Can one hear the shape of a room ?

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Outline

Problem formulation

Can one hear the shape of a room ?

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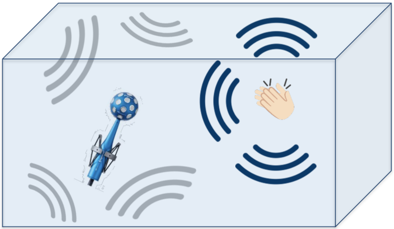
In other words, given:

→ an initial sound impulse (Dirac in time and 3D space)

→ discrete-time, multichannel, low-pass measurements of the room response (RIR)

can we reconstruct the positions of the walls, floor and ceiling ?



Model

The pressure field $p(\mathbf{r}, t)$ in the room is solution to the following wave equation with boundary conditions:

$$\begin{cases} \frac{1}{c^2}\frac{\partial^2}{\partial t^2}p(\mathbf{r}, t) - \Delta p(\mathbf{r}, t) = a_0\delta\left(\mathbf{r} - \mathbf{r}^{\mathrm{src}}\right)\delta(t) & \mathbf{r} \in \Omega \\ \mathbf{n}(\mathbf{r}) \cdot \nabla p(\mathbf{r}, t) + \frac{1}{c}\frac{\partial}{\partial t}\beta(\mathbf{r}, t) * p(\mathbf{r}, t) = 0 & \mathbf{r} \in \partial\Omega \end{cases} \quad (1)$$

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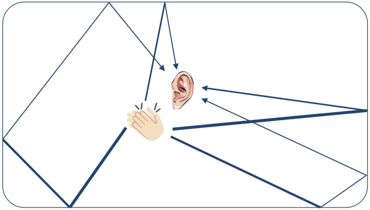
Simplification:

- polyhedric room geometry
- perfectly reflective (rigid) walls
- frequency independant walls



$\rightarrow$ the Image Source Model (ISM)

Image sources

Each reflection path is associated to an image source

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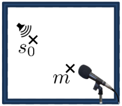

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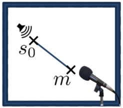

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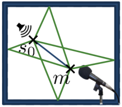

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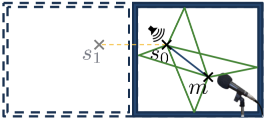

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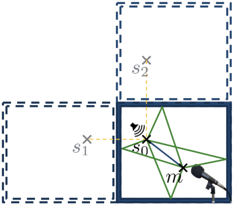

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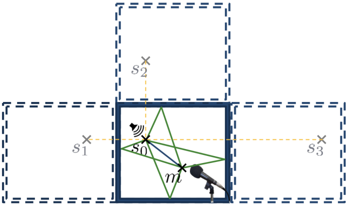

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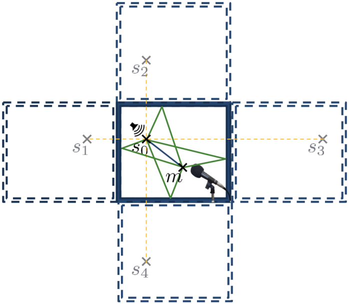

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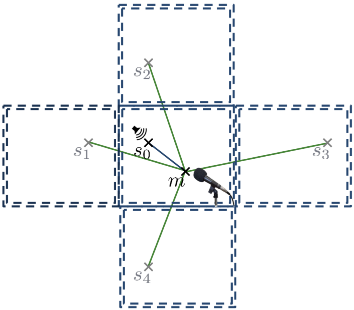

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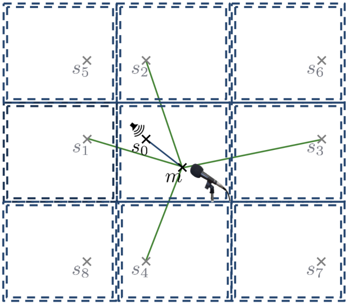

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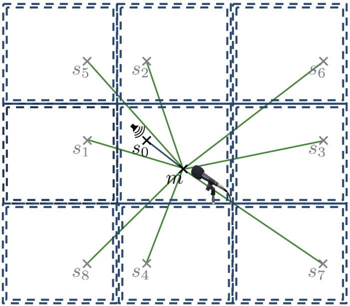

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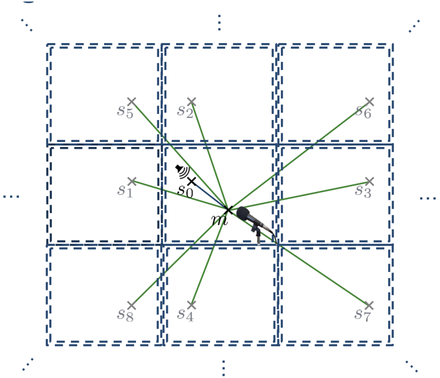

The first order sources are reflections of the source with respect to the walls

$\longrightarrow$ knowing the position of the source and the first order image sources yields the room geometry

Image sources model (ISM)

Perfectly specular reflections translate in Neumann boundary conditions:

$$\begin{cases} \frac{1}{c^2}\frac{\partial^2}{\partial t^2}p(\mathbf{r},t) - \Delta p(\mathbf{r},t) = a_0\delta\left(\mathbf{r}-\mathbf{r}^{\mathrm{src}}\right)\delta(t) & \mathbf{r}\in\Omega \\ \mathbf{n}(\mathbf{r})\cdot\nabla p(\mathbf{r},t) = 0 & \mathbf{r}\in\partial\Omega \end{cases} \tag{2}$$

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When considering a cuboid room, the solution writes:

$$p(\mathbf{r},t) = A \sum_{k=0}^{+\infty} \frac{\delta(\|\mathbf{r} - \mathbf{r}_k^{\text{src}}\| - ct)}{\|\mathbf{r} - \mathbf{r}_k^{\text{src}}\|} \tag{3}$$

where the $\mathbf{r}_k^{\text{src}}$ are the locations of the **image sources** and A is a constant.

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p is then solution to the free field wave equation:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} p(\mathbf{r},t) - \Delta p(\mathbf{r},t) = \sum_{k=0}^{+\infty} \delta\left(\mathbf{r} - \mathbf{r}_k^{\text{src}}\right)\delta(t) \tag{4}$$

Image sources model (ISM)

Extension: consider the free-field wave equation:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}p(\mathbf{r},t) - \Delta p(\mathbf{r},t) = \sum_{k=0}^{K} a_k \delta\left(\mathbf{r} - \mathbf{r}_k^{\mathsf{src}}\right)\delta(t) \tag{4}$$

The general solution of (4) is given by:

$$p(\mathbf{r},t) = \sum_{k=0}^{K} a_k \frac{\delta\left(t - \left\|\mathbf{r} - \mathbf{r}_k^{\mathrm{src}}\right\|_2 / c\right)}{\left\|\mathbf{r} - \mathbf{r}_k^{\mathrm{src}}\right\|}. \tag{5}$$

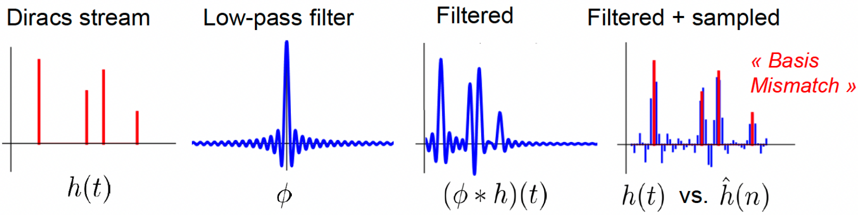
- an absorption factor $\alpha_i, 1 \leq i \leq 7$ is associated to each wall
- the amplitude a for an order n image source is given by :

$$a = \prod_{l=0}^{n} \sqrt{1 - \alpha_{n_l}} \tag{6}$$

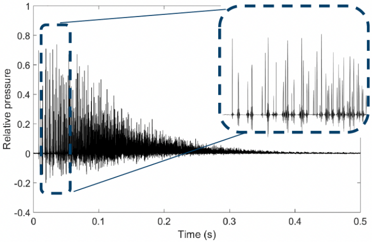
where n_l is the index of the wall corresponding to the l-th reflection

Jont B Allen and David A Berkley, *Image method for efficiently simulating small-room acoustics*, The Journal of the Acoustical Society of America, vol. 65, no. 4, pp. 943–950, 1979

Measured Room Impulse Response signal



Full synthetic RIR (specular only)



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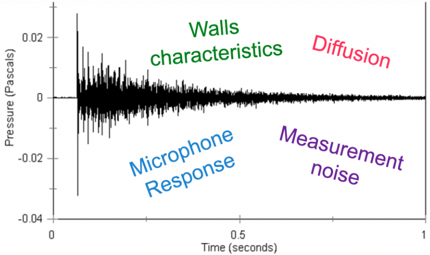

Audio reconstruction

Note: The RIR embeds the acoustical properties of the room. Given a solution G (Green function) of the wave equation with impulse source term:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}p(\mathbf{r},t) - \Delta p(\mathbf{r},t) = \delta\left(\mathbf{r} - \mathbf{r}^{\mathrm{src}}\right)\delta(t) \qquad (7)$$

$t \mapsto G(\mathbf{r}, t)$ corresponds to the RIR observed at position $\mathbf{r}$

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$t \mapsto G(\mathbf{r},t)$ corresponds to the RIR observed at position $\mathbf{r}$
A person speaking at position $\mathbf{r}^{\mathrm{src}}$ can be modelled by the equation:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}p(\mathbf{r},t) - \Delta p(\mathbf{r},t) = \delta\left(\mathbf{r} - \mathbf{r}_k^{\mathrm{src}}\right)h(t) \tag{8}$$

The solution of this last equation is given by a time convolution with G :

$$p(r,t) = \left(G(r,.) * h(.)\right)(t) \tag{9}$$

$\rightarrow$ we can simulate how a sound would play in a room by convolving with a RIR No Reverb abs0.1 abs0.01

Audio measurements

In practice a RIR is measured by real microphones:

- time convolution with a filter κ
- time discretization by a sampling frequency f_s
- recorded at different locations by an array of microphones (multichannel RIR)

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The idealized multichannel RIR is given by:

$$x_{m,n} := (\kappa * p(\mathbf{r}_m^{mic}, \cdot))(n/f_s) = \sum_{k=0}^{K} a_k \frac{\kappa\left(n/f_s - \|\mathbf{r}_m^{\mathrm{mic}} - \mathbf{r}_k^{\mathsf{src}}\|/c\right)}{\|\mathbf{r}_m^{\mathrm{mic}} - \mathbf{r}_k^{\mathsf{src}}\|} \tag{10}$$

where $\kappa : t \mapsto \mathrm{sinc}(\pi f_s t)$ is the ideal low-pass filter at the microphones frequency of sampling.

Outline

General formulation

- we want to reconstruct a **3D** measure $\psi = \sum_i a_i \delta_{\mathbf{r}_i}$
- we only have access to a vector of observations via a linear operator Γ (with kernel γ) : $\mathbf{x} = \Gamma\psi = \int_r \gamma(r) d\psi(r) \in R^{N \times M}$

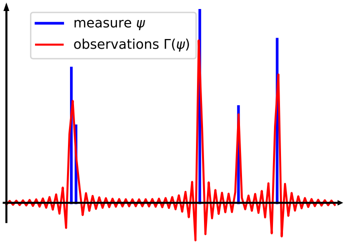

Figure: Example of 1D measure and its observation

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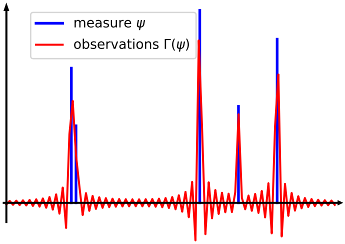

Figure: Example of 1D measure and its observation

Idea: consider a relaxed optimization problem over the entire space of Radon measures of $(\mathbb{R}^3)$:

$$\min_{\psi\in\mathcal{M}(\mathbb{R}^3)} \underbrace{\frac{1}{2}\|\mathbf{x} - \Gamma\psi\|_2^2}_{\text{data compliance}} + \underbrace{\lambda\|\psi\|_{\text{TV}}}_{\text{regularization}} \qquad \text{(BLASSO)}$$

Vincent Duval and Gabriel Peyré, *Exact Support Recovery for Sparse Spikes Deconvolution*, Foundations of Computational Mathematics, vol. 15, no. 5, pp. 1315-1355, 2015

Room Response

How can we express the RIR as the image of an operator Γ ?

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}p(\mathbf{r},t) - \Delta p(\mathbf{r},t) = \psi(\mathbf{r})\delta(t) \tag{11}$$

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For a given source distribution ψ, the solution to the wave equation (11) is given by a convolution with a Green function:

$$p(\mathbf{r},t) = \int_{r'} \frac{\delta\left(t - \left\|\mathbf{r}-\mathbf{r}'\right\|_2 / c\right)}{4\pi \left\|\mathbf{r}-\mathbf{r}'\right\|_2}\psi(\mathbf{r}')d\mathbf{r}'. \tag{12}$$

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The multi-channel response $\mathbf{x}$ measured by the microphones is:

$$x_{m,n} := (\kappa * p(\mathbf{r}_m^{mic}, \cdot))(n/f_s) = \int_{\mathbf{r}\in\mathbb{R}^3} \frac{\kappa\left(n/f_s - \left\|\mathbf{r}_m^{\mathrm{mic}} - \mathbf{r}\right\|_2 / c\right)}{4\pi\left\|\mathbf{r}_m^{\mathrm{mic}} - \mathbf{r}\right\|_2}\psi(\mathbf{r})\, d\mathbf{r} \tag{13}$$

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Room operator

The multi-channel response measured by the microphone $\mathbf{x}$ is:

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We define the linear operator

$$\begin{array}{rccl} \Gamma: & \mathcal{M}(\mathbb{R}^3) & \longrightarrow & \mathbb{R}^{N\times M} \\ & \psi & \mapsto & \mathbf{x} = \left(\int_{r\in\mathbb{R}^3} \frac{\kappa(n_j/f_s - \|r - r_{m_j}^{\mathrm{mic}}\|_2/c)}{4\pi\|r - r_{m_j}^{\mathrm{mic}}\|_2} d\psi(r) \right)_{1\le j\le N\times M} \end{array} \tag{14}$$

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The multi-channel response measured by the microphone $\mathbf{x}$ is:

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In particular, if $\psi = \sum_{k=0}^{K} a_k \delta_{\mathbf{r}_k^{\mathrm{src}}}$ (the measure defined by the image sources), $\mathbf{x} = \Gamma\psi$ is the multichannel RIR.

Room operator

Issue: the kernel $\gamma_{m,n} : r \mapsto \frac{\kappa(n_j/f_s - \|r - r_{m_j}^{\text{mic}}\|_2/c)}{4\pi\|r - r_{m_j}^{\text{mic}}\|_2}$ presents singularities at each microphone location

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$\rightarrow$ we perform a surgery around each microphone

$\rightarrow$ Γ is restricted to $\mathcal{M}(\mathbb{R}^3_\varepsilon)$ where $\mathbb{R}^3_\varepsilon = \mathbb{R}^3 \setminus \cup_m B(\mathbf{r}^{\text{mic}}_m, \varepsilon)$

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Note : the BLASSO optimization problem remains convex

$$\min_{\psi \in \mathcal{M}(\mathbb{R}^3_\varepsilon)} \underbrace{\frac{1}{2}\|\mathbf{x} - \Gamma\psi\|_2^2}_{\text{data compliance}} + \underbrace{\lambda\|\psi\|_{\text{TV}}}_{\text{regularization}} \tag{15}$$

Outline

The Frank-Wolfe algorithm

Let $\mathcal{D}$ denote a weakly compact convex set of a Banach space and $f : \mathcal{D} \to \mathbb{R}$, a convex, differentiable real-valued function. The Frank–Wolfe algorithm solves numerically the optimization problem

$$\inf_{x \in \mathcal{D}} f(x)$$

Algorithm

- Initialization: Let x_0 denote any point in $\mathcal{D}$.
- Step 1: direction-finding. Find s_k solving

$$\inf_{s \in \mathcal{D}} Df(x_k) \cdot s$$

  Interpretation: minimize the linear approximation of the problem given by the first-order Taylor approximation of f around x_k.
- Step 2: step size determination. find α minimizing $\alpha \mapsto f(x_k + \alpha(s_k - x_k))$ subject to $0 \leq \alpha \leq 1$.
- Step 3: update. Set $x_{k+1} \leftarrow x_k + \alpha(s_k - x_k)$ and $k \leftarrow k + 1$ and go to Step 1.

The Sliding-Frank-Wolfe algorithm

Application of Frank-Wolfe to

$$\min_{\psi \in \mathcal{M}(\mathbb{R}^3_\varepsilon)} T_\lambda(\psi) \coloneqq \frac{1}{2}\|\mathbf{x} - \Gamma\psi\|_2^2 + \lambda\|\psi\|_{\mathsf{TV}}. \tag{16}$$

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Consider instead

$$\min_{(t,\psi) \in C} T_\lambda(t,\psi) \coloneqq \frac{1}{2}\|\mathbf{x} - \Gamma\psi\|_2^2 + t\lambda \tag{17}$$

where $C = \{(t,\psi) \in \mathbb{R}_+ \times \mathcal{M}(\mathbb{R}^3_\varepsilon),\ \|\psi\|_{\mathsf{TV}} \leq t \leq \frac{\|\mathbf{x}\|_2}{2\lambda}\}$

The Sliding-Frank-Wolfe algorithm

Application of Frank-Wolfe to

$$\min_{\psi \in \mathcal{M}(\mathbb{R}^3_\varepsilon)} T_\lambda(\psi) := \frac{1}{2}\|\mathbf{x} - \Gamma\psi\|_2^2 + \lambda\|\psi\|_{\mathrm{TV}}. \tag{16}$$

Consider instead

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where $C = \{(t,\psi) \in \mathbb{R}_+ \times \mathcal{M}(\mathbb{R}^3_\varepsilon),\ \|\psi\|_{\mathrm{TV}} \leq t \leq \frac{\|\mathbf{x}\|_2}{2\lambda}\}$
Main idea : construct a new Dirac at each iteration based on the residual, by maximizing :

$$\eta_\lambda^k : \mathbf{r} \mapsto \frac{1}{\lambda}\Gamma^*(\mathbf{x} - \Gamma\psi^k)(\mathbf{r}) \tag{18}$$

Q. Denoyelle, V. Duval, G. Peyré and E. Soubies, *The Sliding Frank-Wolfe Algorithm and its Application to Super-Resolution Microscopy*, Inverse Problems, vol. 36, no 1, p. 014001, 2019

The Sliding-Frank-Wolfe algorithm

- Initialization: let $\psi^0 = 0$ the null measure.
- k-th iteration: let $\psi^k(\mathbf{r}^k) = \sum_i^{N_k} a_i^k \delta_{\mathbf{r}_i^k}$ ($\mathbf{r}_i^k$ pairwise distinct);
  - Step 1 (spike-finding): Find $\mathbf{r}_*^k$ solving the optimization problem

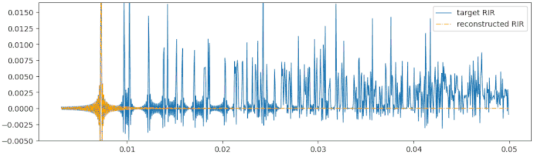
$$\sup_{\mathbf{r}\in\mathbb{R}^3_\varepsilon} |\eta_\lambda^k(\mathbf{r})|, \quad \eta_\lambda^k(\mathbf{r}) = \frac{1}{\lambda}\Gamma^*(\mathbf{x} - \Gamma\psi^k)(\mathbf{r}).$$

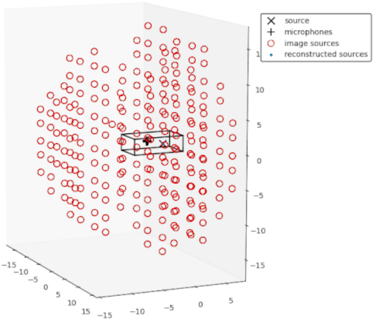
    If $\|\eta_\lambda^k(\mathbf{r})\|_2 \leq 1$, we are done. Else:
  - Step 2 (amplitude optimization): find $\psi^{k+1/2} = \sum_i^{N_k} a_i^{k+1/2}\delta(\mathbf{r} - \mathbf{r}_i^k) + a_{N_k+1}^{k+1/2}\delta(\mathbf{r} - \mathbf{r}_*^k)$ solving

$$\inf_{a_i^{k+1/2}\geq 0} \frac{1}{2}\|\mathbf{x} - \Gamma\psi^{k+1/2}\|_2^2 + \lambda\|\psi^{k+1/2}\|_{TV}.$$

  - Step 3 (sliding): find ψ^{k+1} minimizing locally the criterion wrt $(a, \mathbf{r})$ using as initial point $(a^{k+1/2}, \mathbf{r}^{k+1/2})$.
  - Eventually remove zero amplitudes Dirac masses from ψ^{k+1}.

Numerical results





Iteration 1

Experiments

Simulated experimental setup:

- compact spherical array of 32 microphones (scaled eigenmike with radius 4.2*cm*, 8.4*cm*, etc.)
- random room sizes ($2 \times 2 \times 2$m$\rightarrow 10 \times 10 \times 5$m), random sources and microphone locations
- synthetic noisy RIR cut at 50ms for the observations
- study the impact of the sampling frequency, the noise, the array radius

Numerical results

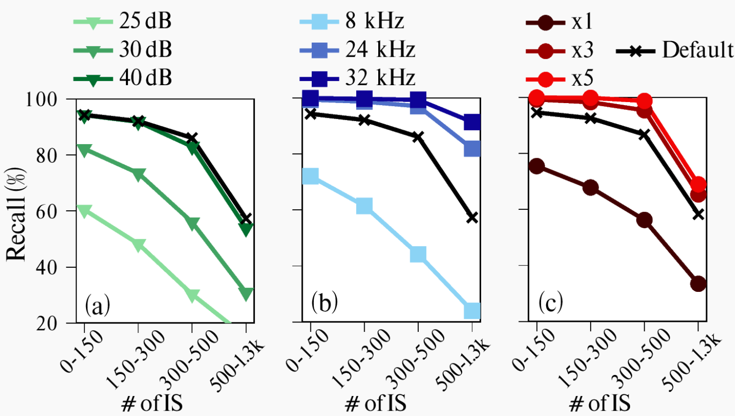


Figure: Recall over a room dataset for varying noise ratios (PSNR), sampling frequencies and spherical microphone array diameter

Default parameters: noiseless, $f_s = 16$kHz, $d = 16.8$cm

T. Sprunck, A. Deleforge, Y. Privat, and C. Foy. *Gridless 3D Recovery of Image Sources from Room Impulse Responses*, IEEE Signal Processing Letters, vol. 29, pp. 2427-2431 2022

Outline

Order 1 method and downsides

Known data: coordinates of the microphones $\mathbf{r}_m^{\mathrm{mic}}$ and the image sources $\mathbf{r}_m^{\mathrm{src}}$ **in the referential of the antenna**.

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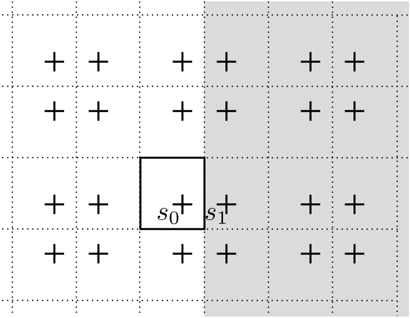

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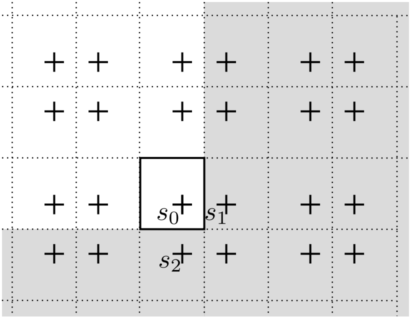

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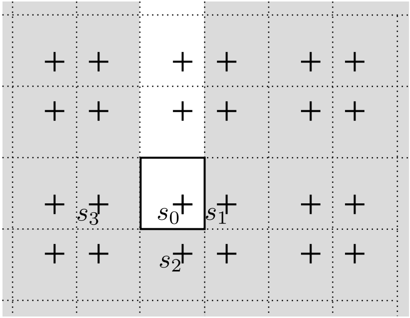

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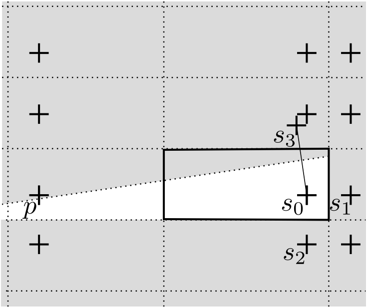
Issues: False positives and imprecise reconstruction



Three steps method

Input data: coordinates of the microphones $\mathbf{r}_m^{\text{mic}}$ and the image sources $\mathbf{r}_k^{\text{src}}$ **in the referential of the antenna**.

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Idea: use the full reconstructed image sources cloud
→ exploit the grid structure of the image sources
→ increased robustness to inaccuracies/false positives

Step 1: find the orientation of the room in the antenna referential

Step 2: project the image sources on the three room axes (normals to the walls)

Step 3: apply a 1D clustering algorithm on each axis to find the source/walls distances on each axis

Step 1: finding the orientation

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$$\max_{\|u\|=1} L(u) := \sum_{i,j} f(u, \mathbf{r}_i^{\mathrm{src}} - \mathbf{r}_j^{\mathrm{src}}) \tag{19}$$

$$\forall u, v \in \mathbb{R}^3, \quad f(u, v) = \begin{cases} 1 & \text{if } u \perp v \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

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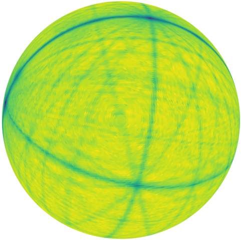
In practice, we use the following approximation:

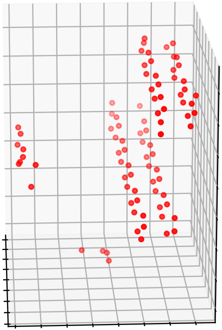
$$\forall u, v \in \mathbb{R}^3, \quad f_\sigma(u, v) = e^{-\frac{1}{2\sigma^2}\left(\frac{u.v}{\|u\|\|v\|}\right)^2} \tag{21}$$

Step 1: finding the orientation

- maximize f_σ over the unit sphere to find a first basis vector e_1
- consider the normal plane to e_1 and maximize the same function over the unit circle to find a second basis vector e_2
- get the last vector $e_3 = e_1 \times e_2$



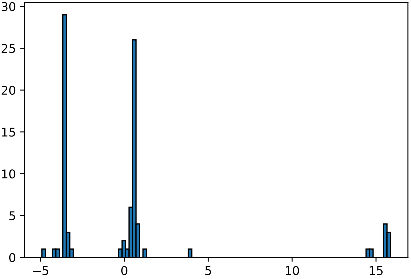
(a) Cost function values



(b) Image sources

Step 2 and 3: clustering projection and deducing dimensions

- compute the clusters of the projection on each e_i to get the room dimensions
- identify the central cluster (cluster containing the source) on each axis
- each dimension is the half of the distance between the two clusters surrounding the central cluster



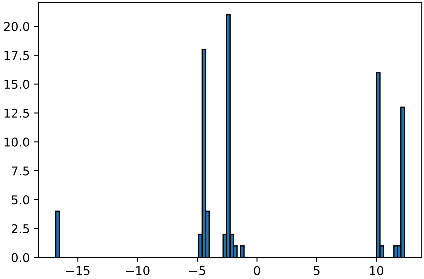


Figure: Projections on e_1, e_2

Conclusion and perspectives

The proposed method offers significant advantages for room geometry reconstruction :

- gridless, direct estimation of continuous 3D IS positions from discrete RIRs
- high precision recovery of low order image sources
- robustness to noise
- does not require prior information on the room properties

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The proposed method offers significant advantages for room geometry reconstruction :

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Some of the areas that remain to explore in this model:

- joint estimation of the source-microphone response κ
- taking into account the source and microphone directivities
- application to real data
- generalization to non-rectangular room shapes

Conclusion and perspectives

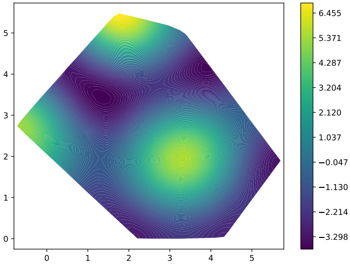
Another approach to handle non-rectangular geometries:

$$\inf_{\Omega \in \mathcal{O}_\ell^{\mathrm{adm}}} \sum_{m=1}^{M} \sum_{n=1}^{N} (\widetilde{p}_\Omega(k_n, \mathbf{r}_m^{\mathrm{mic}}) - \widetilde{x}_{m,n})^2 \, dt \tag{22}$$

where $\tilde{p}_\Omega$ is subject to the Helmholtz equation:

$$\begin{cases} \Delta \widetilde{p}(\mathbf{r}) + k_n^2 \widetilde{p}(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{r}^{\mathrm{src}}) & \mathbf{r} \in \Omega \\ \mathbf{n}(\mathbf{r}) \cdot \nabla \widetilde{p}(\mathbf{r}, t) = 0 & \mathbf{r} \in \partial\Omega \end{cases} \tag{23}$$

and $\mathcal{O}_\ell^{\mathrm{adm}} =$ set of polygons with at most ℓ vertices.



Thank you for your attention.

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A. Deleforge, C. Foy, Y. Privat, and T. Sprunck. *Can one hear the shape of a room? (article in preparation)*,