Thermodynamics and Heat Transfer Assignment Week 3
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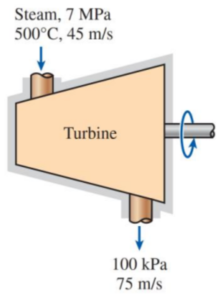
1. Steam enters an adiabatic turbine steadily at 7 MPa, 500°C, and 45 m/s and leaves at 100 kPa and 75 m/s. If the power output of the turbine is 5 MW and the isentropic efficiency is 77 percent, determine (a) the mass flow rate of steam through the turbine, (b) the temperature at the turbine exit, and (c) the rate of entropy generation during this process.?
(20 Points)



Exercise 1:

Assumptions: steady conditions, $\Delta KE = \Delta PE = 0$

$P_1 = 7\,MPa$, $T_1 = 500^0C$, $v_1 = 45\,m/s$

$P_2 = 100\,kPa$, $V_2 = 75\,m/s$, $\eta_T = 77\%$, $\dot{W}_a = 5MW$

a) Determine $\dot{m}$

$$\left.\begin{array}{l} P_1 = 7MPa \\ T_1 = 500\,°C \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_1 = 3411.4\ kJ/kg \\ s_1 = 6.8\ kJ/kg\cdot K \end{array}\right.$$

$$\left.\begin{array}{l} P_2 = 100kPa \\ S_{2S} = S_1 = 6.8\,kJ/kg\cdot K \end{array}\right\} \gg \{\, h_{2S} = 2466.6\,kJ/kg$$

The power output if the expansion was isentropic would be:

$$\dot{W}_S = \frac{\dot{W}a}{\eta_T} = \frac{5000\,kW}{77\%} = 6493.5\,kW$$

Energy balance on the turbine: $\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_{2S} + \frac{V_2^2}{2}\right) + \dot{W}_S$

Dimensions: $kJ/kg = 10^3\ m^2/s^2$

$$\Rightarrow \dot{m}\left(3411.4\,kJ/kg + 10^{-3}\left(\frac{45m/s}{2}\right)^2\right) = \dot{m}\left(2466.6\frac{kJ}{kg} + 10^{-3}\frac{(75m/s)^2}{2}\right) + 6494\,kW$$

$\Rightarrow \dot{m} = 6.886\ kg/s$ (answer)

b) Find T_2 ?

Energy balance for actual process: $\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}_a$

$$\Rightarrow 6.886\left(3411.4 + 10^{-3}\frac{45^2}{2}\right) = 6.886\left(h_2 + 10^{-3}\frac{75^2}{2}\right) + 5000$$

$\Rightarrow h_2 = 2683.48\ kJ/kg$

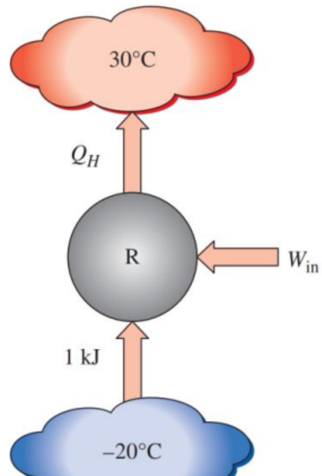
Other properties at exis state: $\left.\begin{array}{l} P_2 = 100kPa \\ h_2 = 2683.48kJ/kg \end{array}\right\} \gg \left\{\begin{array}{l} T_2 = 103.7°C \text{ (answer)} \\ S_2 = 7.3817\ kJ/kg\cdot K \end{array}\right.$

c) Since the turbine is adiabatic, the entropy generation is

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1) = 6.886\frac{kg}{s}\left(7.3817 - 6.8\right)\frac{kJ}{kg\cdot K}$$

$= 4.0056\ kW/K$ (answer)

2. A refrigerator with a coefficient of performance of 4 transfers heat from a cold region at –20°C to a hot region at 30°C. Calculate the total entropy change of the regions when 1 kJ of heat is transferred from the cold region. Is the second law satisfied? Will this refrigerator still satisfy the second law if its coefficient of performance is 6? **(20 Points)**



Exercise 2:

Assumptions: The fridge aperates steadily, $\Delta KE = \Delta PE \approx 0$

$T_C = -20°C$, $T_H = 30°C$, $COP_R = 4$, $Q_L = 1\,kJ$

- Heat transferred to the hot region: $Q_H = Q_L\left(1 + \frac{1}{COP_R}\right)$

$\Rightarrow Q_H = 1\,kJ\left(1 + \frac{1}{4}\right) = 1.25\,kJ$

When COP = 4, the entropy change of everything is

$$\Delta S_{total} = \Delta S_H + \Delta S_L = \left(\frac{Q_H}{T_H}\right) + \left(\frac{Q_L}{T_L}\right) = \frac{1.25\,kJ}{(30 + 273K)} + \frac{-1\,kJ}{(-20 + 273K)}$$

$\Rightarrow \Delta S_{total} = 0.1728 \times 10^{-3}\ \frac{kJ}{K} > 0 \Rightarrow$ at COP = 4, the Second Law is satisfied

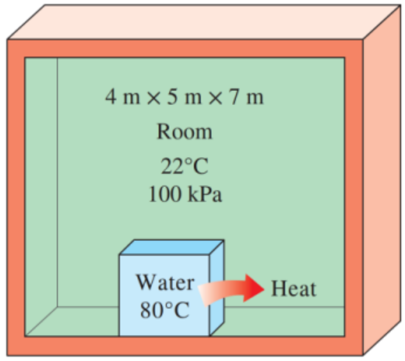
- At COP = 6: $Q_H = 1\,kJ\left(1 + \frac{1}{6}\right) = 1.1667\,kJ$

The entropy change of everything:

$$\Delta S_{total} = \frac{Q_H}{T_H} + \frac{Q_L}{T_L} = \frac{1.1667\,kJ}{(30 + 273)K} + \frac{-1\,kJ}{(-20 + 273)K} = -0.102 \times 10^{-3}\ \frac{kJ}{K} < 0$$

$\Rightarrow$ For COP = 6, the thermaldynamic process is not possible. Seoond Law is violated

3. One ton of liquid water at 80°C is brought into a well-insulated and well-sealed 4-m × 5-m × 7-m room initially at 22°C and 100 kPa. Assuming constant specific heats for both air and water at room temperature, determine (a) the final equilibrium temperature in the room and (b) the total entropy change during this process, in kJ/K. **(20 Points)**



Exercise 3:

Assumptions: $\Delta KE = \Delta PE = 0$, $W = 0$

a) Find equilibrium temperature in the room

Volume of the room: $V_1 = 4m \times 5m \times 7m = 140 m^3$

Mass of the air: $m_{air} = \frac{P_1 V_1}{R T_1} = \frac{100\,kPa \cdot 140\,m^3}{0.287\,\frac{kPa}{kg \cdot K}(22+273)K} = 165.36\,kg$

The energy balance: $E_{in} - E_{out} = \Delta E_{system}$

$$0 = \Delta U = \Delta U_{water} + \Delta U_{air}$$

or $[mc(T_2 - T_1)]_{water} + [mc_v(T_2 - T_1)]_{air} = 0$

$\Rightarrow 1000\,kg \times 4.18\,\frac{kJ}{kg°C} \times (T_2 - 80°C) + 165.36\,kg \times 0.718\,\frac{kJ}{kg°C} \times (T_2 - 22°C) = 0$

$\Rightarrow T_2 = 78.398°C \approx 78.4°C$ (answer)

b) Total entropy changed during this process

Change in air entropy: $\Delta S_{air} = mc_v \ln\left(\frac{T_2}{T_1}\right) + mR\ln\left(\frac{V_2}{V_1}\right)$ $V_1 = V_2 \Rightarrow \ln(1) = 0$

$\Rightarrow \Delta S_{air} = 165.36\,kg \times 0.718\,\frac{kJ}{kg°K} \ln\left(\frac{(78.4+273)K}{(22+273)K}\right) = 20.78\,kJ/K$

Change in water entropy: $\Delta S_{water} = mc\ln\left(\frac{T_2}{T_1}\right)$

$\Rightarrow \Delta S_{water} = 1000\,kg \times 4.18\,\frac{kJ}{kg°K} \ln\left(\frac{(78.4+273)K}{(80+273)K}\right) = -18.99\,kJ/K$

$\Rightarrow$ Total entropy change: $\Delta S_{total} = \Delta S_{air} + \Delta S_{water} = 20.78 - 18.99$

$\Rightarrow \Delta S_{total} = 1.79\,kJ/K$ (answer)

4. The compression ratio of an ideal dual cycle is 14. Air is at 100 kPa and 300 K at the beginning of the compression process and at 2200 K at the end of the heat-addition process. Heat transfer to air takes place partly at constant volume and partly at constant pressure, and it amounts to 1520.4 kJ/kg. Assuming variable specific heats for air, determine (a) the fraction of heat transferred at constant volume and (b) the thermal efficiency of the cycle. Comparing the thermal efficiency if varying the compression ratio from 12 to16 (12, 13, 14, 15, 16). **(20 Points)**

Exercise 4:

Assumptions: ideal gas with variable specific heats. $\Delta KE = \Delta PE \approx 0$

a) Determine fraction of heat transferred at constant volume

$T_1 = 300K \rightarrow \begin{cases} u_1 = 214.07 \text{ kJ/kg} \\ v_1 = 621.2 \end{cases}$, $\frac{v_2}{v_1} = 14$ (compression ratio)

$v_{r2} = \frac{v_2}{v_1} v_{r1} \rightarrow \begin{cases} T_2 = 823.1K \\ u_2 = 611.2 \text{ kJ/kg} \end{cases}$; $T_3 = 2200K \rightarrow \begin{cases} h_3 = 2503.2 \text{ kJ/kg} \\ v_{r3} = 2.012 \end{cases}$

$q_{in} = 1520.4 \text{ kJ/kg}$, $u_2 = 611.2 \text{ kJ/kg}$, $h_3 = 2503.2 \text{ kJ/kg}$, $T_x = 1300K$

$\Rightarrow h_x - u_x = 371.6 \text{ kJ/kg}$, $h_x = 1395.97$, $u_x = 1022.82 \text{ kJ/kg}$

Thus, $q_{x-2,in} = u_x - u_2 = 1022.82 - 611.2 = 411.62 \text{ kJ/kg}$

and fraction of heat transfer $= \frac{q_{x-2,in}}{q_{in}} = \frac{411.62 \text{ kJ/kg}}{1520.4 \text{ kJ/kg}} = 27.073\%$ (answer)

b) Determine the thermal efficiency of the cycle

Based on the ideal gas equation: $\frac{P_3 v_3}{T_3} = \frac{P_x v_x}{T_x} \Rightarrow \frac{V_3}{V_x} = \frac{T_3}{T_x} = r_c$

$\Rightarrow r_c = \frac{2200K}{1300K} = 1.6923$ (compression ratio)

$v_{r4} = \frac{v_4}{v_3} v_{r3} = \frac{v_4}{1.6923 \, v_2} v_{r3} = \frac{r}{1.6923} v_{r3} = \frac{14}{1.6923} \times 2.012 = 16.6448$

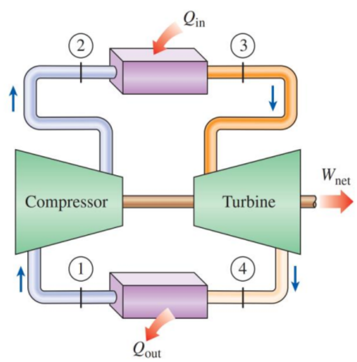
Interpolating A-17 table, we have $u_4 = 886.3 \text{ kJ/kg}$

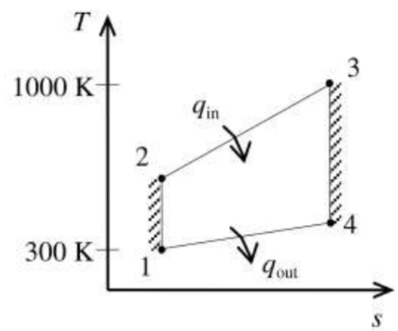
$\Rightarrow q_{out} = u_4 - u_1 = 886.3 - 214.07 = 672.23 \text{ kJ/kg}$

$\Rightarrow \eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{672.23 \text{ kJ/kg}}{1520.4 \text{ kJ/kg}} = 55.78\%$ (answer)

c) The thermal efficiency η_{th} increases as r_c increases from 12 to 16 as η_{th} can be represented by temperature difference. ΔT increases when the gas is more compressed, resulting in higher η_{th}

5. A simple ideal Brayton cycle operates with air with minimum and maximum temperatures of 27°C and 727°C. It is designed so that the maximum cycle pressure is 2000 kPa and the minimum cycle pressure is 100 kPa. Determine the net work produced per unit mass of air each time this cycle is executed and the cycle's thermal efficiency. Use constant specific heats at room temperature. **(20 Points)**





Exercise 5:

Assumptions: steady conditions, ideal gas, $\Delta KE = \Delta PE \approx 0$

- Using isentropic relations for an ideal gas

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300\,K \times \left(\frac{2000\,kPa}{100\,kPa}\right)^{\frac{1.4-1}{1.4}} = 706.064\,K$$

Similarly: $T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1000\,K \times \left(\frac{100\,kPa}{2000\,kPa}\right)^{\frac{1.4-1}{1.4}} = 424.89\,K$

Applying 1^{st} law to constant-pressure heat addition process 2-3 produces

$$q_{in} = (h_3 - h_2) = c_p (T_3 - T_2) = 1.005 \frac{kJ}{kg \cdot K} \times (1000 - 706.064)\,K = 295.4\ kJ/kg$$

Similarly: $q_{out} = (h_4 - h_1) = c_p (T_4 - T_1) = 1.005 \frac{kJ}{kg \cdot K} \times (424.89 - 300)\,K = 125.55\ kJ/kg$

- The network production: $W_{net} = q_{in} - q_{out} = 295.4 - 125.55 = 169.85\ kJ/kg$ (answer)
- The thermal efficiency of this cycle: $\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{169.85\ \frac{kJ}{kg}}{295.4\ \frac{kJ}{kg}} = 57.5\ \%$ (answer)

6. **Your free feedback on the third week and time spent on this learning exercise.** (This does not affect the grading)

Entropy is an interesting concept that governs how the universe works. I find this week's knowledge to be useful and applicable to real world's problems.
I spent 5 hours on this exercise.