IIT Madras

ONLINE DEGREE

Statistics for Data Science -1

Lecture 3.5: Describing Numerical Data- Percentiles

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Percentiles

- The sample $100p$ percentile is that data value having the property that at least $100p$ percent of the data are less than or equal to it and at least $100(1-p)$ percent of the data values are greater than or equal to it.

[1] Figure source: Mann, P. S. (2007). Introductory statistics. John Wiley & Sons.

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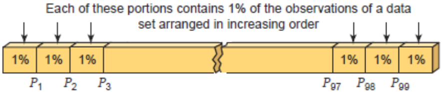

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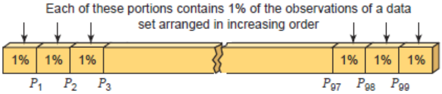

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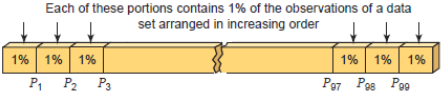

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- If two data values satisfy this condition, then the sample $100p$ percentile is the arithmetic average of these values.
- Median is the 50^{th} percentile.

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2. If np is not an integer, determine the smallest integer greater than np. The data value in that position is the sample 100p percentile.
3. If np is an integer, then the average of the values in positions np and $np + 1$ is the sample 100p percentile.

Example

Let $n = 10$

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0.25	2.5	47
0.5	5	(61+66)/2=63.5
0.75	7.5	68
1	10	79

Computing percentile using googlesheets-PERCENTILE function

Step 1 Paste the dataset in a column.

Step 2 In a blank cell enter PERCENTILE(data, percentile), where data indicates the range of data for which percentile needs to be computed, and percentile is the decimal form of the desired percentile.

- For example if the data is in cell A1:A10, and we are interested in computing the 90^{th} percentile, then enter PERCENTILE(A1:A10,0.9) in a blank cell.

Computing percentile using googlesheets-algorithm

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Step 1 Arrange data in increasing order.

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$rank = percentile \times (n - 1) + 1$ where n is total number of observations in the dataset

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Step 4 Compute the ordered data value $x_{[i]}$ corresponding to the integer part rank.

- The ordered data value corresponding to integer part rank of 3, $x_{[3]}$ is 47.

Computing percentile using googlesheets-algorithm-contd

Step 5 The percentile value is given by the formula

$$Percentile = x_{[i]} + fractional\ part \times \left[x_{[i+1]} - x_{[i]}\right]$$

- $Percentile = 47 + 0.25 \times [58 - 47] = 47 + 0.25 \times 11 = 47 + 2.75 = 49.75$

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In other words, the quartiles break up a data set into four parts with about 25 percent of the data values being less than the first(lower) quartile, about 25 percent being between the first and second quartiles, about 25 percent being between the second and third(upper) quartiles, and about 25 percent being larger than the third quartile.

The Five Number Summary

- Minimum
- Q_1: First Quartile or lower quartile
- Q_2: Second Quartile of Median
- Q_3: Third Quartile or upper quartile
- Maximum

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- IQR for the example
  - First quartile, $Q_1 = 49.75$
  - Third quartile, $Q_3 = 68$
  - $IQR = Q_3 - Q_1 = 18.25$

Section summary

- Definition of percentiles.
- How to compute percentiles.
- Definition of quartile.
- Five-number summary.
- Interquartile range as a measure of dispersion.

Summary

1. Frequency tables
    1.1 Frequency table for discrete data.
    1.2 Frequency table for continuous data.
2. Graphical summaries
    2.1 Histograms.
    2.2 Stem-an-leaf plot.
3. Numerical summaries
    3.1 Measures of central tendency
        3.1.1 Mean, Median, Mode
    3.2 Measures of dispersion
        3.2.1 Range, Variance, Standard deviation
    3.3 Percentiles
        3.3.1 Interquartile range as a measure of dispersion.