Warm-up 1. *Positive integer* 221 *can be expressed as sum of two squares in two different ways* $221 = 10^2 + 11^2 = 5^2 + 14^2$. *Find the lowest positive integer with this property.*

Warm-up 2. *Determine whether there exist rational numbers* r *and* q *such that* $r^2 + q^2 = = 15$.

Problems

1. Prove that there are infinitely many integers of the form $4m + 1$ that are not a sum of two perfect squares.

2. Prove that if n is a sum of two squares, then $2n$ is also a sum of two squares.

3. The product of two numbers, each of which is a sum of two squares, is itself a sum of two squares.

4. Prove that if number n is divisible by 5 and n is a sum of two squares, then $n/5$ is also a sum of two squares.

5. Prove that if $a^2 + b^2$ is divisible by a prime $p = 4m + 3$, then p divides both a and b.

6. Prove that if $a^2 + b^2$ is divisible by 21, then $a^2 + b^2$ is also divisible by 441.

7. Prove that for any integer m there exists a multiple of m that is not a sum of two squares.

Sums of two squares (homework)

1. Prove that if $a^2 + b^2$ is divisible by 3^{2021}, then both a^2 and b^2 are divisible by 3^{2022}.

2. Prove that if n is a sum of two squares, then $5n$ is also a sum of two squares.

3. Express 17×29 as a sum of two squares.

4. Can 19×29 be expressed as a sum of two squares?

5. Prove that if number n is divisible by 13 and n is a sum of two squares, then $n/13$ is also a sum of two squares.

6. Prove that there are infinitely many primes of the form $4m + 3$.