

Test-6, March 24  
Level 3, from 14:00 to 18:20

**Problem 1.** In triangle  $ABC$  with  $\angle B = 60^\circ$ ,  $O$  is the circumcenter, and  $L$  is the foot of an angle bisector of angle  $B$ . The circumcircle of triangle  $BOL$  meets the circumcircle of  $ABC$  at point  $D \neq B$ . Prove that  $BD \perp AC$ .

**Problem 2.** Let  $a, b, c$  be positive real numbers satisfying  $a^2 + b^2 + c^2 = 3abc$ . Prove that

$$\frac{a}{b^2c^2} + \frac{b}{c^2a^2} + \frac{c}{a^2b^2} \geq \frac{9}{a+b+c}.$$

**Problem 3.** Prove that if  $a$  is an integer then  $2a^2 - 1$  has no divisors of the form  $b^2 + 2$  with  $b$  integer.

**Problem 4.** In each field of an  $n \times n$  table an integer is written. A *path* of this table is a sequence of distinct fields such that its first field is in the first row, its last field is in the  $n$ -th row, and every two consecutive fields have a common side. We say that this table is  $m$ -good if it has a path in which the sum of the elements is divisible by  $m$ . Find all pair  $(n, m)$  of positive integers such that every  $n \times n$  table is  $m$ -good.