Problem 5.1. Let a and b are divisors of n with $a > b$. Prove that $a > b + \frac{b^2}{n}$.

Problem 5.2. Do there exist 3 real numbers a, b and c such that the following inequalities hold simultaneously

$$|a| < |b - c|, \quad |b| < |c - a|, \quad |c| < |a - b|.$$

Problem 5.3. Prove that for $n \geq 1$ the following inequality holds

$$1 + \frac{5}{6n - 5} \leq 6^{1/n} \leq 1 + \frac{5}{n}.$$

Problem 5.4. Let a, b, c are positive and less than 1. Prove that

$$1 - (1 - a)(1 - b)(1 - c) > k,$$

where $k = max(a, b, c)$.

Problem 5.5. Let $x, y, z \geq 0$ and $x + y + z = 3$. Prove that

$$\sqrt{x} + \sqrt{y} + \sqrt{z} \geq xy + xz + zx.$$

Problem 5.6. Let $a, b, c > 0$. Prove that

$$\frac{a + b}{a^2 + b^2} + \frac{b + c}{b^2 + c^2} + \frac{c + a}{c^2 + a^2} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Problem 5.7. Let P be a point inside $\triangle ABC$ such that $\angle PBA = \angle PCA$. Draw $PD \perp AB$ at D and $PE \perp AC$ at E. Show that the perpendicular bisector of DE passes through the midpoint of BC.