— Combinatorics for L4 —

— January 4, 2020 — Triangulations (1) —

Definition.

A *triangulation* of a (not necessarily convex) polygon is a dissection of this polygon with some of its diagonals into triangles in such a way that the diagonals do not intersect (they can share endpoints).

Problem 6.

A. Prove that every triangulation of an n-gon has exactly $n-3$ diagonals.

B. Prove that every triangulation of an n-gon has exactly $n-2$ triangles.

Problem 7.

A. Prove that every polygon has at least one diagonal.

B. Prove that every polygon has at least one triangulation.

Problem 8.

A. Prove that for every $n \geq 3$ there exists an n-gon admitting exactly one triangulation.

B. Prove that for every positive integer k there exists a polygon, which admits exactly k diffferent triangulations.

Problem 9.

A. Prove that vertices of a triangulated polygon can be colored with three colors in such a way that every two connected vertices have different colors.

B. Prove that triangles of a triangulated polygon can be colored with two colors in such a way that every two triangles sharing a side have different colors.

Problem 10.

Consider a triangulated convex n-gon with $n \geq 4$. Prove that there are two more triangles whose exactly two sides are sides of the n-gon than triangles whose all sides are diagonal of the n-gon.

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— January 4, 2020 — Triangulations (2) —

Problem 11.

The vertices of a convex polygon are colored by at least three colors such that no two consecutive vertices have the same color. Prove that one can dissect the polygon into triangles by diagonals that do not cross and whose endpoints have different colors.

Problem 12.

Each vertex of a convex n-gon ($n \geq 4$) is to be colored black or white. Determine number of colorings for which there exists a triangulation with all diagonals having endpoints of different colors.

Problem 13.

A. At each vertex of a triangulated n-gon an even number of diagonals meet. Prove that n is divisible by 3.

B. Does there exist a triangulation of an n-gon with $n \geq 4$ such that at each vertex the number of diagonals is divisible by 3?

Problem 14.

A. Given is a triangulated regular n-gon. Prove that at most one of the triangles is acute.

B. Given is a triangulated regular n-gon, where $n \geq 5$ is odd. Prove that at least three triangles are isosceles.

C. Determine all $n \geq 3$ for which there exists a triangulation of a regular n-gon in which all triangles are isosceles.

Problem 15.

Let P be a regular $(4k+2)$-gon. A diagonal of P is called *good* if its endpoints divide the boundary of P into two parts, each composed of an odd number of sides of P. The sides of P are also called good. Suppose P has been dissected into triangles by $4k-1$ diagonals, no two of which have a common point in the interior of P. Find the maximum number of isosceles triangles having two good sides that could appear in such a configuration.