May Camp - 2021
Algebra L2 Counting

Warm-up 1. *Work out the sum*

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots+\frac{1}{99\cdot 101}.$$

Warm-up 2. *Simplify*

$$1\cdot 1!+2\cdot 2!+3\cdot 3!+\ldots+n\cdot n!.$$

Problems

1. Let $p>2$ be a prime number and

$$A_k=1^k+2^k+\ldots+(p-1)^k,\qquad 1\leq k\leq p-2.$$

Prove that A_k is divisible by p.

2. Let $p>2$ be a prime number and

$$A_k=1^k+2^k+\ldots+(p-1)^k,\qquad 1\leq k\leq p.$$

Prove that $A_3, A_5, \ldots, A_{p-2}, A_p$ are divisible by p^2.

3. Let $p>3$ be a prime number and

$$A_k=1^k+2^k+\ldots+(p-1)^k.$$

Prove that p^5 divides $p^2A_{p-1}-2A_p$.

Homework

1. Simplify the sum

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\ldots+\frac{1}{99\cdot 100\cdot 101}.$$

2. Let u_n be a sequence such that $u_1=1$, $u_2=1$, and $u_{n+2}=u_{n+1}+u_n$ for every $n\in\mathbb{N}$. Simplify the sum

$$u_1+u_2+u_3+\ldots+u_n$$

3. Let $p>5$ be a prime number, k an odd integer such that $3<k<p$, and

$$A_k=1^k+2^k+\ldots+(p-1)^k.$$

Prove that p^4 divides $kpA_{k-1}-2A_k$.