Competition Preparation for Saudi Arabia Team 2021: Level 4

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Homework: Week 3

Problems:

1. (Easy) A path on a graph of length k is a collection of k edges such that each edge is connected to the next edge. Show that the complete graph of n vertices may be decomposed into $n-1$ paths of lengths $1, 2, \ldots, n-1$.

2. (Medium) Initially, we have three numbers: $0, 1, \sqrt{2}$. In each move you are allowed to multiply the difference of two of the numbers by a rational number and add it to the (remaining) third number (leaving the first two numbers unchanged). Is it possible to eventually obtain numbers: $0, 2, \sqrt{2}$?

3. (Difficult) Two magicians perform the following trick for the audience. A person from the audience selects randomly 5 cards and gives it to one of the magicians on stage, while the second magician is kept isolated. The first magician takes one card and places in sequence the remaining cards on the table in 4 slots. The first magician then leaves and then the second magician enters and based on the cards on the table and their ordering guesses the card that was taken by the first magician. How can this trick be achieved?