Email training, N6

September 29- October 5, 2019

Problem 6.1. Find 8 positive integers $n_1, n_2, \ldots, n_8$ such that we can express every integer n with $|n| < 2019$ as $a_1 n_1 + \ldots + a_8 n_8$ with each $a_i = 0, \pm 1$.

Problem 6.2. Find the minimum value of the expression

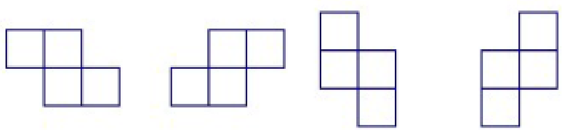
$$|\ x - 1\ | + |\ 2x - 1\ | + |\ 3x - 1\ | + \ldots + |\ 119x - 1\ |\ .$$

Problem 6.3. Find all primes p such that $p^2 + 11$ has exactly six different divisors (including 1 and the number itself).

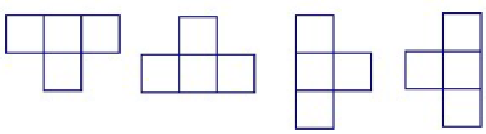
Problem 6.4. Find the number of odd coefficients of the polynomial $(x^2 + x + 1)^{33}$.

Problem 6.5. The integers a and b have the property that for every nonnegative integer n, the number $2^n a + b$ is a perfect square. Show that $a = 0$.

Problem 6.6. Can we number the squares of an 8×8 board with the numbers $1, 2, \ldots, 64$ so that any four squares with any of the following shapes



have sum divisible by 4? Can we do it for the following shapes?



Problem 6.7. Let ABC is an isosceles triangle with $AB = AC = 2$. There are 100 points $P_1, P_2, \ldots, P_{100}$ on the side BC. Denote $m_i = AP_i^2 + BP_i \cdot CP_i$. Find the value of $m_1 + m_2 + \ldots + m_{100}$.

Problem 6.8. Let a and b be two sides of a triangle. How should the third side c be chosen so that the points of contact of the incircle and excircle with side c divide that side into three equal segments?(The excircle corresponding to the side c is the circle which is tangent to the side c and the extensions of the sides a and b .)

Solution submission deadline October 5, 2019