Test 1

August 26, 2020

Problem 1. *Let* x, y, z *be positive real numbers with* $x+y+z \geq 1$*. Prove that* $\frac{x\sqrt{x}}{y+z} + \frac{y\sqrt{y}}{z+x} + \frac{z\sqrt{z}}{x+y} \geq \frac{\sqrt{3}}{2}$

Problem 2. *An* 2020×2020 *board is filled with "+" and "-". One is allowed to take a turn and change the signs of every element of a chosen row or column. It is known that it is possible to take several turns and make all the elements of the board "+". Prove that one is able to make all elements of the board "+" by taking no more than* 2020 *turns.*

Problem 3. *Find all the triples of positive integers* (x, y, p) *so that* p *is prime and* $x(y^2 - p) + y(x^2 - p) = 5p$*.*

Problem 4. *Let* a, b, c *be real positive numbers satisfying* $a + b + c = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$*. Prove that* $\frac{1}{(2a+b+c)^2} + \frac{1}{(a+2b+c)^2} + \frac{1}{(a+b+2c)^2} \leq \frac{3}{16}$*.*