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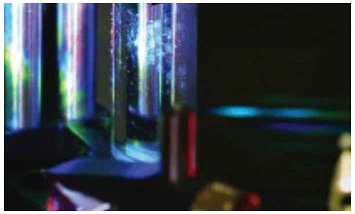

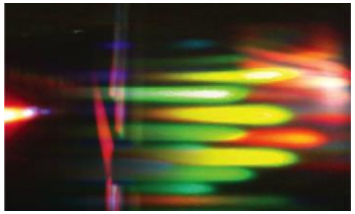

COMP70058 Computer Vision

Lecture 4 – The Hough Transform

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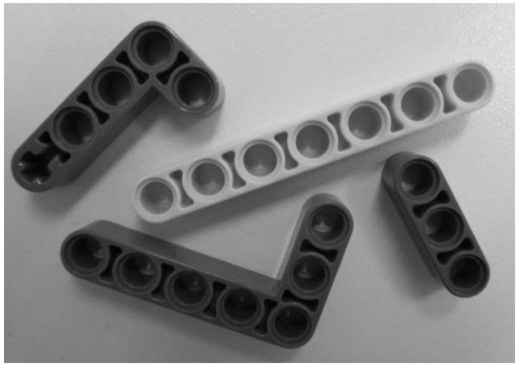
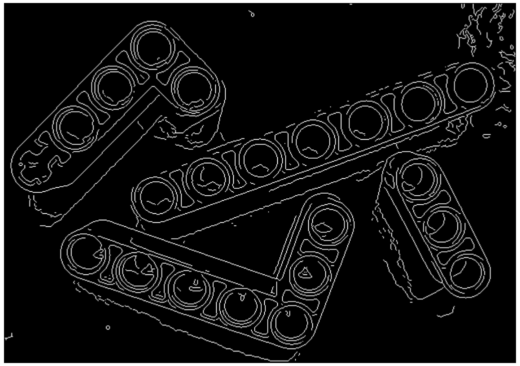
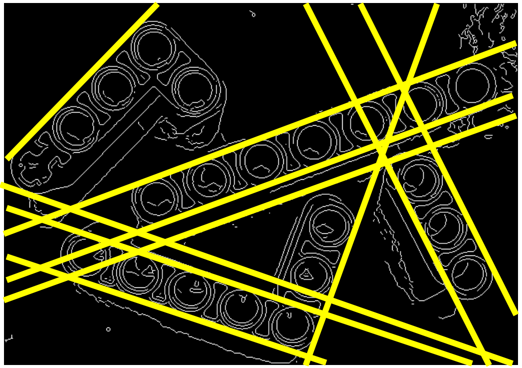
- Hough transform for straight lines
- Side lobes and bias
- Heuristics in the Hough Transform
- Hough transform for extracting higher order shapes
- Adaptive Hough transform

Introduction

- One important way that we recognise shapes is through mathematical functions that describe the boundary curves
- How can we get a parametric representation of extracted edges?
- A well-known method of utilising this kind of information in computer vision is the Hough Transform, named after Paul Hough who patented the method in 1962
- Given a set of edge points, the output of the Hough transform is a parametric model which represents these points.

Note for complex mathematical functions, although theoretically possible to utilise Hough Transform, computationally the method is normally restricted to first and second order boundary equations (straight lines and conic sections) due to the computational cost involved.

Line parameterisation

- Slope intercept form

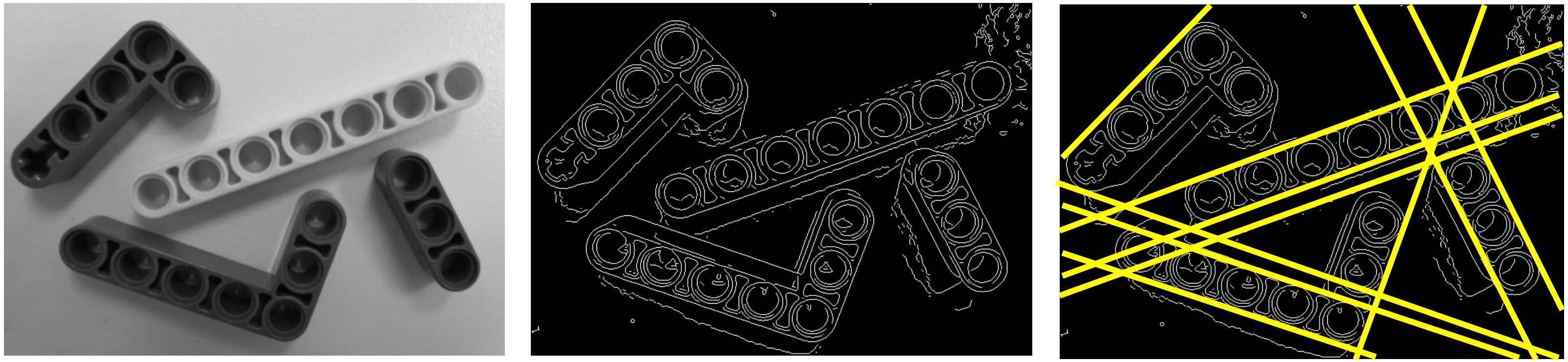
$$y = mx + b$$

- Double intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

- Normal form

$$x\cos(\theta) + y\sin(\theta) = \rho$$

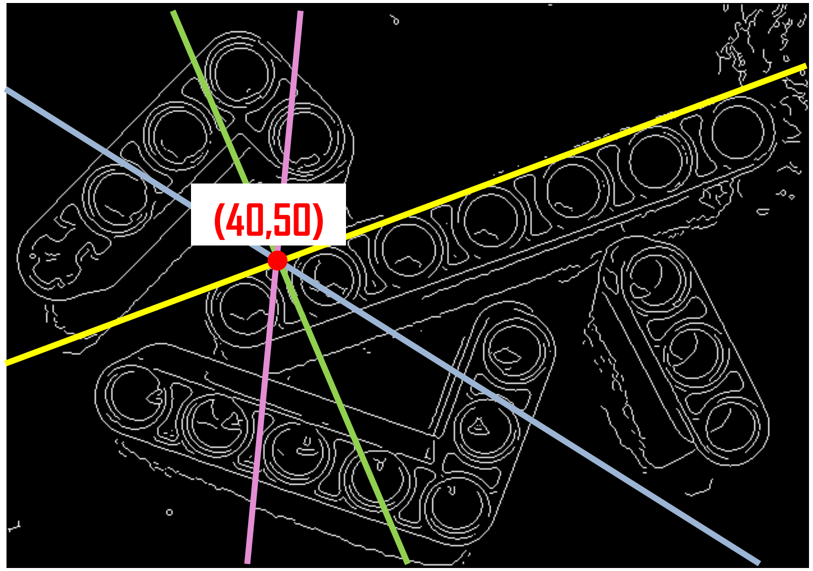


The Hough Transform

- If we consider the normal Cartesian formulation of a straight line:

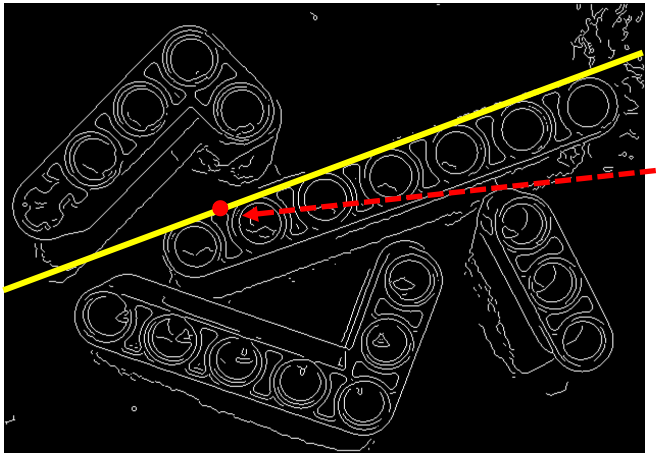
$$y - mx - c = 0$$

- We can regard the two constants m and c as parameters defining the line
- If we choose one point on the Cartesian axis (x, y) it can be considered to belong to a whole family of lines defined by different values of m and c.
- One point (x_i, y_i) in Cartesian space will correspond to a line in the (m, c) space with equation $y_i - mx_i = c$.

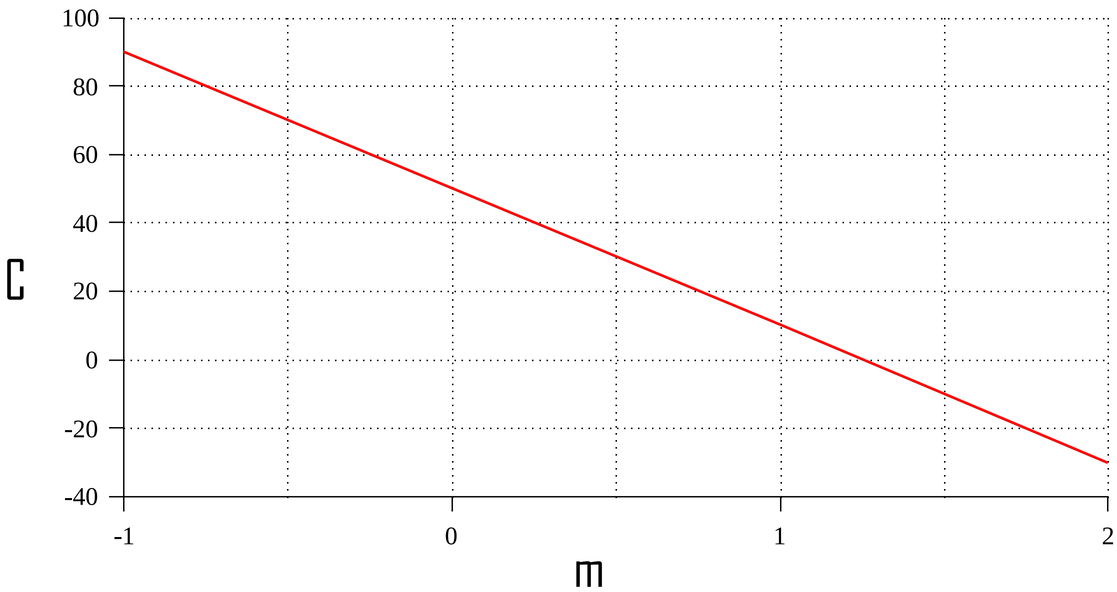


$$y = mx + c$$

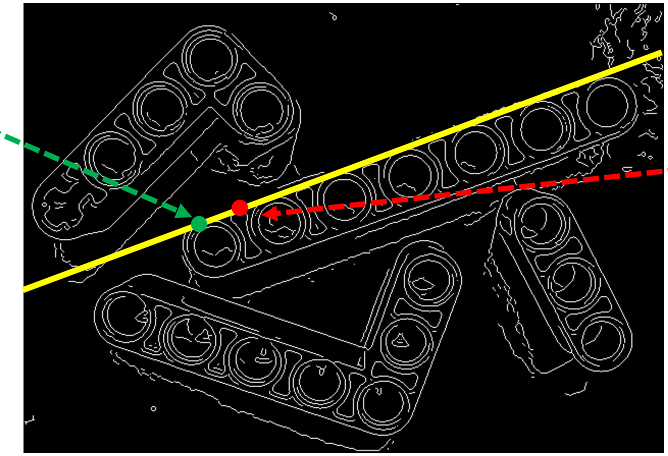
$$50 = 40m + c$$



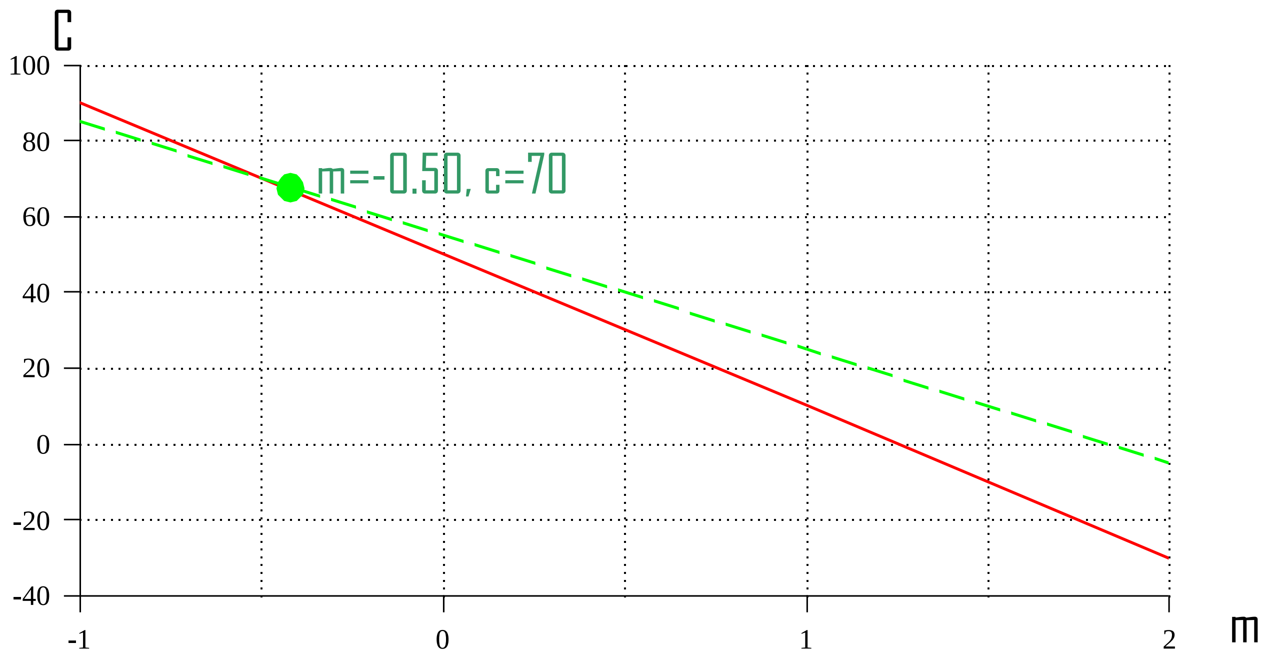
$$50 = 40m + c$$



$$55 = 30m + c$$

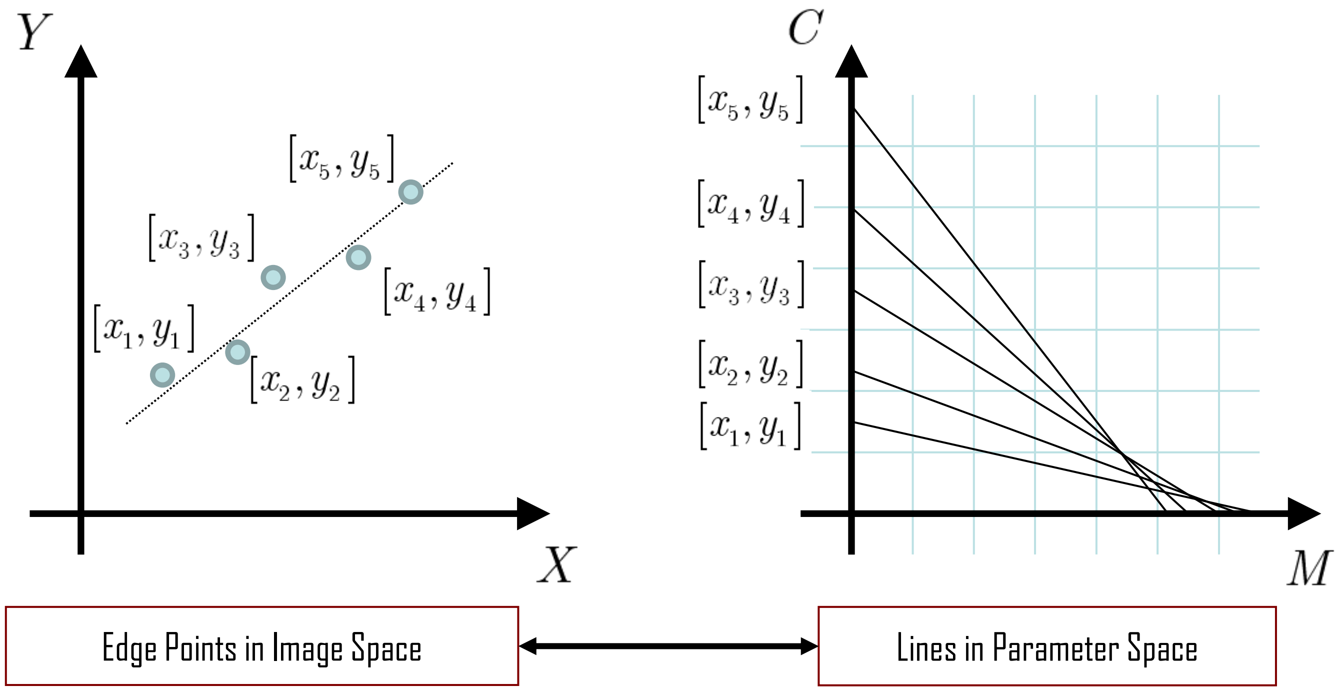


$$50 = 40m + c$$

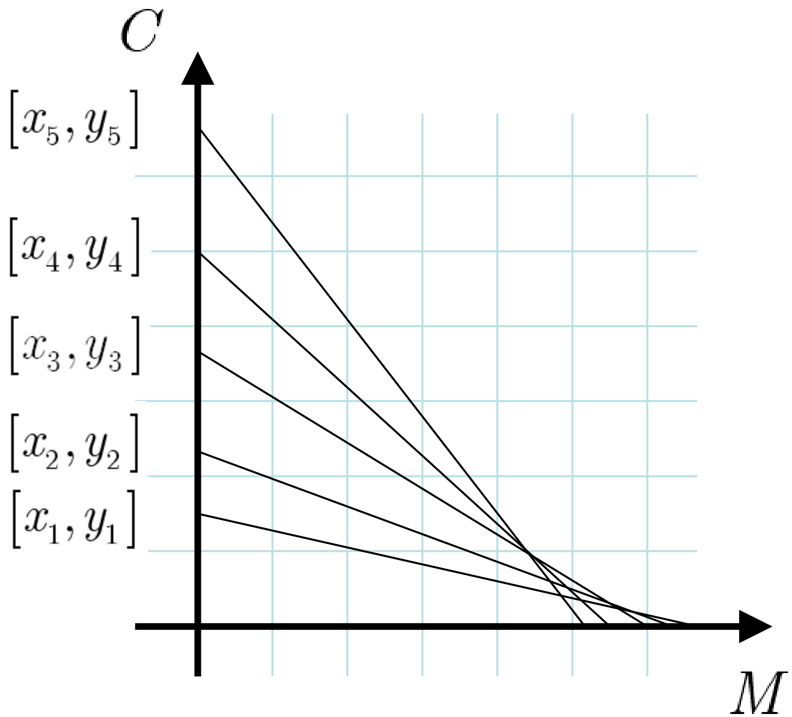


The Hough Transform for Straight Lines

- Thus, if we take a set of points in Cartesian space, this corresponds to a set of lines in m-c space



Problem with m-c Representation



$$\boxed{y - mx - c = 0}$$

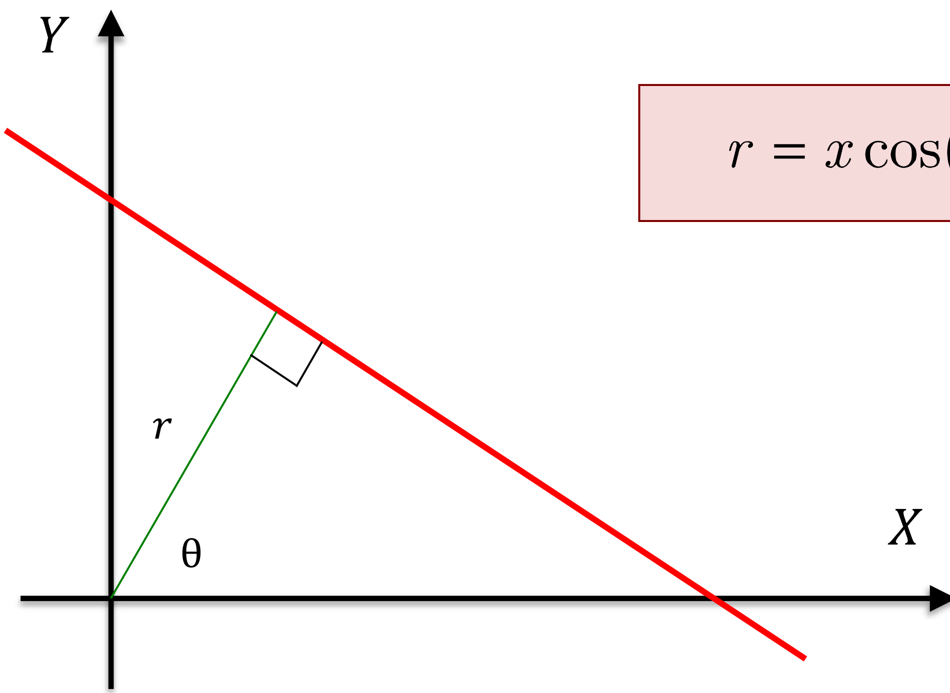
$$\infty > m > -\infty$$

The slope parameter m covers an infinite range.

If those points were co-linear, it is easy to see that all the lines meet in a single point, and that point defines the slope and offset of line. In practice, we may have many lines, so the technique is to divide up the m-c space into small areas, and count the number of lines which cross each. **The [m,c] value at the centre of the area with the largest number of lines is used as the estimate of the most likely line in Cartesian space.** Unfortunately, if we consider all the possible lines that can appear in an image, the slope parameter m covers an infinite range. For that reason the [m,c] parameterization is difficult to use.

r-θ Representation

- A line can also be represented by its shortest distance from the origin (r) and its orientation (θ)
- This is a second, perhaps more useful parameterisation



$$r = x\cos(\theta) + y\sin(\theta)$$

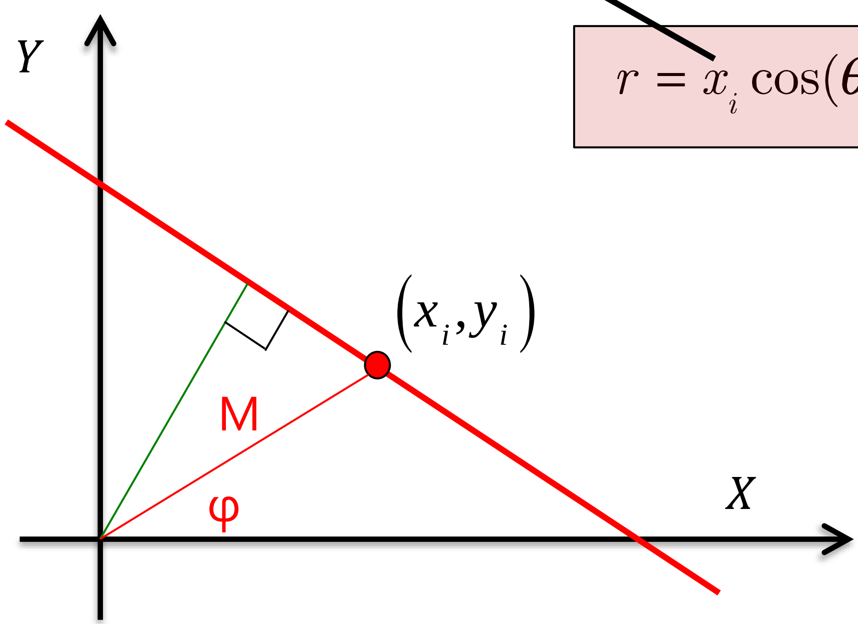
r-θ Representation

- What corresponds to a point (x_i, y_i) in r-θ space?
- We make a substitution

$$M = \sqrt{(x_i^2 + y_i^2)} \text{ and define } \cos(\varphi) = \frac{x_i}{M}, \quad \sin(\varphi) = \frac{y_i}{M}$$

$$r = \boxed{M\cos(\varphi)}\cos(\theta) + \boxed{M\sin(\varphi)}\sin(\theta) = M\cos(\theta - \varphi)$$

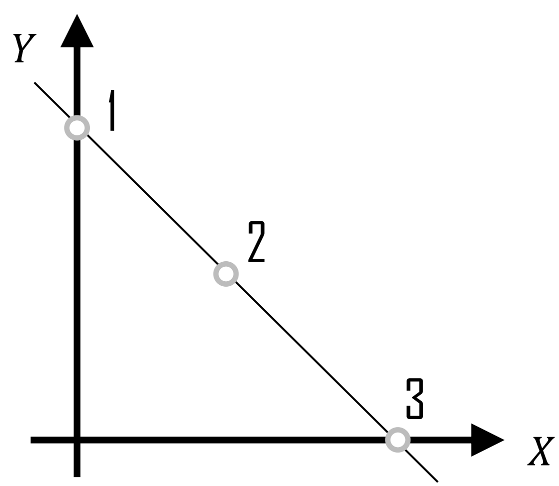
$$\boxed{r = x_i \cos(\theta) + y_i \sin(\theta)}$$



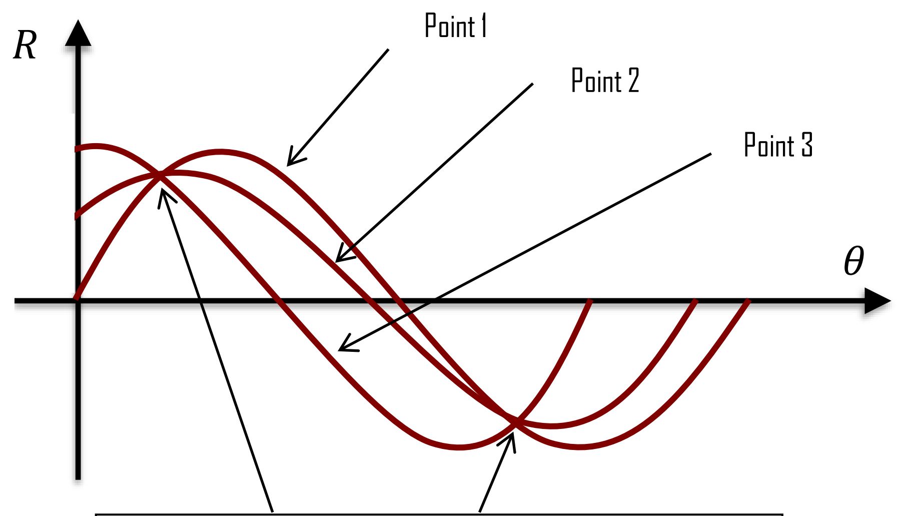
So, this time a point in Cartesian space corresponds to a **sine wave** in r-θ space, and we are searching for the point where the majority of these sinusoids intersect.

The Hough Transform in r-θ space

- This can be considered a relation between the co-ordinates (x, y) of some point in the edge image, and the values of the parameters [r,θ] which define a line;
- As before, in any practical case, we are seeking not to identify a single intersection, but a cluster of intersections
- We therefore must quantise the parameters into discrete values



Points transform to sinusoids in r- θ space



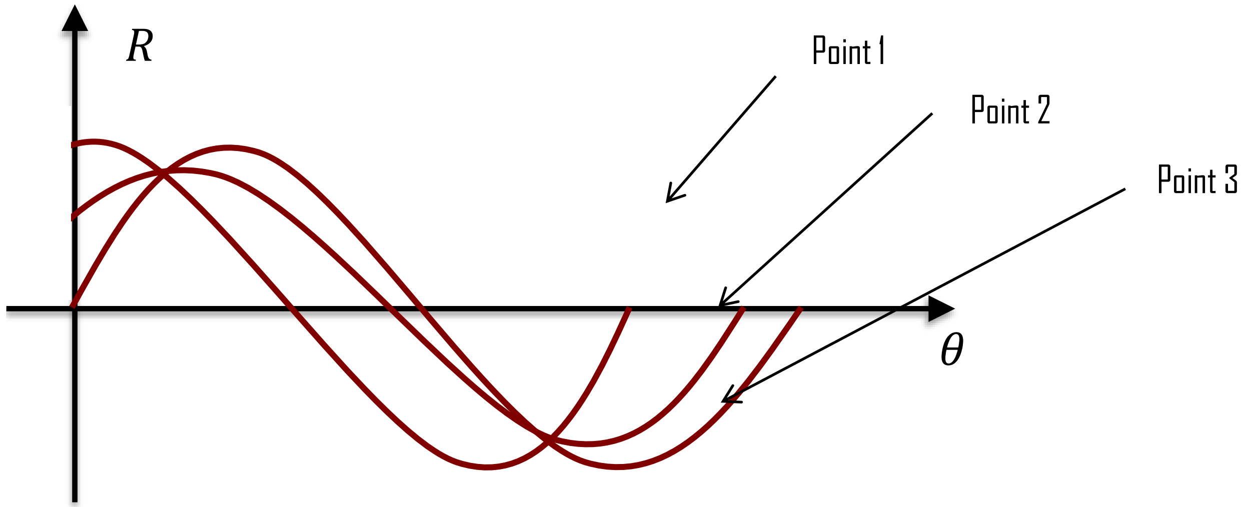
Intersection points estimate the line equation

The Hough Transform in r-θ space

- The main advantage of the parameterisation is that quantisation is easy
- Not all the parameter space need be considered
- The sinusoids all have the same period, and therefore we can limit θ to the range of [0,2π] without loss of generality
- The range [0,2π] can be divided into equal angles denoted:

$$\theta_i, 2\theta_i, 3\theta_i, \ldots, 2\pi$$

- It is also possible to limit the range over which r is considered.



Computing the Hough Transform

- Step 1: Set up an **accumulator array** (histogram) in which the rows correspond to the quantised values of r, and the columns to the values of θ.

- Step 2: For each edge point of the image we can compute a value of r for each θ in our range. This is then rounded to the nearest value represented in the array, and the corresponding entry in the histogram array is incremented. Thus each point in the edge map transforms into a number of points in the quantised (r, θ) space, or we can say that **each point "votes" for a number of lines**.

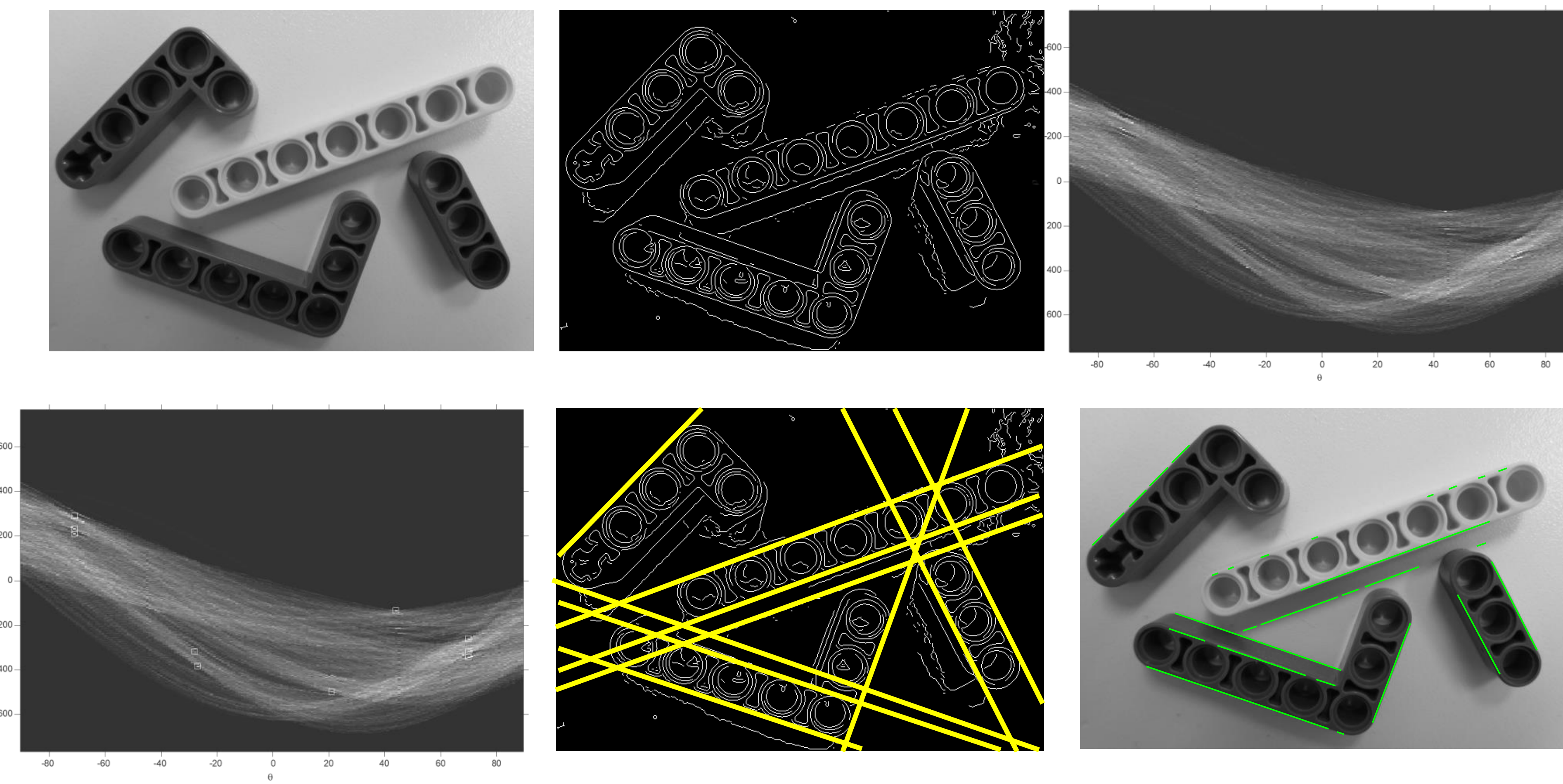
- Step 3: Once we have processed all the points in the edge image, we look through the histogram array, and each place which contains a sufficiently large number of votes defines the equation of a line in the image. Note that the accuracy and the computation time for the Hough transform are dependent on the size of the accumulator array actually used.

r

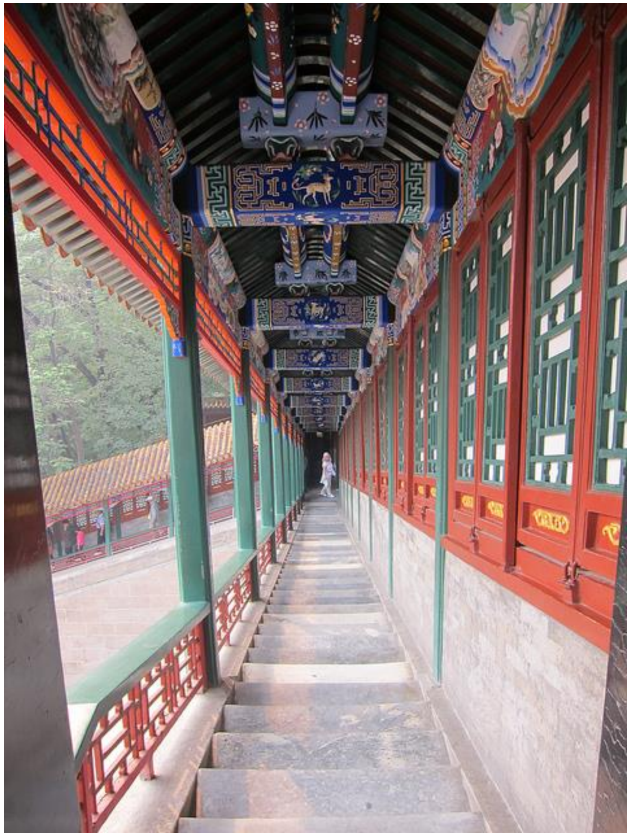
	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		
1								1	
									1

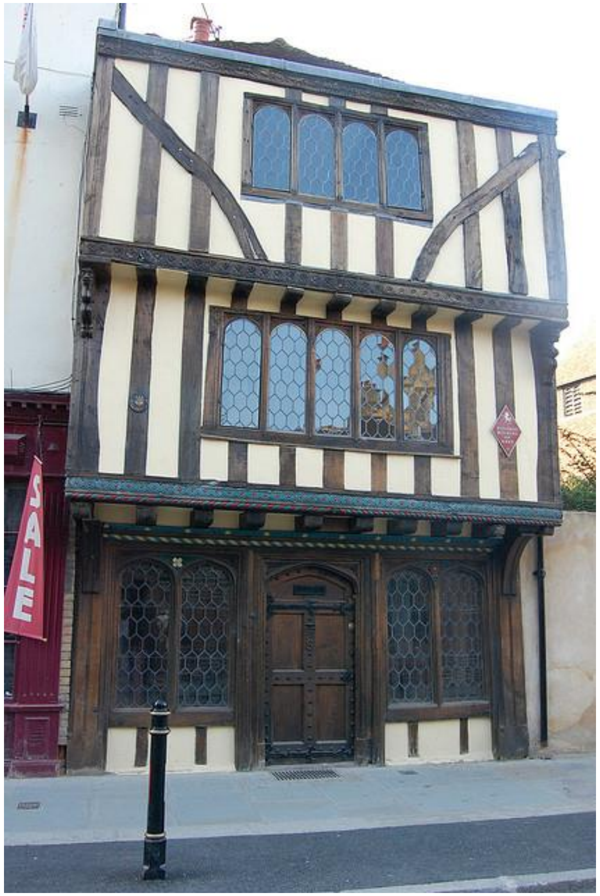
θ

Hough Transform – In Action



Hough Transform – In Action



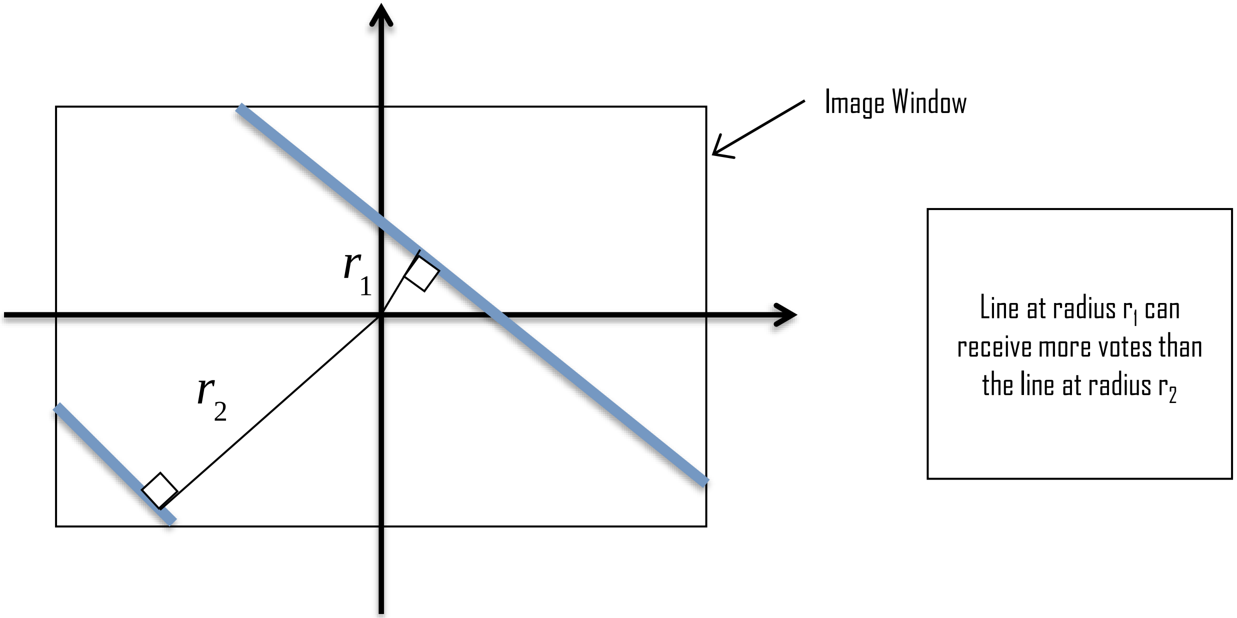


Side Lobes

- One inherent problem comes from the fact that each point in the edge map votes for more than one line in the parameter space
- In fact, for the ideal case where we have one straight line, there will be one main peak in the histogram, and then several smaller peaks, reducing in magnitude as the distance from the main peak increases.
- These secondary peaks are called side lobes
- They cause a problem when there are several lines in the image. The combined effect of the side lobes can be to create a false peak in the histogram

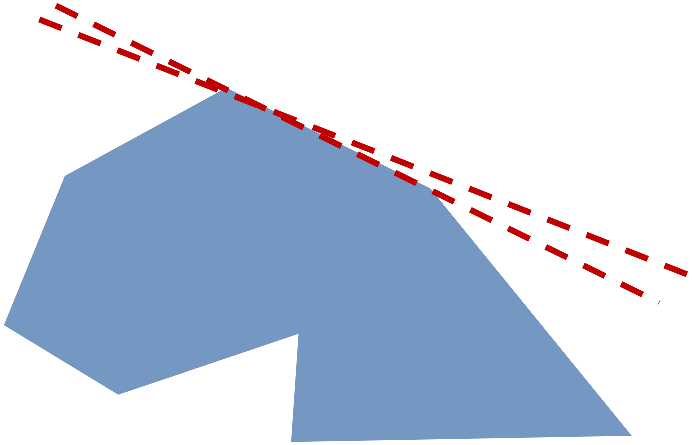
Bias in Hough Transform

- A similar problem to side lobes is inherent bias in the parameterisation
- This results from the fact that a finite space is used, rather than an infinite one
- Using the r-θ method, it is clear to see that lines with small values of r in a finite image will usually be longer than those with larger values of r, which are more readily clipped by the edges of the image
  - Hence, for large r, a histogram point for a significant line will have fewer votes than one with small r
- Bias may be removed by either adopting non linear quantisation of the parameter space, or by measuring the histogram for a random sample of edge points, and normalising the image histogram accordingly



Use of Heuristics for Hough Transform

- Some have proposed heuristics for reducing the effects of noise, and of side lobes
- One common idea is to eliminate points from the parameter space if they do not agree with the edge point direction
  - Limit the selection of θ and ensure it corresponds to the edge direction
  - Thus, for point (x_i, y_i) with edge point direction $\varphi(x_i,y_i)$ we assume that it will belong to a line with direction $\theta=90-\varphi(x_i,y_i)$ and consider only points in the range $\theta+\Delta\theta$, where $\Delta\theta$ is a small quantity, corresponding to three or four quantised levels of θ.



This appealing idea is really only successful if the image consists of well defined straight lines, as say an engineering drawing, or a photograph of the Barbican Centre, may do. Any more complex an image will be over filtered by this method.

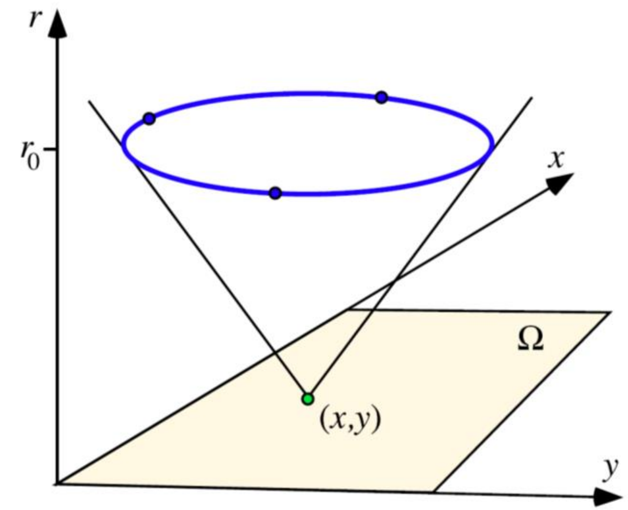
Hough Transform for Higher Orders - Circle

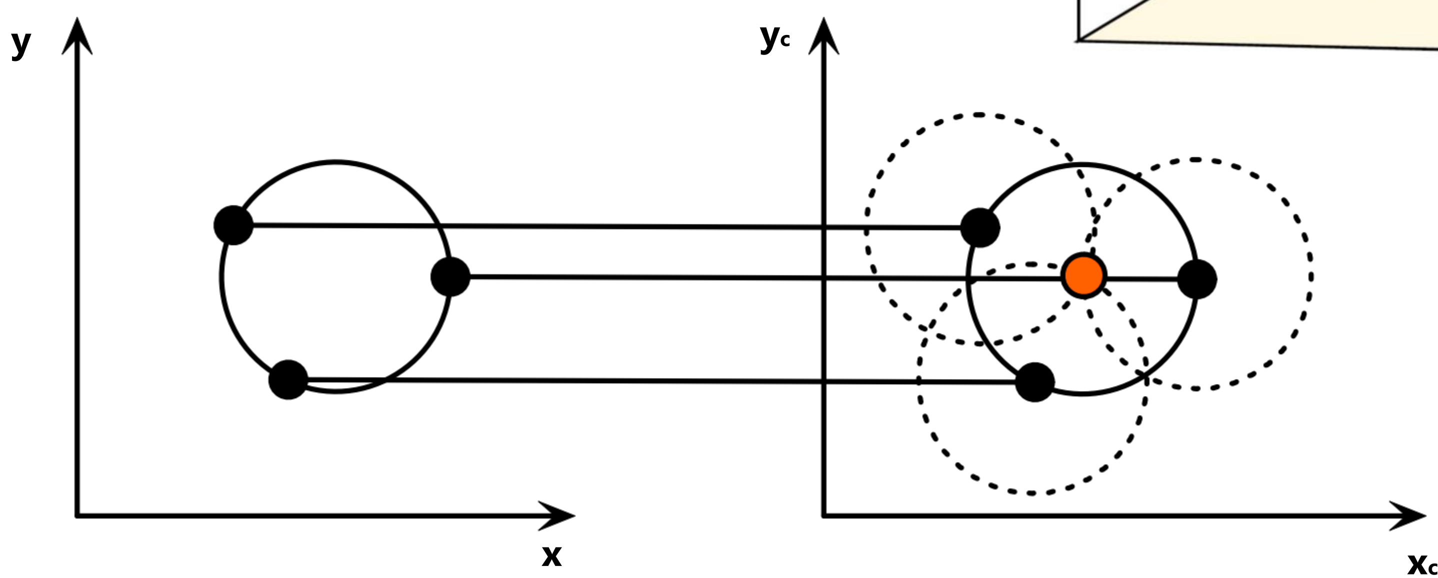
- A circle with radius R and centre (x_c,y_c):

$$\left(\frac{x - x_c}{R}\right)^2 + \left(\frac{y - y_c}{R}\right)^2 = 1$$

can be parameterised as:

$$x = x_c + R\ \cos\theta$$
$$y = y_c + R \sin\theta$$

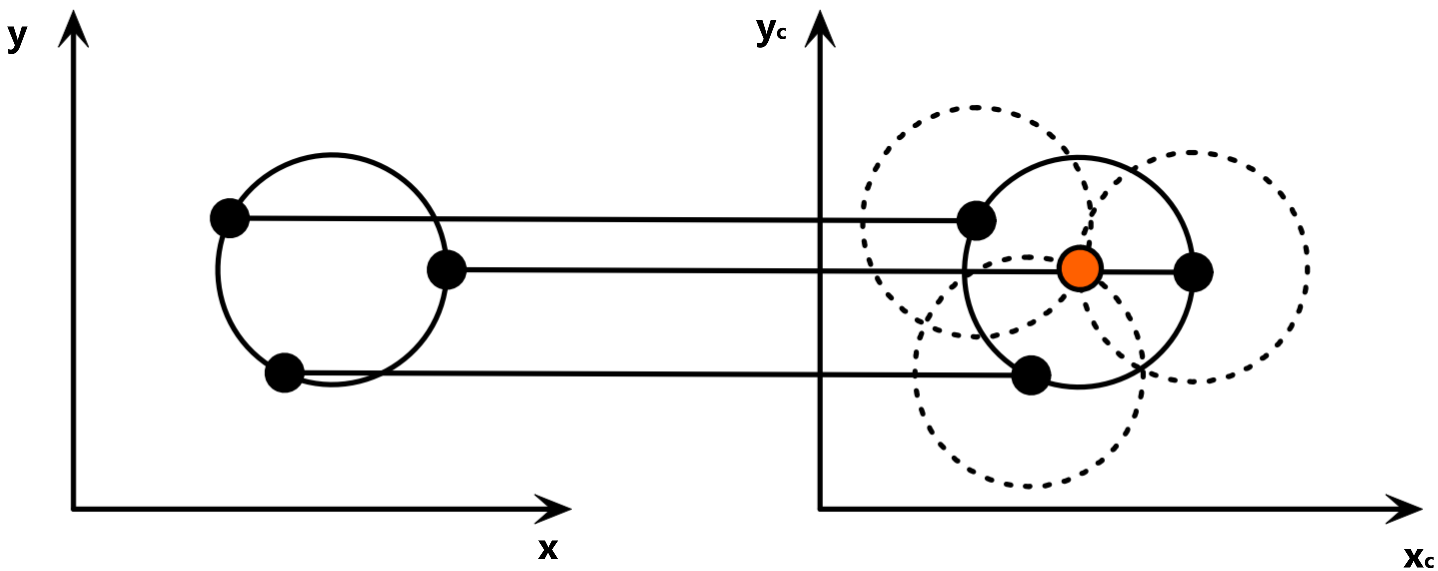




Hough Transform for Higher Orders - Circle

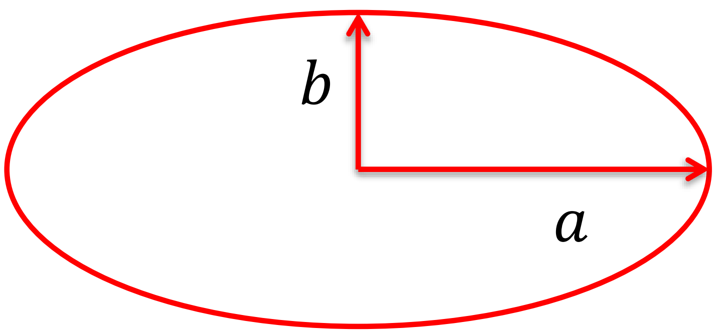
- The process of the circle parameter estimation can be divided into two stages:
  - The first stage is fixing radius then find the optimal centre of circles in a 2D parameter space.
  - The second stage is to find the optimal radius in a one dimensional parameter space.

$$x = x_c + R\ \cos\theta$$
$$y = y_c + R \sin\theta$$



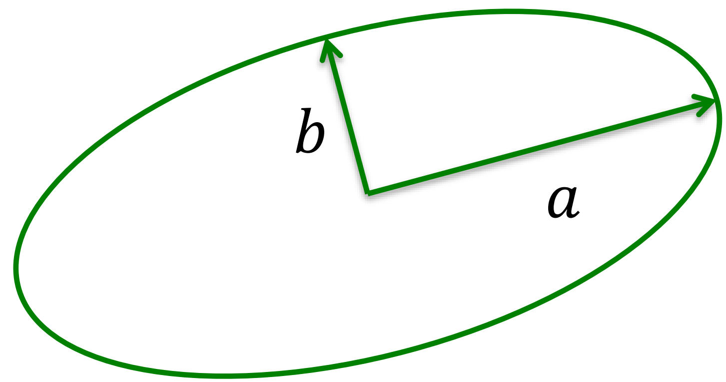
Hough Transform for Higher Orders - Ellipses

An ellipse at the origin and parallel to the axes is characterised by an equation:



$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

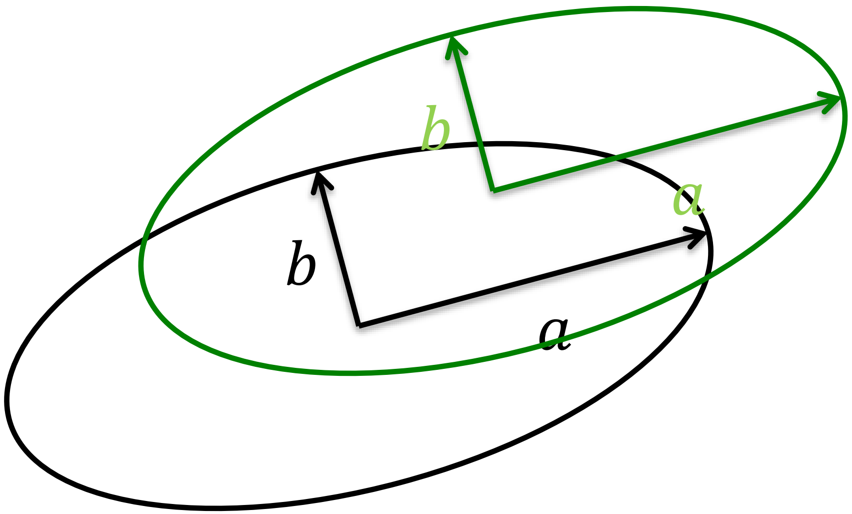
In general we need to consider the rotated ellipse:



$$\left(\frac{x\cos\theta - y\sin\theta}{a}\right)^2 + \left(\frac{x\sin\theta + y\cos\theta}{b}\right)^2 = 1$$

Hough Transform for Ellipses

And we need the ellipse to be at any position in the image (x_c, y_c) :

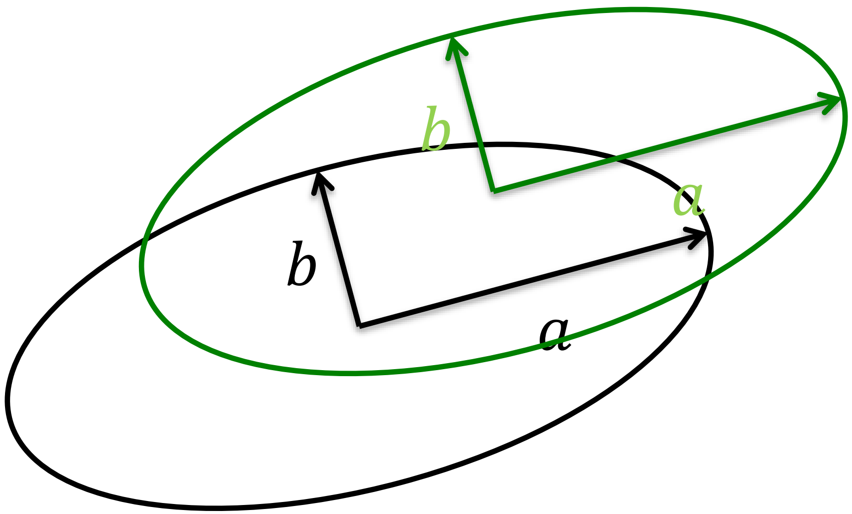


$$\left(\frac{(x - x_c)\cos\theta - (y - y_c)\sin\theta}{a}\right)^2 + \left(\frac{(x - x_c)\sin\theta + (y - y_c)\cos\theta}{b}\right)^2 = 1$$

- What parameters do we need to find?

What Parameters Do We Need to Find?

$$a, b, x_c, y_c, \theta$$



- Thus the histogram array will be five dimensional
- If we adopt the same strategy used for the straight line Hough transform and we quantize each parameter into n levels, we would need a n^5 size array.
  - If n=100, that would be 10,000,000,000, i.e., over 10Gb
- Accordingly, the usual approach is to decompose the problem into two parts namely the identification of the centre, followed by detection of the major and minor axes and the angle

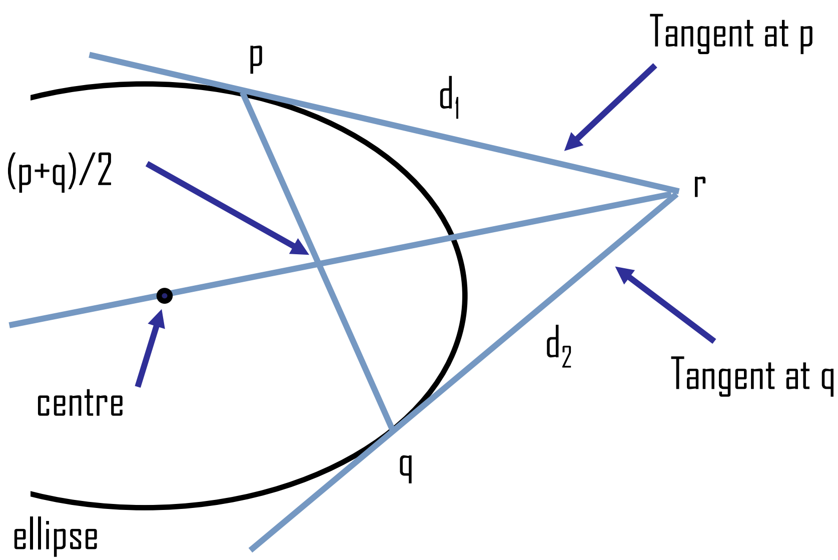
How to Simplify the Process?

$$a, b, x_c, y_c, \theta$$

- **Step 1:** Find the centre (x_c, y_c)
- **Step 2:** Estimate the ratio between $h=a/b$ and θ
- **Step 3:** Find a and b

Finding the Centre

- One way to find the centre is to use the symmetry property of the ellipse. We can apply the Hough method by finding all pairs of edge points with similar gradient, and calculating their mid points. A 2D histogram is constructed to count these mid points and the maxima are selected as candidates for the ellipse centre.
  - This method fails if part of the ellipse is obscured in the image
- An alternative strategy is to construct the centre from the tangent directions of a number of edge points



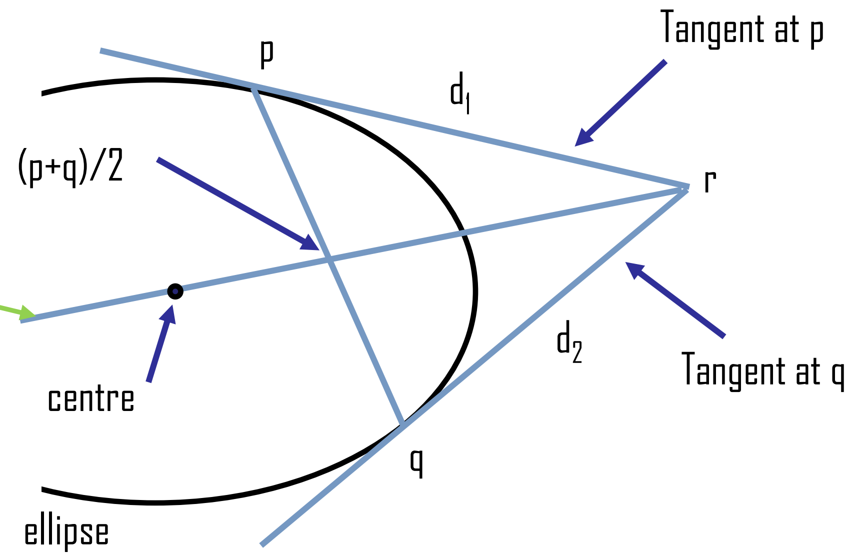
$$r = p + ad_1 = q + bd_2$$

$$d_1 = \left(g_y(p), -g_x(p)\right)$$
$$d_2 = \left(g_y(q), -g_x(q)\right)$$

Finding the Centre

$$c - r = m\left(\frac{1}{2}(p+q) - r\right)$$

$$\boxed{c = \frac{1}{2}m(p+q) + (1-m)r}$$



Separating this equation into the two Cartesian components, we can construct a two dimensional histogram in x-y, by stepping through y, computing the value of m from the y-component equation, then substituting the value into the x-component equation to calculate x and hence determine which histogram location to estimate. Again, we need to carry out the process for each pair of edge points in the edge map, after which the histogram maxima give us an estimate of the ellipse centre. The corresponding construction method for the circle is rather simpler.

Finding $h=a/b$ and θ

- The further three parameters may now be estimated directly from the equation:

$$\left(\frac{x\cos\theta - y\sin\theta}{a}\right)^2 + \left(\frac{x\sin\theta + y\cos\theta}{b}\right)^2 = 1$$

- with all points first translated to the estimated centre (x_c, y_c)

- However, we still require a histogram array of size n^3 for quantization into n levels. A quantization into 128 levels still requires an array with 2M entries.
- If we rearrange the equation by multiplying by a^2 we get:

$$(x\cos\theta - y\sin\theta)^2 + \left(\frac{a}{b}\right)^2 (x\sin\theta + y\cos\theta)^2 = a^2$$

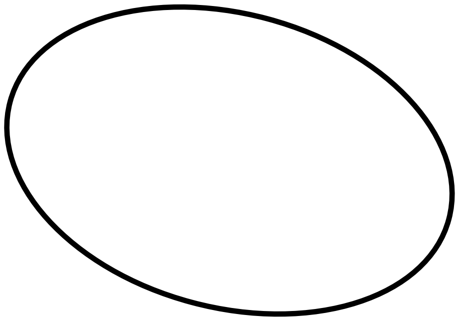
Finding h=a/b and θ

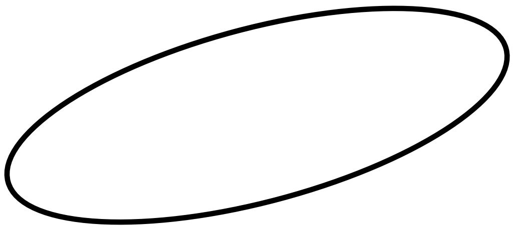
Letting h=a/b and differentiating we get

$$2(x\cos\theta - y\sin\theta)(\cos\theta - y'\sin\theta) + 2(x\sin\theta + y\cos\theta)(\sin\theta + y'\cos\theta)h^2 = 0$$

Note y'=dy/dx can be obtained from the edge direction.

h controls the aspect ration of the ellipse

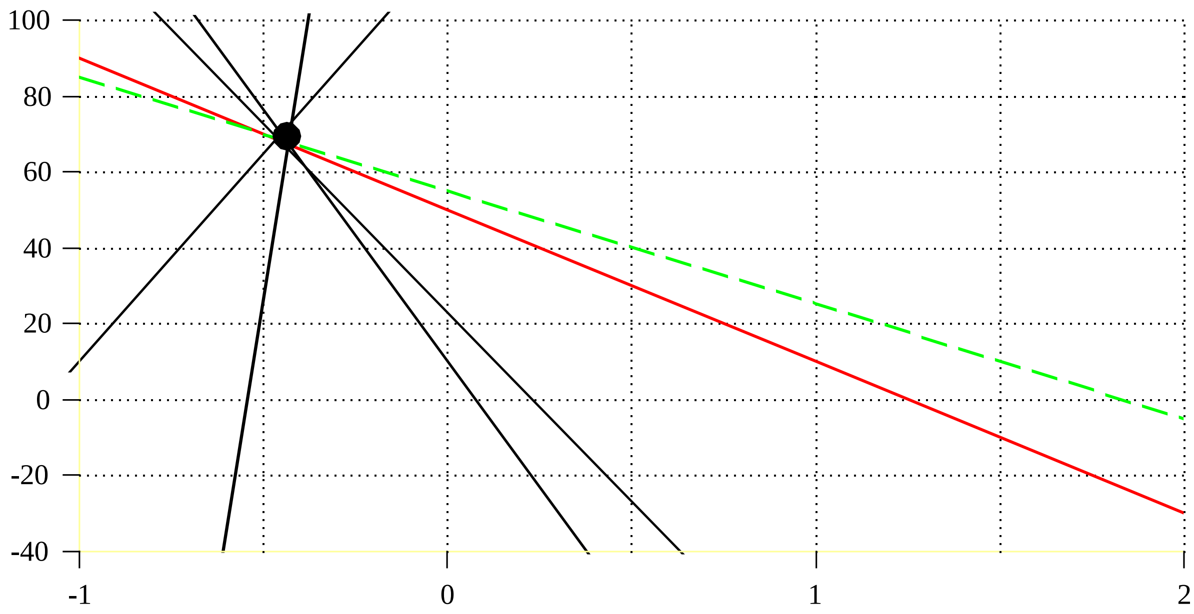




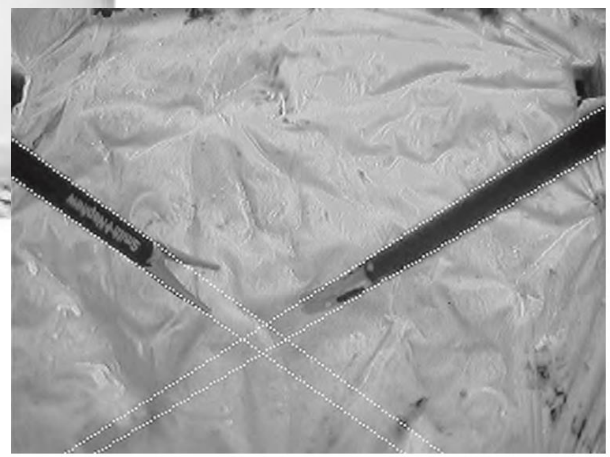
- This equation has now only two parameters (θ, h), which can be estimated by the normal Hough method without any additional computation
- In practice, it will be necessary to place limits on h, that will reflect the degree to which the ellipses can be squashed
- Once these two parameters have been estimated, the value of a^2 can be determined by using a one dimensional Hough transform with the original equation

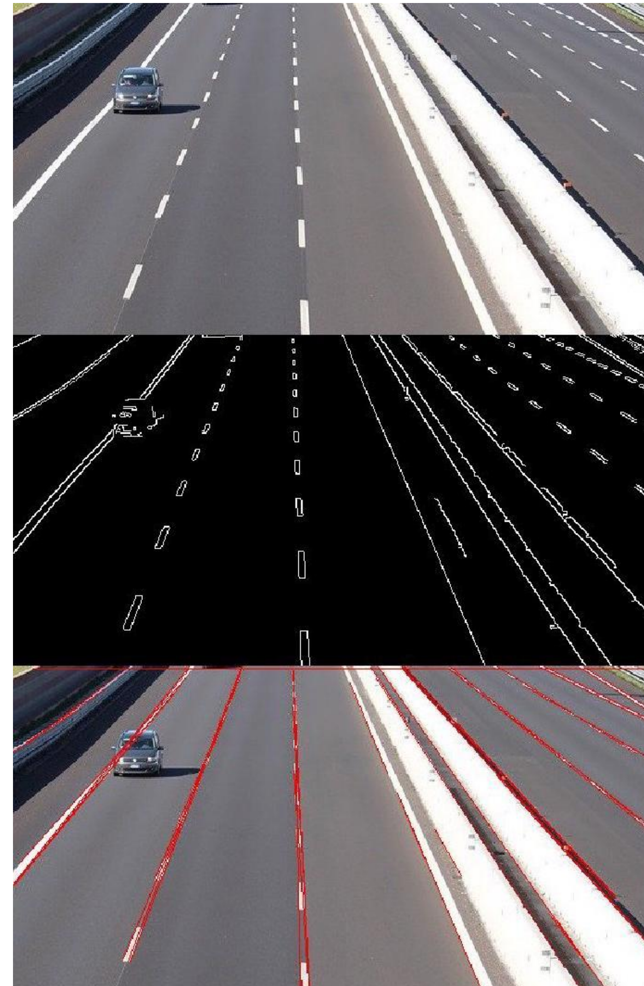
Adaptive Hough Transform

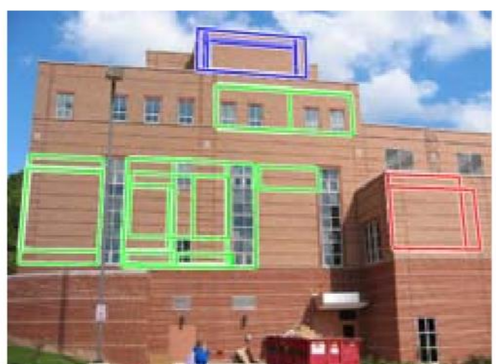
- A computational strategy, particularly useful for dealing with cases where three or more parameters need to be estimated, is to iteratively increase the resolution of the quantization
- This strategy reduces considerably the computation time and memory requirement, but has the disadvantage that thresholding is required, and consequently the correct histogram point may be hidden at high resolution by combinations of side lobes.



Applications of Hough Transform







Conclusions

- Hough transform for straight lines
- Side lobes and bias
- Heuristics in the Hough Transform
- Hough transform for extracting higher order shapes
- Adaptive Hough transform