Level 2 E-training, week 6

Due to 23:59, Friday, 16 October 2020

Problem 1. For which non-equilateral triangles we have collinearity of the circumcenter, orthocenter, and incenter?

Problem 2. For which $p \in \mathbb{P}$ we have the following implication

$$p|a^{p^2} - 1 \Rightarrow p^4|a^{p^2} - 1$$

holding for every $a \in \mathbb{N}$?

Problem 3. Find all reals x satisfying the equation

$$7^x + 9^x + 11^x = 13^x + 14^x$$

Problem 4. Let $n \geq 4$ be an even positive integer. Construct a set $\mathcal{S}$ of n points on the plane such that the perpendicular bisector of any 2 points of $\mathcal{S}$ passes through another point of $\mathcal{S}$.

Problem 5. Construct an infinite family of nonsimilar triangles ABC such that the median from A, the altitude from B, and the internal bisector of $\angle C$ are concurrent.

Problem 6. Prove that $ord_{3^{n+1}}(2) = 2 \cdot 3^n$ for every $n \in \mathbb{N}$.

Problem 7. Each grid point of a cartesian plane is colored with one of three colors, whereby all three colors are used. Show that one can always find a right-angled triangle, whose three vertices have pairwise different colors.

Problem 8. Let $a_1, a_2, \ldots, a_{2020} \in \{1, 2, \ldots, 2020\}$ such that

$$a_{a_{a_n}} = n \ \ \forall\, 1 \leq n \leq 2020$$

Show that $a_n = n$ for some $1 \leq n \leq 2020$.