

# Aalto University

## *School of Engineering*

MEC-E8007 Thin-Walled Structures  
Lecture 1. Design, Loads and Discretization  
**Jani Romanoff**



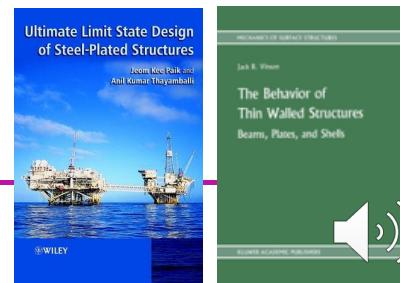
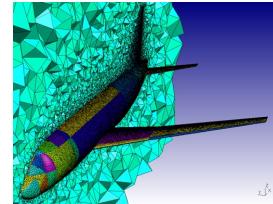
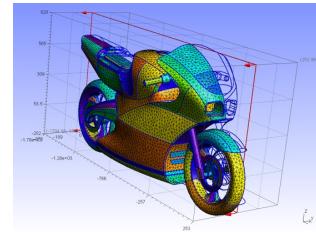
In order to analyze and design thin-walled structures, we must understand the context in which the analysis and design are to be carried out. Here it is intended to introduce a wide range of topics covering different aspects of thin-walled, that is lightweight, structural assemblies.

As a reader you are expected to have open mind for learning and dedication to this learning process. It is also expected that you have the basic knowledge of solid mechanics, materials science and most of all Finite Element Method. Moreover, you know how to use FEM to analyze simple structures like beams, plates and shells making the thin-walled structural assemblies.

The material is build based on the assumption that you know all this before and we can focus on really the specific features of thin-walled structures. Otherwise the learning experience will be a tough one. If you know the stuff, it is guaranteed that you learn a lot. One student said few years ago, that after the course there is the understanding how many things can go wrong. So the responsibility of the designer became clear. The fundamentals are important, but they are not enough if you do not understand how they are in reality. That is are the assumptions we have to make in order to derive the prevailing (differential) equations for the analysis valid in practice? Or is the (analytical or) numerical solution accurate? The first question relates to the modeling in terms of differential equations, while second much more to the analytical or numerical solution of the prevailing models. In design we have to be absolutely sure that these issues are properly treated as our focus must be on the feasibility and functionality of the design.

# Contents

- The aim of the lecture is to understand the issues affecting the analysis and the discretization of the thin-walled structures using the Finite Element Analysis
- Motivation
- Definition of thin-walled structure
- Analysis, evaluation, decision making
- Design philosophies
- Failure modes
- Loads
- Discretization
- Analysis types
- Building the models
- Literature:
  1. Paik, J.K. and Thayamballi, A.K., "Ultimate Limit State Design of Steel-Plated Structures", Wiley
  2. Ventsel, E., "Thin plates and shells: theory, analysis and applications", CRC Press
  3. Vinson, J.R., "The behavior of thin-walled structures", Kluwer Academic Publishing



The aim of the lecture is to understand the issues affecting the analysis and the discretization of the thin-walled structures using the Finite Element Method and Analysis.

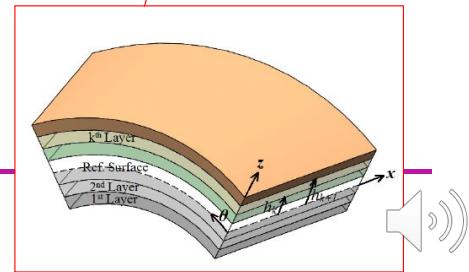
As in all lectures we first go through the motivation for the lecture and make interfaces to the related subjects. Then we discuss about important assumptions and by doing these we give you the definition of thin-walled structure in terms of physics and mathematics. This definition is good to remember throughout the course. What is also important is to define the differences between the analysis, evaluation and decision making which are all parts of the thin-walled structures design process. Similarly, we need to define the different design philosophies as the way these are defined has direct impact on the final results. All thin-walled structures have certain failure modes that contribute to the strength and maximum loads the structures can take. The loads can be mechanical and controllable or natural and stochastic. In finite element analysis, key issue is the Discretization of our finite element model and this is affected by the type of analysis we are about to perform. We conclude the lecture by few examples on how to build the models in practice.

**About the Literature:** The lecture slides are enough to give you an overview of the subject. However, the enclosed articles are good as they show you how is the science around the subject. This is what defines the current developments of the field and as M.Sc. or D.Sc. graduates you should understand that the field is developing constantly and you need to be able to follow these developments. These are always written before the textbooks, which are written when the field or topic is mature enough to be put to the format of a book.

The books given here are good for showing you the wide range of topics associated with thin-walled structures. The book of Vinson is really about analysis of **idealized** thin-walled structures. Alternative to this could be the book of Ventsel. The book of Paik and Thayamballi is good if you want to understand the complexity of design of these structures when you account for manufacturing and materials, that is **non-idealized structures**.

## Thin-Walled Structures

- Thickness of the shell plating significantly smaller than the global dimensions  $t/L \ll 1$
- In many vehicles own weight is often critical and lightweight solutions are needed, e.g.
  - Ship side shell
  - Airplane skin
  - Shell plating of cars
- The global stiffness is obtained by membrane action, local bending stiffness can be ignored
- For the local stiffness we need some special techniques, e.g.
  - Stiffeners
  - Laminates and sandwich structures
- Large thin-walled structures are often designed for long life cycles, e.g. 10-100 years



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The definition of a thin-walled structure mathematically is often given in format where the thickness of the shell plating significantly smaller than the global dimensions  $t/L \ll 1$ . The global dimension can be here for example the diameter of the fuselage of the airplane or the soda can, both of them being thin-walled and lightweight structures with high strength and large capacity for payload, i.e.  $m_{\text{payload}}/m_{\text{structure}}$  is large. Often the thickness to length,  $t/L$ -ratio, is given for the initial and idealized geometry in which the physics of the problem or the manufacturing or operation induced flaws are not accounted for. If the physics are accounted for, the definition can change. Example of this is buckling, in which the buckling wave length defines this characteristic length,  $L$ . I have enclosed an article about this to the additional reading (Romanoff et al, 2020). On the other hand the corrosion experienced during the lifetime of the thin-walled structure, may decrease the characteristics of the thickness.

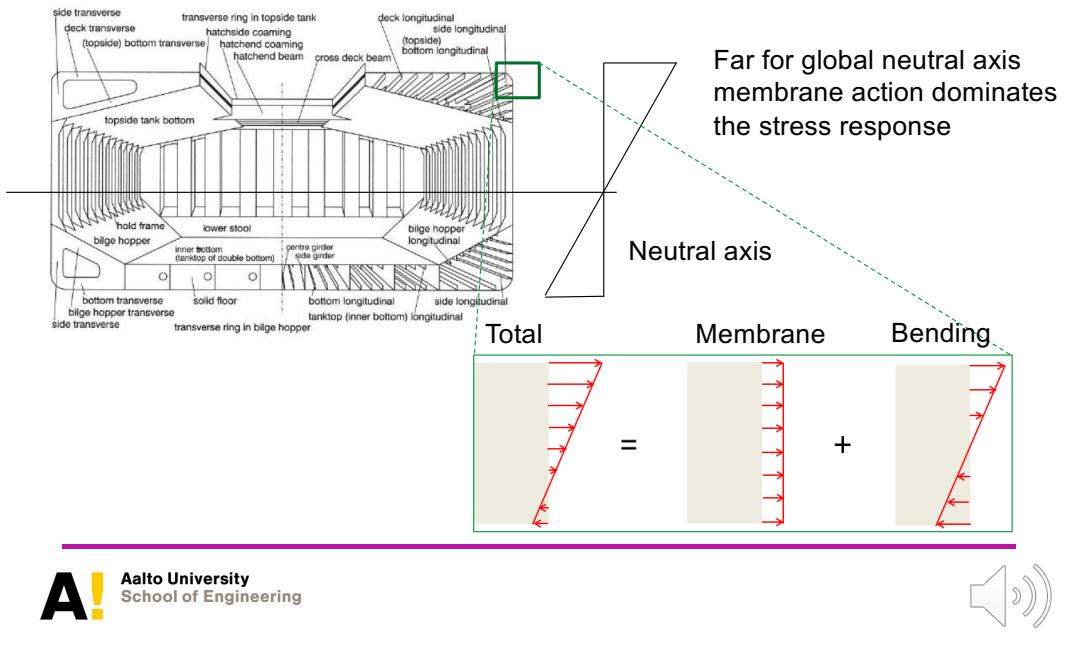
Why do we need thin-walled structures? In many vehicles own weight is often critical and lightweight solutions are needed so that the payload is maximized and the own weight of the structure is minimized without sacrificing the safety. Examples of such weight critical applications are ship side shell with buoyancy carrying the total weight consisting of structural and payload weight, airplane skin with the lift generated by the wings compensated by the total weight and the shell plating of cars in which the main purpose of it is to create aerodynamic and weather protection, esthetics and to contribute to the collision strength.

In thin-walled structures, the main idea is that the global stiffness is obtained by membrane action of the beam, plate or shell and the local bending stiffness can be ignored from the analysis. That means that the stiffness terms associated with the product of Young's modulus,  $E$ , the area of the cross section,  $A$ , and the distance (squared) from so-called reference plane,  $d$  (and  $d^2$ ), is much higher than the contribution from second moment of area,  $I$ , and Young's modulus,  $EAd^2 \gg EI$ . This is the reason thin-walled structures are efficient. Large diameter is beneficial to have with "thin wall".

Of course in reality the structures are also loaded with point loads (point loads are in practice important to avoid in design, unless you use local strengthening) and pressure load (e.g. aerodynamic loads) that call for increased local stiffness. For this we need some special techniques, for example: stiffeners that increase the stiffness of the shell plating or laminates and sandwich structures in which we exploit the ideas of the high membrane stiffness far away from the reference plane locally.

One should also remember that the large thin-walled structures are often designed for long life cycles, e.g. 10-100 years. This means in practice that one size does not fit all. There are have numerous load (and boundary) conditions that affect the design and these should be properly accounted for.

## Membrane vs. Bending Action

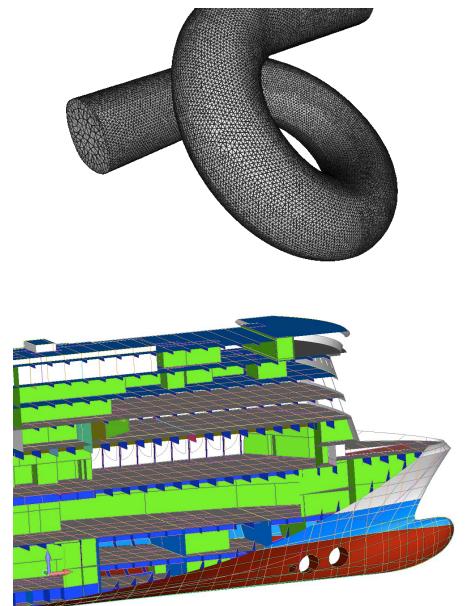


The idea of thin-walled structure is presented here. The example is ship hull girder (bulk carrier), but the same principle works for bridges, soda cans and fuselages of airplanes. As the "hull girder" bends globally and the walls are thin, they experience practically constant stress far from the neutral axis. The small "inclined" part of the total stress, that varies over the wall thickness at the top most or bottom deck is practically negligible and does not contribute more than few percent to the total stress. The thinner is the section, the smaller is the error introduced by this simplification. The thinner is the wall, the more effective it also is. Thus, we really aim to make this assumption valid when minimizing the weight of the system.

This idea is told here in terms of vertical bending, i.e. over the horizontal axis. In principle the girders can bend in both horizontal and vertical directions and also experience torsion to different degrees and their total load effect can be estimated by the superposition of the obtained responses for these basic modes of deformation. We should remember here that the superposition assumes linearity in prevailing equations. Thus, if we go to non-linear region, the two load effects must be applied at the same time to obtain more realistic responses.

# Motivation

- Finite Element Analysis enables strength assessment of structures having
  - Complex shape
  - Complex material behavior
  - Complex kinematics, including non-linearity
- The problem is that the solution is discrete:
  - Solved only at certain locations called *nodes*
  - The physical model between the locations is described by *elements* (beams, plates, shells, solids) with certain assumptions
- Due to discrete nature of the problems
  - We end in matrix operations – matrix size depends on the discretization – influences computation time – time is limited
  - Accuracy depends on the discretization – what is enough for the accuracy
  - Analysis type also affects the discretization, dense mesh required for
    - Fatigue to capture the stress gradients
    - Buckling to capture the buckling modes
    - Etc

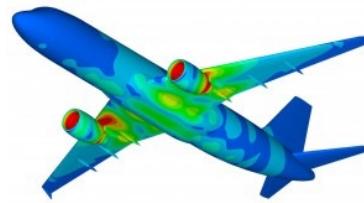


Traditionally the analysis of thin-walled structures have been executed by using dedicated beam, plate and shell theories and their analytical solutions. However, thanks to the development of our computational capabilities, simulation based design has practically replaced these analytical approaches as this way we can handle more complex interactions between the structural elements, but also cope with more complex material, load and geometry definitions. Today, the Finite Element Analysis (FEA) enables strength assessment of structures having **complex shape, material behavior and kinematics, including non-linearity, under various load and boundary conditions with ease.**

The problem with the FEA is that the solution is discrete. The system equations are solved only at certain locations called *nodes* and the physical model between the locations/nodes is described by *elements* (beams, plates, shells, solids) with certain assumptions and prevailing differential equations. These differential equations are solved in FEA context often in weak or strong forms to obtain approximations to the equation system simulating our engineering problem. Due to the discrete nature of the problems we end in matrix operations. The matrix size depends on the discretization. The matrix size,  $N \times N$ , with  $N$  being the number of degrees of freedom, influences computation time as the solution is carried out by inverting the stiffness matrix or by iterations in which the number of mathematical operations is either proportional to the  $N^2$  or  $N$ . As the time is limited in design, it is of high importance to find a model that is small enough and accurate enough at the same time. Too small model leads to too inaccurate solution, while too large model is too expensive. What is too much or too little depends on the type of problem we are solving (e.g. fatigue, buckling) and time available for pre-processing, analysis and post-processing. Due to increased computational power the analysis time is not always the bottle-neck of the design, but rather making the numerical models and assessing the obtained results. These stages can be very difficult to automatize if the spectrum of different geometries, topologies, materials and load and boundary conditions is large.

## Analysis, Evaluation, Decision Making

- Design process involves several steps
- Analysis
  - Computing or experimenting the responses (displacements, stresses, eigenmodes...) of the structure for relevant load sets
- Evaluation
  - Assessment of the obtained levels of responses to the critical values, e.g. limit states, rule values
- Decision making
  - Optimization and selecting the design that will be manufactured



The design process of any structural system contains several steps and tasks that have different meanings.

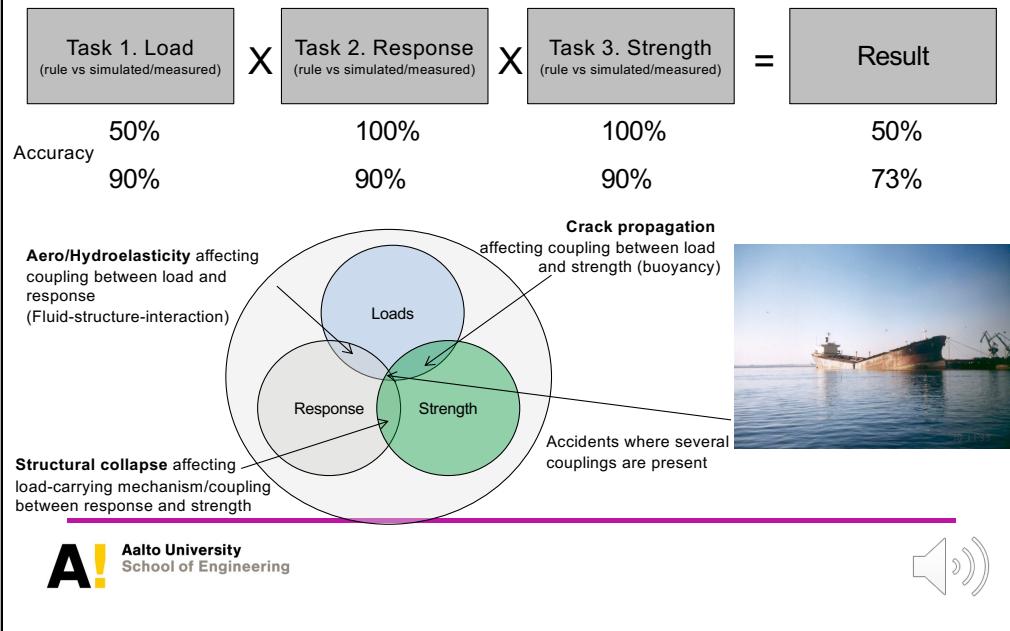
**Analysis** is all about analyzing the structural system responses for given load and boundary conditions. It is about computing or experimenting the responses in terms of displacements, stresses and eigenmodes etc of the structure for relevant load cases and sets.

When the responses are known, **evaluation** steps can be carried out. The idea is to compare obtained response levels with the critical values. Critical values can be set by authorities (e.g. comfort), they can be computed by using limit state analysis to derive the critical stress values before the system fails (e.g. buckling stress) or these can be for example tested values for materials or sub-components of the structural system.

When evaluation is completed, the design process proceeds to the **decision making** in which for example system optimization and selection of the design that will be manufactured can be done. This often requires several rounds of analysis and evaluation before final decision can be made.

## Analysis

All design tasks and assumptions must be in balance



In the analysis it is extremely important that the assumptions we make in **load assessment, response calculations** and in **strength evaluation** are in balance.

The simple schematical computation show that the accuracy is dependent in all assumptions we make and it is now effective to be accurate in some of the sections in the structural design and then take other sections more lightly. So for example taking rule loads, but making very detailed analysis of responses and strength can lead to worse situation than making accurate assessment of the loads, with the expenses of accuracy in responses and strength and trying to balance the accuracy on all of these three sections of the design.

Why such simplifications are made? As structural engineers, we cannot master everything. Yet we aim to do the best we can in what we know. Unless we can collaborate with people from other fields of specialization, we very easily aim to do some things much better than others. This may lead to bias in the way we think about our design.

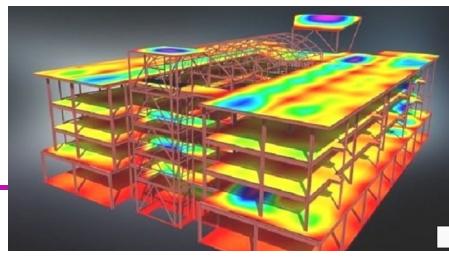
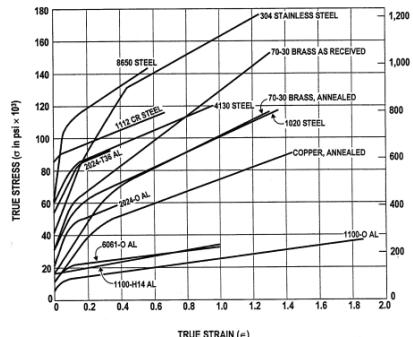
Typical flaws are due to the fact that it is good to simplify the design. The design process is much more effective in terms of design if we can separate the load analysis from responses and strength. If the system behaves linearly, such simplification are justified and very effective. However, problems such as aero- or hydroelasticity couple the loads and responses and the problems have to be solved as coupled fluid-structure-interaction problems. Same goes for crack propagation, which may lead to changes in the integrity of the structural system (e.g. ship split to floating bodies) or progressive failure of a structural system which changes the load carrying mechanism of the system.

There are several examples of such couplings and we have selected here a case article of Bazant about the World Trade Center collapse to demonstrate the type of coupling we may have in complex structural systems.

## Evaluation

For which materials the stresses are acceptable?

- When we know the structural responses, we can select the right materials (grades) to meet the strength and stiffness criteria
- Stiffness is defined by Young's modulus
- Strength is defined by yield and ultimate strength
- There are also criteria related to fire, accidents etc...



When we know the structural responses in terms of displacements, stresses or vibration modes, we can select the right materials to meet the strength and stiffness criteria. The right materials may be seemingly the same in terms of stiffness, e.g. Young's modulus of a steel, but their strength can very based on selected material grade which can be influenced also by the temperature, moisture and other environmental factors.

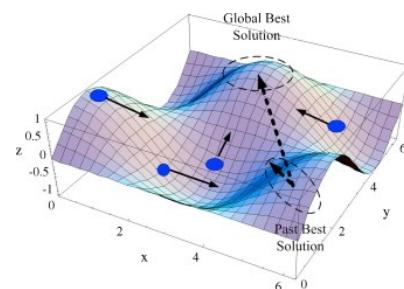
**Stiffness** is defined by Young's modulus and it can be considered constant with very little variation in its properties. It is important for applications where for example the deformations must be limited. **Strength** is defined by yield and ultimate strength of the material and typically the variation in material properties is much larger than in stiffness. Strength is important when the maximum load carrying capacity of the structure is to be defined as strength and cross-section area together form the load-carrying capacity (Force=stress\*area). As the area increases the weight, modifying strength is often the way to increase the load-carrying mechanism of the thin-walled structure.

In evaluation we can also use other criteria such as resistance to fire loads, accidents, just to mention a few.

## Decision Making

### Which design is optimal?

- Once our solutions meet all design criteria, our task is to select the best feasible design
- As analysis is done for several criteria and objectives, with numerical models, we often need direct optimization methods
- Direct optimization methods often aim to simulate natural selection
  - Genetic Algorithms simulating evolution
  - Particle Swarm Optimization simulating swarm behavior



When analysis and evaluation steps are completed once, they can be executed for several design alternatives. Once our **feasible** solutions meet all design criteria, our task is to select the best design. The best can be based on single or multiple objectives, such as mass, price, safety, sustainability or their combinations. In practice, the analysis is done with FEA and mathematically using some optimization algorithm suitable for structural optimization.

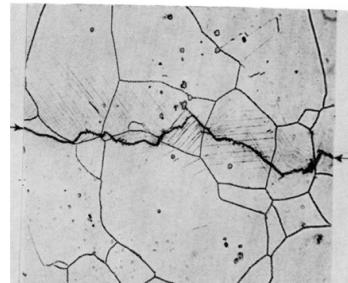
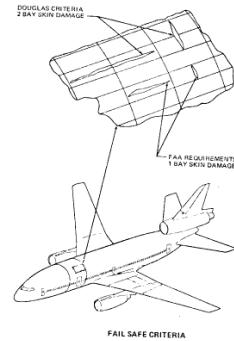
**Gradient-based methods** require the function value, but also the gradient value at the point where we are about to evaluate our current design and its position in the design space. True optimum can be reached only seldomly as pure design space may contain several local optima in which the gradient based algorithm stops as it has found “optimum” locally. The remedy is to start optimization from several places and trying to find the best “local optimum” from all of those found. Still the downside remains that the gradient should be taken for each design variable and the larger is the design space in terms of design variables, the more derivatives must be defined numerically. This means that with difference method, we need two points per variable to compute the gradient and 3 points to compute the curvature, which tells if our point is at local minimum or maximum. One point being here the function value itself it means that we need 1 or 2 additional function evaluations around each change of variable. As there are often numerous variables, the gradient based method causes “unnecessary design changes” and increase computational time.

More robust, but also often more time consuming approach is to use so-called direct **global optimization algorithms**. These work with only function values and do not need the additional analyses. Instead they require a lot of design alternatives to find the true optimum. Genetic algorithms mimic the evolution from nature. The idea is to create a generation of designs, let them mutate, be parents for the next generation and to make other evolutionary processes to produce better designs for the next generations of designs. As we have seen from the nature, it has been very effective of optimizing the natural designs to meet the requirements of this design space over several generations. Particle swarm algorithms instead exploit the ideas of group dynamics in the way that if there is social aspects in the swarm, its members are sharing their performance and experiences with each other. The idea is here as if we build on the ideas of others, we can find a way to constantly improve our designs. There are also several other direct optimization methods.

In many cases the best combination is obtained by combining both gradient and direct methods. Often the challenge is not the solution of the problem, but defining the variables, objectives and constraints in a meaningful way that takes into account the life cycle of the product and for example the true performance and potential of the materials and production we exploit.

## Design Philosophies

- Safe life
  - Here extremely low risk of failure is accepted, e.g. the structure will never experience detectable crack
  - Leads often to large structural weight
- Fail-safe
  - Mean that the structure will not endanger lives or properties when it fails
  - There is a back up plan, for example a crack arrester (stiffener etc.)
- Damage tolerance
  - The structure can sustain defects until repair can be done
  - Critical crack length during voyage etc

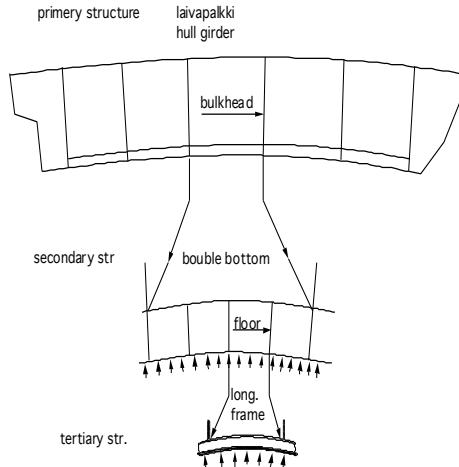


There are several Design Philosophies we can use in the design that result in different outcomes. The outcomes vary on many aspects, for example if our design is conservative or unconservative, if we are able to account for all necessary aspects and limit states and thus, the following statements should be taken mostly as guidelines and differences between the ways to thinking in design.

The **Safe life** concept is based on the idea that we allow extremely low risk of failure, meaning that the structure will never experience detectable crack in practice unless the probability of failure is reached. This design principle leads often to large structural weight as the stress levels we allow must be lower than those based on other philosophies. **Fail-safe** concepts means that the structure will not endanger lives or properties when it fails. This means that there is always a back up plan, for example a crack arrester (stiffener etc.) that restrains the failure to expand to the critical size. **Damage tolerance** concept means that the structure can sustain defects until repair can be done. For example, we may detect a crack in our structure, but if we know the load history and crack propagation rates, we can make an estimate how the structure can be safely operated until the next repair. Naturally, this leads often to smallest weight as we exploit the potential of the structure to full extent, but it also requires the most from us as engineers, as we must be much more aware of the situations as in the two other design philosophies.

## Division of Structural Response into Different Levels (different length-scales)

- Classical simplification in order to simplify the structural analysis is to divide the response assessment to three levels
  - Primary level (hull girder)
  - Secondary level (e.g. larger unit such as double bottom)
  - Tertiary level (e.g. plate, stiffener)
  - ...up to level N (e.g. lattice materials)
- Design rules often follow this principle
  - Analysis of tertiary structures
  - Analysis of secondary structures
  - Check longitudinal strength



Sometimes due to the fact that we have numerous length-scales involved in design ranging from bridges at kilometer length-scale to tailored materials at micron-scale, we need to consider separation of length-scales as we can not form the finite element model to include all of these at the same time. It would be computationally impossible task to solve in practice. This is why the division of structural response into different levels (different length-scales) is often done. The levels are based on the geometry commonly, but it could be based also on the responses.

Classical simplification in order to simplify the structural analysis is to divide the response assessment:

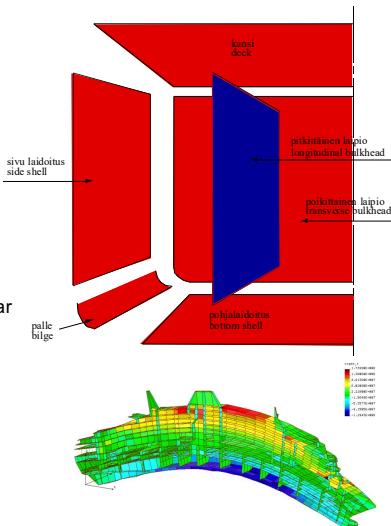
- Primary level that accounts for deformations of the whole structural assembly (e.g. hull girder)
  - Secondary level that accounts for deformations of any larger units of the structure such as double bottom or cargo holds
  - Tertiary level in which the smallest structural units deform (e.g. plate, stiffener)
  - ...
  - ...up to level N which can be lattice materials
- Design rules often follow this principle in order of:
    - Analysis of tertiary structures
    - Analysis of secondary structures
    - Check primary strength

and superposition principle is used to compute the combined effect of all of these responses. There are few remarks here. This is about structural assessment, not materials design. In multi-scale modeling of materials we expand the ideas to materials which are often more random than the "deterministic" structures are. We also have to remember in superposition that linearity must be a valid assumption and that if the loads are environmental they may not reach their maximum values at the same time (e.g. bending moment of primary structures is at different phase with the pressure on the plating).

## Main Elements of Primary Level

- The girder consists of main elements which form closed compartments
  - Deck
  - Bottom
  - Sides
  - Bulkheads
- In certain structures some of the main elements are missing
  - Airplanes: transverse bulkheads
  - Container ships: main deck due to cargo hatches
- These elements carry mainly the hull girder bending, shear and torsion loads as membranes
- These primary loads and resulting stresses can break the structure
  - the secondary and tertiary responses often cause often only local failures (of course hull girder collapse can start from these)

$$I \approx \sum Ad^2$$



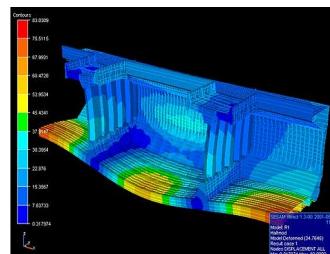
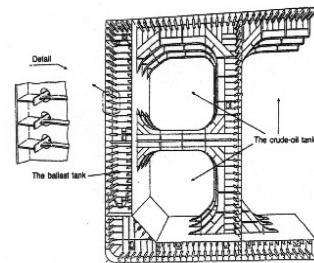
The girder consists of main elements which form closed compartments. These are the decks that carry vertical (e.g. gravity) loads, the sides that carry horizontal loads (e.g. dynamic pressure) and bulkheads/internal walls that split the compartments to smaller rooms. In certain thin-walled structures some of the main elements are missing:

- Airplanes, trains, roro-ships: transverse bulkheads due to the operational requirements related to moving of passengers or cargo inside the compartment.
- Container ships, heavy cargo trucks, trains etc where the main deck is missing due to vertical loading and unloading. It should be noted here that the cargo hatches are not connected in rigid enough way to the rest of the girder to close the section.

These main elements carry mainly the girder bending, shear and torsion loads as membranes due to the fact that the wall thickness,  $t$  is small in comparison to the characteristic dimension of the deformations,  $L$ . These primary loads and resulting stresses can break the structure, while the secondary and tertiary responses often cause often only local failures. It should be realized that of course the local failures can expand and result in the failure of the entire girder. That is the girder collapse can start from these local failures.

## Secondary Level

- Secondary level consists of parts that can deform in larger scale
  - Double skinned decks and bottom, double side
  - Web frames, secondary girders, grillages
- In certain structures some of the main elements of these responses are dominating
- Transfers load between primary and tertiary
  - Acts as boundary condition for level 3 response evaluation
  - Acts as internal load for level 1 response evaluation

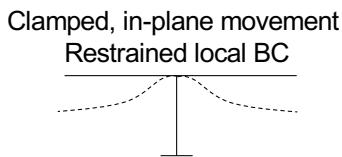
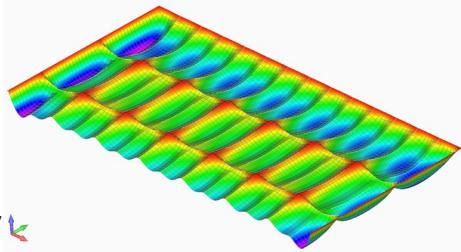


Secondary level consists of parts that can deform in larger scale and the main problem in that scale is that it acts as the load transfer length-scale between primary and tertiary responses. Thus, the idea of this length-scale is to act as the length-scale that defines the “zero” displacements for assessment at the boundary of tertiary structures. This allows us to focus on tertiary level on smaller structural units.

Typical examples of these are any larger grillages (made from secondary girders) and double-skinned structures such as decks, bulkheads and sides in which both membrane and bending effects are present. Typically these deformations can be critical if one cargo hold is loaded while the neighboring ones are not. This causes one hold to deform more than the others, while the deformations of others cannot be omitted either. Thus, the length-scale separation is not always clear.

## Tertiary Level

- Tertiary level consists of local parts such as
  - Beams
  - Plating
- Can be assessed using basic beam and plate theory and simple load and boundary conditions.



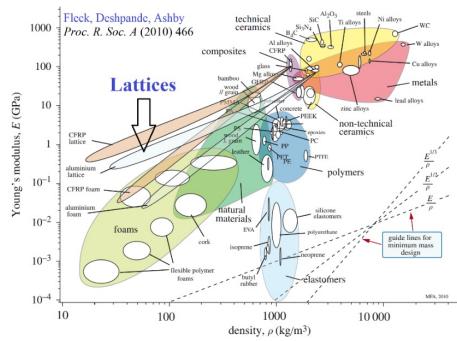
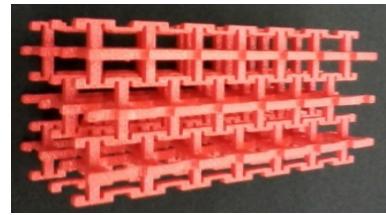
The tertiary level consists of local parts such as beams, plates and shells which can be assessed by using corresponding structural theories and simple definitions on load and boundary conditions.

Few remarks are important here. The loading if preferably of pressure type and the responses are mainly bending-induced and point loads are always a challenge for thin structures and to FEA (some elements do not behave well under point loads). Note that several point loads create a load equal to the pressure load according to superposition principle. The large deflection non-linear structural theories can be used to account for the so-called von Karman strains that invoke membrane effects when the deflection,  $w$ , is in the order of plate thickness  $0.5\text{--}1.0t$ , and thus make the structure stiffer and stronger than the linear theories predict. In lightweight design this is important.

The second thing is about boundary conditions. We often exploit clamped boundary condition with in-plane movements restrained. This gives a stiffer structure than simply supported boundary condition would give and thus the structure becomes lighter. Clamped boundary conditions are used as the the loading is often continuous over several “characteristic spans”, thus the reason for clamping is symmetry of loading rather than the rotation stiffness of the supporting stiffer structures. Actually, we can never make the supports infinitely stiff (due to 3D deformations, which due to elasticity make the cross-section to deform at the supports). That would result in overly heavy structure. If the loading is not continuous, the simple support is conservative and more accurate assumption than clamping. The in-plane movement restrained allows membrane effects to occur at large deflection. Again this is justified if the structure is continuous and we assume that it remains intact.

## Material Levels

- As the accuracy of production technology increases, we can design regular structures also at material scale
- In principle we can obtain ultralightweight materials
- The "structural" analysis must be carried out in smaller and smaller length scales

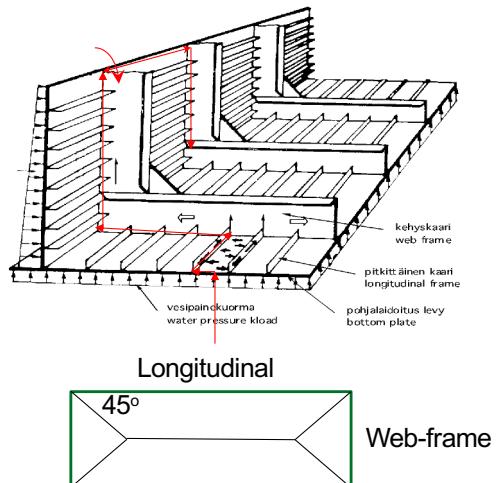


The ideas of the 3 length-scales and their separation can be extended basically infinitely. The main limitation is the production technology we have. As the accuracy of production technology increases, we can design regular structures also at material scale. In principle we can obtain ultralightweight materials by exploiting the ideas of structural lightweight design at the material length-scales. This is computationally exhaustive task as the number of layers of "structural modeling" increase and always the length-scale separation is not clear.

## Load Transfer

### Example of Pressure on Bottom Plating

- Thin-walled structures are designed to have optimal load-carrying mechanism
- As structural elements are 1- or 2-dimensional, the load-carrying mechanism is too, e.g.
  - Pressure on plate supported by longitudinals and web frames
  - Line load,  $p^*$ 's, on longitudinals supported by web frames
  - Point loads on web frames supported by side structures
- In 2-dimensional structures, if failure occurs in one of the directions, the load can still transfer in the other direction



The load transfer between the length-scales is important. We go through this by an example of pressure acting on the bottom plating of a ship and how this tertiary level loading transfers to primary level loading. We should keep in mind that the thin-walled structures are designed to have optimal load-carrying mechanism and this example highlights this logic. As the structural elements used in thin-walled structures are 1- or 2-dimensional, the load-carrying mechanism is too.

Pressure,  $p$ , acting on the plate is supported by longitudinal stiffeners and web frames that act as "hard lines" to the plate. These lines are the boundary conditions for the plate and as the vertical stiffness of the supporting lines is much higher than on the plate, this restrains the deformations in the vertical direction. If the loading is continuous, over the neighboring plates too, this results in zero rotation in zero rotation along these lines and the boundary condition is clamped (and pinned). So we know the pressure, we know the distance between the "hard lines", we know the boundary conditions and the plate theory and we can solve the problem for bending induced stresses and deflections by considering this small plated element, separated from the entire structure.

The supports for the plate, i.e. web-frames and longitudinal stiffeners take the load as a **line load**, with magnitude  $p^*$ 's. The spacing,  $s$ , is used here to describe the distance between the web-frames or stiffeners. As the longitudinal stiffeners are supported by the stiffer web frames, they induce additional point loads to the web-frames. (In order to not count things many times, envelope of loading can be used, in which we reduce the  $s$  by using the envelope with 45 degree). The web-frames are in turn supported by the side structures (primary level) that take the web-frame loading as point load. When all web-frame loads are taken together we get the girder level

load distribution.

## Limit States

- Limit states define the load-carrying capability of the structure
- These should be always compared against the load
- **Serviceability:** The function of the structure cannot be carried out as intended
- **Fatigue:** cumulative damage in structural details due to fluctuating stresses typically below the yield strength of the material
- **Ultimate:** maximum load the structure can take
- **Accidental:** unexpected events, e.g. collisions, explosions



Hungry horse look (serviceability)  
• Can increase the flow resistance  
• Esthetics (looks bad)  
• Initial imperfection for buckling



Limit state is an important concept as it defines the capacity that our structure has against the loads. So we use the concept at the evaluation phase of the design. There are many ways to categorise the limit states, but one way to look at these is the following:

**Serviceability:** The function of the structure cannot be carried out as intended. This class includes typically issues such as vibration dosage which can be converted to accumulated vibration amplitude or excessive deformations that for example affect the resistance of the skin of the thin-walled structure, are bad for the visual effects such as esthetics or acts as initial imperfections reducing the buckling strength.

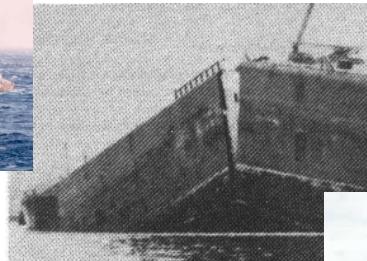
**Fatigue:** The cumulative damage in structural details due to fluctuating stresses typically below the yield strength of the material. This is one of the most common and challenging limit states we must assess as, it is often highly localized, there are numerous details where fatigue problems may occur due to different load and manufacturing induced load combinations over long life cycle.

**Ultimate:** The ultimate limit state corresponds to the maximum load the structure can take for example when loaded in wrong due to cargo, storms or/and their combined effect. The ultimate strength in tension is related to fracture of the entire structure, and in compression and shear, typically due to buckling. Ultimate strength is often a process of sequential failures and a highly non-linear problem.

**Accidental:** Accidental limit state relates to unexpected events, such as collisions, explosions and fires. Unfortunately, it is very difficult to define these cases before they actually occur. Good example of standardized approach is the collision safety of cars, which is assessed for basic load cases defined by industry standards. Yet we have to remember that we design structures for the predefined tests, not for the unexpected "collisions" we may see in real operations.

Often these limit states are coupled and the failures we see are simply due to interaction of these basic modes of limit states.

## Limit States – Fatigue and Fracture

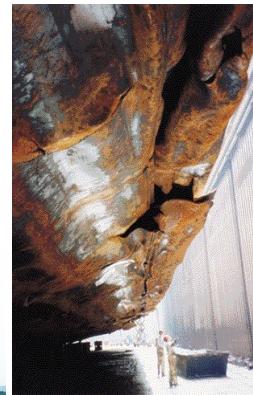


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Here are examples of fatigue and fracture induced decoupled and coupled failures. The top left and bottom right figures represent cases in which there has been significant fatigue damage experienced due to unconventional materials and/or loading (higher strength steels than we are used to and island hopping in flight profile) while the middle case shows that bad manufacturing can decrease the material properties so much that the failure happens by wrong loading sequence at the harbor.

## Limit States – Ultimate and Accidental Limit States



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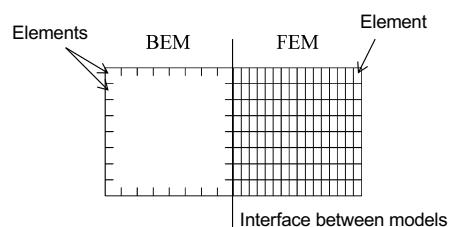
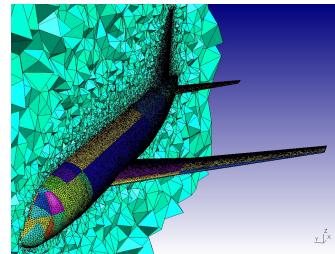


Here we see examples of the ultimate and accidental limit states. The hoods of the cars and the bilge of the ship have **buckled** and **then fractured** due to unusual loading, while otherwise the operational conditions are normal in terms of material. This means the temperature and moisture are “normal”.

The middle figure on the other hand shows an event in which temperature affects for sure the way the material properties are. This can mean a loss of structural integrity in fuel tanks that lead to expansion of the fire to final explosion.

## Load modelling

- Loads on structures can be created by:
  - Static and Moving masses
  - Hydro- or aerodynamics
  - Magnetism, temperature and moisture,
  - Etc.
- Often the structure has finite size and can be assessed with discretized *Finite Element Model* (FEM)
- Often the domain/medium causing the loading can be considered as infinite at least to some direction(s) and then *Boundary Element Method* is often the most efficient solution method
- The problem is how to couple these two numerical solution schemes and carry out the analysis for large structure with sufficient accuracy. Difficulties arise due to
  - Solving Navier-Stokes equations for large systems
  - Solving elasticity equations for large systems
  - Iterating between the two solutions

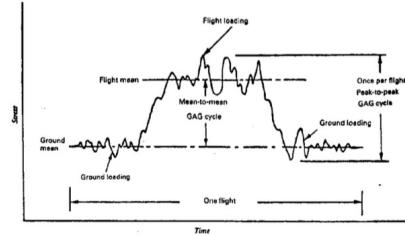


Loads on structures can be created by static and moving (accelerations) masses, hydro- or aerodynamics, magnetism, temperature and moisture and the time frame of these can have multiple time spans (temporal length-scale) ranging from fraction of a second to years. Also some of the loads are human-controlled, well-known and deterministic, while we also tend to have random loads which are less easy to control as they are caused by the nature.

The fact is that often the structure has finite size and can be assessed with discretized *Finite Element Model* (FEM) with certain size, while the domain/medium causing the loading can be considered as infinite at least to some direction(s) and then *Boundary Element Method* is often the most efficient solution method for the loads. The problem is how to couple these two numerical solution schemes and carry out the analysis for large structure with sufficient accuracy. Difficulties arise due to solving full Navier-Stokes equations and elasticity (and plasticity) equations for large systems at the same time. So it may be beneficial to decouple these two analyses. In case the problems cannot be decoupled, then iterations between the two different numerical schemes is needed.

## Stochastic Loads

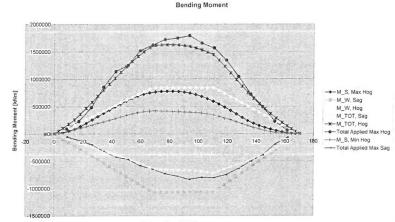
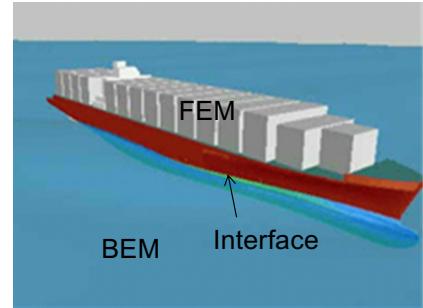
- Stochastic or random loads (i.e. vibrations) occur in a variety of applications of mechanical and civil engineering
  - the dynamics of a vehicle on an irregular road surface
  - the variation in time of thermodynamic variables in municipal waste incinerators due to fluctuations in heating value of the waste
  - the vibrations of an airplane flying through turbulence
  - the response of off-shore structures to random wave loading
- Basically all environmental loads are dynamic and irregular
  - Wave,
  - Wind
  - Current
- Stochastic approach is needed for defining load cases for design
  - Ultimate load cases
  - Fatigue load spectrum



Stochastic or random loads (i.e. vibrations) occur in a variety of applications of mechanical and civil engineering examples being the dynamics of a vehicle on an irregular road surface, the variation in time of thermodynamic variables in municipal waste incinerators due to fluctuations in heating value of the waste, the vibrations of an airplane flying through turbulence and the response of offshore structures to random wave loading. These environmental loads are dynamic and irregular and stochastic approach is needed for defining load cases for design. The two very different cases to consider is the ultimate load case which often requires non-linear analysis of coupled fluid structure interaction problem and the fatigue spectrum case in which we must run many load cases that we may experience during the life time of our structure.

## Loads

- The environment is often stochastic instead of deterministic
  - Instead of accurate values probability to certain condition can be only given
  - Spectrum (wave, wind) can be used to describe the environment when it is not changing
  - In practice it is not feasible to carry out the FEM-BEM for all environmental conditions and headings during lifetime of the structure
- However, there are rule loads and methods that simplify the reality to sufficient accuracy level
  - Analysis of given extreme event in time or frequency domain, FEM-BEM
  - Assuming rigid body (ship, building, aeroplane) in BEM and calculating the pressures and accelerations to FEM (both are needed in dynamics analysis, pressures + inertia)
  - Predefined quasi-static load distribution
- Application of rule-based loads is relatively easy
  - Based often on bending moment and shear force
  - The question is how to model this in 3D FEM – cargo hold models + rigid links
- Balancing the models might become practical problem



The environment is often stochastic instead of deterministic which means that instead of accurate values only a probability to certain load condition can be only given. This also means that the spectrum (wave, wind) can be used to describe the environment when it is not changing over time too much. This requires that we have a stationary process. In practice it is not feasible to carry out the FEM-BEM for all environmental conditions and headings during lifetime of the structure and instead we can use the rule loads and methods that simplify the reality to sufficient accuracy level. This means in practice that we focus the analysis for given extreme event in time or frequency domain by using the coupled FEM-BEM techniques. We can also assume a rigid body in BEM and calculating the pressures and accelerations to FEM as both of these are needed in dynamic analysis, in format of pressures and inertia loads. The simplest way to do this, is to use a predefined quasi-static load distribution that represents the reality to sufficient degree. The challenges in these simplified load introduction methods is the selection of the representative case for structure with complex load-carrying mechanism, balancing the models might become practical problem and also that the application of the loads may require iterations between the FEM and BEM meshes or introduction of rigid links etc additional elemental techniques.

# Loads coupling between FEM and CFD

Table 1. Algorithm for the iterative coupling of BEM/FEM

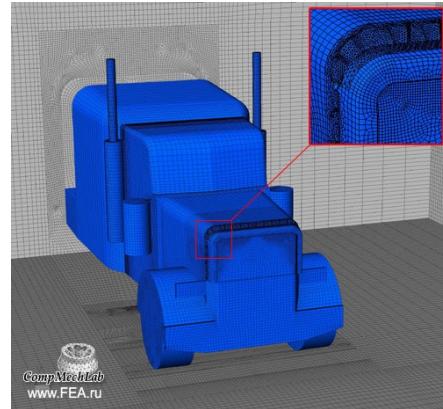
<b>1. Initial Calculations:</b>
1.1. The Problem domain is subdivided into two sub-domains that are well behaved and solvable, each one being modeled by the FEM and the BEM.
1.2. Time-steps for each sub-domain are selected: $F\Delta t$ and $B\Delta t$
1.3. Initial prescribed values are chosen to the FEM node forces at the interface surface, for example, $\int_F^t \mathbf{F}_{(0)}^{At} = 0$ .
1.4. Standard initial calculations related to the FEM and BEM are considered, for instance, the calculation of the matrices A, B, etc.
<b>2. Time-step loop:</b>
2.1. Initial time attribution: $Bt = B\Delta t$ and $Ft = 0$ .
2.2. Beginning of the evaluations at each time-step: $Ft = Ft + F\Delta t$
2.3. BEM pre-iterative processing: if $Ft > Bt$ then
2.3.1. $Bt = Bt + B\Delta t$
2.3.2. Evaluation of the BEM previous time influence vector $\mathbf{Q}_B^{At}$
2.4. Iterative loop:
2.4.1. Solve the FE problem. Obtain the displacements at the interface: $\int_F^t \mathbf{U}_{(k+2)}^{At}$ .
2.4.2. Adoption of a relaxation parameter $\alpha$ in order to ensure and/or speed up convergence: $\int_F^t \mathbf{U}_{(k+1)}^{At} = \alpha \int_F^t \mathbf{U}_{(k+2)}^{At} + (1 - \alpha) \int_F^t \mathbf{U}_{(k)}^{At}$ .
2.4.3. Time extrapolation of $\int_F^t \mathbf{U}_{(k+1)}^{At}$ in order to obtain $\int_F^t \mathbf{U}_{(k+1)}^{At}$ . Since the interpolation function $\phi_u^n(t)$ (equation 7) is usually considered as being linear, one has: $\int_B^t \mathbf{U}_{(k+1)}^{At} = (\int_F^t \mathbf{U}_{(k+1)}^{At} - \rho_B^t \mathbf{U}^{At-\Delta t}) / (1 - \beta)$ .
2.4.4. Solve the BE problem. Obtain the tractions at the interface: $\int_B^t \mathbf{P}_{(k+1)}^{At}$ .
2.4.5. Time interpolation of $\int_B^t \mathbf{P}_{(k+1)}^{At}$ in order to obtain $\int_F^t \mathbf{P}_{(k+1)}^{At}$ . Since the interpolation function $\phi_p^n(t)$ (equation 8) is usually considered as being piecewise constant, one has: $\int_F^t \mathbf{P}_{(k+1)}^{At} = \int_B^t \mathbf{P}_{(k+1)}^{At}$ .
2.4.6. From the tractions at the interface, obtain the FEM nodal forces: $\int_F^t \mathbf{F}_{(k+1)}^{At}$ .
2.4.7. Check for convergence (usual nonlinear convergence check procedures, namely residual, displacement and/or energy convergence checks, can be maintained). Go back to 2.4.1 if convergence has not been achieved.
2.5. Actualization (and impression) of results related to the FEM.
2.6. If $Ft + F\Delta t > Bt$ then: actualization (and impression) of results related to the BEM.
2.7. Go back to 2.2 until the final time-step is reached.
<b>3. End of calculation.</b>



In order to carry out the coupled problem numerically we need to first make sure that our numerical models behave well before we couple them. This includes checking the relevant time steps and spatial discretization, which in turn means that we need to assess for example the eigenfrequencies of the two systems. When the models are well-behaving, then we can couple them. In practice both the pre-processing and the solutions require iterations. The coupling should be ideally two-ways, but numerous models utilize only one way coupling to reduce the computational burden.

## Discretization

- Nowadays discretization is not a problem in terms mesh creation, computer does this
- It is a problem when defining accuracy for the analysis, it is affected by the element
  - Type (beam, shell, solid...)
  - Shape (rectangular, triangle,...)
  - Shape functions (linear, parabolic, NURBS...)
  - Implementation (numerics, coding...)
- Convergence must be guaranteed
  - Meshes with different sizes
  - Analytical solutions
  - Physical experiments
- Different authorities/societies have set guidelines for meshing in FEA



Nowadays discretization is not a problem in terms mesh creation as computer does this automatically. It is a problem when defining accuracy for the analysis as it is affected by the element type (beam, shell, solid...), shape (rectangular, triangle,...), formulation and shape functions (linear, parabolic, NURBS...) and the actual implementation to the software (numerical method, coding...). For the designer it is very important that the convergence of the numerical solution is guaranteed. This can be done by considering meshes with different sizes, different elements and implementations, comparison with analytical solutions for simple benchmark cases and by physical experiments of the actual structures in the laboratory or at full-scale. Different authorities/societies have set guidelines for meshing in FEA. It is also good to check some strength and benefits of different meshing strategies, see next slides.

# Discretization

## Element Selection

Table 1. Discretization levels

Level	Accuracy	Comments
3D solid model mapping all structural details	Very high accuracy, detailed strain information available, low discretization error	Fine mesh required (<1mm) to maintain element aspect ratio, very high computational cost (solving time: months), infeasible for large complex structures
3D shell model mapping all structural details	High accuracy, simplified strain information in thickness direction available, reasonable discretization error	Smallest structural member defines the minimum element size (~bulb width), high computational cost (solving time: weeks), feasible for large complex structures
3D shell model with beam elements for structural details	Accuracy depends on the correspondence between the shell and the beam element (DOF) and the level of deformation to be expected, simplified strain information available, potential risk for discretization error	Smallest structural member is modeled with beam elements, moderate computational cost (solving time: days), feasible for large complex structures
3D shells with enriched functionality (homogenized plates, super elements, macro elements)	Accuracy depends on capabilities of the enrichment, local strain information needs to be obtained from the enrichment, correct element orientation is vital	Element size depends on enrichment, typically one stiffened panel is one element, one element between decks, small computational cost (solving time: hours), standard for complex structures



Here is an example of a practical advice on how to select the element type or combination of elements. We can approach the problems always with solids and get very accurate results, but the resulting fine mesh tends to become very large and time-consuming and resource intensive for practical use. Therefore, we tend to use shell elements, shells with beams or homogenized beams, plates and shells to reduce the computational burden with some level of inaccuracy. This is always really a balancing act between accuracy and cost.

# Discretization

## Guidelines, Good Practises

The image consists of two side-by-side panels. The left panel is a white document cover for 'SSC-387 GUIDELINE FOR EVALUATION OF FINITE ELEMENTS AND RESULTS'. It features a large stylized 'SC' logo at the bottom. Below the logo is a small note: 'This document has been approved for public release and may be distributed outside the DoD'.

The right panel is a screenshot of a website for NAFEMS (Computational Structural Mechanics Engineering Analysis and Simulation). The page shows a navigation bar with links like 'home', 'about', 'membership', 'training', 'events', 'learning', 'publications', 'resource', 'regional groups', 'technical groups', 'news', 'partners', 'contact', 'projects', and 'NCC 2012 Conference'. A sub-menu for 'technical groups' is open, showing options such as 'analysis management', 'CAD/CAM integration', 'composites', 'computational fluid dynamics', 'computational structural mechanics', 'contact', 'members', 'mesh generation', 'dynamics and testing', 'surrogate modeling', 'geotechnics', 'high performance computing', 'multi-body dynamics', 'multiphysics', 'stochastics', 'simulation data management', and 'technique briefs group'. The main content area displays a 'Technical Groups' section with a red header and a list of groups, including 'Computational Structure Mechanics Working Group' and 'Chairman'. At the bottom of the page, there is a footer with the text 'website by duotone design | powered by duotone'.

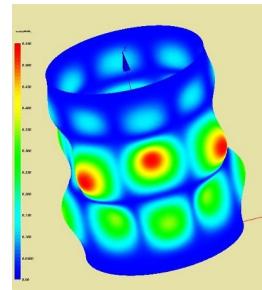
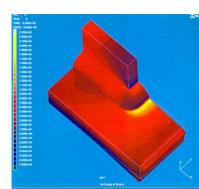
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Here are few examples of guidelines and bets practices on how to get acquainted with the FEA in practice. There are field specific committee such as the one on the left and more broad communities such as the one on the right. Typically it is good to get acquainted with both as while the field-specific really guides you to the deep end of your field, the broad communities may bring you ideas on how to renew the practices in your field.

## Analysis types

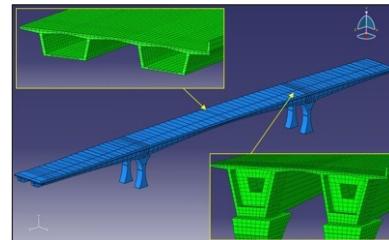
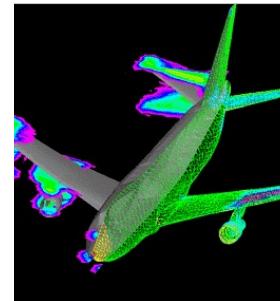
- Static
  - Fatigue simulations (stress concentration, identification of critical areas,...)
  - Stress analysis (yield, max stresses,...)
  - Deformations (max and min deflections,...)
- Dynamic analyses
  - Vibrations (eigenfrequencies and modes),
  - Forced vibrations (machinery induced, wave induced)
  - Impact (collisions, grounding, explosion)
- Collapse
  - Buckling and post buckling
  - Instability (bifurcation)
- Etc



The type of analyses can be split to static, dynamic and their combinations such as collapse. Static analyses typically cover fatigue simulations (stress concentration, identification of critical areas,...), stress analysis (yield, max stresses,...) and analysis of deformations for serviceability (max and min deflections,...). The dynamic analyses can be split to fundamental vibrations (eigenfrequencies and modes), forced vibrations (machinery induced, wave induced) and transient or impact (collisions, grounding, explosion) analyses. Collapse in terms of buckling and post buckling or instability may contain either static or dynamic analysis type depending on the type of non-linearities present in the problem and on the objectives of the analysis. Of course we can expand this spectrum to cover thermal, magnetic and many other types of analyses.

## Building the Models

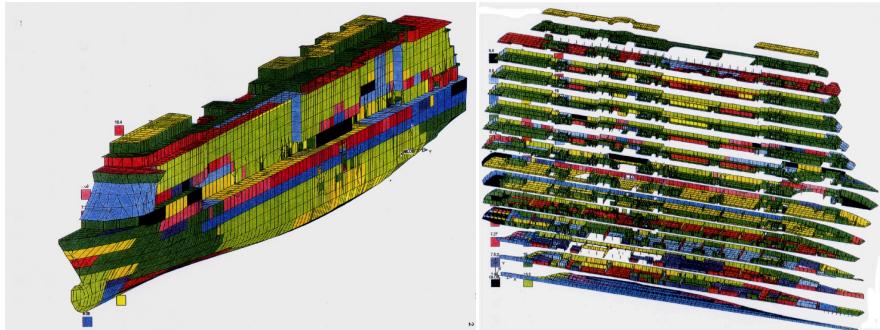
- From scratch
  - Direct modeling with FE pre-processor
  - You know what you get
  - Problem, “we have this small change in plan” ☺
- From CAD tools (NapaSteel, CATIA, ProEngineer,...)
  - Easy to use and effective
  - Easy to handle changes and discuss with other design disciplines (manufacturing, systems design)
  - Interfaces to solvers are not always complete (element types missing)



The next sections present some of the ways on how to build the models. These can be done from scratch with direct modeling with FE pre-processor. Then you know exactly what you get from meshing and modeling, but the downside is the time investment and the fact that in practical design we have to do the modeling many times. Instead, many CAD tools offer automatic options for this which are easy to use and effective, it is easy to handle changes and discuss with other design disciplines (manufacturing, systems design). The downside of this is that there are interfaces to solvers are not always complete (element types missing) and the mesh quality may not be acceptable for different limit states. For example in fatigue assessment, the element type, size and shape, need to follow certain rules at the location of interest.

## Building the Models

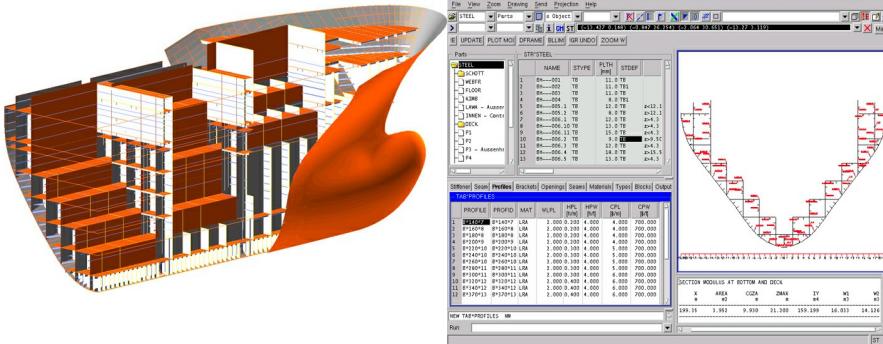
3D Finite Element Model of a cruise ship created by hand due to lot of details and highly curved shapes



This is an example of a complex model which requires a lot of hand work and experience from the analyst. The reason is first of all that we need to know the purpose and analysis type of this model. What we see is a lot of details in the geometry, which we can either neglect or take into account – this is where we need the experience and understanding of the problem. While we know the required accuracy, then we need to select the element size and type in the way that it fits its purpose. Typically building these models takes days or weeks from a experienced person.

## Building the Models

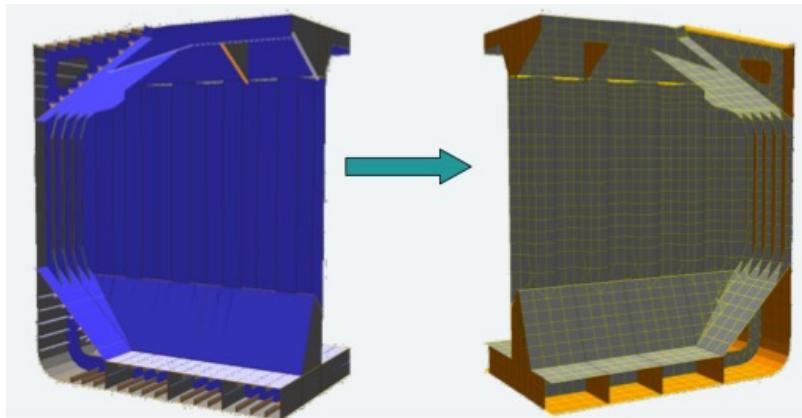
### NAPA-steel hull CAD model to FEA (automatic) with input given in tables and parametric way



The other way is to exploit the CAD model we have from engineering in our hands. There we typically have defined the geometry exactly, we know the materials and scantlings and we can basically export the model directly to FEA.

## Building the Models

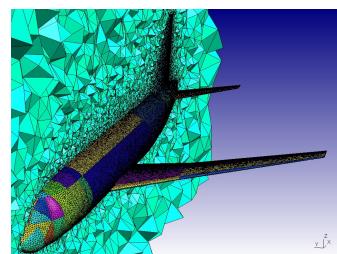
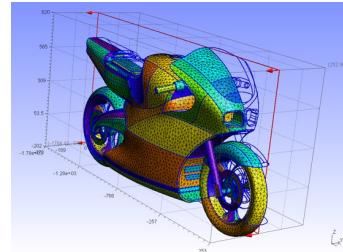
NAPA-steel hull CAD model to FEA (automatic)  
with given mesh parameters (element shape, size)



Of course the same principles can be applied to the sub models of the entire thin-walled structure in a way that you simply cut from the large model a part to be analyzed further and with the FEA export you create the FE model automatically. Here the export typically requires that you define the mesh control options, so that it becomes feasible.

## Summary

- The aim of the lecture was to understand the issues affecting the analysis and the discretization of the thin-walled structures using the Finite Element Analysis
- Thin-walled structure is a lightweight, structure that often has thickness of the shell orders of magnitude smaller than global dimensions
- Analysis, evaluation and decision making coupled to different design philosophies and failure modes, load types and discretization of the numerical model creates a complex and highly-motivating environment for life-long-learning



The aim of the of this session was to understand the issues affecting the analysis and the discretization of the thin-walled structures using the Finite Element Analysis. By definition, thin-walled structure is a lightweight structure that often has thickness of the shell orders of magnitude smaller than global dimensions. Analysis, evaluation and decision making coupled to different design philosophies and failure modes, load types and discretization of the numerical model creates a complex and highly-motivating environment for life-long-learning.