Problem 1F. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1$$

for all $x, y \in \mathbb{R}$.

Problem 2F. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(f(x) + y) = f(f(x) - y) + 4f(x)y$$

for all $x, y \in \mathbb{R}$.

Problem 3F. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x + f(x + y)) + f(xy) = x + f(x + y) + yf(x)$$

for all $x, y \in \mathbb{R}$.

Problem 4F. Determine all functions $f : [1, \infty) \to [1, \infty)$ satisfying

(1) $f(x) \leqslant 2(1 + x)$ for all $x \in [1, \infty)$;

(2) $xf(x + 1) = f(x)^2 - 1$ for all $x \in [1, \infty)$.

Problem 5F. Prove that there does not exist a function $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x)^2 \geqslant f(x + y)(f(x) + y)$$

for all $x, y \in \mathbb{R}^+$.

Problem 6F. Determine all functions $f : \mathbb{N} \to \mathbb{N}$ such that for all $x, y \in \mathbb{N}$ there is a non-degenerated triangle with side lengths

$$x, \quad f(y) \quad \text{and} \quad f(y + f(x) - 1).$$

Problem 7F. Let $f : \mathbb{R} \to \mathbb{R}$ be a function satisfying

$$f(x + y) \leqslant yf(x) + f(f(x))$$

for all $x, y \in \mathbb{R}$. Prove that $f(x) = 0$ for all $x \leqslant 0$.