Test 1

Level 2, November 30

Problem 1.1. Prove that for every polynomial $P(x)$ there exist polynomials $Q(x)$ and $R(x)$ such that $P(x) = Q(x^2) + R((x+1)^2)$.

Problem 1.2. Find all quadruples (p, a, b, x) that satisfy the equality $p^2 + 4^a 9^b = x^2$, where p is a prime and a, b, x nonnegative integers.

Problem 1.3. Given is an equilatereal triangle ABC. Points D, E, F lie on sides BC, CA, AB, respectively, and satisfy $AF = BD$ and $DF = EF \neq DE$. Prove that $\angle CDE = 90°$.

Problem 1.4. Two players, A and B play the following game. On a $1 \times n$ board, where fields are labeled in order from 1 to n, a coin is placed at position k. Players take turns moving the coin, with player A starting first. Each player can move a coin one or two fields in either direction, with the restriction that the coin cannot move onto a field it had already occupied. The player unable to make a move loses. For which values of (n, k) does which player have a winning strategy?