— Combinatorics for L3 —

— January Camp, 2022 — Graph Theory (3): Finding Cycles —

Warm-up.

- Prove that if $|V(G)| \geq 6$, then in G or $\overline{G}$ there is a triangle.
- Prove that a graph of minimum degree $\delta \geq 2$ has a cycle of length at least $\delta + 1$.
- Prove that if a simple graph on $2n$ vertices has more than n^2 edges, then it contains a triangle.

13. Graph G with at least 5 vertices has the following properties: every vertex has at least one non-neighbor and every two non-neighbors have exactly one common neighbor. Prove that G contains an induced 5 cycle on 5 vertices (i.e. a cycle whose diagonals are not edges).

14. Given is a simple n-vertex graph G with $n \geq 3$ with the following property: the sum of degrees in every pair of non-neighbors is at least n. Prove that in G there is a cycle of length n.

15. Given is an n-vertex graph G with $n \geq 5$ such that the complement graph $\overline{G}$ is triangle-free. Prove that in G there is a cycle of length at least $n/2$.

16. Given is an $1 \times n$ board with some cells colored black. We call a rectangle consisting of whole cells *odd* if it contains an odd number of black cells. Determine the largest possible number of odd rectangles.