Level 2- Test 3

20 November 2020, 1:00-5:00

Problem 1. In the right triangle ABC we have $\angle C = 30^\circ$ and A is right. Let ω be the circle passing through A and tangent to BC at its midpoint. Suppose that ω cuts AC again at N and the circumcircle of ABC again at M. Show that $MN \perp BC$.

Problem 2. Let a, b, c be positive integers with $gcd(a, b, c) = 1$ and

$$a^2 + b^2 + c^2 = 2(ab + bc + ca)$$

Show that a, b and c are all perfect squares.

Problem 3. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that

$$n^2 - 1 \leq f(n) \cdot f(f(n)) \leq n^2 + n$$

for every $n \in \mathbb{N}$.

Problem 4. Inside a 7×7 square there are 6 points. Prove that the distance between some pair of points is at most $5\sqrt{2}$.