Geometric Fundamentals

A Few Practice Problems From 2020 Olympiads

Alex Song

1. (Czech-Polish-Slovak 2020) Let $ABCD$ be a parallelogram whose diagonals meet at P. Denote by M the midpoint of AB. Let Q be a point such that QA is tangent to the circumcircle of MAD and QB is tangent to the circumcircle of MBC. Prove that points Q, M, P are collinear.

2. (IberoAmerican 2020) Let ABC be an acute scalene triangle such that $AB < AC$. The midpoints of sides AB and AC are M and N, respectively. Let P and Q be points on the line MN such that $\angle CBP = \angle ACB$ and $\angle QCB = \angle CBA$. The circumscribed circle of triangle ABP intersects line AC at D ($D \neq A$) and the circumscribed circle of triangle AQC intersects line AB at E ($E \neq A$). Show that lines BC, DP, and EQ are concurrent.

3. (USAMTS 2020) Let ABC be a triangle with $AB < AC$. T is the point on $\overline{BC}$ such that $\overline{AT}$ is tangent to the circumcircle of $\triangle ABC$. Additionally, H and O are the orthocenter and circumcenter of $\triangle ABC$, respectively. Suppose that $\overline{CH}$ passes through the midpoint of $\overline{AT}$. Prove that $\overline{AO}$ bisects $\overline{CH}$.

4. (Cono Sur 2020) Let ABC be an acute triangle such that $AC < BC$ and ω its circumcircle. M is the midpoint of BC. Points F and E are chosen in AB and BC, respectively, such that $AC = CF$ and $EB = EF$. The line AM intersects ω in $D \neq A$. The line DE intersects the line FM in G. Prove that G lies on ω.

5. (Global Quarantine Math Olympiad 2020) Let ABC be an acute scalene triangle, with the feet of A, B, C onto BC, CA, AB being D, E, F respectively. Let W be a point inside ABC whose reflections over BC, CA, AB are W_a, W_b, W_c respectively. Finally, let N and I be the circumcenter and the incenter of $W_aW_bW_c$ respectively. Prove that, if N coincides with the nine-point center of DEF, the line WI is parallel to the Euler line of ABC.

More problems here: https://sites.google.com/site/imocanada/2017-winter-camp