Email training, N1

August 25- 31, 2019

Problem 1.1. Let x_1 and x_2 are the roots of the equation $x^2 + 5x - 11$. Find a quadratic polynomial which roots are x_1x_2 and $x_1^2x_2^2$.

Problem 1.2. Simplify

$$\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}.$$

Problem 1.3. Find all positive integers n for which $n^2 + 3n$ is perfect square.

Problem 1.4. Find all integer solutions to the equation

$$x^2 - 6xy + 13y^2 = 100.$$

Problem 1.5. Find the number of 7-digit positive integers that all digits are ordered in
a) strictly increasing order,
b) strictly decreasing order.

Problem 1.6. A triple $(1, 1, 1)$ is given. On each step one chooses 2 of them and increases by 1. Is it possible after some steps get numbers $(2016, 2016, 2016)$.

Problem 1.7. There are four points A, B, C, D on the plane, such that any three points are not collinear. Prove that in the triangles ABC, ABD, ACD and BCD there is at least one triangle which has an interior angle not greater than 45^o.

Problem 1.8. Triangles ABC and ABD are isosceles with $AB = AC = BD$, and AC intersects BD at E. Also AC is perpendicular to BD. Find $\angle C + \angle D$ on degrees.

Solution submission deadline August 31, 2019