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Created, developed, and
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Orthogonal Decomposition

The orthogonal decomposition of a vector \mathbf{y} in \mathbb{R}^n is the sum of a vector in a subspace W of \mathbb{R}^n and a vector in the orthogonal complement W^\perp to W .

The orthogonal decomposition theorem states that if W is a subspace of \mathbb{R}^n , then each vector \mathbf{y} in \mathbb{R}^n can be written uniquely in the form

$$\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z},$$

where $\hat{\mathbf{y}}$ is in W and \mathbf{z} is in W^\perp . In fact, if $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_p\}$ is any orthogonal basis of W , then

$$\hat{\mathbf{y}} = \frac{\mathbf{y} \cdot \mathbf{u}_1}{\mathbf{u}_1 \cdot \mathbf{u}_1} \mathbf{u}_1 + \frac{\mathbf{y} \cdot \mathbf{u}_2}{\mathbf{u}_2 \cdot \mathbf{u}_2} \mathbf{u}_2 + \dots + \frac{\mathbf{y} \cdot \mathbf{u}_p}{\mathbf{u}_p \cdot \mathbf{u}_p} \mathbf{u}_p,$$

and $\mathbf{z} = \mathbf{y} - \hat{\mathbf{y}}$.

Geometrically, $\hat{\mathbf{y}}$ is the orthogonal projection of \mathbf{y} onto the subspace W and \mathbf{z} is a vector orthogonal to $\hat{\mathbf{y}}$

SEE ALSO:
Fredholm's Theorem, LU Decomposition, QR Decomposition

This entry contributed by Viktor Bengtsson

REFERENCES:
Golub, G. and Van Loan, C. *Matrix Computations*, 3rd ed. Baltimore, MD: Johns Hopkins University Press, 1996.

Referenced on Wolfram|Alpha: Orthogonal Decomposition

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orthogonal decomposition

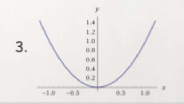
THINGS TO TRY:
= orthogonal decomposition
= 5:1 odds
= factoradic form of the permutation (3 1 2 5 4)

Check Your Answers
(and the steps)

100%

1. $x = -2 \pm i\sqrt{2}$

2. $x = 7, \quad y = 3$

3. 

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