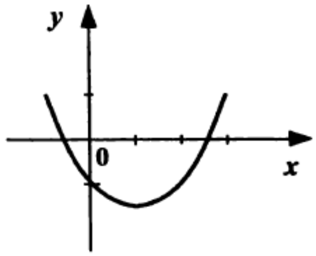
Email training, N1

September 11-16

Problem 1.1. Below is the graph of function $y = ax^2 + bx + c$. Define the signs of a, b and c.



Problem 1.2. Solve the system of equations

$$\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{5}{6} \\ \frac{1}{x^3} + \frac{1}{y^3} = \frac{35}{216} \end{cases}$$

Problem 1.3. Prove that $1^9 + 2^9 + 3^9 + \ldots + 16^9$ is divisible by 17.

Problem 1.4. Find all three digit integers $\overline{abc}$ that satisfy to the equation

$$\overline{abc} = 11(a^2 + b^2 + c^2).$$

Problem 1.5. There are 85 cubes in the shop, each of them painted in one color. Prove that either there exist 10 cubes all of them having the same color, either there exist 10 cubes all of them having different colors.

Problem 1.6. Is it possible to write 7 integers around the table with total sum equal to 1000, such that the difference of two neigbor numbers by absolute value is equal to

a) 1,
b) 2.

Problem 1.7. Prove that the sum of the distances from any point of the base of an isosceles triangle to its sides is equal to the altitude drawn to either of the sides.

Problem 1.8. In triangle ABC the bisector of the angle at B meets AC at D and the bisector of the angle at C meets AB at E. These bisectors intersect at O and the lengths of OD and OE are equal. Prove that either $\angle BAC = 60^o$ or triangle ABC is isosceles.

Solution submission deadline 15:00, September 16, 2022

Send the solution as single PDF file to imo20etraining@gmail.com