Problem 1A. Let $x_1, x_2, \ldots, x_n$ be real numbers such that for every $i \in \{1, 2, \ldots, n\}$ there exists a positive integer k such that $x_i + x_{i+1} + \cdots + x_{i+k-1} \geqslant 0$ (indices are taken modulo n). Prove:

$$x_1 + x_2 + \cdots + x_n \geqslant 0.$$

Problem 2A. We say that a sequence of nonnegative real numbers $a_1, a_2, \ldots, a_k$ is *embeddable* in the interval $[b, c]$ if there exist numbers $x_0, x_1, \ldots, x_k \in [b, c]$ such that $|x_i - x_{i-1}| = a_i$ for $i = 1, 2, \ldots, k$. The sequence is *normalized* if its terms do not exceed 1. Given a positive integer n, prove that every normalized sequence of length $2n + 1$ is embeddable in the interval $[0, 2 - \frac{1}{2^n}]$.

Problem 3A. For a positive integer $n \geqslant 2$, let $C(n)$ the least positive real constant such that there exist n real numbers $x_1, x_2, \ldots, x_n$, which are not all zero, and satisfy:

(i) $x_1 + x_2 + \cdots + x_n = 0$;

(ii) for all $i = 1, 2, \ldots, n$ we have $x_i \leqslant x_{i+1}$ or $x_i \leqslant x_{i+1} + C(n)x_{i+2}$ (the indices are taken modulo n).

Prove:

a) $C(n) \geqslant 2$ for all n;

b) $C(n) = 2$ if and only if n is even.

Problem 4A. Let $n \geqslant 3$ be a positive integer and let $(a_1, a_2, \ldots, a_n)$ be a strictly increasing sequence of n positive real numbers with sum equal to 2. Let X be a subset of $\{1, 2, \ldots, n\}$ such that the value of

$$\left| 1 - \sum_{i \in X} a_i \right|$$

is minimised. Prove that there exists a strictly increasing sequence of n positive real numbers $(b_1, b_2, \ldots, b_n)$ with sum equal to 2 such that

$$\sum_{i \in X} b_i = 1.$$

Problem 5A. Find all integers $n > 3$ with the following property: for all real numbers $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ satisfying $|a_k| + |b_k| = 1$ for $1 \leqslant k \leqslant n$, there exist $x_1, x_2, \ldots, x_n$, each of which is either -1 or 1, such that

$$\left| \sum_{k=1}^{n} x_k a_k \right| + \left| \sum_{k=1}^{n} x_k b_k \right| \leqslant 1.$$

Problem 6A. For a sequence $x_1, x_2, \ldots, x_n$ of real numbers, we define its price as

$$\max_{1 \leqslant i \leqslant n} |x_1 + \cdots + x_i|.$$

Given n real numbers, Dave and George want to arrange them into a sequence with a low price. Diligent Dave checks all possible ways and finds the minimum possible price D. Greedy George, on the other hand, chooses x_1 such that $|x1|$ is as small as possible; among the remaining numbers, he chooses x_2 such that $|x_1 + x_2|$ is as small as possible, and so on. Thus, in the i-th step he chooses xi among the remaining numbers so as to minimise the value of $|x_1 + x_2 + \cdots + x_i|$. In each step, if several numbers provide the same value, George chooses one at random. Finally he gets a sequence with price G. Find the least possible constant c such that for every positive integer n, for every collection of n real numbers, and for every possible sequence that George might obtain, the resulting values satisfy the inequality $G \leqslant cD$.

Problem 7A. Let $a_0, a_1, a_2, \ldots$ be a sequence of real numbers such that $a_0 = 0$, $a_1 = 1$, and for every $n \geqslant 2$ there exists $1 \leqslant k \leqslant n$ satisfying

$$a_n = \frac{a_{n-1} + a_{n-2} + \ldots + a_{n-k}}{k}.$$

Find the maximal possible value of $a_{2018} - a_{2017}$.