Preparation for Saudi Arabia Team 2021
May/June Session: Level 3

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Lesson 2
Markings and Colorings

Problems:

1. A 5×5 board has a single field taken off. For which fields taken out will it be possible to cover the remaining board with 3×1 rectangles? Solve the same problem for a 7×7 board.

2. A $m \times n$ board, where m is an odd number, is covered in 2×2 squares and L-tetraminoes. Find the smallest possible number of L-tetraminoes.

3. Let there be n light-bulbs in one row and let the light-bulb in the k-th position be ON and all others OFF. In one move, we're allowed to select three consecutive light-bulbs and change their states (turn them ON if they're OFF, and turn them OFF if they're ON). For which values of n and k can we reach a state where all the light-bulbs are OFF?

4. On a 29×29 board 99 2×2 squares are placed without overlap. Prove that at least one more 2×2 square can be placed without overlap.

5. A spider is a figure that can move up to 2 spaces diagonally or vertically and only 1 space horizontally. On an $(2n + 1) \times (2n + 1)$ board find the minimum number of moves required for the spider to visit every square at least once. The spider has visited a square if it is places initially on it, if it moves to it, or if it is the square between the initial and final square of a move when the spider has moved 2 spaces. The initial position of the spider can be freely chosen.

6. Can a 10×10 board be covered by L-tetraminoes?

7. A 5×100 table is divided into 500 unit square cells, where n of them are coloured black and the rest are coloured white. Two unit square cells are called adjacent if they share a common side. Each of the unit square cells has at most two adjacent black unit square cells. Find the largest possible value of n.