# Exercise 12

Let X be the design matrix:

X =

To approximate the probability as an F distribution we will use equation 9 with its’ associated notation from the notes where required:



From the R output we can determine that the F-Test value was 6.65 with 2 and 14 degrees of freedom which has the associated probability of p=0.009. Therefore there is overwhelming evidence to reject the null hypothesis that . Thus gives extremely strong evidence that Beta one is not equal to zero and/or Beta two is not equal to zero.

(b)

Using R to compare the two models we can see from the output that there is no evidence to reject the null hypothesis: (p=0.6081)

(c)

Performing the appropriate approximation to the F-Test which has 2 and 14 degrees of freedom yields a F-Value of 38.773 and an associated probability of p=0.0000002. Therefore there is extremely strong evidence to reject the null hypothesis: and accept that

(d)

Performing the appropriate approximation to the F-Test which has 2 and 14 degrees of freedom yields a F-Value of 11.329 and an associated probability of p=0.0012 Therefore there is strong evidence to reject the null hypothesis: and accept that:

# Code

*library*(stringr)  
  
tractor\_data <- *read.table*("tractor data.txt", header = TRUE)  
  
tractor\_linear\_model <- *lm*(tractor\_data$Maint ~ tractor\_data$Age)  
tractor\_log\_model <- *lm*(tractor\_data$Maint ~ tractor\_data$Age + *I*(*log*(tractor\_data$Age)))  
  
*writeLines*("Test H\_0: Beta\_1 = Beta\_2 = 0")  
*anova*(tractor\_log\_model)  
f\_b1\_equals\_b2\_equals\_zero <- ((1099635 + 23240)/2)/(1182167/14)  
*str\_glue*("The F value for beta one equals beta two equals zero is: {f\_b1\_equals\_b2\_equals\_zero}")  
p\_b1\_equals\_b2\_equals\_zero <- 1 - *pf*(f\_b1\_equals\_b2\_equals\_zero, 2, 14)  
*str\_glue*("The probability that beta one equals beta two equals zero is: {p\_b1\_equals\_b2\_equals\_zero}")  
  
*writeLines*("\n\n")  
  
*writeLines*("Test H\_0: Beta\_2 = 0")  
*anova*(tractor\_linear\_model, tractor\_log\_model)  
*writeLines*("\n\n")  
  
*writeLines*("Test H\_0: Beta\_2 = 3")  
*summary*(tractor\_log\_model)  
C <- *rbind*(*c*(0, 0, 1))  
X <- *cbind*(*rep*(1, *length*(tractor\_data$Age)), tractor\_data$Age, *I*(*log*(tractor\_data$Age)))  
  
*#values obtained from tractor\_log\_model$coefficients*beta\_hat <- *c*(436.6988, 45.6270, 196.2627)  
  
  
c <-*c*(3)  
CB\_less\_c <- C%\*%beta\_hat - c  
D <- *solve*(*t*(X)%\*%X)  
G <- *solve*(C%\*%D%\*%*t*(C))  
  
f\_b2\_equals\_3 <- (1/(2\*290.6))\**t*(CB\_less\_c)%\*%G%\*%CB\_less\_c  
  
*str\_glue*("The F value that beta two equals three is: {f\_b2\_equals\_3}")  
p\_b2\_equals\_three <- 1 - *pf*(f\_b2\_equals\_3, 2, 14)  
*str\_glue*("The probability that beta two equals three is: {p\_b2\_equals\_three}")  
  
*writeLines*("\n\n")  
  
  
*writeLines*("Test H\_0: Beta\_1 = Beta\_2")  
C <- *rbind*(*c*(0, 1, -1))  
X <- *cbind*(*rep*(1, *length*(tractor\_data$Age)), tractor\_data$Age, *I*(*log*(tractor\_data$Age)))  
  
*#values obtained from tractor\_log\_model$coefficients*beta\_hat <- *c*(436.6988, 45.6270, 196.2627)  
  
  
c <-*c*(0)  
CB\_less\_c <- C%\*%beta\_hat - c  
D <- *solve*(*t*(X)%\*%X)  
G <- *solve*(C%\*%D%\*%*t*(C))  
  
f\_b2\_equals\_b1 <- (1/(2\*290.6))\**t*(CB\_less\_c)%\*%G%\*%CB\_less\_c  
  
*str\_glue*("The F value that beta two equals beta\_1 is: {f\_b2\_equals\_b1}")  
p\_b2\_equals\_b1 <- 1 - *pf*(f\_b2\_equals\_b1, 2, 14)  
*str\_glue*("The probability that beta two equals beta one is: {p\_b2\_equals\_b1}")

# Code Output

## Test H\_0: Beta\_1 = Beta\_2 = 0

Analysis of Variance Table

Response: tractor\_data$Maint

Df Sum Sq Mean Sq F value Pr(>F)

tractor\_data$Age 1 1099635 1099635 13.0226 0.002849 \*\*

I(log(tractor\_data$Age)) 1 23240 23240 0.2752 0.608056

Residuals 14 1182167 84440

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Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

The F value for beta one equals beta two equals zero is: 6.64891254788875

The probability that beta one equals beta two equals zero is: 0.00933254133476336

## Test H\_0: Beta\_2 = 0

Analysis of Variance Table

Model 1: tractor\_data$Maint ~ tractor\_data$Age

Model 2: tractor\_data$Maint ~ tractor\_data$Age + I(log(tractor\_data$Age))

Res.Df RSS Df Sum of Sq F Pr(>F)

1 15 1205407

2 14 1182167 1 23240 0.2752 0.6081

## Test H\_0: Beta\_2 = 3

Call:

lm(formula = tractor\_data$Maint ~ tractor\_data$Age + I(log(tractor\_data$Age)))

Residuals:

Min 1Q Median 3Q Max

-396.28 -168.28 -35.23 111.79 541.29

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 436.70 262.95 1.661 0.119

tractor\_data$Age 45.63 168.11 0.271 0.790

I(log(tractor\_data$Age)) 196.26 374.11 0.525 0.608

Residual standard error: 290.6 on 14 degrees of freedom

Multiple R-squared: 0.4871, Adjusted R-squared: 0.4139

F-statistic: 6.649 on 2 and 14 DF, p-value: 0.009333

The F value that beta two equals three is: 38.773241682738

The probability that beta two equals three is: 1.95616095843842e-06

## Test H\_0: Beta\_1 = Beta\_2

The F value that beta two equals beta\_1 is: 11.329018819857

The probability that beta two equals beta one is: 0.00118498201823014