Email training, N3

September 25 - October 1

Problem 3.1. Let $S(x)$ be the sum of digits of x. Solve the equation

$$x + S(x) + S(S(x)) = 2023.$$

Problem 3.2. Find the maximum possible value of $x^6 + y^6$ if it's known that $x^2 + y^2 = 1$.

Problem 3.3. Solve the inequality

$$\frac{2x^2 - 5x - 2}{3x - x^2 - 7} \leq 1.$$

Problem 3.4. Let $S(n)$ be the sum of divisors of n (for example $S(6) = 1 + 2 + 3 + 6 = 12$). Find all n for which $S(2n) = 3S(n)$.

Problem 3.5. Is it possible to write numbers (each once) from 1 to 10 on edges and vertices of triangular pyramid in such a way, that any number on the edge is the arithmetical mean of the numbers written on the endpoints of that edge.

Problem 3.6. Let numbers $(1, 2, 3, 4)$ are given. On each step one chooses 2 neighboring numbers (first and fourth numbers are considered as neighboring) and increases by 1. Is it possible after some steps get numbers $(2021, 2022, 2023, 2022)$?

Problem 3.7. -

لدينا الدائرتان ω_1, ω_2 مركزيهما O_1, O_2 على الترتيب يتقاطعان في A, B بحيث O_1 يقع على ω_2. النقطة P
تقع على ω_1. رسم PB, PA فقطع O_1, O_2 في X, Y على الترتيب. إذا كانت النقطة C هي نقطة تقاطع O_1O_2
مع ω_2. أثبت أن الشكل الرباعي $XPYC$ متوازي الأضلاع.

Problem 3.8. -

لدينا $ABCD$ رباعيا. E هي نقطة تقاطع المستقيمين DC, AB. إذا كان $\angle DAC = \angle CAB = 60^\circ$ و
$AB = BD - AC$. اثبت أن $\angle ADB = 2\angle BEC$.

Solution submission deadline October 1, 2022