Email training, N8

Level 4, November 1-7

Problem 8.1. Find all pairs of integers (m, n) such that

$$\binom{n}{m} = 1984.$$

Problem 8.2. Prove that for any natural number $n > 1$, the number $2^n - 1$ does not divide $3^n - 1$.

Problem 8.3. Let u_n be the least common multiple of the first n terms of a strictly increasing sequence of positive integers $a_1, a_2, a_3, \ldots, a_{1000}$. Prove that

$$\sum_{k=1}^{1000} \frac{1}{u_k} \leq 2.$$

Problem 8.4. Let $\sigma(n)$ denote the sum of the divisors of n. Prove that there exist infinitely many integers n such that $\sigma(n) > 3n$. Prove also that $\sigma(n) < n(1 + \log_2 n)$.

Problem 8.5. Let $\sigma(n)$ denote the sum of divisors of n. Show that $\sigma(n) = 2^k$ if and only if n is a product of Mersenne primes, i.e., primes of the form $2^k - 1$.

Problem 8.6. Let $a_1 = 1,\ a_{n+1} = a_n + [\sqrt{a_n}]$. Find all n for which a_n is a perfect square.

Problem 8.7. Given a quadrilateral $ABCD$, the external angle bisectors of $\angle CAD$, $\angle CBD$ intersect at P. Show that if $AD + AC = BC + BD$, then $\angle APD = \angle BPC$.

Solution submission deadline November 7, 2021

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