

Number Theory

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Problems – April 5

1. Find all primes p for which $p^2 - 2$, $p^2 + 6$ and $p^2 + 10$ are also primes.
2. By $[x]$ we denote the integer part of x . Find all positive integers for which $[\frac{n^2}{5}]$ is a prime number.
3. How many pairs of positive integers (x, y) are there for which $[x, y] = 20!$?
4. Find the largest positive integer whose digits are all nonzero and distinct, and that is divisible by the product of its digits.
5. We are given $n \geq 3$ consecutive odd three-digit numbers. Prove that these n numbers can be ordered in a sequence b_1, b_2, \dots, b_n so that the number $\overline{b_1 b_2 \dots b_n}$, obtained by writing these numbers one after another in the decimal system, be composite.
6. Can all numbers greater than 10^{100} be written as the sum of a prime and a perfect square?
7. Determine all prime numbers p such that both $\frac{p+1}{2}$ and $\frac{p^2+1}{2}$ are perfect squares.
8. Find all positive integers n for which (a) $n(n-10)$; (b) $n^3 - n$ is a perfect square.
9. Find all positive integers n for which $n \cdot 2^n + 4$ is a perfect square.