Test 1

Level 3, November 28

Problem 1.1. Prove that it is possible to pick 9 numbers among numbers $1, 2, \ldots, 500$ such that the members of any non-empty subset of these 9 numbers has a sum which is not an $n-$th power of some number (for any $n > 1$).

Problem 1.2. Let ABC be an acute triangle with $AB < AC$. Let I be its incenter and Γ be its circumcircle. Let M be the midpoint of BC, K the midpoint of arc BC not containing A, L the midpoint of arc BC containing A and J the reflection of I with respect the line KL. The line LJ cuts Γ again at T (different from L). The line TM cuts Γ again at S (different from T). Prove that the points S, I, M and K are concyclic.

Problem 1.3. Let $a1$, a_2, a_3, $\ldots$, a_{10} and b_1, b_2, $\ldots$, b_{10} be a real numbers such that the roots of these 10 polynomials

$$x^2 + a_1x + b_1, \quad x^2 + a_2x + b_2, \quad ,\ldots\ldots \quad x^2 + a_{10}x + b_{10}$$

are all integer numbers ±1, ±2,..., ±10 (in some order).

a) What is the maximum amount of odd values among a_1, b_1, a_2, b_2, $\ldots$, a_{10}, b_{10}?

b) FInd the minimum and maximum values of the sum $b_1 + b_2 + \ldots + b_{10}$.

Problem 1.4. Mohammed has a chess board, colored white and black in the usual way and mentioned a point, which lies strictly inside one of the 64 cells. Ahmed can draw any closed broken line without self-intersections and ask Mohammed whether his point is inside or outside the broken line. How many questions does Ahmed need to ask so that he can find out whether the point is in a black or in a white cell?