BALKAN MATHEMATICAL OLYMPIAD

TEAM SELECTION TEST

Day 2 , October 16, 2020

Problem 5. Find all prime numbers a and b such that

$$20a^3 - b^3 = 1.$$

Problem 6. Let $1 \leq x, y, z \leq 2$. Find the minimal and maximal possible values for

$$\frac{3y}{z} + \frac{2x}{y} + \frac{z}{x}.$$

Problem 7. 100 couples are invited to a traditional Moldovan dance. The 200 people stand in a line, and then in a step, two of them (not necessarily adjacent) may swap positions. Find the least C such that whatever the initial order, they can arrive at an ordering where everyone is dancing next to their partner in at most C steps.

Problem 8. Let H is the orthocenter of an acute triangle ABC. Let o is the center of ω which is the circumcircle of ABC. The line AO intersects ω at point D. Let D_1, D_2 and H_2, H_3 be the symmetrical points of the points D and H with respect to the lines AB and AC respectively. Let ω_1 be the circumcircle of the triangle AD_1D_2. Suppose that the line AH intersects again ω_1 at the point U, the line H_2H_3 intersects the segments D_1D_2 at the points K and the line DH_3 intersects the segment UD_2 at the point L. Prove that one of the intersection points of the circumcircles of the triangles D_1KH_2 and UDL lies on the line KL.