November Camp - 2019
Algebra L4 - Revision

Revision 1. *Let P and Q be polynomials and $P(Q(x)) = Q(P(x))$ for every $x \in \mathbb{R}$. Find as many pairs of P and Q as you can.*

Revision 2. *Polynomials $A(x)$, $B(x)$, $C(x)$, $D(x)$ satisfy the equation*

$$A(x^5) + xB(x^5) + x^2C(x^5) = (1 + x + x^2 + x^3 + x^4)D(x) \quad \textit{for all } x \in \mathbb{R}.$$

Find all possible values of $A(1)$.

Revision 3. *A sequence $a_1, a_2, \ldots, a_n$ is called k-balanced if*

$$a_1 + a_{k+1} + \ldots = a_2 + a_{k+2} + \ldots = \ldots = a_k + a_{2k} + \ldots$$

Suppose the sequence $a_1, a_2, \ldots, a_{50}$ is k-balanced for $k = 3, 5, 7, 11, 13, 17$. Prove that a_i are all zeros.

Revision 4. *Suppose we have the following polynomial with integer coefficients*

$$P(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_0.$$

If there exists a prime number p such that the following three conditions all apply:

- *p divides each a_i for $0 \leq i < n$,*
- *p does not divide a_n,*
- *p^2 does not divide a_0,*

then $P(x)$ is irreducible.