DB_Nov02_01

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2022-11-02

Question 1: For the matrix $M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}$ find the following:

```
M = matrix(c(1,1,1,1,1,2,3,4,1,3,6,10,1,4,10,20),nrow = 4 ,byrow = T)
M

##      [,1] [,2] [,3] [,4]
## [1,]    1    1    1    1
## [2,]    1    2    3    4
## [3,]    1    3    6   10
## [4,]    1    4   10   20
```

Question 1(a): Trace of M.

```
tr(M)

## [1] 29
```

Quetion 1(b): Rank of M.

```
R(M)

## [1] 4
```

Question 1(c): Inverse of M.

```
inv(M)

##      [,1] [,2] [,3] [,4]
## [1,]    4   -6    4   -1
## [2,]   -6   14  -11    3
## [3,]    4  -11   10   -3
## [4,]   -1    3   -3    1
```

Question 1(d): Determinant of M using the functions `Det(A)` and `cofactor(A, i, j)`.

```
Det(M)

## [1] 1

d = 0
for (i in 1:4){
  d = d + M[1,i]*cofactor(M, 1, i)
}
d
```

```
## [1] 1
```

Question 2: Apply Gram-Schmidt orthogonalization to the following sequene of vectors: $(1,2,0), (8,1,-6), (0,0,1)$.

```
V = matrix(c(1,2,0,8,1,6,0,0,1), nrow = 3, byrow = F)
GM = GramSchmidt(V, normalize = F, verbose = F, tol = sqrt(.Machine$double.ep
s))
GM

##      [,1] [,2]       [,3]
## [1,]    1    6 -0.4444444
## [2,]    2   -3  0.2222222
## [3,]    0    6  0.5555556
```

Question 3: Check whether the following vectors are independent:

3.(i) $(2,3,1), (1,0,4), (2,4,1), (0,3,2)$.

```
M1 = matrix(c(2,3,1,1,0,4,2,4,1,0,3,2), nrow = 3, byrow = F)
R(M1)

## [1] 3
```

Rank of the matrix formed by the given vectors(3) are less than number of vectors(4); hence the given vectors are not independent.

3.(ii) $(1,3,-1,0), (2,9,-1,3), (4,5,6,11), (1,-1,2,5), (3,-2,6,7)$.

```
M2 = matrix(c(1,3,-1,0,2,9,-1,3,4,5,6,11,1,-1,2,5,3,-2,6,7), nrow = 4, byrow
= F)
R(M2)

## [1] 4
```

The given vectors are independent.

3.(iii) $(1,1,0), (3,0,1), (5,2,1)$.

```
M3 = matrix(c(1,1,0,3,0,1,5,2,1), nrow = 3, byrow = F)
R(M3)

## [1] 2
```

The given vectors are not independent.

3.(iv) $(1,4,1,7), (3,-5,2,3), (2,-1,6,9), (-2,3,1,6)$.

```
M4 = matrix(c(1,4,1,7,3,-5,2,3,2,-1,6,9,-2,3,1,6), nrow = 4, byrow = F)
R(M4)

## [1] 4
```

The given vectors are independent.

Question 4: Find the rank and basis of the row space, column space, null space and the left null space of the following matrices:

4.(i) $A = \begin{bmatrix} 1 & -1 & 13 & 2 \\ 2 & -1 & 15 & 1 \\ 3 & -1 & 17 & 0 \\ 1 & -1 & -1 & -3 \end{bmatrix}$.

```
A1 = matrix(c(1,-1,13,2,2,-1,15,1,3,-1,17,0,1,-1,-1,-3), nrow = 4, byrow = T)
R(A1)

## [1] 3

rref(A1)

##      [,1] [,2] [,3]       [,4]
## [1,]    1    0    0 -1.7142857
## [2,]    0    1    0  0.9285714
## [3,]    0    0    1  0.3571429
## [4,]    0    0    0  0.0000000

t(A1)

##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    1
## [2,]   -1   -1   -1   -1
## [3,]   13   15   17   -1
## [4,]    2    1    0   -3

rref(t(A1))

##      [,1] [,2] [,3] [,4]
## [1,]    1    0   -1    0
## [2,]    0    1    2    0
## [3,]    0    0    0    1
## [4,]    0    0    0    0

null(t(A1))

##               [,1]
## [1,] -4.082483e-01
## [2,]  8.164966e-01
## [3,] -4.082483e-01
## [4,]  9.714451e-17

null(A1)

##            [,1]
## [1,]  0.7721873
## [2,] -0.4182681
## [3,] -0.1608724
## [4,]  0.4504426
```

- Rank of A is 3.
- Column 4 of `rref(A1)` is linear combination of the other 3 columns. Hence, the column space of A is spanned by (1, 2, 3, 1), (-1, -1, -1, -1), (13, 15, 17, -1).
- Column 3 of `rref(t(A1))` is linear combination of the other 3 columns. Hence, the row space of A is spanned by (1, -1, 13, 2), (2, -1, 15, 1), (1, -1, -1, -3).
- The basis of null space of A is the vector given by `null(t(A1))`; hence rank is 1.
- The basis of the left null space of A is the vector given by `null(A1)`; hence rank is 1.

4.(ii) $A = \begin{bmatrix} 5 & 10 & 15 & 20 & 25 & 30 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 9 & 11 & 13 & 15 & 17 \\ 50 & 49 & 48 & 47 & 46 & 45 \\ -5 & -6 & -7 & -8 & -9 & -10 \end{bmatrix}$.

```
B1 = matrix(c(5,10,15,20,25,30,1,2,3,4,5,6,7,9,11,13,15,17,50,49,48,47,46,45,
-5,-6,-7,-8,-9,-10), nrow = 5, byrow = T)
R(B1)

## [1] 2

rref(B1)

##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    0   -1   -2   -3   -4
## [2,]    0    1    2    3    4    5
## [3,]    0    0    0    0    0    0
## [4,]    0    0    0    0    0    0
## [5,]    0    0    0    0    0    0

t(B1)

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    5    1    7   50   -5
## [2,]   10    2    9   49   -6
## [3,]   15    3   11   48   -7
## [4,]   20    4   13   47   -8
## [5,]   25    5   15   46   -9
## [6,]   30    6   17   45  -10

rref(t(B1))

##      [,1] [,2] [,3]  [,4]  [,5]
## [1,]    1  0.2    0 -4.28  0.12
## [2,]    0  0.0    1 10.20 -0.80
## [3,]    0  0.0    0  0.00  0.00
## [4,]    0  0.0    0  0.00  0.00
## [5,]    0  0.0    0  0.00  0.00
## [6,]    0  0.0    0  0.00  0.00
```

```
null(t(B1))

##              [,1]        [,2]       [,3]
## [1,] -0.32018842 -0.27151303 0.17219535
## [2,] -0.25097129  0.94591029 0.10248308
## [3,]  0.90817610  0.16683978 0.04252145
## [4,] -0.08514773 -0.02083818 0.07238916
## [5,]  0.04958652 -0.05713713 0.97611358

null(B1)

##             [,1]        [,2]        [,3]        [,4]
## [1,] -0.3451263 -0.34506396 -0.34500158 -0.34493921
## [2,] -0.1480991  0.13469373  0.41748654  0.70027934
## [3,]  0.8967633 -0.08611058 -0.06898448 -0.05185837
## [4,] -0.1188747  0.83969151 -0.20174232 -0.24317615
## [5,] -0.1345126 -0.23450640  0.66549984 -0.43449392
## [6,] -0.1501506 -0.30870430 -0.46725800  0.37418830
```

- The rank of B is 2.
- Columns 3, 4, 5, 6 of `rref(B1)` are some linear combinations of columns 1 and 2; hence a basis of the column space of B would be: (5, 1, 7, 50, -5), (10, 2, 9, 49, -6).
- Columns 2, 4, 5 of `rref(t(B1))` are some linear combinations of columns 1 and 3; hence a basis of the row space of B would be: (5, 10, 15, 20, 25, 30), (7, 9, 11, 13, 15, 17).
- The basis of the null space of B is the 3 column vectors of `null(t(B1))`; hence the rank is 3.
- The basis of the left null space of B is the 4 column vectors of `null(B1)`; hence the rank is 4.

4.(iii) Find the orthogonal basis for the row space of the matrix A in 4(i).

```
#Gram-Schmidt orthogonalization on basis vectors of row space of A
C = matrix(c(1,-1,13,2,2,-1,15,1,1,-1,-1,-3), nrow = 4, byrow = F)
GramSchmidt(C, normalize = F, verbose = F, tol = sqrt(.Machine$double.eps))

##      [,1]       [,2]         [,3]
## [1,]    1  0.8571429 -0.465882353
## [2,]   -1  0.1428571 -1.357647059
## [3,]   13  0.1428571  0.002352941
## [4,]    2 -1.2857143 -0.461176471
```

An orthogonal basis of row space of A is the three column vectors of the matrix above.