Trees

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Definition

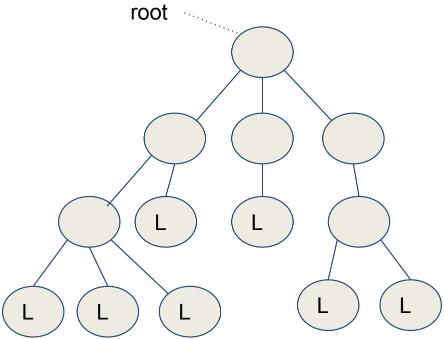
A tree is composed of nodes

Each node can have 0 or more children

A special node called root is the top of the tree

- Computer science trees grow upside-down!

Nodes with 0 children are called leaf nodes



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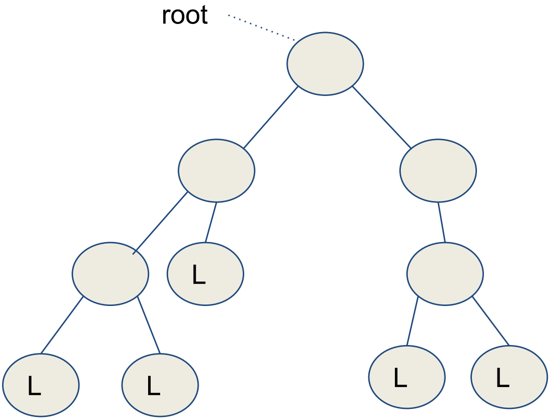
In a binary tree all nodes have no more than 2 children



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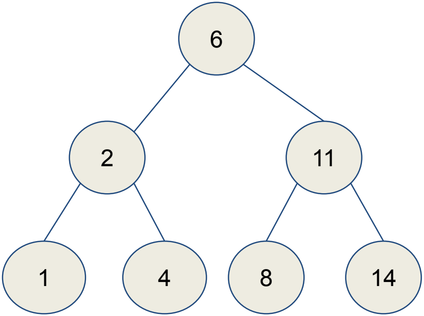
Ordered Binary Trees/Binary Search Tree

An Ordered Binary Tree is a binary tree where

- Each node contains a key
- For each node
  $\text{key}_{\text{left child}} < \text{key}_{\text{this node}}$
  $\text{key}_{\text{right child}} \geq \text{key}_{\text{this node}}$
  or
  $\text{key}_{\text{left child}} \leq \text{key}_{\text{this node}}$
  $\text{key}_{\text{right child}} > \text{key}_{\text{this node}}$
- Furthermore this rule applies to the whole subtree

- Example: anything to the right of the root ≥ 6



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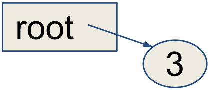
```
class Node {
  public int val;
  public Node left, right;
}
```



Tree must define a Node

```
OrderedBinaryTree.insert(v)
  if root == null
    root = new Node(v)
    return // special case for empty tree
  end
```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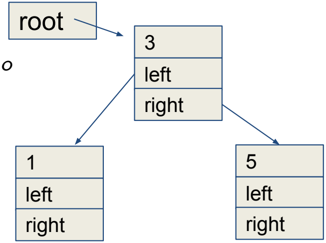
Insert into OrderedBinaryTree, contd

```
p <- root
   while p != null
      if p.val < v // can be <= also
         if p.right == null
            p.right <- new Node(v)
            return
         end
      else
         if p.left == null
            p.left <- new Node(v)
            return
         end
    end
   p <- p.next
```



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There are three classic traversal algorithms for binary trees

- Inorder
- Preorder
- Postorder

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Inorder

```
inorder(node)
  if node == null
    return // termination condition for recursion!
  end
  inorder(node.left)
  do whatever you want to do to this node (like print it)
  inorder(node.right)
end
```


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Preorder

```
preorder(node)
  if node == null
    return // termination condition for recursion!
  end
  do whatever you want to do to this node (like print it)
  preorder(node.left)
  preorder(node.right)
end
```


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Postorder

```
postorder(node)
  if node == null
    return // termination condition for recursion!
  end
  postorder(node.left)
  postorder(node.right)
  do whatever you want to do to this node (like print it)
end
```


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Another way of Terminating Recursive Traversals

The following code requires testing first that root is not null, but it is more efficient

Complexity is the same

```
postorder(node)
  if node.left != null
    postorder(node.left)
  end
  if node.right != null
    postorder(node.right)
  end
  do whatever you want to do to this node (like print it)
end
```


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Expressions and Relationship to Trees

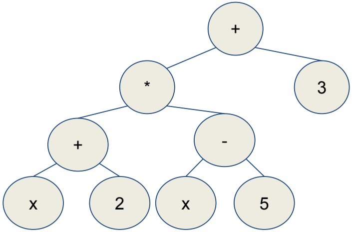
In mathematics we are taught to write expressions inline

$(x+2)(x-5)+3$

The operations the above expression represents have an order

Conventions are defined mathematically

The end result is a tree of operations like this:



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Three Ways of Writing Expressions

Inorder: $(x+2)(x-5)$

Preorder: $* + x2 - x5$

Postorder: $x2 + x5 - *$

All 3 reflect the same tree of operations, traversed using inorder, preorder, postorder

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Sorting Using an Ordered Binary Tree

An ordered binary tree is intrinsically sorted

Just add elements to the tree

Print out the elements using inorder traversal

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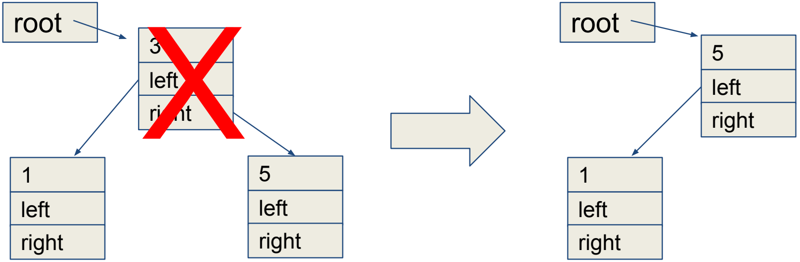
Deletion from an Ordered Binary Tree

To delete, remove a node, then promote one of the children in its place



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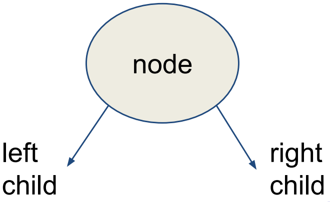
Implementation of an Ordered Binary Tree

```
class OrderedBinaryTree {
private:
  class Node {
  public:
    Node* left;
    Node* right;
    int val;
  };
  Node* root; // an empty tree has root=null
  ...
}
```



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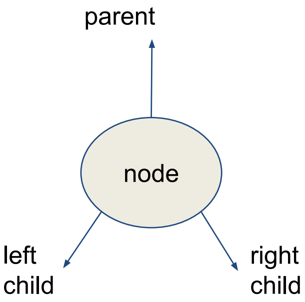
Adding a Parent Pointer

For some applications, if the algorithm is looking at a node it is useful to also be able to find the parent

```
class Node {
public:
  Node* left;
  Node* right;
  Node* parent;
  int val;
};
```



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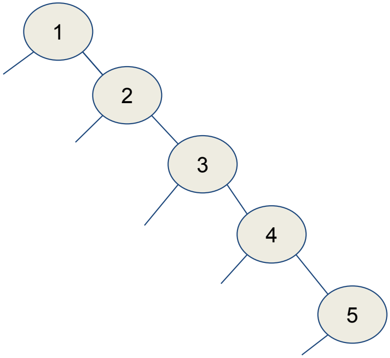
Ordered Binary Trees/Binary Search Trees can get lopsided

- Performance is as bad as lists
- Added overhead of the unused left pointers

Example: Add 1,2,3,4,5 to ordered binary tree

What is the complexity of insertion into a BST?

O(?)



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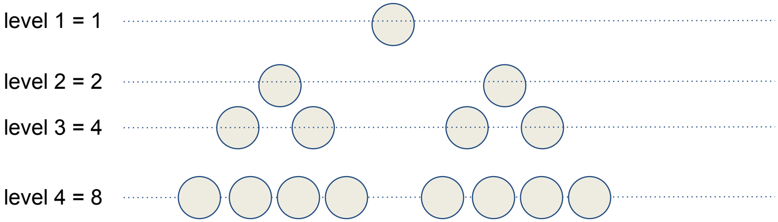
Balanced Case

When a Tree is balanced, n elements fit into $log_2 n$ levels



$$n = 15 \text{ elements, levels} = log_2(n+1) = 4$$

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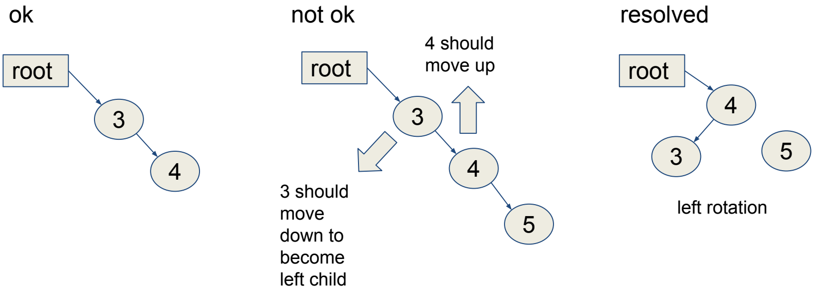
Trees Cannot be Perfectly Balanced

A Tree with one element on a new level is by definition, unbalanced



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Balanced Binary Trees

Ensure trees remain balanced under all possible operation

Insertion and deletion will have to trigger "rebalancing" while not slowing down

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There are a number of implementations

- Red-Black Trees (RB-Trees)
- AVL Trees
- Fibonacci Trees

We will just do one: RB-Trees

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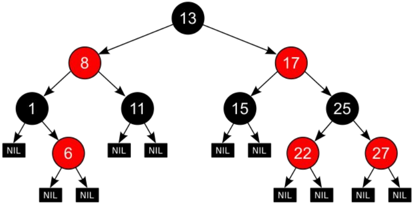
RB Trees

Objective: Maintain a balanced tree no matter the order of insertion

No worse than $2\log_2 n$

Rules

- Each node is assigned a color (red or black, really just binary)
- Red nodes cannot have red children
  - Colors alternate
- Optional: root is black



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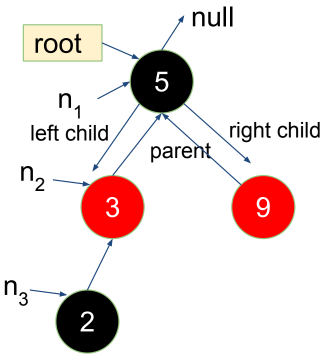
n_1.parent is node that has n as a child. This is null for root

n.grandparent() is n's parent's parent, null if it does not exist

n.uncle is a node's grandparent's other child (not parent)

- If a node has no grandparent, it has no uncle
- n_2 has no uncle
- n_3 uncle is 9



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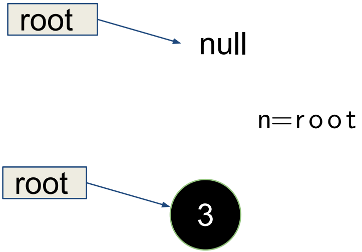
Insertion into RB Tree

```
Start with an empty tree
RBTree.insert(3)
  calls BST.insert(3)
  then RBTree.correct(n)
```




```
case 1:
if n.parent == null
  return // special case, root
```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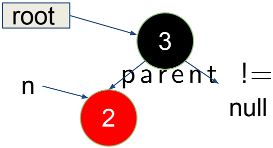
Insertion into RB Tree, part 2

```
RBTree.insert(1)
   calls BST.insert(1)
   then RBTree.correct(n)
```




```
parent != null
```

```
case 2:
if parent.color == BLACK
    return
end
```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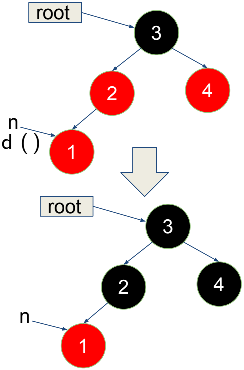
Insertion into RB Tree, part 3

```
RBTree.insert(1)
  calls BST.insert(1)
  then RBTree.correct(n) parent != null
parent is RED
case 3:
  if n.parent.isred() and n.uncle().isred()
    flip color of parents
    flip color of grandparent
    RBTree.correct(grandparent)
recursive
  end
```



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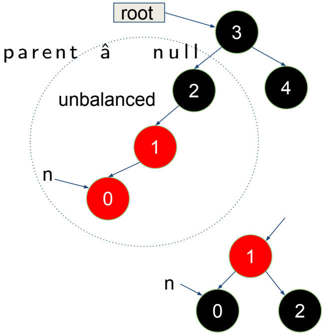
Insertion into RB Tree, part 3

```
RBTree.insert(0)
   calls BST.insert(0)
   then RBTree.correct(n)
parent is RED
[TBD]
```



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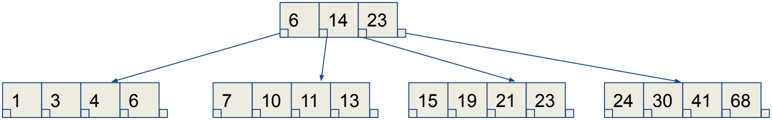
BTrees are a more complicated generalization of RBTree with a higher degree than 2

Used in database systems to try to keep the depth of the tree to a minimum

Shown: BTree of degree 4



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Properties of Disk Access

Hard Disk Drives, and the more modern Solid State Drives (SSD)

- Are Block-oriented devices
- Have high overhead to access a block (latency, and OS call)
- Therefore, try to keep reads to a minimum
- Each block is large, so degree of the BTree can be kept very high (64)

With $n = 64 \ (2^6)$ we can store

Levels	Power of 2	Elements stored
1	2^6	64
2	2^{12}	4096
3	2^{18}	256k
4	2^{24}	16M
5	2^{30}	1 billion

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How BTrees are used in Databases

SQL was developed in the 1970s

Creates tables of data, typically with lookup via BTree

The following table defines a primary key

- The records are stored in a BTree using the key
- The records therefore are in sorted order
- Searching for a specific city by name is O(log n)

```
CREATE TABLE Cities (
name varchar(10) primary key,
population int
)
```


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BTrees in SQL

```
SELECT * FROM cities                                   results sorted by name
SELECT * FROM cities ORDER BY population    requires sorting
SELECT * FROM cities WHERE name='New York' O(log n)
INSERT INTO cities values ('Tokyo', 15000000)    2x slower
```


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Adding an Index

```
CREATE INDEX cities_bypopulation              Second sorting order
ON cities (population);

SELECT * FROM cities ORDER BY population      no sorting

SELECT * FROM cities WHERE population=100    O(log n)
```


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Tries

A Trie is a complete tree with a high degree

- 26 for lowercase English letters
- Has not been used much because storage requirements are high
- Hash maps have dominated tries for most applications

Today, there are applications

- Autocompletion of words
- DNA searches

https://brilliant.org/wiki/tries/

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Code Structure of a Trie

A Trie node has

- boolean stating whether the path is a word or not
- Pointers to the next node

A trie dictionary can contain the root node

- Does not have to be a pointer
- Can never be null

```
class Trie {
  struct Node {
    bool isWord;
    Node* next[26];
  }
  Node root;
public:
  void add(const string& w);
  void remove(const string& w);
  bool contains(const string& w);
  bool startsWith(const string& w)
};
```


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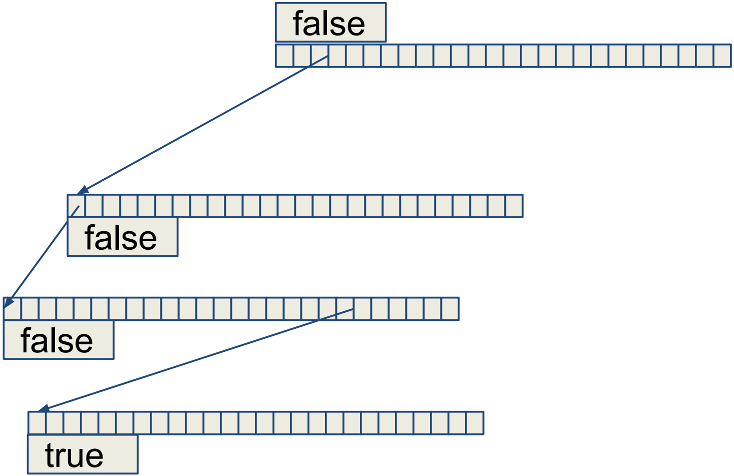
Inserting into a Trie



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Searching for a Word in a Trie

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Searching for a Prefix in a Trie

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