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$c \in \mathbb{R}$

$$P(x) = x^5 - 5x^3 + 4x - c$$

has five distinct real roots:

$$x_1, x_2, \dots, x_5$$

Compute sum of absolute values of coeff's of the polynomial:

$$Q(x) = (x - x_1^2)(x - x_2^2) \cdots (x - x_5^2)$$

$$\begin{aligned} \text{Hint } Q(x^2) &= (x^2 - x_1^2)(x^2 - x_2^2) \cdots (x^2 - x_5^2) = \\ &= (x - x_1)(x + x_1)(x - x_2)(x + x_2) \cdots (x - x_5)(x + x_5) \\ &= \underbrace{(x - x_1)(x - x_2) \cdots (x - x_5)}_{P(x)} \cdot \end{aligned}$$

$$\begin{aligned} &\underbrace{(x + x_1)(x + x_2) \cdots (x + x_5)}_{= -P(-x)} = \\ &= \end{aligned}$$

$$= \boxed{-P(x) - P(-x)}$$

$$P(x) = x^5 - 5x^3 + 4x - c$$

$$= (x^5 - 5x^3 + 4x - c) \cdot (-x^5 + 5x^3 - 4x - c)$$

$$= (x^5 - 5x^3 + 4x - c)(x^5 - 5x^3 + 4x + c) =$$

$$= (x^5 - 5x^3 + 4x)^2 - c^2$$

$$= x^{10} - 10x^8 + 33x^6 - 40x^4 + 16x^2 - c^2$$

$$Q(x) = \Downarrow x^5 - 10x^4 + 33x^3 - 40x^2 + 16x - c$$

$$1 + 10 + 33 + 40 + 16 + c^2 = 100 + c^2$$

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$$x^n \pm x^{n-1} \pm \dots \pm x \pm 1$$

All roots are real?

r_1, r_2, \dots, r_n - are roots

$$\sum_{i=1}^n r_i^2 = -1 \text{ or } 3$$

$$\sum_{i=1}^n r_i = ?$$

$$\sum_{i=1}^n r_i^2 = 3$$

$$\sum_{i=1}^n r_i r_j = -1$$

! AM-GM

$$\sum_{i=1}^n r_i^2 = \left(\sum_{i=1}^n r_i \right)^2 - 2 \sum_{i=1}^n r_i r_j$$

$$1 \pm 2$$

$$\frac{3}{n} = \frac{\sum_{i=1}^n r_i^2}{n} \Rightarrow \sqrt[n]{r_1^2 r_2^2 \dots r_n^2} = ? \underline{1}$$

(+1)²

$$n \leq 3$$

$$F_n = \boxed{n=3}$$

$$r_1^2 = r_2^2 = r_3^2$$

$$\frac{(x-1)(x+1)(x+1)}{(x-1)(x-1)(x+1)}$$

$$\textcircled{n=2}$$

$$\frac{x^2 \pm x - 1}{x \pm 1} \rightsquigarrow$$

□

$$n=1$$

$$x \pm 1$$

$$P(x) = x^n - 2nx^{n-1} + 2n(n-1)x^{n-2} + \dots + a_0$$

has only real roots.

Find these roots

only these are fixed

$$\sum_1^n (x_i - 1)^2 = 4n - 4n + n = n$$

$$r_1, r_2, \dots, r_n \text{ — roots}$$

$$\sum_1^n r_i = 2n$$

$$\sum_1^n r_i r_j = 2n(n-1)$$

$$\sum_1^n r_i^2 = \left(\sum_1^n r_i \right)^2 - 2 \sum_1^n r_i r_j =$$

$$= (2n)^2 - 2 \cdot 2n(n-1) = 4n$$

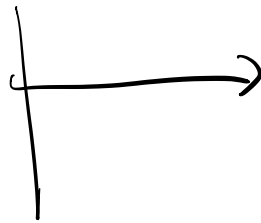
$$\sum_{i=1}^n (r_i - 2)^2 = \sum_{i=1}^n r_i^2 - 4 \sum_{i=1}^n r_i + n \cdot 4$$

$$= \underline{4n - 4 \cdot 2n + 4n = 0}$$

\Downarrow
 all $r_i = 2$

Last Vict Ex

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'regul'
 $f: \mathbb{N} \rightarrow \mathbb{N}$