Test-15, July 15

Level 2, 9:30-13:30

Problem 1. If the real numbers x, y, z, k satisfy $x \neq y \neq z \neq x$ and

$$x^3 + y^3 + k(x^2 + y^2) = y^3 + z^3 + k(y^2 + z^2) = z^3 + x^3 + k(z^2 + x^2) = 2020$$

find the product xyz.

Problem 2. Solve in positive integers the equation

$$2^a 3^b + 9 = c^2$$

Problem 3. Let A, B, C and O be four points in the plane, such that $\angle ABC > 90^{\circ}$ and $OA = OB = OC$. Define the point $D \in AB$ and the line l such that $D \in l, AC \perp DC$ and $l \perp AO$. Line l cuts AC at E and the circumcircle of ABC at F, where F lies between D and E. Prove that the circumcircles of triangles BEF and CFD are tangent at F.

Problem 4. What is the greatest number of chess knights one can put on a 6×6 table so that no two knights can attack each other?

Problem 5. Let $a, b, c \in \mathbb{R}^+$ such that

$$3 \leq a + b + c \leq 6$$

Prove the following inequality

$$\frac{a}{2 + bc} + \frac{b}{2 + ca} + \frac{c}{2 + ab} \geq 1$$

and find all equality cases.