STAT215: Assignment 4

Your Name

Due: March 13, 2020

Problem 1: Consider a Gaussian linear dynamical system (LDS),

$$p(x_{1:T}, y_{1:T}) = \mathcal{N}(x_1 \mid 0, q^2) \left[\prod_{t=2}^{T} \mathcal{N}(x_t \mid a x_{t-1} + b, q^2) \right] \left[\prod_{t=1}^{T} \mathcal{N}(y_t \mid x_t, r^2) \right],$$

for $x_t, y_t \in \mathbb{R}$ for all t, and parameters $a, b \in \mathbb{R}$ and $q^2, r^2 \in \mathbb{R}_+$. Compute the forward filtered distribution $p(x_t \mid y_{1:t})$ in terms of the model parameters and the filtered distribution $p(x_{t-1} \mid y_{1:t-1})$. Solve for the base case $p(x_1 \mid y_1)$. For reference, consult the state space modeling chapters of either the Bishop or the Murphy textbook.

Your solution here.

Problem 2: Sample a time series of length $T = 30$ from the Gaussian LDS in Problem 1 with parameters $a = 1, b = 0, q = 0.1, r = 0.3$. Plot the sample of $x_{1:T}$ as a solid line, and plot the observed $y_{1:T}$ as +'s. Write code to compute the filtered distribution $p(x_t \mid y_{1:t})$ you derived in Problem 1. Then plot the mean of the filtered distribution $\mathbb{E}[x_t \mid y_{1:t}]$ over time as a solid line, and plot a shaded region encompassing the mean ± 2 standard deviations of the filtered distribution. All plots should be on the same axis. Include a legend.

Write your code in a Colab notebook and include a PDF printout of your notebook as well as the raw .ipynb file.

Problem 3: Reproduce Figure 2.5 of Rasmussen and Williams, *Gaussian Processes for Machine Learning*, available at http://www.gaussianprocess.org/gpml/chapters/RW2.pdf. Use a randomly generated dataset as described in the figure caption and surrounding text.

Write your code in a Colab notebook and include a PDF printout of your notebook as well as the raw .ipynb file.