Saudi Arabia 2022 – Math Camp

Level 4 and 4+

Geometry - Miscellaneous problems

Instructor: Regis Barbosa

1. (Canada/1997) The point O is situated inside the parallelogram $ABCD$ so that

$$\angle AOB + \angle COD = 180^\circ$$

Prove that $\angle OBC = \angle ODC$.

2. (IMO/2006) Let ABC be triangle with incenter I. A point P in the interior of the triangle satisfies $\angle PBA + \angle PCA = \angle PBC + \angle PCB$. Show that $AP \geq AI$, and that equality holds if and only if $P = I$.

3. (Japan/2011) Let ABC be a given acute triangle and let M be the midpoint of BC. Draw the perpendicular HP from the orthocenter H of ABC to AM. Show that $AM \cdot PM = BM^2$.

4. (Brazil/2020) Let ABC be a triangle. The ex-circles touch sides BC, CA and AB at points U, V and W, respectively. Let r_U the line passing through U perpendicular to BC. The lines r_V and r_W are defined similarly. Prove that r_U, r_V and r_W are concurrent.

5. (IMO Shortlist/2014) Consider a fixed circle Γ with three fixed points A, B, and C on it. Also, let us fix a real number $\lambda \in (0,1)$. For a variable point $P \notin \{A, B, C\}$ on Γ, let M be the point on the segment CP such that $CM = \lambda \cdot CP$. Let Q be the second point of intersection of the circumcircles of the triangles AMP and BMC. Prove that as P varies, the point Q lies on a fixed circle.