黄瑞轩

PB20111686

(1) $F(p) = \mathcal{L}\left[\frac{1}{2}\sin 2t + \cos 3t\right]$

$= \frac{1}{2}\mathcal{L}[\sin 2t] + \mathcal{L}[\cos 3t]$

$= \frac{1}{2}\cdot\frac{2}{p^2+4} + \frac{p}{p^2+9}$

$= \frac{1}{p^2+4} + \frac{p}{p^2+9}$

(2) $F(p) = \mathcal{L}[e^{3t} - e^{-2t}]$

$= \mathcal{L}[e^{3t}] - \mathcal{L}[e^{-2t}]$

$= \frac{1}{p-3} - \frac{1}{p+2}$

(3) $F(p) = \mathcal{L}[1 - e^{at}]$

$= \mathcal{L}[1] - \mathcal{L}[e^{at}]$

$= \frac{1}{p} - \frac{1}{p-a}$

(4) $F(p) = \frac{a}{a-b}\mathcal{L}[e^{at}] - \frac{b}{a-b}\mathcal{L}[e^{bt}]$

$= \frac{1}{a-b}\left(\frac{a}{p-a} - \frac{b}{p-b}\right)$

(5) $F(p) = \frac{1}{b^2-a^2}\mathcal{L}[\cos at] - \frac{1}{b^2-a^2}\mathcal{L}[\sin bt]$

$= \frac{1}{b^2-a^2}\left(\frac{p}{p^2+a^2} - \frac{b}{p^2+b^2}\right)$

(6) $F(p) = \frac{1}{a^2}\left(a\mathcal{L}[t] - \mathcal{L}[\sin at]\right)$

$= \frac{1}{a}\left(\frac{1}{p^2} - \frac{1}{p^2+a^2}\right)$

(7) $G(p) = \mathcal{L}[\sin 5t] = \frac{5}{p^2+25}$

$F(p) = G(p+2) = \frac{5}{(p+2)^2+25}$

(8) $F(p) = \dfrac{1}{p+3+4i}$

(9) $G(p) = L[e^{5t}] = \dfrac{1}{p-5}$

$F(p) = (-1)\,G'(p) = \dfrac{1}{(p-5)^2}$

(10) $F(p) = \frac{1}{2} L[e^{wt}] + \frac{1}{2} L[e^{-wt}]$

$= \dfrac{1}{2}\left(\dfrac{1}{p-w} + \dfrac{1}{p+w}\right)$

$= \dfrac{p}{p^2-w^2}$

$$L[f(t)\sin wt] = \int_0^{+\infty} f(t)\sin wt\, e^{-pt}\, dt$$

$$= \int_0^{+\infty} f(t)\left[\frac{1}{2i}\left(e^{iwt} - e^{-iwt}\right)\right] e^{-pt}\, dt$$

$$= \frac{1}{2i}\left[\int_0^{+\infty} f(t)\, e^{-(p-iw)t}\, dt - \int_0^{+\infty} f(t)\, e^{-(p+iw)t}\, dt\right]$$

$$= \frac{1}{2i}\left[F(p-iw) - F(p+iw)\right] \quad \#$$

---

(1) $F(p) = \dfrac{1}{2}\left(\dfrac{1}{p+1} - \dfrac{1}{p+3}\right)$

$f(t) = \frac{1}{2} e^{-t} - \frac{1}{2} e^{-3t}$

(3) $F(p) = \dfrac{p+2}{p^2+4p+5} = \dfrac{p+2}{(p+2)^2+1}$

$f(t) = e^{-2t}\cos t$

---

(2) 设 $L[y(t)] = Y(p)$

$L[y'(t)] = pY(p) - y(0) = pY(p)$

$L[y''(t)] = p^2 Y(p) - p\,y(0) - y'(0) = p^2 Y(p)$

原方程两边作拉氏变换：

$$(p^2-p)Y(p) = \frac{1}{p-1} \Rightarrow Y(p) = \frac{1}{(p-1)^2 p} = \frac{1}{p} - \frac{1}{p-1} + \frac{1}{(p-1)^2}$$

作拉氏反变换:

$y(t) = 1 - e^t + te^t$ #

(5) 设 $L[y(t)] = Y(p)$

$L[y'(t)] = pY(p) - y(0) = pY(p) + 1$

$L[y''(t)] = p^2Y(p) - py(0) - y'(0) = p^2Y(p) + p + 2$

对方程两边作拉氏变换:

$$(p^2-1)Y(p) + p + 2 = \frac{4}{p^2+1} + \frac{5p}{p^2+4}$$

$$Y(p) = \frac{4}{(p^2+1)(p^2-1)} + \frac{5p}{(p^2+4)(p^2-1)} - \frac{p+2}{p^2-1}$$

$$= \frac{2}{p^2-1} - \frac{2}{p^2+1} + p\left(\frac{1}{p^2-1} - \frac{1}{p^2+4}\right) - \frac{p}{p^2-1} - \frac{2}{p^2-1}$$

作拉氏反变换:

$y(t) = -2\sin t - \cos 2t$ #

令 $L[y(t)] = Y(p)$, $L[f(t)] = F(p)$

$L[y'(t)] = pY(p) - y(0) = pY(p)$

$L[y''(t)] = p^2Y(p) - py(0) - y'(0) = p^2Y(p)$

方程两边作拉氏变换:

$(p^2+w^2)Y(p) = F(p)$

$$Y(p) = \frac{F(p)}{p^2+w^2}$$

作拉氏反变换得 $y(t) = f * \frac{1}{w}\sin wt = \frac{1}{w}\int_0^t f(u)\sin w(t-u)\,du$ #

记 $\mathcal{L}[f(t)] = F(p)$

$$f(t) = a\sin bt + c\left(\sin bt * f(t)\right)$$

作拉氏变换：

$$F(p) = \frac{ab}{p^2+b^2} + \frac{cb}{p^2+b^2}F(p)$$

$$\Rightarrow F(p) = \frac{ab}{p^2+b^2} \Big/ \left(\frac{p^2+b^2-cb}{p^2+b^2}\right) = \frac{ab}{p^2+b^2-cb}$$

作拉氏反变换：

$$f(t) = \frac{ab}{\sqrt{b^2-cb}} \sin\sqrt{b^2-cb}\, t \qquad \#$$