Saudi Arabia 2022 – Math Camp

Day 2 - Level 4+

Geometry - Miscellaneous problems

Instructor: Regis Barbosa

1. (APMO/2007) Let ABC be an acute angled triangle with $\angle BAC = 60^\circ$ and $AB > AC$. Let I be the incenter and H the orthocenter of the triangle ABC. Prove that $2\angle AHI = 3\angle ABC$.

2. (Russia/2005) In an acute-angled triangle ABC, AM and BN are altitudes. A point D is chosen on arc ACB of the circumcircle of the triangle. Let the lines AM and BD meet at P and the lines BN and AD meet at Q. Prove that MN bisects segment PQ.

3. (IMO/2008) Let H be the orthocenter of an acute-angled triangle ABC. The circle Γ_A centered at the midpoint of BC and passing through H intersects the sideline BC at points A_1 and A_2. Similarly, define the points B_1, B_2, C_1 and C_2. Prove that six points A_1, A_2, B_1, B_2, C_1 and C_2 are concyclic.

4. (Brazil/2012) In a triangle ABC the excenter relative to A is the point of intersection of the external bisectors of $\angle B$ and $\angle C$. Let I_A, I_B and I_C be the excenters relative to A, B and C, respectively, on the scalene triangle ABC. Let X, Y and Z be the midpoints of I_BI_C, I_CI_A and I_AI_B, respectively. The incircle of ABC touches the sides BC, CA and AB at points D, E and F, respectively. Prove that the lines DX, EY and FZ passes through a common point on line OI, where O and I are the circumcenter and incenter of triangle ABC, respectively.

5. (IMO Shortlist/2000) Let O be the circumcenter and H the orthocenter of an acute triangle ABC. Show that there exist points D, E, and F on sides BC, CA, and AB respectively such that $OD + DH = OE + EH = OF + FH$ and the lines AD, BE, and CF are concurrent.

6. (China TST/2005) Let ω be the circumcircle of ΔABC. P is an interior point of ΔABC. A_1, B_1, C_1 are the intersections of AP, BP, CP respectively and A_2, B_2, C_2 are the symmetrical points of A_1, B_1, C_1 with respect to the midpoints of side BC, CA, AB. Show that the circumcircle of $\Delta A_2B_2C_2$ passes through the orthocenter of ΔABC.