$1 \le n \le 2020$

$n = k^2 \pm m$

$0 \le m \le 10$

$45^2 = 2025$

$1, 2, 3, \ldots, 121, 122, \ldots, 131 \longrightarrow 131$

$12^2 \pm 10, 13^2 \pm 10, \ldots, 44^2 \pm 10 \longrightarrow 21 \cdot 33 = 693$

$2026, 2019, \ldots, 2015 \longrightarrow 6$

الكل : 830

$g(f(2)) = 2, \quad g(f(3)) = 0, \quad g(f(4)) = 6$

$g(f(5)) = ?$

2

$f(2), f(3), f(4), f(5)$

متتابعة حسابية

2	0	-2	-4
or	or	or	
-1	1	3	5

الحل: 020

3

حمزة	مابه
30	0
35	6
⋮	⋮
80	60
⋮	⋮
130	120
⋮	⋮
180	180
181	181

أنهو الصفحة 180 سويا

يبدؤون عند 181

6 دقائق

مابه ← 60 ص

حمزة ← 5 ص

الحل : 181

$$5^n \mid 1442! - 1441! - 1440!$$

$$1440!\left(1442 \cdot 1441 - 1441 - 1\right)$$

$$= \underbrace{1440!}_{+358} \times \underbrace{1442}_{+0} \times \underbrace{1440}_{+1}$$

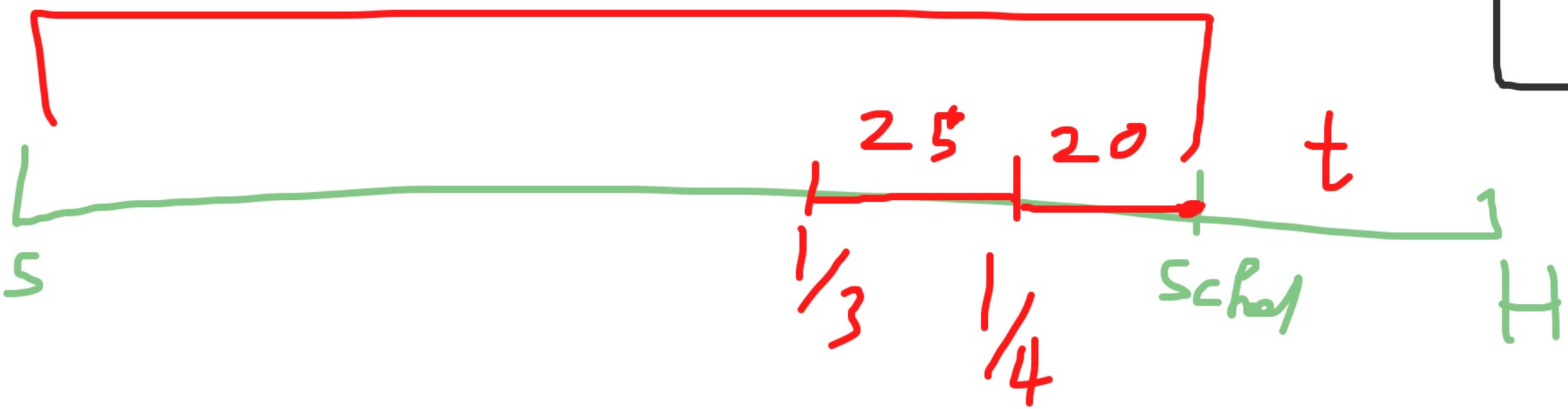
$$\left\lfloor \frac{1440}{5} \right\rfloor + \cdots + \left\lfloor \frac{1440}{5^4} \right\rfloor$$

$$\boxed{\text{الحل : } 359}$$

T

المطلوب :

5

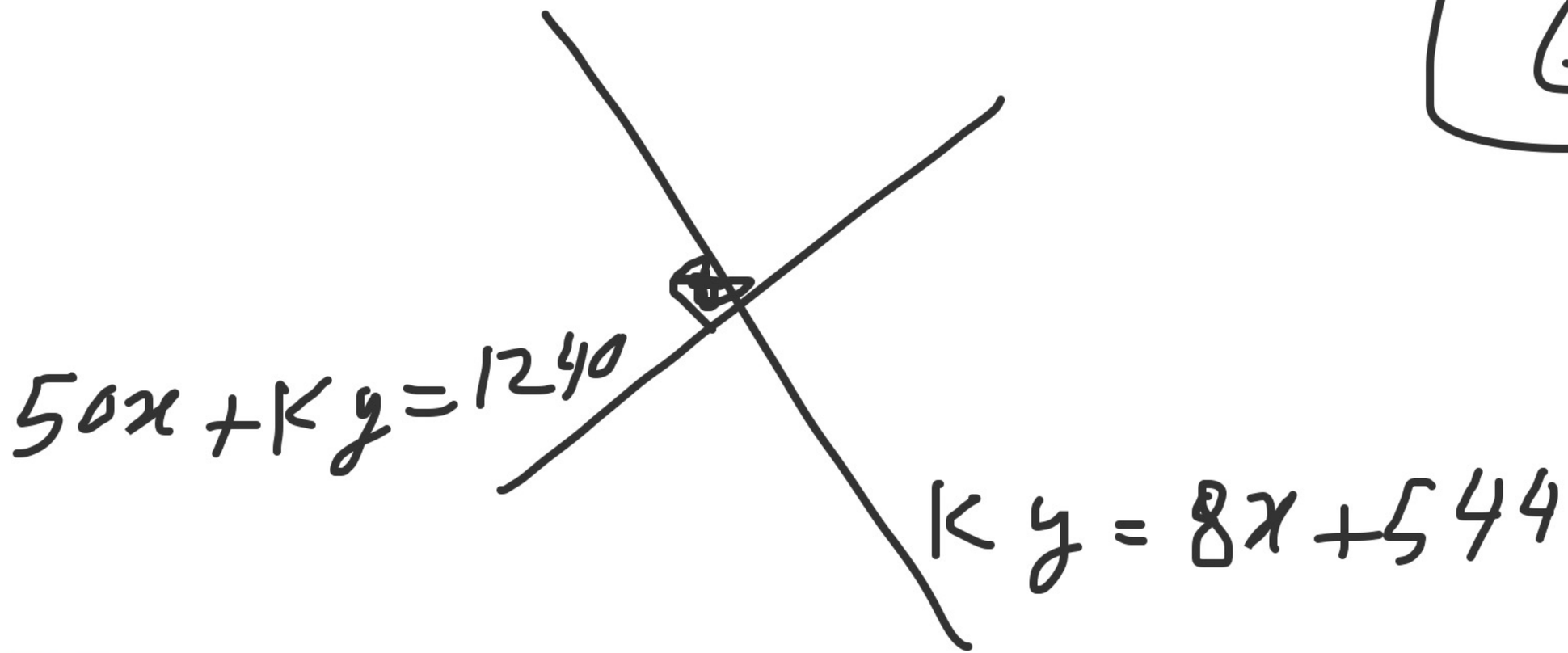


$$\Rightarrow t+20 = \frac{1}{4}(t+T)$$

$$t+45 = \frac{1}{3}(t+T)$$

$$\Rightarrow t+T=300 \longrightarrow T = \boxed{245}$$ الحل :

6

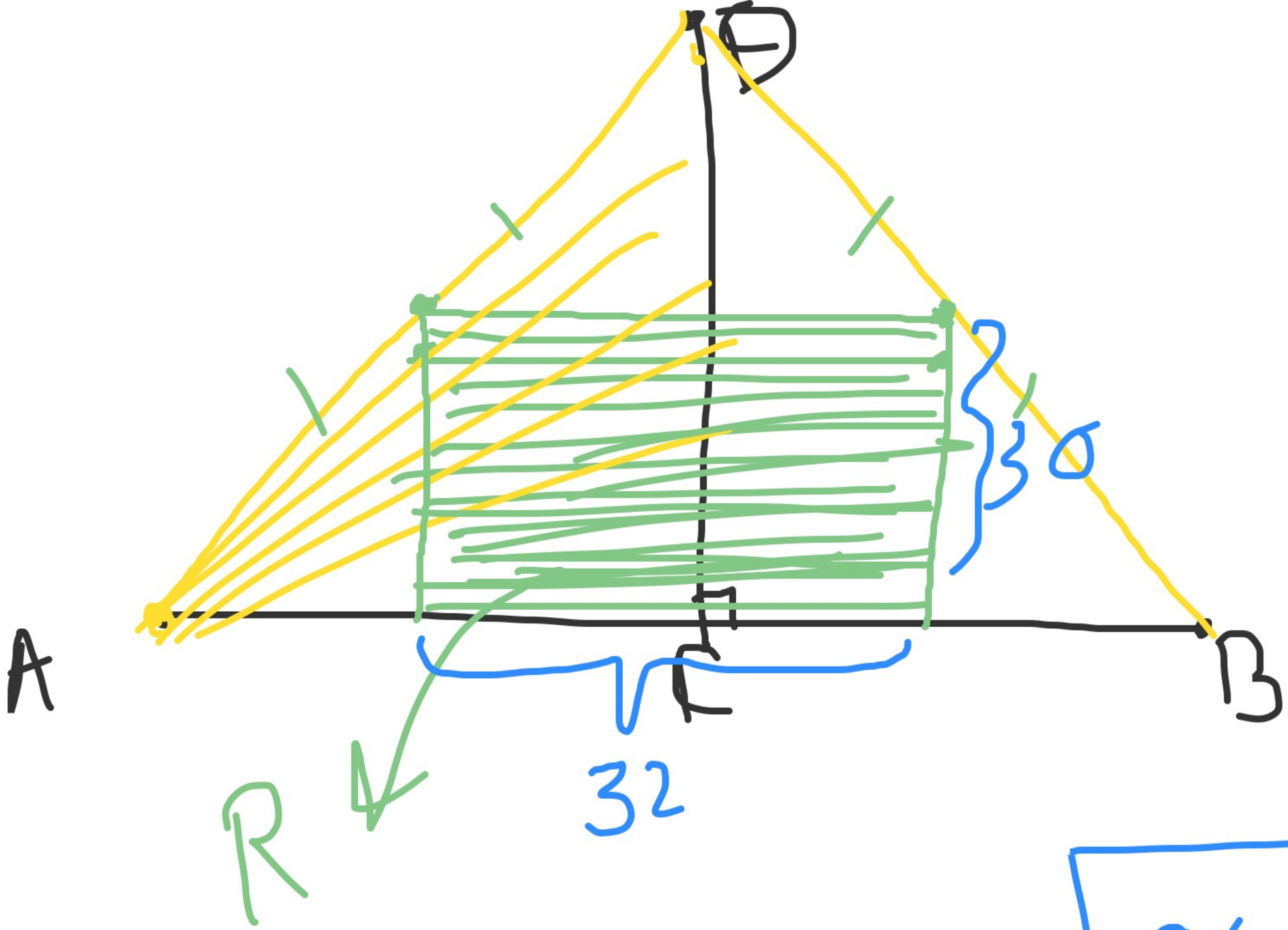


$$\frac{8}{K} \cdot \left(-\frac{50}{K}\right) = -1 \iff K = 20$$

$$\begin{cases} 50m + 20n = 1240 \\ 20n = 8m + 544 \end{cases} \rightarrow (m,n) = (12,32)$$

الحل : 044

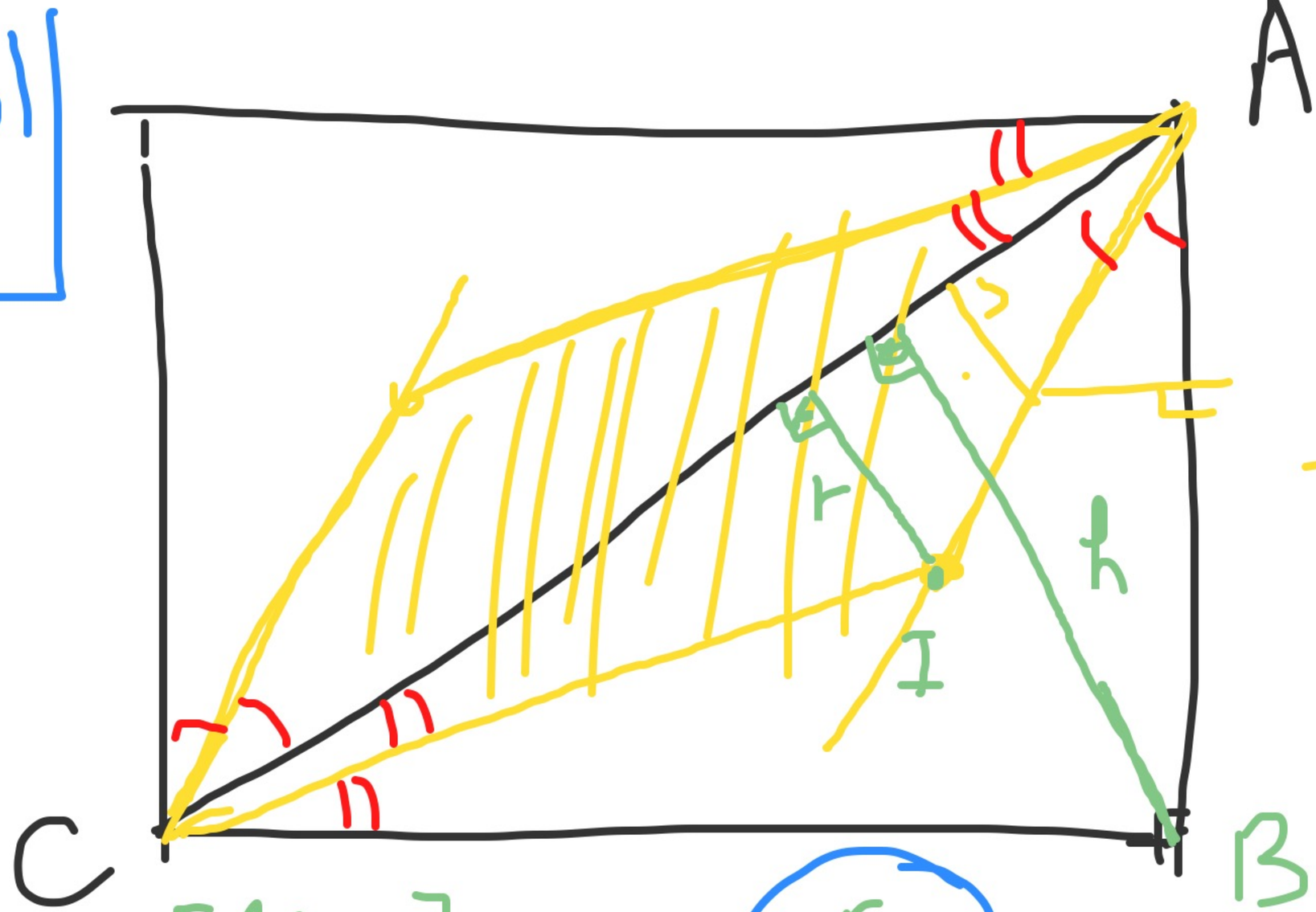
6



الحل: 960

الحل:
017

8



المساحة الغراء

مساحة المستطيل

$$= [AIC]$$

$$r = \frac{[ABC]}{s} = \frac{24}{12} = 2$$

$$\frac{r}{h} = \frac{[AIC]}{[ABC]}$$

$$h = \frac{2[ABC]}{AC} = \frac{48}{10} = \frac{24}{5}$$

$$= \frac{5}{12}$$

$\boxed{9}$

$$P(x) = x(x+1)\cdots(x+239)$$
$$\equiv (x^3 - x)^{80}$$
$$\equiv x^{80}(x^2-1)^{80} \pmod 3$$

Let $1 \leq k \leq 80$; $\binom{81}{k} = \frac{81}{k}\binom{80}{k-1}$

$\Rightarrow k\binom{81}{k} = 3^4\binom{80}{k-1}$, $v_3(k) < 4$

$\Rightarrow 3 \mid \binom{81}{k} = \binom{80}{k-1} + \binom{80}{k}$, $\binom{80}{0} \equiv 1 \ [3]$

$$\Rightarrow \binom{80}{1} \equiv -1 \quad \Rightarrow \binom{80}{2} \equiv 1 \quad \cdots$$

$$\boxed{\binom{p^k-1}{m} \equiv (-1)^m \ [p]}$$

$$\binom{80}{k} \equiv (-1)^k \pmod 3$$

$$\not\equiv 0$$

$$P(x) \equiv x^{80}(x^2-1)^{80} \quad [3]$$

$(\deg P = 240)$

$$\equiv \sum_{k=0}^{80} \binom{80}{k} (-1)^k x^{80+2k}$$

3 على تقبل لا معاملاتها ← $x^{80}, x^{82}, x^{84}, \ldots, x^{240}$

$$\Rightarrow 241 - 81 = \boxed{160}$$: الحل

10

العدد الكلي :

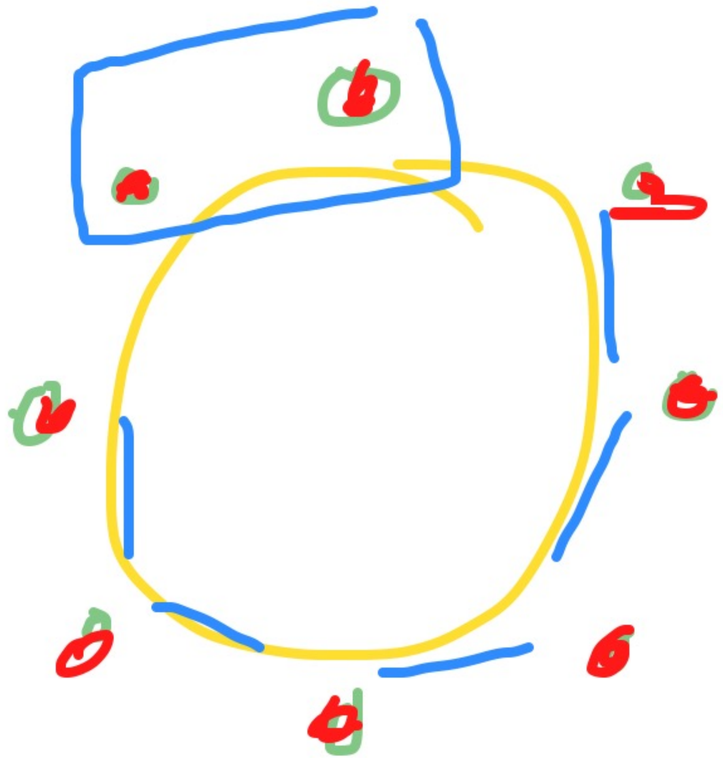
$$\frac{8!}{2!\,2!\,2!\,2!} \rightarrow 2520$$

يوجد زوج :

$$\frac{8\cdot 4\cdot 6!}{2!\,2!\,2!} \rightarrow 2880$$

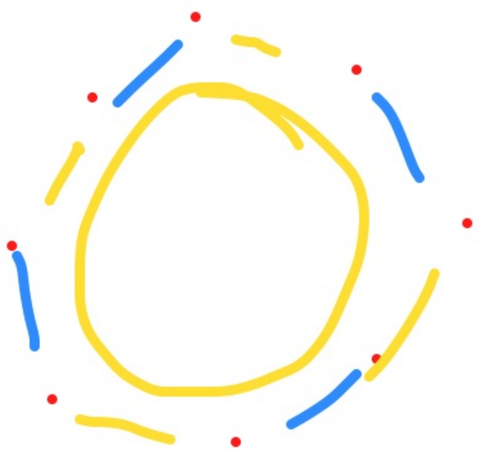
يوجد زوجان :

$$\frac{\binom{4}{2}\cdot 8\cdot 5\cdot 4!}{2!\cdot 2!} \rightarrow 1440$$



يوجد 4 أزواج :

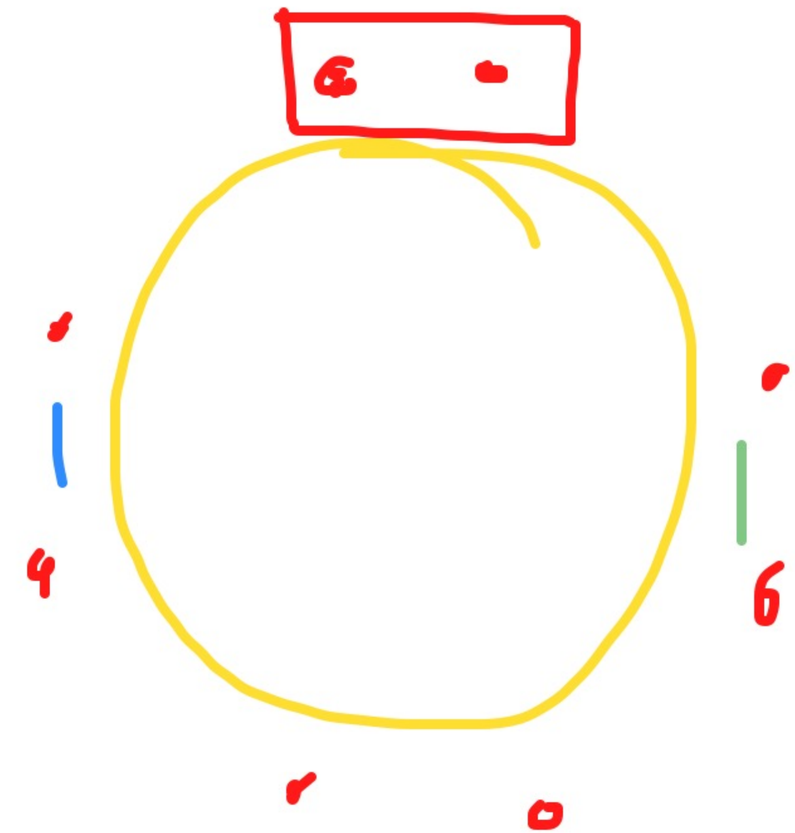
$$2\cdot 4! \rightarrow 48$$



بوجه 3 أزواج:

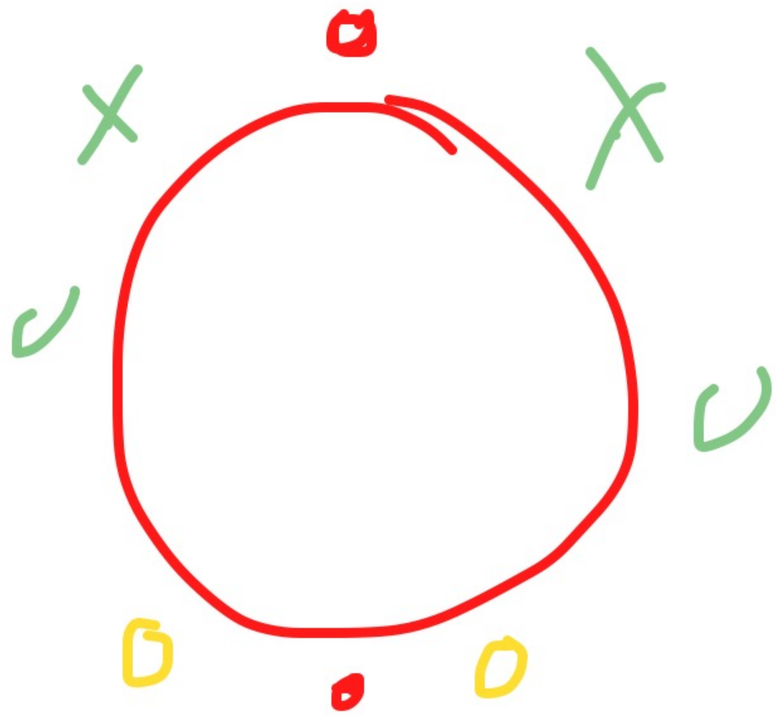
$$\binom{4}{3}\cdot 8\left(2\cdot 3+2\cdot 2+2\right)$$

$$=384$$



$$2520-2880+1440-384+48=\boxed{744}$$

الحل



الفكرة الثالثة

النظر في الفرق
بين الشخصين أصحاب اللون الأحمر.

$$17 \mid \underbrace{3^n - n}_{f(n)}$$

$\text{ord}_{17}(3) = 16$

$$f(n + 16 \cdot 17) \equiv f(n) \quad [17]$$

$$2 \mid 1$$

$$(*) \leftarrow f(n+16) \equiv f(n) + 1 \quad [17]$$

$$0, 1, 2, \ldots\ldots, 271$$

$0, 16, 32, \ldots, 16 \cdot 16 = 256$

$1, 17, 33, \ldots, 16 \cdot 16 + 1 = 257$

$\vdots$

$15, 31, 47, \ldots, 16 \cdot 16 + 15 = 271$

$\{f(0), f(16), \ldots, f(256)\}$ is CRS [17]

$\{f(i), f(16+i), \ldots, f(256+i)\}$ is CRS [17]

$f(0) \equiv 1 \longrightarrow f(256) \equiv 0 \quad r_0 \ [272]$

$f(1) \equiv 3 \longrightarrow f(225) \equiv 0 \quad r_1 \ [272]$

$f(2) \equiv 9 \longrightarrow f(130) \equiv 0$

$f(7) \equiv 10 \longrightarrow f(115) \equiv 0$

$\vdots$

$f(15) \equiv 6 \longrightarrow f(191) \equiv 0 \quad r_{15} \ [272]$

Thus, all possible n's are $n \equiv r_i \ [272]$

12

$$a^3+b^3+c^3 \overset{?}{\geq} 3abc + 6|(a-b)(b-c)(c-a)|$$

$b+c \geq a \geq b \geq c$

$$\Leftrightarrow (a+b+c)\left(\underline{(a-b)^2+(b-c)^2+(c-a)^2}\right) \overset{?}{\geq} 12(a-b)(a-c)(b-c)$$

Let $x = a-b$
$y = b-c$

$$a+b+c > 2a > 2(a-c) \quad \text{— ①}$$

$$\underline{x^2+y^2+(x+y)^2} = 2(x^2+xy+y^2)$$

$$\geq 6xy = 6(a-b)(b-c) \quad \text{— ②}$$

multiply ① × ②, we get the desired result! ∎

(search for the invariant)

$$(a,b,c,d) \longrightarrow \left(\frac{a+b}{\sqrt{2}}, \frac{a-b}{\sqrt{2}}, c, d\right)$$

$$f(a,b) = f\left(\frac{a+b}{\sqrt{2}}, \frac{a-b}{\sqrt{2}}\right)$$

$$2(a^2+b^2) = (a-b)^2 + (a+b)^2$$

$$a^2+b^2+c^2+d^2 = \frac{(a+b)^2}{2} + \frac{(a-b)^2}{2} + c^2+d^2 .$$

if we define $S(a,b,c,d) = a^2+b^2+c^2+d^2$

we see that S is invariant!

$$S(0,3,4,5) = 50$$

(b) $S(1, 2\sqrt{2}, 3\sqrt{2}, 5) = 52$ $\longleftarrow$ impossible

(a) $S(1,2,3,6) = 50$

$0, 3, 4, 5$

$\longrightarrow 2\sqrt{2}, 2\sqrt{2}, 3, 5$

$\longrightarrow \sqrt{2}, 2\sqrt{2}, 2\sqrt{2}, 4\sqrt{2}$

$\longrightarrow 1, 3, 2\sqrt{2}, 4\sqrt{2}$

$\longrightarrow \boxed{1, 3, 2, 6}$

$\frac{a+b}{\sqrt{2}}, \frac{a-b}{\sqrt{2}}$

$\longrightarrow a, b$

(a) ✓

(b) ✗

الواجب

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$$

$$\sum_{cyc}\sqrt{x+yz} \overset{?}{\geq} \sqrt{xyz}+\sum_{cyc}\sqrt{x}$$

Let $a=\frac{1}{x}$, $b=\frac{1}{y}$, $c=\frac{1}{z}$.

$$\Leftrightarrow \sum_{cyc}\sqrt{a+bc} \overset{?}{\geq} 1+\sqrt{ab}+\sqrt{bc}+\sqrt{ca}$$

$$\sum_{cyc} \sqrt{a+bc} = \sum_{cyc} \sqrt{a(a+b+c)+bc}$$

$$= \sum_{cyc} \sqrt{(a+b)(a+c)}$$

$$\overset{\text{Cauchy}}{\geq} \sum_{cyc} (a+\sqrt{bc}) = 1 + \sum_{cyc} \sqrt{ab}$$

$\blacksquare$

AIME

AMC

Good sources for short questions!