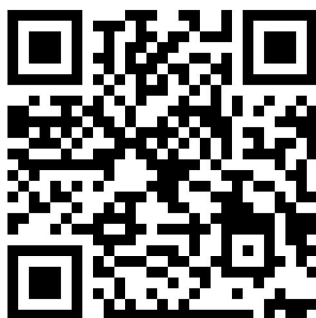


— GEOMETRY FOR L4 —

— NOVEMBER 26, 2021 — OVERVIEW WARM-UP —

First of all let me know a little bit about your history with olympic geometry. Please fill in the short form available under the link forms.gle/UcbmkqdRfyUuypMK9 (you can also use the QR code).



1. Let $ABCDEFGH$ be a regular 7-gon. Lines AB , CE intersect at P . Find $\angle PDG$.
2. Let $ABCDEFGH$ be a regular 7-gon of side length 1. Lines CF and DG intersect at P . Find AP .
3. Let D be a point on the side AB of an equilateral triangle ABC , and let E and F be reflections of D about lines AC and BC , respectively. Denote by P the intersection point of AF and BE . Prove that points D , P , C are collinear.
4. Let ABC be an equilateral triangle. Line ℓ intersects lines BC , CA , AB at points K , L , M , respectively. Prove that there exists a point P satisfying $PK = AK$, $PL = BL$, $PM = CM$.
5. Given is a circle ω and three points A , B , C on this circle. Circle o is tangent to ω and to segments AC , BC at points K , L , respectively. Circles o_1 , o_2 are both tangent to ω and externally to o at K , L , respectively. Prove that there exists a common tangent to o , o_1 and o_2 .
6. Let Ω be the circumcircle of an acute triangle ABC and let D be the midpoint of the arc BC of Ω which does not contain A . Circle ω centered at D is tangent to BC at E . Tangents to ω through A intersect the line BC at points K , L in such a way that B , K , L , C is the order of points on BC . Circle γ_1 is tangent to segments AL , BL and to Ω at M . Circle γ_2 is tangent to segments AK , BK and to Ω at N . Lines KN and LM intersect at P . Prove that $\angle KAP = \angle EAL$.