TEAM SELECTION TEST
INTRERNATIONAL MATHEMATICAL OLYMPIAD
Day 2, May 3, 2021

Problem 1. In a regular 100-gon, 41 vertices are colored black and the remaining 59 vertices are colored white. Prove that there exist 24 convex quadrilaterals Q_1, Q_2, …, Q_{24} whose corners (vertices of the quadrilateral) are vertices of the 100-gon, so that

- the quadrilaterals Q_1, Q_2, …, Q_{24} are pairwise disjoint, and
- every quadrilateral Q_i has three corners of one color and one corner of the other color.

Problem 2. Let ABC be a non isosceles triangle with incenter I and let the circumcircle of the triangle ABC has radius R. Let AL is the external angle bisector of $\angle BAC$ with $L \in BC$. Let K is the point on the perpendicular bisector of BC such that $IL \perp IK$. Prove that $OK = 3R$.

Problem 3. Determine all functions $f\ : \mathbf{Z} \ \to\ \mathbf{Z}$ such that

$$f^{(a^2+b^2)}(a+b) = af(a) + bf(b)$$

for every $a, b \in \mathbf{Z}$. Here, f^n denotes the n^{th} iteration of f, i. e. $f^{(0)}(x) = x$ and $f^{(n+1)}(x) = f(f^{(n)}(x))$ for all $n \geq 0$.