Level 2 E-training, week 1

Due to 23:59, Friday, 11 September 2020

Problem 1. Characterize all positive integers n satisfying that

$$\frac{\varphi(n)}{n} = \frac{24}{35}$$

Problem 2. Let $a, b, c > 0$. Prove that

$$2(a^3+b^3+c^3)\Big(\frac{1}{ca+bc}+\frac{1}{ab+ca}+\frac{1}{bc+ab}\Big) \geq 3(a+b+c)$$

Problem 3. Let ABC be a triangle with centroid G. An arbitrary line through G is drawn, and it meets segments AB, AC at D, E, respectively. Calculate:

$$\frac{AB}{AD}+\frac{AC}{AE}$$

Problem 4. Let S be a set of 100 distinct positive integers such that for any 4 elements $a < b < c < d$ of S, either $d = a + b + c$ or a divides b, c and d. Show that S contains an element that divides all other elements.

Problem 5. The solid $\mathcal{S}$ meets every plane in a disk. Show that $\mathcal{S}$ is a sphere.