— Geometry for L3 —

— November 28, 2021 — Enriching the Picture (2) —

9. In a convex hexagon $ABCDEF$ we have $\angle BCD = \angle EFA = 90^\circ$. Prove that the perimeter of quadrilateral $ABDE$ is at least $2 \cdot CF$.

10. Given is a triangle ABC with $\angle ACB = 120^\circ$. Points P and Q lie on AC and BC, respectively, and satisfy $AP = PQ = QB$. Let M be the midpoint of segment AB. Prove that $\angle PMQ = 90^\circ$.

11. Given is a triangle ABC with $\angle ACB = 90^\circ$. Points D, E, F, belonging to segments BC, CA, AB, respectively, satisfy

$$\angle ADC = \angle BDF, \quad \angle BFD = \angle AFE, \quad \angle AEF = \angle BEC.$$

Prove that $AD + DF = BE + EF$.

12. Point D lies on side BC of an acute triangle ABC, and points E and F are projections of D onto the lines AB and AC, respectively. Point O is the circumcenter of ABC. Prove that the broken line EOF divides ABC into two pieces of equal areas.