$a^{n_0} - 1, a^{m_0} - 1$

$a^{n_0} - 1, a^{n_0} - a^{m_0}$

Function $\nu_p(n)$

1. (Warm up) Let a, m, n be positive integers. Prove that

$$\gcd(a^n - 1, a^m - 1) = a^{\gcd(n,m)} - 1.$$

$/ \; a^{n_0 - m_0} - 1$

Definition 1. *For a prime number p and a nonnegative integer k, write $p^k \,\|\, n$ to mean that $p^k \mid n$ and $p^{k+1} \nmid n$.*

In the case we will say that n is exactly (or fully) divisible by p^k. For example, $5^2 \,\|\, 50$. It means the same as $\nu_5(50) = 2$.

2. Find the number of zeros at the end of $5^{100}!$

3. Let a and b be positive integers such that $a \mid b^2$, $b^2 \mid a^3$, $a^3 \mid b^4$, $b^4 \mid a^5$, and so on. Show that $a = b$.

4. Determine if the product of all integers from $2^{1917} + 1$ to $2^{1991} - 1$ inclusive is a perfect square.

5. Prove that $\frac{1}{n+1}\binom{2n}{n}$ is an integer.

6. Let p be a prime number and p^r divides $\binom{2n}{n}$. Show that $p^r \leqslant 2n$.

7. If $n \geqslant 3$, p is a prime number and $\frac{2n}{3} < p \leqslant n$, then $\binom{2n}{n}$ is not divisible by p. Prove it.