Preparation for Saudi Arabia Team 2021

June Session: Junior Balkan Mathematics Olympiad

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Lesson 2

Diophantine Equations

Problems:

1. Find all integers a, b and c such that:
$$2^a + 3^b + 1 = 6^c.$$
2. Show that there are no positive integers a, b, c and d such that:
$$2^a + 3^b + 5^c = 2014^d.$$
3. Find all positive integer solutions to the equation $3^x + 4^y = 5^z$.
4. Find all triplets (x, y, p) of positive integers such that p is a prime number and:
$$\frac{xy^3}{x+y} = p.$$
5. Find all positive integers x, y, z such that $x^2 = 2^y + 2001^z$.
6. Show that there are no positive integers x and y such that $x^{100} - y^{100} = 100!$.
7. Find all pairs of prime numbers p and q such that:
$$pq | p^q + q^p + 7.$$
8. Find all positive integers x, y, z such that:
$$20^x + 13^y = 2013^z.$$
9. If x and y are integers such that $2x^2 + x = 3y^2 + y$ show that $x - y$ is a perfect square.
10. In nonnegative set of integers solve the equation:
$$(2^{2015} + 1)^x + 2^{2015} = 2^y + 1.$$