Saudi Arabia 2022 – Math Camp

Extra - Level 4+

Combinatorics

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1. There are 30 teams in NBA and every team play 82 games in the year. Bosses of NBA want to divide all teams on Western and Eastern Conferences (not necessary equally), such that number of games between teams from different conferences is half of number of all games. Can they do it?

2. A field has a shape of checkboard 41×41 square. A tank concealed in one of the cells of the field. By one shot, a fighter airplane fires one of the cells. If a shot hits the tank, then the tank moves to a neighboring cell of the field, otherwise it stays in its cell (the cells are neighbors if they share a side). A pilot has no information about the tank, one needs to hit it twice. Find the least number of shots sufficient to destroy the tank for sure.

3. A square is partitioned in $n^2 \geq 4$ rectangles using $2(n-1)$ lines, $n-1$ of which,are parallel to the one side of the square, $n-1$ are parallel to the other side. Prove that we can choose $2n$ rectangles of the partition,such that,for each two of them, we can place the one inside the other (possibly with rotation).

4. There are $n > 1$ cities in the country, some pairs of cities linked two-way through straight flight. For every pair of cities there is exactly one aviaroute (can have interchanges).
Major of every city X counted amount of such numberings of all cities from 1 to n, such that on every aviaroute with the beginning in X, numbers of cities are in ascending order. Every major, except one, noticed that results of counting are multiple of 2016. Prove, that result of last major is multiple of 2016 too.

5. 110 teams participated in a volleyball tournament; every two teams played against each other exactly one game (in volleyball, there are no draws). It happened that in every group of 55 teams there exists a team which has lost to at most four out of the other 54 teams in this group. Prove that in the whole tournament there exists a in this group. Prove that in the whole tournament there exists a in this group. Prove that in the whole tournament there exists a team which has lost to at most four out of the other 109 teams.

6. A point X is chosen within a convex 100–agon such that X is neither on the sides nor on the diagonals of the polygon. Initially the vertices of the polygon are not marked. Pete and Bazil alternately mark the vertices in turns. Pete starts and marks 2 vertices on his first turn. Then, on each turn, each player marks a vertex. If, after one of the players moves, point X appears inside a polygon with all vertices marked, then that player loses the game. Prove that Pete has a winning strategy.