Inequalities on convex functions

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Problem 1. *Compute the first and the second order derivatives of the following functions and comment on the convexity (concavity):* $f(x) = x^2 cos(x), \frac{x^3}{e^x}, x/sin(x), tan(x) + x^3, xlog(x), log(x)^2, log(x^2), \sqrt{x} + \sqrt[3]{x^2}$.

Problem 2. *Let* ABC *be a triangle. Prove that* $sinA + sinB + sinC \leq \frac{3\sqrt{3}}{2}$ *and* $cosAcosBcosC \leq \frac{1}{8}$.

Problem 3. *Let* $x_1, ..., x_n$ *be positive real numbers. Prove that* $(\frac{x_1+...+x_n}{n})^{x_1+...+x_n} \leq {x_1}^{x_1}...{x_n}^{x_n}$.

Problem 4. *Let* x, y, z *be positive real numbers with* $x + y + z = xyz$. *Prove that* $\frac{1}{1+xy} + \frac{1}{1+yz} + \frac{1}{1+zx} \leq \frac{3}{4}$.