PROPERTY OF ANGLE BISECTORS

Problem 1.

Let ABC be a triangle with P is the intersection of median BM and angle bisector CP. Prove that $\dfrac{PC}{PD} - \dfrac{AC}{AB} = 1$.

Problem 2.

Let ABC be a triangle with $AB + AC = 2BC$. Denote I, G as its incenter and centroid. Prove that $IG \parallel BC$.

Problem 3.

Let ABC be a right triangle with $\angle A = 90°$ and BM is angle bisector, G is the centroid and AD is the median. Suppose that $GM \perp AC$. Prove that $BM \perp AD$.

Problem 4.

Let ABC be an equilateral. Take any point E on the opposite ray of AB, denote D as the reflection of E qua BC. Suppose that CD cuts AB at F. Prove that

$$\frac{1}{BC} = \frac{1}{BD} + \frac{1}{BF}.$$

Problem 5.

Let ABC be a triangle with $AB = 2AC$ and $\angle A = 2\angle B$. Calculate the angles of this triangle.

Problem 6.

Let ABC be a triangle with CM is median. Take D on BM such that $BD = 2MD$. Suppose that $\angle MCD = \angle BCD$, prove that ACD is a right triangle.