

COMPUTATIONAL FINANCE: 422

Course Outline

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(Slides courtesy of Daniel Kuhn)

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General Information

- Lecturer: [Panos Parpas](#) (Online/zoom email: p.parpas@imperial.ac.uk)
- Tutorial Helpers:
 - [Daniel Lengyel](#) (d.lengyel19@imperial.ac.uk)
 - [Alexis Laignelet](#) (alexis.laignelet18@imperial.ac.uk)
 - [Conor Mcmeel](#) (c.mcmeel18@imperial.ac.uk)
- Lecture slides and tutorials are available on scientia
- There will be [weekly tutorials](#) and [one assessed coursework](#); active participation is strongly encouraged!

Prerequisites

Required:

- 233 - Computational techniques

for computing students; a similar background is required for the other engineering students

Recommended:

- 343 - Operations research

Students who have not taken this course are assumed to be familiar with mathematical programming (LP/QP)

Aims of the Course

After this course, students should

- understand the **basic concepts** of quantitative finance and financial engineering;
- be aware of the major **decision**, **hedging**, and **valuation problems** in finance, know how to formulate these problems as **mathematical models**, and know several **computational techniques** to solve the arising models;
- be able to read the **technical literature** in computational finance and to undertake independent **self-study** (or research) in the future.

General Remarks

Please note that

- this course does not prepare you for a typical IT job in the financial industry;
- this course discusses tools that would be useful for a quantitative analyst;
- although every effort is made to present the concepts in an intuitive manner, this course may not be suitable for people who experience a discomfort when exposed to mathematical formalism.

Recommended Books I

- D.G. Luenberger, *Investment Science*, Oxford University Press, 1998.
 - Extremely well written; everyone should own that book.
- D.J. Higham, *An Introduction to Financial Option Valuation*, Cambridge University Press, 2004.
 - Very good intro to computational methods; Matlab code available from http://personal.strath.ac.uk/d.j.higham/option_book.html
- J. Hull, *Options, Futures, and other Derivatives*, Prentice Hall, 2012.
 - A classic text on derivatives.
- P. Wilmott, *Derivatives: the Theory and Practice of Financial Engineering*, Wiley, 1998.
 - Also a classic text on derivatives.

Recommended Books II

- P. Boyle and F. Boyle, *Derivatives: the Tools that Changed Finance*, Risk Books, 2001.
 - Very good introductory text; available freely from www.thederivativesbook.com.
- D. Duffie, *Dynamic Asset Pricing Theory*, Princeton University Press, 2001.
 - Standard text for doctoral students and researchers; more difficult to read than the other books in this list.
- T. Crack, *Heard on the Street: Quantitative Questions from Wall Street Job Interviews*, 2009.
 - Just for fun or to prepare for a job interview in a bank.

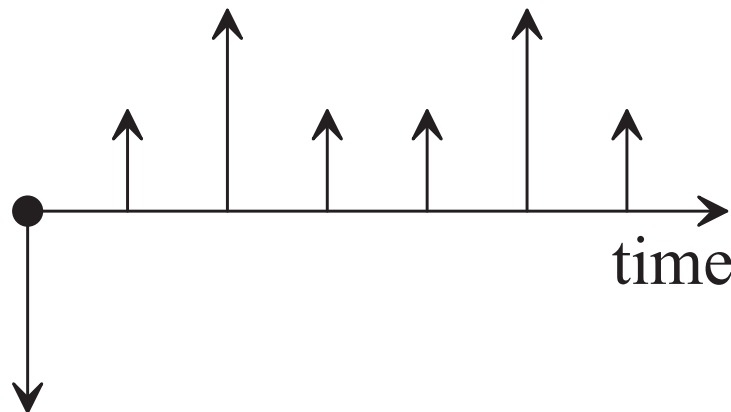
This course is mainly based on the books by Luenberger and Higham.

(Planned) Course Outline

- Introduction
- Mathematical Preliminaries
- The Basic Theory of Interest
- Fixed-Income Securities
- Mean-Variance Portfolio Theory
- The Capital Asset Pricing Model
- General Principles
- Asset Price Dynamics
- Basic Options Theory
- Additional Options Topics

Cash Flow Streams I

- An **investment** is the **current commitment of resources** (e.g. money) in order to achieve **later benefits** (hopefully more money).
- In most situations, the amount of money to be obtained later is **uncertain**.
- Broader interpretation: an **investment** is defined in terms of its resulting **cash flow stream**, that is, the amounts of money that will flow to and from an investor over time.



Cash Flow Streams II

- Which of two given cash flow streams should I prefer?
- How much would I be willing to pay for a given stream?
- Are two streams together worth more to me than the sum of their individual values?
- Given a collection of several cash flow streams, what is the most favorable combination of them?

Sometimes, the timing and the amounts of the cash flows in a stream are not fixed, but can be influenced by the investor.

- ⇒ Determination of suitable management strategies is also part of investment science.
- ⇒ One can view investment science as the tailoring of cash flow streams.

Investments and the Market

- Investment analysis is the process of **examining alternatives** and **deciding which alternative is the most preferable**.
- Investment problems differ from other decision problems in an important respect: they are carried out within the framework of a **financial market**.
- The financial market provides a **basis for comparison**.
- Important aspects are:
 - the **comparison principle**;
 - **arbitrage**;
 - **dynamics**;
 - **risk aversion**.

Example: Financial Option Pricing

Aim of this example:

- Provide a (preliminary) understanding of the basic principles of option pricing.

Learning outcomes:

- Students will be able to describe the characteristics of a European call option and
- to calculate its fair value by using on a binomial lattice model.

Betting on Coin Tosses

Basic proposition #1: You pay £1. I flip a coin.

- If it is heads, you get £3.
- If it is tails, you get nothing.

Basic proposition #2: You pay £1. I flip a coin.

- If it is heads, you get £1.
- If it is tails, you get £1, as well.

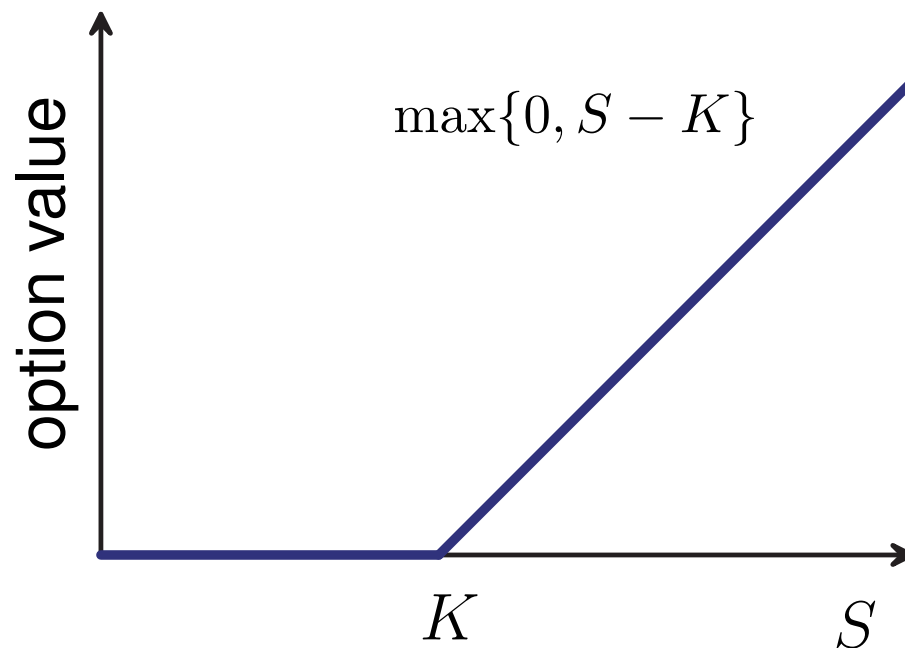
New proposition: I flip the coin twice.

- If at least one flip is heads, you get £9.
- If no flip is heads, you get nothing.

How much is this proposition worth?

European Call Options

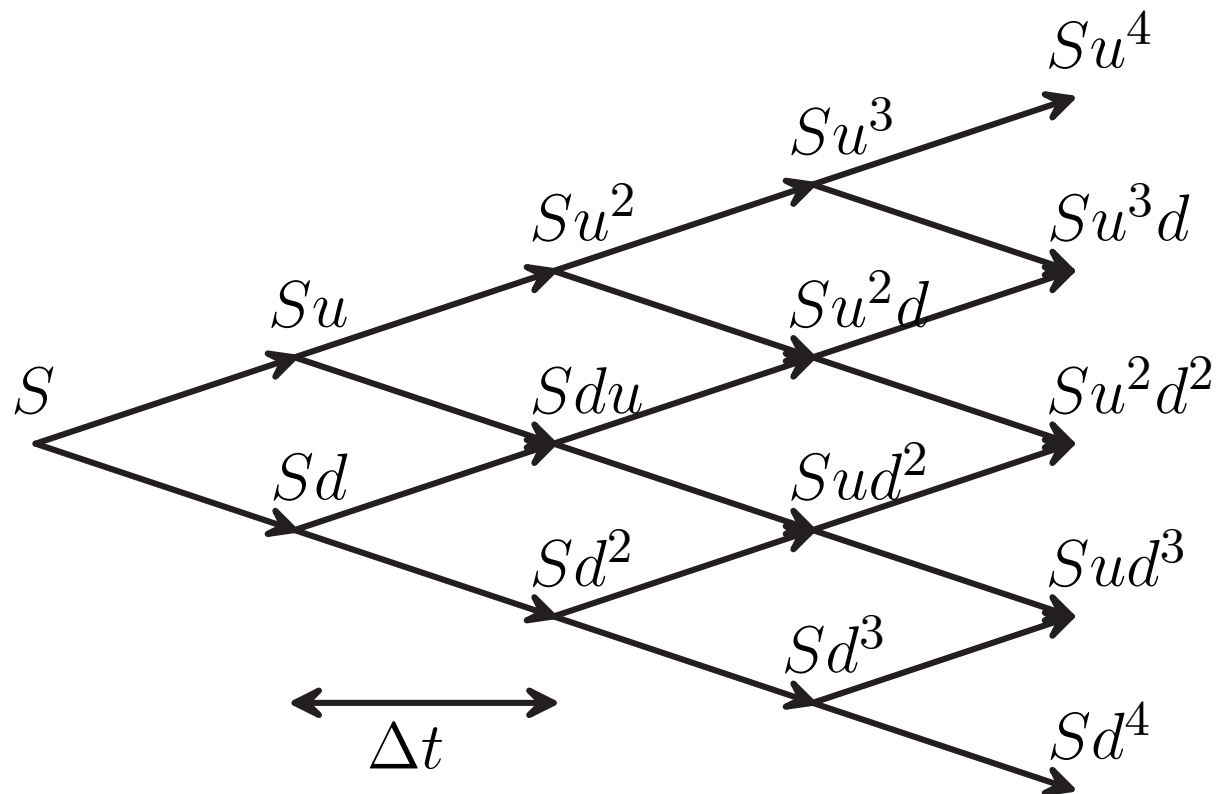
Definition: A **European call option** gives its **holder** the right (but not the obligation) to purchase from the **writer** a specific stock for a prescribed **strike price** at a future time.



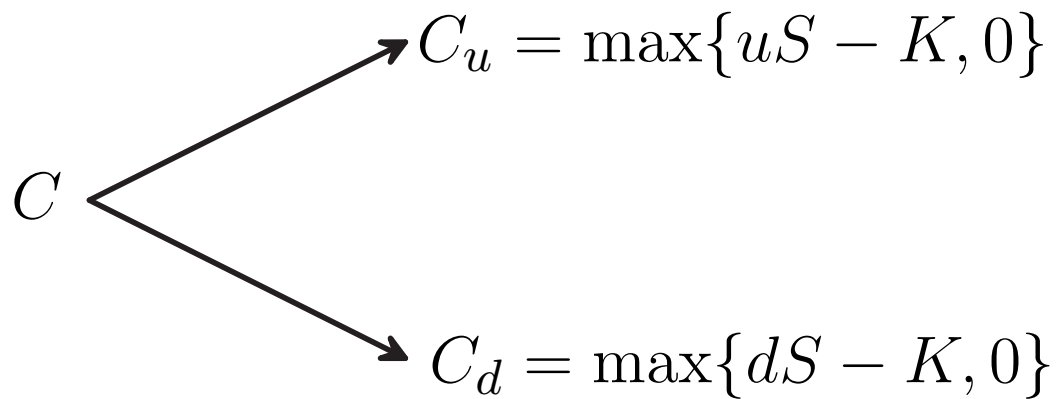
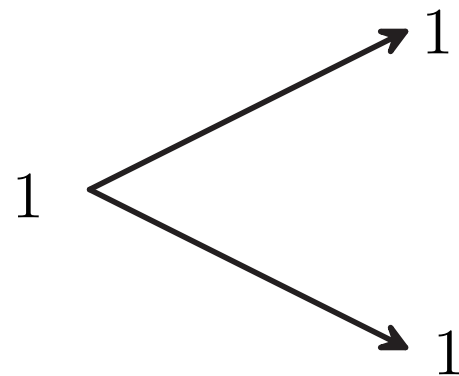
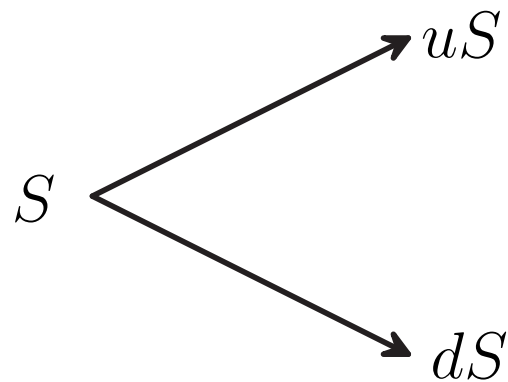
Payoff at expiry (S = stock price, K = strike price)

Binomial Lattice Model

Over a period of length Δt the **stock price** moves either up (with probability p) or down (with probability $1 - p$).



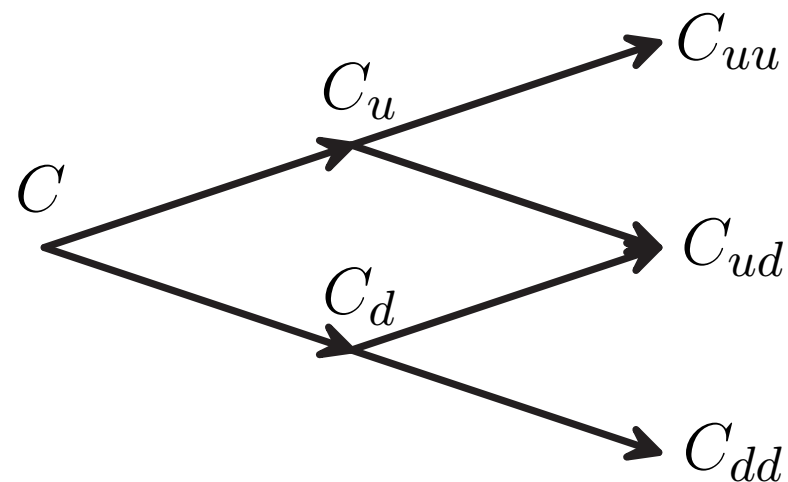
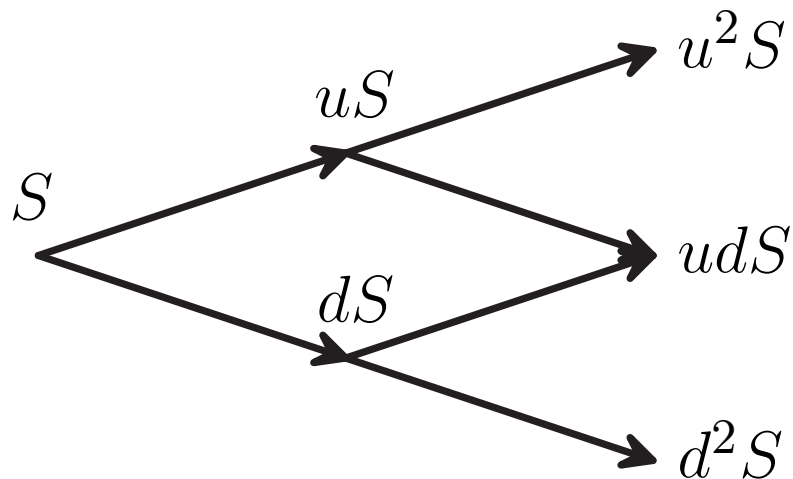
Single-Period Binomial Options Pricing



$$\left. \begin{array}{l} ux + y = C_u \\ dx + y = C_d \end{array} \right\} \Rightarrow C = x + y = \frac{1-d}{u-d}C_u + \frac{u-1}{u-d}C_d$$

Multiperiod Binomial Options Pricing

The one-period solution can be extended to multiperiod options by **working backward one step at a time**.



Terminal condition:

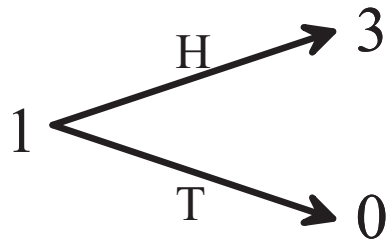
$$C_{uu} = \max\{u^2S - K, 0\}$$

$$C_{ud} = \max\{udS - K, 0\}$$

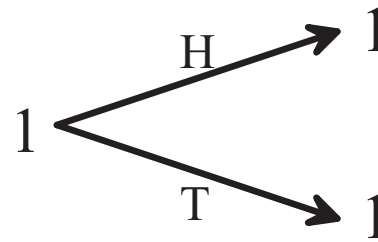
$$C_{dd} = \max\{d^2S - K, 0\}$$

Betting on Coin Tosses (Revisited)

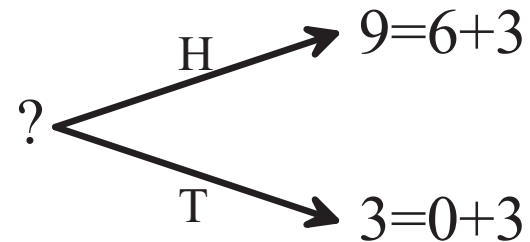
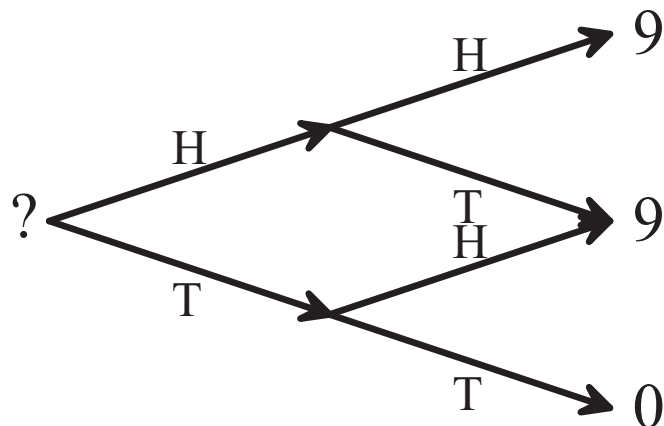
Proposition #1



Proposition #2



New proposition



The value of the new proposition is £5.