$\boxed{40}$ $a, b, c \in \mathbb{R}_+$

$a+b+c = 1$

Prove : $\boxed{a^2+b^2+c^2+2\sqrt{3abc} \leq 1}$

$$\boxed{1^2 = (a+b+c)^2}$$

$$\cancel{a^2}+\cancel{b^2}+\cancel{c^2}+\cancel{2}\sqrt{3abc} \leq (a+b+c)^2 = \cancel{a^2}+\cancel{b^2}+\cancel{c^2}+\cancel{2}(ab+bc+ca)$$

$$\sqrt{3abc} \leq ab+bc+ca \qquad \wedge (2)$$

$$3\cancel{a}bc \leq a^2b^2+b^2c^2+c^2a^2+2abc\underbrace{(a+b+c)}_{1}$$

$$\| \quad a^2b^2+b^2c^2+c^2a^2+\cancel{2abc}$$

$$\underline{abc \leq a^2b^2+b^2c^2+c^2a^2}$$

$$+\begin{cases} a^2b^2+b^2c^2 \geq 2b^2ac \\ a^2b^2+c^2a^2 \geq 2a^2bc \\ c^2a^2+b^2c^2 \geq 2abc^2 \end{cases}$$

$$2(a^2b^2+b^2c^2+c^2a^2) \geq 2abc(a+b+c) = 2abc$$

$\boxed{4)}$

$$\frac{a_1^3+a_2^3}{a_1^2+a_1a_2+a_2^2}+\dots+\frac{a_n^3+a_1^3}{a_n^2+a_na_1+a_1^2}\geq\frac{2(a_1+a_2+\dots+a_n)}{3}$$

postive.

$$\sum_1^1\frac{a^3}{a^2+ab+b^2}=\sum_1^1\frac{b^3}{a^2+ab+b^2}$$

$$\sum_1^1\frac{a^3-b^3}{a^2+ab+b^2}\sum_1^1 a-b=0$$

So
$$\frac{a^3+b^3}{a^2+ab+b^2}\geq\frac{a+b}{3}$$

$$3(a^3+b^3)\geq\cancel{a^3}+a^2b+b^2a+ba^2+ab^2+\cancel{b^3}$$

$$2(a^3+b^3)\geq 2a^2b+2b^2a$$

$$a^3+b^3\geq a^2b+b^2a \quad \Big|: a+b$$

$$a^2-ab+b^2\geq ab \qquad (a-b)^2\geq 0$$

$$(3,0)\geq(2,1)$$

$\vdots$

$\boxed{39}$

$$\frac{a^2b(b-c)}{a+b} + \frac{b^2c(c-a)}{b+c} + \frac{c^2a(a-b)}{c+a} \geq 0$$

$$a, b, c \in \mathbb{R}_+$$

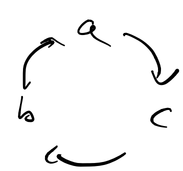
$$a^2b(b-c)(b+c)(c+a) + b^2c(c-a)(a+b)(c+a) +$$

$$+ c^2a(a-b)(a+b)(b+c) \geq 0$$

$$a^2b(b^2-c^2)(c+a)$$

$$\parallel$$

$$a^2b(b^2c + b^2a - c^3 - c^2a)$$



$$\begin{cases} a^2b^3c + a^3b^3 - a^2bc^3 - a^3bc^2 \\ b^2c^3a + b^3c^3 - b^2ca^3 - b^3ca^2 \\ c^2a^3b + c^3a^3 - c^2ab^3 - c^3ab^2 \end{cases}$$

$$a^2b^3c + a^3b^3 - a^2bc^3 - a^3bc^2 + b^2c^3a + b^3c^3 - b^2ca^3 - b^3ca^2 +$$

$$+ c^2a^3b + c^3a^3 - c^2ab^3 - c^3ab^2 \geq 0$$

$a^2\cancel{b^3}c + b^2\cancel{c^3}a + c^2\cancel{a^3}b + a^3b^3 + b^3c^3 + c^3a^3 \geq$

$\geq a^2bc^3 + \cancel{a^3bc^2} + b^2ca^3, \quad + c^2ab^3 + \cancel{ca^3b^2} +$
$\cancel{b^3ca^2}$

$$a^3b^3 + b^3c^3 + c^3a^3 \geq a^3b^2c + a^2bc^3 + a^3c^2$$

$$a^3b^3 + b^3c^3 + c^3a^3 \geq a^3b^2c + a^2bc^3 + ab^3c^2$$

$$+\left\{ a^3b^3 + a^3b^3 + c^3a^3 \geq 3\sqrt[3]{a^9b^6c^3} = \right.$$
$$= 3a^3b^2c$$

$\boxed{\text{AM-GM}}$ D