Mathematical riddles

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1. In a kingdom, there are 100 wizards. The king is testing them as follows. He orders them in a line and puts each a white or black hat, so that each sees the hats of those in front of him, but not his own, or of those behind. Then the wizards try to guess the color of their hat. The wizards can make a plan before the test. At most how many of them can guess right?

2. You are at a market, wanting to buy two chicken - one male and one female. The villager is showing you a box with 12 chicken, but they all look same. However, the villager is somehow able to distinguish them and offers you help: You show him any set of chicken, and he will tell you how many of them are male (without specifying them). At least how many questions do you need in order to determine a pair of chicken you need?

3. We have n boxes put next to each other in a line. In one of them there is a mouse. You try to kill the mouse by throwing bombs in boxes. If the mouse is in the box you throw a bomb in, he dies, otherwise he moves to the adjacent box to his right. From the rightmost box he cannot move. How many bombs do you need to secure killing the mouse?

4. A group of thieves hides their treasure in one of 101 caves, numbered 1 to 101. On daylight the treasure lies in a cave, but at night they move it to one of the two adjacent caves. Ali Baba knows this and enters one cave every day, trying to retrieve the treasure. Does he have a strategy that guarantees him to find the treasure?

5. Seven cards, numbered 1 through 7, have been arbitrarily divided between three people A, B and C, where A and B have got three cards each, and C has got only one. A and B want to share to each other which cards they have, but C is a spy, hears everything they say, and must not know about the location of any card. How can A and B do the job?

6. There is a 7×7 board filled with numbers $1,\dots,49$ and we must guess the arrangement of numbers. In each question, we choose a square consisting of some cells and ask which numbers are inside it. At least how many questions do we need?

7. The numbers from 1 to 100 are ordered in a sequence. With one question we can find out the mutual arrangement of any 50 numbers. Show that 5 questions are enough to find out the order of all 100 numbers.

8. Alice and Bob play a game against Charlie. Charlie starts by placing cards numbered 1 through 100 in a line on the table. Then Alice may choose two cards and switch them (if she wants). Charlie then turns the cards face-to-bottom and Bob comes to the table. Charlie chooses a number and asks Bob to find it. Bob turns he cards one by one, but is given only 50 guesses. Can Alice and Bob win?

9. There is a round table with 30 people sitting, some of which are smart and some are stupid. We only know that at most G of them are stupid. We ask each of them whether their neighbor to the right is smart or stupid. The smart will tell us the truth, but the stupid will give us random answers. For which largest G will we always be able to identify at least one smart person?

10. (a) Two players perform a trick with cards. First player is given five random cards. Upon seeing the cards, he places them on the table in a line so that four cards are face-up and the fifth is face-down. The second player then guesses the hidden card. Prove that they can make a plan so that the trick always succeeds.

    (b) The same question, but now the first player shows off four cards and does not place the fifth card at all.

11. Boys and girls, n^2 of them in total, are placed in an $n \times n$ array. For each row or column, as well as each of the $2(2n-1)$ diagonals, we know the numbers of girls. For which positions in the array are these information always sufficient to determine whether a boy or a girl occupies it?

12. A bat flies along the real line seeking his house which lies at point A. He starts from the origin O and knows he is at least 1 meter away from the house, but he does not know how far it is, and on which direction. Moreover, he is blind, so he won't see it until he bumps into it. Prove that the bat has a strategy that will ensure finding the house flying a total distance not exceeding $9 \cdot OA$. Can the constant 9 be improved?