MDTS-4113/Sem-1/Core 3/Module-1

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Assignment-2

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2.a) The dimension of the whole ~~...~~ $(n\times n)$ matrix space is 'n^2'.

2.b) The dimension of the subspace of diagonal matrices is 'n'.

3.a) [All f such that $f(x^2) = f^2(x)$]

This doesnot form a subspace of V.

For a counter-example, we take the constant function

$$f(x) = 1 \quad \forall x.$$

Clearly, $f \in S$, the set S being all such functions for which $f(x^2) = f^2(x)$

Now take a constant $c \in \mathbb{R}$ st. $c \neq 0, 1$.

Then, $cf(x) = c$, say $g(x) = cf(x)$.

$\therefore g^2(x) = c^2$.

But, $g(x) = c \Rightarrow g(x)$ is a constant function

$\Rightarrow g(x^2) = c$,

hence $g(x^2) \neq g^2(x)$.

$\Rightarrow cf(x) \notin S$ for $c \in \mathbb{R}$; violating constant-multiplacive closure property.

(2)

3.b) [All f such that $f(0) = f(1)$]

We take two functions f and $g \in S$, the linear combination of these two is :- $(\lambda_1 f + \lambda_2 g)$, $(\lambda_1, \lambda_2 \in \mathbb{R})$

Now, $(\lambda_1 f + \lambda_2 g)(0)$

$$= \lambda_1 f(0) + \lambda_2 g(0)$$
$$= \lambda_1 f(1) + \lambda_2 g(1) \quad = (\lambda_1 f + \lambda_2 g)(1) ;$$

Proving the linear combination $(\lambda_1 f + \lambda_2 g) \in S$.

Hence, this forms a subspace.

3.c) [All f such that $f(3) = 1 + f(-5)$]

This does not form a subspace. For a counter-example

we take $f(x)$ such that $f(x) = 1$ for $x = 3$; then $f \in S$
$= 0 \quad \forall x \neq 3$;

But, for $c \in \mathbb{R}$, a constant, $g(x) = cf(x) = c$ for $x = 3$
$= 0 \quad \forall x \neq 3$; $\quad$ -(1)

Put values from (1) in : $g(3) = 1 + g(-5)$ gives $g(-5) = c - 1$

which means $(c-1)$ is not necessarily always 0., hence

$f \in S$ but $cf \notin S$ for $c \in \mathbb{R}$.

3.d) [all f such that $f(-1)=0$]

We take f and g such that $f, g \in S$, and take their linear-combination : $(\lambda_1 f + \lambda_2 g)$, where $\lambda_1, \lambda_2 \in \mathbb{R}$.

$$(\lambda_1 f + \lambda_2 g)(-1)$$
$$= (\lambda_1 f(-1) + \lambda_2 g(-1)) = (0+0) = 0.$$

i.e., for $f, g \in S$, $(\lambda_1 f + \lambda_2 g) \in S$ $\forall$ $\lambda_1, \lambda_2 \in \mathbb{R}$.

Hence, this set forms a subspace.

3.e) [All f that are Continuous]

We know that linear combination of two continuous functions ~~are~~ is also continuous, hence this set also forms a subspace.

4. The column space of $A = \begin{pmatrix} 1 & 7 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ spans x-axis in $\mathbb{R}^3$.

The column space of $B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \\ 0 & 0 \end{pmatrix}$ spans the x-y plane in $\mathbb{R}^3$.

The column space of $C = \begin{pmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \end{pmatrix}$ spans a line passing through origin in $\mathbb{R}^3$.

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5) We take the matrix $A = \begin{pmatrix} 2 & -1 & 3 & 2 \\ -1 & 1 & 1 & -3 \\ 1 & 1 & 9 & -5 \\ 3 & -1 & 0 & -1 \end{pmatrix}$, and try

to calculate its row-echelon-form

$$A = \begin{pmatrix} 2 & -1 & 3 & 2 \\ -1 & 1 & 1 & -3 \\ 1 & 1 & 9 & -5 \\ 3 & -1 & 0 & -1 \end{pmatrix} \xrightarrow{R_1' = R_1/2} \begin{pmatrix} 1 & -1/2 & 3/2 & 1 \\ -1 & 1 & 1 & -3 \\ 1 & 1 & 9 & -5 \\ 3 & -1 & 0 & -1 \end{pmatrix}$$

$$\downarrow \begin{array}{l} R_2' = R_2 + R_1 \\ R_3' = R_3 - R_1 \\ R_4' = R_4 - 3R_1 \end{array}$$

$$\begin{pmatrix} 1 & -1/2 & 3/2 & 1 \\ 0 & +1 & 5 & -4 \\ 0 & 3/2 & 15/2 & -6 \\ 0 & 1/2 & -9/2 & +4 \end{pmatrix} \xleftarrow{R_2' = R_2 \times 2} \begin{pmatrix} 1 & -1/2 & 3/2 & 1 \\ 0 & 1/2 & 5/2 & -2 \\ 0 & 3/2 & 15/2 & -6 \\ 0 & 1/2 & -9/2 & -4 \end{pmatrix}$$

$$\downarrow R_1' = R_1 + R_2/2, \quad R_3' = R_3 - \frac{3R_2}{2}, \quad R_4' = R_4 - R_2/2$$

$$\begin{pmatrix} 1 & 0 & 4 & -1 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -7 & -2 \end{pmatrix}$$

Clearly, at this point we can see $\text{Rank}(A) < 4$, meaning the rows of A are linearly dependent.

Hence, the answer is yes.

~~1~~ ~~2~~

1\. On $\mathbb{R}^n$, define two operations:

$$\underline{\alpha} \oplus \underline{\beta} = \underline{\alpha} - \underline{\beta},$$

$$c \cdot \underline{\alpha} = -c\underline{\alpha}.$$

(i) Checking commutativity >

$$\underline{\alpha} \oplus \underline{\beta} = \underline{\alpha} - \underline{\beta}, \quad \underline{\beta} \oplus \underline{\alpha} = \underline{\beta} - \underline{\alpha}$$

Not satisfied.

(ii) Checking Associativity >

$$(\underline{\alpha} \oplus \underline{\beta}) \oplus \underline{\gamma} = (\underline{\alpha} - \underline{\beta}) \oplus \underline{\gamma} = \underline{\alpha} - \underline{\beta} \oplus \underline{\gamma} = \underline{\alpha} - \underline{\beta} - \underline{\gamma}$$

$$\underline{\alpha} \oplus (\underline{\beta} \oplus \underline{\gamma}) = \underline{\alpha} - (\underline{\beta} \oplus \underline{\gamma}) = \underline{\alpha} - \underline{\beta} + \underline{\gamma}$$

Not satisfied.

$(\forall \lambda_1, \lambda_2 \in F)$

$$(\lambda_1 \lambda_2)\underline{\alpha} = -\lambda_1\lambda_2\underline{\alpha}, \text{ and } \lambda_1(\lambda_2\underline{\alpha}) = \lambda_1 \cdot -\lambda_2\alpha = \lambda_1\lambda_2\alpha.$$

Not satisfied

Checking Additive identity >

$$\underline{\alpha} \oplus \underline{0} = \underline{\alpha} - 0, \text{ but } 0 \oplus \underline{\alpha} = 0 - \underline{\alpha}$$

Not satisfied

Checking Additive Inverse >

If there is no additive identity, there is no additive inverse as well.

Not satisfied

(6).

Checking multiplicative identity >

$1 \cdot \underline{\alpha} = -\alpha \neq \alpha$; hence not satisfied

Checking Distributive properties :>

$(\lambda_1 + \lambda_2) \cdot \underline{\alpha} = -(\lambda_1 + \lambda_2)\underline{\alpha} = -\lambda_1\underline{\alpha} \oplus -\lambda_2 \underline{\alpha}$

$\lambda_1 \underline{\alpha} \oplus \lambda_2 \cdot \underline{\alpha} = -\lambda_1 \underline{\alpha} \oplus -\lambda_2 \underline{\alpha} = -\lambda_1 \underline{\alpha} + \lambda_2 \underline{\alpha}$

Not satisfied

$\cancel{\lambda_1 \cdot (\underline{\alpha}_1 \oplus \underline{\alpha}_2)} = \lambda_1 \cdot (\underline{\alpha} \pm \underline{\beta}) = \lambda_1 \cdot (\underline{\alpha} - \underline{\beta})$

$= -\lambda_1 (\underline{\alpha} - \underline{\beta}) = -\lambda_1 \underline{\alpha} + \lambda_2 \underline{\beta}$

$\lambda_1 \cdot \underline{\alpha} + \lambda_1 \cdot \underline{\beta} = -\lambda_1 \underline{\alpha} \oplus -\lambda_2 \underline{\beta}$

$= -\lambda_1 \underline{\alpha} + \lambda_2 \underline{\beta}$

Only additive distributive property is satisfied.