# Warmers-uppers

## Lesson by Senya, group L4+

Today we will work with quite different problems from combinatorics this time. The only thing that unites these problems below is that they have a quite short solution each.

**Problem 1.** There is a $10 \times 11$ board consisting of 110 unit cells. All of it's cells are divided into pairs. Prove that among these 55 pairs there is one such that the length of the common border between this pair and any other one is at most 2.

**Problem 2.** There is a set $S$ consisting of $n$ elements and there are $[\sqrt{2n}] + 2$ subsets of it. It turned out that the union of any three of these subsets is $S$. Prove that the union of some two of these subsets is $S$.

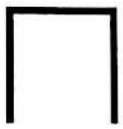
(As always, $[x]$ stands for the integer part of a real number $x$)

**Problem 3.** There is a complete graph on 35 vertices. Two players take turns removing edges of a simple path from it. The players who cannot make a move loses. Which player has a winning strategy?

**Problem 4.** There is a rectangular box that is chock-a-block filled with wooden cubes. Prove that two of the cubes are of the same size.

**Problem 5.** There are 2017 lines in the plane such that no three of them go through the same point. Turbo the snail sits on a point on exactly one of the lines and starts sliding along the lines in the following fashion: she moves on a given line until she reaches an intersection of two lines. At the intersection, she follows her journey on the other line turning left or right, alternating her choice at each intersection point she reaches. She can only change direction at an intersection point. Can there exist a line segment through which she passes in both directions during her journey?

**Problem 6.** Each piece of a "Young physicist" kit is in the form as on the picture below (it consists of three unit segments). Is it possible to make carcass of a $2 \times 2 \times 2$ cube using these pieces? (the carcass consists of 27 points that are connected with unit segments; any two of the points should be connected with at most 1 unit segment).



**Problem 7.** Two cars are going in the same direction on a road, each has speed 60km/hour. Along the road there are a few traffic lights, each of which is green for 1 minute, then red for 1 minute, then again green for 1 minute, etc... (the traffic lights need not be synchronized). Each car must stop next to a traffic light it is red. Prove that if cars start at distance that is strictly bigger than 2 kilometers from each other, then they will never end up at one place.