Theoretical Concepts and Extra Code explanations

A helpful companion to the physicsfun codebase

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Collisions

Basics of Collsions

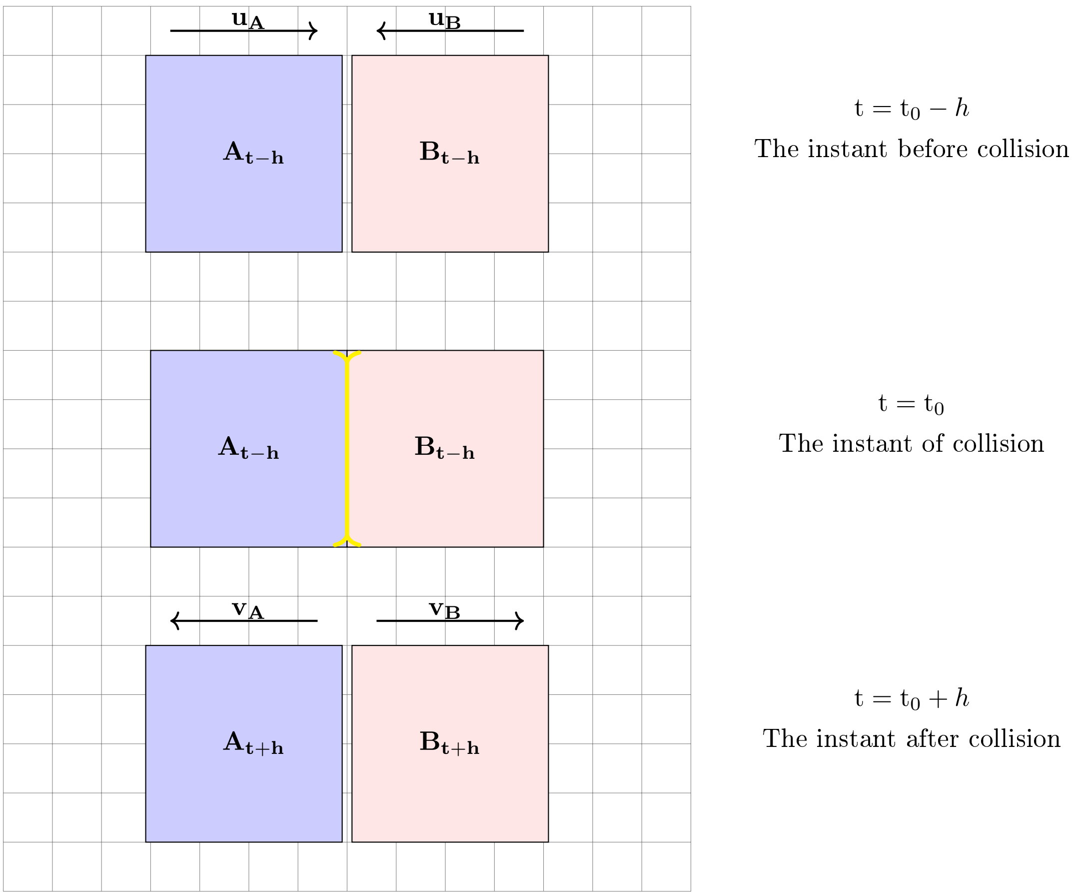

Figure 1: A figure displaying the three interesting moments during a collision

Collisions are simplified as instantanous changes to velocity that occur when two objects touch. The three interesting moments are shown above in figure . Collsions must obey newtons second law, which in turn means that they must conserve momentum. The momentum of an object is given as $p = mv$ where p is the momentum, v the object's velocity and m the object's mass. The total momentum of a system is given as $\sum_i m_i v_i = \sum_i p_i$. This gives us a conservation law over the course of a collision, equation 1

$$\sum_i m_i u_i = \sum_i m_i v_i \tag{1}$$

Where u is velocity pre collision and v velocity post collision. This requirement is however not enough to solve even a simple collision of two objects since it provides us with only one equation to solve for two variables. To deal with this issue we can make a variety of assumptions about the collision which gives us a number of sub cases. The simplest assumption is to assert the preservation of energy called a perfectly elastic collision, you can also assume maximum loss of energy which is an ineleastic collision. Finally there is the inbetween elastic collsision where some energy is lost. For simplicities sake I will only describe collisions in one dimension, and collsions between two objects, then I will describe how these solutions can be expanded to hold for general collisions.

Perfectly Elastic Collisions

The momentum balance equation in equation 1 still holds, and we can introduce a new equation governing the energy of the system. Since the coordinates of the objects remain the same and no energy is produced or lost in the collision the consvervation of energy is really the conservation of kinetic energy. The kinetic energy of a particle or solid object is given by $\mathbf{E_k} = \frac{mv^2}{2}$ or equivilantly $\mathbf{E_k} = \frac{p^2}{2m}$. Solving for two objects we get the system in equation 2.

$$\begin{gathered} m_A u_A^2/2 + m_B u_B^2/2 = m_A v_A^2/2 + m_B v_B^2/2 \\ m_A u_A + m_B u_B = m_A v_A + m_B v_B \end{gathered} \tag{2}$$

The second equation yields $v_B = \frac{-m_A v_A + m_A u_A + m_B u_B}{m_b}$, if inserted into the first this yields equation 3.

$$\begin{aligned} m_A u_A^2/2 + m_B u_B^2/2 &= m_A v_A^2/2 + m_B(\frac{-m_A v_A + m_A u_A + m_B u_B}{m_b})^2/2 \\ m_A u_A^2/2 + m_B u_B^2/2 &= m_A v_A^2/2 + (\frac{m_A^2 v_A^2 + m_A^2 u_A^2 + m_B^2 u_B^2 - 2m_A^2 v_A u_A - 2m_A m_B v_A u_B + 2m_A m_b u_A u_B}{2m_b}) \\ m_A u_A^2/2 + m_B u_B^2/2 &= m_A v_A^2/2 + \frac{m_A^2 v_A^2 - v_A(2m_A^2 u_A + 2m_A m_B u_B)}{2m_b} + \frac{m_A^2 u_A^2 + m_B^2 u_B^2 + 2m_A m_b u_A u_B}{2m_b} \\ (m_A^2 + m_A m_B)v_A^2 - v_A(2m_A^2 u_A + 2m_A m_B u_B) &+ m_A^2 u_A^2 + 2m_A m_b u_A u_B - m_A m_B u_A^2 = 0 \end{aligned} \tag{3}$$

Equation 3 is a quadratic equation which so long as both masses are positive will always have a two real solutions. Assuming u_A is taken to be positive the minimum of the solutions will be v_A. Of note (this is easiest to see from the earlier system in equation 2) if the two masses are the same the two objects will simply trade velocities.

Perfectly Elastic Approximations

Identical Bounce

If two objects with the same mass collide perfectly elastically in the simulation they will clip into each other for up to one frames travel distance. Assuming that we can treat the velocity as constant over this period the size of the overlap will be overlap $= (v_1 - v_2)t_o$ where v_1 and v_2 are the respective velocities, v_1 being chosen as the velocity with greater absolute value, and t_o is the amount of time the overlap has spent forming / the amount of time since the balls should have collided but didn't. After the bounce the objects will have swapped velocities, so the first object will need to be reset to the initial collision position, ie $\text{pos}_1(t) - (v_1)t_o$ and then moved to it's new position according to it's new velocity such that $\text{pos}_1 = \text{pos}_1(t) - v_1 t_o + v_2 t_o$. The same process for the other particle yields $\text{pos}_2 = \text{pos}_2(t) - v_2 t_o + v_1 t_o$. In other words $\text{pos}_1 = \text{pos}_1(t) - \mathit{overlap}$, $\text{pos}_2 = \text{pos}_2(t) + \mathit{overlap}$.