Email training, N1

Level 4, September 13-19, 2021

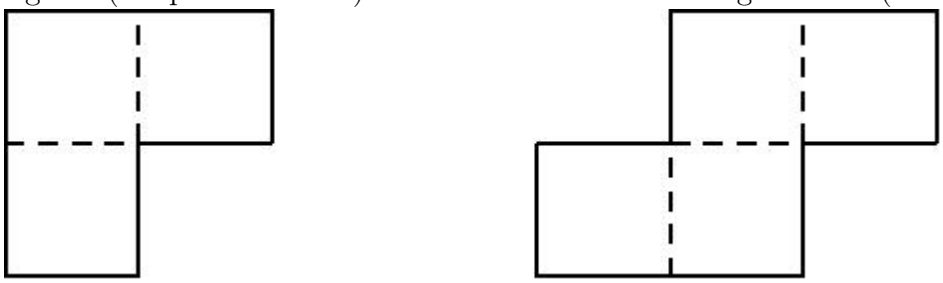
Problem 1.1. Does there exist a polynomial $P(x)$ of second degree with integer coefficients such that the leading coefficient is not divisible by 2022 and all numbers $P(1)$, $P(2)$, ..., $P(2022)$ give different residues mode 2022.

Problem 1.2. Prove that for any 2 positive integers m and n with $(m, n) \neq (1, 1)$ the value of expression

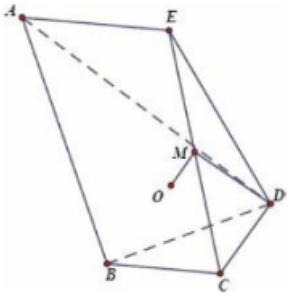
$$\frac{1}{m} + \frac{1}{m+1} + \ldots + \frac{1}{m+n-1}$$

is not an integer.

Problem 1.3. Let m and n are positive integers greater than 3. Determine the least number of figures (see picture below) needed to cover the rectangle of size $(2n-1) \times (2m-1)$.



Problem 1.4. Refer to the diagram below. Let $ABCDE$ is a pentagon such that $BC \parallel AE$, $AB = BC + AE$ and $\angle B = \angle D$. Let M be the midpoint of CE and O be the circumcenter of ΔBCD. Show that if $OM \perp MD$ then $\angle CDE = 2\angle ADB$.



Solution submission deadline September 19, 2021
Submit single PDF file in filename format L4_YOURNAME_week1.pdf
submission email **imo20etraining@gmail.com**