Test 1

Level 2, November 28

Problem 1.1. How many triples (x, y, z) of 6 digit positive integers exists, such that all digits of x, y, z are odd and $x + y = 10z$.

Problem 1.2. Let ABC be an acute triangle, D be the foot of altitude from A to BC. We constructed the two squares $ABKL$, $ACMN$ outside the triangle. Prove that the lines AD, BM, KC are concurrent at one point.

Problem 1.3. Positive numbers a, b satisfy the condition $a + b + \frac{1}{a} + \frac{1}{b} = 5$. Prove that $1 \leq a + b \leq 4$.

Problem 1.4. One has 2022 yellow, 2022 red and 2022 blue sticks. It is known that if we pick three sticks of different color, then it is possible to construct a triangle using these sticks. Prove that there exists a color, which satisfies the following condition: if we pick three sticks of that color, then it is possible to construct a triangle using these sticks.