Day 1 (1)

Problem 1. Find all functions $f : \mathbb{R}/\{0\} \to \mathbb{R}$ satisfying the conditions:

- $f(1) = 1$
- $f(\frac{1}{x+y}) = f(\frac{1}{x}) + f(\frac{1}{y})$
- $(x + y)f(x + y) = xyf(x)f(y)$

for all x, y with $xy(x + y) \neq 0$.

Problem 2. Find all functions $f : \mathbb{Q}^+ \to \mathbb{Q}^+$ satisfying the conditions:

- $f(x + 1) = f(x) + 1$ for all x from $\mathbb{Q}^+$
- $f(x^2) = f^2(x)$ for all x from $\mathbb{Q}^+$

Problem 3. Cauchy Equation (additive function) with monotonicity.
For a function $f : \mathbb{R} \to \mathbb{R}$. If $f(x + y) = f(x) + f(y)$ and f is increasing, then prove $f(x) = f(1)x$ for all $x \in \mathbb{R}$.

Problem 4. $f : \mathbb{R} \to \mathbb{R}$. If f is additive and $f(x^2) = xf(x)$, then prove $f(x) = f(1)x$ for all $x \in \mathbb{R}$.

Problem 5. $f : \mathbb{R} \to \mathbb{R}$. If f is additive and $f(x^2) = f^2(x)$, then prove $f(x) = f(1)x$ for all $x \in \mathbb{R}$.

Problem 6. $f : \mathbb{R}/\{0\} \to \mathbb{R}$. If f is additive and $f(x) = x^2 f(\frac{1}{x})$, then prove $f(x) = f(1)x$ for all $x \in \mathbb{R}/\{0\}$.

Problem 7. Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying the condition:

$$f(x^2 - y) = xf(x) - f(y)$$

for all x, y from $\mathbb{R}$

Problem 8. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x^4 + f(y)) = y + f^4(x) \qquad \forall x, y \in \mathbb{R}$$

Problem 9. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(2x^2 + y + f(y)) = 2y + 2f^2(x) \qquad \forall x, y \in \mathbb{R}$$

Problem 10. Find all functions $f : \mathbb{R}/\{0\} \to \mathbb{R}/\{0\}$ such that

$$f(x + y) = x^2 f(\frac{1}{x}) + y^2 f(\frac{1}{y})$$

Problem 11. $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x + y + xy) = f(x) + f(y) + f(xy)$$

Prove that f is additive.