Email training, N4

Level 2, October 4-10

Problem 4.1. Find the maximum possible value of $x^6 + y^6$ if it's known that $x^2 + y^2 = 1$.

Problem 4.2. Let a, b, c, d be real numbers such that

$$a^4 + b^4 + c^4 + d^4 = 16.$$

Prove the inequality

$$a^5 + b^5 + c^5 + d^5 \leq 32.$$

Problem 4.3. Let $S(n)$ be the sum of divisors of n (for example $S(6) = 1+2+3+6 = 12$). Find all n for which $S(2n) = 3S(n)$.

Problem 4.4. Find all pairs of possitive integers (x, y) such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{lcm(x,y)} + \frac{1}{gcd(x,y)} = \frac{1}{2}.$$

Problem 4.5. Ali has choosen 8 cells of the chessboard 8×8 such that no any two lie on the same line or in the same row (we call it general configuration). On each step Baba chooses 8 cells in general configuration and puts coins on them. Then Ali shows all coins that are out of cells chosen by Ali. If Ali shows even number of coins then Baba wins, otherwise Baba removes all coins and makes the next move. Find the minimal number of moves that Baba needs to guarantee the win.

Problem 4.6. Let $1 \leq r \leq n$. We consider all r-element subsets of $(1, 2, \ldots, , n)$. Each of them has a minimum. Prove that the average of these minima is $\frac{n+1}{r+1}$.

Problem 4.7. Let G be the centroid of ΔABC. Draw a line $XY \parallel BC$ passing through G, intersecting AB, AC at X, Y respectively. Let BG and CX intersect at P, as well as CG and BY intersect at Q. Let M is the midpoint of BC. Prove that $\Delta ABC \sim \Delta MQP$.

Solution submission deadline October 10, 2021

Submit single PDF file in filename format L2_YOURNAME_week4.pdf

submission email **imo20etraining@gmail.com**