Email training, N2

Level 2, September 20-26

Problem 2.1. Let polynomial

$$P(x) = \underbrace{((\ldots((}_{k} x-2)^2-2)^2-\ldots)^2-2)^2$$

is given. Find coefficient at x^2.

Problem 2.2. Let the sequence a_1, a_2, ..., a_n is such that $a_1 = 0$, $|a_2| = |a_1+1|$, $|a_3| = |a_2+1|$, ..., $|a_n| = |a_{n-1}+1|$: Prove that

$$\frac{a_1+a_2+\ldots+a_n}{n} \geq -\frac{1}{2}.$$

Problem 2.3. Prove that for any 2 positive integers m and n with $m > n$ holds the following inequality

$$lcm(m,n) + lcm(m+1,n+1) > \frac{2mn}{\sqrt{m-n}}.$$

Problem 2.4. Do there exist an infinite sequence p_1, p_2, p_3, ... of prime numbers such that for any positive integer n the following condition holds

$$|p_{n+1} - 2p_n| = 1.$$

Problem 2.5. Let convex s-gon is divided to q quadrilaterals such that b of them are not convex. Prove that

$$q \geq b + \frac{s-2}{2}.$$

Problem 2.6. Let positive numbers are written along the circle, such that all of them are less than 1. Prove that one can split the circle to 3 parts such that for each two arcs the sums of numbers written on them differs by at most 1.

Problem 2.7. Let incircle of triangle ABC has center I and touches sides BC, AC and AB at points D, E, F respectively. Let J_1, J_2, J_3 be te ex-centres opposite A, B, C respectively. Let J_2F and J_3E intersect at P, J_3D and J_1F intersect at Q, J_1E and J_2D intersect at R. Show that I is the circumcenter of PQR.

Solution submission deadline September 26, 2021
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