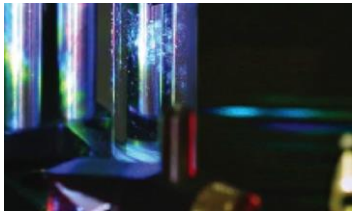
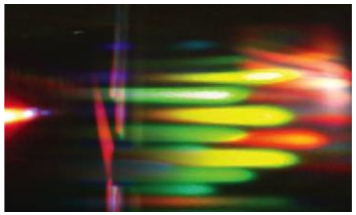
Imperial College London





COMP70058 Computer Vision

Lecture 10 – Feature Descriptors and Matching

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Contents

- Image Processing with Local Features
- Feature Descriptors
- SIFT
- SURF
- Shape Context descriptor
- Feature Descriptor Distance
- Feature Matching

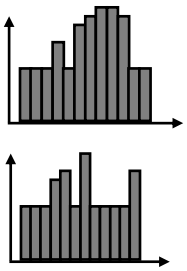
Image Processing with Local Feature

1. **Detection** - Find a set of distinctive key points.

2. **Description** - Extract feature descriptor around each interest point as vector.

$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}] \qquad \mathrm{x}_1$$

$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}] \qquad \mathrm{x}_2$$



3. **Matching** - Compute distance between feature vectors to find correspondence.

$$d(\mathrm{x}_1, \mathrm{x}_2) < T$$

Feature Descriptors

Note for good image features, we require **locality** (small areas are less sensitive to view-dependent deformations), **invariance** (sale, orientation and deformation), **repeatability** (same points can be repetitively identified as required for image sequence tracking), and **distinctiveness** (to ensure high sensitivity and specificity).

- Most feature descriptors can be thought of as templates, histograms (counts), or combinations
- The ideal descriptor should be
  - Robust and Distinctive
  - Compact and Efficient
- Most available descriptors focus on edge/gradient information
  - Capture texture information
  - Color rarely used

SIFT Descriptor

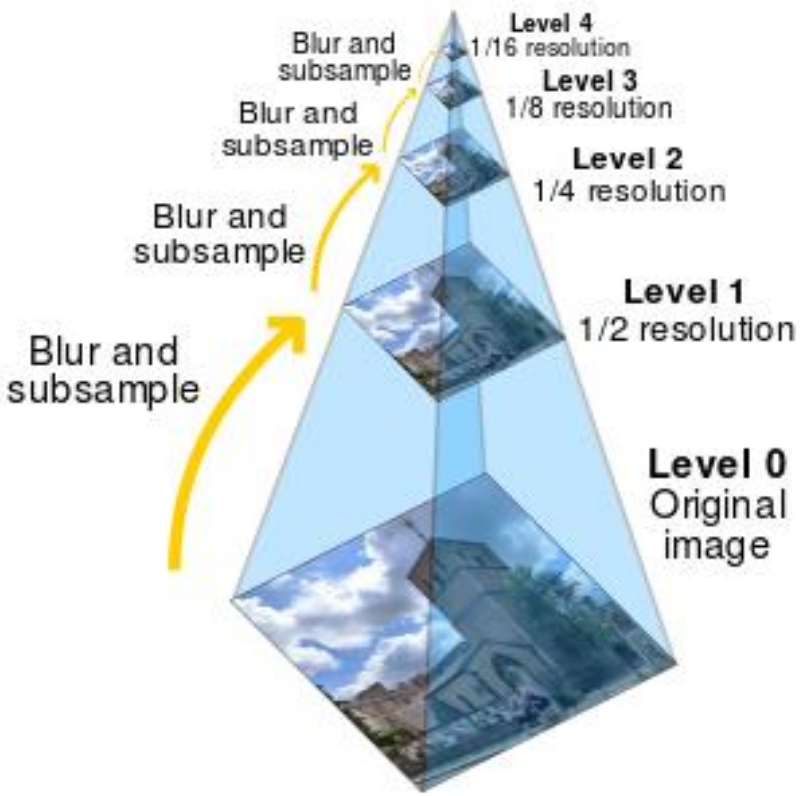
SIFT – Scale Invariant Feature Transform

Step 1: Scale-space Extrema Detection - Detect interesting points using DOG.

Step 2: Keypoint Localization – Determine location and scale at each candidate location, and select them based on stability.

Step 3: Orientation Estimation – Use local image gradients to assign orientation to each localized keypoint. Preserve theta, scale and location for each feature.

Step 4: Keypoint Descriptor - Extract local image gradients at the selected scale around the keypoint and form a representation invariant to local shape distortion and illumination



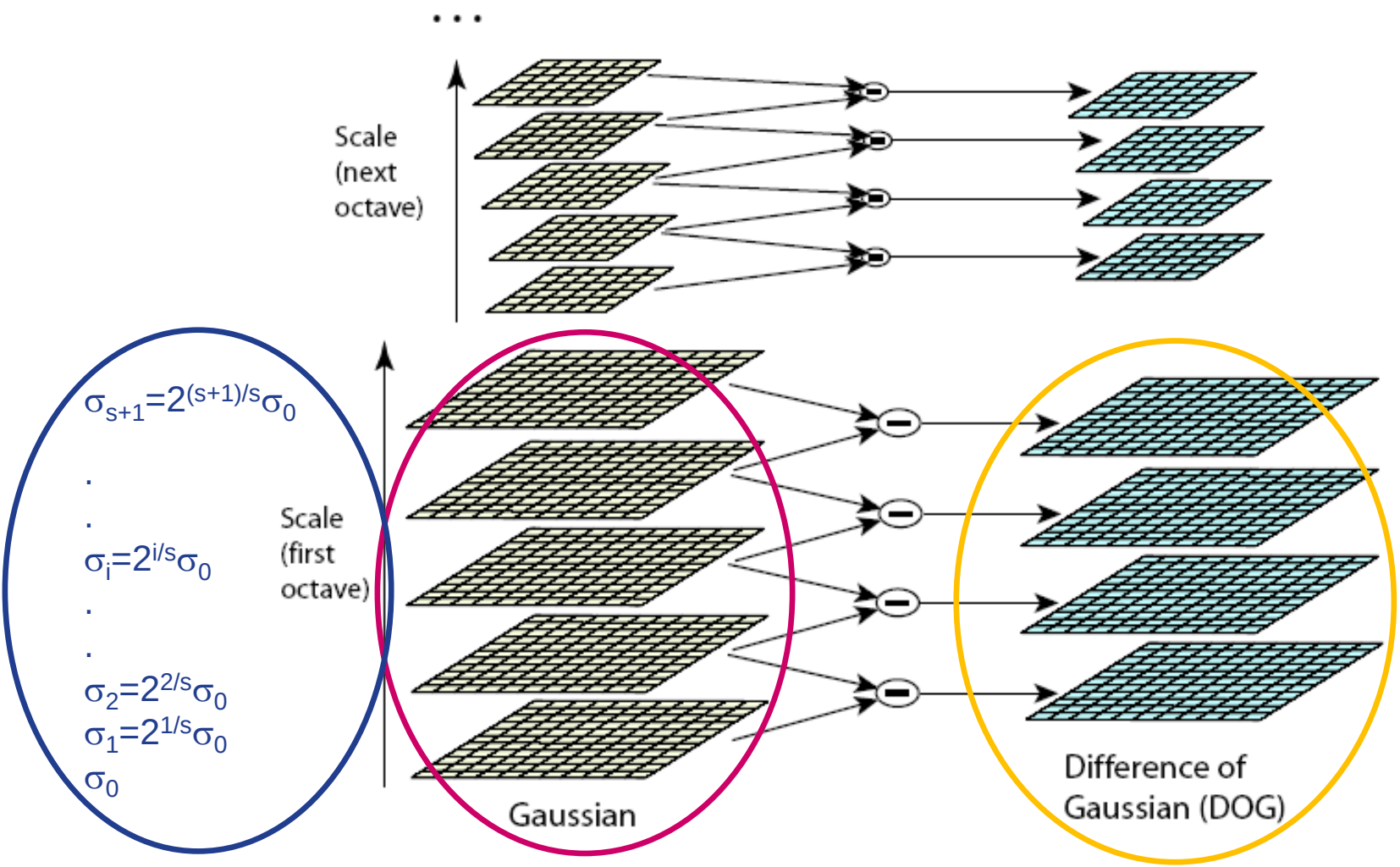
https://cs.brown.edu/courses/csci1430/

D. Lowe. Distinctive image features from scale-invariant keypoints, IJCV 2004.

SIFT Descriptor - Extrema Detection

- Scale space is separated into octaves
  - Octave 1 uses scale σ
  - Octave 2 uses scale 2σ
- In each octave, the initial image is repeatedly convolved with Gaussians to produce a set of scale space images
- Adjacent Gaussians are subtracted to produce the DOG
- Once a complete octave has been processed, the Gaussian image that has twice the initial value of σ (it will be 2 images from the top of the stack) is resampled by taking every second pixel in each row and column to start the next level

SIFT Descriptor - Extrema Detection



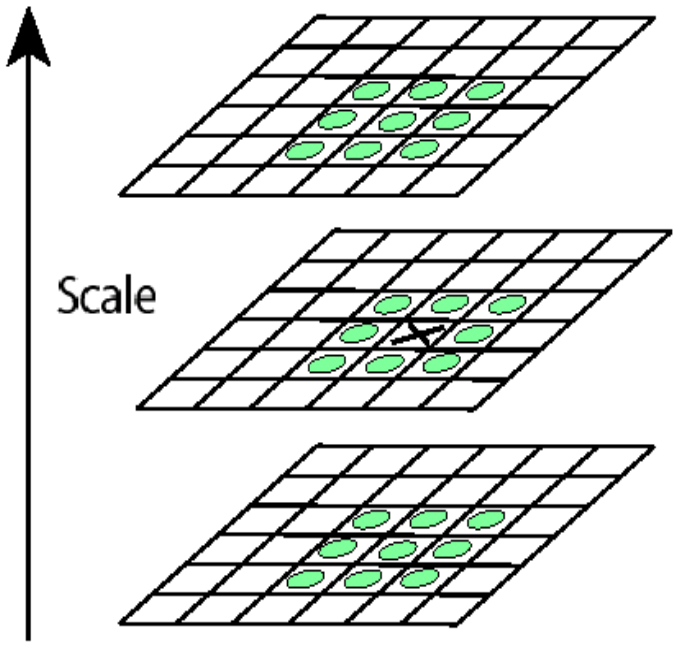
s+2 filters

s+3 images including original

s+2 difference images

SIFT Descriptor - Extrema Detection

- Increasing σ increases robustness, but it is also computationally expensive. σ = 1.6 a good tradeoff
- It has been shown experimentally that the highest repeatability is obtained when sampling 3 scales per octave
- Detect maxima and minima of Difference-of-Gaussian in scale space
- Each point is compared to its 26 neighbors in the current image and in the scales above and below



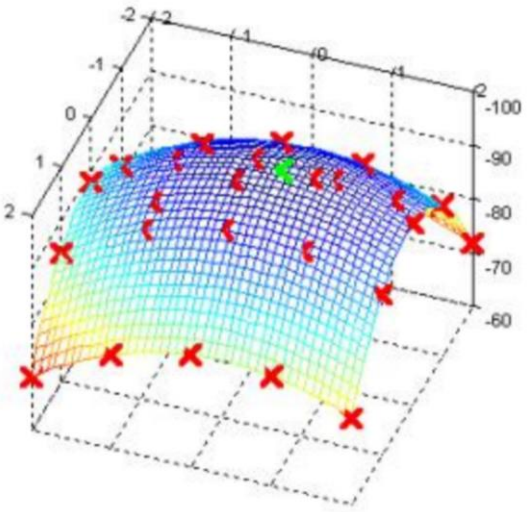
⇒ **List of**
(x, y, s)

SIFT Descriptor - Keypoint Localization

- For each candidate keypoint, interpolation of nearby data is used to accurately determine its position and scale
- This can be done by fitting a quadratic Taylor expansion of the Difference-of-Gaussian scale-space function shifted so that the origin is at the sample point

$$D(\mathrm{X}) = D + \frac{\partial D^T}{\partial \mathrm{X}}\mathrm{X} + \frac{1}{2}\mathrm{X}^T\frac{\partial^2 D}{\partial \mathrm{X}^2}\mathrm{X}$$

where $\mathrm{X} = (x, y, \sigma)^T$



- The offset for the extremum location is determined by taking the derivative of this function with respect to X and setting it to zero

$$0 = \frac{\partial D}{\partial X} + \frac{\partial^2 D}{\partial X^2}\hat{X}$$

SIFT Descriptor - Keypoint Localization

- This equation results in a 3x3 linear system
- Derivatives can be approximated by finite differences
- The solution using Newton's method is:

$$\hat{\mathrm{X}} = -\frac{\partial^2 D^{-1}}{\partial \mathrm{X}^2}\frac{\partial D}{\partial \mathrm{X}}$$

- If the offset X is greater than 0.5 in any dimension, it means that the extremum lies closer to a different sample point and the process is repeated

$$\begin{bmatrix} \frac{\partial^2 D}{\partial \sigma^2} & \frac{\partial^2 D}{\partial \sigma y} & \frac{\partial^2 D}{\partial \sigma x} \\ \frac{\partial^2 D}{\partial \sigma y} & \frac{\partial^2 D}{\partial y^2} & \frac{\partial^2 D}{\partial y x} \\ \frac{\partial^2 D}{\partial \sigma x} & \frac{\partial^2 D}{\partial y x} & \frac{\partial^2 D}{\partial x^2} \end{bmatrix} \begin{bmatrix} \sigma \\ y \\ x \end{bmatrix} = -\begin{bmatrix} \frac{\partial D}{\partial \sigma} \\ \frac{\partial D}{\partial y} \\ \frac{\partial D}{\partial x} \end{bmatrix}$$

$$\frac{\partial D}{\partial \sigma} = \frac{D_{k+1}^{i,j} - D_{k-1}^{i,j}}{2}$$

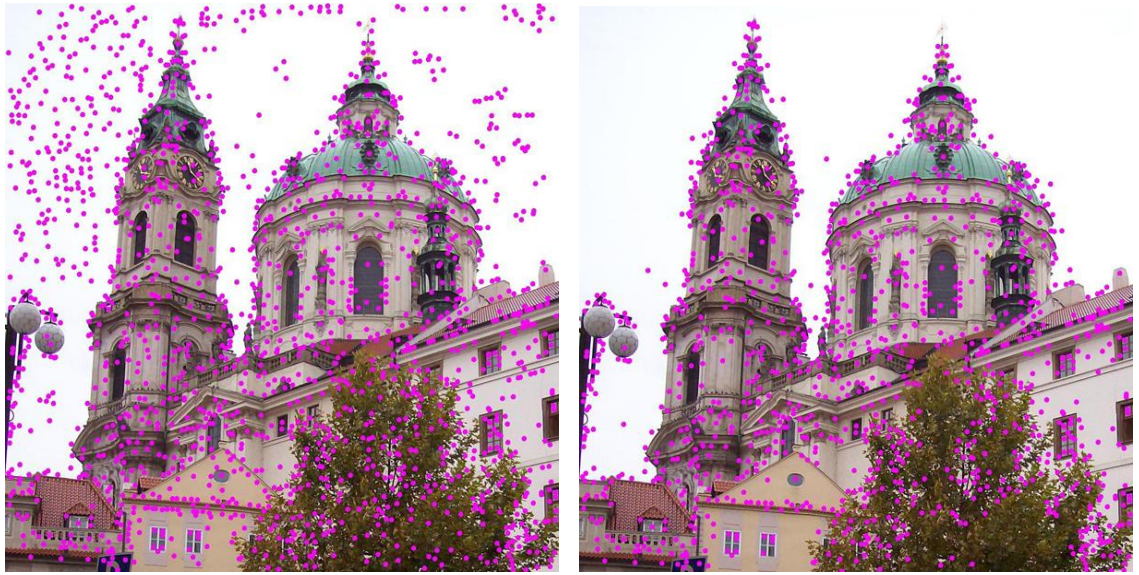
$$\frac{\partial^2 D}{\partial \sigma^2} = \frac{D_{k-1}^{i,j} - 2D_{k}^{i,j} + D_{k+1}^{i,j}}{1}$$

$$\frac{\partial^2 D}{\partial \sigma y} = \frac{(D_{k+1}^{i+1,j} - D_{k-1}^{i+1,j}) - (D_{k+1}^{i-1,j} - D_{k-1}^{i-1,j})}{4}$$

SIFT Descriptor - Keypoint Localization

- Some of the detected keypoints lie along an edge, or they don't have enough contrast. In both cases, they are not as useful as features and should be eliminated
- If the DoG value at an extrema is less than a threshold (e.g 0.03), the keypoint is rejected

$$|D(\hat{\mathbf{x}})| < 0.03$$



https://en.wikipedia.org/wiki/Scale-invariant_feature_transform

SIFT Descriptor - Keypoint Localization

- A keypoint located on an edge in the image will have a large principle curvature across the edge and a low one along it. On the contrary, a well-defined keypoint will have a large principle curvature in both directions. Finding these principal curvatures amounts to solving for the eigenvalues of the second-order Hessian matrix.

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

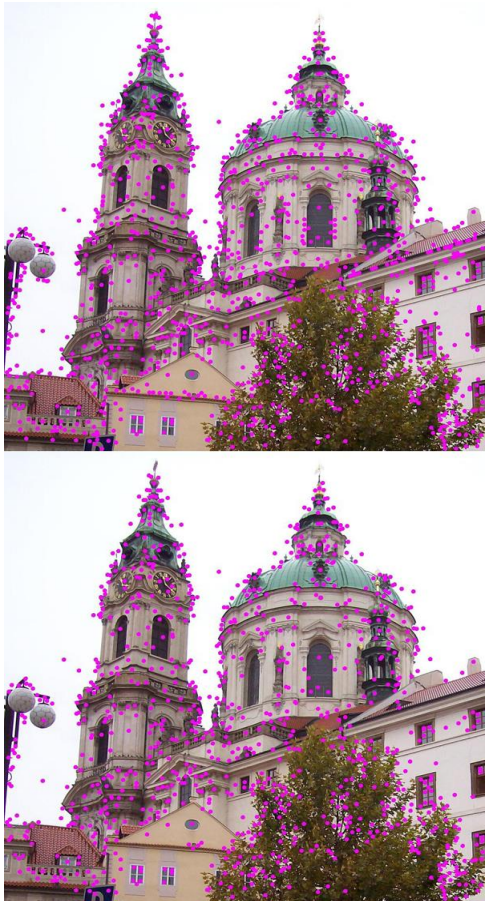
$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r}$$

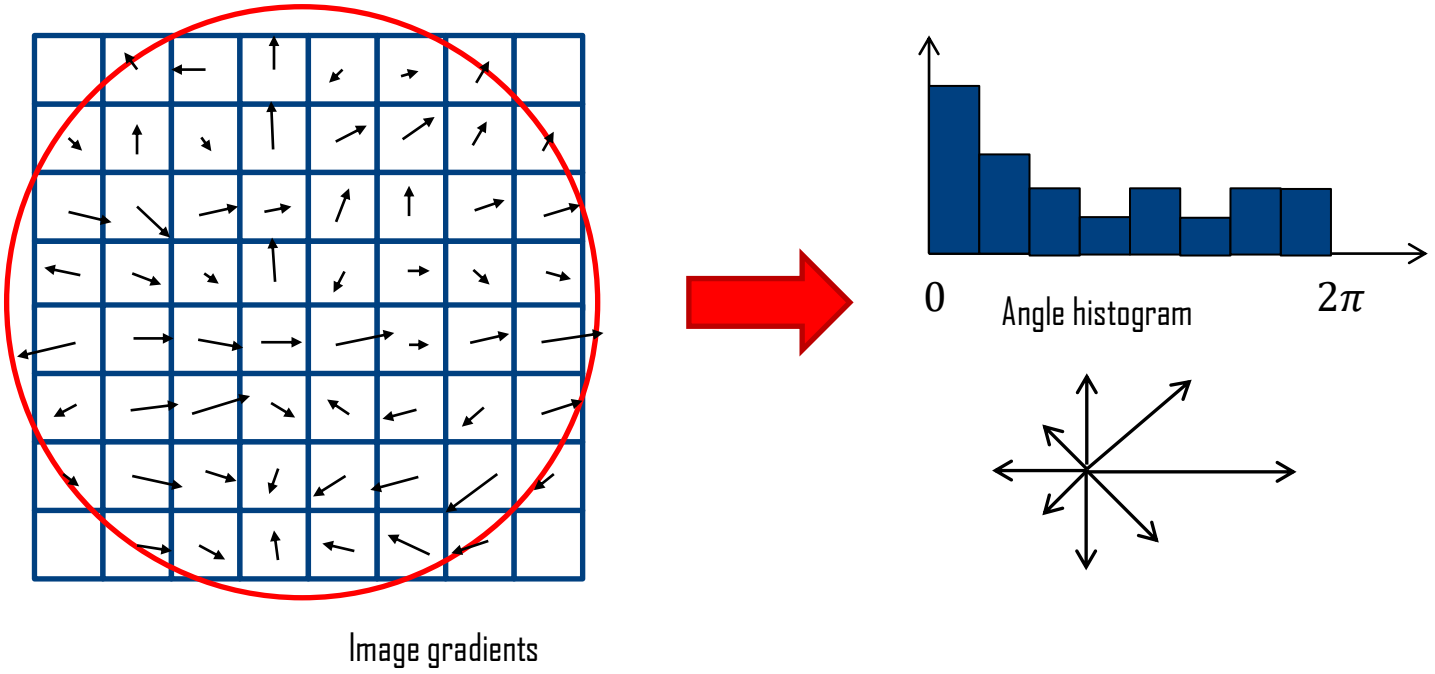
$> (r_{th}+1)^2/(r_{th})$

If the above ratio for a candidate keypoint is larger than a threshold (r_{th}=10), the keypoint is poorly localized and hence rejected.



SIFT Descriptor

- Each keypoint is characertised by location, scale, orientation
- A descriptor is computed for the local image window around each keypoint



SIFT Descriptor

- The Basic Idea
  - 16×16 window around interest point (8×8 shown below)
  - Compute edge orientation (angle of the gradient minus 90° for each pixel)
  - Throw out weak edges (threshold gradient magnitude)
  - Create histogram of surviving edge orientations

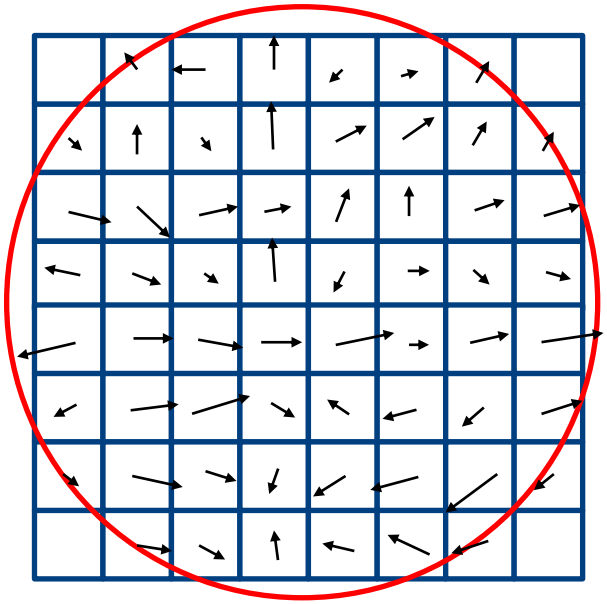
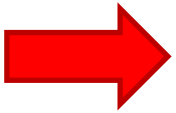
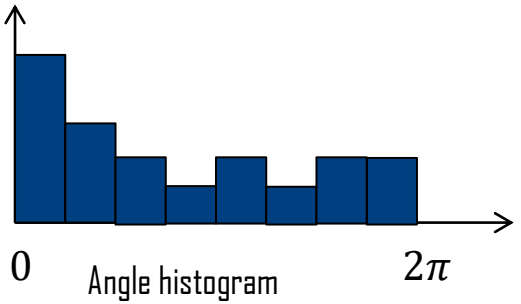
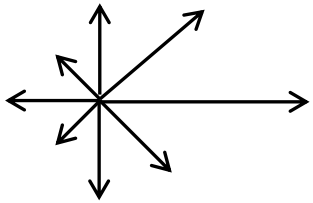


Image gradients







SIFT Descriptor

- Full Version
  - Divide the 16×16 window into a 4×4 grid of cells (8×8 window and 2×2 grid shown below for simplicity)
  - Compute an orientation histogram for each cell
  - 16 cells * 8 orientations = 128 dimensional descriptor

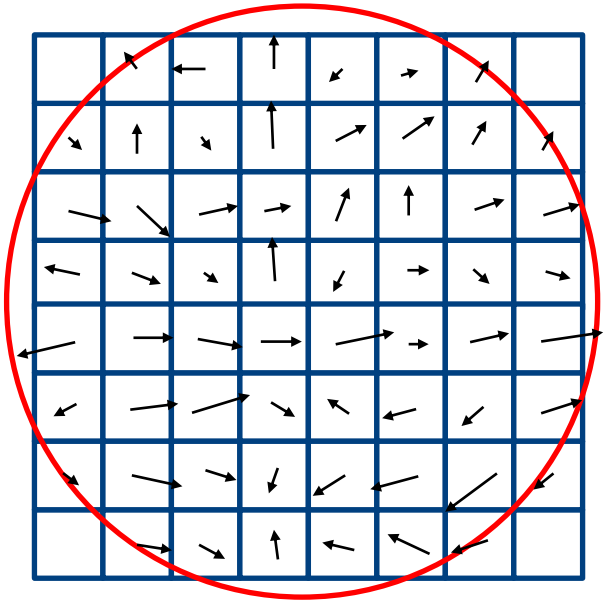
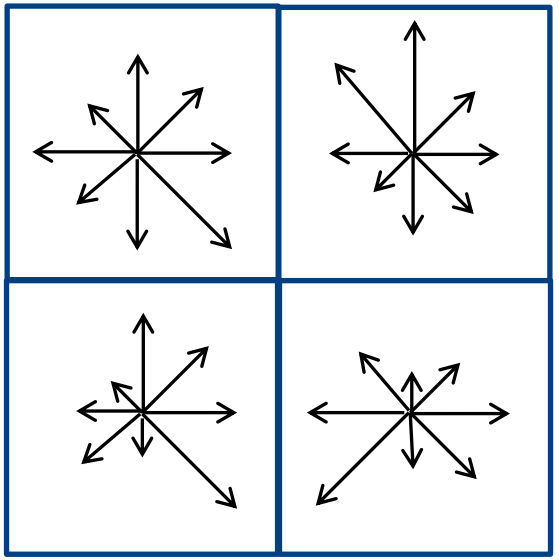


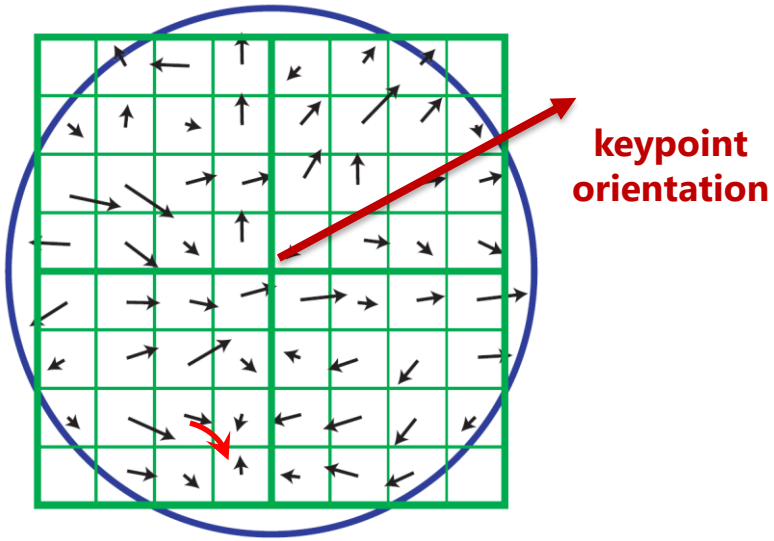
Image gradients



Keypoint descriptor

SIFT Descriptor - Invariance Properties

- To be robust to intensity value changes
  - Use gradient orientations
- To be scale-invariant
  - Estimate the scale using scale-space extrema detection
  - Scale the window size based on that scale at which the point was found
  - Calculate the gradient after Gaussian smoothing with this scale
- To be orientation-invariant
  - Rotate the gradient orientations using the dominant orientation in a neighbourhood.

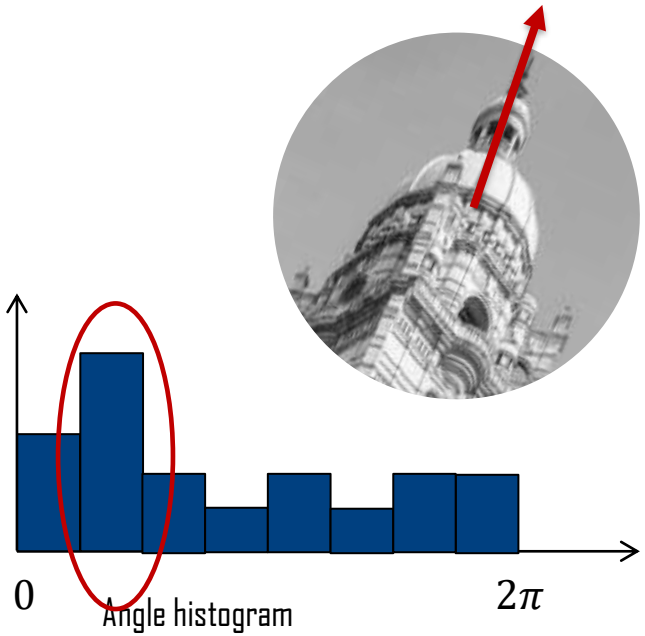


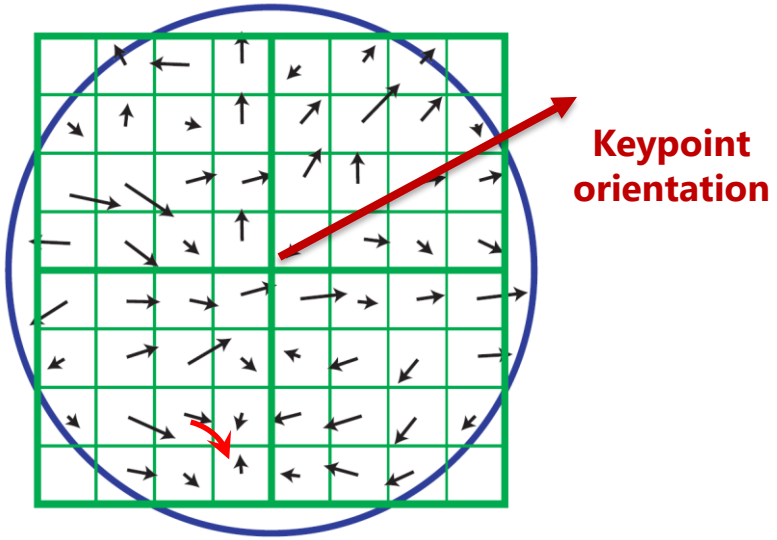
SIFT Descriptor - Orientation Estimation

- To be orientation-invariant the descriptor of a keypoint can be computed relative to the dominant orientation in its neighbourhood

$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$$

$$\theta(x,y) = a\tan 2((L(x,y+1) - L(x,y-1))/(L(x+1,y) - L(x-1,y)))$$





Properties of SIFT Descriptors for Matching

- Robust and can handle changes in viewpoint
  - Up to about 60 degrees out of plane rotation
- Can handle significant changes in illumination
- Fast and efficient, can run in real-time
- Other variations available, e.g., SURF, PCA SIFT, GLOH (gradient location-orientation histogram)

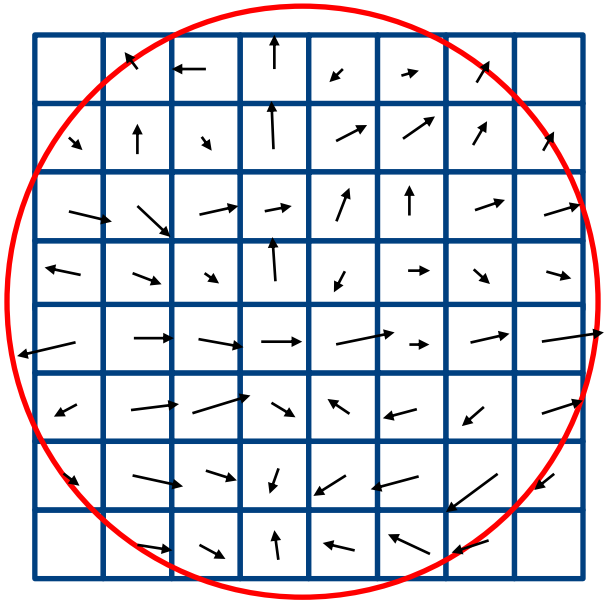
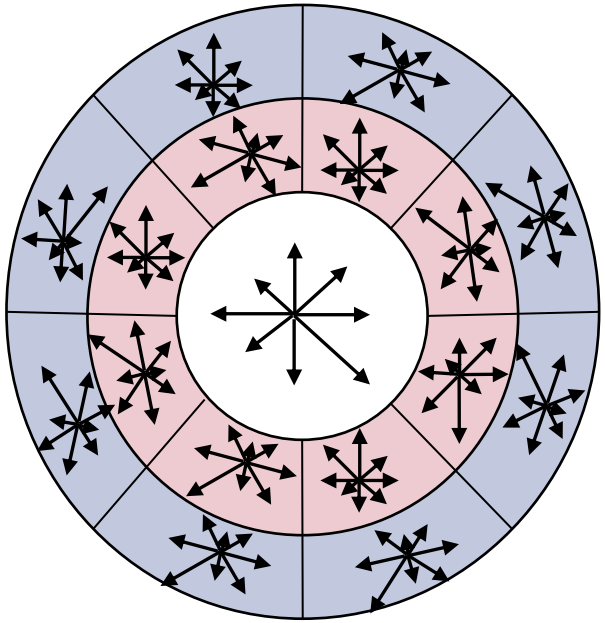


Image gradients

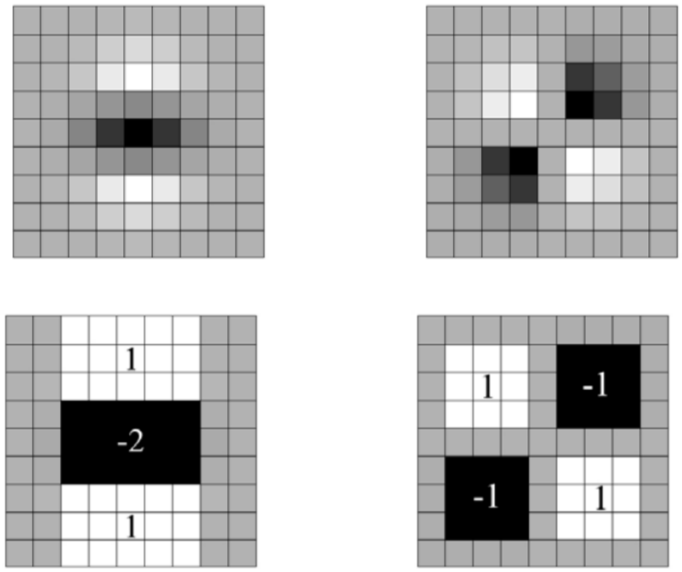


SURF: Speeded Up Robust Features

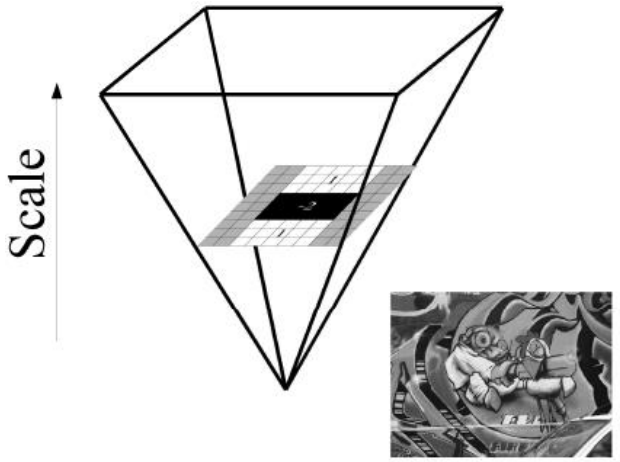
- The SURF feature detector works by applying an approximate Gaussian second derivative mask to an image at multiple scales and using the Hessian matrix to find points of interest

$$\det(H_{approx}) = D_{xx}D_{yy} - (0.9D_{xy})^2$$

- The method is very fast because of the use of integral images for image convolution
- Due to using integral images, filters of any size can be applied at high speed
- A keypoint is detected as a local maximum of the determinant of the Hessian matrix within a $3 \times 3 \times 3$ array similar to keypoint detection in the LoG or DoG scale space

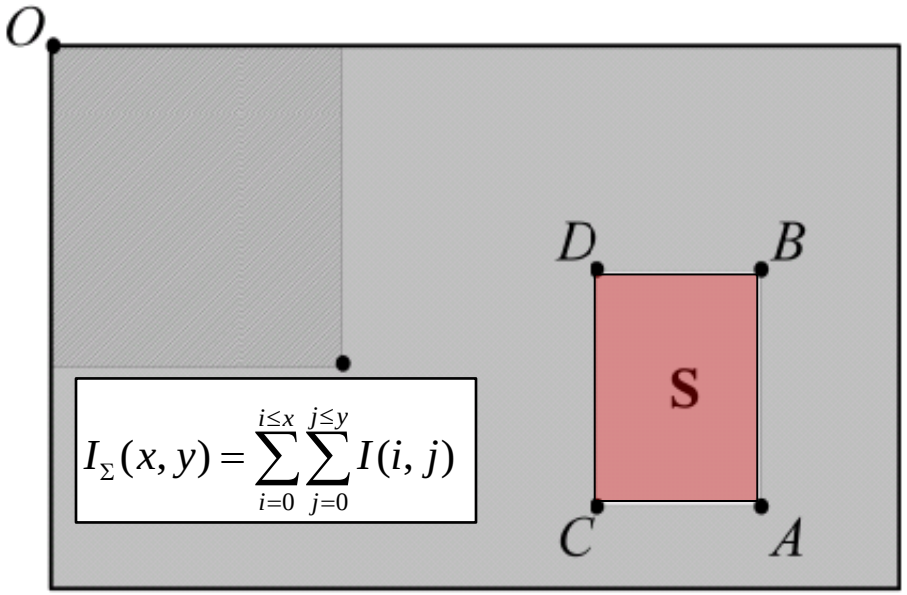


Gaussian second order partial derivatives in y-direction and xy-direction and their box filter approximations



SURF: Speeded Up Robust Features

- The integral image $I_\Sigma(x,y)$ of an image I(x, y) represents the sum of all pixels in I(x,y) of a rectangular region formed by (0,0) and (x,y).
- Using integral images, it takes only four array references to calculate the sum of pixels over a rectangular region of any size.



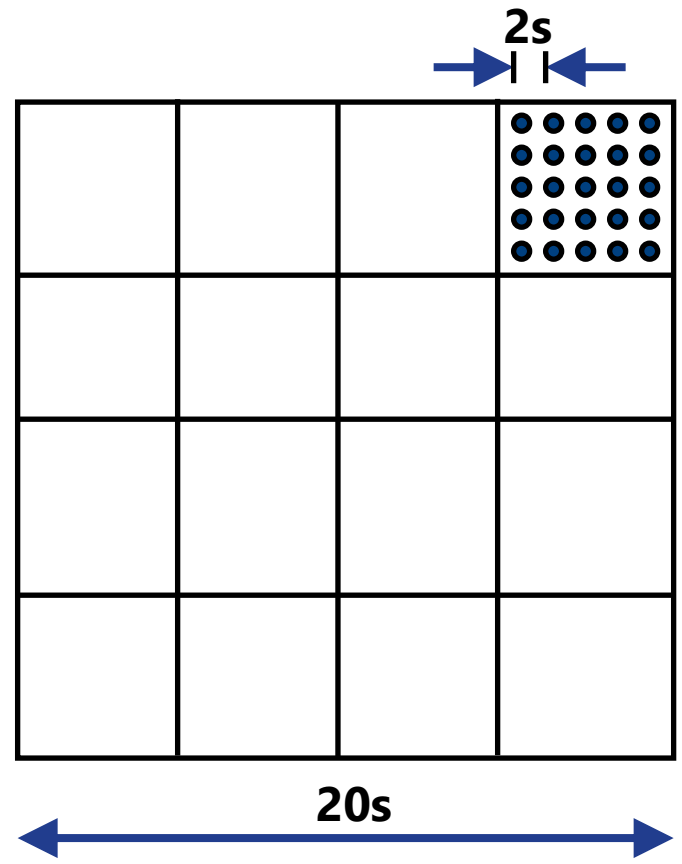
S=A-B-C+D

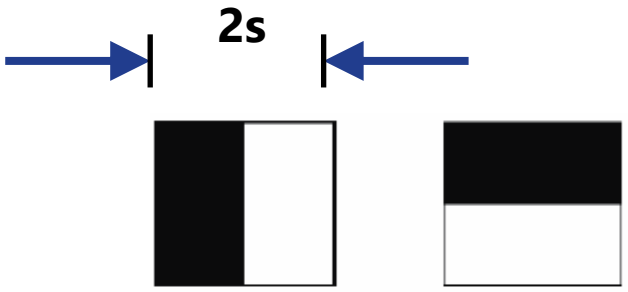
SURF: Speeded Up Robust Features

- To estimate the SURF descriptor of a keypoint:
  - Divide an image window of size 20*scale around the keypoint into 4x4 subregions
  - The Haar wavelet response d_x in the horizontal and d_y in the vertical direction is estimated in each subregion at 5x5 regularly spaced sample points
  - In each subregion:
    - estimate dx and dy at the 25 (5x5) points
    - sum over all 25 points to get 4 values

SURF = (dx, dy, |dx|, |dy|)

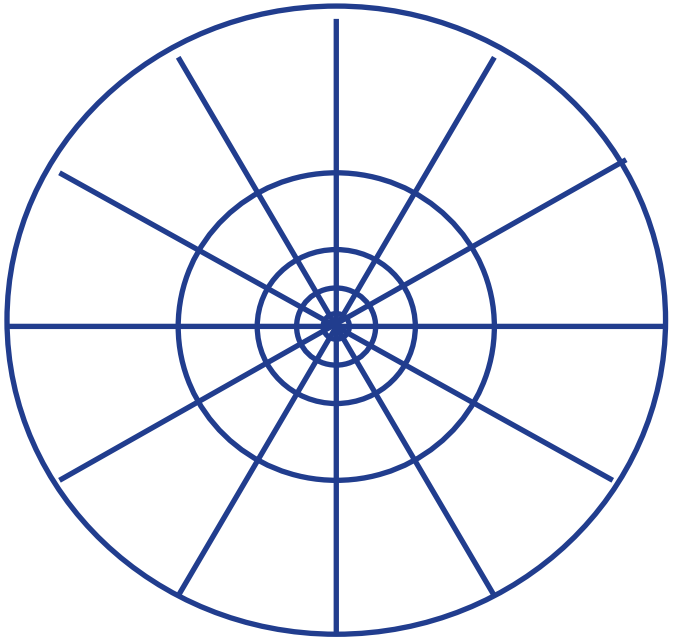
  - This results in a descriptor vector of length 4x16=64
  - 3 times faster than SIFT

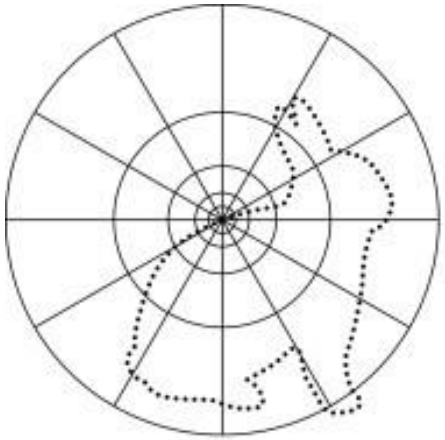


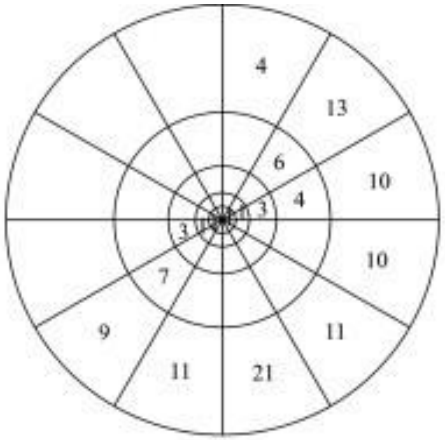


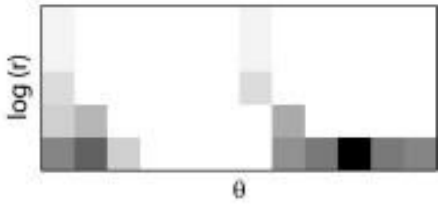
Shape Context Descriptor

- The Shape Context descriptor estimates a 2D histogram of edge point locations and orientations
- The location is quantized into 5 bins using a log-polar coordinate system
- The orientation is quantized in 12 bins
- To estimate the descriptor, the number of points inside each bin are counted
- Log-polar binning provides more precision for nearby points, more flexibility for farther points









Common Detectors/Descriptors

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Overview

This page is focused on the problem of detecting affine invariant features in arbitrary images and on the performance evaluation of region detectors/descriptors.

Affine Covariant Regions

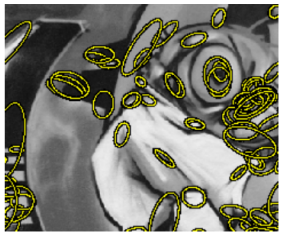


Image 1

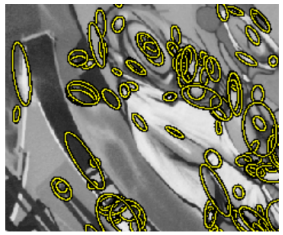


Image 2

Publications

Region detectors

- *Harris-Affine & Hessian Affine*: K. Mikolajczyk and C. Schmid, Scale and Affine invariant interest point detectors. In IJC V 60(1):63-86, 2004. PDF
- *MSER*: J.Matas, O. Chum, M. Urban, and T. Pajdla, Robust wide baseline stereo from maximally stable extremal regions. In BMVC p. 384-393, 2002. PDF
- *IBR & EBR*: T.Tuytelaars and L. Van Gool, Matching widely separated views based on affine invariant regions . In IJCV 59(1):61-85, 2004. PDF
- *Salient regions*: T. Kadir, A. Zisserman, and M. Brady, An affine invariant salient region detector. In ECCV p. 404-416, 2004. PDF
- *All Detectors - Survey*: T. Tuytelaars and K. Mikolajczyk , Local Invariant Feature Detectors - Survey. In CVG, 3(1):1-110, 2008. PDF

Region descriptors

- *SIFT*: D. Lowe, Distinctive image features from scale invariant keypoints. In IJCV 60(2):91-110, 2004. PDF

Performance evaluation

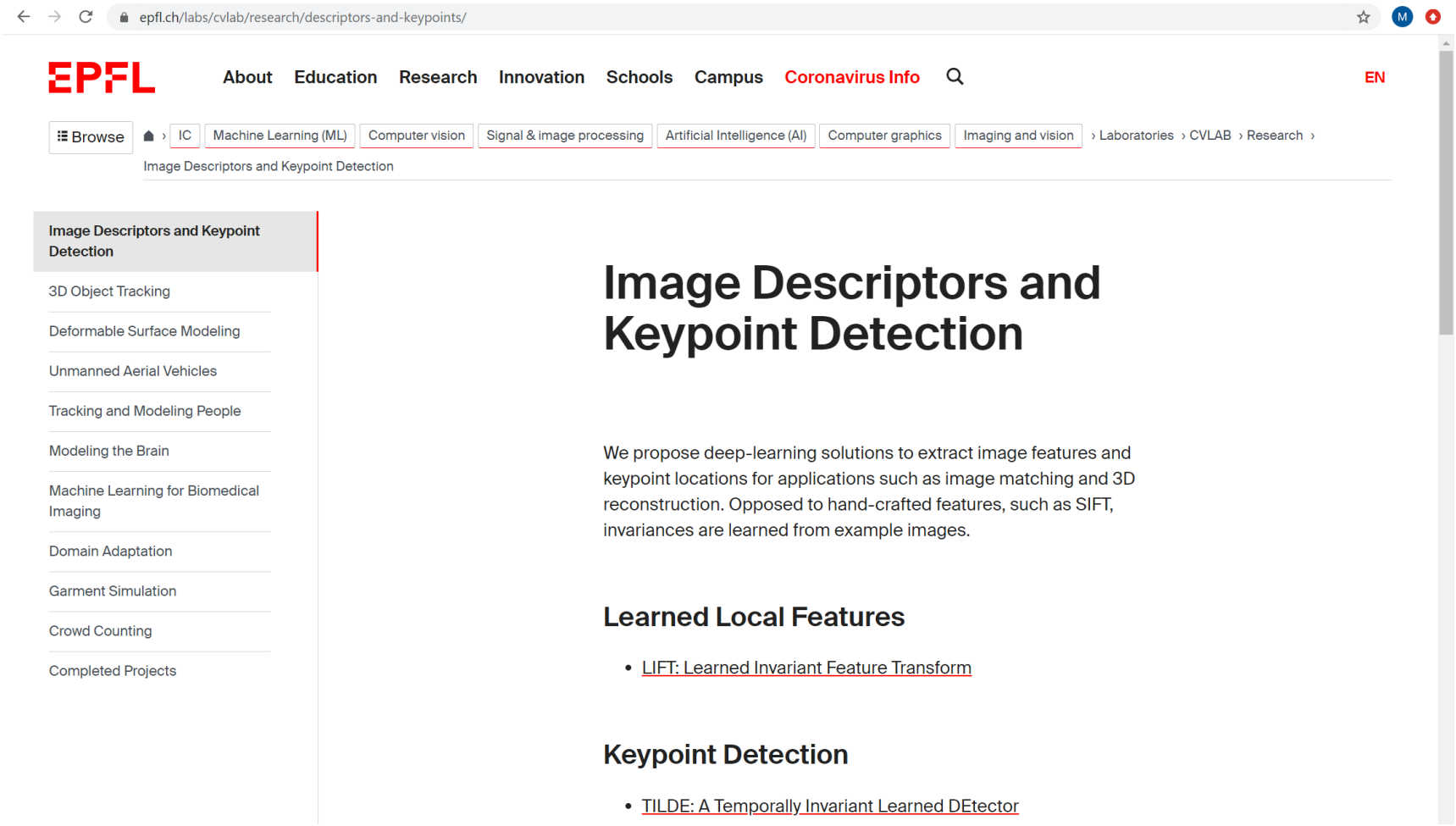
- K. Mikolajczyk, T. Tuytelaars, C. Schmid, A. Zisserman, J. Matas, F. Schaffalitzky, T. Kadir and L. Van Gool, A comparison of affine region detectors. In IJCV 65(1/2):43-72, 2005. PDF
- K. Mikolajczyk, C. Schmid, A performance evaluation of local descriptors. In PAMI 27(10):1615-1630 . PDF

Software

- Region detectors - Linux binaries for detecting affine covariant regions.
- Region descriptors - Linux binaries for computing region descriptors.

http://www.robots.ox.ac.uk/~vgg/research/affine/index.html

Common Detectors/Descriptors



https://www.epfl.ch/labs/cvlab/research/descriptors-and-keypoints/

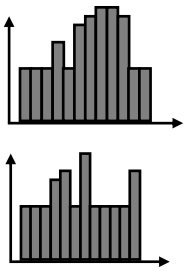
Image Processing with Local Feature

1. **Detection** - Find a set of distinctive key points.

2. **Description** - Extract feature descriptor around each interest point as vector.

$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}] \qquad \mathrm{x}_1$$

$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}] \qquad \mathrm{x}_2$$



3. **Matching** - Compute distance between feature vectors to find correspondence.

$$d(\mathrm{x}_1, \mathrm{x}_2) < T$$

Feature Descriptor Distance

Two feature descriptors **x** an **y** can be compared using any of the following distances:

Quadratic distance

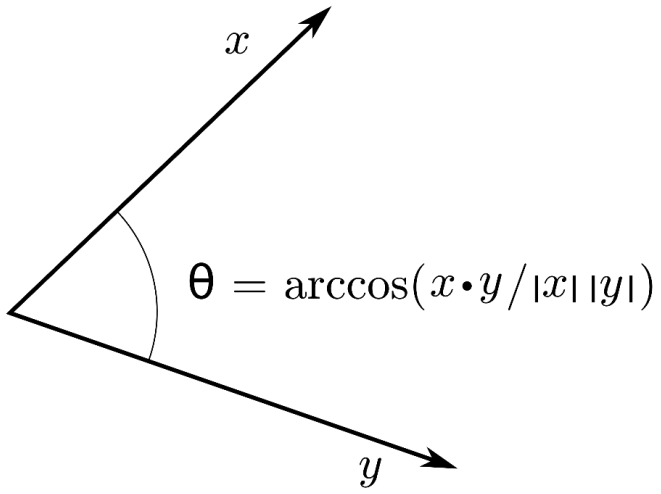
$$D(\boldsymbol{x},\boldsymbol{y}) = \left(\sum_{i=1}^{n} |x_i - y_i|^2\right)$$

x^2 distance

$$D(\boldsymbol{x},\boldsymbol{y}) = \sum_{i=1}^{n} \frac{(x_i - y_i)^2}{x_i + y_i}$$

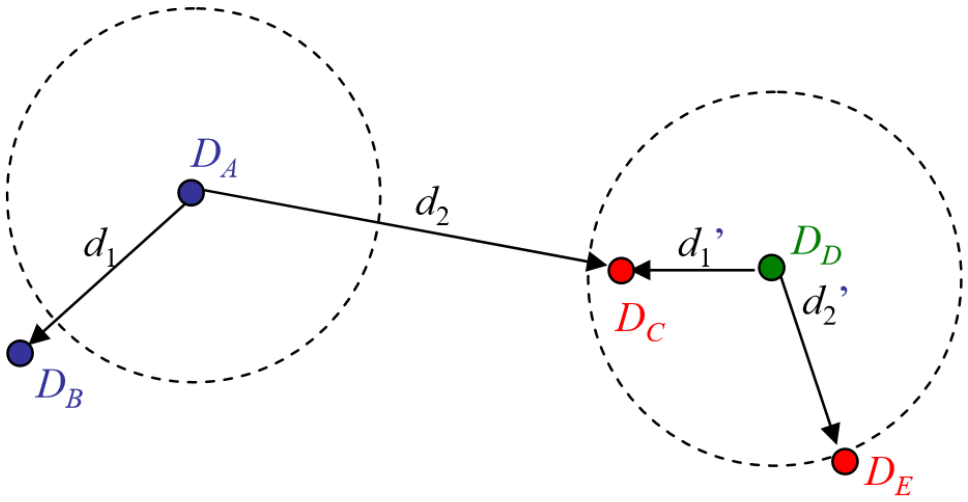
Cosine similarity

$$D(\boldsymbol{x},\boldsymbol{y}) = \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{\|\boldsymbol{x}\|\|\boldsymbol{y}\|} = \frac{x_1y_1 + \cdots + x_ny_n}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}}$$



Feature Matching

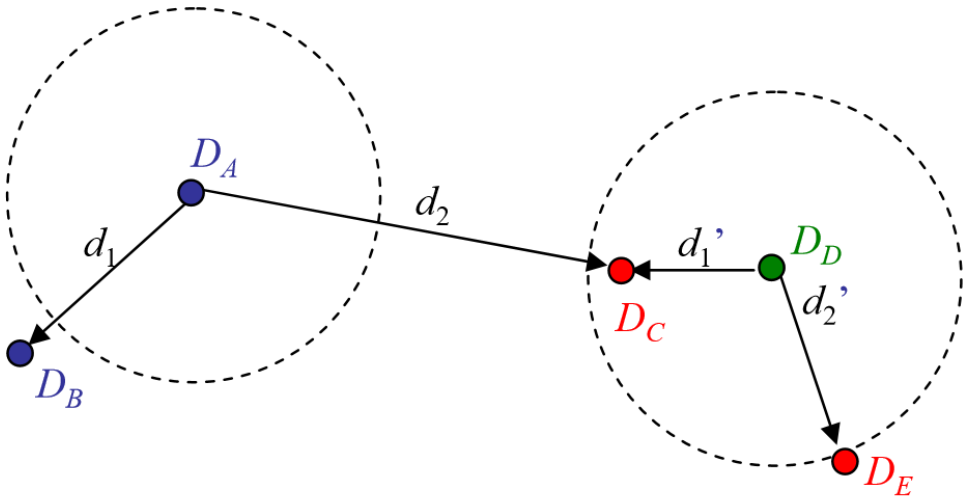
- Estimate the distance of keypoints in the feature space, e.g., Euclidean distance between 128-dim SIFT descriptors
- The simplest matching strategy is to set a **threshold** and to return all matches from other images within this threshold
- A high threshold results in too many incorrect matches being returned (false positives) while a low threshold results in too many correct matches being missed (false negatives)
- The problem with using a fixed threshold is that it is difficult to set



At a fixed distance threshold (dashed circles), descriptor D_A fails to match D_B and D_D incorrectly matches D_C and D_E.

Feature Matching

- A better strategy is to match a keypoint to its **nearest neighbour** in the feature space
- Since some features may have no matches (e.g., due to occlusion), a threshold can still be used to reduce the number of false positives
- Non-distinctive features could have lots of close matches, only one of which is correct



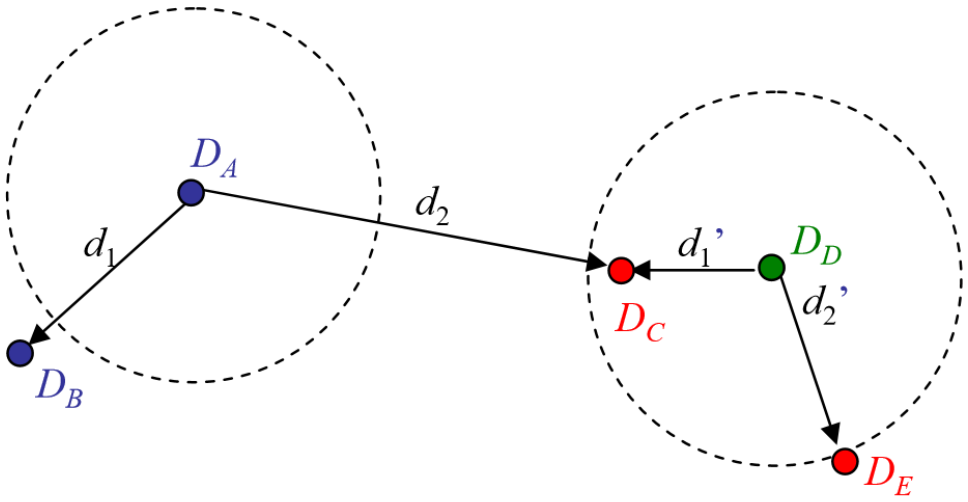
Using the nearest neighbour, descriptor D_A correctly matches D_B but D_D incorrectly matches D_C.

Feature Matching

- Another feature matching approach is the Nearest Neighbour Distance Ratio
- Estimate the distance of a feature vector to its Nearest Neighbour (d_1) and its second Nearest Neighbour (d_2)

$$\mathrm{NNDR} = \frac{d_1}{d_2} = \frac{|D_A - D_B|}{|D_A - D_c|}$$

  - If $d_1 \approx d_2$ $NNDR \approx 1$ the matching is ambiguous and should be rejected
  - If $d_1 << d_2$ $NNDR \rightarrow 0$ the matching is correct



Using NNDR for matching, descriptor correctly matches D_A to D_B and correctly rejects matches for D_D.

Conclusions

- Image Processing with Local Features
- Feature Descriptors
- SIFT
- SURF
- Shape Context descriptor
- Feature Descriptor Distance
- Feature Matching