final_test

June 3, 2020

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
%matplotlib inline
from statsmodels.tsa.stattools import adfuller
from pmdarima import auto_arima
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.tools.eval_measures import rmse
from sklearn.metrics import mean_squared_error
from statsmodels.tsa.arima_model import ARIMA
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.tsa.holtwinters import ExponentialSmoothing

import warnings
warnings.filterwarnings("ignore")
```

1 Analyze the first decade

[2]:
```
df_19 = pd.read_csv("TOTALSA_19.csv", parse_dates=True, index_col="DATE")
df_19.freq="MS"
```

[3]:
```
df_19.head()
```

[3]:
```
            TOTALSA
DATE
1990-01-01   16.308
1990-02-01   14.363
1990-03-01   14.486
1990-04-01   14.281
1990-05-01   14.022
```

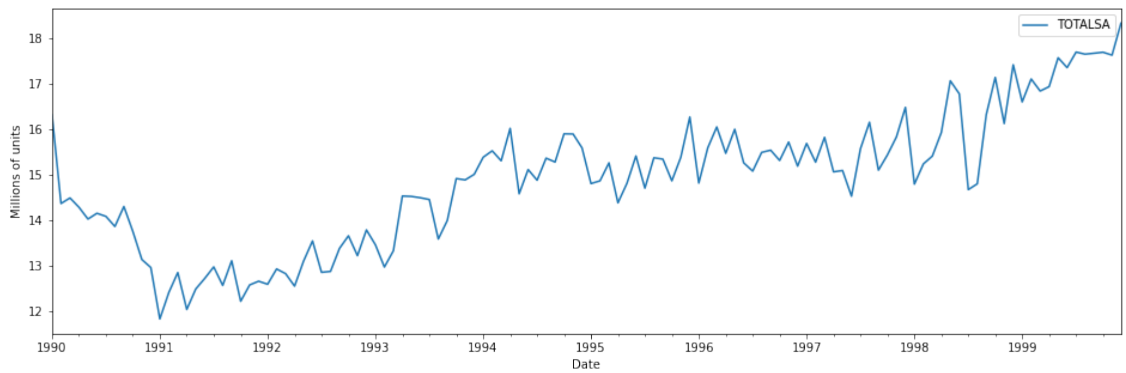
[4]:
```
df_19.tail()
```

[4]:
```
            TOTALSA
DATE
```

```
1999-08-01    17.641
1999-09-01    17.662
1999-10-01    17.684
1999-11-01    17.620
1999-12-01    18.322
```

Let us see the data that we're working with.

[5]:
```
ax = df_19.plot(figsize=(16,5))
plt.ylabel("Millions of units")
plt.xlabel("Date")
plt.show()
```



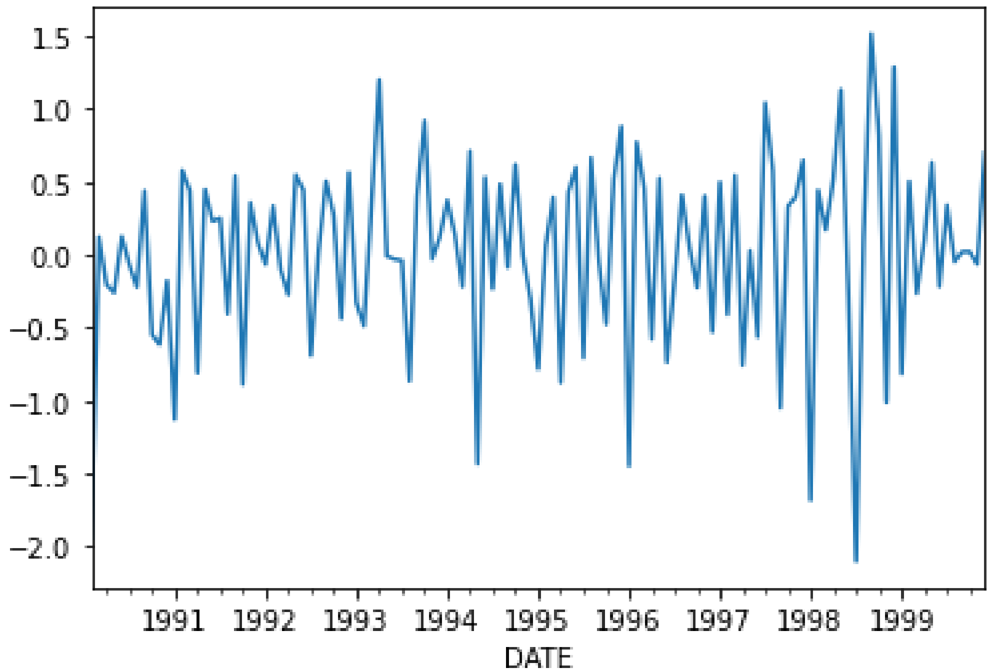
Lets check the adfuller test, to see if the data is stationary.

[6]:
```
adfuller(df_19.TOTALSA)
```

[6]:
```
(0.627662482388793,
 0.9882779821994239,
 5,
 114,
 {'1%': -3.489057523907491,
  '5%': -2.887246327182993,
  '10%': -2.5804808802708528},
 178.24663671468213)
```

They are not, so we suspect that we need differencing. Lets see the result of the adfuller test after differencing by one.

[7]:
```
df_19['d1'] = df_19.TOTALSA.diff(1)
df_19.d1.dropna(inplace=True)
df_19.d1.plot();
```

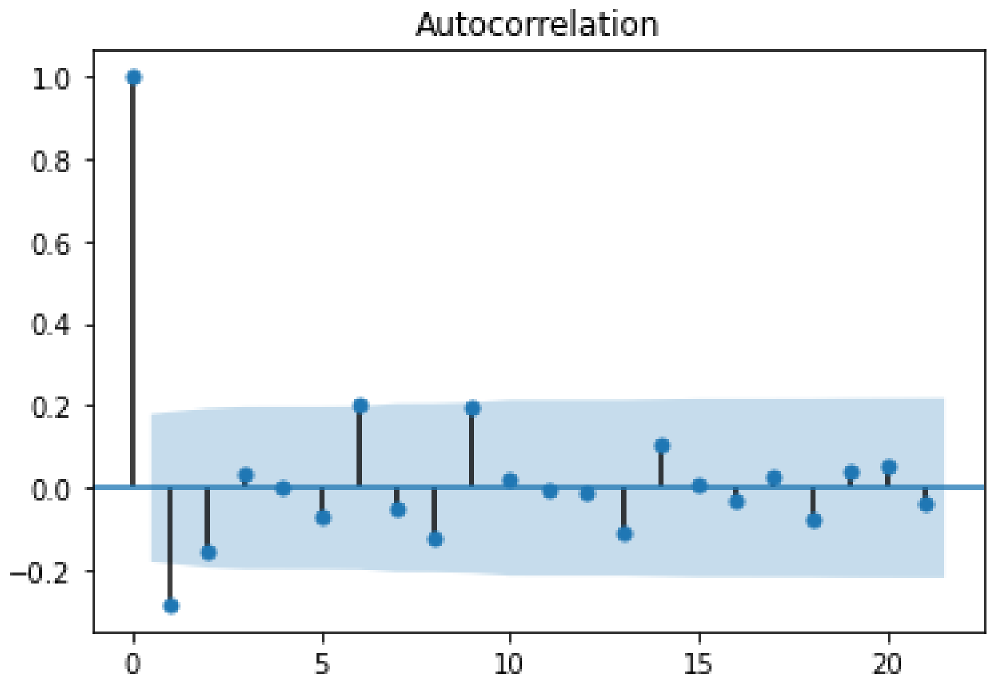


[8]:
```
adfuller(df_19.d1)
```

[8]:
```
(-7.409310416655577,
 7.204209993186997e-11,
 4,
 114,
 {'1%': -3.489057523907491,
  '5%': -2.887246327182993,
  '10%': -2.5804808802708528},
 175.19172244825384)
```

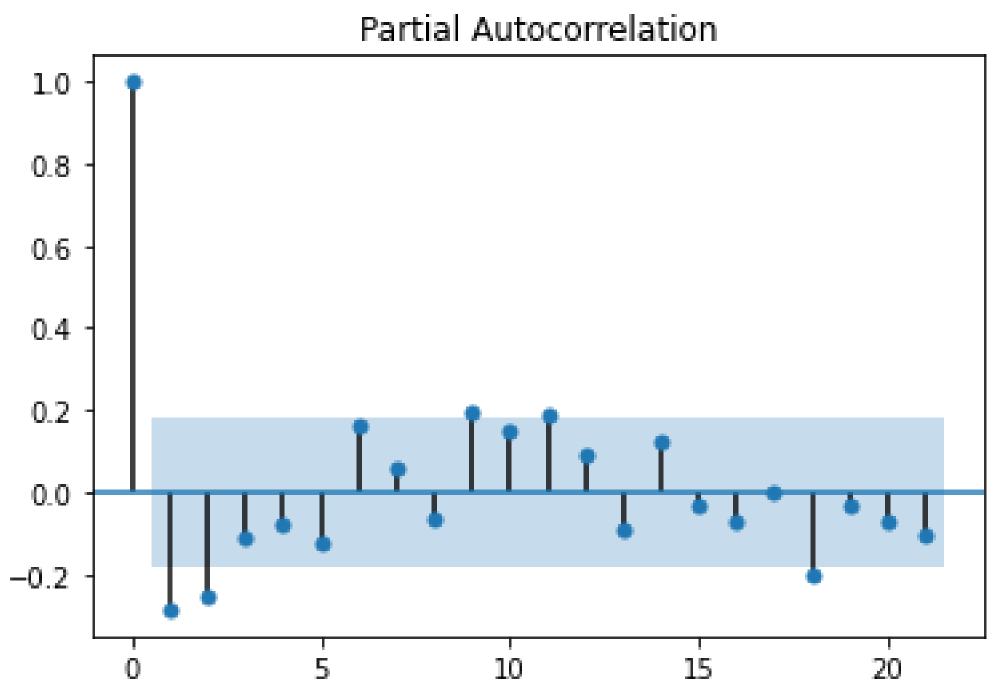
Differencing helped. We cannot use ARMA model, we should aim for the models with differencing. ARIMA or even SARIMA. Let us look on the acf and pacf to get some idea about the order of the model.

[9]:
```
plot_acf(df_19.d1);
```



MA order around 2

[10]:
```
plot_pacf(df_19.d1);
```



AR model around 3.

Lets run the autoarima, and check the results.

[11]:
```
auto_arima(df_19.d1).summary()
```

[11]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  119
Model:               SARIMAX(2, 0, 2)   Log Likelihood                 -97.637
Date:                Wed, 03 Jun 2020   AIC                            207.273
Time:                        01:26:06   BIC                            223.948
Sample:                             0   HQIC                           214.044
                                - 119
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      0.0047      0.009      0.513      0.608      -0.013       0.022
ar.L1          1.2565      0.097     12.950      0.000       1.066       1.447
ar.L2         -0.4963      0.111     -4.461      0.000      -0.714      -0.278
ma.L1         -1.8325      0.525     -3.489      0.000      -2.862      -0.803
ma.L2          0.9975      0.577      1.728      0.084      -0.134       2.129
sigma2         0.2883      0.167      1.727      0.084      -0.039       0.616
================================================================================
===
Ljung-Box (Q):                       25.32   Jarque-Bera (JB):
2.13
Prob(Q):                              0.97   Prob(JB):
0.34
Heteroskedasticity (H):               1.24   Skew:
-0.31
Prob(H) (two-sided):                  0.51   Kurtosis:
3.22
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

Let's check autoarima on original data, not differenced.

[12]:
```
stepwise = auto_arima(
    df_19.TOTALSA,
    start_p=1,
    start_q=1,
    max_p=4,
    max_d=4,
    max_q=4,
    stepwise=False,
    trace=True,
)
stepwise.summary()
```

```
Fit ARIMA(0,1,0)x(0,0,0,1) [intercept=True]; AIC=236.950, BIC=242.509,
Time=0.011 seconds
Fit ARIMA(0,1,1)x(0,0,0,1) [intercept=True]; AIC=217.801, BIC=226.138,
Time=0.041 seconds
Fit ARIMA(0,1,2)x(0,0,0,1) [intercept=True]; AIC=218.121, BIC=229.237,
Time=0.068 seconds
Fit ARIMA(0,1,3)x(0,0,0,1) [intercept=True]; AIC=219.152, BIC=233.048,
Time=0.102 seconds
Fit ARIMA(0,1,4)x(0,0,0,1) [intercept=True]; AIC=219.056, BIC=235.731,
Time=0.159 seconds
Fit ARIMA(1,1,0)x(0,0,0,1) [intercept=True]; AIC=228.078, BIC=236.415,
Time=0.023 seconds
Fit ARIMA(1,1,1)x(0,0,0,1) [intercept=True]; AIC=218.629, BIC=229.745,
Time=0.086 seconds
Fit ARIMA(1,1,2)x(0,0,0,1) [intercept=True]; AIC=219.719, BIC=233.614,
Time=0.146 seconds
Fit ARIMA(1,1,3)x(0,0,0,1) [intercept=True]; AIC=211.741, BIC=228.416,
Time=0.404 seconds
Fit ARIMA(1,1,4)x(0,0,0,1) [intercept=True]; AIC=221.501, BIC=240.955,
Time=0.188 seconds
Fit ARIMA(2,1,0)x(0,0,0,1) [intercept=True]; AIC=220.979, BIC=232.096,
Time=0.044 seconds
Fit ARIMA(2,1,1)x(0,0,0,1) [intercept=True]; AIC=219.078, BIC=232.974,
Time=0.076 seconds
Fit ARIMA(2,1,2)x(0,0,0,1) [intercept=True]; AIC=207.280, BIC=223.954,
Time=0.336 seconds
Fit ARIMA(2,1,3)x(0,0,0,1) [intercept=True]; AIC=212.585, BIC=232.039,
Time=0.353 seconds
Fit ARIMA(3,1,0)x(0,0,0,1) [intercept=True]; AIC=220.695, BIC=234.591,
Time=0.056 seconds
Fit ARIMA(3,1,1)x(0,0,0,1) [intercept=True]; AIC=220.982, BIC=237.657,
Time=0.093 seconds
Fit ARIMA(3,1,2)x(0,0,0,1) [intercept=True]; AIC=223.044, BIC=242.498,
Time=0.131 seconds
Fit ARIMA(4,1,0)x(0,0,0,1) [intercept=True]; AIC=221.262, BIC=237.937,
```

```
Time=0.058 seconds
Fit ARIMA(4,1,1)x(0,0,0,1) [intercept=True]; AIC=222.427, BIC=241.881,
Time=0.124 seconds
Total fit time: 2.515 seconds
```

[12]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  120
Model:               SARIMAX(2, 1, 2)   Log Likelihood                 -97.640
Date:                Wed, 03 Jun 2020   AIC                            207.280
Time:                        01:26:20   BIC                            223.954
Sample:                             0   HQIC                           214.051
                                - 120
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      0.0046      0.009      0.504      0.614      -0.013       0.022
ar.L1          1.2571      0.097     12.988      0.000       1.067       1.447
ar.L2         -0.4964      0.111     -4.472      0.000      -0.714      -0.279
ma.L1         -1.8311      0.314     -5.832      0.000      -2.446      -1.216
ma.L2          0.9958      0.347      2.872      0.004       0.316       1.676
sigma2         0.2879      0.103      2.792      0.005       0.086       0.490
================================================================================
===
Ljung-Box (Q):                       25.31   Jarque-Bera (JB):
2.15
Prob(Q):                              0.97   Prob(JB):
0.34
Heteroskedasticity (H):               1.24   Skew:
-0.31
Prob(H) (two-sided):                  0.50   Kurtosis:
3.23
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

The order of AR and MA match, and we see that we need the differencing.

[13]:
```python
def evaluate_model(model, start, test, model_name, **kwargs):
    """ Function to evaluate predictions against test set.
```

```
    Args:
        model: model to produce predictions
        start: start index of prediction usually len(train)
        test: test data
        model_name: name to rename the model data
        **kwargs: additional parameters for predict function

    Returns:
        predictions
    """
    predictions = model.predict(start=start, end=start+len(test)-1,␣
↪dynamic=False, **kwargs).rename(model_name)
    print(f"MSE for {model_name}: \t{mean_squared_error(test, predictions)}")
    print(f"RMSE for {model_name}: \t{rmse(test, predictions)}")
    return predictions
```

[14]:
```
def plot_results(test, predictions):
    """ Plots the predicted data and test data to visualise the predictions.

    Args:
        test: test data
        prediction: predictions data
    """
    title = "Compare prediction with test set"
    xlabel = "date"
    ylabel="Millions of units"

    ax = test.plot(legend=True, figsize=(12,6), title=title, label="Test data")
    predictions.plot(legend=True)
    ax.autoscale(axis="x", tight=True)
    ax.set(xlabel=xlabel, ylabel=ylabel)
    plt.show()
```

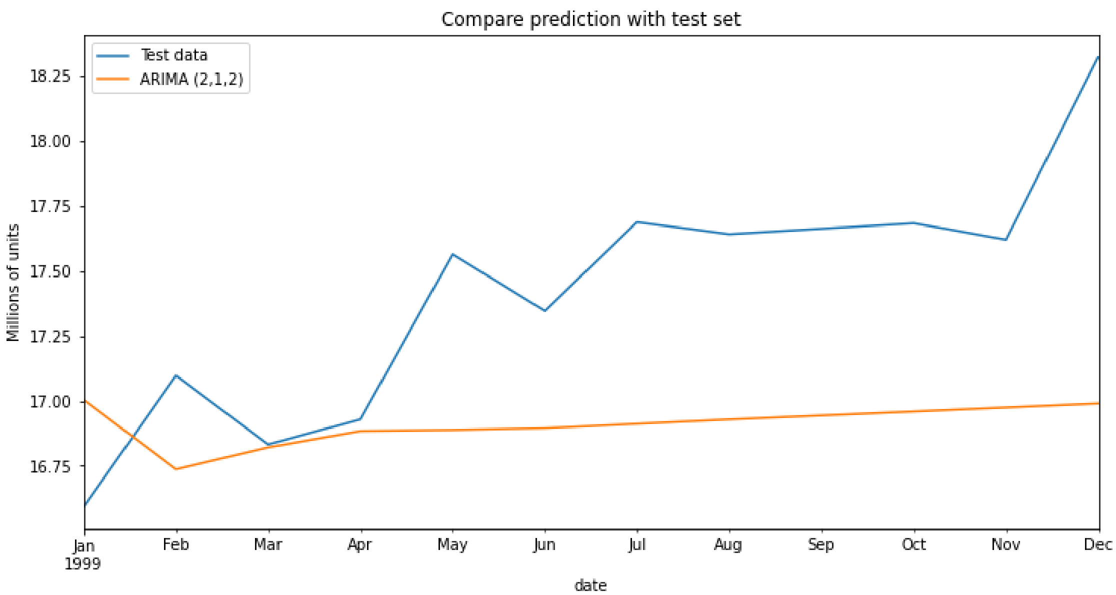
[15]:
```
def forecast_sarima(data, order, seasonal_order=None, n=11):
    """ Forecast fr the data using defined model
    """
    if seasonal_order:
        model = SARIMAX(data, order=order, seasonal_order=seasonal_order).fit()
    else:
        model = SARIMAX(data, order=order).fit()
    return model.predict(
        len(data),
        len(data)+n,
        dynamic=False).rename(f"Forecast {order}x{seasonal_order}"
    )
```

[16]:
```
s_index = -12
train = df_19.iloc[:s_index]
test = df_19.iloc[s_index:]
```

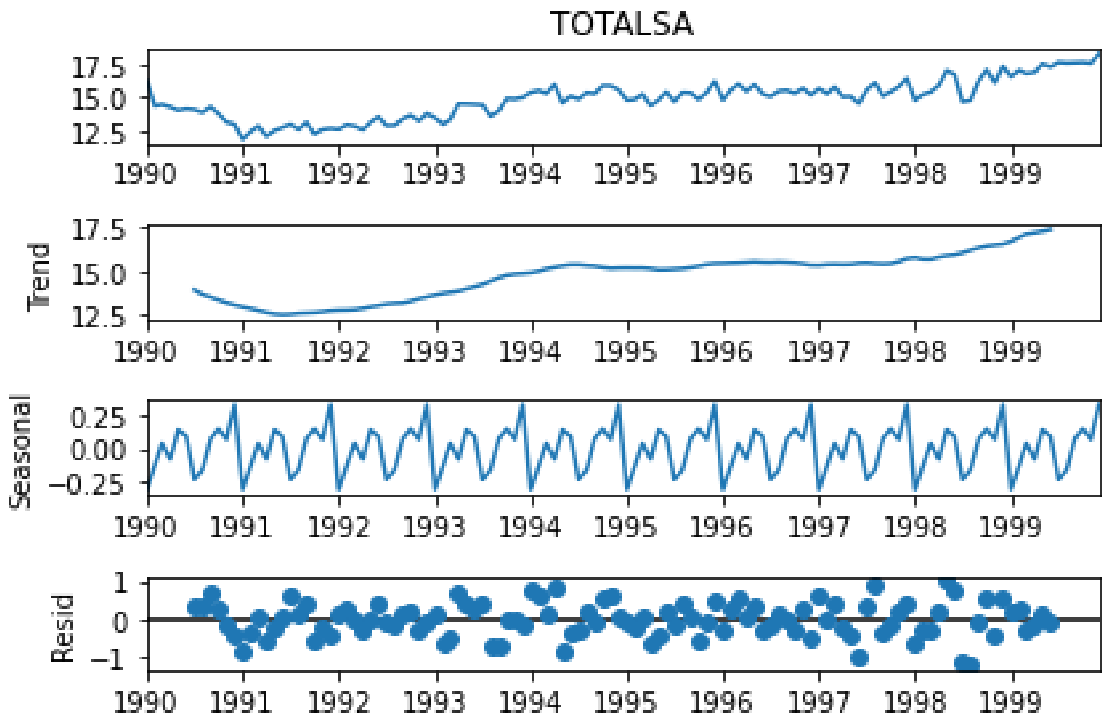
[17]:
```
arima_2_1_2 = ARIMA(train.TOTALSA, order=(2,1,2)).fit()
res_arima_2_1_2 = evaluate_model(arima_2_1_2, len(train), test.TOTALSA, "ARIMA␣
↪(2,1,2)", typ="levels")
```

```
MSE for ARIMA (2,1,2):  0.441958428895752 93
RMSE for ARIMA (2,1,2):         0.6647995403847335
```

[18]:
```
plot_results(test.TOTALSA, res_arima_2_1_2)
```



[19]:
```
resd = seasonal_decompose(df_19.TOTALSA)
resd.plot();
```



We see that there is 1year=19month seasonality, let us try to fit this model. Specyfing **D=1** and **d=1**

[20]:
```
stepwise = auto_arima(
    df_19.TOTALSA,
    m=12,
    d=1,
    D=1,
    start_p=1,
    start_q=1,
    max_p=4,
    max_d=4,
    max_q=4,
    max_P=4,
    max_Q=4,
    stepwise=False,
    trace=True,
)
stepwise.summary()
```

```
Fit ARIMA(0,1,0)x(0,1,0,12) [intercept=True]; AIC=290.159, BIC=295.505,
Time=0.024 seconds
Fit ARIMA(0,1,0)x(0,1,1,12) [intercept=True]; AIC=238.110, BIC=246.129,
Time=0.517 seconds
```

```
Fit ARIMA(0,1,0)x(0,1,2,12) [intercept=True]; AIC=239.859, BIC=250.550,
Time=1.273 seconds
Fit ARIMA(0,1,0)x(0,1,3,12) [intercept=True]; AIC=241.130, BIC=254.494,
Time=1.489 seconds
Fit ARIMA(0,1,0)x(0,1,4,12) [intercept=True]; AIC=243.066, BIC=259.103,
Time=7.052 seconds
Fit ARIMA(0,1,0)x(1,1,0,12) [intercept=True]; AIC=262.236, BIC=270.254,
Time=0.155 seconds
Fit ARIMA(0,1,0)x(1,1,1,12) [intercept=True]; AIC=239.894, BIC=250.585,
Time=0.879 seconds
Fit ARIMA(0,1,0)x(1,1,2,12) [intercept=True]; AIC=241.361, BIC=254.726,
Time=2.536 seconds
Fit ARIMA(0,1,0)x(1,1,3,12) [intercept=True]; AIC=242.847, BIC=258.884,
Time=4.535 seconds
Fit ARIMA(0,1,0)x(1,1,4,12) [intercept=True]; AIC=244.849, BIC=263.559,
Time=3.460 seconds
Fit ARIMA(0,1,0)x(2,1,0,12) [intercept=True]; AIC=250.865, BIC=261.556,
Time=0.427 seconds
Fit ARIMA(0,1,0)x(2,1,1,12) [intercept=True]; AIC=241.219, BIC=254.583,
Time=1.163 seconds
Fit ARIMA(0,1,0)x(2,1,2,12) [intercept=True]; AIC=242.866, BIC=258.903,
Time=1.187 seconds
Fit ARIMA(0,1,0)x(2,1,3,12) [intercept=True]; AIC=244.785, BIC=263.495,
Time=4.717 seconds
Fit ARIMA(0,1,0)x(3,1,0,12) [intercept=True]; AIC=248.894, BIC=262.258,
Time=0.957 seconds
Fit ARIMA(0,1,0)x(3,1,1,12) [intercept=True]; AIC=243.155, BIC=259.192,
Time=4.549 seconds
Fit ARIMA(0,1,0)x(3,1,2,12) [intercept=True]; AIC=244.861, BIC=263.570,
Time=2.576 seconds
Fit ARIMA(0,1,0)x(4,1,0,12) [intercept=True]; AIC=244.889, BIC=260.926,
Time=1.940 seconds
Fit ARIMA(0,1,0)x(4,1,1,12) [intercept=True]; AIC=244.404, BIC=263.114,
Time=4.376 seconds
Fit ARIMA(0,1,1)x(0,1,0,12) [intercept=True]; AIC=274.752, BIC=282.771,
Time=0.065 seconds
Fit ARIMA(0,1,1)x(0,1,1,12) [intercept=True]; AIC=222.489, BIC=233.180,
Time=0.818 seconds
Fit ARIMA(0,1,1)x(0,1,2,12) [intercept=True]; AIC=223.606, BIC=236.971,
Time=2.012 seconds
Fit ARIMA(0,1,1)x(0,1,3,12) [intercept=True]; AIC=225.476, BIC=241.513,
Time=4.570 seconds
Fit ARIMA(0,1,1)x(0,1,4,12) [intercept=True]; AIC=226.983, BIC=245.693,
Time=7.630 seconds
Fit ARIMA(0,1,1)x(1,1,0,12) [intercept=True]; AIC=242.651, BIC=253.342,
Time=0.245 seconds
Fit ARIMA(0,1,1)x(1,1,1,12) [intercept=True]; AIC=223.658, BIC=237.022,
Time=1.512 seconds
```

```
Fit ARIMA(0,1,1)x(1,1,2,12) [intercept=True]; AIC=225.016, BIC=241.053,
Time=2.938 seconds
Fit ARIMA(0,1,1)x(1,1,3,12) [intercept=True]; AIC=227.397, BIC=246.107,
Time=4.791 seconds
Fit ARIMA(0,1,1)x(2,1,0,12) [intercept=True]; AIC=236.666, BIC=250.030,
Time=0.620 seconds
Fit ARIMA(0,1,1)x(2,1,1,12) [intercept=True]; AIC=225.574, BIC=241.611,
Time=1.835 seconds
Fit ARIMA(0,1,1)x(2,1,2,12) [intercept=True]; AIC=227.437, BIC=246.147,
Time=3.051 seconds
Fit ARIMA(0,1,1)x(3,1,0,12) [intercept=True]; AIC=234.113, BIC=250.150,
Time=1.381 seconds
Fit ARIMA(0,1,1)x(3,1,1,12) [intercept=True]; AIC=227.259, BIC=245.969,
Time=4.657 seconds
Fit ARIMA(0,1,1)x(4,1,0,12) [intercept=True]; AIC=229.450, BIC=248.160,
Time=2.920 seconds
Fit ARIMA(0,1,2)x(0,1,0,12) [intercept=True]; AIC=269.963, BIC=280.654,
Time=0.133 seconds
Fit ARIMA(0,1,2)x(0,1,1,12) [intercept=True]; AIC=222.036, BIC=235.401,
Time=1.193 seconds
Fit ARIMA(0,1,2)x(0,1,2,12) [intercept=True]; AIC=223.701, BIC=239.738,
Time=3.274 seconds
Fit ARIMA(0,1,2)x(0,1,3,12) [intercept=True]; AIC=225.458, BIC=244.168,
Time=4.587 seconds
Fit ARIMA(0,1,2)x(1,1,0,12) [intercept=True]; AIC=243.047, BIC=256.411,
Time=0.502 seconds
Fit ARIMA(0,1,2)x(1,1,1,12) [intercept=True]; AIC=223.730, BIC=239.767,
Time=2.361 seconds
Fit ARIMA(0,1,2)x(1,1,2,12) [intercept=True]; AIC=225.729, BIC=244.439,
Time=20.844 seconds
Fit ARIMA(0,1,2)x(2,1,0,12) [intercept=True]; AIC=236.836, BIC=252.873,
Time=2.206 seconds
Fit ARIMA(0,1,2)x(2,1,1,12) [intercept=True]; AIC=225.560, BIC=244.270,
Time=4.076 seconds
Fit ARIMA(0,1,2)x(3,1,0,12) [intercept=True]; AIC=234.114, BIC=252.823,
Time=1.748 seconds
Fit ARIMA(0,1,3)x(0,1,0,12) [intercept=True]; AIC=271.701, BIC=285.065,
Time=0.329 seconds
Fit ARIMA(0,1,3)x(0,1,1,12) [intercept=True]; AIC=223.501, BIC=239.538,
Time=1.394 seconds
Fit ARIMA(0,1,3)x(0,1,2,12) [intercept=True]; AIC=225.124, BIC=243.833,
Time=3.656 seconds
Fit ARIMA(0,1,3)x(1,1,0,12) [intercept=True]; AIC=242.866, BIC=258.903,
Time=0.492 seconds
Fit ARIMA(0,1,3)x(1,1,1,12) [intercept=True]; AIC=225.241, BIC=243.951,
Time=3.470 seconds
Fit ARIMA(0,1,3)x(2,1,0,12) [intercept=True]; AIC=238.129, BIC=256.839,
Time=9.446 seconds
```

```
Fit ARIMA(0,1,4)x(0,1,0,12) [intercept=True]; AIC=254.374, BIC=270.411,
Time=0.939 seconds
Fit ARIMA(0,1,4)x(0,1,1,12) [intercept=True]; AIC=223.083, BIC=241.793,
Time=2.067 seconds
Fit ARIMA(0,1,4)x(1,1,0,12) [intercept=True]; AIC=237.615, BIC=256.325,
Time=1.288 seconds
Fit ARIMA(1,1,0)x(0,1,0,12) [intercept=True]; AIC=287.222, BIC=295.241,
Time=0.045 seconds
Fit ARIMA(1,1,0)x(0,1,1,12) [intercept=True]; AIC=231.779, BIC=242.470,
Time=0.841 seconds
Fit ARIMA(1,1,0)x(0,1,2,12) [intercept=True]; AIC=232.685, BIC=246.049,
Time=1.727 seconds
Fit ARIMA(1,1,0)x(0,1,3,12) [intercept=True]; AIC=234.330, BIC=250.367,
Time=2.093 seconds
Fit ARIMA(1,1,0)x(0,1,4,12) [intercept=True]; AIC=236.328, BIC=255.038,
Time=15.469 seconds
Fit ARIMA(1,1,0)x(1,1,0,12) [intercept=True]; AIC=254.853, BIC=265.544,
Time=0.223 seconds
Fit ARIMA(1,1,0)x(1,1,1,12) [intercept=True]; AIC=232.800, BIC=246.164,
Time=1.014 seconds
Fit ARIMA(1,1,0)x(1,1,2,12) [intercept=True]; AIC=234.163, BIC=250.200,
Time=2.745 seconds
Fit ARIMA(1,1,0)x(1,1,3,12) [intercept=True]; AIC=236.184, BIC=254.894,
Time=2.558 seconds
Fit ARIMA(1,1,0)x(2,1,0,12) [intercept=True]; AIC=244.644, BIC=258.008,
Time=0.562 seconds
Fit ARIMA(1,1,0)x(2,1,1,12) [intercept=True]; AIC=234.401, BIC=250.438,
Time=2.083 seconds
Fit ARIMA(1,1,0)x(2,1,2,12) [intercept=True]; AIC=236.236, BIC=254.946,
Time=1.826 seconds
Fit ARIMA(1,1,0)x(3,1,0,12) [intercept=True]; AIC=242.446, BIC=258.483,
Time=2.424 seconds
Fit ARIMA(1,1,0)x(3,1,1,12) [intercept=True]; AIC=236.398, BIC=255.108,
Time=5.225 seconds
Fit ARIMA(1,1,0)x(4,1,0,12) [intercept=True]; AIC=238.450, BIC=257.159,
Time=2.468 seconds
Fit ARIMA(1,1,1)x(0,1,0,12) [intercept=True]; AIC=272.262, BIC=282.953,
Time=0.130 seconds
Fit ARIMA(1,1,1)x(0,1,1,12) [intercept=True]; AIC=222.765, BIC=236.129,
Time=1.177 seconds
Fit ARIMA(1,1,1)x(0,1,2,12) [intercept=True]; AIC=224.277, BIC=240.314,
Time=3.055 seconds
Fit ARIMA(1,1,1)x(0,1,3,12) [intercept=True]; AIC=226.042, BIC=244.752,
Time=5.579 seconds
Fit ARIMA(1,1,1)x(1,1,0,12) [intercept=True]; AIC=243.768, BIC=257.132,
Time=0.389 seconds
Fit ARIMA(1,1,1)x(1,1,1,12) [intercept=True]; AIC=224.317, BIC=240.354,
Time=1.694 seconds
```

```
Fit ARIMA(1,1,1)x(1,1,2,12) [intercept=True]; AIC=225.777, BIC=244.487,
Time=3.415 seconds
Fit ARIMA(1,1,1)x(2,1,0,12) [intercept=True]; AIC=237.448, BIC=253.485,
Time=1.157 seconds
Fit ARIMA(1,1,1)x(2,1,1,12) [intercept=True]; AIC=226.151, BIC=244.860,
Time=3.116 seconds
Fit ARIMA(1,1,1)x(3,1,0,12) [intercept=True]; AIC=234.723, BIC=253.433,
Time=1.941 seconds
Fit ARIMA(1,1,2)x(0,1,0,12) [intercept=True]; AIC=271.875, BIC=285.239,
Time=0.179 seconds
Fit ARIMA(1,1,2)x(0,1,1,12) [intercept=True]; AIC=223.814, BIC=239.851,
Time=1.270 seconds
Fit ARIMA(1,1,2)x(0,1,2,12) [intercept=True]; AIC=225.455, BIC=244.165,
Time=2.950 seconds
Fit ARIMA(1,1,2)x(1,1,0,12) [intercept=True]; AIC=244.283, BIC=260.320,
Time=0.681 seconds
Fit ARIMA(1,1,2)x(1,1,1,12) [intercept=True]; AIC=225.489, BIC=244.199,
Time=1.337 seconds
Fit ARIMA(1,1,2)x(2,1,0,12) [intercept=True]; AIC=238.584, BIC=257.294,
Time=1.696 seconds
Fit ARIMA(1,1,3)x(0,1,0,12) [intercept=True]; AIC=272.403, BIC=288.440,
Time=0.290 seconds
Fit ARIMA(1,1,3)x(0,1,1,12) [intercept=True]; AIC=225.907, BIC=244.617,
Time=1.625 seconds
Fit ARIMA(1,1,3)x(1,1,0,12) [intercept=True]; AIC=235.418, BIC=254.128,
Time=1.493 seconds
Fit ARIMA(1,1,4)x(0,1,0,12) [intercept=True]; AIC=253.970, BIC=272.680,
Time=0.790 seconds
Fit ARIMA(2,1,0)x(0,1,0,12) [intercept=True]; AIC=277.410, BIC=288.102,
Time=0.068 seconds
Fit ARIMA(2,1,0)x(0,1,1,12) [intercept=True]; AIC=225.431, BIC=238.795,
Time=0.733 seconds
Fit ARIMA(2,1,0)x(0,1,2,12) [intercept=True]; AIC=226.657, BIC=242.694,
Time=2.174 seconds
Fit ARIMA(2,1,0)x(0,1,3,12) [intercept=True]; AIC=228.600, BIC=247.310,
Time=3.410 seconds
Fit ARIMA(2,1,0)x(1,1,0,12) [intercept=True]; AIC=243.982, BIC=257.346,
Time=0.288 seconds
Fit ARIMA(2,1,0)x(1,1,1,12) [intercept=True]; AIC=226.686, BIC=242.723,
Time=0.959 seconds
Fit ARIMA(2,1,0)x(1,1,2,12) [intercept=True]; AIC=228.203, BIC=246.913,
Time=2.892 seconds
Fit ARIMA(2,1,0)x(2,1,0,12) [intercept=True]; AIC=238.791, BIC=254.828,
Time=0.749 seconds
Fit ARIMA(2,1,0)x(2,1,1,12) [intercept=True]; AIC=228.653, BIC=247.363,
Time=3.304 seconds
Fit ARIMA(2,1,0)x(3,1,0,12) [intercept=True]; AIC=237.078, BIC=255.787,
Time=1.487 seconds
```

```
Fit ARIMA(2,1,1)x(0,1,0,12) [intercept=True]; AIC=269.555, BIC=282.919,
Time=0.158 seconds
Fit ARIMA(2,1,1)x(0,1,1,12) [intercept=True]; AIC=223.060, BIC=239.097,
Time=1.770 seconds
Fit ARIMA(2,1,1)x(0,1,2,12) [intercept=True]; AIC=224.805, BIC=243.515,
Time=3.050 seconds
Fit ARIMA(2,1,1)x(1,1,0,12) [intercept=True]; AIC=242.623, BIC=258.660,
Time=0.482 seconds
Fit ARIMA(2,1,1)x(1,1,1,12) [intercept=True]; AIC=224.827, BIC=243.537,
Time=1.384 seconds
Fit ARIMA(2,1,1)x(2,1,0,12) [intercept=True]; AIC=237.769, BIC=256.479,
Time=1.727 seconds
Fit ARIMA(2,1,2)x(0,1,0,12) [intercept=True]; AIC=254.832, BIC=270.869,
Time=0.560 seconds
Fit ARIMA(2,1,2)x(0,1,1,12) [intercept=True]; AIC=213.961, BIC=232.671,
Time=1.548 seconds
Fit ARIMA(2,1,2)x(1,1,0,12) [intercept=True]; AIC=244.615, BIC=263.325,
Time=0.824 seconds
Fit ARIMA(2,1,3)x(0,1,0,12) [intercept=True]; AIC=257.877, BIC=276.587,
Time=0.680 seconds
Fit ARIMA(3,1,0)x(0,1,0,12) [intercept=True]; AIC=272.136, BIC=285.501,
Time=0.101 seconds
Fit ARIMA(3,1,0)x(0,1,1,12) [intercept=True]; AIC=224.311, BIC=240.348,
Time=1.298 seconds
Fit ARIMA(3,1,0)x(0,1,2,12) [intercept=True]; AIC=225.998, BIC=244.708,
Time=1.958 seconds
Fit ARIMA(3,1,0)x(1,1,0,12) [intercept=True]; AIC=243.693, BIC=259.730,
Time=0.340 seconds
Fit ARIMA(3,1,0)x(1,1,1,12) [intercept=True]; AIC=226.016, BIC=244.726,
Time=1.468 seconds
Fit ARIMA(3,1,0)x(2,1,0,12) [intercept=True]; AIC=238.416, BIC=257.126,
Time=0.943 seconds
Fit ARIMA(3,1,1)x(0,1,0,12) [intercept=True]; AIC=270.983, BIC=287.020,
Time=0.171 seconds
Fit ARIMA(3,1,1)x(0,1,1,12) [intercept=True]; AIC=224.905, BIC=243.614,
Time=1.873 seconds
Fit ARIMA(3,1,1)x(1,1,0,12) [intercept=True]; AIC=244.620, BIC=263.330,
Time=0.610 seconds
Fit ARIMA(3,1,2)x(0,1,0,12) [intercept=True]; AIC=272.345, BIC=291.055,
Time=0.263 seconds
Fit ARIMA(4,1,0)x(0,1,0,12) [intercept=True]; AIC=273.427, BIC=289.464,
Time=0.130 seconds
Fit ARIMA(4,1,0)x(0,1,1,12) [intercept=True]; AIC=225.708, BIC=244.418,
Time=1.454 seconds
Fit ARIMA(4,1,0)x(1,1,0,12) [intercept=True]; AIC=245.356, BIC=264.066,
Time=0.442 seconds
Fit ARIMA(4,1,1)x(0,1,0,12) [intercept=True]; AIC=272.978, BIC=291.688,
Time=0.225 seconds
```

```
Total fit time: 261.704 seconds
```

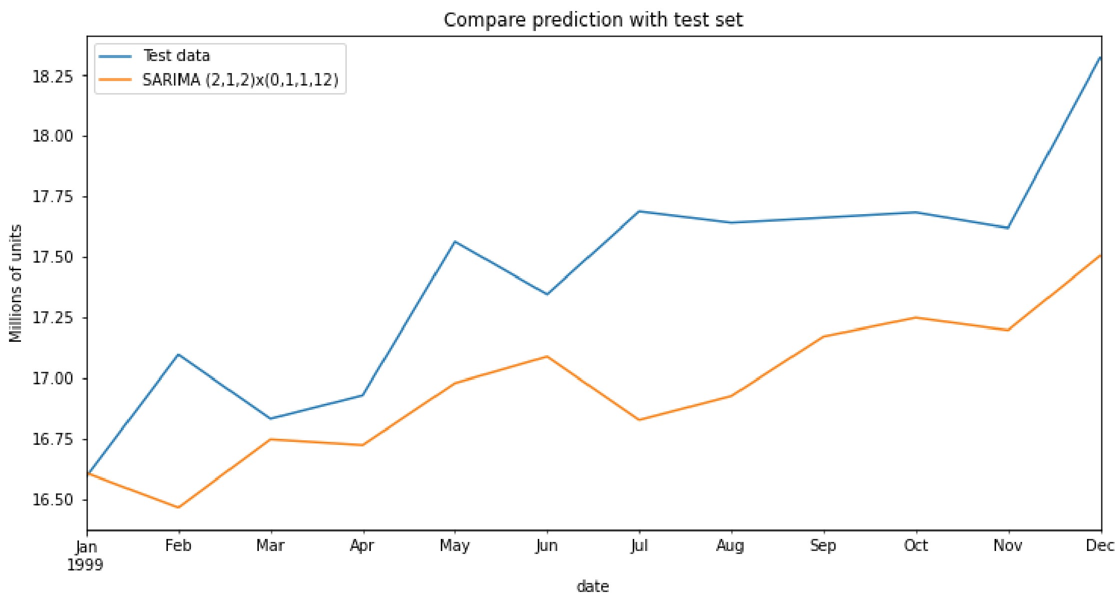
```
[20]: <class 'statsmodels.iolib.summary.Summary'>
"""
                                     SARIMAX Results
================================================================================
============
Dep. Variable:                                    y   No. Observations:
120
Model:             SARIMAX(2, 1, 2)x(0, 1, [1], 12)   Log Likelihood
-99.981
Date:                              Wed, 03 Jun 2020   AIC
213.961
Time:                                      01:32:29   BIC
232.671
Sample:                                           0   HQIC
221.546
                                              - 120
Covariance Type:                                opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      0.0037      0.003      1.252      0.211      -0.002       0.010
ar.L1          1.2517      0.120     10.433      0.000       1.017       1.487
ar.L2         -0.4559      0.110     -4.144      0.000      -0.671      -0.240
ma.L1         -1.7903      0.056    -31.901      0.000      -1.900      -1.680
ma.L2          0.9428      0.050     18.892      0.000       0.845       1.041
ma.S.L12      -0.9914      3.000     -0.330      0.741      -6.871       4.888
sigma2         0.2933      0.858      0.342      0.732      -1.388       1.974
================================================================================
===
Ljung-Box (Q):                       32.01   Jarque-Bera (JB):
0.20
Prob(Q):                              0.81   Prob(JB):
0.91
Heteroskedasticity (H):               1.27   Skew:
-0.10
Prob(H) (two-sided):                  0.48   Kurtosis:
3.06
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

[21]:
```
arima_2_1_2_seasonal_0_1_1 = SARIMAX(
    train.TOTALSA,
    order=(2,1,2),
    seasonal_order=(0,1,1,12)
).fit()
res_arima_2_1_2_seasonal_0_1_1 = evaluate_model(
    arima_2_1_2_seasonal_0_1_1,
    len(train),
    test.TOTALSA, "SARIMA (2,1,2)x(0,1,1,12)")
```

```
MSE for SARIMA (2,1,2)x(0,1,1,12):      0.28194837904814896
RMSE for SARIMA (2,1,2)x(0,1,1,12):     0.5309881157315566
```

[22]:
```
plot_results(test.TOTALSA, res_arima_2_1_2_seasonal_0_1_1)
```



What if we let the autoarima to determin the "D" parameter?

[23]:
```
stepwise = auto_arima(
    df_19.TOTALSA,
    m=12,
    d=None,
    D=None,
    start_p=1,
    start_q=1,
    max_p=4,
    max_q=4,
    max_d=4,
    max_P=4,
```

```
    max_D=4,
    max_Q=4,
    stepwise=False,
    trace=True,
    n_jobs=-1
)
stepwise.summary()
```

```
Total fit time: 29.738 seconds
```

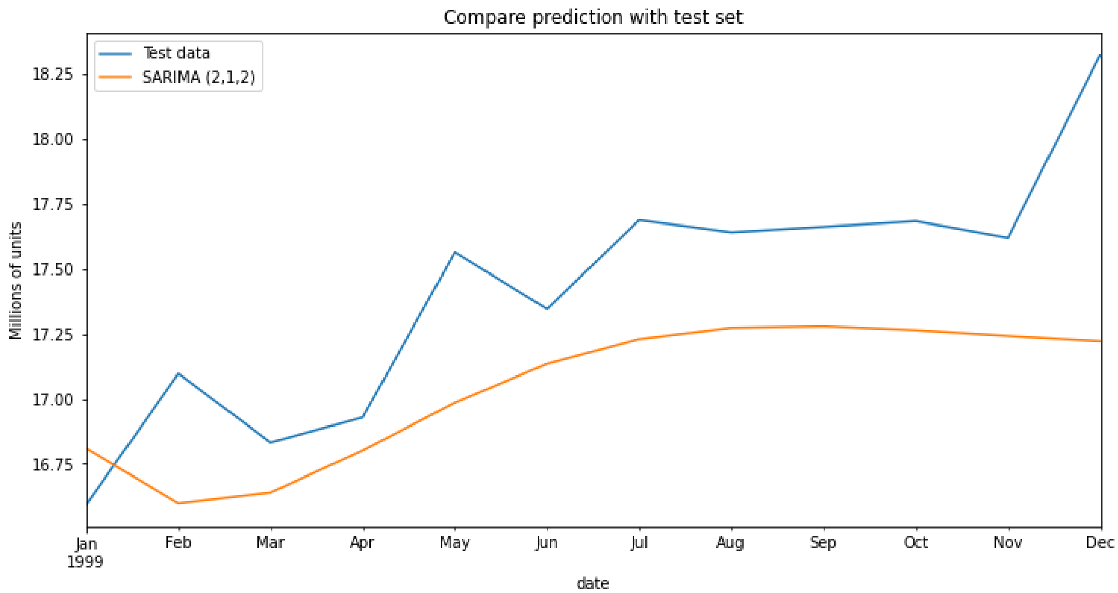
[23]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  120
Model:               SARIMAX(2, 1, 2)   Log Likelihood                 -97.640
Date:                Wed, 03 Jun 2020   AIC                            207.280
Time:                        01:33:31   BIC                            223.954
Sample:                             0   HQIC                           214.051
                                - 120
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      0.0046      0.009      0.504      0.614      -0.013       0.022
ar.L1          1.2571      0.097     12.988      0.000       1.067       1.447
ar.L2         -0.4964      0.111     -4.472      0.000      -0.714      -0.279
ma.L1         -1.8311      0.314     -5.832      0.000      -2.446      -1.216
ma.L2          0.9958      0.347      2.872      0.004       0.316       1.676
sigma2         0.2879      0.103      2.792      0.005       0.086       0.490
================================================================================
===
Ljung-Box (Q):                       25.31   Jarque-Bera (JB):
2.15
Prob(Q):                              0.97   Prob(JB):
0.34
Heteroskedasticity (H):               1.24   Skew:
-0.31
Prob(H) (two-sided):                  0.50   Kurtosis:
3.23
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

[24]:
```
sarima_2_1_2_model = SARIMAX(train.TOTALSA,order=(2,1,2)).fit()
res_sarima_2_1_2 = evaluate_model(sarima_2_1_2_model, len(train), test.TOTALSA,
 ↪"SARIMA (2,1,2)")
```

```
MSE for SARIMA (2,1,2):         0.22900061704753993
RMSE for SARIMA (2,1,2):        0.478540089279404
```

[25]:
```
plot_results(test.TOTALSA, res_sarima_2_1_2)
```

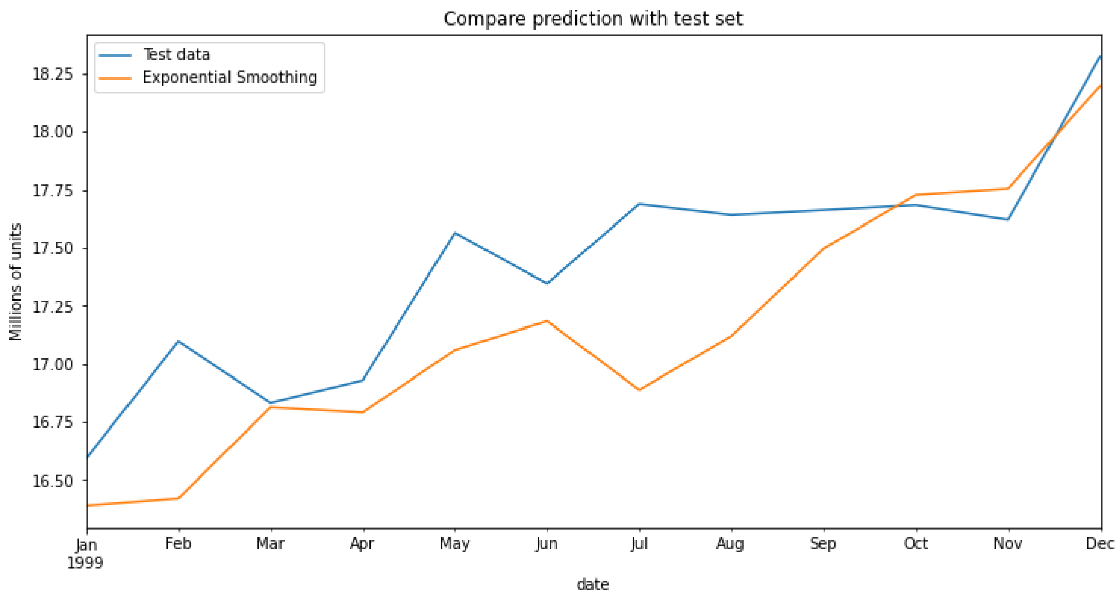


How about exponential Smoothing method?

[26]:
```
hw_es = ExponentialSmoothing(
    train.TOTALSA,
    trend='mul',
    seasonal='mul',
    seasonal_periods=12
).fit()
res_hw_es = hw_es.forecast(12).rename("Exponential Smoothing")
print(f"MSE for Exponential Smoothing: \t{mean_squared_error(test.TOTALSA,
 ↪res_hw_es)}")
print(f"RMSE for Exponential Smoothing: \t{rmse(test.TOTALSA, res_hw_es)}")
```

```
MSE for Exponential Smoothing:  0.14785876957616004
RMSE for Exponential Smoothing:         0.3845240819196634
```

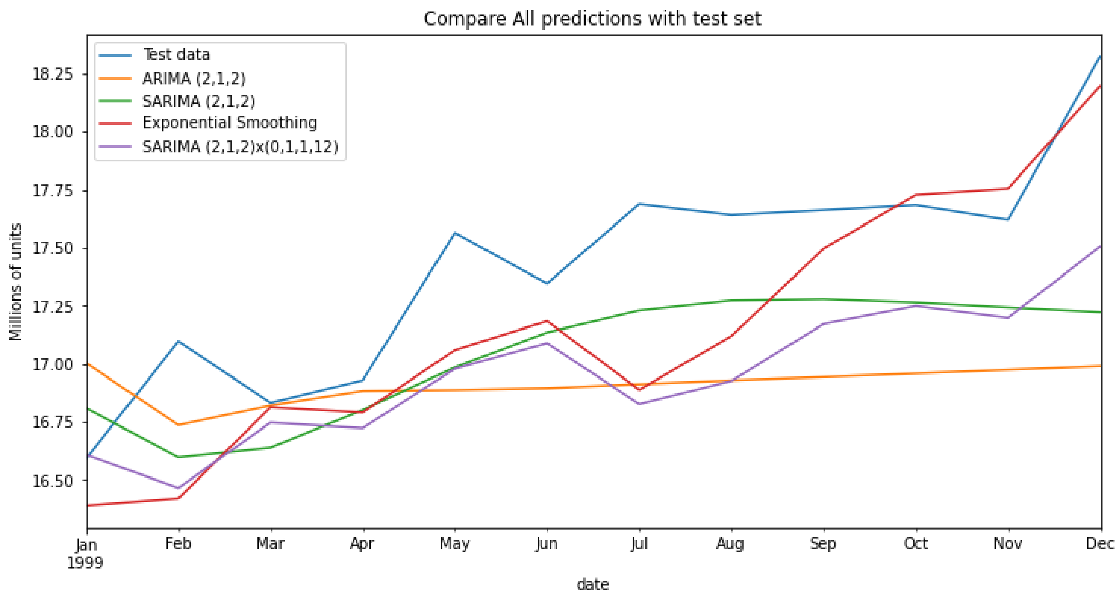
[27]:
```
plot_results(test.TOTALSA, res_hw_es)
```



Model	MSE	RMSE
ARIMA (2,1,2)	0.441	0.664
SARIMAX (2,1,2)x(0,1,1,12)	0.281	0.530
SARIMAX (2,1,2)	0.229	0.478
Exponential Smoothing	**0.147**	**0.3845**

[28]:
```
# All on one
title = "Compare All predictions with test set"
xlabel = "date"
ylabel="Millions of units"

ax = test.TOTALSA.plot(legend=True, figsize=(12,6), title=title, label="Test␣
↪data")
res_arima_2_1_2.plot(legend=True)
res_sarima_2_1_2.plot(legend=True)
res_hw_es.plot(legend=True)
res_arima_2_1_2_seasonal_0_1_1.plot(legend=True)
ax.autoscale(axis="x", tight=True)
ax.set(xlabel=xlabel, ylabel=ylabel)
plt.show()
```



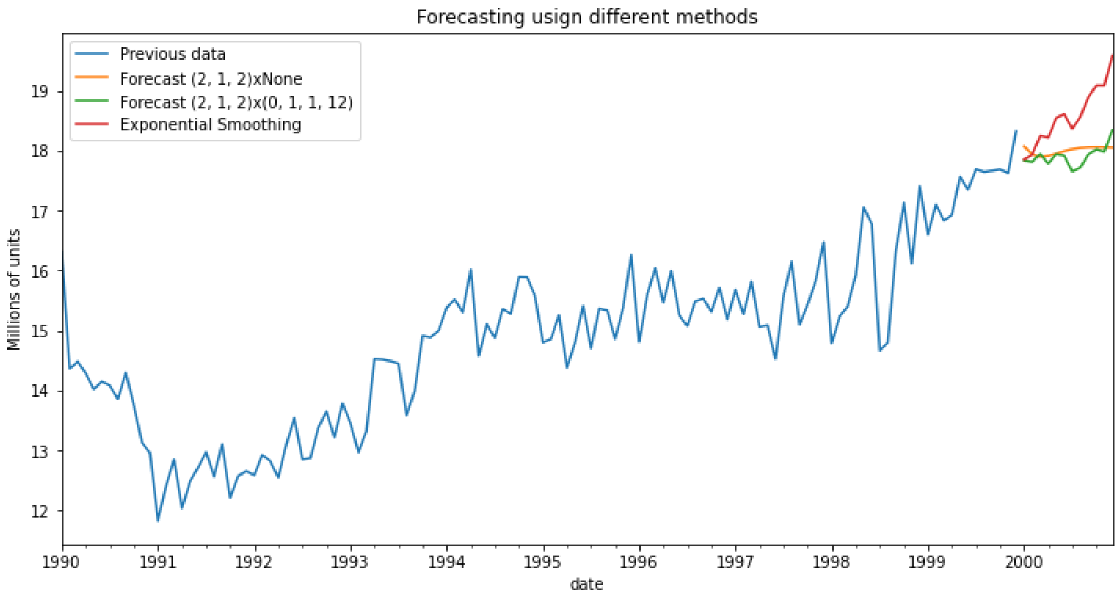
1.0.1 Forecasting

[29]:
```
models = [
    dict(
        order=(2,1,2)
    ),
    dict(
        order=(2,1,2),
        seasonal_order=(0,1,1,12)
    )
]
forecasts = [forecast_sarima(df_19.TOTALSA, **i) for i in models]
```

[30]:
```
fcast_es = ExponentialSmoothing(
    df_19.TOTALSA,
    trend='mul',
    seasonal='mul',
    seasonal_periods=12
).fit()
forecasts.append(fcast_es.forecast(12).rename("Exponential Smoothing"))
```

[31]:
```
# All on one
title = "Forecasting usign different methods"
xlabel = "date"
ylabel="Millions of units"
```

```
ax = df_19.TOTALSA.plot(legend=True, figsize=(12,6), title=title,␣
 ↪label="Previous data")
for f in forecasts:
    f.plot(legend=True)
ax.autoscale(axis="x", tight=True)
ax.set(xlabel=xlabel, ylabel=ylabel)
plt.show()
```



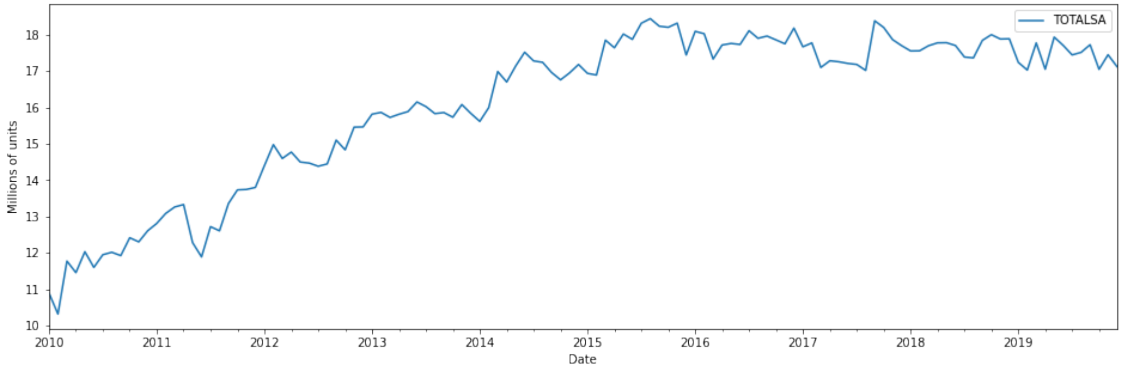
2 Second deacade

[32]:
```
df_21 = pd.read_csv("TOTALSA_21.csv", index_col="DATE", parse_dates=True)
df_21.freq="MS"
df_21.head()
```

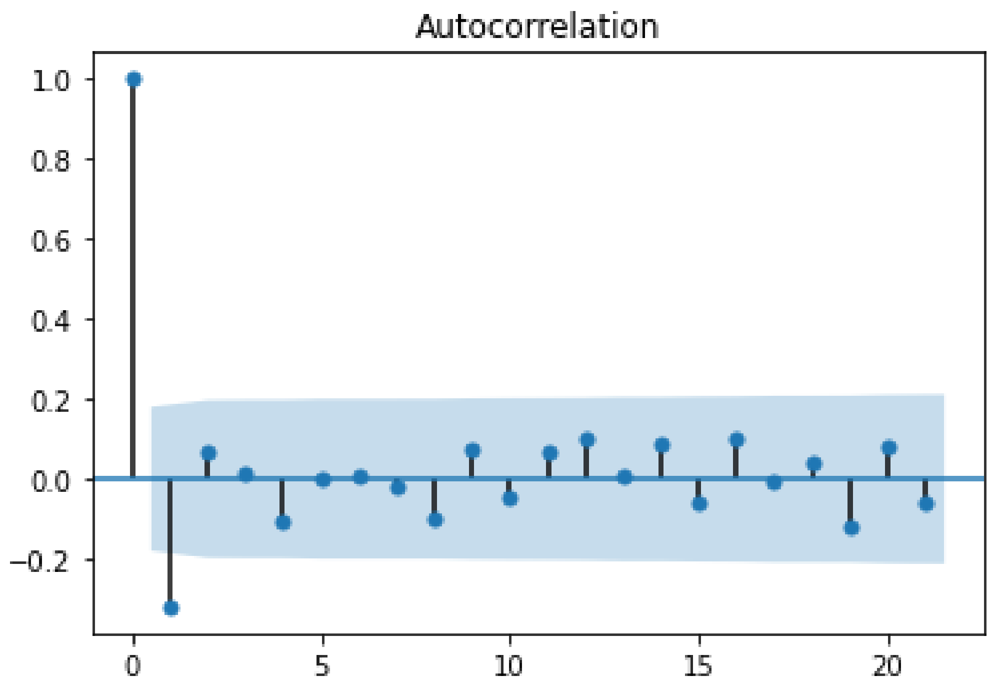
[32]:

DATE	TOTALSA
2010-01-01	10.893
2010-02-01	10.315
2010-03-01	11.772
2010-04-01	11.454
2010-05-01	12.030

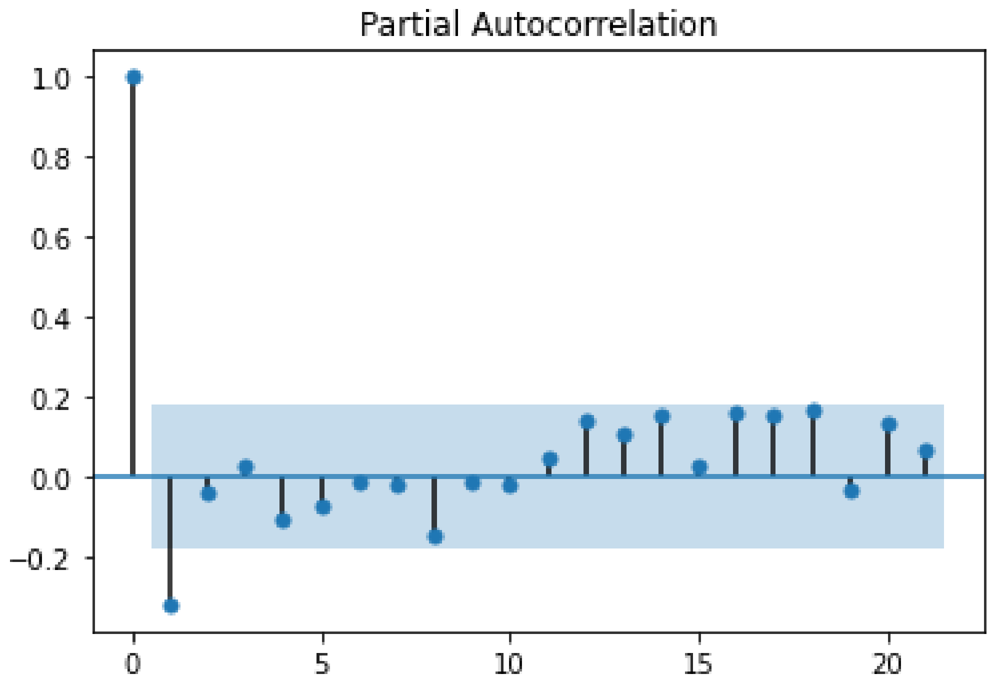
[33]:
```
ax = df_21.plot(figsize=(16,5))
plt.ylabel("Millions of units")
plt.xlabel("Date")
plt.show()
```



[34]:
```
plot_acf(df_21.TOTALSA.diff(1).dropna());
```



[35]:
```
plot_pacf(df_21.TOTALSA.diff(1).dropna());
```



Look like we sould use the same model, (2,1,2)

[36]:
```
s_index = -12
train = df_21.iloc[:s_index]
test = df_21.iloc[s_index:]
```

Automatic seasonal detection

[37]:
```
stepwise = auto_arima(
    df_19.TOTALSA,
    m=12,
    start_p=1,
    start_q=1,
    max_p=4,
    max_q=4,
    max_d=4,
    max_P=4,
    max_D=4,
    max_Q=4,
    stepwise=False,
    trace=True,
    n_jobs=-1
)
stepwise.summary()
```

```
Total fit time: 23.219 seconds
```

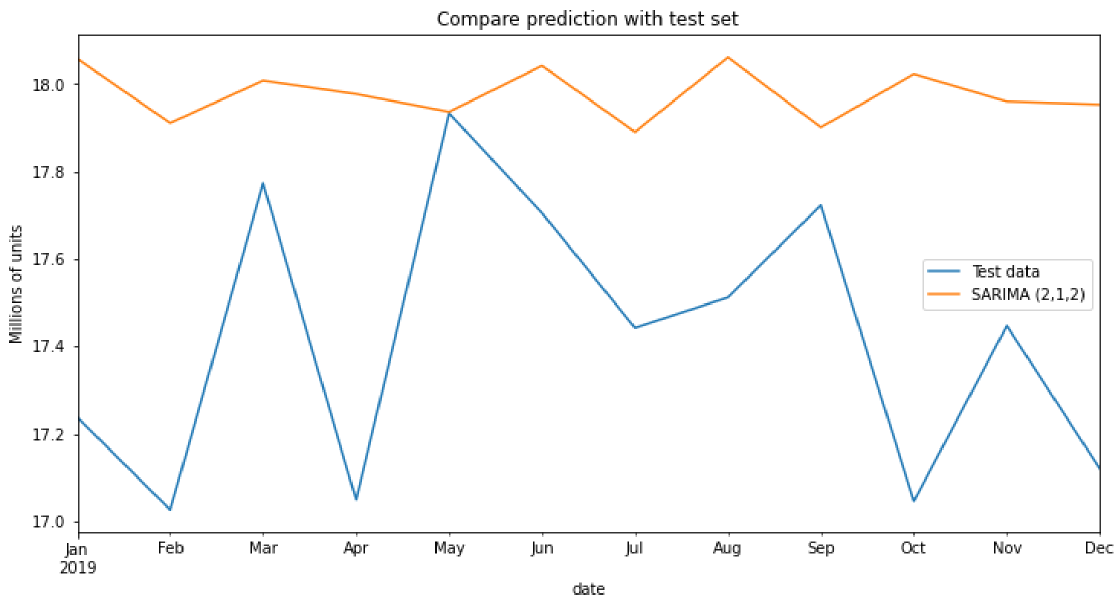
[37]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  120
Model:               SARIMAX(2, 1, 2)   Log Likelihood                 -97.640
Date:                Wed, 03 Jun 2020   AIC                            207.280
Time:                        01:36:42   BIC                            223.954
Sample:                             0   HQIC                           214.051
                                - 120
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      0.0046      0.009      0.504      0.614      -0.013       0.022
ar.L1          1.2571      0.097     12.988      0.000       1.067       1.447
ar.L2         -0.4964      0.111     -4.472      0.000      -0.714      -0.279
ma.L1         -1.8311      0.314     -5.832      0.000      -2.446      -1.216
ma.L2          0.9958      0.347      2.872      0.004       0.316       1.676
sigma2         0.2879      0.103      2.792      0.005       0.086       0.490
================================================================================
===
Ljung-Box (Q):                       25.31   Jarque-Bera (JB):
2.15
Prob(Q):                              0.97   Prob(JB):
0.34
Heteroskedasticity (H):               1.24   Skew:
-0.31
Prob(H) (two-sided):                  0.50   Kurtosis:
3.23
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

[38]:
```python
sarima_2_1_2 = SARIMAX(train.TOTALSA, order=(2,1,2)).fit()
res_sarima_2_1_2 = evaluate_model(sarima_2_1_2, len(train), test.TOTALSA,␣
 ↪"SARIMA (2,1,2)")
```

```
MSE for SARIMA (2,1,2):         0.4108134503938932
RMSE for SARIMA (2,1,2):        0.6409473070338101
```

[39]:
```python
plot_results(test.TOTALSA, res_sarima_2_1_2)
```



Fixed **D=1** and **d=1**

[40]:
```
auto_arima(
    df_19.TOTALSA,
    m=12,
    d=1,
    D=1,
    start_p=1,
    start_q=1,
    max_p=4,
    max_q=4,
#     max_d=4,
    max_P=4,
#     max_D=4,
    max_Q=4,
    stepwise=False,
    trace=True,
    n_jobs=-1
).summary()
```

```
Total fit time: 58.293 seconds
```

[40]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                                      SARIMAX Results
===============================================================================
============
Dep. Variable:                                              y   No. Observations:
```

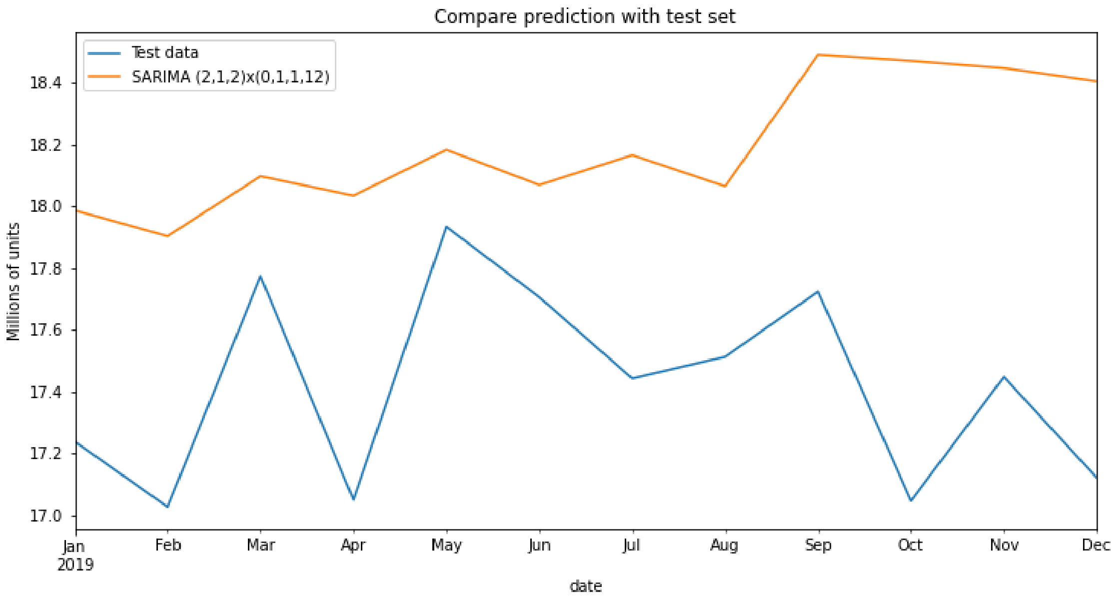
```
120
Model:              SARIMAX(2, 1, 2)x(0, 1, [1], 12)   Log Likelihood
-99.981
Date:                               Wed, 03 Jun 2020   AIC
213.961
Time:                                       01:38:23   BIC
232.671
Sample:                                            0   HQIC
221.546
                                               - 120
Covariance Type:                                 opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      0.0037      0.003      1.252      0.211      -0.002       0.010
ar.L1          1.2517      0.120     10.433      0.000       1.017       1.487
ar.L2         -0.4559      0.110     -4.144      0.000      -0.671      -0.240
ma.L1         -1.7903      0.056    -31.901      0.000      -1.900      -1.680
ma.L2          0.9428      0.050     18.892      0.000       0.845       1.041
ma.S.L12      -0.9914      3.000     -0.330      0.741      -6.871       4.888
sigma2         0.2933      0.858      0.342      0.732      -1.388       1.974
================================================================================
===
Ljung-Box (Q):                      32.01   Jarque-Bera (JB):
0.20
Prob(Q):                             0.81   Prob(JB):
0.91
Heteroskedasticity (H):              1.27   Skew:
-0.10
Prob(H) (two-sided):                 0.48   Kurtosis:
3.06
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
"""
```

[41]:
```
sarimax_2_1_2_ses_0_1_1 = SARIMAX(train.TOTALSA, order=(2,1,2),␣
 ↪seasonal_order=(0,1,1,12)).fit()
res_sarimax_2_1_2_ses_0_1_1 = evaluate_model(
    sarimax_2_1_2_ses_0_1_1,
    len(train),
    test.TOTALSA,
    "SARIMA (2,1,2)x(0,1,1,12)"
)
```

```
MSE for SARIMA (2,1,2)x(0,1,1,12):      0.7237103232176859
RMSE for SARIMA (2,1,2)x(0,1,1,12):     0.8507116569188916
```

[42]:
```
plot_results(test.TOTALSA, res_sarimax_2_1_2_ses_0_1_1)
```



What will happen if we increase the ORDER of **P**?

[43]:
```
sarimax_2_1_2_ses_1_1_1 = SARIMAX(train.TOTALSA, order=(2,1,2),␣
 ↪seasonal_order=(1,1,1,12)).fit()
res_sarimax_2_1_2_ses_1_1_1 = evaluate_model(
    sarimax_2_1_2_ses_1_1_1,
    len(train),
    test.TOTALSA,
    "SARIMA (2,1,2)x(1,1,1,12)"
)
```

```
MSE for SARIMA (2,1,2)x(1,1,1,12):      0.8413441581081381
RMSE for SARIMA (2,1,2)x(1,1,1,12):     0.9172481442380453
```

It's worse.

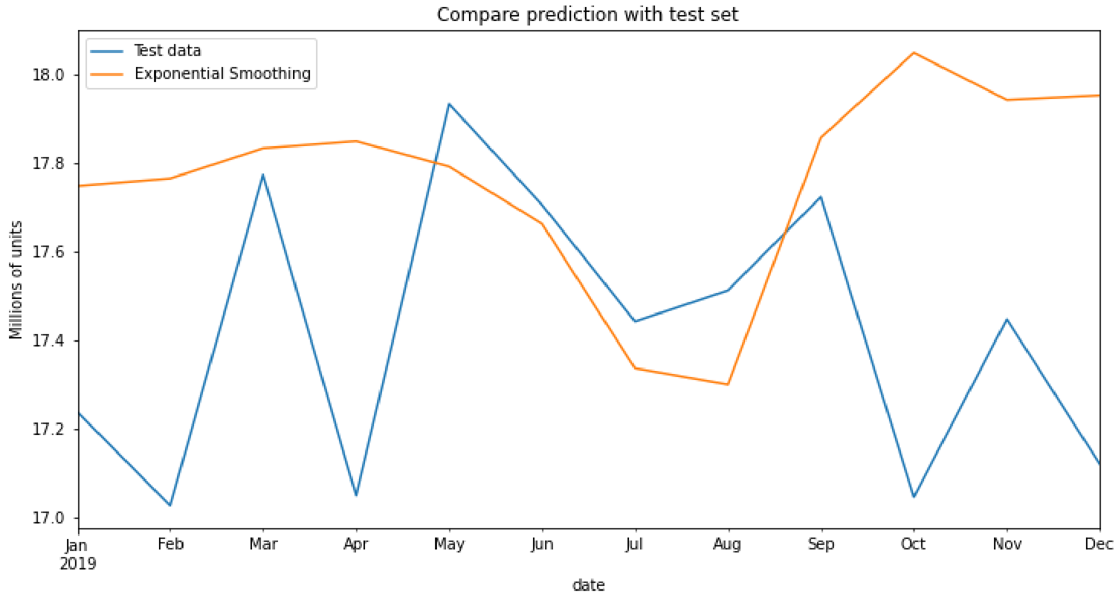
Lets try Exponential Smoothing

[44]:
```
hw_es2 = ExponentialSmoothing(
    train.TOTALSA,
    trend='mul',
    seasonal='mul',
    seasonal_periods=12
).fit()
```

```
res_hw_es2 = hw_es2.forecast(12).rename("Exponential Smoothing")
print(f"MSE for Exponential Smoothing: \t{mean_squared_error(test.TOTALSA,␣
 ↪res_hw_es)}")
print(f"RMSE for Exponential Smoothing: \t{rmse(test.TOTALSA, res_hw_es)}")
```

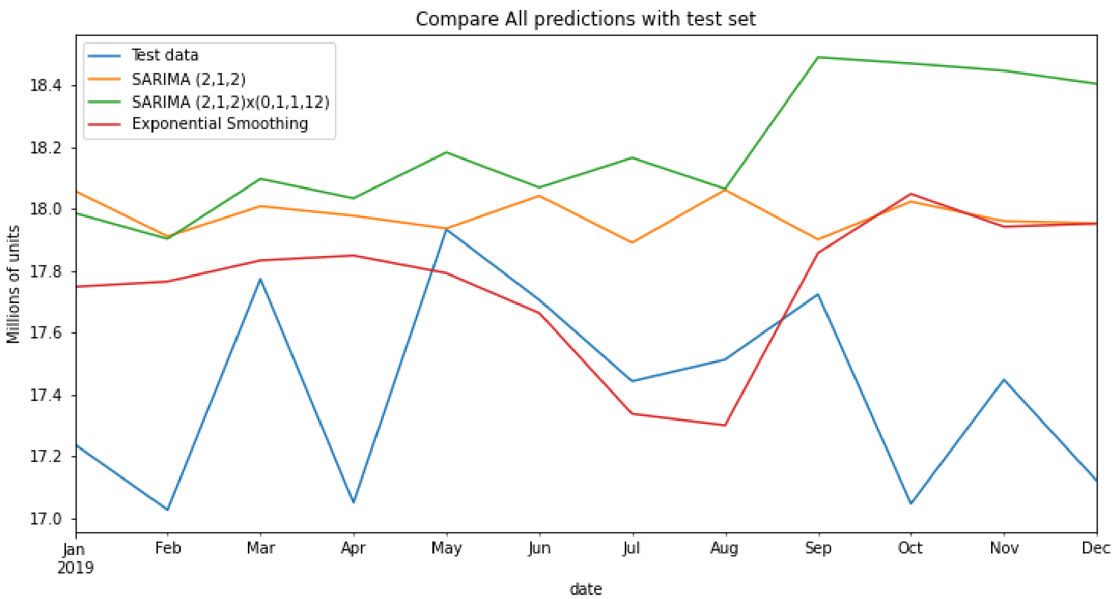
```
MSE for Exponential Smoothing:  0.44475911684650704
RMSE for Exponential Smoothing:         0.6669026292094724
```

[57]:
```
plot_results(test.TOTALSA, res_hw_es2)
```



[58]:
```
# All on one
title = "Compare All predictions with test set"
xlabel = "date"
ylabel="Millions of units"

ax = test.TOTALSA.plot(legend=True, figsize=(12,6), title=title, label="Test␣
 ↪data")
res_sarima_2_1_2.plot(legend=True)
res_sarimax_2_1_2_ses_0_1_1.plot(legend=True)
res_hw_es2.plot(legend=True)
ax.autoscale(axis="x", tight=True)
ax.set(xlabel=xlabel, ylabel=ylabel)
plt.show()
```



Model	MSE	RMSE
SARIMA (2,1,2)	**0.410**	**0.640**
SARIMA (2,1,2)x(0,1,1,12)	0.723	0.850
Exponential Smoothing	0.444	0.667

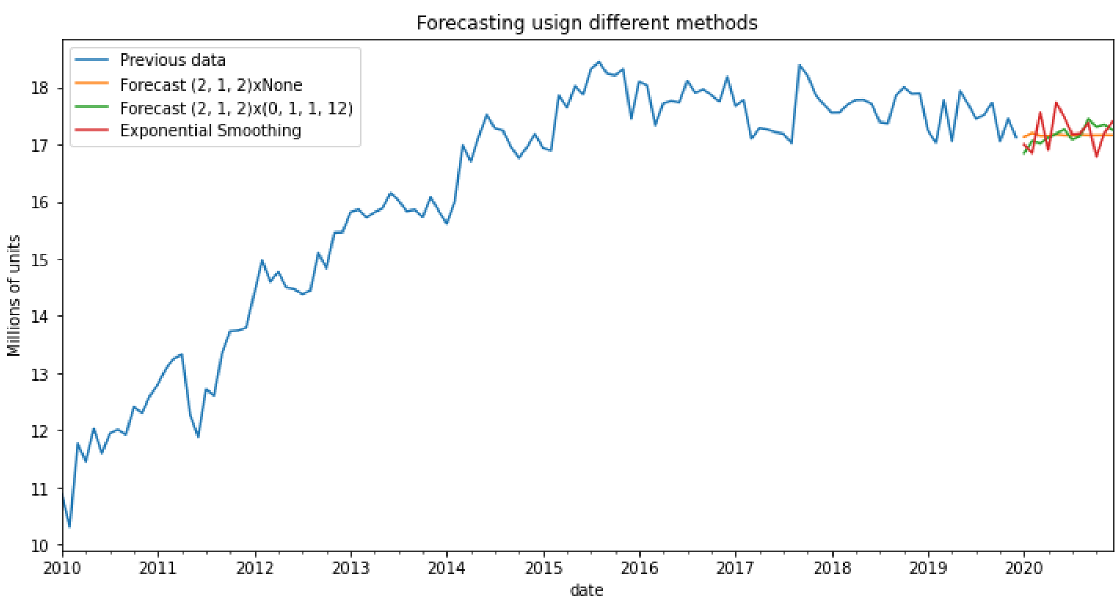
[45]:
```
sarima_models = [
    dict(
        order=(2,1,2)
    ),
    dict(
        order=(2,1,2),
        seasonal_order=(0,1,1,12)
    )
]

forecasts = [forecast_sarima(df_21.TOTALSA, **i) for i in models]
```

[46]:
```
fcast_es = ExponentialSmoothing(
    df_21.TOTALSA,
    trend='mul',
    seasonal='mul',
    seasonal_periods=12
).fit()
forecasts.append(fcast_es.forecast(12).rename("Exponential Smoothing"))
```

[47]:
```
# All on one
title = "Forecasting usign different methods"
xlabel = "date"
ylabel="Millions of units"

ax = df_21.TOTALSA.plot(legend=True, figsize=(12,6), title=title,␣
 ↪label="Previous data")
for f in forecasts:
    f.plot(legend=True)
ax.autoscale(axis="x", tight=True)
ax.set(xlabel=xlabel, ylabel=ylabel)
plt.show()
```



For the first decade the best model, was ExponentialSmoothing. Adding the seasonality, wasn't decreasing the error.

For the second decade Exponential Smoothing model and SARIMA(2,1,2) has comparable errors.