概统第十一次作业　PB20111686　黄禄城.

Ex.1 (4.46)

设 X_i 是第 i 个基金经理月收益，$1\le i\le 20$. $X_i\sim N(50,10)$.

则 $E(\sum_i X_i)=\sum_i EX_i=1000$，又 $i\neq j$，X_i, X_j 互相独立，

则 $Var(\sum_i X_i)=\sum_i Var X_i=200$

Ex.2. (4.47)

设两个收益分别为 X, Y. 则

$E(X+Y)=EX+EY=2\times 1.05\times 2=4.2$

$Var(X+Y)=E(X+Y)^2-(E(X+Y))^2$

$=EX^2+2EXY+EY^2-4.2^2$

$X=2(1+r)=2+2r$

$EX^2=4E(r^2+2r+1)=4Er^2+8Er+4=4\int_{-0.1}^{0.2} r^2\frac{1}{0.3}dr+8\times 0.05+4=4.44$

Y同证，略

$VarX=EX^2-(EX)^2=0.03$

$\sigma_{XY}=\rho_{XY}\sigma_X\sigma_Y=\rho_{XY}\sqrt{VarX\,VarY}=6\times 10^{-3}$

$=EXY-EXEY$

故 $EXY=4.416$.

$Var(X+Y)=0.072$

Ex. 3 (4.63)

(1) $Cov(\alpha X+\beta Y,\alpha X-\beta Y)=E(\alpha^2X^2-\beta^2Y^2)-E(\alpha X+\beta Y)E(\alpha X-\beta Y)$

$=\alpha^2EX^2-\beta^2EY^2-(\alpha EX+\beta EY)(\alpha EX-\beta EY)$

$=\alpha^2EX^2-\beta^2EY^2-\alpha^2(EX)^2+\beta^2(EY)^2$

$=\alpha^2VarX-\beta^2VarY=(\alpha^2-\beta^2)\sigma^2$

(2) $\alpha X+\beta Y$ 和 $\alpha X-\beta Y$ 独立 $\Leftrightarrow Cov(\alpha X+\beta Y,\alpha X-\beta Y)=0$

$\Leftrightarrow \alpha^2=\beta^2\Leftrightarrow|\alpha|=|\beta|$ 时.

EX. 4. (5.13)

(1) X_i ⇒第i个部件正常工作，$1 \le i \le 100$，即 $X_i = \begin{cases} 1, & i正常 \\ 0, & i不正常 \end{cases}$

$S_{100} = \sum_{i=1}^{100} X_i$，即$P(S_{100} \ge 85)$，对于 X_i，$\mu = 1\times 0.9 + 0\times 0.1 = 0.9$，$\sigma = 1^2\times 0.9 - (0.9)^2 = 0.09$

X_i 独立同分布，故 $P(S_{100} \ge 85) = 1 - P(S_{100} \le 85) \approx 1 - \Phi\left(\frac{85 - 100\times 0.9}{\sqrt{100\times 0.1\times 0.9}}\right) = 1 - \Phi\left(-\frac{5}{3}\right)$

$= 0.95$

(2) 即求 $P(S_n \ge 0.8n)$，由上，

$P(S_n \ge 0.8n) \approx 1 - \Phi\left(\frac{0.8n - 0.9n}{\sqrt{n\times 0.9\times 0.1}}\right) = 1 - \Phi\left(-\frac{\sqrt{n}}{3}\right) \ge 0.95$

$\Rightarrow \Phi\left(-\frac{\sqrt{n}}{3}\right) \le 0.05$

而 $\Phi(-1.64) \approx 0.05 \Rightarrow n \ge 24.2$

取 $n = 25$