Test-14, July 8

Level 2, 9:30-13:30

Problem 1. Let AB be a diameter of a circle ω and center O , OC a radius of ω perpendicular to AB, and M be a point of the segment OC. Let N be the second intersection point of line AM with ω and P the intersection point of the tangents to ω at points N and B. Prove that points M, O, P, N are concyclic.

Problem 2. Let $n > 1$ be an integer. Show that

$$(n^3 + 2n^2 + n)^n > 4^n(n!)^3$$

Problem 3. Find the maximum number of different integers that can be selected from the set $\{1, 2, ..., 2020\}$ so that no two exist that their difference equals to 24.

Problem 4. Solve in positive integers:

$$\frac{1}{x^2} + \frac{y}{xz} + \frac{1}{z^2} = \frac{1}{1441}$$

Problem 5. Let I be the incenter and AB be the shortest side of the scalene triangle ABC. Points P, Q lie on the line AB such Q, A, B, P are in this order and satisfy $IC = IP = IQ$. Let D be the tangency point of the A-excircle of the triangle ABC with BC. Let E be the reflection of C with respect to the point D. Prove that $PE \perp CQ$.