

Preparation for Saudi Arabia Team 2021
June Session: Junior Balkan Mathematics Olympiad

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Lesson 6

Polynomial Diophantine Equations

Problems:

1. Find all prime numbers p, q, r and s such that:

$$pq + pr + qr = 12s + 1.$$

2. Find all integer solutions to the equation:

$$x^2 = y^2(x + y^4 + 2y^2).$$

3. Find all pairs of integers (m, n) , such that:

$$(m + n^2)(m + 1) = 4mn.$$

4. Find all prime number p such that there exist positive integers n, x and y such that $p^n = x^3 + y^3$.

5. Find all integers a and b such that $2^a + 17 = b^4$.

6. Determine all triples (m, n, p) satisfying

$$n^{2p} = m^2 + n^2 + p + 1,$$

where m and n are positive integers and p is a prime number.

7. Determine all prime numbers $p_1, p_2, \dots, p_{12}, p_{13}, p_1 \leq p_2 \leq \dots \leq p_{12} \leq p_{13}$, such that:

$$p_1^2 + p_2^2 + \dots + p_{12}^2 = p_{13}^2$$

and one of the primes is equal to $2p_1 + p_9$.

8. Show that there are no positive integers x and y such that $x^7 + 7 = y^2$.

9. Find all integers x and y such that:

$$x^2 + x = y^3 + y^2 + y.$$