JBMO Test 2

Time: 4.5 hours

Problem 1. Find all triples (p, q, r) of prime numbers that satisfy

$$p + q = (p - q)^r.$$

Problem 2. Let $x_1, x_2, \ldots, x_n$, $n > 1$, be distinct integer numbers. What is the least possible value of

$$(x_1 - x_2)^2 + (x_2 - x_3)^2 + \ldots + (x_{n-1} - x_n)^2 + (x_n - x_1)^2 ?$$

Problem 3. There are $n \geq 5$ people in a group in which every pair of strangers has exactly one common friend and nobody is friends with everyone else. Prove that one can select five people from this group to be seated by a round table in such a way that everybody will take place between their friends.

Problem 4. In a non-isosceles triangle ABC points O and I denote circumcenter and incenter, respectively. Point B' which is symmetric to point B with respect to line OI, lies inside the angle ABI. Prove that the tangents to the circumcircle of the triangle $BB'I$ at points B' and I intersect on the line AC.