Olympiad style inequalities

August 25, 2020

Problem 1. *Let* $a_2, ..., a_n$ *be positive reals, so that* $a_2...a_n = 1$. *Prove that* $(1+a_2)^2(1+a_3)^3...(1+a_n)^n > n^n$.

Problem 2. *Let* a, b, c *be real numbers with* $abc = 1$. *Prove that* $(a+\frac{1}{b})^2 + (b+\frac{1}{c})^2 + (c+\frac{1}{a})^2 \geq 3(a+b+c+1)$.

Problem 3. *Let* a, b, c *be positive numbers with* $a+b+c = 3$. *Find the minimum value of the expression* $A = \frac{2-a^3}{a} + \frac{2-b^3}{b} + \frac{2-c^3}{c}$.

Problem 4. *Let* a, b, c *be positive real numbers. Prove that* $\frac{8}{(a+b)^2+4abc} + \frac{8}{(b+c)^2+4abc} + \frac{8}{(c+a)^2+4abc} + a^2 + b^2 + c^2 \geq \frac{8}{a+3} + \frac{8}{b+3} + \frac{8}{c+3}$.

Problem 5. *Let* a, b *be two distinct real numbers and* c *be a real positive number such that* $a^4 - 2019a = b^4 - 2019b = c$. *Prove that* $-\sqrt{c} < ab < 0$.