

problem 1: $3^{n-1} + 5^{n-1} \mid 3^n + 5^n - 3(3^{n-1} + 5^{n-1})$

$$\Rightarrow 3^{n-1} + 5^{n-1} \mid 5^{n-1}(2), \quad \gcd(3^{n-1} + 5^{n-1}, 5) = 1$$

$$\Rightarrow 3^{n-1} + 5^{n-1} \mid 2 \Rightarrow n = 1$$

problem 2: $d_n \mid 100+n^2, 100+(n+1)^2$

$$\Rightarrow d_n \mid 2n+1 \Rightarrow n \equiv -\frac{1}{2} \pmod{d_n}$$

$$\Rightarrow n^2 \equiv \frac{1}{4} \pmod{d_n}$$

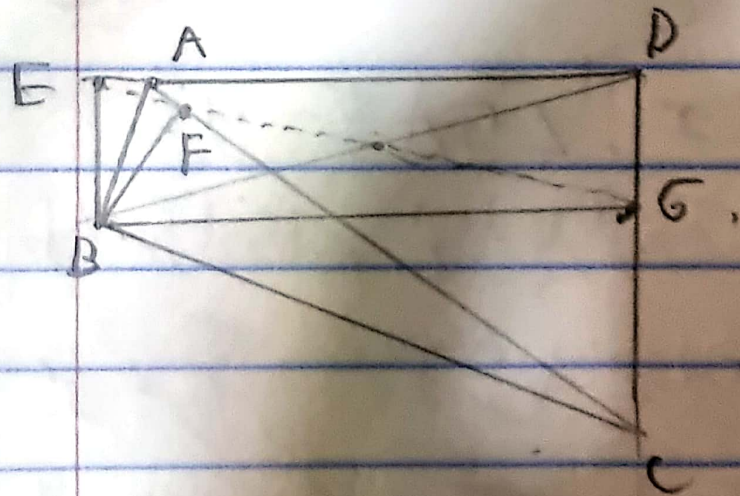
$$\Rightarrow 100 + \frac{1}{4} \equiv 0 \pmod{d_n} \Rightarrow d_n \mid 401$$

$$\Rightarrow d_n \leq 401$$

$$n=200 \Rightarrow 100+n^2 = 100 \times 401$$

$$100+(n+1)^2 = 101 \times 401$$

problem 7:



افزونی G منقطه B على DC

منه E, F, G على مستقيم واحد

$\triangle E D G$ متساوي الاضلاع

EG يسقط بمثلث BD

لان EF يسقط بمثلث BD

problem 4 :

$$(n+1)^3 - n^3 - n^3 + (n-1)^2 + k^3$$

$$k^3 \equiv k \pmod{6} \quad 6 \mid k(k-1)(k+1)$$

$$= 6n + k^3 \equiv 6n + k \pmod{6} \quad k \in \{0, 1, 2, 3, 4, 5\}$$

problem 5: $\phi(m) : 2$ for all $m > 2$

$\phi(n) \leq 2n$ for n even (equality when $n=2^a$)

because $\phi(2^k a) = \phi(2^k) \phi(a) = 2^{k-1} \phi(a)$

If $m = 2^a \Rightarrow \phi(m) = 2^{a-1} \Rightarrow \omega(m) = a$

If $m \neq 2^a \Rightarrow \phi(m) : 2 \quad \phi_2(m) \leq \frac{1}{2} \phi(m) < \frac{1}{2} m$

$\phi_3(m) \leq \frac{1}{2} \phi_2(m) \leq \frac{1}{4} \phi(m) < \frac{1}{4} m$

$\Rightarrow \phi_a(m) < \frac{1}{2^{a-1}} \times m < 2 \quad \left(\begin{array}{l} \phi_k(m) = 1 \\ k < a \end{array} \right)$ (افزونی عدم وجود)

$\Rightarrow \phi_a(m) = 1 \Rightarrow m \leq a$

problem 6 : if $4 \times m \quad \sqrt{\frac{m}{4}} = k \quad m > 12$

المتطيل $4 \times k$ إذا كانت المتطيل أصغر من

$$m > 4(k-1) = 4k - \min(4, k) \Rightarrow \text{المتطيل الأصغر}$$

أي $3 \mid m$ $4 \mid m$ $5 \mid m$

if $5 \times m$ similarly take $5 \times \sqrt{\frac{m}{5}} \quad (\sqrt{\frac{m}{5}} > 12)$

if $7 \times m$ similarly take $7 \times \sqrt{\frac{m}{7}} \quad (\sqrt{\frac{m}{7}} > 12)$

$5 \mid m, 7 \mid m$

افرض P_n هو اقل عدد لا يقسم m

$5 \leq P_{n-2} \leq P_{n-1}$

$$\Rightarrow \sqrt{\frac{m}{P_n}} > \frac{m}{P_n} > \frac{12 P_{n-2} \times P_{n-1}}{P_n}$$

$4P_{n-2} > 2P_{n-1} > P_n$ (Bertrand)

$$\Rightarrow 8 P_{n-2} \times P_{n-1} > P_n^2$$

$$\Rightarrow \sqrt{\frac{m}{P_n}} > \frac{12 P_{n-2} \times P_{n-1}}{P_n} > P_n$$

بأنه المتطيل $P_n \times \sqrt{\frac{m}{P_n}}$ إذا كانت المتطيل أصغر

$$m > P_n \times \sqrt{\frac{m}{P_n}} - P_n \Rightarrow \text{المتطيل الأصغر}$$

problem 3: $x = a-1$ $y = b-1$ $z = c-1$ $x, y, z > 0$

$$xyz \mid (x+1)(y+1)(z+1) - 1 = xyz + xy + yz + zx + x + y + z$$

$$xyz \mid xy + yz + zx + x + y + z \Rightarrow xyz \leq xy + yz + zx + x + y + z$$

Assume $x \geq y \geq z$

$$\text{if } z \geq 4 \Rightarrow xyz \geq 4xy \geq xy + yz + zx + 4x > xy + yz + zx + x + y + z$$

$$z = 1 \Rightarrow xy \mid 2x + 2y + 1$$

$$\Rightarrow x \mid 2y + 1 \quad y \mid 2x + 1 \quad \text{if } x = y \Rightarrow y \mid 1 \Rightarrow \boxed{x = y = 1}$$

$$x \equiv \frac{y-1}{2} \pmod{y}, \quad 2y+1 \geq x \geq y$$

$$x = 2y+1 \Rightarrow y=1 \Rightarrow x=3 \Rightarrow (x, y, z) = (3, 1, 1) \quad (3 \mid 4 \cdot 2 \cdot 2 - 1)$$

$$2y \geq x > y \Rightarrow x = \frac{3y-1}{2}$$

$$\frac{3y-1}{2} \mid 2y+1 \Rightarrow 3y-1 \mid 4y+2 \Rightarrow 3y-1 \mid y+3$$

$$3y-1 \leq y+3 \Rightarrow y \leq 2 \quad y=1 \Rightarrow x=1, \quad y=2 \Rightarrow x = \frac{5}{2}$$

$$z = 2 \Rightarrow 2xy \mid 3x + 3y + 2 \Rightarrow 2xy \leq 3x + 3y + 2$$

$$\text{if } y \geq 4 \Rightarrow 2xy \geq 8x \geq 3x + 3y + 2x > 3x + 3y + 2$$

$$y = 2 \Rightarrow 4x \mid 3x + 8 \Rightarrow x \mid 8, \quad 4 \mid 3x$$

$$x = 4 \text{ or } 8$$

$$4 \cdot 2 \cdot 2 \mid 5 \cdot 3 \cdot 3 - 1 = 44$$

$$8 \cdot 2 \cdot 2 \mid 9 \cdot 3 \cdot 3 - 1 = 80$$

$$y = 3 \Rightarrow \underset{E}{xyz} \mid \underset{E}{(x+1)(y+1)(z+1)} - 1$$

$$z = 3 \Rightarrow 3xy \mid 4x + 4y + 3, \quad x \geq y \geq 3 \Rightarrow 3xy \leq 4x + 4y + 3$$

$$3xy \geq 9x \geq 4x + 4y + 3, \quad 4x + 4y + 3$$

$$\text{if } x = y = 3 \Rightarrow 27 \mid 64 - 1$$

(a, b, c)

$= (2, 2, 2)$

or $(4, 2, 2)$

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