1. Find in terms of n short form of the sum $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n!$.

2. Find in terms of n short form of the sum $\displaystyle\sum_{i=1}^{n} \frac{1}{(3i-1)(3i+2)}$.

3. Find in terms of n short form of the sum $\displaystyle\sum_{i=1}^{n} \frac{1}{i(i+1)(i+2)}$.

4. Sequence $a_1, a_2, a_3, \ldots,$ satisfies $a_1 = \frac{1}{2}$ and

$$a_1 + a_2 + \ldots + a_n = n^2 a_n \quad \text{for } n = 1, 2, 3, \ldots.$$

Compute a_n for all $n = 1, 2, 3, \ldots$.

5. Sequence $a_0, a_1, a_2, \ldots$ satisfies $a_0 = 1$, $a_1 = 2$ and

$$n(n+1)a_{n+1} = n(n-1)a_n - (n-2)a_{n-1} \quad \text{for all } n \geqslant 1.$$

Compute sum $\frac{a_0}{a_1} + \frac{a_1}{a_2} + \ldots + \frac{a_{50}}{a_{51}}$.

6. Seqeunce a_n of positive numbers satisfies for each natural n inequality $a_n^2 \leqslant a_n - a_{n+1}$. Prove that for each natural n holds $a_n \leqslant \frac{1}{n}$.

7. Sequence $a_0, a_1, \ldots, a_n$ satisfies

$$a_0 = \frac{1}{2} \quad \text{and} \quad a_k = a_{k-1} + \frac{a_{k-1}^2}{n} \quad \text{for } k = 1, 2, \ldots, n.$$

Prove that

a) $a_n < 1$;

b) $1 - \frac{1}{n} < a_n$.

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