Email training, N4

October 2-8

Problem 4.1. Let a, b, c are solutions of equation $x^3 + x^2 - 3x - 1 = 0$. Construct an equation which roots are $a + 1$, $b + 1$ and $c + 1$.

Problem 4.2. Let a, b and c are pairwise different numbers. Solve the system of equations

$$\begin{cases} z + ay + a^2x + a^3 = 0 \\ z + by + b^2x + b^3 = 0 \\ z + cy + c^2x + c^3 = 0. \end{cases}$$

Problem 4.3. Solve equation in integers

$$x! + 13 = y^2.$$

Problem 4.4. Let numbers $x_1, x_2, \ldots, x_n$ are given and each of them is equal either $+1$ or -1. Prove that if

$$x_1x_2 + x_2x_3 + \ldots + x_nx_1 = 0$$

then n is divisible by 4.

Problem 4.5. Chess king has started from some cell and by passing over each cell exactly ones came back to original position. Prove that the king has done even number of diagonal moves.

Problem 4.6. Let k is given and numbers from 1 to 100 are written on the board. Ali erases from the board arbitrary k numbers. Is it true that Bob may choose k numbers written on the board, which sum is equal to 100. Consider cases when a) $k = 8$, b) $k = 9$.

Problem 4.7. . Let ABCD be a quadrangle, $|AD| = |BC|$, $\angle A + \angle B = 120^o$ and let P be a point exterior to the quadrangle such that P and A lie at opposite sides of the line DC and the triangle DPC is equilateral. Prove that the triangle APB is also equilateral.

Problem 4.8. In triangle ABC, $\angle A = 96^o$. Extend BC to an arbitrary point D. The angle bisectors of angle ABC and ACD intersect at A_1, and the angle bisectors of A_1BC and A_1CD intersect at A_2, and so on. The angle bisectors of A_4BC and A_4CD intersect at A_5 . Find the size of $\angle A_5$ in degrees.

Solution submission deadline October 8, 2022