Statistical Inference Project Part 1

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Instructions

1. Show the sample mean and compare it to the theoretical mean of the distribution.
2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
3. Show that the distribution is approximately normal.

Loading Libraries

```
library("data.table")
library("ggplot2")
```

Task

```
# set seed for reproducability
set.seed(31)

# set lambda to 0.2
lambda <- 0.2

# 40 samples
n <- 40

# 1000 simulations
simulations <- 1000

# simulate
simulated_exponentials <- replicate(simulations, rexp(n, lambda))

# calculate mean of exponentials
means_exponentials <- apply(simulated_exponentials, 2, mean)
```
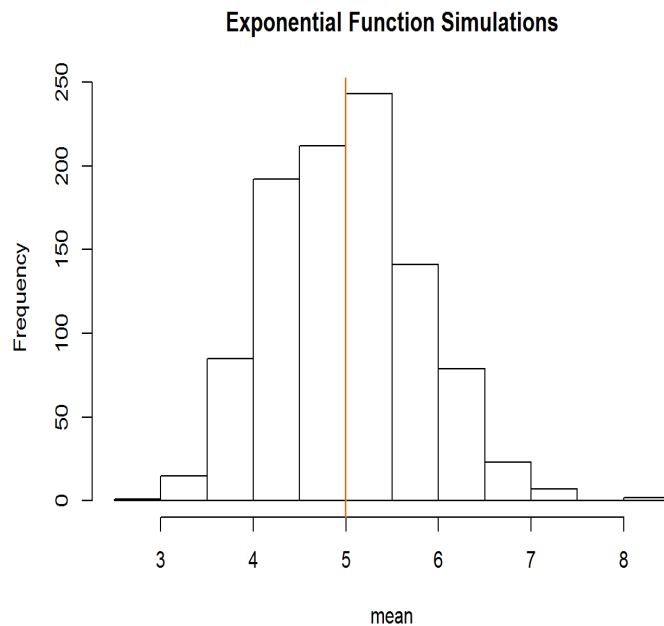
Question 1

Show where the distribution is centered at and compare it to the theoretical center of the distribution.

```
analytical_mean <- mean(means_exponentials)
analytical_mean
## [1] 4.993867
# analytical mean
theory_mean <- 1/lambda
theory_mean
## [1] 5
# visualization
hist(means_exponentials, xlab = "mean", main = "Exponential Function Simulations")
abline(v = analytical_mean, col = "red")
abline(v = theory_mean, col = "orange")
```

Exponential Function Simulations

The analytics mean is 4.993867 the theoretical mean 5. The center of distribution of averages of 40 exponentials is very close to the theoretical center of the distribution.

Question 2

Show how variable it is and compare it to the theoretical variance of the distribution..

```
# standard deviation of distribution
standard_deviation_dist <- sd(means_exponentials)
standard_deviation_dist
## [1] 0.7931608
# standard deviation from analytical expression
standard_deviation_theory <- (1/lambda)/sqrt(n)
standard_deviation_theory
## [1] 0.7905694
# variance of distribution
variance_dist <- standard_deviation_dist^2
variance_dist
## [1] 0.6291041
# variance from analytical expression
variance_theory <- ((1/lambda)*(1/sqrt(n)))^2
variance_theory
## [1] 0.625
```

Standard Deviation of the distribution is 0.7931608 with the theoretical SD calculated as 0.7905694. The Theoretical variance is calculated as $((1 / ??) * (1/???n))^2 = 0.625$. The actual variance of the distribution is 0.6291041

# Question 3

Show that the distribution is approximately normal.

```
xfit <- seq(min(means_exponentials), max(means_exponentials), length=100)
yfit <- dnorm(xfit, mean=1/lambda, sd=(1/lambda/sqrt(n)))
hist(means_exponentials,breaks=n,prob=T,col="orange",xlab = "means",main="Density of means",ylab="density")
lines(xfit, yfit, pch=22, col="black", lty=5)
```

Density of means
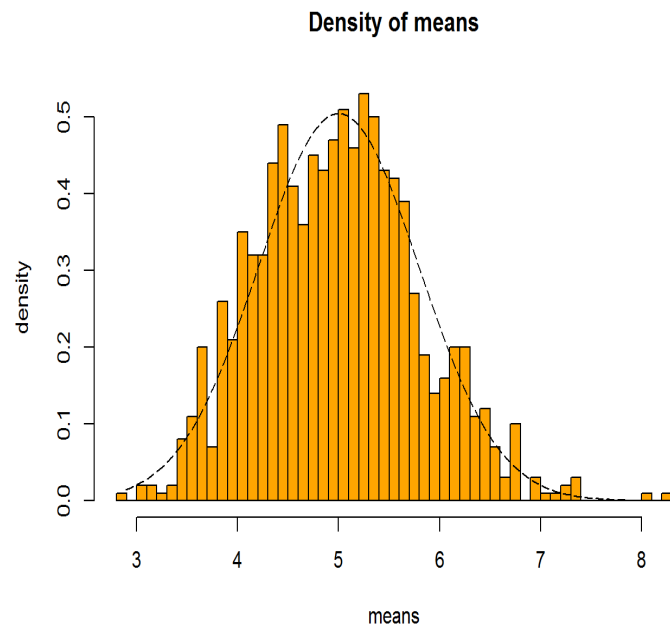

```
# compare the distribution of averages of 40 exponentials to a normal distribution
qqnorm(means_exponentials)
qqline(means_exponentials, col = 2)
```

Normal Q-Q Plot
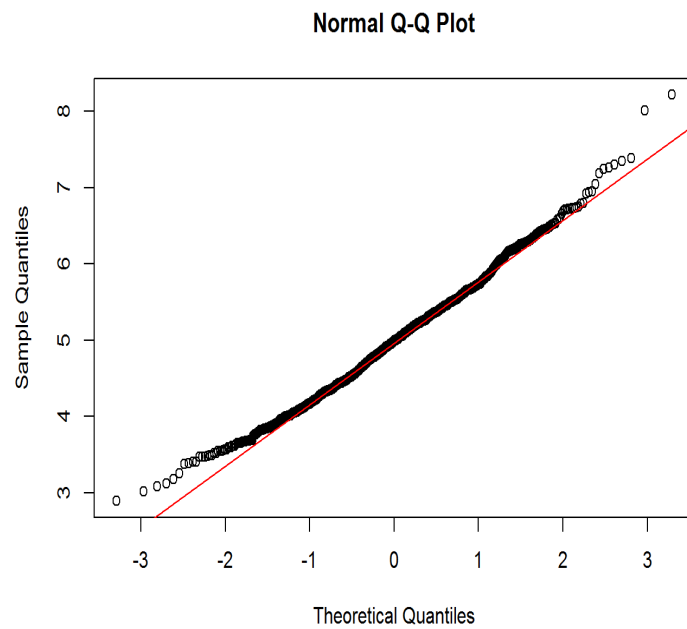
Due to Due to the central limit theorem (CLT), the distribution of averages of 40 exponentials is very close to a normal distribution.