

# Thermodynamics and Heat Transfer

Lecture 12: Steady Heat Conduction

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# STEADY HEAT CONDUCTION IN PLANE WALLS

The energy balance for the wall can be expressed as:

$$\left( \begin{array}{l} \text{Rate of} \\ \text{heat transfer} \\ \text{into the wall} \end{array} \right) - \left( \begin{array}{l} \text{Rate of} \\ \text{heat transfer} \\ \text{out of the wall} \end{array} \right) = \left( \begin{array}{l} \text{Rate of change} \\ \text{of the energy} \\ \text{of the wall} \end{array} \right)$$

or

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt}$$

steady operation

(3-1)

So,  $\dot{Q}_{\text{in}} = \dot{Q}_{\text{out}} \rightarrow \dot{Q}_{\text{cond, wall}} = \text{constant}$

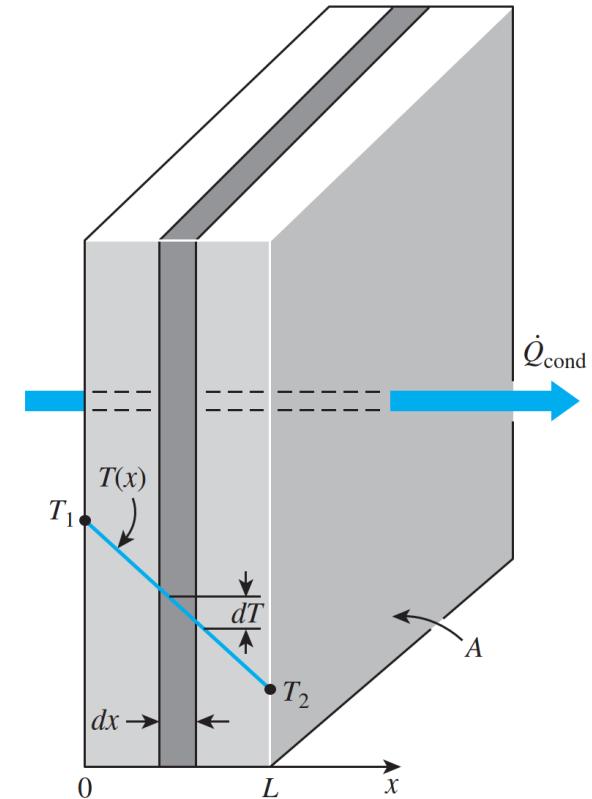
$$\dot{Q}_{\text{cond, wall}} = -kA \frac{dT}{dx} \quad (\text{W})$$
(3-2)

Separating the variables in the preceding equation and integrating from  $x = 0$ , where  $T(0) = T_1$ , to  $x = L$ , where  $T(L) = T_2$ , we get

$$\int_{x=0}^L \dot{Q}_{\text{cond, wall}} dx = - \int_{T=T_1}^{T_2} kA dT$$

Performing the integrations and rearranging gives

$$\dot{Q}_{\text{cond, wall}} = kA \frac{T_1 - T_2}{L} \quad (\text{W})$$
(3-3)



**FIGURE 3-2**

Under steady conditions, the temperature distribution in a plane wall is a straight line.

# Thermal Resistance Concept

Equation 3–3 for heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \quad (\text{W}) \quad (3-4)$$

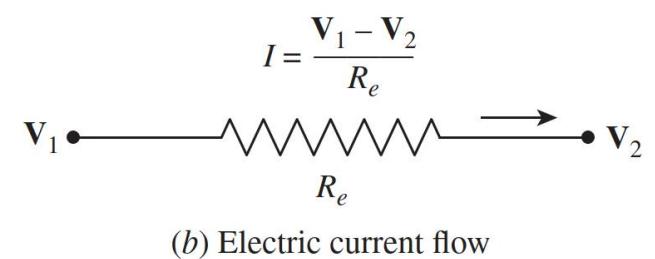
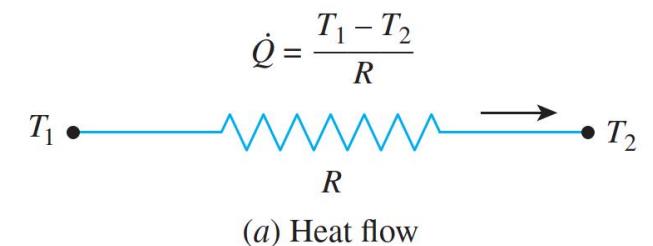
where

$$R_{\text{wall}} = \frac{L}{kA} \quad (\text{K/W}) \quad (3-5)$$

is the *thermal resistance* of the wall against heat conduction or simply the **conduction resistance** of the wall. Note that the thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium. Note that thermal resistance can also be expressed as  $R_{\text{wall}} = \Delta T / \dot{Q}_{\text{cond, wall}}$ , which is the ratio of the driving potential  $\Delta T$  to the corresponding transfer rate  $\dot{Q}_{\text{cond, wall}}$ .

This equation for heat transfer is analogous to the relation for *electric current flow*  $I$ , expressed as

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e} \quad (3-6)$$



**FIGURE 3–3**  
Analogy between thermal and electrical resistance concepts.

## The convection resistance of the surface:

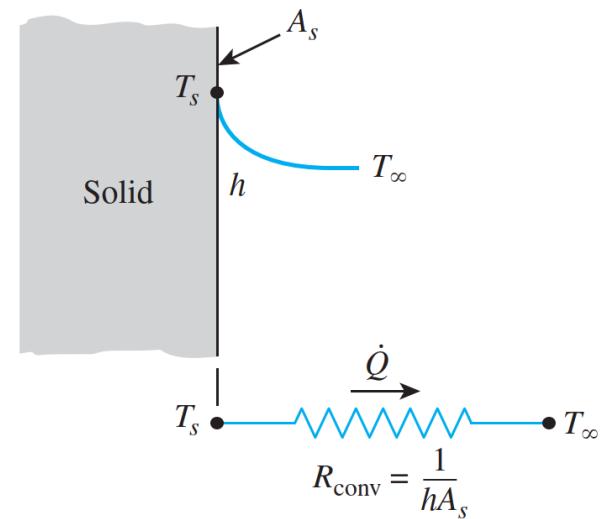
Consider convection heat transfer from a solid surface of area  $A_s$  and temperature  $T_s$  to a fluid whose temperature sufficiently far from the surface is  $T_\infty$ , with a convection heat transfer coefficient  $h$ . Newton's law of cooling for convection heat transfer rate  $\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$  can be rearranged as

$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \quad (\text{W}) \quad (3-7)$$

where

$$R_{\text{conv}} = \frac{1}{hA_s} \quad (\text{K/W}) \quad (3-8)$$

is the thermal resistance of the surface against heat convection, or simply the convection resistance of the surface.



**FIGURE 3-4**

Schematic for convection resistance at a surface.

# Radiation resistance

When the wall is surrounded by a gas, the *radiation effects*, which we have ignored so far, can be significant and may need to be considered. The rate of radiation heat transfer between a surface of emissivity  $\varepsilon$  and area  $A_s$  at temperature  $T_s$  and the surrounding surfaces at some average temperature  $T_{\text{surr}}$  can be expressed as

$$\dot{Q}_{\text{rad}} = \varepsilon \sigma A_s (T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}} A_s (T_s - T_{\text{surr}}) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}} \quad (\text{W}) \quad (3-9)$$

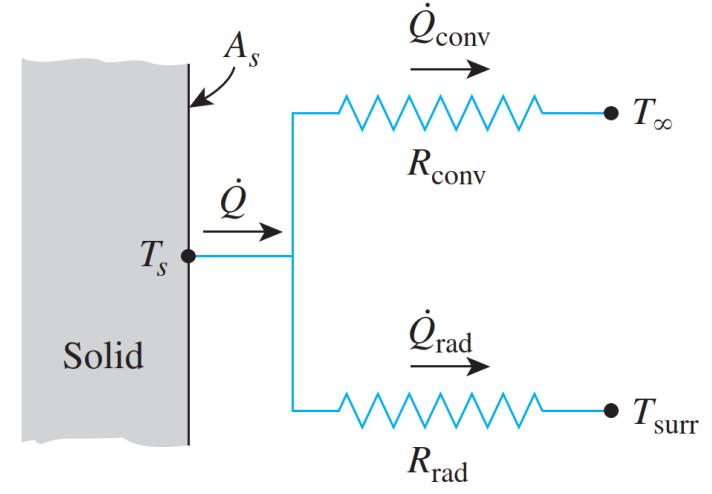
where

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_s} \quad (\text{K/W}) \quad (3-10)$$

is the *thermal resistance* of a surface against radiation, or the **radiation resistance**, and

$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s (T_s - T_{\text{surr}})} = \varepsilon \sigma (T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \quad (\text{W/m}^2 \cdot \text{K}) \quad (3-11)$$

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} \quad (\text{W/m}^2 \cdot \text{K})$$



$$\dot{Q} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

**FIGURE 3-5**

Schematic for convection and radiation resistances at a surface.

# Thermal Resistance Network

Now consider steady one-dimensional heat transfer through a plane wall of thickness  $L$ , area  $A$ , and thermal conductivity  $k$  that is exposed to convection on both sides to fluids at temperatures  $T_{\infty 1}$  and  $T_{\infty 2}$  with heat transfer coefficients  $h_1$  and  $h_2$ , respectively, as shown in Fig. 3-6. Assuming  $T_{\infty 2} < T_{\infty 1}$ , the variation of temperature will be as shown in the figure. Note that the temperature varies linearly in the wall, and asymptotically approaches  $T_{\infty 1}$  and  $T_{\infty 2}$  in the fluids as we move away from the wall.

Under steady conditions we have

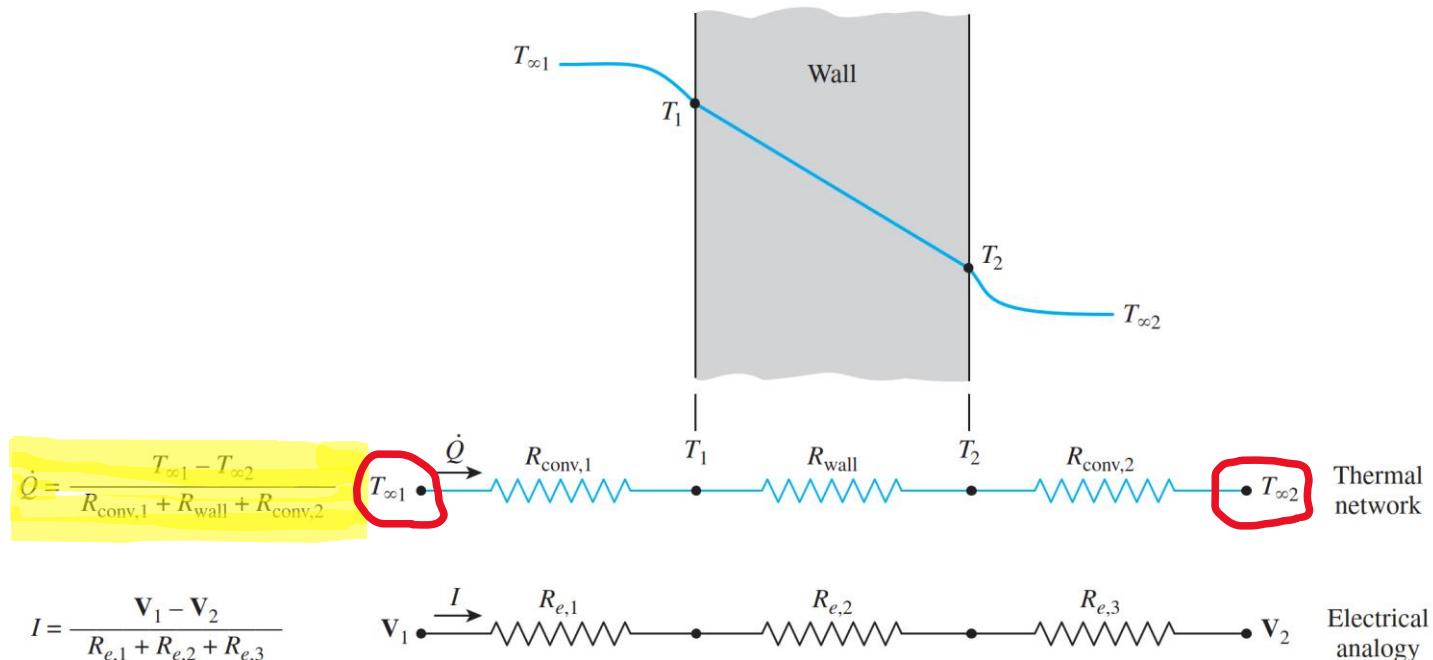


FIGURE 3-6

The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

$$Q_{\text{conv,in}} = Q_{\text{cond}} = Q_{\text{conv,out}}$$

$$\left( \begin{array}{l} \text{Rate of} \\ \text{heat convection} \\ \text{into the wall} \end{array} \right) = \left( \begin{array}{l} \text{Rate of} \\ \text{heat conduction} \\ \text{through the wall} \end{array} \right) = \left( \begin{array}{l} \text{Rate of} \\ \text{heat convection} \\ \text{from the wall} \end{array} \right)$$

or

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA \frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2}) \quad (3-13)$$

which can be rearranged as

$$\begin{aligned} \dot{Q} &= \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A} \\ &= \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv},2}} \end{aligned} \quad (3-14)$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \quad (\text{W}) \quad (3-15)$$

where

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{wall}} + R_{\text{conv},2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad (\text{K/W}) \quad (3-16)$$

It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA \Delta T \quad (\text{W}) \quad (3-18)$$

where  $U$  is the **overall heat transfer coefficient** with the unit  **$\text{W/m}^2\cdot\text{K}$** . The overall heat transfer coefficient is usually used in heat transfer calculations associated with **heat exchangers** (Chapter 11). It is also used in heat transfer calculations through windows (Chapter 9), commonly referred to as U-factor. A comparison of Eqs. 3-15 and 3-18 reveals that

$$UA = \frac{1}{R_{\text{total}}} \quad (\text{W/K}) \quad (3-19)$$

### Multilayer Plane Walls:

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \quad (3-21)$$

where  $R_{\text{total}}$  is the *total thermal resistance*, expressed as

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv},1} + R_{\text{wall},1} + R_{\text{wall},2} + R_{\text{conv},2} \\ &= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \end{aligned} \quad (3-22)$$

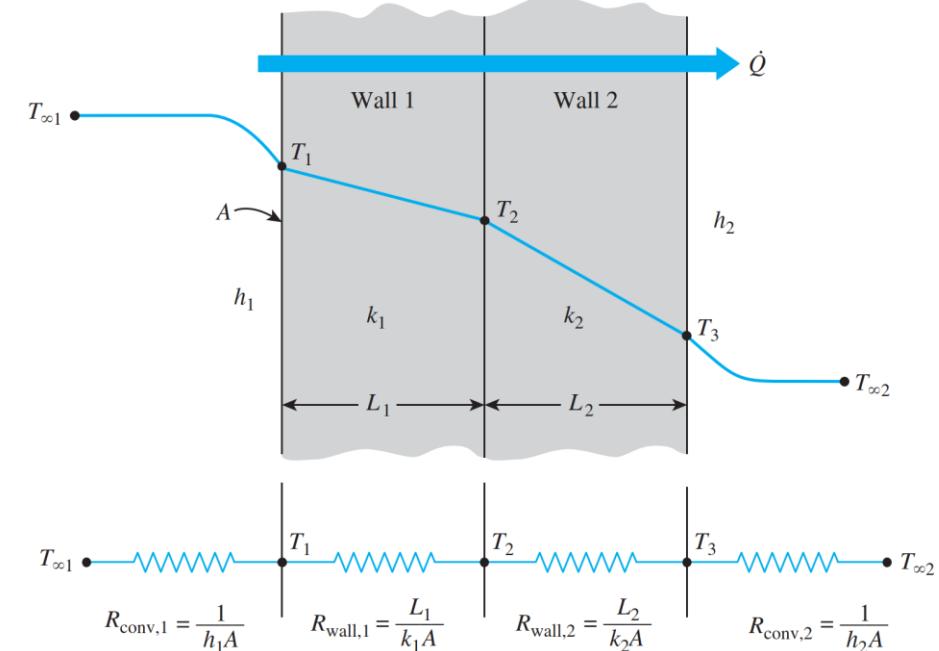


FIGURE 3-9

The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

# EXAMPLE 3–1 Heat Loss through a Wall

Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is  $k = 0.9 \text{ W/m}\cdot\text{K}$  (Fig. 3–11). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be  $16^\circ\text{C}$  and  $2^\circ\text{C}$ , respectively. Determine the rate of heat loss through the wall on that day.

**Properties** The thermal conductivity is given to be  $k = 0.9 \text{ W/m}\cdot\text{K}$ .

**Analysis** Noting that heat transfer through the wall is by conduction and the area of the wall is  $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$ , the steady rate of heat transfer through the wall can be determined from Eq. 3–3 to be

$$\dot{Q} = kA \frac{T_1 - T_2}{L} = (0.9 \text{ W/m}\cdot\text{C})(15 \text{ m}^2) \frac{(16 - 2)^\circ\text{C}}{0.3 \text{ m}} = 630 \text{ W}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

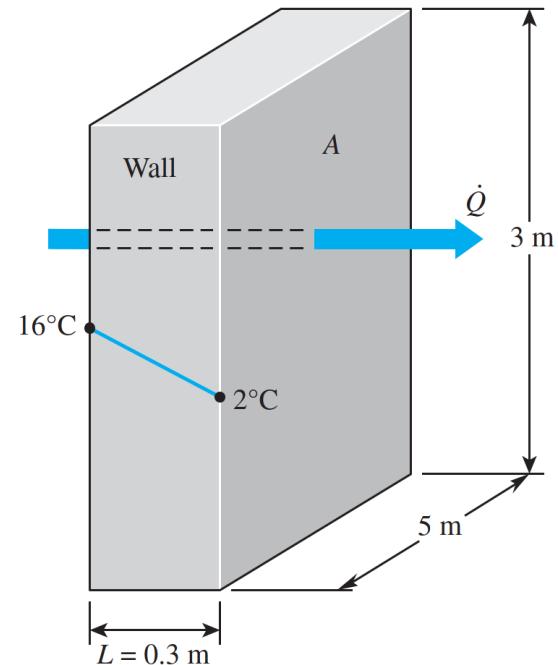
$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

where

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3 \text{ m}}{(0.9 \text{ W/m}\cdot\text{C})(15 \text{ m}^2)} = 0.02222^\circ\text{C/W}$$

Substituting, we get

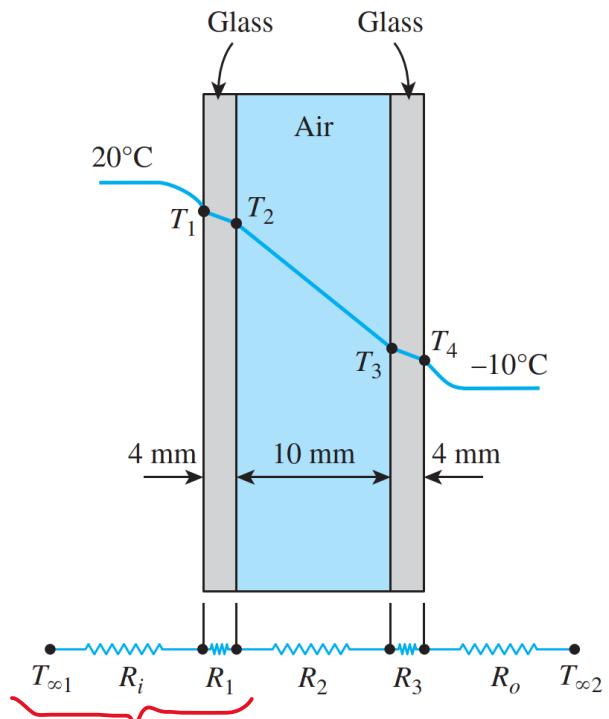
$$\dot{Q} = \frac{(16 - 2)^\circ\text{C}}{0.02222^\circ\text{C/W}} = 630 \text{ W}$$



**FIGURE 3–11**  
Schematic for Example 3–1.

### EXAMPLE 3-3 Heat Loss through Double-Pane Windows

Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass ( $k = 0.78 \text{ W/m}\cdot\text{K}$ ) separated by a 10-mm-wide stagnant air space ( $k = 0.026 \text{ W/m}\cdot\text{K}$ ). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at  $20^\circ\text{C}$  while the temperature of the outdoors is  $-10^\circ\text{C}$ . Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be  $h_1 = 10 \text{ W/m}^2\cdot\text{K}$  and  $h_2 = 40 \text{ W/m}^2\cdot\text{K}$ , which includes the effects of radiation.



**FIGURE 3-13**

Schematic for Example 3-3.

$$\dot{Q}_i = \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}}$$

**Analysis** This example problem is identical to the previous one except that the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that enclose a 10-mm-wide stagnant air space. Therefore, the thermal resistance network of this problem involves two additional conduction resistances corresponding to the two additional layers, as shown in Fig. 3-13. Noting that the area of the window is again  $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$ , the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv}, 1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004 \text{ m}}{(0.78 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.00427^\circ\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01 \text{ m}}{(0.026 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.3205^\circ\text{C/W}$$

$$R_o = R_{\text{conv}, 2} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.02083^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv}, 1} + R_{\text{glass}, 1} + R_{\text{air}} + R_{\text{glass}, 2} + R_{\text{conv}, 2} \\ &= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 \\ &= 0.4332^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.4332^\circ\text{C/W}} = 69.2 \text{ W}$$

which is about one-fourth of the result obtained in the previous example. This explains the popularity of the double- and even triple-pane windows in cold climates. The drastic reduction in the heat transfer rate in this case is due to the large thermal resistance of the air layer between the glasses.

The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty 1} - \dot{Q} R_{\text{conv}, 1} = 20^\circ\text{C} - (69.2 \text{ W})(0.08333^\circ\text{C/W}) = 14.2^\circ\text{C}$$

# GENERALIZED THERMAL RESISTANCE NETWORKS

The *thermal resistance* concept or the *electrical analogy* can also be used to solve steady heat transfer problems that involve parallel layers or combined series-parallel arrangements. Although such problems are often two- or even three-dimensional, approximate solutions can be obtained by assuming one-dimensional heat transfer and using the thermal resistance network.

Consider the composite wall shown in Fig. 3–19, which consists of two parallel layers. The thermal resistance network, which consists of two parallel resistances, can be represented as shown in the figure. Noting that the total heat transfer is the sum of the heat transfers through each layer, we have

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \quad (3-29)$$

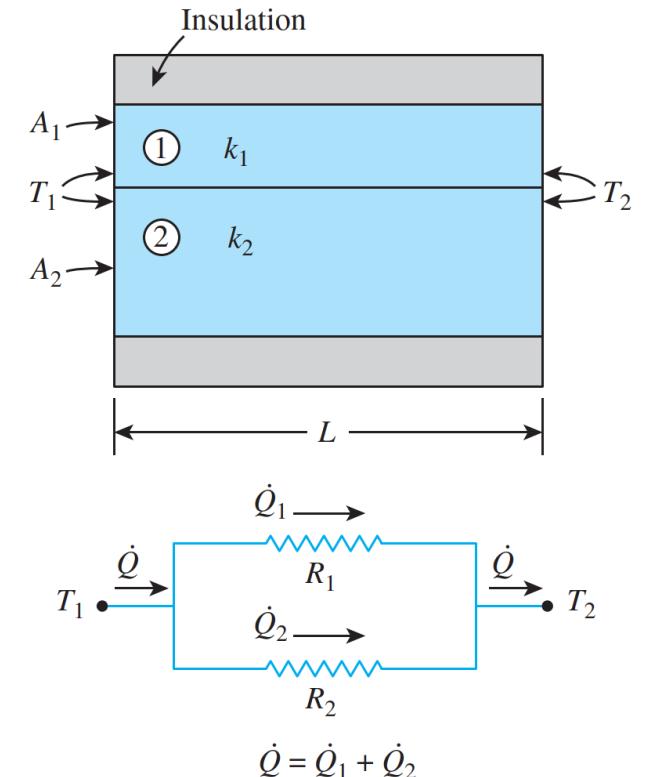
Utilizing electrical analogy, we get

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}} \quad (3-30)$$

where

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} \quad (3-31)$$

since the resistances are in parallel.



**FIGURE 3–19**  
Thermal resistance  
network for two parallel layers.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}}$$

(3-32)

where

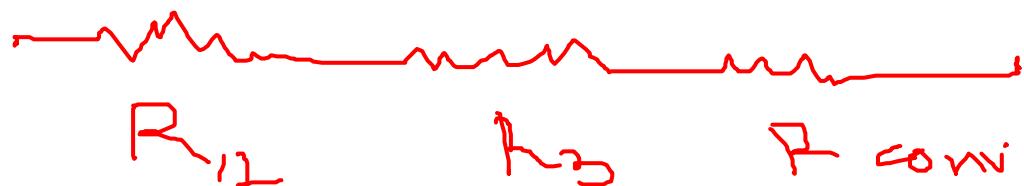
$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}}$$

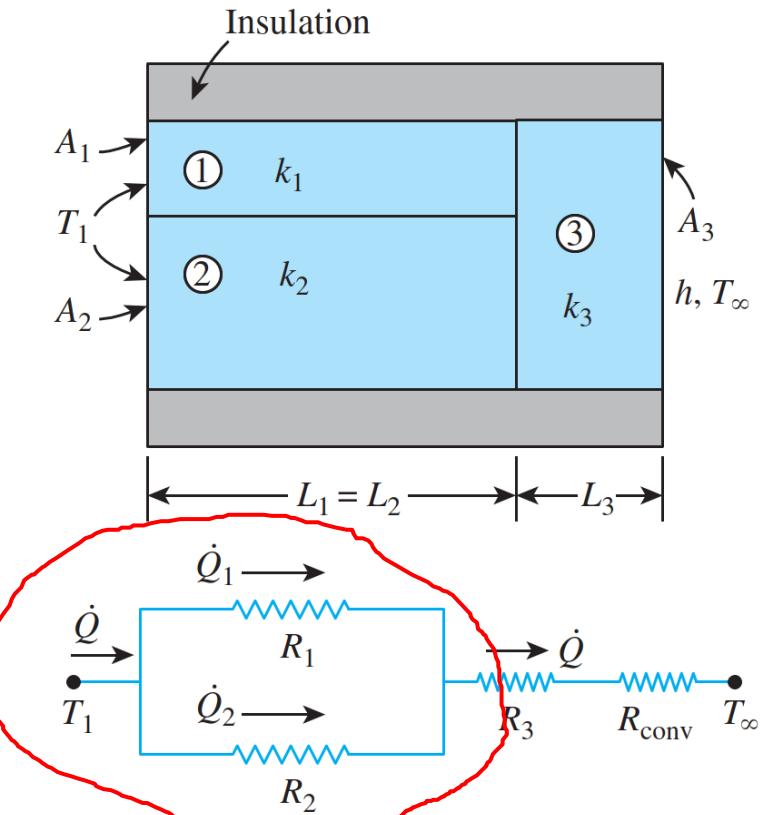
(3-33)

and

$$R_1 = \frac{L_1}{k_1 A_1} \quad R_2 = \frac{L_2}{k_2 A_2} \quad R_3 = \frac{L_3}{k_3 A_3} \quad R_{\text{conv}} = \frac{1}{h A_3}$$

(3-34)





**FIGURE 3-20**

Thermal resistance network for combined series-parallel arrangement.

### EXAMPLE 3–6 Heat Loss through a Composite Wall

A 3-m-high and 5-m-wide wall consists of long 16-cm  $\times$  22-cm cross section horizontal bricks ( $k = 0.72 \text{ W/m}\cdot\text{K}$ ) separated by 3-cm-thick plaster layers ( $k = 0.22 \text{ W/m}\cdot\text{K}$ ). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ( $k = 0.026 \text{ W/m}\cdot\text{K}$ ) on the inner side of the wall, as shown in Fig. 3–21. The indoor and the outdoor temperatures are  $20^\circ\text{C}$  and  $-10^\circ\text{C}$ , respectively, and the convection heat transfer coefficients on the inner and the outer sides are  $h_1 = 10 \text{ W/m}^2\cdot\text{K}$  and  $h_2 = 25 \text{ W/m}^2\cdot\text{K}$ , respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

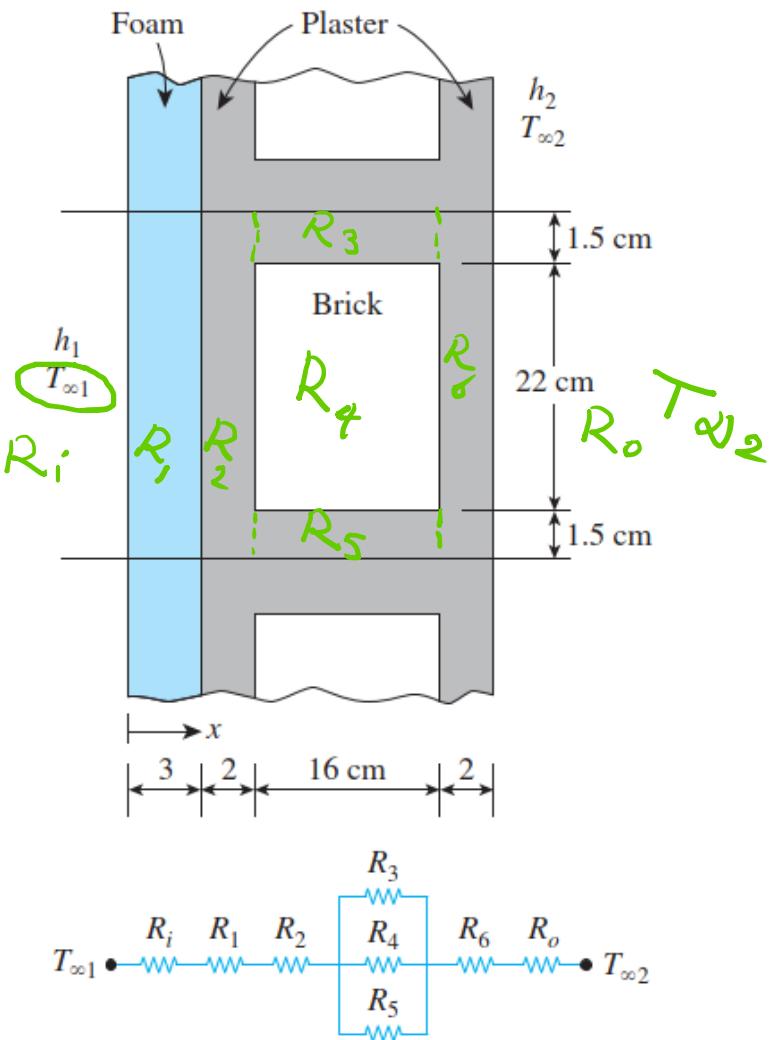
**Analysis** There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

$$R_i = R_{\text{conv}, 1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(0.25 \times 1 \text{ m}^2)} = 0.40^\circ\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03 \text{ m}}{(0.026 \text{ W/m}\cdot\text{K})(0.25 \times 1 \text{ m}^2)} = 4.62^\circ\text{C/W}$$

$$\begin{aligned} R_2 &= R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.22 \text{ W/m}\cdot\text{K})(0.25 \times 1 \text{ m}^2)} \\ &= 0.36^\circ\text{C/W} \end{aligned}$$

$$\begin{aligned} R_3 &= R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.22 \text{ W/m}\cdot\text{K})(0.015 \times 1 \text{ m}^2)} \\ &= 48.48^\circ\text{C/W} \end{aligned}$$



**FIGURE 3–21**  
Schematic for Example 3–6.

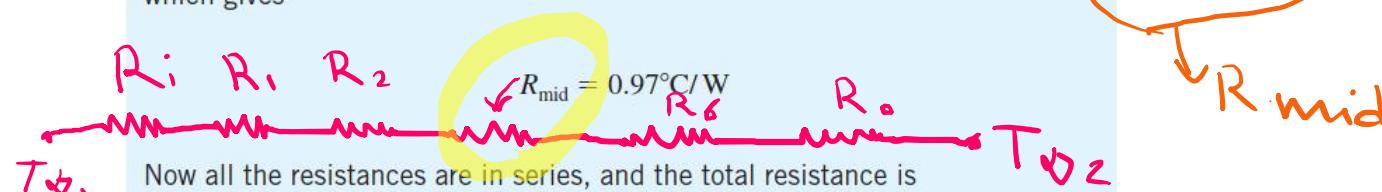
$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.72 \text{ W/m}\cdot\text{K})(0.22 \times 1 \text{ m}^2)} = 1.01^\circ\text{C/W}$$

$$R_o = R_{\text{conv}, 2} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2\cdot\text{K})(0.25 \times 1 \text{ m}^2)} = 0.16^\circ\text{C/W}$$

The three resistances  $R_3$ ,  $R_4$ , and  $R_5$  in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03 \text{ W}/^\circ\text{C}$$

which gives



Now all the resistances are in series, and the total resistance is

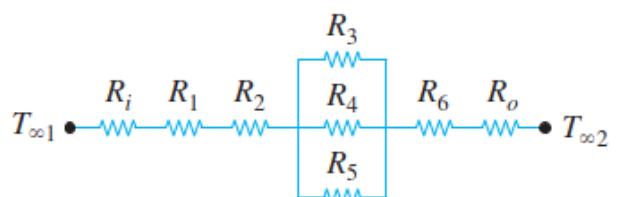
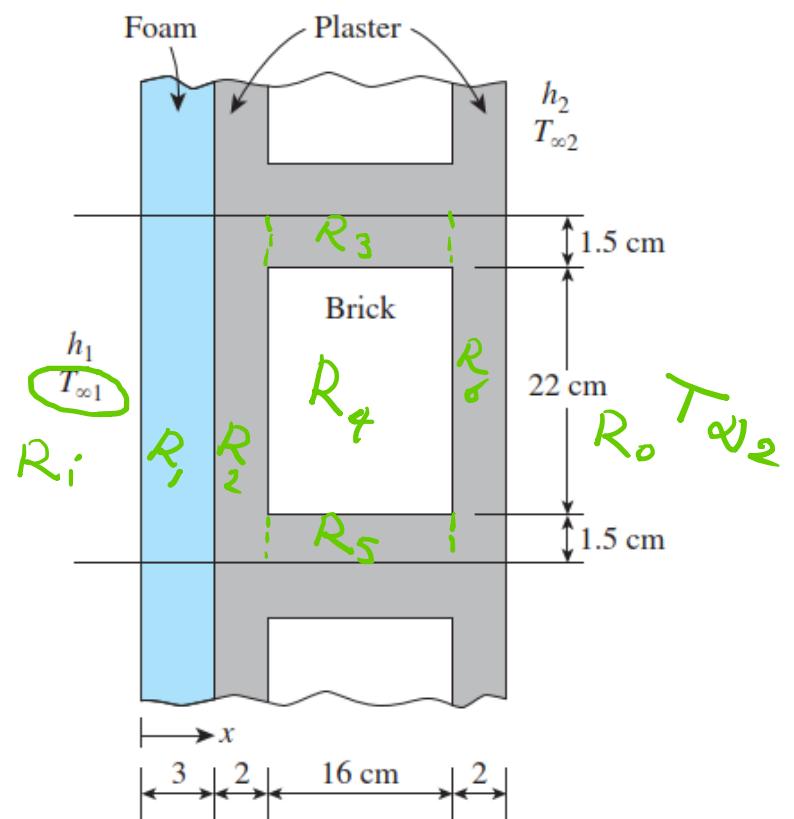
$$\begin{aligned} R_{\text{total}} &= R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o \\ &= 0.40 + 4.62 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.87^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{6.87^\circ\text{C/W}} = 4.37 \text{ W} \quad (\text{per } 0.25 \text{ m}^2 \text{ surface area})$$

or  $4.37/0.25 = 17.5 \text{ W per m}^2 \text{ area}$ . The total area of the wall is  $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$ . Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{total}} = (17.5 \text{ W/m}^2)(15 \text{ m}^2) = 263 \text{ W}$$



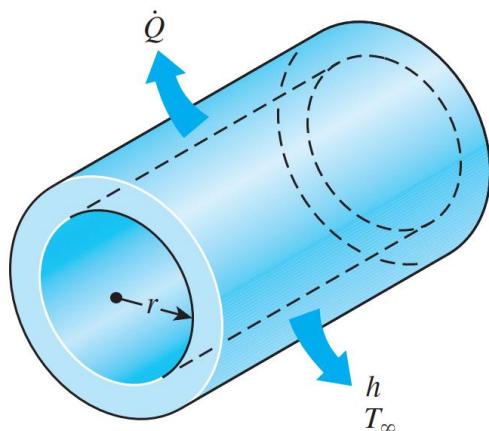
**FIGURE 3-21**  
Schematic for Example 3-6.

# HEAT CONDUCTION IN CYLINDERS AND SPHERES

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \quad (\text{W}) \quad (3-38)$$

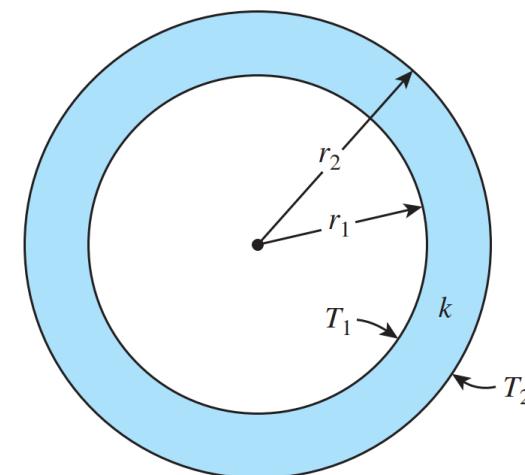
where

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi L k} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}} \quad (3-39)$$



**FIGURE 3-23**

Heat is lost from a hot-water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.



**RE 3-24**

A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures  $T_1$  and  $T_2$ .

(3-42)

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

where

$$R_{\text{conv},i} = \frac{1}{h_i A_i}$$

*convection* *conduction*

$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{cyl}} + R_{\text{conv}, 2}$$

$$= \frac{1}{(2\pi r_1 L) h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L) h_2}$$

*convection 2*

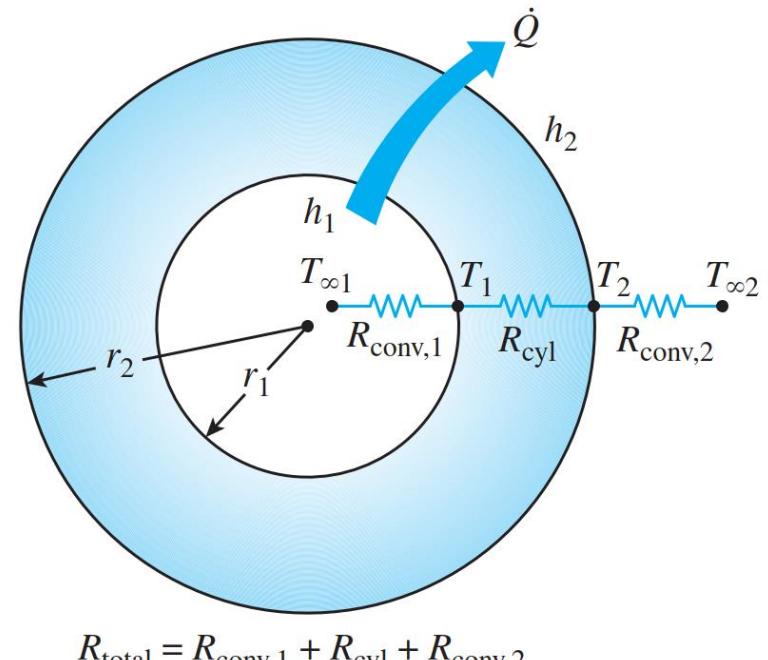
(3-43)

for a cylindrical layer, and

$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{sph}} + R_{\text{conv}, 2}$$

$$= \frac{1}{(4\pi r_1^2) h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2) h_2}$$

(3-44)

**FIGURE 3-25**

The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

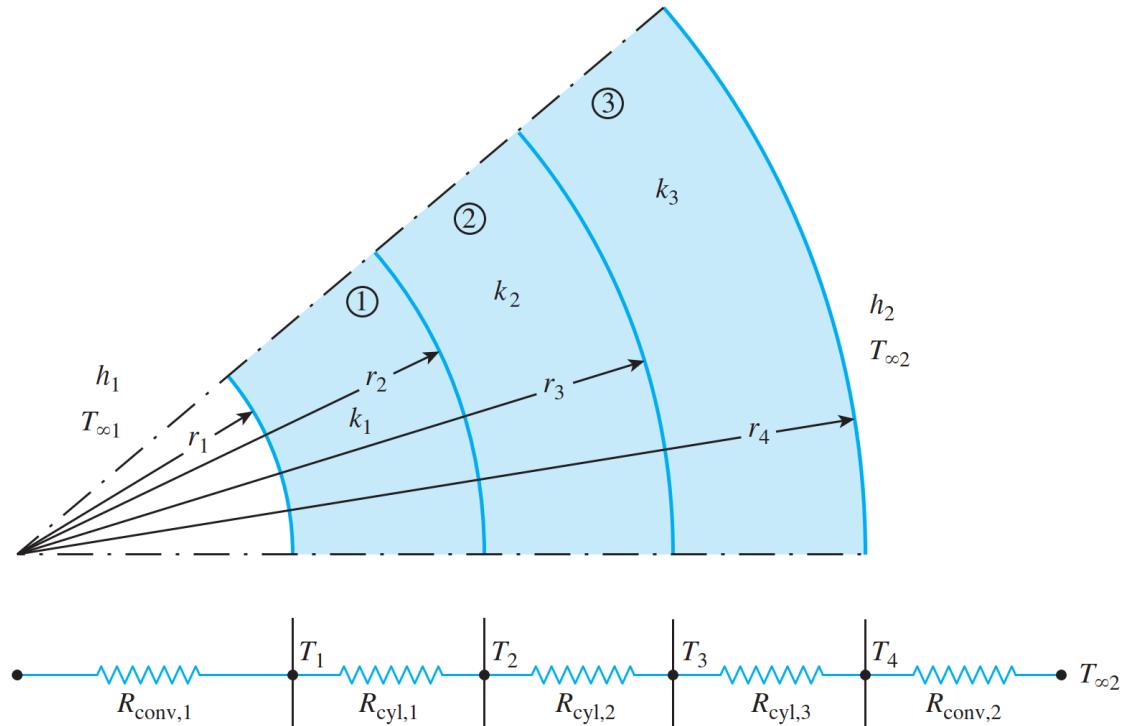
for a *spherical* layer. Note that  $A$  in the convection resistance relation  $R_{\text{conv}} = 1/hA$  is the *surface area at which convection occurs*. It is equal to  $A = 2\pi rL$  for a cylindrical surface and  $A = 4\pi r^2$  for a spherical surface of radius  $r$ . Also note that the thermal resistances are in series, and thus the total thermal resistance is determined by simply adding the individual resistances, just like the electrical resistances connected in series.

# Multilayered Cylinders and Spheres

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \quad (3-45)$$

where  $R_{\text{total}}$  is the *total thermal resistance*, expressed as

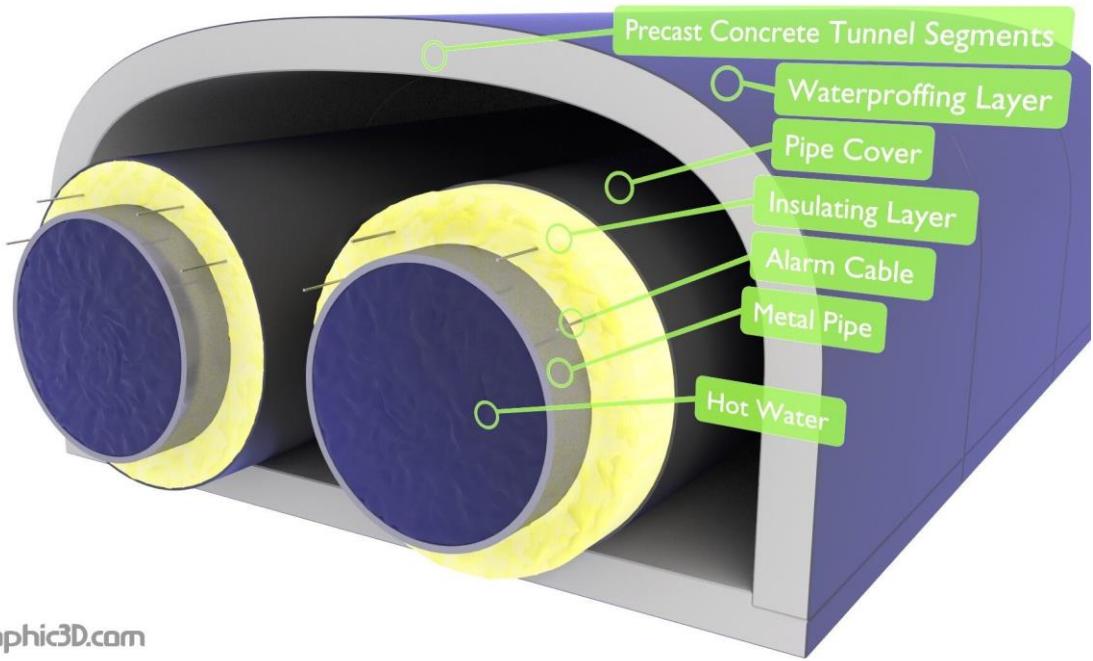
$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{cyl}, 1} + R_{\text{cyl}, 2} + R_{\text{cyl}, 3} + R_{\text{conv}, 2} \\ = \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4} \quad (3-46)$$



**FIGURE 3-26**

The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

# District Heating



Source: <http://artgraphic3d.com/3d-models/sewerage/2844-tunnel-district-heating-pipes-steam-3d-15.html>

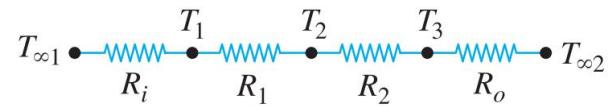
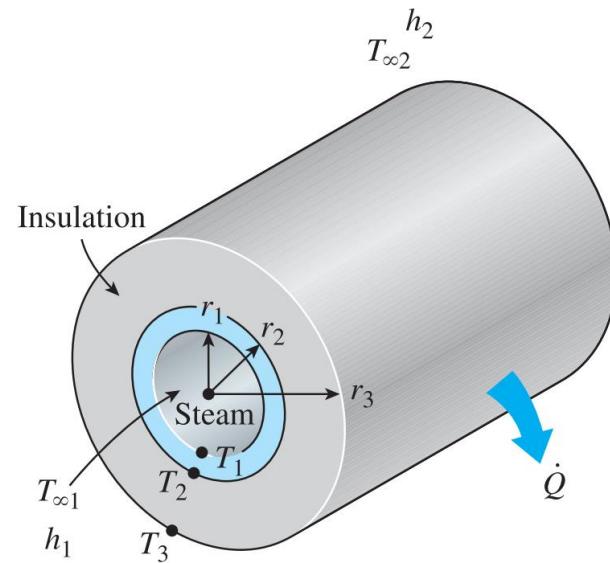


Credit: Photographed by [Mike1024](#)

### EXAMPLE 3–8 Heat Loss through an Insulated Steam Pipe

Steam at  $T_{\infty 1} = 320^\circ\text{C}$  flows in a cast iron pipe ( $k = 80 \text{ W/m}\cdot\text{K}$ ) whose inner and outer diameters are  $D_1 = 5 \text{ cm}$  and  $D_2 = 5.5 \text{ cm}$ , respectively. The pipe is covered with 3-cm-thick glass wool insulation with  $k = 0.05 \text{ W/m}\cdot\text{K}$ . Heat is lost to the surroundings at  $T_{\infty 2} = 5^\circ\text{C}$  by natural convection and radiation, with a combined heat transfer coefficient of  $h_2 = 18 \text{ W/m}^2\cdot\text{K}$ . Taking the heat transfer coefficient inside the pipe to be  $h_1 = 60 \text{ W/m}^2\cdot\text{K}$ , determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

**Properties** The thermal conductivities are given to be  $k = 80 \text{ W/m}\cdot\text{K}$  for cast iron and  $k = 0.05 \text{ W/m}\cdot\text{K}$  for glass wool insulation.



**FIGURE 3–29**

Schematic for Example 3–8.

**Analysis** The thermal resistance network for this problem involves four resistances in series and is given in Fig. 3–29. Taking  $L = 1 \text{ m}$ , the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2$$

$$A_3 = 2\pi r_3 L = 2\pi(0.0575 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv}, 1} = \frac{1}{h_1 A_1} = \frac{1}{(60 \text{ W/m}^2 \cdot \text{K})(0.157 \text{ m}^2)} = 0.106^\circ\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80 \text{ W/m}\cdot\text{K})(1 \text{ m})} = 0.0002^\circ\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05 \text{ W/m}\cdot\text{K})(1 \text{ m})} = 2.35^\circ\text{C/W}$$

$$R_o = R_{\text{conv}, 2} = \frac{1}{h_2 A_3} = \frac{1}{(18 \text{ W/m}^2 \cdot \text{K})(0.361 \text{ m}^2)} = 0.154^\circ\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61^\circ\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)^\circ\text{C}}{2.61^\circ\text{C/W}} = 121 \text{ W} \quad (\text{per m pipe length})$$

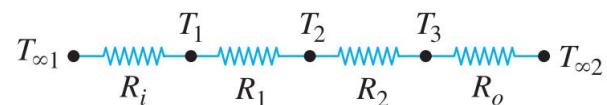
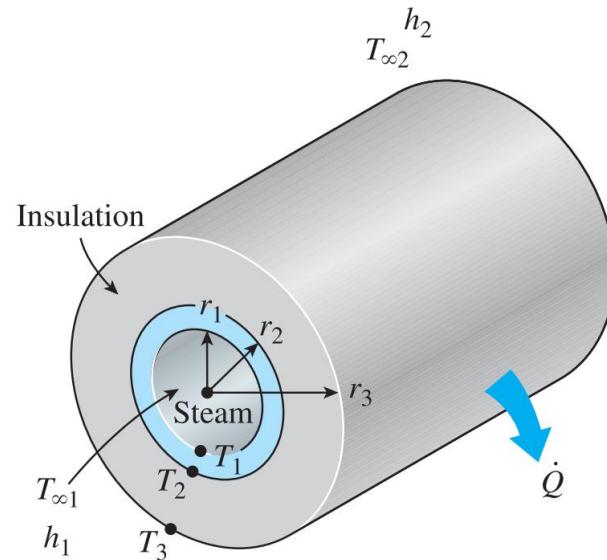
The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length  $L$ .

The temperature drops across the pipe and the insulation are determined from Eq. 3–17 to be

$$\Delta T_{\text{pipe}} = \dot{Q} R_{\text{pipe}} = (121 \text{ W})(0.0002^\circ\text{C/W}) = 0.02^\circ\text{C}$$

$$\Delta T_{\text{insulation}} = \dot{Q} R_{\text{insulation}} = (121 \text{ W})(2.35^\circ\text{C/W}) = 284^\circ\text{C}$$

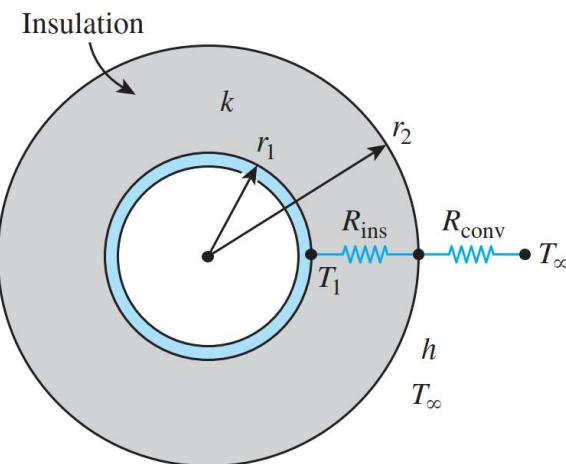
That is, the temperatures between the inner and the outer surfaces of the pipe differ by  $0.02^\circ\text{C}$ , whereas the temperatures between the inner and the outer surfaces of the insulation differ by  $284^\circ\text{C}$ .



**FIGURE 3–29**  
Schematic for Example 3–8.

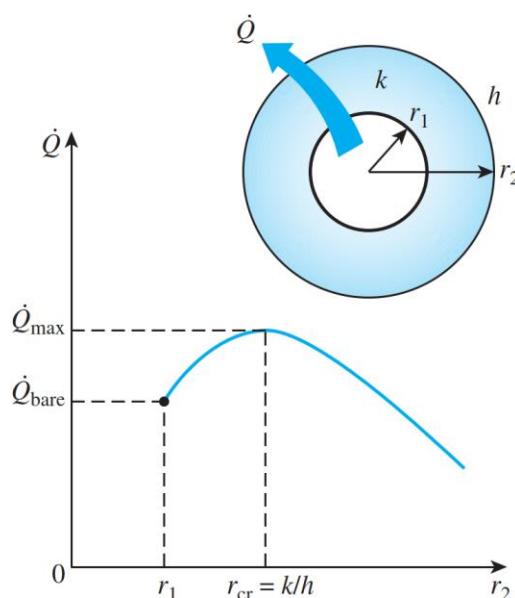
# CRITICAL RADIUS OF INSULATION

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{h(2\pi r_2 L)}} \quad (3-49)$$



**FIGURE 3–30**

An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.



**FIGURE 3–31**

The variation of heat transfer rate with the outer radius of the insulation  $r_2$  when  $r_1 < r_{\text{cr}}$ .

The variation of  $\dot{Q}$  with the outer radius of the insulation  $r_2$  is plotted in Fig. 3–31. The value of  $r_2$  at which  $\dot{Q}$  reaches a maximum is determined from the requirement that  $d\dot{Q}/dr_2 = 0$  (zero slope). Performing the differentiation and solving for  $r_2$  yields the **critical radius of insulation** for a cylindrical body to be

$$r_{\text{cr, cylinder}} = \frac{k}{h} \quad (\text{m}) \quad (3-50)$$

Note that the critical radius of insulation depends on the **thermal conductivity of the insulation  $k$**  and the **external convection heat transfer coefficient  $h$** . The rate of heat transfer from the cylinder increases with the addition of insulation for  $r_2 < r_{\text{cr}}$ , reaches a maximum when  $r_2 = r_{\text{cr}}$ , and starts to decrease for  $r_2 > r_{\text{cr}}$ . Thus, insulating the pipe may actually increase the rate of heat transfer from the pipe instead of decreasing it when  $r_2 < r_{\text{cr}}$ .

### EXAMPLE 3–9 Heat Loss from an Insulated Electric Wire

A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is  $k = 0.15 \text{ W/m}\cdot\text{K}$ . Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at  $T_\infty = 30^\circ\text{C}$  with a heat transfer coefficient of  $h = 12 \text{ W/m}^2\cdot\text{K}$ , determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

**Properties** The thermal conductivity of plastic is given to be  $k = 0.15 \text{ W/m}\cdot\text{K}$ .

**Analysis** Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = W_e = \mathbf{V}\mathbf{I} = (8 \text{ V})(10 \text{ A}) = 80 \text{ W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 3–32. The values of these two resistances are

$$A_2 = (2\pi r_2)L = 2\pi(0.0035 \text{ m})(5 \text{ m}) = 0.110 \text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_2} = \frac{1}{(12 \text{ W/m}^2\cdot\text{K})(0.110 \text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15 \text{ W/m}\cdot\text{K})(5 \text{ m})} = 0.18^\circ\text{C/W}$$

and therefore

$$R_{\text{total}} = R_{\text{plastic}} + R_{\text{conv}} = 0.76 + 0.18 = 0.94^\circ\text{C/W}$$

Then the interface temperature can be determined from

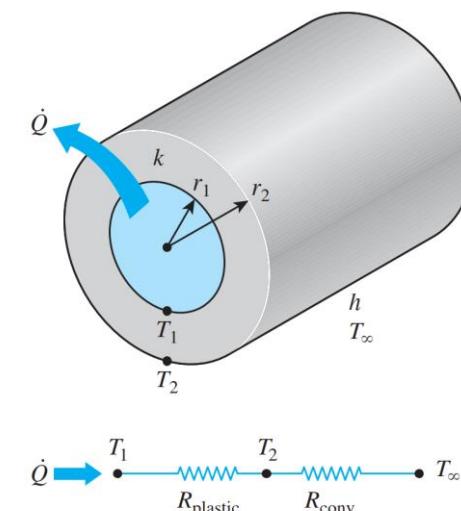
$$\begin{aligned} \dot{Q} &= \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}} \\ &= 30^\circ\text{C} + (80 \text{ W})(0.94^\circ\text{C/W}) = 105^\circ\text{C} \end{aligned}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

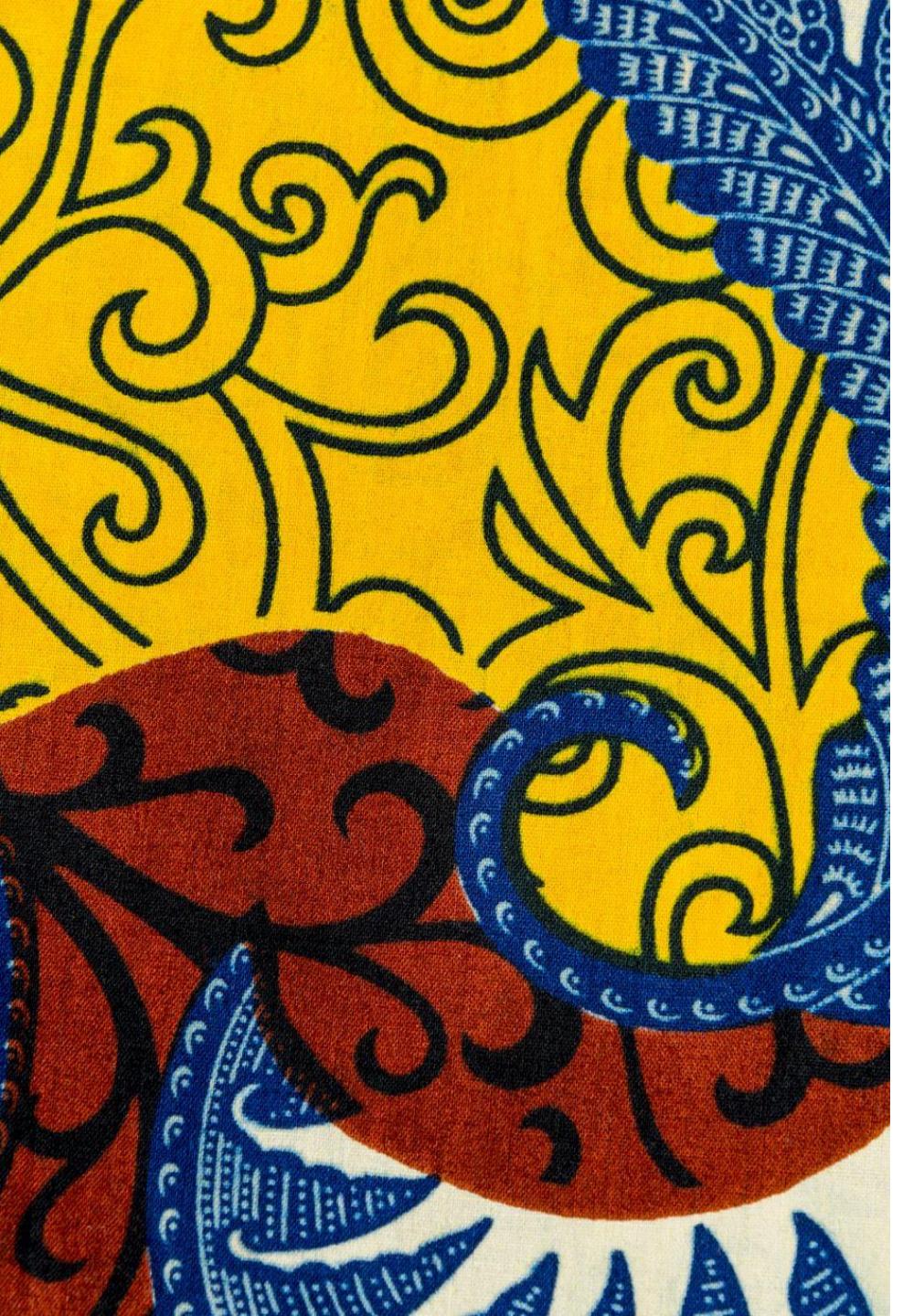
To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 3–50 to be

$$r_{\text{cr}} = \frac{k}{h} = \frac{0.15 \text{ W/m}\cdot\text{K}}{12 \text{ W/m}^2\cdot\text{K}} = 0.0125 \text{ m} = 12.5 \text{ mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer  $\dot{Q}$  will *increase* when the interface temperature  $T_1$  is held constant, or  $T_1$  will *decrease* when  $\dot{Q}$  is held constant, which is the case here.



**FIGURE 3–32**  
Schematic for Example 3–9.



# EES

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- If  $T_1 = 105^\circ\text{C}$ , plot the variation of heat transfer rate with the outer radius of the insulation  $r_2$ .



# Thank you

Ali Khosravi