Inequalities

August 25, 2020

Problem 1. *Let a, b, c be positive reals. Prove that $(a^2b + b^2c + c^2a)(ab^2+bc^2+ca^2) \geq 9a^2b^2c^2$. For which values of a, b, c the equality is held?*

Problem 2. *Let a, b, c be positive reals. Prove that $a^5 + b^5 + c^5 \geq a^3bc + b^3ca + c^3ab \geq abc(ab + bc + ca)$. For which values of a, b, c the equality is held?*

Problem 3. *Let a, b, c be positive reals with $abc = 1$. Prove that $a + b + c \leq a^2 + b^2 + c^2$. For which values of a, b, c the equality is held?*

Problem 4. *Let a, b, c be positive reals. Prove that $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$. For which values of a, b, c the equality is held?*

Problem 5. *Let a, b, c, d be positive reals. Prove that $\frac{1}{a} + \frac{1}{b} + \frac{4}{c} + \frac{16}{d} \geq \frac{64}{a+b+c+d}$. For which values of a, b, c, d the equality is held?*

Problem 6. *Let $u_1, u_2, ..., u_{2020}$ be numbers satisfying $u_1 + ... + u_{2020} = 0$ and $u_1^2 + ... + u_{2020}^2 = 1$. Let a be the minimal number and b be the maximal number in the set $\{u_1, ..., u_{2020}$. Prove that, $ab \leq -\frac{1}{2020}$. When does the equality hold?*