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MEC-E8007 Thin-Walled Structures

Lecture 2. Materials for thin-walled structures

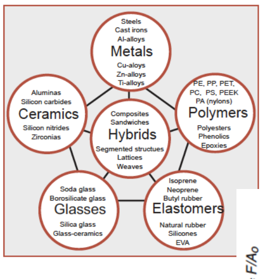
Jani Romanoff

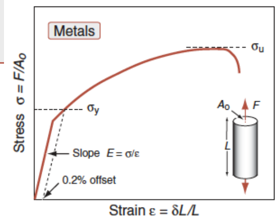
Aalto University
School of Engineering

Materials are essential part of the design of thin-walled structures. The spectrum of materials and structural solutions and their combinations are basically infinite, but their practical applications limit the amount of choices we have. Thus, it is important to understand how materials are selected, modelled and tested in the framework of thin-walled structures.

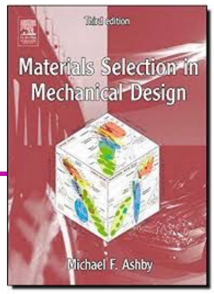
Contens

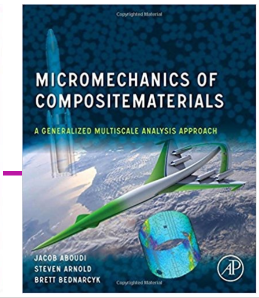
- The aim of the lecture is to understand the importance of material selection and design in the design of thin-walled structures
- Motivation
- Material selection
- Definitions for stress and strain
- Generalized Hooke’s law
- Non-linear material behavior
- Failure criteria: von Mises, Tsai-Wu,…
- Literature:
  1. Ashby,M.F., “Materials Selection in Mechanical Design”
  2. Reddy, JN, “Mechanics of laminated composite plates and shells: theory and analysis”
  3. Jones, N., “Structural Impact”
  4. Aboudi, J., Arnold, S. and Bednarcyk, B., “Micromechanics of Composite Materials – A Generalised Multiscale Analysis Apporach”, AP.





Ashby,M.F., “Materials Selection in Mechanical Design”





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The aim of the lecture is to understand the importance of material selection and design in the design of thin-walled structures

The motivation is given first and then we visit the basic concept of material selection as given in many engineering programs at the B.Sc. level. In order to treat the mechanics of thin-walled structures properly, we need to revisit the definitions of stress and strain and generalized Hooke’s law to be able handle various limit states accurately. After this we focus on the non-linear behavior as this is often needed in the analysis of lightweight structures which are expected to be safe also in extreme situations. This requires knowledge on the various failure criteria used for both isotropic and anisotropic materials.

Motivation

- Combination of proper materials and structural geometry enables lightweight design
- It is of fundamental importance to understand the mechanical properties of materials
- The relation between load (stress) and response (strain) at material level is called constitutive behavior
- In thin-walled structures we need to look at both elastic (recoverable) and plastic (permanent) material behavior
- In the analysis the terminology is important as FE codes work with very clear definitions of the material behavior

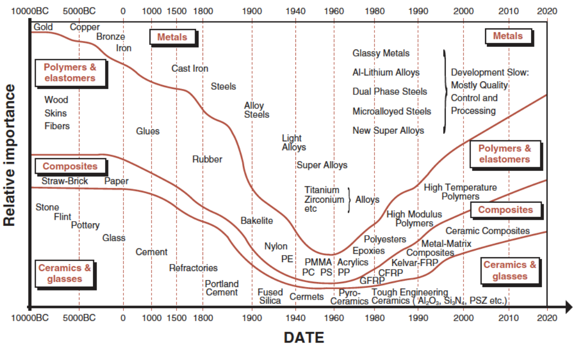


Figure 1.1 The evolution of engineering materials with time. "Relative importance" is based on information contained in the books listed under "Further reading", plus, from 1960 onwards, data for the teaching hours allocated to each material family in UK and US Universities. The projections to 2020 rely on estimates of material usage in automobiles and aircraft by manufacturers. The time scale is non-linear. The rate of change is far faster today than at any previous time in history.

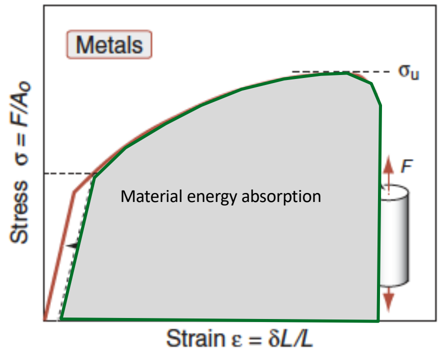
Ashby,M.F., "Materials Selection in Mechanical Design"

The proper combination of materials and structural geometry enables lightweight design. This means that the structural or material optimization alone can be used to improve the performance, but true gains can be only achieved when materials and structures are designed at the same time. For this purpose, it is of fundamental importance to understand the mechanical properties of materials beyond the elastic design which is often introduced in the elementary solid mechanics courses. In thin-walled structures we need to look at both elastic (recoverable) and plastic (permanent) material behavior.

The relation between load (stress) and response (strain) at material level is called constitutive behavior. One should keep in mind that the stress is extremely difficult issue to measure in practice for the entire stress-strain path, while load is easy. The same goes for strain measurement via displacements, this is easier to measure in practice. So it is very important to make a difference between stress and strain and not to mix these. In the analysis the terminology is important as FE codes work with very clear definitions of the material behavior.

Material Capacity

- The product of stress and strain is equal to energy absorbed by the specific material volume
- Often we allow material to yield locally as we know that there is plastic capacity in the material
- This means that in case we exceed the allowable stress, material capacity will make the structure safe
- Due to this effect it is important to know the entire stress strain curve of the material used



Maximising the deforming volume

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When we talk about the **capacity** of the material, it is important to think about the product of stress and strain that is equal to energy absorbed by the specific material volume. With this idea we understand that the larger is the product the more energy produced by the **load** we can tolerate with our design. As we aim to minimize the structural weight, we either want to minimize the density of the material or the volume it occupies.

Often we allow material to yield locally as we know that there is plastic capacity in the material in addition to the elastic which is typical used in stress-based design. This means that in case we exceed the allowable (e.g. yield) stress, material (e.g. plastic) capacity will make the structure safe. If material does not posses plastic capacity we must knock-down the allowable elastic stress to make sure that there is elastic reserve. Due to this effect it is important to know the entire stress strain curve of the material used, which is given in the figure with the grey area.

Material selection

- In general material selection is function of many different aspects and stakeholders
  - Function, shape, manufacturing process
  - Supplier, subcontractor, manufacturer, lifetime service
- Materials have number of properties that affect this selection, this is a topic of its own and one can become expert on this field
- For analysis of thin-walled structures, we need to understand the interface between materials science and technology and structural engineering

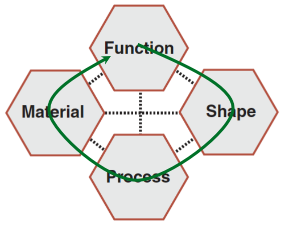

Figure 2.6 The central problem of materials selection in mechanical design: the interaction between function, material, shape and process.

Ashby,M.F., “Materials Selection in Mechanical Design”

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In general material selection is function of many different aspects and stakeholders, such as function, shape, manufacturing process and/or supplier, subcontractor, manufacturer, lifetime service provider. This combination of stakeholders narrows down the practical choice of materials and structural geometries effectively. We should keep in mind that the manufacturing processes are often material dependent and require significant investments.

Materials have also a number of properties that affect material selection, this is a topic of its own and one can become expert on this field if studying material science and engineering. For analysis of thin-walled structures, we need to understand the interface between materials science and technology and structural engineering. We need to understand the connection between material, function, shape and process in order to make optimal designs in practice. Otherwise we create theoretical solutions which may be infeasible in practice.

Material selection

Table 3.1 Basic design-limiting material properties and their usual SI units*

Class	Property	Symbol and units	
General	Density	ρ	(kg/m^3 or Mg/m^3)
	Price	C_m	($/kg)
Mechanical	Elastic moduli (Young's, shear, bulk)	E, G, K	(GPa)
	Yield strength	σ_y	(MPa)
	Ultimate strength	σ_u	(MPa)
	Compressive strength	σ_c	(MPa)
	Failure strength	σ_f	(MPa)
	Hardness	H	(Vickers)
	Elongation	ε	(–)
	Fatigue endurance limit	σ_e	(MPa)
	Fracture toughness	K_{1C}	(MPa.m$^{1/2}$)
	Toughness	G_{1C}	(kJ/m^2)
	Loss coefficient (damping capacity)	η	(–)
Thermal	Melting point	T_m	(C or K)
	Glass temperature	T_g	(C or K)
	Maximum service temperature	T_{max}	(C or K)
	Minimum service temperature	T_{max}	(C or K)
	Thermal conductivity	λ	(W/m.K)
	Specific heat	C_p	(J/kg.K)
	Thermal expansion coefficient	α	(K^{-1})
	Thermal shock resistance	ΔT_s	(C or K)
Electrical	Electrical resistivity	ρ_e	(Ω.m or $\mu\Omega$.cm)
	Dielectric constant	ε_d	(–)
	Breakdown potential	V_b	(10^6 V/m)
	Power factor	P	(–)
Optical	Optical, transparent, translucent, opaque	Yes/No	
	Refractive index	n	(–)
Eco-properties	Energy/kg to extract material	E_f	(MJ/kg)
	CO_2/kg to extract material	CO_2	(kg/kg)
Environmental resistance	Oxidation rates	Very low, low, average,	
	Corrosion rates	high, very high	
	Wear rate constant	K_A	MPa^{-1}

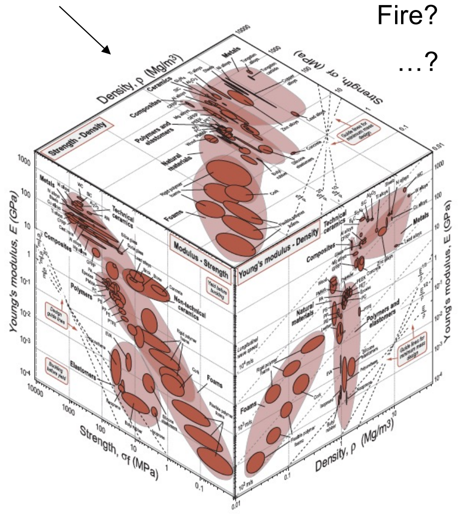
Strength

Ductility?

Fire?

…?

Stiffness



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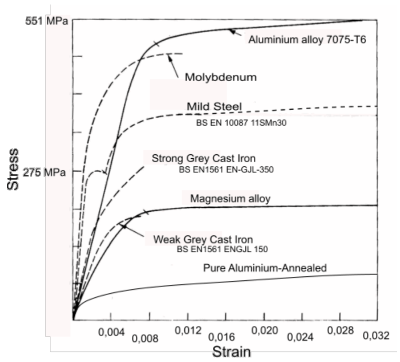
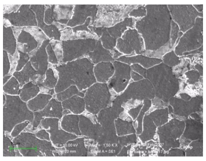
Typical material properties for thin-walled structures analysis are the mechanical properties and density of the material. Mechanical properties are characterized through stress-strain curve and we tend to compute in FEA always first the **displacements**, then the strains and finally **stresses**. From stresses we are interested on the strength. Often the focus is on ultimate strength, i.e. stress that breaks the material. For this process it is important that we know both stiffness and strength of the material. For special limit states we may be also interested on thermal, electrical or optical properties. Today even more on environmental properties. Ashby-diagram is classical way to present these **mechanical** properties as a function of the density (**general material property**).

Metallic Materials for Thin-walled Structures

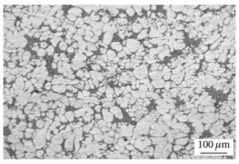
- Metals are the most common material for the thin-walled structures, due to their availability, processability, price…
- The stress-strain behavior is controlled my material microstructure
- Microstructure will be affected by the processing, e.g. welding
- The stress-strain behavior should be always related to the density of the material, i.e. energy/volume, stiffness/volume etc.



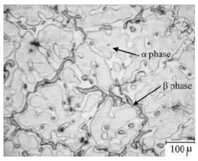
Steel



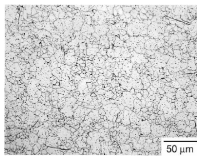
Aluminium



Magnesium



Titanum



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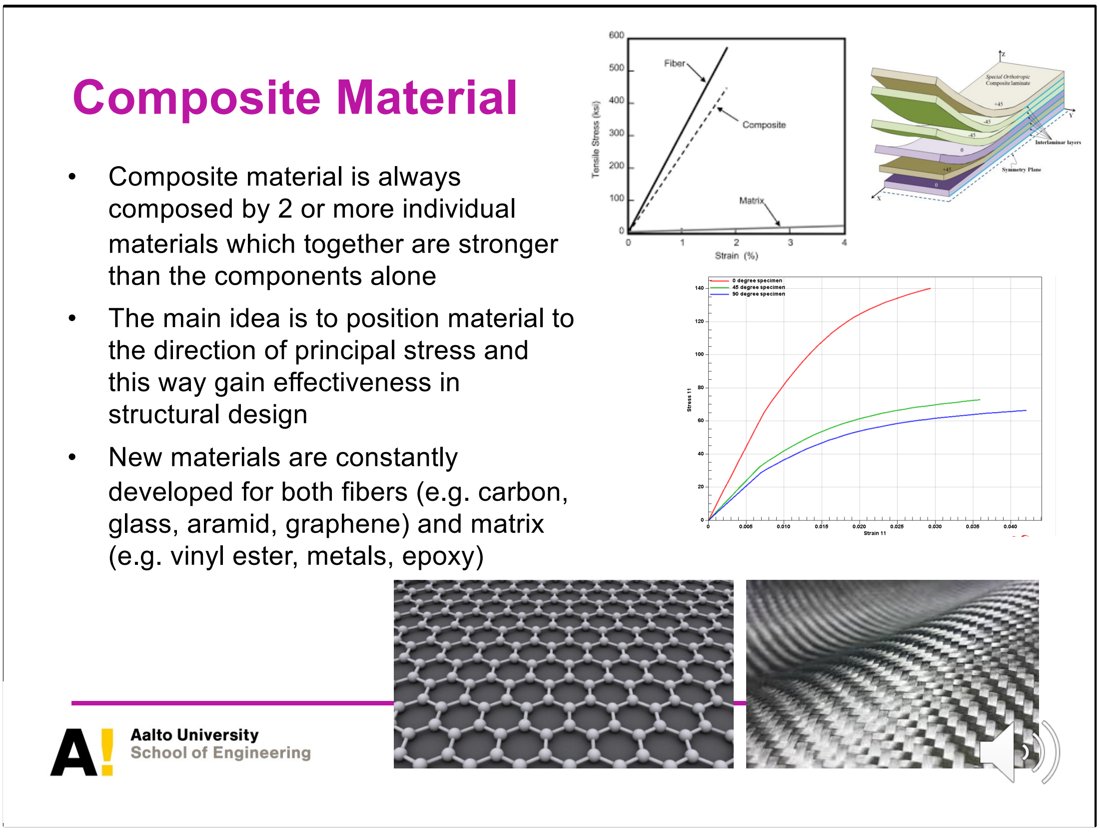
Metals are the most common material for the thin-walled structures, due to their availability, processability, price and due to the fact that the facilities that produce thin-walled structures are often optimized for metallic products. Metals are also good from sustainability viewpoint and they possess a stress-strain curve with good plastic capacity. The stress-strain behavior is controlled my material microstructure, which can be affected by processing, e.g. welding (often worsening the properties) or by heat treatments (often improving the properties). This should be kept in mind when selecting the process for metallic structures manufacturing (e.g. riveting used as joining method to not to alter the properties). The stress-strain behavior should be always related to the density of the material, i.e. energy/volume, stiffness/volume as only this way we become aware of the weight effectiveness of the material. This can be seen in Ashby-diagram as materials which are in top right corner if they are effective and bottom left corner if they are ineffective. One should remember that Ashby-diagram is for non-processed (manufactured) base materials which are in the conditions as initially intended. Most typical metallic materials for thin-walled structures are steel, aluminum, magnesium and titanium.



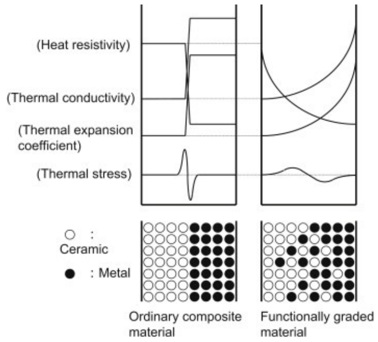
Composite materials have during recent decades become very popular material choice due to their excellent strength-to-weight and stiffness-to-weight ratios. Composite material is always composed by 2 or more individual materials which together are stronger than the components alone.

The main idea in composite material is to position material to the direction of principal stress and this way gain effectiveness in structural design. Due to this fact the definition of composite material and composite structure may sometimes be confusing and the difference between the two is not clear. One way to look at the definition is that material is a bulk entity with larger scatter in properties than in the deterministic structure in which the scatter is much smaller. In practice this means that we are not exactly sure where the fibers are in the material, but we know where the stiffeners or layers are “exactly”.

New materials are constantly developed for both fibers (e.g. carbon, glass, aramid, graphene) and matrix (e.g. vinyl ester, metals, epoxy) this way making the design space of composite materials practically infinite.

Functionally-Graded Material

- Functionally-graded materials combine the benefits of metallic and composite materials
- The material phases vary smoothly over the thickness, making the material stress concentration smaller
- The strong and stiff material phases can be positioned at the locations where needed, e.g. material surface experiencing bending strains and stresses



Material structures and properties of ordinary composite and functionally graded materials

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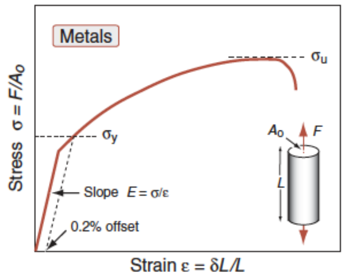
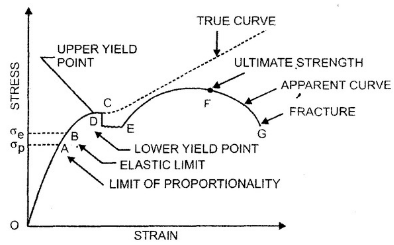
Functionally-graded materials combine the benefits of metallic and composite materials in the sense that the material phases vary smoothly over the thickness (or in the domain material occupies), making the material stress concentration smaller and this way increasing the strength (not stiffness) or the material.

The strong and stiff material phases can be positioned at the locations where needed, e.g. material surface experiencing high bending strains and stresses or at the notches. This is of course challenging in practice as the positioning must be very accurate. The analogy to structures is that typically at the stress concentration we aim to introduce a bracket or smoothen the geometry to obtain smoother stress field with reduced stress peak.

Definitions for stress and strain

- Stress is ”scaled” representation of the ”force” the material can take
- Strain is the ”scaled” elongation of the corresponding “deformation” or displacement
- The material relation for different materials is given in format of stress strain curves
  - Size effects on components become less important
  - Is used as input for structural models
- It is important to understand the differences between
  - Engineering values: small deformations
  - True values: significant deviation from engineering definition when deformations are large





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The different definitions of stress and strain are important to understand from the viewpoint of experimentally derived materials properties to be used as part of the Finite Element Analysis. By definition, the stress is a ”scaled” representation of the ”force” the material can take. The scaling is done over the area. This is why both direction of the “force” and the ”normal of the area” are exploited in stress definitions. Strain is the ”scaled” elongation of the corresponding “deformation” or displacement. Scaling is done based on the testing length of the tested specimen. With the scaling we get rid of the length-scale in the sense that the material properties have smaller scatter as the size of the specimen does not affect the results. (Stress-strain curve can be constant, force-displacement depends on specimen size).

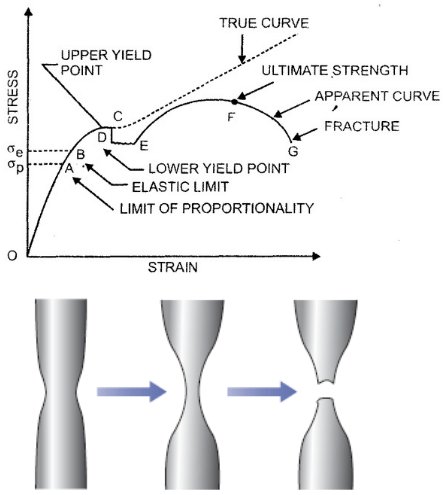
The material relation for different materials is given in format of stress strain curves. Due to the scaling, the size effects on components become less important, however, there remains a size effect due to the material microstructure, which can be large in comparison to the specimen size. This is why, we can never get rid of the scaling issue completely unless the length-scale separation is large between the consecutive length-scales. Stress-strain curve is used as input for structural models.

The area and the length in stress and strain respectively can be defined based on initial or deformed geometry. While the first case is easy to define, the latter requires constant update of the area or length during the experiment. Therefore, it is important to understand the differences between:

- **Engineering values**: small deformations, the initial and current values are the same
- **True values**: significant deviation from engineering definition when deformations are large

Definitions for stress and strain

- Engineering values:
  - Small deformations, **initial configuration**
  - Piola-Kirchhoff stress tensor
  - Infinitesimal strain tensor
- True values:
  - Significant deviation from engineering definition when deformations are large, **current configuration**
  - Cauchy stress tensor
  - Green-Lagrange strain tensor



necking leading to fracture

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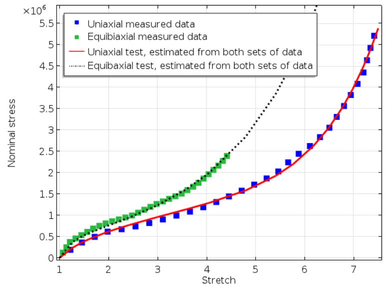
The engineering values for stress and strain are defined based on the initial, undeformed geometry of the test specimen. The (displacement) gauge length is used as the reference length and the initial area of the cross-section as the as the reference area. This means that we work with small deformations and this is why initial configuration is sufficient. The literature uses terms **Piola-Kirchhoff** stress tensor and **Infinitesimal** strain tensor in this case. Often this definition is enough for the serviceability and fatigue limit states of large thin-walled structures (locally we may need the true values) with the FEA.

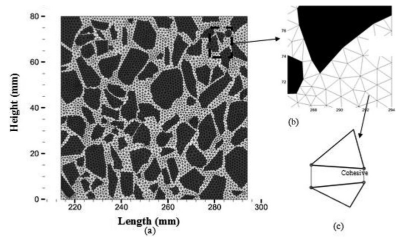
The true values of stress and strain are based on the current geometry of the tested specimen. When plastic deformations are significant, significant deviation from engineering definition is observed in this case for example due to the necking in which the material get locally thinner in one location in the specimen, while other locations may possess geometry similar to the undeformed state. This happens at large deformations. The corresponding terms for the stress and strain are **Cauchy** stress tensor and the **Green-Lagrange** strain tensor. When for example ultimate and accidental limit states are to be assessed with FEA, we need these definitions.

We should also note that while in the engineering stress-strain curve the stress may decrease for increasing strain, in true stress-strain curve, a monotonic increase is observed.

Generalised Hooke’s Law for Elastic Range

- Materials that have stress strain relation only dependent on the current state are called ”elastic”
- Hyper elastic materials are dependent on both initial and current configuration
- Homogenous material contains particles that are similar throughout the body while heterogeneous contain differences (function of position)
- We can tailor the materials for the structures if we know the direction of principal stresses (position material in the right place)





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Generalized Hooke’s law is important for the assessment of the elastic range of the material, especially when we deal with materials which are anisotropic. Materials that have stress strain relation only dependent on the current state are called ”elastic”. Hyper elastic materials are dependent on both initial and current configuration. Homogenous material contains particles that are similar throughout the body while heterogeneous contain differences (function of position). We can tailor the materials for the structures if we know the direction of principal stresses (position material in the right place). If we align for example the fibers along the direction of principal stress, we can always get stronger material with small weight than if we have random orientation of fibers.

Generalised Hooke's Law for Elastic Range

- In general the stress is function of the strain and initial stress state as

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46}\\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56}\\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}\end{bmatrix} \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{Bmatrix} + \begin{Bmatrix}\sigma_1^0\\ \sigma_2^0\\ \sigma_3^0\\ \sigma_4^0\\ \sigma_5^0\\ \sigma_6^0\end{Bmatrix}$$

- We often also need the compliance of this, i.e. $[S]=[C]^{-1}$

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16}\\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26}\\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36}\\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46}\\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56}\\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}\end{bmatrix} \begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{Bmatrix} + \begin{Bmatrix}\varepsilon_1^0\\ \varepsilon_2^0\\ \varepsilon_3^0\\ \varepsilon_4^0\\ \varepsilon_5^0\\ \varepsilon_6^0\end{Bmatrix}$$

- These can be simplified under specific symmetry conditions as follows

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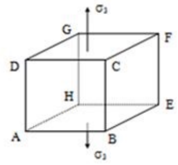
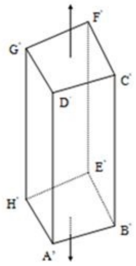
Generally, we have stresses and strain in 6 directions, meaning that we have 3 normal stresses and strains and 3 shear stresses and strains. In order to describe stress based on known strains and initial (e.g. residual stress) we need the 6x6-material matrix, i.e. constitutive law and the 6 strains and 6 initial stress components. Generally the constitutive matrix is fully occupied. The inverse matrix is called compliance matrix. If that is used, we need to know the stress state which is generally much more difficult to define than strain state and the initial strains. These matrixes can be simplified under specific symmetry conditions as follows.

Generalised Hooke's Law for Elastic Range

- Monoclinic materials (same value of elastic constant for every **pair** of coordinate systems that are mirror images with respect to each other)

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix}$$





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Monoclinic materials (same value of elastic constant for every pair of coordinate systems that are mirror images with respect to each other) reduce the number of independent terms in the stiffness matrix, in the way that we only need to consider the upper right or lower left hand corner of the stiffness matrix, which results in 21 material constants of which 8 are zero.

Generalised Hooke's Law for Elastic Range

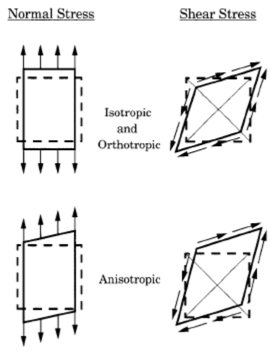
- Orthotropic materials (three mutually orthogonal material planes)

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}$$

- Isotropic materials (no preferred direction of the material)

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \Lambda \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}$$

$$E_1 = E_2 = E_3 = E, \quad G_{12} = G_{13} = G_{23} \equiv G, \quad \nu_{12} = \nu_{23} = \nu_{13} \equiv \nu$$



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For the orthotropic materials in which there is three mutually orthogonal material planes, the constitutive matrix is simplified even more 9 independent and nonzero material constant. When further simplified, we reduce the independent material constants to 2, giving us the isotropic material.

The key issue in these definitions is the deformations that occur in other directions when stretched in principal direction. The more complex is the material model, the more deformations will appear. For isotropic and orthotropic materials the normal stresses and shear stresses are decoupled, while in anisotropic case these are coupled.

As stated earlier, we have the load and the response. When loaded in one way (either by stress or strain), the specimen will respond to this based on the "material design" of the specimen. All these responses must be accounted for when we define the constitutive matrix.

Generalised Hooke's Law for Elastic Range

- Orthotropic materials (three mutually orthogonal material planes)

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}$$

- The compliance matrix is

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}$$

$$C_{11} = \frac{S_{22}S_{33} - S_{23}^2}{S} \qquad C_{12} = \frac{S_{13}S_{23} - S_{12}S_{33}}{S}$$

$$C_{22} = \frac{S_{33}S_{11} - S_{13}^2}{S} \qquad C_{13} = \frac{S_{12}S_{23} - S_{13}S_{22}}{S}$$

$$C_{33} = \frac{S_{11}S_{22} - S_{12}^2}{S} \qquad C_{23} = \frac{S_{12}S_{13} - S_{23}S_{11}}{S}$$

$$C_{44} = \frac{1}{S_{44}} \qquad C_{55} = \frac{1}{S_{55}} \qquad C_{66} = \frac{1}{S_{66}}$$

$$S = S_{11}S_{22}S_{33} - S_{11}S_{23}^2 - S_{22}S_{13}^2 - S_{33}S_{12}^2 + 2S_{12}S_{23}S_{13}$$

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}; \quad \frac{\nu_{31}}{E_3} = \frac{\nu_{13}}{E_1}; \quad \frac{\nu_{32}}{E_3} = \frac{\nu_{23}}{E_2}$$

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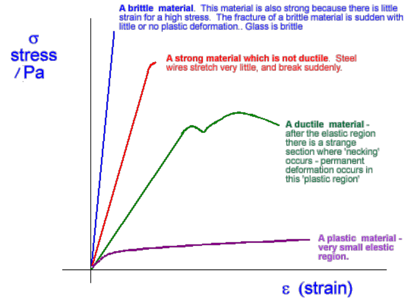
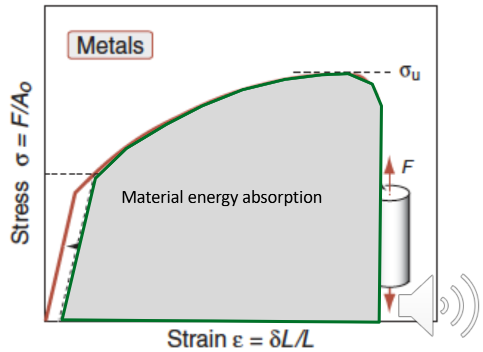
For the design of thin-walled structures one of the most commonly used formulations is the orthotropic material definition as with this we can create optimal materials for our application cases. The compliance matrix is the inverse of the constitutive matrix and can be written as the function of material constants in each 3 directions (3 Young's modulus, 3 Poisson's ratio and 3 shear modulus). We should also note that due to the symmetries certain mathematical conditions must be valid with the material constants.

Non-linear material behaviour

- Most materials (e.g. metals) have significant plastic range that can be used in the design for example as ”safety margin”
- The plastic behaviour is complex, material dependent process that contains
  - Damage initiation and propagation at single or multiple places
  - Thermodynamic effects
  - Different hardening behaviours
  - Strain rates
  - Etc.
- These processes are modelled through continuum (average stress and strain) in Representative Volume Element (RVE) of the material
- Nowadays multi-scale modelling is a new tool to simulate these behaviours instead of experimenting





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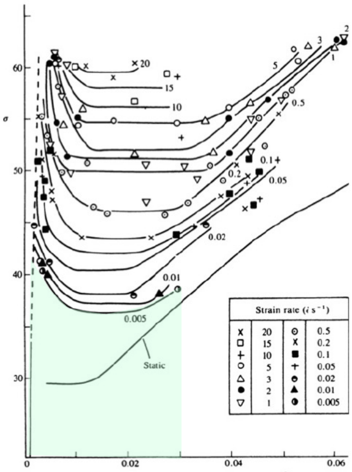
Most materials (e.g. metals) have significant plastic range that can be used in the design for example as ”safety margin” for unexpected events. If our design is fully known in terms of certainty in loads, this safety margin is not needed. Do we have any of such cases in practice? The plastic behavior is complex, material dependent process that contains damage initiation and propagation at single or multiple places, thermodynamic effects in the way that material that plasticizes creates heat and due to the fact different hardening behaviors alter the material microstructure and void formation in the material.

These processes are modelled in practice through continuum (average stress and strain) descriptions. Representative Volume Element (RVE) of the material is considered to be a representative block of material which is large enough to represent the variations in material properly, yet small enough to satisfy the definition of material in continuum sense (material point is really an infinitesimal point). Nowadays multi-scale modelling is a new tool to simulate these behaviors instead of experimenting. With these models we can simulate the effect of microstructural evolutions to the plasticity we see at the continuum level. These simulations require significant computational capacity.

Non-linear material behaviour



Jones, N., “Structural Impact”

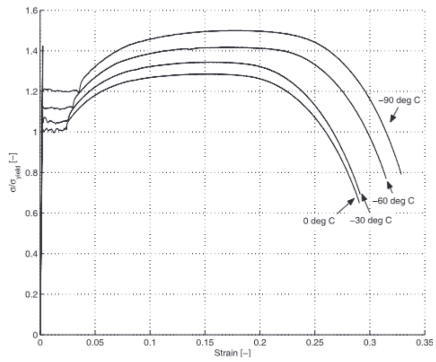


Fig. 7. Normalized tensile test results for SZT.

Ehlers, S. and Ostby, E., “Increased crashworthiness due to arctic conditions – The influence of sub-zero temperature”, Marine Structures, Volume 28, Issue 1, August 2012, Pages 86-100

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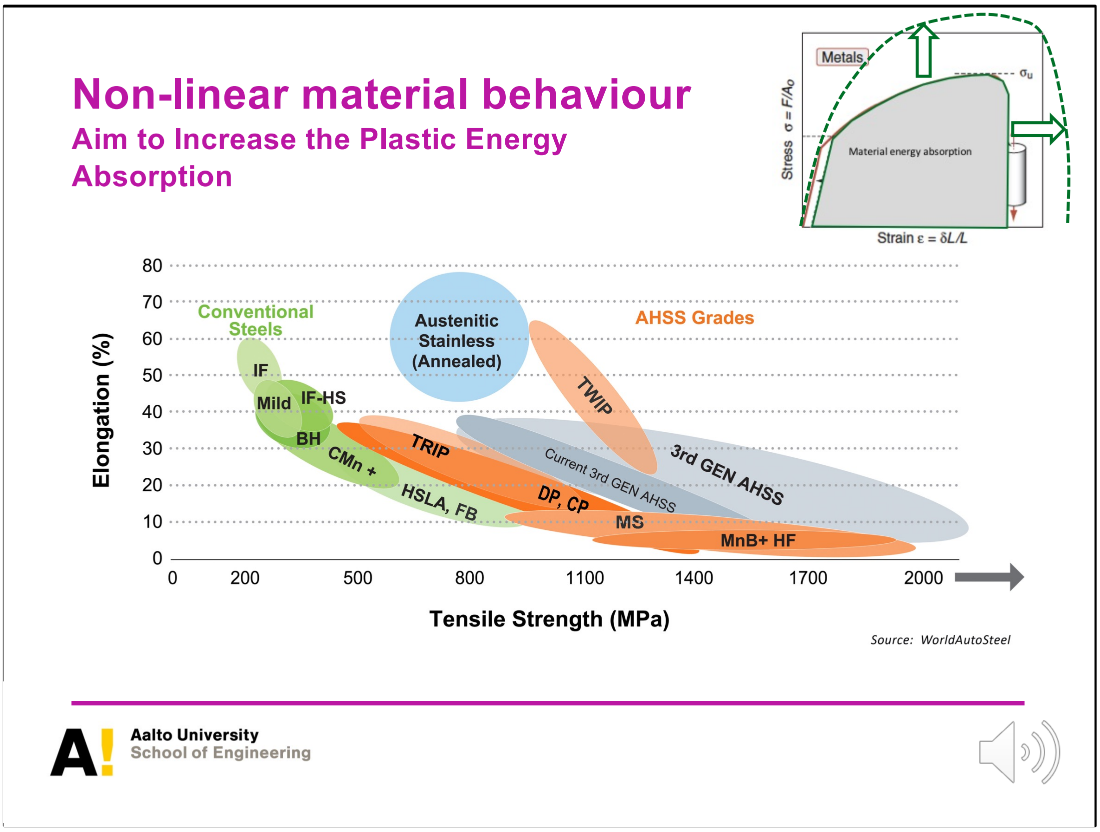
Here we see how complex the stress strain behaviour of a material (metal in this case) can be due to the strain rate or temperature effects. Similar problems can be identified in some composites for moisture effects.

We have to remember here the “reserve capasity” of the plastic region. So for example on the figure in the left we see that the increasing strain rate increases the stresses at plastic regions and in some cases decrease the elongation of fracture. This effect can be accounted with so-called Cowper-Symonds formula that describes the effect of strain rate to the stress-strain curve.

Similarly on the right we see how the temperature descrease increases the area below the stress strain curve in static tensile test of a arctic steel. There are also studies indicating the opposite behaviour.

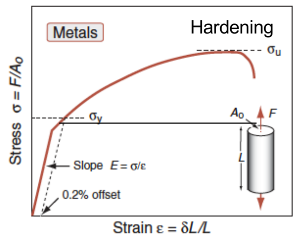


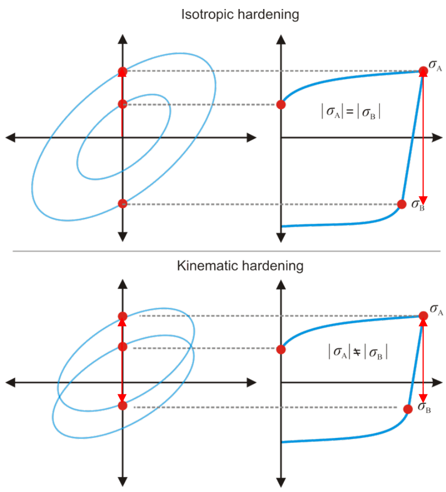
The idea of the multiscale modeling is to numerically optimize materials in the way that for example the plastic capasity of the material can be increased. Generally, in metals if we increase the strength (tensile stress), we decrease the elongation to failure (tensile strain). The “banana” curve demonstrates this effect for different families of steels. The figure also shows that with certain processes we are able to increase the strenght and elongation at the same time, increasing the energy per volume the material can absorb. This means that the microstructural mechanisms of deformations must be manipulated in beneficial manner. Simulations are cheaper way to do this than actual experiments.

Non-linear material behavior

Evolvement of Plasticity in Continuum Description

- Plasticity is often modelled by yield surfaces (e.g. von Mises)
- Hardening behaviour can be modelled by increasing the size and/or by moving the yield surface





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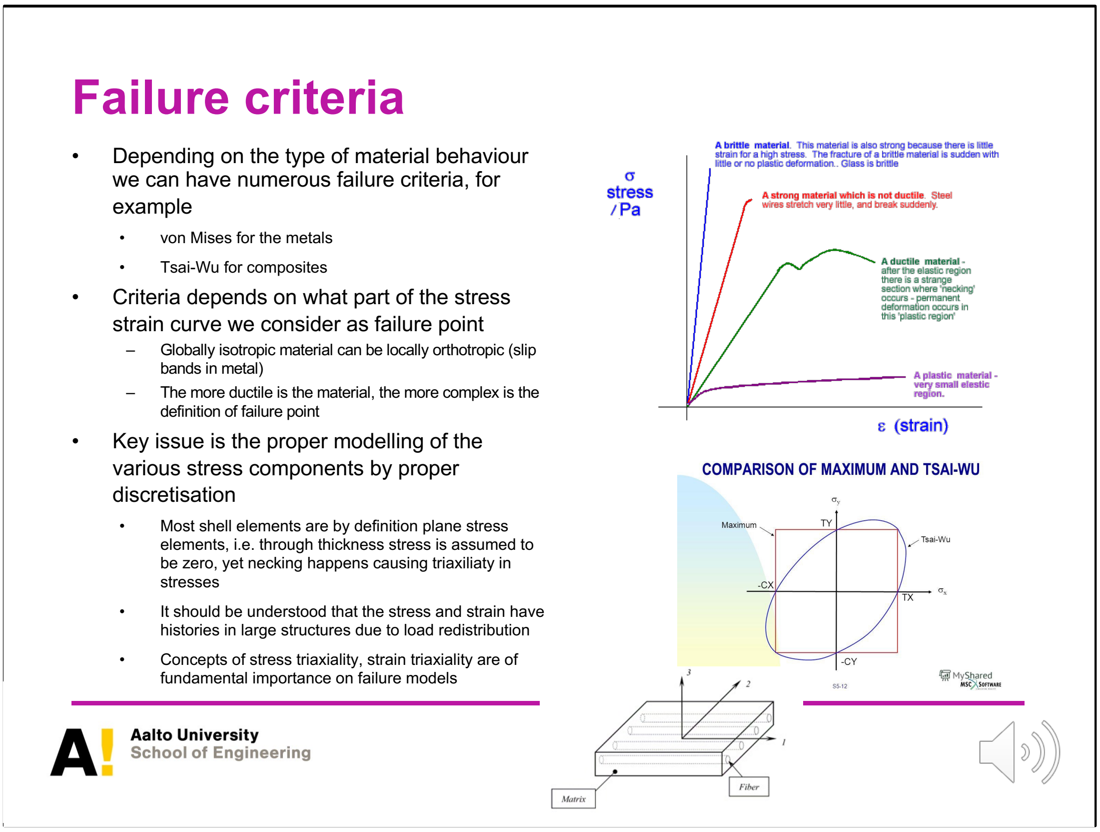
In isotropic materials, the plasticity is often modelled by yield surfaces (e.g. von Mises). The hardening behavior can be modelled by increasing the size and/or by moving the yield surface. Sometimes we do both in order to model the physics of the problem correctly. Isotropic hardening means that the size of the yield surface is increasing, meaning in practice that if we unload the specimen and reload it again either to same or opposite direction, we are working on the elastic responses until we reach the same stress level as we did before the unloading. When we move beyond this stress level, the specimen plasticizes more. In kinematic hardening the size of the yield surface does not change but its’ position in the stress space does. This means that after the unloading, if we move to the opposite direction, the range is reduced before plasticization starts again. Isotropic hardening is acceptable for cyclic loads that remain under control (high-cycle fatigue) or we load the structure once to excessive load (ultimate strength). It is also often the easiest to implement into the material models. Kinematic hardening models allow for fully cyclic behavior, but miss the effects of cyclic hardening and softening (low-cycle fatigue). This is why also combined mechanisms have been formulated.



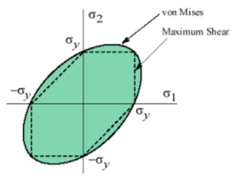
Depending on the type of material behavior we can have numerous failure criteria define mathematically such as von Mises for the metals or Tsai-Wu for composites. The applicability of the criteria depends on what part of the stress strain curve we consider as failure point. Generally metals are isotropic close to the yield point even though we know that they are slightly better in milling direction than normal to that. As we move towards ultimate strength of metal in which the specimen fractures, the behavior becomes more orthotropic. Another issue is the size of the RVE where we assess phenomena. Globally isotropic material can be locally orthotropic (slip bands in metal). The more ductile is the material, the more complex is the definition of failure point.

Key issue is the proper modelling of the various stress components by proper discretization in FEA. Most shell elements are by definition plane stress elements, i.e. through thickness stress is assumed to be zero, yet necking happens causing triaxiliaty in stresses. Continuum shells can in principle model the evolution of triaxiality, but the more accuracy we want to get, the closer we end up using through thickness solid element mesh. In thin-walled structure this increases the size of the computational models rapidly as t/l-ratio is very small. It should be understood that the stress and strain have histories in large structures due to load redistribution. That is even tough the primary level load (girder) seems to have monotonic increase in load and response, the material can be unloaded due to the changes in the load-carrying mechanism. Buckling as a phenomena tends to change the load carrying mechanism of the structural assembly.

Concepts of stress triaxiality, strain triaxiality are of fundamental importance on failure models. In classical continuum mechanics, stress triaxiality is the relative degree of hydrostatic stress in a given stress state. It is commonly used as a triaxiality factor, which is the ratio of the hydrostatic stress, to the von Mises equivalent stress.

Failure criteria by multi-scale modelling

- Today we can identify experimentally the material microstructure very accurately
- Based on 2D- or 3D-maps we can create FE-models of the microstructure
- Within the FE-models, we can have crystal plasticity and atomistic models modelling the material behavior at ever-smaller scales
- This discipline is called multi-scale modeling where we aim to solve/optimize the material properties computationally
- Basic understanding of the continuum mechanics, numerical methods and their limitations are needed when using this method



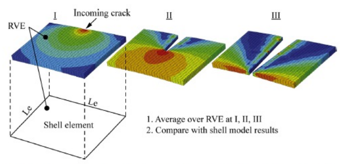


Fig. 4. Principle for comparing the solid and shell results in the RVE at different levels of deformation.

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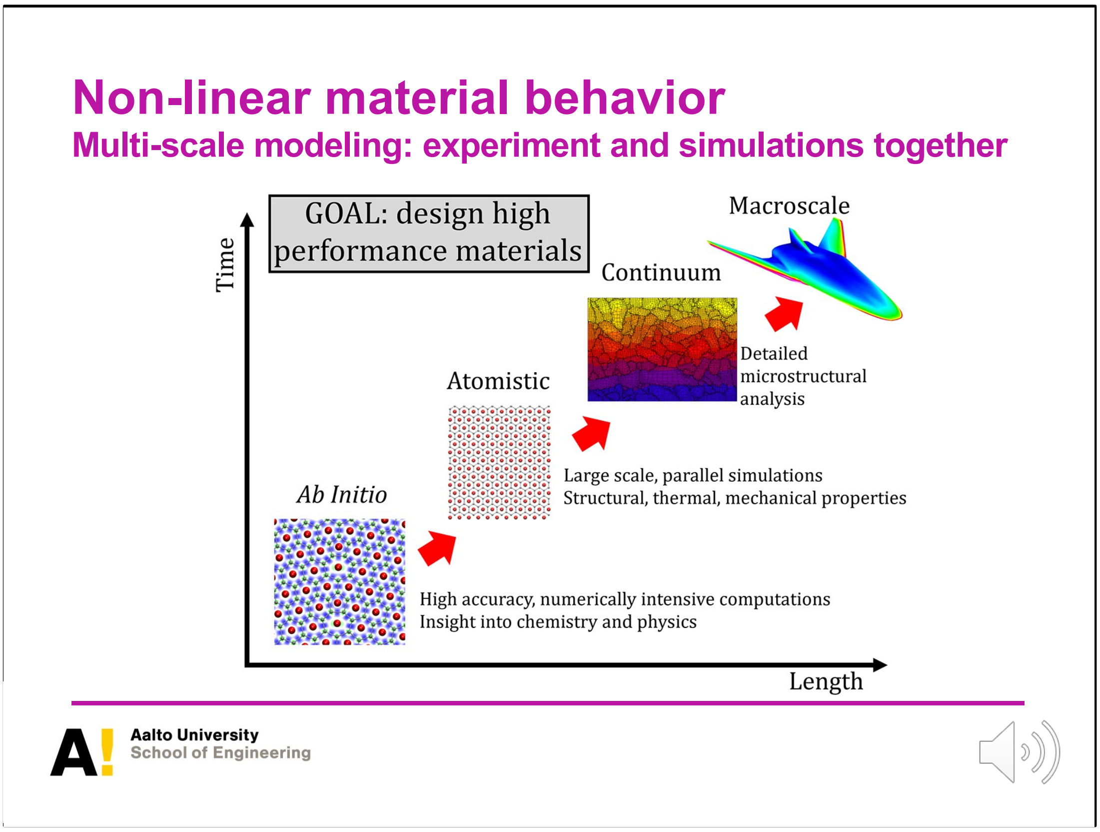
Multiscale modeling is evolving field of materials engineering with a lot of potential to develop better materials for structural engineers. Today we can identify experimentally the material microstructure very accurately and use the obtained 2D- or 3D-maps to create computational models of the microstructure. These models can be for example Multiphysics models that are solved with the Finite Element Method. Within the structural FE-models, we can also use crystal plasticity and atomistic models to simulate the material behavior at ever-smaller scales. Basic understanding of the continuum mechanics, numerical methods and their limitations are needed when using this method as we operate very much at the interfaces of basic physical phenomena. In principle with these models we can computationally create the failure criteria for our materials to be used in continuum models or get rid of the criteria totally by coupling the structural model to the simulated material model. In case we do that, we must understand that the assumptions of classical continuum mechanics are not necessarily valid. As an example the RVE can also bend when crack propagates through it under uniform tension, while we allow only stretching and shearing in our classical continuum models. Non-classical continuum mechanics is dealing with these types of extensions.



When we talk about the multi-scale modeling, we need to understand that there is spatial and temporal length-scales involved with the problem. Spatial refers to the length dimension which can range from distance between the atoms to the full length of the aircraft fuselage. Temporal length-scale refers to time and phenomena related to the movement due to the dynamics (quantum up to the classical structural mechanics). While the bottom left end is dealing with physics and chemistry, the top right end is dealing with the structural problems. Moving from bottom left to top right is called homogenization (averaging) while the opposite process is called localization (finding distributions that contain the same average properties). Localization is mathematically ill-posed problem which means that there are several feasible solutions and finding the right one mathematically is very difficult task.

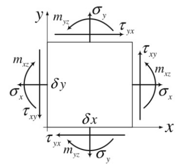
Multi-scale modelling terminology

- Volume averaging over representative volume element (RVE)
  - Smallest unit that gives the right behavior
  - Can be looked at in terms of geometry, statistics,…
- Scale transition is needed to make sure that the micro- and macro-scales are equivalent, e.g.
  - Energy, Hill-Mandel relation
  - Stress uniform in RVE, Reuss approximation, lower bound
  - Strain uniform in RVE, Voigt approximation, upper bound
- The actual equivalency is always between upper and lower bounds
- In cases where the damage is large in comparison to the element size, the classical continuum mechanics may fail and we need to use non-classical continuum mechanics (to include strain-gradient due to bending)

$$\sigma_{ij}^* = \frac{1}{V}\int_V \sigma_{ij}\,dV \qquad \varepsilon_{ij}^* = \frac{1}{V}\int_V \varepsilon_{ij}\,dV$$

$$0 = \int_V \sigma_{ij}\left(\varepsilon_{ij} - \varepsilon_{ij}^*\right)dV$$

$$0 = \int_V \varepsilon_{ij}\left(\sigma_{ij} - \sigma_{ij}^*\right)dV$$



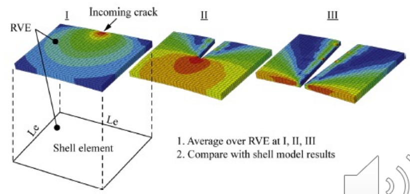


Fig. 4. Principle for comparing the solid and shell results in the RVE at different levels of de… mation.

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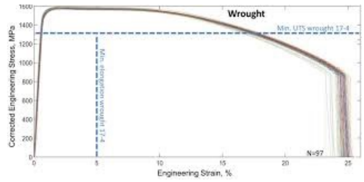
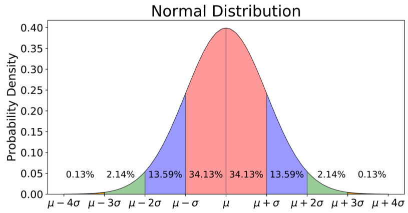
Homogenization process means volume averaging over representative volume element (RVE). RVE is the smallest unit that gives the right behavior, yet it must be large enough to represent the physics correctly in terms of statistics. The definition of the size can be therefore be based on physical size or statistics. Size scaling means that there is for example 100000 grains or composite fibers in the RVE. Statistics scaling means that while changing the material distribution slight, the results in terms of resulted stress-strain curve do not.

Scale transition is needed to make sure that the micro- and macro-scales are equivalent. This can be based on energy (Hill-Mandel relation), stress uniform in RVE (Reuss approximation, lower bound of stiffness) or strain uniform in RVE (Voigt approximation, upper bound of the stiffness). In ideal case, all of these three criterion are satisfied at the same time. The actual equivalency is always between upper and lower bounds. In cases where the damage is large in comparison to the element size, the classical continuum mechanics may fail and we need to use non-classical continuum mechanics (to include strain-gradient due to bending).

Statistical Properties

- The stress-strain curves exploited in design are often results of statistical analysis
- The better is the material quality, the less is the statistical variation
- The material can follow several different probability laws:
  - Gaussian for ductile materials, springs in parallel –analogy
  - Weibull for brittle materials, springs in series –analogy
  - Something between for most of the materials (fishnet approach)
- The design values are often associated to certain level of probability of failure





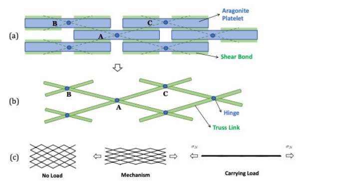


Figure 2: a) Schematic illustration of original nacre micro-structure; b) Equivalent fishnet structure; c) Deforming mechanism of fishnet

Luo and Bazant (2017)

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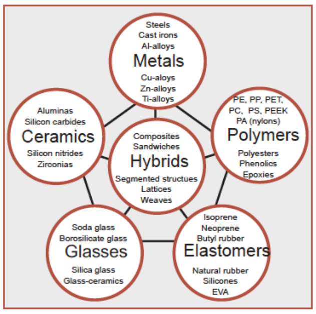
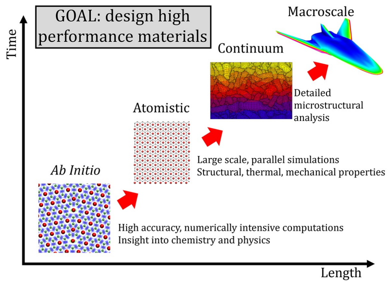
Whether measured or simulated, the stress-strain curves exploited in design are often results of a statistical analysis, meaning that there is numerous measurements or simulations done before the design values are extracted. This means that the values are never exactly the same between the experiments (physical or numerical). The better is the material quality, the less is the statistical variation. This is why composites are typically more expensive than metals. We want to make sure that the material properties are what they should be in the known direction, while in metals we assume that the material is isotropic and the orientation is not of interest. Controlling the direction is expensive.

The material can follow several different probability laws. Gaussian probability law is typical for ductile materials and this can be though through springs in parallel –analogy. If one ductile spring fails and it has neighboring ductile springs, these springs will experience higher stress, but at slightly higher strains than before the failure. If the spring about to fail has Gaussian strength distribution, so will be the result of all springs. Weibull distribution is typical for brittle materials, and this can be modelled through springs in series –analogy. In this model the basic idea is that we have a chain with links each having a Gaussian distribution. The entire chain fails when the weakest link fails. Thus, the probability of the chain failing is a function of the size of the chain but also weakest link. Mathematically, this process leads to Weibull distribution. In reality, most of the materials are something between these two cases (fishnet approach can model this). The design values are often associated to certain level of probability of failure, e.g. 2.3%.

Summary

- Material selection and design are fundamental issues in the design of thin-walled structures
- For the material selection it is important to understand the different definitions of stress and strain, i.e. engineering vs true
- Generalized Hooke's law can be used to define different classes of materials from monoclinic to isotropic in elastic range
- Non-linear material behavior is characterized by damage evolution over different length-scales that can be investigated numerically and experimentally using continuum mechanics and multi-scale modeling





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Material selection and design are fundamental issues in the design of thin-walled structures and typically the best solutions are obtained by combing the benefits of both aspects. For the material selection it is important to understand the different definitions of stress and strain, i.e. engineering vs true and how these are used in FEA context. Generalized Hooke's law can be used to define different classes of materials from monoclinic to isotropic in elastic range. Non-linear material behavior is characterized by damage evolution over different length-scales that can be investigated numerically and experimentally using continuum mechanics and multi-scale modeling.