Hints for Homework Problems

12 June, 2020

Let q be fixed prime number. Show that there exists infinitely many primes of the form $qm + 1$

Hint: We previously proved that $\frac{(p^p-1)}{p-1}$ prime deviosrs are of the form $pk + 1$. Try to do a similar thing on $\frac{(A^p-1)}{A-1}$.

$Calculate\ ord_{3^n}(2)\ for\ any\ n \in \mathbb{N}.$

Hint: $ord_{3^n}(2) = \varphi(3^n)$

Find all pairs of prime numbers (p,q) *such that*

$$\frac{(2p^2-1)^q+1}{p+q} \quad and \quad \frac{(2q^2-1)^p+1}{p+q} \quad are \; integers$$

Hint: $2p^2 - 1 \equiv 2q^2 - 1 \ (mod \ p + q)$.

Let k, n be positive integers greater than 1. Prove that if there exists natural number a such that $k|2^a + 1$, $n|2^a - 1$ then there is no naturat number b satisfying $k|2^b + 1$, $n|2^b - 1$.

Hint: look at the summary of last lecture