Day 1

Problem 1. Let $n \geq 2$ be some fixed positive integer and suppose that $a_1, a_2, ..., a_n$ are positive real numbers satisfying $a_1 + a_2 + ... + a_n = 2^n - 1$.
Find the minimum possible value of

$$\frac{a_1}{1} + \frac{a_2}{1+a_1} + \frac{a_3}{1+a_1+a_2} + ... + \frac{a_n}{1+a_1+a_2+...+a_{n-1}}$$

Problem 2. Consider the following system of 10 equations in 10 real variables $v_1, v_2, ..., v_{10}$

$$v_i = 1 + \frac{6v_i^2}{v_1^2 + v_2^2 + ... + v_{10}^2} \quad (i = 1, 2, ..., 10).$$

Find all 10-tuples $(v_1, v_2, ..., v_{10})$ that are solutions of this system.

Problem 3. Let n be an odd positive integer, and let $x_1, x_2, ..., x_n$ be non-negative real numbers. Show that

$$\min_{i=1,\ldots,n} (x_i^2 + x_{i+1}^2) \leq \max_{j=1,\ldots,n} (2x_j x_{j+1})$$

where $x_{n+1} = x_1$.

Problem 4. Three different non-zero real numbers a, b, c satisfy the equations $a + \frac{2}{b} = b + \frac{2}{c} = c + \frac{2}{a} = p$, where p is a real number. Prove that $abc + 2p = 0$.

Problem 5. The sequence (a_n) is defined as: $a_1 = 1007$ and $a_{i+1} \geq a_i + 1$.
Prove the inequality:

$$\frac{1}{2016} > \sum_{i=1}^{2016} \frac{1}{a_{i+1}^2 + a_{i+2}^2}$$

Problem 6. Positive integers $a_1, a_2, ..., a_{2020}$ are pairwise distinct, and define $a_{2021} = a_1, a_{2022} = a_2$. Prove that there exists an integer $1 \leq n \leq 2020$ such that $a_n^2 + a_{n+1}^2 \geq a_{n+2}^2 + n^2 + 3$.

Problem 7. Positive reals $a_1, a_2, ...$ satisfy:

- $a_{n+1} = a_1^2 \cdot a_2^2 \cdot ... \cdot a_n^2 - 3 \quad \forall n$ positive integers
- $\frac{1}{2}(a_1 + \sqrt{a_2 - 1})$ is a positive integer.

Prove that $\frac{1}{2}(a_1 \cdot a_2 \cdot ... \cdot a_n + \sqrt{a_{n+1} - 1})$ is a positive integer.