

Number Theory

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Problems – April 26

1. Find all primes p for which $3^p + 7p - 4$ is a perfect square.
2. What is the largest power of 5 dividing $2^{299} + 2^{199} - 1$?
3. Find all positive integers n for which n^2 divides $2^n + 1$.
4. Find all positive integers x, y and n for which $x^3 + y^3 = 3^n$.
5. Find all pairs of primes p, q for which $5pq + 1$ is a fifth power.
6. Find at least one pair (a, b) of positive integers such that neither of a, b and $a + b$ is divisible by 7, but $(a + b)^7 - a^7 - b^7$ is divisible by 7^7 .
7. Let $n > 2$ be a positive integer. Prove that the number $2^{2^n - 1} - 2^n - 1$ is composite.
8. Find all primes p and positive integers m, n such that $(p - 1)^m + 1 = p^n$.
9. Prove that there exists an odd positive integer a such that $2^n + a$ is composite for all nonnegative integers n .
10. Given a positive integer n , find the GCD of all numbers of the form $a^n + (a + 1)^n + (a + 2)^n$, where a goes over all positive integers.
11. Let a and b be different positive integers. Prove that there is a positive integer n such that $a^n - b^n$ is not a perfect power.
12. Let p be a prime number. Prove that there exists a prime number q such that $x^p \equiv p \pmod{q}$ has no solutions.