Test-10, June 04

Level 3, from 09:00 to 13:10

Problem 1. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation

$$a_{n+2} + pa_{n+1} + qa_n = 0.$$

Prove that the sequences:

a) $a_{n+1}^2 + pa_n a_{n+1} + qa_n^2$
b) $a_{n+1}^2 - a_n a_{n+2}$,

are geometric progressions with quotient q.

Problem 2. Let n be a positive integer. A spider travels on a $(2n+1)\times(2n+1)$ board. In every move, it can travel two squares vertically (up or down), two squares diagonally or one square horizontally (left or right). Find the minimum number of moves the spider needs to pass through every single square. For each move, the spider is considered to have visited the starting square, the ending square and the square between the starting and ending square when the spider moves two squares. The starting point can be arbitrarily chosen.

Problem 3. Find all prime numbers p such that $\frac{p^2-p-2}{2}$ is a perfect cube.

Problem 4. Let ABC be a triangle, and H is its orthocenter. Let AH intersects BC at D. Let K is the symmetric of D with respect to H. If K is the midpoint of AD, Prove that $\angle BKC = 90^o$.