Problem HW1. Let a, b and c be positive integers such that

$$\frac{a}{b^2} + \frac{b^2}{2c} + \frac{c^3}{a}$$

is also a positive integer and $v_2(a) < v_2(c) \leqslant 2v_2(b)$. Prove that $v_2(a) + v_2(b) + v_2(c) + 1$ is divisible by 5.

Problem HW2. Let k and n be positive integers. Can the number

$$S(n,k) = \frac{1}{n} + \frac{1}{n+13} + \ldots + \frac{1}{n+13k}$$

be a positive integer?

Problem HW3. Let p and q be coprime positive integers. Prove that for all positive integers $n > m$ the following inequality holds

$$\mathrm{LCM}(qm+p, q(m+1)+p, \ldots, qn+p) \geqslant m\binom{n}{m}.$$