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MEC-E8007 Thin-Walled Structures
Lecture 11. Buckling and Ultimate Strength Analysis
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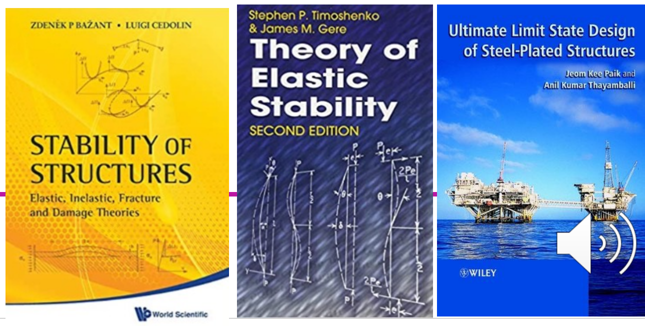
In thin-walled structures the buckling strength precides the ultimate strength. This is why we treat the issues together. Assessing these if of fundamental importance when we think about the allowable stress levels within the structures.

Contents

- The aim is to understand the methods to assess the buckling and ultimate strength of large complex structures using FEM
- Motivation
- Definition of the buckling and ultimate strength
- Global collapse
- Load-end-shortening curves for structures
- Example: Ultimate strength of hull girder
- Fracture
- Literature

1. Timoshenko, S.P. and Gere, J.M. The theory of Elastic Stability, Dover
2. Bazant, Z. and Cedolin, L., “Stability of Structures”, World Scientific
3. Paik, J.K. and Thayamballi, A.K., ”Ultimate Limit State Design of Steel-Plated Structures”, Wiley



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The aim is to understand the methods to assess the buckling and ultimate strength of thin-walled, large and complex structures using FEM. For this we need to define of the buckling and ultimate strength and look at things both in local and global length-scales and understand the nature of failure. This is why we also discuss about the load-end-shortening curves for structures which are analogous to the stress strain curves of materials. The structure looses its integrity when fracture occurs.

Motivation
Case Erika

- The Erika, which broke up off the coast of France causing an ecological disaster, was an accident waiting to happen
- The Erika was one of a batch of eight sister ships built with successive yard numbers from 283-290 at Kasado Dockyard in Japan 1974-1976.
- They were built to minimum scantlings and the workmanship was poor, according to a source who has inspected them all at different times.
- The lightweights are 10-15% less than in many of similar ships. Three of these vessels have suffered substantial structural failure that if circumstances had been a little different, would have resulted in an Erika-style disaster.
- You would think that three sisters out of eight would have been enough of a wake-up call but shipping doesn't work like that.
- Ultimate strength analysis gives the design margin, i.e. Difference between design point and total collapse of structure (1.15-1.5)

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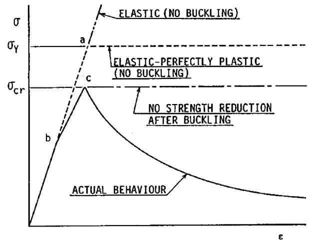
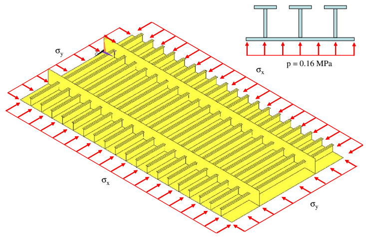
The cargo ship Erika, which broke up off the coast of France causing an ecological disaster, was an accident waiting to happen. The Erika was one of a batch of eight sister ships built with successive yard numbers from 283-290 at Kasado Dockyard in Japan 1974-1976. They were built to minimum scantlings based on existing structural design rules and the workmanship was poor, according to a source who has inspected them all at different times. The lightweights are 10-15% less than in many of similar ships which of course means that the payload is significantly higher than in these competitive designs. Three of these vessels have suffered substantial structural failure that if circumstances had been a little different, would have resulted in an Erika-style disaster. You would think that three sisters out of eight would have been enough of a wake-up call but shipping doesn't work like that. Payload matters. Ultimate strength analysis gives the design margin, i.e. difference between design point and total collapse of structure, the ratio is typically 1.15-1.5.

Definition of Buckling and Ultimate Strength

- All structures have maximum load that they can carry, this is called *ultimate strength*
- As this point is approached typically the parts of the structure start to fail
  - Buckling of plate
  - Local yielding
  - Rupture
- ***Buckling*** is the point where structure suddenly looses it's stability (from in-plane to out-of-plane), geometrical non-linearity
- ***Yielding*** is when material behavior becomes non-linear
- ***Rupture*** is typically combination of material and geometrical non-linearity





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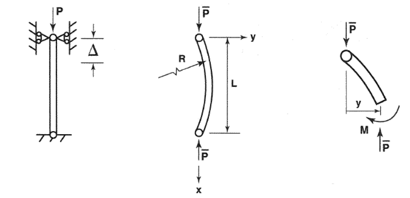
All structures have maximum load that they can carry, this is called ultimate strength (capacity). As this point is approached typically the parts of the structure start to fail in the sequence of buckling, local yielding, gross-yielding and rupture; in healthy design there is significant capacity left when one of these stages is passed. That is in stiffened panel, the plate buckles before the supporting stiffeners buckle and the stiffeners buckle, before the supporting web frames buckle. Buckling is the point where structure suddenly looses it's stability (from in-plane to out-of-plane) and often the stages around this point experience significant geometrical non-linearity. Both before and after. Yielding is when material behavior becomes non-linear and rupture/fracture is typically combination of material and geometrical non-linearity and the final stages of the load-carrying, defining the capacity and ultimate strength.

Derivation of buckling equation for a column

- We start with beam equation and perturbate it a bit with with moment are of axial load

$$EI\frac{d^2y}{dx^2} = M = F \cdot y \;\rightarrow\; \frac{d^2y}{dx^2} = \frac{F}{EI} \cdot y$$



a Actual Support b Idealized Support c Free-Body Diagram

- which gives

$$y'' - k^2 \cdot y = 0 \qquad k^2 = \frac{F}{EI}$$

- The general solution to this differential equation is of the form

$$y = A\,sin\,kx + B\,cos\,kx$$

- We can find the coefficients from the boundary conditions

$$y(0) = 0 \;\rightarrow\; B = 0; \quad y(L) = 0 \;\rightarrow\; A\,sin\,kL = 0$$

- so either $A = 0$ or $sin\,kL = 0$

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Simple example of the buckling is the eigen solution of Euler buckling of a column. The bifurcation point is defined by looking the the balance between the internal stress resultant moment and the external axial load offset by the slightly deflected beam whic creates the external moment. These must balance and we look at the load in which the balance is found. The solution for the differential equation can be found from many text books of solid mechanics and structural stability.

Derivation of buckling equation

- If A=0 then there is no amplitude and no buckling, which leaves

$$sin\, kL = 0 \;\rightarrow\; kL = n\pi \;\; for\, n = 1,2, \ldots$$

- When we return the substitution

$$\frac{F}{EI}L^2 = n^2\pi^2 \;\;\rightarrow\;\; F = \frac{n^2\pi^2 EI}{L^2}$$

- Thus the buckling load depends on the buckling shape, but the lowest buckling load will be for *n* = 1, which gives Euler column equation

$$F_E = \frac{\pi^2 EI}{L^2}$$

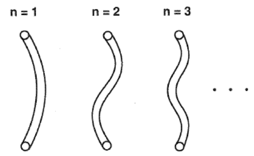

Figure 2-3 Buckling Mode Shapes

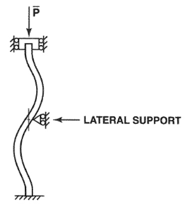

Figure 2-20 Buckling of an Axially Loaded Bar with Lateral Support

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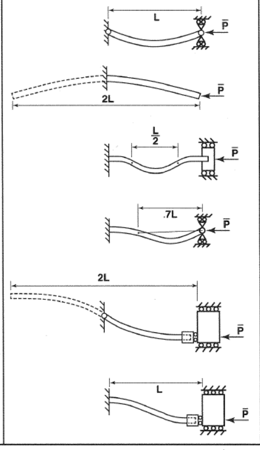
The predicted buckling load depends on the buckling shape, but the lowest buckling load will be for n = 1, which gives Euler column equation.

Influence of boundary conditions

- Assumption previously was that deformation will be 'one half-wave' sinusoidal shape between the supports
- If there is some rotational restraint at the support, the shape will be different (and consequentially the buckling load)
- This is conveniently resolved by determining the 'effective length' of the column, measuring again the span of 'one half-wave'

$$F_E = \frac{\pi^2 EI}{L_e^2}$$

Boundary Conditions*	Buckling Load	Buckled Shape and Relation to that of a Pinned-Pinned Bar	Effective Length
Pinned-Pinned	$\pi^2 \frac{EI}{L^2}$		L
Fixed-Free	$\frac{\pi^2}{4} \frac{EI}{L^2}$		2L
Fixed-Fixed	$4\pi^2 \frac{EI}{L^2}$		$\frac{L}{2}$
Fixed-Pinned	$2.04\pi^2 \frac{EI}{L^2}$		.7L
Pinned-Guided	$\frac{\pi^2}{4} \frac{EI}{L^2}$		2L
Fixed-Guided	$\pi^2 \frac{EI}{L^2}$		L



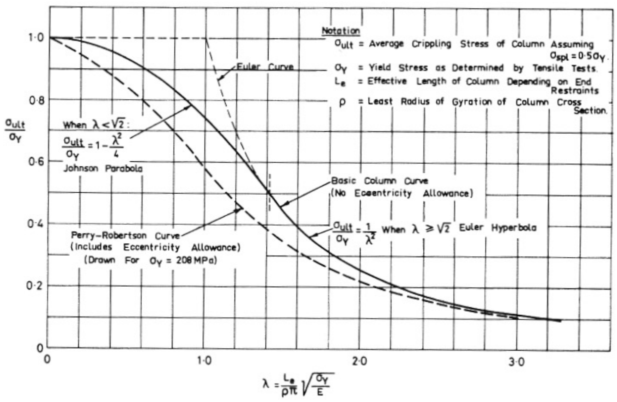
* 'Guided' has the special definition in Sections 2.3.6 and 2.3.7 (y' = 0, but y ≠ 0).

Jones 2006

The boundary conditions affect the Euler buckling load. Assumption previously was that deformation will be 'one half-wave' sinusoidal shape between the supports. If there is some rotational restraint at the support, the shape will be different (and consequentially the buckling load). This is conveniently resolved by determining the 'effective length' of the column, measuring again the span of 'one half-wave'

Validity of Euler buckling formula

- Experiments have shown that Euler´s equation gives over-estimation of column/plate buckling strength when it goes over $\sigma_y/2$
- The parabola equation was taken as the lower bound of the experimental values



Jonhson's parabola, valid when $\sigma_{el} > \sigma_y/2$

$$\sigma_c = \sigma_y\left(1 - \frac{\lambda^2}{4}\right) = \sigma_y\left(1 - \frac{\sigma_y}{4\sigma_{el}}\right)$$

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Experiments have shown that Euler´s equation gives over-estimation of column/plate buckling strength when it goes over half of material yield strength. This is as the plasticity of the section starts to cut the predicted linear stress over the beam cross-section (kinematics). The Johnson parabola equation was taken as the lower bound of the experimental values observed.

Classification Society Approach
DNV, Pt. 3. Ch.1, Sec. 13, C100

C. Stiffeners and Pillars

C 100 General

101 Methods for calculating the critical buckling stress for the various buckling modes of axially compressed stiffeners and pillars are given below. Formulae for the ideal elastic buckling stress σ_{el} are given. From this stress the critical buckling stress σ_c may be determined as follows:

$$\sigma_c = \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

compressive stresses, supporting bulkhead stiffeners, pillars, cross ties, panting beams etc., the ideal elastic lateral buckling stress may be taken as:

$$\sigma_{el} = 0.001\ E\ \frac{I_A}{A l^2} \quad (\text{N/mm}^2)$$

I_A = moment of inertia in cm^4 about the axis perpendicular to the expected direction of buckling
A = cross-sectional area in cm^2.

When calculating I_A and A, a plate flange equal to 0.8 times the spacing is included for stiffeners. For longitudinals supporting plate panels where elastic buckling is allowed, the plate flange shall not be taken greater than the effective width, see B207 and Appendix A.

Where relevant t_k shall be subtracted from flanges and web plates when calculating I_A and A.

The critical buckling stress is found from 101.

The formula given for σ_{el} is based on hinged ends and axial force only.

Theory (SI-units)

$$\sigma_{el} = \pi^2 \frac{EI}{L_e^2 A}$$

$$\sigma_c = \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right)$$

Notation: $\sigma_y = \sigma_f$

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This is how the result is presented in design codes. We have the Euler buckling load corrected by the Jonhsson parabola. Thus the design rule accounts theoretical derivations, but also the physical corrections from experiments. Yet the equations are closed form. Thus, if you want to go beyond the rules, you must understand the background of the rules and make physically more justified and correct assumptions.

Derivation of buckling equation for plates

- In columns we started with

$$\frac{d^2y}{dx^2} = \frac{F \cdot y}{EI}$$

- The same principle is valid for plates, but we add second dimension and change stiffness parameter

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \cdot \left(q + N_x \cdot \frac{\partial^2 w}{\partial x^2} + 2 \cdot N_{xy} \cdot \frac{\partial^2 w}{\partial x \partial y} + N_y \cdot \frac{\partial^2 w}{\partial y^2} \right)$$

- Now we assume deflection for simply supported plate as

$$w(x,y) = C_{mn} \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a} \right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b} \right)$$

- and inserting back to differential equation we get

$$\frac{m^4 \cdot \pi^4}{a^4} + 2 \cdot \frac{m^2 \cdot n^2 \cdot \pi^4}{a^2 \cdot b^2} + \frac{n^4 \cdot \pi^4}{b^4} = -\frac{1}{D} \cdot \left[N_x \cdot \frac{m^2 \cdot \pi^2}{a^2} + \lambda N_x \cdot \frac{n^2 \cdot \pi^2}{b^2} \right] \Leftrightarrow N_x = -D \frac{\left[\frac{m^4 \cdot \pi^4}{a^4} + 2 \cdot \frac{m^2 \cdot n^2 \cdot \pi^4}{a^2 \cdot b^2} + \frac{n^4 \cdot \pi^4}{b^4} \right]}{\left[\frac{m^2 \cdot \pi^2}{a^2} + \lambda \cdot \frac{n^2 \cdot \pi^2}{b^2} \right]}$$

The Euler buckling can be applied also to plates. So we consider the plate bending equation and derive the double series solution for the eigen value of the load.

Derivation of buckling equation

- The buckling stress can be obtained from here by minimizing $\sigma_x = N_x/t$ with respect to *n* or *m*. When *n*=1 and we have uniaxial compression and only one half wave in y-direction and this simplifies to

$$\sigma_x = \frac{N_x}{t} = -\frac{Et^3}{12(1-\nu^2)}\frac{1}{t}\frac{\left[\frac{m^4 \cdot \pi^4}{a^4} + 2 \cdot \frac{m^2 \cdot n^2 \cdot \pi^4}{a^2 \cdot b^2} + \frac{n^4 \cdot \pi^4}{b^4}\right]}{\left[\frac{m^2 \cdot \pi^2}{a^2} + \lambda \cdot \frac{n^2 \cdot \pi^2}{b^2}\right]} = -\frac{Et^2}{12(1-\nu^2)}\frac{\frac{\pi^2}{b^2}\left[\frac{b^2}{a^2} \cdot m^2 + 1\right]^2}{\left[\frac{m^2 \cdot \pi^2}{a^2}\right]}$$

- which gets minimum when *m=a/b* (take derivative with respect to *m*, *m=a/b-> k*=4). The end result is Bryan's equation

$$\sigma = k \cdot \frac{\pi^2 D}{b^2 t} = k \cdot \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

- where k is obtained from design curves and depends on boundary conditions. The inelastic buckling is taken into account with Johnson's parabola

$$\sigma_c = \sigma_{ty} - \frac{\sigma_{ty}^2}{4}\frac{\lambda^2}{\pi^2 E} = \sigma_{ty}(1 - \frac{\sigma_{ty}}{4\sigma_E})$$

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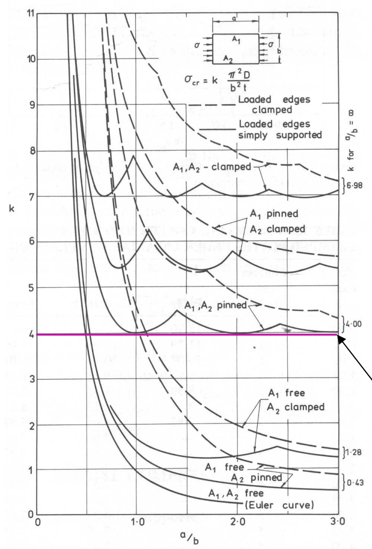
The buckling stress can be obtained from here by minimizing the stress with respect to n or m. When n=1 and we have uniaxial compression this simplifies to and only one half wave in y-direction and we can get rid of the n-dependency.

The resulting equation gets minimum when *m=a/b* (take derivative with respect to *m*, *m=a/b-> k*=4). The end result is Bryan's equation where k is obtained from design curves and depends on boundary conditions, this is called the buckling coefficient of the bifurcation buckling problem and the coefficient is dependent on the boundary conditions, load-type etc. The inelastic buckling is taken into account with Johnson's parabola, in which the elastic buckling stress is corrected based on the yield stress of the material.

Buckling strength using buckling coefficient



$$\sigma_{el} = k\frac{\pi^2 D}{b^2 t} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

$$\sigma_{el} = 3.6E\left(\frac{t}{b}\right)^2$$

$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

Simply supported, *n*=1

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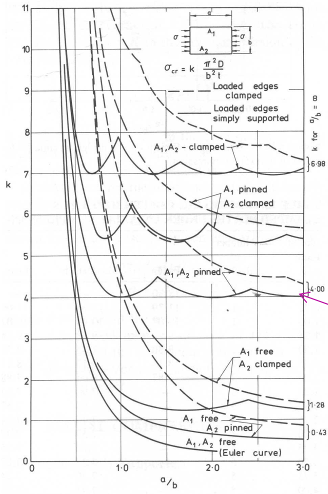
This is what we see to happen to the buckling load for different amount of buckling half waves in the two directions. We use in design of course the minimum values.

Equations in classification rules



SECTION 13
BUCKLING CONTROL

A. General

A 100 Introduction

101 This section covers the requirements for buckling control of:

— plating subject to in-plane compressive and or shear stresses
— axially compressed stiffeners and pillars
— panel ultimate strength.

102 The buckling strength requirements are related to:

— longitudinal hull girder compression and shear stresses based on design values of still water and wave bending moments and shear forces
— axial forces in pillars, supporting bulkheads and panting beams based on the rule loads
— axial and shear forces in primary girders based on the rule loads.

A 200 Definitions

201 Symbols:

t = thickness in mm of plating
s = shortest side of plate panel in m
l = longest side of plate panel in m
= length in m of stiffener, pillar etc.
E = modulus of elasticity of the material
= $2.06 \cdot 10^5$ N/mm^2 for steel
σ_{el} = the ideal elastic (Euler) compressive buckling stress in N/mm^2
σ_f = minimum upper yield stress of material in N/mm^2, and shall not be taken less than the limit to the yield point given in Sec.2 B201.
τ_{el} = the ideal elastic (Euler) shear buckling stress in N/mm^2
σ_c = the critical compressive buckling stress in N/mm^2
τ_c = the critical shear stress in N/mm^2
σ_a = calculated actual compressive stress in N/mm^2
τ_a = calculated actual shear stress in N/mm^2

η = stability (usage) factor = $\frac{\sigma_a}{\sigma_c} = \frac{\tau_a}{\tau_c}$

z_n = vertical distance in m from the baseline or deckline to the neutral axis of the hull girder, whichever is relevant
z_a = vertical distance in m from the baseline or deckline to the point in question below or above the neutral axis, respectively
f_1 = material factor
= 1.0 for NV-NS steel [1)]
= 1.08 for NV-27 steel [1)]
= 1.28 for NV-32 steel [1)]
= 1.39 for NV-36 steel [1)]
= 1.43 for NV-40 steel. [1)]

1) For details see Sec.2 B and C .

B. Plating

B 100 General

101 Local plate panels between stiffeners may be subject to uni-axial or bi-axial compressive stresses, in some cases also combined with shear stresses. Methods for calculating the critical buckling stresses for the various load combinations are given below.

102 Formulae are given for calculating the ideal compressive buckling stress σ_{el}. From this stress the critical buckling stress σ_c may be determined as follows:

$$\sigma_c = \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

103 Formulae are given for calculating the ideal shear buckling stress τ_el. From this stress the critical buckling stress τ_c may be determined as follows:

$$\tau_c = \tau_{el} \quad \text{when} \quad \tau_{el} < \frac{\tau_f}{2}$$

$$= \tau_f\left(1 - \frac{\tau_f}{4\tau_{el}}\right) \quad \text{when} \quad \tau_{el} > \frac{\tau_f}{2}$$

τ_f = yield stress in shear of material in N/mm^2
= $\frac{\sigma_f}{\sqrt{3}}$.

B 200 Plate panel in uni-axial compression

201 The ideal elastic buckling stress may be taken as:

$$\sigma_{el} = 0.9\,k\,E\left(\frac{t - t_k}{1000 s}\right)^2 \quad (\text{N/mm}^2)$$

For plating with longitudinal stiffeners (in direction of compression stress):

$$k = k_l = \frac{8.4}{\psi + 1.1} \quad \text{for } (0 \le \psi \le 1)$$

For plating with transverse stiffeners (perpendicular to compression stress):

$$k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1} \quad \text{for } (0 \le \psi \le 1)$$

c = 1.21 when stiffeners are angles or T-sections
= 1.10 when stiffeners are bulb flats
= 1.05 when stiffeners are flat bars
c = 1.3 when the plating is supported by floors or deep girders.

For longitudinal stiffened double bottom panels and longitudinal stiffened double side panels the c-values may be multiplied by 1.1.

ψ is the ratio between the smaller and the larger compressive stress assuming linear variation, see Fig.1.

and again we see the correspondence between the rules and the theory. What we see here is really that the buckling equation is the same as the theory indicated and the buckling coefficient remains to be solved.

Extensions to cover multi-axial loads

B 200 **Plate panel in uni-axial compression**

201 The ideal elastic buckling stress may be taken as:

$$\sigma_{el} = 0.9\,k\,E\left(\frac{t - t_k}{1000s}\right)^2 \quad (\mathrm{N/mm^2})$$

$$\sigma_{el} = 3.6E\left(\frac{t}{b}\right)^2$$

For plating with longitudinal stiffeners (in direction of compression stress):

$$k = k_l = \frac{8.4}{\psi + 1.1} \quad \text{for } (0 \le \psi \le 1)$$

For plating with transverse stiffeners (perpendicular to compression stress):

$$k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1} \quad \text{for } (0 \le \psi \le 1)$$

c = 1.21 when stiffeners are angles or T-sections
= 1.10 when stiffeners are bulb flats
= 1.05 when stiffeners are flat bars
c = 1.3 when the plating is supported by floors or deep girders.

B 300 **Plate panel in shear**

301 The ideal elastic buckling stress may be taken as:

$$\tau_{el} = 0.9\,k_t\,E\left(\frac{t - t_k}{1000s}\right)^2 \quad (\mathrm{N/mm^2})$$

$$k_t = 5.34 + 4\left(\frac{s}{l}\right)^2$$

B 500 **Plate panel in bi-axial compression and shear**

501 For plate panels subject to bi-axial compression and in addition to in-plane shear stresses the interaction is given by:

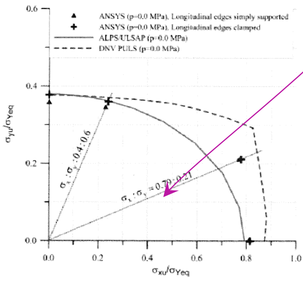
$$\frac{\sigma_{ax}}{\eta_x \sigma_{cx} q} - K\frac{\sigma_{ax}\sigma_{ay}}{\eta_x \eta_y \sigma_{cx}\sigma_{cy} q} + \left(\frac{\sigma_{ay}}{\eta_y \sigma_{cy} q}\right)^n \le 1$$

σ_{ax}, σ_{ay}, σ_{cx}, σ_{cy}, η_x, η_y, K and n are as given in 401.

$$q = 1 - \left(\frac{\tau_a}{\tau_c}\right)^2$$

τ_a and τ_c are as given in 303.

Only stress components acting simultaneously shall be inserted in the formula, see also 401.



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Of course in reality the loading in plates can be caused by compression in two directions and the shear. So in order to solve the buckling problem, we need so called buckling interaction equations where the combined effect is modelled. As these are eigen value solutions corrected by some empiral tests, we cannot account for stabilizing tensile stresses.

Local buckling and ultimate strength

- Bifurcation buckling (Euler buckling) models the collapse of perfect structure
  - Eigenvalue problem
  - Problem is to find the lowest eigenmode in design
- Real structures have initial imperfections and material non-linearity
  - Requires non-linear analysis
  - Give typically lower strength than the bifurcation solution

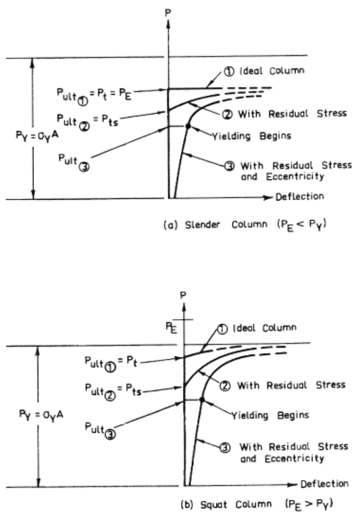

Figure 11.1 Typical load deflection diagrams.

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The challenges with previous approaches is that bifurcation buckling (Euler buckling) models the collapse of perfect structure, i.e. eigenvalue problem is solved and problem is to find the lowest eigenmode for design. Real structures have initial imperfections and material non-linearity, which in requires non-linear analysis and results typically in lower strength than the bifurcation solution. Thus, we must understand the range of validity of the linear buckling solution.

Basic differences in load-carrying behavior between columns, plates and shells

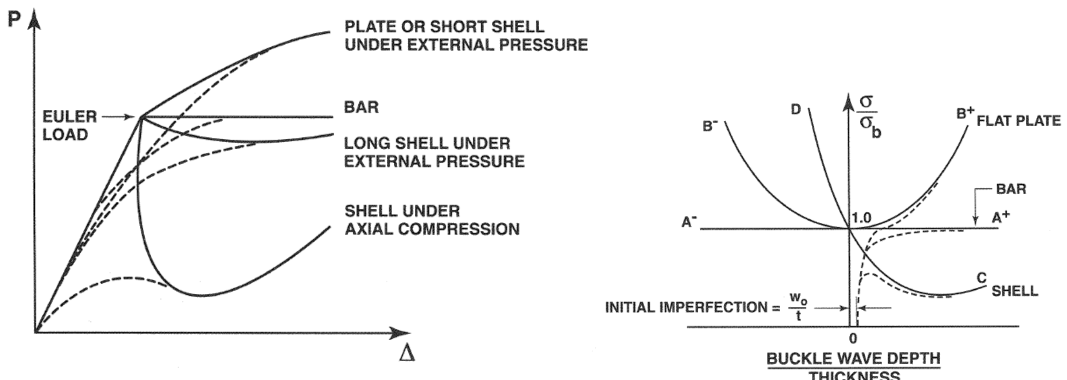

Figure 5-2 Contrast Between Bar, Plate, and Shell Behavior

Jones 2006

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So depending on whether we have a beam, plate or a shell our results are significantly different in terms of load-carrying capacity. At the buckling load, the beams typically loose the load carrying capacity for increasing strain. Meanwhile, in short shells and plates, the is significant post-buckling resistance. Another aspect is the sharpness of changes. The more perfect is the structure, the sharper is the bifurcation point. Real structures always have fairly smooth transitions, which may be difficult to model mathematically.

Influence of initial geometric imperfections

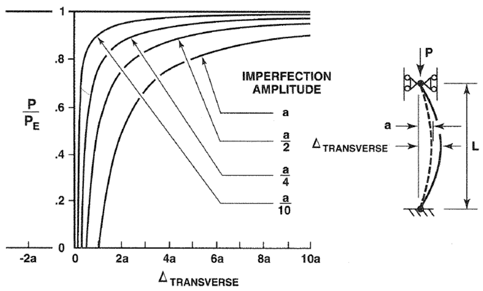

Figure 2-63 Load versus Total Transverse Deflection at Bar Mid-Length

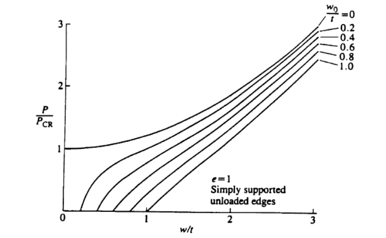

Fig. 5. Load—out of plane deflection curves for simply supported square plates.

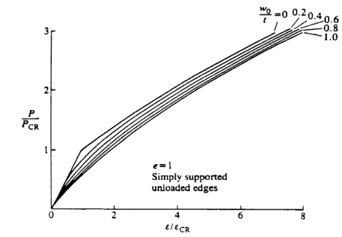

Fig. 6. Load end displacement curves for square simply supported plates.

Jones 2006

So as we see here the load-carrying mechanism is smaller in initially deflected structure if the deflection increases. The shape of initial imperfection may also stiffen the structure.

Influence of residual stress

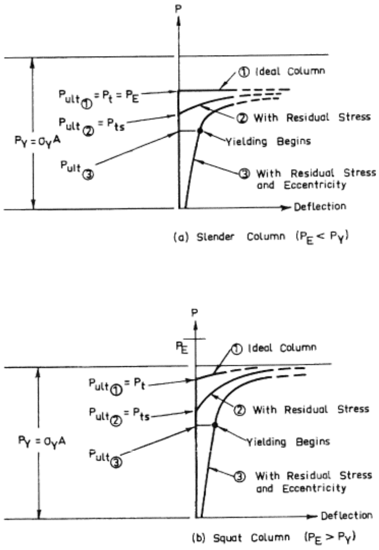

Figure 11.1 Typical load deflection diagrams.

Hughes and Paik 2010

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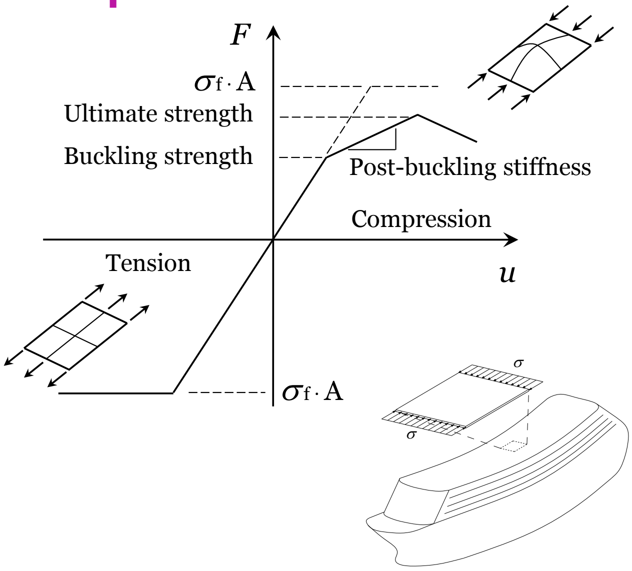
We also know that the residual stresses often decrease the buckling strength. We could consider them as ”initial” stress state in our beam, plate and shell models that are superimposed to the stresses caused by the deformation. We can also have beneficial pre-stress state that increases the laod-carrying capacity. However, real structures tend to find the minimum ”positive” effect and you really have to design for positive pre-stress states if you want to use them for your benefits.

Plate tension vs. compression

- In large thin-walled structures many plates experience both compressive and tensile stresses as load fluctuates due to global deformations
- In compression and shear the failure is often due to buckling or plastic buckling
- In tension we end up dealing with yielding or fracture
- Load-end-shortening curve is structural stress strain curve



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Post-buckling stiffness depends on slenderness. For very slender plates the ULS much larger than buckling strength. In large thin-walled structures many plates experience both compressive and tensile stresses as load fluctuates due to global deformations. In compression and shear the failure is often due to buckling or plastic buckling. In tension we end up dealing with yielding or fracture. Load-end-shortening curve is structural stress strain curve.

Failure modes

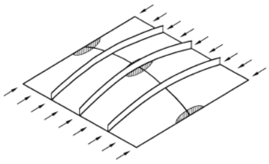


Figure 15.1a Collapse mode I: Overall collapse of the plating and stiffeners as a unit (shaded areas represent yielded regions).

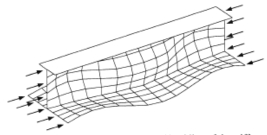


Figure 15.1d Collapse Mode IV: Local buckling of the stiffener web (after the buckling collapse of the plating between the stiffeners).

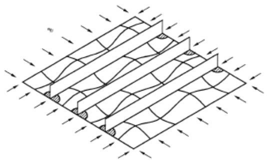


Figure 15.1b Collapse mode II: Biaxial compressive collapse (shaded areas represent yielded regions).

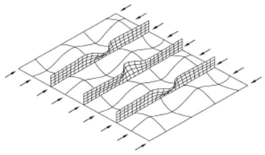


Figure 15.1e Collapse mode V: Flexural–torsional buckling of the stiffeners (after the buckling collapse of the plating between the stiffeners).

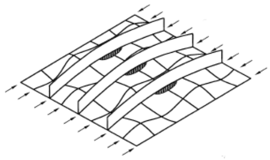


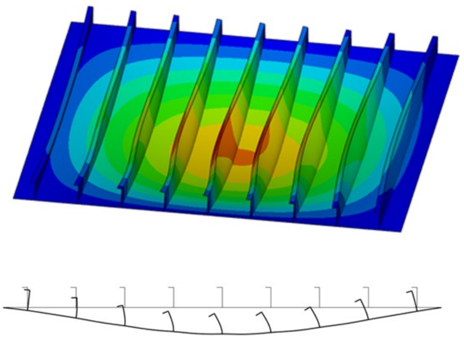
Figure 15.1c Collapse mode III: Beam-column type collapse (shaded areas represent yielded regions).

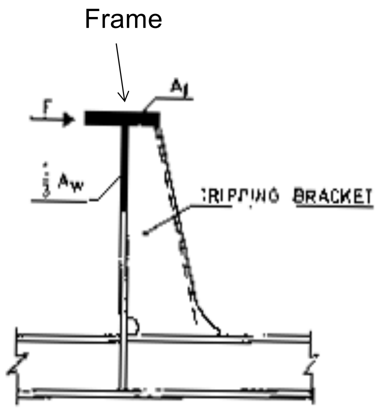
Paik & Thayamballi, 2003a

As stated earlier, buckling can initiate in numerous ways. We should always aim to initiate this at the structures which are not intended to support the other structures. This is as if the stiffener looses its stability, so will loose the supported plate.

Tripping failure mode

AKA flextural-torsional buckling of stiffeners:





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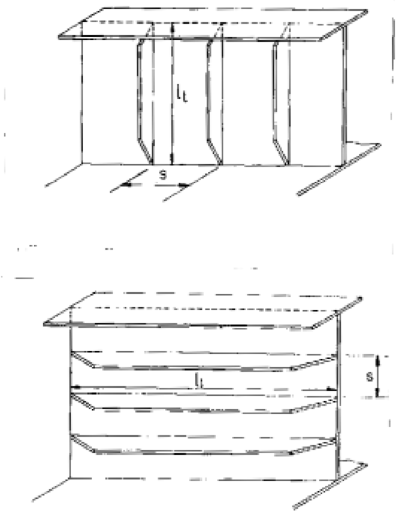
Tripping as a failure mode, means that the stiffener “trips” sideways and this way affects the rotation restraint it produces. So for example this can mean that the “buckling coefficient” for the plate between the stiffeners change from that of clamped to that of a simply supported plate. As this reduces the plate buckling strength a lot, we need to check tripping in our analysis.

Buckling of web



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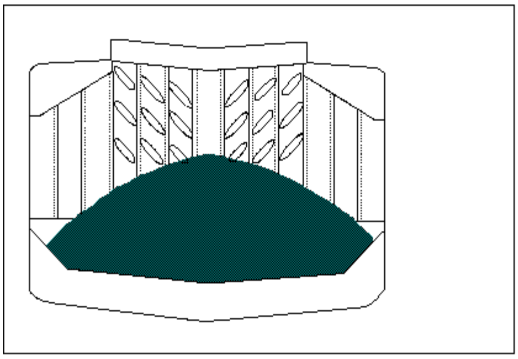
In order to prevent the buckling of shear webs in plated beam of wall structures, we may have to add shear buckling constrains in format of additional stiffeners to highly loaded plates.

Shear buckling



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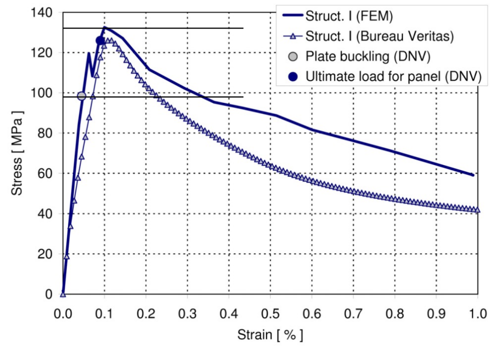
So shear buckling may happen at the support of beams in which the shear forces are highest or plated structures where sehar stresses are high. Not the 45 degree angle which is due to the fact that shear is characterised by both negative and positive at +/- 45 degrees. The compressive stress cause the buckling to the plate.

Ultimate Strength of Panels
Plate vs. Panel Ultimate Strength

- Usually buckling should follow the order
  - Plate
  - Stiffener
  - Panel
- This is commonly assumed and some rules are based on the assumption
- If the order changes, the whole failure process changes
- This affects the safety factors in design



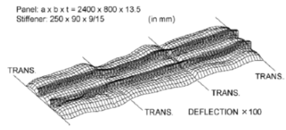


Fig. 3. Real distribution of initial deflection or so-called thin-horse mode initial deflection [3].

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Usually buckling should follow the order: Plate, stiffener and panel. This is commonly assumed and some rules are based on the assumption. If the order changes, the whole failure process changes and this will affect the safety factors in design.

Local buckling and ultimate strength in panels

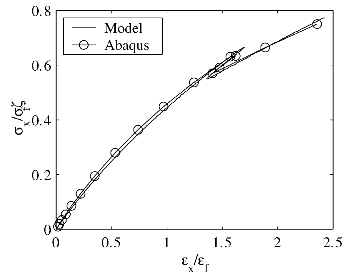
- Geometrical non-linearity of panels can be modelled with homogenized plates, unless there is local failure

$$\begin{bmatrix} \Delta N_1 \\ \Delta N_2 \\ \Delta N_3 \\ \Delta M_1 \\ \Delta M_2 \\ \Delta M_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & Q_{11} & Q_{12} & Q_{13} \\ C_{21} & C_{22} & C_{23} & Q_{21} & Q_{22} & Q_{23} \\ C_{31} & C_{32} & C_{33} & Q_{31} & Q_{32} & Q_{33} \\ Q_{11} & Q_{21} & Q_{31} & D_{11} & D_{12} & D_{13} \\ Q_{12} & Q_{22} & Q_{32} & D_{21} & D_{22} & D_{23} \\ Q_{13} & Q_{23} & Q_{33} & D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \Delta\varepsilon_3 \\ \Delta\kappa_1 \\ \Delta\kappa_2 \\ \Delta\kappa_3 \end{bmatrix}$$

- Local failure requires in addition non-linear ABD-matrix

$$C_{ij} = C_{ij}^{\mathrm{L}} + C_{ij}^{\mathrm{NL}}$$
$$D_{ij} = D_{ij}^{\mathrm{L}} + D_{ij}^{\mathrm{NL}}$$
$$Q_{ij} = Q_{ij}^{\mathrm{L}} + Q_{ij}^{\mathrm{NL}}$$

- Approach works for
  - Stiffened plates
  - Cellular sandwich panels
  - Composites



- This type on model gives effective stiffness for the global analysis, i.e. load-end-shortening relation (structural σε-relation)

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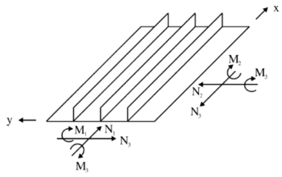
Geometrical non-linearity of panels can be modelled with homogenized plates, unless there is local failure, for example local plate buckling between the stiffeners or tripping of the stiffeners. Local failure requires in addition non-linear ABD-matrix as the stiffness is reduced due to local collapse. Approach works for stiffened plates, cellular sandwich panels and composites. This type on model gives effective stiffness for the global analysis, i.e. load-end-shortening relation (structural stress strain relation)

Analysis methods, PULS

- Global & local buckling and post-buckling of stiffened panels
- Non-linear plate theory (large strains) for global buckling and set of nonlinear stiffness coefficients accounting for local buckling
- Stresses at critical points are calculated and when it reaches yielding, ultimate strength is declared



$$\begin{bmatrix} \Delta N_1 \\ \Delta N_2 \\ \Delta N_3 \\ \Delta M_1 \\ \Delta M_2 \\ \Delta M_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & Q_{11} & Q_{12} & Q_{13} \\ C_{21} & C_{22} & C_{23} & Q_{21} & Q_{22} & Q_{23} \\ C_{31} & C_{32} & C_{33} & Q_{31} & Q_{32} & Q_{33} \\ Q_{11} & Q_{21} & Q_{31} & D_{11} & D_{12} & D_{13} \\ Q_{12} & Q_{22} & Q_{32} & D_{21} & D_{22} & D_{23} \\ Q_{13} & Q_{23} & Q_{33} & D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \Delta \varepsilon_1 \\ \Delta \varepsilon_2 \\ \Delta \varepsilon_3 \\ \Delta \kappa_1 \\ \Delta \kappa_2 \\ \Delta \kappa_3 \end{bmatrix}$$

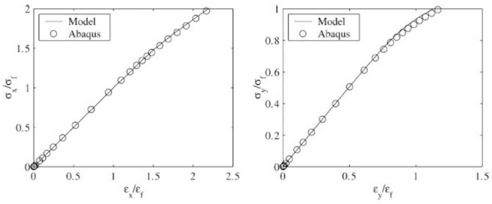


Fig. 7. Load–average strain response due to pure global buckling for stiffened panel subjected to axial compression (left) and transverse compression (right).

Byklum et al. 2004

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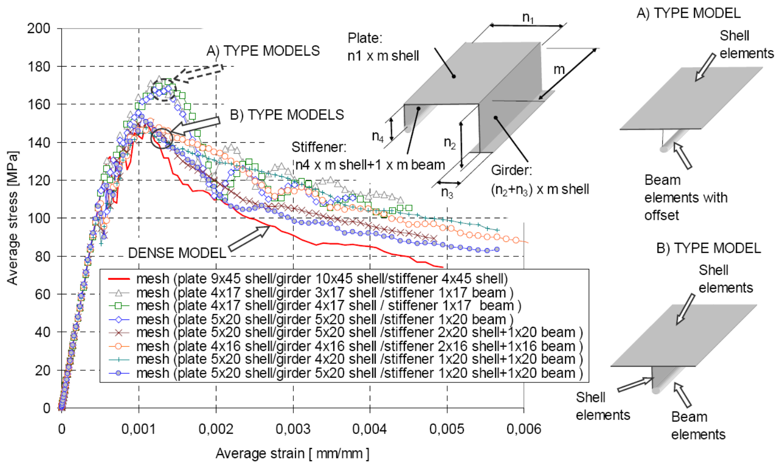
An example of this is the Panel Ultimate Limit State (PULS) code of DNV developed for thin-walled structures. Global & local buckling and post-buckling of stiffened panels si the aim of the code. Non-linear plate theory (large strains) for global buckling and set of nonlinear stiffness coefficients accounting for local buckling is derived. Stresses at critical points are calculated and when it reaches yielding, ultimate strength is declared. Thus, the model cannot account for the material non-linearity. The apporach is analytical and the some of the FE-implementations are also able to handle the material non-lineairy.

Discretization and choice of elements in FEM analysis



Note! Yield Stress of 235MPa is not reached!

Naar 2006

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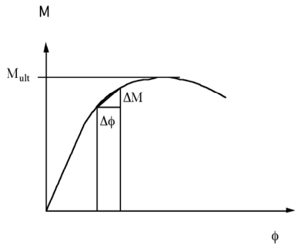
The way the discretization is done in FEA affect the ultimate strength a lot as the discretization affects the possible buckling modes but also the stress localization and the development of plasticity.

Global Collapse of the Structure

- Typically the structure fails progressively meaning that parts of it fail before the total collapse
- Therefore the Moment-Curvature or Force-Displacement relation becomes non-linear
- Typically in thin-walled structures buckling is the first failure in compression, while in tension it is yielding
- Due to differences of scales the methods to assess ultimate strength are
  - Effective material description, i.e. load-end-shortening for local analysis & beam theory or FEM for local analysis
  - Idealized Structural Unit Method, ISUM
  - Finite Element Method
- Since structure is large and prototype cannot be tested, we need more than one method to be sure about the results



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Typically the thin-walled structure fails progressively meaning that parts of it fail before the total collapse. Therefore the Moment-Curvature or Force-Displacement relation becomes non-linear. Typically in thin-walled structures buckling is the first failure in compression, while in tension it is yielding. Due to differences of scales the methods to assess ultimate strength are:

- Effective material description, i.e. load-end-shortening for local analysis & beam theory or FEM for local analysis
- Idealized Structural Unit Method, ISUM
- Finite Element Method and the Intelligent Finite Element Method

Since structure is large and prototype cannot be tested, we need more than one method to be sure about the results when doing the actual design. So we can use some method with different discretization schemes or compare the results of different methods to gain confidence of simulated results.

Design moment vs. ultimate moment of large structure

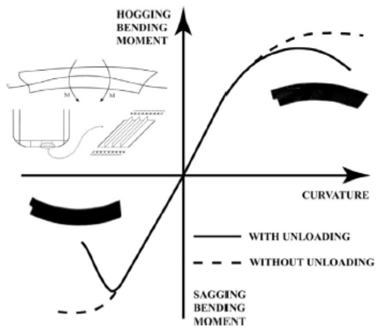
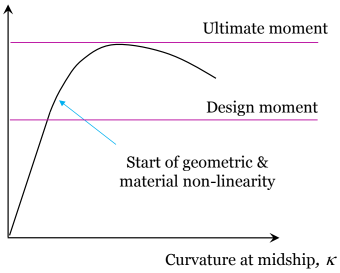


Fig. 1. Typical moment–curvature relationship.

$$M_y = \int_A \sigma_x \cdot d_{NA} \cdot da$$



Here are the examples of design curves and definitions of design adn ultimate moments (or loads). The safety factor is the multiplier of design load that gives the ultimate load. Often the design load is defined by linear elastic methods, while the ultimate strength is obtained trhough testing or non-linear FEA.

Smith’s method

Numerical procedure:

- Divide the cross-section into small elements
- Compute load-displacement curves for each element (see right)
- Increment vertical curvature (displacements are calculated assuming plane-sections remain plane and bending occurs around instantaneous neutral axis)
- Element stresses are obtained from displacements (strains)
- Element stresses are integrated over cross-section to obtain bending moment

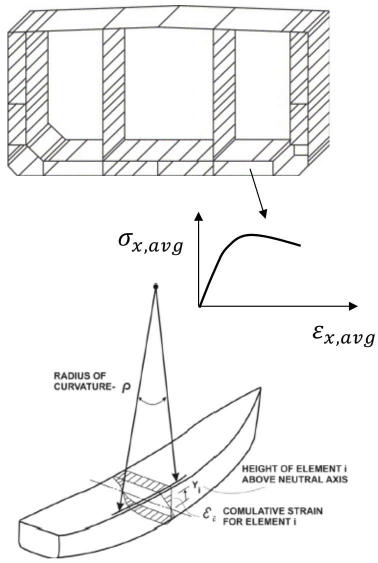


Fig. 2. Ship hull girder bending concept (Smith’s method).

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Smith’s method is a classical analytical and non-linear method to obtain the moment curvature relationship for thin-walled structure. Numerical procedure is such that first we divide the cross-section into small elements which do not interact but take the loads as strains defined by the girder kinematics (EB-beam theory). Then we compute load-displacement curves for each element in compression and tension. When we know these two, we introduce an increment to the vertical curvature (displacements are calculated assuming plane-sections remain plane and bending occurs around instantaneous neutral axis) and we compute the element stresses are obtained from displacements (strains). Element stresses are integrated over cross-section to obtain bending moment. So we exploit the same steps as when deriving the differential equations for CLT, FSDT, TSDT and equivalent beams and plates, but we allow the structural stress-strain behavior to be non-linear. Constitutive law is non-linear.

The ultimate strength behavior of tanker Smith’s Method + Idealized Structural Unit Method (ISUM)

Paik JK & Thayamballi AK, 2003b

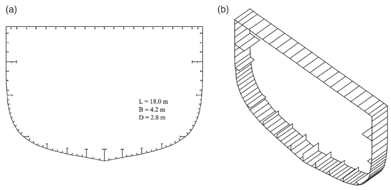
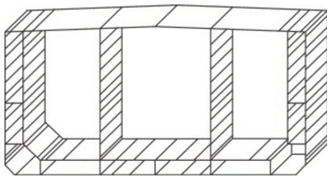
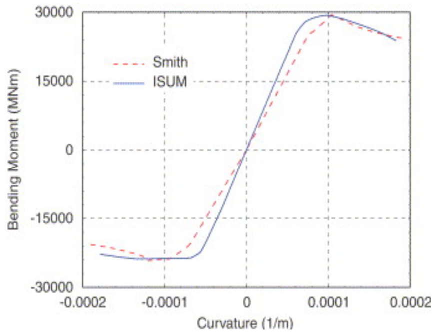


Fig. 18. (a) Mid-section of the Dow frigate test structure. (b) ALPS/HULL model for the progressive collapse analysis of the Dow frigate test structure.

ISUM = simplified non-linear FEM (time savings)

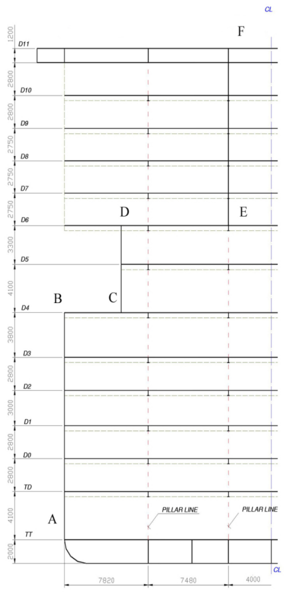




With this process we can obtain very accurate results for the girder resistance.

Example of Ultimate Strength of Post-Panamax Passenger Ship (Naar, 2006) – Progressive Failure

- The conceptual structure of a post-Panamax passenger ship was considered as a prismatic beam with a length of 273 m.
- The use of the prismatic beam instead of the non-prismatic one was justified because the aim of the investigation was to reveal the collapse process of the hull girder more explicitly.
- The distributed load with a cosine-shape was applied on the hull girder ensuring self-balance.
- This load shape complies also well with the description required by Classification Societies, as the maximum value of the bending moment was at amidships and that of the shear force at a quarter length measured from the ends



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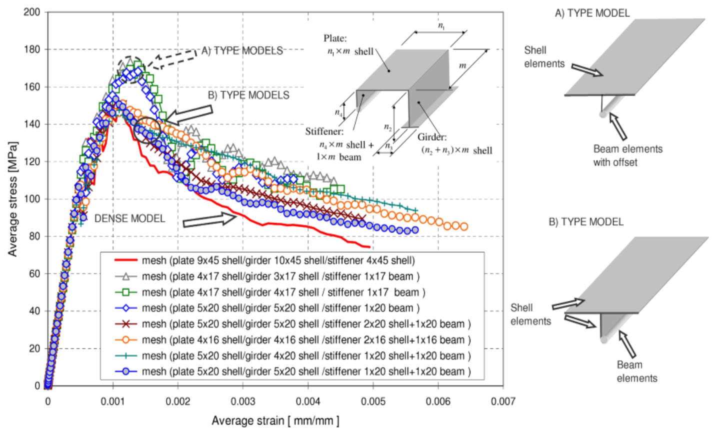
Sometimes the girder kinematics are not as clearly defined as we assumed on the Smith's method. An example of deviation from here is the case study of Naar 2006. The conceptual structure of a post-Panamax passenger ship was considered as a prismatic beam with a length of 273 m. The use of the prismatic beam instead of the non-prismatic one was justified because the aim of the investigation was to reveal the collapse process of the hull girder more explicitly. The distributed load with a cosine-shape was applied on the hull girder ensuring self-balance. This load shape complies also well with the description required by Classification Societies, as the maximum value of the bending moment was at amidships and that of the shear force at a quarter length measured from the ends

Load-End-Shortening of Decks

Naar, H., "Ultimate Strength of Hull Girder for Passenger Ships", Doctoral Thesis, Helsinki University of Technology, 2006.

Note!
Yield Stress of 235MPa is not reached!



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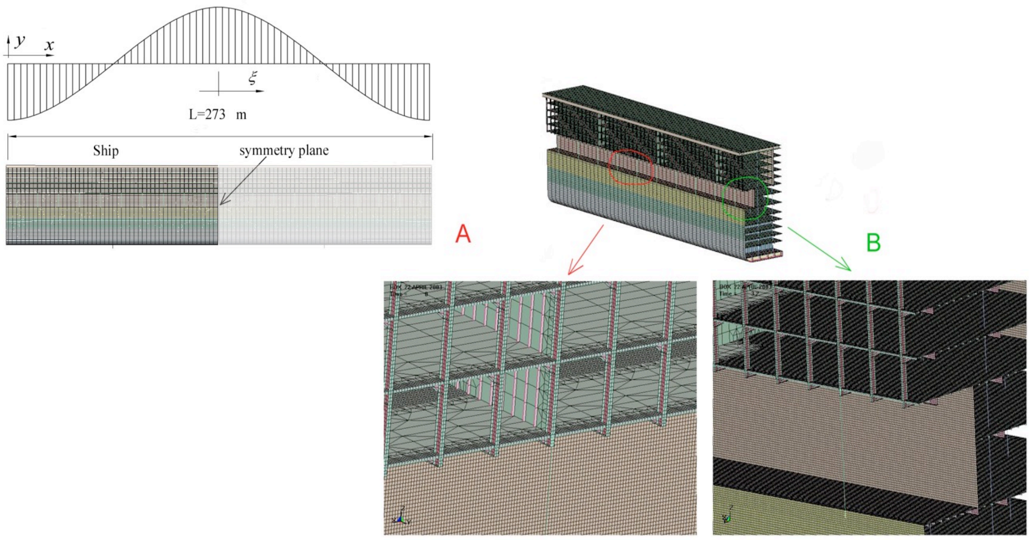
The FE-submodels were optimised in terms of accuracy and time spent for the solution.

Non-Linear FEM - Load and Mesh

Naar, H., ”Ultimate Strength of Hull Girder for Passenger Ships”, Doctoral Thesis, Helsinki University of Technology, 2006.



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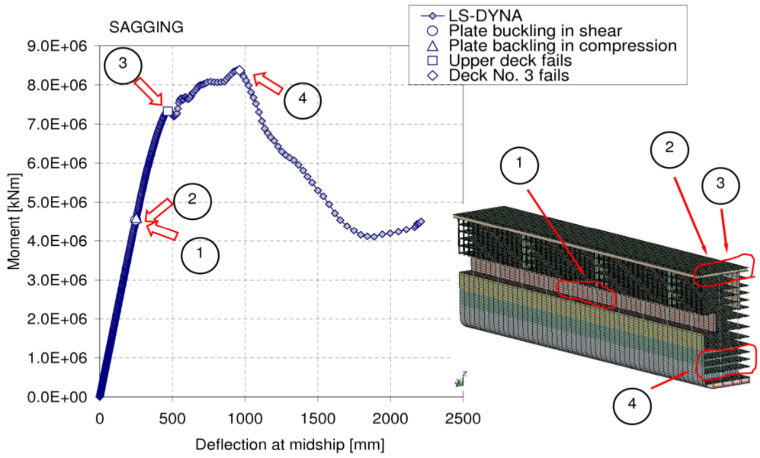
The mesh was extremly fine, yet knot fine enough. Still today we cannot solve this size of a problem in adequate time. It takes in desktop weeks to solve the probelm until the ultimate strength. Often the analysis stops there.

Moment vs. Deflection Sagging

Note! The sign convention of bending moment is not standard
and first failure is shear buckling of recess, then buckling of top most deck, failure of the deck…



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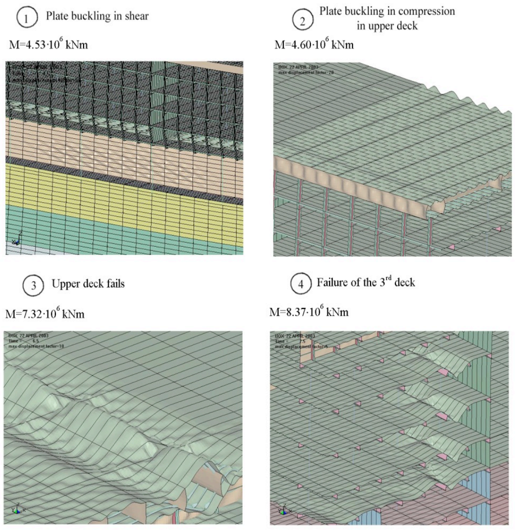
Here we see the sequence of failures starting from shear buckling of recess and proceeding to the deck buckling. When the top and bottom fibres fail, the bending moment collapses at the product of stress and deck area making the most of bending moment collapses.

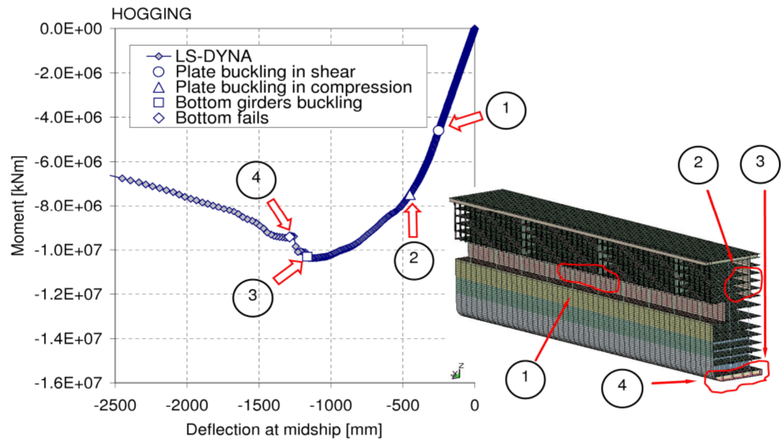
Failure Process
Sagging



This is how the different stages of the failure are. So first we experience shear buckling in the side shell of the recess area, which is followed by the buckling at the top most deck. Then the entire deck experiences the ultimate strength and as the deck looses the load-carrying capacity, the damage progresses to the lower decks. This is very much in-line with beam theory thinking, in which the top most decks are the most heavily loaded ones in bending.

Moment vs. Deflection
Hogging

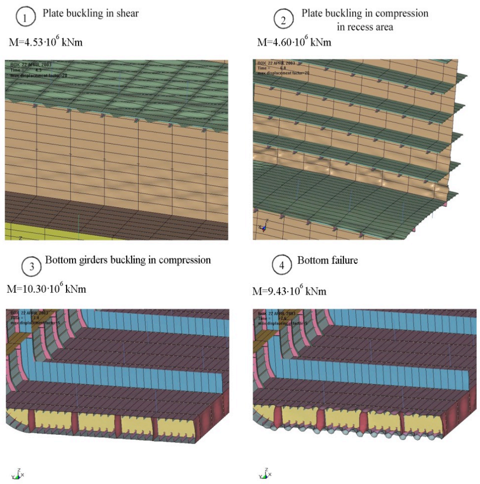
Note! The sign convention of bending moment is not standard



In sagging the situation is slightly different as the double bottom is strengthened against grounding damages. So, now the damage is much more local, but also much more damaging from the overall perspective as the failing structural unit has larger load carrying capacity than the following internal decks.

The point is that when we loose a strong load-carrying structure from a beam, the beam behavior changes drastically.

Failure Process
Hogging



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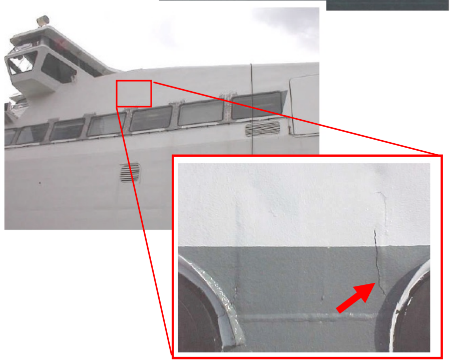
When we change the load-direction and remember that the hull girder is not symmetric, then of course the failure process changes too. Again we start from shear, then get a compressive failure at the recess area, after which the bottom fails very rapidly.

Fracture

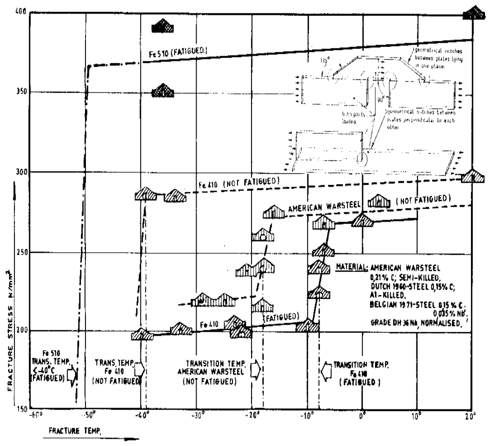
- Today, most fractures in thin-walled structures are due to fatigue, but it can occur with single very high load cycle
  - Structural optimisation aiming cost-efficient structure
  - Application of high strength steel
- Fatigue crack at critical length can break the ship in unstable, brittle fashion

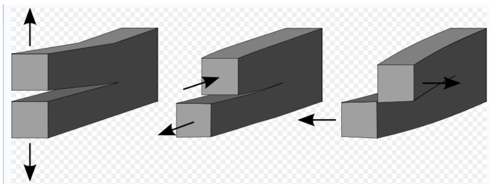


Today, most fractures in thin-walled structures are due to fatigue, but it can occur with single very high load cycle. This is especially the case when structural optimization aiming to cost-efficient structure is used or we use application of high strength materials. Fatigue crack at critical length can break the ship in unstable, brittle fashion.

Brittle Fracture

- The requirements for structural steels in shipbuilding were created after number of brittle fractures during 1950's
  - From riveted to welded structures
  - About 1000 from built 5000 ships had failures
- The main reasons were
  - Unkilled steel material
  - Bad constructions (notch effect)
  - Poor welding and plate cutting
- Brittle fracture occurs when ductile properties of the material are lost → low amount of external energy is needed for crack propagation
- The factors causing brittle fracture (transition from ductile to brittle)
  - Low temperature
  - Impact type loading
  - Triaxial stress state





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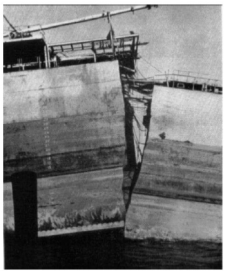
The requirements for structural steels in shipbuilding were created after number of brittle fractures during 1950's. Manufacturing was changed from riveted to welded structures and as a result about 1000 from built 5000 ships had failures. This is why in airspace still rivets are considered. The main reasons for failures were bad steel material, bad constructions (notch effect) and poor welding and plate cutting. Brittle fracture occurs when ductile properties of the material are lost meaning that a low amount of external energy is needed for crack propagation. The factors causing brittle fracture (transition from ductile to brittle) are low temperature, speed of strains and loading and triaxial stress state.

Brittle vs. Ductile Fracture

- Brittle fracture can be controlled with proper material selection – result of bad quality
- Ductile fracture usually means that the fracture does not propagate extremely fast – typical in ship collisions and groundings, car crash



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Brittle fracture can be controlled with proper material selection as it is often result of bad quality in materials or material properties after manufacturing. Ductile fracture usually means that the fracture does not propagate extremely fast and this is typical in ship collisions and groundings and car crashes when proper actions are taken in manufacturing of materials and structures.

Summary

- Ultimate strength of assessment of large complex structures is important part of the design of thin-walled structures
- Buckling can be considered as the initiation of progressive failure process that leads to complete failure if loading increases
- Ultimate strength is the maximum load that the structure can take
  - Fracture occurs under tensile stresses
  - Plastic buckling and folding under compressive stresses

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