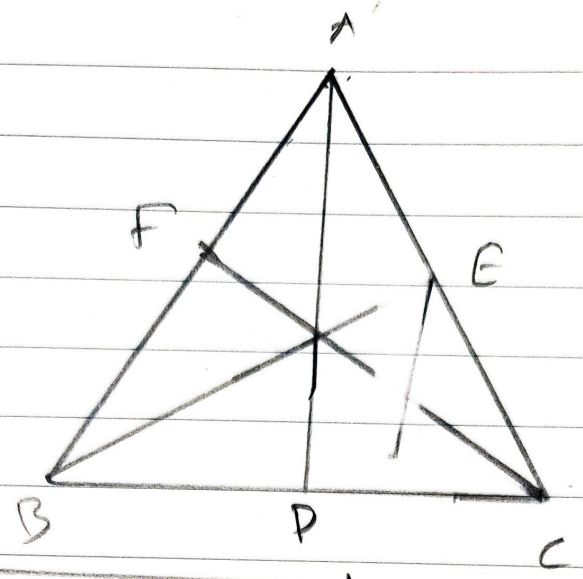
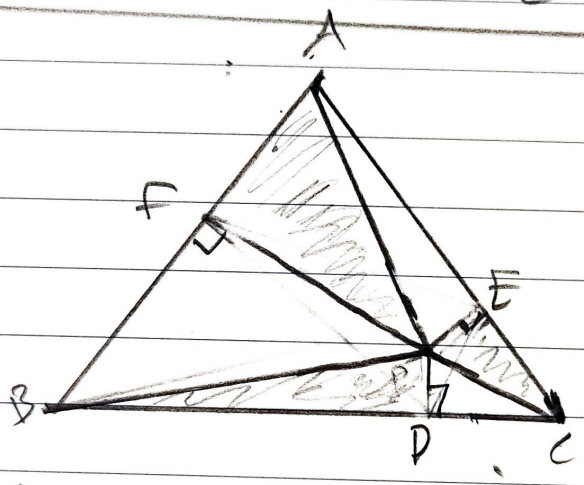
26



a)



$\triangle ABC = \triangle APB + \triangle APC + \triangle BPC$

$= \frac{1}{2} PF \cdot AB + \frac{1}{2} PE \cdot AB + \frac{1}{2} PD \cdot AB$

$\frac{2\triangle ABC}{AB}$ $= PF + PE + PD = h$

b)

22)

$$Ax = AW = AD + Dx = AB + BW$$

$$\parallel \qquad\qquad \parallel$$

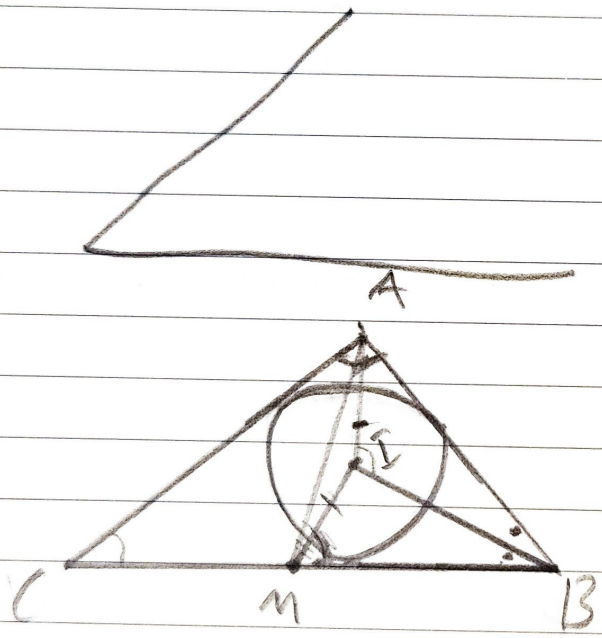
$$AD + Dy = AB + BZ$$

$$Cy = CZ$$

$$+$$

$$AD + DC = AB + BC$$

23)

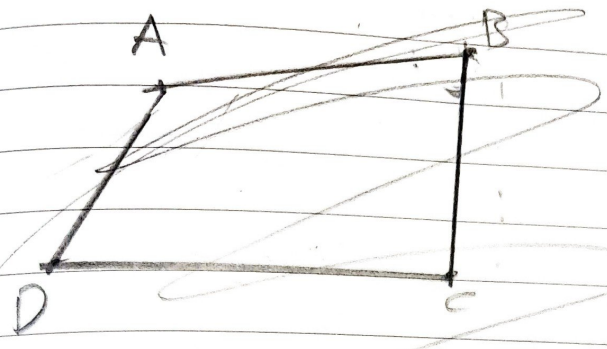


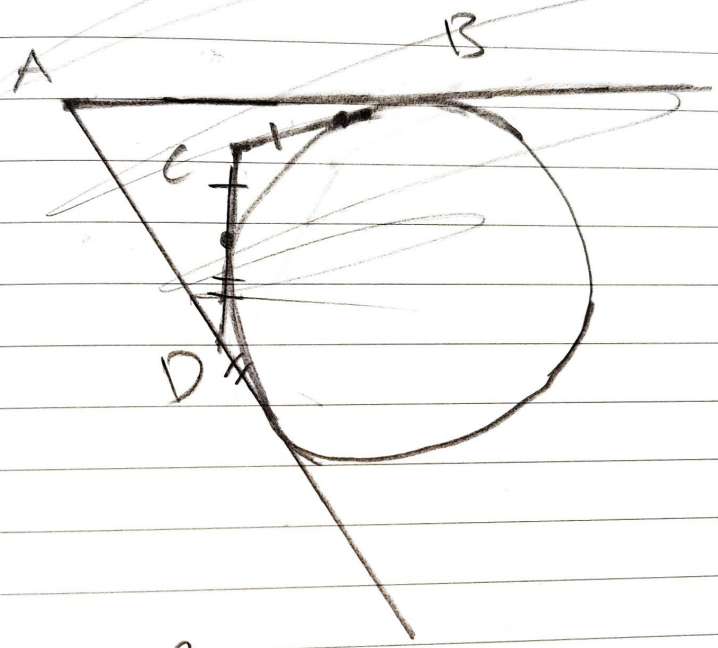
AIMC ▱

$\angle IMB = \angle CAI = \angle IAB \Rightarrow \triangle AIB \cong MIB$

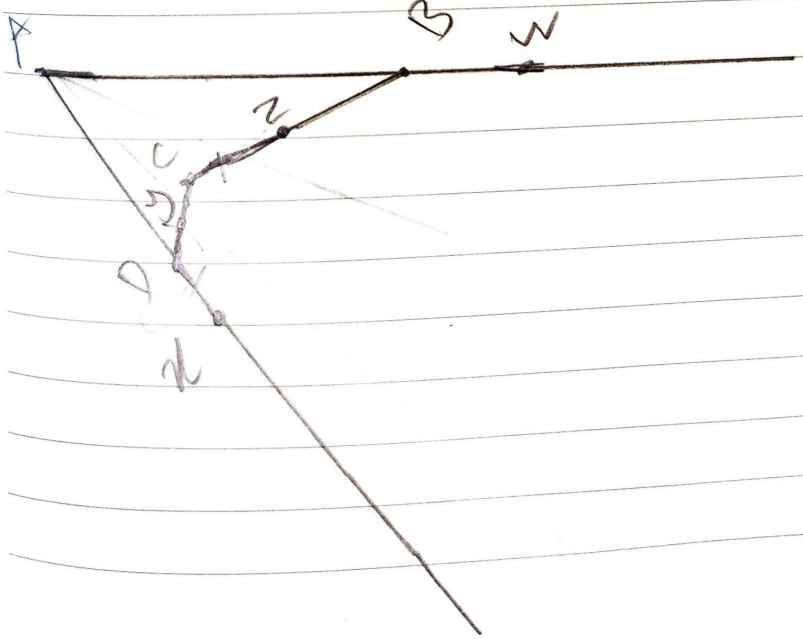
$AB = BM \Rightarrow 2AB = BC$

22)





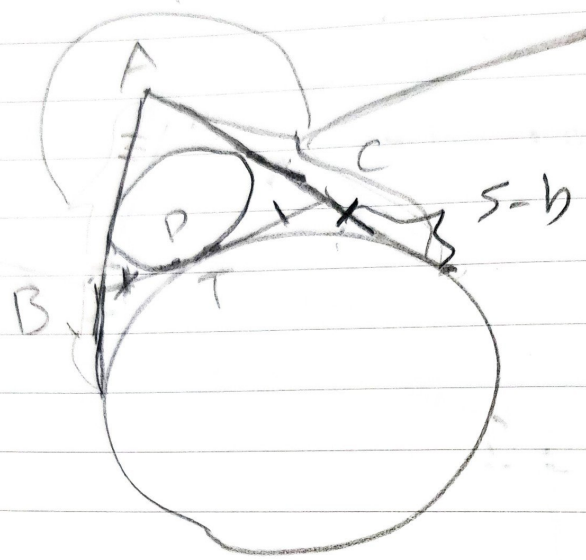
22)



MIB

20)

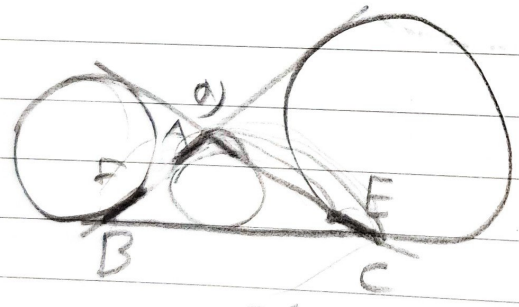
$t = b + c + BT + TC \Rightarrow a + b + c \Rightarrow = s$



$BD = s - b = CT$

---

21)



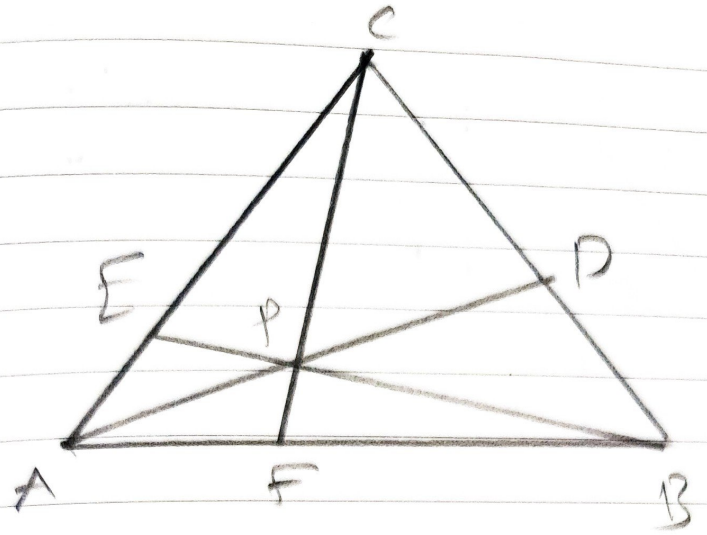
b) $AE + AF = BC$

$AE = b - EC$

$AF = c - FB$

$= b + c - 2\left(\frac{-a+b+c}{2}\right) = b + c + a - b - c = a = BC$

18)



$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

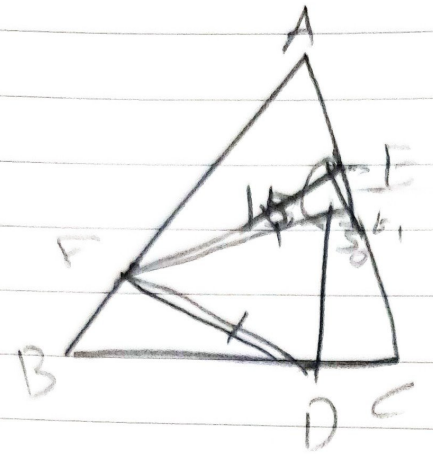
1) P exist

$$\frac{AF}{FB} = \frac{[APC]}{[BPC]} \qquad \frac{BD}{DC} = \frac{[APB]}{[APC]} \qquad \frac{CE}{EA} = \frac{[BPC]}{[APB]}$$

by multiplying

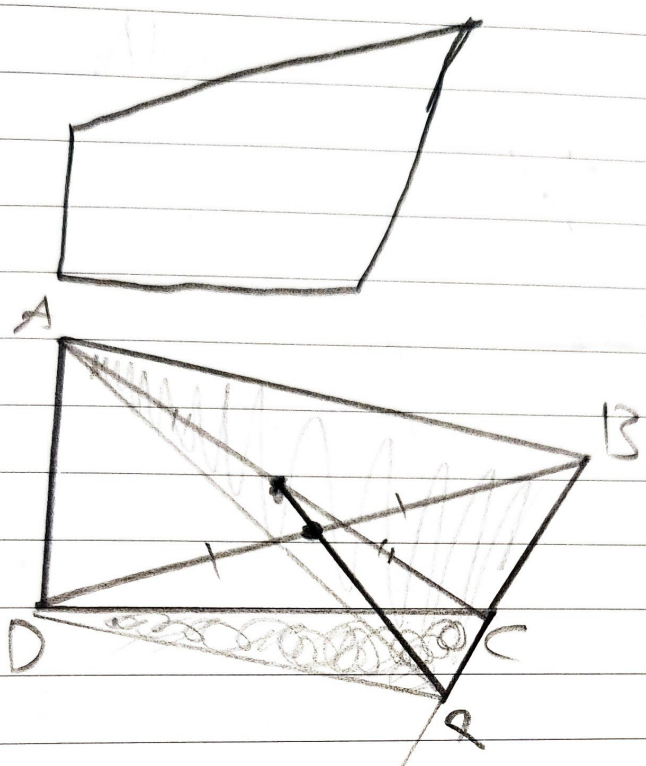
2)

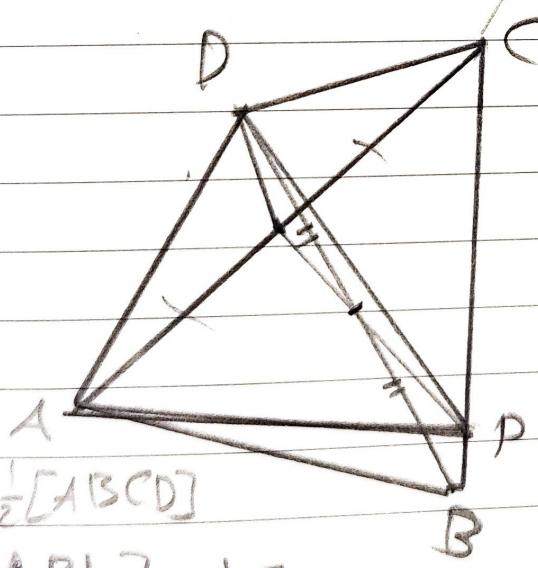
Suppose



E_1 s.t $AE_1 = BF = CD$

17)





$[ADP] = \frac{1}{2}[ABCD]$

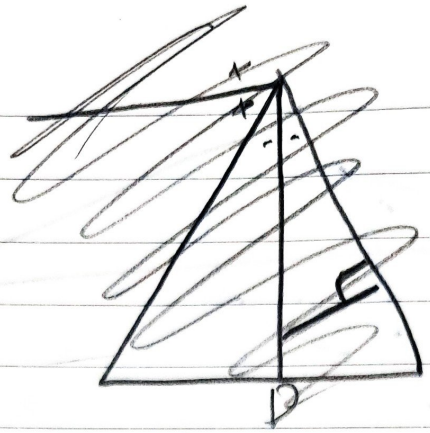
$[DAPK] = \frac{1}{2}[DAPC]$

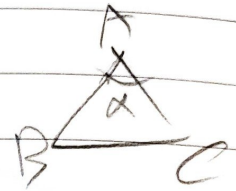
$[DKP] \overset{?}{=} \frac{1}{2}[ABP]$

$\parallel$

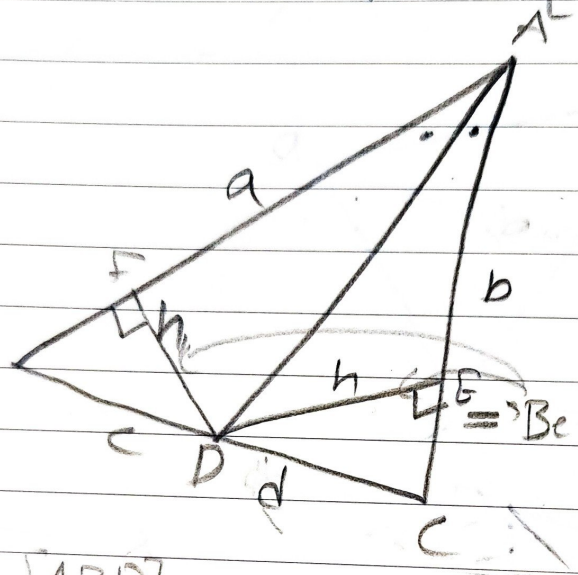
$[BKP] = \frac{1}{2}[ABP$

16)





$[A] = \sin\alpha \cdot AB \cdot AC$



1 Firstway

$\Rightarrow$ Because $\Delta AED \cong \Delta AFD$

$$\frac{c}{d} = \frac{[ABD]}{[ACD]}$$

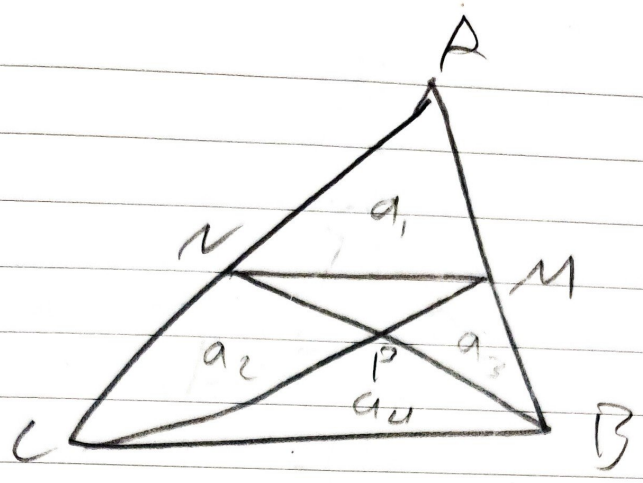
$$\frac{a}{b} = \frac{\frac{ha}{2}}{\frac{hb}{2}} = \frac{[ABD]}{[ACD]} = \frac{c}{d}$$

2 way

$$\frac{[ABD]}{[ACD]} = \frac{\frac{1}{2} a \cdot h}{\frac{1}{2} b \cdot h_1} = \frac{a}{b}$$

$h = h_1 \Rightarrow \angle BAD = \angle DAC$

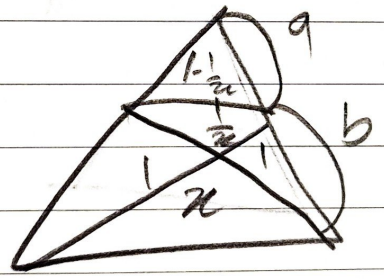
15)



if $a_1 = a_2 = a_3$

$PM = PC$ and $PB = PN \Rightarrow BCNM$

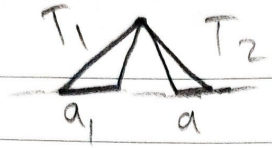
so



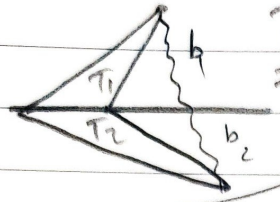
$$\frac{1-\frac{1}{x}}{1+\frac{1}{x}} = \frac{a}{b} = \frac{2}{1+x} \Rightarrow x = 3$$

$\sqrt{5}-2$

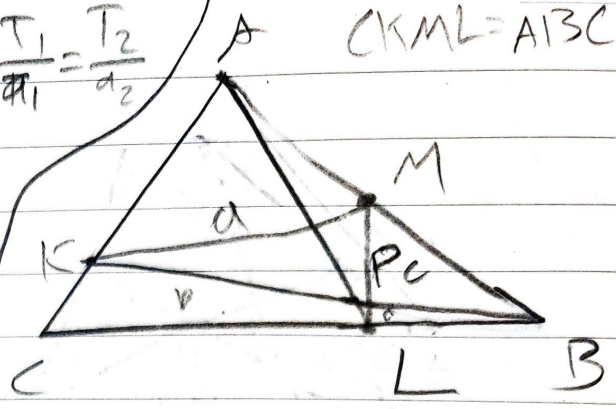
1)



$$\frac{T_1}{a_1} = \frac{T_2}{a_2}$$



$$\frac{T_1}{T_2} = \frac{b_1}{b_2}$$



$CKML = ABC$

a) prove that $b = c$

$$\frac{AK}{KC} = \frac{[ABK]}{[KBC]} = \frac{a+c}{b+d}$$

$$\frac{CL}{LB} = \frac{[ACL]}{[ALB]} = \frac{a+b}{c+d}$$

$$\Rightarrow (b+a)(b+d) = (c+d)(c+a)$$

$$b^2 + ab + bd + ad = c^2 + ac + ad + dc$$

$$\Leftrightarrow (b-c)(b+c) + ab - ac + bd - dc = 0$$

$$\Leftrightarrow (b-c)(b+c) + a(b-c) + d(b-c) = 0$$

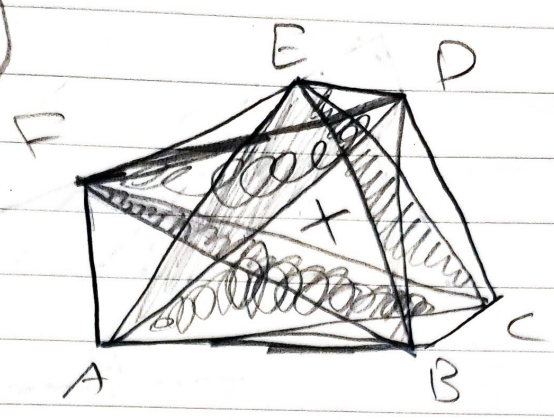
$$(b-c)(a+b+c+d) = 0 \Rightarrow b = c$$

b) $[CKML] = [ABC] - \frac{1}{2}[AKB] - \frac{1}{2}[BLA]$

$$= a+b+c+d - \frac{1}{2}(a+c) - \frac{1}{2}(b+d)$$

$$= \frac{1}{2}(a+b+c+d) = \frac{1}{2}[ABC]$$

(2)



(3)

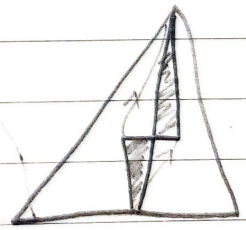
$[AMC] = [BMC] \neq [PRN] = [QRN]$

$= [PNRMC] = [QNRMC]$

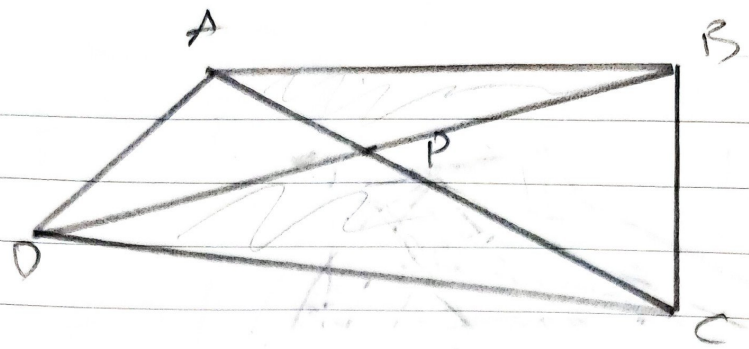
$[PNC] = \frac{1}{2}[ABC] = [PNRMC]$

$\quad\quad\quad\quad \parallel$

$\quad\quad\quad [QNC]$



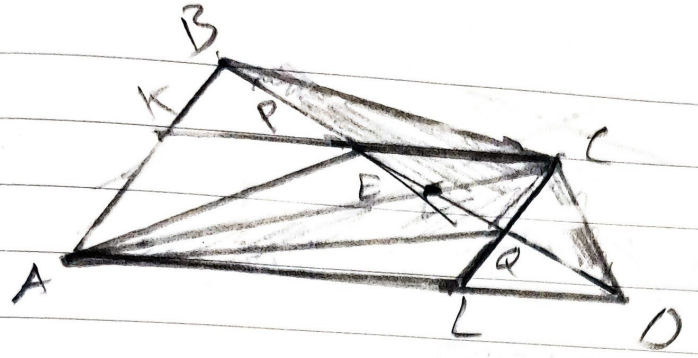
$\Rightarrow CR \parallel MN \Rightarrow PQ \perp CR$



$$\frac{BP}{PD} = \frac{[BCP]}{[CDP]} = \frac{[ABP]}{[ADP]}$$

$$[BCP] \cdot [ADP] = [ABP] \cdot [CDP]$$

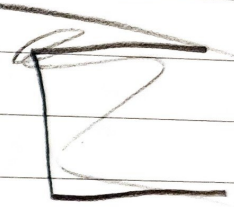
10)



$[ACQ] = [BCQ]$ $CQ \parallel AB$

$[CED] = [BEA]$

11)



$\sqrt{[ABCD]} \geq \sqrt{[ABP]} + \sqrt{[CDP]}$

$[ABCD] \geq [ABP] + [CDP] + 2\sqrt{[ABP]\cdot[CDP]}$

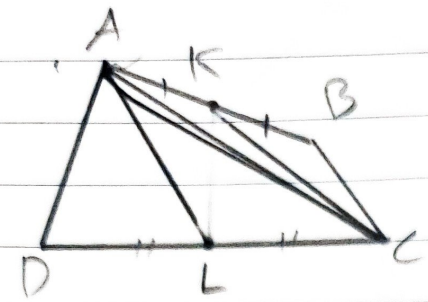
$[BCP] + [ADP] \geq 2\sqrt{[ABP]\cdot[CDP]}$

$[BCP]^2 + [ADP]^2 + 2[BCP]\cdot[ADP] \geq 4[ABP]\cdot[CDP]$

$[ADP]^2 + [BCP]^2 \geq 2[ADP]\cdot[BCP]$ true from AM-GM

equality when $[ADP] = [BCP]$ when $ABCD$ is parallel

8)



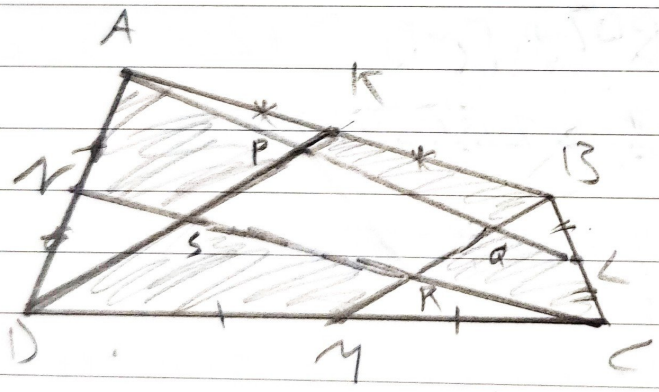
$[ACL] = [ADL] = \frac{1}{2}[ACD]$

$[CAK] = [CKB] = \frac{1}{2}[CAB]$

$+$

$[AKCL] = \frac{1}{2}[ABCD]$

9)



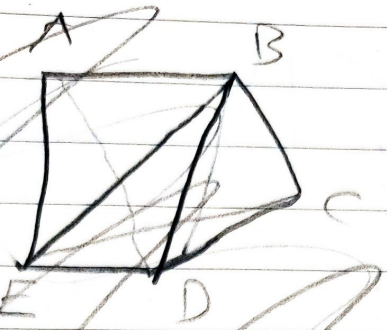
$2[PQRS] + [\text{The Shadded}] = [ABCD]$

$-[ABCD]$

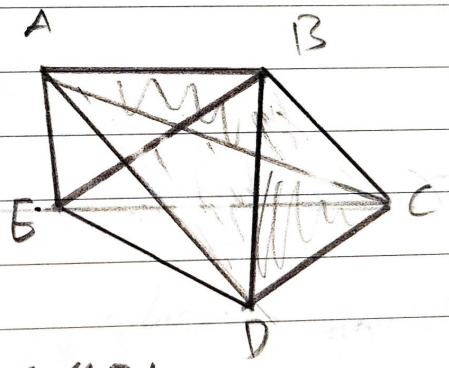
$[PQRS] - ([APK] + [BQL] + [CRM] + [DSN]) = 0$

$\Leftrightarrow [PQRS] = [APK] + [BQL] + [CRM] + [DSN]$

7)



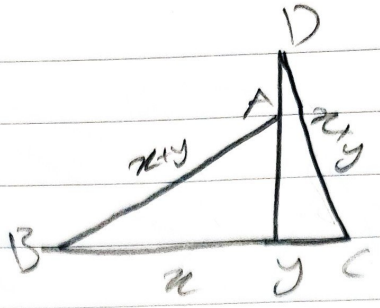
[△AED] = [△BDE] (same hight and base ED)



$AB \parallel CE$ $BC \parallel DA$

[BCD] = [BCA] = [ABE]

6)



$$|BE - CE| < AD\sqrt{3}$$

$$|x-y| \overset{?}{<} \sqrt{3}(DE - AE)$$

$$\Leftrightarrow \quad |x-y| \overset{?}{<} \sqrt{3}\left(\sqrt{(x+y)^2+y^2} - \sqrt{(x+y)^2+x^2}\right)$$

$$x^2+y^2-2xy \overset{?}{<} 3\left(x(x+2y)+y(y+2x)-2\sqrt{xy(x+2y)(y+2x)}\right)$$

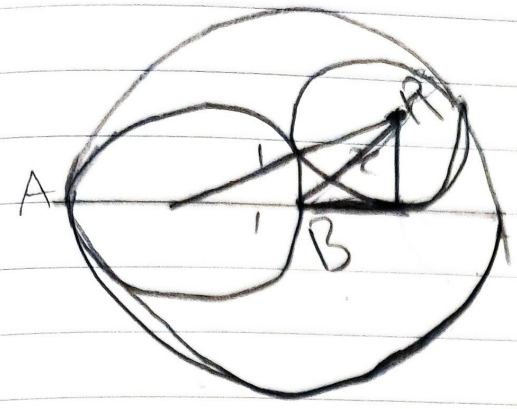
$$6\sqrt{xy(x+2y)(y+2x)} \overset{?}{<} 2x^2+2y^2+14xy$$

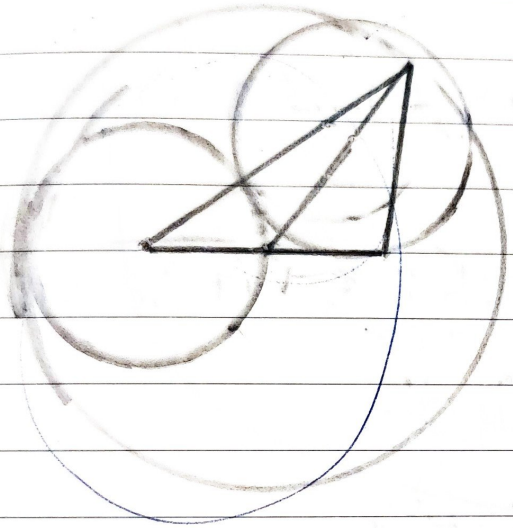
$$9xy(x+2y)(y+2x) < x^4+y^4+49x^2y^2+2x^2y+14x^3y+14xy^3$$

$$9x^2y^2+18xy^3+18x^3y+36x^2y^2 < x^4-4x^3y+6x^2y^2-4xy^3+y^4$$

$$0 < (x-y)^4$$

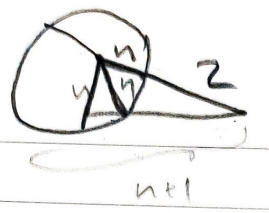
5)





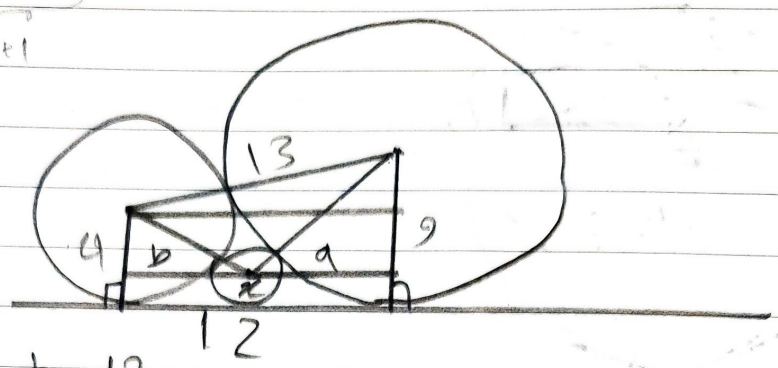
Geometrya

تریکونت



$\frac{2n+1}{n+1} = \cancel{2(n+1)+2}$

4)



$a+b=12$

$(x+9)^2 = a^2+(9-x)^2 \Rightarrow a^2=36x \Rightarrow a=6\sqrt{x}$

$(x+4)^2 = b^2+(4-x)^2$

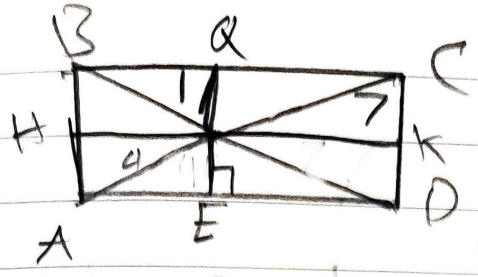
$x^2+8x+16 = b^2+x^2+16-8x$

$16x = b^2 \Rightarrow b = 4\sqrt{x}$

$6\sqrt{x}+4\sqrt{x}=12$

$x=\frac{36}{25}$

2)



$PD^2 = PK^2 + PE^2$

$PK^2 = 49 - PQ^2 = 49 - 1 + PH^2 = 49 - 1 + 16 - PE^2 = 64 - PE^2$

$\Rightarrow PK^2 + PE^2 = 64 \Rightarrow PD = 8$

3)

from POP $\cancel{2\cdot(n+2) = (n+1)(n+1-AD)}$

$\cancel{2n+3 = n^2 + 2n - AD - nAD + 1}$

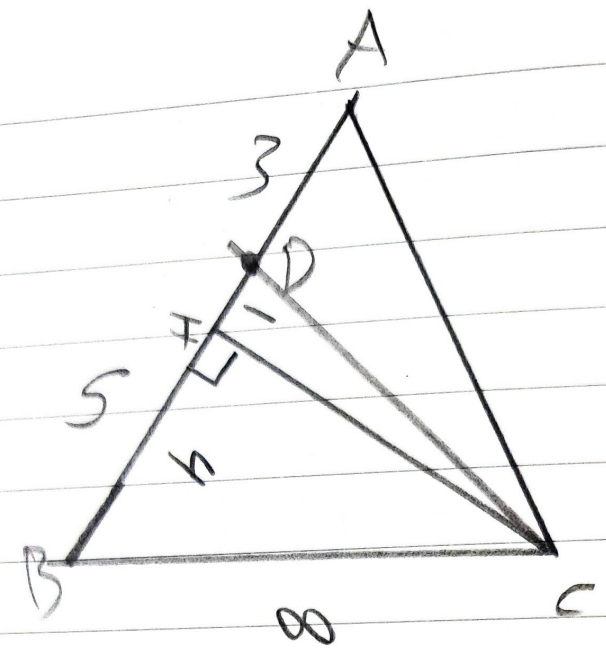
$\cancel{AD(n+1) = n^2 - 3}$

$2(2n+2) = (n+1)BD$

$4n+4 = n\cdot BD + BD$

$BD = 4$

$AD = n-3$



$$CD^2 = HD^2 + CH^2$$

$$CH^2 = 8^2 - 4^2 =$$

$$CD^2 = 1 + 64 - 16 = 49 \Rightarrow CD = 7$$