Email training, N1

Level 3, September 13-19, 2021

Problem 1.1. Characterize all positive integers $n > 1$ for which the expression

$$(n-1)! \cdot \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n-1}\right)$$

is not divisible by n.

Problem 1.2. Let $x(n)$ be the biggest prime divisor of n. Prove that there exist infinitely many number n such that $x(n) < x(n+1) < x(n+2)$.

Problem 1.3. Let the sequence a_1, a_2, ..., a_{20} is the permutation of integers 1, 2, ..., 20. Find the maximum possible value of

$$\min\{|a_2 - a_1|, |a_3 - a_2|, \ldots, |a_{20} - a_{19}|, |a_1 - a_{20}|\}.$$

Problem 1.4. Two triangles ABC and $A_1B_1C_1$ are symmetric about the center of their common incircle of radius r. Prove that the product of the areas of the triangles ABC, $A_1B_1C_1$ and the six other triangles formed by the intersecting sides of the triangles ABC and $A_1B_1C_1$ is equal to r^{16}.

Solution submission deadline September 19, 2021
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submission email **imo20etraining@gmail.com**