## Fundamentals of Kalman Filtering: A Practical Approach

Paul Zarchan and Howard Musoff Charles Stark Draper Laboratory, Inc. Cambridge, Massachusetts

## Volume 190 PROGRESS IN ASTRONAUTICS AND AERONAUTICS

**Paul Zarchan, Editor-in-Chief** Charles Stark Draper Laboratory, Inc. Cambridge, Massachusetts

Published by the American Institute of Aeronautics and Astronautics, Inc. 1801 Alexander Bell Drive, Reston, Virginia 20191-4344

## **Table of Contents**

Preface		ix
Acknowledgr	nents	xv
Chapter 1.	Numerical Basics	1
-	Introduction	1
	Simple Vector Operations	1
	Simple Matrix Operations	3
	Numerical Integration of Differential Equations	13
	Noise and Random Variables	19
	Gaussian Noise Example	23
	Calculating Standard Deviation	26
	White Noise	28
	Simulating White Noise	30
	State-Space Notation	33
	Fundamental Matrix	34
	Summary	38
	References	38
	TO TO TO TO THE TOTAL THE TOTAL TO THE TOTAL THE TOTAL TO THE TOTAL TH	50
Chapter 2.	Method of Least Squares	41
	Introduction	41
	Overview	41
	Zeroth-Order or One-State Filter	42
	First-Order or Two-State Filter	46
	Second-Order or Three-State Least-Squares Filter	50
	Third-Order System	56
	Experiments with Zeroth-Order or One-State Filter	59
	Experiments with First-Order or Two-State Filter	64
	Experiments with Second-Order or Three-State Filter	71
	Comparison of Filters	78
	Accelerometer Testing Example	80
	Summary	89
	References	90
Chapter 3.	Recursive Least-Squares Filtering	91
	Introduction	91
	Making Zeroth-Order Least-Squares Filter Recursive	91
	Properties of Zeroth-Order or One-State Filter	93
		103
	=	112
	•	124
		128

## TABLE OF CONTENTS

Chapter 4.	Polynomial Kalman Filters	129
	Introduction	129
	General Equations	129
	Derivation of Scalar Riccati Equations	
	Polynomial Kalman Filter (Zero Process Noise)	
	Comparing Zeroth-Order Recursive Least-Squares and Kalman	
	Filters	136
	Comparing First-Order Recursive Least-Squares and Kalman Filters.	139
	Comparing Second-Order Recursive Least-Squares and Kalman Filters.	142
	Comparing Different-Order Filters	148
	Initial Covariance Matrix	
	Riccati Equations with Process Noise	
	Example of Kalman Filter Tracking a Falling Object	
	Revisiting Accelerometer Testing Example	
	Summary	
	•	
	References	182
Chapter 5.	Kalman Filters in a Nonpolynomial World	183
	Introduction	183
	Polynomial Kalman Filter and Sinusoidal Measurement	183
	Sinusoidal Kalman Filter and Sinusoidal Measurement	194
	Suspension System Example	203
	Kalman Filter for Suspension System	207
	Summary	218
	References	
Chapter 6.	Continuous Polynomial Kalman Filter	219
onapier of	Introduction	
	Theoretical Equations	
	Zeroth-Order or One-State Continuous Polynomial Kalman	
	Filter	221
	Filter	227
	Filter	232
	Transfer Function for Zeroth-Order Filter	
	Transfer Function for First-Order Filter	
	Transfer Function for Second-Order Filter	
	Filter Comparison	
	Summary	
	References	
Chanter 7	Extended Volmon Filtering	257
Chapter 7.	Extended Kalman Filtering	
	Introduction	
	Theoretical Equations	257

	TABLE OF CONTENTS	xix
	Drag Acting on Falling Object	259
	First Attempt at Extended Kalman Filter	261
	Second Attempt at Extended Kalman Filter	274
	Third Attempt at Extended Kalman Filter	284
	Summary	291
	References	291
Chapter 8.	Drag and Falling Object	293
	Introduction	293
	Problem Setup	293
	Changing Filter States	309
	Why Process Noise Is Required	311
	Linear Polynomial Kalman Filter	
	Summary	
	References	329
Chapter 9.	Cannon-Launched Projectile Tracking Problem	331
onapter >t	Introduction	
	Problem Statement	
	Extended Cartesian Kalman Filter	
	Polar Coordinate System	
	Extended Polar Kalman Filter	
	Using Linear Decoupled Polynomial Kalman Filters	
	Using Linear Coupled Polynomial Kalman Filters	
	Robustness Comparison of Extended and Linear Coupled	370
	Kalman Filters	385
	Summary	. 393
	Reference	394
Chapter 10.	Tracking a Sine Wave	. 395
<b>-</b>	Introduction	
	Extended Kalman Filter	
	Two-State Extended Kalman Filter with a Priori Information	
	Alternate Extended Kalman Filter for Sinusoidal Signal	
	Another Extended Kalman Filter for Sinusoidal Model	
	Summary	
	References	
Chamtau 11	Catallita Navigation	. 443
Chapter 11.	Satellite Navigation	
	Introduction	
	Problem with Perfect Range Measurements	
	Estimation Without Filtering	
	Linear Filtering of Range	. 453
	Using Extended Kalman Filtering	
	Using Extended Kalman Filtering with One Satellite	. 465
	Using Extended Kalman Filtering with Constant Velocity Receiver	474
	Received	, <del>+</del> /4

	Single Satellite with Constant Velocity Receiver	479
	Receiver	493
	Variable Velocity Receiver and Single Satellite	505
	Summary	513
	References	513
	References	313
Chapter 12.	Biases	
	Introduction	515
	Influence of Bias	515
	Estimating Satellite Bias with Known Receiver Location	519
	Estimating Receiver Bias with Unknown Receiver Location	
	and Two Satellites	525
	Estimating Receiver Bias with Unknown Receiver Location	500
	and Three Satellites	533
	Summary	
	Reference	547
Chapter 13.	Linearized Kalman Filtering	549
-	Introduction	549
	Theoretical Equations	549
	Falling Object Revisited	552
	Developing a Linearized Kalman Filter	
	Cannon-Launched Projectile Revisited	
	Linearized Cartesian Kalman Filter	
	Summary	583
	References	
Chapter 14.	Miscellaneous Topics	587
Chapter 14.	Transduction	587
* .	Introduction	
	Sinusoidal Kalman Filter and Signal-to-Noise Ratio	
	When Only a Few Measurements Are Available	
	Detecting Filter Divergence in the Real World	
	Observability Example	
	Aiding	
	Summary	
	References	646
Appendix	Fundamentals of Kalman-Filtering Software	647
	Software Details	
	MATLAB®	648
	True BASIC	654
	Peference	662