

Email training, N6
Level 2, October 18-24

Problem 6.1. Let $a^2 + b^2 > a + b$ with $a > 0$ and $b > 0$. Prove that

$$a^3 + b^3 > a^2 + b^2.$$

Problem 6.2. Let the sequence x_n is given such that $0 < x_1 < 1$ and $x_{k+1} = x_k - x_k^2$ for all $k \geq 1$. Prove that for all n one has

$$x_1^2 + x_2^2 + \dots + x_n^2 < 1.$$

Problem 6.3. Find the maximum value of expression $\sqrt{x^2 + y^2}$ if it's known that

$$\{-4 \leq y - 2x \leq 2, \quad 1 \leq y - x \leq 2\}.$$

Problem 6.4. Prove that for any numbers $a, b, c > 0$ the following inequality holds

$$\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab} \geq \frac{2}{a} + \frac{2}{b} - \frac{2}{c}.$$

Problem 6.5. Prove the inequality

$$\sqrt{a+1} + \sqrt{2a-3} + \sqrt{50-3a} \leq 12.$$

Problem 6.6. Let the parabola $y = x^2 + px + q$ is given, which intersects coordinate axes in 3 different points. Consider the circumcircle of the triangle having vertices these 3 points. Prove that there is a point that belongs to that circle, regardless of values p and q . Find that point.

Problem 6.7. Let I be the incenter of $\triangle ABC$. Let AI is extended and intersects the circumcircle of $\triangle ABC$ at P . Draw $ID \perp BP$ at D and $IE \perp CP$ at E . Show that: $ID + IE = AP \sin \angle BAC$.

Solution submission deadline October 24, 2021
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submission email imo20etraining@gmail.com