Thermodynamics and Heat Transfer Assignment Week 4
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Problem 1: A supply of geothermal hot water is used as the energy source in an ideal Rankine cycle with R-134a, as shown in Fig. 1. Saturated vapor R-134a leaves the boiler at a temperature of 85 °C, and the condenser temperature is 40 °C. Calculate the thermal efficiency of this cycle. Compare your results with NH_3 as working fluid (15 points).

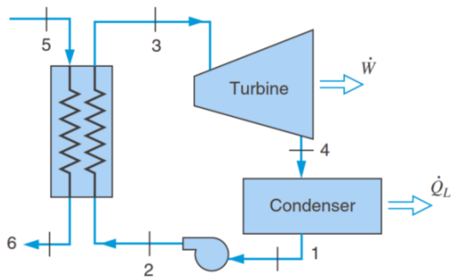

Figure 1

Exercise 1:

- For pump $(q=0)$: $w_{pump,in} = h_2 - h_1 = v(P_2 - P_1)$

  $T_2 = 85°C \Rightarrow P_2 = 2926.2$ kPa

  $T_1 = 40°C \Rightarrow P_1 = 1017$ kPa, $v = 0.000873$ (m³/kg), $h_1 = 256.54$ kJ/kg

  $\Rightarrow w_{pump,in} = 0.000873 \times (2926.2 - 1017) = 1.67$ kJ/kg

  $\Rightarrow h_2 = w_{pump,in} + h_1 = 1.67 + 256.54 = 258.21$ kJ/kg

- For boiler: $(w=0)$: $T_2 = 85°C$ saturated vapor $\Rightarrow h_3 = 428.10$ kJ/kg

  $\Rightarrow q_{in} = h_3 - h_2 = 428.10 - 258.21 = 169.89$ kJ/kg

- For turbine: $(q=0)$, isentropic $\Rightarrow s_4 = s_3 = 1.6782$ at $T_2 = 85°C$

  $T_1 = 40°C$ sat. liquid $\Rightarrow s_f = 1.1909$ kJ/k·K

  $T_1 = 40°C$ evap. $\Rightarrow s_{fg} = 0.5214$ kJ/k·K

  $$\Rightarrow x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{1.6782 - 1.1909}{0.5214} = 0.9346$$

  $T_1 = 40°C$ sat. liquid $\Rightarrow h_f = 256.24$ kJ/kg

  evap. $\Rightarrow h_{fg} = 163.28$ kJ/kg

  $\Rightarrow h_4 = h_f + x_4 h_{fg} = 256.24 + 0.9346 \times 163.28$
  $= 409.14$ kJ/kg

- Thus, $q_{out} = h_4 - h_1 = 409.14 - 256.54 = 152.6$ kJ/kg

  Thermal efficiency of the cycle is:

  $$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{152.6}{169.89} = 0.1017 = 10.17\% \text{ (answer)}$$

Doing the same calculations with NH_3, we have $\eta_{th} NH_3 = 88.66\%$

$\Rightarrow NH_3$ as a working fluid is more thermal efficient than R-134a

Problem 2: A utility runs a Rankine cycle with a water boiler at 3.0 MPa, and the cycle has the highest and lowest temperatures of 450∘C and 60∘C, respectively. Find the plant efficiency and the efficiency of a Carnot cycle with the same temperatures (10 points).

Exercise 2:

- For saturated water, we have: $T_1 = 60°C$ and $x_1 = 0$

$\Rightarrow v_1 = 0.001017\ m^3/kg$, $h_1 = 251.11\ kJ/kg$, $P_1 = 19.941\ kPa$

At $P_2 = 3\ MPa$ and $T_2 = 450°C \Rightarrow h_3 = 3344\ kJ/kg$, $s_3 = 7.0833\ kJ/kgK$

- Isentropic process $\Rightarrow w_{pump,in} = h_2 - h_1 = v_1(P_2 - P_1)$

$w_{pump,in} = 0.001017 \times (3\times10^3 - 19.941) = 3.03\ kJ/kg$

and $h_2 = w_{pump,in} + h_1 = 3.03 + 251.11 = 254.14\ kJ/kg$

- For boiler ($w = 0$): $T_2 = 450°C \Rightarrow h_3 = 3344\ kJ/kg$

$\Rightarrow q_{in} = h_3 - h_2 = 3344 - 254.14 = 3089.86\ kJ/kg$

- For turbine: ($q = 0$), isentropic $\Rightarrow s_4 = s_3 = 7.0833\ (kJ/kg\text{-}K)$

$T_1 = 60°$ sat. liquid $\Rightarrow s_f = 0.8311\ kJ/k\text{-}K$

$T_1 = 60°$ evap. $\Rightarrow s_{fg} = 7.0784\ kJ/k\text{-}K$

$$\Rightarrow x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{7.0833 - 0.8311}{7.0784} = 0.8833$$

$T_1 = 60°C$ sat. liquid $\Rightarrow h_f = 251.11\ kJ/kg$

evap. $\Rightarrow h_{fg} = 2358.48\ kJ/kg$

$\Rightarrow h_4 = h_f + x_4 h_{fg} = 251.11 + 0.8833 \times 2358.48$

$= 2334.36\ kJ/kg$

- Thus, $q_{out} = h_4 - h_1 = 2334.36 - 251.11 = 2083.25\ kJ/kg$
- Thermal efficiency of the Rankine cycle:

$$\eta_{th(Rankine)} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2083.25}{3089.86} = 0.3257 = 32.57\%$$

- Thermal efficiency of the Carnot cycle:

$$\eta_{th(Carnot)} = 1 - \frac{T_1}{T_2} = 1 - \frac{(60 + 273.15)K}{(450 + 273.15)K} = 53.93\%$$

$\Rightarrow$ Carnot cycle is more efficient than the Rankine cycle

Problem 3: Consider the ammonia Rankine-cycle power plant shown in Fig. 2. The plant was designed to operate in a location where the ocean water temperature is 25°C near the surface and 5°C at some greater depth. The mass flow rate of the working fluid is 1000 kg/s (15 points).

a. Determine the turbine power output and the pump power input for the cycle.

b. Determine the mass flow rate of ocean water through each heat exchanger.

c. What is the thermal efficiency of this power plant?

Exercise 3:

a) Determine W_{in} and W_{out} for the cycle. Liquid is ammonia

Isentropic process $\Rightarrow$ at $T_1 = 20°C \Rightarrow s_2 = s_1 = 5.0860\ kJ/kg\text{-}K$ and $h_1 = 1460.2\ kJ/kg$

$T_3 = 10°C$, sat. liquid $\Rightarrow s_f = 0.8779\ kJ/kg\text{-}K$

$T_3 = 10°C$, evap $\Rightarrow s_{fg} = 4.3266\ kJ/kg\text{-}K$

$$\Rightarrow x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{5.086 - 0.8779}{4.3266} = 0.9726$$

$T_3 = 10°C$, sat. liquid $\Rightarrow h_f = 226.97\ kJ/kg$

evap. $\Rightarrow h_{fg} = 1225.1\ kJ/kg$

$\Rightarrow h_2 = h_f + x_2 h_{fg} = 226.97 + 0.9726 \times 1225.1 = 1418.5\ kJ/kg$

$\Rightarrow w_{in} = h_1 - h_2 = 1460.2 - 1418.5 = 41.7\ kJ/kg$

$\Rightarrow$ Turbine power input: $W_{in} = \dot{m}\, w_{in} = 1000\ kg/s \times 41.7\ kJ/kg$

$= 41700\ kW$ (answer)

- For pump $(q = 0)$: $w_{out} = v_3 (P_4 - P_3)$

At $T_1 = 20°C \Rightarrow P_4 = 857.5\ kPa$

$T_3 = 10°C \Rightarrow P_3 = 615.2\ kPa$, $v_3 = 0.0016$

$\Rightarrow w_{out} = 0.0016(857.5 - 615.2) = 0.3877\ kJ/kg$

$\Rightarrow$ Turbine power output: $W_{out} = \dot{m}\, w_{out} = 1000\ kg/s \times 0.3877\ kJ/kg$

$= 387.7\ kW$ (answer)

b) Determine $\dot{m}$ through each heat exchange

- For heat exchange from $T_1 = 20°C$ to $T_2 = 10°C$

The net heat transfer by ammonia: $\dot{Q}_{12} = \dot{m}(h_2 - h_1)$

$\Rightarrow \dot{Q}_{12} = 1000\ kg/s\,(1418.5 - 226.97) = 1191530\ kW$

Specific heat capacity of water: $c = 4.184\ kJ/kg\text{-}C°$

$$\Rightarrow \dot{m}_{12} = \frac{\dot{Q}_{12}}{c\Delta T} = \frac{1191530\ kW}{4.184(7°C - 5°C)} = 141850\ kg/s \text{ (answer)}$$

- For heat exchange from cold to hotter water

We have $h_4 = h_3 + w_{out} = 226.97 + 0.3877 = 227.36\ kJ/kg$

$\Rightarrow \dot{Q}_{34} = \dot{m}(h_1 - h_4) = 1000\ kg/s\,(1460.2 - 227.36)$

$= 1232640\ kW$

$$\Rightarrow \dot{m}_{34} = \frac{\dot{Q}_{34}}{c\Delta T} = \frac{1232640\ kW}{4.184(25 - 23)°C} = 147304\ kg/s \text{ (answer)}$$

c) Thermal efficiency: $$\eta_{th} = \frac{\Delta \dot{W}}{\Delta \dot{Q}} = \frac{41700 - 387}{1232640} = 33.51\% \text{ (answer)}$$

Problem 4: A small power plant produces steam at 3 MPa, 600 °C in the boiler. It keeps the condenser at 45°C by transfer of 10 MW out as heat transfer. The first turbine section expands to 500 kPa, and then flow is reheated followed by expansion in the low-pressure turbine. Find the reheat temperature so that the turbine output is saturated vapor. For this reheat, find the total turbine power output and the boiler heat transfer (10 points).

Exercise 4:

- $T_1 = 45°C$, saturated water $\Rightarrow v_1 = 0.00101\ m^3/kg$, $h_1 = 188.42\ kJ/kg$
  $P_1 = 9.593\ kPa$
- $T_2 = 600°C$, $P_2 = 3\ MPa \Rightarrow h_2 = 3682.34\ kJ/kg$, $s_2 = 7.5084\ kJ/kg\text{-}K$
- $T_1 = 45°C$, saturated vapor $\Rightarrow h_3 = 2583.19\ kJ/kg$, $s_3 = 8.1647\ kJ/kg\text{-}K$
- For pump ($q = 0$): $w_{pump,in} = v_1(P_2 - P_1) = 0.00101(3000 - 9.59)$
  $= 3.02\ kJ/kg$
  $\Rightarrow h_{in} = h_1 + w_{pump,in} = 188.42 + 3.02 = 191.44\ kJ/kg$
- For high pressure turbine; $s_{HP} = s_2 = 7.5084\ kJ/kg\text{-}K$
  $\Rightarrow h_{HP} = 3093.26\ kJ/kg$, $T_{HP} = 314°C$
- For low pressure turbine: $s_{LP} = s_3 = 8.1647\ kJ/kg$, $T_{LP} = 529°C$ (answer)
- For condenser: $q_{out} = h_3 - h_1 = 2583.19 - 188.42 = 2394.77\ kJ/kg$
  Mass rate flow: $\dot{m} = \frac{\dot{Q}}{q} = \frac{10 \times 10^3}{2394.77} = 4.176\ kg/s$
- Total turbine power output: expand to 500 kPa $\Rightarrow h_{LP} = 3547.55\ kJ/kg$
  $\Sigma \dot{W}_{out} = \dot{m} w_{out} = \dot{m}(h_2 - h_3 + h_{LP} - h_{HP})$
  $= 4.176\ kg/s\,(3682.34 - 2583.19 + 3547.55 - 3093.26)\ kJ/kg$
  $= 6487\ kW$ (answer)
- The boiler heat transfer
  $\Sigma \dot{Q} = \dot{m}(h_2 - h_{in} + h_{LP} - h_{HP})$
  $= 4.1726\ kg/s\,(3682.34 - 191.44 + 3547.55 - 3093.26)\ kJ/kg$
  $= 16475.11\ kW$ (answer)