Day 1

July 6, 2022

Problem 1.1 Yousif has a 13×9 board, such that 7 random cells are colored black and the rest of the squares (110) are colored white, in each move Yousif chooses a 2×2 square inside the board and switches each of its 4 cells (from black to white or from white to black), can the board be completely colored white after a finite number of moves?

Problem 1.2 Hamoody and Abdulkareem play a game. Hamoody writes 37 distinct positive integers on the board such that their sum is less than 1000. Then Abdulkareem walks in, Hamoody wins if Abdulkareem can't find two numbers written on the board that add up to 37, otherwise Abdulkareem wins. Which player has a winning strategy and why?

Problem 1.3 How many ways are there to form a word containing k letters only using the letters 'a' and 'b', such that after each letter 'a' there is at least one letter 'b'? Solve the problem for

(a) $k = 4$ **(b)** $k = 5$ **(c)** $k \geq 6$

Day 2

July 7, 2022

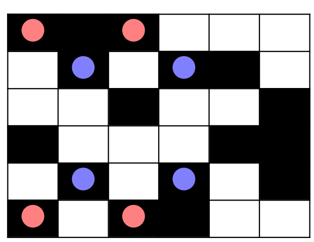
Problem 2.1 A convex polygon P lies strictly inside another convex polygon Q. Prove that the perimeter of Q is greater than the perimeter of P.

Problem 2.2 Miseed and Moath play a game. The numbers from 1 to 100 are written on the board. Moath then erases k numbers from the board. Then Miseed walks in, Miseed wins if he can find k numbers on the board that add up to 100, otherwise Moath wins. Who has a winning strategy for

(a) $k = 9$ **(b)** $k = 8$ and why?

Problem 2.3 Ahmed and Amer play a game on a 6×6 board. Initially all the cells are colored white, then Amer picks k cells and colors them black. Then Ahmed walks in, Ahmed wins if he can find 4 black squares that form the corners of a rectangle or a square (as shown in the figure below), otherwise Amer wins. Who has a winning strategy for

(a) $k = 16$ **(b)** $k = 17$ and why?

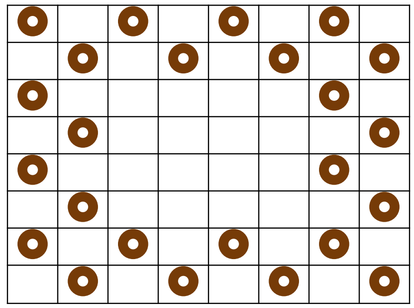


Day 3

July 8, 2022

Problem 3.1 Tournament of Towns Junior A-Level Fall 2005 Omar starts with an 8×8 board and places a rook on each cell. Step by step, Omar removes a rook ♖ if it attacks an odd number of other rooks ♖. What is the maximal number of rooks that can be removed by Omar?

Problem 3.2 Several checkers are placed on a board. Each turn, a checker may jump diagonally over an adjacent piece if the opposite square is empty. If a checker is jumped over in this way, it is removed from the board. Is it possible to make a sequence of such jumps to remove all but one checker from the board shown below?



Problem 3.3 MOSP 2007 In a $n \times n$ square, each of the numbers $1, 2, ..., n$ appear exactly n times. Show that there is a row or column that contains at least $\sqrt{n}$ distinct numbers.

Day 4

July 16, 2022

Problem 4.1 Can the vertices of a regular 30-gon be labeled with numbers $1, 2, \ldots, 30$ in such a way that the sum of labels of every pair of neighboring vertices is a perfect square?

Problem 4.2 Denote A_n as the number of strings of length n only using letters 'a' and 'b', such that neither 'aba' nor 'bab' appears. Denote B_n as the number of strings of length n only using letters 'a' and 'b', such that neither 'aa' nor 'bb' appears. Prove that $A_n = 2B_{n-1}$.

Problem 4.3 A rectangle R is tiled with rectangles such that each these rectangle has an integer side. Prove that R also has an integer side.

Day 5

July 17, 2022

Problem 5.1 Elyas and Lukasz play a game on a $2n \times 2n$ board. Elyas covers the board by dominos. Then Lukasz walks in, lukasz wins if he can cut the board into two parts by a straight line that doesn't cut any domino, otherwise Elyas wins. Who has a wining strategy for

(a) $n = 2$ **(b)** $n \geq 4$ **(c)** $n = 3$ and why?

Problem 5.2 Find the number of sets $(a_1, a_2, \ldots, a_n)$ such that $a_i \in \mathbb{Z}$ and $0 \leq a_i \leq a_j \leq m \; \forall \; i \leq j$.

Problem 5.3 Cono Sur 2011 Let Q be a $2n + 1 \times 2n + 1$ board. Some of its cells are colored black in such a way that every 2×2 board of Q contains at most 2 black cells. Find the maximum amount of black cells that the board may have.

Day 6

July 18, 2022

Problem 6.1 Safwat places a knight onto an 8×8 board. Then Smbat moves the knight, Then Safwat makes a move, but he may not place it on a square visited before, and so on. The loser is the one who cannot move, Who has a winning stratagy and why?

Problem 6.2 Tournament of Towns 1988 An infinite chessboard has the shape of the first quadrant. Is it possible to write a positive integer into each square, such that each row and each column contains each positive integer exactly once?

Problem 6.3 Czech-Polish-Slovak Match 2018 In a 2×3 rectangle there is a poly-line of length 36, which can have self-intersections. Show that there exists a line parallel to two sides of the rectangle, which intersects the other two sides and intersects the poly-line in fewer than 10 points.

Day 7

July 19, 2022

Problem 7.1 Tournament of Towns Junior A-Level Spring 2002 Dominik and Nikola play a game on a 99×99 board. Dominik controls two white chips in the bottom left and top right corners. Nikola controls two black chips in the bottom right and top left corners. The players move alternately (Dominik starts). In each move, a player moves one of his chips to an adjacent square (by side). Dominik wins if he can make his two white chips adjacent (by side). Can Nikola prevent Dominik from winning?

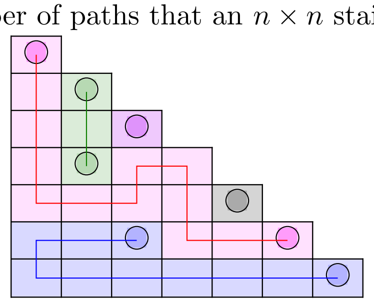
Problem 7.2 Tournament of Towns Junior A-Level Fall 2001 Let $n \geq 3$ be an integer. Each row in an $(n-2) \times n$ array consists of the numbers $1, 2, \ldots, n$ in some order, and the numbers in each column are all different. Prove that this array can be expanded into an $n \times n$ array such that each row and each column consists of the numbers $1, 2, \ldots, n$.

Problem 7.3 Romania JBMO TST 2011 We consider an $n \times n$ ($n \geq 2$) grid. Determine all the values of $k \in \mathbb{N}$ for which we can write a real number in each square such that the sum of the n^2 numbers is positive, while the sum of the numbers from any $k \times k$ square is negative.

Day 8

July 21, 2022

Problem 8.1 Consider an $n \times n$ staircase, which consists of the squares on or below the main diagonal of an $n \times n$ grid. A path is a sequence of distinct squares, every two consecutive of which share an edge. What is the minimum number of paths that an $n \times n$ staircase can be partitioned into?



Problem 8.2 Over half of the squares of a 7×7 board are occupied by rooks. Prove that there is a rook that is ”encircled” (there is a rook attacking it from each of the four directions).

Problem 8.3 Can three faces of a $8 \times 8 \times 8$ cube having a common vertex be covered with 64 1×3 strips (The strips can be folded along the edges of the cube)?

Day 9

July 22, 2022

Problem 9.1 Tournament of Towns Junior A-Level Spring 2001 Several non-intersecting diagonals divide a convex polygon into triangles. At each vertex the number of triangles adjacent to it is written. Can you always reconstruct all the diagonals using these numbers if the diagonals are erased?

Problem 9.2 Tournament of Towns Junior A-Level Fall 2001 Dusan places a rook on any square of an empty 8×8 chessboard. Then he places additional rooks one rook at a time, each attacking an odd number of rooks which are already on the board. What is the maximum number of rooks Dusan can place on the chessboard?

Problem 9.3 In an $n \times n$ grid, at least $2n$ squares are marked. Prove that there is a sequence $P_1, P_2, \ldots, P_k$ of centers of marked squares such that the segments P_iP_{i+1} alternate between horizontal and vertical for all $1 \le i \le k$ where $P_{k+1} = P_1$.

Day 10

July 23, 2022

Problem 10.1 In the three dimensional lattice plane nine points are marked. Prove that the midpoint of the segment connecting some two of the marked points also lies on the lattice grid.

Problem 10.2 Tournament of Towns Junior A-Level Fall 2013 On a table, there are 11 piles of ten stones each. Marwan and Hamza play the following game. In turns they take $1, 2$ or 3 stones at a time: Marwan takes stones from any single pile while Hamza takes stones from different piles but no more than one from each. Marwan moves first. The player who cannot move, loses. Which of the players, Marwan or Hamza, has a winning strategy?

Problem 10.3 All Russian Olympiad 2011 Omar and his 99 students play a game on an infinite cell grid. Omar starts first, then each of the 99 students makes a move, then Omar and so on. On one move the person can color one unit segment on the grid. A segment cannot be colored twice. Omar wins if, at some moment, there is a 1×2 or 2×1 rectangle such that each segment from it's border is colored, but the segment between the two adjacent squares isn't colored. Can Omar grantee a win?

Day 11

September 12, 2022

Problem 11.1 The "Minesweeper" game is played on a 10×10 board. Some cells have a "mine" placed on them, while in the other cells, there is written a number indicating how many of the neighboring cells (that share a vertex) contain a mine. Prove that if we "reverse" the board (put mines on the cells, where there was no mines initially, and fill the numbers similarly in the cells where were mines) then the sum of numbers in the board will remain the same.

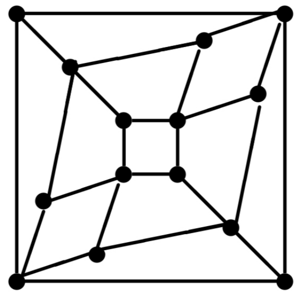
Problem 11.2 On an 8×8 screen with unit pixels. Initially, there are at least 50 pixels which are "on". In any 2×2 square, as soon as there are 3 pixels which are "off", the fourth pixel turns off automatically. Prove that the whole screen can never be totally "off".

Problem 11.3 AIME 1989 Let S be a subset of $\{1, 2, 3, \ldots, 2022\}$ such that no two members of S differ by 4 or 7. What is the largest number of elements S can have?

Day 12

September 15, 2022

Problem 12.1 The following figure shows a road map connecting 14 cities. Is there a path passing through each city exactly once?



Problem 12.2 Baltic Way 1998 Can we tile a 13×13 table from which we remove the central unit square using only 1×4 or 4×1 rectangles?

Problem 12.3 IMO shortlist 1998, Romanian TST 1997 Let p, q, r be distinct prime numbers and let

$$A = \{p^a q^b r^c \mid 0 \leq a, b, c \leq 7\}$$

Find the least $n \in \mathbb{N}$ such that for any $B \subset A$ where $|B| = n$, has elements x and y such that x divides y.

Day 13

September 17, 2022

Problem 13.1

Problem 13.2

Problem 13.3