Email training, N3

September 8-14, 2019

Problem 3.1. Find all triples (a, b, c) such that $a = (b + c)^2$, $b = (c + a)^2$ and $c = (a + b)^2$.

Problem 3.2. Find all integer solutions of the equation

$$1 + x + x^2 + x^3 + x^4 = y^4.$$

Problem 3.3. Three prime numbers p, q, r and a positive integer n are given such that the numbers

$$\frac{p+n}{qr}, \frac{q+n}{rp}, \frac{r+n}{pq}$$

are integers. Prove that $p = q = r$.

Problem 3.4. $a_1, a_2, ..., a_{100}$ are permutation of $1, 2, ..., 100$. $S_1 = a_1, S_2 = a_1 + a_2, ..., S_{100} = a_1 + a_2 + ... + a_{100}$. Find the maximum number of perfect squares from S_i

Problem 3.5. Is it possible to put positive integers in the cells of the table 7×7 such that the sum of number in any square 2×2 and any square 3×3 is an odd number.

Problem 3.6. The natural numbers from 1 to 50 are written down on the blackboard. At least how many of them should be deleted, in order that the sum of any two of the remaining numbers is not a prime?

Problem 3.7. In the triangle ABC one has $\angle A = 96^o$. The segment BC is extended to an arbitrary point D. The angle bisectors of angles ABC and ACD intersect at A_1, and the angle bisectors of A_1BC and A_1CD intersect at A_2 and so on... the angle bisectors of A_4BC and A_4CD intersect at A_5. Find the size of BA_5C in degrees.

Problem 3.8. Let $ABCD$ is a parallelogram. A point M is drawn on the line AB such that $\angle MAD = \angle AMO$, where O is the point of intersection of the diagonals of the parallelogram. Prove that $MD = MC$.

Solution submission deadline September 14, 2019