

Problem 1.1. Prove that for any positive integer n at least one coefficient of the polynomial $(x^4 + x^3 - 3x^2 + x + 2)^n$

is negative.

Problem 1.2. Let polynomial

$$P(x) = \underbrace{((\dots((x-2)^2 - 2)^2 - \dots)^2 - 2)^2}_k$$

is given. Find coefficient at x^2 .

Problem 1.3. Let $f(x) = x^2 - 6x + 5$. Draw on the plane the set of pairs (x, y) that satisfy to the following system on inequalities

$$\begin{cases} f(x) + f(y) \leq 0 \\ f(x) - f(y) \geq 0 \end{cases}.$$

Problem 1.4. Prove that for any 2 positive integers m and n with $m > n$ holds the following inequality

$$lcm(m, n) + lcm(m+1, n+1) > \frac{2mn}{\sqrt{m-n}}$$

where $lcm(a, b)$ is the least common multiplier of a and b (for example $lcm(6, 8) = 24$).

Problem 1.5. Let convex s -gon is divided to q quadrilaterals such that b of them are not convex. Prove that

$$q \geq b + \frac{s-2}{2}.$$

Problem 1.6. Let positive numbers are written along the circle, such that all of them are less than 1. Prove that one can split the circle to 3 parts such that for each two arcs the sums of numbers written on them differs by at most 1.

Problem 1.7. Let the triangle ABC is given and D, E, F are on sides BC, AC, AB , respectively, such that

$$\frac{BD}{CD} = \frac{CE}{AE} = \frac{AF}{BF}.$$

Show that if the circumcircle of ABC and DEF coincide, then ABC is equilateral.

Solution submission deadline September 26, 2021
Submit single PDF file in filename format L3-YOURNAME_week2.pdf
submission email **imo20etraining@gmail.com**