Introduction to Symbolic AI

Logic Exercises 1: Propositional Logic

Syntax

1. Which of the following are propositional formulas strictly, according to Definition 1.1? For those that are not, say why not.

   i. $(\to p \wedge \neg q\ r)$
   ii. p
   iii. $p \vee q \vee r \vee \top$
   iv. $((p \to q) \vee ((\neg p) \to r))$
   v. $4 \wedge 3 = 7$
   vi. p and q
   vii. $\neg r$
   viii. $(\neg r \to (p \to ((q \vee r) \to (p \wedge \neg s))))$
   ix. $(\neg(p))$
   x. $(\neg\neg\top)$
   xi. $)$
   xii. $(\neg(\neg(\neg(\neg(\neg p)))))$

2. Apply the bracket-removing conventions in the notes (removing outer brackets, the binding conventions, and the right-associativity of the binary operators) to remove all possible brackets from the following formulas without changing their reading.

   E.g., we can abbreviate $(p \to (q \leftrightarrow (\neg r)))$ to $p \to (q \leftrightarrow \neg r)$. But $p \to q \leftrightarrow \neg r$ is going too far: the lack of a relation of binding strength between $\to$ and $\leftrightarrow$ means that we would not know how to read it (see slides 14–15).

   For this exercise, you should bracket repeated occurrences of $\to$ and $\leftrightarrow$. (Even where, given the right-associativity, of these connectives, strictly it would be possible to remove them.)

   i. $((\neg p) \wedge (\neg q))$
   ii. $(p \wedge (q \to r))$
   iii. $(\neg((\neg p) \leftrightarrow ((\neg q) \vee (\neg r))))$
   iv. $(\neg(p \wedge q))$
   v. $(\neg(\neg p))$
   vi. $(((\neg p) \wedge q) \to r)$
   vii. $((\neg(p \wedge q)) \to r)$
   viii. $(p \vee ((\neg q) \leftrightarrow ((\neg(r \wedge (\neg p))) \to \bot)))$

3. Draw the construction trees and list all subformulas of the following formulas. Use the binding conventions to disambiguate them. (Normally I'd put more brackets in (v).) Which of the subformulas here are literals? Which are clauses?

   i. $p \to q \wedge r$
   ii. $\neg p \wedge q \leftrightarrow r \vee s$
   iii. $(p \wedge q) \vee r \to \neg(p \to r)$
   iv. $\neg p \to (p \to r \vee s)$
   v. $\neg\neg\top \leftrightarrow (\neg\neg\bot \wedge \top \to \neg p)$
   vi. $\neg\neg\neg\neg\bot$

Semantics and translation

4. Suppose that p is **t**, q is **t** and r is **f** in a certain situation. Use truth-tables to decide which of the following evaluate to **t** and which to **f** in this situation.

   i. $(p \to q) \to \neg q$
   ii. $(\neg p \to (\neg q \wedge r)) \vee q$
   iii. $(((p \vee \neg r) \wedge \neg q) \to p) \to (\neg q \wedge \neg r)$

5. Translate the following into propositional logic, stating to which sentences your propositional atoms (p, q,. . .) correspond.

i. Priya will work hard and get good grades, or she'll be in the dramatic society.
ii. Yukio likes Yasunari, but Akira too.
iii. Unless you're catching the bus, you should either check the forecast or carry an umbrella.
iv. If Ahmed remembered to do his chores, then things are clean but not neat; if he forgot and Nourah did them, then things are neat but not clean.
v. If Cosima and Richard are lovers, Minna will find out.
vi. CSG is responsible only if the computer was installed since January and is running Linux.
vii. Anna and Herb aren't both unhappy.
viii. If the suspect fled with the stolen vehicle, then she's already over the border—provided the border patrol wasn't vigilant; but if the suspect didn't flee in the stolen vehicle, she obviously didn't get far.
ix. All bachelors are male and unmarried, though they aren't necessarily Catholic priests.
x. Thales, an Ionian philosopher, predicted the solar eclipse of the 28th of March, 585 BCE.
xi. I know that if you're coming, then Andreas and Berthold are coming too.
xii. Johannes and Julia are both doctors if and only if neither of them is in love with the other.
xiii. If it's snowing then the bus isn't coming and it's cold.

6. Let p represent *I revised for the exams*, q represent *I was drunk the night before the exams*, and r represent *I passed the exams.* Find *natural* English expressions for the following:

i. $p \wedge \neg q$
ii. $\neg r$
iii. $r \to (p \vee \neg q)$
iv. $q \leftrightarrow r$
v. $\neg r \to \neg p$
vi. $\neg(\neg p \wedge r)$

7. Let $\mathcal{A}$ be $\{p, q, r\}$ and $v : \mathcal{A} \to \{\mathbf{t}, \mathbf{f}\}$ be such that $v(p) = \mathbf{f}$, $v(q) = \mathbf{t}$ and $v(r) = \mathbf{f}$. Use direct argument to determine the truth-values of:

i. $(p \leftrightarrow q) \leftrightarrow (\neg q \to p)$
ii. $(r \vee (q \to p)) \to \neg(\neg p \to (q \to (r \leftrightarrow (\neg(p \wedge q)))))$
iii. $\neg\neg p \vee r \vee (p \wedge \neg p) \vee \neg q$

8. Consider a set of objects labelled $a, b, c, \ldots$ placed on a 3×3 grid:

a		d
c	f	b
	e	

The following atomic formulas represent properties of the objects:

- $[x$ *next-to* $y]$ means (that is, it is true if) x and y are adjacent (horizontally or vertically, but not diagonally);
- $[x$ *sees* $y]$ means x and y are in the same row or the same column;
- $[x$ *left-of* $y]$ means x is in a column to the left of the column of y
- $[x$ *above* $y]$ means x is in a row above the row of y.

For the placements shown in the figure, which of the following evaluate to true, and why?

i. $[a$ *sees* $b] \leftrightarrow [b$ *sees* $c]$
ii. $[b$ *next-to* $d] \vee [b$ *next-to* $e]$
iii. $\neg([a$ *left-of* $f] \wedge [f$ *above* $a])$

iv. $\neg([e \textit{ left-of } d] \to \neg[d \textit{ next-to } c]) \to \neg[a \textit{ sees } e]$

v. $([e \textit{ sees } d] \vee [f \textit{ sees } e]) \to \neg([b \textit{ above } e] \leftrightarrow [b \textit{ next-to } c])$

Place the 6 objects $a, \ldots, f$ on the grid so that **all** the formulas above are true.

Now place the 6 objects so that all the formulas above are *false*.
(**Hint**: work out which basic relationships need to be true or false. E.g., to make (iii) false requires $[a \textit{ left-of } f]$ and $[f \textit{ above } a]$ to be true. There are many correct solutions.)

Arguments, equivalence, validity

9. Which of the following are true? In each case, either give a direct argument to show that premise $\models$ conclusion, or (if premise $\not\models$ conclusion) specify a situation in which the premise is true and the conclusion false.

   i. $p \wedge q \models p$
   ii. $p \vee q \models p$
   iii. $p \to q \models q \to p$
   iv. $p \to q \models \neg q \to \neg p$
   v. $(p \wedge q) \vee (r \wedge s) \models (p \vee r) \wedge (q \vee s)$
   vi. $(p \vee r) \wedge (q \vee s) \models (p \wedge q) \vee (r \wedge s)$

10. Use direct argument to show that the following formulas are logically equivalent:

    i. $\bot \vee p$ and p
    ii. $\top \vee p$ and $\top$
    iii. $p \wedge \top$ and p
    iv. $\bot \to p$ and $\top$
    v. $p \vee q$ and $(p \to q) \to q$
    vi. $p \leftrightarrow (q \leftrightarrow r)$ and $(p \leftrightarrow q) \leftrightarrow r$

11. Let $A_1, \ldots, A_n, B$ be any propositional formulas. Express (define) $A_1, \ldots, A_n \models B$ using (i) validity, (ii) satisfiability, (iii) logical equivalence.

12. Prove the following:

    i. For any propositional formula A: $\top \models A$ if and only if A is valid.
    ii. For any propositional formulas $A_1, \ldots, A_n, B$: $(\bigwedge A_i) \wedge \neg B$ is unsatisfiable if and only if $A_1, \ldots, A_n \models B$.