November camp – KSA training 2022. Level 4 & 4+.

MAXIMUM & MINIMUM PARAMETERS IN INEQUALITY

Problem 1. Find the least value of k such that

$$k(a^2+1)(b^2+1)(c^2+1) \ge (a+b+c)^2$$

for all real numbers a,b,c.

Problem 2. Find the least value of k such that

$$(a^3-3a+1)^2+(b^3-3b+1)^2+(c^3-3c+1)^2+k(ab+bc+ca+6) \ge 2022$$

for all a,b,c are real numbers satisfying $a^2+b^2+c^2=6$.

Problem 3.

a) Find the least value of $k>0$ such that for all $x,y,z>0$ and $xy+yz+zx=3$, the following inequality is true:

$$(x+k)(y+k)(z+k) \ge (1+k)^3.$$

b) Find the least value of $k>0$ such that for all $a,b,c>0$ and $a+b+c=ab+bc+ca$, the following inequality is true:

$$\frac{1}{(a+k)(b+k)}+\frac{1}{(b+k)(c+k)}+\frac{1}{(c+k)(a+k)} \le \frac{3}{(1+k)^2}.$$

Problem 4.

a) Find the least value of k such that $\varphi(n)\sigma(n) < kn^2$ for all positive integer $n>1$.

b) Find the least value of c such that for all $n \in \mathbb{Z}^+$ then $\{n\sqrt{17}\} > \dfrac{1}{cn}$.

Problem 5. Find the maximum integer k such that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+k\frac{(a-b)(b-c)(c-a)}{(a+b)(b+c)(c+a)} \ge 3$$

for all positive real numbers a,b,c.

Problem 6. For $a,b,c>0$, denote $m=\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{a}, n=\dfrac{a}{c}+\dfrac{c}{b}+\dfrac{b}{a}$. Find all possible values of k such that the following inequality is true for all m,n defined as above:

$$(m-n)k^2+(2m-n-3)k-8m+6n+6 \le 0.$$

Problem 7*. Find the least value of $k>0$ such that

$$|x-ky|+|y-kz|+|z-kx| \ge 7$$

for all real numbers x,y,z satisfying $x^2+y^2+z^2=21$.