Competition Preparation for Saudi Arabia Team 2021: Level 4

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Homework: Week 2

Problems:

1. (Easy) Several square napkins of equal size are laid out on a rectangular table such that their edges are parallel to the edges of the table. It is possible for the napkins to overlap with each other. Show that for each positive integer n we can hammer some nails into the table such that each napkin is nailed into the table exactly n times.

2. (Medium) Cover the following sets with mutually disjoint line segments (Open sets are denoted with '(' and ')' and closed sets are denoted with '[' and ']'):

   (a) Closed triangular area without one point in the interior: $[ABC] \setminus E,\ E \in (ABC)$.

   (b) Closed triangle: $[ABC]$.

   (c) Closed n-gon: $[A_1A_2 \dots A_n]$.

   (d) Closed circle: $[k]$.

   (e) Open triangular area: (ABC).

   (f) Open n-gon: $(A_1A_2 \dots A_n)$.

   (g) Open circular area: (k).

3. (Difficult) A simple graph with $n > 3$ vertices (points) such that no vertex has a degree of $n-1$ is called a *good* graph if for every pair of vertices, connected of not, there exists a unique vertex connected to both of them.

   (a) If two vertices on a good graph are not connected, prove that they have the same degree.

   (b) Can there be a good graph with 2021 vertices?

   (A simple graph is one such that each pair of vertices is connected at most once, and no vertex is connected to itself. A degree of a vertex is the number of vertices connected to this vertex.)