Saudi Arabia – Math Camp

Day 4 (Part 1) - Level 4

Geometry - Inversion

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Theorem. (Feuerbach's theorem) The nine-point circle of triangle is tangent to the incircle and all three excircles.

More Problems

11. (Brazil/2009) Let ABC be a triangle and O its circumcenter. The lines AB and AC intersect the circumcircle of OBC again at points $B_1 \neq B$ and $C_1 \neq C$, respectively. The lines BA and BC intersect the circumcircle of OAC again at points $A_2 \neq A$ and $C_2 \neq C$, respectively. The lines CA and CB intersect the circumcircle of OAB again at points $A_3 \neq A$ and $B_3 \neq B$, respectively. Prove that the lines A_2A_3, B_1B_3 and C_1C_2 have a common point.

12. (IMO/1985) A circle with center O passes through points A and C and intersects the sides AB and BC of the triangle ABC at points K and N, respectively. The circumscribed circles of the triangles ABC and KBN intersect at two distinct points B and M. Prove that $\angle OMB = 90°$.

13. (Cono Sur/2010) The incircle of triangle ABC touches sides BC, AC, and AB at D, E, and F respectively. Let ω_a ,ω_b and ω_c be the circumcircles of triangles EAF, DBF, and DCE, respectively. The lines DE and DF cut ω_a at $E_a \neq E$ and $F_a \neq F$, respectively. Let r_A be the line E_aF_a. Let r_B and r_C be defined analogously. Show that the lines r_A, r_B, and r_C determine a triangle with its vertices on the sides of triangle ABC.

14. (Japan/2009) Let Γ be the circumcircle of a triangle ABC. A circle with center O touches to line segment BC at P and touches the arc BC of Γ which doesn't have A at Q. If $\angle BAO = \angle CAO$, then prove that $\angle PAO = \angle QAO$.

15. (IMO/2015) Let ABC be an acute triangle with $AB > AC$. Let Γ be its circumcircule, H its orthocenter and F the foot of the altitude from A. Let M be the midpoint of BC. Let Q be the point on Γ such that $\angle HQA = 90°$, and let K be the point on Γ such that $\angle HKQ = 90°$. Assume that the points A, B, C, K and Q are all different, and lie on Γ in this order.

Prove that the circumcircles of triangles KQH and FKM are tangent to each other.