**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:

*μ* = 45

*σ* = 8

Actual time given by the company to collect car from drop off is 1h = 60 mins

Expected time to work is 60-10= 50 mins as the service manager starts after 10 mins of drop off

So x =50

P(x>50)=?

As it is normally distributed

Stats.norm.cdf(x,loc=mean,scale=std)

Or

(x-*μ)/σ convert to z score and find the z value from the z table pdf*

1-stats.norm.cdf(50, loc=45,scale=8)

0.26598552904870054

Ans is 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

False, because the probability of employees older than 44 is 15.87% where as the probability of employees with age between 38 and 44 is 34.13% which is more than the probability of employees older than 44

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

True because the expected value of employees with age under 30 is 36.48 which is approx. 36

Ans: *μ* = 38

*σ* =6

n=400

1. More employees at the processing center are older than 44 than between 38 and 44.

P(x>44)=?

P(38<x<44)?

P(x>44)= round(1-stats.norm.cdf(44, loc=38,scale=6),4)

=0.1587

P(38<x<44) =round( stats.norm.cdf(44, loc=38,scale=6) – (1- stats.norm.cdf(38, loc=38,scale=6)),4)

= 0.3413

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

P(x<30)=?

P(x<30) = round( stats.norm.cdf(30, loc=38,scale=6),4)

= 0.0912

Total employees is 400

To calculate no. of employees which fall under 30 age

P(x<30)\* Total employees

= .0912\*400

=36.48 (~ 36)

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

we know that X1 and X2 are two independent Random variable and from the above statements these are approximately normally distributed

so when we perform arithmetic operation of the RV each value in the RV undergoes arithmetic operation say

ex: a+b = *N*(μ1, σ12)+ *N*(μ2, σ22) = *N*(μ1+ μ2, σ12+ σ22)

x1+x2 ~ *N*(μ, σ2)+ *N*(μ, σ2) ~ N(2μ, 2σ2)

2 *X*1 ~ 2\* *N*(μ, σ2) ~ N(2 μ, 2σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

μ = 100

σ=20

Z = 0.99

Probability of getting RV value between a and b is 0.99

Probability of getting RV value outside a and b is 1- 0.99 = .01

As it is symmetric so both side left over portion probabilities we need to calculate

Probability towards left from a = -0.005 (ie. 0.01/2).

Probability towards right from b = +0.005 (ie. 0.01/2).

Here Z1 = -(round( stats.norm.ppf(.005), 4)) = -(-2.57583)= 2.57583

And Z2 = round(stats.norm.ppf(.005),4) = -2.57583

Z = (X- μ)/ σ

X1 = (Z1\* σ)+ μ

=2.57583\*20+100

`=151.5166

Second value

X2 = (Z2\* σ)+ μ

=-2.57583\*20+100

= 48.4834

Option D is correct

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

{99.009 , 980.991 } in rupees million is the range of the annual profit of a company

1. Specify the 5th percentile of profit (in Rupees) for the company

5% profit is 169.90875 rupee millions

1. Which of the two divisions has a larger probability of making a loss in a given year?

Std deviation of division 2 is high that is 4 so it tend to make more loss

And also when we consider 0% loss for both the division and calculate via python

Divison 1 = round( stats.norm.cdf(0, loc=5,scale=3),4) = 0.0478 = 4.78%

division 2 = round( stats.norm.cdf(0, loc=7,scale=4),4) = 0.0401 = 4.01 %

From the above value for the 0% loss which is nothing but 100% gain we can say that Division 2 has less probability of making 0%loss i.e 100% gain

Ans:

Assume that $1 = Rs. 45

Division 1 :

μ1 = 5, σ12 = 32

Division 2:

μ 2 = 7, σ22 = 42

company = Division 1 + Division 2

company mean μ= μ1 +  μ2 = 7+5 = 12$ = 12\*45 = 540 rupees millions

company variance σ2 = σ12 + σ22 = 32+42 = 25 $ million

company std deviation σ = sqrt(25) = 5$ = 225 ruppes millions

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Consider let the 95% probability falls btwn a and b

Probability of a = from 95% area remaining area is 5% as it is symmetric area toward left side is 2.5% and probability is = .025

Probability of b = As it is normal distribution 95% area toward right side is 2.5% and probability is

95+2.5 =97.5% = .975

Z1 = Z(.975) = round(stats.norm.ppf(0.975),5) = 1.95996

Z2 = Z(.025) round(stats.norm.ppf(0.025),5) = -1.95996

Z = (X- μ)/ σ

X1 = (Z2\* σ)+ μ

= -1.95996\*225 +540

=99.009 rupee million

X2 = (Z1\* σ)+ μ

= 1.95996\*225 +540

=980.991 rupee million

So range is {99.009 , 980.991 }

Or another method we can use stats.norm.interval(.95,loc=540,scale=255)

(99.00810347848784, 980.9918965215122)

1. Specify the 5th percentile of profit (in Rupees) for the company

5% = .05

Z(.05) = round(stats.norm.ppf(0.05),5) = -1.64485

Z = (X- μ)/ σ

X = (Z(.05) \* σ)+ μ

= -1.64485\*225+540

=169.90875