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MEC-E8007 Thin-Walled Structures
Lecture 10. Vibration Analysis
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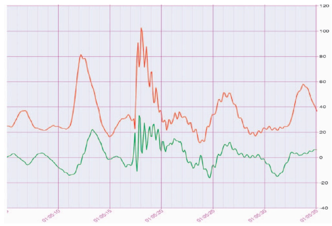
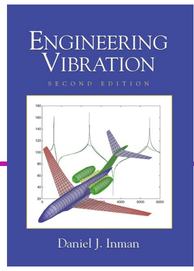
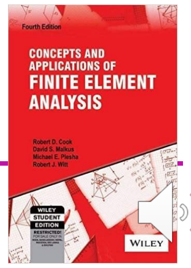
When the stuctural models with kinematic assumptions have been defined we constrain the possible modes of vibration. On the other hand vibrations are important both from comfort and fatigue aspects as they cause vibration dosage to people and contribute to the fatigue damage.

Contens

- The aim of the lecture is to understand why vibration analyses are needed, what are the fundamental problems in analysis of large structures. The aim is to apply the knowledge gained to perform calculations on large structure.
- Motivation
- Vibration analysis types
- Wake
- Modal superposition
- Vibratory analysis in multiple scales
- Springing and Whipping
- Examples
- Literature
  1. Cook, R.D., Malkus, D.S. and Plesha, M.E., "Concepts and Applications of the Finite Element Analysis, 3rd Edition", John Wiley and Sons,
  2. Inman, D.J., Engineering Vibration, 2nd edition







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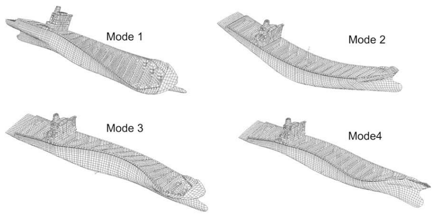
The aim of the lecture is to understand why vibration analyses are needed, what are the fundamental problems in analysis of thin-walled large structures. The aim is to apply the knowledge gained to perform calculations on large structure.

Motivation

- Vibrations are everywhere and they can cause
  - Discomfort
  - Fatigue strength problems
  - Excessive deformations in resonance
- Vibrations cannot be omitted in structures which are exposed to wide spectrum of loads
  - Wind
  - Waves
  - Machinery
  - Impacts
- Global analyses are needed to make sure that the vibration response levels do not become too large



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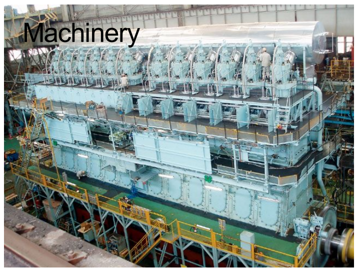
Vibrations are everywhere and they can cause discomfort to passengers and crew, fatigue strength problems due accumulated damage and excessive deformations in resonance. Vibrations cannot be omitted in structures which are exposed to wide spectrum of loads coming from environment such as wind, waves and ice, but also manmade loads such as operational loads and machinery-induced static and impact type of loads. Global analyses are needed to make sure that the vibration response levels do not become too large at the level of the entire system. Good example of this is the airplane taking off. When departing from the ground, we often feel vibrations of the entire fuselage and then during the flight mostly only local vibrations. Similar phenomena are present in other thin-walled structures such as bridges and ships.

Vibration Analysis Types

- Vibration analyses can be
  - Eigenfrequency analyses
  - Forced vibration analyses
- Vibrations may occur due to various excitations
  - Wind
  - Machinery and systems
  - Wave-induced
  - Impact
  - Propulsion
- Global (girder) vibrations
  - Vertical bending
  - Horizontal bending
  - Torsion
  - Longitudinal
- Local vibrations
  - Decks and bulkheads
  - Superstructure
  - Etc.
- Important issues on vibrations
  - Mass and stiffness distribution – ineffective mass, e.g. cars with suspension
  - Added mass of vibrating fluid
  - Effective mass of cargo (cars with suspension, dead load, liquid, etc)
  - Damping
- FE or analytical methods are used for the analysis



There are many different types of vibration analyses depending on the objectives of the analyses. Vibration analyses can be for example eigenfrequency or forced vibration analyses in which the first focuses on understanding the vibration characteristics of the system, while the second is focused on responses under known loading. Both time and frequency domain analyses can be considered here. Vibrations may occur due to various excitations from environment or by the operations. Vibrations may occur at different length-scales for example the Tacoma bridge girder, in which we see mixed vertical bending, horizontal bending and torsion with eigenfrequencies all being close to each other and active in this type of forces global vibrations. Local vibrations can happen at the decks and bulkheads, that is at the tertiary level or at the level of larger units such as superstructure of a ship vibrating as sub-entity, i.e. at the intermediate length-scales. Here we really have to put emphasis to the length-scale interaction. We cannot use the same ideas of superposition we could in static responses, but we really have to account also the interaction in dynamic sense. If our model is kinematically constrains (e.g. CLT, FSDT, TSDT) or our mesh is too coarse, the all relevant vibration modes may not be captured.

Important issues on vibrations is for example the level of discretization, but also the true mass and stiffness distribution, with properly accounted ineffective masses, e.g. cars with suspension, added mass of vibrating fluid and effective masses of cargo (cars with suspension, dead load, liquid, etc). Damping is often very complex and sometimes also exactly unknown. Both analytical and FE methods can be used.

Superposition of Eigenmodes

- Typically the structures are flexible and they are vibrating with the medium around it
  - Added mass due to vibrating fluid to equation of motion

    $$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F(t)\}$$

  - Starting point are the "dry" eigenmodes

    $$\left|[K]-\omega^2[M]\right|=0.$$

- Global eigenmodes include both
  - rigid body
  - deformation modes

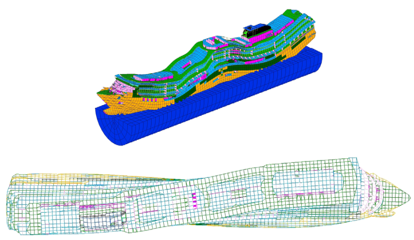


Fig. 4. Vertical and torsional wet bending modes of the hull girder.

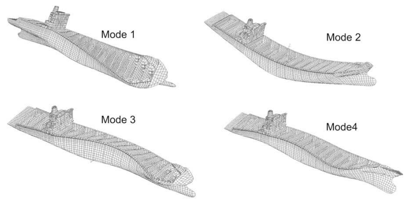
- Displacement {y} can be written as product of eigenmode [ϕ(x)] and generalized coordinate {p(t)} as

  $$\{y(x,t)\}= [\phi(x)]\ \{p(t)\}$$

- Undamped beam, to obtain eigenfrequencies ω_i and shapes ϕ_i we formulate eigenvalue problem:

  $$\left([K]-\omega^2[M]\right)\{\phi\}=0.$$

- As the discretization is done on global model and includes certain amount of dofs the question is how to extract the global modes from mixture of global and local



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When eigenmodes are known, one of the most typical ways to obtain the responses for forced vibrations is to use the superposition of eigenmodes principle. There the FE mesh is fixed in size and we have extracted the eigenmodes of this mesh. The number of eigenmodes is a function of the discretization. That is for infinite mesh density, we will have infinite eigenmodes. The basic idea is to use FEA, and thus we have only some of the eigenmodes of the system in reality. What is missing can be due to the mesh size or kinematic constraints we use for example in ESL beam, plate and shell formulations. The eigenmodes can be extracted accounting the added mass of cargo or surrounding fluid medium or excluding it. These eigenmodes include both rigid body and deformation eigen modes. We are interested on the deformations. We should remember that the eigenmodes are always scaled, i.e. their amplitudes are relative and often the maximum displacement is scaled to value of 1.

Superposition of Eigenmodes

- Substitution of {y} as product of eigenmodes [ϕ(x)] and generalized coordinate {p(t)} and its' derivatives gives:

  $$[\mathrm{M}][\phi(x)]\{\ddot{p}(t)\}+[\mathrm{C}][\phi(x)]\{\dot{p}(t)\}+[\mathrm{K}][\phi(x)]\{p(t)\}=\{F(t)\}$$

- Multiplying from left with eigenmode i transpose gives

  $$\{\phi_i\}^T[\mathrm{M}][\phi(x)]\{\ddot{p}(t)\}+\{\phi_i\}^T[\mathrm{C}][\phi(x)]\{\dot{p}(t)\}+\{\phi_i\}^T[\mathrm{K}][\phi(x)]\{p(t)\}=\{\phi_i\}^T\{F(t)\}$$

- Due to orthogonality

  $\{\phi_i\}^{\mathrm{T}}[\mathrm{M}]\{\phi_j\}=0$

  $\{\phi_i\}^{\mathrm{T}}[\mathrm{C}]\{\phi_j\}=0$, when $i \neq j$

  $\{\phi_i\}^{\mathrm{T}}[\mathrm{K}]\{\phi_j\}=0$

- We can simplify this to

  $\{\phi_i\}^{\mathrm{T}}\,[\mathrm{M}]\,\{\phi_i\}\,\ddot{p}_i + \{\phi_i\}^{\mathrm{T}}\,[\mathrm{C}]\,\{\phi_i\}\,\dot{p}_i + \{\phi_i\}^{\mathrm{T}}\,[\mathrm{K}]\,\{\phi_i\}\,p_i = \{\phi_i\}^{\mathrm{T}}\,\{F(t)\}$

- From which the generalized coordinate can be solved
- The modes are now uncoupled and can be solved.

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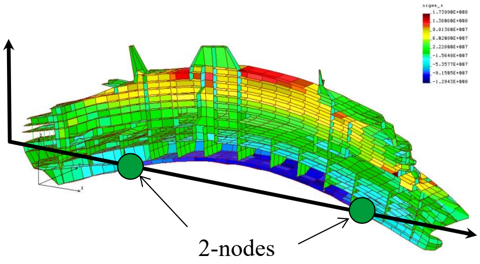
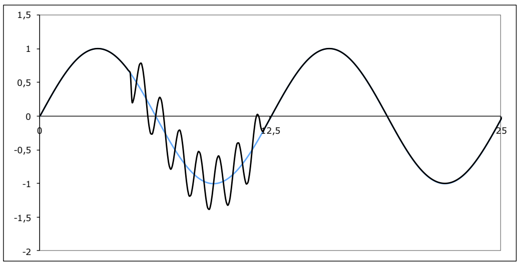
Due to scaling, we often exploit so called generalized coordinate system from which we can by multiplying the equation of motion term by term from left obtain a new equation of motion which is orthogonal meaning that the modes representing certain load frequency components get uncoupled in the equation of motion. Then we can solve “each” row of this system of equations.

Springing vs. Whipping

- The resonant global vibration due to the global load wave loading is called *springing* (2 or 3-nodes bending)
  - Structure with low natural frequency, i.e. low stiffness to mass ratio
  - "sideways" springing is often called swinging
  - Calculations suggest that springing may also contribute to the extreme response for some ships, but springing vibrations are generally more important for fatigue, up to 50%.
- When transient load causes hull girder vibrations the phenomena is called *whipping*
  - Slamming causes impact load
  - Due to this load vibrations occur
  - Uncomfortable for crew and passengers





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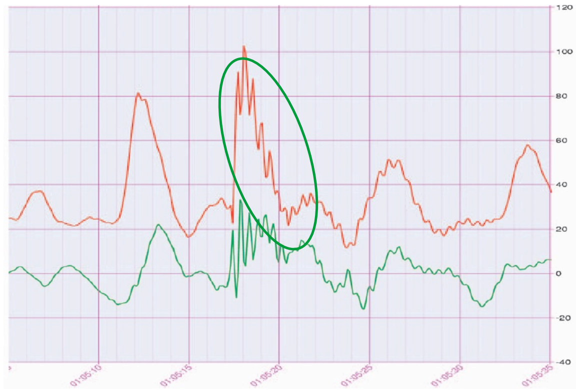
Springing and whipping can be used to describe responses that can become critical at thin-walled structures. The resonant global vibration due to the global load (wave) loading is called springing (2 or 3-nodes bending). This typically happens in thin-walled structure with low natural frequency, i.e. low stiffness to mass ratio at the global, girder, level. Sometimes "sideways" springing is often called swinging. Calculations suggest that springing may also contribute to the extreme response for some ships, but springing vibrations are generally more important for fatigue, up to 50% of the observed damage.

When transient, impact, load causes girder vibrations the phenomena is called whipping. For example wave slamming causes impact load and due to this vibrations occur. Similar is the take off of an airplane and transient fuselage vibrations.

Exmaple of Whipping
Wave bending Moment + Impact Response

- the time evolution of the stresses caused by the vertical bending moment, following severe slamming event



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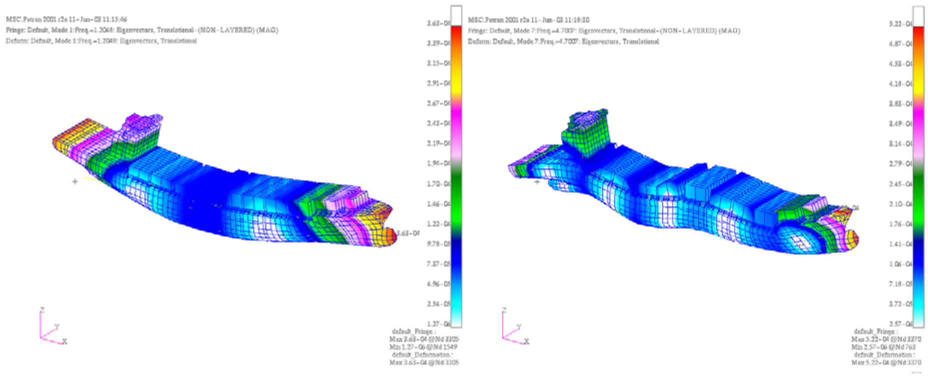
So here we see variation of container ship stresses at the main deck due to the whipping responses.

What we see is the large peaks which are due to the ship meeting the waves as usual, but also the whipping oscillations in the circled peak after which we get micro-fluctuaitions to the repsonses that are on top of the macro-fluctuations. These micro-vibrations damp out after a while.

Example
Hull Girder Vibration



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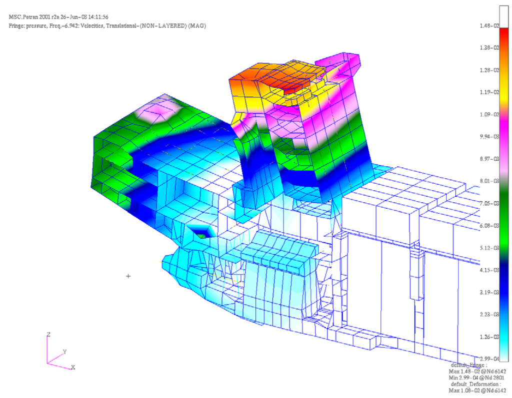
Here we see an example of the lowest eigenmodes of the hull girder. On the left the 2-node vibration, node used here to describe the point along longitudinal axis with zero deflection and on the right we see a case with more nodes. The more there are “nodes” the more energy the mode takes and thus in reality we are only interested about the low energy modes in design. Of course exception is resonance frequencies, bu there the question is about energy available for vibrations. This means often high frequencies, wake local structures as the energy is small to wake these.

Example
Superstructure Vibration

Classification societies provide the maximum accelerations and velocities for each class and ship space.

E.g. a passenger spaces in a ”comfort-class” ship need to have all velocities under 2 mm/sec



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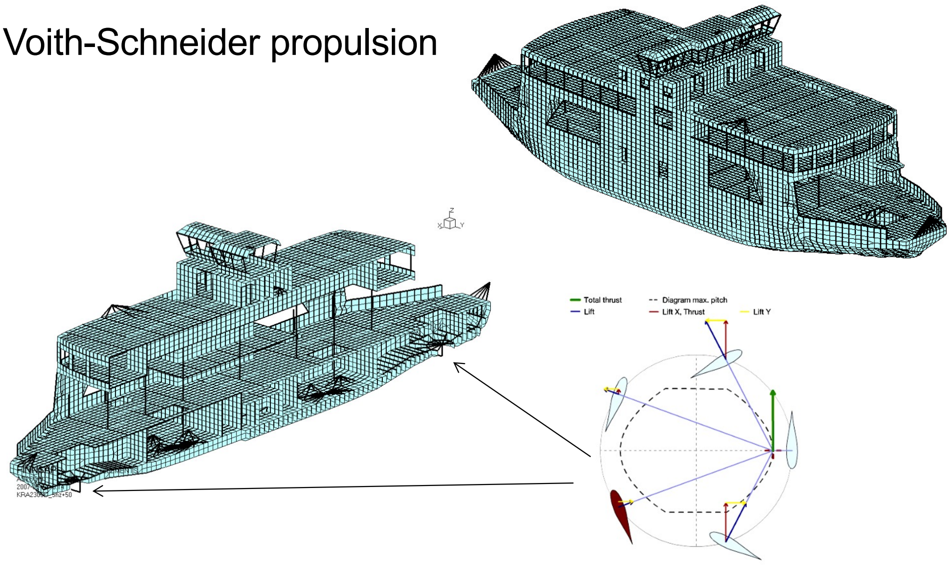
Authorities, such as classification societies provide the maximum accelerations and velocities for each class and ship space. For example a passenger spaces in a ”comfort-class” ship need to have all velocities under 2 mm/sec

Example of Interacting Vibrations – Ferry

- Voith-Schneider propulsion



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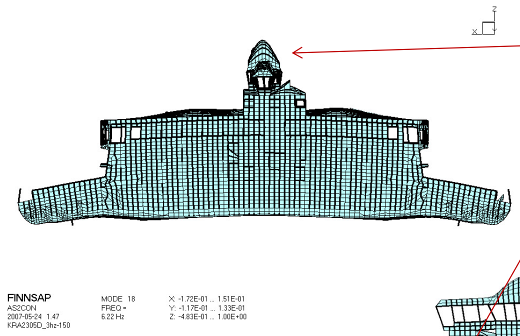
Here is an example of the load by propulsion which is due to the new type of propulsion units that operate in the flow field arround the ship hull and the propeller.

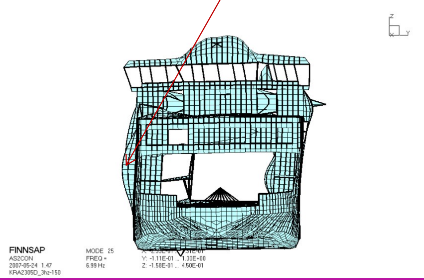
Example Ferry

- **2-node global bending at 6.22 Hz**



Local
vibrations

- **Racking Mode at 6.99 Hz**



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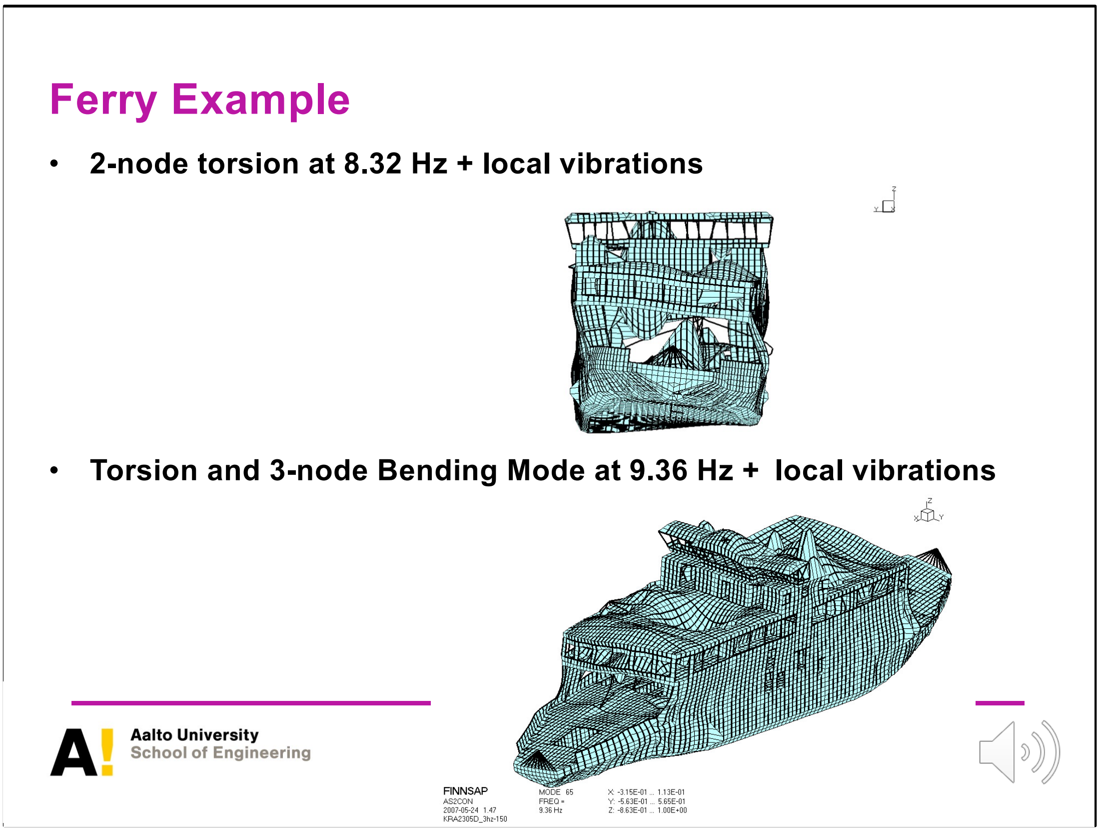
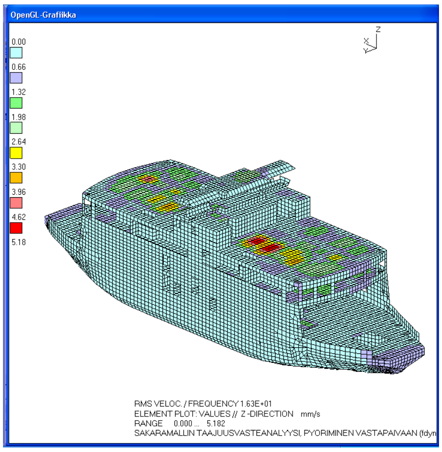
The global modes of the structure are fairly close and only minor local vibrations can be seen at the low frequencies. This indicates that the problems are difficult to solve by stiffening as the entire structure deforms. This means that the main dimensions or the loading should be changed.

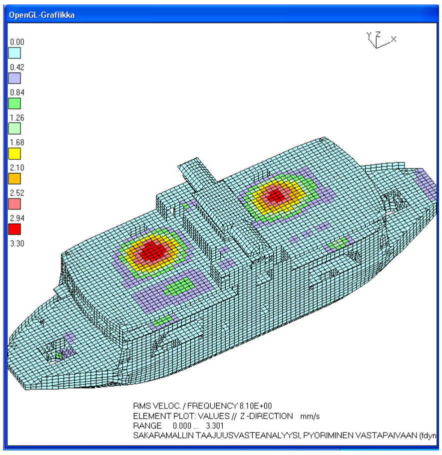


When frequency is increased the global and local modes start to interact and this requires better discretization to make sure that the coupling is properly accounted for.

Forced vibrations response



16,3 Hz



8,1 Hz

Z-direction velocities

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When we know the eigenfrequencies and modes we can use the modal superposition to obtain the results for forced vibrations (spectrum loading).

Practical Aspects

- Mass distribution affect the results considerably and this is often uncertain knowledge during project – weight management is important

- You will get as many modes as there are degrees of freedom, if
  – Interest is on global modes, you might have to look for these from set of local modes when mesh is very fine
  – Interest is on global modes, the mesh is typically very coarse and you do not have any info on local modes
  – Sometimes these modes are close and interact to make our engineering interesting…

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Mass distribution affect the results considerably and this is often uncertain knowledge during project and therefore the weight management is important. Another fact is that you will get as many modes as there are degrees of freedom in your FE-model. If the interest is on global modes, you might have to look for these from a set of local modes when mesh is very fine. If the interest is on global modes, the mesh is typically very coarse and you do not have any info on local modes. For example if you use the ESL models, you may miss some local modes. Sometimes these modes are close and interact to make our engineering interesting. We can think this through the concept of micro-inertia which means that when ever we homogenize or make the structure equivalent in some way, there is some inertia on the length-scale smaller than we consider that may interact with the modes we have. Thus, we may overestimate the eigenfrequency due to this.

Role of FE mesh size

- In large structures, when we use for example:
  - orthotropic material properties,
  - offset beams or
  - smeared/homogenized plates

  we reduce the number of degrees of freedom of our FE-model through

- This means that we also can have less eigenfrequencies and modes due to these simplifications:
  - If we model the structure properly, we do not need the “missed” modes and frequencies
  - If we model the structure incorrectly, we miss important modes.

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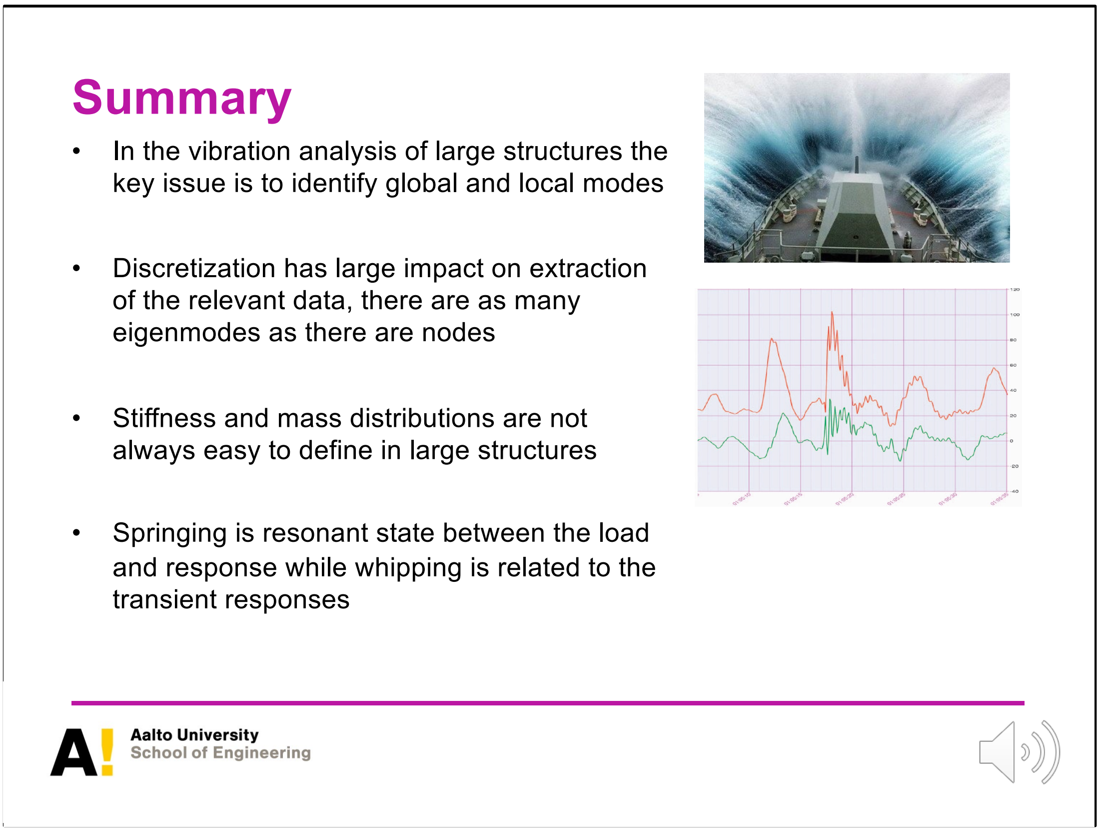
In large structures, when we use for example: orthotropic material properties, offset beams or smeared/homogenized plates
we reduce the number of degrees of freedom of our FE-model through kinematics or material models. This means that we also can have less eigenfrequencies and modes due to these simplifications that we must solve. If we model the structure properly, we do not need the “missed” modes and frequencies. If we model the structure incorrectly, we miss important modes. In the latter case, there is a way to bring in the missed modes by microinertia and other computational techniques. What we can also do is sub-modeling in which we solve the dynamic problem in frequency bands.



In the vibration analysis of large structures the key issue is to identify global and local modes

Discretization has large impact on extraction of the relevant data, there are as many eigenmodes as there are nodes

Stiffness and mass distributions are not always easy to define in large structures

Springing is resonant state between the load and response while whipping is related to the transient responses