Email training, N4

Level 4, October 4-10

Problem 4.1. Let a, b, c, d be real numbers such that

$$a^4 + b^4 + c^4 + d^4 = 16.$$

Prove the inequality

$$a^5 + b^5 + c^5 + d^5 \leq 32.$$

Problem 4.2. Consider the positive numbers $x_1, x_2, \ldots, x_n$ such that

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} \frac{1}{x_i}.$$

Prove that

$$\sum_{i=1}^{n} \frac{1}{n-1+x_i} \leq 1.$$

Problem 4.3. Find all pairs of possitive integers (x, y) such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{lcm(x,y)} + \frac{1}{gcd(x,y)} = \frac{1}{2}.$$

Problem 4.4. Find all integer numbers m and n such that

$$(5 + 3\sqrt{2})^m = (3 + 5\sqrt{2})^n.$$

Problem 4.5. Let $1 \leq r \leq n$. We consider all r-element subsets of $(1, 2, \ldots, , n)$. Each of them has a minimum. Prove that the average of these minima is $\frac{n+1}{r+1}$.

Problem 4.6. Twenty children are queueing for ice cream that is sold at SR5 per cone. Ten of the children have a SR5 coin, the others want to pay with a R10 bill. At the beginning, the ice cream man does not have any change. How many possible arrangements of the twenty kids in a queue are there so that the ice cream man will never run out of change?

Problem 4.7. Given ΔABC, D is a point on BC and P is on AD. A line ℓ is passing through D intersects AB, PB at M, E respectively, and intersects AC extended and PC extended at F, N respectively. Let $DE = DF$. Prove that $DM = DN$.

Solution submission deadline October 10, 2021

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