0.1 Functional Equations from 2021 Olympiads

Problem 0.1.1. *Determine all functions $f : \mathbb{R} \to \mathbb{R}$ which satisfy the relationship*

$$f(xf(y) - f(x)) = 2f(x) + xy, \text{ for any } x, y \in \mathbb{R}.$$

Problem 0.1.2. *Determine all $f : \mathbb{R} \to \mathbb{R}$ such that*

$$f(xf(y) + y^3) = yf(x) + f(y)^3$$

Problem 0.1.3. *Find all functions $f, g : \mathbb{R} \to \mathbb{R}$ such that satisfies*

$$f(x^2 - g(y)) = g(x)^2 - y$$

for all $x, y \in \mathbb{R}$.

Problem 0.1.4. *Find all functions $f \colon \mathbb{R} \to \mathbb{R}$ that satisfy the inequality*

$$f(y) - \left(\frac{z-y}{z-x}f(x) + \frac{y-x}{z-x}f(z)\right) \leq f\left(\frac{x+z}{2}\right) - \frac{f(x)+f(z)}{2}$$

for all real numbers $x < y < z$.