$\boxed{34}$

$(n-1)^3 \mid n^{2n} - n^{n+2} + n^n - 1$ for any $n \geq 2$

$x^{2n} - x^{n+2} + x^n - 1$

$f'(x) = 2n \cdot x^{2n-1} - (n+2)x^{n+1} + nx^n$ $\qquad f'(1) = 2n - (n+2) + n \neq 0$

$\downarrow$

$\boxed{33}$ $\qquad \boxed{f(x) = x^{2n} - n \cdot x^{n+1} + n \cdot x^{n-1} - 1}$

$(n-1)^3 \mid f(n)$

$f(x) = (x-1)^3 \cdot (\text{something})$

$f(1) = f'(1) = f''(1) = 0$

$f(1) = 1^{2n} - n \cdot 1^{n+1} + n \cdot 1^{n-1} - 1 = 1 - 1 + 1 - 1 = 0$

$f'(x) = 2n \cdot x^{2n-1} - n(n+1)x^n + n(n-1)x^{n-2}$

$f'(1) = 2n - n(n+1) + n(n-1) = 2n - n^2 - n + n^2 - n = 0$

$f''(x) = 2n \cdot (2n-1)x^{2n-2} - n(n+1)n\, x^{n-1} + n(n-1)(n-2)x^{n-3}$

$f''(1) = 2n(2n-1) - n(n+1)(n-1) + n(n-1)(n-2)$

$= 4n^2 - 2n - n^2(n+1) + n^3 - 3n^2 + 2n$

$= 4n^2 - 2n - n^3 - n^2 + n^3 - 3n^2 + 2n = 0$

Take

$\boxed{33}$

$f(x) := x^3 + ax^2 + bx + c$ r, s, t

$c \neq 0$ has three distinct real roots

Prove

$Q(x) := x^3 - bx^2 + acx - c^2$ d, e, f

has also three real roots.

r, s, t — roots of f

$r + s + t = -a$
$rs + st + rt = b$
$rst = -c$

$d + e + f = b$
$de + df + ef = ac$
$def = c^2$

rs, st, rt

rt, rs, st are roots of Q

What is $rt + rs + st$

$rt \cdot rs + rt \cdot st + st \cdot rs = rst(r+s+t) = ac$

$rt \cdot rs \cdot st =$

$$P(x) = a_n x^n + \boxed{\cdots + a_1 x + a_0}$$

$$P(x) = a_n x^n + Q(x)$$

(35)

$$P(x)^3 = P(x^3)$$

this functional equation!

$$a_n \neq 0$$

$$\boxed{P(x) = a_n x^n}$$

Compare Coefficients!

$$f = g$$

$$\Updownarrow$$

$$\boxed{a_n = b_n}$$

$$\left(a_n x^n + Q(x)\right)^3 = a_n x^{3n} + Q(x^3)$$

$\boxed{\deg Q = k}$ $\quad$ $a_k \neq 0$

$$a_n^3 x^{3n} + Q(x)^3 + 3a_n x^n Q^2(x) + 3a_n^2 x^{2n} Q(x)$$

$$= a_n x^{3n} + Q(x^3) \qquad x^{2n+k}$$

$$Q(x^3) - Q(x)^3 = 3a_n x^n Q^2(x) + 3a_n^2 x^{2n} Q(x) +$$

$\boxed{n+2k < 2n+k}$ $\quad + \; a_n^3 x^{3n} - a_n x^{3n}$ $\quad < \deg 3n$

$\deg$ LHS $\le 3k$ $\qquad$ RHS

$\boxed{a_n^3 - a_n = 0}$ $\rightarrow$ $\boxed{a_n^3 = a_n}$

$$\Downarrow$$

$$a_n \in \{1, -1\}$$

$\boxed{2n+k \geq 3k}$

$$3a_n^2 \cdot a_k = 0 \qquad x^n$$

$$\Downarrow \qquad -x^n$$

$$a_n,\ a_k = 0$$

$P(x^3) = P(x)^3$ ✓ x^4

$x^{34} = x^{34}$

$-x^{34} = (-x^4)^3$ ✓ $\square$

COMPARING DEGRES AND COEFF'S OF LHS, RHS

NEXT WEEK

3 POLY PROBLEMS
3 INAQUALITIES
} TRY