Email training, N6

Level 3, October 18-24

Problem 6.1. Prove the inequality
$$\sqrt{a+1}+\sqrt{2a-3}+\sqrt{50-3a}\leq 12.$$

Problem 6.2. Let the parabola $y = x^2 + px + q$ is given, which intersects coordinate axes in 3 different points. Consider the circumcircle of the triangle having vertices these 3 points. Prove that there is a point that belongs to that circle, regardless of values p and q. Find that point.

Problem 6.3. Find all integer polynomials P for which $(x^2+6x+10)P^2(x)-1$ is the square of an integer polynomial.

Problem 6.4. a) Find the minimum number of elements that must be deleted from the set $\{1, 2 \ldots, 2018\}$ such that the set of the remaining elements does not contain two elements together with their product. b) Does there exist, for any k, an arithmetic progression with k terms in the infinite sequence
$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots.$$

Problem 6.5. Prove that not all zeros of a polynomial of the form $x^n+2nx^{n-1}+2n^2x^{n-2}+\ldots$ can be real.

Problem 6.6. Let the polynomial $P(x) = a_{2n}x^{2n} + a_{2n-1}x^{2n-1} + \ldots + a_1x + a_0$ with all coefficients $a_i \in [100, 101]$ is given. Find the minimal possible value of n for which $P(x)$ has a root.

Problem 6.7. Let ABC be a right-angled triangle with $\angle A = 90^o$ and let AD is an altitude of the triangle ABC. Let J, K be the incenters of the triangles ABD, ACD respectively. Let JK intersects AB, AC at E, F respectively. Prove that $AE = AF$.

Solution submission deadline October 24, 2021

Submit single PDF file in filename format L3_YOURNAME_week6.pdf

submission email **imo20etraining@gmail.com**