— Geometry for L4 —

— November 26, 2021 — Overview Warm-up —

First of all let me know a little bit about your history with olympic geometry. Please fill in the short form available under the link `forms.gle/UcbmkqdRfyUuypMK9` (you can also use the QR code).

1. Let $ABCDEFG$ be a regular 7-gon. Lines AB, CE intersect at P. Find $\angle PDG$.

2. Let $ABCDEFG$ be a regular 7-gon of side length 1. Lines CF and DG intersect at P. Find AP.

3. Let D be a point on the side AB of an equilateral triangle ABC, and let E and F be reflections of D about lines AC and BC, respectively. Denote by P the intersection point of AF and BE. Prove that points D, P, C are collinear.

4. Let ABC be an equilateral triangle. Line ℓ intersects lines BC, CA, AB at points K, L, M, respectively. Prove that there exists a point P satisfying $PK = AK$, $PL = BL$, $PM = CM$.

5. Given is a circle ω and three points A, B, C on this circle. Circle o is tangent to ω and to segments AC, BC at points K, L, respectively. Circles o_1, o_2 are both tangent to ω and externally to o at K, L, respectively. Prove that there exists a common tangent to o, o_1 and o_2.

6. Let Ω be the circumcircle of an acute triangle ABC and let D be the midpoint of the arc BC of Ω which does not contain A. Circle ω centered at D is tangent to BC at E. Tangents to ω through A intersect the line BC at points K, L in such a way that B, K, L, C is the order of points on BC. Circle γ_1 is tangent to segments AL, BL and to Ω at M. Circle γ_2 is tangent to segments AK, BK and to Ω at N. Lines KN and LM intersect at P. Prove that $\angle KAP = \angle EAL$.