

Inequalities on convex functions

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Problem 1. Compute the first order derivatives of the following functions: $f(x) = x^2 \sin(x), \frac{x^4}{\cos(x)}, x/\tan(x), \cot(x) + x^5, x^2 \log(x)^2, \log(x)^3, \sqrt[5]{x} + \sqrt[3]{x^2}$.

Problem 2. Let n be an integer. Prove that $\sqrt{1^2+1} + \dots + \sqrt{n^2+1} \geq \frac{n}{2}\sqrt{n^2+2n+5}$.

Problem 3. Prove that if k is a non-negative integer then $2^{3^k} + 1$ is divisible by 3^{k+1} but is not divisible by 3^{k+2} .

Problem 4. Prove that if a, b, m, n are positive integers and $\gcd(a, b) = 1$, then $(a^m - b^m, a^n - b^n) = a^{(m,n)} - b^{(m,n)}$. Under which conditions on m and n we can claim that $(a^m + b^m, a^n + b^n) = a^{(m,n)} + b^{(m,n)}$?

Problem 5. It is known that $a > b > 0$ and $ab(a+b)$ is divisible by $a^2 + ab + b^2$. Prove that $a - b > \sqrt[3]{ab}$.