15

$a, b, c, d \in \mathbb{R}_+$ s.t. $a, c > 1$

$b, d < 1$

$$\boxed{\frac{a}{ab+c+1} + \frac{b}{bc+d+1} + \frac{c}{cd+a+1} + \frac{d}{da+b+1} > 1.}$$

$a > 1$

$b < 1$ $\rightsquigarrow$ $(a-1)(b-1) < 0$

$$\boxed{ab+1 < a+b}$$

$ab+1$

$$+\begin{cases} \dfrac{a}{ab+c+1} > \dfrac{a}{a+b+c} > \dfrac{a}{a+b+c+d} \\[2ex] \dfrac{b}{bc+d+1} > \dfrac{b}{a+b+c+d} \\[2ex] \dfrac{c}{cd+a+1} > \dfrac{c}{a+b+c+d} \\[2ex] \dfrac{d}{da+b+1} > \dfrac{d}{a+b+c+d} \end{cases}$$

$$\frac{a}{ab+c+1} + \frac{b}{bc+d+1} + \frac{c}{cd+a+1} + \frac{d}{da+b+1} >$$

$$> \frac{a+b+c+d}{a+b+c+d} = 1.$$

$\boxed{16}$ $F, G, H \in \mathbb{R}[x] \leftarrow$ real coeff.

$\deg(F), \deg(G), \deg(H) \leq 2n+1$

$\downarrow$ odd mult

i) $F(x) \leq G(x) \leq H(x) \quad \forall x \in \mathbb{R}$

ii) $\exists \; x_1, x_2, \dots, x_n$ distinct

$\boxed{F(x_i) = H(x_i)} \quad \forall i \in \{1 \dots n\}$.

iii) $\exists \; x_0 \notin \{x_1 \dots, x_n\}$:

$$F(x_0) + H(x_0) = 2G(x_0)$$

Prove:

$$F(x) + H(x) = 2G(x) \quad \forall x \in \mathbb{R}$$

Proof

$$H(x) - G(x) \geq 0 \quad \forall x$$

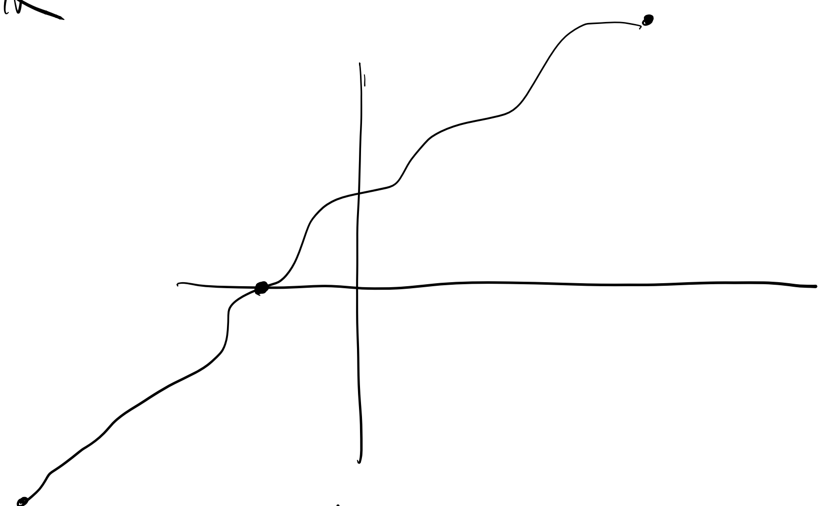
$$G(x) - F(x) \geq 0 \quad \forall x$$

$\deg H-G$, $\deg G-F$ is even $\leq 2n$

GENERAL Q.

What can we say about degree of nonnegative poly?

$$\forall_{x \in \mathbb{R}} \quad P(x) \geq 0 \qquad P(x) \approx A x^{2n+1}$$



$\Downarrow$

deg P = even.

$$\boxed{\deg(F+H-2G) \overset{\deg}{=} (F-G+H-G) \leq 2n}$$

$(H-G)((x_i)) = 0$

$(H-G)$ take non negative values.

x_i - root, what is multiplicity of x_i?

$H(x_i) - G(x_i))$

$$H \geq G \geq F$$

$$H(x_i) = F(x_i) = G(x_i)$$

x_i - root of $H-G$

$F-G$

$$P(x) = (x-2)^1(x-3)^1(x-5)^2$$

if $P(x) = (x-x_0)^{\boxed{k}} \cdot G(x)$

$$G(x_0) \neq 0$$

$$mult_{x_0}^{P}(P) = k$$

$\underline{x^2-2x+1}$ $\underline{(x-1)^2}$

so $\text{mult}_{x_i}(H-G)$ – even
$F-G$

. . . .

- $H-G \geq 0$
  $G-F \geq 0$ $\implies$ $\deg(H-G,\ G-F)$ even

<u>Fakt</u> Poly of odd degree takes negative value!

- $(F-H)(x_i) = 0$

  But $F \leq G \leq H$

  $F(x_i) = G(x_i) = H(x_i)$

so x_i are roots of $G-F$ and $H-G$

n – roots.

- $\deg(G-F)$ $\leq 2n+1$
  $\deg(H-G)$

  becos $\deg G, F, H \leq 2n+1$

  but are even so

  $\deg(G-F), \deg(H-G) \leq 2n$!

- $G-F$ have roots $x_1, x_2, \dots, x_n$
  $H-G$

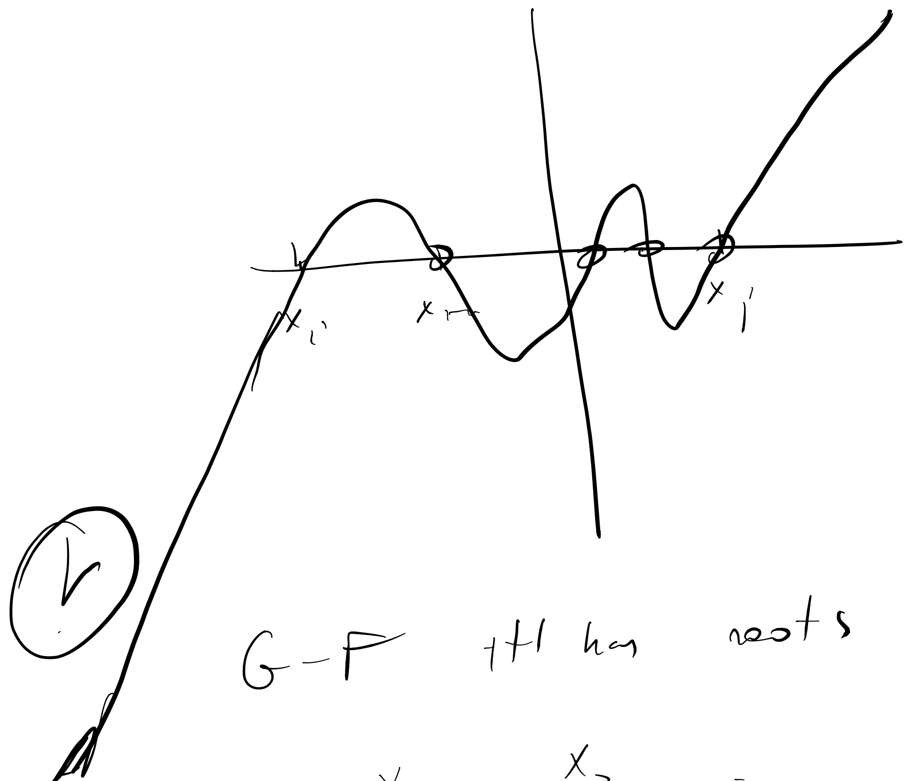
and multiplicity of x_i is even
why?

if not

$$(G-F) = (x-x_1)^{2k+1} \cdot Z(x)$$

$$(G-F) = (x-x_1)^{\alpha_1} \cdot \dots \cdot (x-x_n)^{\alpha_n} =$$

$$= (x-x_i) \cdot \dots \cdot (x-x_j) \cdot (\text{Positive poly})$$



$G-F$ $t+l$ has roots

$x_1, \quad x_2 \quad - \quad - \quad , \quad x_4$

$\geq 2 \quad \geq 2 \quad \geq 2$

total degree $\leq 2n$

$x_1 \; - \; - \; - \; x_4$

$\Downarrow$

$2 \qquad 2$

x_i is double root of $G-F$

$H-G$

$F+H-2G = (F-G)+(H-G)$

deg $\leq 2n$

x_i is double root of $F+H-2G$

degree $\leq 2n$

$\exists\ x_0$ differ. root of $\boxed{F+H-2G \equiv 0}$

Poly with more roots than degree is 0 !

17, 18 at Home,