Email training, N8

Level 2, November 1-7

Problem 8.1. Let a be a real number such that numbers $a^2 + a$ and $a^3 + a$ are rationals. Prove that a is rational.

Problem 8.2. Let $P(x)$ and $Q(x)$ be monic polynomials of degree 10 with real coefficients such that equation $P(x) = Q(x)$ does not have a solution in real numbers. Prove that equation $P(x+1) = Q(x-1)$ has a solution in real numbers.

Problem 8.3. Find all positive integers n for which $n^5 + n^4 + 1$ is a prime.

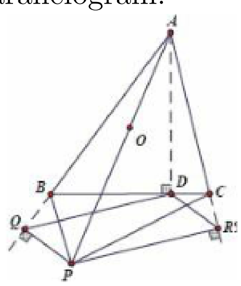
Problem 8.4. Let n be a positive integer. Prove that there exists positive integers a and b, such that

$$a^2 + a + 1 = (n^2 + n + 1)(b^2 + b + 1).$$

Problem 8.5. The circle is divided into 6 sectors and each sector contains exactly 1 coin. At each step Ali allowed to move two coins to the neighbor sectors (two sectors are called neighbor if they have a common side). Decide if it's possible to have all coins in the same sector.

Problem 8.6. Scientists participate to the conference. It occurs that any two scientists that have the same number of friends, have no common friend. Prove that there exists a scientists that has exactly 1 friend.

Problem 8.7. Given a non-isosceles acute angled triangle ΔABC, let O is its circumcenter. Let P is a point on AO extended such that $\angle BPA = \angle CPA$. Refer to the diagram on the below. Draw $PQ \perp AB$ at Q, $PR \perp AC$ at R and $AD \perp BC$ at D. Show that $PQDR$ is a parallelogram.



Solution submission deadline November 7, 2021

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