1. (IMO Shortlist 1982) A box has p white balls and q black balls. Beside the box is a large pile of black balls. Two balls are removed from the box. If they have the same colour, a black ball from the pile is put into the box. If they have different colours, then white ball is put back into the box. Repeat this process until only 1 ball remains in the box. What is the probability that the last ball is white?

2. Consider an $m \times n$ rectangle. What is the minimum number of 1×1 cells that must be coloured such that there is no way to put an L-tromino onto uncoloured squares?

3. Nine 1×1 cells of a 10×10 square are infected. Each time unit, any cell with at least two infected (meaning sharing an edge) neighbours becomes infected. Show that no matter how much time evolves, all 100 squares do not become infected.

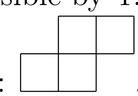
4. Write m A's and n B's on the board, with $m, n \geq 1$. Repeatedly erase 2 letters and write a new letter: if both letters removed are the same, then write an A and if both letters removed are different than write B. Continue until only one letter remains. Show that the final letter does not depend on the order of these operations.

5. Seven vertices of a cube are marked by 0, and one is marked by 1. You repeated select an edge, and add an integer to both numbers corresponding to that edge.

   a) Prove that it is not possible to (simultaneously) obtain the number 0 at all 8 vertices.

   b) Same as (a), but replace the word "integer" with "real number".

6. (IMO Shortlist 1984)

   (a) Is it possible to label the squares of an 8×8 chessboard using the numbers $1, 2, \cdots, 64$ (each once) such that, in any 4 squares making up a "zig-zag"-tetronimo (rotations are allowed), the sum of the 4 numbers is divisible by 4? (For your viewing pleasure, here is a "zig-zag" tetronimo, as seen in the wild:  .)

   (b) Same as previous, but with "zig-zag" replaced with "T". (Our photographer was unable to track down a T-tetronimo to shoot (with a camera).)

7. (IMO Shortlist 1989) A positive integer is written in each square of an $m \times n$ board. The allowed move is to add an integer k to each of two adjacent (sharing an edge) in such a way that no negative numbers are obtained. Find a necessary and sufficient condition for it to be possible to make all the numbers into 0.

8. (IMO Shortlist 1976) Let $I = (0, 1]$, and fix $a \in (0, 1)$. Define $T : I \to I$ by $T(x) = x + (1 - a)$ whenever $0 < x \leq a$ and $T(x) = x - a$ whenever $a < x \leq 1$. Let $J \subseteq I$ be an interval (that isn't just a singleton, depending on your definition of "interval"). Prove that $T^n(J) \cap J \neq \emptyset$ for some $n \geq 1$.

9. (Moscow Olympiad, 1995) We have four congruent right-angled triangles. In one step, you may take any triangle and cut it in two with the altitude from the right angle. Prove that two of your triangles will always be congruent.

10. (IMO 1993, #3) Given an infinite chessboard, a square of area n^2 is covered with coins. A move is a jump of a coin over another adjacent (horizontally or vertically) coin and to remove the coin that was jumped over. The goal of the game is to end up with only a single coin remaining. For which n is it possible to win?

11. (IMO Shortlist 2014 - it's at the end because I'm only vaguely sure how to solve it.) There are n circles drawn on a piece of paper in such a way that any two circles intersect in two points, and no three circles pass through the same point. Ginger the snail starts at a point on a single, initially moving clockwise. When she hits the intersection of two circles, she starts moving along the new circle, this time anti-clockwise. When she reaches a new intersection, she starts moving along the new circle, this time clockwise, and so on. Suppose that Ginger's path fully covers every circle. Prove that n is odd.