Mock Olympiad #3

1. Let $n \in \mathbb{N}$. Find the number of permutations $(a_1, a_2, \ldots, a_n)$ of $\{1, 2, \ldots, n\}$ for which $k \mid 2(a_1 + a_2 + \ldots + a_k)$, for every $1 \leq k \leq n$.

2. Find all functions $f : R^+ \to R^+$ such that for any $x, y, z \in R^+$ with $x + y \geq z$,
$$f(x + y - z) + f(2\sqrt{xz}) + f(2\sqrt{yz}) = f(x + y + z).$$

3. *(IMO Short list 2001, N4)* Let $p \geq 5$ be a prime number. Prove that there exists an integer a with $1 \leq a \leq p - 2$ such that neither $a^{p-1} - 1$ nor $(a + 1)^{p-1} - 1$ is divisible by p^2.