Email training, N13

February 2-12, 2020

Problem 13.1. Find all positive integers a and b for which

$$\frac{a^{2020}+b}{ab}$$

is an positive integer.

Problem 13.2. Let $0 \le x \le 2$, $0 \le y \le 3$ and $x+y+z=11$. Find the maximal possible value for expression xyz.

Problem 13.3. Find all integers x and y for which $x^3+y^3=x+y+xy$.

Problem 13.4. Determine all pairs of distinct real numbers (x, y) such that both of the following are true:

$$x^{100}-y^{100}=2^{99}(x-y)$$
$$x^{200}-y^{200}=2^{199}(x-y)$$

Problem 13.5. Let a, b, c, d be real numbers with $0 \leqslant a, b, c, d \leqslant 1$. Prove that

$$ab(a-b)+bc(b-c)+cd(c-d)+da(d-a) \leqslant \frac{8}{27}.$$

When equality holds?

Problem 13.6. Determine the minimal value of

$$\left(x+\frac{1}{y}\right)\left(x+\frac{1}{y}-2020\right)+\left(y+\frac{1}{x}\right)\left(y+\frac{1}{x}-2020\right),$$

where x and y vary over the positive reals.

Solution submission deadline February 12, 2020