Test 2

Level 4, December 2

Problem 2.1. Define $a_0 = 2$ and $a_{n+1} = a_n^2 + a_n - 1$. Prove that a_n is coprime to $2n + 1$.

Problem 2.2. Given is an acute triangle ABC with $BC < CA < AB$. Points K and L lie on segments AC and AB and satisfy $AK = AL = BC$. Perpendicular bisectors of segments CK and BL intersect line BC at points P and Q, respectively. Segments KP and LQ intersect at M. Prove that $CK + KM = BL + LM$.

Problem 2.3. In an $n \times n$ table for $n \geq 2$ at least $\lceil n(\sqrt{n} + \frac{1}{2}) \rceil$ fields have been colored. Prove that there exist 4 colored fields whose centers form (the vertices of) a rectangle or a square.

Problem 2.4. Find all integer numbers a, b, c such that

$$x(x-a)(x-b)(x-c)+1$$

can be expressed as product of two polynomials (non constant) with integer coefficients.