Orthogonal Decomposition

The orthogonal decomposition of a vector $\mathbf{y}$ in $\mathbb{R}^n$ is the sum of a vector in a subspace W of $\mathbb{R}^n$ and a vector in the orthogonal complement $W^\perp$ to W.

The orthogonal decomposition theorem states that if W is a subspace of $\mathbb{R}^n$, then each vector $\mathbf{y}$ in $\mathbb{R}^n$ can be written uniquely in the form

$$\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z},$$

where $\hat{\mathbf{y}}$ is in W and $\mathbf{z}$ is in $W^\perp$. In fact, if $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_p\}$ is any orthogonal basis of W, then

$$\hat{\mathbf{y}} = \frac{\mathbf{y}\cdot\mathbf{u}_1}{\mathbf{u}_1\cdot\mathbf{u}_1}\mathbf{u}_1 + \frac{\mathbf{y}\cdot\mathbf{u}_2}{\mathbf{u}_2\cdot\mathbf{u}_2}\mathbf{u}_2 + \ldots + \frac{\mathbf{y}\cdot\mathbf{u}_p}{\mathbf{u}_p\cdot\mathbf{u}_p}\mathbf{u}_p,$$

and $\mathbf{z} = \mathbf{y} - \hat{\mathbf{y}}$.

Geometrically, $\hat{\mathbf{y}}$ is the orthogonal projection of $\mathbf{y}$ onto the subspace W and $\mathbf{z}$ is a vector orthogonal to $\hat{\mathbf{y}}$

SEE ALSO:
Fredholm's Theorem, LU Decomposition, QR Decomposition

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REFERENCES:

Golub, G. and Van Loan, C. *Matrix Computations, 3rd ed.* Baltimore, MD: Johns Hopkins University Press, 1996.

Referenced on Wolfram|Alpha: Orthogonal Decomposition

CITE THIS AS:
Bengtsson, Viktor. "Orthogonal Decomposition." From *MathWorld*--A Wolfram Web Resource, created by Eric W. Weisstein. http://mathworld.wolfram.com/OrthogonalDecomposition.html