TEAM SELECTION TEST

Day 2 , August 23, 2020

Problem 5. Real numbers a and b satisfy $a^3 + b^3 - 6ab = -11$. Prove that

$$-\frac{7}{3} < a + b < -2.$$

Problem 6. Find all prime numbers p and nonnegative integers $x \neq y$ such that

$$x^4 - y^4 = p(x^3 - y^3).$$

Problem 7. In a certain city there are n straight streets, such that every two streets intersect, and no three streets pass through the same intersection. The City Council wants to organize the city by designating the main and the side street on every intersection. Prove that this can be done in such way that if one goes along one of the streets, from its beginning to its end, the intersections where this street is the main street, and the one where it is not, will appear in alternating order.

Problem 8. Let ABC be a right-angled triangle with $\angle A = 90^o$. Let K be the midpoint of BC, and let $AKLM$ be a parallelogram with centre C. Let T be the intersection of the line AC and the perpendicular bisector of BM. Let ω_1 be the circle with centre C and radius CA and ω_2 be the circle with centre T and radius TB. Prove that one of the points of intersection of ω_1 and ω_2 is on the line LM.