Test2 - P1

if $a < b \Rightarrow x^a > x^b$

Now: $1 > x$ $x_0 > x_1$

$x < x^x < 1$ $x_1 < x_2 < x_0$

$1 > x^x > x^{x^x} > x$ $x_0 > x_2 > x_3 > x_1$

$x_1 < x_3 < x_4 < x_2 < x_0$

$x_0 > x_2 > x_4 > x_5 > x_3 > x_1$

$x_1 < x_3 < x_5 < x_6 < x_4 < x_2 < x_0$

then by an easy induction we prove that:

$$x_0 > x_2 > \cdots > x_{2k} > x_{\underline{2k+1}} > \underbrace{x_{2k-1} > \cdots > x_1}$$

if $n = 2k$ (even)

$\Rightarrow$ # of indices $= k = \lfloor \frac{n}{2} \rfloor$

if $n = 2k+1$ (odd)

$\Rightarrow$ # of indices $= k = \lfloor \frac{n}{2} \rfloor$

Thus, the answer is $\lfloor \frac{n}{2} \rfloor$

$\blacksquare$

Test2 - Problem2

Answer: Yes.

Example1:

0	1	3
2	4	7
5	6	8

0	0	0	0
0	0	1	2
1	1	2	2
2	1	2	2

Example 2:

1	0	2
3	4	5
6	8	7

0	0	0	0
1	0	0	2
0	2	2	1
2	2	2	2

Test 2 - P4

$$n^k + 1 = (n-2)!$$

Solution 1

try small cases, we get $(n,k) = (4,0)$
$(5,1)$

if $n \notin \mathbb{P}$ $\quad n \geq 6$

$\Rightarrow n = ab$ s.t $n > a > b > 1$

$\Rightarrow a, b < n-1 \Rightarrow n = ab \mid (n-2)!$

$\Rightarrow n \mid (n-2)! - n^k$ (clearly $k > 0$

$\Rightarrow n \in \mathbb{P}$ and $4 \mid (n-2)!$

$$\Rightarrow \quad n^k \equiv -1\,[4] \Rightarrow k \text{ odd}$$

$$v_2(n^k+1) = v_2(n+1) = v_2((n-2)!)$$

$$v_2((n-2)!) = v_2(n-2) + \cdots + v_2(1)$$

$$\geq v_2\left(\frac{n+1}{2}\right) + v_2(4)$$

$$\forall\, n > 7 \quad \left(\because\ n-2 \geq \frac{n+1}{2} > 4 \right)$$

but $v_2\left(\frac{n+1}{2}\right) + v_2(4)$

$$= v_2(n+1) + 1 \qquad ↯ \quad \blacksquare$$

Solution 2 | try small cases,

we get $(n,k) = (5,1), (4,0)$

now $n > 5$; n is clearly a prime. $\Rightarrow$ $n-1$ composite

$n-1 > 4 \Rightarrow n-1 \mid (n-2)!$

$$n^k - 1 = (n-2)! - 2$$

$n-1 \mid LHS \Rightarrow n-1 \mid RHS$

$\Rightarrow n-1 \mid 2$ $\lightning$. $\blacksquare$