JUNIOR BALKAN MATHEMATICAL OLYMPIAD

TEAM SELECTION TEST

Day 1 , August 22, 2020

Problem 1. Find all triples (p, q, r) of prime numbers such that all of the following numbers are integers

$$\frac{p^2+2q}{q+r}, \qquad \frac{q^2+9r}{r+p}, \qquad \frac{r^2+3p}{p+q}.$$

Problem 2. Let A and B be two non-empty, disjoint subsets of $X = \{1, 2, 3, \ldots, 11\}$ with $A \cup B = X$. Let P_A be the product of all elements of A and let P_B be the product of all elements of B. Find the minimum and maximum possible value of $P_A + P_B$ and find all possible equality cases.

Problem 3. Let ABC be a non-isosceles triangle with incenter I. Let D be a point on the segment BC such that the circumcircle of BID intersects the segment AB at $E \neq B$, and the circumcircle of CID intersects the segment AC at $F \neq C$. The circumcircle of DEF intersects AB and AC at the second points M and N respectively. Let P be the point of intersection of IB and DE, and let Q be the point of intersection if IC and DF. Prove that the three lines EN, FM and PQ are parallel.

Problem 4. We have a group of n kids. For each pair of kids, at least one has sent a message to the other one. For each kid A, among the kids to whom A has sent a message, exactly 25% have sent a message to A. How many possible two-digit values of n are there.