Problem 1C. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x^3 + y^3) = (x + y)(f(x)^2 - f(x)f(y) + f(y)^2)$$

for all $x, y \in \mathbb{R}$. Prove that $f(2021x) = 2021f(x)$ for all $x \in \mathbb{R}$.

Problem 2C. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x)f(yf(x)) = f(x + y)$$

for all $x, y \in \mathbb{R}^+$ (here $\mathbb{R}^+$ denotes the set of all positive real numbers).

Problem 3C. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x + f(y)) = f(x + y) + f(y)$$

for all $x, y \in \mathbb{R}^+$ (here $\mathbb{R}^+$ denotes the set of all positive real numbers).

Problem 4C. Determine all functions $f, g : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x + g(y)) = xf(y) - yf(x) + g(x)$$

for all $x, y \in \mathbb{R}$.

Problem 5C. Determine all functions $f, g : \mathbb{R} \to \mathbb{R}$ satisfying

(1) $f(xg(y + 1)) + y = xf(y) + f(x + g(y))$, for all $x, y \in \mathbb{R}$;

(2) $f(0) + g(0) = 0$.

Problem 6C. Determine all functions $f : \mathbb{Q} \to \mathbb{Q}$ satisfying

$$f(xf(x) + y) = f(y) + x^2$$

for all $x, y \in \mathbb{Q}$.

Problem 7C. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x) + f(y) = (f(f(x)) + f(f(y)))\, f(xy) \qquad \text{for all } x, y \in \mathbb{R}^+,$$

and there are only finitely many elements of the image of f that are the image of at least originals (i.e. there are only finitely many $b \in \mathbb{R}^+$ such that the set $\{a \in \mathbb{R}^+ : f(a) = b\}$ has at least two elements).