Email training, N1

September 11-16, 2022

Problem 1.1. Solve the equation

$$x(7-x)(7+x^2) = 12(x+1)^2.$$

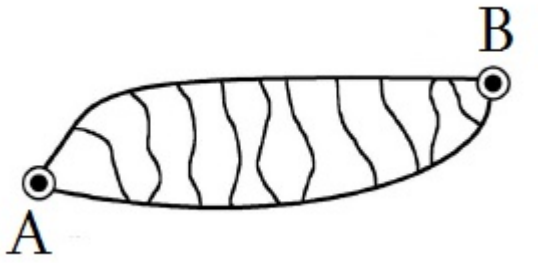
Problem 1.2. Find m for which equations $x^3 + mx + 1 = 0$ and $x^4 + mx^2 + 1 = 0$ have common root.

Problem 1.3. Find digits x and y if it's known that $\overline{12x3y4}$ is divisible by 599.

Problem 1.4. Balayan 10 - 6A Prove that if $a + b$ and ab are divisible by c, then $a^6 + b^6$ is divisible by c^3.

Problem 1.5. Let points A, B, C and D are given and no three of them lie on the same line. Prove that all triangles ABC, BCD, CDA and DAB can't be acute at the same time.

Problem 1.6. In how many ways it is possible to go from A to B without passing the same road twice.



Problem 1.7. In triangle ABC the bisector of the angle at B meets AC at D and the bisector of the angle at C meets AB at E. These bisectors intersect at O and the lengths of OD and OE are equal. Prove that either $\angle BAC = 60^o$ or triangle ABC is isosceles.

Problem 1.8. In triangle ABC one has $\angle ABC = \angle ACE = 40^o$. Let BD bisects $\angle ABC$, with D located on AC. Prove that $BD + DA = BC$.

Solution submission deadline 15:00, September 16, 2022
Send the solution as single PDF file to imo20etraining@gmail.com