INEQUALITY ON THE INTERVAL

- Trick: if $x \in [a,b]$ then $(x-a)(x-b) \le 0$.
- For two consecutive integers $a, a+1$ then $(x-a)(x-a-1) \ge 0$ for all $x \in \mathbb{Z}$.

Problem 1. Let a,b,c be real numbers in $[1;3]$ and $a+b+c=7$. Find the maximum value of the following expressions:

a) $T = a^2 + b^2 + c^2$.

b) $T = a^3 + b^3 + c^3$.

c) $T = a^5 + b^5 + c^5$.

d) $T = a^n + b^n + c^n$ for any positive integer n.

Problem 2.

a) Let $x, y, z \in [1;2]$ such that $x^2 + y^2 + z^2 = 6$. Find the minimum value of

$$N = x + y + z.$$

b) Let $x, y, z \in [0;1]$, find the maximum value of

$$M = x + y^2 + z^3 - xy - yz - zx.$$

Problem 3. Let a,b,c be real numbers in $[1;3]$ and $a+b+c=6$. Find the maximum and minimum value of

$$P = a^2 + b^2 + c^2.$$

Problem 4. On the plane, there are 66 points and 16 lines. Denote m as the number of pairs (a,b) such that the point a belongs to the line . Prove that $m \le 159$.

Problem 5. Let $a,b,c \in [0;2]$, find the maximum value of

$$M = a^3 + b^3 + c^3 - 3abc.$$

Problem 6. Let $x, y \in [1,2]$, find the minimum value of

$$T = \frac{x+2y}{x^2+3y+5} + \frac{2x+y}{y^2+3x+5} + \frac{1}{4(x+y-1)}.$$

Problem 7*. Let a,b,c be real numbers in $[1;3]$ and $a+b+c=6$.

a) Prove that

$$\frac{(ab+bc+ca)^2+72}{ab+bc+ca} \le \frac{abc}{2} + \frac{160}{11}.$$

b) Find the maximum value of

$$P = a^4 + b^4 + 10c^2 - \frac{13}{\sqrt{4(a^3+b^3)+13c^2+5}}.$$