Email training, N7
Level 2, October 25-31

Problem 7.1. Find the biggest integer n such that $n^3 + 100$ is divisible by $n + 10$.

Problem 7.2. Find the number of positive integers n letss than 10000, for which $2^n - n^2$ is divisible by 7.

Problem 7.3. Let a be an odd integer and m is such that $2^m | a + 1$ and $2^{m+1} \nmid a + 1$. Prove that for any positive integer k one has
$$2^{k+m+1} | (2a + 1)^{2^k} - 1 \quad \text{and} \quad 2^{k+m+2} \nmid (2a + 1)^{2^k} - 1$$

Problem 7.4. Find all positive integers n such that
$$3^{n-1} + 5^{n-1} | 3^n + 5^n.$$

Problem 7.5. Find all pairs of positive integers a, b such that
$$\frac{a^2 + b}{b^2 - a} \quad \text{and} \quad \frac{b^2 + a}{a^2 - b}$$
are both integers

Problem 7.6. The numbers in the sequence $101, 104, 109, 116, \ldots$ are of the form $a_n = 100 + n^2$, where $n = 1, 2, 3, \ldots$. For each n, let d_n be the greatest common divisor of a_n and a_{n+1}. Find the maximum value of d_n as n ranges through the positive integers.

Problem 7.7. Let one has a quadrilateral $ABCD$ with $AB = AD$ and $BC \neq CD$. Let CA bisects $\angle BCD$. Prove that A, B, C, D are concyclic.

Solution submission deadline October 31, 2021
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submission email **imo20etraining@gmail.com**