TEAM SELECTION TEST
JUNIOR BALKAN MATHEMATICAL OLYMPIAD
Day 1, May 2, 2021

Problem 1. Find all positive integers a, b, c and prime number p such that

$$73p^2 + 6 = 9a^2 + 17b^2 + 17c^2.$$

Problem 2. In a circle O, there are six points, A, B, C, D, E, F in a counterclockwise order. $BD \perp CF$, and CF, BE, AD are concurrent. Let the perpendicular from B to AC be M, and the perpendicular from D to CE be N. Prove that $AE \parallel MN$.

Problem 3. Consider the sequence $a_1, a_2, a_3, \ldots$ defined by $a_1 = 9$ and

$$a_{n+1} = \frac{(n+5)a_n + 22}{n+3}$$

for $n \geq 1$. Find all positive integers n for which a_n is a perfect square.

Problem 4. Let F is the set of all sequences $\{(a_1, a_2, ..., a_{2020})$ with $a_i \in \{-1, 1\}\}$ for all $i = 1, 2, ..., 2020$. Prove that there exists a set S, such that $S \subset F$, $|S| = 2020$ and for any $(a_1, a_2, ..., a_{2020}) \in F$ there exists $(b_1, b_2, ..., b_{2020}) \in S$, such that $\Sigma_{i=1}^{2020} a_i b_i = 0$.