Test-6, March 28

Level 3

Problem 1. Decide whether there exist three distinct, non zero real numbers a, b, c such that there are exactly two equal numbers among

$$\frac{a+b}{a^2+ab+b^2}, \quad \frac{b+c}{b^2+bc+c^2}, \quad \frac{c+a}{c^2+ca+a^2}.$$

Problem 2. In acute-angled triangle ABC, BH is the altitude of the vertex B. The points D and E are midpoints of AB and AC respectively. Suppose that F be the reflection of H with respect to ED. Prove that the line BF passes through circumcenter of ABC.

Problem 3. The sequence $(a_n)n \geq 0$ is given by $a1 = 1$ and

$$a_{n+1} = a_n + \frac{1}{a_n}$$

Show that $a_{2021} > 60$

Problem 4. An invisible tank is on a 100×100 table. A cannon can fire at any 60 cells of the board after that the tank will move to one of the adjacent cells (by side). Then the progress is repeated. Can the cannon guarantee to shoot the tank?