Saudi Arabia 2022 – Math Camp

Day 3 - Level 4+

Geometry - Miscellaneous problems

Instructor: Regis Barbosa

0. (IMO/2015) Let ABC be an acute triangle with $AB > AC$. Let Γ be its circumcircule, H its orthocenter and F the foot of the altitude from A. Let M be the midpoint of BC. Let Q be the point on Γ such that $\angle HQA = 90°$, and let K be the point on Γ such that $\angle HKQ = 90°$. Assume that the points A, B, C, K and Q are all different, and lie on Γ in this order.

Prove that the circumcircles of triangles KQH and FKM are tangent to each other.

1. (Tuymaada/2012) Point P is taken in the interior of the triangle ABC, so that

$$\angle PAB = \angle PCB = \frac{1}{4}(\angle A + \angle C)$$

Let L be the foot of the angle bisector of $\angle B$. The line PL meets the circumcircle of ΔAPC at point Q. Prove that QB is the angle bisector of $\angle AQC$.

2. (IMO Shortlist/2005) Let $ABCD$ be a parallelogram. A variable line g through the vertex A intersects the rays BC and DC at the points X and Y, respectively. Let K and L be the A-excenters of the triangles ABX and ADY. Show that the angle $\angle KCL$ is independent of the line g.

3. (Russia/2013) Let ω be the incircle of a triangle ABC, and let I be its incenter. Let Γ be the circumcircle of the triangle AIB. Denote by X and Y the two points of intersection of ω and Γ. Denote by Z the point of intersection of the common tangents to ω and Γ. Prove that the circumcircles of the triangles ABC and XYZ are tangent to each other.

4. (RMM/2012) Let ABC be a triangle and let I and O denote its incenter and circumcenter respectively. Let ω_A be the circle through B and C which is tangent to the incircle of the triangle ABC; the circles ω_B and ω_C are defined similarly. The circles ω_B and ω_C meet at a point A_0 distinct from A; the points B_0 and C_0 are defined similarly. Prove that the lines AA_0, BB_0 and CC_0 are concurrent at a point on the line IO.