**Problem 1C.** Let $x_1, x_2, \ldots, x_n$ be real numbers such that $|x_i| \leqslant \frac{n+1}{2}$ for $1 \leqslant i \leqslant n$ and $|x_1 + x_2 + \cdots + x_n| = 1$. Prove that there exists a permutation $(y_1, y_2, \ldots, y_n)$ of the numbers $x_1, x_2, \ldots, x_n$ such that

$$|x_1 + 2x_2 + \cdots + nx_n| \leqslant \frac{n+1}{2}.$$

**Problem 2C.** Let $p, q, n$ be positive integers such that $p+q < n$. Let $(x_0, x_1, \ldots, x_n)$ be an $(n+1)$-tuple of integers satisfying the following:

a) $x_0 = x_n = 0$;

b) for each $i$ with $1 \leqslant i \leqslant n$ either $x_i - x_{i-1} = p$ or $x_i - x_{i-1} = -q$.

Prove that there exists a pair $(i, j)$ of distinct indices with $(i, j) \neq (0, n)$, such that $x_i = x_j$.

**Problem 3C.** Let $a_0, a_1, a_2, \ldots$ be a sequence of real numbers such that $a_0 = 0$, $a_1 = 1$, and for every $n \geqslant 2$ there exists $1 \leqslant k \leqslant n$ satisfying

$$a_n = \frac{a_{n-1} + a_{n-2} + \ldots + a_{n-k}}{k}.$$

Find the maximal possible value of $a_{2018} - a_{2017}$.

**Problem 4C.** An infinite sequence $a_0, a_1, a_2, \ldots$ of real numbers satisfies the condition

$$a_n = |a_{n+1} - a_{n+2}| \quad \text{for all } n \geqslant 0,$$

with $a_0$ and $a_1$ positive and distinct. Can this sequence be bounded?

**Problem 5C.** Let $n$ be fixed positive integer. Find the maximum possible value of

$$\sum_{1 \leqslant r < s \leqslant 2n} (s - r - n) x_r x_s,$$

where $-1 \leqslant x_i \leqslant 1$ for all $i = 1, 2, \ldots, 2n$.

**Problem 6C.** Let $n \geqslant 2$ be a positive integer and $a_1, a_2, \ldots, a_n$ be real numbers such that

$$a_1 + a_2 + \cdots + a_n = 0.$$

Define the set $A$ by

$$A = \{(i, j) \,:\, 1 \leqslant i < j \leqslant n,\ |a_i - a_j| \geqslant 1\}.$$

Prove that, if $A$ is not empty, then

$$\sum_{(i,j)\in A} a_i a_j < 0.$$

**Problem 7C.** Find all positive integers $n \geqslant 3$ for which there exist real numbers $a_1, a_2, \ldots, a_n$ ($a_{n+1} = a_1$, $a_{n+2} = a_2$) such that

$$a_i a_{i+1} + 1 = a_{i+2}$$

for $i = 1, 2, \ldots, n$.