Email training, N4

October 2-8

Problem 4.1. Let a, b, c are solutions of equation $x^3+x^2-3x-1=0$. Construct an equatil which roots are $a+1$, $b+1$ and $c+1$.

Problem 4.2. Let a, b and c are pairwise different numbers. Solve the system of equations

$$\begin{cases} z+ay+a^2x+a^3=0 \\ z+by+b^2x+b^3=0 \\ z+cy+c^2x+c^3=0. \end{cases}$$

Problem 4.3. Solve equation in integers

$$x!+13=y^2.$$

Problem 4.4. Let numbers $x_1, x_2, \ldots, x_n$ are given and each of them is equal either $+1$ or -1. Prove that if

$$x_1x_2+x_2x_3+\ldots+x_nx_1=0$$

then n is divisible by 4.

Problem 4.5. In the cells of infinite grid are written positive integers such, that each number is equal to the arithmetical mean of the 4 neighbor numbers. Prove that all numbers are equal.

Problem 4.6. Let 10 pairwise different numbers are written on the board. Ali writes on his paper the square of difference ($(a-b)^2$) for all possible pairs , and Bob writes on his paper the absolute value of difference of squares ($|a^2-b^2|$) for all possible pairs. May it happen that Ali and Bob have the same collection of numbers?

Problem 4.7. In triangle ABC, $\angle A = 96^o$. Extend BC to an arbitrary point D. The angle bisectors of angle ABC and ACD intersect at A_1, and the angle bisectors of A_1BC and A_1CD intersect at A_2, and so on. The angle bisectors of A_4BC and A_4CD intersect at A_5 . Find the size of $\angle A_5$ in degrees.

Problem 4.8. Let $ABCD$ is a parallelogram. A point M is found on the side AB or its extension such that $\angle MAD = \angle AMO$ where O is the point of intersection of the diagonals of the parallelogram. Prove that $MD = MC$.

Solution submission deadline October 8, 2022