— COMBINATORICS FOR L3 —

— JANUARY CAMP, 2022 — GRAPH THEORY (1): INTRODUCING GRAPHS —

WARM-UP.

- Make sure you are familiar with the following notions: GRAPH, EDGE, VERTEX, SIMPLE GRAPH, DEGREE, NEIGHBORHOOD, PATH, CYCLE
- Draw all simple graphs on up to 4 vertices.
- Prove that the number of different n-vertex graphs is at least $2^{\binom{n}{2}}/n!$.
- How do graphs of maximum degree 2 look like?

✓ **1.** Can the vertices of a regular 30-gon be labeled with numbers 1, 2, ..., 30 in such a way that the sum of labels of every pair of neighboring vertices is a perfect square?

✓ **2.** In a soccer tournament there are $2n$ teams ($n \geq 2$). On the first day n matches were played, with each team having played exactly one game. Similarly on the second day every team played exactly one game (with a different team). Prove that after two days one can still find n teams such that no two of them played with each other.

✓ **3.** At the party there are n people. Initially everyone had exactly 3 friends among others, but during the party some people got to know each other and as a result when the party ended everyone had exactly 4 friends. Find all numbers n for which such situation is possible.

✓ **4.** At a party there are 99 people.

(a) Suppose that among every three of them there is one who knows the other two. Prove that there is at least one person who knows everyone else at the party.

(b) Suppose that among every four of them there is one who knows the other three. Prove that there are at least 96 people who know everyone else at the party.

✓ **5.** Plane is divided with horizontal and vertical lines into unit squares and some of these squares (finitely many) are painted black. Suppose that each black square is adjacent to exactly 2 other black squares. Determine all possible numbers of squares painted black.

✓ **6.** In a group everyone has exactly d friends and every two strangers have exactly one common friend. Prove that the size of the group is not larger than d^2+1.