November camp – KSA training 2022. Level 4 & 4+.

INEQUALITY ON INTEGER SEQUENCES

Problem 1. Let (u_n) be the strictly increasing integer sequence such that among 2022 consecutive positive integers, there exist at least one term in (u_n). Prove that

$$\frac{1}{u_2^2-u_1^2}+\frac{1}{u_3^2-u_2^2}+\cdots+\frac{1}{u_{n+1}^2-u_n^2} \text{ is unbounded.}$$

Problem 2. Let (u_n) be the strictly increasing integer sequence such that

i) $u_1>1$ and the terms of this sequences are pairwise coprime.

ii) $\frac{1}{\sqrt{u_1u_2}}+\frac{1}{\sqrt{u_3u_4}}+\cdots+\frac{1}{\sqrt{u_{2n-1}u_{2n}}}$ is unbounded.

Prove that this sequence contains infinitely many primes.

Problem 3. Let (u_m) be the strictly increasing integer sequence and there exist some constant c such that Positive

$$u_{2m}+u_{2m-1}=c\cdot u_m \text{ for all } n\in\mathbb{Z}^+.$$

Find the least value of c.

Problem 4. For some positive integer a, consider the sequence (u_n) definied by

$$u_1=a,\ u_{n+1}=u_n+f(u_n) \text{ for } n=1,2,\ldots$$

with $f(x)$ is the product of all of digits of x. Prove that there exist N such that $u_n=u_N$ for any $n\geq N$.

Problem 5. Let (u_n) be a sequence such that $u_1=2^{2023}-2$ and for any $n\geq 1$,

$$u_{n+1}=\varphi(u_n)+1, \text{ with } \varphi(x) \text{ is Euler function.}$$

Prove that $u_{2^{2022}}$ is a prime bigger than 3.

Problem 6. Consider two integer arithmetic progressions $(a_n),(b_n)$ such that in each of them, the first term and the difference are coprime. Suppse that there exist infinitely many positive integer n such that one of two products

$$(a_n^2+a_{n+1}^2)(b_n^2+b_{n+1}^2),\ (a_n^2+b_n^2)(a_{n+1}^2+b_{n+1}^2)$$

is perfect square. Prove that $a_n=b_n,\forall n$.

Problem 7*. Consider the sequence (a_n) definded by $a_1=a_2=1$ and

$$a_n=a_{a_{n-1}}+a_{n-a_{n-1}} \text{ for all } n\geq 3.$$

Prove that $a_{2n}\leq 2a_n$ for any n.