Level 2- Test 2

17 November 2020, 1:00-5:00

Problem 1. Let $0 < x < 1$. The sequence $x_0, x_1, x_2, \ldots$ is given by $x_0 = 1$ and $x_{n+1} = x^{x_n}$ for every $n \geq 0$. Now fix an $n > 1$, find the number of indices $k < n$ satisfying $x_k < x_n$.

Problem 2. A board 4×4 is divided into 16 unit squares. Can we fill each cell by a number in $\{0, 1, 2\}$ in such a way that the sums of integers in each 2×2 square of the board are different?

Problem 3. Points X, Y lie on arc BC of (ABC) not containing A such that $\angle BAX = \angle CAY$. Let M be the midpoint of AX. Prove that $BM + CM > AY$.

Problem 4. Find all pairs (n, k) of nonnegative integers satisfying

$$n^k + 1 = (n-2)!$$