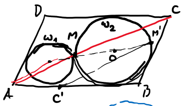
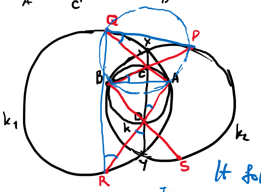
1



H_M: $\omega_1 \to \omega_2$
$AB \to CD$
$AD \to BC$
$A \to C$

2.



$CB \cdot CP = CX \cdot CY = CA \cdot CQ$

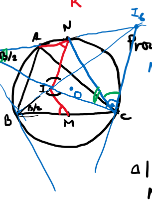
$\angle AQP = \angle ABP = \angle ABC = \angle ADC$
$= \angle ARQ$, $\underline{\text{if}}$ $QR \parallel XY$

$\mathcal{H}_A$: $k \to k_1$
$C \to Q$
$D \to R$
$CD \to QR$

It follows that QR is the common tangent.

3.

$AB < AC$
I = incenter
M = midpoint of BC
N = midpoint of $\widehat{BAC}$



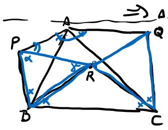
Prove that $\angle ANI = \angle BMI$.

N = midpoint of $I_b I_c$

$\angle I_c CN = \angle BCN - \angle BCI = 90° - \frac{\alpha}{2} - \frac{\gamma}{2} = \frac{\beta}{2}$
$\angle CNI_c = \angle CNA = 180° - \beta \Rightarrow \angle NI_cC = \frac{\beta}{2}$
$\Rightarrow$ N is the circumcenter of $\triangle CI_bI_c$

$\triangle II_bI_c \sim \triangle ICB$ (equal angles)
$N \longleftrightarrow M$
$\Rightarrow \triangle INI_c \sim \triangle IMB \Rightarrow \angle INI_c = \angle IMB$.

4.



$\triangle APB \sim \triangle AQC \sim \triangle BRC$
$AP = PB$, $AQ = QC$, $BR = RC$

Prove that $APRQ$ is a parallelogram

$\frac{CB}{CR} = \frac{CA}{CQ}$, $\angle BCR = \angle ACQ \Rightarrow$

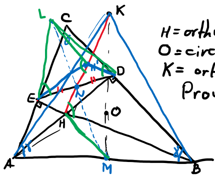
$\frac{CB}{CA} = \frac{CR}{CQ}$, $\angle BCA = \angle RCQ \Rightarrow \triangle ABC \sim \triangle QRC$

Analogously, $\triangle PRR \sim \triangle ABC$

$\left.\begin{array}{l}\angle APR = 180° - 2x - \alpha \\ \angle PAQ = \alpha + 2x\end{array}\right\}$ $AQ \parallel RP$

$AP \parallel QR$ similarly.

5.



H = orthocenter
O = circumcenter
K = orthocenter of $\triangle ABD$
Prove that HK bisects DE.

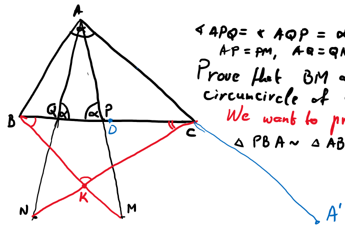
$\angle AOB = 2\gamma \Rightarrow \angle AKB = 180° - 2\gamma$, $\angle KAB = \angle KBA = \gamma$
M = midpoint of AB.

$\triangle ABC \sim \triangle DEC$
$M \longleftrightarrow N$ = midpoint N of DE (we want to show that $K \in HN$)
$H \longleftrightarrow H'$ = orthocenter of $\triangle DEC$: $DHEH'$ is a parallelogram,
$K \longleftrightarrow L$ such that $\angle LDE = \angle LED = \gamma$; $LN \perp DE$

We want to show that $\angle MHK = \angle MHN$. Instead, we have $\angle MHK = \angle NH'L$
So we need $\angle NHM = \angle NH'L$, and this is equivalent to M and L being symmetric wrt. N.

$\angle MDE = 90° - \angle DBE = 90° - (90° - \gamma) = \gamma = \angle LDE \Rightarrow \triangle MDE \cong \triangle LDE$. End.

6.



$\angle APQ = \angle AQP = \alpha$
$AP = PM$, $AQ = QN$

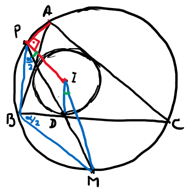
Prove that BM and CN meet on the circumcircle of ABC.

We want to prove that $\angle BKC = 180° - \alpha$.

$\triangle PBA \sim \triangle ABC \sim \triangle QAC$.

$\angle ADB = \angle ACN$
$\angle ADC = \angle ABM$
$\Rightarrow \angle ACN + \angle ABM = 180°$

7.



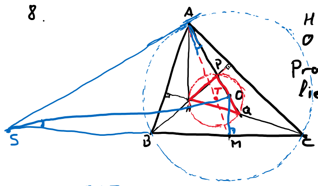
M = midpoint of $\widehat{BC}$
Prove that $\angle API = 90°$.

$\triangle MDB \sim \triangle MBP$, so $MD \cdot MP = MB^2 = MI^2$
$\Rightarrow \triangle MDI \sim \triangle MIP$

Thus $\angle MPI = \angle MID = \frac{\beta - \gamma}{2}$

$\angle MPA = \angle MBA = \frac{\alpha}{2} + \beta \Rightarrow \angle API = \frac{\alpha}{2} + \beta - \frac{\beta - \gamma}{2} = 90°$.

8.



H = orthocenter
O = circumcenter

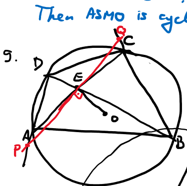
Prove that the circumcenter T of HPQ lies on the median from A in $\triangle ABC$.

$\triangle HPQ \sim \triangle ABC$
$T \longleftrightarrow O$
$A \longleftrightarrow S$

AH is tangent to circle HPQ.
AS = tangent to circle ABC

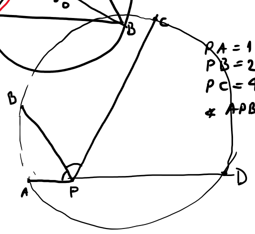
$\angle PAT = \angle BSO$; Redefine M as the intersection of AT and BC.
Then $ASMO$ is cyclic. $\angle OMS = 180° - \angle OAS = 90° \Rightarrow M$ = midpoint.

9.



The perp. to OE at E meets AD at P, BC at Q.
Prove that $EP = EQ$.

10.



$PA = 1$
$PB = 2$
$PC = 4$
$\angle APB = \angle BPC$

Circle ABC meets AP at D.
Find AD.