中国科学技术大学
University of Science and Technology of China
地址：中国 安徽 合肥市金寨路96号 邮编：230026
电话：0551-63602184 传真：0551-63631760 Http://www.ustc.edu.cn

赵乐君 SA24225465

18. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++" Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers and logical connectives. The universe of discourse for quantifiers consists of all students at your school.

i) There is a student at your school who can speak Russian and knows C++.

Answer: $\exists x (P(x) \wedge Q(x))$.

ii) There is a student at your school who can speak Russian but doesn't know C++.

Answer: $\exists x (P(x) \wedge \neg Q(x))$.

iii) Every student at your school either can speak Russian or knows C++.

Answer: $\forall x (P(x) \vee Q(x))$.

iv) No student at your school can speak Russian or knows C++.

Answer: $\forall x (\neg P(x)) \wedge (\neg Q(x))$. $\Leftrightarrow \neg \exists x (P(x) \vee Q(x))$

20. What rule of inference is used in each of these arguments.

i) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

Answer: Addition rule $(p \rightarrow (p \vee q))$

ii) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.

Answer: Simplification rule $((p \wedge q) \rightarrow p)$

iii) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.

Answer: Modus ponens. $([(p \rightarrow q) \wedge p] \rightarrow q)$

iv) If ~~it~~ it snows today, the university will close. The university is not closed today. Therefore, it didn't snow today.

Answer: Modus tollens. $([\neg q \wedge (p \rightarrow q)] \rightarrow \neg p)$

v) If I go swimming, then I will stay in the sun too long, then I will sunburn. Therefore. If I go swimming, then I will sunburn.

Answer: Hypothetical syllogism. $([(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r))$

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赵乐君 SA24225465 2/2

22. Let A, B and C be sets. Show that.

i) $A\cup(B\cup C) = (A\cup B)\cup C$

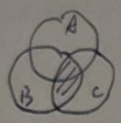
$B\cup C$

$A\cup B$

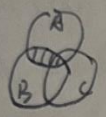
$A\cup(B\cup C)$, $(A\cup B)\cup C$

Therefore, $A\cup(B\cup C) = (A\cup B)\cup C$
This is Associative law

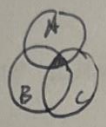
ii) $A\cap(B\cap C) = (A\cap B)\cap C$



$B\cap C$



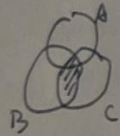
$A\cap B$



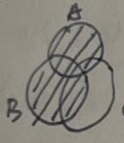
$A\cap(B\cap C)$, $(A\cap B)\cap C$

Therefore, $A\cap(B\cap C) = (A\cap B)\cap C$
This is Associative law.

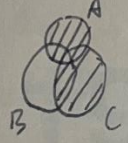
iii) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$



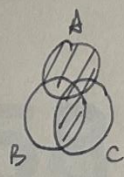
$B\cap C$



$A\cup B$



$A\cup C$



$A\cup(B\cap C)$, $(A\cup B)\cap(A\cup C)$

Therefore, ~~AA~~ $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$
This is Distributive law.

24. Determine whether the function $f: \mathbb{Z}\times\mathbb{Z} \to$ is onto.

i) $f(m,n) = m+n$ is onto

ii) $f(m,n) = m^2+n^2$ is NOT onto. $(m^2+n^2) \in N$. $m\in\mathbb{Z}$. $n\in\mathbb{Z}$.

iii) $f(m,n) = m$ is onto.

iv) $f(m,n) = |n|$ is NOT onto $|n| \in N$. $m\in\mathbb{Z}$. $n\in\mathbb{Z}$.

v) $f(m,n) = m-n$ is onto.

26. What are the values of these sums?

i) $\sum_{k=1}^{5}(k+1) = 20$

ii) $\sum_{j=0}^{4}(-2)^j = 11$

iii) $\sum_{i=1}^{10} 3 = 30$

iv) $\sum_{j=0}^{8}(2^{j+1}-2^j) = \sum_{j=0}^{8} 2^j = 2^9-1 = 511$