

## Exercise 4

(e) Discuss briefly on the results

Let's see the results of our fit on the data.

→ LS estimator: Fairly good fit, with weights above  $\theta_3$  ~~begin~~ fluctuating between positive and negative values at large values. This is a sign of overfit meaning our model is heavily unconfident about new data and presents great variance as an estimator.

→ Ridge Estimator: Here we maintain a fairly good fit for values  $10^{-4}$ ,  $10^{-3}$ ,  $10^{-2}$  and we see a great penalization on the terms above the 2<sup>nd</sup> degree. Theta (3<sup>+</sup>) values are relatively small compared to the lesser terms. However at  $\lambda_2 = 0,1$  we see that the penalization interferes with the quality of the fit, leading to worse results.

→ Lasso Estimation: In our lasso regression estimation we see a good fit at  $\ell_1 = 5 \cdot 10^{-4}$  and  $\ell_1 = 10^{-4}$  with a strict penalization of terms other than  $\theta_0$  and  $\theta_2$ , a hard assumption that our model is of  $y = \theta_2 x_2^2 + \theta_0$  form. At values greater than that our model seems to over-penalize the fit, leading from a poor-fitted model to a complete ~~0~~ ~~variance~~ 0-variance straight line at  $\ell_1 = 0,1$ .

## Exercise 6

(ii) Comment briefly on the results.

In exercise 5 we proved by example that the fit of a LSE on a random dataset, after some trials approaches the true ~~estimated~~ values, under of course the assumption of a proper fit. That means our  $\hat{\theta}$  estimator ~~follows a distribution~~ eventually converges to its dataset-generating value at high  $N$  (a lot of trials).

Some fits may estimate higher, some lower but their mean "cancels-out" the deviation between them, leading to a better estimate. Also we proved that  $l_2$  penalization (Ridge Reg.) is not always a solution. We need to pick a sensible value of  $l_2$  in order to get good results. As I plotted in my Jupyter Notebook, a relatively low range of such values (2-40) gives the best results.