TRAIN

- Divisibility
- Diophantine equations (mod's, algebraic techniques, LFF, FT, $4k+1$)
- CRT

$f: \mathbb{N} \to \mathbb{N}$

with NT bakers

2+2=4

Δ

Problem 1 $a, b \in \mathbb{Z}_+$, p, q — distinct primes

Mods

$aq \equiv 1 \mod p$

$bp \equiv 1 \mod q$

Prove: $\frac{a}{p} + \frac{b}{q} > 1$.

$$aq \equiv 1 \pmod p \iff \boxed{p \mid aq - 1}$$

$$A \equiv B \pmod C \iff \boxed{C \mid A - B}$$

5 is divisible by 5

$$\boxed{5 \equiv 0 \pmod 5}$$

$a, b, c \in \mathbb{Z}_+$

$$\frac{a\sqrt{2} + b}{b\sqrt{2} + c} \in \mathbb{Q}$$

$$\Downarrow$$

$$a + b + c \mid ab + bc + ca$$

$aq \equiv 1 \mod p$

$bp \equiv 1 \mod q$

Try to find N : $p \mid N$, $q \mid N$.

$pq \mid N$

$N \geq pq$

$p \mid aq - 1$

$q \mid bp - 1$

$$\boxed{\begin{aligned} q &\mid aq + bp - 1 \\ p &\mid aq + bp - 1 \end{aligned}}$$

$$\boxed{N = aq + bp - 1}$$

$$\boxed{pq \mid aq + bp - 1}$$

$2 \mid 4$

$4 \mid 4$ $\leadsto$ $2 \cdot 4 \mid 4$

$$\frac{aq + bp - 1}{pq} \geq 1 \leadsto \frac{aq}{pq} + \frac{bp}{pq} - \frac{1}{pq} \geq 1$$

$$\boxed{\frac{a}{p}+\frac{b}{q} \geq 1+\frac{1}{pq} > 1}$$

$\boxed{P2}$ $a, b, c \in \mathbb{Z}_+$

$$\frac{a\sqrt{2}+b}{b\sqrt{2}+c} \in \mathbb{Q}$$

$$\Downarrow$$

$$a+b+c \mid ab+bc+ca$$

Proof

$$\frac{a\sqrt{2}+b}{b\sqrt{2}+c} \in \mathbb{Q}$$

$$\Updownarrow$$

$$\frac{(a\sqrt{2}+b)(b\sqrt{2}-c)}{(b\sqrt{2}+c)(b\sqrt{2}-c)} \in \mathbb{Q}$$

$$\frac{2ab - ac\sqrt{2} + b^2\sqrt{2} - b}{2b^2-c^2} \in \mathbb{Q}$$

$$\omega = \frac{2ab + (b^2 - ac)\sqrt{2} - b}{2b^2 - c^2} \in \mathbb{Q}$$

$$\underbrace{(b^2 - ac)}_{\substack{\| \\ 0}}\sqrt{2} = \underbrace{\omega \cdot (2b^2 - c^2) + b - 2ab}_{\substack{\cap \\ \mathbb{Q}}}$$

$$\boxed{b^2 = ac}$$

$a+b+c \mid ab + bc + ca =$

$ab + bc + b^2 = b(a+b+c)$!

Suppose not;

$$\sqrt{2} = \boxed{\frac{\omega(2b^2 - c^2) + b - 2ab}{b^2 - ac}}$$

$$\cap$$

$$\mathbb{Q}$$

$\Downarrow$

$\sqrt{2}$ is rational

$\gcd(p, q) = 1$

Suppose $\sqrt{2} = \frac{p}{q} \rightsquigarrow q\sqrt{2} = p \rightsquigarrow$

$\to \boxed{2q^2 = p^2}$

$\Rightarrow 2 \mid p^2 \longrightarrow \boxed{2 \mid p}$ $\lightning$

$p = 2p_1$

$2q^2 = (2p_1)^2 = 4p_1^2 \quad | : 2$

$q^2 = 2p_1^2$

$2 \mid q^2 \rightsquigarrow \boxed{2 \mid q}$

$\sqrt{n}$ is irrational iff n is not square $\square$

rationals

irrationals

$\frac{\cancel{6}}{\cancel{4}} = \frac{3}{2}$

p

$\forall$ n $\quad \dfrac{a+n}{b+n}$ is reducible

$\Downarrow$

$a=b$

PRIME

WLOG: $a>b$

Let n be such that $a+n = p$

$\boxed{n := p-a}$, $p>a$

P. Condition

$$\frac{a+n}{b+n} = \frac{p}{b+n} \implies p \mid b+n$$

$p \mid b+n = b+p-a \rightsquigarrow \boxed{p \mid b-a}$!!!

Infinitely many primes p s.t

$p \mid b-a$

$\Downarrow$

$b-a=0 \rightsquigarrow b=a$

$b-a$ — has only finitely many discontinuities.