WARMUP COMBINATORICS PROBLEMS

First, here are some warmup warmup problems:

(1) (Taiwan preliminary) Let a board game has 10 cards: 3 skull cards, 5 coin cards and 2 blank cards. We put these 10 cards downward and shuffle them and take cards one by one from the top. Once 3 skull cards or coin cards appears we stop. What is the possibility of it stops because there appears 3 skull cards?

(2) (Brazil EGMO TST) The duchess is a chess piece such that the duchess attacks all the cells in two of the four diagonals which she is contained(the directions of the attack can vary to two different duchesses). Determine the greatest integer n, such that we can put n duchesses in a table 8×8 and none duchess attacks other duchess.

(3) How many sets of 10 distinct positive integers sum to at most 2021?

(4) (Russia) On a circle there're 1000 marked points, each colored in one of k colors. It's known that among any 5 pairwise intersecting segments, endpoints of which are 10 distinct marked points, there're at least 3 segments, each of which has its endpoints colored in different colors. Determine the smallest possible value of k for which it's possible.

(5) (Putnam 2013) Recall that a regular icosahedron is a convex polyhedron having 12 vertices and 20 faces; the faces are congruent equilateral triangles. On each face of a regular icosahedron is written a nonnegative integer such that the sum of all 20 integers is 39. Show that there are two faces that share a vertex and have the same integer written on them.

Now, here are some warmup problems:

(1) (CHKMO) There is a table with n rows and 18 columns. Each of its cells contains a 0 or a 1. The table satisfies the following properties:
   (a) Every two rows are different.
   (b) Each row contains exactly 6 cells that contain 1.
   (c) For every three rows, there exists a column so that the intersection of the column with the three rows (the three cells) all contain 0.

   What is the greatest possible value of n?

(2) (China TST) Given positive integers n and k, $n > k^2 > 4$. In a $n \times n$ grid, a k-group is a set of k unit squares lying in different rows and different columns. Determine the maximal possible N, such that one can choose N unit squares in the grid and color them, with the following condition holds: in any k-group from the colored N unit squares, there are two squares with the same color, and there are also two squares with different colors.

(3) (Ukraine) There are 101 not necessarily different weights, each of which weighs an integer number of grams from 1 g to 2020 g. It is known that at any division of these weights into two heaps, the total weight of at least one of the piles is no more than 2020. What is the largest number of grams can weigh all 101 weights?

(4) (HK TST) On the table there are 20 coins of weights $1, 2, 3, \ldots, 15, 37, 38, 39, 40$ and 41 grams. They all look alike but their colours are all distinct. Now Miss Adams knows the weight and colour of each coin, but Mr. Bean knows only the weights of the coins. There is also a balance on the table, and each comparison of weights of two groups of coins is called an operation. Miss Adams wants to tell Mr. Bean which coin is the 1 gram coin by performing some operations. What is the minimum number of operations she needs to perform?

Finally, here are some warm problems:

(1) A coin's weighting (probability of landing on heads) is picked uniformly at random in $[0, 1]$. The coin is then tossed 10 times. What is the probability that it lands on heads exactly 5 times?

(2) (Russia) Each girl among 100 girls has 100 balls; there are in total 10000 balls in 100 colors, from each color there are 100 balls. On a move, two girls can exchange a ball (the first gives the second one of her balls, and vice versa). The operations can be made in such a way, that in the end, each girl has 100 balls, colored in the 100 distinct colors. Prove that there is a sequence of operations, in which each ball is exchanged no more than 1 time, and at the end, each girl has 100 balls, colored in the 100 colors.

(3) (OMO) Every two residents of a city have an even number of common friends in the city. One day, some of the people sent postcards to some of their friends. Each resident with odd number of friends sent exactly one postcard, and every other - no more than one. Every resident received no more than one postcard. Prove that the number of ways the cards could be sent is odd.

(4) (Japan) Let n be a positive integer. Find all integers k among $1, 2, \ldots, 2n^2$ which satisfy the following condition: There is a $2n \times 2n$ grid of square unit cells. When k different cells are painted black while the other cells are painted white, the minimum possible number of 2×2 squares that contain both black and white cells is $2n - 1$.