1. Reasoning. 28.03.2020

1. In the cells of the square table 10×10 one placed 0's and 1's, and it is known that out of any four rows of the table, some two coincide. Prove that the table has two identical columns.

2. 15 magazines lie casually on a table, completely closing it (magazines can lie on top of each other and hang from the table). Prove that you can remove 7 magazines so that the remaining cover at least 8/15 of the table.

3. 41 different natural numbers less than 1000 are given. It is known that among any three of them there are two such that their product equals perfect square. Prove that among these numbers there is an exact square.

4. There are 25 points on the plane. It is known that out of any three of them there are two, the distance between which does not exceed 1.
a) Prove that two circles of radius one can cover all these points.
b) Prove that two rectangles 1×2 can cover all these points.

5. 99 students came to the camp, and all who came had the same non-zero number of friends among the rest. A group of students with the property "any of those who are not in this group are familiar with someone from this group" will be called *popular*. Prove that from any popular group containing more than 49 children, you can choose a popular group containing exactly 49 children.

6. 2018 students study at the school, and 25 types of cakes are sold in the school cafeteria. Every student loves pies of at least one kind; at least one schoolboy likes each pie. A group of schoolchildren will be called omnivorous if it contains lovers of all types of pies. A control group is a group of schoolchildren in which there is at least one representative of each omnivorous group. It turned out that it is impossible to remove a non-empty set of students from the control group G so that it remains the control. Prove that there is a kind of pies that everyone in G loves.

7. One or more pieces of clothing are hanging on a clothesline. Each piece of clothing is held up by either 1, 2 or 3 clothespins. Let a_1 denote the number of clothespins holding up the first piece of clothing, a_2 the number of clothespins holding up the second piece of clothing, and so forth. You want to remove all the clothing from the line, obeying the following rules:
(i) you must remove the clothing in the order that they are hanging on the line;
(ii) you must remove either 2, 3 or 4 clothespins at a time, no more, no less;
(iii) all the pins holding up a piece of clothing must be removed at the same time.
Find the number of sequences $a_1, a_2, \ldots, a_{2019}$ of any length for which all the clothing can be removed from the line.

8. There are several boys and girls in the city, some couples are familiar. It turned out that in any set D of 8 girls there is a (possibly empty) subset D' such that any boy who is familiar with all the girls from D' is also familiar with at least one more girl from D . Prove that in any set M of 300 boys there is a (possibly empty) subset M' such that any girl who is familiar with all the boys in M' is also familiar with at least one more boy from M.