# Aalto University
## *School of Engineering*

MEC-E8007 Thin-Walled Structures

Lecture 9. Fatigue Analysis and Sub-modelling

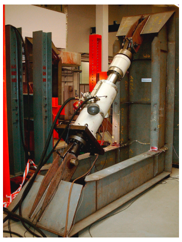
**Jani Romanoff**

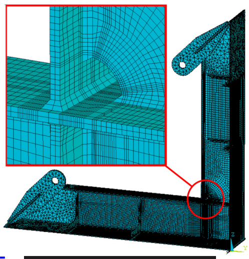
Aalto University
School of Engineering

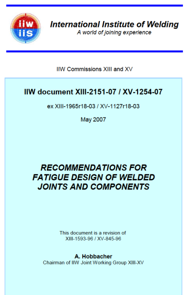
Now when the basic structural elements such as beams, plates and shells are covered, we can start to analyze the obtained results with strength criteria. As the structural models presented give elastic stress and deflections, we first focus on fatigue strength in which elastic stress analysis is often sufficient, but we need to be able to assess the stresses in very small details. This is why we also present the concept of sub modeling.

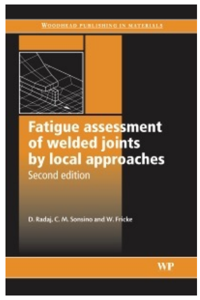
# Contens

- The objective is to understand the fatigue assessment procedure of large complex structures and apply the knowledge to practical design problems
- Motivation
- Definitions
- Analysis types: full spectral, simplified, screening
- Structural Stress analysis method
- Notch stress method
- Uncertainty in fatigue assessment
- Literature
  1. Radaj, D., Sonsino, C.M. and Fricke, W., Fatigue Assessment of Welded Joints by Local Approaches2nd Edition, Woodhead Publishing
  2. Hobbacher, A., “Recommendations for Fatigue Design of Welded Joints and Components”, International Institue of Welding, XIII-1593-96/XV-845-96, May 2007.








Aalto University
School of Engineering

The objective is to understand the fatigue assessment procedures of large complex thin-walled structures and apply the knowledge to practical design problems. This is important as the fatigue is most often the bottle neck of the design. In order to do the analysis we need to define the basic assumptions, methods used and concepts used specifically in thin-walled structures. For example the nominal and structural stress methods are lightweight in comparison to notch stress method when it comes to work load, but the notch stress method is most generic. The as we deal with cumulative damage over the life time, we must understand the ideas of the spectral analysis, the simplified life time analysis and screening which is important for large structures. Finally, we discuss about uncertainties.

# Motivation for Fatigue Assessment

- Fatigue is present in all structures loaded by cyclic loads
  - Waves,
  - Winds,
  - Operations,
  - Machinery

- Fatigue can initiate anywhere in the structure and should be traceable and critical locations should be improved in design and construction

- Finite Element Analysis is the tool for modern fatigue assessment of structures

Aalto University
School of Engineering

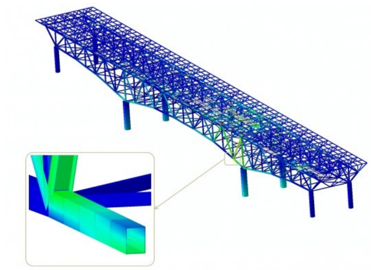
Fatigue is present in all structures loaded by cyclic loads whether the loads are deterministic or random. The load can be caused waves, winds, road surface, normal and abnormal operations and for example rotating machinery. As a process, fatigue can initiate anywhere in the structure and should be traceable and critical locations should be improved in design and construction. What is critical location in system loaded by random loading and containing enormous amount of “hot spots” called stress concentrations is really a major challenge in design. Finite Element Analysis is the tool for modern fatigue assessment of structures and there is a lot of open literature on how to use it properly to design for fatigue, unfortunately there is not one single approach that works always.

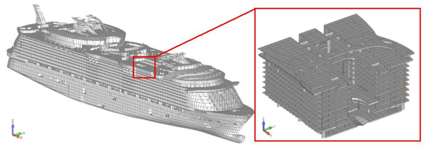
# Motivation for Sub-models

- Static global analysis is needed to understand how the structure behaves
  - Understanding the load-carrying mechanism
  - Identification of critical locations
  - Decision how structure can be improved (optimization)
  - Create load cases for the local analyses
  - Etc

- Static local analyses are needed in detail design
  - Strength of details (buckling, yield, fatigue)
  - To create the design basis for details and validate the details
  - Etc

- Static FE analyses can be easily 100-1000 for large complex structure





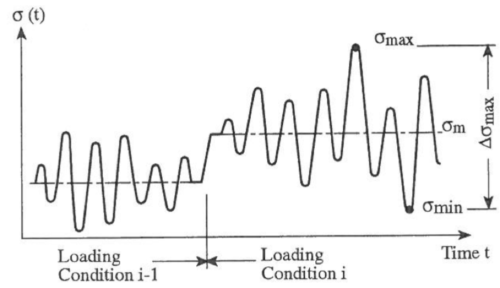
Aalto University
School of Engineering

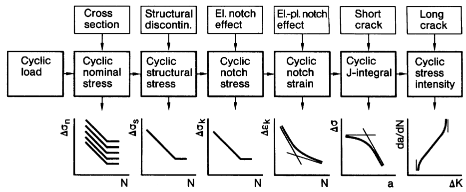
The advantages of sub-modelling become very obvious also when we think of how structural analysis are generally performed. Global analysis is commonly required to understand how the load-carrying mechanism works. Towards this goal, global model of the structure is usually built with several levels of simplifications. One alternative, for instance, is the equivalent plate we learned in the previous lecture. While this global modelling does not give us the exact level of stress, we are usually capable of identifying the most critical locations of the structure in terms of stresses which require a further insight and analyses.

Therefore, in detail design analysis these critical locations are further checked to confirm their compliance with all the requirements and quarantee their safety.

# Definitions

- Fatigue is cumulative damage process which leads to rupture of the structure
- All load cycles contribute to fatigue in damage accumulation can be taken into account for example using Miner's sum
- There are numerous methods to assess fatigue which have different level of
  - Accuracy
  - Complexity
- In large complex structure the problem is to identify the fatigue critical locations and to assess the fatigue strength



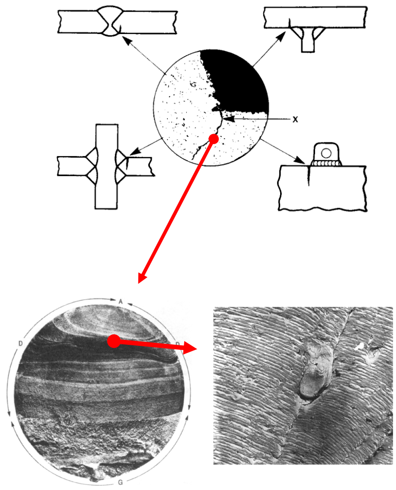

**Aalto University**
School of Engineering

Fatigue is cumulative damage process which leads to rupture of the structure unless preventive actions are taken. One one hand the allowing some damage allows lightweight design, on the other restricting it totally leads to heavy weight especially if we deal with random loading which would require in this case very high safety margins. The challenge is also that small errors in fatigue stress assessment can lead to much larger errors in the fatigue life as the phenomena is related in logarithmic manner and the slope of the SN-curve (stress, life).

All load cycles contribute to fatigue in damage accumulation can be taken into account for example using Miner's sum. There is the so-called fatigue limit after which in principle damage does not occur, however, in practice we have seen that even after that limit damage can be observed to develop. There are numerous methods to assess fatigue which have different level of accuracy and complexity in usage. In large complex structure the practical problem is to identify the fatigue critical locations and to assess the fatigue strength in these.

## Fatigue Phenomenom

- Fatigue is material failure due to time varying stresses:
  - Stress below of yield limit
  - Accumulated damage, i.e. progressive process during several millions of load cycles
- Fatigue is localised and initiates at geometrical discontinuities
- Fatigue failure composed of
  - Crack initiation
  - Propagation
  - Final fracture



Aalto University
School of Engineering

Fatigue is material failure due to time varying stresses, often the stress which is below of yield limit of the material. It is also a process of accumulated damage, i.e. progressive process during several millions of load cycles. Fatigue is localized and initiates at geometrical discontinuities this is why sub modeling is needed. Fatigue failure consists of several stages which vary in length in thin-walled structures due to many possible load-carrying mechanisms. Crack initiation phase happens at the very small-scale and is basically impossible to observe by bare eye. This is about cyclic plasticity and damage evolution at the very small length-scales. Crack propagation stage is what we typically model and this process can be seen also in practice by bare eye. Final fracture happens when the crack is long enough for the given material. This is when the collapse of the structure happens.

# Fatigue Terminology

- Main parameter affect on the fatigue is stress (S) range

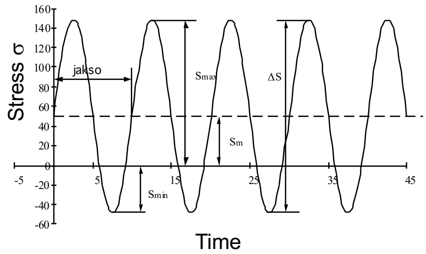
  $\Delta\sigma = \sigma_{max} - \sigma_{min}$

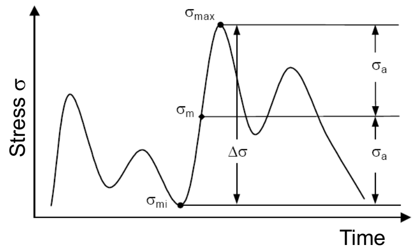
- It is equal to two times of stress amplitude

  $\Delta\sigma = 2 \cdot \sigma_a$

- Mean stress gets

  $\sigma_m = (\sigma_{max} + \sigma_{min}) / 2$



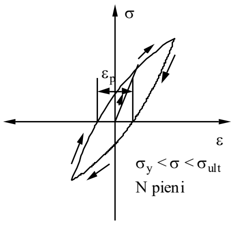




Aalto University
School of Engineering

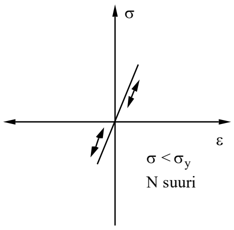
The terminology is important to understand the prevailing physics and dependencies of different fatigue related parameters. The most important parameter is the stress range which is the difference between the maximum and minimum stresses (note that the signs must be correct here). The means stress is the average of the maximum and minimum components and the amplitude half of the range.

# Types of Fatigue

- Low cycle fatigue (L)
  - High stress range, maximum stress over yield limit
  - Number of load cycles is small
  - Submarine
- High cycle fatigue (H)
  - Maximum stress in elastic range
  - Number of load cycles is high
  - Welded structures such as ship




Low cycle fatigue



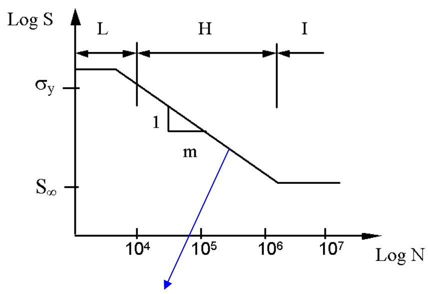

High cycle fatigue

Aalto University
School of Engineering

The terms low and high cycle fatigue relates to the number of cycles observed before the failure. We have to remember here the logarithmic scale of cycles which means that the processes are completely different at these two extremes. Low cycle fatigue is associated with high stress range, maximum stress being often over yield limit at large volume of the material and resulting in small number of load cycles before failure. An example of this is a pressure vessel, such as LNG tanks or submarines. High cycle fatigue is associated with the maximum stress in elastic range, except in infinitely small material volume where the damage accumulates. This results in high number of load cycles before failure and typical examples are welded structures such as ship hulls. The hysteresis in material behavior is noticeable in low cycle fatigue, but not in the high cycle fatigue.

## Fatigue Strength

- Fatigue strength of structures is described with the help of S-N (Wöhler) curve
  - Number of load cycles N is presented as a function of stress range in double logarithms scale
- S-N combined of
  - Plastic part; L= low cycle fatigue
  - Linear part; H = high cycle fatigue
  - Constant part; I = infinite life




$NS^m = C$

or

$m \log S = -\log N + \log C$

C = Fatigue capacity

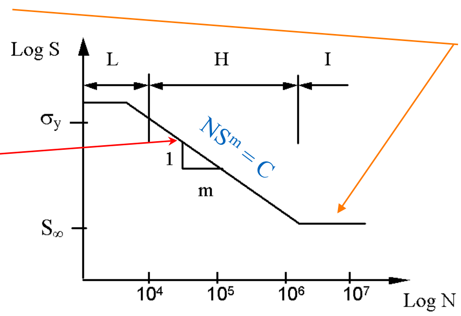
m = Slope of S-N curve

Aalto University
School of Engineering

Fatigue strength of structures is described with the help of SN (Wöhler) curve in which the number of load cycles N is presented as a function of stress range in double logarithms scale. The curve consists of three parts, i.e. plastic part associated with the low cycle fatigue, linear part associated with the high cycle fatigue and the constant part with the infinite life. Today, the constant part has been questioned for existence thanks to high frequency testing machines that show that the slope is not zero at this range, but different from linear part.

# Principle of Fatigue Design

- Design based on constant part (I) i.e. endurance limit
  - Infinite fatigue life
  - A high weight of structure
- Design based on linear range (H)
  - Finite fatigue life
  - Fatigue crack has to be observed
  - Low weight of structure
- Fatigue design is based on the latter case
  - Feature of cyclic loads
  - Number of load cycles is high
  - Structures are weight critical




Endurance limit $S_\infty$

*Maximum value of stress range without material failure for infinite life $N \rightarrow \infty$.*

Aalto University
School of Engineering

Design based on constant part (I) i.e. endurance limit results in infinite fatigue life and a high weight of structure, while the linear range (H) results in finite fatigue life in which fatigue crack has to be observed before it is critical causing final failure and this results in low weight of structure. This is why the fatigue design is based on the latter case as we typically have random loading, number of load cycles is high and thin-walled structures are weight critical.

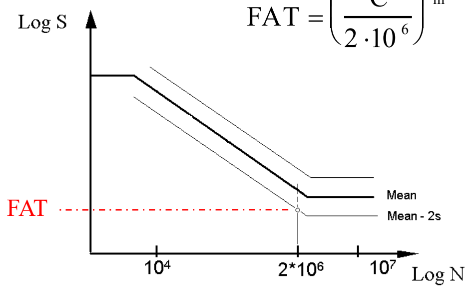
## Design SN-Curve

- Values of fatigue capacity
  - Based on experiments
  - For welded joint, m=3
  - Value of C at failure probability of 2,5 %
- In design rules, structures are categorised using FAT classes
  - FAT = stress range at number of load cycles of $2*10^6$.

$$NS^m = C$$

$$\log C_{97,5\%} = \log C_{50\%} - 2s$$

$$FAT = \left(\frac{C}{2 \cdot 10^6}\right)^{\frac{1}{m}}$$





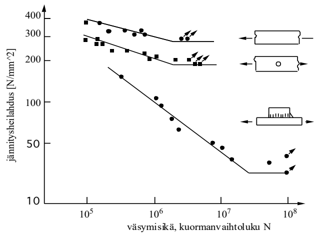

Design SN-curves are derived experimentally and thus the results represent the reality. When the design curve is defined, the failure probability of 2.5% is often considered and thus the resulting curve is below the average. Typically, the reference stress range corresponds $2*10^6$ cycles and the slope of m=3 is defined from there so that 97.5% of the test results are above the resulting design curve.

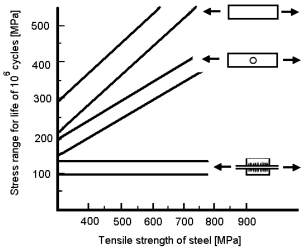
In thin-walled structures the “real slope” can be higher than m=3, for example m=5 for high-quality welds. It should be noted that often we still assume m=3 when deriving the design curve. This can lead to numerical scatter in fatigue design curve which means that we cannot exploit the capacity of the material to full extent.

# Main Factors

- Load history
  - Stress range, maximum stress, load frequency
- Geometry of structure and joint
  - Production technology
- Material
- Environment

Typical Random Load Histories



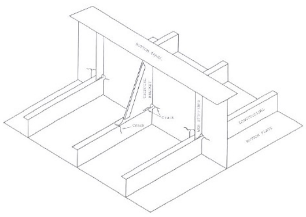



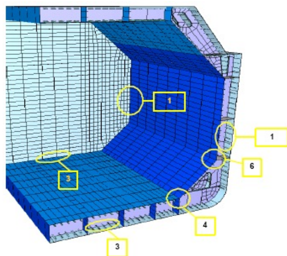

Aalto University
School of Engineering

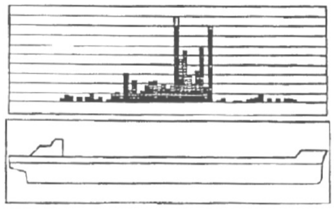
Load history in terms of stress range, maximum stress, load frequency affect the fatigue strength a lot with the geometry of structure and joint to be investigated. Fatigue strength is also affected by the production technology, material used and the environment the component is operated. For example corrosion and moisture can affect negatively the fatigue strength.

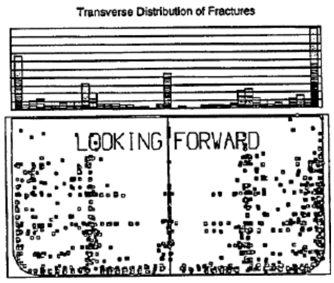
# Typical Fatigue Failures

- Fatigue failures exist typically in
  - Geometrical discontinuity i.e. intersection of structural member
  - Welded joints












Fatigue failures exist typically in geometrical discontinuity i.e. intersection of structural member, such as welded joints and highly loaded sections such as (hull) girder sections undergoing maximum bending moments or shear forces. This is why screening method is needed to identify the worst combination of high load and geometrical discontinuity.

# Definitions for Stress

- Stress state has influence on the damage
  - Membrane stress
  - Bending stress
  - Non-linearity in stress
  - Direction of the stress

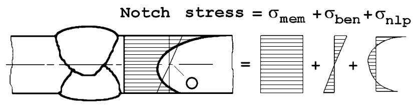
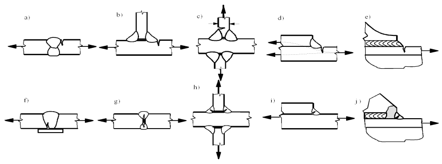



**Fig. (2.2)-1** Non-linear stress distribution separated to stress components

- The type of welded joint has influence on the fatigue strength
- Complex loading and its' sequence have an effect on fatigue strength
- Etc.

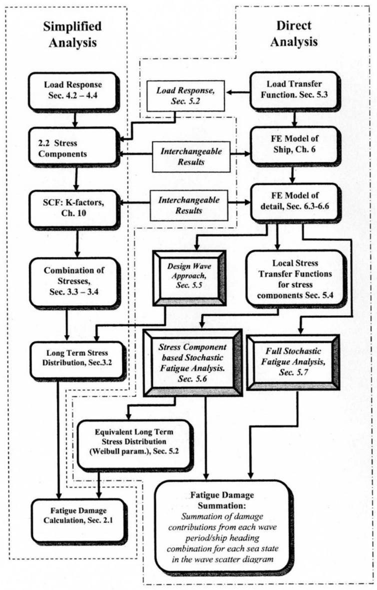


Aalto University
School of Engineering

Stress state has influence on the damage and therefore we must be able to distinguish between membrane and bending stress in beam, plate and shell elements, but also with the non-linearity in stress for example due to curved shapes, stress concentrations etc that the classical structural models cannot predict. Direction of the stress is of high importance as fatigue is mainly observed under tensile stresses. The type of welded joint has influence on the fatigue strength and this is why the experiments are often carried out separately for different joint types. Complex loading and its' sequence have an effect on fatigue strength, that is if there is rough storm at the start or end of design life the end result will be different. Overload at the beginning can have beneficial effects to the observed fatigue life, due to residual stress relaxation.

# Analysis Types

- The analysis type depends on the
  - Required accuracy
  - Time available
  - Tools available
  - Knowledge available
- Analysis needs balanced knowledge about the load and the strength
  - Simplified vs. spectral analysis of loads
  - Beam analysis vs. Finite Element Analysis of the fatigue strength
- Often the analysis steps are
  - Screening the fatigue critical locations based on nominal stresses
  - Structural and/or notch stress analysis of the detail

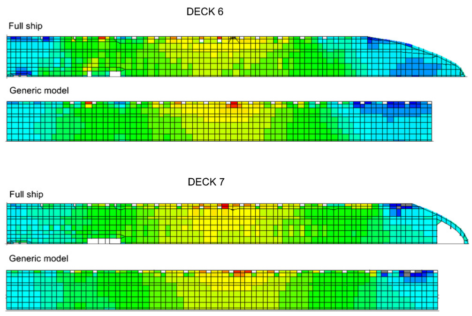


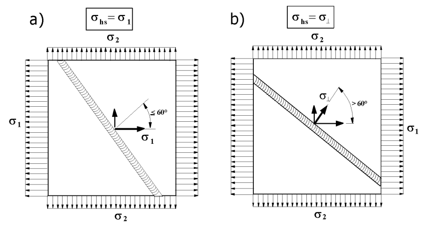



The fatigue analysis type depends on the required accuracy and time and tools available. It should be also noted that the methods require a significant amount of knowledge and skills due to the uncertainties associated with the assumptions and their validity. Too many engineers take this methods as ”accurate” because they are ”standards” and this will often lead to design failures. It is not really the flaw of the method, but it is the flaw due to lazyness of the designer to check the background. Thus, the analysis needs balanced knowledge about the load and the strength. This is the case for example with the simplified vs. spectral analysis of loads, in which the first assumes certain long-term distribution of loads, while the second actually models the life time of operations in details. Another example is beam analysis vs. Finite Element Analysis of the fatigue strength in which the first is able to model only 1D estimates of the membrane and bending stresses, while the second all stress components depending on the level of idealisation used. Often the analysis steps are 1. Screening the fatigue critical locations based on nominal stresses and 2. Structural and/or notch stress analysis of the detail.

# Screening

- The idea is to identify the critical parts
  - Maximum load
  - Stress concentration due to opening
  - Etc
- Interests is on the combined effect
- Nominal stress approach is often enough
- Direction of the 1st principal stress to the detail needs to be taken into account



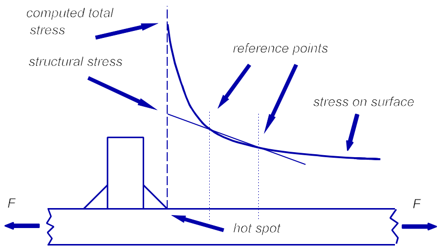

Aalto University
School of Engineering

In screening the idea is to identify the critical parts from the thin-walled structure where the maximum load and the stress concentration for example due to openings meet in critical way. The possible reasons for this combination are many and the intrest is here really on the combined effect. Therefore, the nominal stress approach is often enough fot he screening purposes when complemented the with knowledge about the joint types at this region. We look at the direction of the 1st principal stress to the detail as this way we know how the detail is loaded. The capasity of the detail is taken from the corresponding SN-curves.

# Structural Stress Method (Hot-Spot)

## Basic Idea

- The idea of the method is to get the structural stress without geometrical non-linearity
- When structural stress is extrapolated to the critical location fatigue strength is achieved from corresponding SN-curve




Aalto University
School of Engineering

The structural or hot spot stress method is perhaps the most common method for fatigue assessment. The idea of the method is to get the structural stress without geometrical non-linearity for the detail. When structural stress is extrapolated to the critical location fatigue strength is achieved from corresponding SN-curve.

# Structural Stress Method (Hot-Spot) Extrapolation Rules and Modeling

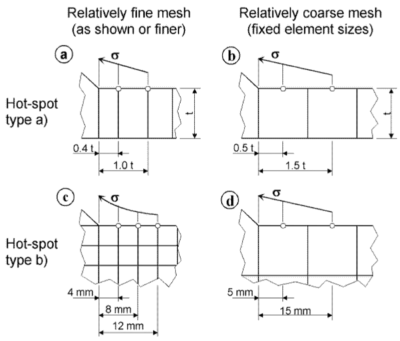
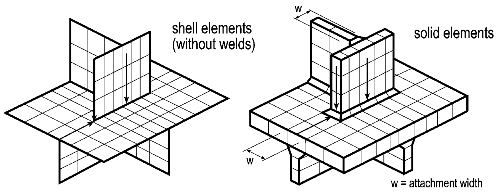



**Fig. (2.2)-12:** Reference points at different types of meshing




Aalto University
School of Engineering

The extrapolation rules are dependent on the element type (shell or solids) we are about to use. We can extrapolate along the surface of the specimen or through the thickness.

# Hot-Spot
## IIW Recommendations for Extrapolation

**Tab. 2.2.-2:** Recommended meshing and extrapolation (see also fig. (2.2)-12)

| Type of model and weld toe | | Relatively coase models | | Relatively fine models | |
|---|---|---|---|---|---|
| | | Type a | Type b | Type a | Type b |
| Element size | Shells | t x t<br>max t x w/2*) | 10 x 10 mm | ≤0.4 t x t or<br>≤0.4 t x w/2 | ≤ 4 x 4 mm |
| | Solids | t x t<br>max t x w | 10 x 10 mm | ≤0.4 t x t or<br>≤0.4 t x w/2 | ≤ 4 x 4 mm |
| Extra-polation points | Shells | 0.5 t and 1.5 t<br>mid-side points**) | 5 and 15 mm<br>mid-side points | 0.4 t and 1.0 t<br>nodal points | 4, 8 and 12 mm<br>nodal points |
| | Solids | 0.5 and 1.5 t<br>surface center | 5 and 15 mm<br>surface center | 0.4 t and 1.0 t<br>nodal points | 4, 8 and 12 mm<br>nodal points |

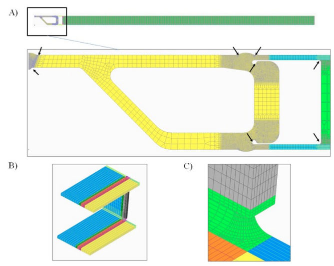
*) w = longitudinal attachment thickness + 2 weld leg lengths
**) surface center at transverse welds, if the weld below the plate is not modelled (see left part of fig. 2.2-11)

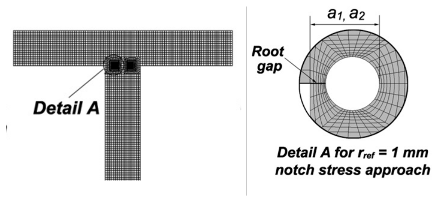
Aalto University
School of Engineering

So the IIW recommendations for the meshing is based on element size and type, but also the extrapolation points selected for the picking of the reference stresses either from the element surface, element itself or the nodal points.

# Notch Stress Method

- Notch stress method aims to calculate the fatigue effective stress at notch tip
- Artificial rounding is introduced to model macro-support effect
- Extremely tedious, but flexible method for fatigue assessment of details such as weld profiles etc



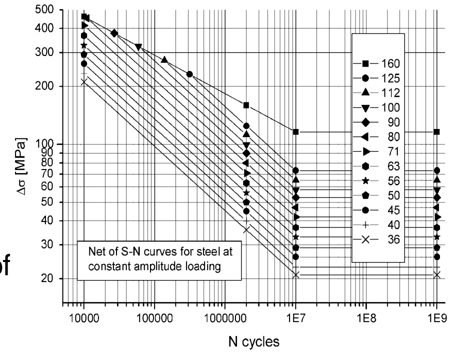



Notch stress method aims to calculate the fatigue effective stress at notch tip. Artificial rounding is introduced to model macro-support effect of the material. The method is extremely tedious, but flexible method for fatigue assessment of details such as weld profiles. Here we need solid elements for the analysis, which make the analysis of thin-walled structures very costly due to the slenderness of the thin wall.

## Fatigue Classes

- Depending on the selected fatigue assessment method the SN-curves are different
  - Nominal stress
  - Hot-Spot stress
  - Notch stress

- Typically the design curves relate to the 97.7% probability of survival




Aalto University
School of Engineering

Depending on the selected fatigue assessment method the SN-curves are different for the different methods: i.e. nominal, Hot-Spot / structural and notch stress. Typically the design curves relate to the 97.7% probability of survival. What we can see here is different levels for different details.

# Uncertainty in Fatigue Assessment

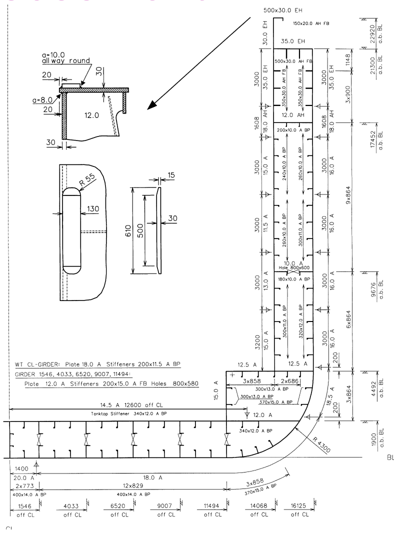
- Fatigue is very complex problem and analysis requires holistic approach

- This benchmark with top experts show how difficult the topic is on fixed problem

Table 1
Comparison of results from comparative study

| Rules and guidelines | Type of stress approach | Histogram of nominal stress ranges | | | | SCF (hot spot/weld) | Design S–N curve | | | | Fatigue life (yr) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\Delta\sigma_{max}$ (MPa) | $n_{max}$ | $\xi$ | $\sigma_m$[a] (MPa) | | $\Delta\sigma_R$ (MPa) | $N_q$ | $m$ | $r$ | |
| ABS [7] | Nominal | 318.7 | $5 \times 10^7$ | 0.81 | — | —/— | 49.9 | $10^7$ | 3 | 5 | 8.9 |
| | Hot spot | 318.7 | $5 \times 10^7$ | 0.81 | — | 1.736/— | 80.3 | $10^7$ | 3 | 5 | 7.0 |
| BV/RINA [10] | Notch | 278[b] | $2.8 \times 10^7$ | 0.943 | — | 1.63/1.84 | 142.6 | $10^7$ | 3 | 5 | 6.0 |
| | | 136[c] | $2.8 \times 10^7$ | 0.943 | — | | | | | | |
| DNV [11] | Notch | 233.0 | $6.65 \times 10^7$ | 0.93 | — | 1.47/1.5 | 142.2 | — | 3 | — | 20.6 |
| GL [12,18] | Nominal | 209.2 | $5 \times 10^7$ | 1.0 | 104.9 | —/— | 50 | $5 \times 10^6$ | 3 | 5 | 13.4[d] |
| | Hot spot | 209.2 | $5 \times 10^7$ | 1.0 | 104.9 | 1.9/— | 110[f] | $5 \times 10^6$ | 3 | 5 | 20.7[d] |
| KR [14] | Hot spot | 278.8 | $5.64 \times 10^7$ | 0.943 | | 1.66/— | 91.3 | $10^7$ | 3 | 5 | 6.5 |
| LR [15] | Hot spot[g] | 210[e] | $5.7 \times 10^7$ | — | — | 1.81[g] | 124[g] | $10^7$ | 3 | 5 | 12.0 |
| NK [16] | Hot spot | 281.5 | $10^8$ | 1.0 | 108.9 | 2.15/— | 95 | $5 \times 10^6$ | 3 | 5 | 1.8 |
| RS [17] | Hot spot | 199.0 | $5 \times 10^7$ | 0.88 | 113.8 | 1.80/— | 100 | $5 \times 10^6$ | 3.5 | 5.5 | 15.2[h] |

[a] Mean stress only given if it affects life.
[b] Part from head seas.
[c] Part from oblique seas.
[d] Estimated from usage factor.
[e] Read from $10^{-8}$ probability of exceedence.
[f] Including 10% increase for exact stress analysis.
[g] Hot spot with some embedded notch stress effects.
[h] Life 13.2 years for corrosion wastage by 0.5% of section modulus every year.

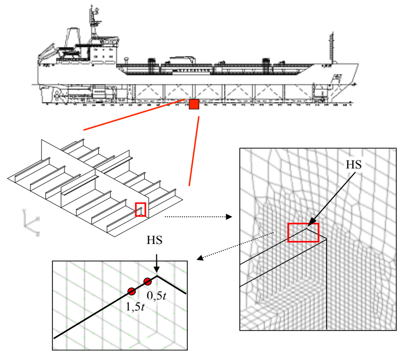


Aalto University
School of Engineering

Fatigue is very complex problem and analysis requires holistic approach in which the loads, responses and strength(s) are assessed in balanced manner. This benchmark with top experts show how difficult the topic is on fixed problem and how the error may accumulate based on the modeling assumptions. The results vary in life 10x, so even though the errors may see small in stresses they can be large in life.

# Sub-modeling

- Large structures have number of details that affect the strength
  - Hot spots for fatigue
  - Openings with stress concentrations and reduced stiffness
  - Etc
- All these cannot be included into the FE-model since the matrix size increases too much
- Special techniques are needed to reduce the problem size
- These techniques have many names which are often conflicting and not standardized
- Therefore it is important to understand the background of these



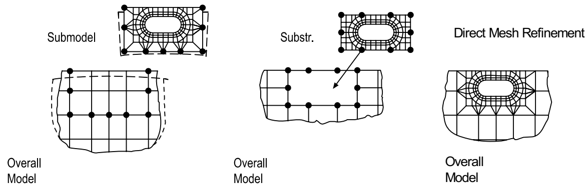

Aalto University
School of Engineering

The usual goal of a structural analysis is to obtain the required results, e.g. the strength of a global structure or a detail under a given loading condition, in a reasonably short time, respectively at low computational cost. The accuracy of the simulation however is directly linked with the discretization length or in other words, element size used in the model. Therefore, the more accurate results we desire the more refined mesh we need to use. Problem is that computational time increases with the decrease of element size. Essentially, a brute force approach would be to locally refine the mesh in a critical location to achieve the level of required accuracy which would however drive our simulation time beyond reasonable levels. Another problem with that is that we always are not aware where these critical locations which require mesh refinement are located. In addition, detailed modelling of these local structures in a global model can considerably add to the modelling effort, which for a full three-dimensional (3D) finite element model of a cruise ship can already be several months.

Therefore, special techniques are needed to reduce the problem size. Here, we generalize those under the umbrella of sub-modelling approaches. Nevertheless,  These techniques have many names which are often conflicting and not standardized. The reason is that different authors use different designation although underlying differences between approaches are small. Therefore it is important to understand the background of these.

# Definitions for the Sub-Modeling

- Submodel
  - Two (or several) different FE-meshes are solved: global and local
  - Global solves the loading, local solves the stress
  - Typically the information is one way, global to local (sometimes stiffness may be reduced)
- Superelement & substructure
  - The global mesh is fine mesh but the global matrix is split to smaller matrixes called superelements
  - The key question is the interface split
  - Information goes two ways
- Direct mesh refinement
  - The local mesh is included into the global mesh at selected location
  - If repeated many times the size of the matrix increases considerably

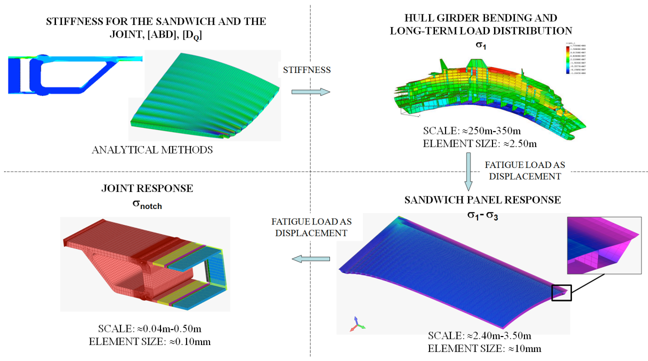



Aalto University
School of Engineering

In order to assess stresses in local details we use sub-modelling techniques. First one is sub-modelling. Here, the coarse global model will be used to obtain the global behaviour, e.g. displacements and stresses, in the vicinity of the structural details as shown in the figure. Hence, the local model can be solved with the boundary conditions obtained from the global model. This is typically a unidirectional approach as local stress information is not transported back to global model. Superelement and substructure use either a fine global mesh which is split to smaller matrixes called superelements. In these models, the key question is the interface split and to understand that the information goes two ways. In direct mesh refinement the local mesh is included into the global mesh at selected location obtained for example via screening. In this case, if repeated many times the size of the matrix increases considerably and very rapidly.

# Sub-Models
## Process

- The analysis of stress concentration requires information flow from global analysis towards details
- In global model the stiffness has to be right to capture the load-carrying mechanism correctly
  1. Sub-model(s) of the important detail(s) for stiffness
  2. Global strength analysis for the nodal displacements
  3. Nodal displacements as load for the local model
  4. Local stress analysis
- These scales interact and the key question is how successfully this is done

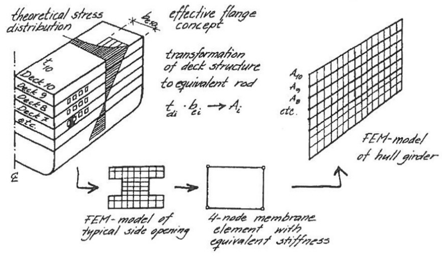



Aalto University
School of Engineering

Lets go through the actual process of sub-modelling. The analysis of stress concentration requires information flow from global analysis towards details. In this example we have a sandwich panel for which stiffness is determined using some pre-processing, it can be an analytical model or FE model, it does not matter. In the global model, this stiffness is assigned to equivalent plate model we learned from the previous lecture. Now we can find the displacements from relation of F=k*U as we know the global loads. These displacement are then used as the boundary conditions for the local sandwich panel model in picture 3, which is then solved for local stresses. Clearly, the success of the whole approach depends on how successfully information flows from one level to another level.

# Sub-Models
## Stiffness

- The “equivalent” element is needed
  - Membrane
  - Plate
  - Shell
- Depending on the formulation the stiffness need to be derived numerically
- In these cases analytical solutions are rare and often not very accurate in general case
- The local detail can be easily changed without changing the global mesh of complex structure – only the stiffness needs to be recalculated

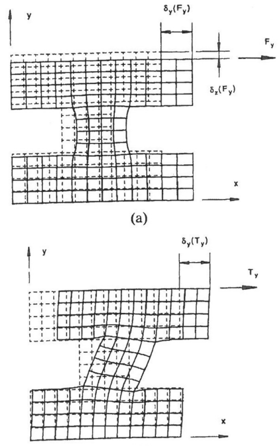



Aalto University
School of Engineering

Lets take a closer look on how we actually make the initial transition from local to global. For that step we needed the stiffnes of the equivalent plate which in current case was a sandwich panel. Naturally, instead of explicitly including this sandwich panel in the global model we want to replace this with a simple shell element that has the equivalent stiffness properties to that of the sandwich panel. For simple structures and linear elastic range, the stiffness can be derived analytically, but generally we have to revert to numerical analysis. Nevertheless, in any case the local model of the detail can be easily changed without changing the global mesh of complex structure – only the stiffness needs to be recalculated. This saves a lot of time.

# Sub-Models
## Stiffness

In-plane stretch directions y & z



(a)

(b)

In-plane shear yz

Plane Stress "Membrane"

$$\begin{bmatrix} \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{33} \end{bmatrix} \begin{bmatrix} \sigma_y \\ \sigma_z \\ \tau_{yz} \end{bmatrix} \quad (1)$$

$$S_{11} = 1/E_y \quad (2)$$

$$S_{12} = -\nu_{yz}/E_y = -\nu_{zy}/E_z = S_{21} \quad (3)$$

$$S_{22} = 1/E_z \quad (4)$$

$$S_{33} = 1/G_{yz} \quad (5)$$

1. Give load $\sigma_x$, $\sigma_y$, $\tau_{xy}$
2. Measure strain $\varepsilon_x$, $\varepsilon_y$, $\gamma_{xy}$
3. Calculate stiffness $s_{ij}$



This is an example how we can find the stiffness for a sub-model. We construct the stiffness matrix by imposing a certain type of load to our sub-structure and check the resulting deformations. As a result, we are able to solve for stiffness of this particular sub-structure. This is very much similar to what we do when deriving the compliance matrix for materials.

# Sub-Models
## Stiffness

Plane Stress "Membrane"

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{zx}}{E_z} & 0 \\ -\frac{\nu_{xz}}{E_x} & \frac{1}{E_z} & 0 \\ 0 & 0 & \frac{1}{G_{xz}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix} \qquad \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\sigma_x}{E_x} - \frac{\nu_{zx}\sigma_z}{E_z} \\ \frac{\sigma_z}{E_z} - \frac{\nu_{xz}\sigma_x}{E_x} \\ \frac{\tau_{xz}}{G_{xy}} \end{Bmatrix}$$

Case 1, equivalent orthotropic properties

$$\sigma_x \neq \sigma_z = \tau_{xz} = 0,$$

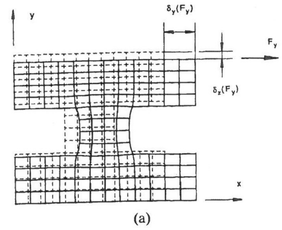
$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\sigma_x}{E_x} \\ \frac{\nu_{xz}\sigma_x}{E_x} \\ 0 \end{Bmatrix} \Rightarrow \quad E_x = \frac{\sigma_x}{\varepsilon_x} \quad \nu_{xz} = \frac{E_x\varepsilon_z}{\sigma_x}$$

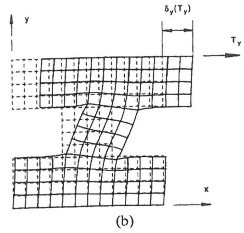
Case 2, equivalent orthotropic properties

$$\sigma_z \neq \sigma_x = \tau_{xz} = 0$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} -\frac{\nu_{zx}\sigma_z}{E_z} \\ \frac{\sigma_z}{E_z} \\ 0 \end{Bmatrix} \Rightarrow \quad E_z = \frac{\sigma_z}{\varepsilon_z} \quad \nu_{zx} = -\frac{E_z\varepsilon_x}{\sigma_z}$$

In-plane stretch directions y & z





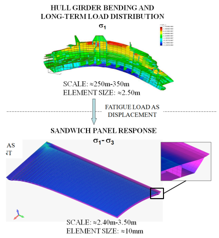
In-plane shear yz

Aalto University
School of Engineering

So what we do in practice is that we expose the RVE to some kind of basic loading and measure the responses. Comparison of load and response component by component gives us the homogenised stiffness matrix.

# Sub-Models

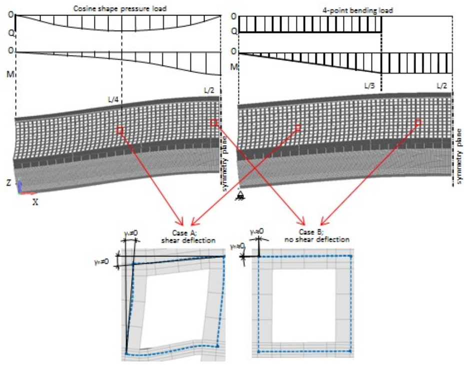
## Global to Local - Displacements




- The global FE mesh gives the response as nodal displacements

- These can be used as loads for the local analyses

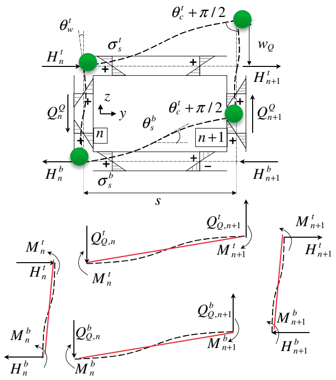
- This involves mapping displacements from coarse mesh to fine mesh
  - Linear interpolation
  - High order interpolation
  - Special attention has to be paid on warping of the cross-section in shear

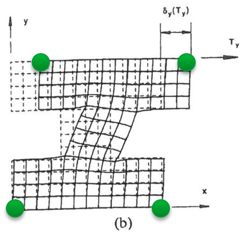



**Aalto University**
School of Engineering

This slide gives an insight to third step where we make the transition back from global to local model. From the global FE solution we get the response as nodal displacements in certain fixed points defined by our discretization. These displacements are then used as loads for the local analyses. Notice however, that one important aspect in this mapping from global to local is interconnectivity between coarse global model and detailed local model. In other words, the global model response, for instance at 3 nodes, must be transferred to local model that descibes the same geometry with 6 nodes. This step includes interpolation of displacement values.

# Sub-Models
## Global to Local - Displacements






When high-order polynomial in approximated with linear distribution, high-order derivatives are lost

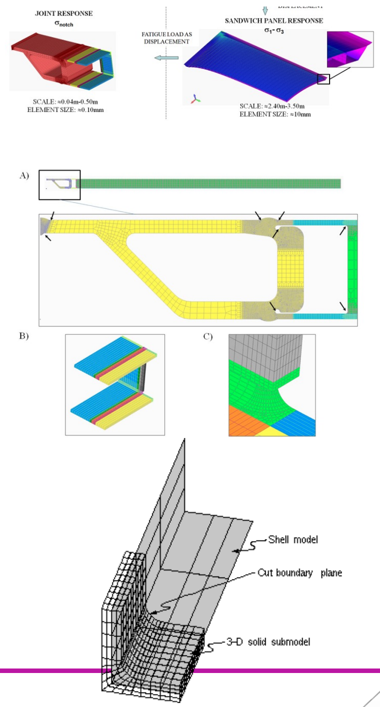
$$M_{g,avg}(y) = -D_g \frac{d^2 w_{g,avg}(y)}{dy^2}.$$

**Aalto University**
School of Engineering

The procedure of transfering displacements from global to local model is usually embedded in commercial software packages. Nevertheless, it is important to understand the basics of the approach. For instance, the selection of appropriate interpolation functions is relevant. The selection of appropriate function needs to be justified and analyst must undestand what kind of information is lost when preferring simpler lower order polynomials to more expensive lower order ones.

# Sub-Models
## Stress Analysis

- Depending on the analysis type the minimum element size can be very small
  - Notch stress analysis for fatigue 0.1-0.2mm
  - Displacements as load through multiple scale transformations
- Special attention to be paid on shell-solid and/or beam-solid coupling
  - Warping
  - Distance to critical location which should be unaffected by the element type transition

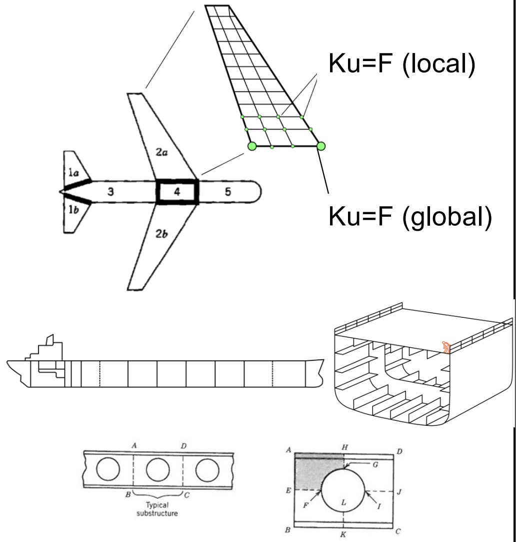



Aalto University
School of Engineering

This is the final step in our sub-modeling technique, which involves the stress analysis of local structural detail. Depending on the analysis type the minimum element size can be very small. Notice further than several scale transitions can be made instead of one, in other words, submodels of submodel are made.  Essentially, in a sub-sub model we can also use solid elements for better representation of through thickness stresses. In that case, attention is paid to shell to solid or beam to solid coupling.

# Superelement and Domain Decomposition

- In super-elements the all details are included to the global mesh
- The system matrix is solved in parts, i.e. based on super-elements
  - Interface nodes are assigned a stiffness
  - The solution is carried out with these “large elements”
  - The interface displacements and forces act as loading to internal nodes
- Problem is to “balance the model numerically” – iterations are needed
- Nowadays some solvers have solution time proportional to $N^1$ instead of $N^2$ (N=number of dofs)
  - Optimization based – minimizes the total potential energy
  - Savings are lost due to this technique




Aalto University
School of Engineering

Lets move on to superelements now. Superelements can be considered as assemblies of primitive elements, also mesh units, which are presented to program users as individual elements. Therefore, superelements can be used to divide the original structure into components so as to manage a large FE analysis project. However, in contrast to submodelling, this substructuring does not break the information flow between different components and thus, these components are often denoted as superelements. Example is the sub structuring of the fuselage into parts, whereas the system matrix is solved in parts, or in other words, based on the super-elements.

# Superelement and Domain Decomposition

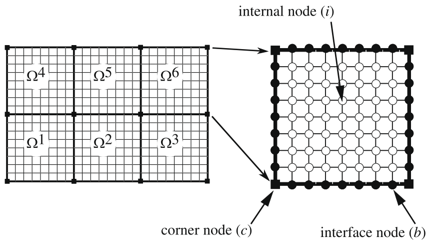



**Fig. 5.** Classification of the subdomain nodes.

$$K^s_{rr}u^s_r + K^s_{rc}L^s_c u_c + B^{s\,T}_r \lambda = f^s_r \quad \text{for } s = 1, \dots N_s \tag{6}$$

$$\sum_{s=1}^{N_s} L^{s\,T}_c K^s_{cr} u^s_r + \sum_{s=1}^{N_s} L^{s\,T}_c K^s_{cc} L^s_c u_c = \sum_{s=1}^{N_s} L^{s\,T}_c f^s_c \tag{7}$$

$$\sum_{s=1}^{N_s} B^s_r u^s_r = 0 \tag{8}$$

**Procedure.** In brief, a substructure is a ‘‘superelement,’’ that is, a single element with many nodes on its boundary and many interior d.o.f. The name ‘‘macroelement’’ is also appropriate. The process is that of condensation and recovery, as described in Sections 8.1 and 8.2. Indeed, elements in Fig. 8.1-1 are substructures having few d.o.f. After the division of a structure into substructures has been selected, static analysis proceeds as follows.

1. Evaluate [**k**] and {**r**} for each substructure, where [**k**] and {**r**} pertain to all d.o.f. of the substructure. Eliminate internal d.o.f. by condensation; that is, apply Eq. 8.1-3. The condensed [**k**] and {**r**} pertain to only the boundary d.o.f. $\{\mathbf{d}_r\}$ of the substructure, which may be called ‘‘attachment’’ d.o.f.
2. Assemble substructures by connecting attachment nodes (i.e., nodes shared by substructures). Thus generate structural equations $[\mathbf{K}_m]\{\mathbf{D}_m\} = \{\mathbf{R}_m\}$, in which $\{\mathbf{D}_m\}$ contains the attachment d.o.f. of all substructures. (Attachment nodes on mating boundaries of adjacent substructures must match in physical placement and in orientation of their d.o.f.) Solve for $\{\mathbf{D}_m\}$.
3. For each substructure, extract from $\{\mathbf{D}_m\}$ the attachment d.o.f. $\{\mathbf{d}_r\}$ of that substructure. Use Eq. 8.1-2 to compute interior d.o.f. $\{\mathbf{d}_c\}$. Now all d.o.f. of the substructure are known. Hence, stress calculation proceeds in the usual way.

Clearly this is a finite element process in which elements have many internal d.o.f. and are given the name ‘‘substructures.’’ It differs from a standard finite element process in that one does not form a single stiffness matrix that operates on *all*

Aalto University
School of Engineering

The main idea of the superelement or macroelement is that by condensation, static or dynamic, we can take out the “internal nodes” and only solve the problem based on the nodes at the superelement boundaries.

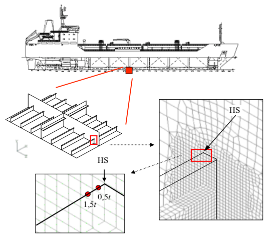
That is we, compute the macroelement stiffness matrices, solve the overall assembly with the macroelement stiffness matrices and the post-process the micro responses based on macro solution.

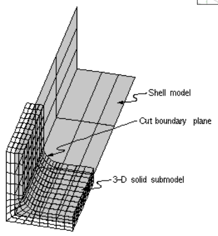
This helps us to reduce the number of degrees of freedom in our model and the fact that we do not have a single large stiffness matrix to operate with.

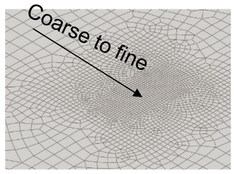
# Direct Mesh Refinement

- Most accurate and time consuming way is to model the detail in global mesh
  - Requires knowledge of the critical location (post-accident ☺ )
  - Check that numerical problems do not arise because of different element sizes
  - Analysis type defines the refinement (in fatigue Hot-Spot 0.5t & 1.5t; Notch stress 1mm radius,…)

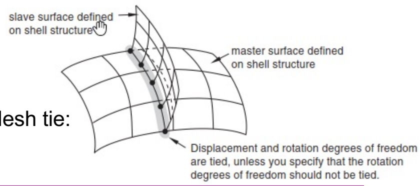
- Special care must be carried out with shell-solid coupling
  - Warping
  - Distance to critical location which should be unaffected by the element type transition




Mesh tie:

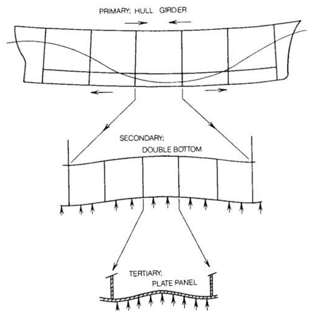


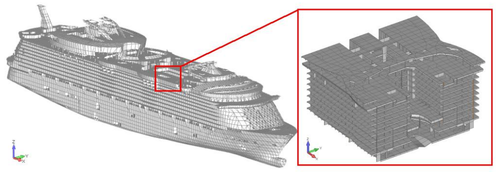

Aalto University
School of Engineering

The third technique for obtaining local stresses was the so called direct mesh refinement, which is in a sense a brute force approach as it is the most time consuming. As already briefly mentiond before, it is efficient when analyst is aware of the critical locations where he wants to assess the local stresses, but generally this is not the case. Furthermore, analyst must take care of the mesh transitions. One option is to use mesh grading from coarse to fine. In that case care has to be taken to ensure the quality of the mesh. Alternatively, when even more rapid mesh density transitions are required special mesh tie constraints can be used to tie two surfaces together for the duration of the simulation.

## Modelling Principles and Simplifications

- There is large scale difference between structures
  - Primary: hull/bridge girder
  - Secondary: intermediate structures such as double bottom, deck
  - Tertiary: stiffeners and plates, welds
- To include all these to the models is not always feasible since this involved a lot of manipulation on large FE-meshes, time-consuming
- Often it is assumed that the scales can be analyzed separately



Aalto University
School of Engineering

Besides those modelling techniques we can use also common sense to simplify our structural problem. In analysis of ship structures, the structural response is divided to three levels: primary level concerning the entire ship hull, secondary level concerning the intermediate large structural components such as double bottoms and decks, and tertiary level elements that are the building blocks for the entire structure such as stiffeners and plates. As we have discussed, including all these to the models is not always feasible since this involves a lot of manipulation on large FE-meshes and is time-consuming. Therefore, uncoupling these different length scales becomes a feasible option and each scale can be analyzed separately.

# Modelling Principles and Simplifications

- The scale separation is not always justified
  - If the structural details change considerably the load carrying mechanism can change too
  - The more complex is the structural behavior, the more you should pay attention to this

- However scale separation is powerful assumption which makes the overall analysis more efficient

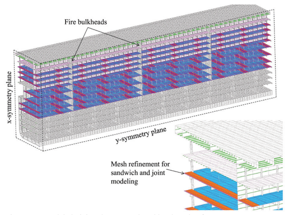



Fig. 2 Global Finite Element mesh and local mesh refinement.

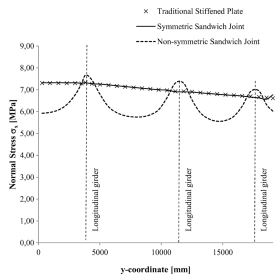


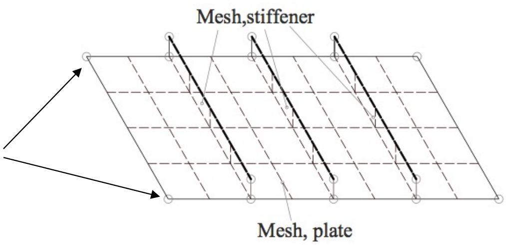

Fig. 6 Comparison of normal stress $\sigma_x$ in the mid-plane of Deck 9.

**Aalto University**
School of Engineering

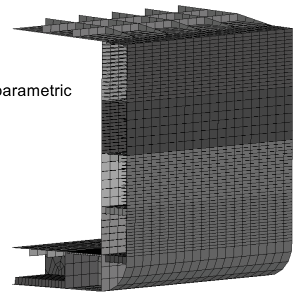
But again, the limitations of the scale separation approach must be understood by the analyst. Nevertheless, scale separation is powerful assumption which makes the overall analysis more efficient

# Sub-Models & Parametrization

- Parametric models require definition of the basic structural unit based on design parameters
  - Plate thicknesses,
  - Stiffener height and flange
- Usually master nodes are defined which define the lines, areas and volumes to be meshed
- Slave nodes are created automatically by stating number of them along certain lines
- Elements are created automatically by the FE-software
- When parametric models are assembled, global models can be achieved with ease
- Suits optimization, be careful on the accuracy of the results!




Ship section made from parametric panels

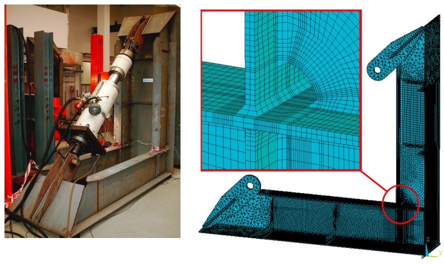


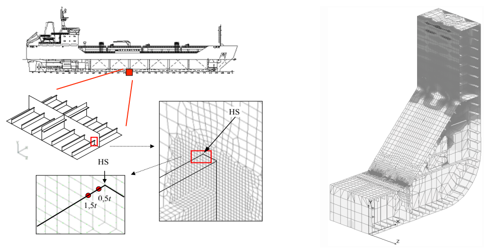
Aalto University
School of Engineering

In this example, parametric modelling was used to optimize ship side structure for crashworthiness. The so-called master nodes define the geometry of the ship cross section, but rest of the stiffened panel parameters are variables during optimization. This incudes plate thickness, number of stiffeners, stiffener type and mesh size.

# Summary

- Fatigue is a major limit state in modeling of thin-walled structures
- The analysis includes different types from: full spectral long-term analysis to simplified and from detailed sub-modeling approach to global screening approach
- The discretization depends on the strength assessment methods
- Significant uncertainty is present in fatigue assessment
- There are different definitions for the sub-modeling
- Sub-model technique is decoupled micro-macro approach
- Substructure technique & domain decomposition iterates the edge displacement between micro and macro models
- Direct Mesh refinement includes the detailed micro model directly in macro model





Aalto University
School of Engineering

Fatigue is a major limit state in modeling of thin-walled structures

The analysis includes different types from: full spectral long-term analysis to simplified and from detailed sub-modeling approach to global screening approach

The discretization depends on the strength assessment methods

Significant uncertainty is present in fatigue assessment

There are different definitions for the sub-modeling

Sub-model technique is decoupled micro-macro approach

Substructure technique & domain decomposition iterates the edge displacement between micro and macro models

Direct Mesh refinement includes the detailed micro model directly in macro model