Email training, N5

October 9-15

Problem 5.1. Does there exist a numerical system (for example we are using base 10 numerical system) in which the following relations hold

a. $3 + 4 = 10$ and $3 \cdot 4 = 15$,
b. $2 + 3 = 5$ and $2 \cdot 3 = 11$.

Problem 5.2. Five numbers are written on the board. Mohammed calculates the sum of all pairs and gets the results 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121. Find the numbers written on the board.

Problem 5.3. Find all positive integers m and n for which $1! + 2! + 3! + \cdots + n! = m^2$.

Problem 5.4. Let a, b, c, be a positive integer such that $a^2+b^2 = c^2$. Prove that $\frac{1}{2}(c-a)(c-b)$ is a perfect square.

Problem 5.5. Let numbers 1, 2, 3, ..., 19, 20 are written on the board. At each step one may erase any two numbers a and b and write the number $a + b$–1. Which number will be written on the board after 19 steps.

Problem 5.6. Let 100 points are drawn on the plane such that the distance between 2 any points is less than 1. Also it's known that for any three points A, B and C the triangle ABC is not acute. Prove that there exists a circle of radius 0.5 which contains in it's interior all 100 points.

Problem 5.7. -

In square *ABCD*, *M* is the midpoint of *AD* and *N* is the midpoint of *MD*.
Prove that $\angle NBC = 2\angle ABM$

Problem 5.8. -

$\triangle ABC$ is an isosceles triangle with $AB = AC = 2$. There are 100 points $P_1 + P_2 + \ldots + P_{100}$ on the side *BC*. Write $m_i = AP_i^2 + BP_i \times P_iC \; (i = 1,2,3,\ldots,100)$.
Find the value of $m_1 + m_2 + \ldots + m_{100}$.

Solution submission deadline October 15, 2022