— Combinatorics for L3 —

— March 24, 2019 — Some nice problems —

28. Cells of a 10×10 chessboard are painted in a standard manner. Can one choose five black and five white cells in such a way that each row and each column contains exactly one chosen cell?

29. In a chess competition took part $2n$ humans and $2n+1$ aliens. It turned out that every human played against the same number of aliens. Prove that some two aliens played against the same number of humans.

30. Is it possible to label vertices of a regular 30-gon with numbers from 1 to 30 in such a way that the sum of labels of every pair of adjacent vertices is a perfect square?

31. In a badminton tournament $n \geq 3$ people took part, each two played exactly one match and there were no tie games. After the tournament all participants took places by a round table in such a manner that everyone won with their right neighbor. Prove that there exist participants A, B, C such that A defeated B, B defeated C, and C defeated A.

32. Each point of a plane was colored with one of three colors. It turned out that every segment has the following properties:

- if its endpoints are of the same color, then its midpoint also have this color,
- if its endpoints have different colors, then its midpoint is of the third color.

Prove that whole plane is colored with the same color.

33. Non-intersecting diagonals (possibly sharing endpoints) divide a convex n-gon into triangles in such a way at each n-gon's vertex an odd number of triangles meet. Prove that n is a multiple of 3.

34. Each positive integer is painted with some color. It is known that for all pairs a, b of integers greater than 1 the numbers $a+b$ and ab have the same color. Prove that all numbers greater than 4 are of the same color.