CQF Exam One

June 2022 Cohort

Instructions

All questions must be attempted. Full workings should be provided to obtain maximum credit. Books and lecture notes may be referred to. Help from others is not permitted.

- Computation can be implemented on Excel spreadsheets or coded in Python/other languages. Each plot must have a brief explanation. Submission must include Excel file(s) or Python code(s). All code/Excel workings to be uploaded including the PDF report as ONE zip file named LASTNAME CODE.zip

- Submission of Exam report in word document or Excel files only or Python notebook only will receive a deduction in marks, particularly where there is unnecessary output. Absence of summary tables will receive a deduction. Submission in multiple pdf/image files will result in a delay and/or fail. Complex submissions will be graded with/after Extensions.

Portal, upload and logistical questions should be directed to CQFProgram@fitchlearning.com and clarifying questions to Kannan.Singaravelu@fitchlearning.com. Tutor is unable to re-explain calculation or confirm correct numerical answers or required sections.

A. Products, Strategies and Pricing

1. Solve the following problems:

   (a) A stock is currently priced at \$80. At the end of four months, the stock price is either \$75 or \$85 (the range gives information about volatility and ultimately, delta). By constructing a portfolio, find the value of a four-month European put option with strike \$80. Assume continuous compounding and interest rates of 5%.

   (b) On July 1, 2009 a company enters into a forward to buy 10 million Japanese Yen on January 1, 2010. On September 1, 2009, it enters into an off-setting forward to sell 10 million Japanese Yen on January 1, 2010. Describe the payoff from this strategy.

   (c) Suppose the price of silver is \$8 per ounce. The storage costs are \$0.36 per ounce per year payable quarterly in advance. Assuming that interest rates are 10% per annum for all maturities, calculate the futures price of silver for delivery in 9 months.

2. A European put option is traded at \$4 with the underlying price at \$95. Time to expiry is one month and the strike is \$100. The risk-free interest rate (continuously compounded) is 3% per annum. Suppose you are an arbitrageur, devise a trading strategy to explore arbitrage opportunities. What is the minimum profit of your arbitrage portfolio?

3. Implement the multi-step binomial method as described in the Binomial Method lecture with the following variables and parameters: stock $S = 100$, interest rate $r = 0.05$ (continuously compounded) for a call option with strike $E = 100$, and maturity $T = 1$.

   (a) Using four time steps, calculate the value of the option for a range of volatilities and plot the results.

   (b) Then, fix volatility at $\sigma = 0.2$ and plot the value of the option as the number of time steps of the tree increases NTS = 4, 5, ...

Hint: This is a computational problem.

B. Portfolio Optimisation

The investment universe is composed of a set of following 4 assets:

Asset	μ	σ
A	0.04	0.07
B	0.08	0.12
C	0.12	0.18
D	0.15	0.26

With a correlation structure

$$R = \begin{pmatrix} 1 & 0.2 & 0.5 & 0.3 \\ 0.2 & 1 & 0.7 & 0.4 \\ 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.4 & 0.9 & 1 \end{pmatrix}$$

Denote the column vector of asset weights by $\mathbf{w}$, the column vector of asset returns μ, and the covariance matrix by Σ.

1. Compute the covariance matrix Σ.

2. Consider the following optimization:

$$\min_{\mathbf{w}} \frac{1}{2}\mathbf{w}^T\Sigma\mathbf{w}$$

   Subject to constraints

$$\mathbf{w}^T\,\mathbf{1} = 1$$

$$\mathbf{w}^T\mu = 0.1$$

   - Explain in plain English what this optimization does.
   - Solve this optimization using the Lagrangian method.
   - Compute the standard deviation of this optimal portfolio.
   - On a graph of expected returns plotted against standard deviation, identify this optimal portfolio.

3. Now, consider a less constrained optimization:

$$\min_{\mathbf{w}} \frac{1}{2}\mathbf{w}^T\Sigma\mathbf{w}$$

   Subject to a constraint

$$\mathbf{w}^T\,\mathbf{1} = 1$$

   - Explain in plain English what this optimization does.
   - Solve this optimization using the Lagrangian method.
   - Compute the return and standard deviation of this optimal portfolio.
   - On a graph of expected returns plotted against standard deviation, identify and name this optimal portfolio.

C. Empirical Value at Risk

For this section, use FTSE100 index returns data provided in order to calculate VaR on a rolling basis and provide charts.

1. Calculate the 99%/10day Value at Risk for an investment in the market index using a sample standard deviation.

    - Calculate the rolling 21-day sample standard deviation.
    - Timescale of the standard deviation is 'daily' regardless of how many days are in the sample. Project from 1-day to 10-day using the additivity of variance $\sigma_{10D} = \sqrt{10 \text{ x } \sigma^2}$.
    - Using constant mean for the whole dataset scaled over 10-day as $\mu_{10D} = \mu \text{ x } 10$ and Normal factor for 99% confidence calculate the VaR measure as

$$\text{VaR} = \mu_{10D} + \sigma_{10D} \text{ x Factor}$$

2. Calculate the 99%/10day Value at Risk for an investment in the market index using a GARCH.

    - Assume a GARCH(1,1) model has been estimated from the long-term data as follows:

$$\sigma^2 = 0.000001 + 0.047u_{t-1}^2 + 0.9466\sigma_{t-1}^2$$

    where $\omega = 0.000001$, $\alpha = 0.047$, and $\beta = 0.9466$.

    - Apply the same time-scaling rule and VaR formula as above. Set the initial value of σ_0^2 equal to the sample variance for all returns. Then, take past variance and return for $t-1$ from the row above to set up a recursive GARCH calculation as demonstrated in the Lecture.

3. Calculate the percentage of VaR breaches for both measures, using 10-day period on the rolling basis. Are the breaches independent in time and independent of the level of VaR (volatility)?

    - Assume that a breach occurs when a realised 10-day index return $u_{10D} = ln\ (S_t/S_{t-10})$ is below the VaR quantity. The risk measure assumed to be known and fixed at the start of each period.

    - Example: 20/08/2009 is the first day at which VaR computation is available, compare that figure to the index return realised in 10 days. Repeat. Total number of comparisons is N = 978.

Hint: This is a computational problem. The question about VaR breaches being an iid process can be answered by a brief analysis of observations.

*** END OF EXAM ***