TEAM SELECTION TEST
JUNIOR BALKAN MATHEMATICAL OLYMPIAD
Day 2, May 19, 2021

Problem 1. Let $(a_n)_{n\geq 1}$ be a sequence given by $a_1 = 45$ and $a_n = a_{n-1}^2 + 15a_{n-1}$ for $n > 1$. Prove that the sequence contains no perfect squares.

Problem 2. In a triangle ABC the point K on the median BM such that $CM = CK$. It turned out that $\angle CBM = 2\angle ABM$. Show that $BC = KM$.

Problem 3. We have $n > 2$ nonzero integers such that everyone of them is divisible by the sum of the other $n-1$ numbers, Show that the sum of the n numbers is precisely 0.

Problem 4. Let us call a set of positive integers *nice* if the number of its elements equals to the average of its numbers. Call a positive integer n an *amazing* number if the set $\{1, 2, , ..., n\}$ can be partitioned into nice subsets.
(a) Prove that every perfect square is amazing
(b) Show that there are infinitely many positive integers which are not amazing.