Competition Preparation for Saudi Arabia Team 2021: Level 4

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Homework: Week 4

Problems:

1. (Easy) In a table tennis tournament with $2n + 1$ contestants, each pair of contestants played each other exactly once and none of the games resulted in a tie. We will call a triangle any collection of 3 contestants A, B and C, such that A defeated B, B defeated C and C defeated A. Let T be the number of triangles that occurred on the tournament. (Two different triangles may overlap in zero, one or two contestants.)

   (a) Find the smallest possible value of T.

   (b) Find the largest possible value of T.

2. (Medium) A unit square is partitioned into triangles. Prove that the sum of the perimeters of the triangles is at least $4 + 2\sqrt{2}$. When does equality hold?

3. (Difficult) On a 2022×2022 board we place on each square a coin. In each move we remove a coin which is adjacent to a positive even number of coins as long as at least one such coin exists. Find k, the lowest possible number of coins we can obtain and prove that if we end up with k coins then no two coins are adjacent. A square is adjacent to another square if it has at least one common vertex. (In the class we solved the problem for a 2021×2021 board).