Test 2

September 5, 2020

Problem 1. *Let S be a set of 100 positive integers, so that if we choose 4 distinct elements of S, then among these 4 we can find one, which either divides the other three numbers, or is equal to the sum of other three numbers. Prove that S has an element, which divides all elements of S.*

Problem 2. *Find the minimal n, for which the number $n^2 - n + 11$ is product of four (not necessarily distinct) prime numbers.*

Problem 3. *Let a, b, c, d be positive real numbers with $abcd = 1$. Prove that*

$$\frac{1}{a^3 + b + c + d} + \frac{1}{a + b^3 + c + d} + \frac{1}{a + b + c^3 + d} + \frac{1}{a + b + c + d^3} \leq \frac{a + b + c + d}{4}$$

Problem 4. *Solve the equation with positive integers:*

$$45^x - 6^y = 2019^z$$