

Email training, N6
Level 4, October 18-24

Problem 6.1. Prove the inequality

$$\sqrt{a+1} + \sqrt{2a-3} + \sqrt{50-3a} \leq 12.$$

Problem 6.2. Let the parabola $y = x^2 + px + q$ is given, which intersects coordinate axes in 3 different points. Consider the circumcircle of the triangle having vertices these 3 points. Prove that there is a point that belongs to that circle, regardless of values p and q . Find that point.

Problem 6.3. Find all integer polynomials P for which $(x^2 + 6x + 10)P^2(x) - 1$ is the square of an integer polynomial.

Problem 6.4. a) Find the minimum number of elements that must be deleted from the set $\{1, 2, \dots, 2018\}$ such that the set of the remaining elements does not contain two elements together with their product. b) Does there exist, for any k , an arithmetic progression with k terms in the infinite sequence

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n}, \dots$$

Problem 6.5. Prove that not all zeros of a polynomial of the form $x^n + 2nx^{n-1} + 2n^2x^{n-2} + \dots$ can be real.

Problem 6.6. Let the polynomial $P(x) = a_{2n}x^{2n} + a_{2n-1}x^{2n-1} + \dots + a_1x + a_0$ with all coefficients $a_i \in [100, 101]$ is given. Find the minimal possible value of n for which $P(x)$ has a root.

Problem 6.7. Let AD is the altitude of the triangle ABC . Let J, K be the incenters of the triangles ABD, ACD respectively. Let JK intersects AB, AC at E, F respectively. Prove that $AE = AF$ if and only if $AB = AC$ or $\angle A = 90^\circ$.

Solution submission deadline October 24, 2021
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submission email **imo20etraining@gmail.com**