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# *School of Engineering*

MEC-E8007 Thin-Walled Structures
Lecture 7. Equivalent Beams with Offset
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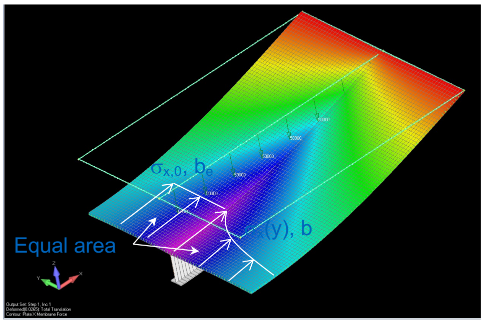
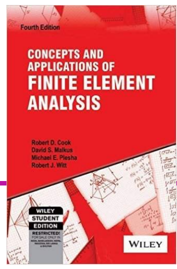
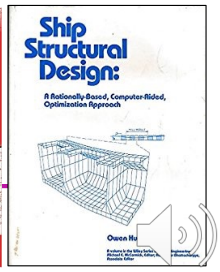
Equivalent beams with offset are one of the mostly used structural parts of thin-walled structures, just after plate and shells. Their mechanics is based on the elementary beam theory, but this is not enough to account the interaction with the plating. This is why we need to revisit the context.

# Contens

- The aim of the lecture is to understand how beam elements can be used to reduce the size of the FE-models and learn to apply beam element modeling to large complex structures
- Motivation
- Contents
  - Effective breadth concept and usage in FEA
  - Effective beam length modeling due to brackets
  - Offset beam modeling
  - Equivalent stiffness properties for beams
  - Examples
- Literature
  1. Hughes, O. And Paik, J.K., ”Ship Structural Analysis and Design - Chapter 6 – ”Frame Analysis”, SNAME, 2010.
  2. Cook, R.D., Malkus, D.S. and Plesha, M.E., “Concepts and Applications of the Finite Element Analysis, 3rd Edition”, John Wiley and Sons, 1989, pp. 220-221.

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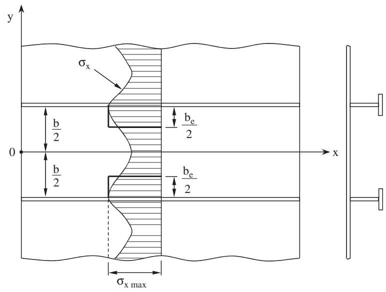
The aim of the lecture is to understand how beam elements can be used to reduce the size of the FE-models and learn to apply beam element modeling to large complex structures. As stated, the interaction between the plate and the beam is important for the analysis of thin-walled structures and therefore, effective breadth concept and usage in FEA is important to revisit, see the book of Hughes for this. For the effective beam length modeling due to use of brackets, that reduce the stresses at beam ends, the same book is good. For the FE-implementation of the the offset, it is recommended to visit the book of Cook, Malkus and Plesha.

# Motivation

- Many large & complex structures are large frameworks made from beams that include both
  - In-plane behaviour
  - Out-of plane behaviour
- Often these include attached plating which bends together with the beam
  - Adds stiffness to the beam
  - Takes stresses due to beam bending
- It is not feasible to model beam web and flange with shell or solid elements – high computational cost
- The size reduction can be done by
  - Beam analysis only: include the plate through effective breadth to beam element stiffness, small models, in-accuracy from plate response modelling
  - Offset beams: slightly larger models, inaccuracy from web and flange modelling

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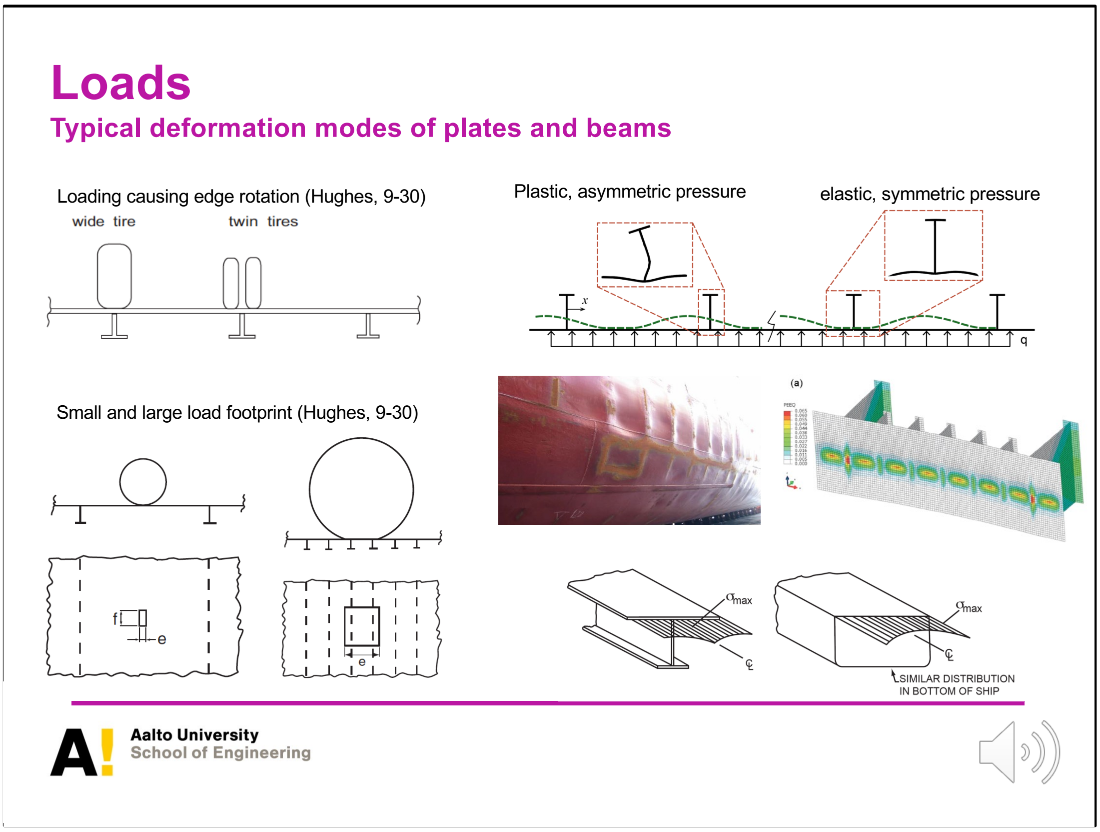
Many large and complex structures are large frameworks made from beams that include both in-plane and out of plane behaviors. This means they are compressed, bended and twisted. Often these include attached plating which bends together with the beam and the interaction must be considered correctly as it adds stiffness to the beam, takes stresses due to beam bending and also changes the stress distribution. So for thin-walled structures there are beneficial effects that are however a bit more complex to account for than the basic beam theory allows us to do. In practice it is not feasible to model beam web and flange with shell or solid elements, this leads in practice to high computational costs, can lead to bad accuracy due to distorted elements (aspect ratio) in thin-walled structures and requires also considerable amount of additional pre- and post-processing time. The size reduction can be done by much more effectively by extending the beam analysis only; include the plate through effective breadth to beam element stiffness, small models, in-accuracy from plate response modelling; or by using offset beams: slightly larger models, inaccuracy from web and flange modelling.



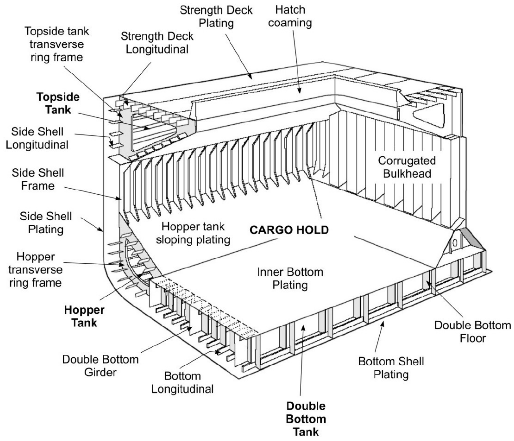

When beam is attached to the plating, the load is often carried out by the plating that the system of beams is supporting. When considering this, we can have the pure bending of beams in cases where the loading is symmetric with respect to beam axis or combined bending and torque when it is unsymmetric with respect to the beam axis.

Another fact is that the wide flange to the offset beam is not fully efective due to shear lag phenomenom (analogy to the fibre ends in composite fiber-matrix interaction). This is why we cannot assume that the full stiffener spacing can be used to calculate the section modulus of the beam. We need to compute the correct width of the beam somehow.

# Beam or Stiffener Bending

- Here first beam bending differential equation is derived, steps
  1. Kinematics, i.e. displacement relations
  2. Relation between displacement and strain
  3. Constituve equations, i.e. stress vs. strain
  4. Integration of stress resultants, N, M, Q
  5. Equilibrium equations, $M_{ext}=M_{int}$

- The same steps apply to
  – beams and plates
  – with and without shear deformation
  – with and without non-linearity

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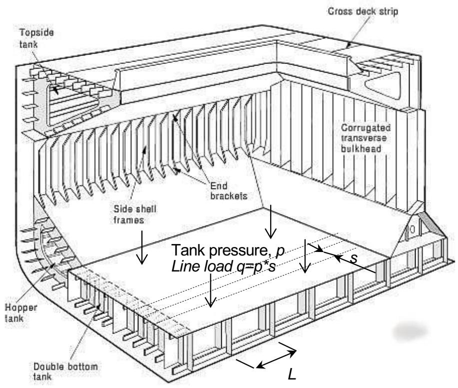
As done before with plates, we need to think about the 5 step process to derive the beam differential equation. That is define the kinematics, define the strains, use the constitutive equations to obtain the stresses, integrate through the thickness to obtain the stress resultant and check the equilibrium with the external loading.

# Beam or Stiffener Bending

## Idealization of the Structure

- Thin-walled structure
  - Stiffeners are spaced at *s* and have certain cross-section with second moment of area $I_{stiff}$
  - The pressure *p* acts on the plating supported by the stiffeners
  - Each stiffener carry a lineload which can be approximated as $q=p*s$
  - Web frames support the stiffeners, thus the span is *L*
- When stiffener bends
  - Tensile and compressive stresses are induced which vary linearly across the stiffener height away from the supports – **beam theory captures only these linear stresses**
  - Plate bends with it and contributes to the second moment of area $I=I_{plate}+I_{stiff}+I_{steiner}$

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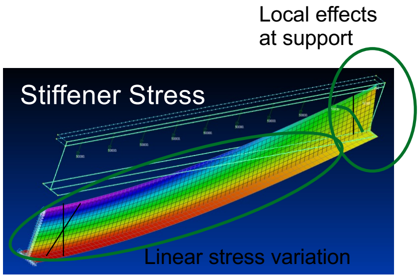
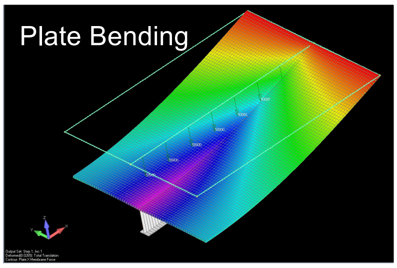
In thin-walled structure, locally the stiffeners are spaced at distance, *s*, and have certain cross-section with second moment of area $I_{stiff}$. The pressure, *p*, acts on the plating supported by the stiffeners and each stiffener carry a line load which can be approximated as q=p*s. Web frames support the stiffeners, thus the span is L, which is the same as the web-frame spacing.

When stiffener bends both tensile and compressive stresses are induced which vary linearly across the stiffener height away from the supports – beam theory captures only these linear stresses and locally at the supports, the effects are not possible to obtain and this is why sometimes effective length is needed. More importantly the part of the plate bends with the stiffener and contributes to the second moment of area of the combined plate-stiffener system, that is $I=I_{plate}+I_{stiff}+I_{steiner}$. However, which part of the plate is effective depends on various aspects and this is why we need to revisit the beam theory assumptions in this context again.

# Beam or Stiffener Bending
## Idealization of the Structure

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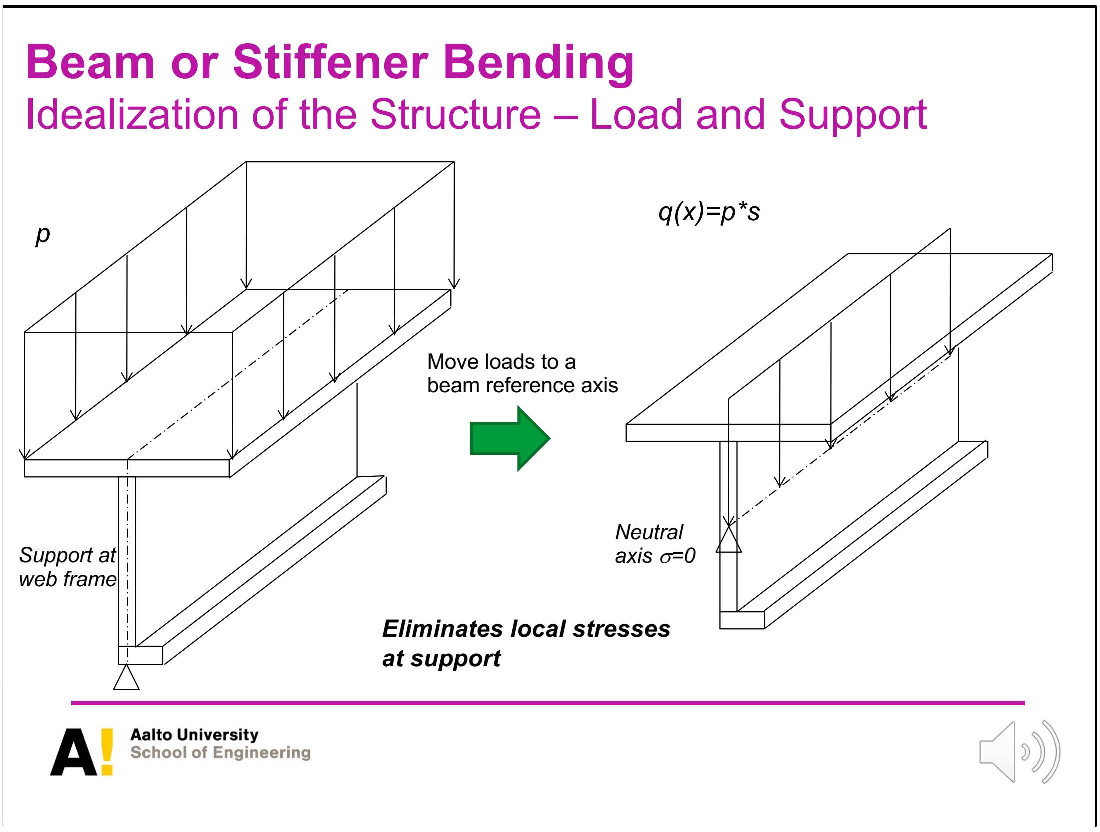
This is to highlight these two phenomena better. We have constructed two finite element models with shell elements, in first case a T-beam and in the second T-beam with the plate. Thus, the first case demonstrates pure beam bending with higher order modeling based on plane stress elements, while second models the interaction. As we can see from the figure on the left, the stiffener stresses are linear at the mid-span of bended beam through the thickness, but at the support due to the fact that support is not at the neutral axis the stress distribution is affected. We should stop here to think, why not in the neutral axis? In practice it would be very difficult to support the beam from there. It is easier to support it from top or bottom end. In the second example we see that the stress is not uniform in the attached plating but decays from the interface between the plating and the stiffener towards the plate free edges. This is a fact due to the plane stress boundary condition at the free edge, where the shear stresses must vanish. So there must be decay of shear. This is called shear lag.



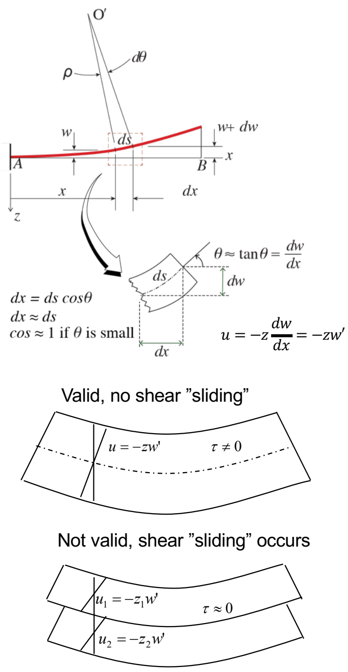

So with these in mind we can back to the beam theory. Beam theory is essential a 1D equivalent single layer or line theory (ESL). This means that the loads, boundary conditions and displacements are defined at this ESL. So even when the pressure acts on the plating. We need to reduce it first to the line on the plate and then downwards to the ESL. ESL can be here the neutral axis or any other reference plane that we select. The same goes for boundary conditions, we need to move them to the ESL. Due to this movement in normal stress in the thickness direction of the beam vanishes and the linearity in stress distribution through the thickness becomes a reality also at the support, where we saw that the stress distribution is non-linear. So we question here the assumptions of the kinematics in the sense that we say that w=w(x) and there is not any local deformations present.

# Beam or Stiffener Bending

## Kinematics

- Notation
  - Deflection, i.e. z-displacement *w*,
  - In-plane, i.e. x-displacement *u*,

- Linear change of *u* in thickness direction – valid when shear deformation is not important
  - Holds for stiffener and plate bending when steel or aluminum is used, L/h>10
  - Not valid for extremely wide flanges, e.g. web-frames
  - Typically not valid for composites and sandwich structures

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So the kinematics are now as they were in Kirchhoff plate theory:

1) Straight lines perpendicular to mid surface remain straight after deformation
2) The transverse normal do not experience deformation
3) Transverse normal remain perpendicular to mid surface

Linear change of u in thickness direction – valid when shear deformation is not important. So this holds, when stiffener and plate bending when steel or aluminum is used, L/h>10. It is not valid for extremely wide flanges, e.g. web-frames where due to shear lag in plating we have also variation of the stresses in width direction. This is not valid for composites and sandwich structures which have the high shear deformation due to low shear stiffness.

# Beam or Stiffener Bending

## Strain definitions and Stresses

Strain from displacement:

$$u = -z \cdot \frac{dw}{dx} \Rightarrow \varepsilon_x = \frac{du}{dx} = -z \cdot \frac{d^2 w}{dx^2}$$

Hooke's law and uniaxial stress state:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y + \sigma_z}{E} \qquad \varepsilon_x = \frac{\sigma_x}{E}$$

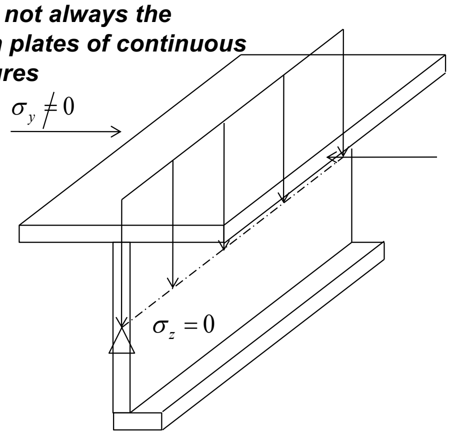
$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x + \sigma_z}{E} \longrightarrow \varepsilon_y = -\nu \frac{\sigma_x}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu \frac{\sigma_x + \sigma_y}{E} \qquad \varepsilon_z = -\nu \frac{\sigma_x}{E}$$

NOTE! Change of breadth or height is not
Considered in the following, i.e. problem is
Linearized based on initial state

$$\sigma_x = E\varepsilon_x = -Ez \frac{d^2 w}{dw^2}$$

***This is not always the case in plates of continuous structures***

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So when we know these assumptions, we can use the constitutive model and strain definition the stress state. What we should note here is the fact that in continuous structures, such as plates, due to continuity, the plate cannot shrink and the stress normal to the beam axis is not zero. So we need to correct the Hooke's law in the way that we allow this stress component to occur. In practice this can be done by increasing the Young's modulus of the plate at maximum to that of plane stress, that is $E^*=E/(1-v^2)$ which in most metals would lead to 9% increase.

# Equilibrium

## Internal (response of the beam)

Normal force and moment is obtained from integration over thickness:

$$N_x = \int_A \sigma_x dA = -\int_h Ebz \frac{d^2w}{dx^2} dz$$

$$M_x = \int_A \sigma_x z dA = -\int_h Ebz^2 \frac{d^2w}{dx^2} dz$$

Constant curvature and Young's modulus in beam gives:

$$N_x = -E \frac{d^2w}{dx^2} \int_h bzdz$$

$$M_x = -E \frac{d^2w}{dx^2} \int_h bz^2 dz$$

Neutral axis, i.e. location of zero stress $\sigma_x$

$$N_x = -E \frac{d^2w}{dx^2} \int_e^{h-e} bzdz = 0 \Rightarrow \int_e^{h-e} bzdz = 0$$

$$M_x = -E \frac{d^2w}{dx^2} \underbrace{\int_e^{h-e} bz^2 dz}_{=I} = -EI \frac{d^2w}{dx^2}$$

## External (load on the beam)

Starting point is the beam element equilibrium equations

$$\uparrow -qdx + Q - (Q + dQ) = 0$$

$$\lrcorner M + qdx\frac{dx}{2} + (Q + dQ)dx - (M + dM) = 0$$

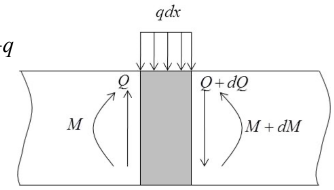
Simplifying these give
(NOTE! dx*dy is very small)

$$\uparrow -qdx = dQ \Leftrightarrow -q = \frac{dQ}{dx}$$

$$\lrcorner - \underbrace{qdx\frac{dx}{2}}_{\approx 0} + Qdx + \underbrace{dQdx}_{\approx 0} - dM = 0 \Leftrightarrow Q = \frac{dM}{dx}$$

Thus, we obtain the well-known relation

$$\frac{d^2M}{dx^2} = \frac{dQ}{dx} = -q$$

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After this we can perfrom the normal beam theory formulations, i.e. do the ingeration of stress resultants and balancing them with the external loading.

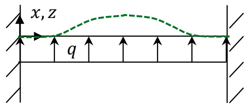
# Equilibrium
## Differential Equation

External equilibrium $\frac{d^2M}{dx^2} = \frac{dQ}{dx} = -q$

Internal equilibrium $M_x = -EI\frac{d^2w}{dx^2}$

$$\boxed{\frac{d^2M}{dx^2} = -EI\frac{d^4w}{dx^2} = -q}$$

Solution for clamped beam with constant line load




$$w(x) = \frac{qL^4}{24EI}\left(\frac{x^2}{L^2} - 2\frac{x^3}{L^3} + \frac{x^4}{L^4}\right)$$

$$M(x) = -EI\frac{d^2w}{dx^2} = -\frac{qL^2}{2}\left(\frac{1}{6} - \frac{x}{L} + \frac{x^2}{L^2}\right)$$

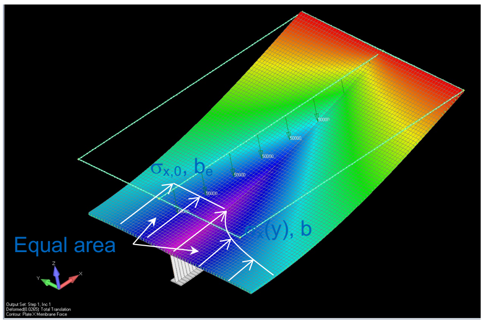


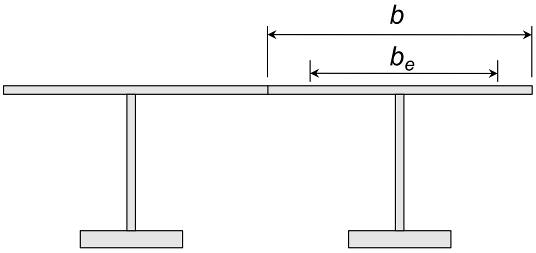
When this is done, we end up in the differential equation of the beam which we can solve for example analytically or with the FEA. So basically, we went through the same stuff as in elementary solid mechanics courses. However, we also showed some of the assumptions which are violated when stiffener and plate are interacting.

# Effective Breadth Concept

- The deck plating is not full effective when the spacing of web frames or girders is large
  - Caused by shear lag
  - Should not be mixed with *effective width* due to buckling of the plate
- Effective breadth can be used to evaluate the flange for the girder
  - Stress at the intersection between beam and plate
  - Equal areas for real and idealized stress distributions

$$b_e\sigma_{x,0} = 2\int_0^b \sigma_x dy \Leftrightarrow b_e = \frac{2\int_0^b \sigma_x dy}{\sigma_{x,0}}$$



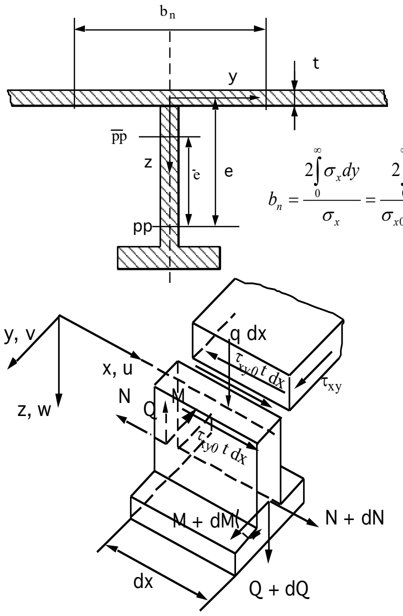






The plating is not full effective when the spacing of web frames or girders is large, and as stated before this is due to the shear lag. The effective breadth concept should not be mixed with **effective width** which is due to buckling of the plate. Same ideas are there, but there is different physical reasons and formulations. **Effective breadth** can be used to evaluate the flange for the girder and it means that we must model the stress at the intersection between beam and plate and how it decays properly. We set the stress at the intersection as the acting stress and try to find a breadth that produces the same net force than the decaying distribution produces for the flange.

# Effective Breadth

- We consider a membrane-type of plate (bending stiffness neglible) attached to the beam (e.g. T-beam)
  - Beam carries the vertical load *q*
  - Only membrane stresses at the plate are considered
- When this assembly bends, the bending stiffness is
  - Larger than that of beam alone
  - Smaller than that considering the full web frame spacing
- The combined effect is called effective breadth and it must be accounted in the analysis since is affects
  - The deflection
  - The normal and shear stress
- Effective breadth varies along beam axis but average value can be derived for design purposes
  - Distance between zero moments (DNV)
  - Regression (Paik, 2008)




$$b_n = \frac{2\int_0^\infty \sigma_x dy}{\sigma_x} = \frac{2\int_0^\infty \sigma_x dy}{\sigma_{x0} - \nu\sigma_{y0}}$$

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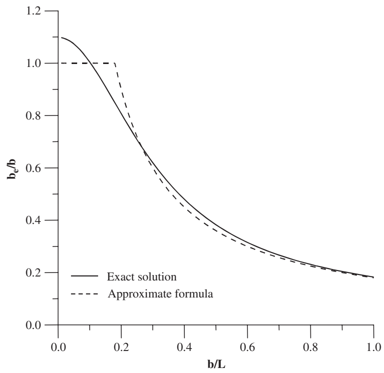
In the solution of this we a plane stress problem and a membrane-type of plate (bending stiffness negligible) attached to the beam (e.g. T-beam). We state that the beam carries the vertical load q and only membrane stresses at the plate are considered. When these two are attached the stress must be equal at the interface. When this assembly bends, the bending stiffness is larger than that of beam alone and smaller than that considering the full web frame spacing. So the true value is somewhere between these two extremes. The combined effect is called effective breadth and it must be accounted in the analysis since is affects the stiffness of the system but also the stress state in beneficial way. The solution can be derived by plane stress analysis or by using simplified methods. Schade has analytically derived the plane stress solutions for typical examples. One very useful approach is presented by DNV where we take the analytical beam solution relevant for our case and define the distance between zero moments from there. Another approach is the regression by Paik (2008).

# Effective Breadth

Paik (2008)

$$b_n = \frac{2\int_0^\infty \sigma_x dy}{\sigma_x} = \frac{2\int_0^\infty \sigma_x dy}{\sigma_{x0} - \nu\sigma_{y0}}$$

$$\frac{b_e}{b} = \begin{cases} 1.0 & \text{for } \frac{b}{\omega} \leqslant 0.18, \\ \frac{0.18}{(b/L)} & \text{for } \frac{b}{\omega} > 0.18. \end{cases}$$

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What we do here is check the breadth to length ratio of the beam plate assembly and then select the $b_e/b$-ratio from the graph. This graph is a result of regression analysis of numerous plane stress FE-analyses.

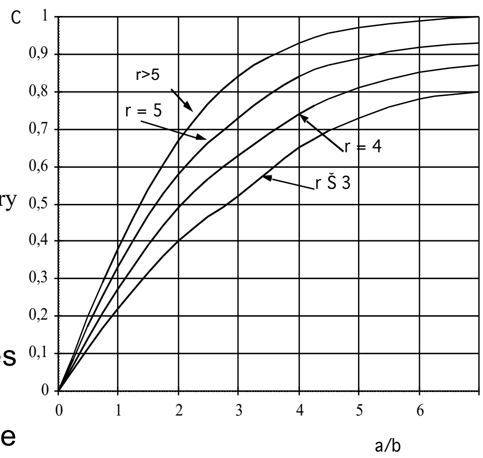
# Effective Breadth
## For example DNV

- The effective breadth is evaluated with use of:

  $b_e = C\ b.$

- where
  - ***b*** is web frame spacing,
  - ***C*** factor that depends on load and boundary conditions

- *C* is defined with,
  - ***a*** distance between zero values at bending moment diagram
  - ***r*** is number of point loads in the web frame

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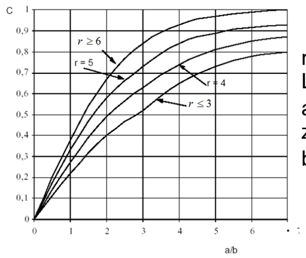
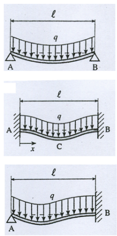
The alternative approach is from DNV where you need to derive from beam theory solution the distance between zero bending moments, a, and also the number of point loads in the beam. If the beam has uniform load, it has infinite points loads and we use the top most curve. So based on the a/b-ratio and number of point loads, we can select the C-factor that we use to multiply the stiffener spacing.

# Effective Breadth
## Boundary Conditions and Loading




r = Number of Point Loads Across Span
a = distance between zero bending moment
b = web frame spacing

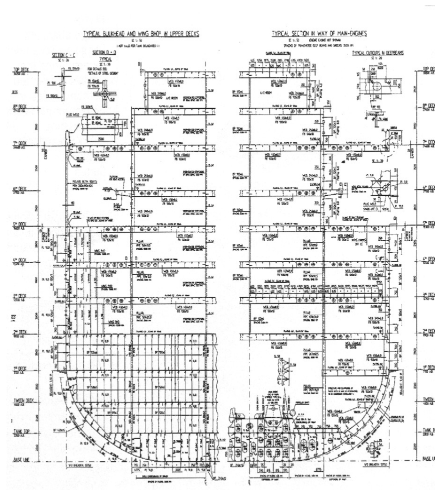


$$M = \frac{qlx}{2}\left(1 - \frac{x}{l}\right)$$

$$M = -\frac{ql^2}{2}\left(\frac{1}{6} - \frac{x}{l} + \frac{x^2}{l^2}\right)$$

$$M = \frac{qlx}{2}\left(\frac{3}{4} - \frac{x}{l}\right)$$

Appropriate BC's based on symmetry of loading and pillars



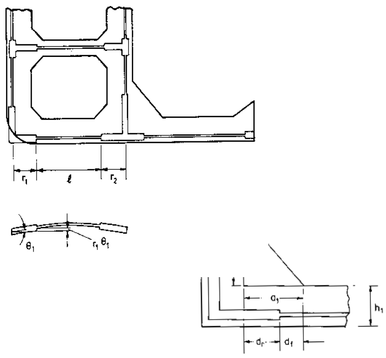


This is about the use of bending moment diagrams of beam theory to compute the distance between zero bending moments. So first we idealize our real structure to these basic cases. We must think that is the loading continuous over the supports so that we can assume symmetry at the supports and clamped-clamped case or something else, i.e. simply supported or simply supported and clamped. When we have this defined, we must solve the roots of the polynomial describing the bending moment distribution. The distance of the roots is the distance of zero bending moments. The we use the graph to compute the C-value.

# Equivalent Beams
## Rigid Ends

- When the web frame dimensions are such that at the nodal region stiffness is very high, beam element with infinite rigidity is used
  - End lengths $r_1$ and $r_2$,
  - For example area of brackets,
- In local coordinate system the stiffness is $[k] = [S]^T [k^*] [S]$ where $[k^*]$ is obtained by subsituting real lenght $L$ with elastic length $l$.
- The displacements are obtained by multiplication with:

$$[S]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & r_1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -r_2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

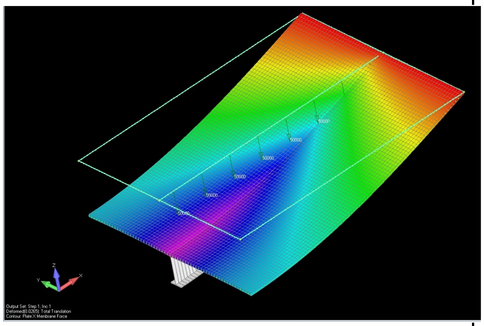


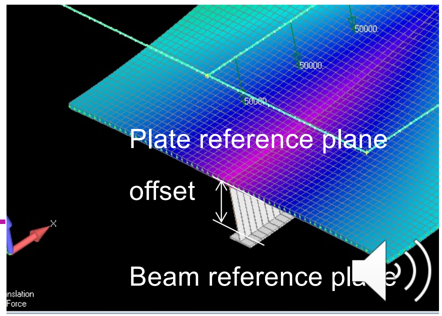


In thin-walled structures the web frame dimensions are such that at the nodal region stiffness is very high and the beam length tends to be longer in our models than it really should be. This results in too high deflections. To account this effect, a beam element with infinite rigidity is used at the locations where we have stress reducing brackets etc. Mathematically the end lengths $r_1$ and $r_2$ can be used to describe the slightly different continuity and geometry conditions we have at these locations.

# Offset Beam Element Modelling

- Sometimes the plate-beam interaction is so complex that both plate and beam need to be modeled
- Since the stiffener is one-sided the neutral axis of the stiffener is far from the plate mid-surface
  - When stiffener stiffness are calculated the profile alone is considered
  - The mathematical reference plane is at different location as the plate reference plane
  - Beam offset is used to “map” the stiffener response in the true location
- Be careful there, due to large offset the error becomes significant in this type of modelling (proportional to *offset*$^2$)



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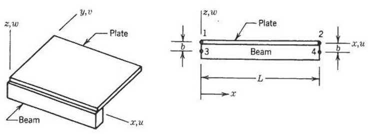
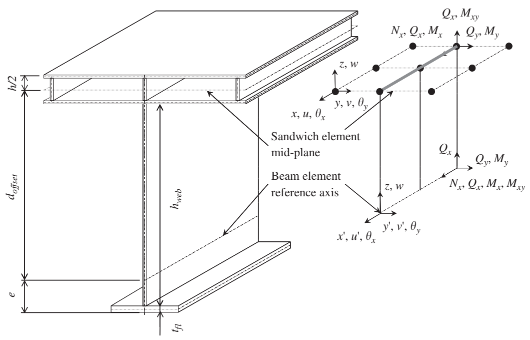
Sometimes the plate-beam interaction is so complex that both plate and beam need to be modeled. This means that the effective length and breadth definitions take more time than the more accurate the modeling and instead we go with offset beams. Since the stiffener is one-sided the neutral axis of the stiffener is far from the plate mid-surface. When stiffener stiffness are calculated the profile alone is considered instead of the entire system like it was done with effective breadth. The mathematical reference plane of the beam is at different location as the plate reference plane meaning that these should be properly attached. Beam offset is used to “map” the stiffener response in the true and correct location. Be careful there, due to large offset the error becomes significant in this type of modelling (proportional to offset$^2$). The recommendation is that the offset is computed, not set visually, as the error of offsetting increases to the second power of the offset.

# Offset Beam Element Modelling

$$\begin{Bmatrix} u_3 \\ w_3 \\ \theta_3 \end{Bmatrix} = [\mathbf{T}_\ell] \begin{Bmatrix} u_1 \\ w_1 \\ \theta_1 \end{Bmatrix}, \quad \text{where} \quad [\mathbf{T}_\ell] = \begin{bmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \{\mathbf{r}\} &= [\mathbf{T}]^T\{\mathbf{r}'\} \\ [\mathbf{k}] &= [\mathbf{T}]^T[\mathbf{k}'][\mathbf{T}] \end{aligned} \quad \text{where} \quad \underset{6\times 6}{[\mathbf{T}]} = \begin{bmatrix} \mathbf{T}_\ell & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_\ell \end{bmatrix}$$

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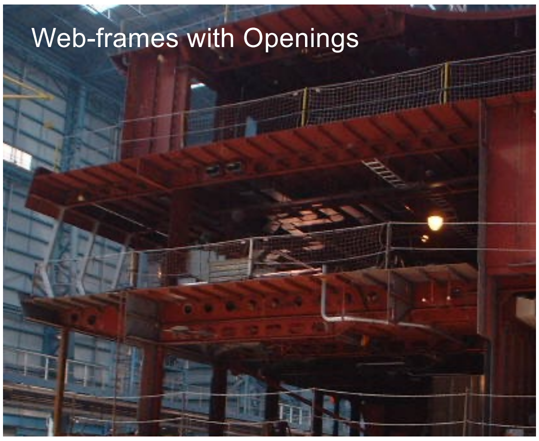
So in practice this is done mathematically by multiplying the true stiffness matrix of the beam from left and right with the transpose of offset matrix and offset matrix itself. For displacements left hand side multiplication is enough. In modeling of the geometry this is done by selecting the reference planes of the beam and plate and by thinking that where is the stiffener starting from with respect to the plate. Often it starts from the lower surface of the plate, yet we model the geometrical mid-planes. Thus make sure that the offset distance accounts half of the plate thickness.

# Equivalent Stiffness Properties for the Beams

- Beam section properties calculated as in basic strength of materials ($A$, $A_{shear}$, $I$, $J$,…)
  - Consider all directions
  - Check that the beam is assembled in right way (normal plot, section plot)
- Often the beams have openings due to system integration
- This reduces the shear stiffness of the beams
  - Shear-deflection might become important
  - Shear deformation and stiffness is important as the L/h-ratio decreases (<10 is critical)
- There are many ways to define the shear stiffness for web with openings
  - Analytical equations for predefined opening shapes
  - FE-modeling based on sub-analyses

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Beam section properties calculated as in basic strength of materials (A, $A_{shear}$, I, J,…) and we should consider all deformations and directions as we often have unsymmetric loading causing torsion, the beams are also loaded in-plane and so on in large structures. Also note that if we have openings in beams, then the shear area is reduced which affects the shear stiffness of the beams. Often the beams have openings due to system integration and these are the places where shear stiffness of the beam gets the reduction. Shear-deflection might become important in some applications due to this and the L/h-ratio may not tell the full truth (L/h<10 is critical). There are many ways to define the shear stiffness for web with openings, including analytical or FE-based with sub-analyses of the webs with openings.

# Examples

## Web-Frame Analysis of a Deck – Beams & Shells

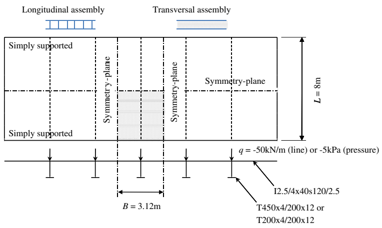


Fig. 3. Case description.

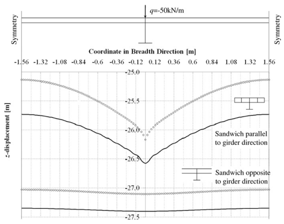


**Fig. 4.** Comparison of the deflection at $x=L/2$. Dots denote the equivalent modeling and solid line 3D-FEM. $h_{web}=450$ mm.

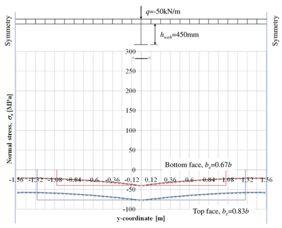


**Fig. 5.** Comparison of the x-direction normal (membrane) stresses at $x=L/2$ and mid-plane of plate elements. Dots denote the present method and solid line 3D-FEM. Sandwich panel parallel to girder direction and girder web height is $h_{web}=450$ mm.

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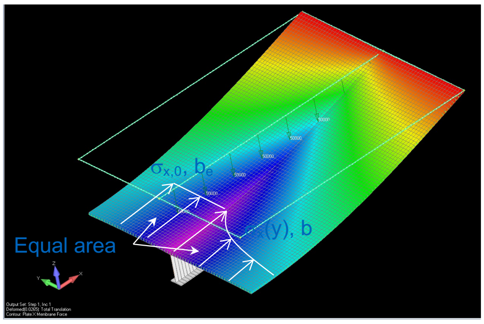
Here we have an example of a grillage of properly modelled offset. As you can see the 3D-shell and ESL-plate-beam-model are in perfect agreement when offset-neutral axis is set to the bottom of the plate. The stresses are exact in both longitudinal and transverse directions. We also see that the effective breadth would be different for the top and bottom surfaces of the hollow plate as they are at different hights (linear stress distribution, z-coordinate dependency).

## Summary

- Beam elements can be used to reduce the size of the FE-models significantly
- Depending on the level of model reduction one can model the entire plate stiffener assembly with beam elements or use offset beams
- Openings within the webs can be modelled by equivalent stiffness properties

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