TEAM SELECTION TEST
INTRERNATIONAL MATHEMATICAL OLYMPIAD
Day 1, May 2, 2021

Problem 1. For a non-empty set $\mathcal{T}$ denote by $p(\mathcal{T})$ the product of all elements of $\mathcal{T}$. Does there exist a set $\mathcal{T}$ of 2021 elements such that for any $a \in \mathcal{T}$ one has that $p(\mathcal{T}) - a$ is an odd integer. Consider two cases:

1. All elements of $\mathcal{T}$ are irrational numbers.
2. At least one element of $\mathcal{T}$ is a rational number.

Problem 2. Find all positive integers n, such that n is a perfect number and $\varphi(n)$ is power of 2.

NOTE 1. Positive integer n is called perfect if the sum of all its positive divisors is equal to $2n$.

NOTE 2. $\varphi(n)$ is the number of positive divisors of n.

Problem 3. Let I and I_A be the incenter and the A-excenter of an acute-angled triangle ABC with $AB < AC$. Let the incircle meets BC at D, and the line AD meets BI_A and CI_A at E and F, respectively. Prove that the circumcircles of triangles AID and I_AEF are tangent to each other.