Email training, N3

September 25 - October 1

Problem 3.1. Prove that for all $n \geq 4$ the following inequalities hold $n! > 2^n$ and $2^n \geq n^2$.

Problem 3.2. It is known that $a < 1$, $b < 1$ and $a + b \geq 0.5$. Prove that $(1-a)(1-b) \leq \frac{9}{16}$.

Problem 3.3. Let a and b are divisors of n with $a > b$. Prove that $a > b + \frac{b^2}{n}$.

Problem 3.4. Read the proof of Bernouli inequality. Conclude that $8^{91} > 7^{92}$ and for $n \geq 1$ the following inequality holds

$$1 + \frac{5}{6n-5} \leq 6^{1/n} \leq 1 + \frac{5}{n}.$$

(https://www.youtube.com/watch?v=7BZWeWZoVcY).

Problem 3.5. Prove that for any posotive integer $n \geq 3$ the following inequality holds

$$\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} > \frac{3}{5}.$$

Problem 3.6. Let a, b, c are positive and less than 1. Prove that

$$1 - (1-a)(1-b)(1-c) > k,$$

where $k = max(a, b, c)$.

Problem 3.7. -

If the lengths of three sides of a triangle are consecutive positive integers, and its perimeter is less than or equal to 100, how many such acute triangles are there?

Problem 3.8. -

In the triangle *ABC*, $\angle A = 70°$, *D* is on the side *AC*, and the angle bisector of $\angle A$ intersects *BD* at *H* such that *AH* : *HE* = 3 : 1 and *BH* : *HD* = 5 : 3. Find $\angle C$ in degrees.

Solution submission deadline October 1, 2022