Plan:

- $\boxed{50 \text{ and } 51}$ ✓ (similar) ~ 15:00
- $\boxed{52 \quad 53}$

f eq + seq.

$\boxed{50}$ $1 < a, b, c < 2$

$$\frac{b\sqrt{a}}{4b\sqrt{c} - c\sqrt{a}} + \frac{c\sqrt{b}}{4c\sqrt{a} - a\sqrt{b}} + \frac{a\sqrt{c}}{4a\sqrt{b} - b\sqrt{c}} \geq 1$$

$\neq 0$

denominators are positive!

$$4b\sqrt{c} - c\sqrt{a} >$$
$$\geq 4b - c\sqrt{a} > 4b - 2\cdot\sqrt{2} > 4 - 2\sqrt{2} > 0$$

$$(\quad) + (\quad) + (\quad) \geq 1$$

$$\frac{a}{a+b+c} + \frac{b}{a+b+c} + \frac{c}{a+b+c} = 1$$

It is enough to prove that

$a, b, c \in (1,2)$

$a \in (1,2)$

$$\frac{b\cancel{\sqrt{a}}}{4b\sqrt{c} - c\sqrt{a}} \geq \frac{\cancel{a}\sqrt{a}}{a+b+c}$$

$$ba + b^2 + bc \geq 4b\sqrt{ac} - ca$$

✓ $\boxed{a, b, c > 0}$

$$\boxed{ba + b^2 + bc + ca \geq 4b\sqrt{ac}}$$

$\mathbin{\rotatebox[origin=c]{90}{$\geq$}}$ AM-GM

$$4\sqrt[4]{ba \cdot b^2 \cdot bc \cdot ca} = 4\sqrt[4]{b^4a^2c^2} = 4b\sqrt{ac}.$$

Try to solve in similar way

$\boxed{51}$

$a, b, c \in (0,1)$

$$\boxed{\frac{a}{bc+1} + \frac{b}{ca+1} + \frac{c}{ab+1} < 2}$$

So maybe try

$$\frac{\cancel{a}}{bc+1} \leq \frac{2\cancel{x}}{a+b+c} \quad \ldots -$$

$$a+b+c \leq 2bc + 2$$

$a \leq 2bc + 2 - b - c =$

$= bc + 1 + (b-1)(c-1)$

$(a, b, c) \in (0,1)$

$a < 1 < 1 + bc + \underbrace{\underbrace{(b-1)}_{<0}\underbrace{(c-1)}_{<0}}_{>0}$

$\boxed{52}$

$$\boxed{\begin{array}{l} f: \mathbb{R} \to \mathbb{R} \\ \bullet\ f(x) + f(y) \geq xy \quad \forall x, y \in \mathbb{R} \\ \bullet\ \forall x \in \mathbb{R}\ \exists y \in \mathbb{R} \text{ s.t. : } \\ \qquad f(x) + f(y) = xy \end{array}}$$

Move to evening