Email training, N1

September 11-16, 2022

Problem 1.1. Solve the equation

$$x(7-x)(7+x^2) = 12(x+1)^2.$$

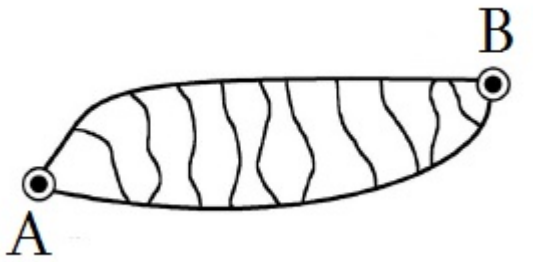
Problem 1.2. Solve the equation

$$27^x - 8^x = 3(18^x - 12^x).$$

Problem 1.3. Prove that 1280000401 is composite (don't use calculator).

Problem 1.4. Prove that if $a+b$ and ab are divisible by c, then a^6+b^6 is divisible by c^3.

Problem 1.5. In how many ways it is possible to go from A to B without passing the same road twice.



Problem 1.6. There are 2017 coins on a table. For $i = 1, 2, 3, \ldots, 2017$ in succession, one must turn over exactly i coins. Prove that it is always possible either to make all of the coins fae up or to make all of the coins face down, but not both.

Problem 1.7. In triangle ABC, AH is an altitude (H is on BC) and BE is a bisector (E is on AC) . We are given that angle $\angle BEA = 45^o$. Prove that $\angle EHC = 45^o$.

Problem 1.8. The bisector of angle BAD in the parallelogram $ABCD$ intersects the lines BC and CD at the points K and L respectively. It is known that $ABCD$ is not a rhombus. Prove that the centre of the circle passing through the points C, K and L lies on the circle passing through the points B, C and D.

Solution submission deadline 15:00, September 16, 2022
Send the solution as single PDF file to imo20etraining@gmail.com