Test 2
Level 2, December 1

Problem 2.1. Positive integers a, b, c are given. It is known that $\frac{c}{b} = \frac{b}{a}$, and the number $b^2 - a - c + 1$ is a prime. Prove that a and c are double of a squares of positive integers.

Problem 2.2. Solve the system of equations with real numbers:

$$\begin{cases} 1 + x_1x_2x_3 = 0, \\ 1 - x_2x_3x_4 = 0, \\ 1 + x_3x_4x_5 = 0, \\ 1 + x_{2019}x_{2020}x_1 = 0, \\ 1 - x_{2020}x_1x_2 = 0. \end{cases}$$

Problem 2.3. Several boxes of weight no more than 1 kg weigh 36 kg in total. Your friend can carry at most 4 kg at a time. Prove that he can carry all the weights in no more than 11 times.

Problem 2.4. Let $ABCD$ be a trapezoid with $AB \parallel CD$ and $AB + CD = AD$. Its' diagonals intersect at E. A line passing through the point E and parallel to the bases of the trapezoid cuts AD at F. Prove that $\angle BFC = 90^\circ$.