Problem 1

$P(x) = ax^2 + bx + c$

Assume $2022 \nmid a \Rightarrow \exists\, p \mid 2022,\ p \nmid a$

$P(x)$ is full residue class mod 2022

So it is full residue class mod p

but $(p, a) = 1 \Rightarrow \exists\ K = \frac{-b}{a} \bmod p$

$\Rightarrow P(K) = a\left(\frac{-b}{a}\right)^2 + \left(\frac{-b}{a}\right)\cdot b + c \equiv c \equiv P(0) \bmod p$

but that is contradiction because we have bijection

if $K \equiv 0 \bmod p \Rightarrow b \equiv 0 \bmod p$

$\Rightarrow P(x) \equiv ax^2 + c \bmod p$

$\Rightarrow P(x) \equiv P(-x) \bmod p \Rightarrow x \equiv -x \bmod p \Rightarrow p = 2$

$\Rightarrow a = 1011 \Rightarrow P(x) = 1011x^2 + 2x + c$

and that is true because $P(x)$ is full residue class mod 1011

and $P(x) \equiv x + c \bmod 2$

problem 2 if $n \le m \Rightarrow \frac{1}{m} + \dots + \frac{1}{m+n-1} < m\left(\frac{1}{m}\right) = 1$

$\left(\frac{1}{m+k} < \frac{1}{m}\right)$

by Bertrand: $\exists$ prime $m < p < m+n$

Assume p is the biggest prime

if $2p \in \{m, m+1, \dots, m+n-1\}$

$\Rightarrow$ by Bertrand $\exists$ prime $p < q < 2p$ $\lightning$

$\Rightarrow \frac{1}{p}$ is the onld one has a number divisible by p

$\Rightarrow \frac{1}{m} + \dots + \frac{1}{m+n-1} = \frac{a}{b} + \frac{1}{p} = \frac{pa+b}{pb} \notin \mathbb{Z}$

$(p \nmid b)$ $(p \nmid b)$

problem 3:

coloring like this :→

(2n-1)

(2m-1)

1	2	1	2	....
3	4	3	4	--
1	2	1	2	--
⋮	⋮	⋮	⋮	

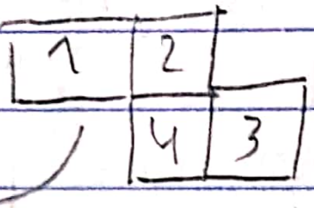
There are:

mn ones

$m(n-1)$ twos

$n(m-1)$ threes

$(n-1)(m-1)$ fours



this piece always takes 1, 2, 3, 4

so the number of pieces less than number of fours $= (n-1)(m-1)$

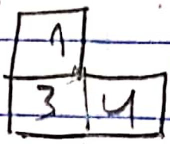
Assume we used this piece $(n-1)(m-1)-a$ times

remaining:

$m+n-1+a$ ones , $n-1+a$ twos

$m-1+a$ threes , a fours

this piece takes three different numbers ←



if we cover the rest by it

$\Rightarrow 2 \cdot$ones $\leq$ twos + threes + fours

(because each piece has almost 1 one and At least 2 others)

$\Rightarrow (m+n-1+a)2 \leq m+n-2+3a \Rightarrow m+n \leq a$

since we want to cover it with the least number of pieces we need the most number of ()

$(n-1)(m-1)-a \geq (n-1)(m-1)-m-n$

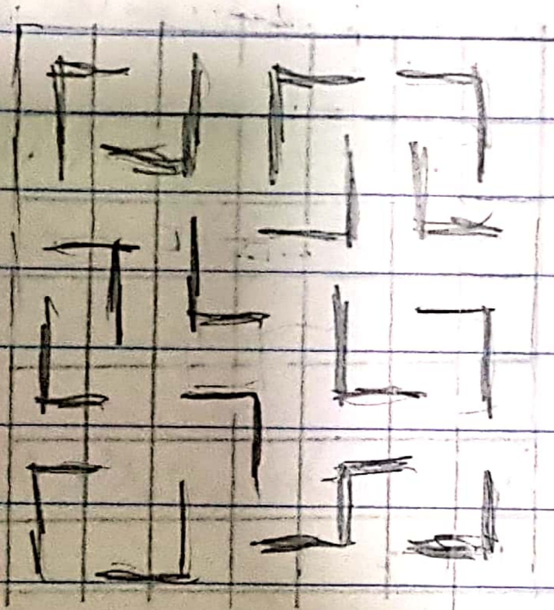
And the number of () is $(2m+2n-1)$

problem 3:

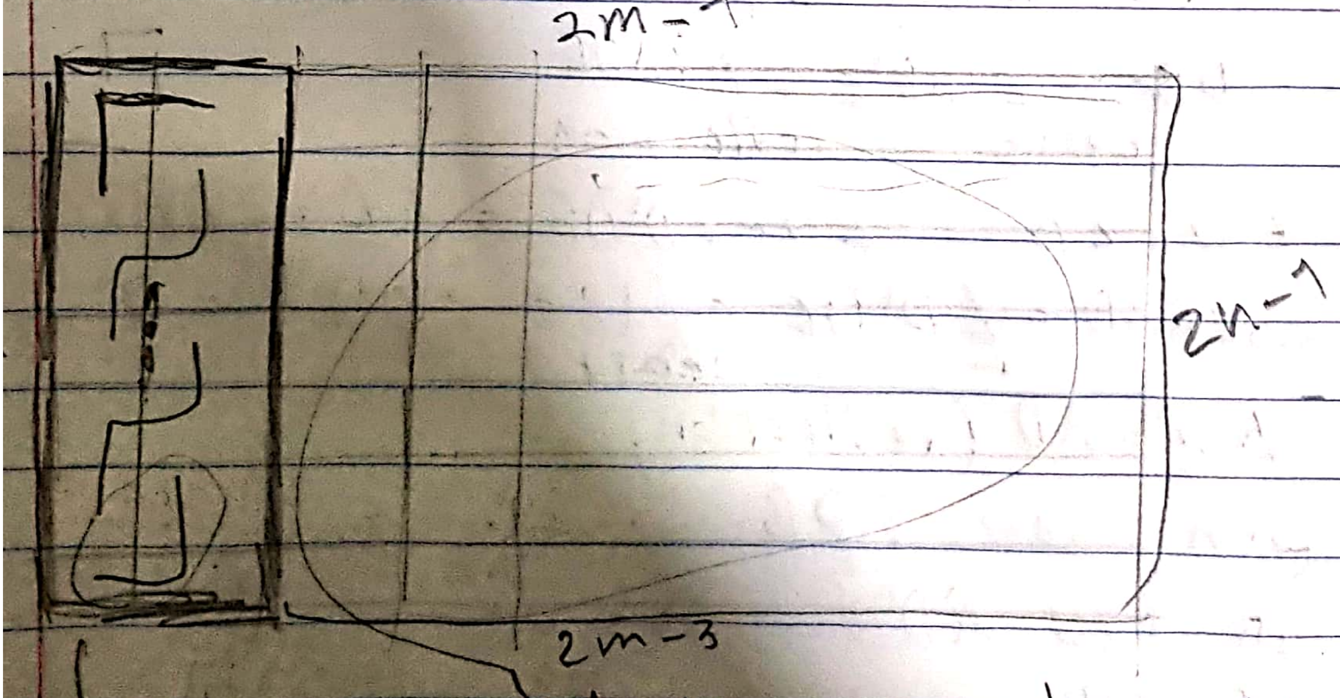
example, by induction:

number of (⊞) is $(2m+2n-1)$ $(m,n>3)$

base case: 7×7 $(⊞) = 15$



Assume $m \geqslant n$, induction, $(m,n) \to (m-1,n)$



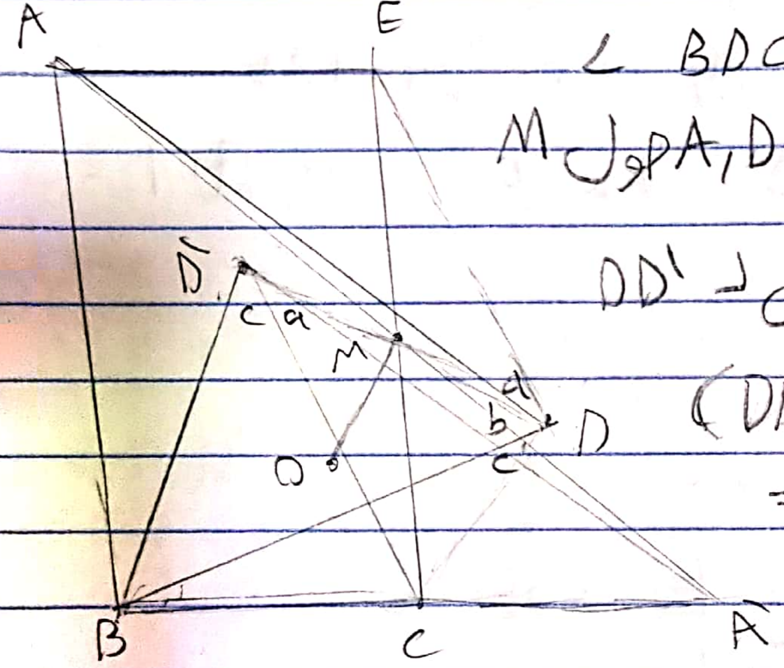
2 (⊞)   by induction $2(m-1)+2n-1$ (⊞

$(⊞) = 2(m-1)+2n-1+2 = 2n+2m-1$

Problem 4:

افرض: $\angle EDA = a$, $\angle ADB = b$

$\angle BDC = c$



افرض A', D' انعكاسات A, D حول M

M هو العامود المنصف لـ DD'

إذن D' على الدائرة (DBC)

$\Rightarrow \angle BD'C = c$

$A'CD'$ هو انعكاس AED حول M:

$\Rightarrow \angle A'D'C = \angle EDA = a$

$\Rightarrow \angle BD'A' = c + a$

$A'C = AE$, $AE + BC = AB \Rightarrow A'B = AB$

$\angle DBC \underset{\text{cyclic}}{=} \angle DD'C \underset{\text{reflection}}{=} \angle D'DE$

$\angle D = \angle B \Rightarrow \angle D - \angle D'DE = \angle B - \angle DBC$

$\Rightarrow \angle ABD = \angle D'DC \underset{\text{cyclic}}{=} 180 - \angle D'BC = x$

$A'D' = AD$ (reflection)

sin law $ADB \Rightarrow \frac{\sin b}{AB} = \frac{\sin 180 - x}{AD}$

sin law $A'D'B \Rightarrow \frac{\sin a+c}{A'B} = \frac{\sin x}{A'D'}$

$$\left.\begin{array}{l} AB = A'B \\ AD = A'D' \\ \sin x = \sin 180 - x \end{array}\right\} \begin{array}{l} \Rightarrow \sin a+c = \sin b \\ a+b+c = \angle D < 180 \\ \Rightarrow a+c = b \Rightarrow b = \frac{\angle D}{2} \end{array}$$