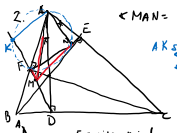
2.



$\sphericalangle MAN = \sphericalangle DAC = 90^\circ - \gamma = \sphericalangle DFC = \sphericalangle CFE$
$= \frac{1}{2} \sphericalangle DFE$

AK symmetric to AN w.r.t. AM

$\sphericalangle NAK = \sphericalangle DFE \Rightarrow$ ANFK is cyclic

FA bisects $\sphericalangle KFN \Rightarrow AK = AN$

$\Rightarrow \triangle AKM \cong \triangle ANM \Leftrightarrow \sphericalangle KMA = \sphericalangle NMA$

1.

E = nine-point center, $\sphericalangle ABP = \sphericalangle ACP = \alpha$. Prove that $x = 90^\circ - 2\alpha$
= circumcenter of JKL

Where are points X on BC with $\sphericalangle AXP = \alpha$?

$PM \parallel EK$, $PN \parallel EL$: $PM = PN$ because $EK = EL$
$MN \parallel KL \parallel BC$

$\sphericalangle PBN = x$

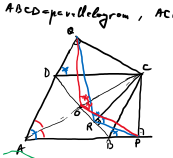
$\sphericalangle PXN = x$, where X is the mirror of C in the perp. bis. of MN

Then X is on the circle NPB.

$A - x = \sphericalangle CBP = \sphericalangle PNX = \sphericalangle PMC = \sphericalangle EKC = \sphericalangle EKJ + \sphericalangle JKC = 90^\circ - \gamma + \alpha$

$x = A - 90^\circ + \gamma - \alpha = 90^\circ - 2\alpha$.

3. ABCD = parallelogram, $AC \cap BD = O$. Prove that O lies on the circumcircle of $\triangle PQR$

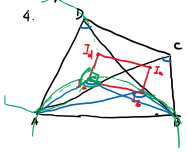


$\sphericalangle PRQ = 2 \sphericalangle A$

$\sphericalangle POQ = \sphericalangle PAC + \sphericalangle CDQ = 2 \sphericalangle PAC + 2 \sphericalangle CAQ$
$= 2 \sphericalangle A$

$\sphericalangle PAC = \sphericalangle DBC$
$\sphericalangle DAB = \sphericalangle BCD$
$EA \cdot EC = EB \cdot ED$
$FA \cdot FD = FB \cdot FC$

4.



I_a, I_b, I_c, I_d = incenters of $\triangle DAB$, $\triangle ABC$, $\triangle BCD$, $\triangle CDA$

Prove that they form a rectangle.

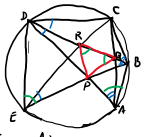
$\sphericalangle AI_bB = 90^\circ + \frac{1}{2} \sphericalangle ACB$
$\sphericalangle AI_aB = 90^\circ + \frac{1}{2} \sphericalangle ADB$ $\Rightarrow ABI_bI_a$ is cyclic.

$\sphericalangle AI_aI_b = 180^\circ - \sphericalangle ABI_b = 180^\circ - \frac{1}{2} \sphericalangle B$.

$\sphericalangle BI_aI_d = 180^\circ - \frac{1}{2} \sphericalangle D$

$\sphericalangle A I_aI_b + \sphericalangle AI_aI_d = 270^\circ \Rightarrow \sphericalangle I_bI_aI_d = 90^\circ$

5.



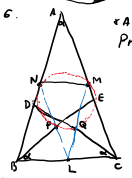
$AB = BC$, $CD = DE$

Prove that PQR is isosceles.

DEPR, ABQP are cyclic

$\Rightarrow \sphericalangle PQR = \sphericalangle PAB = \sphericalangle PED = \sphericalangle PAR$.

6.



$\sphericalangle A < \sphericalangle B, \sphericalangle C$

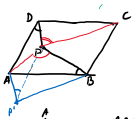
Prove that the midpoints of AB, AC, BE, CD lie on a circle.

NP and MQ meet at the midpoint L of BC

? $LP \cdot LN = LQ \cdot LM$.

$LP \cdot LN = LB^2 = \frac{1}{4} BC^2$ because $\triangle LBP \sim \triangle LNB$
$\parallel$
$LQ \cdot LM$.

7.

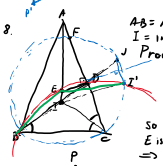


Prove that $\sphericalangle APB + \sphericalangle CPD = 180^\circ$
(Given that $\sphericalangle ABP = \sphericalangle ADP$)

$\overrightarrow{PP'} = \overrightarrow{DA} = \overrightarrow{CB}$

APBP' is cyclic $\Rightarrow \sphericalangle APB + \sphericalangle AP'B = 180^\circ =$
$= \sphericalangle APB + \sphericalangle CPD$.

8.



$AB = AC$, $D = BI \cap AC$

I = incenter, I' = its reflection in AC

Prove that I' lies on the circle BDE

$\triangle AII' \sim \triangle ABC$

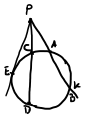
$\triangle DCI \sim \triangle DBC \Rightarrow DI \cdot DB = DC^2$

F, J = reflections of C, I in D $\Rightarrow DJ \cdot DB = DC \cdot DF$

So BCJF is cyclic. I' lies on the same circle because CFJI' is an isosceles trapezoid

E is the circumcenter

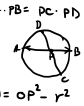
$\Rightarrow BE = EI'$



Where is E?
— on the perp. bis. of BI'
— on the ext. bisector of $\sphericalangle BDI'$

So, E is on the circle BDI'

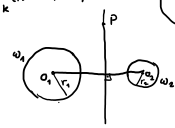
$0 < PA \cdot PB = PC \cdot PD = PE^2$ = power of point P in circle k : $P_k(P)$



$PA \cdot PB = PC \cdot PD < 0$

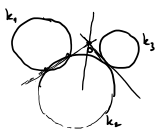
$P_k(P) = OP^2 - r^2$

$P_k(P) = PA \cdot PB = -PA^2 = OP^2 - r^2$



$P_{\omega_1}(P) = P_{\omega_2}(P)$ ← Where are such points P?

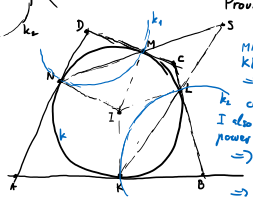
The locus of such points P is „radical axis" of ω_1 and ω_2 = a line $\perp O_1O_2$



O = radical center of k_1, k_2, k_3

The pairwise rad. axes of k_1, k_2, k_3 all pass through O.

9.



Prove that $IS \perp BD$.

MN = rad. axis of k, k_1

KL = rad. axis of k, k_2

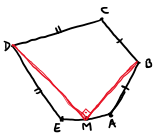
$\Rightarrow$ S is the rad. center of k_1, k_2, k

I also has the same power in k_1, k_2

$\Rightarrow IS$ = rad. axis of k_1, k_2

$\Rightarrow IS \perp BD$

10. ABCDE



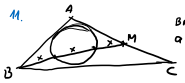
$AB = BC$

$CD = DE$

$\sphericalangle ABC + \sphericalangle CDE = 180^\circ$

Prove that $\sphericalangle BMD = 90^\circ$

11.



BM = median.

$a : b : c = ?$

Continue tomorrow at 3:30 pm