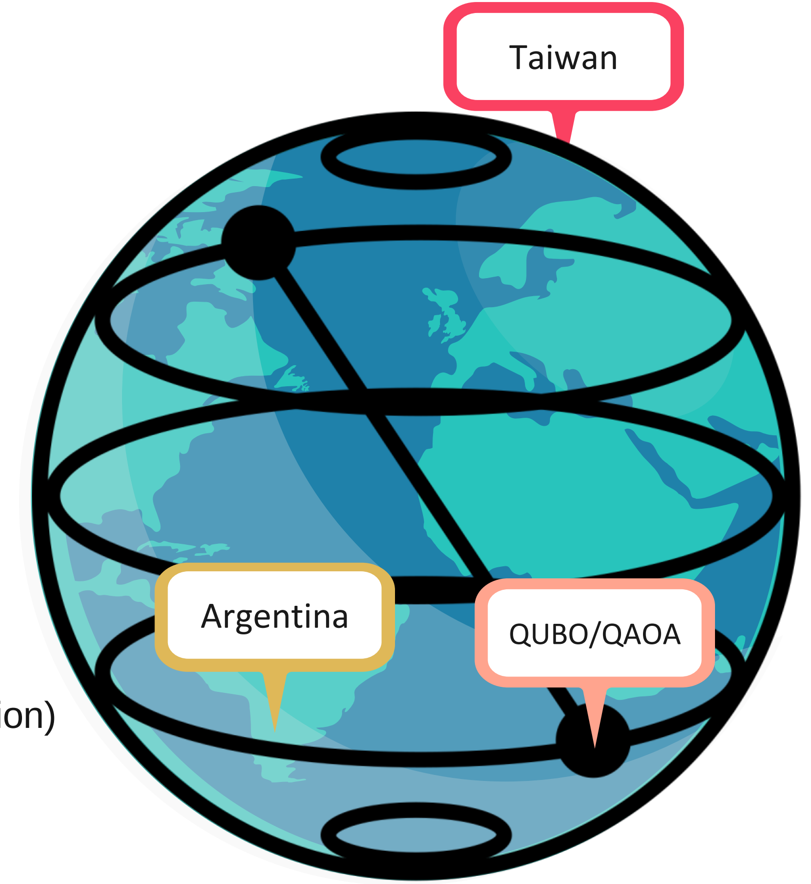
Traveling Salesman Problem



Quantum Mapper team:
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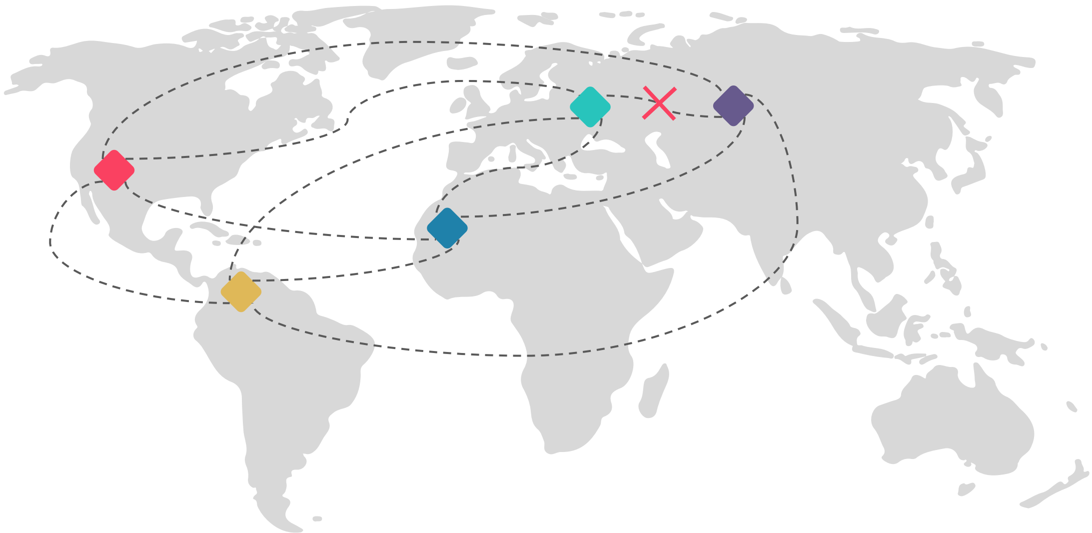
Chun-Tse Li (presenter, coding)
Yu-Yuan Yang(presenter, coding)

Traveling Salesman Problem (TSP)

You want to visit every site.

Goal: Minimize the traveling cost (distance …) around the world.

TSP to Mathematical Matrix (min. distance)



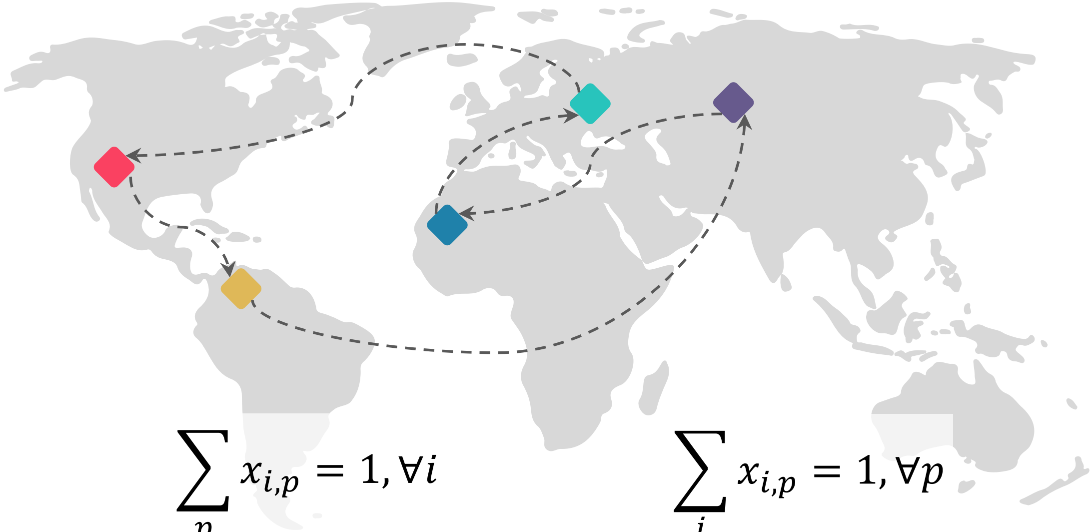
$w_{i,j}$ (j ↓, i →)					
		10^6	10	30	34
			8	20	26
				7	13
					6

Distance matrix

$$Objective\ function:\ C(X) = \sum_{i,j} w_{i,j} \sum_{p} x_{i,p}\, x_{j,p+1}$$

$w_{i,j}$: distance weight
$x_{i,p}$: decision variable ∈{0 or 1}
p: time order

TSP to Mathematical Matrix (constraint)



$x_{i,p}$	D1	D2	D3	D4	D5
◆	0	1	0	0	0
◆	0	0	0	1	0
◆	0	0	1	0	0
◆	1	0	0	0	0
◆	0	0	0	0	1

Valid solution
(D: Day; 1: choose to go; 0: not to go)

$$\sum_p x_{i,p} = 1, \forall i$$

	D1	D2	D3	D4	D5
◆	0	1	0	0	0
◆	0	1	0	1	0
◆	0	0	1	0	0
◆	1	0	0	0	0
◆	0	0	0	0	1

Invalid solution:
one day at two places

$$\sum_i x_{i,p} = 1, \forall p$$

	D1	D2	D3	D4	D5
◆	0	1	0	0	0
◆	0	0	1	1	0
◆	0	0	0	0	0
◆	1	0	0	0	0
◆	0	0	0	0	1

Invalid solution:
visiting the same place twice

TSP to Mathematical Matrix (overall)

$$Overall\ objective\ function:$$

$$C(X) = \sum_{i,j} w_{i,j} \sum_{p} x_{i,p}\, x_{j,p+1} + A \sum_{p} \left(1 - \sum_{i} x_{i,p}\right)^2 + A \sum_{i} \left(1 - \sum_{p} x_{i,p}\right)^2$$

$w_{i,j}$: distance weight
$x_{i,p}$: decision variable ∈{0 or 1}
p: time order
A: hyperparameter (make sure the constraints work)

Task:

min C(X)

Solution:

vector of decision variables, X (that is, optimal path)

Solve TSP objective function (QUBO)

$$C(X) = \sum_{i,j} w_{i,j} \sum_{p} x_{i,p}\, x_{j,p+1} + A \sum_{p} \left(1 - \sum_{i} x_{i,p}\right)^2 + A \sum_{i} \left(1 - \sum_{p} x_{i,p}\right)^2$$

Objective function

For TSP.

$$= \sum_{i,j,p,p'} w_{(i,p),(j,p')}\, x_{i,p}\, x_{j,p'}\, \delta_{p',p+1} - 2A \sum_{i,p} x_{i,p} + A \sum_{i,j,p,p'} x_{i,p}\, x_{j,p'}\, \delta_{i,j} - 2A \sum_{i,p} x_{i,p} + A \sum_{i,j,p,p'} x_{i,p}\, x_{j,p'}\, \delta_{p,p'}$$

$\delta_{i,j}$: delta function = {if i == j: 1; else: 0}

Variable recombination

In order to transform the QUBO form.

Quantum unconstrained binary opimization (QUBO)

$$= \sum_{k,l} Q_{k,l}\, x_k\, x_l + \sum_{k} C_k x_k$$

$$x_k = \frac{1+\sigma_k}{2}, \qquad x_l = \frac{1+\sigma_l}{2}$$

Ising model

$$H_{cost} = \sum_{k,l} J_{k,l}\, \sigma_k\, \sigma_l + \sum_{k} h_k \sigma_k$$

QUBO + Ising model

Prepare for QAOA

Solve TSP objective function (Workflow)

Combinatorial Optimization

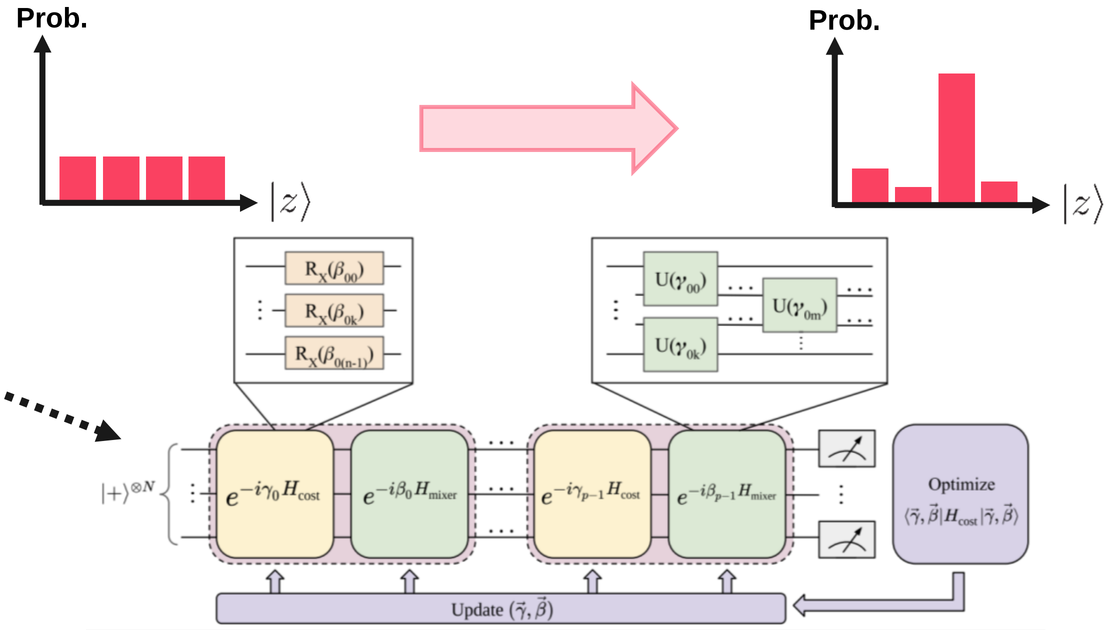
Ising Model Formulation

Parameterized Quantum Circuit

Quantum Hardware

Classical Optimizer

Solution

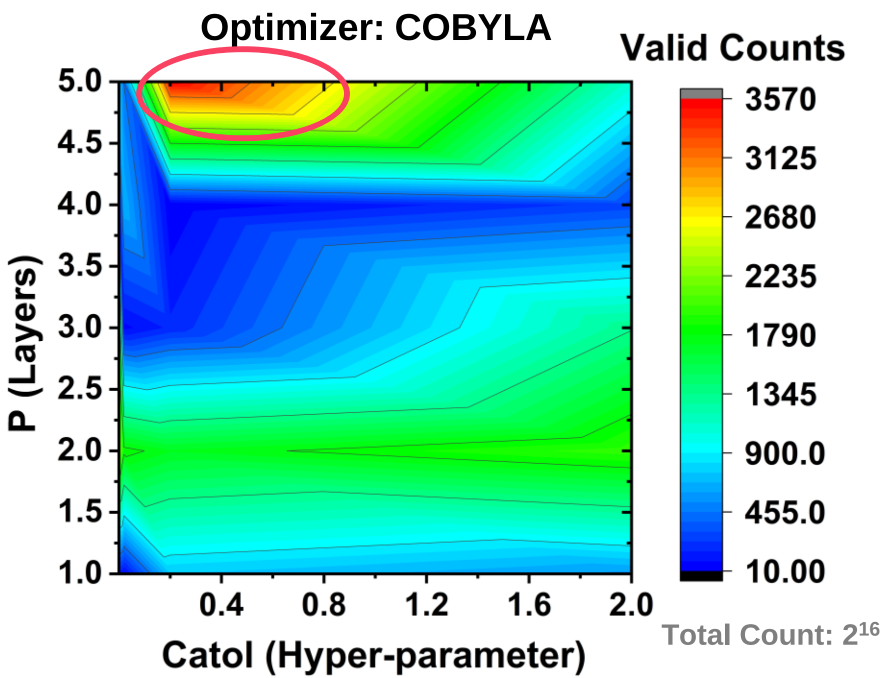


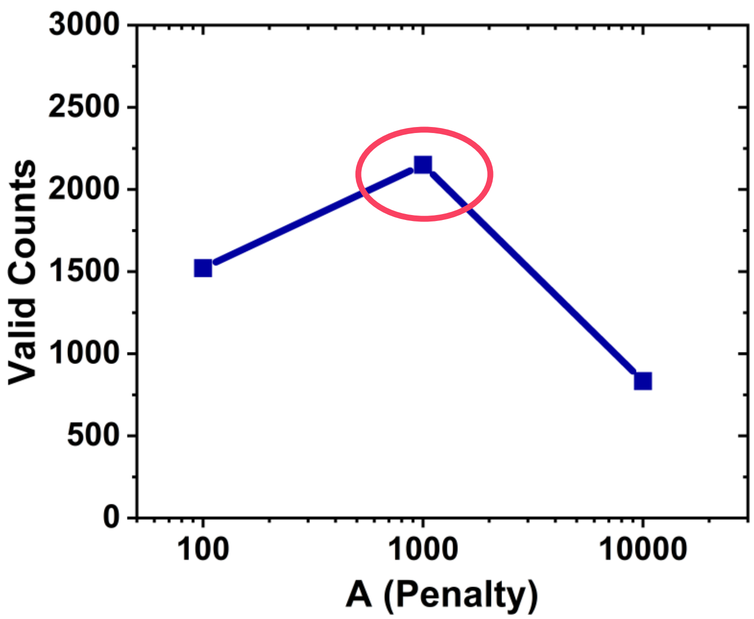
Cost Unitary: $U_{cost}(\gamma_0) = e^{-i\gamma_0 H_{cost}}$

Mixer Unitary: $U_{mixer}(\beta_0) = e^{-i\beta_0 H_{mixer}}$ $\quad H_{mixer} = \sum_{j=1}^{n} \sigma_j^x$

Azad, U., Behera, B. K., Ahmed, E. A., Panigrahi, P. K., & Farouk, A. (2022). Solving Vehicle Routing Problem Using Quantum Approximate Optimization Algorithm. *IEEE Transactions on Intelligent Transportation Systems*.

Performance Analysis

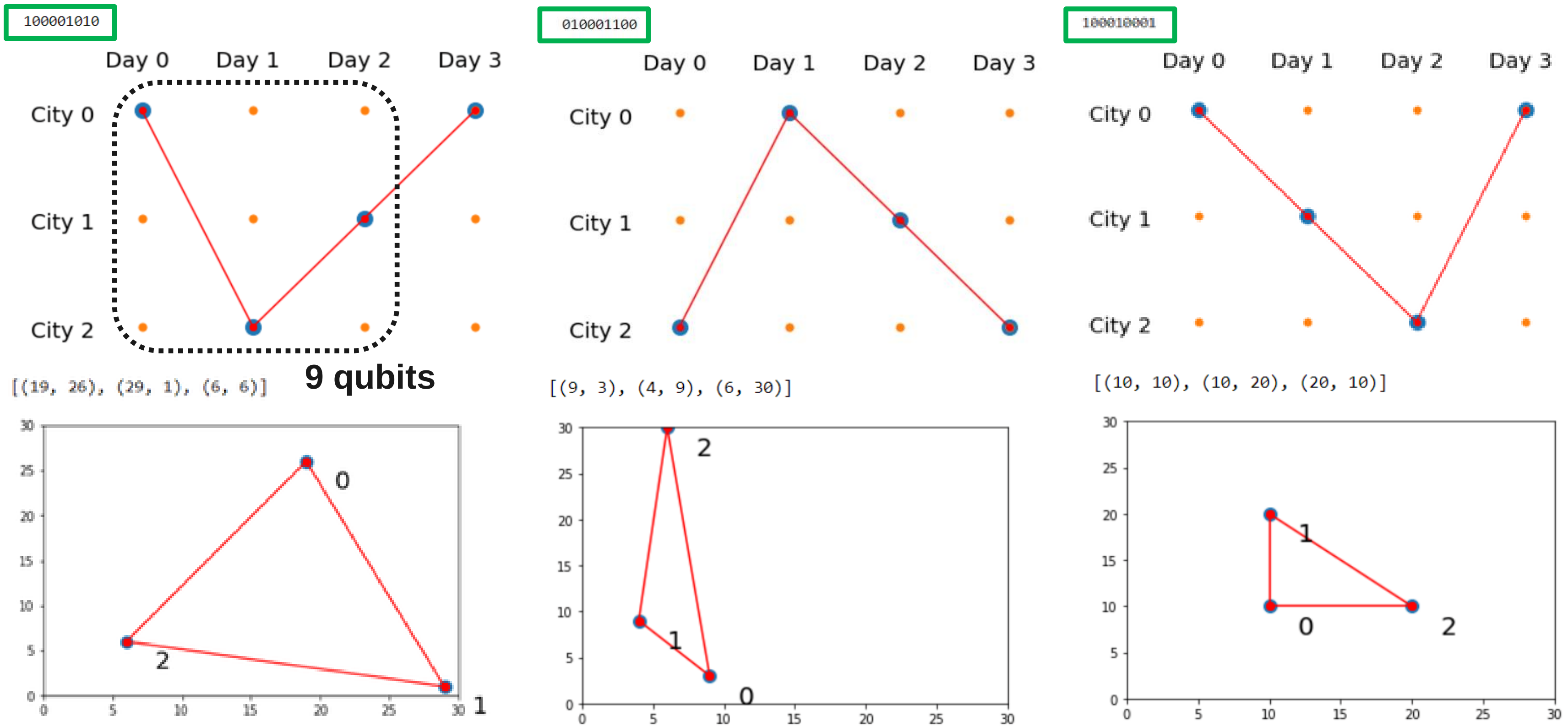




Hyper-parameter should be optimized for different depth of circuits

The values of the penalty may distort the energy profile → stuck in local minima

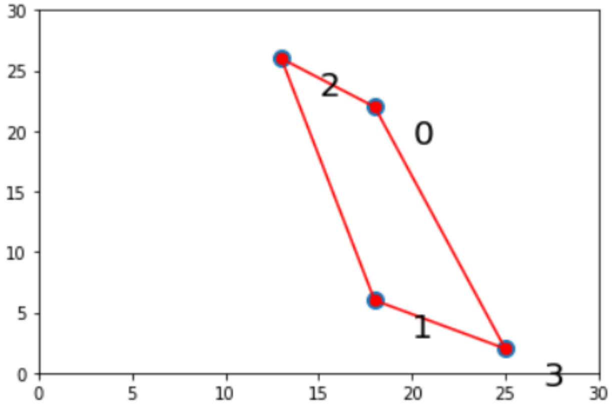
Solving Different Graphs (9-qubit)



Our QAOA model is able to solved TSP with different graphs

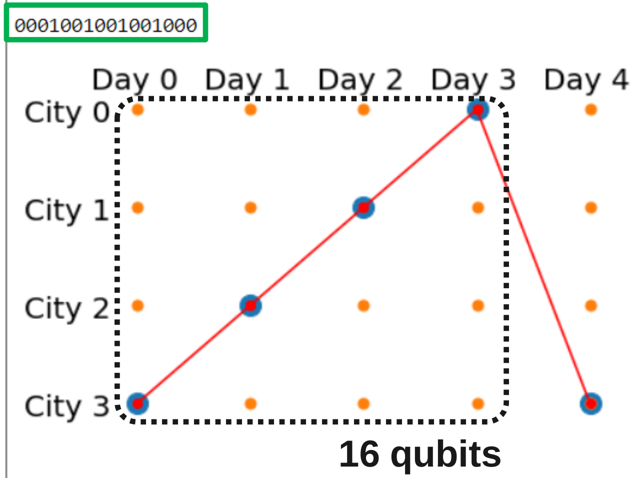
Solving Different Graphs (16-qubit)

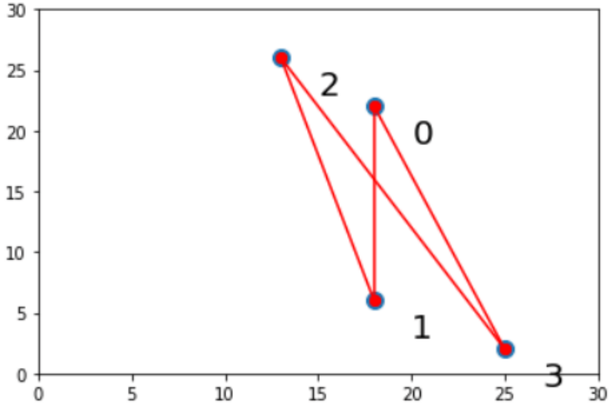
Brute-force Search



Ground state solution

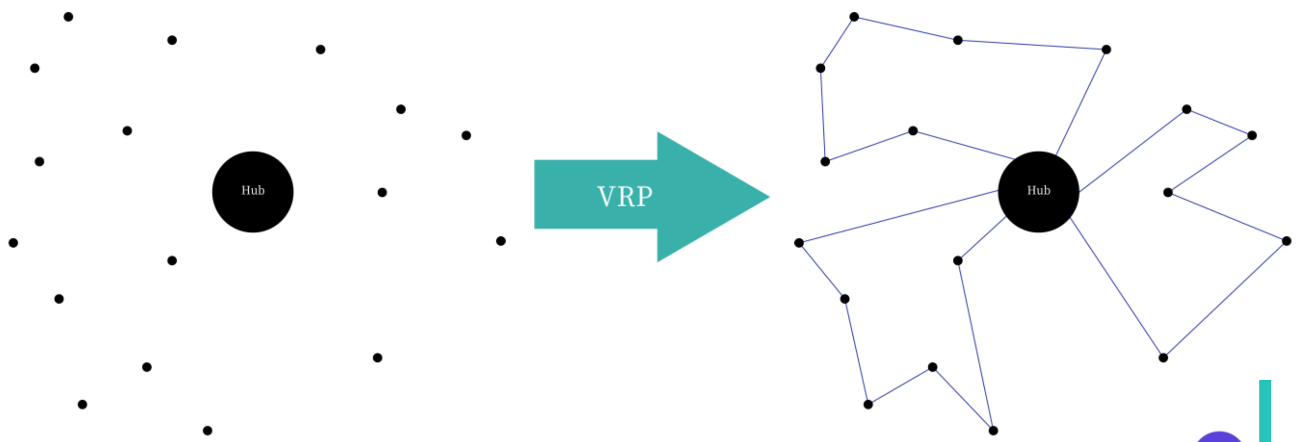
Search by QAOA





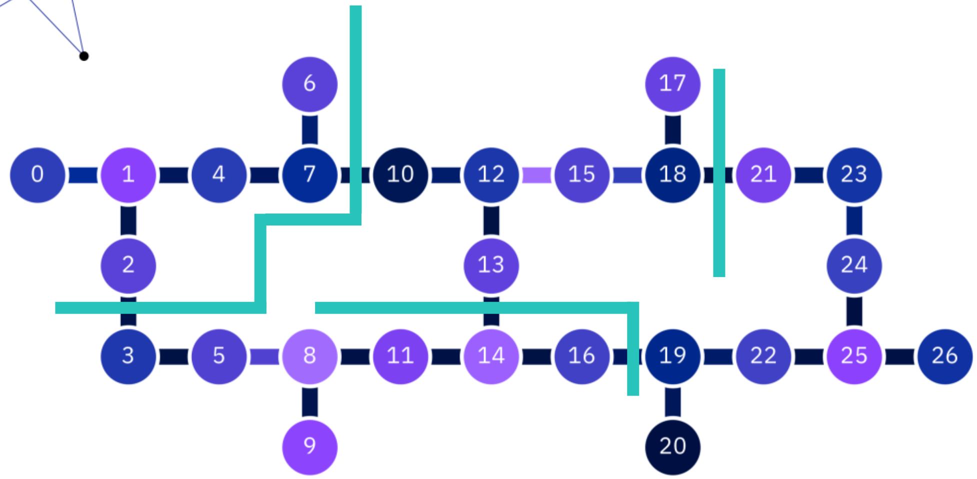
The solution follows the constraints but is not optimal (hyper-parameters need to be tuned…)

Further application: Vehicle Routing Problem (VRP)



Split a larger graphs to multiple sub-graphs.

Parallel computing on single or multiple quantum computers.



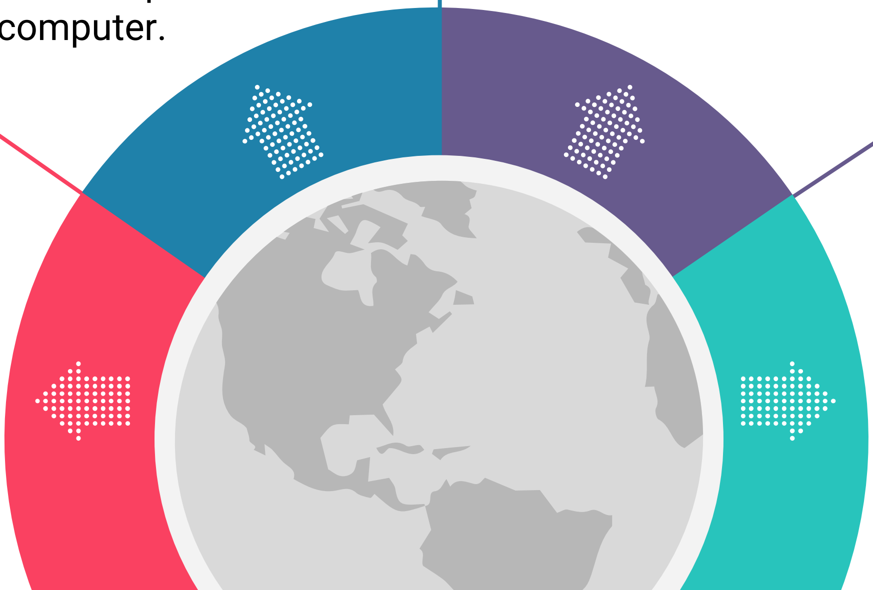
Our Contributions

Originality and Uniqueness

Fine-tune the hyperparameter of QAOA. Enable to scale up with more powerful quantum computer.

Preliminary results

Penalty is a crucial hyperparameter in TSP.



Usefulness and Complexity

Enable to deal with the different city maps. Further application: vehicle routing problem.

Quantum Community Benefit

Rebuild the TSP solution with QUBO-Ising-QAOA method from scratch. Provide the codes publicly in detail.

Thank for listening.