# Geometry Lvl 4

AoMP TEAM

July 2022

This file has 10 geometry problems with different difficulty levels and with different ideas.

The problems are sorted by difficulty level (P1 IMO, P1+ IMO, P2 IMO, P2+ IMO, P3 IMO). In my opinion solve the problems from the easiest to the hardest and if you can't solve any problem in one of the difficulty levels don't skip that level.

## P1 IMO

**1** Sharygin Let $I$ be the incenter of triangle $ABC$, and $M, N$ be the midpoints of arcs $ABC, BAC$ of it's circuncircle. prove that points $M, I, N$ are collinear if and only if

$$AC + CB = 3AB.$$

**2** Let $ABC$ be an acute triangle. A line perpendicular on $BC$ cuts $BC, AC, AB$ in $D, E, F$. Prove that the orthocenter of triangles $ABC, AEF$ and $D$ are collinear.

## P1+ IMO

**3** Let $ABCDEF$ convex hexagon such that the triangles $ACE, BDF$ have the same centroid $(G)$.The points $X, Y, Z$ lie on $AD, BE, CF$ such that

$$\frac{AX}{DX} = \frac{EY}{YB} = \frac{CZ}{ZF}.$$

Prove that the point $G$ is the centroid of the triangle $XYZ$.

**4** RMM Let $ABC$ be an acute triangle $(AB \neq AC)$ with incenter and circumcenter $I, O$. Let the incircle touch $BC, AC, AB$ at $D, E, F$. Assume that the line through $I$ parallel to $EF$, the line through $D$ parallel to $AO$, and the altitude from $A$ are concurrent. Prove that the concurrency point is the orthocenter of the triangle $ABC$.

## P2 IMO

**5** Ukraine TST Let $ABC$ be an acute triangle with circumcenter and centroid $O, G$, The perpendicular bisectors of the segments $GA, GB, GC$ intersect at points $D, E, F$. Prove that $O$ is the centroid of the triangle $DEF$.

**6** NSMO Let $ABC$ be a triangle. The circle $(O)$ passes through the points $B, C$ and cuts $AB, AC$ again at $D, E$. $H$ is the intersection of the segments $BE$ and $CD$. Points $F, G$ lie on $AB, AC$ such that

$$AD = BF,\ AE = CG.$$

$K$ is the circumcenter of triangle $AFG$. Prove that $AK$ parallel to $HO$.

## P2+ IMO

**7** Iran TST Let $ABC$ be a triangle with circumcenter $O$. Points $X, Y$ lie on $AB, AC$ such that reflection of $BC$ on $XY$ is tangent to the circumcircle of triangle $AXY$. Prove that the circumcircle of triangle $AXY$ tangent to the circumcircle of triangle $BOC$.

**8** Iran TST Let $ABC$ be a triangle,arbitrary points $P, Q$ lie on side $BC$ such that $BP = CQ$ and $P$ lies between $B, Q$. The circumcircle of triangle $APQ$ intersects sides $AB, AC$ at $E, F$. The point $T$ is the intersection of $EP, FQ$. Two lines passing through the midpoint of $BC$ and parallel to $AB, AC$ intersect $EP, FQ$ at points $X, Y$. Prove that the circumcircle of triangle $TXY$ and triangle $APQ$ are tangent to each other.

## P3 IMO

**9** Let $ABC$ be an acute triangle with centroid $G$, orthocenter $H$. The line passing through $G$ and the projection of $A$ on $BC$ cuts the circumcircle of triangle $ABC$ at $A'$. Defined $B', C'$ similarly. Prove that $AA', BB', CC', GH$ are concurrent.

**10** Ukraine TST Let $ABC$ be an acute triangle with incircle $\Gamma$ touch sides $BC, CA, AB$ at $D, E, F$. Let $I_a$ be the excenter opposite to $A$ in triangle $ABC$. Define $G$ as the centroid of triangle $DEF$. Let $H$ be the projection of $D$ on $EF$. Prove that $GH, I_aD, \Gamma$ intersect in one point.