— Combinatorics for L4 —

— January Camp, 2022 — Domino Tilings —

Warm-up.

- Calculate the number of domino tilings of a $2 \times n$ rectangle.
- Prove that the number of domino tilings of a $2n \times 2n$ board is of the form $m2^n$ for some integer m.
- Prove that m defined above is a perfect square.
- Prove that m defined above is odd.

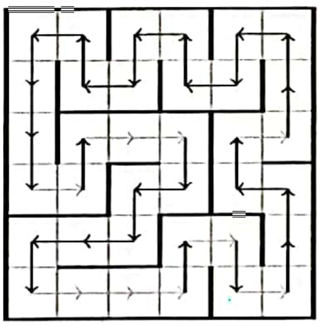
6. Calculate the number of ways to build a $2 \times 2 \times n$ *chimney* from $1 \times 2 \times 2$ *bricks.*

7. Prove that for every $n \geq 1$ the number of ways to build a $2 \times 2 \times 2n$ chimney from $1 \times 1 \times 2$ bricks is a perfect square.

8. Prove that for every $n \geq 1$ the number of domino tilings of an $n \times (n+1)$ rectangle is odd.

9. Let n be a positive integer. Determine the smallest positive integer k with the following property: it is possible to mark k cells on a $2n \times 2n$ board so that there exists a unique partition of the board into dominoes, none of which contains two marked cells.

10. The picture presents a *maze* (closed loop visiting each cell exactly once) on a 6×6 board with the following property: in order to traverse the maze once, one has to perform equally many steps in each of the four directions.



Can you design a maze with this property on an 8×8 board?