# Email training, N11
November 26 - December 7, 2019

**Problem 10.1.** Construct a polynomial with integer coefficients with root $\sqrt{2} + \sqrt{3}$.

**Problem 10.2.** Find the coefficient of $x^{100}$ after opening the brackets and grouping terms of $(1 + x + x^2 + \ldots + x^{100})^3$.

**Problem 10.3.** Five numbers are written on the board. Mohammed calculates the sum of all pairs and gets the results 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121. Find the numbers written on the board.

**Problem 10.4.** Find all positive integers $m$ and $n$ for which $1! + 2! + 3! + \cdots + n! = m^2$.

**Problem 10.5.** Let $a$, $b$, $c$, be a positive integer such that $a^2 + b^2 = c^2$. Prove that $\frac{1}{2}(c - a)(c - b)$ is a perfect square.

**Problem 10.6.** Are there exists integers $a, b$ such that $a^2 + b$ and $a + b^2$ are consecutive integers?

**Problem 10.7.** Let numbers 1, 2, 3, ..., 19, 20 are written on the board. At each step one may erase any two numbers $a$ and $b$ and write the number $a + b$–1. Which number will be written on the board after 19 steps.

**Problem 10.8.** Let 100 points are drawn on the plane such that the distance between 2 any points is less than 1. Also it's known that for any three points $A$, $B$ and $C$ the triangle $ABC$ is not acute. Prove that there exists a circle of radius 0.5 which contains in it's interior all 100 points.

**Problem 10.9.** 10 players participate to chess tournament. Each day they play 5 games - one game each. After 9 days all chess players have been played with each other. It occurs that the most of games are played between the players from Jeddah. Prove that every day at least two players from Jeddah have been played with each other.

**Problem 10.10.** The numbers 1, 1 are written on the board. At each step between two neighbor numbers Aziz writes their sum. Below are the results after first three steps.

$$1, 2, 1$$
$$1, 3, 2, 3, 1$$
$$1, 4, 3, 5, 2, 5, 3, 4, 1$$

Find the total sum of numbers written after 100-th step.

Solution submission deadline December 7, 2019