Email training, N15

February 24-March 1, 2020

Problem 15.1. Is it true that for any nonzero rational numbers a and b one can find integers m and n such that the number $(am+b)^2+(a+nb)^2$ is an integer?

Problem 15.2. Do there exists a positive integer n such that $n!+223$ is a perfect cube.

Problem 15.3. Let S is the set of numbers n^2+n+1, for all positive integers n. Prove that there exist infinitely many pairwise different integers $a, b, c, d \in S$ such that $\frac{a}{b}=\frac{c}{d} \in S$.

Problem 15.4. Find the smallest integer n such that there exists a table with 2 rows and n columns such that each cell contains one of the integers 1, 50 and 100 such that the sum in both rows are equal and the sum of the squares in both rows are different.

Problem 15.5. For comprime positive integers x and y find all possible integer values for expresion $\frac{x+1}{y}+\frac{y+1}{x}$.

Problem 15.6. For a positive integer n write down all its positive divisors in increasing order: $1=d_1<d_2<\ldots<d_k=n$. Find the smallest positive integer n, if it is known that $n=d_{19}\cdot d_{20}$.

Problem 15.7. Let 2020 numbers are written on blackboard. It is known that it doesn't matter how do you split them into 1010 pairs, there are two pairs such that their sums are equal. Find the maximal possible number of different values written on the blackboard.

Solution submission deadline March 1, 2020