Problem 1B. Find all triples of positive rational numbers (m, n, p) such that the numbers

$$m + \frac{1}{np}, \quad n + \frac{1}{pm}, \quad p + \frac{1}{mn} \qquad \text{are integers.}$$

Problem 2B. Find all pairs of positive integers (a, b) such that $a^{b^2} = b^a$.

Problem 3B. Prove that the function $f : \mathbb{N} \to \mathbb{Z}$ defined with

$$f(n) = n^{2021} - n\,!$$

is injective.

Problem 4B. Let $S = \{a_1, a_2, \dots, a_r\}$ be a set of positive integers, and $\mathcal{P}_k$ the set of all subsets of S with k elements. For a set A we denote $\gcd(A) = \gcd(\{a : a \in A\})$. Prove that

$$\operatorname{lcm}(a_1, a_2, \dots, a_r) = \prod_{i=1}^{r} \prod_{A \in \mathcal{P}_i} \gcd(A)^{(-1)^{i-1}}.$$

Problem 5B. Let $n \geqslant 2018$ be a positive integer and let $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n$ be pairwise distinct positive integers not greater than $5n$. Suppose that the sequence

$$\frac{a_1}{b_1}, \frac{a_2}{b_2}, \dots, \frac{a_n}{b_n}$$

forms an arithmetic progression. Prove that the terms of this sequence are equal.

Problem 6B. Let $f \in \mathbb{Q}[X]$ be such that $\deg(f) \geqslant 2$. We define the sequence $f^0(\mathbb{Q}) = \mathbb{Q}$, $f^1(\mathbb{Q}) = f(\mathbb{Q})$ and $f^{n+1}(\mathbb{Q}) = f(f^n(\mathbb{Q}))$ for $n \geqslant 1$. Prove that the set

$$F = \bigcap_{n \geq 0} f^n(\mathbb{Q}) \qquad \text{is finite.}$$