JUNIOR BALKAN MATHEMATICAL OLYMPIAD
TEAM SELECTION TEST
Test 2, June 3, 2022

Problem 1.1. Find all pairs of positive prime numbers (p, q) such that

$$p^5 + p^3 + 2 = q^2 - q.$$

Problem 1.2. Let x, y are real numbers such that $x + y \geq 0$. Find the minimum value of $K = x^5 + y^5 - x^4y - xy^4 + x^2 + 4x + 7$. For which values of x, ydoes the expression K get its minimum value?

Problem 1.3. Let BB_1 and CC_1 be the altitudes of acute-angled triangle ABC, and A_0 is the midpoint of BC. Lines A_0B_1 and A_0C_1 meet the line passing through A and parallel to BC in points P and Q. Prove that the incenter of triangle PA_0Q lies on the altitude of triangle ABC.

Problem 1.4. You plan to organize your birthday party, which will be attended either by exactly m persons or by exactly n persons (you are not sure at the moment). You have a big birthday cake and you want to divide it into several parts (not necessarily equal), so that you are able to distribute the whole cake among the people attending the party with everybody geting cake of equal mass (however, one may get one big slice, while others several small slices - the sizes of slices may differ). What is the minimal number of parts you need to divide the cake, so that it is possible, regardless of the number of guests.