Test 1

Levels 4 and 4+, November 28

Problem 1.1. Prove that it is possible to pick 20 numbers among numbers $1, 2, ..., 10000$ such that the members of any non-empty subset of these 20 numbers has a sum which is not an $n-$th power of some number (for any $n > 1$).

Problem 1.2. Let ABC be an acute triangle with $AB < AC$. Let K be the midpoint of arc BC of the circumcircle of ABC that does not contain A and P the midpoint of side BC. The points I_B and I_C are the excenters related to B and C, respectively. Let Q be the reflection of K with respect the point A. Prove that P, Q, I_B and I_C are concyclic.

Problem 1.3. Find all functions $f : R \to R$ such that

$$f\Big(2xf(x) - 2f(y)\Big) = 2x^2 - y - f(y)$$

for all $x, y \in R$.

Problem 1.4. Do there exist 100 points on the plane such that the pairwise distances between them are pairwise distinct consecutive integer numbers larger than 2022?