Email training, N16

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Andy Liu - hungarian olympiads 1934-P1

Problem 16.1. Let n be an integer and let

$$A = \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot 2n}.$$

Prove that for some positive integer k the value of $2^k A$ is an integer.

Problem 16.2. Let $n > 1$. Prove that

$$\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n^2-1} + \frac{1}{n^2} > 1.$$

Problem 16.3. We are given an infinite set of rectangles in the plane, each with vertices of the form $(0,0)$, $(0,m)$, $(n,0)$ and (n,m), where m and n are positive integers. Prove that there exists two rectangles in the set such that one contains another.

Problem 16.4. Prove that for any integers $1 \leq m \leq n$ the value of

$$\frac{gcd(m,n)}{n}\binom{n}{m}$$

is an integer.