INTERNATIONAL MATHEMATICAL OLYMPIAD
TEAM SELECTION TEST
Day 4, April 20, 2022

Problem 10. There are a) 2022, b) 2023 plates placed around a round table and on each of them there is one coin. Alice and Bob are playing a game that proceeds in rounds indefinitely as follows. In each round, Alice first chooses a plate on which there is at least one coin. Then Bob moves one coin from this plate to one of the two adjacent plates, chosen by him. Determine whether it is possible for Bob to select his moves so that, no matter how Alice selects her moves, there are never more than two coins on any plate.

Problem 11. Find all positive integers n with the following property: the k positive divisors of n have a permutation $(d_1, d_2, \ldots, d_k)$ such that for every $i = 1, 2, \ldots, k$ the number $d_1 + d_2 + \ldots + d_i$ is a perfect square.

Problem 12. Let A, B, C, D be points on the line ℓ in that order and $AB = CD$. Denote (P) as some circle that passes through A, B with its tangent lines at A, B are a, b. Denote (Q) as some circle that passes through C, D with its tangent lines at C, D are c, d. Suppose that a cuts c, d at K, L respectively; and b cuts c, d at M, N respectively. Prove that four points K, L, M, N belong to a same circle (ω) and the common external tangent lines of circles $(P), (Q)$ meet on (ω).