Final test

September 7, 2020

Problem 1. *Each cell of the $n \times n$ board contains either a "+" or a "-", so that the total number of "+" and "-" in the cells is equal. Prove that either there exist two rows with equal number of "+", or there exist two columns with equal number of "+".*

Problem 2. *a) It is known that the first digits of natural integers x, x^2, x^3 are the same. Is it true that the first digit of x is equal to 1? (e. g. the first digit of 257 is 2)*
b) It is known that the first digits of natural integers $x, x^2, ..., x^{2020}$ are the same. Is it true that the first digit of x is equal to 1?

Problem 3. *Let p be a prime. A collection of $p+2$ (not necessarily distinct) integers is interesting, if the sum of any of p numbers is divisible by any of the remaining two numbers. Find all the interesting collections.*

Problem 4. *An N-gon with equal sides (but not necessarily regular) lies strictly inside a circle C. Then each side of N-gon is continued till intersection with C. Thus, we have $2N$ line segments, which lie outside the N-gon. Prove that it is possible to color some of these $2N$ line segments with red, and the other ones with blue, so that the total length of the blue line segments is equal to the total length of red line segments.*