Thermodynamics and Heat Transfer

- Chapter 10: Vapor and Combined Power Cycles

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Learning Outcomes

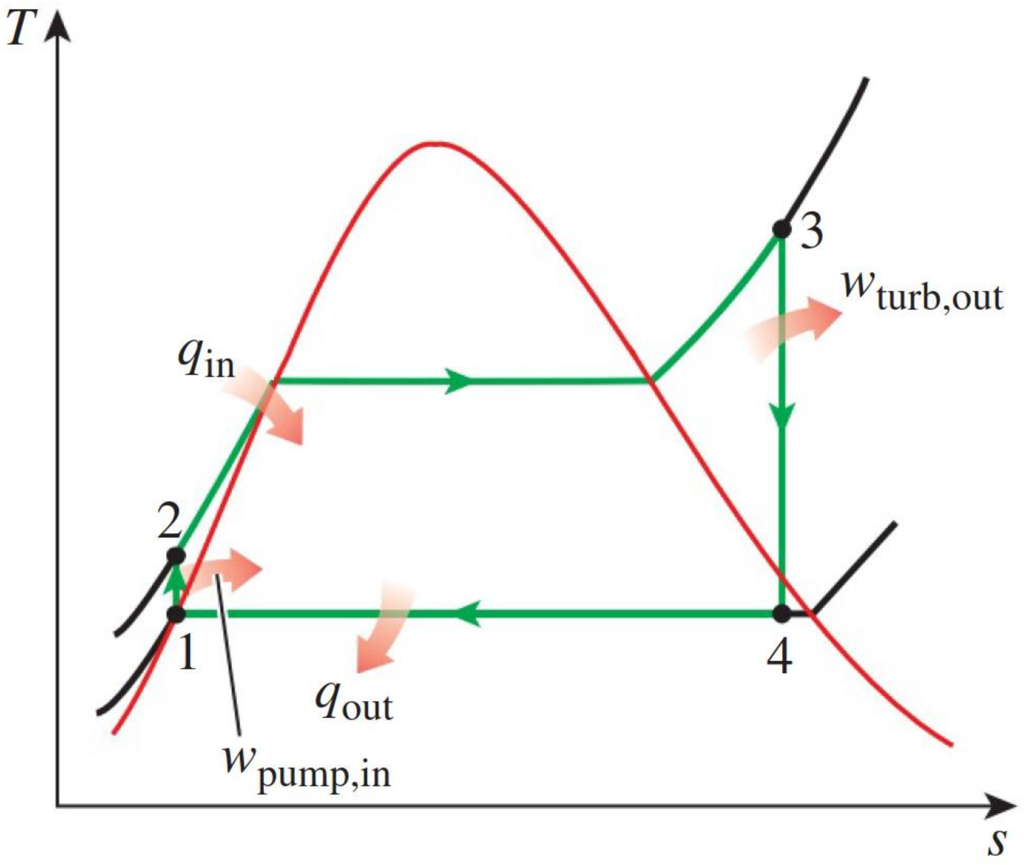
At the end of this topic:

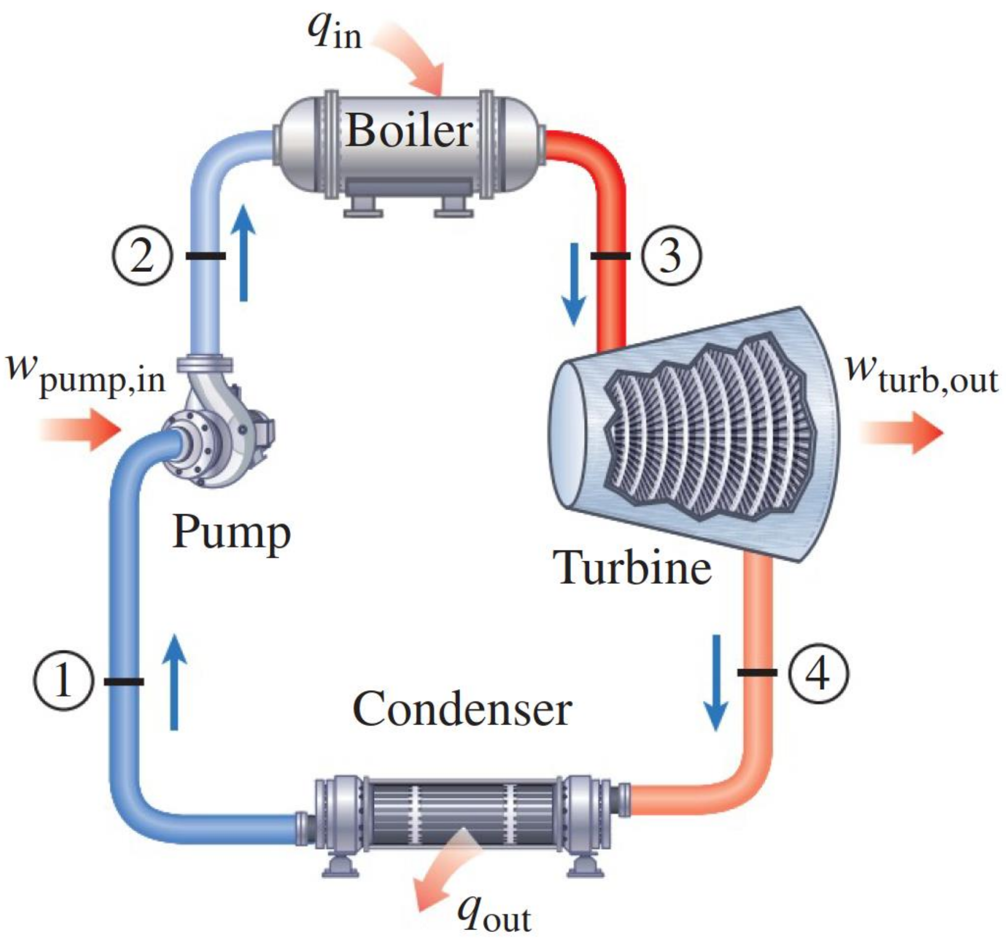
- You will be able to understand how power is generated by thermal power plants.
- You will be able to design a simple and complex thermal power plant.
- You will be able to design a combined gas-vapor power plant.

Power Generation

- Video credit: Energy 101; https://youtu.be/20Vb6hlLQSg?list=RDCMUCji_WmcDFPP1ZKutEl4MuYA

RANKINE CYCLE: THE IDEAL CYCLE FOR VAPOR POWER CYCLES



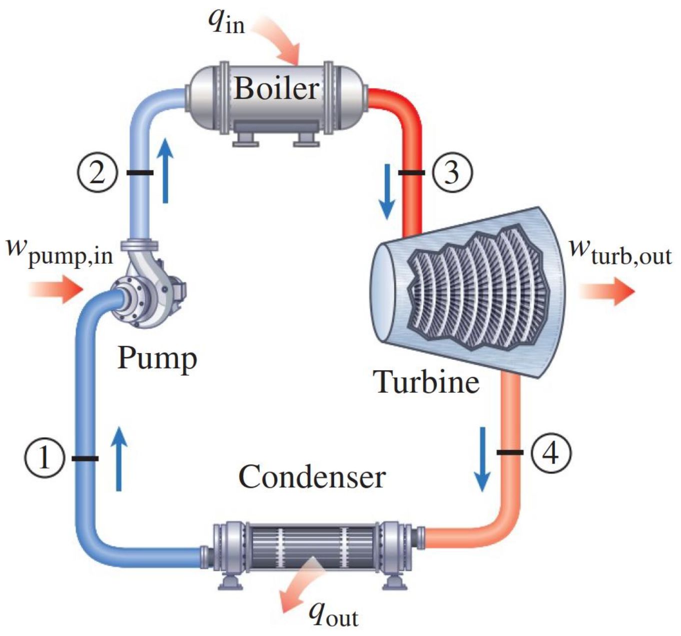


1-2 Isentropic compression in a pump
2-3 Constant pressure heat addition in a boiler
3-4 Isentropic expansion in a turbine
4-1 Constant pressure heat rejection in a condenser

Energy Analysis of the Ideal Rankine Cycle

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_e - h_i \quad \text{(kJ/kg)} \tag{10–1}$$

The boiler and the condenser do not involve any work, and the pump and the turbine are assumed to be isentropic. Then the conservation of energy relation for each device can be expressed as follows:



Pump ($q = 0$): $$w_{pump,in} = h_2 - h_1 \tag{10–2}$$

or,

$$w_{pump,in} = v(P_2 - P_1) \tag{10–3}$$

where

$$h_1 = h_{f\,@\,P_1} \quad \text{and} \quad v \cong v_1 = v_{f\,@\,P_1} \tag{10–4}$$

Boiler ($w = 0$): $$q_{in} = h_3 - h_2 \tag{10–5}$$

Turbine ($q = 0$): $$w_{turb,out} = h_3 - h_4 \tag{10–6}$$

Condenser ($w = 0$): $$q_{out} = h_4 - h_1 \tag{10–7}$$

The *thermal efficiency* of the Rankine cycle is determined from

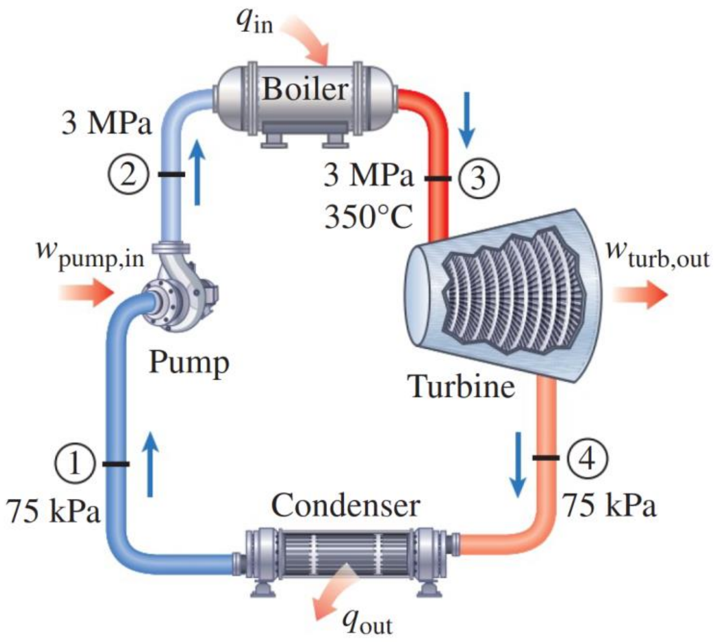
$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} \tag{10–8}$$

where

$$w_{net} = q_{in} - q_{out} = w_{turb,out} - w_{pump,in}$$

EXAMPLE 10–1 The Simple Ideal Rankine Cycle

Consider a steam power plant operating on the simple ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 75 kPa. Determine the thermal efficiency of this cycle.



State 1: $\left.\begin{array}{l} P_1 = 75 \text{ kPa} \\ \text{Sat. liquid} \end{array}\right\}$ $h_1 = h_{f\,@\,75\text{ kPa}} = 384.44 \text{ kJ/kg}$; $v_1 = v_{f\,@\,75\text{ kPa}} = 0.001037 \text{ m}^3\text{/kg}$

State 2: $P_2 = 3 \text{ MPa}$

$s_2 = s_1$

$$w_{\text{pump,in}} = v_1(P_2 - P_1) = (0.001037 \text{ m}^3\text{/kg})[(3000 - 75) \text{ kPa}]\left(\frac{1 \text{ kJ}}{1 \text{ kPa}\cdot\text{m}^3}\right)$$

$$= 3.03 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{\text{pump,in}} = (384.44 + 3.03) \text{ kJ/kg} = 387.47 \text{ kJ/kg}$$

State 3: $\left.\begin{array}{l} P_3 = 3 \text{ MPa} \\ T_3 = 350°\text{C} \end{array}\right\}$ $h_3 = 3116.1 \text{ kJ/kg}$; $s_3 = 6.7450 \text{ kJ/kg}\cdot\text{K}$

State 4: $P_4 = 75 \text{ kPa}$ (sat. mixture)

$s_4 = s_3$

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.7450 - 1.2132}{6.2426} = 0.8861$$

$$h_4 = h_f + x_4 h_{fg} = 384.44 + 0.8861(2278.0) = 2403.0 \text{ kJ/kg}$$

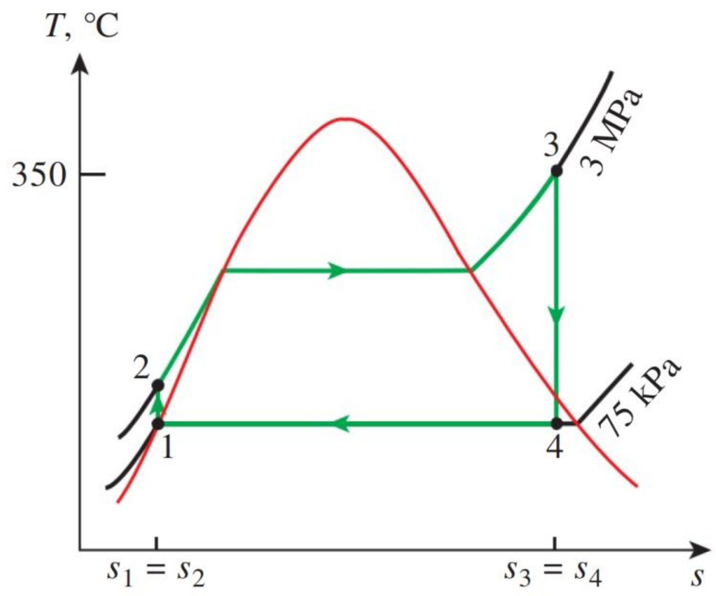
Thus,

$$q_{\text{in}} = h_3 - h_2 = (3116.1 - 387.47) \text{ kJ/kg} = 2728.6 \text{ kJ/kg}$$

$$q_{\text{out}} = h_4 - h_1 = (2403.0 - 384.44) \text{ kJ/kg} = 2018.6 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2018.6 \text{ kJ/kg}}{2728.6 \text{ kJ/kg}} = \mathbf{0.260} \text{ or } \mathbf{26.0\%}$$



The thermal efficiency could also be determined from

$$w_{\text{turb,out}} = h_3 - h_4 = (3116.1 - 2403.0) \text{ kJ/kg} = 713.1 \text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{turb,out}} - w_{\text{pump,in}} = (713.1 - 3.03) \text{ kJ/kg} = 710.1 \text{ kJ/kg}$$

or

$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = (2728.6 - 2018.6) \text{ kJ/kg} = 710.0 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{710.0 \text{ kJ/kg}}{2728.6 \text{ kJ/kg}} = \mathbf{0.260} \text{ or } \mathbf{26.0\%}$$

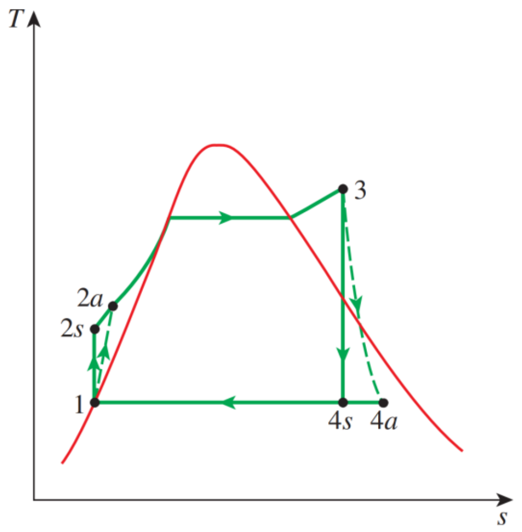
DEVIATION OF ACTUAL VAPOR POWER CYCLES FROM IDEALIZED ONES

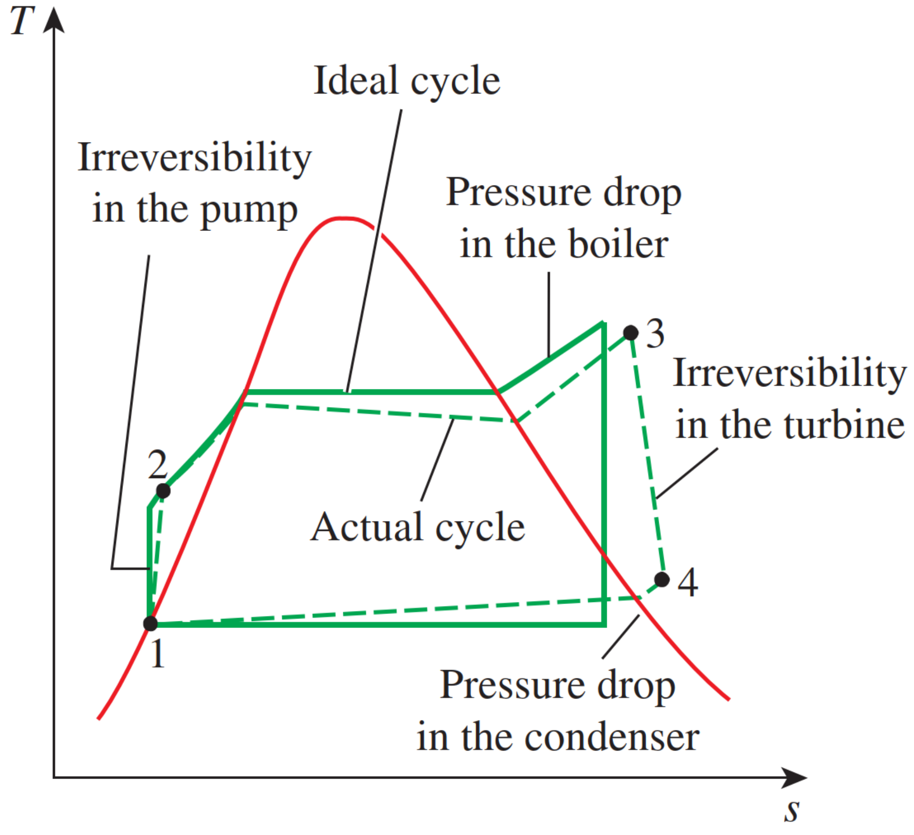
The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as:

$$\eta_P = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \qquad \textbf{(10–10)}$$

and

$$\eta_T = \frac{w_a}{w_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}} \qquad \textbf{(10–11)}$$





EXAMPLE 10–2 An Actual Steam Power Cycle

A steam power plant operates on the cycle shown in Fig. 10–5. If the isentropic efficiency of the turbine is 87 percent and the isentropic efficiency of the pump is 85 percent, determine (*a*) the thermal efficiency of the cycle and (*b*) the net power output of the plant for a mass flow rate of 15 kg/s.

Pump work input:

$$w_{\text{pump,in}} = \frac{w_{s,\text{pump,in}}}{\eta_P} = \frac{v_1(P_2 - P_1)}{\eta_P}$$

$$= \frac{(0.001009\ \text{m}^3/\text{kg})[(16{,}000 - 9)\ \text{kPa}]}{0.85}\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right)$$

$$= 19.0\ \text{kJ/kg}$$

Turbine work output:

$$w_{\text{turb,out}} = \eta_T w_{s,\text{turb,out}}$$

$$= \eta_T(h_5 - h_{6s}) = 0.87(3583.1 - 2115.3)\ \text{kJ/kg}$$

$$= 1277.0\ \text{kJ/kg}$$

Boiler heat input: $q_{\text{in}} = h_4 - h_3 = (3647.6 - 160.1)\ \text{kJ/kg} = 3487.5\ \text{kJ/kg}$

Thus,

$$w_{\text{net}} = w_{\text{turb,out}} - w_{\text{pump,in}} = (1277.0 - 19.0)\ \text{kJ/kg} = 1258.0\ \text{kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1258.0\ \text{kJ/kg}}{3487.5\ \text{kJ/kg}} = \mathbf{0.361}\ \text{or}\ \mathbf{36.1\%}$$

(*b*) The power produced by this power plant is

$$\dot{W}_{\text{net}} = \dot{m}\, w_{\text{net}} = (15\ \text{kg/s})(1258.0\ \text{kJ/kg}) = \mathbf{18.9\ MW}$$

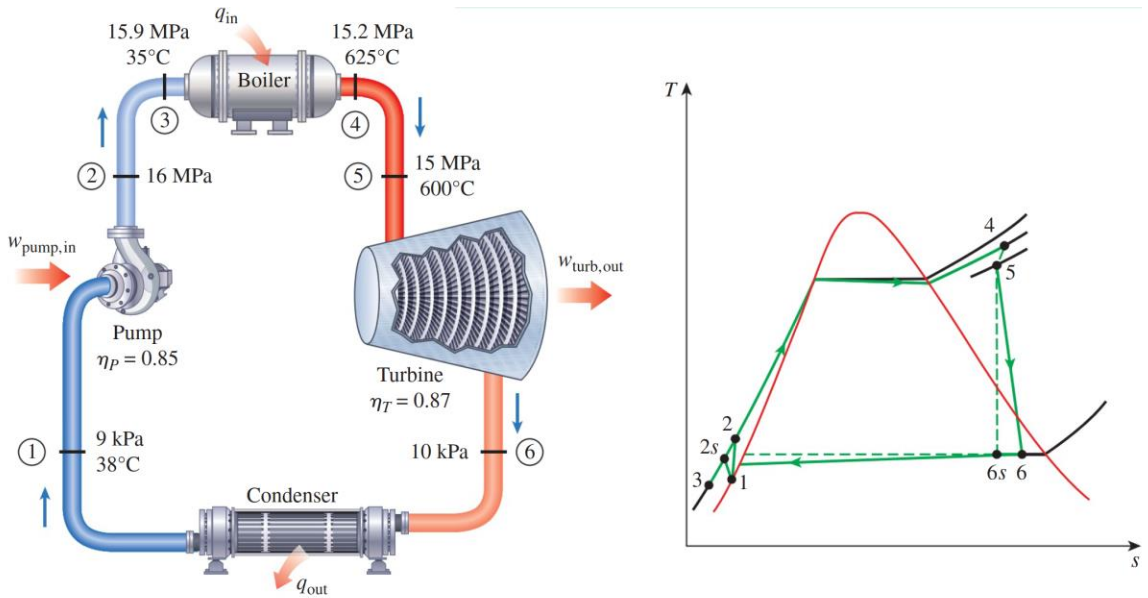


FIGURE 10–5
Schematic and *T-s* diagram for Example 10–2.

THE IDEAL REHEAT RANKINE CYCLE

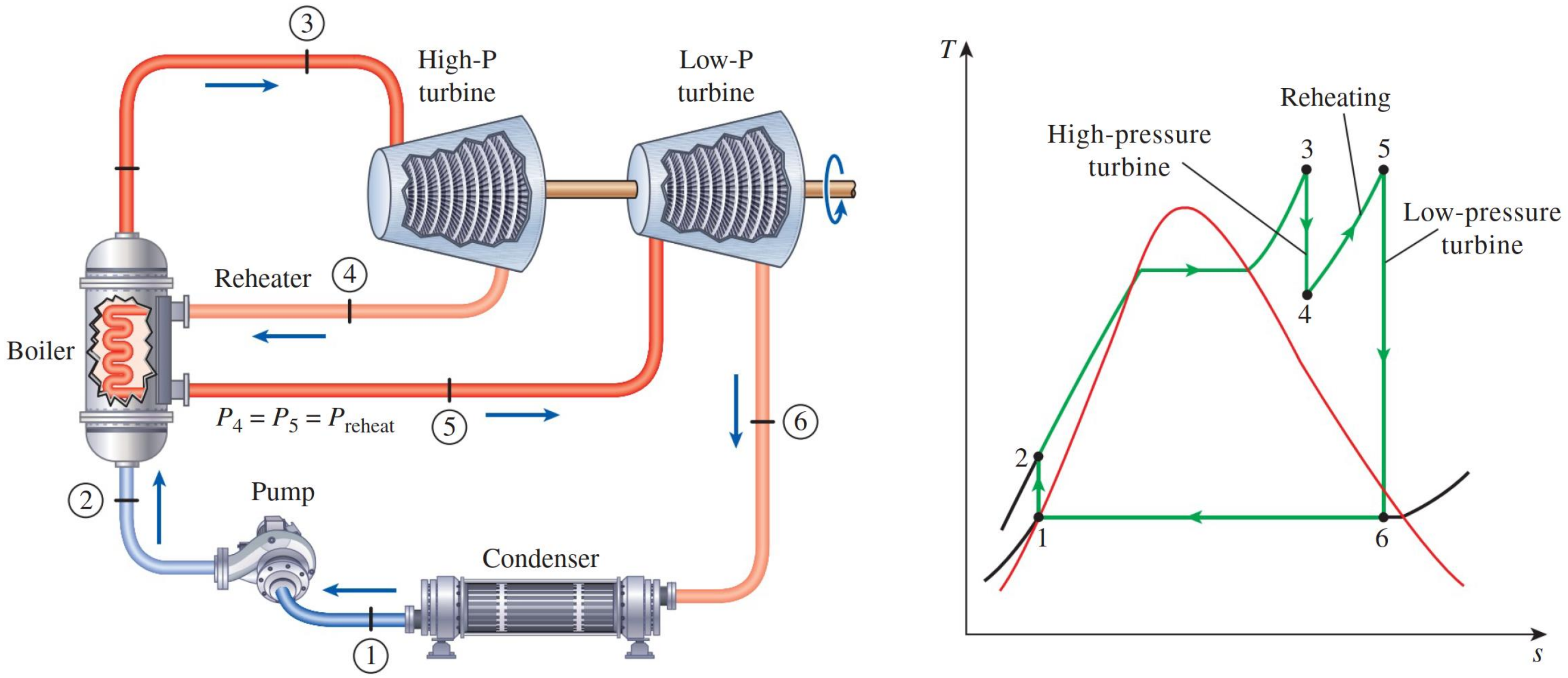


FIGURE 10–11
The ideal reheat Rankine cycle.

$$q_{in} = q_{primary} + q_{reheat} = (h_3 - h_2) + (h_5 - h_4) \qquad \textbf{(10–12)}$$

and

$$w_{turb,out} = w_{turb,I} + w_{turb,II} = (h_3 - h_4) + (h_5 - h_6) \qquad \textbf{(10–13)}$$

EXAMPLE 10–4 **The Ideal Reheat Rankine Cycle**

Consider a steam power plant operating on the ideal reheat Rankine cycle. Steam enters the high-pressure turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. If the moisture content of the steam at the exit of the low-pressure turbine is not to exceed 10.4 percent, determine (*a*) the pressure at which the steam should be reheated and (*b*) the thermal efficiency of the cycle. Assume the steam is reheated to the inlet temperature of the high-pressure turbine.

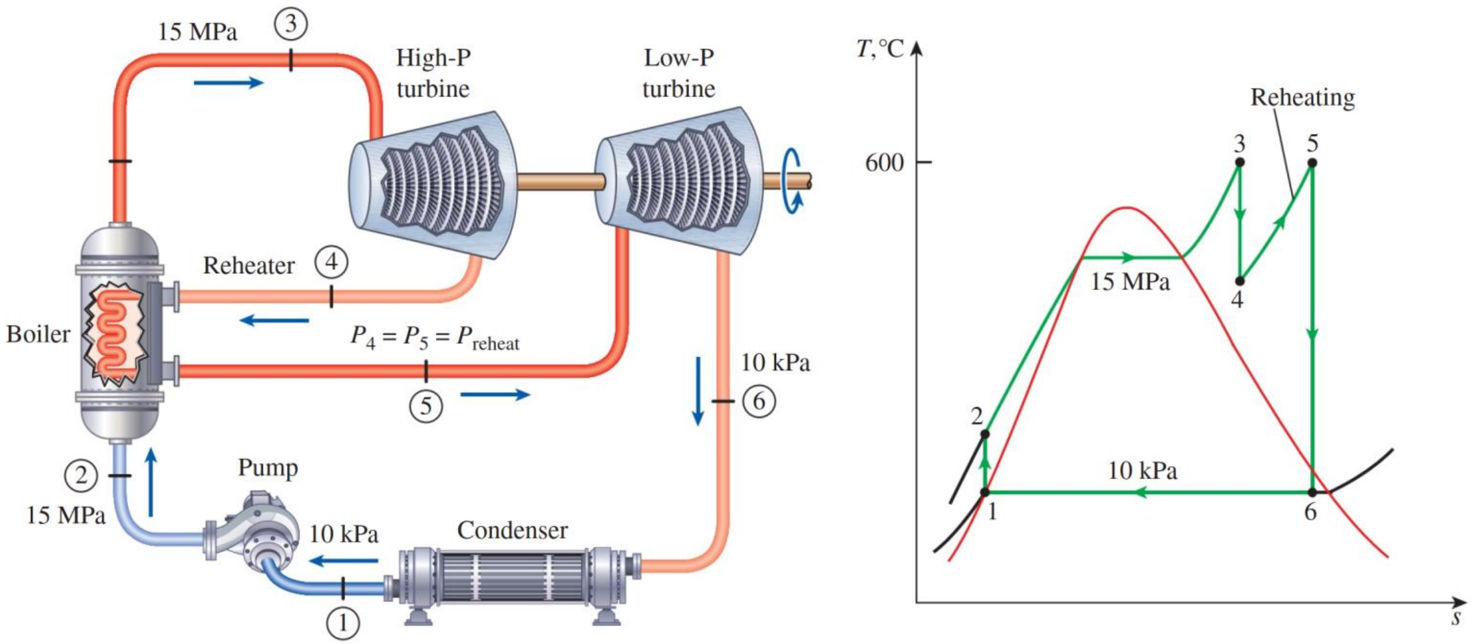


FIGURE 10–13
Schematic and *T-s* diagram for Example 10–4.

(a) The reheat pressure is determined from the requirement that the entropies at states 5 and 6 be the same:

State 6:

$$P_6 = 10 \text{ kPa}$$

$$x_6 = 0.896 \quad \text{(sat. mixture)}$$

$$s_6 = s_f + x_6 s_{fg} = 0.6492 + 0.896(7.4996) = 7.3688 \text{ kJ/kg·K}$$

Also,

$$h_6 = h_f + x_6 h_{fg} = 191.81 + 0.896(2392.1) = 2335.1 \text{ kJ/kg}$$

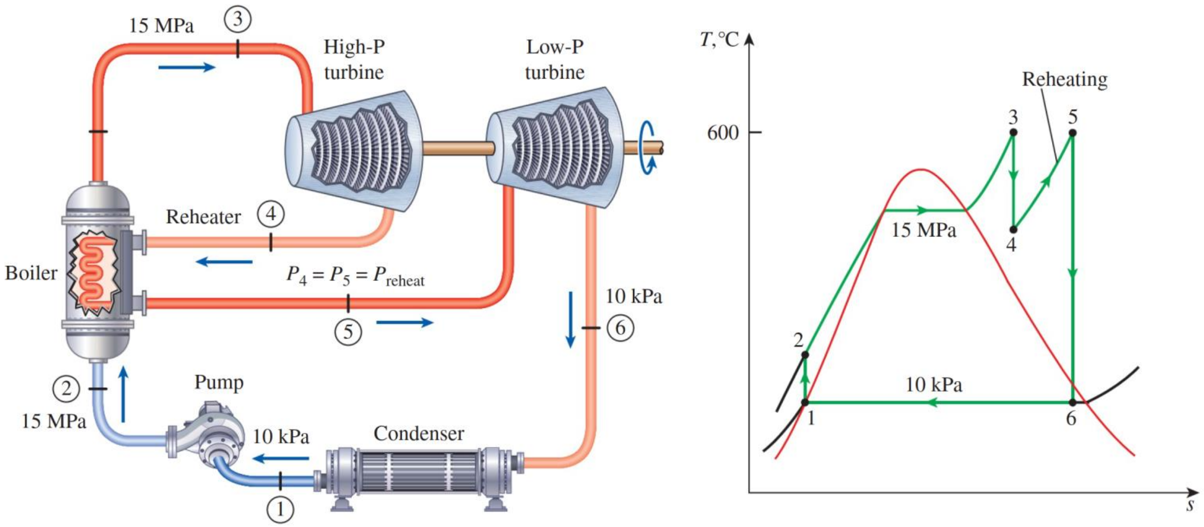


FIGURE 10–13
Schematic and *T-s* diagram for Example 10–4.

Thus,

State 5: $\left.\begin{array}{l} T_5 = 600°\text{C} \\ s_5 = s_6 \end{array}\right\} \begin{array}{l} P_5 = \mathbf{4.0 \text{ MPa}} \\ h_5 = 3674.9 \text{ kJ/kg} \end{array}$

Therefore, steam should be reheated at a pressure of 4 MPa or lower to prevent a moisture content above 10.4 percent.

(b) To determine the thermal efficiency, we need to know the enthalpies at all other states:

State 1: $\left.\begin{array}{l} P_1 = 10 \text{ kPa} \\ \text{Sat. liquid} \end{array}\right\} \begin{array}{l} h_1 = h_{f @ 10 \text{ kPa}} = 191.81 \text{ kJ/kg} \\ v_1 = v_{f @ 10 \text{ kPa}} = 0.00101 \text{ m}^3\text{/kg} \end{array}$

State 2:

$$P_2 = 15 \text{ MPa}$$

$$s_2 = s_1$$

$$w_{\text{pump,in}} = v_1(P_2 - P_1) = (0.00101 \text{ m}^3\text{/kg}) \times [(15{,}000 - 10)\text{kPa}]\left(\frac{1 \text{ kJ}}{1 \text{ kPa·m}^3}\right)$$

$$= 15.14 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{\text{pump,in}} = (191.81 + 15.14) \text{ kJ/kg} = 206.95 \text{ kJ/kg}$$

State 3: $\left.\begin{array}{l} P_3 = 15 \text{ MPa} \\ T_3 = 600°\text{C} \end{array}\right\} \begin{array}{l} h_3 = 3583.1 \text{ kJ/kg} \\ s_3 = 6.6796 \text{ kJ/kg·K} \end{array}$

State 4: $\left.\begin{array}{l} P_4 = 4 \text{ MPa} \\ s_4 = s_3 \end{array}\right\} \begin{array}{l} h_4 = 3155.0 \text{ kJ/kg} \\ (T_4 = 375.5°\text{C}) \end{array}$

Thus

$$q_{\text{in}} = (h_3 - h_2) + (h_5 - h_4)$$

$$= (3583.1 - 206.95) \text{ kJ/kg} + (3674.9 - 3155.0) \text{ kJ/kg}$$

$$= 3896.1 \text{ kJ/kg}$$

$$q_{\text{out}} = h_6 - h_1 = (2335.1 - 191.81) \text{ kJ/kg}$$

$$= 2143.3 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2143.3 \text{ kJ/kg}}{3896.1 \text{ kJ/kg}} = \mathbf{0.450} \text{ or } \mathbf{45.0\%}$$

THE IDEAL REGENERATIVE RANKINE CYCLE

Open Feedwater Heaters

An open (or direct-contact) feedwater heater is basically a mixing chamber, where the steam extracted from the turbine mixes with the feed-water exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure. The schematic of a steam power plant with one open feedwater heater (also called single-stage regenerative cycle) and the T-s diagram of the cycle are shown in Fig. 10–15.

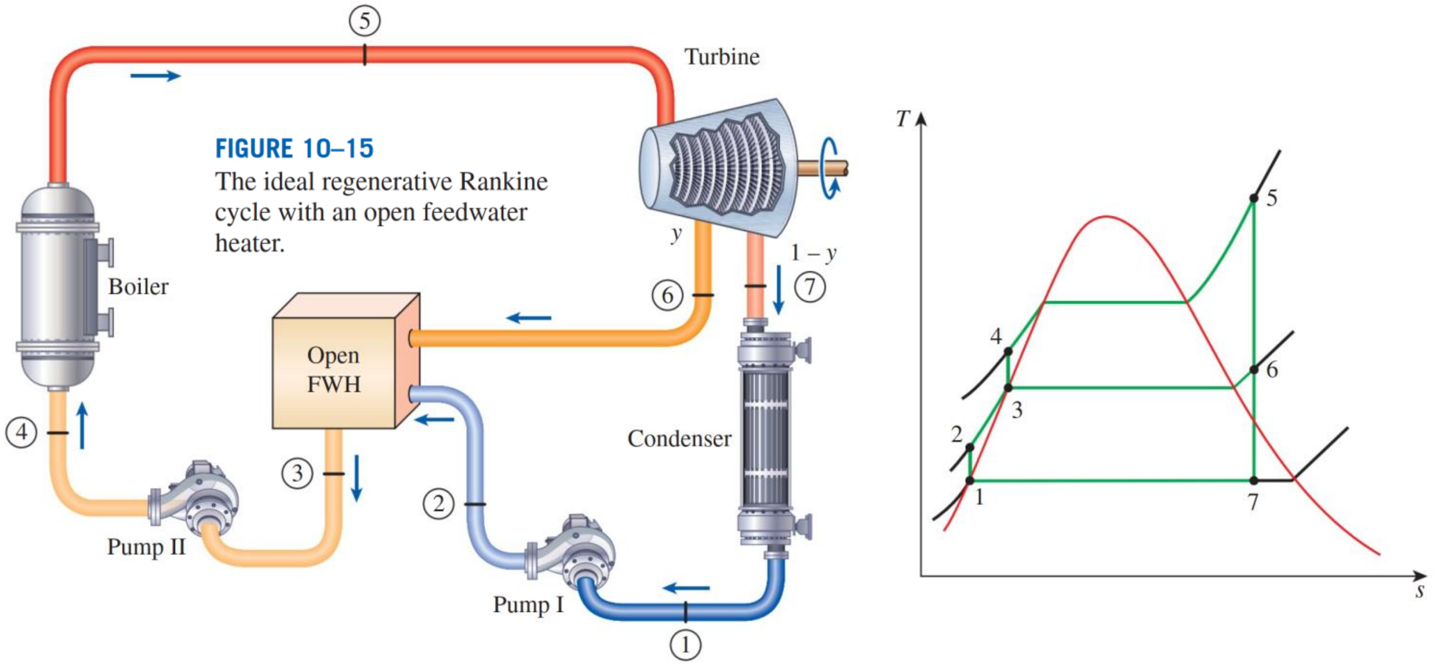


FIGURE 10–15
The ideal regenerative Rankine cycle with an open feedwater heater.

$$q_{in} = h_5 - h_4 \qquad \textbf{(10–14)}$$

$$q_{out} = (1 - y)(h_7 - h_1) \qquad \textbf{(10–15)}$$

$$w_{turb,out} = (h_5 - h_6) + (1 - y)(h_6 - h_7) \qquad \textbf{(10–16)}$$

$$w_{pump,in} = (1 - y)w_{pump\ I,in} + w_{pump\ II,in} \qquad \textbf{(10–17)}$$

where

$$y = \dot{m}_6/\dot{m}_5 \qquad \text{(fraction of steam extracted)}$$

$$w_{pump\ I,in} = v_1(P_2 - P_1)$$

$$w_{pump\ II,in} = v_3(P_4 - P_3)$$

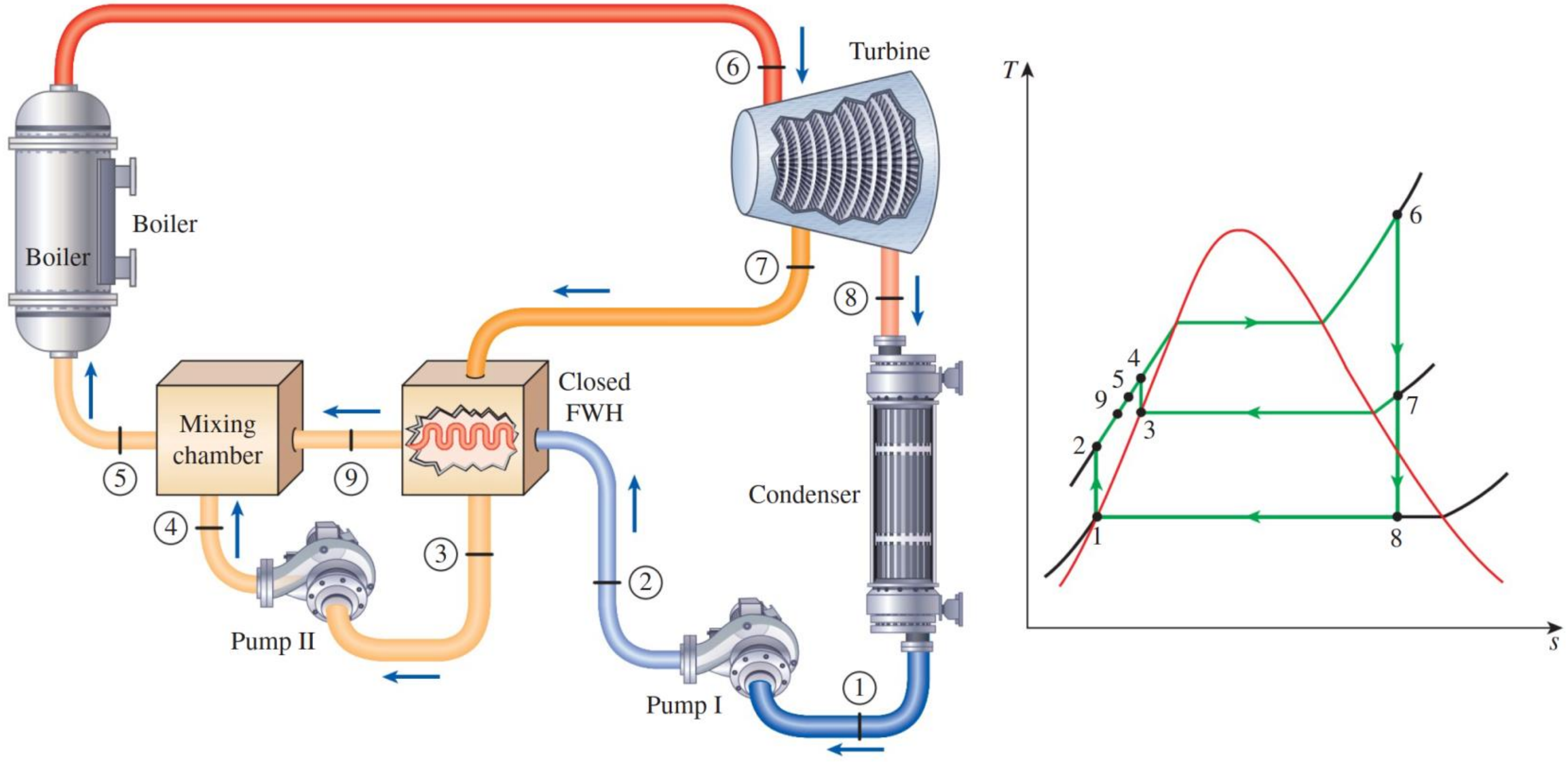


FIGURE 10–16
The ideal regenerative Rankine cycle with a closed feedwater heater.

Closed Feedwater Heaters

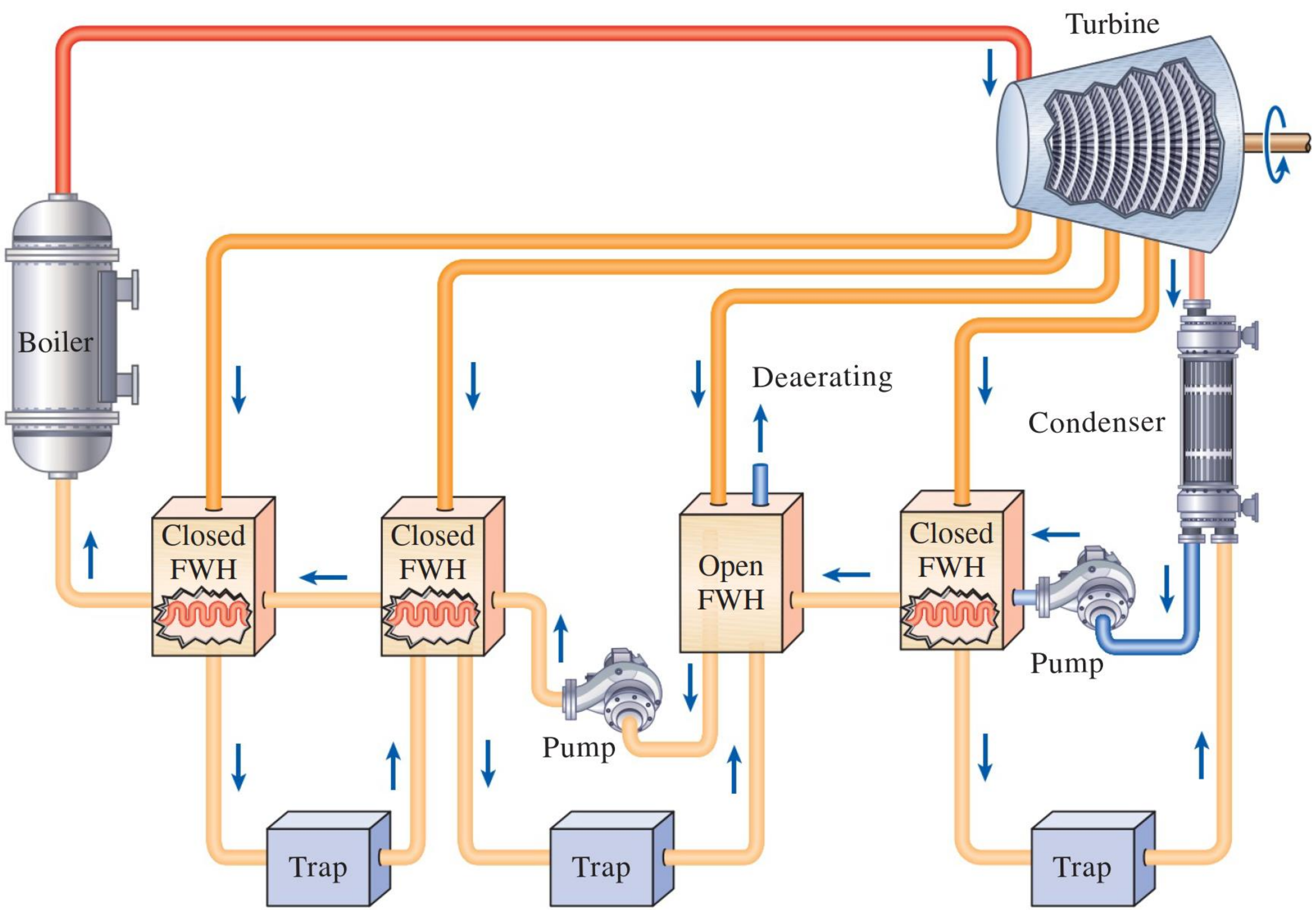


FIGURE 10–17
A steam power plant with one open and three closed feedwater heaters.

EXAMPLE 10–5 The Ideal Regenerative Rankine Cycle

Consider a steam power plant operating on the ideal regenerative Rankine cycle with one open feedwater heater. Steam enters the turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. Some steam leaves the turbine at a pressure of 1.2 MPa and enters the open feedwater heater. Determine the fraction of steam extracted from the turbine and the thermal efficiency of the cycle.

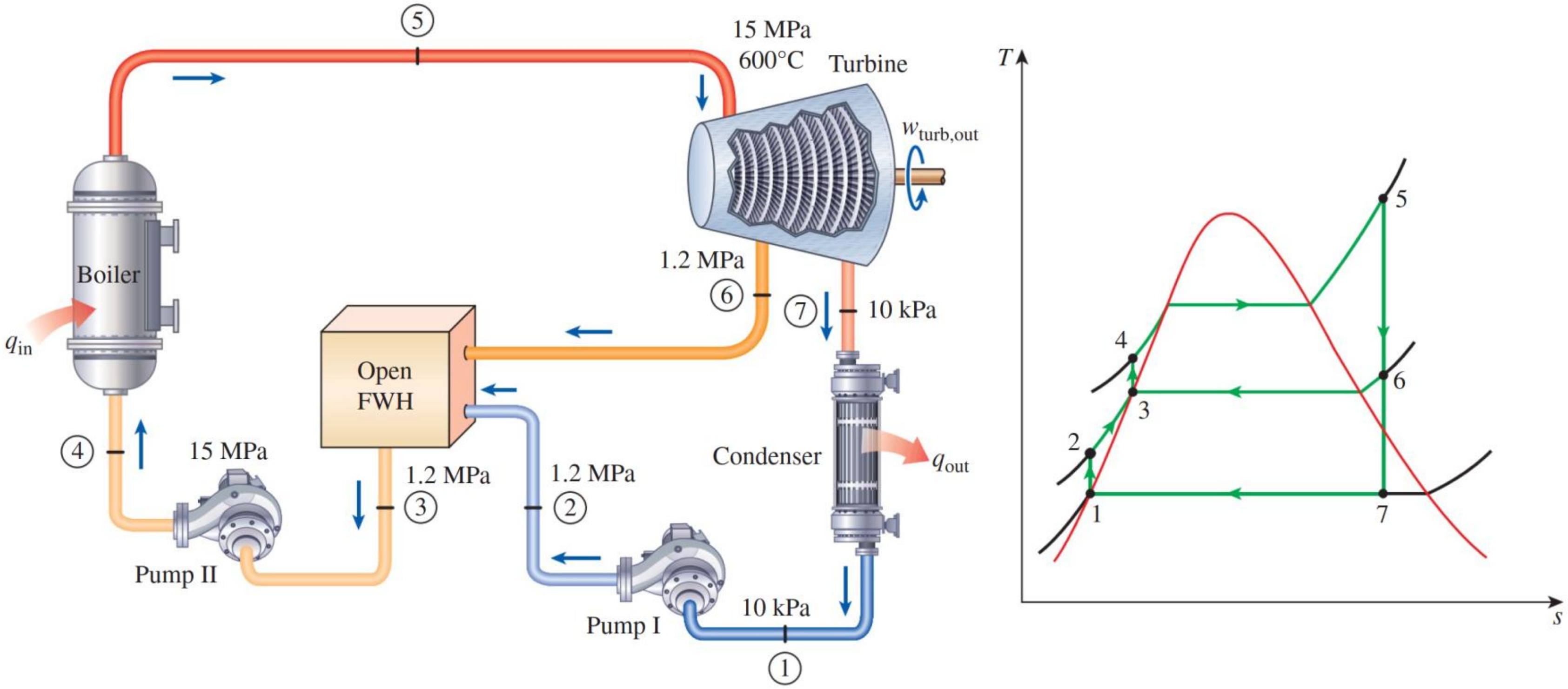


FIGURE 10–18
Schematic and *T-s* diagram for Example 10–5.

State 1: $\left.\begin{array}{l} P_1 = 10 \text{ kPa} \\ \text{Sat. liquid} \end{array}\right\}$ $\begin{array}{l} h_1 = h_{f\ @\ 10\text{ kPa}} = 191.81 \text{ kJ/kg} \\ v_1 = v_{f\ @\ 10\text{ kPa}} = 0.00101 \text{ m}^3\text{/kg} \end{array}$

State 2: $P_2 = 1.2$ MPa

$s_2 = s_1$

$$w_{\text{pump I,in}} = v_1(P_2 - P_1) = (0.00101 \text{ m}^3\text{/kg})[(1200 - 10) \text{ kPa}]\left(\frac{1 \text{ kJ}}{1 \text{ kPa}\cdot\text{m}^3}\right)$$

$$= 1.20 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{\text{pump I,in}} = (191.81 + 1.20) \text{ kJ/kg} = 193.01 \text{ kJ/kg}$$

State 3: $\left.\begin{array}{l} P_3 = 1.2 \text{ MPa} \\ \text{Sat. liquid} \end{array}\right\}$ $\begin{array}{l} v_3 = v_{f\ @\ 1.2\text{ MPa}} = 0.001138 \text{ m}^3\text{/kg} \\ h_3 = h_{f\ @\ 1.2\text{ MPa}} = 798.33 \text{ kJ/kg} \end{array}$

State 4: $P_4 = 15$ MPa

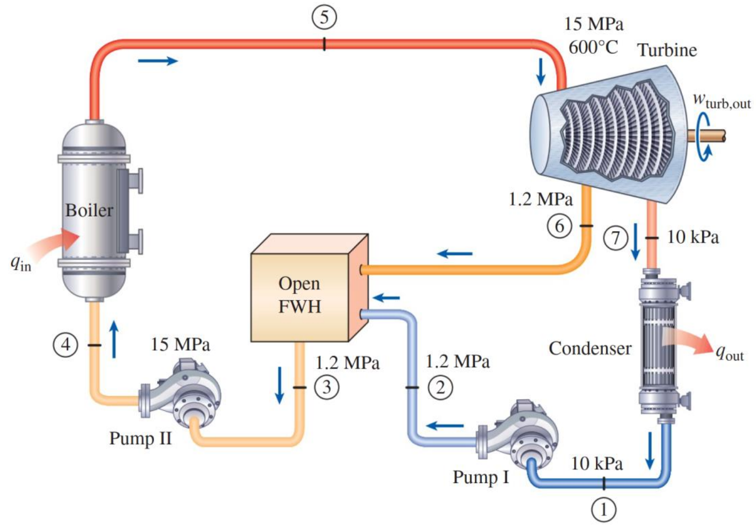
$s_4 = s_3$

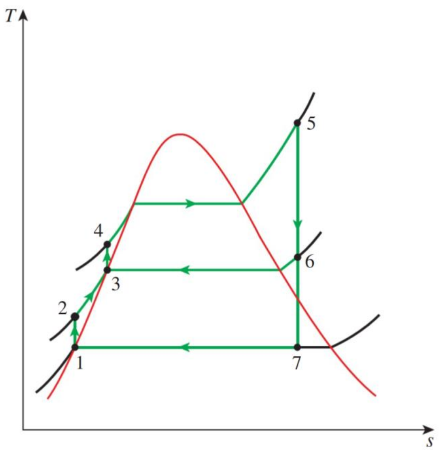
$$w_{\text{pump II,in}} = v_3(P_4 - P_3)$$

$$= (0.001138 \text{ m}^3\text{/kg})[(15{,}000 - 1200) \text{ kPa}]\left(\frac{1 \text{ kJ}}{1 \text{ kPa}\cdot\text{m}^3}\right)$$

$$= 15.70 \text{ kJ/kg}$$

$$h_4 = h_3 + w_{\text{pump II,in}} = (798.33 + 15.70) \text{ kJ/kg} = 814.03 \text{ kJ/kg}$$





State 5: $\left.\begin{array}{l} P_5 = 15 \text{ MPa} \\ T_5 = 600°\text{C} \end{array}\right\}$ $\begin{array}{l} h_5 = 3583.1 \text{ kJ/kg} \\ s_5 = 6.6796 \text{ kJ/kg}\cdot\text{K} \end{array}$

State 6: $\left.\begin{array}{l} P_6 = 1.2 \text{ MPa} \\ s_6 = s_5 \end{array}\right\}$ $\begin{array}{l} h_6 = 2860.2 \text{ kJ/kg} \\ (T_6 = 218.4°\text{C}) \end{array}$

State 7: $P_7 = 10$ kPa

$$s_7 = s_5 \quad x_7 = \frac{s_7 - s_f}{s_{fg}} = \frac{6.6796 - 0.6492}{7.4996} = 0.8041$$

$$h_7 = h_f + x_7 h_{fg} = 191.81 + 0.8041(2392.1) = 2115.3 \text{ kJ/kg}$$

The energy analysis of open feedwater heaters is identical to the energy analysis of mixing chambers. The feedwater heaters are generally well insulated ($\dot{Q} = 0$), and they do not involve any work interactions ($\dot{W} = 0$). By neglecting the kinetic and potential energies of the streams, the energy balance reduces for a feedwater heater to

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}} \longrightarrow \sum_{\text{in}} \dot{m}h = \sum_{\text{out}} \dot{m}h$$

or

$$yh_6 + (1 - y)h_2 = 1(h_3)$$

where y is the fraction of steam extracted from the turbine ($= \dot{m}_6/\dot{m}_5$). Solving for y and substituting the enthalpy values, we find

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{798.33 - 193.01}{2860.2 - 193.01} = \mathbf{0.2270}$$

Thus,

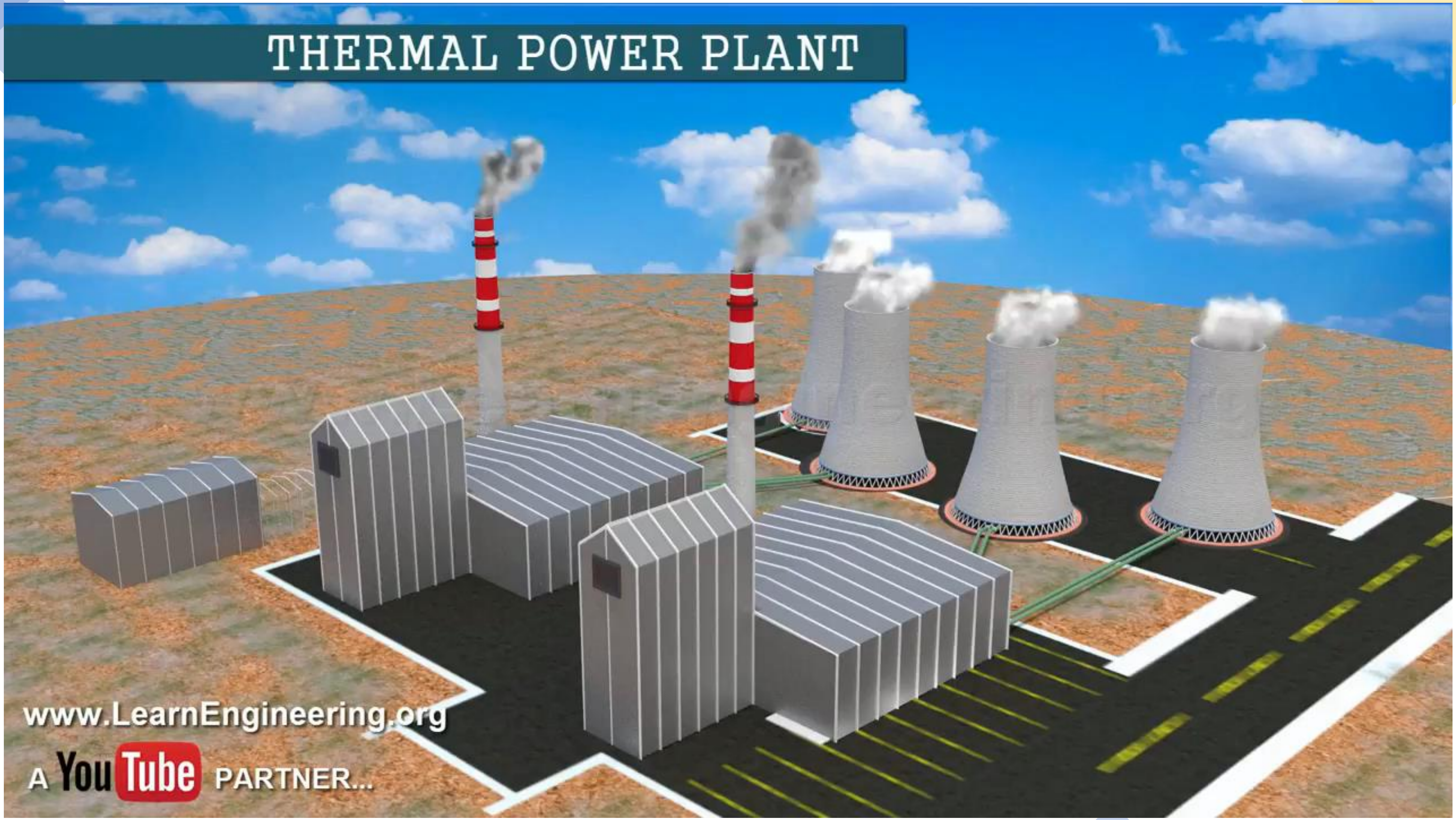
$$q_{\text{in}} = h_5 - h_4 = (3583.1 - 814.03) \text{ kJ/kg} = 2769.1 \text{ kJ/kg}$$

$$q_{\text{out}} = (1 - y)(h_7 - h_1) = (1 - 0.2270)(2115.3 - 191.81) \text{ kJ/kg}$$

$$= 1486.9 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1486.9 \text{ kJ/kg}}{2769.1 \text{ kJ/kg}} = \mathbf{0.463} \text{ or } \mathbf{46.3\%}$$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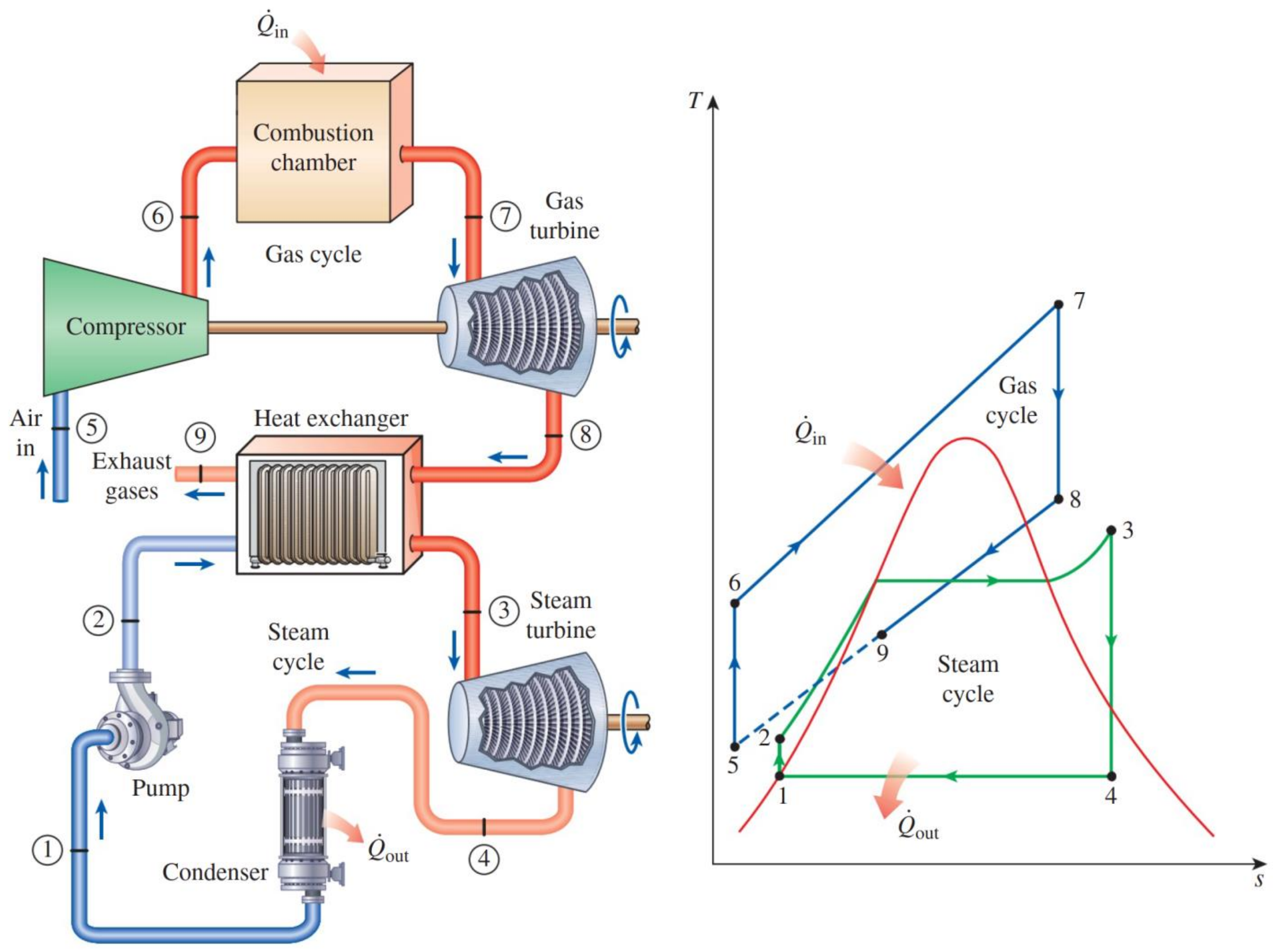


FIGURE 10–25
Combined gas–steam power plant.

COMBINED GAS–VAPOR POWER CYCLES

EXAMPLE 10–9 A Combined Gas–Steam Power Cycle

Consider the combined gas–steam power cycle shown in Fig. 10–26. The topping cycle is a gas-turbine cycle that has a pressure ratio of 8. Air enters the compressor at 300 K and the turbine at 1300 K. The isentropic efficiency of the compressor is 80 percent, and that of the gas turbine is 85 percent. The bottoming cycle is a simple ideal Rankine cycle operating between the pressure limits of 7 MPa and 5 kPa. Steam is heated in a heat exchanger by the exhaust gases to a temperature of 500°C. The exhaust gases leave the heat exchanger at 450 K. Determine (*a*) the ratio of the mass flow rates of the steam and the combustion gases and (*b*) the thermal efficiency of the combined cycle.

SOLUTION A combined gas–steam cycle is considered. The ratio of the mass flow rates of the steam and the combustion gases and the thermal efficiency are to be determined.

Analysis The *T-s* diagrams of both cycles are given in Fig. 10–26. The gas-turbine cycle alone was analyzed in Example 9–6, and the steam cycle in Example 10–8*b*, with the following results:

Gas cycle: $h'_4 = 880.36$ kJ/kg $(T'_4 = 853\text{ K})$

$q_{\text{in}} = 790.58$ kJ/kg $w_{\text{net}} = 210.41$ kJ/kg $\eta_{\text{th}} = 26.6\%$

$h'_5 = h_{@\,450\text{ K}} = 451.80$ kJ/kg

Steam cycle: $h_2 = 144.78$ kJ/kg $(T_2 = 33°\text{C})$

$h_3 = 3411.4$ kJ/kg $(T_3 = 500°\text{C})$

$w_{\text{net}} = 1331.4$ kJ/kg $\eta_{\text{th}} = 40.8\%$

(*a*) The ratio of mass flow rates is determined from an energy balance on the heat exchanger:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_g h'_5 + \dot{m}_s h_3 = \dot{m}_g h'_4 + \dot{m}_s h_2$$

$$\dot{m}_s(h_3 - h_2) = \dot{m}_g(h'_4 - h'_5)$$

$$\dot{m}_s(3411.4 - 144.78) = \dot{m}_g(880.36 - 451.80)$$

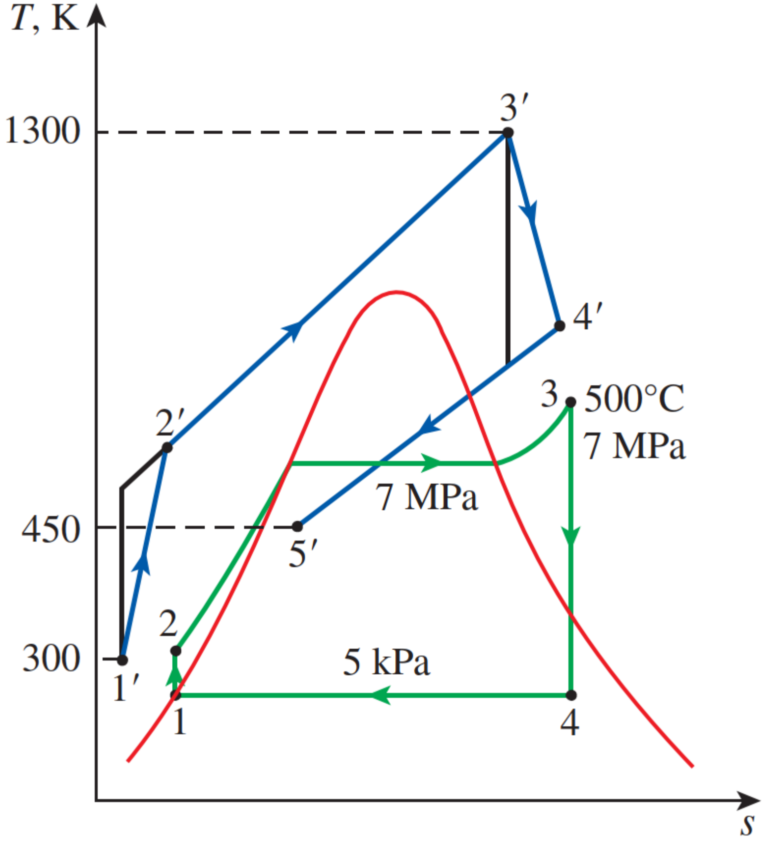


FIGURE 10–26
T-s diagram of the gas–steam combined cycle described in Example 10–9.

Thus,

$$\frac{\dot{m}_s}{\dot{m}_g} = y = \mathbf{0.131}$$

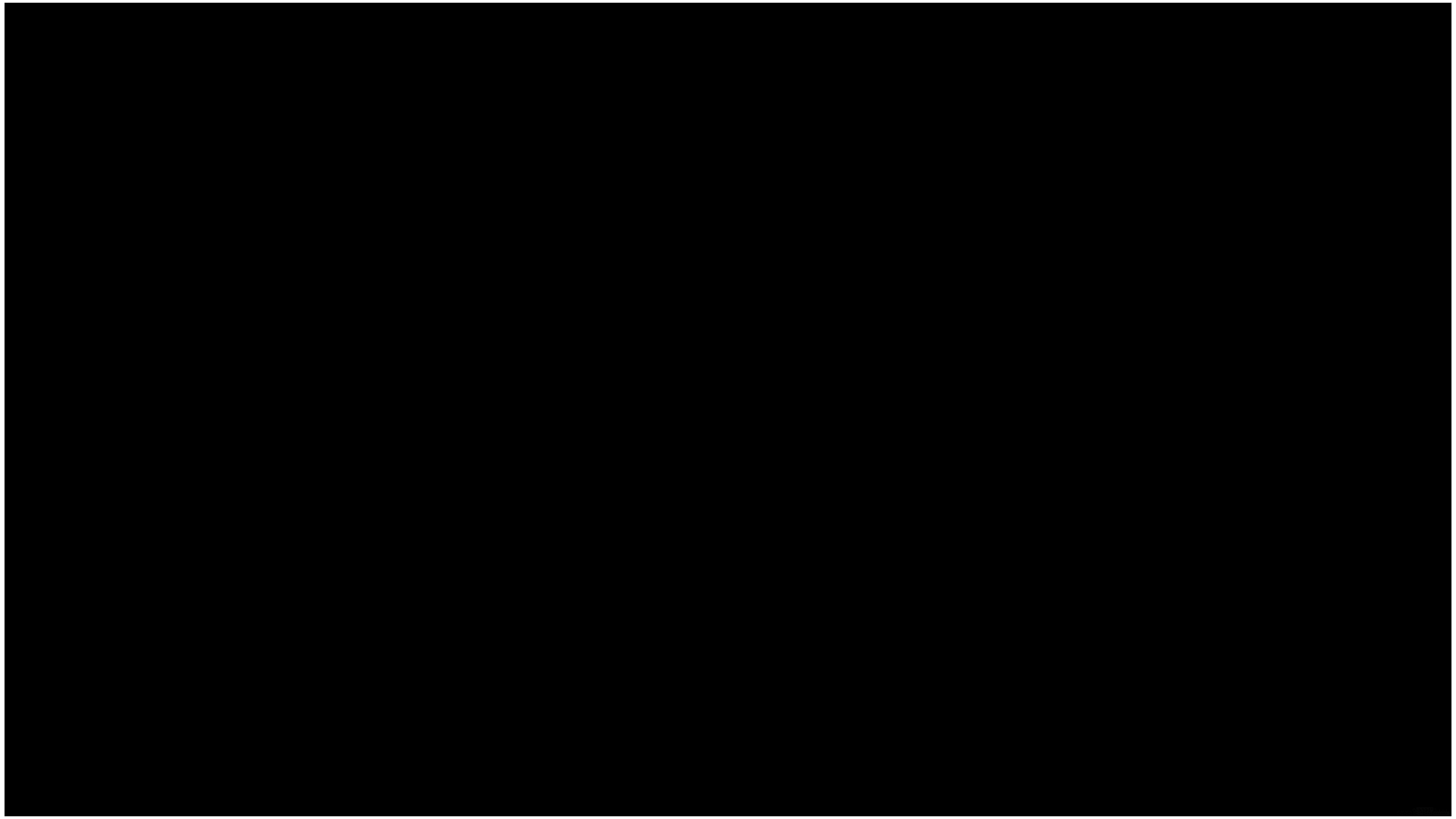
That is, 1 kg of exhaust gases can heat only 0.131 kg of steam from 33 to 500°C as they are cooled from 853 to 450 K. Then the total net work output per kilogram of combustion gases becomes

$$\begin{aligned} w_{\text{net}} &= w_{\text{net,gas}} + y w_{\text{net,steam}} \\ &= (210.41\ \text{kJ/kg gas}) + (0.131\ \text{kg steam/kg gas})(1331.4\ \text{kJ/kg steam}) \\ &= 384.8\ \text{kJ/kg gas} \end{aligned}$$

Therefore, for each kg of combustion gases produced, the combined plant will deliver 384.8 kJ of work. The net power output of the plant is determined by multiplying this value by the mass flow rate of the working fluid in the gas-turbine cycle.

(*b*) The thermal efficiency of the combined cycle is determined from

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{384.8\ \text{kJ/kg gas}}{790.6\ \text{kJ/kg gas}} = \mathbf{0.487} \text{ or } \mathbf{48.7\%}$$



COMBINED GAS–VAPOR POWER CYCLES

Thank you

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