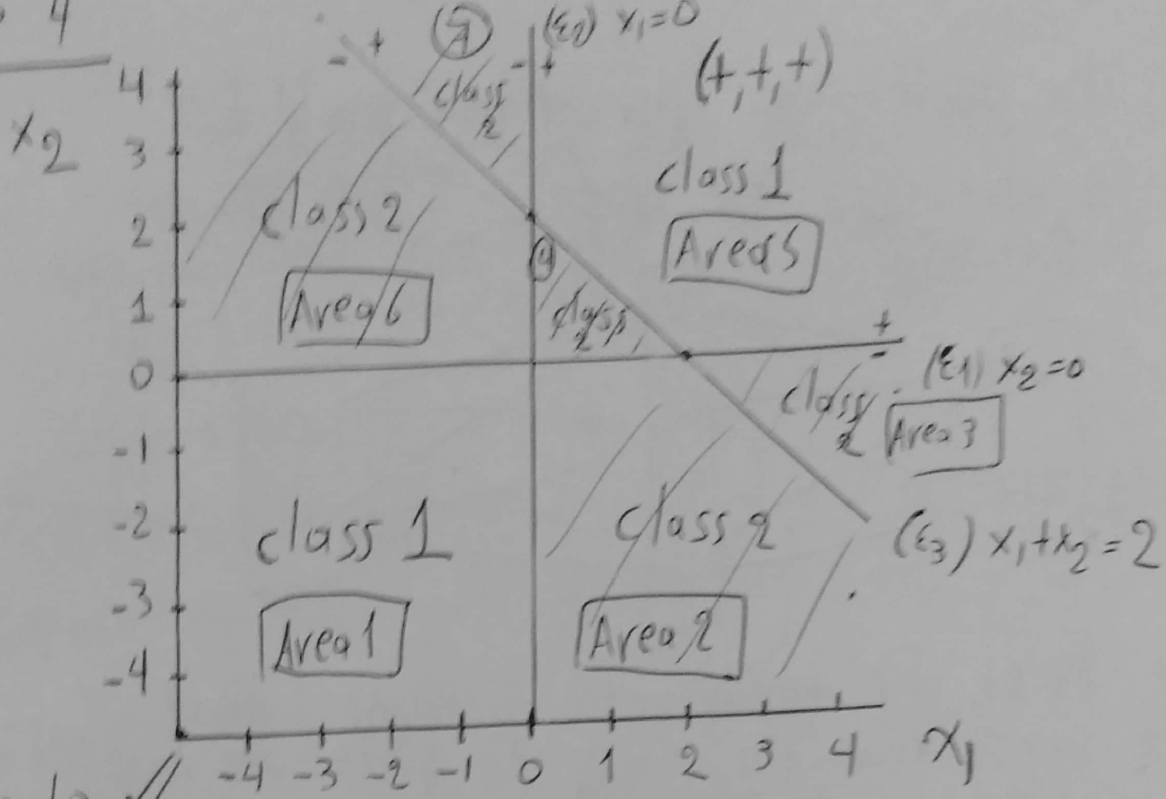
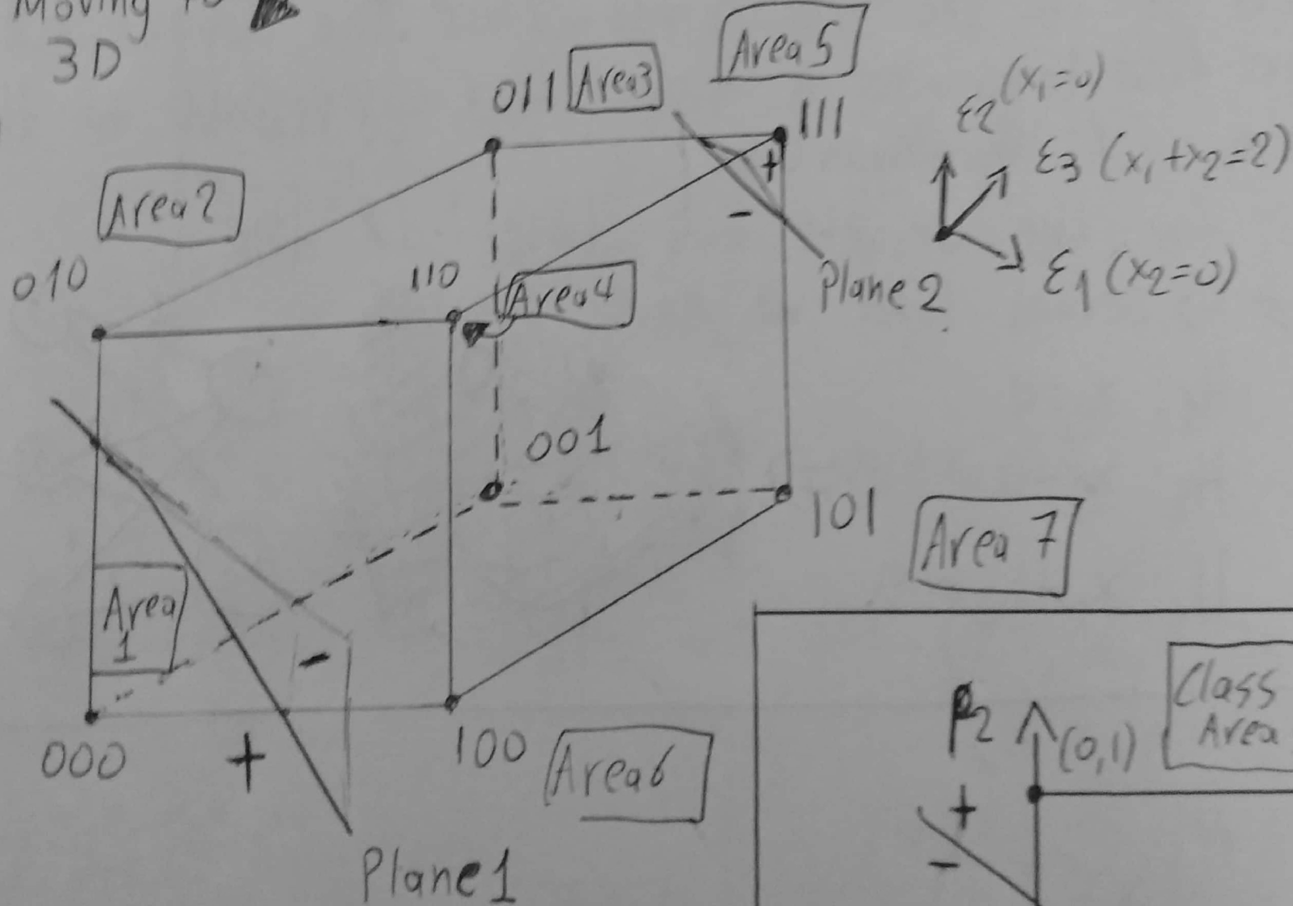
Exersize 4

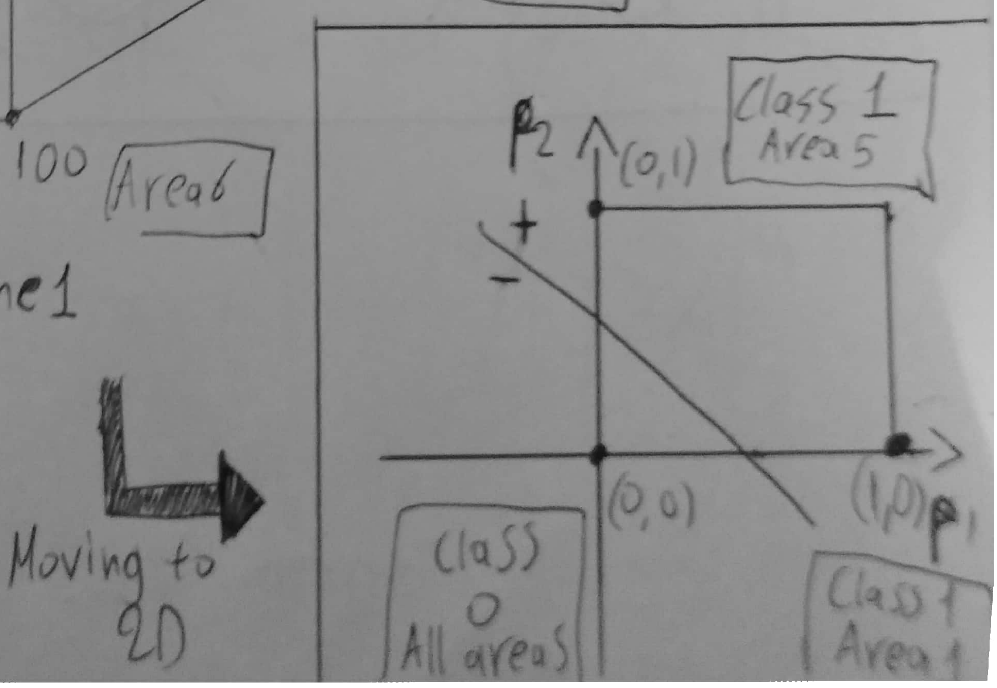
(i)



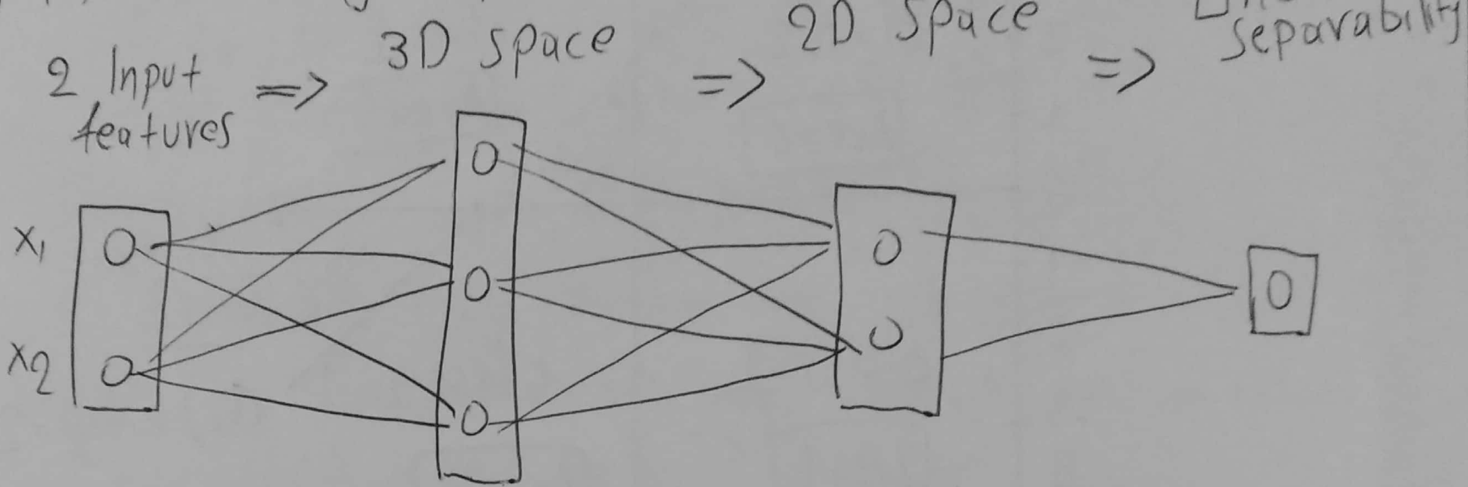
Moving to 3D

(ii)





(ii) The multi-layer perceptron would be:



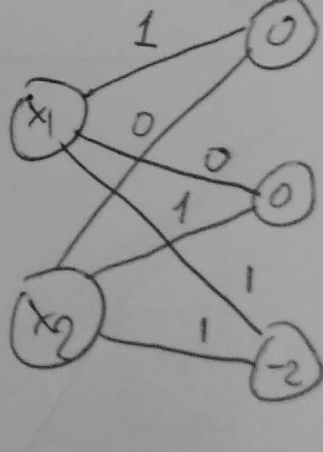
We start by taking into account that the points from the initial 2D space are mapped to a 3D hypercube in the transformed space. The output of the first layer is a triplet that corresponds to the edge of the hypercube.

Thus for the weights and biases of the first -> second layer we have:

$H_1: x_1 = 0$

$H_2: x_2 = 0$ $\Rightarrow$

$H_3: x_1 + x_2 = 2$



Exersize 4 (cont)

For the next transition we move from the 3D hypercube to the 2D space in order to linearly separate the points. That happens with the use of two planes. Lets use the points $(0,0,\frac{1}{3})$ $(0,\frac{1}{3},0)$ and $(\frac{1}{3},0,0)$ at random to get a plane for separating the point $(0,0,0)$. The equation that describes that point is:

$$P_1: \quad \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ 0 & 0 & 1/3 & 1 \\ 0 & 1/3 & 0 & 1 \\ 1/3 & 0 & 0 & 1 \end{vmatrix} = x_1 \begin{vmatrix} 0 & 1/3 & 1 \\ 0 & 0 & 1 \\ 1/3 & 0 & 1 \end{vmatrix} - x_2 \begin{vmatrix} 0 & 1/3 & 1 \\ 1/3 & 0 & 1 \\ 0 & 0 & \end{vmatrix} + x_3 \begin{vmatrix} 0 & 0 & 1 \\ 1/3 & 0 & 1 \\ 0 & 1/3 & 1 \end{vmatrix}$$

$$- \begin{vmatrix} 0 & 0 & 1/3 \\ 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{9}x_1 + \frac{1}{9}x_2 + \frac{1}{9}x_3 = \frac{1}{27} \Rightarrow x_1 + x_2 + x_3 = \frac{1}{3}$$

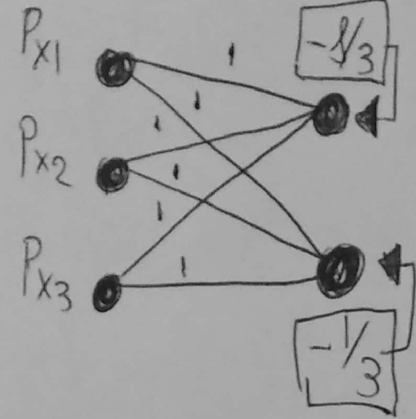
P_2 for the $(1,1,1)$ point we get $(1,1,\frac{2}{3})$ $(1,\frac{2}{3},1)$ $(\frac{2}{3},1,1)$

$$P_2: \quad \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ 2/3 & 1 & 1 & 1 \\ 1 & 2/3 & 1 & 1 \\ 1 & 1 & 2/3 & 1 \end{vmatrix} = x_1 \begin{vmatrix} 1 & 1 & 1 \\ 2/3 & 1 & 1 \\ 1 & 2/3 & 1 \end{vmatrix} - x_2 \begin{vmatrix} 2/3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2/3 & 1 \end{vmatrix} + x_3 \begin{vmatrix} 2/3 & 1 & 1 \\ 1 & 2/3 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

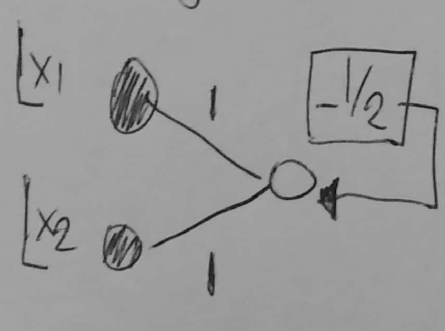
$$- \begin{vmatrix} 2/3 & 1 & 1 \\ 1 & 2/3 & 1 \\ 1 & 1 & 2/3 \end{vmatrix} = 0 \Rightarrow \frac{1}{9}x_1 + \frac{1}{9}x_2 + \frac{1}{9}x_3 = \frac{8}{27} \Rightarrow$$

$$\Rightarrow x_1 + x_2 + x_3 = 8/3$$

That gives the weights between 2^{nd} and 3^{rd} layer:



Finally for the last transition from the 2D plane to a 1D class (1/0) we choose again arbitrarily a line that can split out the (0,0) from the (1,0) and (0,1) points. We choose $x_1 + x_2 = 1/2$ so the weights of the last layer are:



The whole network is