Email training, N5

September 22-28, 2019

Problem 5.1. Prove the inequality
$$\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\frac{1}{6}-\ldots-\frac{1}{99}+\frac{1}{100}>\frac{1}{5}.$$

Problem 5.2. Show that for all $n \geq 1$ one has
$$\frac{1}{1^2}+\frac{1}{2^2}+\ldots+\frac{1}{n^2}<2.$$

Problem 5.3. Prove that for any numbers a, b, $c>0$ the following inequality holds
$$\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\geq\frac{2}{a}+\frac{2}{b}-\frac{2}{c}.$$

Problem 5.4. How many integer solutions has the following inequality
$$\left(x-\frac{1}{2}\right)^1\cdot\left(x-\frac{3}{2}\right)^3\cdot\ldots\cdot\left(x-\frac{2017}{2}\right)^{2017}<0.$$

Problem 5.5. Find the maximum value of expression $\sqrt{x^2+y^2}$ if it's known that
$$\{-4\leq y-2x\leq 2,\quad 1\leq y-x\leq 2\}.$$

Problem 5.6. Quadrilateral $ABCD$ is given such that
$$\angle DAC=\angle CAB=60^o,$$
and
$$AB=BD-AC.$$
Lines AB and CD intersect each other at point E. Prove that $\angle ADB=2\angle BEC$.

Problem 5.7. There are $n>2$ lines on the plane in general position; Meaning any two of them meet, but no three are concurrent. All their intersection points are marked, and then all the lines are removed, but the marked points are remained. It is not known which marked point belongs to which two lines. Is it possible to know which line belongs where, and restore them all?

Problem 5.8. Find all quadrilaterals $ABCD$ such that all four triangles DAB, CDA, BCD and ABC are similar to one-another.

Solution submission deadline September 28, 2019