Problems Set

27 June, 2020

23. Let p be a prime. Show that there are infinitely many positive integers n such that p divides $2^n - n$.

24. Let n be an integer greater than three. Prove that $1! + 2! + \cdots + n!$ cannot be a perfect power.

25. Let k be an odd positive integer. Prove that

$$(1 + 2 + \cdots + n) \mid (1^k + 2^k + \cdots + n^k)$$

for all positive integers n.

26. Let p be a prime greater than 5. Prove that $p - 4$ cannot be the fourth power of an integer.

27. For a positive integer n, prove that

$$\sigma(1) + \sigma(2) + \cdots + \sigma(n) \leq n^2.$$

28. Determine all finite nonempty sets S of positive integers satisfying

$$\frac{i + j}{\gcd(i, j)}$$

is an element of S for all i and j (not necessarily distinct) in S.

29. Knowing that 2^{29} is a nine-digit number all of whose digits are distinct, without computing the actual number determine which of the ten digits is missing. Justify your answer.

30. Prove that for any integer n greater than 1, the number $n^5 + n^4 + 1$ is composite.

31. The product of a few primes is ten times as much as the sum of the primes. What are these (not necessarily distinct) primes?

32. A 10-digit number is said to be *interesting* if its digits are all distinct and it is a multiple of 11111. How many interesting integers are there?