— COMBINATORICS FOR JBMO —

— JUNE 2019 — PROBLEM SET #1 — MINIMUM, MAXIMUM... —

REMINDER. If you want to prove that a number n is *maximal* with respect to certain property, you need to provide two arguments:

- why numbers less than n cannot satisfy the property;
- why n does satisfy it.

Typically the first part consists of some counting argument, and the second one of constructing an example. Completing one of them might be (and usually is) helpful when it comes to the other one!

1. Each entry of a 5×5 array is a real number from the interval $[-1,1]$. It is known that in every 2×2 subsquare some three out of four numbers have sum zero. What is the maximal possible sum of all entries in the array?

2. On a table there are 25 piles, each initially consisting of one token. Alice wants to merge all of them into one big pile of 25 tokens. She will do it using 24 operations of merging two of the existing piles. Whenever Alice merges a pile of a tokens with a pile of b tokens, she receives exactly $a\cdot b$ candies from Bob. What is the maximal possible number of candies Alice could have after performing all 24 operations?

3. Into each cell of a 9×9 array one of the integers -1, 0, 1 should be put in such a way that each row has non-negative sum and each column has non-positive sum. What is the minimal possible number of 0's used?

4. Initially each cell of a 7×7 board is occupied by a rabbit. All rabbits, at once, hop onto one of neighboring cells. What is the minimum possible number of occupied cells after these 49 simultaneous jumps?

5. We are given a 10×10 sheet of paper with one corner 2×2 square removed. What is the maximal possible number of rectangles of dimensions 1×6 or 2×3 that can be cut out of this sheet?

6. Given is a group in which everyone has exactly 3 friends and every two strangers have exactly one common friend. What is the maximum possible number of people in this group?

7. (JBMO 2009) Each one of 2009 distinct points in the plane is coloured in blue or red, so that on every blue-centered unit circle there are exactly two red points. Find the greatest possible number of blue points.

8. (JBMO 2016) A 5×5 table is called regular f each of its cells contains one of four pairwise distinct real numbers,such that each of them occurs exactly one in every 2×2 subtable. The sum of all numbers of a regular table is called the total sum of the table.With any four numbers,one constructs all possible regular tables,computes their total sums and counts the distinct outcomes.Determine the maximum possible count.