Functional Programming for BDA - List 2 Maps, folds and lambda expressions

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Treat exercises as a warm up. Submit tasks 1-4 only.

Exercise 1. Implement functions

- a) a(x,y) = x + y;
- b) $b(x,y) = \operatorname{const}_x(y) = x;$
- c) $\pi_2(x,y) = y;$
- d) id(x) = x;

using lambda calculus.

Exercise 2. Simplify the following expression:

$$(\xspace x \to \yspace y \to x + 2 * y)(x + 2 * y).$$

Tip: Rename colliding variables (it's called α -conversion) and apply $(\x \to \y \to x + 2*y)$ to the expression (x + 2*y) (it's called β -reduction).

Exercise 3. Let f and g be functions. Express f.g (the composition of f and g) in the lambda calculus. What is the type of (.) as a function?

Exercise 4. Let p1 = $\xy \rightarrow x$ and p2 = $\xy \rightarrow y$. Calculate p1.p2 and p2.p1.

Exercise 5. Calculate the output of foldl ($^{\wedge}$) 1 [2,2,2] and foldr ($^{\wedge}$) 1 [2,2,2].

Exercise 6. Calculate the output of (head $map (x y \rightarrow x+2*y) [7,11,13]) 5.$

Task 1. Express map via foldr. Use lambda expressions.

Task 2. Implement a function that for a given list of integers returns the sum of squares of its even members. Use fold.

Task 3. Implement a function that for a given list of natural numbers calculates how many members of the list are prime. It should be reasonably fast. Use fold.

Task 4. Implement a function that for a given natural number n calculates the approximation of e, i.e. $\sum_{k=0}^{n} \frac{1}{k!}$. It should do so fast - in a linear time.