Email training, N1

Level 2, September 13-19

Problem 1.1. Let integers x and y are such that $5x + 7y = 111$. Prove that $a + b$ is even.

Problem 1.2. Is it possible to put signs $+$ and $-$ instead of $*$'s to get correct expression

$$1 * 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 * 10 = 0.$$

Problem 1.3. Find the number of 3 digit positive integers, such that all digits are even.

Problem 1.4. During the contest 10 students all together have solved 35 problems. It's known that some student solved exactly 1 problem, there is a students that solved exactly 2 problems and there is a student that solved exactly 3 problems. Prove that there is a student that solved at least 5 problems.

Problem 1.5. Recover missing digits

$$1 * \cdot * 1 = 1 * *1.$$

Problem 1.6. Which number is bigger $(n-1)! \cdot (n+1)$ or $n! \cdot n$.

Problem 1.7. Let AA', BB' and CC' are the altitudes of the triangle ABC. Let A_1 and A_2 are the projections of A' on AB and AC, respectively, B_1 and B_2 are the projections of B' on BC and BA, as well as C_1 and C_2 are the projections of C' on CA and CB. Prove that:

- $B_2C_1 \parallel BC$,
- The hexagon $A_1B_2C_1A_2B_1C_2$ is cyclic.

Solution submission deadline September 19, 2021
Submit single PDF file in filename format L2_YOURNAME_week1.pdf
submission email **imo20etraining@gmail.com**