More combinatorics

September 6, 2020

Problem 1. *In each cell of an $n \times n$ board is written a single number. It is known, that the sum of maximal two numbers of each row is equal to a and the sum of maximal two numbers of each column is equal to b. Prove that $a = b$.*

Problem 2. *Let a, b be integers so that $\frac{a^2+b^2}{ab-1} = k$ is an integer. Prove that $k = 5$.*

Problem 3. *In each cell of the 10×10 board a positive number is written. The sum of the integer of each row is equal to 100. It is allowed to interchange two numbers, if they are on the same row. One is making several operations, after which we are going to sum up the maximal numbers of each of 10 columns. Prove that it is possible to make operations in a way that sum is going to be less than 300.*

Problem 4. *In each cell of the 11×11 board is written either a "+" or a "-", in a way that the total number of "+" is even. Moreover, in each 2×2 square the number of "+" is even. Prove that the number of "+" on the main diagonal is also even.*

Problem 5. *Several distinct positive integers are written on the board. The sum of any of two numbers is a perfect power of 2. Find the maximal number of integers, which can be written on the board.*

Problem 6. *Is it possible to mark several cells of the 19×19 board, so that in each 10×10 board there is different number of marked cells?*

Problem 7. *Two classes consist of m and n students respectively ($m \neq n$). The classes play a ping-pong tournament. The first game is played by some two players from different teams, while the other students stand in a row. Then, after the game ends, the student from the front of the row substitutes his teammate, and latter moves to the end of the row. Prove that at some point, each pair of students from different teams will play a game.*