Problem 3.1. Find all positive integers n such that the products of its digits is equal $n^2 - 10n - 22$.

Problem 3.2. Prove there exist infinitely many positive integers divisible by 2021 and each of them containing the same number of digits 0, 1, ..., 9.

Problem 3.3. Find all values of a for which the equation $x^3 + ax^2 + 56x - 4 = 0$ has 3 roots forming consecutive terms of a geometric progression.

Problem 3.4. Let $f(x) = \frac{9^x}{9^x+3}$. Evaluate the sum

$$\sum_{k=0}^{2021} f\left(\frac{k}{2021}\right).$$

Problem 3.5. One cuts a grid of size 8×8 by a straight line. Find the maximal possible number of cells that are cut by the line.

Problem 3.6. In the cells of the grid 10×10 are written positive integers, all of them less than 11. It is known that the sum of 2 numbers written in the cells having common vertex is a prime number. Prove that there are 17 cells containing the same number.

Problem 3.7. Given a non-isosceles acute angled triangle ΔABC where O is the midpoint of BC. Let the circle with diameter BC, intersects AB, AC at D, E respectively. Let the angle bisectors of $\angle A$ and $\angle DOE$ intersect at P. If the circumcircles of ΔBPD and ΔCPE intersect at P and Q, show that Q lies on BC.

Solution submission deadline October 3, 2021
Submit single PDF file in filename format L3_YOURNAME_week3.pdf
submission email **imo20etraining@gmail.com**

1