$\leftarrow$ A
start

$$\cancel{\sqrt{2-x}+\sqrt{x-3}+\sqrt{3x-2}=\sqrt{3+2x}}$$

$$\sqrt{2-x}+\sqrt{1-3x}+\sqrt{3-2x}=\sqrt{2+3x}+\sqrt{3+x}+\sqrt{1+2x}$$

$x=0$ ✓

$x>0$: $\sqrt{2+3x}>\sqrt{1-3x}$
$\sqrt{3+x}>\sqrt{2-x}$
$\sqrt{1+2x}>\sqrt{3-2x}$
×

$x<0$: $\sqrt{2+3x}<\sqrt{1-3x}$
$\sqrt{3+x}<\sqrt{2-x}$
$\sqrt{1+2x}<\sqrt{3-2x}$
×

$$a+\frac{1}{b}=b+\frac{1}{c}=c+\frac{1}{a}$$

$a+\frac{1}{b}=c+\frac{1}{a}$ $\Downarrow$ $a-c=\frac{a-b}{ab}$

$b+\frac{1}{c}=c+\frac{1}{a}$ $\Downarrow$ $b-c=\frac{a-c}{ac}$

$b+\frac{1}{c}=a+\frac{1}{b}$ $\quad b-a=\frac{b-c}{bc}$

×

$$(a-c)(b-c)(a-b)=\frac{(a-b)(a-c)(b-c)}{a^2b^2c^2}$$

$$\Rightarrow |abc|=1$$

hölder

$$\cancel{(\overset{n}{1+\dots+1})(\overset{n}{1+\dots+1})(x_1^3+\dots+x_n^3) \geq (x_1+x_2+\dots+x_n)^3}$$

$$\cancel{x_1^3+x_2^3+\dots+x_n^3 \geq (x_1+x_2+\dots+x_n)^2}$$

$a \quad -1$

$b \quad 1$

$c \quad 2$

$-1a+b+2c=20 \quad \times 2$

$+$

$a+b+4c=100 \quad \times 1$

$\geq -a+b+8c \geq$

$\parallel$

$-a+b+8c \geq -a+3b+8c$

(min b)

max when $b=0$

$-a+8c=140$

$a=20, \; c=20$

max 140

$-a+b+8c \leq -a+b+2c$

min when $c=0$

so $a=40, b=60$

min $=20$

$$1 - x + x^2 - x^3 + \dots - x^{19} + x^{20} - x^{21} = a_0 + a_1 y + \dots + a_{21} y^{21}$$

$y = x+1$

$x = y-1$

$$\cancel{(1-x)(1+x^2+x^4+\dots+x^{20})} =$$

$$= \frac{x^{22}+1}{x+1} = \frac{x^{22}+1}{y}$$

$$a_2 = \binom{2}{2} + \binom{3}{2} + \dots + \binom{21}{2} = \binom{22}{3}$$

$$=$$

$$1 - (y-1) + (y-1)^2 \dots$$

$$= \dots + y^2\left(\binom{2}{2} + \binom{3}{2} + \binom{4}{2} \dots \binom{21}{2}\right)$$

$$\frac{1-x^{22}}{1+x} = \frac{1-(y-1)^{22}}{y}$$

$$\Rightarrow a_2 = (\square\, y^3 \text{ in } (y-1)^{22}) = \binom{22}{3}$$

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}=2^n$$

$$\frac{8^x+27^x}{12^x+18^x}=\frac{7}{6}$$

$$\frac{8^x+27^x}{6(2\cdot}$$

$$\frac{x^3+y^3}{x^2y+xy^2}$$

$$\frac{2^{3x}+3^{2x}}{(2^2\cdot 3)^x+(2\cdot 3^2)^x}=\frac{7}{6}=\frac{2^{3x}+3^x}{2}$$

$$\frac{2^{3x}+3^{3x}}{2^{x-1}\cdot 3^{x-1}(2^x+3^x)}$$

$$\left(\frac{3}{2}\right)^x=y$$

$$\frac{1+\left(\frac{3}{2}\right)^{3x}}{\left(\frac{3}{2}\right)^x+\left(\frac{3}{2}\right)^{2x}}=\frac{7}{6}$$

$$\frac{y^3+1}{y^2+y^2}=\frac{7}{6}=\frac{1-y+y^2}{y}$$

$$6y^2-6y+6=-7y \iff 6y^2-13y+6=0 \iff y=\frac{3}{2},\ -\frac{2}{3}$$

$$\Rightarrow x=1,-1$$

min m s.t.

$\forall n$

$$\binom{2n}{n}^{\frac{1}{n}} < m$$

$$\cancel{\frac{(2n)!}{n!\cdot n!} < m^n}$$

$$\cancel{\frac{n+1}{1}\cdot\frac{n+2}{2}\cdot\frac{n+3}{3}\cdots\frac{2n-1}{n-1}\cdot\frac{2n}{n}}$$

$$\cancel{\frac{2n}{n}\cdot\frac{2n-2}{n-1}\cdots\frac{n+1}{}}$$

$$\binom{2n}{0}+\binom{2n}{1}+\cdots+\binom{2n}{2n}=2^{2n}=4^n$$

$$\Rightarrow \binom{2n}{n}<4^n$$

$$\binom{2n}{n}^{\frac{1}{n}}<4 \qquad m=4$$

$$\binom{10}{5}^{\frac{1}{5}}>3$$

$abcde = a+b+c+d+e \qquad a,b,c,d,e \in \mathbb{N}$

$a \geq b \geq c \geq d \geq e$

$a \leq abcde = a+b+c+d+e \leq 5a$

$1 \leq bcde \leq 5$

e	d	c	b
1	1	1	1
1	1	1	2
1	1	1	3
1	1	1	4
1	1	2	2
1	1	1	5

$a = \dfrac{b+c+d+e}{bcde - 1} \rightarrow \text{Max } a = 5$

$a_1 \quad a_2 \quad a_3 \quad a_4$

$3n+n$

$$1 = \frac{a+b+c+d+e}{abcde} = \sum_{cyc} \frac{1}{abcd} \le \frac{1}{de} + \frac{1}{de} + \frac{1}{de} + \frac{1}{e} + \frac{1}{d} = \frac{3+d+e}{de}$$

$3+d+e \ge de$

$(d-1)(e-1) \le 4$

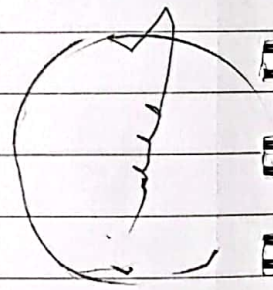
$e \le 5$

$$1 = \frac{x_1 + \dots x_n}{x_1 \cdots x_n} = \sum \frac{1}{x_2 x_3 \cdots x_n} \le \frac{1}{x_n x_{n-1}} + \overbrace{\frac{1}{x_n x_{n-1}} + \dots + \frac{1}{x_n} + \frac{1}{x_{n-1}}}^{n-2 \text{ times}}$$

$$= \frac{n-2+x_n+x_{n-1}}{x_n x_{n-1}}$$

$\Leftrightarrow \quad n-2 \ge x_n x_{n-1} - x_n - x_{n-1}$

$n-1 \ge (x_n-1)(x_{n-1}-1)$



$n \ge x_n$

$\parallel$

"=" when

$n \cdot x_{n-2} = x_{n-2} + n + n - 2$

$\Rightarrow x_{n-2} = 2, \; x_1 = x_2 = 1$

$$\frac{n+2}{n!(1+n+1+(n+2)(n+1))} = \frac{n+2}{n!(n+2)^2} = \frac{1}{(n+2)(n!)} = \frac{n+1}{(n+2)^2}$$

$$\frac{1}{(n-1)!} - \frac{1}{n!}$$

$$\frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n-1}{n!} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} \cdots - \frac{1}{2022!}$$

$$= \frac{1}{2} - \frac{1}{2022!}$$

$$\frac{2005^x + 2006^x + 2007^x + 2008^x + 1^x}{2021^x} = \frac{2021^x + 2022^x}{2021^x}$$

$$\left(\frac{2005}{2021}\right)^x + \left(\frac{2006}{2021}\right)^x + \left(\frac{2007}{2021}\right)^x + \left(\frac{2008}{2021}\right)^x = 1 + \left(\frac{2022}{2021}\right)^x$$

For $x<0$ RHS is decreasing and LHS is increasing and RHS >2021 and LHS <2

For $x>0$ RHS is increasing and LHS is decreasing so there is only 1 solution

$P(\alpha) = 0 \Rightarrow P(x) \vdots x - \alpha$

$x^5 + x^4 + 3 : x^3 + 2x$

$x^2(x^3 + 2x) = x^5 + 2x^3$

$$
\begin{array}{l|l}
 & x^2 + x - 2 \\
\hline
x^5 + x + 3 & x^3 + 2x \\
\hline
x^5 + 2x^3 & \\
\hline
x^4 - 2x^3 + 3 & \\
x^4 + 2x^2 & \\
\hline
-2x^3 - 2x^2 + 3 & \\
-2x^3 - 4x & \\
\hline
-2x^2 + 4x + 3 &
\end{array}
$$

$\Rightarrow x^5 + x^4 + 3 = (x^3 + 2x)(x^2 + x - 2) - 2x^2 + 4x + 3$

$\deg R < \deg Q, \quad \deg P = \deg Q + \deg S$

$$
\begin{array}{l|l}
P(x) & Q(x) \\
\hline
 & S(x) \\
R(x) &
\end{array}
$$

$P = Q \cdot S + R$

$P \in \mathbb{Z}[x]$, $Q \in \mathbb{Z}[x]$ ± 1 monic $\Rightarrow R, S \in \mathbb{Z}[x]$

Bezou

$$P(x) = a_n x^n + \dots + a_1 x + a_0$$
$$P(\alpha) = a_n \alpha^n + \dots + a_1 \alpha + a_0$$

$$a_n(x^n - \alpha^n) + \dots + a_1(x - \alpha)$$
$$= (x-\alpha)(a_n x^{n-1} + a_n x^{n-2}\alpha + \dots + a_n \alpha^{n-1} + a_{n-1} x^{n-2} + \dots + a_1)$$

$\Rightarrow P(x) = (x-\alpha) \cdot Q(x) + P(\alpha)$

if $P(\alpha) = 0 \Rightarrow P(x) \vdots (x-\alpha)$

$P(x) - P(\alpha) = (x-\alpha) \cdot Q(x)$

$P(x) - P(\alpha) \vdots (x-\alpha)$

$P \in \mathbb{Z}[x]$ $a, b \in \mathbb{Z}$

$$\Rightarrow \frac{P(a) - P(b)}{a-b} \in \mathbb{Z} = Q(x)$$

$P \in \mathbb{Z}[x]$

$P(2) \vdots 6$ $\overset{?}{\Rightarrow}$ $P(5) \vdots 6$
$P(3) \vdots 6$

$P(2)+P(3)=P(5)$ but $P(2) \vdots 6$ and $P(3) \vdots 6 \Rightarrow P(5) \vdots 6$

$P(5)-P(2) \vdots 3 \Rightarrow P(5) \vdots 3$
$P(5)-P(3) \vdots 2 \Rightarrow P(5) \vdots 2$
$\Downarrow$
$P(5) \vdots 6$

$ax^3+bx^2+cx+d \in \mathbb{Z}[x]$

$\forall n \in \mathbb{N}$ $P(n) \vdots 5 \overset{?}{\Rightarrow} a,b,c \vdots 5$

$a+b+c \vdots 5$

$-a+b-c \vdots 5$ +

$b \vdots 5$
$\Downarrow$
$a+c \vdots 5$

$8a+2c \vdots 5$
$4a+c \vdots 5$
$\Downarrow$
$3a \vdots 5 \Rightarrow a \vdots 5 \Rightarrow c \vdots 5$

$27a + 9b = -40$ but $-40 \nmid 3$

$p(x) = a_n x^n + \dots + a_0 \in \mathbb{Z}[x]$

$p\left(\frac{s}{t}\right) = 0 \qquad a_0 \mid s$

$a_n \mid t$

$$\frac{1, 2, -1, 2}{3, 1, -3, -1} \text{ is roots}$$

$P(x) \in \mathbb{Z}[x]$.

$p(d) = 1 \qquad P(a) = P(b) = P(c) = 0$

$\searrow$ No Exist

~~$P(x) = (x-a)(x-b)(x-c)$~~

$1 = P(d) - P(a) \vdots x - a$

$1 \vdots x - b$

$1 \vdots x - c \quad \Rightarrow |x-a| = |x-b| = |x-c| = 1$

so WLOG $a = b$

$\deg P = \deg Q \quad P, Q \in \mathbb{Z}[a]$

العوامل مبدلة

$$\begin{cases} 3x^2+2x+4 \\ 4x^2+3x+2 \end{cases}$$

$P(2022) - Q(2022) \vdots 43$

$P(a)=b,\ P(b)=c,\ P(c)=a \overset{?}{\Rightarrow} a=b=c$

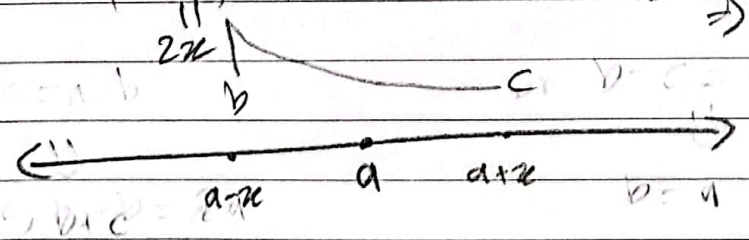
if $a=b$ then $a=b=c$, assume $a \neq b \neq c \neq a$

$a-b \mid P(a)-P(b) = b-c$

$a-b \mid b-c,\ b-c \mid c-a,\ c-a \mid a-b$

$\Rightarrow a-b \geq b-c \geq c-a \geq a-b \geq b-c \geq c-a$

$\Rightarrow |b-c| = |c-a| = |a-b| = x \quad \Rightarrow x=0 \Rightarrow a=b=c$



$$\begin{array}{cccccc} & & & 7 & & \\ & & & \downarrow & & \\ \times & & \boxed{3} & \boxed{a_2} & \boxed{7} & \\ & & \boxed{9} & 3 & \boxed{1} & \\ \hline & & 3 & \boxed{a_1} & \boxed{a_7} & \\ & \boxed{} & \boxed{a_8} & 3 & \boxed{a_1} & 0 \\ 3 & \boxed{3} & \boxed{} & 3 & 0 & 0 \\ \hline \end{array}$$

$$\cancel{= (100a_1 + 30 + a5)\, a_3}$$

---

n even

$$\cancel{(x)^n + x^n + \dots + 2^n + x^n + \dots + 2^n = 3x^n + \dots + 2^{n+1}}$$

$a^2 + b^2 + c^2 \neq 0$ $\frac{r}{s} = 0$

n odd

$$\left(\frac{r}{s}\right)^n + \left(\frac{r+2s}{s}\right)^n + \left(\frac{r-2s}{s}\right)^n = 0$$

$\Leftrightarrow$ $r^n + (r+2s)^n + (r-2s)^n = 0$

$\Leftrightarrow r^n + 2\left(r^n + \binom{n}{2} r^{n-2}(2s)^2 + \binom{n}{4} r^{n-4}(2s)^4 + \dots + \binom{n}{1} \cdot r \cdot (2s)^{n-1}\right) = 0$

$\Leftrightarrow 3r^{n-1} + 2\binom{n}{2} r^{n-3}(2s)^2 + \dots + 2\binom{n}{1}(2s)^{n-1} = 0$

$r = 2k \Rightarrow s = 1$ or 3

$\Downarrow$ $3 \mid r$

حرف

$$P(x) = x^n + (2+x)^n + (2-x)^n$$

$$\left(\frac{r}{s}\right)^n + \left(\frac{r+2s}{s}\right)^n + \left(\frac{2s-r}{s}\right)^n = 0$$

$$r^n + (2s+r)^n + (2s-r)^n = 0$$

$$= \cancel{1 + 2\binom{n}{2}(2s)^{n-3} r^2 + 2\binom{n}{4}(2s)^{n-5} r^4 + \dots + 2\binom{n}{1} + r^{n-1} = 0}$$

$$= r^n + r^n + \binom{n}{1} r^{n-1} (2s)^1 + \binom{n}{2} r^{n-2} (2s)^2 + \dots + (2s)^n +$$

$$- r^n + \binom{n}{1} r^{n-1} (2s) - \binom{n}{2} r^{n-2} (2s)^2 + \dots + (2s)^n$$

$$= r^n + 2\left((2s)^n + (2s)^{n-2} r^2 \binom{n}{2} + \dots + \binom{n}{1} \cdot 2s \cdot r^{n-1}\right) = 0$$

$r = 2k \Rightarrow 2^2 k = r \Rightarrow s \vdots 2$

$f(1,1) = 2, \quad f(m+1,n) = f(m,n) + m, \quad f(m,n) - n$

$f(2,1) = f(1,1) + 1 = 3$

$f(x,y) = 2021$

$f(2,1) = 2+1$
$f(3,1) = 2+(1+2)$
$f(4,1) = 2+(1+2+3)$

$f(x,1) = 2 + \frac{x(x-1)}{2}$

$f(x,2) = f(x,1) - 1$
$f(x,3) \quad f(x,2) - 2$

$f(x,y) = 2 + \frac{x(x-1)}{2} - \frac{y(y-1)}{2} = 2021$

$\Rightarrow x^2 - xy^2 + y = 2 \cdot 2019$

$(x-y)(x+y-1) = 2 \cdot 2019$

cases

$$x_n = \frac{x_{n-1} x_{n-2}}{2x_{n-2} - x_{n-1}}$$

$$x_3 = \frac{x_2 x_1}{2x_1 - x_2}$$

$$\frac{1}{x_n} = \frac{2}{x_{n-1}} - \frac{1}{x_{n-2}}$$

$$\frac{1}{x_n} = y_n$$

$$y_n = 2y_{n-1} - y_{n-2}$$

$$y - y_{n-1} = y_{n-1} - y_{n-2}$$

تابع خطی

$$\Rightarrow y_n = \text{constant} \Rightarrow x_1 = x_2 \neq 0$$

$$x^3 - 3x = \sqrt{x+2}$$

$$x^6 + 9x^2 - 2x^4 = x + 2$$

$$x^6 - 2x^4 + 9x^2 - x + 2 = 0$$

$$(x-2)(x^4 + 9x) = x^6 - 2x^4 + 9x^2 - 18x$$

$x(x-3) > x$ for $x > 2$ $\quad x^3 > 4x$

$x \overset{?}{>} \sqrt{x+2}$

$(x+1)(x-2)$

$x = 2 \cdot \cos\alpha$

$8\cos^3\alpha - 6\cos\alpha = \sqrt{2(1+\cos\alpha)} \quad 2\cos^2\frac{\alpha}{2}$

$2\cos 3\alpha = 2\cos\frac{\alpha}{2}$

$$17 > \sum_{k=1}^{80} \frac{1}{\sqrt{k}} > 16$$

$\left(\frac{1}{k(k+5)}\right) = \frac{1}{5}\left(\frac{1}{k} - \frac{1}{k+5}\right)$

$\cancel{\frac{\sqrt{1}}{1} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3} + \dots + \frac{\sqrt{80}}{80}}$

$$\sum \frac{2}{\sqrt{k+1} + \sqrt{k}}$$

$$\sum \frac{2(\sqrt{k+1} - \sqrt{k})}{1}$$

$= 2(\sqrt{81} - 1) = 16 > 16$ $\quad$ but $\sum \frac{2}{\sqrt{k}} > \sum \frac{2}{\sqrt{k+1} + \sqrt{k}}$

$P(x) = X^n + a_1 x^{n-1} + \dots + a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$

if y is a solution $\Rightarrow y \mid a_n \Rightarrow y = p_i \cdot t \quad , p_i \nmid t$

$\Leftrightarrow (p_i t)^n + a_1 (p_i t)^{n-1} + \dots + a_{n-2}(p_i t)^2 + a_{n-1} p_i t + a_n = 0$

$i = n \Rightarrow s = p_n \cdot t$

$p^2 \mid p_1 p_2 \cdots p_n \cdot t + p_1 p_2 \cdots p_n$

$\Rightarrow p^2 \mid t+1$

$p_n^3 \mid a_{n-2}(x^2 + p_{n-1}x + p_{n-1}p_n)$

$p_n^3 \mid p_n^2 t^2 + p_n p_{n-1} t + p_{n-1} p_n$

$p_n^2 \mid p_n t + p_{n-1} t + p_{n-1}$

$x^2 = 2^x + 3^x + 6^x$

$x = -1$ is a solution

if a is a solution for $a \geq 1$

then $a^2 \geq 3 \Leftrightarrow a \geq \sqrt{3}$

$2^a \geq 1 + a > a$

$6^a > (2^a)^2 > a^2$

$\Rightarrow 6^a + 3^a + 2^a > a^2$ ↯

Bernouli inequality

$(1+x)^a \geq 1 + ax$ "=" $a \geq 1$, $x > -1$

$(2^x+1)(3^x+1) \geq 1 + x^2$

$(2+x)(2+2x) > 1 + x^2$ ↯

$x > 0 \Rightarrow LHS > 3 \Rightarrow x^2 > 3 \Rightarrow x > 1$

$\Rightarrow LHS > 11 \Rightarrow x^2 > 11 \Rightarrow x > 3 \Rightarrow LHS > 250$

$\Rightarrow x^2 > 250 \Rightarrow x > 15$

$x \leq 3 \Rightarrow 2^x + 3^x + 6^x > x^2$

$x > 3$

$x > 3$

$$2^x + 3^x + 6^x \geq 2^{\lfloor x \rfloor} + 3^{\lfloor x \rfloor} + 6^{\lfloor x \rfloor} = (1+1)^{\lfloor x \rfloor} + (1+2)^{\lfloor x \rfloor} + (1+5)^{\lfloor x \rfloor} \geq 1 + \lfloor x \rfloor +$$

$$+ \frac{\lfloor x \rfloor (\lfloor x \rfloor - 1)}{2} + 1 + 2\lfloor x \rfloor + \frac{\lfloor x \rfloor (\lfloor x \rfloor + 1)}{2} \cdot 2^2 + 1 + 3\lfloor x \rfloor + \frac{\lfloor x \rfloor (\lfloor x \rfloor - 1}{2} \cdot 3^2 =$$

$$= 3 + 6\lfloor x \rfloor + 13 \cdot \frac{\lfloor x \rfloor (\lfloor x \rfloor - 1)}{2} \geq 3 + 6x - 6 + \frac{13}{2}(x-2)(x-1) > x^2$$

$$2(2^x - 1)x^2 + (2^{x^2} - 2)x$$

$$x^2(2^x - 1) + x(2^{x^2-1} - 1) - (2^x - 1) = 0$$

$$(x^2 - 1)(2^x - 1) = -x(-2^{x^2-1} + 1)$$

$$\frac{2^x - 1}{x} + \frac{2^{x^2-1} - 1}{x - 1} = 0$$

$$f(a) = \frac{2^a - 1}{a}$$

$$\Rightarrow f(x) - f(x-1) = 0$$

$$f(a) > 0 \Rightarrow x = 0,\ x^2 - 1 = 0 \Rightarrow x = 0, 1, -1$$

$$x^7 = (1+y)(1+y^2)(1+y^4)$$
$$y^7 = (1+x)(1+x^2)(1+x^4)$$

$x, y > 0 \quad x \geq y$

$$1 + x + x^3 + x^2 \dots + x^2 = y^7 \leq$$

$x_1 = y_1 = \sqrt{3}$

$x_{n+1} = x_n + \sqrt{x_n^2 + 1}$ $\qquad y_n = \frac{1}{a_n}$ $\qquad y_{n+1} = \frac{y_n}{1 + \sqrt{1 + y_n^2}}$

$\Rightarrow$

$$\frac{1}{a_{n+1}} = \frac{\frac{1}{a_n}}{1 + \sqrt{\frac{1}{a_n^2} + 1}}$$

$$a_{n+1} = a_n + a_n\sqrt{\frac{1}{a_n^2} + 1}$$

$a_1 = \frac{1}{\sqrt{3}} \qquad a_{n+1} = a_n + \sqrt{1 + a_n^2}$

$\Downarrow$

$$a_2 = \frac{\sqrt{3}}{3} + \sqrt{1 + 3} = \frac{\sqrt{3}}{3} + \frac{2}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow a_{n+1} = x_n \Rightarrow y_{n+1} = \frac{1}{x_n}$$

$$x_n y_{n+1} = \frac{1}{x_n}\left(x_n + \sqrt{x_n^2 + 1}\right) = 1 + \sqrt{1 + \frac{1}{x_n^2}}$$

$$\Rightarrow \quad 1 < \sqrt{1 + \frac{1}{x_n^2}} < 2$$

$$1 < 1 + \frac{1}{x_n^2} < 4$$

$$0 < \frac{1}{x_n^2} < 3$$

$$0 < \frac{1}{x_n} < \sqrt{3} \checkmark \qquad x_n > \sqrt{3} \Rightarrow \frac{1}{x_n} <$$

$\cdot\ x=0$ ↯

$\cdot\ x=y \Rightarrow 1+x+x^3 \dots + x^6 = \frac{x^7-1}{x-1} = 0$ ↯

$\cdot\ x<0<y$ ↯

$\cdot\ x<y<0 \Rightarrow x<y<-1$

$$y^7 > 1+y+y^2+y^3+y^4+y^5+y^6+y^7 = x^7$$

$$\frac{y^7-1}{y-1} < 0 \quad ↯$$

$$1+x+x^2+\dots+x^6+1+y+y^2+\dots+y^6=0$$

$$\underbrace{\begin{cases} \frac{x^7-1}{x-1}, & x\neq 1 \\ 7, & x=1 \end{cases}}_{+} + \underbrace{\begin{cases} \frac{y^7-1}{y-1}, & y\neq 1 \\ 7, & y=1 \end{cases}}_{+} = 0$$

↯

$$7x_{n+1} = x_{n-1} + \frac{1}{x_n}$$

$$7x_{n+1}x_n = \underbrace{x_n x_{n-1}}_{a_n} + 1$$

$$7(x_{n+1}x_{n-1}) = \underbrace{x_n x_{n-1} - 1}_{a_n}$$

$$7a_{n+1} = a_n$$

$$7$$

$$\cancel{a_{n+1} = \frac{a_n+1}{7}}$$

$\cancel{\text{orbitis periodic}}$

$$\cancel{\Rightarrow a_n = \frac{a_n+1}{7}}$$

$$\cancel{7a_n = a_n+1}$$

$$\cancel{a_n = 1}$$

$$\cancel{\Rightarrow x_n x_{n+1} = 1}$$

$$\cancel{\Rightarrow x_0 x_1 = 1}$$

$$a_{n+1} = \frac{a_n}{7}$$

$$\Rightarrow a_n = \frac{a_n}{7}$$

$$\Rightarrow a_n = 0$$

$$x_0 x_1 = a_1 + 1 = 1$$

A end

$a+b+a^2+b^2+ab-1=0$

$a^2+b^2+ab-a-b+1=0$

$2a^2+2b^2-2ab-2a-2b+2=0$

$a^2-2ab+b^2+a^2-2a+1+b^2-2b+1=0$

$(a-b)^2+(a-1)^2+(b-1)^2=0$

$\Leftrightarrow a=b=0$

$\Rightarrow x=1$