Saudi Arabia 2022 – Math Camp

Day 4 (Part 2) - Level 4+

Geometry - Miscellaneous problems

Instructor: Regis Barbosa

1. (EGMO/2016) Two circles, ω_1 and ω_2, of equal radius intersect at different points X_1 and X_2. Consider a circle ω externally tangent to ω_1 at a point T_1, and internally tangent to ω_2 at a point T_2. Prove that lines X_1T_1 and X_2T_2 intersect at a point lying on ω.

2. (Brazil/2009) Let ABC be a triangle and O its circumcenter. The lines AB and AC intersect the circumcircle of OBC again at points $B_1 \neq B$ and $C_1 \neq C$, respectively. The lines BA and BC intersect the circumcircle of OAC again at points $A_2 \neq A$ and $C_2 \neq C$, respectively. The lines CA and CB intersect the circumcircle of OAB again at points $A_3 \neq A$ and $B_3 \neq B$, respectively. Prove that the lines A_2A_3, B_1B_3 and C_1C_2 have a common point.

/ 3. (IMO Shortlist/2012) In an acute triangle ABC the points D, E and F are the feet of the altitudes through A, B and C respectively. The incenters of the triangles AEF and BDF are I_1 and I_2 respectively; the circumcenters of the triangles ACI_1 and BCI_2 are O_1 and O_2 respectively. Prove that I_1I_2 and O_1O_2 are parallel.

/ 4. (USA TST/2006) Let ABC be a triangle. Triangles PAB and QAC are constructed outside of triangle ABC such that $AP = AB$ and $AQ = AC$ and $\angle BAP = \angle CAQ$. Segments BQ and CP meet at R. Let O be the circumcenter of triangle BCR. Prove that $AO \perp PQ$.