KSA Training – May camp

GEOMETRY FOR LEVEL 2

Session 1. **Trainer: Le Phuc Lu (Vietnam)**

Problem 1. Let $ABCD$ be a quadrilteral with $\angle A = \angle B = 90^\circ$, $AB = AD$. Denote E as the midpoint of AD, suppose that $CD = BC + AD, AD > BC$. Prove that

$$\angle ADC = 2\angle ABE.$$

Problem 2. Let ABC be a an acute, non-isosceles triangle with altitudes BD, CE. The perpendicular bisector of CE cuts the line DE at T and cuts AC at S. Prove that BT, BS are symmetric with respect to the angle bisector of $\angle ABC$.

Problem 3. Let ABC be an acute, non-isosceles triangle with AD, BE, CF are altitudes and d is the tangent line of the circumcircle of triangle ABC at A. The line throught H and parallel to EF cuts DE, DF at Q, P respectively. Prove that d is tangent to the ex-circle with respect to vertex D of triangle DPQ.

GEOMETRY FOR LEVEL 2

Session 2. **Trainer: Le Phuc Lu (Vietnam)**

Problem 1. Let ABC be an acute, non-isosceles triangle inscribed in (O) and BB', CC' are altitudes. Denote E, F as the intersections of BB', CC' with (O) and D, P, Q are projections of A on BC, CE, BF. Prove that the perpendicular bisectors of PQ bisects two segments AO, BC.

Problem 2. Given a semicircle (ω) of diameter AB and center O, let C, D are two distinct points on that (ω) such that ray AC meets ray BD at K lying outside (ω). The line passes through O, parallel to AC cuts CD at T, cuts KB at S and cuts (ω) at P. Take M on BT such that $TB = TM$, take E on AD such that $EA = EB$.

1) Prove that AB is tangent to (KBM) and D, E, O, T are concyclic.

2) Prove that AP is the angle bisector of angle SAT.

Problem 3. Let ABC be an acute, non-isosceles triangle with centroid G. Take D, E on AB, AC respectively such that G is the orthocenter of triangle ADE. Denote O as circumcenter of ADE and M, N as the midpoints of DE, BC.

1) Prove that $OMNG$ is parallelogram.

2) Prove that $MN \perp BC$.

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GEOMETRY FOR LEVEL 2

Session 3. **Trainer: Le Phuc Lu (Vietnam)**

Problem 1. Let ABC be an acute, non-isosceles triangle with altitude AD and orthocenter H. Denote O_1, O_2 as the centers of circle pass through B, C respectively and external tangent to the circle of diameter AD. Prove that O_1O_2 bisects HD.

Problem 2. Let ABCD be a parallelogram with T as the intersection of two diagonals. A circle (ω) of center O, passes through D and is tangent to BD. Suppose that (ω) cuts the segment CD at E, cuts ray AD at F such that points B, E, F are collinear. Prove that $\angle ATD = \angle BOD$.

Problem 3. Let ABC be a triangle right at A with incenter I. Denote M as midpoint of AB and IM cuts AC at Q. Circle (I) is tangent to BC, AC at D, E respectively. Take P on DE such that $AP \perp BC$. Prove that $AP = AQ$.

Problem 4. *(about the IMO TST)* Let ABC be a non-isosceles triangle with incenter I and let R be the circumradius of this triangle. Denote AL as the external angle bisector of angle BAC with L on BC. Let K be a point on perpendicular bisector of BC such that $IL \perp IK$. Prove that $OK = 3R$.