— Geometry for L3 —

— November 27, 2021 — Enriching the Picture —

Which elements should we add to the picture? General hints:

- two equilateral triangles sharing a vertex produce lots of congruencies;
- parallelograms produce lots of equalities (of both angles and segments);
- it's good to have equal segments sharing a point, or even overlapping;
- if you have right triangle, midpoint of its hypotenuse is always nice to look at.

1. Given is an acute triangle ABC with $\angle ACB = 60^\circ$. Segments AK and BL are its altitudes. Prove that $AB = 2 \cdot KL$.

2. Given is a convex hexagon $ABCDEF$ with ABC and DEF being equilateral triangles. Prove that $AD + CF \geq BE$.

3. In triangle ABC point D is the midpoint of side AB and point E is the midpoint of segment CD. Prove that if $\angle CAE = \angle BCD$, then $AC = CD$.

4. Given is a triangle ABC with $AC = BC$. Points D and E belong to AC and BC, respectively, and satisfy $AB = BD = DE$ and $AD = CE$. Find $\angle ACB$.

5. Given is a triangle ABC with $AC = BC$. Points D and E belong to AC and BC, respectively, and satisfy $\angle EAB = \angle ABD = \angle ACB$ and $AE + ED + DB = AC$. Find $\angle ACB$.

6. Point P lies inside an equilateral triangle ABC and satisfies $\angle APB = 150^\circ$. Prove that $AP^2 + BP^2 = CP^2$.

7. Given is a convex pentagon $ABCDE$ whose all sides are equal and in which $\angle ABC + \angle CDE = 180^\circ$. Segments AD and BE intersect at P. Prove that the length of CP is the same as the side length of the pentagon.

8. Let ABC be an equilateral triangle. Line ℓ intersects lines BC, CA, AB at points K, L, M, respectively. Prove that there exists a point P satisfying $PK = AK$, $PL = BL$, $PM = CM$.