

Number Theory

Instructor: Dušan Djukić

Problems – April 28

1. Find all primes p and positive integers m, n such that $(p-1)^m + 1 = p^n$.
2. Prove that there exists an odd positive integer a such that $2^n + a$ is composite for all nonnegative integers n .
3. Given a positive integer n , prove that there exists a positive integer $m > n^n$ such that $m+n$ divides $n^m - m^n$.
4. Solve the equation $3^a - 2^b = 1$ in positive integers a, b .
5. Find all positive integers a, b such that $7^a = 3 \cdot 2^b + 1$.
6. Find all positive integers a, b, c such that $2^a + 3^b = 5^c$.
7. Solve the equation $2^a - 5^b = 7$ in positive integers a, b .
8. Solve the equation $2^a - 5^b = 3$ in positive integers a, b .
9. Given a positive integer n , find the GCD of all numbers of the form $a^n + (a+1)^n + (a+2)^n$, where a goes over all positive integers.
10. Let a and b be different positive integers. Prove that there is a positive integer n such that $a^n - b^n$ is not a perfect power.
11. Let p be a prime number. Prove that there exists a prime number q such that $x^p \equiv p \pmod{q}$ has no solutions.