Email training, N16

March 8-10, 2020

Problem 16.1. Prove that for prime p with $p > 2$ the value of expression $\left[(2+\sqrt{5})^p\right] - 2^{p+1}$ is divisible by p.

Problem 16.2. Prove that for any integers $1 \leq m \leq n$ the value of

$$\frac{gcd(m,n)}{n}\binom{n}{m}$$

is an integer.

Problem 16.3. Find all positive integer values of a and b for which

$$\frac{b}{4}\sqrt{\frac{2a-b}{2a+b}}$$

is positive integer as well.

Problem 16.4. The girth of a graph is the length of the smallest polygon in the graph. Let G be a graph with girth 5 for which all vertices have degree $\geq d$. Show that G has at least d^2+1 vertices.