Test 2

Level 2, December 2

Problem 2.1. Find the number of integer solutions of the equation

$$\left[\frac{x}{7}\right] = \left[\frac{x}{12}\right] + \left[\frac{x}{17}\right].$$

$[x]$ *is the largest integer not exceeding* x.

Problem 2.2. Point P lies inside parallelogram $ABCD$ and satisfies $PC = BC$. Prove that the line joining midpoints of segments AP and CD is perpendicular to BP.

Problem 2.3. Let n be a natural number. Find the number of permutations of the set $\{1, 2, \ldots, n\}$ such that for each $i = 1, 2, \ldots, n$, the first i numbers in the permutation are not larger than $i + 1$. For example, there are 4 such permutations for $n = 3$: $\{1, 2, 3, \}$, $\{2, 1, 3\}$, $\{1, 3, 2\}$ and $\{2, 3, 1\}$.

Problem 2.4. The sequence (a_k) is given by $a_1 = \frac{1}{2}$ and $a_{n+1} = 1 - a_1 a_2 \cdots a_n$ for every $n \geq 1$. Prove that $a_{100} > 0.99$.