Combinatorics

C1 A convex polygon is dissected into isosceles triangles by several non-intersecting diagonals. Prove that this polygon has two sides of equal lengths.

C2 There are 2018 boxes numbered from 1 to 2018, some of which contain stones. Two players, A and B, play alternately, starting with A. A move consists in selecting a non-empty box i, taking one or more stones from that box and putting them in box $i+1$. If $i = 2018$, the selected stones are eliminated. The player who removes the last stone wins. If there is exactly one stone in each box, who wins?

C3 In a country there are two-way non-stop flights between some pairs of cities. Any city can be reached from any other by a sequence of at most 100 flights. Moreover, any city can be reached from any other by a sequence of an even number of flights. What is the smallest d for which one can always claim that any city can be reached from any other by a sequence of an even number of flights not exceeding d?

Number Theory

N1 Do there exist two distinct powers of 2 with the same number of digits such that one can be obtained by permuting the digits of the other?

N2 There is an infinite set S of natural numbers such that for any two different numbers $a, b \in S$, the set S also contains at least one of the two numbers $a^b - 2$ or $a^b + 2$. Prove that there is at least one composite number in S.

N3 A nonnegative integer is called *uphill* if its decimal digits are non-decreasing from left to right (0 is considered to be uphill). A polynomial $P(n)$ has rational coefficients and $P(n)$ is an integer for every uphill number n. Is it necessarily true that $P(n)$ is an integer for all integers n?

Algebra

A1 A set S of real numbers satisfies:

- If a is in S and $a \neq 0$ then $1/a$ is in S; and
- If a and b are in S then $a+b$ and $a-b$ are in S.

If $x \neq 0$ is in S, prove that x^{2017} is also in S.

A2 Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(xy + f(x)) = xf(y) + f(x) \quad \text{for all } x, y \in \mathbb{R}$$

A3 Let $Q(x)$ be a quadratic polynomial having two distinct real zeros. Prove that there is a non-constant monic polynomial $P(x)$ with real coefficients such that all coefficients of the polynomial $Q(P(x))$ except the leading one have absolute value less than 0.001.

Geometry

G1 Let ABC be an acute triangle and D be on side BC such that $AB = AD$. The circumcircle of triangle ABD intersects AC at the point K. If L is the intersection of DK and the perpendicular from B to AC, prove that $CL = BC$.

G2 An equilateral triangle ABC is inscribed into a circle Ω and circumscribed about a circle ω. The points P and Q are chosen on the sides AC and AB, respectively, so that the segment PQ is tangent to ω. A circle Ω_b centered at P passes through B, and a circle Ω_c centered at Q passes through C. Prove that the circles Ω, Ω_b and Ω_c have a common point.

G3 Given a finite set S of points in a plane, prove that it is possible to remove at most one point from the set and partition the remaining points into two sets A, B each with a smaller largest distance than the original set (the largest distance of a set T is the length of the largest line segment between points in T).