Email training, N2

Level 2, September 20-26

Problem 2.1. Let x_1 and x_2 are the roots of the equation $x^2 + 5x - 11$. Find a quadratic polynomial which roots are x_1x_2 and $x_1^2x_2^2$.

Problem 2.2. Simplify

$$\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}.$$

Problem 2.3. Find all positive integers n for which $n^2 + 3n$ is perfect square.

Problem 2.4. Find all integer solutions to the equation

$$x^2 - 6xy + 13y^2 = 100.$$

Problem 2.5. Find the number of 7-digit positive integers that all digits are ordered in
a) strictly increasing order,
b) strictly decreasing order.

Problem 2.6. A triple $(1, 1, 1)$ is given. On each step one chooses 2 of them and increases by 1. Is it possible after some steps get numbers $(2022, 2022, 2022)$.

Problem 2.7. Let $ABCD$ be a cyclic quadrilateral. Let extensions of BA and CD intersect at X, extensions of AD and BC intersect at Y. Let the angle bisector of $\angle X$ intersects AD and BC at E and F, respectively, the angle bisector of $\angle Y$ intersects AB and CD at G and H respectively. Prove that $EGFH$ is a rhombus.

Solution submission deadline September 26, 2021
Submit single PDF file in filename format L2_YOURNAME_week2.pdf
submission email **imo20etraining@gmail.com**