INTERNATIONAL MATHEMATICAL OLYMPIAD
TEAM SELECTION TEST

Day 1, April 16, 2022

Problem 1. Let (a_n) be the integer sequence which is defined by $a_1 = 1$ and

$$a_{n+1} = a_n^2 + n \cdot a_n,$$

for all $n \geq 1$. Let S be the set of all primes p such that there exists and index i such that $p | a_i$. Prove that the set S is an infinite set and it isn't equal to the set of all primes.

Problem 2. Let $ABCD$ be a quadrilateral inscribed in a circle Ω. Let the tangent to Ω at D intersect the rays BA and BC at points E and F, respectively. A point T is chosen inside the triangle ABC so that $TE \parallel CD$ and $TF \parallel AD$. Let $K \neq D$ be a point on the segment DF such that $TD = TK$. Prove that the lines AC, DT and BK intersect at one point.

Problem 3. Find all non-constant functions $f : Q^+ \to Q^+$ satisfying the equation

$$f(ab + bc + ca) = f(a)f(b) + f(b)f(c) + f(c)f(a)$$

for all $a, b, c \in Q^+$.