Problem 1B. Given a sequence $a_1, a_2, \ldots, a_n$ of real numbers. For each $1 \leqslant i \leqslant n$ define

$$d_i = \max\{a_j : 1 \leqslant j \leqslant i\} - \min\{a_j : i \leqslant j \leqslant n\}$$

and let $d = \max\{d_i : 1 \leqslant i \leqslant n\}$.

(a) Prove that for arbitrary real numbers $x_1 \leqslant x_2 \leqslant \ldots \leqslant x_n$,

$$\max\{|x_i - a_i| : 1 \leqslant i \leqslant n\} \geqslant \frac{d}{2}.$$

(b) Show that there exists a sequence $x_1 \leqslant x_2 \leqslant \ldots \leqslant x_n$ of real numbers such that we have equality in part (a).

Problem 2B. Find all integers $n \geqslant 3$ with the following property: for all real numbers $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ satisfying $|a_k| + |b_k| = 1$ for $1 \leqslant k \leqslant n$, there exist $x_1, x_2, \ldots, x_n$, each of which is either -1 or 1, such that

$$\left|\sum_{k=1}^{n} x_k a_k\right| + \left|\sum_{k=1}^{n} x_k b_k\right| \leqslant 1.$$

Problem 3B. Let $x_1, x_2, \ldots, x_n$ be real numbers. Prove that

$$\sum_{i=1}^{n}\sum_{j=1}^{n} |x_i + x_j| \geqslant n \sum_{i=1}^{n} |x_i|.$$

Problem 4B. Let $a_1, a_2, \ldots, a_n$ be real numbers such that $|a_i| \leqslant 1$ for $1 \leqslant i \leqslant n$ and $a_1 + a_2 + \cdots + a_n = 0$. Prove that there exists $k \in \{1, 2, \ldots, n\}$ such that

$$|a_1 + 2a_2 + \cdots + ka_k| \leqslant \frac{2k+1}{4}.$$

Problem 5B. Let $x_1, x_2, \ldots, x_n$ be real numbers such that $|x_i| \leqslant \frac{n+1}{2}$ for $1 \leqslant i \leqslant n$ and $|x_1 + x_2 + \cdots + x_n| = 1$. Prove that there exists a permutation $(y_1, y_2, \ldots, y_n)$ of the numbers $x_1, x_2, \ldots, x_n$ such that

$$|x_1 + 2x_2 + \cdots + nx_n| \leqslant \frac{n+1}{2}.$$

Problem 6B. Let p, q, n be positive integers such that $p+q < n$. Let $(x_0, x_1, \ldots, x_n)$ be an $(n+1)$-tuple of integers satisfying the following:

a) $x_0 = x_n = 0$;

b) for each i with $1 \leqslant i \leqslant n$ either $x_i - x_{i-1} = p$ or $x_i - x_{i-1} = -q$.

Prove that there exists a pair (i, j) of distinct indices with $(i, j) \neq (0, n)$, such that $x_i = x_j$.

Problem 7B. Let $a_0, a_1, a_2, \ldots$ be a sequence of real numbers such that $a_0 = 0$, $a_1 = 1$, and for every $n \geqslant 2$ there exists $1 \leqslant k \leqslant n$ satisfying

$$a_n = \frac{a_{n-1} + a_{n-2} + \ldots + a_{n-k}}{k}.$$

Find the maximal possible value of $a_{2018} - a_{2017}$.