Problem 1B. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x)f(y) = f(y)f(xf(y)) + \frac{1}{xy}$$

for all $x, y \in \mathbb{R}^+$ (here $\mathbb{R}^+$ denotes the set of all positive real numbers).

Problem 2B. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$(f(x) + f(z))(f(y) + f(t)) = f(xy - zt) + f(xt + yz)$$

for all $x, y, z, t \in \mathbb{R}$.

Problem 3B. Let $f : \mathbb{R} \to \mathbb{R}$ be a non-constant function that satisfies

$$f(x)f(x - y) + f(y)f(x + y) = f(x)^2 + f(y)^2 \quad \text{for all } x, y \in \mathbb{R}.$$

Prove that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

Problem 4B. Let $f : \mathbb{R} \to \mathbb{R}$ be a function that satisfies

$$f(xy + x + y) = f(xy) + f(x) + f(y) \quad \text{for all } x, y \in \mathbb{R}.$$

Prove that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

Problem 5B. Determine all functions $g : \mathbb{R} \to \mathbb{R}$ satisfying

$$g(x + y) + g(x)g(y) = g(xy) + g(x) + g(y)$$

for all $x, y \in \mathbb{R}$.

Problem 6B. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x^3 + y^3) = (x + y)(f(x^2) - f(x)f(y) + f(y^2))$$

for all $x, y \in \mathbb{R}$. Prove that $f(2021x) = 2021f(x)$ for all $x \in \mathbb{R}$.

Problem 7B. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x)f(yf(x)) = f(x + y)$$

for all $x, y \in \mathbb{R}^+$ (here $\mathbb{R}^+$ denotes the set of all positive real numbers).