— Combinatorics for L4 —

— January 4, 2020 — Triangle packing —

Definition.

Clique or *complete graph* on n vertices, denoted by K_n, is a graph in which every two vertices are neighbors. Graph K_3 is called a *triangle*.

Problem 1.

A. Prove that if edges of K_n can be decomposed into pairwise edge-disjoint triangles, then either $n-1$ or $n-3$ is divisible by 6.

B. Find the decomposition of K_7 into triangles.

C. Find the decomposition of K_{3^n} into triangles, where $n \geq 1$.

Definition.

Anticlique or *independent set* on n vertices, denoted by A_n, is a graph in which no two vertices are neighbors. *Cycle* on n vertices, denoted by C_n, is a graph whose vertices can be denoted by v_i, $i = 1, 2, \ldots, n$, in such a way that only edges $v_i v_{i+1}$ exist (where $v_{n+1} := v_1$). *Complete bipartite graph* on $m+n$ vertices, with parts of sizes m and n, denoted by $K_{m,n}$, is a graph in which edge uv exists iff vertices u and v belong to different parts.

Problem 2.

A *triangle swap* is an operation of replacing A_3 with K_3 or vice versa in the graph.

A. Find all n for which C_n can be obtained from A_n by triangle-swapping.

B. Find all n for which C_n can be obtained from K_n by triangle-swapping.

C. Find all n for which A_{2n} can be obtained from $K_{n,n}$ by triangle-swapping.

Problem 3.

For every k find maximum number of edge-disjoint tringles that can be packed in $K_{k,k,k}$ (complete balanced tripartite graph).

Problem 4.

Consider graph $K_{k,k}$ with one part replaced with a clique (all edges added in that part). For every k find maximum number of edge-disjoint triangles that can be packed in this graph.

Problem 5.

Prove that if K_n can be decomposed into pairwise edge-disjoint triangles, then so does K_{2n+1}.