Email training, N2

September 18-23

Problem 2.1. Let x_1 and x_2 are the roots of the equation $x^2 + 5x - 11$. Find a quadratic polynomial which roots are x_1x_2 and $x_1^2x_2^2$.

Problem 2.2. Simplify

$$\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}.$$

Problem 2.3. Find all positive integers n for which $n^2 + 3n$ is perfect square.

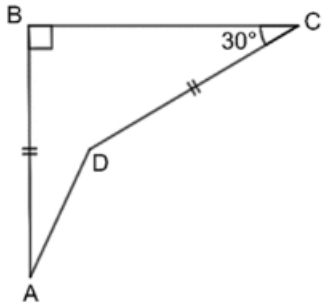
Problem 2.4. Find all integer solutions to the equation

$$x^2 - 6xy + 13y^2 = 100.$$

Problem 2.5. Find the number of 7-digit positive integers that all digits are ordered in
a) strictly increasing order,
b) strictly decreasing order.

Problem 2.6. A triple $(1, 1, 1)$ is given. On each step one chooses 2 of them and increases by 1. Is it possible after some steps get numbers $(2016, 2016, 2016)$.

Problem 2.7. -



على الشكل المجاور: لدينا $AB = CD$ ، $BC = 2AD$،
وكذلك $\angle BCD = 30°$.
إذا كان $BC \perp BA$ فأوجد قياس $\angle BAD$.

Problem 2.8. -

لدينا $\triangle ABC$ مثلث فيه $\angle A = 60°$. النقاط M, N, K تقع على أضلاعه BC, AC, AB على الترتيب
بحيث $BK = KM = MN = NC$. إذا كان $AN = 2AK$، فأوجد كل من $\angle B, \angle C$.

Solution submission deadline 15:00, September 23, 2022
Send the solution as single PDF file to imo20etraining@gmail.com
Filename format: Name_Level_Week.pdf, for example Smbat_L1_Week2.pdf