Firstly: 42 (Hölder) | $\boxed{22:45}$

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$a_1, a_2, \dots, a_n, b_1, \dots, b_n \in \boxed{\mathbb{R}_+}$

$p, q > 1$ real numbers:

$$\boxed{\frac{1}{p} + \frac{1}{q} = 1}$$

then:

$$\sum_{j=1}^{n} a_j b_j \le \left(\sum_{j=1}^{n} a_j^p\right)^{\frac{1}{p}} \left(\sum_{j=1}^{n} b_j^q\right)^{\frac{1}{q}}$$

" = " $\boxed{\frac{a_i^p}{b_i^q} = \text{const}}$

Generalise

a_{ij} : $i = 1, \dots, k$, $j = 1, \dots, k$ $\in \mathbb{R}_+$

$p_1, p_2, \dots, p_k \in \mathbb{R}_+$ s.t. $\sum_{i=1}^{k} \frac{1}{p_i} = 1$.

$$\sum_{j=1}^{n} \prod_{i=1}^{k} a_{ij} \le \prod_{i=1}^{k} \left(\sum_{j=1}^{n} a_{ij}^{p_i}\right)^{\frac{1}{p_i}}$$

$$\frac{a_1^p}{b_1^q} = \frac{a_2^p}{b_2^q} = \dots = \frac{a_n^p}{b_n^q}$$

$$\frac{a_{ij}^{p_i}}{a_{ij}^{q_i}} = \text{const} \ldots$$

$p, q > 1$ real numbers:

$$\boxed{\frac{1}{p} + \frac{1}{q} = 1}$$

then:

$$\sum_{j=1}^{n} a_j b_j \leq \left(\sum_{j=1}^{n} a_j^p \right)^{\frac{1}{p}} \left(\sum_{j=1}^{n} b_j^q \right)^{\frac{1}{q}}$$

"=" holds iff

$$\frac{a_1^p}{b_1^q} = \frac{a_2^p}{b_2^q} = \ldots = \frac{a_n^p}{b_n^p}.$$

a_{ij}: $\quad i = 1, \ldots k \quad \in \mathbb{R}_+$
$\quad\quad j = 1 \ldots k$

$_1, p_2, \ldots, p_k \in \mathbb{R}_+$ s.t $\displaystyle\sum_{i=1}^{k} \frac{1}{p_i} = 1$.

$$\sum_{j=1}^{n} \prod_{i=1}^{k} a_{ij} \leq \prod_{j=1}^{k} \left(\sum_{j=1}^{n} a_{ij}^{p_i} \right)^{\frac{1}{p_i}}$$

$$\left(a_{11}^{p_1}+a_{12}^{p_1}+\dots+a_{1n}^{p_1}\right)^{\frac{1}{p_1}} \cdot \dots \cdot \left(a_{n1}^{p_n}+a_{n2}^{p_n}+\dots+a_{nn}^{p_n}\right)^{\frac{1}{p_n}}$$

$$\geq \sum_{j=1}^{n} \prod_{i=1}^{k} a_{ij}$$

$$\frac{a_{ij}^{p_i}}{a_{ji}^{p_i}} = \frac{a_{ij}^{p_j}}{a_{ji}^{q_j}} \quad , \dots$$

$$a_{ij}^{p_i} \quad \dots$$

vecktor. $\left(a_{11}^{p_1}, a_{12}^{p_1}, \dots\dots a_{1n}^{p_1}\right)$

$$v_j := \left(a_{i1}^{p_i}, \dots\dots, a_{in}^{p_i}\right)$$

$$v_k := \left(a_{k1}^{p_k}, \dots\dots, a_{kn}^{p_k}\right)$$

42

$$S = \frac{x_1 x_2 \dots x_n}{(a+x_1)(x_1+x_2)\dots(x_n+b)} \leq$$

$$(a+x_1) \geq 2\sqrt{ax_1} \dots 2\sqrt{x_n b} \geq$$

Hölder

$$(a+x_1)(x_1+x_2)\dots(x_n+b) \geq \left(\sqrt[n+1]{ax_1 \dots x_n} + \sqrt[n+1]{b x_1 \dots x_n}\right)^{n+1}$$

$$= \left(\sqrt[n+1]{x_1 \dots x_n}\left(\sqrt[n+1]{a} + \sqrt[n+1]{b}\right)\right)^{n+1} =$$

$$= x_1 \dots x_n \left(\sqrt[n+1]{a} + \sqrt[n+1]{b}\right)^{n+1}$$

$$S \leq \frac{x_1 \dots x_n}{x_1 \dots x_n\left(\sqrt[n+1]{a} + \sqrt[n+1]{b}\right)^{n+1}} = \boxed{\frac{1}{\left(\sqrt[n+1]{a} + \sqrt[n+1]{b}\right)^{n+1}}}$$

$$\frac{x_1}{a} = \frac{x_2}{x_1} = \frac{x_3}{x_2} = \dots = \frac{x_n}{x_{n-1}} = \frac{b}{x_n} \quad \dots$$

$$x_k = a \cdot \left(\frac{b}{a}\right)^{k(n+1)} \quad \dots$$

43

$$a_i = \sqrt{c+2}\,(x_i + x_{i+1})$$

$$b_i = 2\sqrt{x_i^2 + c x_i \cdot x_{i+1} + x_{i+1}^2}$$

$$\boxed{a_i^2 - b_i^2} = ?\,?\,?\,?\,?$$

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$$a_i = \sqrt{c+2}\,(x_i + x_{i+1})$$

$$b_i = 2\sqrt{x_i^2 + c x_i \cdot x_{i+1} + x_{i+1}^2}$$

$$a_i^2 - b_i^2 = (c-2)(x_i - x_{i+1})^2$$

$$0 = \sum_1^{l} (a_i - b_i) = \sum_1^{l} \frac{a_i^2 - b_i^2}{a_i + b_i} = \sum_1^{l} \frac{(c-2)(x_i - x_{i+1})^2}{a_i + b_i}$$

$$\sum_1^{l} \frac{\cancel{(c-2)}(x_i - x_{i+1})^2}{a_i + b_i} = 0$$

$$\Downarrow$$

$\boxed{c=2}$

$c \neq 2$

$\forall_i \quad x_i = x_{i+1} \quad i$

x_1 —— x_n

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$$\boxed{a, b, c, d \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$$

$$\sin(a) + \sin(b) + \sin(c) + \sin(d) = 1$$

$$\cos(2a) + \cos(2b) + \cos(2c) + \cos(2d) \geq \frac{10}{3}$$

$$\Rightarrow \quad a, b, c, d \in \left[0, \frac{\pi}{6}\right]$$

$x = \sin(a)$, $y = \sin(b)$, $z = \sin(c)$, $u = \sin(d)$

$x, y, z, u \in [-1, 1]$

$x + y + z + u = 1$

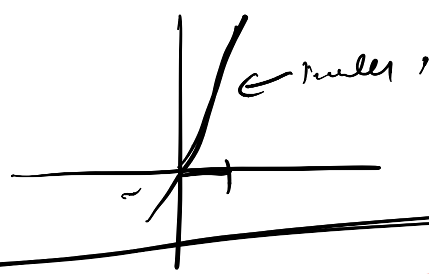
$\cos(2a) = 1 - 2\sin^2(a) = 1 - 2x^2$

$$1 - 2x^2 + 1 - 2y^2 + 1 - 2z^2 + 1 - 2u^2 \geq \frac{10}{3}$$

$$\boxed{x^2 + y^2 + z^2 + u^2 \leq \frac{1}{3}}$$

Look at the statement: $a, b, c, d \in [0, \frac{\pi}{6}]$

$\boxed{\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}}$ and sin is increasing



Every then $\boxed{x, y, z, u \in [0, \frac{1}{2}]}$

A-M-QM

$$\sqrt{\frac{x^2+y^2+z^2}{3}} \geq \frac{x+y+z}{3}$$

$$\Downarrow$$

$$\frac{1}{3} \geq x^2+y^2+z^2+u^2 \geq \frac{(x+y+z)^2}{3}+u^2 =$$

$$= \frac{(1-u)^2}{3}+u^2$$

$$\Downarrow$$

$$\boxed{u^2+\frac{(1-u)^2}{3} \leq \frac{1}{3}}$$

Sim.

$$x^2+\frac{(1-x)^2}{3} \leq \frac{1}{3} \qquad \text{etc ..}$$

$$3u^2+\cancel{1}-2u+u^2 \leq \cancel{1}$$

$$4u^2-2u \leq 0$$

$$2u^2-u \leq 0$$

$$u(2u-1) \leq 0$$

$$u \in \left[0, \tfrac{1}{2}\right]$$

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