

# Assignment 1

August 24, 2020

**Problem 1.** Let  $C(n)$  be the number of prime divisors of  $n$ . (For example,  $C(10) = 2$ ,  $C(11) = 1$ ,  $C(12) = 2$ ). Prove or disprove: there are infinitely many pairs of positive integers  $(a, b)$  such that  $C(a + b) = C(a) + C(b)$ .

**Problem 2.** The "Minesweeper" game is played on  $10 \times 10$  board. In some of the cell, there are some mines, while in the other cells, there is written a number, indicating how many of the neighboring cells contain a mine (neighboring cells are the ones, which share a vertex). Prove that if we "reverse" the board - that is put mines on the cells, where were no mines initially and fill the numbers similarly in the cells, where were mines, then the sum of numbers in the board will remain the same.

**Problem 3.** There are 20 students in the class. There were several class events organized, so that in each event at least one student participates. Prove that there exists an event, so that every student of that event participated in at least  $1/20$  part of all events. Can  $1/20$  be substituted by a better estimate?