Problem 1. Let $, b, c$ be distinct real numbers. Prove that at least two of the following equations have a real root:

$$(x - a)(x - b) = x - c, \qquad (x - c)(x - a) = x - b, \qquad (x - c)(x - b) = x - a.$$

Problem 2. Let O is the circumcenter of triangle ABC and K is the circumcenter of triangle AOC. The lines AB and BC meet the circle AOC again at M and N respectively. Let L is the reflection of K in the line MN. Let BL intersects AC at D. Find the angle BDA.

Problem 3. We consider positive integers n having at least six positive divisors. Let the positive divisors of n be arranged in a sequence $(d_i)_{1 \le i \le k}$ with

$$1 = d_1 < d_2 < \cdots < d_k = n \quad (k \ge 6).$$

Find all positive integers n such that

$$n = d_5^2 + d_6^2.$$

Problem 4. Each positive integer is to be colored red or blue such that the sum of any red number and any blue number is red and the product of any red number and any blue number is blue. In such a coloring 1 is red and 77 is blue. What color is 2015?