A1. Given three reals such that $a_1, a_2, a_3 > 1$ and $S = a_1 + a_2 + a_3$. Provided $\frac{a_i^2}{a_i - 1} > S$ for every $i = 1, 2, 3$ prove that

$$\frac{1}{a_1 + a_2} + \frac{1}{a_2 + a_3} + \frac{1}{a_3 + a_1} > 1$$

A2. Are there 2000 not necessarily distinct nonzero real numbers such that for any group of 1000 of these numbers there is a polynomial with these numbers as its roots, with lead coefficient 1 and its other coefficients a permutation of the remaining 1000 numbers?

A3. The function $f : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 0}$ satisfies the following properties for all $a, b \in \mathbb{R}^{\geq 0}$:

1. $f(a) = 0$ if and only if $a = 0$
2. $f(ab) = f(a)f(b)$
3. $f(a + b) \leq 2 \max\{f(a), f(b)\}$

Prove that for all $a, b \in \mathbb{R}^{\geq 0}$ we have $f(a + b) \leq f(a) + f(b)$.

C1. There are several cities in a country. Some pairs of cities are connected by two-way flights each operated by one of N companies. Each company serves exactly one flight from each city, and it is possible to travel between any two cities through a sequence of flights. During a financial crisis, $N - 1$ flights from different companies have been cancelled. Prove that it is still possible to travel between any two cities.

C2. Let x be an irrational number between 0 and 1 and $x = 0.a_1a_2a_3\cdots$ its decimal representation. For each $k \geq 1$, let $p(k)$ denote the number of distinct sequences $a_{j+1}a_{j+2}\cdots a_{j+k}$ of k consecutive digits in the decimal representation of x. Prove that $p(k) \geq k + 1$ for every positive integer k.

C3. Consider a square of sidelength n and $(n+1)^2$ interior points. Prove that we can choose 3 of these points so that they determine a triangle of area at most $\frac{1}{2}$.

N1. Let n, m be integers greater than 1, and let $a_1, a_2, \ldots, a_m$ be positive integers not greater than n^m. Prove that there exist positive integers $b_1, b_2, \ldots, b_m$ not greater than n, such that

$$\gcd(a_1 + b_1, a_2 + b_2, \ldots, a_m + b_m) < n,$$

where $\gcd(x_1, x_2, \ldots, x_m)$ denotes the greatest common divisor of $x_1, x_2, \ldots, x_m$.

N2. Let $p_1, p_2, \ldots, p_n$ be $n \geq 3$ pairwise different prime numbers. Suppose there is some positive integer r such that the remainder on dividing $\prod_{i \neq k} p_i$ by p_k is r for all $1 \leq k \leq n$. Prove that $r \leq n - 2$.

N3. Let $a > 1$ be a positive integer. Let M be the set of positive integers m such that all of the prime divisors of $a^m - 1$ are less than 10^{2016}. Prove that M is finite.

G1. Let M be the midpoint of side BC of triangle ABC and let AL be the bisector of the angle A. The line passing through M perpendicular to AL intersects the side AB at the point D. Prove that $AD + MC$ is equal to half the perimeter of triangle ABC.

G2. A triangle ABC is inscribed in a circle S. Let A_0 and C_0 be the midpoints of the arcs BC and AB on S, not containing the opposite vertex, respectively. The circle S_1 centered at A_0 is tangent to BC, and the circle S_2 centered at C_0 is tangent to AB. Prove that the incenter I of $\triangle ABC$ lies on a common tangent to S_1 and S_2.

G3. In convex hexagon $ABCDEF$ all sides have equal length and

$$\angle A + \angle C + \angle E = \angle B + \angle D + \angle F$$

Prove that the diagonals AD, BE, CF meet at a common point.