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Limit State Analyses in Design of Thin-Walled Marine Structures—Some Aspects on Length-Scales

This paper gives an overview of the factors that affect the strength and structural design of advanced thin-walled marine structures with reduced plate thickness or alternative topologies to those used today in the marine industry. Due to production-induced initial deformations and resulting geometrical non-linearity, the classical division between primary, secondary, and tertiary responses becomes strongly coupled. Volume-averaged, non-linear response of structural element can be used to define the structural stress-strain relation that enables analysis at the next, larger, length scale. This, today's standard homogenization process, needs to be complemented with localization, where the stresses are assessed at the details, such as welds for fatigue analysis. Due to this, the production-induced initial distortions need to be considered with high accuracy. Another key question is the length-scale interaction in terms of continuum description. Non-classical continuum mechanics are needed when consecutive scales are close. Strain-gradients are used to increase the accuracy of the kinematical description of beams, plates, and shells. The paper presents examples of stiffened and sandwich panels covering limit states such as fatigue, non-linear buckling, and fracture. [DOI: 10.1115/1.4045371]

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Introduction

Lightweight design is essential for marine structures. The trend toward sustainable use of natural resources has strengthened the position of steel as structural material. The fuel efficiency requirements of ships call for alternative structural topologies with reduced plate thicknesses. However, the strength assessment causes challenges as the related design limits and criteria are at their infancy. The major issues are the increased initial distortions and changes in the production process that can lead, for example, changes in weld geometry and strength properties. Initial deformations in slender structure require geometrically non-linear structural analyses. On the other hand, the stress-based fatigue assessment methods are typically related to certain assumptions in weld geometry and plate thickness. Thus, design methods need to be developed further to allow the implementation of these improved structures to practice.

Due to production-induced initial deformations and resulting geometrical non-linearity, the classical division for linear structures, between primary, secondary and tertiary responses become coupled. In this coupling, there are two major issues. One is the process of homogenization and localization of stresses at the level of the structural member, e.g., stiffened panel. Homogenized properties are needed at the larger length scale to accelerate the analysis times, e.g., transition from panel to hull girder level (see, e.g., Refs. [1–11]). This volume-averaged structural stress–strain relation can be used to assess responses at larger scale that allows, for example, investigations on strength and load-carrying mechanism at this level. However, often the failure initiates at the lower length scales and is affected by the multiaxiality of the loading. Therefore, the localization process is needed to estimate the stresses at lower scale, when the responses at larger scale are known (see, e.g., Refs. [12–14]). Another key question is the length-scale interaction in terms of continuum description. In the homogenization–localization process, a fundamental assumption is that the length scales are clearly separable, i.e., $l_{\text{primary}} \gg l_{\text{secondary}} \gg l_{\text{tertiary}}$, where l is the characteristic length of deformation or stress at the corresponding scale. In marine structures, this assumption can be violated. Therefore, non-classical continuum mechanics [15,16] are needed. Strain-gradients and couple stresses are used to increase the accuracy of the kinematical description of beams, plates, and shells [17–20] based on equivalent single layer theory (ESL).

This paper gives an overview of the factors that affect the strength and structural design of advanced thin-walled marine structures with reduced plate thickness or alternative topologies to those used today in the marine industry. First, some challenges and solutions in the response prediction are presented that allow reliable transition between the length scales. Then, we present the same for ultimate strength assessment where we limit ourselves to ductile fracture and non-linear buckling, where the load-end-shortening curves are derived for tension and compression, respectively, in volume-average sense. Next, we present the fatigue assessment where the localization of stresses is important. We show the similarities and extensions to the theories and approaches that have been utilized over recent decades in analysis and design of marine structures. The examples of this paper are selected from stiffened and sandwich panels made from steel.

Response Prediction

One of the main obstacles for the introduction of thin-walled structures to ship structures is due to the production-induced initial distortions and residual stresses, and residual stresses are omitted here to clarify the concepts. Structural design is carried out often by clearly splitting the response evaluation to primary, secondary, and tertiary levels. When the scales are clearly separated in terms of characteristic length of stress or displacement, i.e., $l_{\text{primary}} \gg l_{\text{secondary}} \gg l_{\text{tertiary}}$, one can assess the stress–strain relation for larger scale by using the actual geometry and material of the structure at the lower scale and by computing the relation under certain load and boundary conditions at the edges of the model, i.e., by utilizing representative volume element (RVE). This process of homogenization is widely used in marine technology as the concept of load-end-shortening curves and in materials science as the concept of multi-scale modeling.

It is clear that the non-linear response depends on the adopted load and boundary conditions at the edges of the RVE as well as the initial imperfections and residual stresses. One fundamental assumption in homogenization is the periodicity assumption, i.e., $f(x) = f(x + l_{\text{scale}})$. This means that the deformation and stress at the opposite edges of the RVE must be equal. In marine structures, with geometrically complex shape and topology, this assumption can get violated easily. In order to fix the problem, extended, non-local continuum theories have been developed where the strain at the point is not dependent only on the strain at the same points, but also on its gradients. In the couple stress-based theory, the first gradient of deformation is included for the strain description. This gradient is evaluated at the unit cell reference point, located at the center of the unit cell. In this location, the homogenized and periodic solutions are assumed to be equal. In practice, this means that the RVE can be exposed to in-plane bending in addition to the classical pure tensile and shear components. This allows improved accuracy in response predictions for a wider range of applications. The example for this is shown in Fig. 1, where a beam in three-point bending is computed using continuum-based modified couple stress Timoshenko beam theory as given in Ref. [17] and compared with the three-dimensional-finite element analysis (3D-FEA) of the periodic beam. As the figure shows, the deflection and stress is in excellent agreement even though the scales are close, i.e., $l_{\text{secondary}} = 4l_{\text{tertiary}}$. It is also seen that as the rotation stiffness (and shear stiffness) approaches zero, the classical Timoshenko beam theory fails to predict correctly the deflections while the enhanced theory predicts it correctly.

When continuum description is accurate, the next issue to tackle is transition between the scales when the details are included for the analysis, see the simple linear example from Fig. 2. There, the lightweight design is made by the steel sandwich panels, which calls for asymmetric, single-sided, joining when production issues are highlighted. From the theory of thin-walled structures, it is known that the membrane action of the panel should dominate over the bending when the panel is far from the neutral axis of the ship. Then, the only design parameter that should define the stiffness of the structure, i.e., load-end-shortening curve, is the product of Young's modulus, E , and the cross-sectional area, A , of the stiffened panel, i.e., EA . Figure 2 shows that even though the asymmetry is very local and only induced by the joint between panels (distance

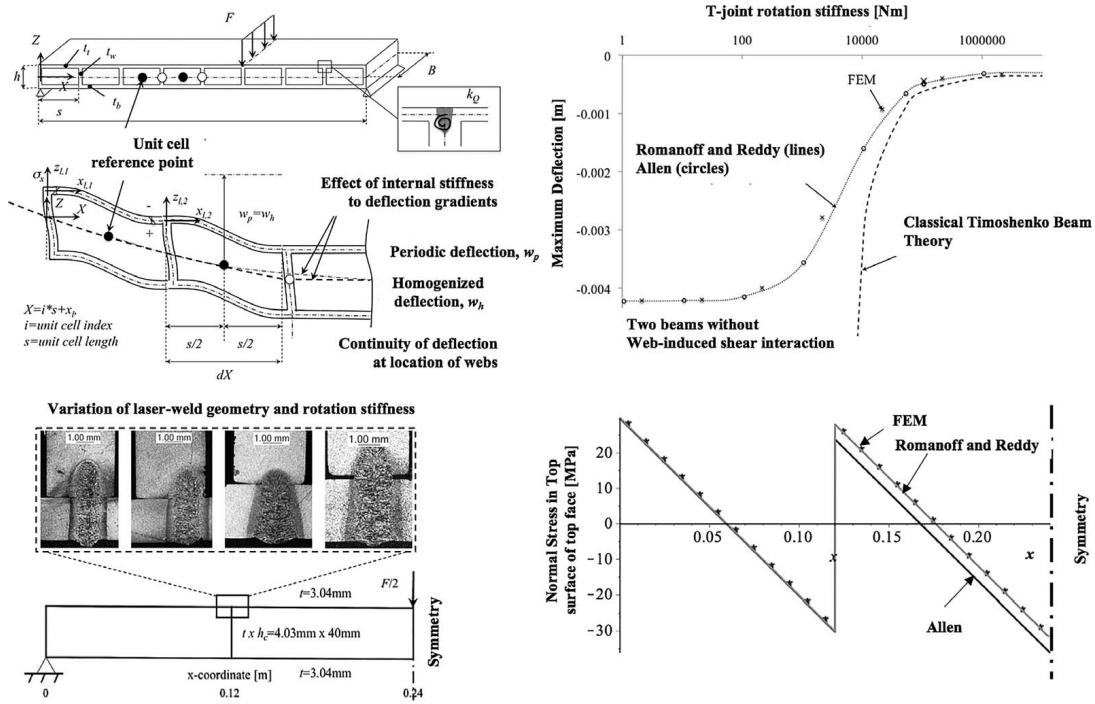


Fig. 1 Top left: Example of periodic beam and its kinematics with periodic and homogenized solutions. Bottom left: Reasons for variation in T-joint rotation stiffness. Right: Comparison of the accuracy of deflection and stress prediction with classical and couple stress-based Timoshenko beam theory (adapted from Refs. [19,20]).

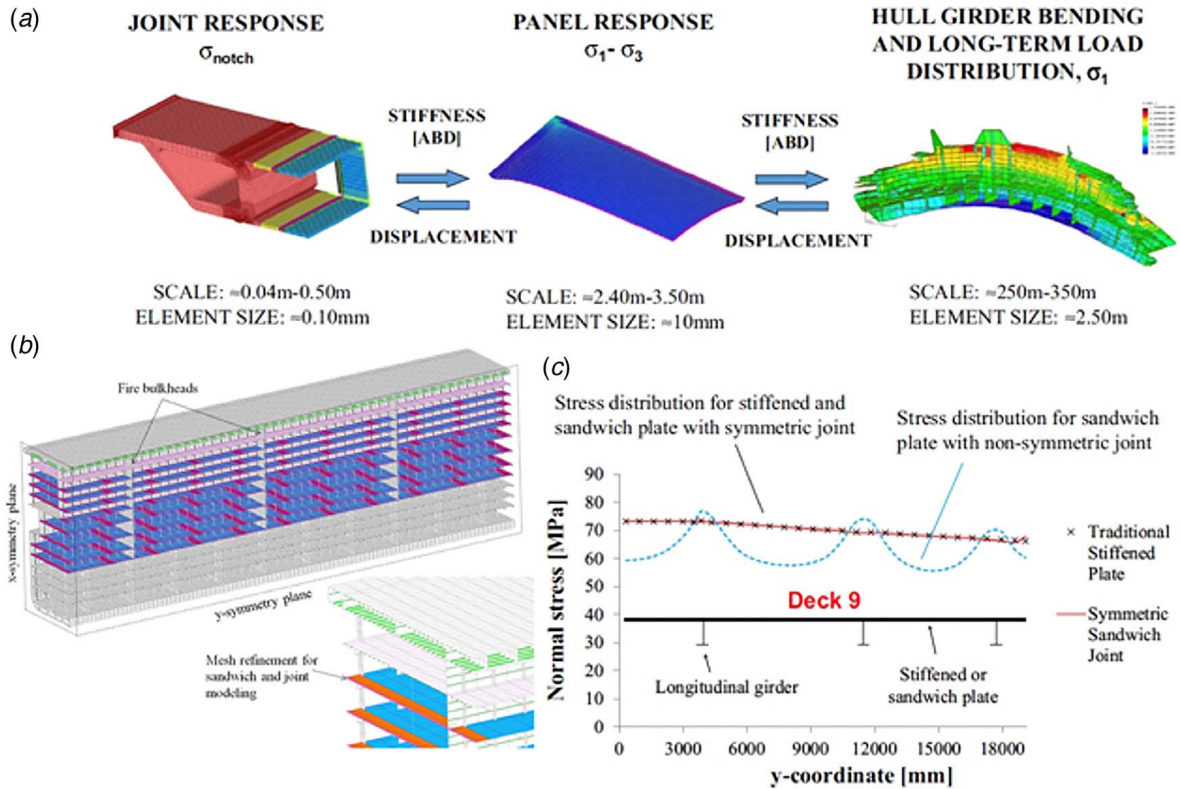


Fig. 2 (a) Multilevel analysis approach for the fatigue strength and response assessment of thin-walled structures from joint through panel to hull girder level. (b) modeling non-linear joints in global model using equivalent single layer theory, and (c) the difference between nominal stress states in panels with symmetric and non-symmetric joints (all panels with equal cross-sectional area), adapted from Ref. [21]

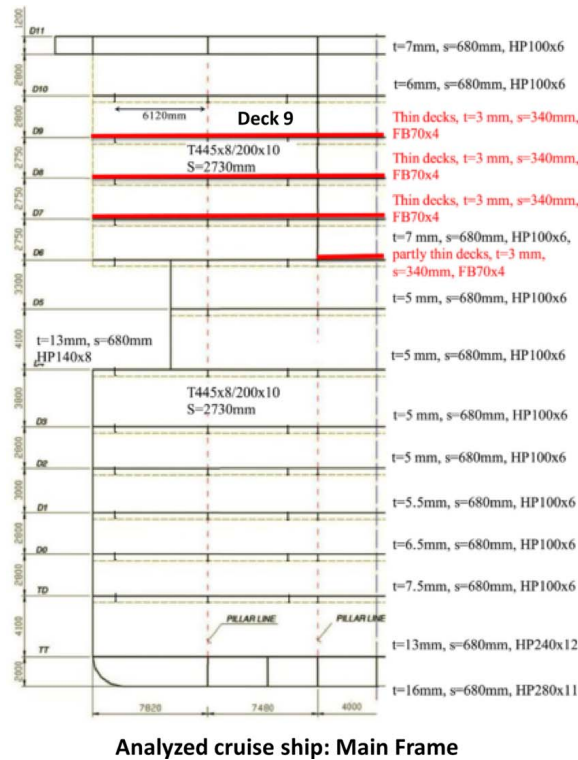
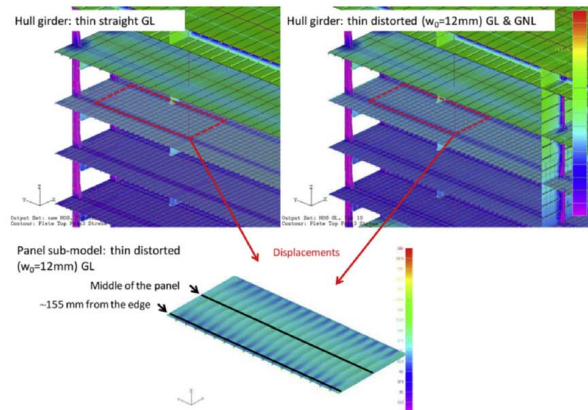


Fig. 3 Influence of distorted thin decks to load-carrying mechanism of a prismatic passenger ship, adapted from Ref. [21]

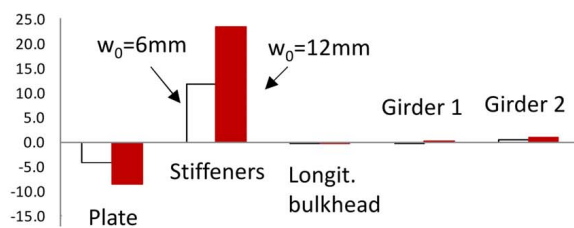
around $l_{secondary} = 10^1$ m), the response of the entire hull girder is affected when normal stresses due to primary bending are assessed ($l_{primary} = 30l_{secondary}$). This phenomenon is of course affected by the level of asymmetry of the panel and more specifically the coupling between membrane and bending, i.e., ABD-matrix, of the stiffened panel element (A = extension, B = extension-bending coupling, and D = bending). The example given here has strong coupling which is very local and periodic in the hull girder. This could be identified as *topological periodicity* in otherwise prismatic structure. In order to handle this in structural analysis, the full ABD-matrix should be used instead of the intuitive and often used only A-matrix. This adds computational efforts but is needed for the accuracy.

It is clear for this linear case that the continuum description results in accurate responses, but the level of modeling details must be right to get reliable results. The situation changes when we extend the investigation to geometrically non-linear two-scale analysis in the same ship but use traditional stiffened panels, see Fig. 3. Now, the initial panel geometry is prismatic, but the production-induced initial imperfections introduce *geometrical periodicity* to the panel, where the length of periodicity is in the order of magnitude of stiffener spacing i.e., $l_{scale} \approx 100$ m [22,23]. The initial imperfections are assumed to have sinusoidal shape as commonly assumed in the analysis of marine structures and the amplitude is varied. It is seen that the load shifts away from the plates to stiffeners as the amplitude increases, but this shift is modest in comparison with the sandwich panel case and seen to have a significant effect only in the cases beyond current IACS recommendations. The same phenomena are seen in Fig. 1(b), but caused in that case due to periodicity in the location of joints. The key issue in both cases is that the nominal load level, i.e., membrane stress, at the decks is not uniform, has periodicity due to membrane-bending coupling, and this periodicity is affected by the boundary conditions for both membrane and bending action.

Global versus local panel analysis



Difference from thin straight % (Deck 9)



Ultimate Strength

In ultimate strength, the key issue is the structural stress-strain relation, i.e., the load-end-shortening curve. From the viewpoint of the response of a larger scale, the focus is on volumetric average response. Roughly idealizing situation, the tensile and compressive responses are needed for each structural element in the hull girder as is done in Smith's method. However, as the response is two-dimensional, we have made attempts here to formulate these approaches directly to the plate-level model. This limits the investigations to ductile fracture in tension and post-buckling until first-fiber yield in compression. This is due to the fact that in periodic plates, the yielding is caused by stress resultants, which can change in magnitude non-proportionally as the applied load increases. This situation is due to the membrane-bending coupling. On the other hand, the ABD-stiffness matrix is affected by yielding. Thus, further research is needed on modeling of the full two-way coupling.

Ductile fracture is affected by the stress triaxiality and shell element size. Recent investigations from collision and grounding research and on material failure [24–28] are used by Körgešaar et al. [29] to formulate the structural stress-strain relation to the panel level. We limit the investigations to the panel-level membrane responses. As Fig. 4 shows, the panel-level responses can be obtained very accurately with non-linear extension stiffness matrix, i.e., A-matrix, even in cases where there is a deck opening. The anisotropic, non-linear, A-matrix in this case is computed using analytical equations and rule-of-mixtures.

In compression the extension matrix, A-matrix, is not enough as the panel fails locally, for example, by buckling of the plates inside RVE. This means that the full ABD-matrix is needed where all terms can be non-linear. Instead of using the analytical approaches, this matrix is derived numerically using sub-models and periodic boundary conditions that follow assumptions of classical continuum mechanics, see Fig. 5. In this approach, the unit cell of the panel is

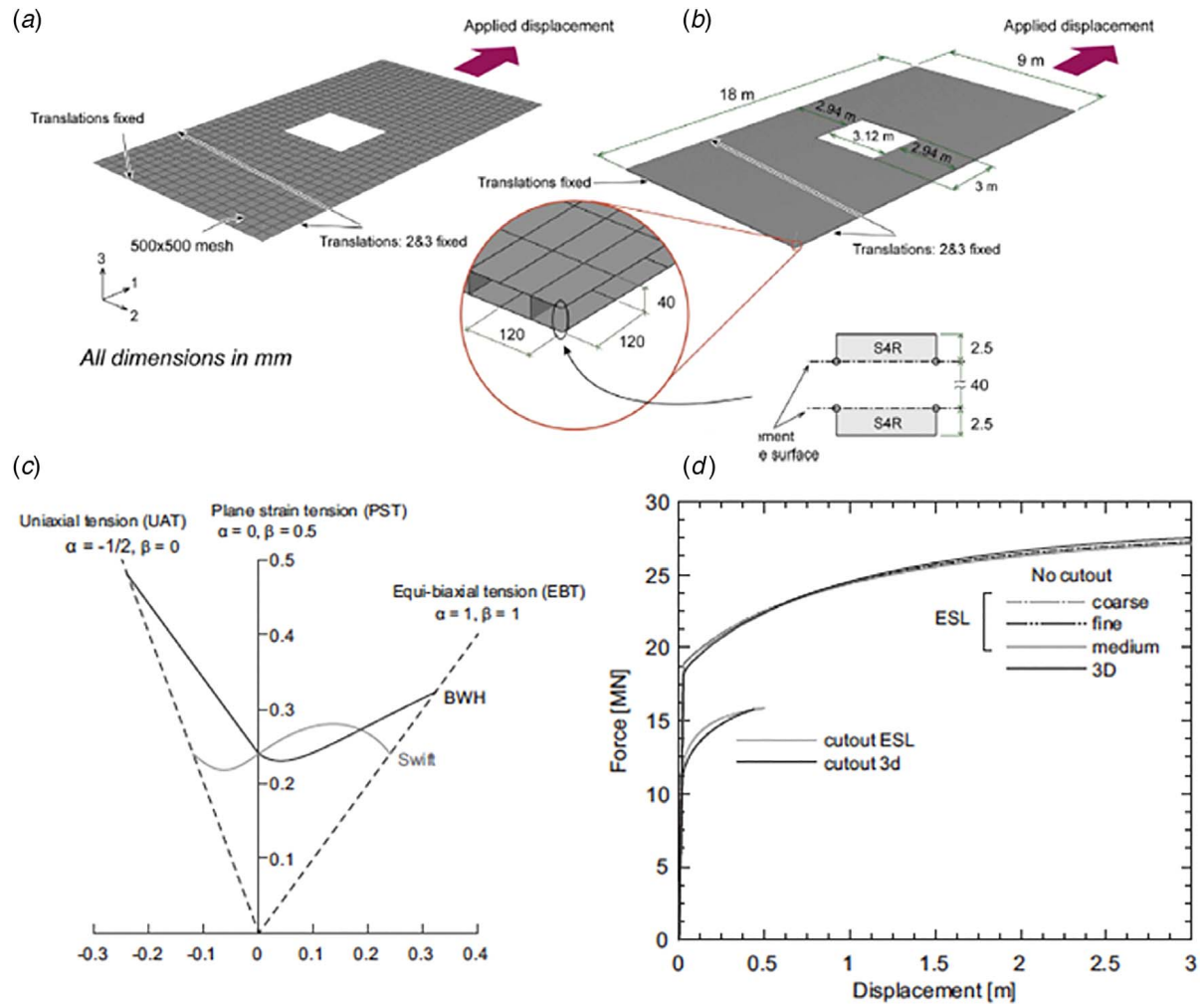


Fig. 4 Modeling fracture in ship deck with openings using (a) ESL, (b) 3D-FEA, (c) local fracture criterion, and (d) comparison between ESL and 3D for deck with and without rectangular cutout (reproduced from Ref. [29])

exposed to membrane and bending strains according to first-order shear deformation theory. The stress resultants are evaluated at unit cell borders. The comparison of strain and curvatures and the resulting stress resultants give the full ABD-matrix. It should be noted that the sequence of applying membrane and bending strains will have an effect on the stiffnesses due to unit cell buckling. This makes the ABD-matrix non-symmetric which increases the computational efforts. Furthermore, as the compressive surface in RVE is shortened and tensile elongated, the RVE changes shape and size for changing load. Therefore, we obtain in this case the third type of length-scale interaction, what we call here *progressive periodicity*. As the results in Fig. 5 show, the accuracy of the panel post-buckling response prediction is very accurate. However, the true test of the method is in analysis of large structures, where the primary, secondary, and tertiary scales are coupled all at the same time. For this type of investigation, we present in Fig. 6, a typical benchmark example of a box-beam in four-point bending [32–35] with a web-core steel sandwich panel as shell structures [31].

Figure 6 presents that when full coupling between the scales is considered the local, normalized load-end-behavior of the deck that buckles is very accurately predicted by the equivalent single-layer mesh and that the full two-scale coupling is not always needed and linear micro-structural modeling is enough when unit cells are sturdy in comparison with macro-scale responses. However, it should be mentioned that the eigenvalue buckling

might wrongly predicted if the corner regions are not properly modeled (i.e., overlapping material). It should also be mentioned here that the computational savings are enormous as the solution with ESL requires around 64 times less memory than 3D-FEA, and the analysis is carried out in hours rather than in days (in the case of 3D-FEA). This computational saving is due to the fact that local failure defines the required time step in FEA which significantly lowers the computational speed at the moment of local failure; this relates to characteristic lengths of buckling, i.e., $l_{secondary} \gg l_{tertiary}$, where the effect of tertiary buckling length is included to pre-computed load-end-shortening curve.

Fatigue Analysis

While ultimate strength analysis focuses on the volumetric average and load-end-shortening curve, in the fatigue assessment detail-level response is the decisive factor. In the structural analysis, the preference is on stress-based methods with material linearity assumption being valid. This assumption is justified as the stress is at maximum mildly non-linear, only at the very small volume of the structure when the design is focused on medium-high-cycle and high-cycle fatigue regions. Another assumption commonly utilized is that the secondary and primary responses are accurately obtained by the undistorted, initial, idealized geometry of the

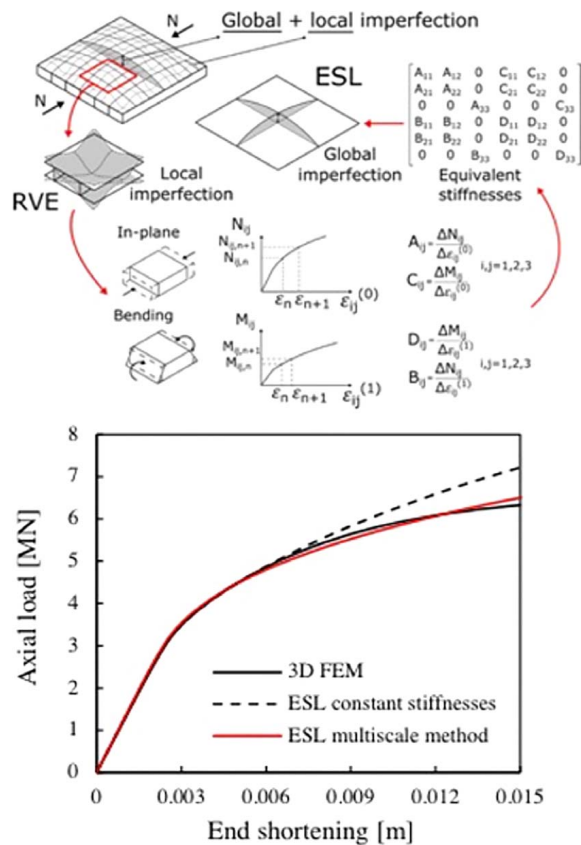


Fig. 5 Modeling buckling in ship deck with openings using 3D shell element and equivalent single-layer modeling (reproduced from Ref. [30])

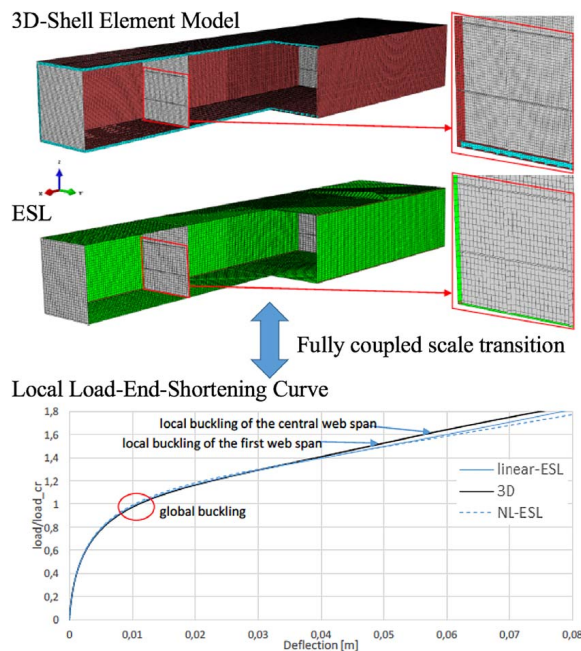
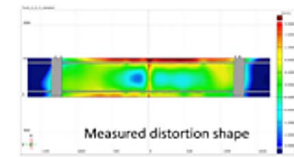
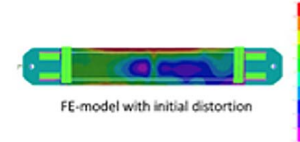


Fig. 6 Fully coupled geometrically non-linear analysis between primary, secondary, and tertiary scales (reproduced from Ref. [31])

Full-field geometry and distortion measurements of a structure



Modification of idea structure to consider manufacturing induced distortions



Geometrically non-linear analysis of initially distorted structure

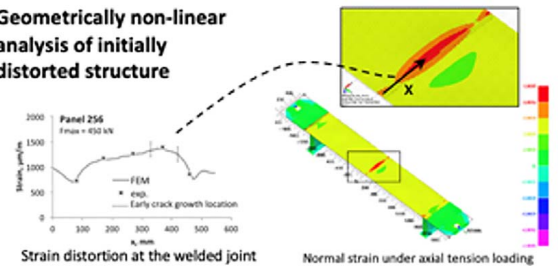


Fig. 7 Failure locations for full-scale panels and corresponding FEA (reproduced from Ref. [23])

panel and the weld. However, when the thickness of the plates is reduced and geometry of the welds changes (e.g., to stake-welds), this assumption becomes one of the sources of uncertainty to the analysis [21,22,36–41]; this result is indicated also in Figs. 2 and 3. Therefore, research has been carried out by (see Fig. 7) measuring the actual shapes of the produced structures, building very detailed shell-element-based finite element models, and carrying out the detailed comparison between measured and computed stresses at fatigue critical locations.

The investigations show that by this sequence [23,42], the stress and displacement responses are accurately captured and the scatter between fatigue tests at full- and component scales is reduced to practically non-existent (see Fig. 8). The challenge, however, from the viewpoint of structural design is that the periodicity of the distorted plate is the valid assumption only for deflections, but not necessarily on slopes and higher order derivatives of the displacement field (i.e., $d^n x/dx^n$, $n = 1, 2, \dots$). This means that the homogenization for stresses, which are based on higher order derivatives, can lead to erroneous results unless also the derivatives are periodic. Due to this localization process, the use of trigonometric functions to describe initial imperfections becomes questionable. In real structures, this condition is almost always violated and 3D-shell element models are needed when fatigue is assessed.

Conclusions and Discussion

This paper gave an overview of the factors that affect the strength and structural design of advanced thin-walled marine structures with reduced plate thickness [22,23,38,40–42] or alternative topologies to those used today in the industry [43–45]. Due to production-induced initial deformations and resulting geometrical non-linearity, the classical division between primary, secondary, and tertiary responses become strongly coupled. This calls for structural design methods that link the traditionally separate field of fatigue [21,46] and ultimate limit state assessments [2,9] under one umbrella.

The ultimate strength is often assessed by using the volume-averaged, non-linear response of structural elements [2,9,47,48].

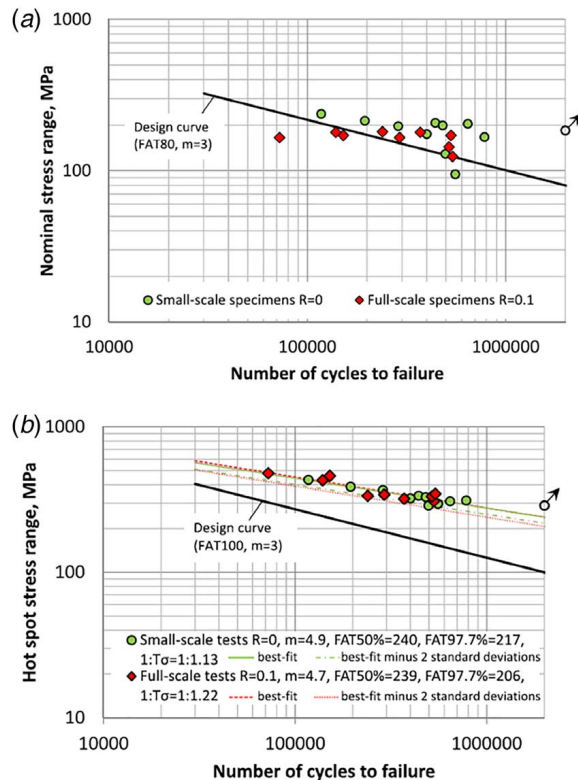


Fig. 8 Stress state assessment at the laser-hybrid welds and complexity of the stress state based on geometry and the loading mode (reproduced from Refs. [23,42])

This structural stress-strain relation enables analysis at the next, larger, length scale. In the analysis of thin-walled structures, this standard homogenization process needs to be complemented with localization, where the stresses are assessed at the details, such as welds for fatigue analysis [12–14,37]. Analogous to multi-scale modeling, the field of engineering science that develops fast in material science [12,13,49], these two processes are needed to be fully coupled to move to the next level of engineering computations. This also requires that the production-induced initial distortions need to be considered with much higher accuracy than in the case of before. With today's measuring and simulations tools, the actual geometry of the produced structure can be measured in detail and fed to the finite element analysis to assess the structural response, for example, in ship's service [23]. In this type of work, it is essential to understand the length scales associated with the structural assessment. The paper discussed *topological* (spacing of joints), *geometrical* (initial distortions), and *progressive* (developing distortions) length-scale interactions. In the cases where homogenization is used to reduce the size of computational models it is important to understand these effects in terms of continuum description. Non-classical continuum mechanics is needed when the order of magnitude of the consecutive scales is not clearly separable [15–19,50]. Strain-gradients and couple stresses were used to increase the accuracy of the kinematical description of beams, plates, and shells where the consecutive length scales are close [19,51,52]. As train-gradients are also widely used to explain localization of plasticity, the formulations could be extended to this direction too [19,50,53–56]. All this is left for future work.

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Nomenclature

f = any function
 l = characteristic length
 N = normal force
 M = bending moment
 A = extension stiffness matrix
 B = extension-bending coupling stiffness matrix
 D = bending stiffness matrix

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