**Problem 2.1.** Let $x_1$ and $x_2$ are the roots of the equation $x^2 + 5x - 11$. Find a quadratic polynomial which roots are $x_1x_2$ and $x_1^2x_2^2$.

**Solution 2.1.** Since $x_1$ and $x_2$ are solution of $x^2 + 5x - 11 = 0$ the, according to Vieta theorem one has $x_1 + x_2 = -5$ and $x_1x_2 = -11$. If $x_1x_2$ and $x_1^2x_2^2$ are solution of $x^2 + ax + b = 0$ then $-a = x_1x_2 + x_1^2x_2^2 = x_1x_2(x_1x_2 + 1) = -110$ and $b = x_1^3x_2^3 = -1331$. So

$$x^2 - 110x - 1331 = 0$$

**Answer:** $x^2 - 110x - 1331 = 0$.

**Problem 2.2.** Simplify

$$\frac{\sqrt{2} + \sqrt{6}}{\sqrt{2 + \sqrt{3}}}.$$

**Solution 2.2.**

$$\frac{\sqrt{2} + \sqrt{6}}{\sqrt{2 + \sqrt{3}}} = \frac{\sqrt{2} + \sqrt{6}}{\sqrt{2 + \sqrt{3}}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2 + 2\sqrt{3}}{\sqrt{4 + 2\sqrt{3}}}$$

$$= \frac{2(1 + \sqrt{3})}{\sqrt{1 + 2\sqrt{3} + 3}} = \frac{2(1 + \sqrt{3})}{1 + \sqrt{3}} = 2:$$

**Problem 2.3.** Find all positive integers $n$ for which $n^2 + 3n$ is perfect square.

**Solution 2.3.** Note that

$$n^2 < n^2 + 3n < n^2 + 4n + 4 = (n + 2)^2.$$

Since $n^2 + 3n$ is perfect square, then it is equal to $(n + 1)^2$.

$$n^2 + 3n = n^2 + 2n + 1,$$

$$n = 1.$$

**Answer:** $n = 1$.

**Problem 2.4.** Find all integer solutions to the equation

$$x^2 - 6xy + 13y^2 = 100.$$

**Solution 2.4.** Rewrite the equation in the following form

$$x^2 - 6xy + 13y^2 = (x - 3y)^2 + (2y)^2 = 100.$$

Since 100 can be written as sum of squares in the following ways

$$100 = 8^2 + 6^2 = 10^2 + 0^2$$

, therefore we have the following options

$$x - 3y = 8, 2y = 6$$
$$x - 3y = -8, 2y = 6$$
$$x - 3y = 6, 2y = 8$$
$$x - 3y = -6, 2y = -8$$
$$x - 3y = 10, 2y = 0$$
$$x - 3y = -10, 2y = 0$$
$$x - 3y = 0, 2y = 10$$

$$x - 3y = 0, 2y = -10$$

From these cases we get solutions

**Answer:** $(17, 3), (1, 3), (18, 4), (-18, -4), (10, 0), (-10, 0), (15, 5), (-15, -5)$.

**Problem 2.5.** Find the number of 7-digit positive integers that all digits are ordered in
a) strictly increasing order,
b) strictly decreasing order.

**Solution 2.5.** a) part is equivalent to write expression 123456789 and remove any 2 digits. It can be done in $\binom{9}{2}$ ways.

b) part is equivalent to write expression 9876543210 and remove any 3 digits. It can be done in $\binom{10}{3}$ ways.

**Answer:** a) $\binom{9}{2}$, b) $\binom{10}{3}$.

**Problem 2.6.** A triple $(1, 1, 1)$ is given. On each step one chooses 2 of them and increases by 1. Is it possible after some steps get numbers $(2016, 2016, 2016)$.

**Solution 2.6.** Note that the sum of written numbers is equal $1 + 1 + 1 = 3$. After first step the total sum will be equal $3 + 2 = 5$, after second step $5 + 2 = 7$ and so on, after $n$th step the total sum will be $3 + 2n$ which is odd. However, at the end we want to have 3 numbers which equal 2016, so their sum must be $3 \cdot 2016 = 6048$ which is even. We have already shown that the sum must be always odd.

**Answer:** Not possible.