1

$$a_1^n + a_2^n + \dots + a_n^n \geq n \cdot a_1 a_2 \dots a_k .$$ (1)

$a_1, a_2, \dots, a_k$

$$\sum_{a_1 \dots a_k}^{!} (n, \underbrace{0, 0, \dots 0}_{k-1}) = (n-1)! \left(a_1^n + a_2^n + \dots + a_n^n\right)$$

$$\sum_{a_1 \dots a_k}^{!} (\underbrace{1, 1, \dots 1}_{n}) = n! \, a_1 a_2 \dots a_k$$

$$(n, \underbrace{0, 0, 0 \dots 0}_{k-1}) \succ (\underbrace{1, 1 \dots 1}_{n})$$ By Muirhead

$$\boxed{a_1^n \dots + a_n^n \geq n a_1 \dots a_n}$$

$n + 0 + 0 \dots 0 = 1 + 1 + \dots 1$

$n \geq 1$

$n + 0 \geq 1 + 1$

$\vdots$

2

$$A = \underbrace{\sum\nolimits^{!} (n, \underbrace{0, 0 \ldots 0}_{n-1})} = (n-1)! \left(a_1^n + a_2^n + \cdots + a_n^n\right)$$

$$B = \underbrace{\sum\nolimits^{!} (1, 1, \ldots, 1)} = n! \cdot a_1 \cdots a_n$$

$= 3!$

$b^3 \quad a^3 b^0 c^0$

$b^3 \quad a^3 c^0 b^0$

$b^3 \quad b^3 a^0 c^0$

$b^3 \quad b^3 c^0 a^0$

$c^3 \quad c^3 a^0 b^0$

$c^3 = c^3 b^0 a^0$

$(3, 0, 0)$

$= \boxed{2(a^3 + b^3 + c^3)} = (3-1)!$

$(n, 0, \ldots 0) \succ (1, 1, 1, \ldots 1)$

so

$A \geq B$ ∎

$xyz = 1$ $\quad \mathbb{R}_+$

$$x^{10} + y^{10} + z^{10} \geq x^9 + y^9 + z^9 \quad \cdot\, x^{\alpha}y^{\alpha}z^{\alpha} \quad \sqrt[3]{xyz}$$ (3)

$$= x^{9+\alpha}y^{\alpha}z^{\alpha} + x^{\alpha}y^{9+\alpha}z^{\alpha} + x^{\alpha}y^{\alpha}z^{9+\alpha}$$

$\boxed{A \succ B}$

$(a_1, a_2, \dots, a_n) \in \mathbb{Q}$, $\quad (b_1, b_2, \dots, b_n)$

$a_1 + a_2 + \dots + a_n = b_1 + b_2 + \dots + b_n$

$a_1 \geq b_1$

$a_1 + a_2 \geq b_1 + b_2$

$\vdots$

$(10, 0, 0) \succ (9, 0, 0)$

$xyz = 1 \Rightarrow x^{\alpha}y^{\alpha}z^{\alpha} = 1$ for orice α

$(10, 0, 0) \succ (9+\alpha, \alpha, \alpha)$

$10 = 9 + \alpha + \alpha + \alpha \rightarrow \alpha = \frac{1}{3}$

$$x^{10} + y^{10} + z^{10} \geq \sum x^{9\frac{1}{3}} y^{\frac{1}{3}} z^{\frac{1}{3}}$$

$\sum_1 (10, 0, 0) = 2(x^{10} + y^{10} + z^{10})$

$$\cancel{\frac{1}{2}} \sum_1 (10, 0, 0) \geq \cancel{\frac{1}{2}} \sum_1 \left(9\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right)$$

Muirhead!

$\boxed{abc=1}$

(4)

3 $$\frac{1}{a^3(b+c)}+\frac{1}{b^3(c+a)}+\frac{1}{c^3(a+b)} \geq \frac{3}{2}$$

$\ldots$:)

$\left(\underline{b^3c^3(a+b)(c+a)}+a^3c^3(b+c)(a+b)+a^3b^3(b+c)(c+a)\right)' \geq \frac{3}{2}\, \underset{\overset{\shortparallel}{1}}{a^3b^3c^3}\,(a+b)(b+c)(c+a)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \shortparallel$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \left(\sum' a^2b+ab^2+2abc\right)$

$b^3c^3(ac+a^2+bc+ab)$

$\shortparallel$

$ab^3c^4+a^2b^3c^3+b^4c^4+ab^4c^3$

$\underline{LHS}: \sum'\underbrace{(4,3,1)}_{8}+\frac{1}{2}\sum'\underbrace{(4,4,0)}_{8}+\frac{1}{2}\sum'\underbrace{(3,3,2)}_{8}$ $\qquad \frac{1}{6}\sum'(1,1,1)$

$\underline{RHS}: \frac{3}{2}\left(\sum'(2,1,0)+2abc\right)=\frac{3}{2}\sum'\underbrace{(2,1,0)}_{3}+3abc$ $\qquad \frac{1}{2}\sum'(1,1,1)$

$(abc)^{\alpha}\,RHS \qquad (2+\alpha,1+\alpha,\alpha) \qquad (1+\alpha,1+\alpha,1+\alpha)$

$3+3\alpha=8$

$\overset{\shortparallel}{1} \quad (abc)^{\frac{5}{3}}\,RHS=\frac{3}{2}\sum'\left(\frac{11}{3},\frac{8}{3},\frac{5}{3}\right)+\frac{1}{2}\sum'\left(\frac{8}{3},\frac{8}{3},\frac{8}{3}\right)$ $\alpha=\frac{5}{3}$

LHS: $\sum_1'(4,3,1) + \frac{1}{2}\sum_1'(4,4,0) + \frac{1}{2}\sum_1'(3,3,2)$

Clear ?

RHS: $\frac{\cancel{3}\,1}{2}\sum_1'\left(\frac{11}{3},\frac{8}{3},\frac{5}{3}\right) + \frac{1}{2}\sum_1'\left(\frac{8}{3},\frac{8}{3},\frac{8}{3}\right)$

(5)

LHS $\geq$ RHS

$\sum_1'(4,3,1) \geq \sum_1'\left(\frac{11}{3},\frac{8}{3},\frac{5}{3}\right)$

$4+3+1 = \frac{11}{3}+\frac{8}{3}+\frac{5}{3}$

$4 \geq \frac{11}{3}$

$7\cancel{12} = 4+3 \geq \frac{11}{3}+\frac{8}{3} = \frac{19}{3} = 6\frac{1}{3}$

$\sum_1'(4,4,0) \geq \sum_1'\left(\frac{8}{3},\frac{8}{3},\frac{8}{3}\right)$

$(4,4,0) \geq \left(\frac{11}{3},\frac{8}{3},\frac{5}{3}\right)$

Ue ~~Ø~~ felt wit

$\frac{1}{2}\sum_1'(3,3,2) \geq \frac{1}{2}\left(\frac{11}{3},\frac{8}{3},\frac{5}{3}\right)$

$3+3+2 = \frac{11}{3}+\frac{8}{3}+\frac{5}{3}$

$3 \geq \frac{11}{3}$

$(3,3,2) \geq \left(\frac{8}{3},\frac{8}{3},\frac{8}{3}\right)$

$= 2\frac{1}{3}$

IMO Shortl. 200..

35

$\boxed{xyz = 1}$

$$\frac{x^3}{(1+y)(1+z)} + \frac{y^3}{(1+z)(1+x)} + \frac{z^3}{(1+z)(1+x)} \geq \frac{3}{4}$$

6

$(\alpha_1, \alpha_2 \dots \alpha_n)$

$\alpha_n \in \mathbb{R}_{\geq 0}$

$(\frac{1}{3}, \frac{1}{3})$ $(-\frac{1}{2})$.

$(\sqrt{2}, 1,)$

IMO 2005 $\boxed{3}$ $x, y, z > 0$, $xyz \geq 1$ Prove

$$\frac{x^5 - x^2}{x^5 + y^2 + z^2} + \frac{y^5 - y^2}{y^5 + z^2 + x^2} + \frac{z^5 - z^2}{z^5 + x^2 + y^2} \geq 0.$$

~ 100 m -

$xyz = 1$

$$\frac{x^3}{(1+y)(1+z)} + \frac{y^3}{(1+z)(1+x)} + \frac{z^3}{(1+z)(1+x)} \geq \frac{3}{4} \qquad xyz = 1$$

⑦

$$\cancel{4\left(x^3(1+z)(1+x) + y^3(1+\right.}$$

$$4\left(x^3(1+x) + y^3(1+y) + (1+z)z^3\right) \geq 3(1+x)(1+y)(1+z)$$

$$\| \qquad\qquad\qquad\qquad \|$$

$$4\Big(\underbrace{x^3+y^3+z^3}_{\frac{1}{2}\Sigma'(3,0,0)} + \underbrace{x^4+y^4+z^4}_{\frac{1}{2}\Sigma'(4,0,0)}\Big) \geq 3\Big(\underbrace{1}_{\frac{1}{6}(0,0,0)} + \underbrace{x+y+z}_{\frac{1}{2}\Sigma'(1,0,0)} + \underbrace{xy+yz+zx}_{\frac{1}{2}\Sigma'(1,1,0)} + \underbrace{xyz}_{\frac{1}{6}(1,1,1)}\Big)$$

$$\cancel{2}\,\Sigma'(3,0,0) + \cancel{2}\,\Sigma'(4,0,0) \geq \cancel{\tfrac{1}{2}(0,0,0)} + \frac{3}{2}\Sigma'(1,0,0) + \cancel{\tfrac{3}{2}}\,\boxed{(1,1,0)} + \cancel{\tfrac{1}{2}(1,1,1)}$$

$$\frac{1}{2}(0,0,0) = \left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right)$$

$$(4,0,0) \succ \left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right)$$

$$(1,1,0) = \left(1\tfrac{2}{3}, 1\tfrac{2}{3}, \tfrac{2}{3}\right)$$

$$(1,1,0)$$

$$(1+\alpha, 1+\alpha, \alpha)$$

$$2+3\alpha = 4 \qquad \alpha = \frac{2}{3}$$

$$\sum_1 (4,3,1) + \frac{1}{2}\sum_1 (4,4,0) + \frac{1}{2}\sum_1 (3,3,2) \geq$$

$$\geq \frac{3}{2}\sum_1 \left(\frac{11}{3}, \frac{8}{3}, \frac{5}{3}\right) + \frac{1}{2}\sum_1 \left(\frac{8}{3}, \frac{8}{3}, \frac{8}{3}\right)$$

$xyz=1$

⑧

$(4,3,1) \succ \left(\frac{11}{3}, \frac{8}{3}, \frac{5}{3}\right)$ $\quad$ $(3,3,2) \succ \left(\frac{8}{3}, \frac{8}{3}, \frac{8}{3}\right)$.

$(4,4,0) \succ \left(\frac{11}{3}, \frac{8}{3}, \frac{5}{3}\right)$

$$\frac{1}{2}\sum_1 (3,0,0) + \frac{1}{2}\sum_1 (\cancel{4},0,0) \geq \frac{1}{2}\sum_1 (0,0,0) + \cancel{\frac{3}{2}\sum (1,0,0)} + \frac{3}{2}\sum_1 (1,0,0) +$$

$$+ \frac{1}{2}\sum_1 (1,1,1)$$

$$\geq \frac{3}{2}\sum_1 (1,0,0)$$

$$\frac{3}{2}(3,0,0) \geq \frac{3}{2}\sum_1 \left(1\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$$

$(4,0,0)$

$1,0,0 \succ (2,1,1)$

$=$

$1+\alpha, \alpha\ \alpha$

$1+3\alpha = 4$

$\alpha = 1$

R

$$\frac{x^5-x^2}{x^5+y^2+z^2}+\frac{y^5-y^2}{y^5+z^2+x^2}+\frac{z^5-z^2}{z^5+x^2+y^2}\geq 0$$

LHS $(x^5-x^2)(y^5+z^2+x^2)(z^5+x^2+y^2) =$

$= (x^5-x^2)(y^5z^5+y^5x^2+y^7+z^7+z^2x^2+z^2y^2+x^2z^5+x^4+x^2y^2) =$

$$\boxed{\begin{aligned}= & \cancel{x^5y^5z^5}+\cancel{x^7y^5}+x^5y^7+\cancel{x^5z^7}+\cancel{x^7z^2}+\cancel{x^5y^2z^2}\;x^7z^5+x^9+\cancel{x^7y^2}- \\ & -\cancel{x^2y^5z^5}-\cancel{x^4y^5}-\cancel{x^2y^7}-x^4z^2-\cancel{x^2z^2y^2}-\cancel{x^4z^5}-\cancel{x^6}-x^4y^2\end{aligned}}$$

$\cancel{3x^5y^5z^5}+\left(x^7y^5+x^5z^7+x^7z^5+x^5y^7\right)\quad \cancel{x^7x^2+x^7y^2}-x^2y^7$

$x^5y^2z^2-x^2y^5z^5$

Total

$$\Big(2\cancel{x^5y^5z^5}+2\Big(\sum_{cyc}x^7y^5\Big)+\cancel{\left(x^7z^2+x^7y^2+y^7x^2+y^7z^2+z^7x^2+z^7y^2\right)}-$$

$$-x^2y^7-y^2z^7-z^2x^7+$$

$$+\cancel{\left(x^5y^2z^2-x^2y^5z^5\right)}+\left(y^5x^2z^2-y^2x^5z^5\right)+\left(z^5x^2y^2-z^2x^5y^5\right)$$

$$-3x^2y^2z^2-\sum_{cyc}x^4y^5-\cancel{\sum_{cyc}x^6}-\cancel{\sum_{cyc}x^4y^2}\Big)$$

$$\cancel{x^5y^5z^5 + 2\sum x^7y^5 +}$$

$$\sum_{cyc} (x^5-x^2)(y^5+z^2+x^2)(z^5+x^2+y^2) =$$

$$\cancel{x^9} + 2x^7y^5 + 2x^7z^5 - \cancel{x^6} + 2x^5y^7 + \cancel{3x^5y^5z^5} - \cancel{x^5y^5z^2} - x^5y^4 -$$

$$- x^5y^2z^5 + x^5y^2z^2 + 2x^5z^7 - x^5z^4 - x^4y^5 - x^4y^2 - x^4z^5 -$$

$$x^4z^2 - x^2y^5z^5 + x^2y^5z^2 - x^2y^4 + x^2y^2z^5 - 3x^2y^2z^2 - x^2y^4 +$$

$$\cancel{y^9} + 2y^7z^5 - \cancel{y^6} + 2y^5z^7 - y^5z^4 - y^4z^5 - y^4z^2 - y^2z^4 + \cancel{z^9} - \cancel{z^6}$$

$$\sum_{cyc} x^9 - \sum_{cyc} x^6 + 2\sum_{cyc} x^7y^5 + 3x^5y^5z^5 -$$

$$- \sum_{cyc} x^5y^5z^2 - \sum_{cyc} x^5y^4 - \sum_{cyc} x^4y^2$$

$$\frac{1}{2}\sum (9,0,0) + 2\sum (7,5,0) + \frac{1}{2}\sum (5,5,5) \geq$$

$$\geq \frac{1}{2}\sum (6,0,0) + \frac{1}{2}\sum (5,5,2) + \sum (5,4,0) + \sum (4,2,0)$$

$$\boxed{\Sigma'(7,5,0) + \Sigma'(5,2,2) \geq 2\,\Sigma'(4,2,0)}$$

Tmcl   Tmcl

$$(7,5,0) + (5,2,2) \geq 2\left(\frac{7+5}{2}, \frac{5+2}{2}, \frac{0+2}{2}\right) =$$

$$= 2(6, 3{,}5, 1) \geq 2(5{,}5, 3{,}5, 1{,}5) \geq$$

$$(6, 3{,}5, 1) \geq (5{,}5, 3{,}5, 1{,}5) \quad \text{why}$$

$$\geq 2(4,2,0) \quad \text{why?}$$

$$6+3{,}5+1 = 5{,}5+3{,}5+1{,}5$$

$$\|$$

$$10{,}5 = 10{,}5$$

$$6+3{,}5 \geq 5{,}5+3{,}5$$

$$9{,}5 \geq 9$$

$$(4,2,0) \leq (4,2,0)(x_4\,2)^{4,5} =$$

$$= (5{,}5, 3{,}5, 1{,}5).$$

RHS: $\cancel{(x^5+y^2+z^2)}$

$$(x^5+y^2+z^2)(y^5z^5+y^5x^2+y^7+z^7+z^2x^2+z^2y^2+x^2z^5+x^4+x^2y^2) =$$

$$= \Big(\cancel{x^5y^5z^5} + \cancel{x^7y^5} + \cancel{x^5y^7} + \cancel{x^5z^7} + \cancel{x^7z^2} + \cancel{x^5z^2y^2} + \cancel{x^7z^5} + x^9 + \cancel{x^7y^2} +$$

$$+ \cancel{y^7z^5} + \cancel{y^7x^2} + y^9 + \cancel{y^2z^7} + \boxed{y^2z^2x^2} + \cancel{y^4z^2} + x^2y^2z^5 + \cancel{y^6x^4} + x^8y^4 +$$

$$+ \cancel{z^2y^5z^5} + z^2y^5x^2 + \cancel{z^2y^7} + z^9 + \cancel{z^4x^2} + \cancel{z^4y^2} + \cancel{x^2z^7} + z^2x^4 + \boxed{z^2x^2y^2} \Big)$$

y^5z^7

$$\cancel{y^5z^2 = \sum_{cyc} x^7y^5}$$

$$\geq 2\sum_{cyc} x^4y^2 + \sum_{cyc} x^6 + \sum_{cyc} x^4y^5 + \sum_{cy}(5,5,2) +$$

$$2x^5y^5z^5 + \sum_{cyc} x^7y^5$$

$x, \ldots n+1) \geq$ $\qquad xyz \geq 1$

$$\cancel{\sum_{cyc}' (9,0,0)} + \cancel{4}\,\cancel{3}\,1 \cancel{\sum' (7,5,0)} + \sum' (5,2,2) + \cancel{\sum' (5,5,5)} \geq$$

$$\geq \cancel{\sum' (6,0,0)} + \cancel{\sum' (5,5,2)} + \cancel{2 \sum' (5,4,0)} + 2 \sum' (4,2,0) + \cancel{\sum' (2,2,2)}.$$

$\boxed{xyz \geq 1}$

$$(9,0,0) \succ (7,1,1) = x^7yz + xy^7z + xyz^7 \geq x^6, y^6, z^6$$

$\underline{xyz \geq 1}$

$$\sum' (9,0,0) \succ \sum' (7,1,1) \succ \sum' (6,0,0)$$

$$\sum' (7,5,0) \geq \cancel{\sum'} (5,5,2) \qquad \boxed{xyz \geq 1}$$

$$2 \sum' (7,5,0) \geq 2 \sum' (6,5,1) \geq 2 \sum' (5,4,0)$$

$$\cancel{3} \sum' (5,5,5) \geq \sum' (2,2,2) \qquad xyz \geq 1$$

We left with $\quad \sum' (7,5,0) + \sum' (5,2,2) \geq 2 \sum' (4,2,0)$

$\cancel{\frac{1}{2}(9,0,0)} + 2(7,5,0) + \cancel{\frac{1}{2}(5,5,5)} \geq$ $\quad x_{42} \geq 1$

$\geq \cancel{\frac{1}{2}(6,0,0)} + (5,4,0) + (4,2,0) + \cancel{\frac{1}{2}(2,2,2)}$

$(9,0,0) \geq (7,1,1) \geq (6,0,0)$

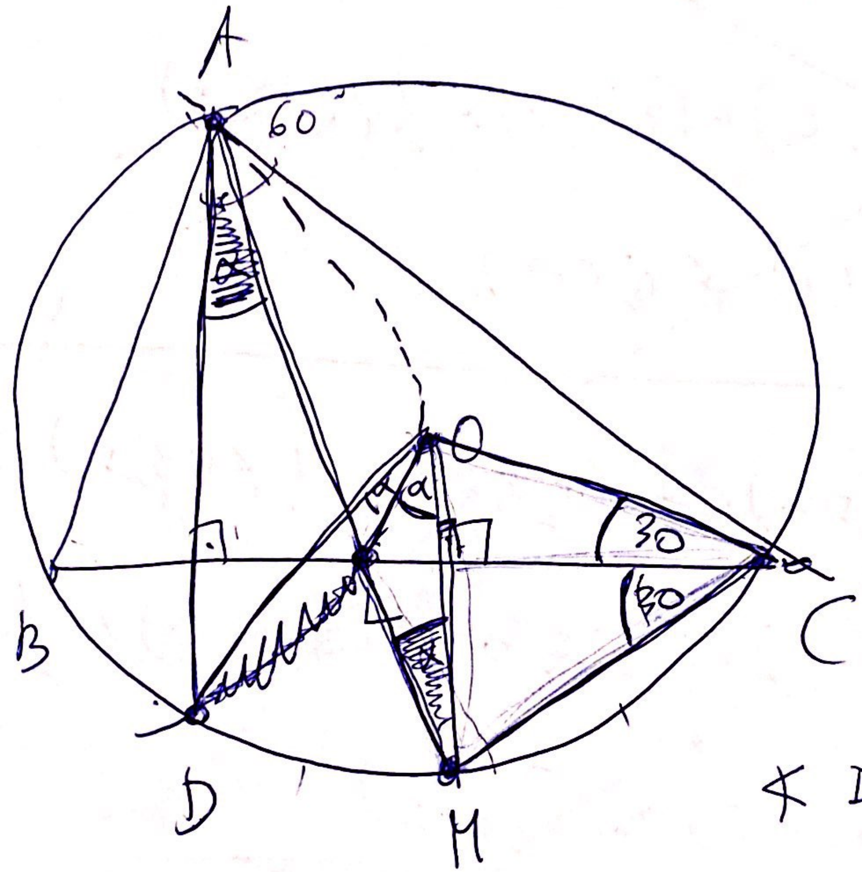
$\cancel{\frac{1}{2}(9,0,0)} + \frac{1}{2}\cancel{(7,5,0)} + \frac{1}{2}(5,2,2) + \cancel{\frac{1}{2}(5,5,5)} \geq$

$\geq \cancel{\frac{1}{2}(6,0,0)} + \cancel{\frac{1}{2}(5,5,2)} + \cancel{(5,4,0)} + (4,2,0) + \cancel{\frac{1}{2}(2,2,2)}$

$(7,5,0) \geq (6,5,1) \geq (5,4,0)$

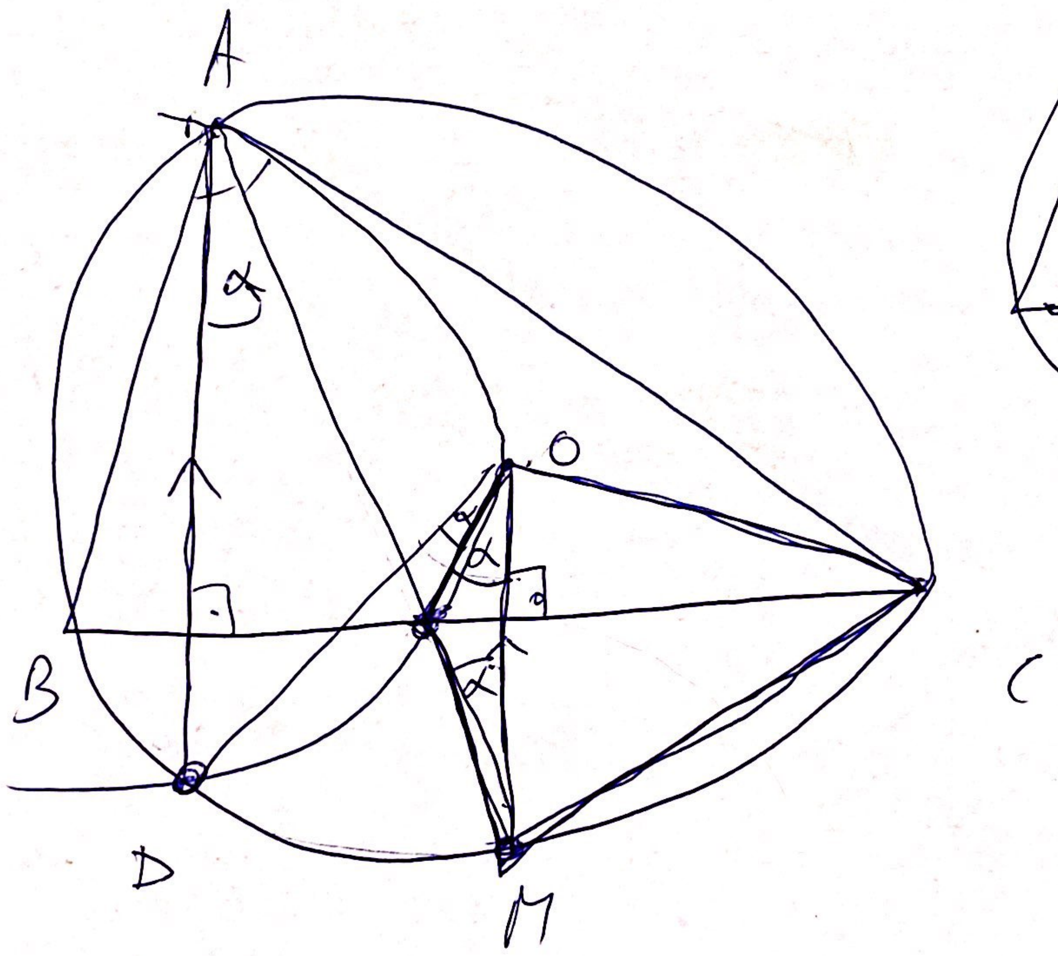
$\boxed{\frac{1}{2}(7,5,0) + \frac{1}{2}(5,2,2) \geq 2(4,2,0)}$

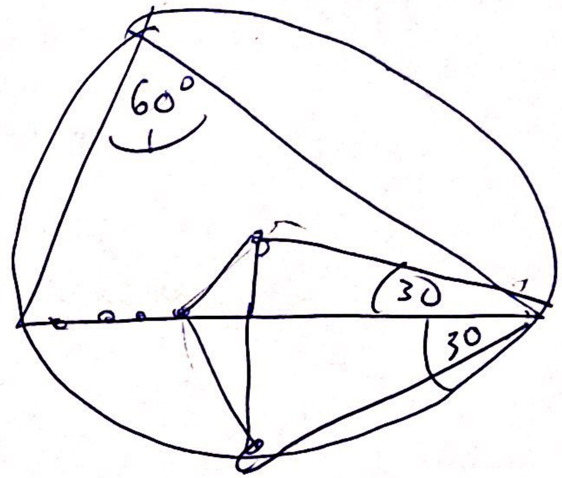
$AD \perp BC$



$\measuredangle DOM = ?$

$DOL = \alpha$





DOM = 2α