Polynomials 1 (L3)

1. (Revision) Let p be a prime number. Determine whether $3^p + 7p - 4$ can be a perfect square.

2. Let P be a polynomial. It is given that the equation $P(x) = x$ has no real solutions. Is it possible that the equation $P(P(x)) = x$ has a solution?

3. Let P be a polynomial and $P(0) = P(1)$. Show that there are two points A and B on the graph of P such that line segment AB is horizontal and $AB = \frac{1}{4}$.

4. Let P be a polynomial such that $P(x) \in \mathbb{Z}$ for every $x \in \mathbb{Z}$. Determine whether every coefficient of P is integer.

5. Let P be a polynomial such that $P(x) \in \mathbb{Q}$ for every $x \in \mathbb{Q}$. Determine whether P can have irrational coefficients.

6. A polynomial P of the nth degree has n distinct real roots. Find what the maximum possible number of its coefficients can be zeros.