

Language: English

Friday, September 11, 2020

Problem 1. Find all triples (a, b, c) of real numbers that satisfy the system of equations:

$$a+b+c=rac{1}{a}+rac{1}{b}+rac{1}{c}$$
 and $a^2+b^2+c^2=rac{1}{a^2}+rac{1}{b^2}+rac{1}{c^2}$

Problem 2. Let $\triangle ABC$ be a right-angled triangle with $\angle BAC = 90^{\circ}$ and let E be the foot of the perpendicular from A on BC. Let $Z \neq A$ be a point on the line AB with AB = BZ. Let (c) be the circumcircle of the triangle $\triangle AEZ$. Let D be the second point of intersection of (c) with C and let C be the antidiametric point of C with respect to C. Let C be the point of intersection of the lines C and C and C and C are conception.

Problem 3. Alice and Bob play the following game: Alice picks a set $A = \{1, 2, ..., n\}$ for some natural number $n \ge 2$. Then starting with Bob, they alternatively choose one number from the set A, according to the following conditions: initially Bob chooses any number he wants, afterwards the number chosen at each step should be distinct from all the already chosen numbers and should differ by 1 from an already chosen number. The game ends when all numbers from the set A are chosen. Alice wins if the sum of all the numbers that she has chosen is composite. Otherwise Bob wins. Decide which player has a winning strategy.

Problem 4. Find all prime numbers p and q such that

$$1 + \frac{p^q - q^p}{p + q}$$

is a prime number.

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Time: 4 hours and 30 minutes Each problem is worth 10 points