INTERNATIONAL MATHEMATICAL OLYMPIAD
TEAM SELECTION TEST

Day 3, April 19, 2022

Problem 7. For every integer $n \geq 1$ consider the $n \times n$ table with entry $\left\lfloor \frac{i \cdot j}{n+1} \right\rfloor$ at the intersection of row i and column j, for every $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n$. Determine all integers $n \geq 1$ for which the sum of the n^2 entries in the table is equal to $\frac{n^2(n-1)}{4}$.

Problem 8. Let n and k be two integers with $n > k \geq 1$. There are $2n+1$ students standing in a circle. Each student S has $2k$ neighbours - namely, the k students closest to S on the right and the k students closest to S on the left. Suppose that $n+1$ of the students are girls, and the other n are boys. Prove that there is a girl with at least k girls among her neighbours.

Problem 9. Prove that there are only finitely many quadruples (a, b, c, n) of positive integers such that

$$n! = a^{n-1} + b^{n-1} + c^{n-1}.$$