Email training, N9

October 20 - 29, 2019

Problem 9.1. Positive real numbers x, y are such that their product is bigger than their sum. Prove that their sum is bigger than 4.

Problem 9.2. Prove that for positive a, b, c holds

$$\frac{a}{\sqrt{a+b}}+\frac{b}{\sqrt{b+c}}+\frac{c}{\sqrt{c+a}}>\sqrt{a+b+c}.$$

Problem 9.3. Let $a, b, c > 1$ be integers such that $\gcd(a-1, b-1, c-1) > 1$. Prove that $abc-1$ is not a prime.

Problem 9.4. Prime numbers p, q, r, s satisfy $p > q > r > s$ and $p-q = q-r = r-s$. Prove that $18 \mid p-s$.

Problem 9.5. Solve the equation

$$\left[x^3\right]+\left[x^2\right]+\left[x\right]=\left\{x\right\}-1,$$

where $[x]$ is the floor function and $\{x\}$ is the rational part function.

Problem 9.6. Calculate the sum

$$\left[\frac{1}{3}\right]+\left[\frac{2}{3}\right]+\left[\frac{2^2}{3}\right]+\left[\frac{2^3}{3}\right]+\ldots+\left[\frac{2^{2019}}{3}\right].$$

Problem 9.7. The circle is divided into 6 sectors and each sector contains exactly 1 coin. At each step Ali allowed to move two coins to the neighbor sectors (two sectors are called neighbor if they have a common side).

Problem 9.8. Proof that if all sides of the triangle are less than 1 then the area of the triangle is less than $\sqrt{3}/4$.

Problem 9.9. Let the triangle ABC is given and let D is the midpoint of BC. Let E and F are two points on the sides AB and AC respectively, such that $\angle EDF = 90^o$. Prove that $BE + CF > EF$.

Problem 9.10. Let the triangle ABC is given and $\angle A : \angle B : \angle C = 1 : 2 : 4$. Prove that

$$\frac{1}{AB}+\frac{1}{AC}=\frac{1}{BC}.$$

Solution submission deadline October 29, 2019