Email training, N4

September 15-21, 2019

Problem 4.1. Prove that for all $n \geq 4$ the following inequalities hold $n! > 2^n$ and $2^n \geq n^2$.

Problem 4.2. It is known that $a < 1$, $b < 1$ and $a + b \geq 0.5$. Prove that $(1-a)(1-b) \leq \frac{9}{16}$.

Problem 4.3. Read the proof of Bernouli inequality. Conclude that $8^{91} > 7^{92}$.
(https://www.youtube.com/watch?v=7BZWeWZoVcY).

Problem 4.4. By using Bernouli inequality prove that for $n \geq 1$ the following inequality holds

$$1 + \frac{5}{6n-5} \leq 6^{1/n} \leq 1 + \frac{5}{n}.$$

Problem 4.5. Let

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n}.$$

Prove that for $n \geq 3$ one has $a_{n+1} \geq a_n$ and based on this conclude that $a_{2019} > \frac{3}{5}$.

Problem 4.6. Let a, b, c are positive and less than 1. Prove that

$$1 - (1-a)(1-b)(1-c) > k,$$

where $k = max(a, b, c)$.

Problem 4.7. In the square $ABCD$ let K is a point on the side BC and the bisector of $\angle KAD$ meets the side CD at point M. Prove that $AK = DM + BK$.

Problem 4.8. Let $ABCD$ is a square, P is an inner point such that $PA : PB : PC = 1 : 2 : 3$. Find $\angle APB$ in degrees.

Solution submission deadline September 21, 2019