Problem 1A. Determine all functions $f : \mathbb{Q}^+ \to \mathbb{Q}^+$ satisfying

$$f(x^2 f(y^2)) = f(x)^2 f(y)$$

for all $x, y \in \mathbb{Q}^+$ (here $\mathbb{Q}^+$ denotes the set of all positive rational numbers).

Problem 2A. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x + f(y) + f(f(z))) = yf\Big(1 + f(f(y))(x + z)\Big)$$

for all $x, y \in \mathbb{R}^+$ (here $\mathbb{R}^+$ denotes the set of all positive real numbers).

Problem 3A. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$xf(x^2)f(f(y)) + f(yf(x)) = f(xy)\Big(f(f(x^2)) + f(f(y^2))\Big)$$

for all $x, y \in \mathbb{R}^+$ (here $\mathbb{R}^+$ denotes the set of all positive real numbers).

Problem 4A. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x)f(y) = f(y)f(xf(y)) + \frac{1}{xy}$$

for all $x, y \in \mathbb{R}^+$ (here $\mathbb{R}^+$ denotes the set of all positive real numbers).

Problem 5A. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$(f(x) + f(z))(f(y) + f(t)) = f(xy - zt) + f(xt + yz)$$

for all $x, y, z, t \in \mathbb{R}$.

Problem 6A. Let $f : \mathbb{R} \to \mathbb{R}$ be a non-constant function that satisfies

$$f(x)f(x - y) + f(y)f(x + y) = f(x)^2 + f(y)^2 \quad \text{for all } x, y \in \mathbb{R}.$$

Prove that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

Problem 7A. Determine all functions $g : \mathbb{R} \to \mathbb{R}$ satisfying

$$g(x + y) + g(x)g(y) = g(xy) + g(x) + g(y)$$

for all $x, y \in \mathbb{R}$.

Problem 8A. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x^3 + y^3) = (x + y)(f(x^2) - f(x)f(y) + f(y^2))$$

for all $x, y \in \mathbb{R}$. Prove that $f(2021x) = 2021f(x)$ for all $x \in \mathbb{R}$.