Email training, N3

Level 4, September 27-October 3

Problem 3.1. Prove there exist infinitely many positive integers divisible by 2021 and each of them containing the same number of digits 0, 1, . . ., 9.

Problem 3.2. Do there exist positive integers m and n such that the decimal representation of 5^m starts with 2^n and the decimal representation of 2^m starts with 5^n.

Problem 3.3. Let $f(x) = \frac{9^x}{9^x+3}$. Evaluate the sum

$$\sum_{k=0}^{2021} f\left(\frac{k}{2021}\right).$$

Problem 3.4. Let the sequence a_i is defined in the following way: $a_1 = m \in Z_+$ and inductively $a_{i+1} = a_i + \left[\sqrt{a_i}\right]$. Prove that the sequence a_i contains infinitely many perfect square.

Problem 3.5. In the cells of the grid 10×10 are written positive integers, all of them less than 11. It is known that the sum of 2 numbers written in the cells having common vertex is a prime number. Prove that there are 17 cells containing the same number.

Problem 3.6. In each cell of a chessboard (sizes 8×8) is put a rock. At each step one can remove from the board one rock which beats an odd number of other rocks (for example in initial configuration top-left rock beats 2 rocks). Find the maximal possible number of rocks one can remove from the board.

Problem 3.7. Given ΔABC where $AB < AC$, M is the midpoint of BC. The circle O passes through A and is tangent to BC at B, intersecting the lines AM, AC at D, E respectively. Let $CF \parallel BE$, intersecting BD extended at F. Let the lines BC and EF intersect at G. Show that $AG = DG$.

Solution submission deadline October 3, 2021

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submission email **imo20etraining@gmail.com**