Data Structures and Algorithms

Lecture 9: Binary Search Trees

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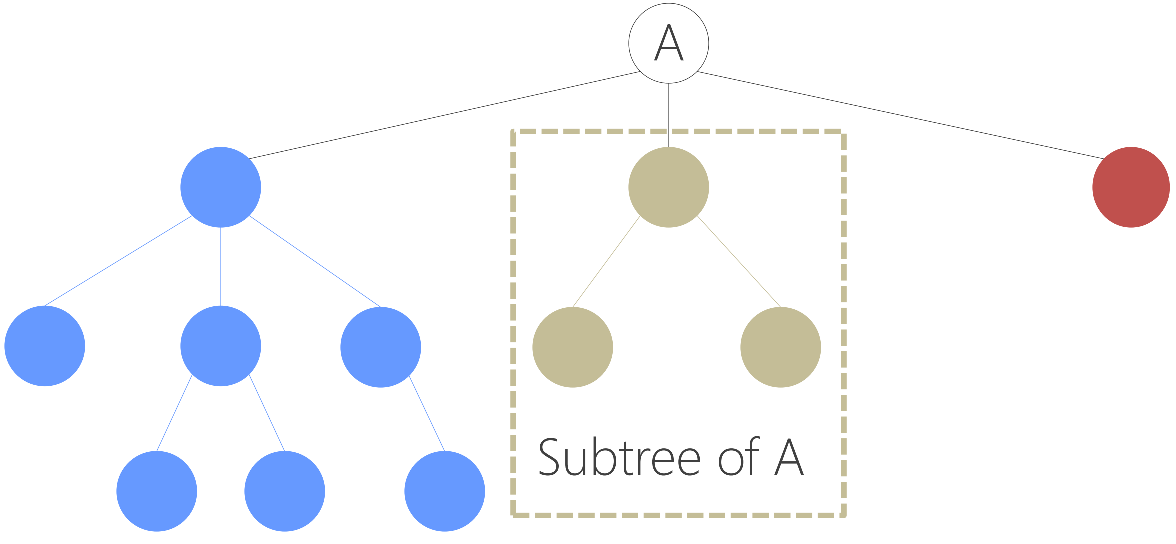
Outline

- Trees
  - Basic Concepts
- Binary Trees
  - Tree Traversal
- Binary Search Trees
  - Find
  - Insert
  - Delete

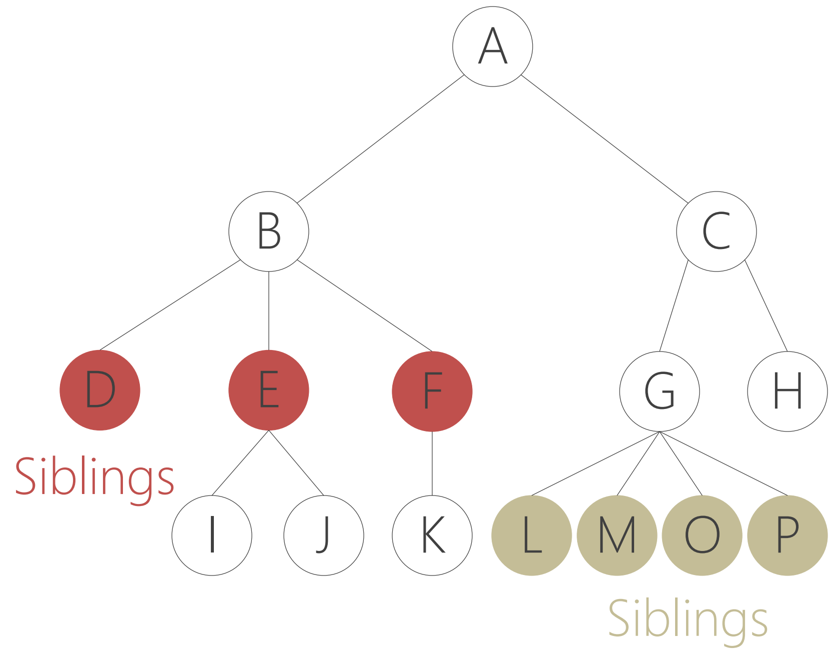
Trees

- A tree is a collection of nodes
  - The collection can be empty
  - (recursive definition) If not empty, a tree consists of a distinguished node r (the root), and zero or more nonempty subtrees T_1, T_2, ..., T_k, each of whose roots are connected by a directed edge from r



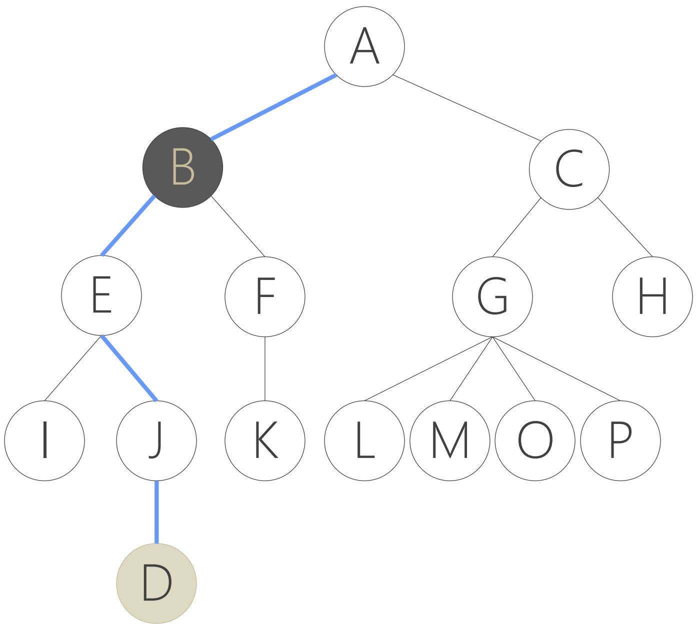
Some Terminologies

- Root and Leaf
- Child and Parent
  - Every node except the root has one parent
  - A node can have an zero or more children
  - A leaf node has no children
- Sibling
  - nodes with same parent



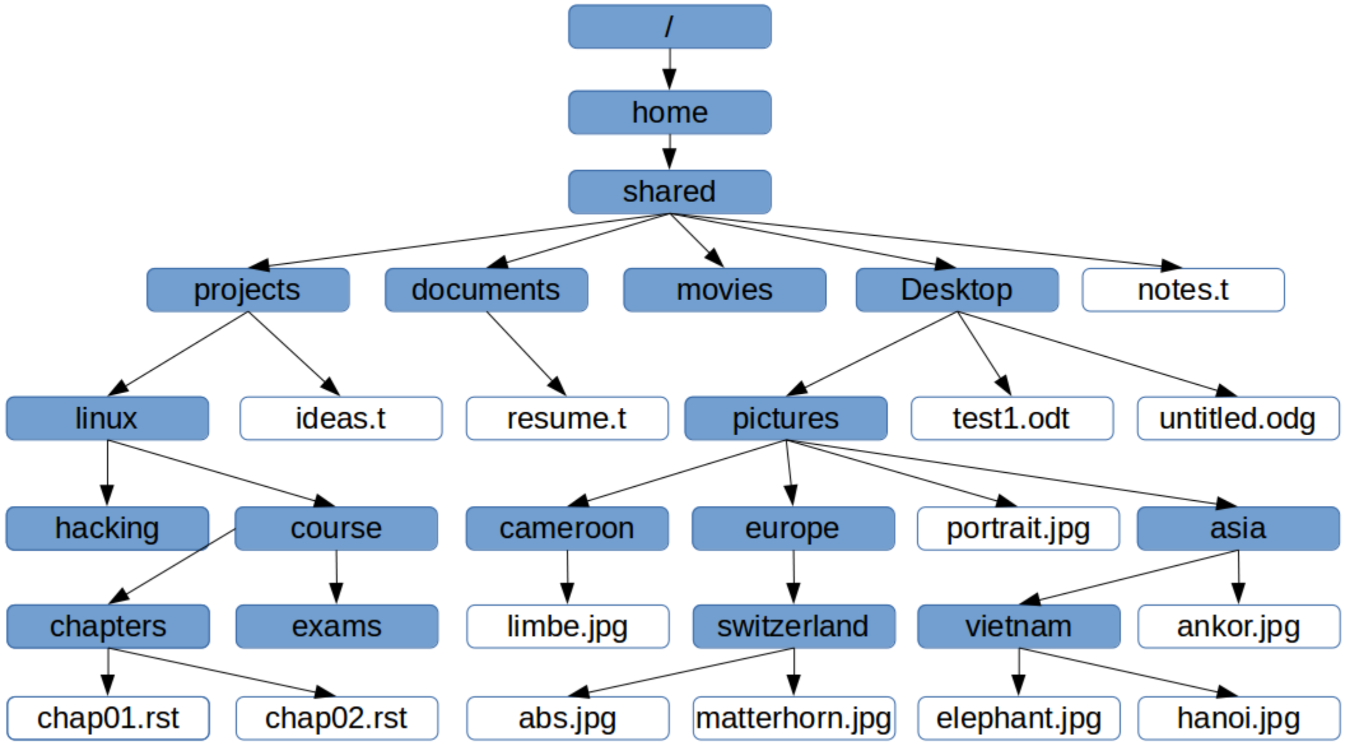
More Terminologies

- Path
  - a sequence of edges
- Length of a path
- Depth of a node
  - length of the unique path to the root
- Height of a node
  - length of the longest path to a leaf
- Tree height
  - the height of the root
  - the depth of the deepest leaf
- Ancestor and descendant
  - If there is a path from n1 to n2
  - n1 is an ancestor of n2, n2 is a descendant of n1
  - Proper ancestor and proper descendant

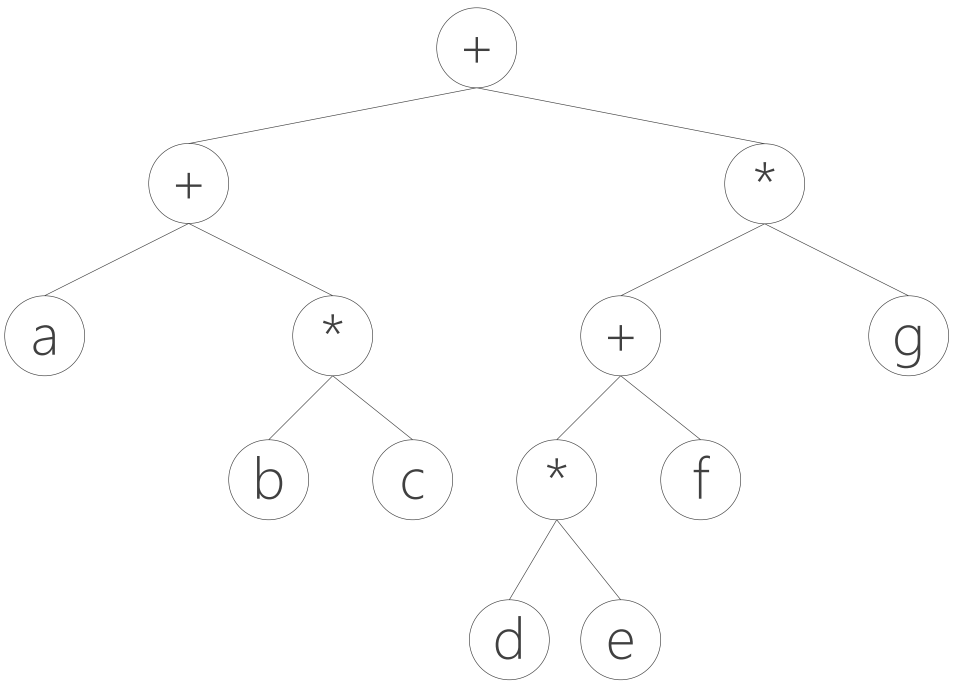


Length of the blue path = 4
Depth(B) = 1
Height(B) = 3
B is D's Ancestor
D is B's Descendant

Example: UNIX Directory



Example: Expression Trees

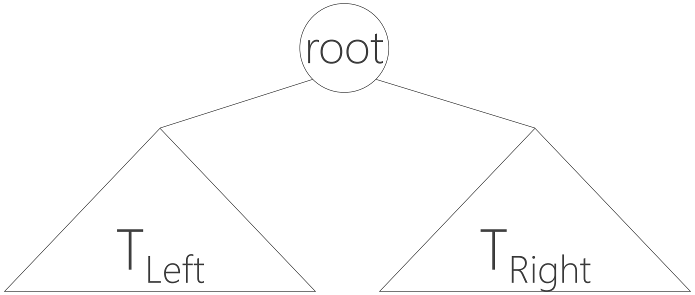


Expression tree for: (a + b*c) + (d*e + f) * g

- Leaves are operands (constants or variables)
- The internal nodes contain operators
- Will not be a binary tree if some operators are not binary

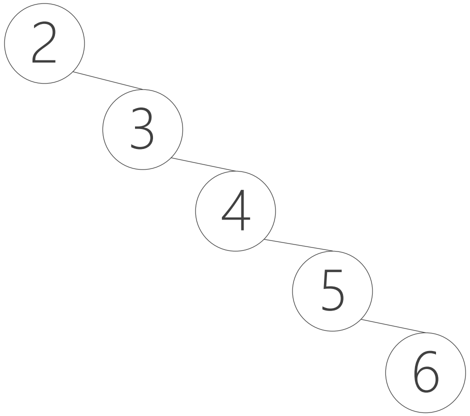
Binary Trees

- A tree in which no node can have more than two children



Generic binary tree

- The depth of an "average" binary tree is considerably smaller than N, even though in the worst case, the depth can be as large as N – 1.



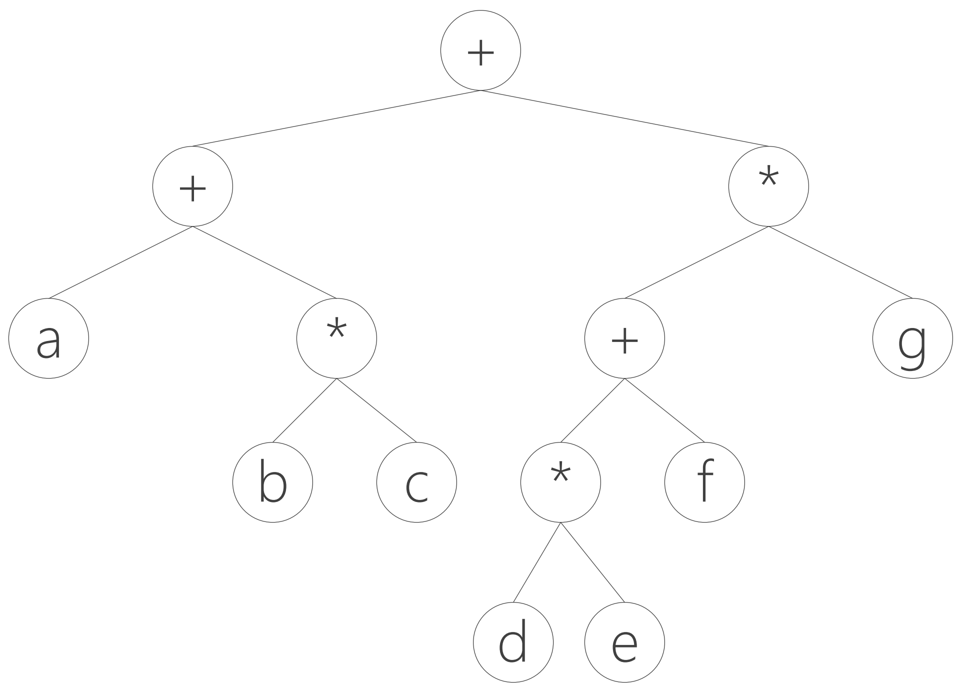
Worst-case binary tree

Binary Tree Traversal

- Three strategies for tree nodes enumeration
- Pre-order traversal
  - Recursive algorithm
  - First visit the root, then the left subtree, then the right
- In-order traversal
  - Recursive algorithm
  - First visit the left subtree, then the root, then the right subtree
- Post-order traversal
  - Recursive algorithm
  - First visit the left subtree, then the right, then the root

Pre-order Traversal

- node, left, right
- prefix expression
  - ++a*bc*+*defg



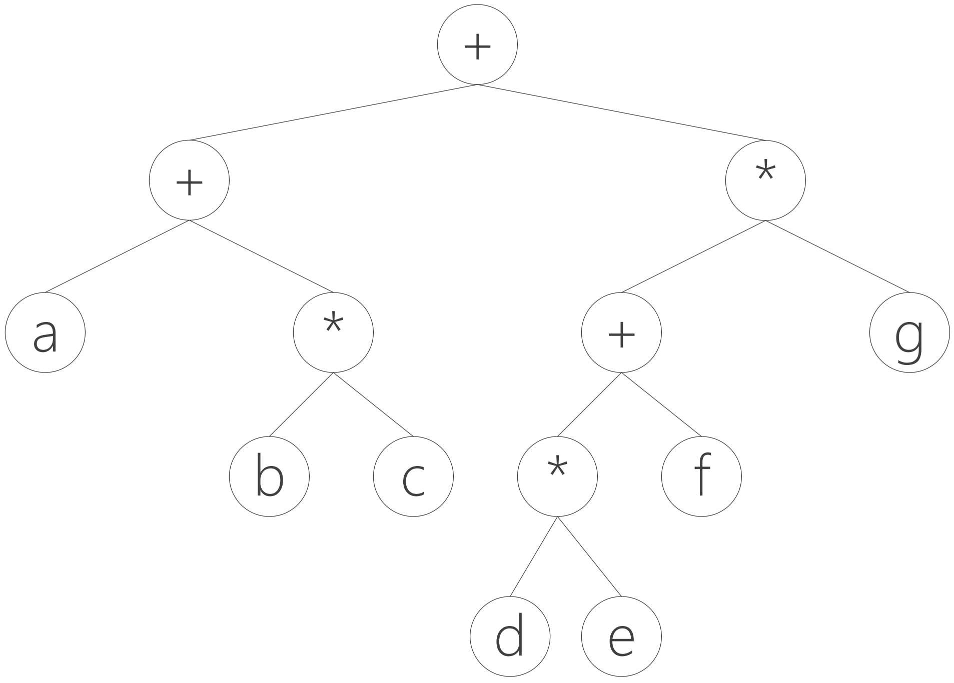
Expression tree for: (a + b*c) + (d*e + f) * g

Post-order Traversal

- left, right, node
- postfix expression
  - abc*+de*f+g*+

In-order Traversal

- left, node, right
- infix exression
  - a+b*c+d*e+f*g



Expression tree for: (a + b*c) + (d*e + f) * g

Pseudo Code for Pre-order, In-order and Post-order

```
PREORDER(root)
1.    IF root = Null
2.        return
3.     PRINT(root)
4.     PREORDER(LEFT(root))
5.     PREORDER(RIGHT(root))
```

```
INORDER(root)
1.    IF root = Null
2.        return
3.     INORDER(LEFT(root))
4.     PRINT(root)
5.     INORDER(RIGHT(root))
```

```
POSTORDER(root)
1.    IF root = Null
2.        return
3.     POSTORDER(LEFT(root))
4.     POSTORDER(RIGHT(root))
5.     PRINT(root)
```

Node Struct of Binary Tree

- Possible operations on the Binary Tree ADT
  - Parent, left, right, sibling, root, etc
- Implementation
  - Because a binary tree has at most two children, we can keep direct pointers to them

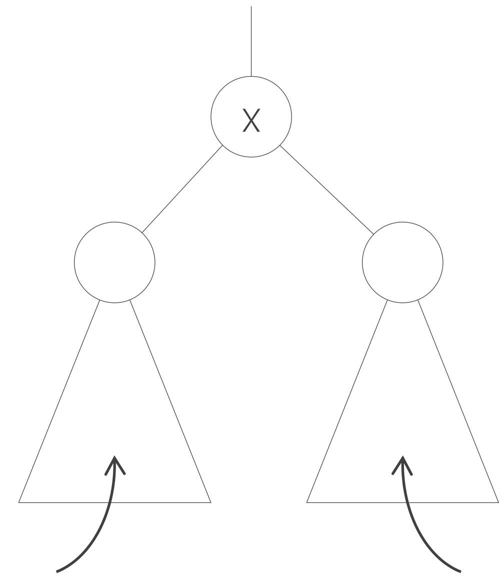
```
typedef struct BinaryNode{
    object data;
    BinaryNode *left;
    BinaryNode *right;
}BinaryNode;
```

A binary tree which offers directed search

BINARY SEARCH TREES

Binary Search Trees (BST)

- Binary search tree property
  - For every node X
  - All the keys in its left subtree are smaller than the key value in X
  - All the keys in its right subtree are larger than the key value in X
- Pre-assumption
  - Objects are stored in tree nodes
    - Book information: ISBN, Title, author, abstract, price, …
  - The keys of the objects are used for search and comparison
    - ISBN



For any node y in this subtree
Key(y)<key(x)

For any node z in this subtree
Key(z)>key(x)

No Duplicates!

For easy demonstration, we store just

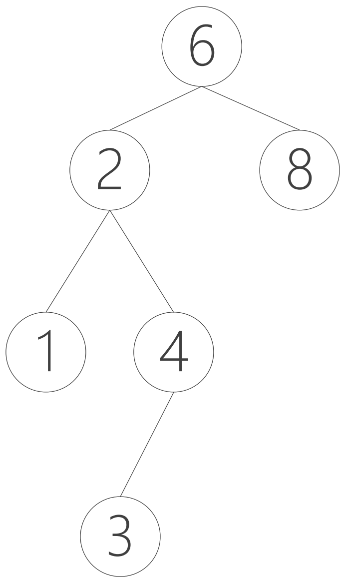
integer keys in the tree nodes, but

be noted that in practice, it is usually

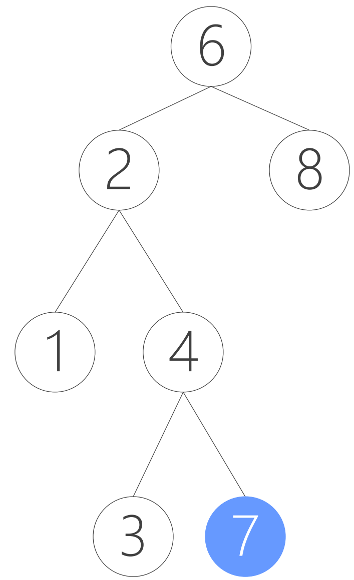
OBJECTS WITH KEYS

that are stored and are later inserted, deleted and searched.

Binary Search Tree Example



A binary search tree

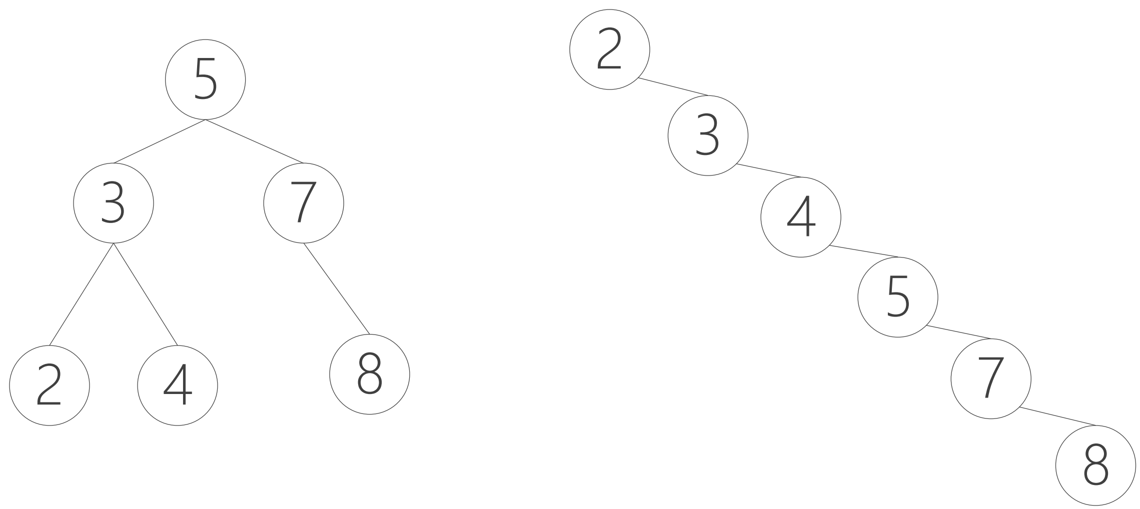


Not a binary search tree

WHY?

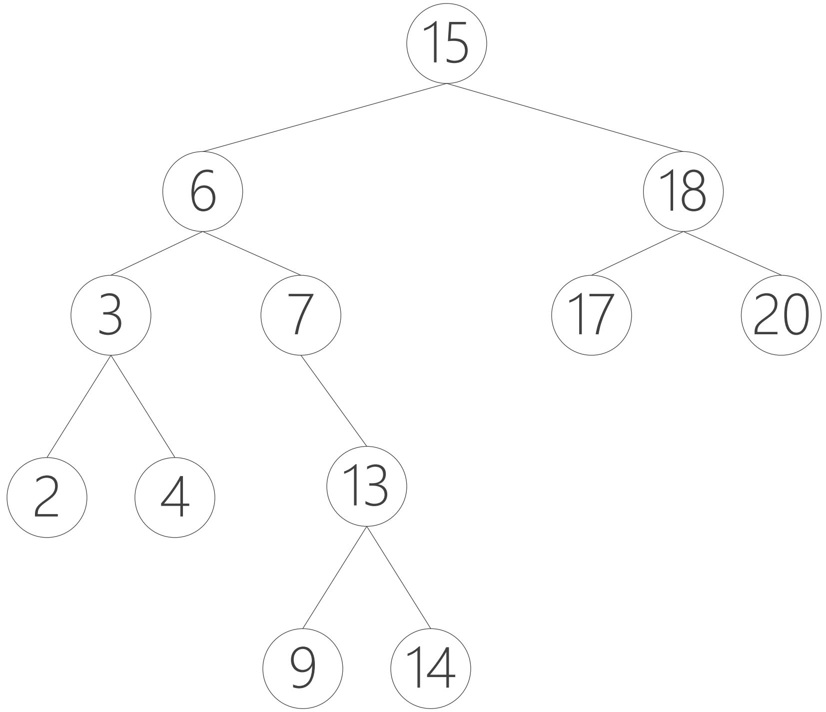
Binary Search Trees

The same set of keys may have different BSTs



- The order of node insertion affects the shape of the tree
- Maximum depth of a node is n-1

In-order Traversal of BST



2, 3, 4, 6, 7, 9, 13, 14, 15, 17, 18, 20

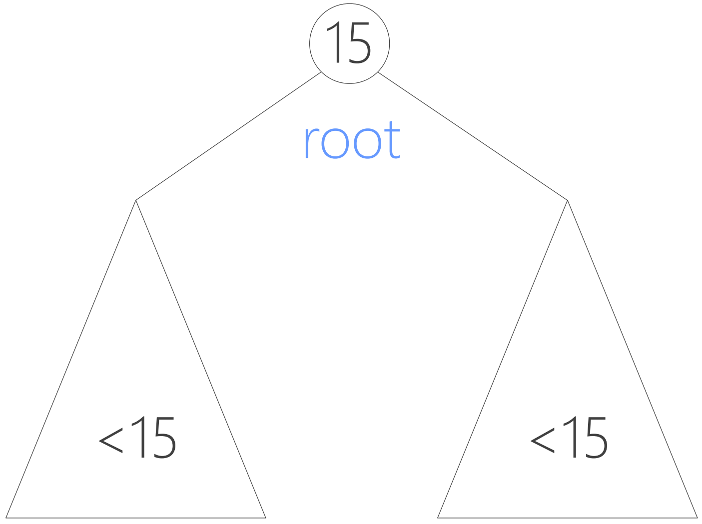
A sorted list!

Operations on Binary Search Trees

- **SEARCH**
- **FINDMIN / FINDMAX**
- **INSERT**
- **DELETE**

Searching BST

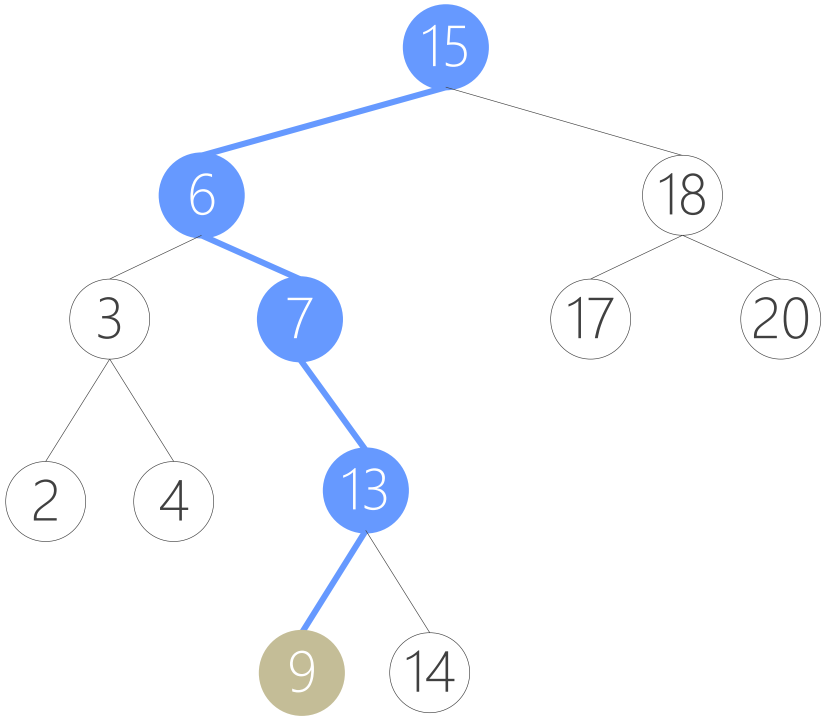
- The current root is 15
  - If we are searching for 15, then we are done.
  - If we are searching for a key < 15, then we should search in the left subtree.
  - If we are searching for a key > 15, then we should search in the right subtree.



Directed Search

Search for 9:

1. Compare 9:15, go left
2. Compare 9:6, go right
3. Compare 9:7, go right
4. Compare 9:13, go left
5. Compare 9:9, found it!



Pseudo Code for Search

- FIND(root, x)
  - Searches the subtree rooted at root
  - Returns a pointer to the node whose key is x
  - Returns Null if no such node exists
- Time complexity: O(tree height)

```
FIND(root, x)
IF root=Null
    return Null
IF root->key=x
    return root
IF root->key>x
    return FIND(root->left, x)
return FIND(root->right, x)
```

findMin / findMax

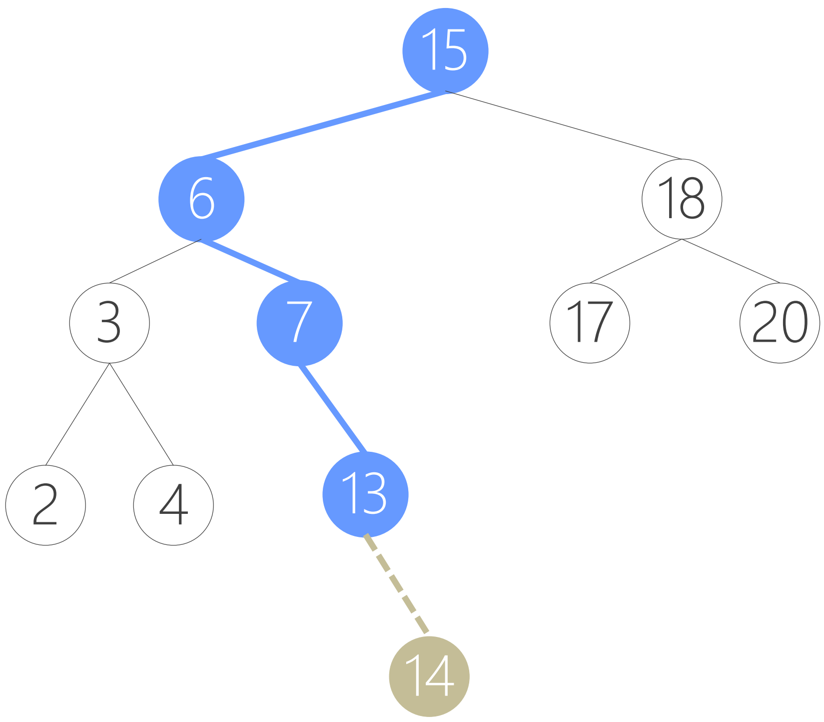
- Goal: return the node containing the smallest (largest) key in the tree
- Algorithm: Start at the root and go left (right) as long as there is a left (right) child. The stopping point is the smallest (largest) element
- Time complexity: O(tree height)

```
FIND-MIN(root)
    IF root=Null
        return Null
    IF root->left=Null
        return root
    return FIND-MIN(root->left)
```

```
FIND-MAX(root)
    IF root=Null
        return Null
    IF root->right=Null
        return root
    return FIND-MAX(root->right)
```

Insertion

- Insert(root, x)
  - Proceed down the tree as you would with a find
  - If x is found, do nothing (reject duplicates)
  - Otherwise, insert x at the last spot on the path traversed
- Time complexity = O(tree height)



Insert(root, 14)

Pseudo Code for Insertion

```
INSERT(root, x)
    IF root=Null
        return root=CREATE-NODE(x)
    IF root->key=x
        return Null
    IF root->key>x
        return INSERT(root->left, x)
    ELSE
        return INSERT(root->right, x)
```

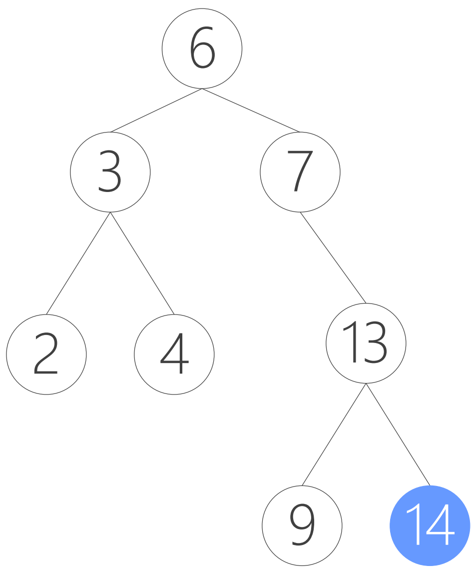
What data structure should `root` be in a C implementation?

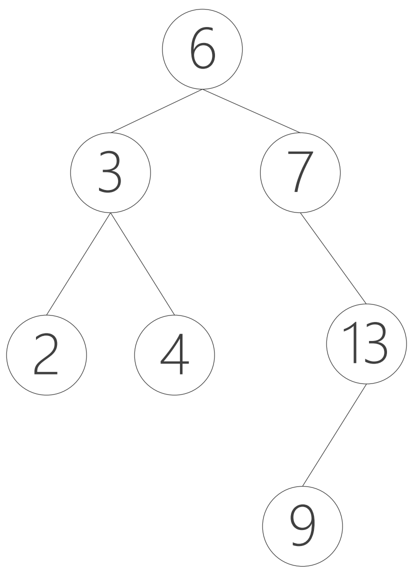
Deletion

- When we delete a node, we need to consider how we take care of the children of the deleted node.
- This has to be done such that the property of the search tree is maintained.

Three Delete Cases

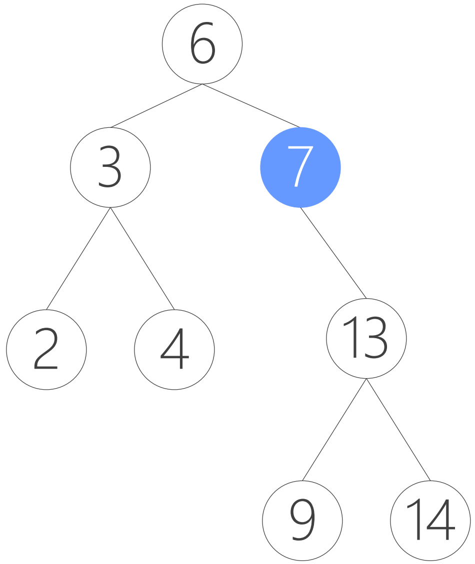
- Case 1: the node is a leaf
  - Delete it immediately

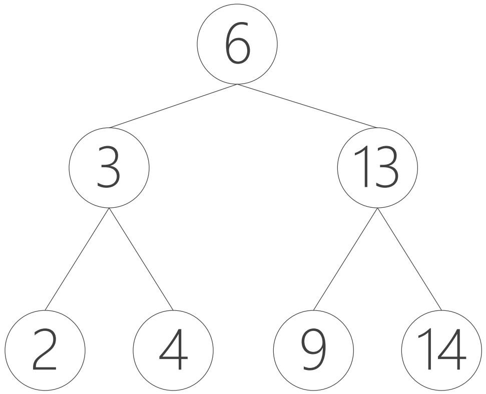




Three Delete Cases

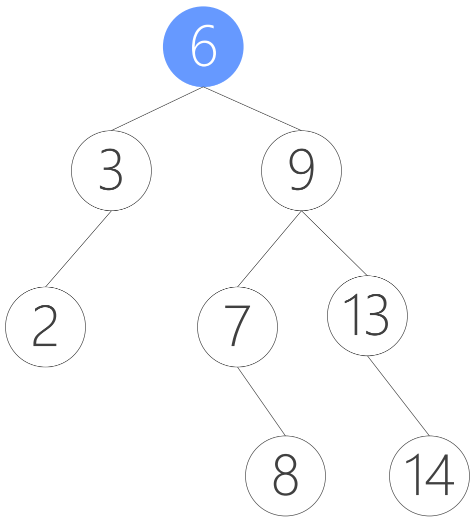
- Case 2: the node has one child
  - Adjust a pointer from the parent to bypass that node

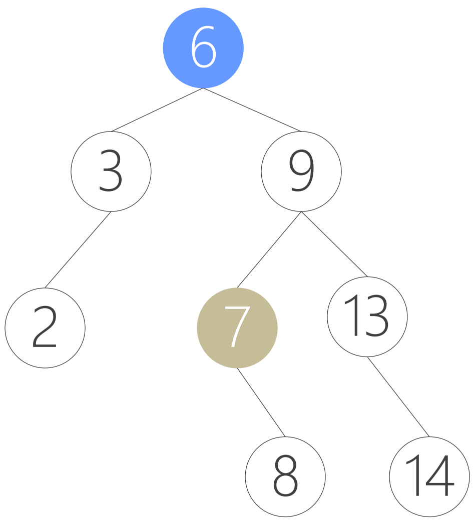


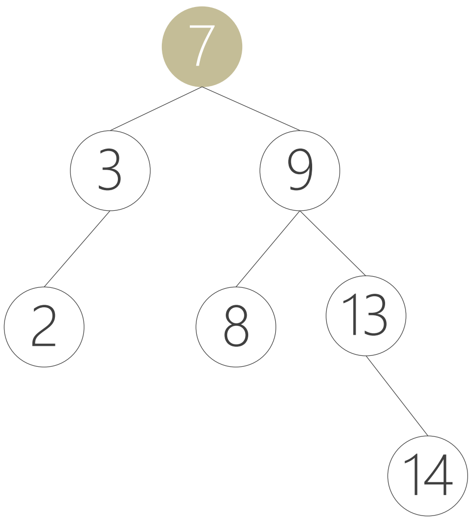


Three Delete Cases

- Case 3: the node has two children
  - Replace that node with the minimum node in the right subtree
  - This invokes delete of that minimum node
    - It's case 1 or 2. WHY?
- Time complexity = O(tree height)







Pseudo Code?

The cost of search, insert and delete are all bounded by the **TREE HEIGHT** which is $O(n)$ in the worst case. And $O(n)$ is

NOT FAST ENOUGH!

Task

- Given BST.h, printTree.cpp and main.cpp, complete BST.cpp
  - BST.h: the header file which defines the data and the methods of a binary search tree
  - printTree.cpp: implements the printTree method defined in BST.h
  - BST.cpp: implements the remaining methods defined in BST.h
    - To be completed by you
    - This is the only file that you are going to modify
    - You may add auxiliary functions if there is a need
  - main.cpp: a main function for testing purpose
- Submit BST.cpp to iSpace.