Problem 1G. Prove that there does not exist a function $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x)^2 \geqslant f(x+y)(f(x)+y)$$

for all $x, y \in \mathbb{R}^+$.

Problem 2G. Determine all functions $f : \mathbb{N} \to \mathbb{N}$ such that for all $x, y \in \mathbb{N}$ there is a non-degenerated triangle with side lengths

$$x, \quad f(y) \quad \text{and} \quad f(y + f(x) - 1).$$

Problem 3G. Let $f : \mathbb{R} \to \mathbb{R}$ be a function satisfying

$$f(x+y) \leqslant yf(x) + f(f(x))$$

for all $x, y \in \mathbb{R}$. Prove that $f(x) = 0$ for all $x \leqslant 0$.

Problem 4G. Prove that there does not exist a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(0) > 0$ and

$$f(x+y) \geqslant f(x) + yf(f(x))$$

for all $x, y \in \mathbb{R}$.

Problem 5G. Determine all functions $f : \mathbb{N} \to \mathbb{N}$

$$f(f(n)^2 + 2f(m)^2) = n^2 + 2m^2$$

for all $n, m \in \mathbb{N}$.

Problem 6G. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that $|f(x)| \leqslant 1$ and

$$f\left(x + \frac{13}{42}\right) + f(x) = f\left(x + \frac{1}{6}\right) + f\left(x + \frac{1}{7}\right)$$

for all $x \in \mathbb{R}$. Prove that f is periodic.