# Email training, N1
August 25- 31, 2019

**Problem 1.1.** Simplify $\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}$.

**Problem 1.2.** Let $a \neq 0$ and let $x_1$ and $x_2$ are the roots of the equation

$$x^2 + ax - \frac{1}{2a^2} = 0.$$

Prove that

$$x_1^4 + x_2^4 \geq 2 + \sqrt{2}.$$

**Problem 1.3.** Find all integer solutions to the equation $x^2 - 6xy + 13y^2 = 100$.

**Problem 1.4.** Prove that $lcm(1, 2, 3, \ldots, 2n) = lcm(n + 1, n + 2, \ldots, 2n)$, where $lcm$ is the least common multiplier.

**Problem 1.5.** In the school more than 90% of the students speak both English and Armenian, more than 90% of the students speak both English and Arabic. Prove that within a students that speak both Armenian and Arabic more than 90% speak English.

**Problem 1.6.** The endpoints of $N$ arcs split the circle into $2N$ equal arcs of length 1. It is known that each arc splits the circle into 2 parts of even length. Prove that $N$ is even.

**Problem 1.7.** Through vertices $B$ and $C$ of triangle $ABC$ are constructed two lines which divide the triangle into four regions (three triangles and one quadrilateral). It is known that three of them have equal area. Prove that one of these three regions is the quadrilateral.

**Problem 1.8.** We are given the square $ABCD$. On sides $AB$ and $CD$ we are given points $K$ and $L$ respectively, and on segment $KL$ we are given point $M$. Prove that the second point of intersection (i.e. the one other than $M$) of the points of intersection of circles circumscribed about triangles $AKM$ and $MLC$ lies on the diagonal $AC$.

Solution submission deadline August 31, 2019