Email training, N4

October 2-8

Problem 4.1. Let a, b are solutions of equation $x^2 + 2x - 13 = 0$. Construct an equation which roots are $a + 1$, $b + 1$.

Problem 4.2. Factorize

$$a^3 + a - 2.$$

Problem 4.3. Solve equation in integers

$$x! + 13 = y^2.$$

Problem 4.4. Let numbers $x_1, x_2, \ldots, x_n$ are given and each of them is equal either $+1$ or -1. Prove that if

$$x_1x_2 + x_2x_3 + \ldots + x_nx_1 = 0$$

then n is divisible by 4.

Problem 4.5. Chess king has started from some cell and by passing over each cell exactly ones came back to original position. Prove that the king has done even number of diagonal moves.

Problem 4.6. Let k is given and numbers from 1 to 100 are written on the board. Ali erases from the board arbitrary k numbers. Is it true that Bob may choose k numbers written on the board, which sum is equal to 100. Consider cases when a) $k = 8$, b) $k = 9$.

Problem 4.7. -

لدينا $VDEF$ فيه EF ^ DH حيث تقع على المستقيم EF . أوجد الفرق بين أكبر وأقل قيمة ممكنة

لمساحة المثلث $VDEF$ ، إذا كان $DH = 21, DF = 35, DE = 60$.

Problem 4.8. -

لدينا $VABC$ فيه $AB =$ [illegible] AC. O هي مركز الدائرة المحيطة له. النقطة M هي منتصف الضلع BC النقطة N M AC . النقطة T تقع في نفس مستوى الدائرة بحيث يكون الشكل $ANBT$ م

$$\angle OMT = \frac{1}{2}\angle BAC$$

Solution submission deadline October 8, 2022