Email training, N3

September 25 - October 1

Problem 3.1. Let a and b are divisors of n with $a > b$. Prove that $a > b + \frac{b^2}{n}$.

Problem 3.2. Do there exist 3 real numbers a, b and c such that the following inequalities hold simultaneously

$$|a| < |b - c|, \quad |b| < |c - a|, \quad |c| < |a - b|.$$

Problem 3.3. Prove that for $n \geq 1$ the following inequality holds

$$1 + \frac{5}{6n - 5} \leq 6^{1/n} \leq 1 + \frac{5}{n}.$$

Problem 3.4. Let a, b, c are positive and less than 1. Prove that

$$1 - (1 - a)(1 - b)(1 - c) > k,$$

where $k = max(a, b, c)$.

Problem 3.5. Let $x, y, z \geq 0$ and $x + y + z = 3$. Prove that

$$\sqrt{x} + \sqrt{y} + \sqrt{z} \geq xy + xz + zx.$$

Problem 3.6. Let $a, b, c > 0$. Prove that

$$\frac{a+b}{a^2+b^2} + \frac{b+c}{b^2+c^2} + \frac{c+a}{c^2+a^2} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Problem 3.7. -

In the triangle ABC the median AM is drawn. Is it possible that the radius of the circle inscribed in the triangle ABM could be twice as large as the radius of the circle inscribed in the triangle ACM?

Problem 3.8. -

A point M is chosen inside the square ABCD in such a way that $\angle MAC = \angle MCD = x$. Find $\angle ABM$.

Solution submission deadline October 1, 2022