TEST 8 – L2

Time: 14:00 – 18:15

Problem 1. Prove that for any nonnegative real numbers x_1, x_2, $\ldots$, x_n the following inequality holds:

$$x_1 + 2x_2 + \ldots + nx_n \leq \binom{n}{2} + x_1 + x_2^2 + \ldots + x_n^n.$$

Problem 2. An 2020×2020 array of real numbers is given. When the sum of the numbers in any row or column is negative, we may switch the signs of all the numbers in that row or column. If this operation is iterated, prove that all of the row or column sums eventually become nonnegative.

Problem 3. Find all primes p, q, r such that

$$p^3 = p^2 + q^2 + r^2.$$

Problem 4. An acute triangle ABC in which $AB < AC$ is given. The bisector of $\sphericalangle BAC$ intersects BC at D. Point M is the midpoint of BC. Prove that the line through centres of circumcircles of triangles ABC and ADM is parallel to AD.