

Supervised vs. Unsupervised Learning

- Supervised learning (**classification**)
 - Supervision: The training data instances and their attributes/features are accompanied by labels indicating the class of the instances.
 - **Predict labels** for testing data instances.
- Unsupervised learning (**clustering**)
 - The class **labels** of training data is **unknown**
 - Given a set of attributes, with the aim of establishing the existence of classes or clusters.

Machine learning types	Data mining tasks
Supervised learning	Classification Regression ...
Unsupervised learning	Clustering Pattern/Association mining ...

Classification: Definition

- Given a collection of records (training set)
 - Each record is by characterized by a tuple (x,y) , where x is the attribute set and y is the class label
 - ◆ x : attribute, predictor, independent variable, input
 - ◆ y : class, response, dependent variable, output
- Task:
 - Learn a model that maps each attribute set x into one of the predefined class labels y

Examples of Classification Task

Task	Attribute set, x	Class label, y
Categorizing email messages	Features extracted from email message header and content	spam or non-spam
Identifying tumor cells	Features extracted from MRI scans	malignant or benign cells
Cataloging galaxies	Features extracted from telescope images	Elliptical, spiral, or irregular-shaped galaxies

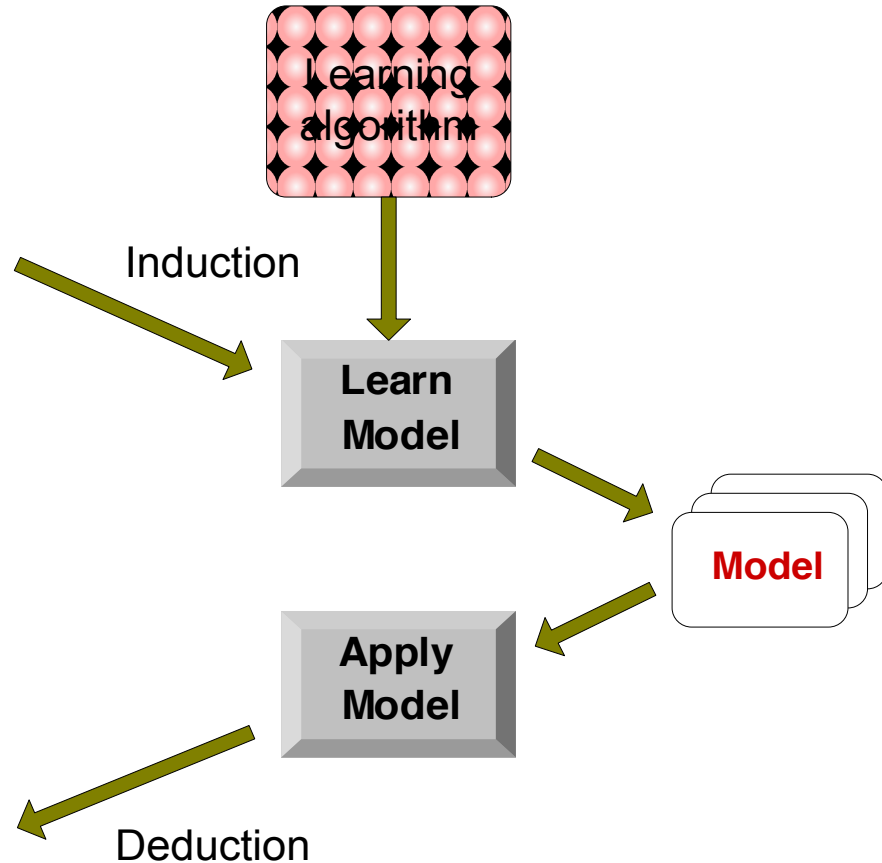
General Approach for Building Classification Model

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



Classification Techniques

- Base Classifiers
 - Decision Tree based Methods
 - Rule-based Methods
 - Nearest-neighbor
 - Neural Networks
 - Naïve Bayes and Bayesian Belief Networks
 - Support Vector Machines
- Ensemble Classifiers
 - Boosting, Bagging, Random Forests

Rule-Based Classifier

- Classify records by using a collection of “if...then...” rules
- Rule: $(Condition) \rightarrow y$
 - where
 - ◆ *Condition* is a conjunctions of attributes
 - ◆ y is the class label
 - *LHS*: rule antecedent or condition
 - *RHS*: rule consequent
 - Examples of classification rules:
 - ◆ $(\text{Blood Type}=\text{Warm}) \wedge (\text{Lay Eggs}=\text{Yes}) \rightarrow \text{Birds}$
 - ◆ $(\text{Taxable Income} < 50\text{K}) \wedge (\text{Refund}=\text{Yes}) \rightarrow \text{Evade}=\text{No}$

Rule-based Classifier (Example)

Name	Body Temperature	Skin Cover	Gives Birth	Aquatic Creature	Aerial Creature	Has Legs	Hibernates	Class Label
human	warm-blooded	hair	yes	no	no	yes	no	Mammals
python	cold-blooded	scales	no	no	no	no	yes	Reptiles
salmon	cold-blooded	scales	no	yes	no	no	no	Fishes
whale	warm-blooded	hair	yes	yes	no	no	no	Mammals
frog	cold-blooded	none	no	semi	no	yes	yes	Amphibians
komodo dragon	cold-blooded	scales	no	no	no	yes	no	Reptiles
bat	warm-blooded	hair	yes	no	yes	yes	yes	Mammals
pigeon	warm-blooded	feathers	no	no	yes	yes	no	Birds
cat	warm-blooded	fur	yes	no	no	yes	no	Mammals
guppy	cold-blooded	scales	yes	yes	no	no	no	Fishes
alligator	cold-blooded	scales	no	semi	no	yes	no	Reptiles
penguin	warm-blooded	feathers	no	semi	no	yes	no	Birds
porcupine	warm-blooded	quills	yes	no	no	yes	yes	Mammals
eel	cold-blooded	scales	no	yes	no	no	no	Fishes
salamander	cold-blooded	none	no	semi	no	yes	yes	Amphibians

- τ_1 : (Gives Birth = no) \wedge (Aerial Creature = yes) \longrightarrow Birds
 τ_2 : (Gives Birth = no) \wedge (Aquatic Creature = yes) \longrightarrow Fishes
 τ_3 : (Gives Birth = yes) \wedge (Body Temperature = warm-blooded) \longrightarrow Mammals
 τ_4 : (Gives Birth = no) \wedge (Aerial Creature = no) \longrightarrow Reptiles
 τ_5 : (Aquatic Creature = semi) \longrightarrow Amphibians

Application of Rule-Based Classifier

- A rule r **covers** an instance x if the attributes of the instance satisfy the condition of the rule

$r_1:$	$(\text{Gives Birth} = \text{no}) \wedge (\text{Aerial Creature} = \text{yes}) \longrightarrow \text{Birds}$
$r_2:$	$(\text{Gives Birth} = \text{no}) \wedge (\text{Aquatic Creature} = \text{yes}) \longrightarrow \text{Fishes}$
$r_3:$	$(\text{Gives Birth} = \text{yes}) \wedge (\text{Body Temperature} = \text{warm-blooded}) \longrightarrow \text{Mammals}$
$r_4:$	$(\text{Gives Birth} = \text{no}) \wedge (\text{Aerial Creature} = \text{no}) \longrightarrow \text{Reptiles}$
$r_5:$	$(\text{Aquatic Creature} = \text{semi}) \longrightarrow \text{Amphibians}$

Name	Body Temperature	Skin Cover	Gives Birth	Aquatic Creature	Aerial Creature	Has Legs	Hibernates
hawk	warm-blooded	feather	no	no	yes	yes	no
grizzly bear	warm-blooded	fur	yes	no	no	yes	yes

The rule r_1 covers a hawk \Rightarrow Bird

The rule r_3 covers the grizzly bear \Rightarrow Mammal

Rule Coverage and Accuracy

- Coverage of a rule:
 - Fraction of records that satisfy the antecedent of a rule
- Accuracy of a rule:
 - Fraction of records that satisfy both the antecedent and consequent of a rule

Tid	Refund	Marital Status	Taxable Income	Class
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

(Status=Single) → No

Coverage = 40%, Accuracy = 50%

How does Rule-based Classifier Work?

r_1 : (Gives Birth = no) \wedge (Aerial Creature = yes) \longrightarrow Birds
 r_2 : (Gives Birth = no) \wedge (Aquatic Creature = yes) \longrightarrow Fishes
 r_3 : (Gives Birth = yes) \wedge (Body Temperature = warm-blooded) \longrightarrow Mammals
 r_4 : (Gives Birth = no) \wedge (Aerial Creature = no) \longrightarrow Reptiles
 r_5 : (Aquatic Creature = semi) \longrightarrow Amphibians

Name	Body Temperature	Skin Cover	Gives Birth	Aquatic Creature	Aerial Creature	Has Legs	Hibernates
lemur	warm-blooded	fur	yes	no	no	yes	yes
turtle	cold-blooded	scales	no	semi	no	yes	no
dogfish shark	cold-blooded	scales	yes	yes	no	no	no

A lemur triggers rule r_3 , so it is classified as a mammal

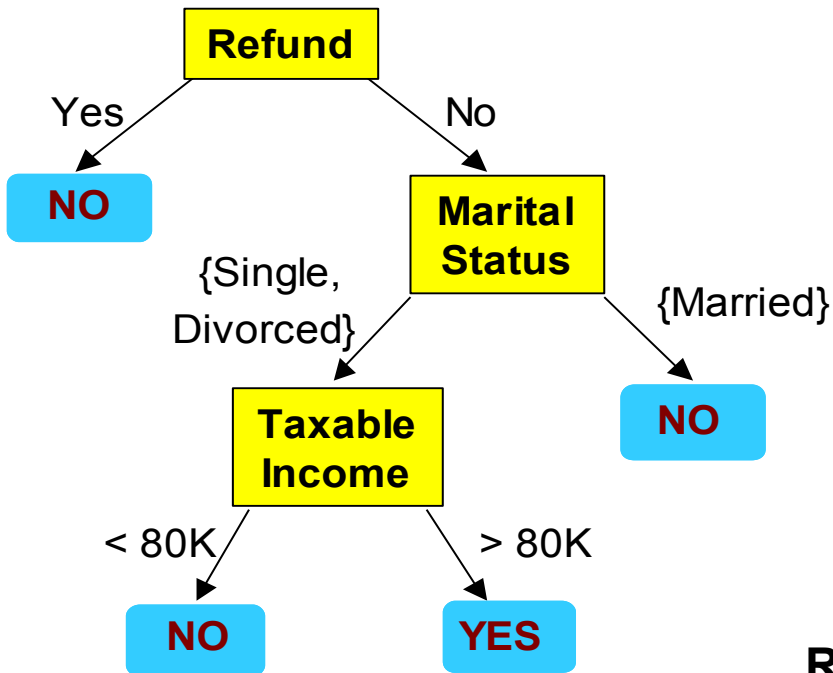
A turtle triggers both r_4 and r_5

A dogfish shark triggers none of the rules

Characteristics of Rule-Based Classifier

- Mutually exclusive rules
 - Classifier contains mutually exclusive rules if the rules are independent of each other
 - Every record is covered by at most one rule
- Exhaustive rules
 - Classifier has exhaustive coverage if it accounts for every possible combination of attribute values
 - Each record is covered by at least one rule

From Decision Trees To Rules



Classification Rules

(Refund=Yes) ==> No

(Refund=No, Marital Status={Single, Divorced}, Taxable Income<80K) ==> No

(Refund=No, Marital Status={Single, Divorced}, Taxable Income>80K) ==> Yes

(Refund=No, Marital Status={Married}) ==> No

Rules are mutually exclusive and exhaustive

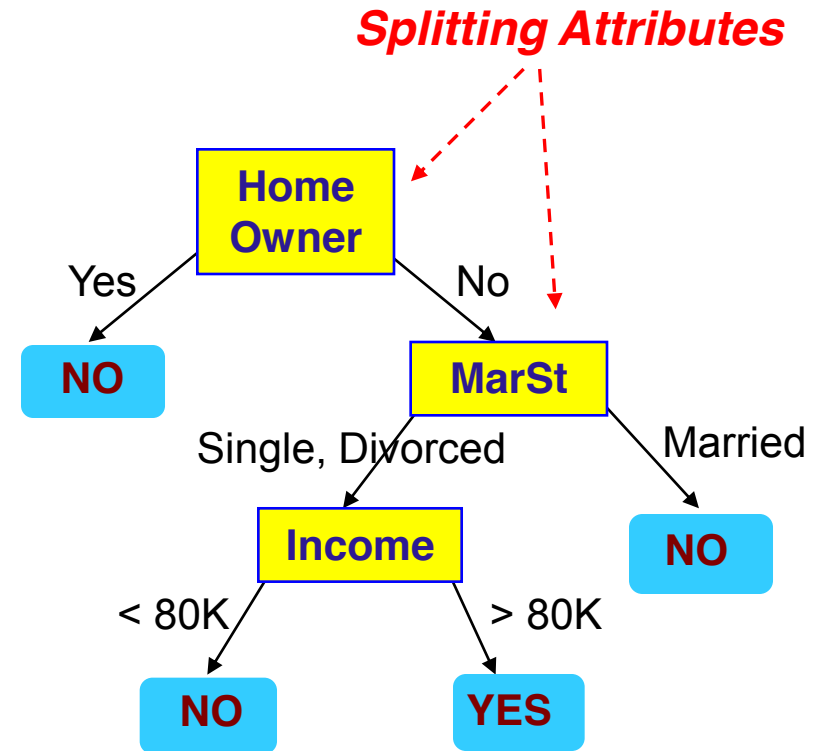
Rule set contains as much information as the tree

Example of a Decision Tree

categorical
categorical
continuous
class

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training Data

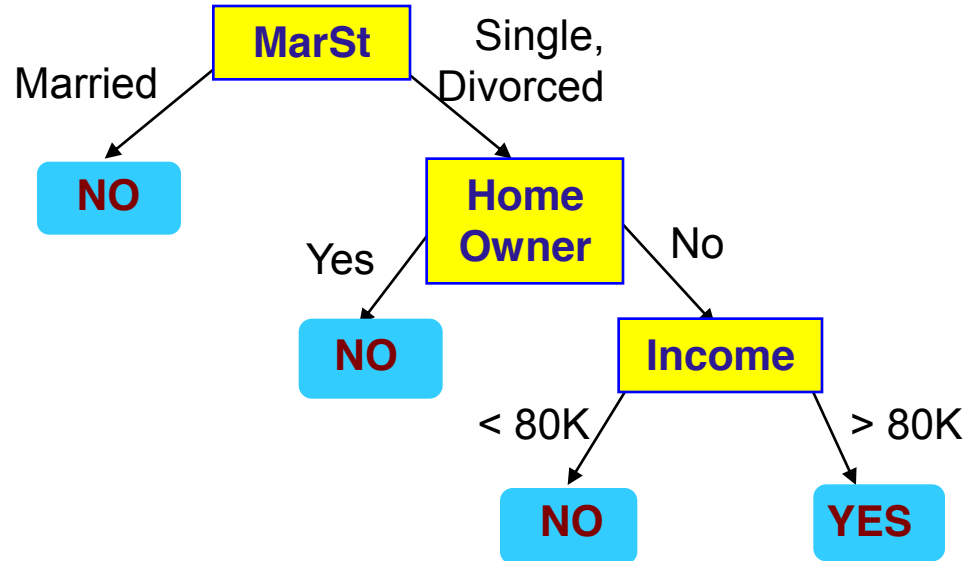


Model: Decision Tree

Another Example of Decision Tree

categorical
categorical
continuous
class

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
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3	No	Single	70K	No
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5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



There could be more than one tree that fits the same data!

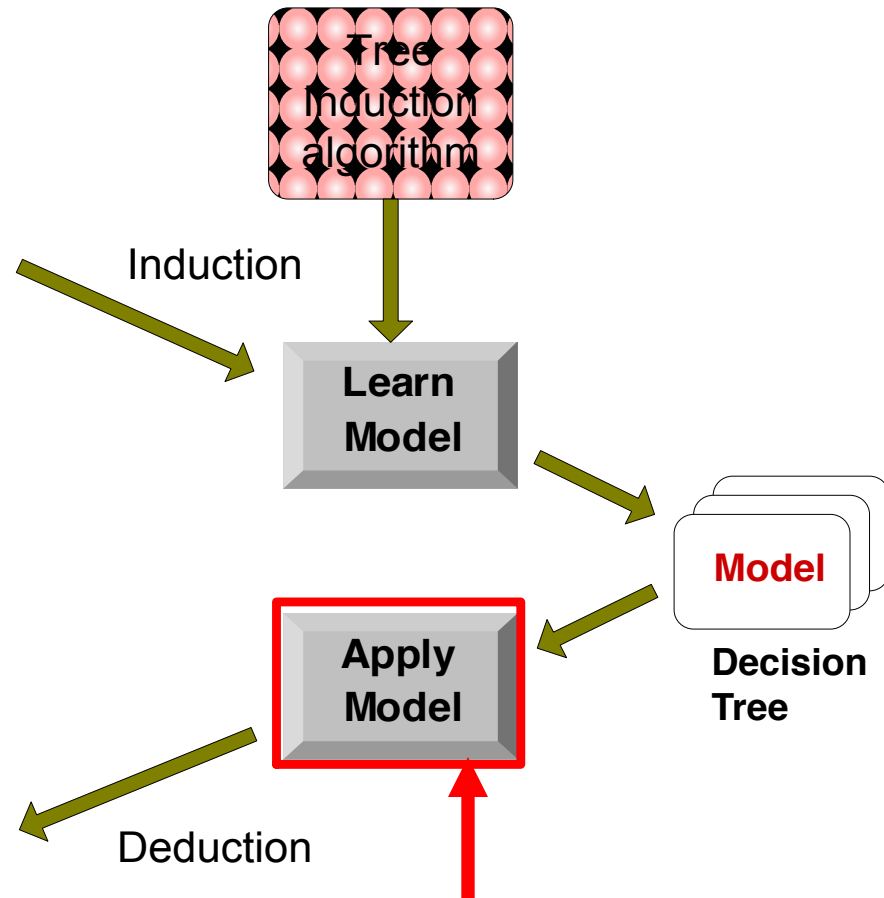
Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
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10	No	Small	90K	Yes

Training Set

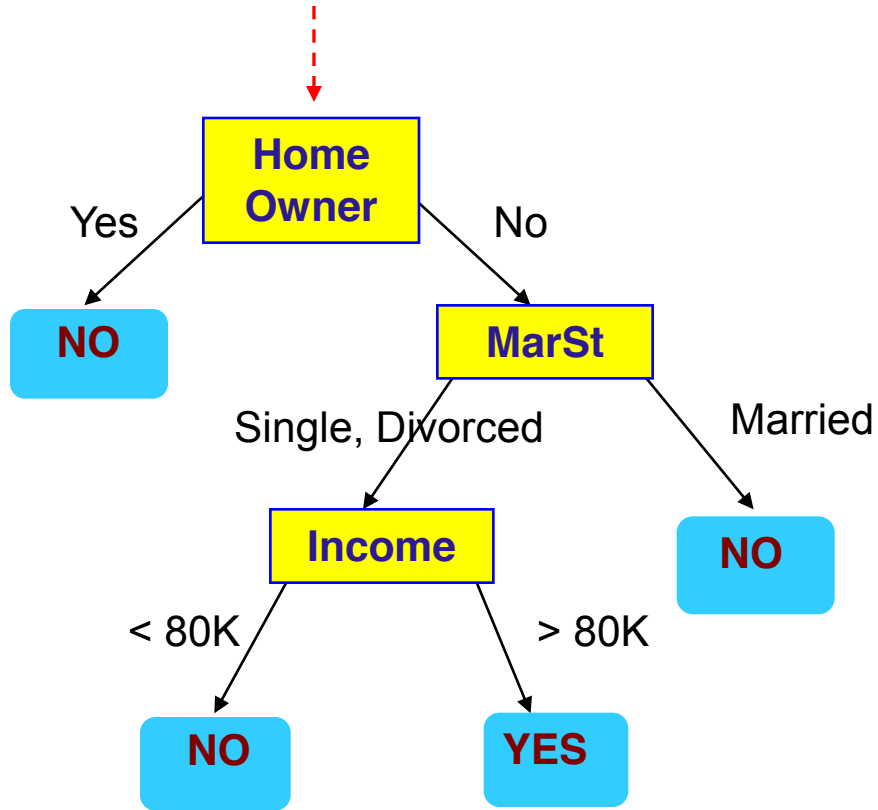
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11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



Apply Model to Test Data

Start from the root of tree.



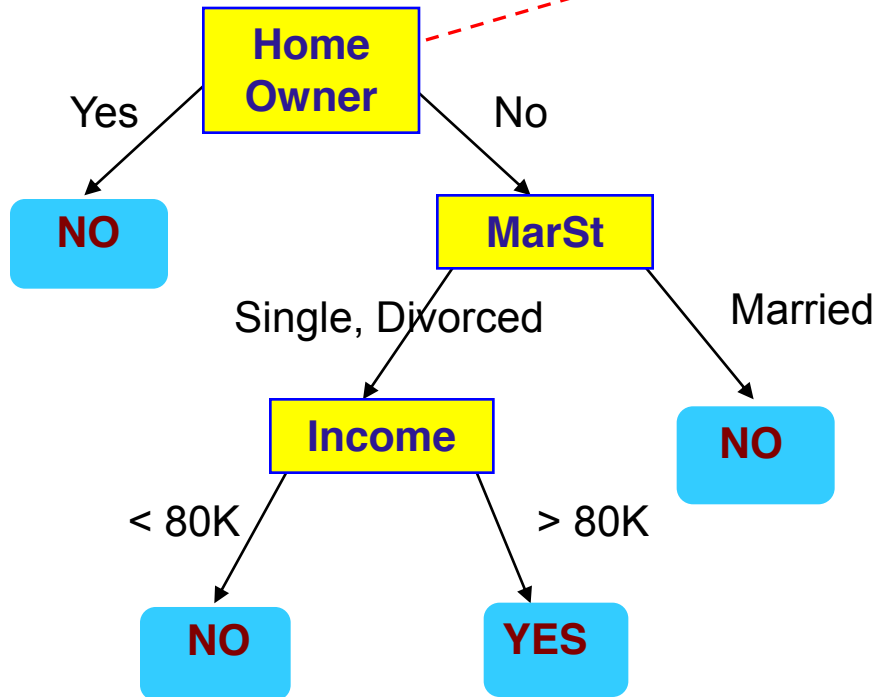
Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?

Apply Model to Test Data

Test Data

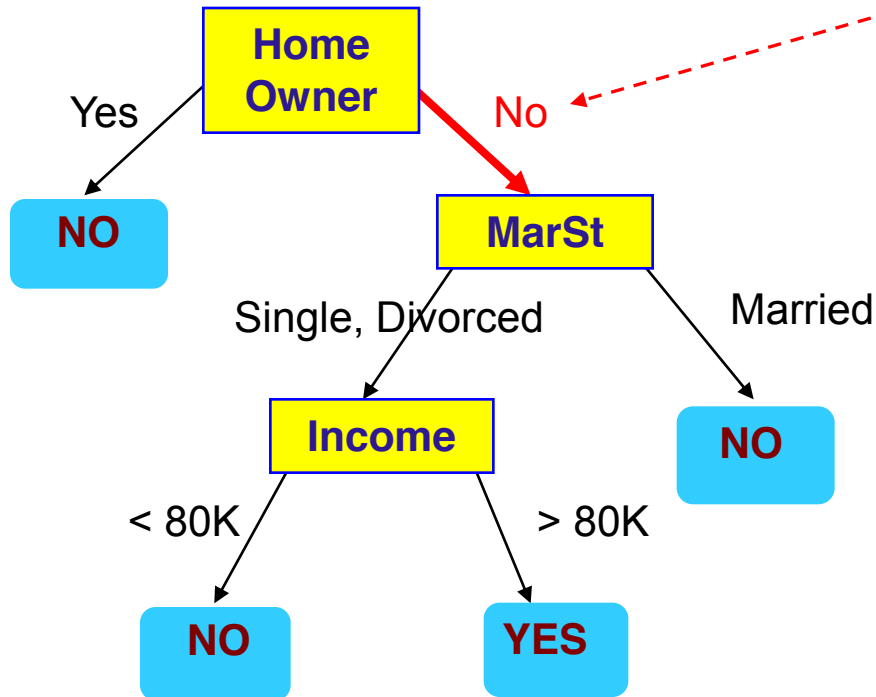
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Apply Model to Test Data

Test Data

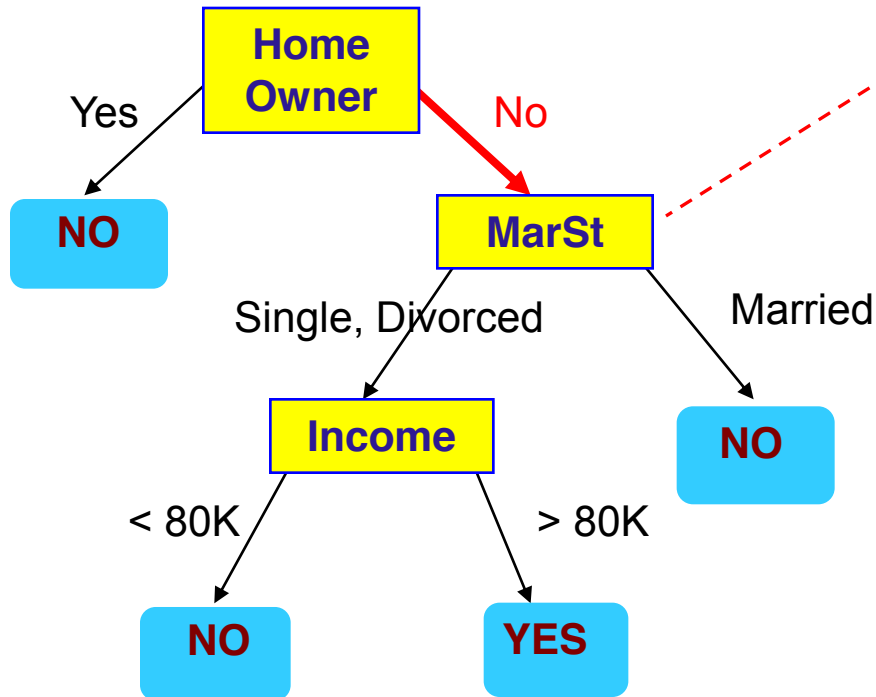
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Apply Model to Test Data

Test Data

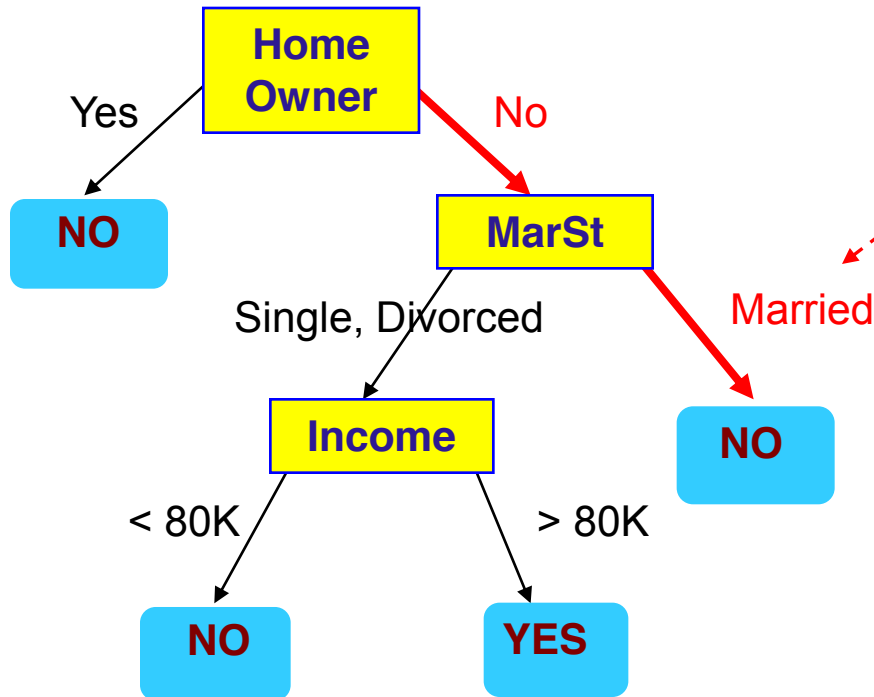
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Apply Model to Test Data

Test Data

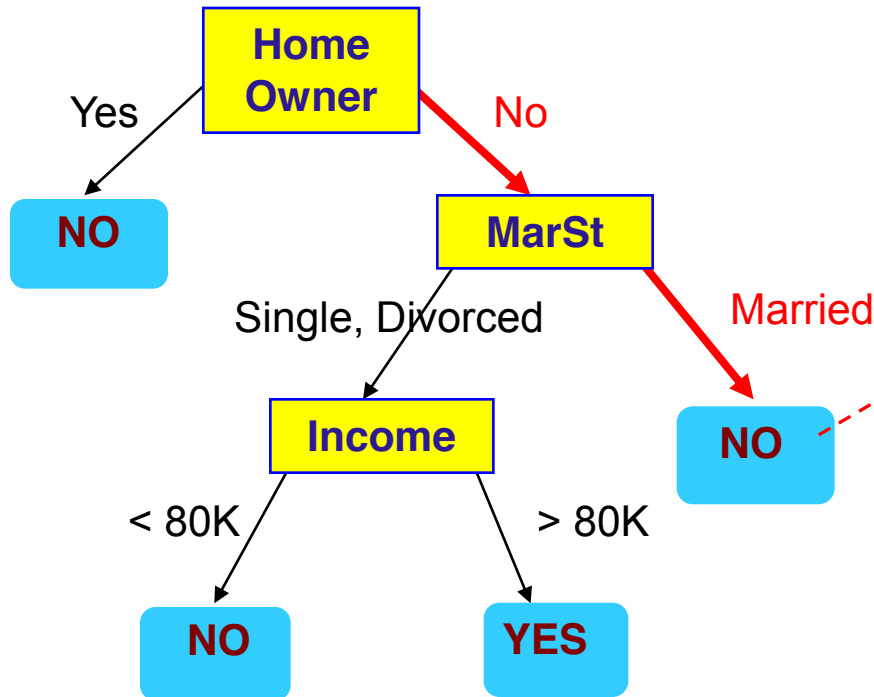
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Apply Model to Test Data

Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Assign Defaulted to
"No"

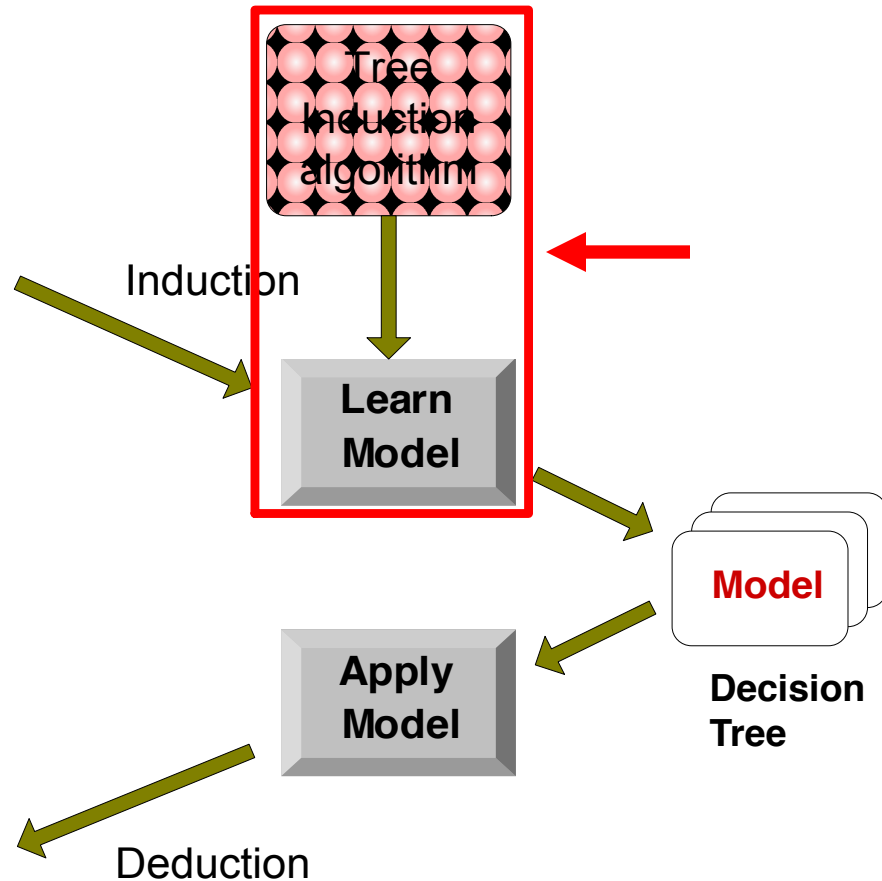
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6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
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Test Set



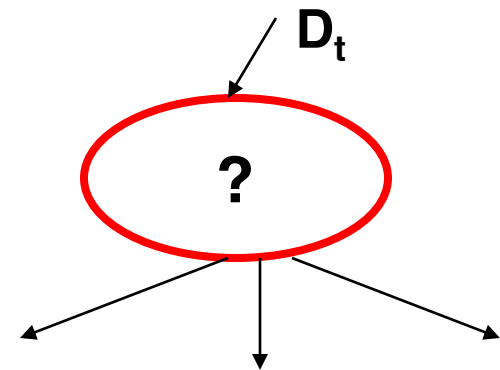
Decision Tree Induction

- Many Algorithms:
 - Hunt's Algorithm (one of the earliest)
 - CART
 - ID3, C4.5
 - SLIQ, SPRINT

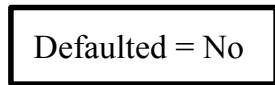
General Structure of Hunt's Algorithm

- Let D_t be the set of training records that reach a node t
- General Procedure:
 - If D_t contains records that belong the same class y_t , then t is a leaf node labeled as y_t
 - If D_t contains records that belong to more than one class, use an attribute test to split the data into smaller subsets. Recursively apply the procedure to each subset.

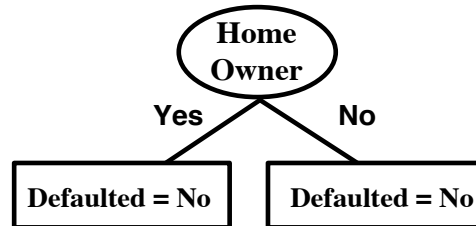
ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



Hunt's Algorithm

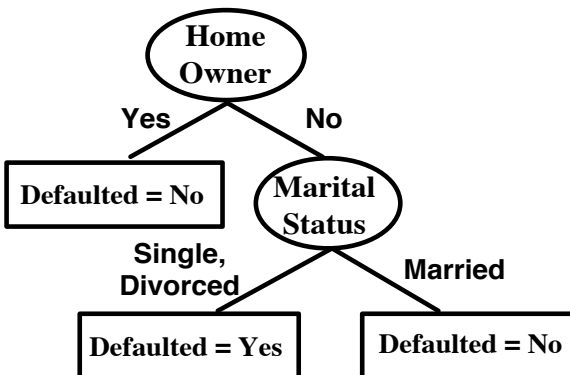


(a)

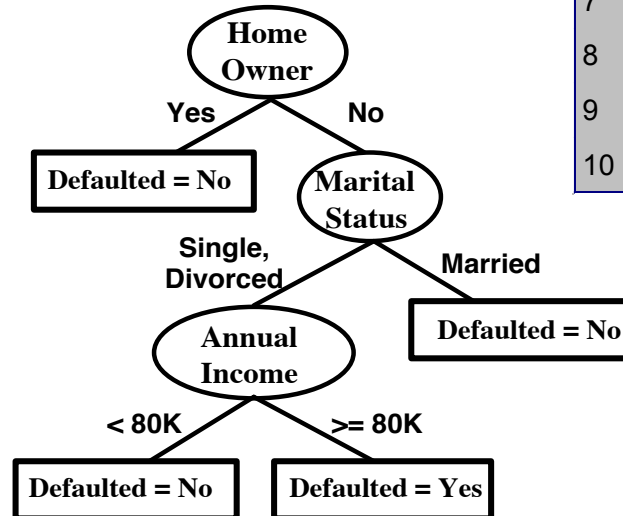


(b)

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



(c)



(d)

Design Issues of Decision Tree Induction

- How should training records be split?
 - Method for specifying test condition
 - ◆ depending on attribute types
 - Measure for evaluating the goodness of a test condition
- How should the splitting procedure stop?
 - Stop splitting if all the records belong to the same class or have identical attribute values
 - Early termination

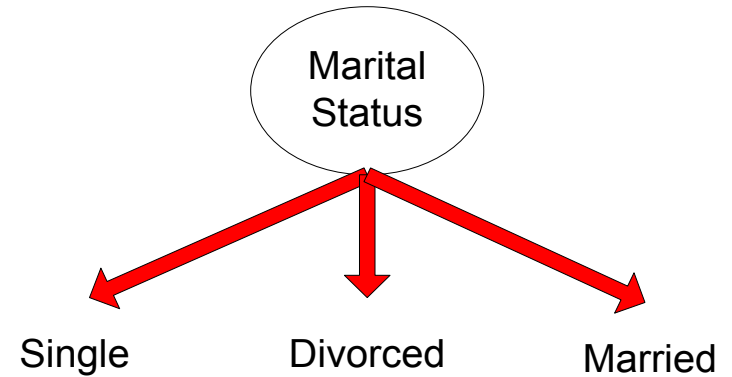
Methods for Expressing Test Conditions

- Depends on attribute types
 - Binary
 - Nominal
 - Ordinal
 - Continuous
- Depends on number of ways to split
 - 2-way split
 - Multi-way split

Test Condition for Nominal Attributes

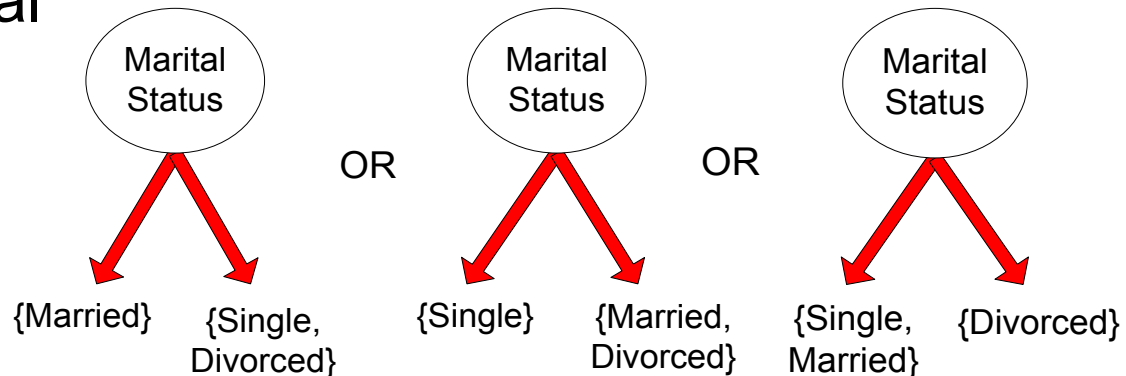
- **Multi-way split:**

- Use as many partitions as distinct values.



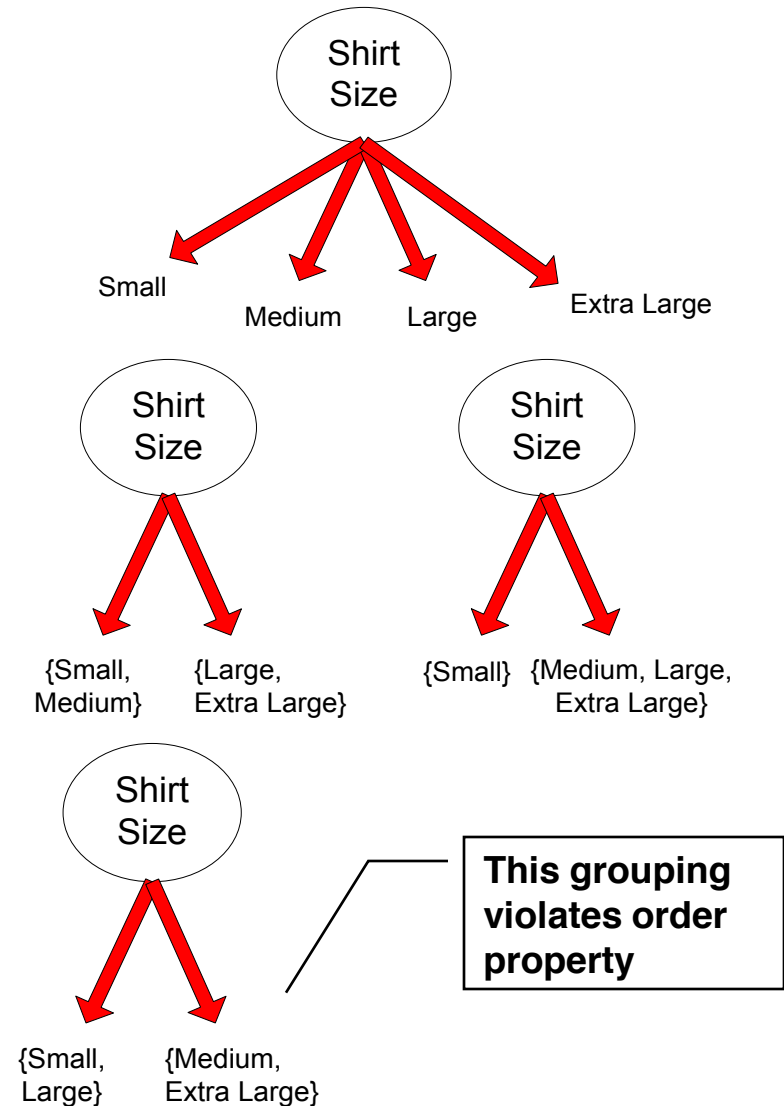
- **Binary split:**

- Divides values into two subsets
- Need to find optimal partitioning.

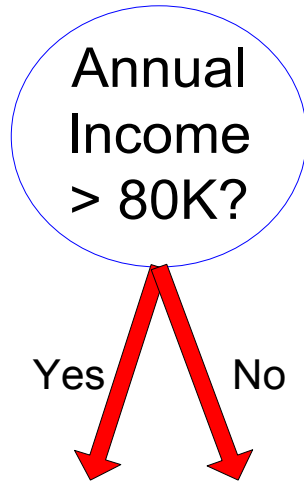


Test Condition for Ordinal Attributes

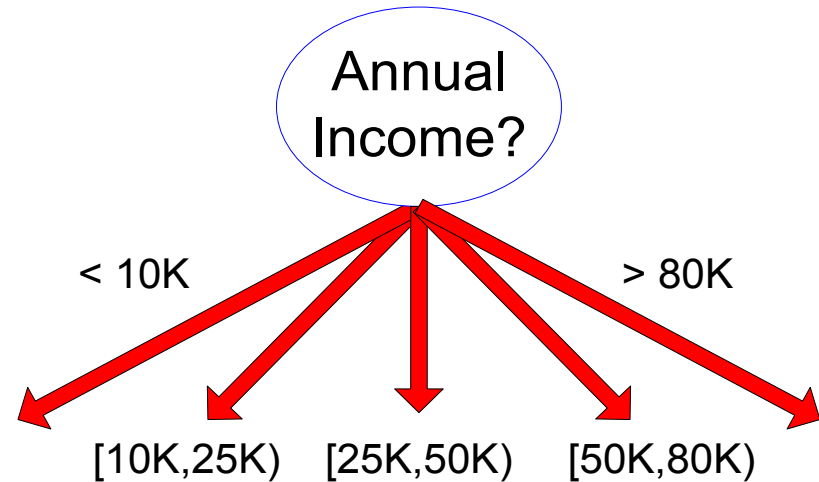
- **Multi-way split:**
 - Use as many partitions as distinct values
- **Binary split:**
 - Divides values into two subsets
 - Need to find optimal partitioning
 - Preserve the order property among attribute values



Test Condition for Continuous Attributes



(i) Binary split



(ii) Multi-way split

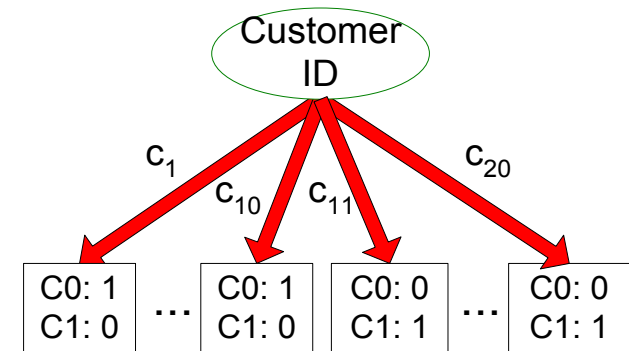
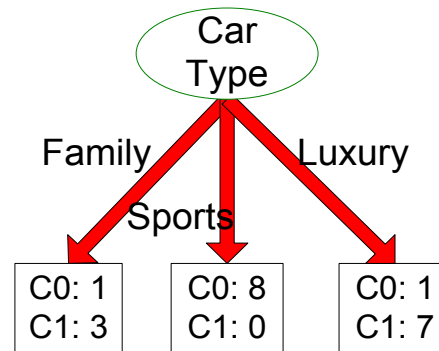
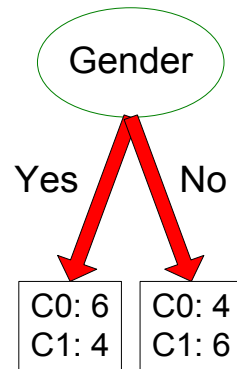
Splitting Based on Continuous Attributes

- Different ways of handling
 - **Discretization** to form an ordinal categorical attribute
 - ◆ Static – discretize once at the beginning
 - ◆ Dynamic – ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.
 - **Binary Decision**: $(A < v)$ or $(A \geq v)$
 - ◆ consider all possible splits and finds the best cut
 - ◆ can be more compute-intensive

How to determine the Best Split

**Before Splitting: 10 records of class 0,
10 records of class 1**

Customer Id	Gender	Car Type	Shirt Size	Class
1	M	Family	Small	C0
2	M	Sports	Medium	C0
3	M	Sports	Medium	C0
4	M	Sports	Large	C0
5	M	Sports	Extra Large	C0
6	M	Sports	Extra Large	C0
7	F	Sports	Small	C0
8	F	Sports	Small	C0
9	F	Sports	Medium	C0
10	F	Luxury	Large	C0
11	M	Family	Large	C1
12	M	Family	Extra Large	C1
13	M	Family	Medium	C1
14	M	Luxury	Extra Large	C1
15	F	Luxury	Small	C1
16	F	Luxury	Small	C1
17	F	Luxury	Medium	C1
18	F	Luxury	Medium	C1
19	F	Luxury	Medium	C1
20	F	Luxury	Large	C1



Which test condition is the best?

How to determine the Best Split

- Greedy approach:
 - Nodes with **pur**er class distribution are preferred
- Need a measure of node impurity:

C0: 5
C1: 5

High degree of impurity

C0: 9
C1: 1

Low degree of impurity

Measures of Node Impurity

- Gini Index

$$GINI(t) = 1 - \sum_j [p(j | t)]^2$$

- Entropy

$$Entropy(t) = -\sum_j p(j | t) \log p(j | t)$$

- Misclassification error

$$Error(t) = 1 - \max_i P(i | t)$$

Finding the Best Split

1. Compute impurity measure (P) before splitting
2. Compute impurity measure (M) after splitting
 - Compute impurity measure of each child node
 - Compute the average impurity of the children (M)
3. Choose the attribute test condition that produces the highest gain

$$\text{Gain} = P - M$$

or equivalently, lowest impurity measure after splitting (M)

Finding the Best Split

Before Splitting:

C0	N00
C1	N01

→ **P**

A?

Yes

No

Node N1

Node N2

C0

N10

C1

N11

C0

N20

C1

N21

M11

M12

M1

B?

Yes

No

Node N3

Node N4

C0

N30

C1

N31

C0

N40

C1

N41

M21

M22

M2

Gain = P – M1 vs P – M2

Measure of Impurity: GINI

- Gini Index for a given node t :

$$GINI(t) = 1 - \sum_j [p(j | t)]^2$$

(NOTE: $p(j | t)$ is the relative frequency of class j at node t).

- Maximum $(1 - 1/n_c)$ when records are equally distributed among all classes, implying least interesting information
- Minimum (0.0) when all records belong to one class, implying most interesting information

C1	0
C2	6
Gini=0.000	

C1	1
C2	5
Gini=0.278	

C1	2
C2	4
Gini=0.444	

C1	3
C2	3
Gini=0.500	

Computing Gini Index of a Single Node

$$GINI(t) = 1 - \sum_j [p(j|t)]^2$$

C1	0
C2	6

$$P(C1) = 0/6 = 0 \quad P(C2) = 6/6 = 1$$

$$Gini = 1 - P(C1)^2 - P(C2)^2 = 1 - 0 - 1 = 0$$

C1	1
C2	5

$$P(C1) = 1/6 \quad P(C2) = 5/6$$

$$Gini = 1 - (1/6)^2 - (5/6)^2 = 0.278$$

C1	2
C2	4

$$P(C1) = 2/6 \quad P(C2) = 4/6$$

$$Gini = 1 - (2/6)^2 - (4/6)^2 = 0.444$$

Computing Gini Index for a Collection of Nodes

- When a node p is split into k partitions (children)

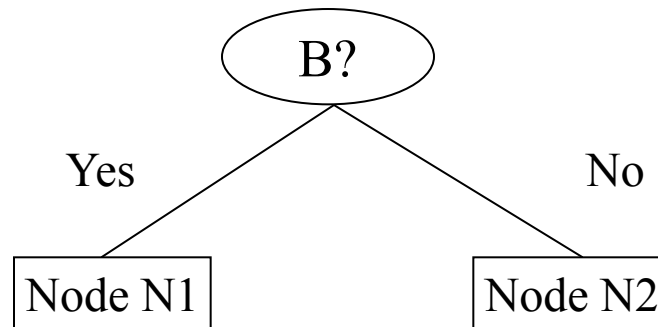
$$GINI_{split} = \sum_{i=1}^k \frac{n_i}{n} GINI(i)$$

where, n_i = number of records at child i ,
 n = number of records at parent node p .

- Choose the attribute that minimizes weighted average Gini index of the children
- Gini index is used in decision tree algorithms such as CART, SLIQ, SPRINT

Binary Attributes: Computing GINI Index

- Splits into two partitions
- Effect of Weighing partitions:
 - Larger and Purer Partitions are sought for.



	Parent
C1	6
C2	6
Gini = 0.500	

$$\begin{aligned}\text{Gini}(N1) &= 1 - (5/6)^2 - (1/6)^2 \\ &= 0.278\end{aligned}$$

$$\begin{aligned}\text{Gini}(N2) &= 1 - (2/6)^2 - (4/6)^2 \\ &= 0.444\end{aligned}$$

	N1	N2
C1	5	2
C2	1	4
Gini=0.361		

$$\begin{aligned}\text{Gini(Children)} &= 6/12 * 0.278 + \\ &\quad 6/12 * 0.444 \\ &= 0.361\end{aligned}$$

Categorical Attributes: Computing Gini Index

- For each distinct value, gather counts for each class in the dataset
- Use the count matrix to make decisions

Multi-way split

	CarType		
	Family	Sports	Luxury
C1	1	8	1
C2	3	0	7
Gini	0.163		

Two-way split
(find best partition of values)

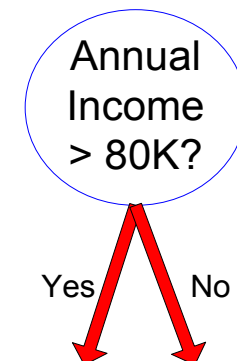
	CarType	
	{Sports, Luxury}	{Family}
C1	9	1
C2	7	3
Gini	0.468	

	CarType	
	{Sports}	{Family, Luxury}
C1	8	2
C2	0	10
Gini	0.167	

Continuous Attributes: Computing Gini Index

- Use Binary Decisions based on one value
- Several Choices for the splitting value
 - Number of possible splitting values = Number of distinct values
- Each splitting value has a count matrix associated with it
 - Class counts in each of the partitions, $A < v$ and $A \geq v$
- Simple method to choose best v
 - For each v , scan the database to gather count matrix and compute its Gini index
 - Computationally Inefficient! Repetition of work.

ID	Home Owner	Marital Status	Annual Income	Defaulted
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
 - Sort the attribute on values
 - Linearly scan these values, each time updating the count matrix and computing gini index
 - Choose the split position that has the least gini index

Sorted Values Split Positions	Cheat	No		No		No		Yes		Yes		Yes		No		No		No		No		
	Annual Income																					
	60		70		75		85		90		95		100		120		125		220			
	55		65		72		80		87		92		97		110		122		172		230	
	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>
Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
Gini	0.420		0.400		0.375		0.343		0.417		0.400		<u>0.300</u>		0.343		0.375		0.400		0.420	

Measure of Impurity: Entropy

- Entropy at a given node t :

$$Entropy(t) = -\sum_j p(j | t) \log p(j | t)$$

(NOTE: $p(j | t)$ is the relative frequency of class j at node t).

- ◆ Maximum ($\log n_c$) when records are equally distributed among all classes implying least information
 - ◆ Minimum (0.0) when all records belong to one class, implying most information
-
- Entropy based computations are quite similar to the GINI index computations

Computing Entropy of a Single Node

$$Entropy(t) = -\sum_j p(j | t) \log_2 p(j | t)$$

C1	0
C2	6

$$P(C1) = 0/6 = 0 \quad P(C2) = 6/6 = 1$$

$$Entropy = -0 \log 0 - 1 \log 1 = -0 - 0 = 0$$

C1	1
C2	5

$$P(C1) = 1/6 \quad P(C2) = 5/6$$

$$Entropy = - (1/6) \log_2 (1/6) - (5/6) \log_2 (1/6) = 0.65$$

C1	2
C2	4

$$P(C1) = 2/6 \quad P(C2) = 4/6$$

$$Entropy = - (2/6) \log_2 (2/6) - (4/6) \log_2 (4/6) = 0.92$$

Computing Information Gain After Splitting

- Information Gain:

$$GAIN_{split} = Entropy(p) - \left(\sum_{i=1}^k \frac{n_i}{n} Entropy(i) \right)$$

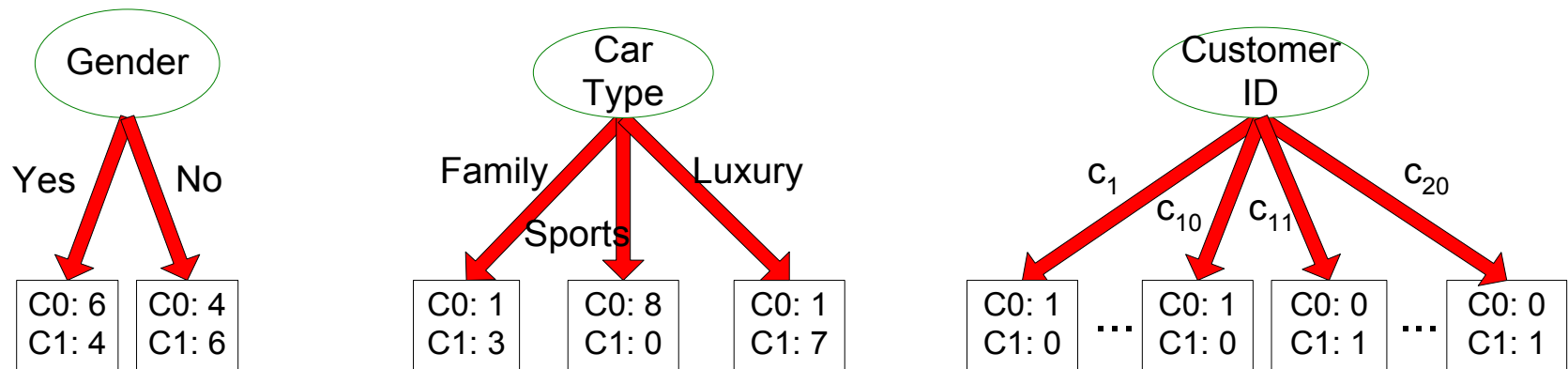
Parent Node, p is split into k partitions;

n_i is number of records in partition i

- Choose the split that achieves most reduction (maximizes GAIN)
- Used in the ID3 and C4.5 decision tree algorithms

Problems with Information Gain

- Info Gain tends to prefer splits that result in large number of partitions, each being small but pure



- Customer ID has highest information gain because entropy for all the children is zero

Gain Ratio

- Gain Ratio:

$$GainRATIO_{split} = \frac{GAIN_{Split}}{SplitINFO} \quad SplitINFO = -\sum_{i=1}^k \frac{n_i}{n} \log \frac{n_i}{n}$$

Parent Node, p is split into k partitions

n_i is the number of records in partition i

- Adjusts Information Gain by the entropy of the partitioning (SplitINFO).
 - ◆ Higher entropy partitioning (large number of small partitions) is penalized!
- Used in C4.5 algorithm
- Designed to overcome the disadvantage of Information Gain

Measure of Impurity: Classification Error

- Classification error at a node t :

$$Error(t) = 1 - \max_i P(i | t)$$

- Maximum ($1 - 1/n_c$) when records are equally distributed among all classes, implying least interesting information
- Minimum (0) when all records belong to one class, implying most interesting information

Computing Error of a Single Node

$$Error(t) = 1 - \max_i P(i | t)$$

C1	0
C2	6

$$P(C1) = 0/6 = 0 \quad P(C2) = 6/6 = 1$$

$$Error = 1 - \max(0, 1) = 1 - 1 = 0$$

C1	1
C2	5

$$P(C1) = 1/6 \quad P(C2) = 5/6$$

$$Error = 1 - \max(1/6, 5/6) = 1 - 5/6 = 1/6$$

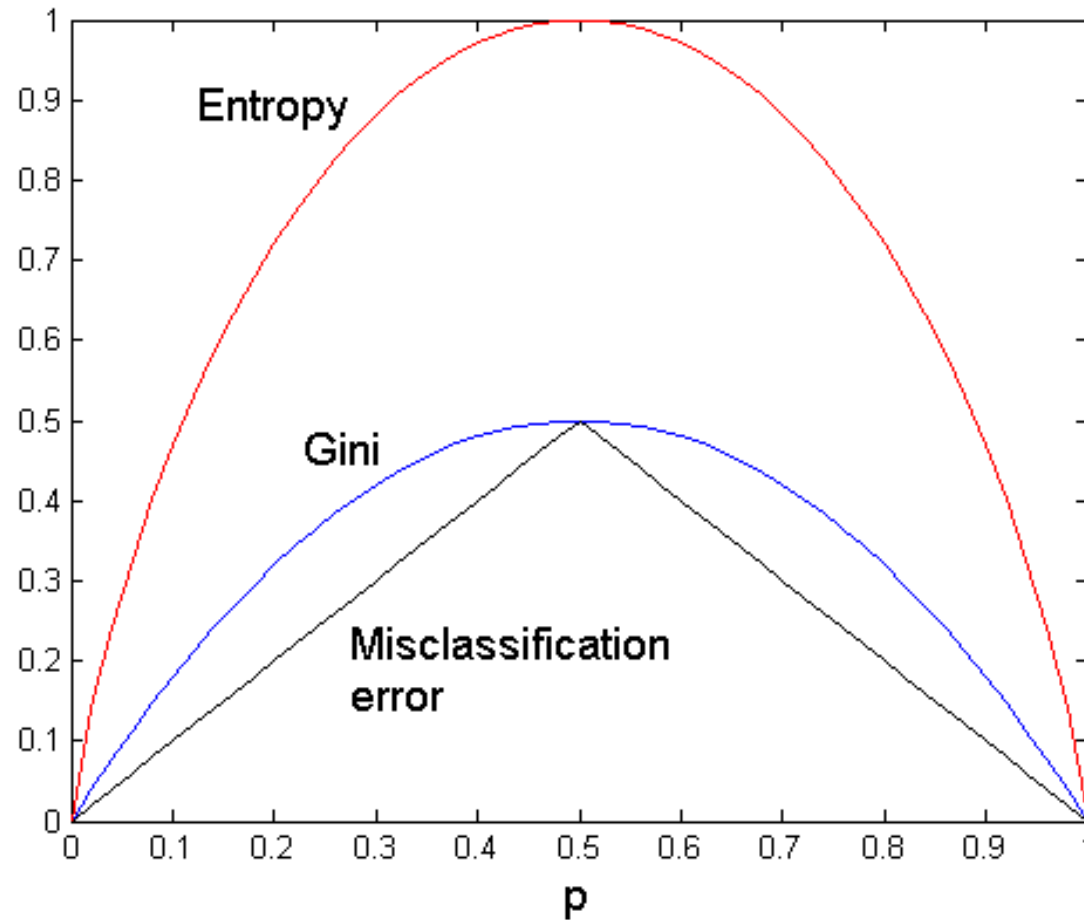
C1	2
C2	4

$$P(C1) = 2/6 \quad P(C2) = 4/6$$

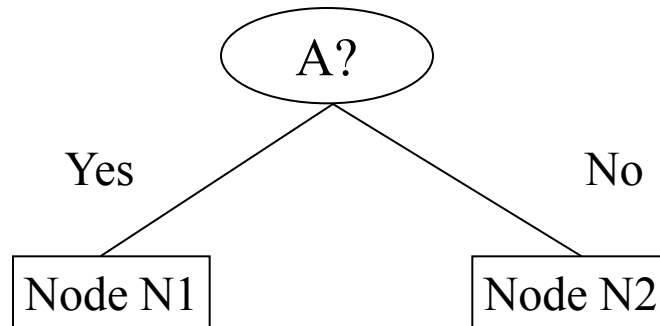
$$Error = 1 - \max(2/6, 4/6) = 1 - 4/6 = 1/3$$

Comparison among Impurity Measures

For a 2-class problem:



Misclassification Error vs Gini Index



	Parent
C1	7
C2	3
Gini = 0.42	

$$\begin{aligned}\text{Gini}(N1) \\ &= 1 - (3/3)^2 - (0/3)^2 \\ &= 0\end{aligned}$$

	N1	N2
C1	3	4
C2	0	3
Gini=0.342		

$$\begin{aligned}\text{Gini}(N2) \\ &= 1 - (4/7)^2 - (3/7)^2 \\ &= 0.489\end{aligned}$$

$$\begin{aligned}\text{Gini(Children)} \\ &= 3/10 * 0 \\ &+ 7/10 * 0.489 \\ &= 0.342\end{aligned}$$

**Gini improves but
error remains the
same!!**

Decision Tree Based Classification

- Advantages:
 - Inexpensive to construct
 - Extremely fast at classifying unknown records
 - Easy to interpret for small-sized trees
 - Accuracy is comparable to other classification techniques for many simple data sets