PROBLEM SET 2

Problem 1.

Let $ABCD$ be a cyclic quadrilateral. Take $F \in AB$, $E \in CD$ such that (ECD) is tangent to AB and (FAB) is tangent to CD. Denote $G = AE \cap DF$, $H = BE \cap CF$. Prove that EF is perpendicular bisector of GH.

Problem 2.

Let ABC be a triangle with (O) is its circumcircle. Take D on the minor arc BC of (O) and the tangent line from D of (O) that intersects BC, CA, AB at M, N, P. Line AM cuts (O) again at K and BN, CP meet at H. Prove that H, K, D are collinear.

Problem 3.

Let ABC be an acute, non-isosceles triangle and (O) is its circumcircle. Denote BD, CE as the altitudes of triangle ABC and AM as the median. Prove that the radical center of circles $(AM), (BE), (CD)$ belongs to (O).

Problem 4.

Let ABC be an acute, non-isosceles triangle with orthocenter H and altitude AD. The circle of diameter BC cuts the segment AD at K. On HB, HC, take S, R such that $BK = BR$ and $CK = CS$. The circumcircle of triangle DRS cuts BC again at T. Prove that

$$\frac{TB^2}{TC^2} = \frac{DB}{DC}.$$

Problem 5.

Let ABC be a triangle with $\angle B = 2\angle C$ and angle bisector BD. The symmedian of vertex B in triangles DAB, BCD cuts the corresponding circumcircle at M, N. Denote P as the reflection of B over C. Prove that the circle (MNP) is tangent to BC and both of circles $(DAB), (BCD)$.

Problem 6.

Let ABC be a triangle and a circle passes through BC cuts AB, AC at F, E. The circle (ABE) cuts CF at M, N (M is between C, F), the circle (ACF) cuts BE at P, Q (P is between B, E). Take R on BE and S on CF such that $\angle ANR = \angle AQS = 90°$. Denote U, V as intersection of pairs (SP, NR) and (RM, QS). Prove that NQ, UV, RS are concurrent.