概率论与数理统计 第十四周作业 PB20111686 黄瑞轩

Ch.6 T21

$X\sim N(\mu_1,\sigma^2)$, $Y\sim N(\mu_2,\sigma^2)$ 且互相独立

$\alpha(\overline{X}-\mu_1)\sim N(0,\frac{\alpha^2\sigma^2}{m})$, $\beta(\overline{Y}-\mu_2)\sim N(0,\frac{\beta^2\sigma^2}{n})$

独立+正态可加 $\Rightarrow \alpha(\overline{X}-\mu_1)+\beta(\overline{Y}-\mu_2)\sim N(0,(\frac{\alpha^2}{m}+\frac{\beta^2}{n})\sigma^2)$

$\frac{(m-1)S_{1m}^2}{\sigma^2}\sim\chi^2(m-1)$, $\frac{(n-1)S_{2n}^2}{\sigma^2}\sim\chi^2(n-1)$

由于 S_{1m}^2 和 S_{2n}^2 独立，故 $\frac{(m-1)S_{1m}^2+(n-1)S_{2n}^2}{\sigma^2}\sim\chi^2(m+n-2)$

令 $W=\frac{\alpha(\overline{X}-\mu_1)+\beta(\overline{Y}-\mu_2)}{\sqrt{\frac{\alpha^2}{m}+\frac{\beta^2}{n}}\,\sigma}$，则 $W\sim N(0,1)$.

$$故\ T=\frac{W}{\sqrt{\frac{(m-1)S_{1m}^2+(n-1)S_{2n}^2}{m+n-2}}\cdot\frac{1}{\sigma}}=\frac{\alpha(\overline{X}-\mu_1)+\beta(\overline{Y}-\mu_2)}{\sqrt{\frac{(m-1)S_{1m}^2+(n-1)S_{2n}^2}{m+n-2}}\cdot\sqrt{\frac{\alpha^2}{m}+\frac{\beta^2}{n}}}\sim t(m+n-2)$$

Ch.7 T6

(1) $EX=\int_0^\theta x\frac{1}{2\theta}dx+\int_\theta^1 x\frac{1}{2(1-\theta)}dx$

$=\frac{1}{2\theta}(\frac{1}{2}\theta^2-0)+\frac{1}{2(1-\theta)}(\frac{1}{2}-\frac{1}{2}\theta^2)$

$=\frac{\theta}{4}+\frac{1+\theta}{4}=\frac{1+2\theta}{4}$

$\overline{X}$ 是 EX 矩估计，故 $\overline{X}=\frac{1+2\hat\theta}{4}$

即 $\hat\theta=\frac{4\overline{X}-1}{2}$.

(2) $E4\overline{X}^2=4E\overline{X}^2$

$EX^2=\int_0^\theta x^2\frac{1}{2\theta}dx+\int_\theta^1 x^2\frac{1}{2(1-\theta)}dx$

$=\frac{2\theta^2+\theta+1}{6}$

$E\overline{X}^2=(E\overline{X})^2+D(\overline{X})$

$=(EX)^2+\frac{1}{n}DX$

$=\frac{4\theta^2+4\theta+1}{16}+\frac{1}{n}(EX^2-(EX)^2)$

$=\frac{\theta^2}{4}+\frac{\theta}{4}+\frac{1}{16}+\frac{1}{n}(\frac{\theta^2}{3}+\frac{\theta}{6}+\frac{1}{6}-\frac{1}{16}-\frac{\theta}{4}-\frac{\theta^2}{4})$

$4E\overline{X}^2$ 显然不等于 θ^2 … 故不是无偏估计

T13

因为是随机的，可以看成独立同分布.

$L(\theta)=(\theta^2)^{n_1}(2\theta(1-\theta))^{n_2}((1-\theta)^2)^{n_3}$

$l=\ln L(\theta)=2n_1\ln\theta+n_2\ln 2\theta(1-\theta)+2n_3\ln(1-\theta)$

$=(2n_1+n_2)\ln\theta+(2n_3+n_2)\ln(1-\theta)+n_2\ln 2$

$l'_\theta=\frac{2n_1+n_2}{\theta}-\frac{2n_3+n_2}{1-\theta}=0\Rightarrow 2n_1+n_2-\theta(2n_1+n_2)=\theta(2n_3+n_2)$

$\Rightarrow 2n_1+n_2=\theta(2n)\Rightarrow\theta=\frac{2n_1+n_2}{2n}$

T19 (1) 当 $0<x<\theta$ 时，$F(x)=\int_{-\infty}^{x} f(x)\,dx = \int_0^x \frac{3x^2}{\theta^3}\,dx = \frac{x^3}{\theta^3}$

$$F(x)=\begin{cases} 0, & x\le 0. \\ \frac{x^3}{\theta^3}, & 0<x<\theta. \\ 1, & x\ge\theta. \end{cases}$$

$F_T(t) = P\left(\max(X_1,X_2,X_3)\le t\right) = P(X_1\le t,\ X_2\le t,\ X_3\le t)$，

X_1,X_2,X_3 独立，故 $F_T(t) = P(X_1\le t)P(X_2\le t)P(X_3\le t) = [F(t)]^3$

$$=\begin{cases} 0, & t\le 0 \\ \frac{t^9}{\theta^9}, & 0<t<\theta \\ 1, & t\ge\theta \end{cases}$$

故 $f_T(t)=\begin{cases} \frac{9t^8}{\theta^9}, & 0<t<\theta \\ 0, & 其他. \end{cases}$

(2) $E(aT) = aET = a\int_0^\theta t\cdot\frac{9t^8}{\theta^9}\,dt = \frac{9a}{\theta^9}\int_0^\theta t^9\,dt = \frac{9a}{\theta^9}\cdot\frac{\theta^{10}}{10} = \frac{9a}{10}\theta = \theta$

故 $a=\frac{10}{9}$.