_1V_1}{v_1} = \frac{A_1V_1p_1}{(\overline{R}/M)T_1} = \frac{(0.1\text{ m}^2)(6\text{ m/s})(10^5\text{ N/m}^2)}{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(290\text{ K})} = 0.72\text{ kg/s}$$

The specific enthalpies h_1 and h_2 can be found from Table A-22. At 290 K, $h_1 = 290.16$ kJ/kg. At 450 K, $h_2 = 451.8$ kJ/kg. Substituting values into the expression for $\dot{W}_{cv}$, and applying appropriate unit conversion factors, we get

$$\begin{aligned}\dot{W}_{cv} &= \left(-180\frac{\text{kJ}}{\text{min}}\right)\left|\frac{1\text{ min}}{60\text{ s}}\right| + 0.72\frac{\text{kg}}{\text{s}}\left[(290.16 - 451.8)\frac{\text{kJ}}{\text{kg}}\right.\\ &\quad \left. + \left(\frac{(6)^2 - (2)^2}{2}\right)\left(\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\right]\\ &= -3\frac{\text{kJ}}{\text{s}} + 0.72\frac{\text{kg}}{\text{s}}(-161.64 + 0.02)\frac{\text{kJ}}{\text{kg}}\\ &= -119.4\frac{\text{kJ}}{\text{s}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = -119.4\text{ kW}\end{aligned}$$ ❷

❶ The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.

❷ In this example $\dot{Q}_{cv}$ and $\dot{W}_{cv}$ have negative values, indicating that the direction of the heat transfer is *from* the compressor and work is done *on* the air passing through the compressor. The magnitude of the power *input* to the compressor is 119.4 kW. The change in kinetic energy does not contribute significantly.

Skills Developed

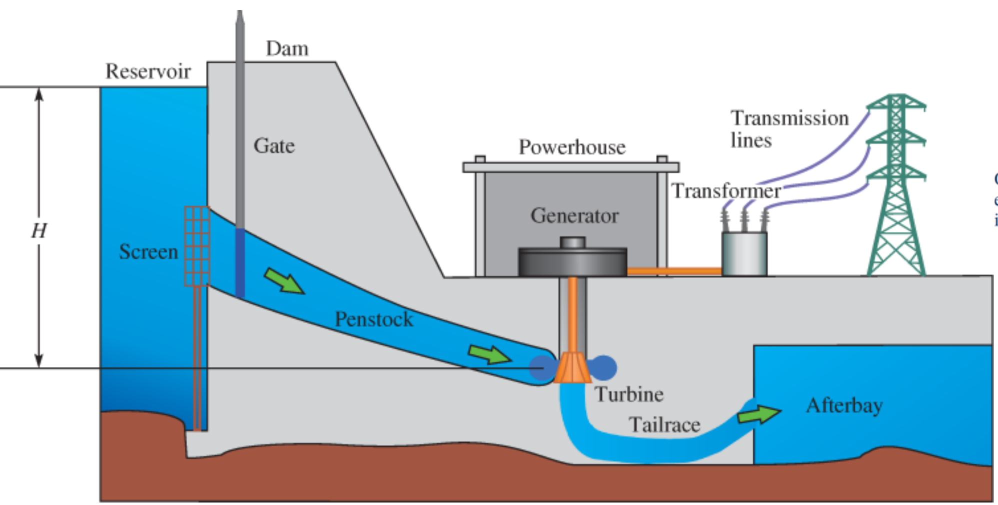
Ability to...

- ❑ apply the steady-state energy rate balance to a control volume.
- ❑ apply the mass flow rate expression, Eq. 4.4b.
- ❑ develop an engineering model.
- ❑ retrieve property data of air modeled as an ideal gas.

Quick Quiz If the change in kinetic energy from inlet to exit were neglected, evaluate the compressor power, in kW, keeping all other data unchanged. Comment. **Ans.** −119.4 kW.

Pumped-Hydro and Compressed-Air Energy Storage

Owing to the dictates of supply and demand and other economic factors, the value of electricity varies with time. Both the cost to generate electricity and increasingly the price paid by consumers for electricity depend on whether the demand for it is on-peak or off-peak. The on-peak period is typically weekdays—for example, from 8 a.m. to 8 p.m., while off-peak includes nighttime hours, weekends, and major holidays. Consumers can expect to pay more for on-peak electricity. Energy storage methods benefiting from variable electricity rates include thermal storage and pumped-hydro and compressed-air storage introduced in the following box.



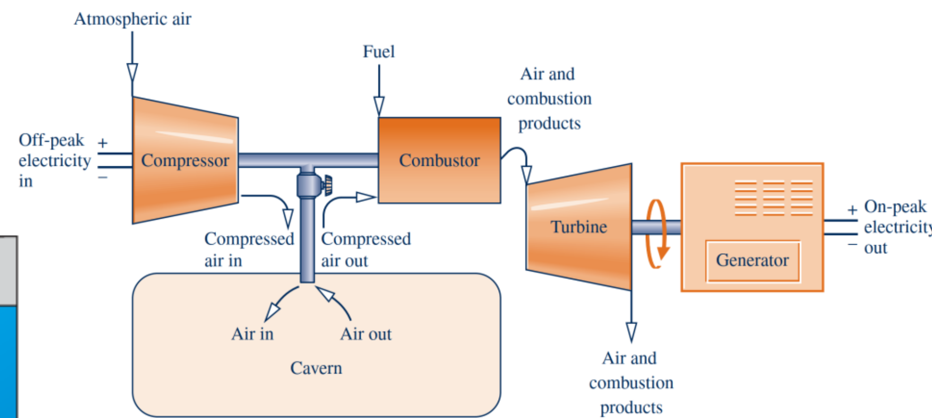


Fig. 4.12 Compressed-air storage.

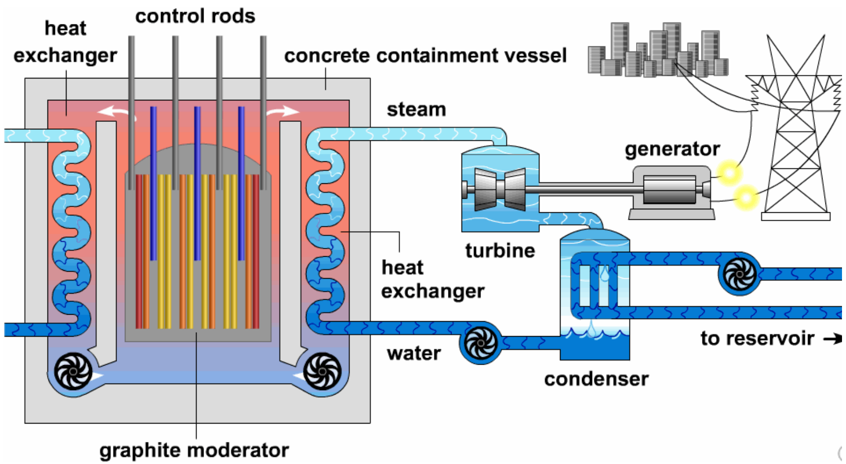
Heat Exchangers

Heat Exchangers

Heat exchangers have innumerable domestic and industrial applications, including use in home heating and cooling systems, automotive systems, electrical power generation, and chemical processing.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

$$0 = \dot{Q}_{cv} + \sum_i \dot{m}_i h_i - \sum_e \dot{m}_e h_e \qquad \text{(a)}$$



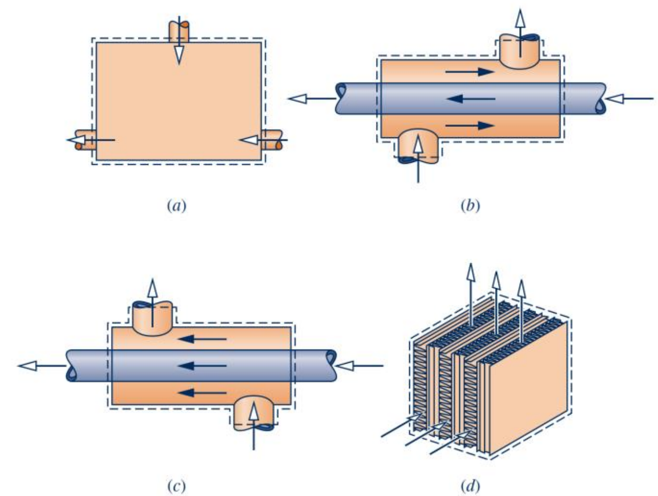


Fig. 4.13 Common heat exchanger types. (*a*) Direct-contact heat exchanger. (*b*) Tube-within-a-tube counterflow heat exchanger. (*c*) Tube-within-a-tube parallel-flow heat exchanger. (*d*) Cross-flow heat exchanger.

EXAMPLE 4.8

Cooling Computer Components

The electronic components of a computer are cooled by air flowing through a fan mounted at the inlet of the electronics enclosure. At steady state, air enters at 20°C, 1 atm. For noise control, the velocity of the entering air cannot exceed 1.3 m/s. For temperature control, the temperature of the air at the exit cannot exceed 32°C. The electronic components and fan receive, respectively, 80 W and 18 W of electric power. Determine the smallest fan inlet area, in cm^2, for which the limits on the entering air velocity and exit air temperature are met.

SOLUTION

Known: The electronic components of a computer are cooled by air flowing through a fan mounted at the inlet of the electronics enclosure. Conditions are specified for the air at the inlet and exit. The power required by the electronics and the fan is also specified.

Find: Determine the smallest fan area for which the specified limits are met.

Schematic and Given Data:

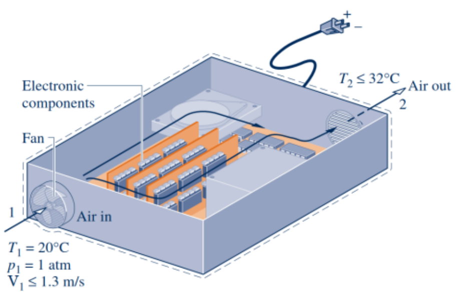


Fig. E4.8

Engineering Model:

1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer from the *outer* surface of the electronics enclosure to the surroundings is negligible. Thus, $\dot{Q}_{cv} = 0$.
3. ❶ Changes in kinetic and potential energies can be ignored.
4. ❷ Air is modeled as an ideal gas with $c_p = 1.005$ kJ/kg · K.

Analysis: The inlet area A_1 can be determined from the mass flow rate $\dot{m}$ and Eq. 4.4b, which can be rearranged to read

$$A_1 = \frac{\dot{m} v_1}{V_1} \tag{a}$$

The mass flow rate can be evaluated, in turn, from the steady-state energy rate balance, Eq. 4.20a.

$$0 = \underline{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \underline{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \underline{g(z_1 - z_2)}\right]$$

The mass flow rate can be evaluated, in turn, from the steady-state energy rate balance, Eq. 4.20a.

$$0 = \underline{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \underline{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \underline{g(z_1 - z_2)}\right]$$

The underlined terms drop out by assumptions 2 and 3, leaving

$$0 = -\dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

where $\dot{W}_{cv}$ accounts for the *total* electric power provided to the electronic components and the fan: $\dot{W}_{cv} = (-80\text{ W}) + (-18\text{ W}) = -98\text{ W}$. Solving for $\dot{m}$, and using assumption 4 with Eq. 3.51 to evaluate $(h_1 - h_2)$

$$\dot{m} = \frac{(-\dot{W}_{cv})}{c_p(T_2 - T_1)}$$

Introducing this into the expression for A_1, Eq. (a), and using the ideal gas model to evaluate the specific volume v_1

$$A_1 = \frac{1}{V_1}\left[\frac{(-\dot{W}_{cv})}{c_p(T_2 - T_1)}\right]\left(\frac{RT_1}{p_1}\right)$$

From this expression we see that A_1 *increases* when V_1 and/or T_2 *decrease*. Accordingly, since $V_1 \le 1.3$ m/s and $T_2 \le 305$ K (32°C), the inlet area must satisfy

$$A_1 \ge \frac{1}{1.3\text{ m/s}}\left[\frac{98\text{ W}}{\left(1.005\dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(305 - 293)\text{K}}\left|\frac{1\text{ kJ}}{10^3\text{ J}}\right|\left|\frac{1\text{ J/s}}{1\text{ W}}\right|\right]\left(\frac{\left(\dfrac{8314\text{ N}\cdot\text{m}}{28.97\text{ kg}\cdot\text{K}}\right)293\text{ K}}{1.01325\times 10^5\text{ N/m}^2}\right)\left|\frac{10^4\text{ cm}^2}{1\text{ m}^2}\right|$$

$$\ge 52\text{ cm}^2$$

For the specified conditions, the smallest fan area is 52 cm^2.

❶ Cooling air typically enters and exits electronic enclosures at low velocities, and thus kinetic energy effects are insignificant.

❷ The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart. Since the temperature of the air increases by no more than 12°C, the specific heat c_p is nearly constant (Table A-20).

Skills Developed

Ability to...

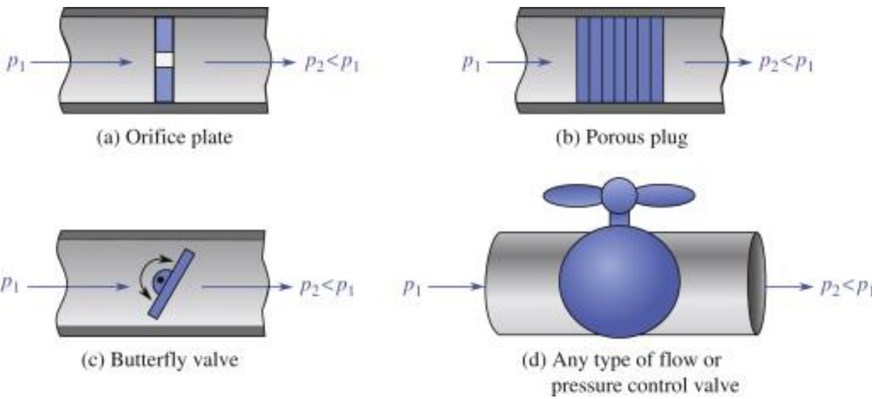
- ❑ apply the steady-state energy rate balance to a control volume.
- ❑ apply the mass flow rate expression, Eq. 4.4b.
- ❑ develop an engineering model.
- ❑ retrieve property data of air modeled as an ideal gas.

Quick Quiz If heat transfer occurs at a rate of 11 W from the outer surface of the computer case to the surroundings, determine the smallest fan inlet area for which the limits on entering air velocity and exit air temperature are met if the total power input remains at 98 W. **Ans.** 46 cm^2.

Throttling Devices

Throttling Devices

A significant reduction in pressure can be achieved simply by introducing a restriction into a line through which a gas or liquid flows. This is commonly done by means of a partially opened valve or a porous plug.



$$0 = \underline{\dot{Q}_{cv}} - \underline{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + \underline{g(z_1 - z_2)}\right]$$

$$0 = (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} \qquad \textbf{(a)}$$

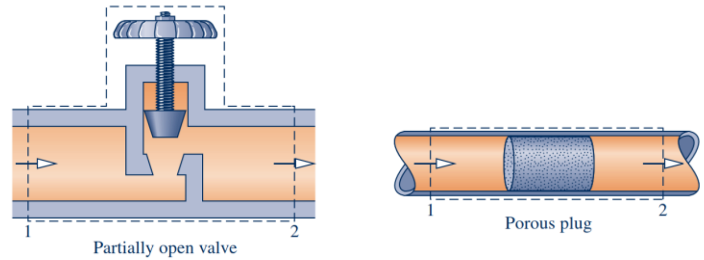


Fig. 4.15 Examples of throttling devices.

$$h_2 = h_1 \qquad (p_2 < p_1)$$

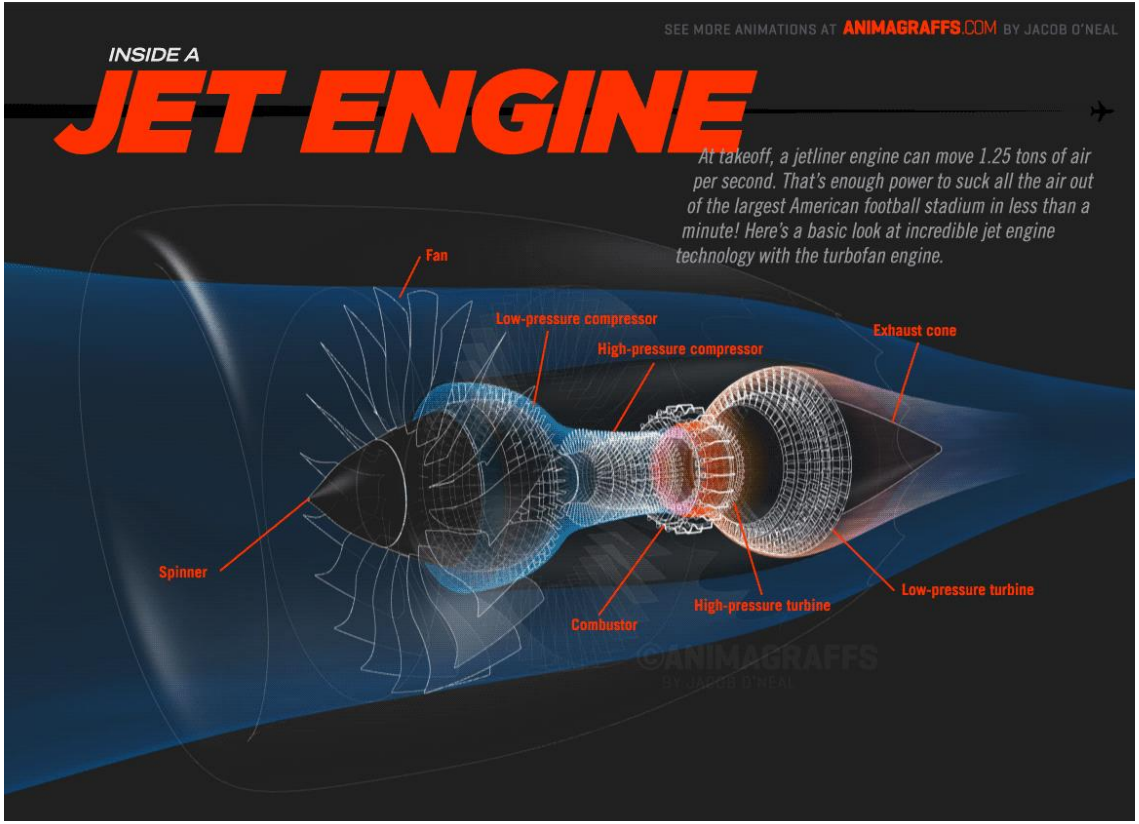
throttling process

When the flow through the valve or other restriction is idealized in this way, the process is called a throttling process.

System Integration

system integration

Engineers often must creatively combine components to achieve some overall objective, subject to constraints such as minimum total cost. This important engineering activity is called system integration.



Transient Analysis

transient operation

Many devices undergo periods of transient operation during which the state changes with time.

First, we place the control volume mass balance in a form that is suitable for transient analysis. We begin by integrating the mass rate balance, from time 0 to a final time t.

$$\int_0^t \left(\frac{dm_{cv}}{dt}\right) dt = \int_0^t \left(\sum_i \dot{m}_i\right) dt - \int_0^t \left(\sum_e \dot{m}_e\right) dt$$

This takes the form

$$m_{cv}(t) - m_{cv}(0) = \sum_i \left(\underline{\int_0^t \dot{m}_i\, dt}\right) - \sum_e \left(\underline{\int_0^t \dot{m}_e\, dt}\right)$$

Introducing the following symbols for the underlined terms

$$m_i = \int_0^t \dot{m}_i\, dt \quad \begin{cases} \text{amount of mass} \\ \text{entering the control} \\ \text{volume through inlet } i, \\ \text{from time 0 to } t \end{cases}$$

$$m_e = \int_0^t \dot{m}_e\, dt \quad \begin{cases} \text{amount of mass} \\ \text{exiting the control} \\ \text{volume through exit } e, \\ \text{from time 0 to } t \end{cases}$$

the mass balance becomes

$$\boxed{m_{cv}(t) - m_{cv}(0) = \sum_i m_i - \sum_e m_e} \qquad (4.23)$$

This states that the change in the amount of mass contained in the control volume equals the difference between the total incoming and outgoing amounts of mass.

The Energy Balance in Transient Analysis

Next, we integrate the energy rate balance, ignoring the effects of kinetic and potential energy.

$$U_{cv}(t) - U_{cv}(0) = Q_{cv} - W_{cv} + \sum_i \left(\underline{\int_0^t \dot{m}_i h_i \, dt} \right) - \sum_e \left(\underline{\int_0^t \dot{m}_e h_e \, dt} \right) \quad (4.24)$$

where Q_{cv} accounts for the net amount of energy transferred by heat into the control volume and W_{cv} accounts for the net amount of energy transferred by work, except for flow work. The integrals shown underlined in Eq. 4.24 account for the energy carried in at the inlets and out at the exits.

For the *special case* where the states at the inlets and exits are *constant with time,* the respective specific enthalpies, h_i and h_e, are constant, and the underlined terms of Eq. 4.24 become

$$\int_0^t \dot{m}_i h_i \, dt = h_i \int_0^t \dot{m}_i \, dt = h_i m_i$$

$$\int_0^t \dot{m}_e h_e \, dt = h_e \int_0^t \dot{m}_e \, dt = h_e m_e$$

Equation 4.24 then takes the following *special* form

$$U_{cv}(t) - U_{cv}(0) = Q_{cv} - W_{cv} + \sum_i m_i h_i - \sum_e m_e h_e \quad (4.25)$$

where m_i and m_e account, respectively, for the *amount* of mass entering the control volume through inlet i and exiting the control volume through exit e, each from time 0 to t.

Another special case is when the intensive properties within the control volume are uniform with position at a particular time t. Accordingly, the specific volume and the specific internal energy are uniform throughout and can depend only on time, that is, $v(t)$ and $u(t)$, respectively.

$$m_{cv}(t) = V_{cv}(t)/v(t) \quad (4.26)$$

$$U_{cv}(t) = m_{cv}(t)u(t) \quad (4.27)$$

EXAMPLE 4.11

Evaluating Heat Transfer for a Partially Emptying Tank

A tank having a volume of 0.85 m^3 initially contains water as a two-phase liquid–vapor mixture at 260°C and a quality of 0.7. Saturated water vapor at 260°C is slowly withdrawn through a pressure-regulating valve at the top of the tank as energy is transferred by heat to maintain constant pressure in the tank. This continues until the tank is filled with saturated vapor at 260°C. Determine the amount of heat transfer, in kJ. Neglect all kinetic and potential energy effects.

SOLUTION

Known: A tank initially holding a two-phase liquid–vapor mixture is heated while saturated water vapor is slowly removed. This continues at constant pressure until the tank is filled only with saturated vapor.

Find: Determine the amount of heat transfer.

Schematic and Given Data:

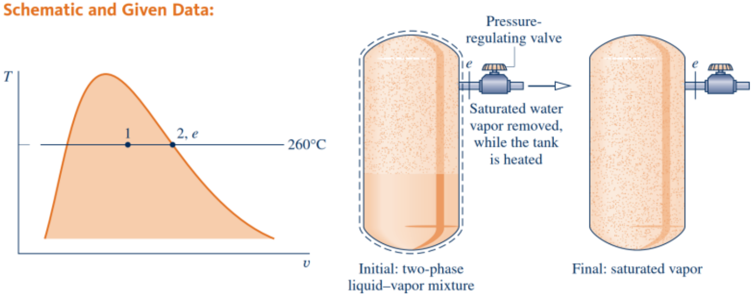


Fig. E4.11

Engineering Model:

1. The control volume is defined by the dashed line on the accompanying diagram.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be neglected.
3. At the exit the state remains constant.
4. ❶ The initial and final states of the mass within the vessel are equilibrium states.

Analysis: Since there is a single exit and no inlet, the mass rate balance Eq. 4.2 takes the form

$$\frac{dm_{cv}}{dt} = -\dot{m}_e$$

With assumption 2, the energy rate balance Eq. 4.15 reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Combining the mass and energy rate balances results in

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + h_e \frac{dm_{cv}}{dt}$$

By assumption 3, the specific enthalpy at the exit is constant. Accordingly, integration of the last equation gives

$$\Delta U_{cv} = Q_{cv} + h_e \Delta m_{cv}$$

Solving for the heat transfer Q_{cv},

$$Q_{cv} = \Delta U_{cv} - h_e \Delta m_{cv}$$

or

❷ $$Q_{cv} = (m_2 u_2 - m_1 u_1) - h_e(m_2 - m_1)$$

where m_1 and m_2 denote, respectively, the initial and final amounts of mass within the tank.

The terms u_1 and m_1 of the foregoing equation can be evaluated with property values from Table A-2 at 260°C and the given value for quality. Thus,

$$\begin{aligned} u_1 &= u_{f1} + x_1(u_{g1} - u_{f1}) \\ &= 1128.4 + (0.7)(2599.0 - 1128.4) = 2157.8 \text{ kJ/kg} \end{aligned}$$

Also,

$$\begin{aligned} v_1 &= v_{f1} + x_1(v_{g1} - v_{f1}) \\ &= 1.2755 \times 10^{-3} + (0.7)(0.04221 - 1.2755 \times 10^{-3}) = 29.93 \times 10^{-3} \text{ m}^3/\text{kg} \end{aligned}$$

Using the specific volume v_1, the mass initially contained in the tank is

$$m_1 = \frac{V}{v_1} = \frac{0.85 \text{ m}^3}{(29.93 \times 10^{-3} \text{ m}^3/\text{kg})} = 28.4 \text{ kg}$$

The final state of the mass in the tank is saturated vapor at 260°C so Table A-2 gives

$$u_2 = u_g(260°\text{C}) = 2599.0 \text{ kJ/kg}, \quad v_2 = v_g(260°\text{C}) = 42.21 \times 10^{-3} \text{ m}^3/\text{kg}$$

The mass contained within the tank at the end of the process is

$$m_2 = \frac{V}{v_2} = \frac{0.85 \text{ m}^3}{\quad} = 20.14 \text{ kg}$$

Substituting values into the expression for the heat transfer yields

$$\begin{aligned} Q_{cv} &= (20.14)(2599.0) - (28.4)(2157.8) - 2796.6(20.14 - 28.4) \\ &= 14{,}162 \text{ kJ} \end{aligned}$$

❶ In this case, idealizations are made about the state of the vapor exiting *and* the initial and final states of the mass contained within the tank.

❷ This expression for Q_{cv} can be obtained by applying Eqs. 4.23, 4.25, and 4.27. The details are left as an exercise.

Skills Developed

Ability to...

- ❑ apply the time-dependent mass and energy rate balances to a control volume.
- ❑ develop an engineering model.
- ❑ retrieve property data for water.

Quick Quiz If the initial quality were 90%, determine the heat transfer, in kJ, keeping all other data unchanged. **Ans.** 3707 kJ.

EXAMPLE 4.12

Using Steam for Emergency Power Generation

Steam at a pressure of 15 bar and a temperature of 320°C is contained in a large vessel. Connected to the vessel through a valve is a turbine followed by a small initially evacuated tank with a volume of 0.6 m^3. When emergency power is required, the valve is opened and the tank fills with steam until the pressure is 15 bar. The temperature in the tank is then 400°C. The filling process takes place adiabatically and kinetic and potential energy effects are negligible. Determine the amount of work developed by the turbine, in kJ.

SOLUTION

Known: Steam contained in a large vessel at a known state flows from the vessel through a turbine into a small tank of known volume until a specified final condition is attained in the tank.

Find: Determine the work developed by the turbine.

Schematic and Given Data:

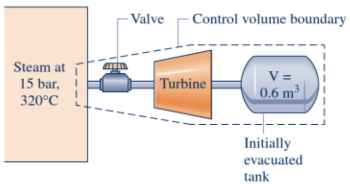


Fig. E4.12

Engineering Model:

1. The control volume is defined by the dashed line on the accompanying diagram.
2. For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. (1) The state of the steam within the large vessel remains constant. The final state of the steam in the smaller tank is an equilibrium state.
4. The amount of mass stored within the turbine and the interconnecting piping at the end of the filling process is negligible.

Analysis: Since the control volume has a single inlet and no exits, the mass rate balance, Eq. 4.2, reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

The energy rate balance, Eq. 4.15, reduces with assumption 2 to

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + \dot{m}_i h_i$$

Combining the mass and energy rate balances gives

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + h_i \frac{dm_{cv}}{dt}$$

Integrating

$$\Delta U_{cv} = -W_{cv} + h_i\,\Delta m_{cv}$$

In accordance with assumption 3, the specific enthalpy of the steam entering the control volume is constant at the value corresponding to the state in the large vessel.

Solving for W_{cv}

$$W_{cv} = h_i \Delta m_{cv} - \Delta U_{cv}$$

ΔU_{cv} and Δm_{cv} denote, respectively, the changes in internal energy and mass of the control volume. With assumption 4, these terms can be identified with the small tank only.

Since the tank is initially evacuated, the terms ΔU_{cv} and Δm_{cv} reduce to the internal energy and mass within the tank at the end of the process. That is,

$$\Delta U_{cv} = (m_2 u_2) - \cancel{(m_1 u_1)}^0, \quad \Delta m_{cv} = m_2 - \cancel{m_1}^0$$

where 1 and 2 denote the initial and final states within the tank, respectively.

Collecting results yields

(2) (3)
$$W_{cv} = m_2(h_i - u_2) \qquad \text{(a)}$$

The mass within the tank at the end of the process can be evaluated from the known volume and the specific volume of steam at 15 bar and 400°C from Table A-4

$$m_2 = \frac{V}{v_2} = \frac{0.6\ \text{m}^3}{(0.203\ \text{m}^3/\text{kg})} = 2.96\ \text{kg}$$

The specific internal energy of steam at 15 bar and 400°C from Table A-4 is 2951.3 kJ/kg. Also, at 15 bar and 320°C, h_i = 3081.9 kJ/kg.

Substituting values into Eq. (a)

$$W_{cv} = 2.96\ \text{kg}\ (3081.9 - 2951.3)\text{kJ/kg} = 386.6\ \text{kJ}$$

(1) In this case idealizations are made about the state of the steam entering the tank and the final state of the steam in the tank. These idealizations make the transient analysis manageable.

(2) A significant aspect of this example is the energy transfer into the control volume by flow work, incorporated in the pv term of the specific enthalpy at the inlet.

(3) This result can also be obtained by reducing Eq. 4.28. The details are left as an exercise.

Skills Developed

Ability to...

- ❑ apply the time-dependent mass and energy rate balances to a control volume.
- ❑ develop an engineering model.
- ❑ retrieve property data for water.

Quick Quiz If the turbine were removed, and the steam allowed to flow adiabatically into the small tank until the pressure in the tank is 15 bar, determine the final steam temperature in the tank, in °C. **Ans.** 477°C.

▸ **EXAMPLE 4.14** ▸

Determining Temperature-Time Variation in a Well-Stirred Tank

A tank containing 45 kg of liquid water initially at 45°C has one inlet and one exit with equal mass flow rates. Liquid water enters at 45°C and a mass flow rate of 270 kg/h. A cooling coil immersed in the water removes energy at a rate of 7.6 kW. The water is well mixed by a paddle wheel so that the water temperature is uniform throughout. The power input to the water from the paddle wheel is 0.6 kW. The pressures at the inlet and exit are equal and all kinetic and potential energy effects can be ignored. Plot the variation of water temperature with time.

SOLUTION

Known: Liquid water flows into and out of a well-stirred tank with equal mass flow rates as the water in the tank is cooled by a cooling coil.

Find: Plot the variation of water temperature with time.

Schematic and Given Data:

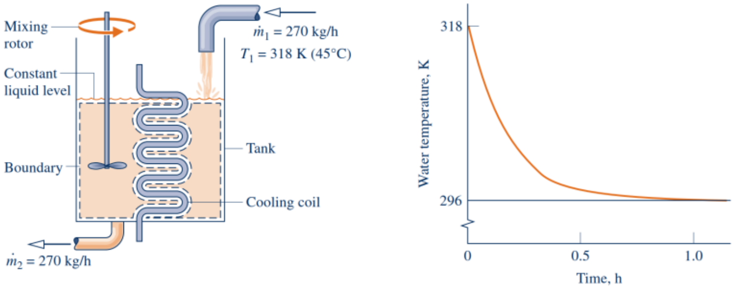


Fig. E4.14

Engineering Model:

1. The control volume is defined by the dashed line on the accompanying diagram.
2. For the control volume, the only significant heat transfer is with the cooling coil. Kinetic and potential energy effects can be neglected.
3. ❶ The water temperature is uniform with position throughout and varies only with time: $T = T(t)$.
4. The water in the tank is incompressible, and there is no change in pressure between inlet and exit.

Analysis: The energy rate balance, Eq. 4.15, reduces with assumption 2 to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

where $\dot{m}$ denotes the mass flow rate.

The mass contained within the control volume remains constant with time, so the term on the left side of the energy rate balance can be expressed as

$$\frac{dU_{cv}}{dt} = \frac{d(m_{cv}u)}{dt} = m_{cv}\frac{du}{dt}$$

Since the water is assumed incompressible, the specific internal energy depends on temperature only. Hence, the chain rule can be used to write

$$\frac{du}{dt} = \frac{du}{dT}\frac{dT}{dt} = c\frac{dT}{dt}$$

where c is the specific heat. Collecting results

$$\frac{dU_{cv}}{dt} = m_{cv}c\frac{dT}{dt}$$

With Eq. 3.20b the enthalpy term of the energy rate balance can be expressed as

$$h_1 - h_2 = c(T_1 - T_2) + v(\cancel{p_1 - p_2}^{0})$$

where the pressure term is dropped by assumption 4. Since the water is well mixed, the temperature at the exit equals the temperature of the overall quantity of liquid in the tank, so

$$h_1 - h_2 = c(T_1 - T)$$

where T represents the uniform water temperature at time t.

With the foregoing considerations the energy rate balance becomes

$$m_{cv}c\frac{dT}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}c(T_1 - T)$$

As can be verified by direct substitution, the solution of this first-order, ordinary differential equation is

$$T = C_1 \exp\left(-\frac{\dot{m}}{m_{cv}}t\right) + \left(\frac{\dot{Q}_{cv} - \dot{W}_{cv}}{\dot{m}c}\right) + T_1$$

The constant C_1 is evaluated using the initial condition: at $t = 0$, $T = T_1$. Finally,

Substituting given numerical values together with the specific heat c for liquid water from Table A-19

$$\begin{aligned} T &= 318\text{ K} + \left[\frac{[-7.6 - (-0.6)]\text{ kJ/s}}{\left(\frac{270}{3600}\frac{\text{kg}}{\text{s}}\right)\left(4.2\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\right]\left[1 - \exp\left(-\frac{270\text{ kg/h}}{45\text{ kg}}t\right)\right] \\ &= 318 - 22[1 - \exp(-6t)] \end{aligned}$$

where t is in hours. Using this expression, we construct the accompanying plot showing the variation of temperature with time.

❶ In this case idealizations are made about the state of the mass contained within the system and the states of the liquid entering and exiting. These idealizations make the transient analysis manageable.

Skills Developed

Ability to...

- ❑ apply the time-dependent mass and energy rate balances to a control volume.
- ❑ develop an engineering model.
- ❑ apply the incompressible substance model for water.
- ❑ solve an ordinary differential equation and plot the solution.

Quick Quiz What is the water temperature, in °C, when *steady state* is achieved? **Ans.** 23°C.

Aalto University
School of Engineering

KEY EQUATIONS

$$\dot{m} = \frac{AV}{v}$$

$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

$$\underset{\text{(mass rate in)}}{\sum_i \dot{m}_i} = \underset{\text{(mass rate out)}}{\sum_e \dot{m}_e}$$

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)$$

$$h_2 = h_1 \qquad (p_2 < p_1)$$