Department of Computer Science and Technology
United International College

Submission Deadline: March 25

COMP3190: Data Structures
WRITTEN ASSIGNMENT 1 — Lists and Algorithm Analysis

Problem 1. (20 marks)
For each pair of $f(n)$ and $g(n)$ below, decide if $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$. Justify your answer using the definitions of these asymptotic notation. Note that more than one of these relations may hold for a given pair; list all correct ones.

(a) $f(n) = \sqrt{n}$ and $g(n) = \sqrt{n + n^2}$.

(b) $f(n) = (\log_3 n)^2$ and $g(n) = \log_2(n^3)$.

(c) $f(n) = 2^n$ and $g(n) = 2^{2n}$.

(d) $f(n) = \log_2(n!)$ and $g(n) = n \log_2 n$.

Problem 2. (20 marks)
Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.

Hint: You can

- *prove a conjecture using its definition*
- *or disprove a conjecture by giving negative examples.*

(a) $f(n) = \Theta(f(n) + 1)$.

(b) $f(n) = \Theta(f(n + 1))$.

Problem 3. (15 marks)
Solve the following recurrence relation: $T(1) = 1, T(n) = 2T(\frac{n}{4}) + n$, where $n > 1$.

Problem 4. (25 marks)
Suppose that each row of an $n \times n$ matrix A consists of 1's and 0's such that, in any row of A,

- all the 1's come before any 0's in that row;
- the number of 1's in the rows is non-decremental.

(a) Describe a method running in $O(n)$ time (not $O(n^2)$ time) for calculating the number of 1's in matrix A. Use either C codes or pseudo codes.

(b) Analyze your algorithm.

1	1	0	0	0
1	1	0	0	0
1	1	1	1	0
1	1	1	1	0
1	1	1	1	1

Table 1: Sample input - 5×5 array A, the output is 17