Email training, N8

Level 3, November 1-7

Problem 8.1. Find all pairs of integers (m, n) such that

$$\binom{n}{m} = 1984.$$

Problem 8.2. Prove that for any natural number $n > 1$, the number $2^n - 1$ does not divide $3^n - 1$.

Problem 8.3. Let u_n be the least common multiple of the first n terms of a strictly increasing sequence of positive integers $a_1, a_2, a_3, \ldots, a_{1000}$. Prove that

$$\sum_{k=1}^{1000} \frac{1}{u_k} \leq 2.$$

Problem 8.4. Let $\sigma(n)$ denote the sum of the divisors of n. Prove that there exist infinitely many integers n such that $\sigma(n) > 3n$. Prove also that $\sigma(n) < n(1 + \log_2 n)$.

Problem 8.5. Let $\sigma(n)$ denote the sum of divisors of n. Show that $\sigma(n) = 2^k$ if and only if n is a product of Mersenne primes, i.e., primes of the form $2^k - 1$.

Problem 8.6. Let $a_1 = 1,\ a_{n+1} = a_n + [\sqrt{a_n}]$. Find all n for which a_n is a perfect square.

Problem 8.7. In an acute angled triangle ΔABC, let D is on BC such that $AD \perp BC$. Let O and H be the circumcenter and orthocenter of ΔABC respectively. The perpendicular bisector of AO intersects BC extended at E. Show that the midpoint of OH is on the circumcircle of ΔADE.

Solution submission deadline November 7, 2021

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