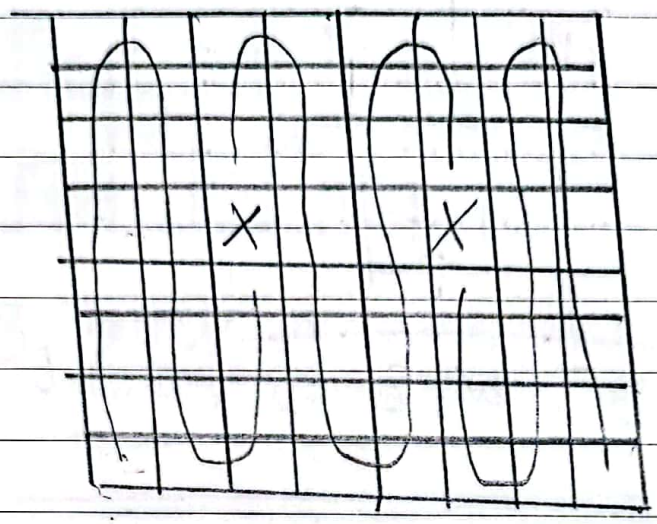
← C start

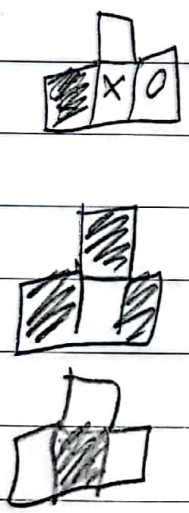
Lucas

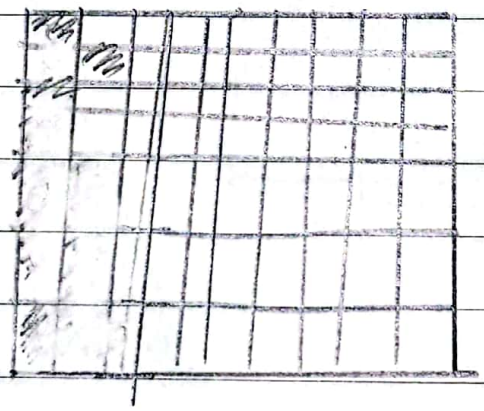
W.2 / Every domino will cover 1 white and 1 black but only 32 white 30

W.3



P.1.4





Black 50

white 50

⇓

25 Tetris

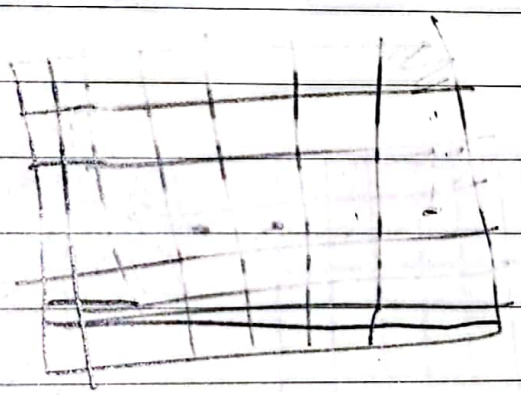
⇓

P.2 Left ⇒ take 1 white

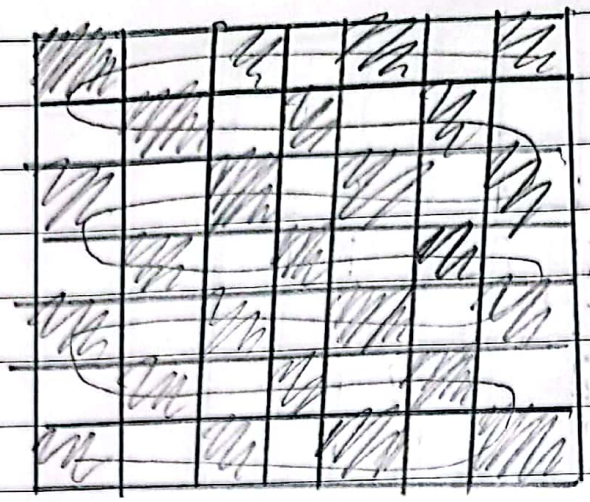
P.3

	B	W
1st floor	25	24
2	24	25
3	25	24
4	24	24
5	25	24
6	24	25
7	25	24

B = W+2 ⇒ cannot

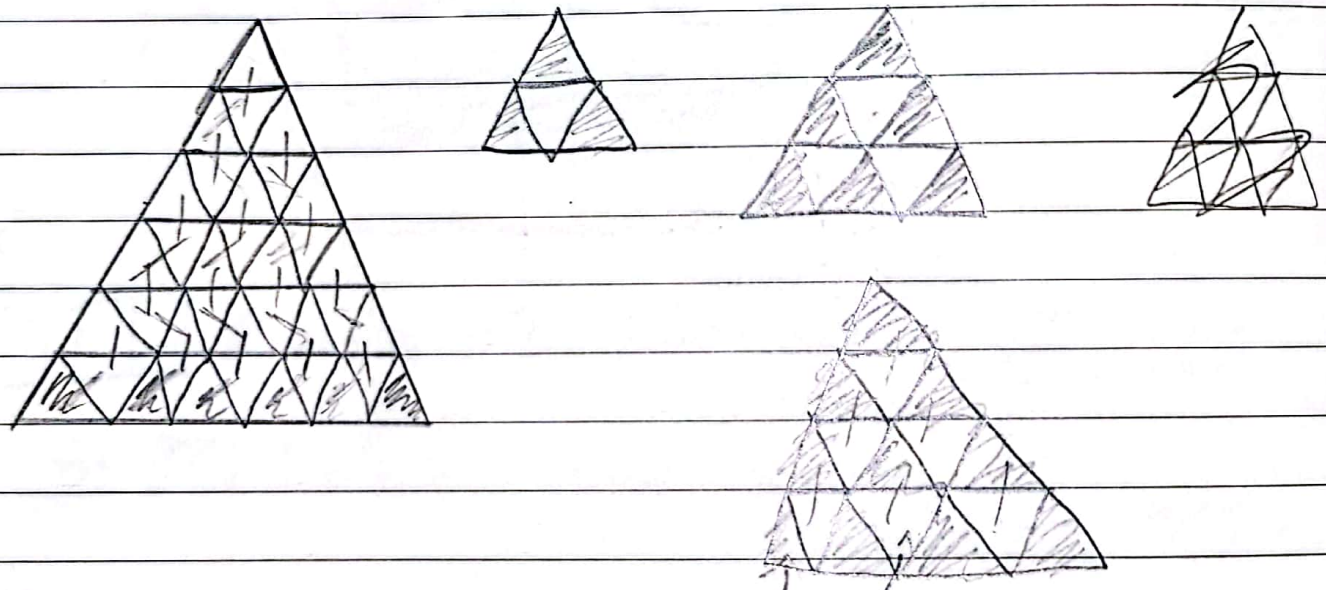


4)



25B 24W => unit should be in B

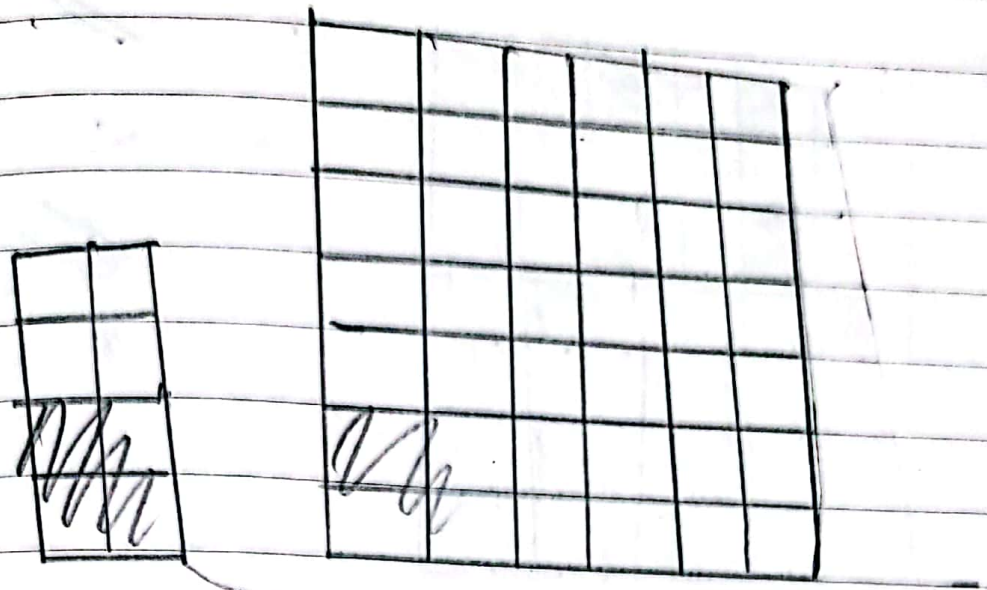
5)



n have $\frac{n(n+1)}{2}$ triangles

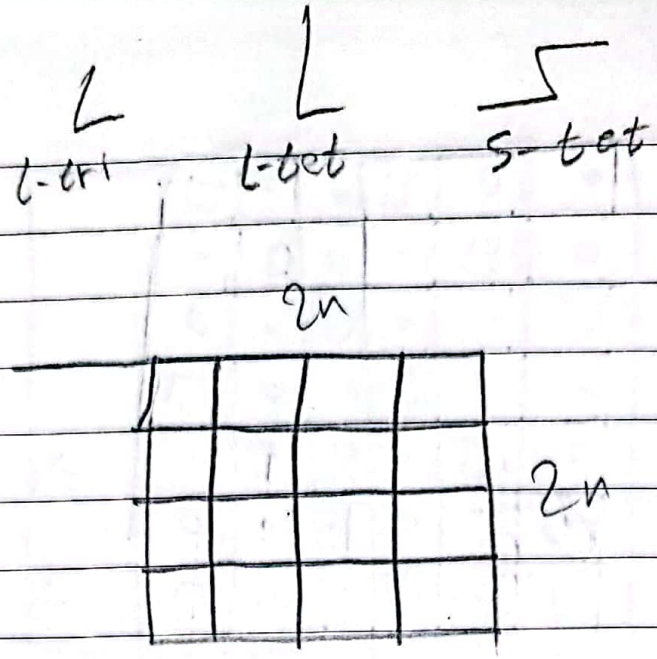
$n^2 - \frac{n(n+1)}{2} = \frac{n(n-1)}{2}$

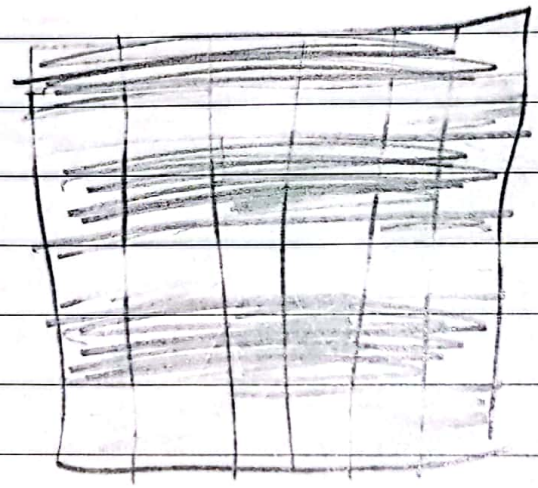
6)



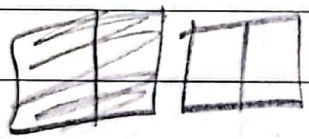
14x8 black

13x8 white but equal in 1x2x4





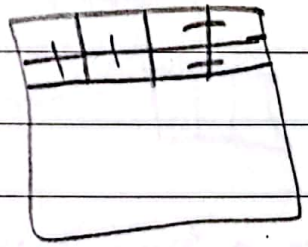
k



2 black or 2 white $2n^2$ times

1 black 1 white $2n^2$ times

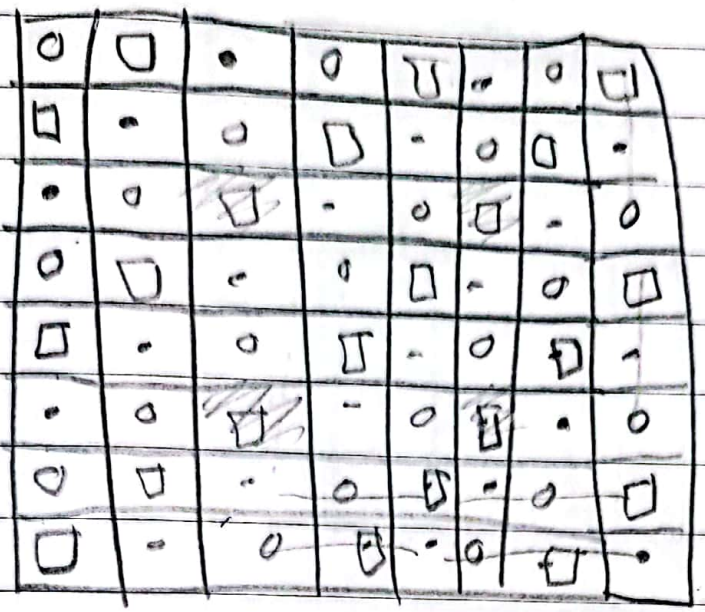
neven



Suppose it is possible

$$\#\blacksquare = \frac{1}{2}(2n)^2$$
$$= 2n^2$$
$$\equiv \#[\blacksquare + \blacksquare] \pmod 2$$
$$\parallel$$
$$n^2$$

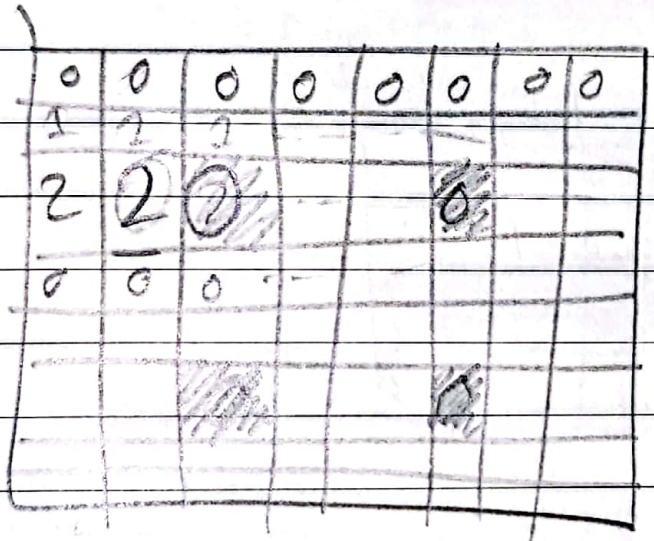
$\lightning$



في المربعات

⇒ تلوينة عطس

⇒



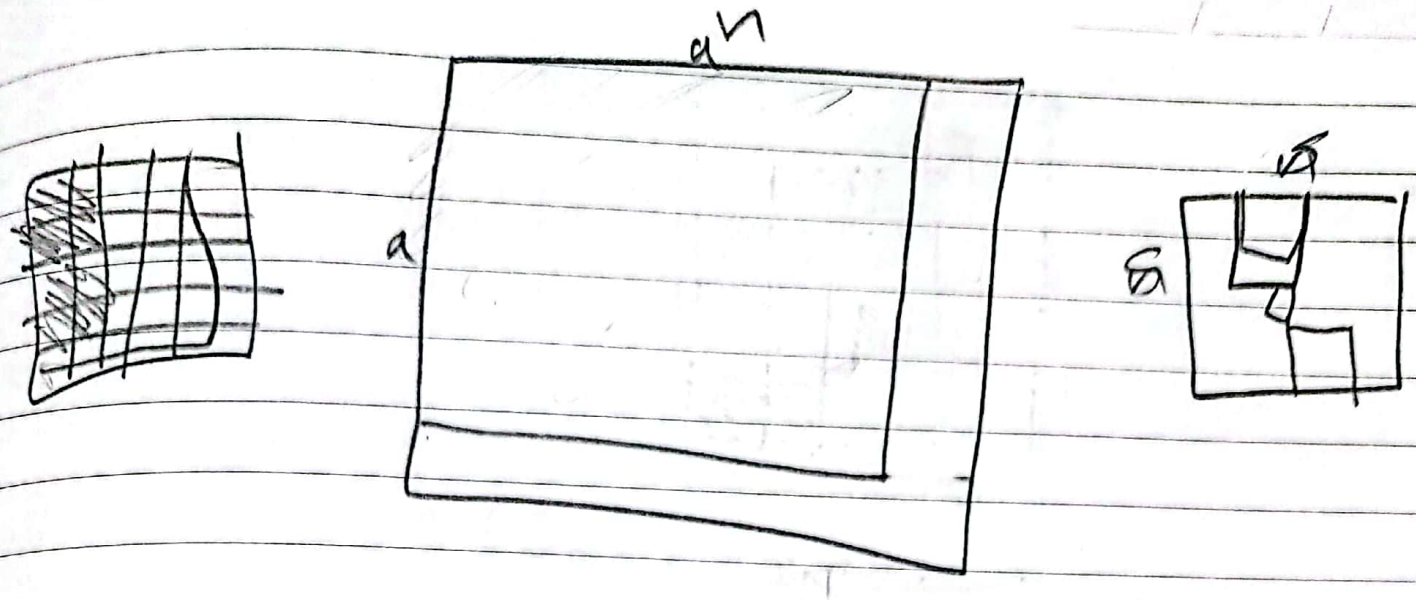
Each covers 0 (mod 3) but board is 2 (mod 3) ⇒ should leave

same but horizontal

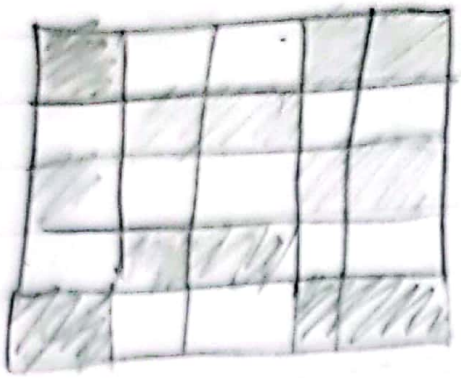
1

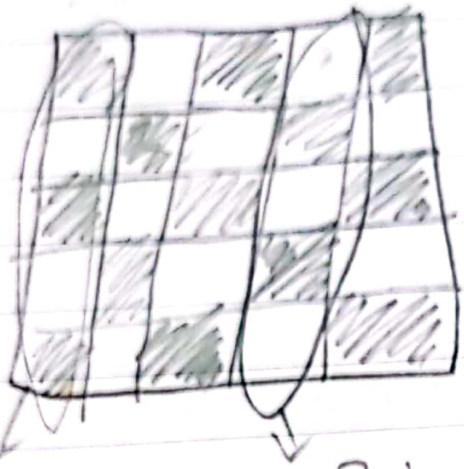
1	2	3	4	1	2	3	4	1	2
2	3	4	1	2	3	4	1	2	3
3	4	1	2	3	4	1	2	3	4
4	1	2	3	4	1	2	3	4	1
1	2	3	4	1	2	3	4	1	2
2	3	4	1	2	3	4	1	2	3
3	4	1	2	3	4	1	2	3	4
4	1	2	3	4	1	2	3	4	1
1	2	3	4	1	2	3	4	1	2
2	3	4	1	2	3	4	1	2	3

1	2	3	4
16	16	16	16
9	10		
25	26		



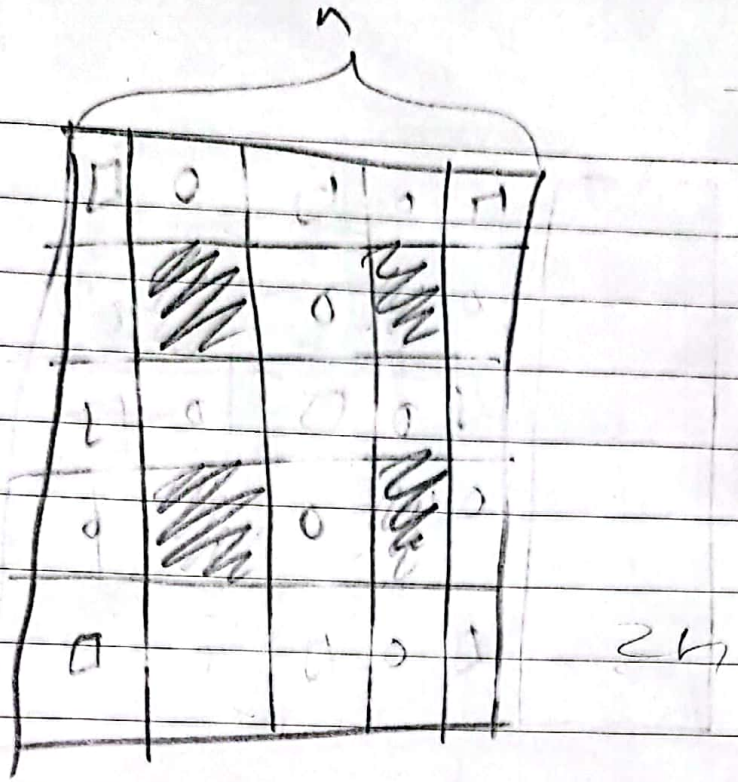
n odd and a even so there is a column and row that you cant fill it so there will be 1 row and 1 colu with 1 compartion so 2n-1 unit square





$$2\left(\frac{n+1}{2}\right)+2\left(\frac{n-2}{2}\right)$$

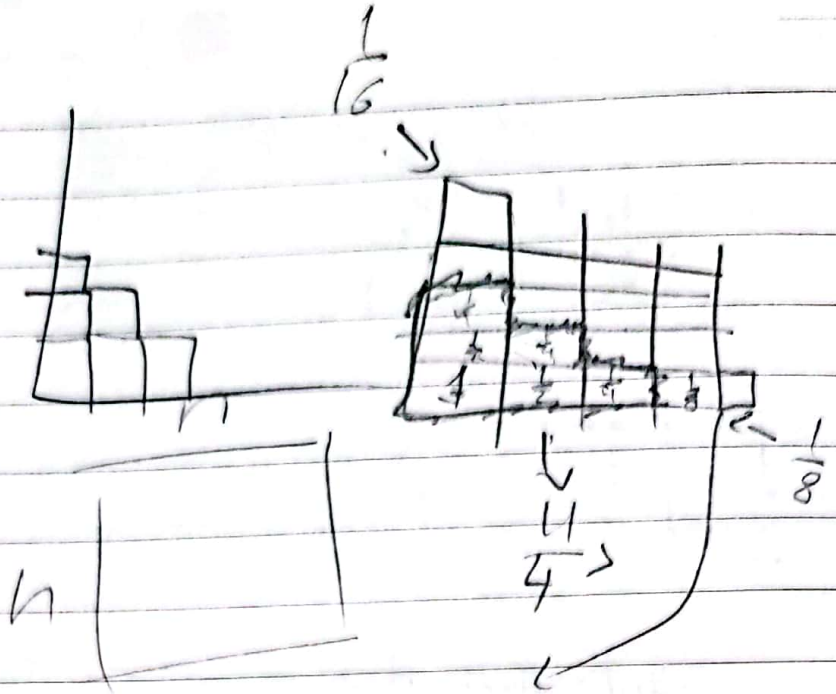
$$= 2n-1$$



$\therefore$ In each $2\left\lfloor \frac{n}{2} \right\rfloor$ # $\leq \left(\frac{n-1}{2}\right)^2$

$\Rightarrow$ cells covered $\leq 4\left(\frac{n-1}{2}\right)^2$

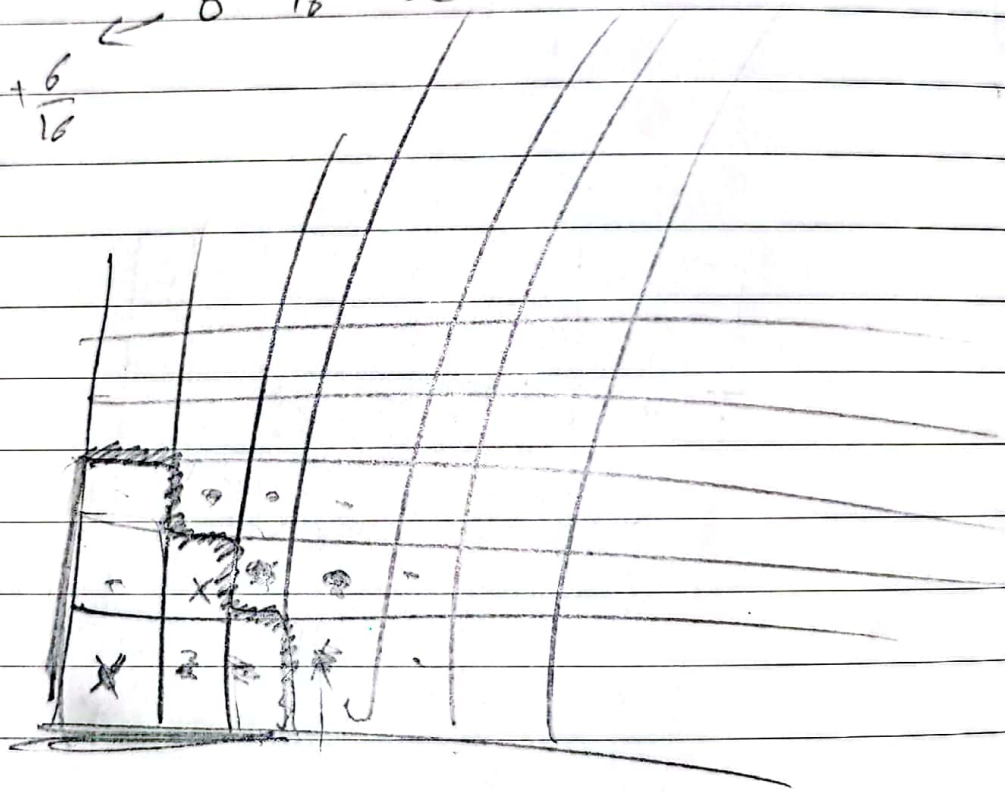
$\Rightarrow$ min $\geq 2n-1$

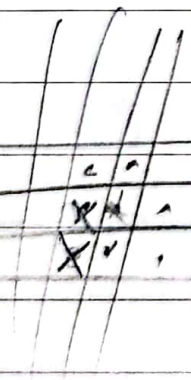


$$S = \frac{4}{8} + \frac{5}{16} + \frac{6}{32} + \dots$$

$$2S = \frac{4}{4} + \frac{5}{8} + \frac{6}{16}$$

$$S = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$$





P2/T4

32 can be filled

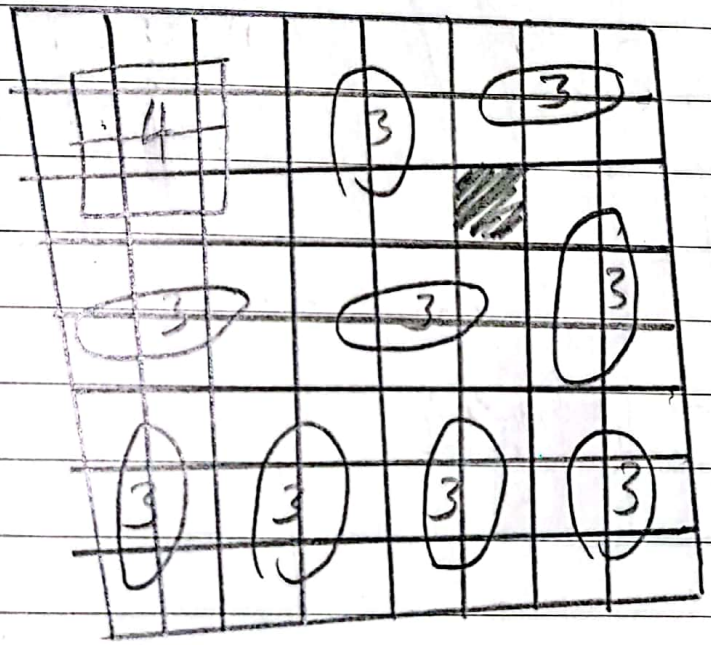
1) suppose 32 is possible

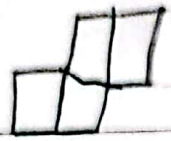
1	2	3	1
2	3	1	2
3	1	2	3
1	3	3	1

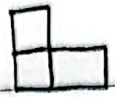
$\#\boxed{1} = \#\boxed{2} = \#\boxed{3}$ in $3\times3\times2$ but not in $8\times8\times2$

32 is not possible

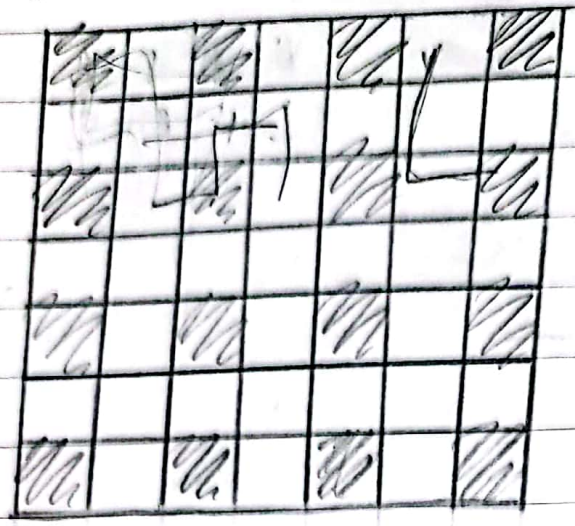
2) 31 مثال



 = 1 ■

 ≤ 1 ■

= a



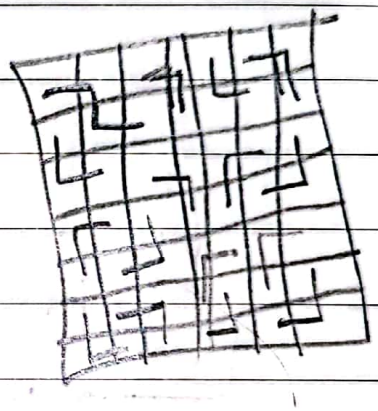
= b

we need ≥ 16 ■ so ≥ tiles

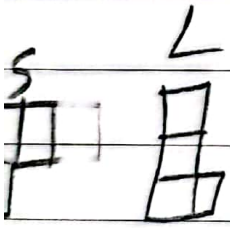
$4a + 3b = 49$

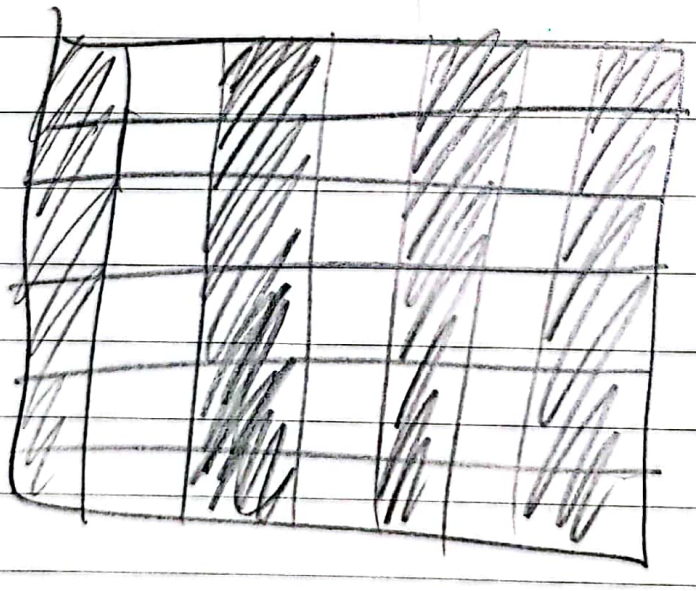
$a + b = 16$

$\Rightarrow a = 1,\ b = 15 \Rightarrow$



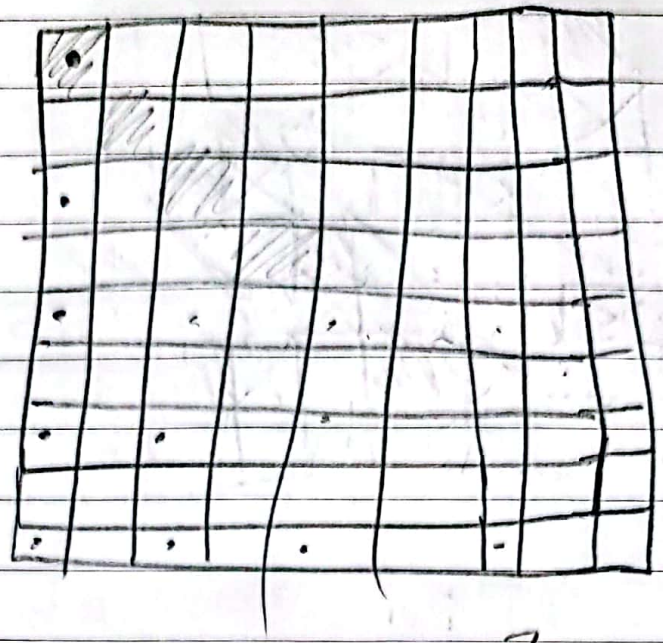
color tile even stripes





of ■ in S is even, # of ■ in L is odd; but # ■ is even

IMO SL C2 2002



				n
$n=1$	$n=3$	5	7	$\left(\frac{n+1}{2}\right)^2$
		9	16	

Dots $\boxed{\cdot}=1$ $\boxed{\cdot}=4$

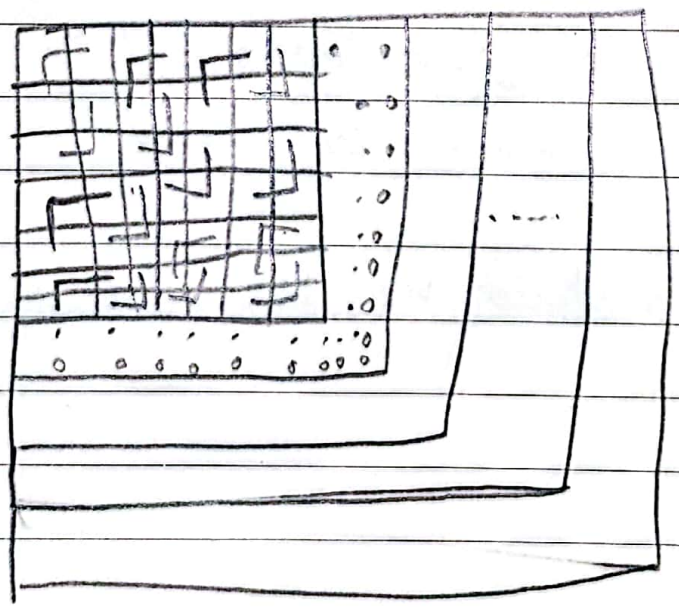
$\Rightarrow$ $\boxed{\cdot}\leq 1\boxed{\cdot}$ $\Rightarrow$ need $\left(\frac{n+1}{2}\right)^2$

Area

$\Rightarrow$ $3\left(\frac{n+1}{2}\right)^2 \leq n^2 \Rightarrow n \geq 7$

induction for $n \geq 7$

base

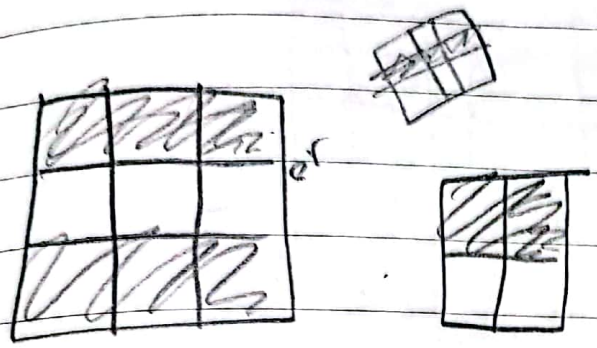


if

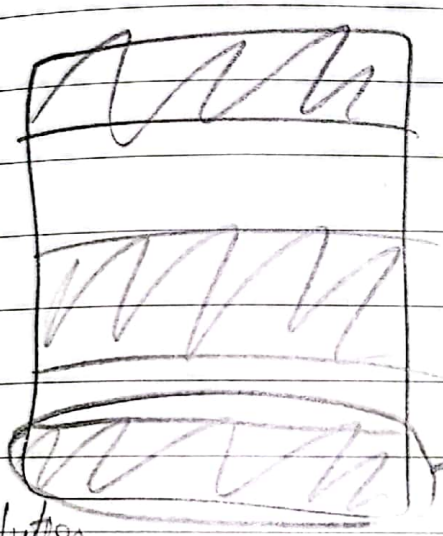
$n = 6k+1$ or $6k+5$

$4x^2 + 9y^2 = 36k^2 + 12k + 1 \equiv 1 \; [3]$

$2x^2 + 3y^2 = 6k+1 \Rightarrow x \equiv 2 \; [3]$



$\Rightarrow 3 \mid \#B - \#W$

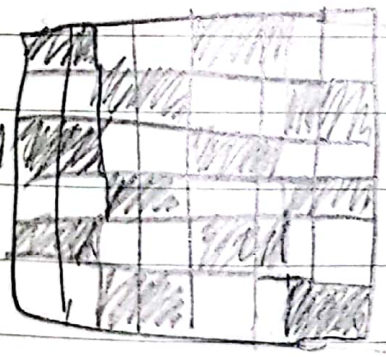


$\Rightarrow 3 \mid \#B - \#W$

$\#B - \#W = \; = 6k+1$ or $6k+5$

If solution

1×1.5 chessboard



2x2 balaced

3x3 balaced

if 2 then $n \times n$ balanced if only 3x2

$\perp$ coprime

b) $2k$ ✓

$2k+1$ and $3 \mid 2k+1 \iff k \equiv 1 [3]$

$2k+1$

	a	c	a	c	a
	b	d	b	d	b
$2k+1$	a	c	a	c	a
	b	d	b	d	b
	a	c	a	c	a

T

k k

$\# \boxed{a} = \# \boxed{b} = \# \boxed{c} = \# \boxed{d}$

$A \to$

d	b	d
c	a	c
d	b	d

$B \to \dots$

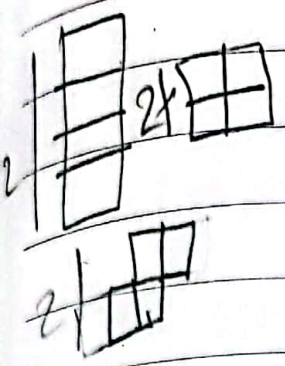
$(k+1)^2 = \# \boxed{a} = T + 4A + 1D + 2B + 2C$

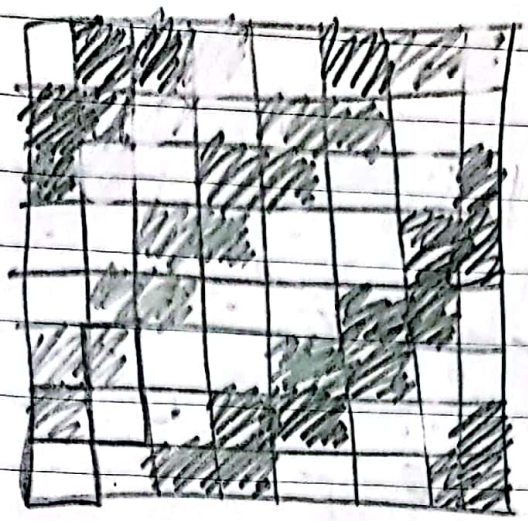
$(k+1) = \# \boxed{b} = T + 2A + 2D +$

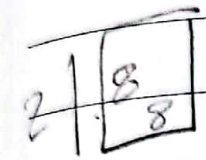
$(k+1) = \# \boxed{c} = T + 2A + 2D +$

$k^2 = \# \boxed{d} = T +$

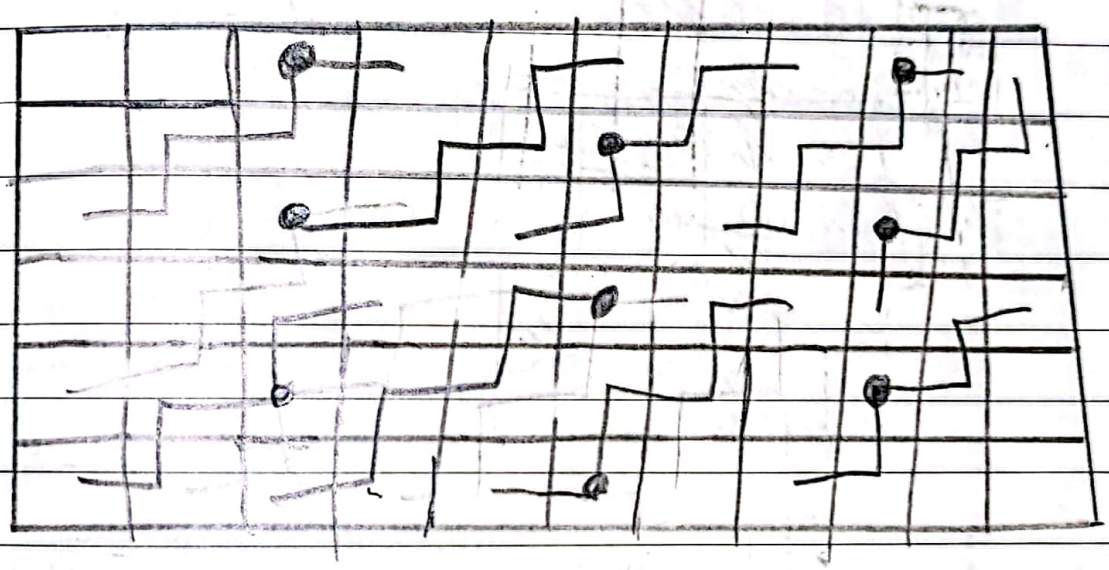
w. 1



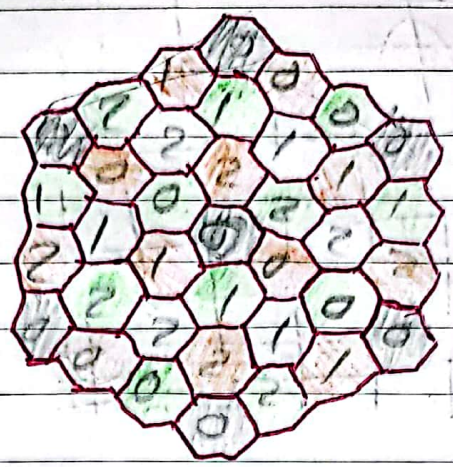




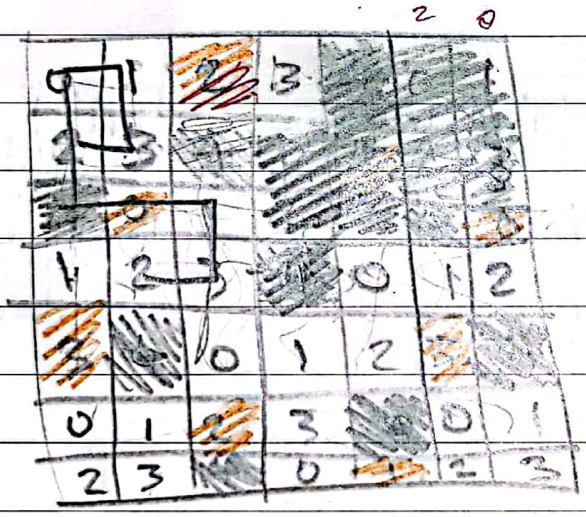
12



13

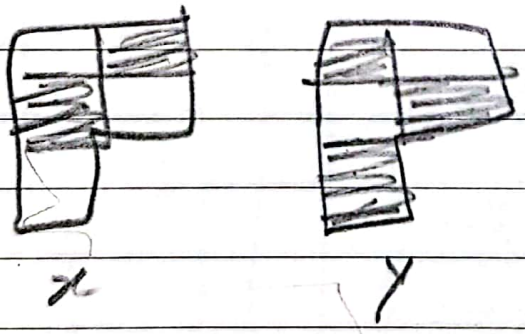


14



$1 \equiv 0 - x \; [5]$

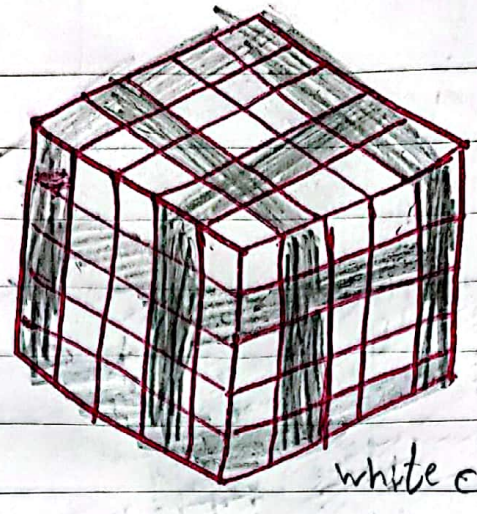
$x \equiv 4 \; [5]$

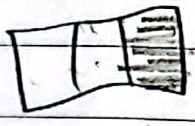


x y

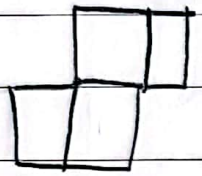
$\cancel{2x + 3y = 25}$

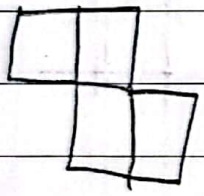
$\cancel{x + y = 10}$

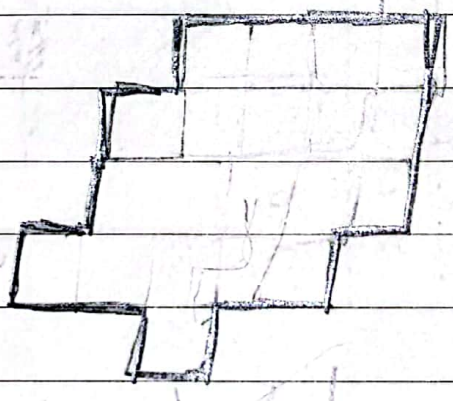




white cells is odd but in every 3x1 there is even number





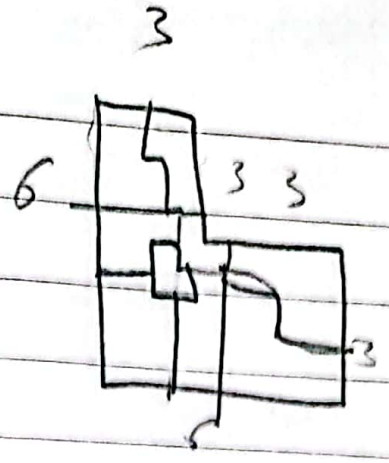


W.2

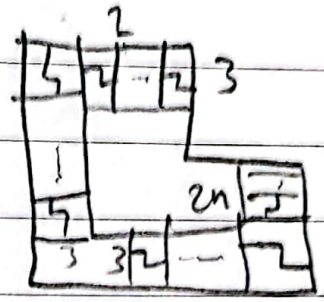
base

$n=1$

$n=3$



step $n \to 2n$ and $2n \to 2n+3$



by induction: You can claim every $n \geq 1$

Suppose you can for all $\leq k$

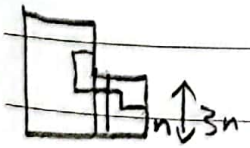
if $k+1$ is even $\frac{k+1}{2} \to k+1$ $\left(\frac{k+1}{2} \leq k\right)$

if $k+1$ is odd, $k-2 \to k+1$ $\forall$ all $k \geq 4$

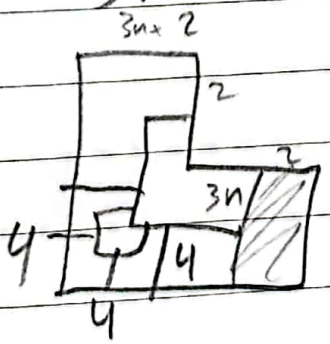
and we checked $k = 1, 3$

other way

$n \to 3n$



$3n \to 3n+2$



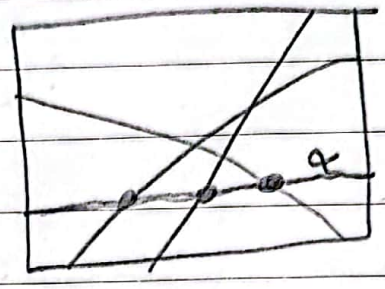
$3n+2 \to 3n+4$

badway

v.4) Ind.

base $n = 0$

ind step: assume k lines gives $\frac{k^2+k+2}{2}$ consider any $k+1$ lines and choose any line α and erase it then by ind. assumption there will be $\frac{k^2+k+2}{2}$ regions.



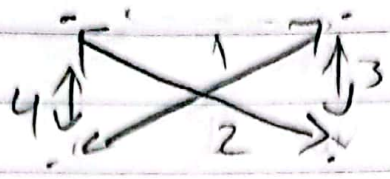
after adding α there will be k intersection with other lines so $k+1$ regions will be halved, so $k+1$ new regions created

$$\Rightarrow \frac{k^2+k+2}{2} + k+1 = \frac{k^2+3k+4}{2} = \frac{(k+1)^2+(k+1)+2}{2}$$

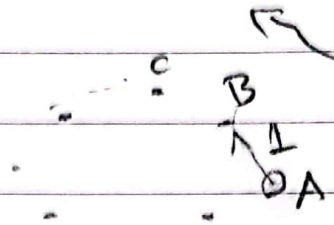
C end

17)

base $n=4$ 4 calls



ind. step each ~~n~~ can have $2n-4$ call, $n+1$ needs $2n-2$?

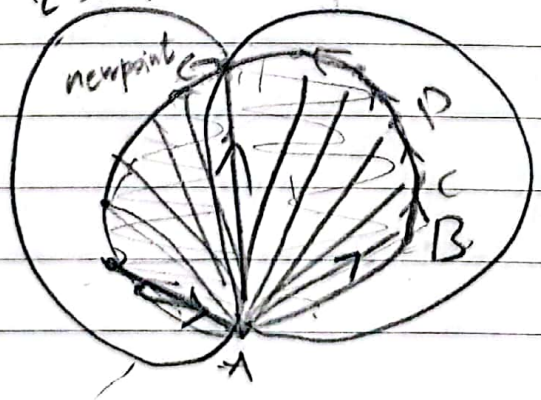


1. A calls B
2. every body without A share calls (n people) ⇒ every body have A
3. B calls A

18)

base $A \to B \to C \to A$

$A \to B, A \to C$

ind.

2 shapes $(n+1)$ ⇒ can make triangle