Project 3: Kalman Filtering and 3D Reconstruction from Factorization

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1. Introduction

Kalman filtering is a common method of feature tracking, meaning a set of feature points are matched from one frame to the next. It is performed recursively by estimating the position and uncertainty of a point from one frame to the next. The 3D structure can also be recovered with the factorization method using the coordinates of tracked points from the Kalman filter process. The following project will perform Kalman filtering on the corners of a polyhedron in a sequence of images, and reconstruct its 3D shape from these tracked features using the factorization method.

2. Feature Tracking with Kalman Filtering

2.1 Theory for the Algorithm

Feature tracking can be performed with a Kalman filter, which is a recursive algorithm that estimates the position and uncertainty of a moving feature point in the next frame. It assumes a linear state model and the uncertainty is zero-mean Gaussian.

Suppose the coordinates of a feature at time interval t are given by $p_t=(x_t,y_t)$, and the velocity is given by $v_t=(v_{x,t},v_{y,t})$. The state at t is given by $s_t=[x_t, y_t, v_{x,t}, v_{y,t}]^t$, which incorporates both position and velocity. The Kalman filter estimates the state vector of t+1 given t.

The Kalman filter performs state prediction using the state model and state updating using the measurement model.

The state model, or temporal part of the system, is linear model given by:

$$s_{t+1}=\phi s_t + w_t \quad (1)$$

In (1), Φ is the state transition matrix and w_t is the system perturbation. If the movement of feature points from one frame to the next is small, it can be assumed to be uniform and the state transition matrix can be parameterized as:

$$\Phi = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The system perturbation is a normal Gaussian distribution given by $w_t \sim N(0,Q)$.

The measurement model is a linear model which represents the spatial system features, given by:

$$z_t = Hs_t + v_t \quad (3)$$

In (3), v_t is the measurement uncertainty and H relates the current state to the current measurement. H is given by:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (4)$$

The measurement uncertainty v_t is a Gaussian normal distribution given by $v_t \sim N(0,R)$.

The Kalman filter algorithm is performed recursively with the following four steps:

1. Predict state and covariance
   The predicted state s_{t+1}^- is obtained from the current state s_t and state transition matrix Φ with the following:

$$s_{t+1}^- = \Phi s_t \quad (5)$$

   The covariance prediction Σ_{t+1}^- is obtained from the state transition matrix Φ, current state covariance matrix Σ_t and Q with the following:

$$\Sigma_{t+1}^- = \Phi \Sigma_t \Phi^t + Q \quad (6)$$

2. Find z_{t+1} through measurement
   The feature detector searches through a determined by the 2x2 upper left submatrix of Σ_{t+1}^-, Σ_{t+1}^{P-}. The center of the search region is the predicted position p_{t+1}^-. The search region is elliptical, given by $(p - p_{t+1}^-)(\Sigma_{t+1}^{P-})^{-1}(p-p_{t+1}^-)^T \leq c^2$, where c=.95. The region was searched using the sum of squared differences correlation method, where a square correlation window is moved through the region to find the matching point.

3. Calculate the gain matrix

The gain matrix K is given by the following:

$$K_{t+1} = \Sigma^-_{t+1} H^T (H\Sigma^-_{t+1} H^T + R)^{-1} \ (8)$$

The gain matrix acts as a weighing factor to scale the contribution of the measurement z_{t+1} and prediction Hs_{t+1}^- to the posterior state estimation s_{t+1}.

4. Find the posterior state and posterior covariance estimation

$$s_{t+1} = s^-_{t+1} + K_{t+1}(z_{t+1} - Hs^-_{t+1}) \ (9)$$

$$\Sigma_{t+1} = (I - K_{t+1}H)\Sigma^-_{t+1} \ (10)$$

The current estimate is recursively conditioned after each prediction and its corresponding measurement. The new prior estimate of a tracked point is the previous posterior estimate.

Kalman filter initialization is used with the first two frames i and i+1 to produce the initial vector $s_0 = [x_0, y_0, v_{x,0}, v_{y,0}]$, where $x_0 = x_{i+1}$, $y_0 = y_{i+1}$, $v_{x,0} = x_{i+1} - x_i$, and $v_{y,0} = y_{i+1} - y_i$. This initial vector is calculated from a set of corresponding detected features in frames i and i+1.

The initial covariance matrix is initialized as:

$$\Sigma_0 = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 25 \end{bmatrix} \ (11)$$

The covariance matrix is initialized to large values, but decreases after a few iterations to a set of stable values.

The system error covariance matrix Q is initialized to:

$$Q = \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \ (12)$$

The values of Q are initialized as such because the standard deviation from positional system error is 4 in the x and y directions, respectively. The standard deviation from the velocity error is 2 pixels per frame.

The measurement error covariance matrix is given as:

$$R = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \quad (13)$$

The values of R are initialized as such because the error for the measurement model can be assumed to be 2 for the x and y directions, respectively.

The Kalman filter process is limited by its assumption of linear state dynamics. In addition, each tracked feature needs its own Kalman filter.

2.2 Experiment Results:

A Kalman filter was used to track each corner point of a moving polyhedron in a sequence of 30 images. The code was implemented in Python 3.

The constant matrices Φ, H, Q and R were initialized to:

$$\Phi = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$R = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

The covariance matrix was initialized to:

$$\Sigma_0 = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 25 \end{bmatrix}$$

In the first two image frames, nine corner points were identified by magnifying the image for each black tape marker and noting the image coordinates with the darkest intensity. These initial feature point sets are shown in the table below. They correspond to the leftmost to rightmost corners of the polyhedron.

X_1	Y_1	X_2	Y_2
118	78	115	78
117	89	114	89
122	158	120	158
232	92	229	91
254	80	251	80
273	79	270	78
273	163	270	163
297	69	295	69
296	138	295	138

Therefore, the initial state vectors s_0 for each point are shown in the following table:

Point	x_0	y_0	$v_{x,0}$	$v_{y,0}$
1	115	78	-3	0
2	114	89	-3	0
3	120	158	-2	0
4	229	91	-3	-1
5	251	80	-3	0
6	270	78	-3	-1

7	270	163	-3	0
8	295	69	-2	0
9	295	138	-1	0

For each point, a Kalman filter was initialized and the point coordinates were tracked from one frame to the next using the algorithm described in section 2.1. For each point, the final state vectors and covariance matrices are shown for each frame.

Point 1:
Frame# 3 X: 114.02758620689656 Y: 77.02758620689656 Vx: -0.972413793103442 Vy: -0.972413793103442
Covariance Matrix:
[100 0 0 0]
[0 100 0 0]
[0 0 25 0]

Frame# 4 X: 111.24240750966317 Y: 77.00220872446162 Vx: -2.785178697233391 Vy: -0.02537748243493354
Covariance Matrix:
[3.88965517 0.   0.68965517 0.   ]
[0.   3.88965517 0.   0.68965517]
[0.68965517 0.   24.68965517 0.   ]

Frame# 5 X: 107.38974280931365 Y: 77.0002029117841 Vx: -3.8526647003495214 Vy: -0.002005812677523977
Covariance Matrix:
[3.67973495 0.   2.0320265 0.   ]
[0.   3.67973495 0.   2.0320265]
[2.0320265 0.   15.79679735 0.   ]

Frame# 6 X: 104.34409565593523 Y: 76.1015314424602 Vx: -3.045647153378411 Vy: -0.8986714693239009
Covariance Matrix:
[3.63252676 0.   1.63790392 0.   ]
[0.   3.63252676 0.   1.63790392]
[1.63790392 0.   12.49632222 0.   ]

Frame# 7 X: 100.45860115682352 Y: 76.01071855701487 Vx: -3.885494499111715 Vy: -0.09081288544533095
Covariance Matrix:
[3.59395662 0.   1.43477724 0.   ]
[0.   3.59395662 0.   1.43477724]
[1.43477724 0.   11.42645575 0.   ]

Frame# 8 X: 95.58452496518302 Y: 76.00114777833844 Vx: -4.874076191640498 Vy: -0.009570778676433633
Covariance Matrix:
[3.57772462 0.   1.3577455 0.   ]
[0.   3.57772462 0.   1.3577455]
[1.3577455 0.   11.06088545 0.   ]

Frame# 9 X: 91.49344319929006 Y: 76.00012353808076 Vx: -4.091081765892966 Vy: -0.0010242402576778886
Covariance Matrix:
[3.57166684 0.   1.32982785 0.   ]
[0.   3.57166684 0.   1.32982785]
[1.32982785 0.   10.93222514 0.   ]

Frame# 10 X: 87.48452784702278 Y: 75.10784329147404 Vx: -4.00891535226728 Vy: -0.8922802466067168

Covariance Matrix:
[3.5694706 0.   1.31979359 0.   ]
[0.   3.5694706 0.   1.31979359]
[1.31979359 0.   10.88638041 0.   ]

Frame# 11 X: 82.59178589181464 Y: 75.01163639609484 Vx: -4.8927419552081375 Vy: -0.09620689537919702
Covariance Matrix:
[3.56868012 0.   1.31619138 0.   ]
[0.   3.56868012 0.   1.31619138]
[1.31619138 0.   10.86996516 0.   ]

Frame# 12 X: 78.49557526063506 Y: 75.0012558752005 Vx: -4.096210631179574 Vy: -0.010380520894344158
Covariance Matrix:
[3.56839611 0.   1.31489815 0.   ]
[0.   3.56839611 0.   1.31489815]
[1.31489815 0.   10.86407649 0.   ]

Frame# 13 X: 73.59316838975566 Y: 75.00013555368366 Vx: -4.902406870879403 Vy: -0.0011203215168364977
Covariance Matrix:
[3.5682941 0.   1.31443379 0.   ]
[0.   3.5682941 0.   1.31443379]
[1.31443379 0.   10.86196254 0.   ]

Frame# 14 X: 69.49578164051682 Y: 75.00001463151837 Vx: -4.0973867492388365 Vy: -0.00012092216529424604
Covariance Matrix:
[3.56825747 0.   1.31426704 0.   ]
[0.   3.56825747 0.   1.31426704]
[1.31426704 0.   10.86120347 0.   ]

Frame# 15 X: 64.59321523250806 Y: 75.00000157932759 Vx: -4.9025664080087665 Vy: -1.3052190780626916e-05
Covariance Matrix:
[3.56824432 0.   1.31420715 0.   ]
[0.   3.56824432 0.   1.31420715]
[1.31420715 0.   10.86093087 0.   ]

Frame# 16 X: 60.49579407055154 Y: 75.00000017047346 Vx: -4.097421161956518 Vy: -1.4088541320234071e-06
Covariance Matrix:
[3.56823959 0.   1.31418564 0.   ]
[0.   3.56823959 0.   1.31418564]
[1.31418564 0.   10.86083297 0.   ]

Frame# 17 X: 55.5932197428092 Y: 75.00000001840102 Vx: -4.902574327742343 Vy: -1.5207243109216506e-07
Covariance Matrix:
[3.56823789 0.   1.31417792 0.   ]
[0.   3.56823789 0.   1.31417792]
[1.31417792 0.   10.86079781 0.   ]

Frame# 18 X: 50.60373624203343 Y: 76.78411853485466 Vx: -4.989483500775769 Vy: 1.7841185164536313
Covariance Matrix:
[3.56823728 0.   1.31417515 0.   ]
[0.   3.56823728 0.   1.31417515]
[1.31417515 0.   10.86078519 0.   ]

Frame# 19 X: 47.38899000435431 Y: 76.08463834526228 Vx: -3.2147462376791154 Vy: -0.6994801895923786
Covariance Matrix:
[3.56823707 0.   1.31417415 0.   ]
[0.   3.56823707 0.   1.31417415]

[1.31417415 0.   10.86078065 0.   ]

Frame# 20 X: 43.47375091789682 Y: 76.90119516706365 Vx: -3.9152390864574897 Vy: 0.8165568218013703
Covariance Matrix:
[3.56823699 0.   1.31417379 0.   ]
[0.   3.56823699 0.   1.31417379]
[1.31417379 0.   10.86077902 0.   ]

Frame# 21 X: 40.374959323877604 Y: 76.98933493096388 Vx: -3.098791594019218 Vy: 0.08813976390023015
Covariance Matrix:
[3.56823696 0.   1.31417366 0.   ]
[0.   3.56823696 0.   1.31417366]
[1.31417366 0.   10.86077844 0.   ]

Frame# 22 X: 37.36429568693214 Y: 76.99884880430315 Vx: -3.0106636369454662 Vy: 0.009513873339273005
Covariance Matrix:
[3.56823695 0.   1.31417362 0.   ]
[0.   3.56823695 0.   1.31417362]
[1.31417362 0.   10.86077823 0.   ]

Frame# 23 X: 33.47108541095955 Y: 76.99987573905692 Vx: -3.893210275972585 Vy: 0.00110269347537672502
Covariance Matrix:
[3.56823695 0.   1.3141736 0.   ]
[0.   3.56823695 0.   1.3141736]
[1.3141736 0.   10.86077815 0.   ]

Frame# 24 X: 30.37467161172698 Y: 77.89204582301788 Vx: -3.0964137992325718 Vy: 0.8921700839609628
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077813 0.   ]

Frame# 25 X: 27.364264632704433 Y: 77.98834734363763 Vx: -3.010406979022548 Vy: 0.09630152061974684
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077812 0.   ]

Frame# 26 X: 24.363141295486635 Y: 77.99874220336707 Vx: -3.0011233372177983 Vy: 0.010394859729444761
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 27 X: 21.36302004162716 Y: 77.9998642324702 Vx: -3.0001212538594757 Vy: 0.0011220291031293073
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 28 X: 18.36300695339947 Y: 78.89204458092341 Vx: -3.000013088227689 Vy: 0.8921803484532091
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 29 X: 16.25506477642216 Y: 78.98834720956249 Vx: -2.1079421769773106 Vy: 0.09630262863908001

Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 30 X: 13.351354179574159 Y: 78.99874218889481 Vx: -2.9037105968480006 Vy: 0.010394979332318144
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Point 2:
Frame# 3 X: 111.08275862068966 Y: 89.0 Vx: -2.91724137931034 Vy: 0.0
Covariance Matrix:
[100 0   0   0]
[0 100   0   0]
[0 0 25   0]

Frame# 4 X: 110.08669243511872 Y: 86.24019878520154 Vx: -0.996066185570939 Vy: -2.7598012147984576
Covariance Matrix:
[3.88965517 0.   0.68965517 0.   ]
[0.   3.88965517 0.   0.68965517]
[0.68965517 0.   24.68965517 0.   ]

Frame# 5 X: 105.46730583878659 Y: 88.74646172576472 Vx: -4.619386596332134 Vy: 2.5062629405631753
Covariance Matrix:
[3.67973495 0.   2.0320265 0.   ]
[0.   3.67973495 0.   2.0320265]
[2.0320265 0.   15.79679735 0.   ]

Frame# 6 X: 101.45347998928918 Y: 88.97426311561514 Vx: -4.013825849497408 Vy: 0.22780138985042697
Covariance Matrix:
[3.63252676 0.   1.63790392 0.   ]
[0.   3.63252676 0.   1.63790392]
[1.63790392 0.   12.49632222 0.   ]

Frame# 7 X: 97.47014873445247 Y: 88.99728298686614 Vx: -3.9833312548367132 Vy: 0.023019871250994584
Covariance Matrix:
[3.59395662 0.   1.43477724 0.   ]
[0.   3.59395662 0.   1.43477724]
[1.43477724 0.   11.42645575 0.   ]

Frame# 8 X: 95.2645116507913 Y: 85.42804220927626 Vx: -2.205637083661159 Vy: -3.569240777589883
Covariance Matrix:
[3.57772462 0.   1.3577455 0.   ]
[0.   3.57772462 0.   1.3577455]
[1.3577455 0.   11.06088545 0.   ]

Frame# 9 X: 91.45899941397575 Y: 85.04607118922473 Vx: -3.8055122368155594 Vy: -0.38197102005152317
Covariance Matrix:
[3.57166684 0.   1.32982785 0.   ]
[0.   3.57166684 0.   1.32982785]
[1.32982785 0.   10.93222514 0.   ]

Frame# 10 X: 86.58864374503948 Y: 85.00496785496885 Vx: -4.87035566893627 Vy: -0.04110333425587953
Covariance Matrix:
[3.5694706 0.   1.31979359 0.   ]

```
[0.         3.5694706 0.         1.31979359]
[ 1.31979359 0.         10.88638041 0.         ]
```

Frame# 11 X: 82.4951191254051 Y: 84.1084370095086 Vx: -4.09352461963438 Vy: -0.8965308454602479
Covariance Matrix:
```
[3.56868012 0.         1.31619138 0.         ]
[0.         3.56868012 0.         1.31619138]
[ 1.31619138 0.         10.86996516 0.         ]
```

Frame# 12 X: 77.59306883137731 Y: 84.90377675009347 Vx: -4.902050294027788 Vy: 0.7953397405848648
Covariance Matrix:
```
[3.56839611 0.         1.31489815 0.         ]
[0.         3.56839611 0.         1.31489815]
[ 1.31489815 0.         10.86407649 0.         ]
```

Frame# 13 X: 73.49575578779903 Y: 84.09754971483906 Vx: -4.097313043578282 Vy: -0.806227035254409
Covariance Matrix:
```
[3.5682941 0.         1.31443379 0.         ]
[0.         3.5682941 0.         1.31443379]
[ 1.31443379 0.         10.86196254 0.         ]
```

Frame# 14 X: 68.59320595047893 Y: 84.90259048985449 Vx: -4.902549837320095 Vy: 0.8050407750154278
Covariance Matrix:
```
[3.56825747 0.         1.31426704 0.         ]
[0.         3.56825747 0.         1.31426704]
[ 1.31426704 0.         10.86120347 0.         ]
```

Frame# 15 X: 64.49579112018988 Y: 84.0974257098119 Vx: -4.097414830289054 Vy: -0.8051647800425883
Covariance Matrix:
```
[3.56824432 0.         1.31420715 0.         ]
[0.         3.56824432 0.         1.31420715]
[ 1.31420715 0.         10.86093087 0.         ]
```

Frame# 16 X: 59.59321858707939 Y: 84.90257565590646 Vx: -4.902572533110487 Vy: 0.8051499460945593
Covariance Matrix:
```
[3.56823959 0.         1.31418564 0.         ]
[0.         3.56823959 0.         1.31418564]
[ 1.31418564 0.         10.86083297 0.         ]
```

Frame# 17 X: 54.603735811260094 Y: 84.98948395016178 Vx: -4.9894827758192974 Vy: 0.08690829425532343
Covariance Matrix:
```
[3.56823789 0.         1.31417792 0.         ]
[0.         3.56823789 0.         1.31417792]
[ 1.31417792 0.         10.86079781 0.         ]
```

Frame# 18 X: 51.38898988704469 Y: 84.99886488986625 Vx: -3.214745924215407 Vy: 0.009380939704470848
Covariance Matrix:
```
[3.56823728 0.         1.31417515 0.         ]
[0.         3.56823728 0.         1.31417515]
[ 1.31417515 0.         10.86078519 0.         ]
```

Frame# 19 X: 46.58169162747716 Y: 84.99987747535718 Vx: -4.807298259567524 Vy: 0.0010125854009261989
Covariance Matrix:
```
[3.56823707 0.         1.31417415 0.         ]
[0.         3.56823707 0.         1.31417415]
[ 1.31417415 0.         10.86078065 0.         ]
```

Frame# 20 X: 43.38661051733024 Y: 84.9999867745969 Vx: -3.1950811101469228 Vy: 0.00010929923972469169
Covariance Matrix:
[3.56823699 0.   1.31417379 0.   ]
[0.   3.56823699 0.   1.31417379]
[1.31417379 0.   10.86077902 0.   ]

Frame# 21 X: 39.4734940853757 Y: 84.9999985724399 Vx: -3.913116431954542 Vy: 1.1797843001204456e-05
Covariance Matrix:
[3.56823696 0.   1.31417366 0.   ]
[0.   3.56823696 0.   1.31417366]
[1.31417366 0.   10.86077844 0.   ]

Frame# 22 X: 37.26699084062419 Y: 84.99999984590808 Vx: -2.2065032447515094 Vy: 1.2734681718029606e-06
Covariance Matrix:
[3.56823695 0.   1.31417362 0.   ]
[0.   3.56823695 0.   1.31417362]
[1.31417362 0.   10.86077823 0.   ]

Frame# 23 X: 33.46058225150393 Y: 85.89205921932363 Vx: -3.8064085891202595 Vy: 0.8920593734155489
Covariance Matrix:
[3.56823695 0.   1.3141736 0.   ]
[0.   3.56823695 0.   1.3141736]
[1.3141736 0.   10.86077815 0.   ]

Frame# 24 X: 31.26559712850826 Y: 85.98834878964966 Vx: -2.194985122995668 Vy: 0.0962895703260358
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077813 0.   ]

Frame# 25 X: 27.46043181383388 Y: 85.99874235945089 Vx: -3.805165314674383 Vy: 0.010393569801223634
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077812 0.   ]

Frame# 26 X: 24.373521654510462 Y: 85.99986424931802 Vx: -3.0869101593234163 Vy: 0.0011218898671359057
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 27 X: 21.364140505513106 Y: 85.99998534696765 Vx: -3.009381148997356 Vy: 0.00012109764962531244
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 28 X: 18.363127897127605 Y: 86.89205765411482 Vx: -3.0010126083855013 Vy: 0.8920723071471741
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 29 X: 16.255077831180603 Y: 87.8804078564664 Vx: -2.108050065947001 Vy: 0.9883502023515831
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]

[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 30 X: 13.351355588714764 Y: 87.98709113263149 Vx: -2.9037222424658395 Vy: 0.10668327616508577
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Point 3:
Frame# 3 X: 118.0551724137931 Y: 155.08275862068965 Vx: -1.9448275862068982 Vy: -2.9172413793103544
Covariance Matrix:
[100 0   0   0]
[0 100   0   0]
[0 0 25   0]

Frame# 4 X: 115.24461623412479 Y: 155.00662617338486 Vx: -2.8105561796683105 Vy: -0.07613244730478641
Covariance Matrix:
[3.88965517 0.   0.68965517 0.   ]
[0.   3.88965517 0.   0.68965517]
[0.68965517 0.   24.68965517 0.   ]

Frame# 5 X: 112.29807741084564 Y: 156.81687211484808 Vx: -2.9465388232791554 Vy: 1.8102459414632222
Covariance Matrix:
[3.67973495 0.   2.0320265 0.   ]
[0.   3.67973495 0.   2.0320265]
[2.0320265 0.   15.79679735 0.   ]

Frame# 6 X: 109.33479062390575 Y: 155.18443222311478 Vx: -2.9632867869398893 Vy: -1.6324398917332985
Covariance Matrix:
[3.63252676 0.   1.63790392 0.   ]
[0.   3.63252676 0.   1.63790392]
[1.63790392 0.   12.49632222 0.   ]

Frame# 7 X: 105.45761883534836 Y: 155.0194702965991 Vx: -3.8771717885573906 Vy: -0.16496192651567299
Covariance Matrix:
[3.59395662 0.   1.43477724 0.   ]
[0.   3.59395662 0.   1.43477724]
[1.43477724 0.   11.42645575 0.   ]

Frame# 8 X: 101.47733648597391 Y: 155.00208494339753 Vx: -3.9802823493744484 Vy: -0.017385353201575526
Covariance Matrix:
[3.57772462 0.   1.3577455 0.   ]
[0.   3.57772462 0.   1.3577455]
[1.3577455 0.   11.06088545 0.   ]

Frame# 9 X: 97.48190625129443 Y: 155.00022440735916 Vx: -3.995430234679475 Vy: -0.0018605360383787684
Covariance Matrix:
[3.57166684 0.   1.32982785 0.   ]
[0.   3.57166684 0.   1.32982785]
[1.32982785 0.   10.93222514 0.   ]

Frame# 10 X: 92.59111378862875 Y: 154.10785416820534 Vx: -4.8907924626656865 Vy: -0.89237023915382
Covariance Matrix:
[3.5694706 0.   1.31979359 0.   ]
[0.   3.5694706 0.   1.31979359]
[1.31979359 0.   10.88638041 0.   ]

Frame# 11 X: 88.49538564551204 Y: 154.9037365965816 Vx: -4.095728143116702 Vy: 0.7958824283762738
Covariance Matrix:
[3.56668012 0.   1.31619138 0.   ]
[0.   3.56668012 0.   1.31619138]
[1.31619138 0.   10.86996516 0.   ]

Frame# 12 X: 84.48517112195955 Y: 155.88168415630133 Vx: -4.0102145235524915 Vy: 0.9779475597197234
Covariance Matrix:
[3.56839611 0.   1.31489815 0.   ]
[0.   3.56839611 0.   1.31489815]
[1.31489815 0.   10.86407649 0.   ]

Frame# 13 X: 80.48410978032605 Y: 154.20310076890712 Vx: -4.001061341633502 Vy: -1.6785833873942124
Covariance Matrix:
[3.5682941 0.   1.31443379 0.   ]
[0.   3.5682941 0.   1.31443379]
[1.31443379 0.   10.86196254 0.   ]

Frame# 14 X: 76.48400997187605 Y: 154.91398355675378 Vx: -4.000099808450003 Vy: 0.710882787846657
Covariance Matrix:
[3.56825747 0.   1.31426704 0.   ]
[0.   3.56825747 0.   1.31426704]
[1.31426704 0.   10.86120347 0.   ]

Frame# 15 X: 71.59194459739071 Y: 154.99071537631633 Vx: -4.892065374485341 Vy: 0.07673181956255348
Covariance Matrix:
[3.56824432 0.   1.31420715 0.   ]
[0.   3.56824432 0.   1.31420715]
[1.31420715 0.   10.86093087 0.   ]

Frame# 16 X: 67.49565691752798 Y: 155.89105728632535 Vx: -4.096287679862726 Vy: 0.9003419100090184
Covariance Matrix:
[3.56823959 0.   1.31418564 0.   ]
[0.   3.56823959 0.   1.31418564]
[1.31418564 0.   10.86083297 0.   ]

Frame# 17 X: 63.485264259566165 Y: 155.09618132838247 Vx: -4.010392657961816 Vy: -0.7948759579428781
Covariance Matrix:
[3.56823789 0.   1.31417792 0.   ]
[0.   3.56823789 0.   1.31417792]
[1.31417792 0.   10.86079781 0.   ]

Frame# 18 X: 58.592083447979725 Y: 155.01038188314092 Vx: -4.89318081158644 Vy: -0.08579944524154826
Covariance Matrix:
[3.56823728 0.   1.31417515 0.   ]
[0.   3.56823728 0.   1.31417515]
[1.31417515 0.   10.86078519 0.   ]

Frame# 19 X: 55.387732192987094 Y: 155.00112062828603 Vx: -3.2043512549926305 Vy: -0.009261254854891376
Covariance Matrix:
[3.56823707 0.   1.31417415 0.   ]
[0.   3.56823707 0.   1.31417415]
[1.31417415 0.   10.86078065 0.   ]

Frame# 20 X: 51.47361514878157 Y: 155.00012096146918 Vx: -3.9141170442055255 Vy: -0.00099966668168525957
Covariance Matrix:

[3.56823699 0.   1.31417379 0.   ]
[0.   3.56823699 0.   1.31417379]
[1.31417379 0.   10.86077902 0.   ]

Frame# 21 X: 48.37494466885574 Y: 155.00001305667325 Vx: -3.098670479925829 Vy: -0.00010790479592515112
Covariance Matrix:
[3.56823696 0.   1.31417366 0.   ]
[0.   3.56823696 0.   1.31417366]
[1.31417366 0.   10.86077844 0.   ]

Frame# 22 X: 45.36429410505789 Y: 156.78411988191388 Vx: -3.010650563797853 Vy: 1.7841068252406274
Covariance Matrix:
[3.56823695 0.   1.31417362 0.   ]
[0.   3.56823695 0.   1.31417362]
[1.31417362 0.   10.86077823 0.   ]

Frame# 23 X: 42.36314447616726 Y: 156.97669773511197 Vx: -3.00114962889063 Vy: 0.19257785319808818
Covariance Matrix:
[3.56823695 0.   1.3141736 0.   ]
[0.   3.56823695 0.   1.3141736]
[1.3141736 0.   10.86077815 0.   ]

Frame# 24 X: 39.363020384741255 Y: 158.78160320739937 Vx: -3.0001240914260023 Vy: 1.8049054722874018
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077813 0.   ]

Frame# 25 X: 35.47094775456282 Y: 159.86848531910405 Vx: -3.8920726301784327 Vy: 1.086882111704682
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077812 0.   ]

Frame# 26 X: 33.266715988970944 Y: 159.98580420483816 Vx: -2.204231765591878 Vy: 0.11731888573410743
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 27 X: 29.460552584575062 Y: 158.2143492234689 Vx: -3.806163404395882 Vy: -1.77145498136926
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 28 X: 27.26559392639647 Y: 158.91519625476676 Vx: -2.1949586581785923 Vy: 0.7008470312978545
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 29 X: 24.35249070406822 Y: 159.8829054547039 Vx: -2.9131032223282496 Vy: 0.9677091999371328
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 30 X: 22.253929644433143 Y: 159.98736072529397 Vx: -2.0985610596350774 Vy: 0.10445527059007986
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Point 4:
Frame# 3 X: 226.08275862068965 Y: 91.97241379310344 Vx: -2.9172413793103544 Vy: 0.972413793103442
Covariance Matrix:
[100 0   0   0]
[0 100   0   0]
[0 0 25 0]

Frame# 4 X: 221.4069574820541 Y: 90.15792379900607 Vx: -4.675801138635535 Vy: -1.8144899940973716
Covariance Matrix:
[3.88965517 0.   0.68965517 0.   ]
[0.   3.88965517 0.   0.68965517]
[0.68965517 0.   24.68965517 0.   ]

Frame# 5 X: 221.03738649622076 Y: 90.01450819256328 Vx: -0.3695709858333487 Vy: -0.14341560644278672
Covariance Matrix:
[3.67973495 0.   2.0320265 0.   ]
[0.   3.67973495 0.   2.0320265]
[2.0320265 0.   15.79679735 0.   ]

Frame# 6 X: 216.5113493583802 Y: 90.89996189416878 Vx: -4.526037137840575 Vy: 0.8854537016054991
Covariance Matrix:
[3.63252676 0.   1.63790392 0.   ]
[0.   3.63252676 0.   1.63790392]
[1.63790392 0.   12.49632222 0.   ]

Frame# 7 X: 211.58182678089938 Y: 90.98943909280682 Vx: -4.92952257748081 Vy: 0.08947719863803627
Covariance Matrix:
[3.59395662 0.   1.43477724 0.   ]
[0.   3.59395662 0.   1.43477724]
[1.43477724 0.   11.42645575 0.   ]

Frame# 8 X: 206.5977203702975 Y: 89.21303568131242 Vx: -4.984106410601868 Vy: -1.7764034114943996
Covariance Matrix:
[3.57772462 0.   1.3577455 0.   ]
[0.   3.57772462 0.   1.3577455]
[1.3577455 0.   11.06088545 0.   ]

Frame# 9 X: 201.60249580255922 Y: 88.13056188198026 Vx: -4.9952245677382905 Vy: -1.0824737993321634
Covariance Matrix:
[3.57166684 0.   1.32982785 0.   ]
[0.   3.57166684 0.   1.32982785]
[1.32982785 0.   10.93222514 0.   ]

Frame# 10 X: 196.60411695636813 Y: 88.01407848386492 Vx: -4.998378846191088 Vy: -0.11648339811533504
Covariance Matrix:
[3.5694706 0.   1.31979359 0.   ]
[0.   3.5694706 0.   1.31979359]
[1.31979359 0.   10.88638041 0.   ]

Frame# 11 X: 191.60468967308793 Y: 88.00151908210913 Vx: -4.999427283280198 Vy: -0.012559401755794397

Covariance Matrix:
[3.56868012 0.   1.31619138 0.   ]
[0.   3.56868012 0.   1.31619138]
[1.31619138 0.   10.86996516 0.   ]

Frame# 12 X: 186.60489439424222 Y: 88.00016394917574 Vx: -4.99979527884571 Vy: -0.0013551329333836293
Covariance Matrix:
[3.56839611 0.   1.31489815 0.   ]
[0.   3.56839611 0.   1.31489815]
[1.31489815 0.   10.86407649 0.   ]

Frame# 13 X: 181.60496781954706 Y: 87.10795332810267 Vx: -4.999926574695166 Vy: -0.892210621073076
Covariance Matrix:
[3.5682941 0.   1.31443379 0.   ]
[0.   3.5682941 0.   1.31443379]
[1.31443379 0.   10.86196254 0.   ]

Frame# 14 X: 176.60499417935924 Y: 86.11959128688771 Vx: -4.999973640187818 Vy: -0.9883620412149554
Covariance Matrix:
[3.56825747 0.   1.31426704 0.   ]
[0.   3.56825747 0.   1.31426704]
[1.31426704 0.   10.86120347 0.   ]

Frame# 15 X: 171.6050036451234 Y: 86.9049685934389 Vx: -4.999990534235849 Vy: 0.7853773065511831
Covariance Matrix:
[3.56824432 0.   1.31420715 0.   ]
[0.   3.56824432 0.   1.31420715]
[1.31420715 0.   10.86093087 0.   ]

Frame# 16 X: 175.5256017794634 Y: 85.20562331295288 Vx: 3.920598134339997 Vy: -1.699345280486014
Covariance Matrix:
[3.56823959 0.   1.31418564 0.   ]
[0.   3.56823959 0.   1.31418564]
[1.31418564 0.   10.86083297 0.   ]

Frame# 17 X: 176.84085245517628 Y: 87.69837308343037 Vx: 1.315250675712889 Vy: 2.492749770477488
Covariance Matrix:
[3.56823789 0.   1.31417792 0.   ]
[0.   3.56823789 0.   1.31417792]
[1.31417792 0.   10.86079781 0.   ]

Frame# 18 X: 177.87488076370076 Y: 89.75156070223073 Vx: 1.0340283085244835 Vy: 2.053187618800365
Covariance Matrix:
[3.56823728 0.   1.31417515 0.   ]
[0.   3.56823728 0.   1.31417515]
[1.31417515 0.   10.86078519 0.   ]

Frame# 19 X: 172.63413905468386 Y: 91.75730176863993 Vx: -5.240741709016902 Vy: 2.0057410664092004
Covariance Matrix:
[3.56823707 0.   1.31417415 0.   ]
[0.   3.56823707 0.   1.31417415]
[1.31417415 0.   10.86078065 0.   ]

Frame# 20 X: 167.60815325304154 Y: 91.08174372866243 Vx: -5.025985801642321 Vy: -0.6755580399775027
Covariance Matrix:
[3.56823699 0.   1.31417379 0.   ]
[0.   3.56823699 0.   1.31417379]

[1.31417379 0.   10.86077902 0.   ]

Frame# 21 X: 162.60534834006913 Y: 93.68500119200694 Vx: -5.002804912972408 Vy: 2.603257463344505
Covariance Matrix:
[3.56823696 0.   1.31417366 0.   ]
[0.   3.56823696 0.   1.31417366]
[1.31417366 0.   10.86077844 0.   ]

Frame# 22 X: 157.60504558072518 Y: 95.75011726066198 Vx: -5.000302759343953 Vy: 2.065116068655044
Covariance Matrix:
[3.56823695 0.   1.31417362 0.   ]
[0.   3.56823695 0.   1.31417362]
[1.31417362 0.   10.86077823 0.   ]

Frame# 23 X: 152.60501290248257 Y: 95.97302746619455 Vx: -5.000032678242604 Vy: 0.22291020553257113
Covariance Matrix:
[3.56823695 0.   1.3141736 0.   ]
[0.   3.56823695 0.   1.3141736]
[1.3141736 0.   10.86077815 0.   ]

Frame# 24 X: 147.60500937582609 Y: 96.88914779992871 Vx: -5.000003526656485 Vy: 0.9161203337341561
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077813 0.   ]

Frame# 25 X: 147.06530517437673 Y: 96.9880345288107 Vx: -0.5397042014493536 Vy: 0.09888672888199324
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077812 0.   ]

Frame# 26 X: 142.54675291152103 Y: 95.21458996633214 Vx: -4.518552262855707 Vy: -1.7734445624785593
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 27 X: 137.59872074822968 Y: 95.91522224073692 Vx: -4.948032163291344 Vy: 0.7006322744047822
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 28 X: 132.60433019625 Y: 97.77496749542448 Vx: -4.994390551979677 Vy: 1.859745254687553
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 29 X: 127.60493568436014 Y: 99.7598282910276 Vx: -4.999394511889861 Vy: 1.984860795603126
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 30 X: 125.28117874853103 Y: 101.75819415373466 Vx: -2.3237569358291097 Vy: 1.998365862707061

Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Point 5:
Frame# 3 X: 250.02758620689656 Y: 77.08275862068966 Vx: -0.972413793103442 Vy: -2.91724137931034
Covariance Matrix:
 [100   0   0   0]
 [0 100   0   0]
 [0   0 25   0]

Frame# 4 X: 246.32247377139703 Y: 76.08669243511872 Vx: -3.7051124354995295 Vy: -0.996066185570939
Covariance Matrix:
[3.88965517 0.   0.68965517 0.   ]
[0.   3.88965517 0.   0.68965517]
[0.68965517 0.   24.68965517 0.   ]

Frame# 5 X: 242.39709836148734 Y: 76.007964287526 Vx: -3.925375409909691 Vy: -0.07872814759272728
Covariance Matrix:
[3.67973495 0.   2.0320265 0.   ]
[0.   3.67973495 0.   2.0320265]
[2.0320265 0.   15.79679735 0.   ]

Frame# 6 X: 238.44635316896333 Y: 76.0008084615543 Vx: -3.9507451925240105 Vy: -0.00715582597169373
Covariance Matrix:
[3.63252676 0.   1.63790392 0.   ]
[0.   3.63252676 0.   1.63790392]
[1.63790392 0.   12.49632222 0.   ]

Frame# 7 X: 233.5749652083152 Y: 75.10565419239802 Vx: -4.871387960648121 Vy: -0.8951542691562793
Covariance Matrix:
[3.59395662 0.   1.43477724 0.   ]
[0.   3.59395662 0.   1.43477724]
[1.43477724 0.   11.42645575 0.   ]

Frame# 8 X: 228.5969856105375 Y: 75.01131379841806 Vx: -4.977979597777704 Vy: -0.0943403939799623
Covariance Matrix:
[3.57772462 0.   1.3577455 0.   ]
[0.   3.57772462 0.   1.3577455]
[1.3577455 0.   11.06088545 0.   ]

Frame# 9 X: 223.60241671863898 Y: 77.67832067832926 Vx: -4.994568891898524 Vy: 2.667006879911199
Covariance Matrix:
[3.57166684 0.   1.32982785 0.   ]
[0.   3.57166684 0.   1.32982785]
[1.32982785 0.   10.93222514 0.   ]

Frame# 10 X: 218.60410842875135 Y: 77.07314329864319 Vx: -4.998308289887632 Vy: -0.6051773796860687
Covariance Matrix:
[3.5694706 0.   1.31979359 0.   ]
[0.   3.5694706 0.   1.31979359]
[1.31979359 0.   10.88638041 0.   ]

Frame# 11 X: 213.60468875294978 Y: 77.00789223310103 Vx: -4.999419675801562 Vy: -0.06525106554215654
Covariance Matrix:
[3.56868012 0.   1.31619138 0.   ]

```
[0.        3.56868012 0.        1.31619138]
[ 1.31619138 0.        10.86996516 0.        ]
```

Frame# 12 X: 208.60489429493495 Y: 76.10877825488367 Vx: -4.999794458014833 Vy: -0.8991139782173576
Covariance Matrix:
```
[3.56839611 0.        1.31489815 0.        ]
[0.        3.56839611 0.        1.31489815]
[ 1.31489815 0.        10.86407649 0.        ]
```

Frame# 13 X: 203.60496780882826 Y: 76.01174104970447 Vx: -4.999926486106688 Vy: -0.09703720517920544
Covariance Matrix:
```
[3.5682941 0.        1.31443379 0.        ]
[0.        3.5682941 0.        1.31443379]
[ 1.31443379 0.        10.86196254 0.        ]
```

Frame# 14 X: 207.5256049669933 Y: 73.32508407960061 Vx: 3.92063715816505 Vy: -2.6866569701038543
Covariance Matrix:
```
[3.56825747 0.        1.31426704 0.        ]
[0.        3.56825747 0.        1.31426704]
[ 1.31426704 0.        10.86120347 0.        ]
```

Frame# 15 X: 204.38055416081448 Y: 73.03508960880892 Vx: -3.14505080617883 Vy: -0.28999447079169727
Covariance Matrix:
```
[3.56824432 0.        1.31420715 0.        ]
[0.        3.56824432 0.        1.31420715]
[ 1.31420715 0.        10.86093087 0.        ]
```

Frame# 16 X: 203.14901774298815 Y: 72.11172811735555 Vx: -1.2315364178263337 Vy: -0.9233614914533632
Covariance Matrix:
```
[3.56823959 0.        1.31418564 0.        ]
[0.        3.56823959 0.        1.31418564]
[ 1.31418564 0.        10.86083297 0.        ]
```

Frame# 17 X: 198.555788470602 Y: 72.01206000883428 Vx: -4.593229272386139 Vy: -0.09966810852127139
Covariance Matrix:
```
[3.56823789 0.        1.31417792 0.        ]
[0.        3.56823789 0.        1.31417792]
[ 1.31417792 0.        10.86079781 0.        ]
```

Frame# 18 X: 193.59969588305307 Y: 72.00130176620038 Vx: -4.9560925875489374 Vy: -0.0107582426338897908
Covariance Matrix:
```
[3.56823728 0.        1.31417515 0.        ]
[0.        3.56823728 0.        1.31417515]
[ 1.31417515 0.        10.86078519 0.        ]
```

Frame# 19 X: 188.60443539139328 Y: 72.00014051362417 Vx: -4.995260491659792 Vy: -0.0011612525762103587
Covariance Matrix:
```
[3.56823707 0.        1.31417415 0.        ]
[0.        3.56823707 0.        1.31417415]
[ 1.31417415 0.        10.86078065 0.        ]
```

Frame# 20 X: 183.6049470170431 Y: 72.00001516714742 Vx: -4.999488374350193 Vy: -0.00012534647675011001
Covariance Matrix:
```
[3.56823699 0.        1.31417379 0.        ]
[0.        3.56823699 0.        1.31417379]
[ 1.31417379 0.        10.86077902 0.        ]
```

Frame# 21 X: 178.60500225650972 Y: 72.00000163715346 Vx: -4.999944760533367 Vy: -1.352999396431187e-05
Covariance Matrix:
[3.56823696 0.   1.31417366 0.   ]
[0.   3.56823696 0.   1.31417366]
[1.31417366 0.   10.86077844 0.   ]

Frame# 22 X: 173.60500822420147 Y: 70.21588170414898 Vx: -4.999994032308251 Vy: -1.784119933004476
Covariance Matrix:
[3.56823695 0.   1.31417362 0.   ]
[0.   3.56823695 0.   1.31417362]
[1.31417362 0.   10.86077823 0.   ]

Frame# 23 X: 168.60500887019086 Y: 70.9153616720453 Vx: -4.999999354010612 Vy: 0.6994799678963091
Covariance Matrix:
[3.56823695 0.   1.3141736 0.   ]
[0.   3.56823695 0.   1.3141736]
[1.3141736 0.   10.86077815 0.   ]

Frame# 24 X: 163.60500894057745 Y: 72.77498254588131 Vx: -4.999999929613409 Vy: 1.8596208738360218
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077813 0.   ]

Frame# 25 X: 158.60500894841132 Y: 72.08365220824652 Vx: -4.999999992166124 Vy: -0.6913303376347955
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077812 0.   ]

Frame# 26 X: 154.4970681851235 Y: 72.00902948328668 Vx: -4.107940763287814 Vy: -0.07462272495983768
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 27 X: 150.48541697666838 Y: 72.00097464932651 Vx: -4.011651208455135 Vy: -0.008054833960173369
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 28 X: 146.48415933633237 Y: 73.78422367594182 Vx: -4.001257640336007 Vy: 1.7832490266153087
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 29 X: 142.4840235856765 Y: 73.08464970290557 Vx: -4.000135750655858 Vy: -0.6995739730362516
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 30 X: 140.26812740419487 Y: 73.00913715362319 Vx: -2.2158961814816394 Vy: -0.07551254928237938
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]

[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Point 6:
Frame# 3 X: 266.11034482758623 Y: 77.02758620689656 Vx: -3.889655172413768 Vy: -0.972413793103442
Covariance Matrix:
[100 0   0   0]
[0 100   0   0]
[0 0 25   0]

Frame# 4 X: 263.249033683048 Y: 76.08227498619547 Vx: -2.861311144538206 Vy: -0.9453112207010861
Covariance Matrix:
[3.88965517 0.   0.68965517 0.   ]
[0.   3.88965517 0.   0.68965517]
[0.68965517 0.   24.68965517 0.   ]

Frame# 5 X: 259.390351544666 Y: 76.00755846395779 Vx: -3.8586821383820507 Vy: -0.07471652223767933
Covariance Matrix:
[3.67973495 0.   2.0320265 0.   ]
[0.   3.67973495 0.   2.0320265]
[2.0320265 0.   15.79679735 0.   ]

Frame# 6 X: 255.44566829388864 Y: 75.10227811077468 Vx: -3.944683250777331 Vy: -0.905280353183116
Covariance Matrix:
[3.63252676 0.   1.63790392 0.   ]
[0.   3.63252676 0.   1.63790392]
[1.63790392 0.   12.49632222 0.   ]

Frame# 7 X: 251.469324062799 Y: 75.01079738192571 Vx: -3.976344231089655 Vy: -0.09148072884896408
Covariance Matrix:
[3.59395662 0.   1.43477724 0.   ]
[0.   3.59395662 0.   1.43477724]
[1.43477724 0.   11.42645575 0.   ]

Frame# 8 X: 246.58567320922248 Y: 74.10823950816385 Vx: -4.883650853576512 Vy: -0.9025578737618645
Covariance Matrix:
[3.57772462 0.   1.3577455 0.   ]
[0.   3.57772462 0.   1.3577455]
[1.3577455 0.   11.06088545 0.   ]

Frame# 9 X: 241.6011991382923 Y: 73.11928242350976 Vx: -4.98447407093019 Vy: -0.9889570846540892
Covariance Matrix:
[3.57166684 0.   1.32982785 0.   ]
[0.   3.57166684 0.   1.32982785]
[1.32982785 0.   10.93222514 0.   ]

Frame# 10 X: 236.60397713709864 Y: 73.01286222019229 Vx: -4.997222001193649 Vy: -0.10642020331746949
Covariance Matrix:
[3.5694706 0.   1.31979359 0.   ]
[0.   3.5694706 0.   1.31979359]
[1.31979359 0.   10.88638041 0.   ]

Frame# 11 X: 231.6046745864527 Y: 72.1092888191984 Vx: -4.99930255064595 Vy: -0.903573400993892
Covariance Matrix:
[3.56868012 0.   1.31619138 0.   ]
[0.   3.56868012 0.   1.31619138]
[1.31619138 0.   10.86996516 0.   ]

Frame# 12 X: 227.49696629200173 Y: 72.9038686829098 Vx: -4.107708294450958 Vy: 0.7945798637114052
Covariance Matrix:
[3.56839611 0.   1.31489815 0.   ]
[0.   3.56839611 0.   1.31489815]
[1.31489815 0.   10.86407649 0.   ]

Frame# 13 X: 222.59331853160555 Y: 72.98962400552097 Vx: -4.903647760396183 Vy: 0.08575532261116336
Covariance Matrix:
[3.5682941 0.   1.31443379 0.   ]
[0.   3.5682941 0.   1.31443379]
[1.31443379 0.   10.86196254 0.   ]

Frame# 14 X: 217.603736767787 Y: 72.99888002635038 Vx: -4.989581763818535 Vy: 0.009256020829411682
Covariance Matrix:
[3.56825747 0.   1.31426704 0.   ]
[0.   3.56825747 0.   1.31426704]
[1.31426704 0.   10.86120347 0.   ]

Frame# 15 X: 212.60486791998966 Y: 72.99987910992968 Vx: -4.998868847797354 Vy: 0.000999083579301896
Covariance Matrix:
[3.56824432 0.   1.31420715 0.   ]
[0.   3.56824432 0.   1.31420715]
[1.31420715 0.   10.86093087 0.   ]

Frame# 16 X: 207.60499239427935 Y: 72.99998695106216 Vx: -4.9998755257103085 Vy: 0.00010784113247552796
Covariance Matrix:
[3.56823959 0.   1.31418564 0.   ]
[0.   3.56823959 0.   1.31418564]
[1.31418564 0.   10.86083297 0.   ]

Frame# 17 X: 202.60500668400195 Y: 74.78411723378368 Vx: -4.9999857102774 Vy: 1.7841302827215202
Covariance Matrix:
[3.56823789 0.   1.31417792 0.   ]
[0.   3.56823789 0.   1.31417792]
[1.31417792 0.   10.86079781 0.   ]

Frame# 18 X: 199.38912706598072 Y: 77.65287525515306 Vx: -3.2158796180212335 Vy: 2.8687580213693877
Covariance Matrix:
[3.56823728 0.   1.31417515 0.   ]
[0.   3.56823728 0.   1.31417515]
[1.31417515 0.   10.86078519 0.   ]

Frame# 19 X: 197.25788417502898 Y: 74.3942941064427 Vx: -2.1312428909517394 Vy: -3.2585811487103626
Covariance Matrix:
[3.56823707 0.   1.31417415 0.   ]
[0.   3.56823707 0.   1.31417415]
[1.31417415 0.   10.86078065 0.   ]

Frame# 20 X: 196.1357769741869 Y: 77.61079736453169 Vx: -1.1221072008420663 Vy: 3.2165032580889914
Covariance Matrix:
[3.56823699 0.   1.31417379 0.   ]
[0.   3.56823699 0.   1.31417379]
[1.31417379 0.   10.86077902 0.   ]

Frame# 21 X: 196.01465587016529 Y: 77.95798917064124 Vx: -0.12112110402162557 Vy: 0.34719180610954936
Covariance Matrix:

[3.56823696 0.   1.31417366 0.   ]
[0.   3.56823696 0.   1.31417366]
[1.31417366 0.   10.86077844 0.   ]

Frame# 22 X: 194.21746349325198 Y: 77.99546531899465 Vx: -1.7971923769133014 Vy: 0.03747614835340585
Covariance Matrix:
[3.56823695 0.   1.31417362 0.   ]
[0.   3.56823695 0.   1.31417362]
[1.31417362 0.   10.86077823 0.   ]

Frame# 23 X: 189.5631769958311 Y: 78.89156975902401 Vx: -4.654286497420884 Vy: 0.8961044400293616
Covariance Matrix:
[3.56823695 0.   1.3141736 0.   ]
[0.   3.56823695 0.   1.3141736]
[1.3141736 0.   10.86077815 0.   ]

Frame# 24 X: 185.49255281193177 Y: 79.88035519276993 Vx: -4.0706241838993265 Vy: 0.9887854337459174
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077813 0.   ]

Frame# 25 X: 187.7293442343209 Y: 83.55532239126211 Vx: 2.236791422389132 Vy: 3.6749671984921832
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077812 0.   ]

Frame# 26 X: 184.4025482666856 Y: 83.05994192330036 Vx: -3.3267959676352916 Vy: -0.49538046796175195
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 27 X: 179.58315518866155 Y: 80.33029246968123 Vx: -4.8193930780240635 Vy: -2.729649453619132
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 28 X: 177.27882774517764 Y: 83.60388896469271 Vx: -2.3043274434839134 Vy: 3.273596495011489
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 29 X: 173.46185993680746 Y: 81.28106576480954 Vx: -3.8169678083701797 Vy: -2.3228231998831745
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 30 X: 173.04985351454465 Y: 80.1382792176781 Vx: -0.4120064222628059 Vy: -1.1427865471314362
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Point 7:
Frame# 3 X: 270.0 Y: 161.05517241379312 Vx: 0.0 Vy: -1.944827586206884
Covariance Matrix:
[100 0 0 0]
[0 100 0 0]
[0 0 25 0]

Frame# 4 X: 266.3202650469354 Y: 159.16454997239094 Vx: -3.67973495306461 Vy: -1.8906224414021722
Covariance Matrix:
[3.88965517 0.   0.68965517 0.   ]
[0.   3.88965517 0.   0.68965517]
[0.68965517 0.   24.68965517 0.   ]

Frame# 5 X: 263.3050271394511 Y: 160.83138030741137 Vx: -3.015237907484277 Vy: 1.6668303350204212
Covariance Matrix:
[3.67973495 0.   2.0320265 0.   ]
[0.   3.67973495 0.   2.0320265]
[2.0320265 0.   15.79679735 0.   ]

Frame# 6 X: 259.4370069414406 Y: 160.98288327256998 Vx: -3.8680201980105267 Vy: 0.15150296515861328
Covariance Matrix:
[3.63252676 0.   1.63790392 0.   ]
[0.   3.63252676 0.   1.63790392]
[1.63790392 0.   12.49632222 0.   ]

Frame# 7 X: 254.57397853787944 Y: 158.31489953901018 Vx: -4.863028403561145 Vy: -2.667983733559794
Covariance Matrix:
[3.59395662 0.   1.43477724 0.   ]
[0.   3.59395662 0.   1.43477724]
[1.43477724 0.   11.42645575 0.   ]

Frame# 8 X: 249.5968799546221 Y: 158.03372047834014 Vx: -4.977098583257344 Vy: -0.2811790606700413
Covariance Matrix:
[3.57772462 0.   1.3577455 0.   ]
[0.   3.57772462 0.   1.3577455]
[1.3577455 0.   11.06088545 0.   ]

Frame# 9 X: 245.49477299584746 Y: 158.89599706355676 Vx: -4.102106958774641 Vy: 0.8622765852166197
Covariance Matrix:
[3.57166684 0.   1.32982785 0.   ]
[0.   3.57166684 0.   1.32982785]
[1.32982785 0.   10.93222514 0.   ]

Frame# 10 X: 240.59250120931262 Y: 158.09661533681177 Vx: -4.90227178653484 Vy: -0.799381726744997
Covariance Matrix:
[3.5694706 0.   1.31979359 0.   ]
[0.   3.5694706 0.   1.31979359]
[1.31979359 0.   10.88638041 0.   ]

Frame# 11 X: 235.60343632267708 Y: 157.11832586198372 Vx: -4.989064886635532 Vy: -0.9782894748280455
Covariance Matrix:
[3.56868012 0.   1.31619138 0.   ]
[0.   3.56868012 0.   1.31619138]
[1.31619138 0.   10.86996516 0.   ]

Frame# 12 X: 230.6047591245517 Y: 157.90484401907295 Vx: -4.998677198125392 Vy: 0.7865181570892332

Covariance Matrix:
[3.56839611 0. 1.31489815 0.]
[0. 3.56839611 0. 1.31489815]
[1.31489815 0. 10.86407649 0.]

Frame# 13 X: 225.6049532191275 Y: 158.88179364690154 Vx: -4.999805905424182 Vy: 0.9769496278285885
Covariance Matrix:
[3.5682941 0. 1.31443379 0.]
[0. 3.5682941 0. 1.31443379]
[1.31443379 0. 10.86196254 0.]

Frame# 14 X: 220.6049926034057 Y: 159.879302012656 Vx: -4.999960615721818 Vy: 0.9975083657544701
Covariance Matrix:
[3.56825747 0. 1.31426704 0.]
[0. 3.56825747 0. 1.31426704]
[1.31426704 0. 10.86120347 0.]

Frame# 15 X: 215.6050034750148 Y: 161.77109164229398 Vx: -4.999989128390894 Vy: 1.8917896296379695
Covariance Matrix:
[3.56824432 0. 1.31420715 0.]
[0. 3.56824432 0. 1.31420715]
[1.31420715 0. 10.86093087 0.]

Frame# 16 X: 210.60500702616014 Y: 161.08323203785346 Vx: -4.9999964488546595 Vy: -0.6878596044405185
Covariance Matrix:
[3.56823959 0. 1.31418564 0.]
[0. 3.56823959 0. 1.31418564]
[1.31418564 0. 10.86083297 0.]

Frame# 17 X: 205.60500826337707 Y: 161.90104344381564 Vx: -4.999998762783065 Vy: 0.8178114059621748
Covariance Matrix:
[3.56823789 0. 1.31417792 0.]
[0. 3.56823789 0. 1.31417792]
[1.31417792 0. 10.86079781 0.]

Frame# 18 X: 200.60500870359118 Y: 162.88137782316855 Vx: -4.999999559785891 Vy: 0.9803343793529109
Covariance Matrix:
[3.56823728 0. 1.31417515 0.]
[0. 3.56823728 0. 1.31417515]
[1.31417515 0. 10.86078519 0.]

Frame# 19 X: 195.60500886124387 Y: 163.8792550796695 Vx: -4.999999842347307 Vy: 0.9978772565009422
Covariance Matrix:
[3.56823707 0. 1.31417415 0.]
[0. 3.56823707 0. 1.31417415]
[1.31417415 0. 10.86078065 0.]

Frame# 20 X: 190.60500891781476 Y: 163.09490746177266 Vx: -4.999999943429117 Vy: -0.7843476178968274
Covariance Matrix:
[3.56823699 0. 1.31417379 0.]
[0. 3.56823699 0. 1.31417379]
[1.31417379 0. 10.86077902 0.]

Frame# 21 X: 185.60500893812622 Y: 160.33406667223932 Vx: -4.999999979688539 Vy: -2.760840789533347
Covariance Matrix:
[3.56823696 0. 1.31417366 0.]
[0. 3.56823696 0. 1.31417366]

[1.31417366 0.   10.86077844 0.   ]

Frame# 22 X: 181.49706818170355 Y: 160.03605941173382 Vx: -4.107940756422664 Vy: -0.29800726050549997
Covariance Matrix:
[3.56823695 0.   1.31417362 0.   ]
[0.   3.56823695 0.   1.31417362]
[1.31417362 0.   10.86077823 0.   ]

Frame# 23 X: 177.48541697548916 Y: 160.89595151640992 Vx: -4.011651206214395 Vy: 0.8598921046761063
Covariance Matrix:
[3.56823695 0.   1.3141736 0.   ]
[0.   3.56823695 0.   1.3141736]
[1.3141736 0.   10.86077815 0.   ]

Frame# 24 X: 172.59210010007587 Y: 160.9887689271715 Vx: -4.893316875413291 Vy: 0.09281741076156891
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077813 0.   ]

Frame# 25 X: 168.49567479409947 Y: 162.7829061810098 Vx: -4.096425305976396 Vy: 1.7941372538383007
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077812 0.   ]

Frame# 26 X: 164.48526657295193 Y: 162.97656672727112 Vx: -4.010408221147543 Vy: 0.19366054626132723
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 27 X: 160.4841431016315 Y: 162.99747059463337 Vx: -4.001123471320426 Vy: 0.02090386736225014
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 28 X: 157.37608106906168 Y: 164.78384544560035 Vx: -3.108062032569819 Vy: 1.786374850966979
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 29 X: 153.47235753491105 Y: 164.08460887643346 Vx: -3.903723534150629 Vy: -0.6992365691668851
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 30 X: 150.37480892598637 Y: 164.90119198255596 Vx: -3.097548608924683 Vy: 0.8165831061224935
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Point 8:

Frame# 3 X: 294.02758620689656 Y: 68.02758620689656 Vx: -0.972413793103442 Vy: -0.972413793103442
Covariance Matrix:
[100 0 0 0]
[0 100 0 0]
[0 0 25 0]

Frame# 4 X: 292.1623412479293 Y: 68.00220872446162 Vx: -1.8652449589672528 Vy: -0.02537748243493354
Covariance Matrix:
[3.88965517 0.   0.68965517 0.   ]
[0.   3.88965517 0.   0.68965517]
[0.68965517 0.   24.68965517 0.   ]

Frame# 5 X: 290.1986506366357 Y: 67.09207122203622 Vx: -1.9636906112936003 Vy: -0.9101375024253997
Covariance Matrix:
[3.67973495 0.   2.0320265 0.   ]
[0.   3.67973495 0.   2.0320265]
[2.0320265 0.   15.79679735 0.   ]

Frame# 6 X: 287.32469772806854 Y: 67.00934622752271 Vx: -2.873952908567162 Vy: -0.08272499451351223
Covariance Matrix:
[3.63252676 0.   1.63790392 0.   ]
[0.   3.63252676 0.   1.63790392]
[1.63790392 0.   12.49632222 0.   ]

Frame# 7 X: 285.2454156519092 Y: 66.10655551448204 Vx: -2.079282076159359 Vy: -0.9027907130406732
Covariance Matrix:
[3.59395662 0.   1.43477724 0.   ]
[0.   3.59395662 0.   1.43477724]
[1.43477724 0.   11.42645575 0.   ]

Frame# 8 X: 282.34752978216136 Y: 65.11849360394598 Vx: -2.8978858697478245 Vy: -0.9880619105360609
Covariance Matrix:
[3.57772462 0.   1.3577455 0.   ]
[0.   3.57772462 0.   1.3577455]
[1.3577455 0.   11.06088545 0.   ]

Frame# 9 X: 279.3603024998169 Y: 65.01275374514711 Vx: -2.987227282344463 Vy: -0.10573985879887005
Covariance Matrix:
[3.57166684 0.   1.32982785 0.   ]
[0.   3.57166684 0.   1.32982785]
[1.32982785 0.   10.93222514 0.   ]

Frame# 10 X: 276.36234131897754 Y: 64.10920520632773 Vx: -2.9979611808393543 Vy: -0.9035485388193791
Covariance Matrix:
[3.5694706 0.   1.31979359 0.   ]
[0.   3.5694706 0.   1.31979359]
[1.31979359 0.   10.88638041 0.   ]

Frame# 11 X: 274.25489892716666 Y: 63.119684321156 Vx: -2.107442391810878 Vy: -0.9895208851717285
Covariance Matrix:
[3.56868012 0.   1.31619138 0.   ]
[0.   3.56868012 0.   1.31619138]
[1.31619138 0.   10.86996516 0.   ]

Frame# 12 X: 271.3512897644132 Y: 63.01291710677463 Vx: -2.903609162753469 Vy: -0.10676721438137093
Covariance Matrix:
[3.56839611 0.   1.31489815 0.   ]

[0.   3.56839611 0.   1.31489815]
[1.31489815 0.   10.86407649 0.   ]

Frame# 13 X: 268.36172357922214 Y: 62.10932984822995 Vx: -2.9895661851910518 Vy: -0.9035872585446825
Covariance Matrix:
[3.5682941 0.   1.31443379 0.   ]
[0.   3.5682941 0.   1.31443379]
[1.31443379 0.   10.86196254 0.   ]

Frame# 14 X: 264.4707997373688 Y: 62.01180094586425 Vx: -3.8909238418533505 Vy: -0.09752890236569556
Covariance Matrix:
[3.56825747 0.   1.31426704 0.   ]
[0.   3.56825747 0.   1.31426704]
[1.31426704 0.   10.86120347 0.   ]

Frame# 15 X: 261.3746384787221 Y: 61.109213897606075 Vx: -3.0961612586467027 Vy: -0.9025870482581766
Covariance Matrix:
[3.56824432 0.   1.31420715 0.   ]
[0.   3.56824432 0.   1.31420715]
[1.31420715 0.   10.86093087 0.   ]

Frame# 16 X: 258.3642602541601 Y: 61.011788605609354 Vx: -3.0103782245619755 Vy: -0.09742529199672134
Covariance Matrix:
[3.56823959 0.   1.31418564 0.   ]
[0.   3.56823959 0.   1.31418564]
[1.31418564 0.   10.86083297 0.   ]

Frame# 17 X: 255.3631405356707 Y: 61.001272470092196 Vx: -3.001119718489406 Vy: -0.010516135517157466
Covariance Matrix:
[3.56823789 0.   1.31417792 0.   ]
[0.   3.56823789 0.   1.31417792]
[1.31417792 0.   10.86079781 0.   ]

Frame# 18 X: 252.3630198565051 Y: 61.00013735135519 Vx: -3.0001206791656045 Vy: -0.0011351187370038929
Covariance Matrix:
[3.56823728 0.   1.31417515 0.   ]
[0.   3.56823728 0.   1.31417515]
[1.31417515 0.   10.86078519 0.   ]

Frame# 19 X: 248.47094764960443 Y: 59.21589633223931 Vx: -3.8920722069006786 Vy: -1.784241019115882
Covariance Matrix:
[3.56823707 0.   1.31417415 0.   ]
[0.   3.56823707 0.   1.31417415]
[1.31417415 0.   10.86078065 0.   ]

Frame# 20 X: 246.26671596789095 Y: 59.91536325396974 Vx: -2.2042316817134804 Vy: 0.6994669217304263
Covariance Matrix:
[3.56823699 0.   1.31417379 0.   ]
[0.   3.56823699 0.   1.31417379]
[1.31417379 0.   10.86077902 0.   ]

Frame# 21 X: 244.24467105063985 Y: 59.99086424507203 Vx: -2.022044917251094 Vy: 0.07550099110229525
Covariance Matrix:
[3.56823696 0.   1.31417366 0.   ]
[0.   3.56823696 0.   1.31417366]
[1.31417366 0.   10.86077844 0.   ]

Frame# 22 X: 241.35023227121553 Y: 59.999013879635946 Vx: -2.89443877942432 Vy: 0.008149634563913821
Covariance Matrix:
[3.56823695 0.   1.31417362 0.   ]
[0.   3.56823695 0.   1.31417362]
[1.31417362 0.   10.86077823 0.   ]

Frame# 23 X: 239.25368586703487 Y: 59.99989355741447 Vx: -2.096546404180657 Vy: 0.0008796777785207155
Covariance Matrix:
[3.56823695 0.   1.3141736 0.   ]
[0.   3.56823695 0.   1.3141736]
[1.3141736 0.   10.86077815 0.   ]

Frame# 24 X: 236.3512053388275 Y: 59.99998851050598 Vx: -2.902480528207377 Vy: 9.495309151219544e-05
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077813 0.   ]

Frame# 25 X: 234.25379090107154 Y: 60.8920579956121 Vx: -2.0974144377559583 Vy: 0.8920694851061199
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077812 0.   ]

Frame# 26 X: 231.35121667646814 Y: 60.988348657555115 Vx: -2.9025742246033985 Vy: 0.09629066194301572
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 27 X: 229.2537921249135 Y: 60.99874234519227 Vx: -2.0974245515546386 Vy: 0.01039368763715487
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 28 X: 227.24327604436894 Y: 60.99986424777892 Vx: -2.010516080544562 Vy: 0.001121902586653789
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 29 X: 225.24214093059987 Y: 60.99998534680151 Vx: -2.0011351137690667 Vy: 0.00012109902258572447
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 30 X: 223.24201840555273 Y: 60.999998418322555 Vx: -2.0001225250471464 Vy: 1.3071521046015278e-05
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Point 9:
Frame# 3 X: 292.0827586206897 Y: 135.08275862068965 Vx: -2.917241379310326 Vy: -2.9172413793103544
Covariance Matrix:

```
[100  0   0   0]
[ 0 100   0   0]
[ 0  0 25   0]
```

Frame# 4 X: 291.0866924351187 Y: 135.00662617338486 Vx: -0.9960661855709532 Vy: -0.07613244730478641
Covariance Matrix:
```
[3.88965517 0.        0.68965517 0.       ]
[0.         3.88965517 0.        0.68965517]
[ 0.68965517 0.       24.68965517 0.      ]
```

Frame# 5 X: 289.1917009080302 Y: 135.0006087353523 Vx: -1.8949915270885072 Vy: -0.00601743803255772
Covariance Matrix:
```
[3.67973495 0.        2.0320265 0.       ]
[0.         3.67973495 0.        2.0320265 ]
[ 2.0320265  0.       15.79679735 0.      ]
```

Frame# 6 X: 287.2224814105337 Y: 134.1015726379534 Vx: -1.9692194974965105 Vy: -0.8990360973988913
Covariance Matrix:
```
[3.63252676 0.        1.63790392 0.       ]
[0.         3.63252676 0.        1.63790392]
[ 1.63790392 0.       12.49632222 0.      ]
```

Frame# 7 X: 285.23462479342436 Y: 134.01072290597548 Vx: -1.987856617109344 Vy: -0.09084973197792579
Covariance Matrix:
```
[3.59395662 0.        1.43477724 0.       ]
[0.         3.59395662 0.        1.43477724]
[ 1.43477724 0.       11.42645575 0.      ]
```

Frame# 8 X: 282.34637426154376 Y: 134.00114824403943 Vx: -2.888250531880601 Vy: -0.009574661936056827
Covariance Matrix:
```
[3.57772462 0.        1.3577455 0.       ]
[0.         3.57772462 0.        1.3577455 ]
[ 1.3577455  0.       11.06088545 0.      ]
```

Frame# 9 X: 280.25254577761945 Y: 133.10775593900223 Vx: -2.0938284839243124 Vy: -0.8933923050371959
Covariance Matrix:
```
[3.57166684 0.        1.32982785 0.       ]
[0.         3.57166684 0.        1.32982785]
[ 1.32982785 0.       10.93222514 0.      ]
```

Frame# 10 X: 277.35072191481623 Y: 133.90378934934296 Vx: -2.9018238628032123 Vy: 0.7960334103407263
Covariance Matrix:
```
[3.5694706  0.        1.31979359 0.       ]
[0.         3.5694706  0.        1.31979359]
[ 1.31979359 0.       10.88638041 0.      ]
```

Frame# 11 X: 275.25364518215054 Y: 133.09751975029243 Vx: -2.0970767326656983 Vy: -0.8062695990505233
Covariance Matrix:
```
[3.56868012 0.        1.31619138 0.       ]
[0.         3.56868012 0.        1.31619138]
[ 1.31619138 0.       10.86996516 0.      ]
```

Frame# 12 X: 272.3511544521343 Y: 133.01052496279377 Vx: -2.9024907300162113 Vy: -0.08699478749866785
Covariance Matrix:
```
[3.56839611 0.        1.31489815 0.       ]
[0.         3.56839611 0.        1.31489815]
[ 1.31489815 0.       10.86407649 0.      ]
```

Frame# 13 X: 269.3617089742058 Y: 133.00113601851245 Vx: -2.989445477928541 Vy: -0.009388944281312206
Covariance Matrix:
[3.5682941 0.   1.31443379 0.   ]
[0.   3.5682941 0.   1.31443379]
[1.31443379 0.   10.86196254 0.   ]

Frame# 14 X: 266.3628592397982 Y: 133.00012262061261 Vx: -2.9988497344075995 Vy: -0.0010133978998396742
Covariance Matrix:
[3.56825747 0.   1.31426704 0.   ]
[0.   3.56825747 0.   1.31426704]
[1.31426704 0.   10.86120347 0.   ]

Frame# 15 X: 263.3629873703718 Y: 133.89207313337926 Vx: -2.9998718694263857 Vy: 0.8919505127666412
Covariance Matrix:
[3.56824432 0.   1.31420715 0.   ]
[0.   3.56824432 0.   1.31420715]
[1.31420715 0.   10.86093087 0.   ]

Frame# 16 X: 261.2550621008846 Y: 132.2042313702045 Vx: -2.1079252694872253 Vy: -1.6878417631747595
Covariance Matrix:
[3.56823959 0.   1.31418564 0.   ]
[0.   3.56823959 0.   1.31418564]
[1.31418564 0.   10.86083297 0.   ]

Frame# 17 X: 258.3513536128767 Y: 132.9141041938903 Vx: -2.903708488007851 Vy: 0.7098728236857994
Covariance Matrix:
[3.56823789 0.   1.31417792 0.   ]
[0.   3.56823789 0.   1.31417792]
[1.31417792 0.   10.86079781 0.   ]

Frame# 18 X: 255.36174756741224 Y: 132.99072834367828 Vx: -2.989606045464484 Vy: 0.07662414978798893
Covariance Matrix:
[3.56823728 0.   1.31417515 0.   ]
[0.   3.56823728 0.   1.31417515]
[1.31417515 0.   10.86078519 0.   ]

Frame# 19 X: 252.36286956454614 Y: 133.89105845721778 Vx: -2.9988780028660926 Vy: 0.9003301135394963
Covariance Matrix:
[3.56823707 0.   1.31417415 0.   ]
[0.   3.56823707 0.   1.31417415]
[1.31417415 0.   10.86078065 0.   ]

Frame# 20 X: 249.36299069749316 Y: 133.096181527319 Vx: -2.999878867052985 Vy: -0.794876929898777
Covariance Matrix:
[3.56823699 0.   1.31417379 0.   ]
[0.   3.56823699 0.   1.31417379]
[1.31417379 0.   10.86077902 0.   ]

Frame# 21 X: 247.2550630183921 Y: 133.0103819074267 Vx: -2.107927679101067 Vy: -0.085799619892299
Covariance Matrix:
[3.56823696 0.   1.31417366 0.   ]
[0.   3.56823696 0.   1.31417366]
[1.31417366 0.   10.86077844 0.   ]

Frame# 22 X: 245.24341322443453 Y: 134.78523910358308 Vx: -2.0116497939575595 Vy: 1.7748571961563755
Covariance Matrix:

[3.56823695 0.   1.31417362 0.   ]
[0.   3.56823695 0.   1.31417362]
[1.31417362 0.   10.86077823 0.   ]

Frame# 23 X: 243.24215573751093 Y: 133.19270007284123 Vx: -2.001257486923606 Vy: -1.5925390307418468
Covariance Matrix:
[3.56823695 0.   1.3141736 0.   ]
[0.   3.56823695 0.   1.3141736]
[1.3141736 0.   10.86077815 0.   ]

Frame# 24 X: 240.34996076783582 Y: 134.8049186647944 Vx: -2.892194969675103 Vy: 1.612218591953166
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077813 0.   ]

Frame# 25 X: 238.25365656112757 Y: 134.08688353579927 Vx: -2.096304206708254 Vy: -0.7180351289951261
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077812 0.   ]

Frame# 26 X: 236.24326141149365 Y: 135.7934967468156 Vx: -2.010395149633922 Vy: 1.7066132110163323
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 27 X: 233.3500801153339 Y: 136.86976911681288 Vx: -2.893181296159753 Vy: 1.0762723699972696
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 28 X: 231.2536694436407 Y: 135.20182430739433 Vx: -2.0964106716932065 Vy: -1.6679448094185432
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 29 X: 229.2432628020602 Y: 135.91384430575323 Vx: -2.010406641580488 Vy: 0.7120199983588975
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

Frame# 30 X: 228.1341987369889 Y: 136.8827595242939 Vx: -1.1090640650712942 Vy: 0.9689152185406726
Covariance Matrix:
[3.56823694 0.   1.31417359 0.   ]
[0.   3.56823694 0.   1.31417359]
[1.31417359 0.   10.86077811 0.   ]

For each frame, the traces of the posterior covariance matrices were averaged and plotted below in Figure 1. With each frame, the trace decreases until it levels off to approximately

28.85, which occurs roughly at frame 8. For each measurement, the area surrounding the predicted coordinates decreases because the Kalman filter predictions become more accurate.
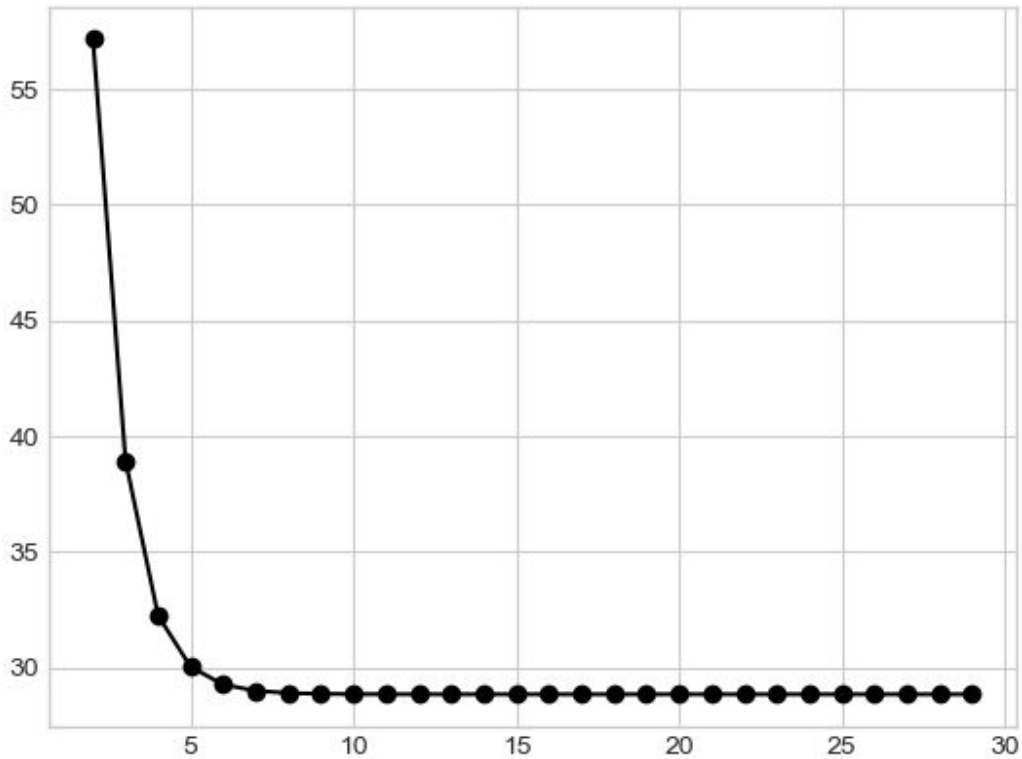

Figure 1: Posterior covariance matrix trace (y-axis) vs. frame (x-axis)

3. Structure from motion and 3D shape recovery

3.1. Theory for the algorithm: Factorization

The 3D shape of an object and its 3D motion relative to the camera can be estimated with the factorization method. This method assumes an orthographic camera model, and the data must consist of n non-coplanar 3D points from N image frames, where N must be no less than three.

Suppose there are n total feature points and N image frames. Each 2D point (j...n) for each image frame (i...N) can be represented as $p_{ij} = (c_{ij}, r_{ij})$, and the centroid of the image points on frame i can be represented as (c^*_i, r^*_i). Each 3D point in the object frame (j...n) for each image frame (i...N) can be represented as $P_{ij} = (x_j, y_j, z_j)$, and P* is the centroid of the 3D points.

The 2D coordinates representing the difference between each feature point and the centroid of all feature points for a given image frame i can be given by:

$$c'_{ij} = c_{ij} - c^*_i \quad (14)$$

$$r'_{ij} = r_{ij} - r^*_i \quad (15)$$

Likewise, suppose the 3D coordinates representing the difference between each 3D feature point and the centroid of all 3D feature points for a given image frame i can be given by:

$$P'_j = P_j - P^* \quad (16)$$

Equations (14), (15) and (16) are related to the rotation matrix $R = [r_1, r_2]^t$ between the camera frame and object frame by the following, due to the assumption of orthographic projection:

$$\begin{pmatrix} c'_{ij} \\ r'_{ij} \end{pmatrix} = \begin{pmatrix} r_{i,1} \\ r_{i,2} \end{pmatrix} P'_j \quad (17)$$

For each image point j in each image frame i, the following equation can be formed from (17):

$$W = RS \quad (18)$$

Here, the 2Nx3 rotation matrix R represents the relative orientation for each of the N image frames to the object frame, and S is a 3xn matrix representing the 3D coordinates for each of the n feature points.

$$R = \begin{pmatrix} r_{1,1} \\ r_{1,2} \\ \vdots \\ r_{N,1} \\ r_{N,2} \end{pmatrix} \quad (19)$$

$$S = \begin{pmatrix} P'_1 & P'_2 & \dots P'_n \end{pmatrix} \quad (20)$$

$$W = \begin{pmatrix} c'_{11} & c'_{12} & \cdots & c'_{1n} \\ r'_{11} & r'_{12} & \cdots & r'_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ c'_{N1} & c'_{N2} & \cdots & c'_{Nn} \\ r'_{N1} & r'_{N2} & \cdots & r'_{Nn} \end{pmatrix} \quad (21)$$

The matrix W is formed from each instance of equations (14) and (15) for each frame and each point. In an ideal case, the rank of W is three because the columns of W are linearly dependent on one another due to the orthographic projection assumption and the rotation matrix should be the same for each point in an image frame. Because image noise may cause the rank of W to be greater than three, the rank theorem can be imposed with SVD. Therefore, W can be decomposed into $W = UDV^T$. Three 3x3 diagonal sub-matrices D', U' and V' can be formed from the three largest diagonal elements of D, U and V. These sub-matrices correspond to the largest singular values of each respective matrix. Next, the matrix W' corresponding to the closest estimate of W which satisfies the rank theorem can be formed from the following:

$$W' = U'D'V'^T \quad (22)$$

Next, W' can be decomposed to obtain estimates for S and R. From (22), matrices R* and S* can be computed as:

$$R^* = U'D'^{\frac{1}{2}} \quad (23)$$

$$S^* = D'^{1/2}V'^T \quad (24)$$

R* and S* can only be determined up to an affine transformation. This is because for any invertible 3x3 ambiguity matrix Q, the solutions are also satisfied by $R = R^*Q$ and $S = Q^{-1}S^*$. The matrix Q can be solved assuming every other successive row of R is orthonormal. This means that,

$$r^*_{i,1}QQ^t r^{*t}_{i,1} = 1 \quad (25)$$
$$r^*_{i,2}QQ^t r^{*t}_{i,2} = 1 \quad (26)$$
$$r^*_{i,1}QQ^t r^{*t}_{i,2} = 0 \quad (27)$$

$A = QQ^t$ can be linearly solved from the previously mentioned constraints. From equations (25), (26), and (27) the following three systems of equations can be formed:

$$\begin{pmatrix} r^*_{i,11} & r^*_{i,12} & r^*_{i,13} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} r^*_{i,11} \\ r^*_{i,12} \\ r^*_{i,13} \end{pmatrix} = 1 \qquad (28)$$

$$\begin{pmatrix} r^*_{i,21} & r^*_{i,22} & r^*_{i,23} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} r^*_{i,21} \\ r^*_{i,22} \\ r^*_{i,23} \end{pmatrix} = 1 \qquad (29)$$

$$\begin{pmatrix} r^*_{i,11} & r^*_{i,12} & r^*_{i,13} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} r^*_{i,21} \\ r^*_{i,22} \\ r^*_{i,23} \end{pmatrix} = 0 \qquad (30)$$

Equations (28), (29), and (30) can be rearranged to form the following:

$$\begin{pmatrix} r^{*2}_{i,11} & 2r^*_{i,11}r^*_{i,21} & 2r^*_{i,11}r^*_{i,13} & r^{*2}_{i,12} & 2r^*_{i,12}r^*_{i,13} & r^{*2}_{i,13} \\ r^{*2}_{i,21} & 2r^*_{i,21}r^*_{i,22} & 2r^*_{i,22}r^*_{i,23} & r^{*2}_{i,22} & 2r^*_{i,22}r^*_{i,23} & r^{*2}_{i,23} \\ r^*_{i,11}r^*_{i,21} & r^*_{i,12}r^*_{i,21}+r^*_{i,11}r^*_{i,22} & r^*_{i,13}r^*_{i,21}+r^*_{i,11}r^*_{i,23} & r^*_{i,12}r^*_{i,22} & r^*_{i,13}r^*_{i,22}+r^*_{i,12}r^*_{i,23} & r^*_{i,13}r^*_{i,23} \end{pmatrix} \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{22} \\ a_{23} \\ a_{33} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad (31)$$

A can be solved using the least square method using N frames and decomposed using Cholesky factorization to obtain Q. Then, given Q, R and S can be found from R=R*Q and S=Q⁻¹S*.

3.2 Experiment and Results

R matrix:
[[-7.57499150e-02 7.77155093e-01 -8.55963921e-01]
 [5.84537878e-01 -4.54903046e-02 -1.88656693e-02]
 [1.25347862e-01 8.80205591e-01 -7.94474032e-01]
 [6.60593900e-01 -2.36495719e-02 2.60667810e-03]
 [-1.46801233e-01 6.84165584e-01 -7.58025493e-01]
 [6.99685589e-01 5.82236351e-02 -2.00693066e-03]
 [-5.86247523e-02 6.81436166e-01 -6.92968282e-01]
 [6.63348732e-01 -5.05385931e-02 -4.87330149e-03]
 [4.57851921e-02 7.03001717e-01 -6.17889612e-01]
 [5.20990035e-01 -5.04662536e-02 4.16674045e-03]
 [1.86829600e-01 7.01527313e-01 -5.40556499e-01]

```
[ 5.88700730e-01 -7.19297055e-02  1.88493279e-02]
[ 3.11782493e-01  6.91050369e-01 -4.65357301e-01]
[ 9.04412399e-01 -1.71168540e-01  1.35283583e-02]
[ 3.94469504e-01  6.42795361e-01 -3.86391583e-01]
[ 8.27391513e-01 -1.83654898e-01  1.95802147e-02]
[ 5.48838729e-01  6.10940634e-01 -3.08887385e-01]
[ 7.77014698e-01 -2.35091855e-01  2.25978677e-02]
[ 5.75562855e-01  6.05995720e-01 -2.30565473e-01]
[ 6.90581791e-01 -1.37474265e-01  2.39263259e-02]
[ 6.61932073e-01  5.70919378e-01 -1.51912608e-01]
[ 7.07683137e-01 -1.27330011e-01  3.08279745e-02]
[ 1.23916723e+00  1.69641871e-01 -1.13286857e-01]
[ 6.59725165e-01  5.47492368e-02  4.64576352e-02]
[ 1.44318879e+00  4.34273323e-02 -4.19994011e-02]
[ 5.24244503e-01  2.93779596e-02  3.86619307e-02]
[ 9.06770145e-01 -5.38562294e-01 -4.02010312e-02]
[ 6.15658741e-01  1.57860900e-01  5.58084577e-02]
[ 3.95437576e-01 -8.87277199e-01 -4.70337987e-03]
[ 2.79210855e-01  1.15944851e-01  3.83490877e-02]
[-1.76466627e-01 -1.12661772e+00  3.45512461e-02]
[-7.47218664e-02  1.69045454e-01  2.17076639e-02]
[-3.03907656e-01 -1.02608713e+00  1.13334393e-01]
[-1.48893617e-01 -5.35222783e-02  6.67208057e-03]
[-3.78011645e-01 -8.81434649e-01  1.90737472e-01]
[-2.58576653e-01  1.05927171e-01  1.26739004e-02]
[-5.12713312e-01 -6.83982875e-01  2.65883542e-01]
[-5.58198767e-01 -1.40519232e-02  6.87890506e-04]
[-5.36658523e-01 -5.15135307e-01  3.38498105e-01]
[-8.58366135e-01 -4.15338600e-03 -8.09745946e-03]
[-3.80267130e-01 -4.89114259e-01  4.12038215e-01]
[-8.78042780e-01  1.11915519e-02 -1.32503770e-02]
[-2.80453607e-01 -4.13573547e-01  4.87056279e-01]
[-8.91144866e-01 -1.95351107e-02 -3.21055649e-02]
[-9.80157339e-01 -3.27405419e-01  5.31821959e-01]
[-1.11324145e+00  1.73437953e-01 -3.36631109e-02]
[-9.23236279e-01 -2.75113015e-01  5.96607523e-01]
[-8.89820423e-01  2.44698155e-01 -2.39591212e-02]
[-6.80513566e-01 -2.81592797e-01  6.66055447e-01]
[-7.66765423e-01  9.12249548e-02 -2.79451224e-02]
[-5.72110614e-01 -1.61389400e-01  7.31597099e-01]
[-1.01198089e+00  1.07992400e-01 -5.15867658e-02]
[-3.43754597e-01 -8.17032090e-02  7.98172950e-01]
[-1.07619943e+00 -8.11343765e-03 -6.25770567e-02]
```

[-4.85685253e-01 -7.32733163e-02 8.36828626e-01]
[-1.17782736e+00 -1.23504089e-01 -7.81716543e-02]]

S matrix:
[[0.93660153   6.91663818 -55.76708465]
 [1.81295516   6.72839992 -54.0785168]
 [0.45353291   6.26352852 -53.77992992]
 [-3.59022089 -2.88073454 -62.02608973]
 [3.80653762 -2.53531926 -64.38479578]
 [-2.46982689 10.08972636 -49.45938518]
 [1.3365849   6.01669713 -67.86411924]
 [3.17373493   3.63177772 -38.86469662]
 [0.95903278   3.05499778 -37.04046826]]

The rotation matrices for every other frame are very similar, so it can be determined that the polyhedron exhibits mostly translational motion with little rotation, and satisfies orthographic projection. The 3D shape can only be recovered up to a scale factor.

The polyhedron was reconstructed from the S matrix, shown below in figure 2.
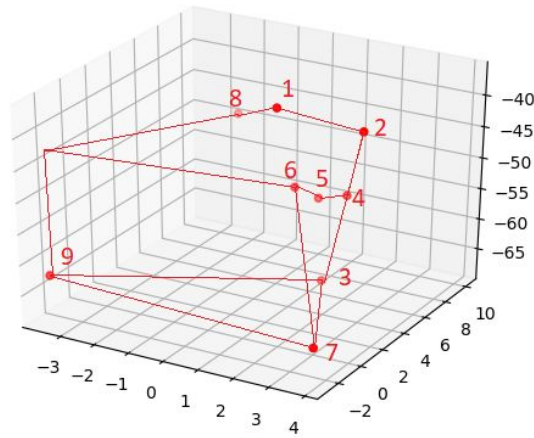

Figure 2: 3D feature points

4. Conclusion

The points were successfully tracked using a Kalman filter and recovered with the factorization method. The feature points representing the polyhedron corners were tracked from one frame to

the next. The orthographic assumption was verified with the results from R. The recovered motion showed that the movement of the polyhedron was mostly translational, because every other R value showed little change. The 3D structure of the polyhedron was recovered up to a scale factor and seemed to be relatively accurate, despite the shape being rotated during plotting.