INTERNATIONAL MATHEMATICAL OLYMPIAD
TEAM SELECTION TEST

Day 2, April 17, 2022

Problem 4. Let $ABCD$ be a parallelogram such that $AC = BC$. A point P is chosen on the extension of the segment AB beyond B. The circumcircle of the triangle ACD meets the segment PD again at Q, and the circumcircle of the triangle APQ meets the segment PC again at R. Prove that the lines CD, AQ and BR are concurrent.

Problem 5. Given a positive integer n, find the smallest value of

$$\left\lfloor \frac{a_1}{1} \right\rfloor + \left\lfloor \frac{a_2}{2} \right\rfloor + \ldots + \left\lfloor \frac{a_n}{n} \right\rfloor$$

over all permutations $(a_1, a_2, \ldots, a_n)$ of $(1, 2, \ldots, n)$.

Problem 6. The kingdom of Anisotropy consists of n cities. For every two cities there exists exactly one direct one-way road between them. We say that a path from X to Y is a sequence of reaods such that one can move from X to Y along this sequence without returning to an already visited city. A collection of paths is called diverse if no road belongs to two or more paths in the collection.

Let A and B be two distinct cities in Anisotropy. Let N_{AB} denote the maximal number of paths in a diverse collection of paths from A to B. Similarly, let N_{BA} denote the maximal number of paths in a diverse collection of paths from B to A. Prove that the equality $N_{AB} = N_{BA}$ holds if and only if the number of roads going out from A is the same as the number of roads going out from B.