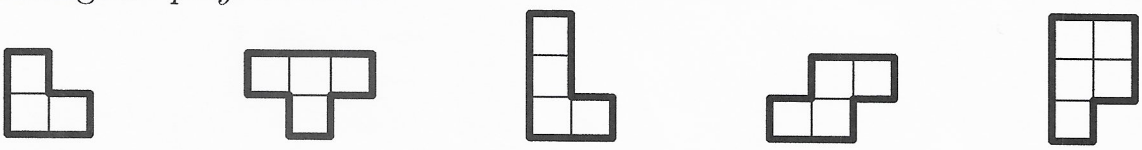
— Combinatorics for L3 —

— March 12, 2019 — Tilings and coverings (1) —

Definition. We say that a figure $\mathcal{F}$ consisting of unit squares (called *cells*) is *tiled with* or *covered by* a set of pieces (each consisting of several cells) if the pieces don't overlap and each k-cell piece covers exactly k whole cells of $\mathcal{F}$.

We distinguish e.g. rectangular pieces $m \times n$ (1×2 is called a *domino*) and some non-rectangular *polyominoes*:



L-trimino T-tetramino L-tetramino S-tetramino P-pentomino

1. Can a 8×8 chessboard with two opposite corner cells removed be tiled with 31 dominoes?

2. Can a 8×8 chessboard with any two cells of different color removed be tiled with 31 dominoes?

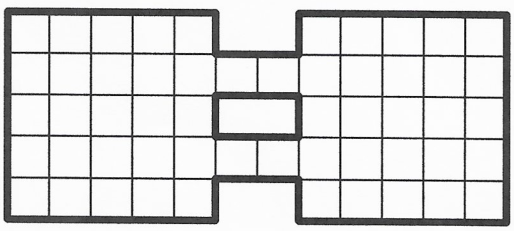
3. Can a 10×10 board be tiled with 25 T-tetraminoes?

4. Can a $7 \times 7 \times 7$ cube be built with 171 bricks of dimensions $1 \times 1 \times 2$ and one unit cube such that the small cube contains the center of the big cube?

5. Initially each cell of an 7×7 board is occupied by a rabbit. All rabbits, simultaneously, hop onto one of neighboring cells. Prove that at least one cell will become empty.

6. (Homework) Initially each cell of an 7×7 board is occupied by a rabbit. All rabbits, simultaneously, hop onto one of neighboring cells. What is the minimum possible number of occupied cells after the jumps?

7. (Homework) Can the 54-cell figure shown in the picture be covered with dominoes?



8. (Homework) One corner cell of a square board of side length $2n+1$ is removed. Determine all n for which the remaining cells can be tiled with dominoes in such a way that exactly half of them is placed horizontally.

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— March 16, 2019 — Tilings and coverings (3) —

11. What is the maximum number of $2\times3\times3$ bricks that fit into a $8\times8\times9$ box?

12. We are at dispose of S-tetraminoes and L-triminoes. For each odd $n\geq7$ determine the least number of such tiles needed to cover an $n\times n$ board.

13. In a park, there grow 10 000 trees planted by a so-called square-cluster method (100 rows of 100 trees each). What is the largest number of trees one has to cut down in order to satisfy the following condition: if one stands on any stump, then no other stump is seen?

14. Can a 10×10 board be tiled with 1×4 rectangles?

15. An 8×8 board is covered with 21 tiles 1×3 and one 1×1 tile. Determine all possible placements of the 1×1 tile.

16. **(Homework)** Find the minimum $n\geq5$ with the following property: from any set of n rooks on a 8×8 chessboard one can take all but five of them away in such a way that neither two of the remaining ones are attacking each other.

17. **(Homework)** A 7×7 board is to be covered with P-pentominoes in such a way that exactly one cell is covered with two tiles and all the remaining — with exactly one tile. Determine all possible placements of the cell that is doubly covered.

Remark. The P-pentominoes can be rotated but they cannot be turned around.

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— March 18, 2019 — Tilings and coverings (4) —

16. Find the minimum $n \geq 5$ with the following property: from any set of n rooks on a 8×8 chessboard one can take all but five of them away in such a way that neither two of the remaining ones are attacking each other.

17. A 7×7 board is to be covered with P-pentominoes in such a way that exactly one cell is covered with two tiles and all the remaining — with exactly one tile. Determine all possible placements of the cell that is doubly covered.

Remark. The P-pentominoes can be rotated but they cannot be turned around.

18. Can a 13×13 board be covered with tiles 1×4 in such a way that only the central cell remains uncovered?

19. Can a 13×13 board be covered with tiles 2×2 and 3×3?

20. Tiles of shapes 1×4, 2×2 and S-tetraminoes are arranged into a square 8×8. Prove that the number of used 2×2 tiles is even.

21. **(Homework)** A big rectangle is subdivided by segments parallel to its sides into small rectangles. It is known that each small rectangle has at least one dimension of integral length. Prove that the big rectangle has at least one dimension of integral length.

22. **(Homework)** Can three faces of a $8 \times 8 \times 8$ cube having a common vertex be covered with 64 strips 1×3? The strips can be folded along the edges of the cube.