

Email training, N4
Level 3, October 4-10

Problem 4.1. Find the maximum possible value of $x^6 + y^6$ if it's known that $x^2 + y^2 = 1$.

Problem 4.2. Let a, b, c, d be real numbers such that

$$a^4 + b^4 + c^4 + d^4 = 16.$$

Prove the inequality

$$a^5 + b^5 + c^5 + d^5 \leq 32.$$

Problem 4.3. Let $S(n)$ be the sum of divisors of n (for example $S(6) = 1 + 2 + 3 + 6 = 12$). Find all n for which $S(2n) = 3S(n)$.

Problem 4.4. Find all pairs of positive integers (x, y) such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{lcm(x, y)} + \frac{1}{gcd(x, y)} = \frac{1}{2}.$$

Problem 4.5. Ali has chosen 8 cells of the chessboard 8×8 such that no any two lie on the same line or in the same row (we call it general configuration). On each step Baba chooses 8 cells in general configuration and puts coins on them. Then Ali shows all coins that are out of cells chosen by Ali. If Ali shows even number of coins then Baba wins, otherwise Baba removes all coins and makes the next move. Find the minimal number of moves that Baba needs to guarantee the win.

Problem 4.6. Let $1 \leq r \leq n$. We consider all r -element subsets of $(1, 2, \dots, n)$. Each of them has a minimum. Prove that the average of these minima is $\frac{n+1}{r+1}$.

Problem 4.7. Given an acute angled triangle $\triangle ABC$ and its circumcenter O . Let AD, BE, CF are altitudes of the triangle. Show that the line segments OA, OF, OB, OD, OC, OE dissect $\triangle ABC$ into three pairs of triangles that have equal areas.

Solution submission deadline October 10, 2021
Submit single PDF file in filename format L3-YOURNAME_week4.pdf
submission email **imo20etraining@gmail.com**