Email training, N12

December 8-14, 2019

Problem 13.1. Find all pairs (x, y) of non-negative integers such that $x^2 + 3y$ and $y^2 + 3x$ are simultaneously perfect squares.

Problem 13.2. Prove that for any sequence of positive integers $a_1, a_2, \ldots, a_n$ there exists a positive integer k, for which

$$s(ka_1) = s(ka_2) \ldots = s(ka_n),$$

where $s(m)$ is the sum of digits of m.

Problem 13.3. Find all real solutions (a, b, c, d) to the equations

$$\begin{cases} a + b + c = d, \\ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{d}. \end{cases}$$

Problem 13.4. Let $f\ : N \to N$ is a strictly increasing $(f(n+1) > f(n))$ function for which

$$f(f(n)) = 3f(n).$$

Evaluate $f(2017)$.

Problem 13.5. On birthday party several married couples came, each of them with from 1 to 10 children. The total number of triples { *father, mother, kid*}, where no two of them are from the same family is exactly 3630. Find the number of children participating to the party.

Problem 13.6. Let 20 black and 20 white balls are placed among the line. On each step player is allowed to swap 2 neighbor balls. Find the minimal number of moves player needs to guarantee position with 20 consecutive black balls.

Solution submission deadline December 14, 2019