Problem 3.1. Find all positive integers n such that the products of its digits is equal $n^2 - 10n - 22$.

Problem 3.2. Prove there exist infinitely many positive integers divisible by 2021 and each of them containing the same number of digits 0, 1, ..., 9.

Problem 3.3. Find all values of a for which the equation $x^3 + ax^2 + 56x - 4 = 0$ has 3 roots forming consecutive terms of a geometric progression.

Problem 3.4. Let $f(x) = \frac{9^x}{9^x + 3}$. Evaluate the sum

$$\sum_{k=0}^{2021} f\left(\frac{k}{2021}\right).$$

Problem 3.5. One cuts a grid of size 8×8 by a straight line. Find the maximal possible number of cells that are cut by the line.

Problem 3.6. In the cells of the grid 10×10 are written positive integers, all of them less than 11. It is known that the sum of 2 numbers written in the cells having common vertex is a prime number. Prove that there are 17 cells containing the same number.

Problem 3.7. Given a quadrilateral $ABCD$, the circumcircle of ΔABC intersects CD, AD at E, F respectively, and the circumcircle ΔACD intersects AB, BC at P, Q respectively. If BE, BF intersect of PQ at X, Y respectively, show that E, F, Y, X are concyclic.

Solution submission deadline October 3, 2021
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submission email **imo20etraining@gmail.com**

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