MDTS - 4113 / Sem 1 / core 3

Assignment - 1

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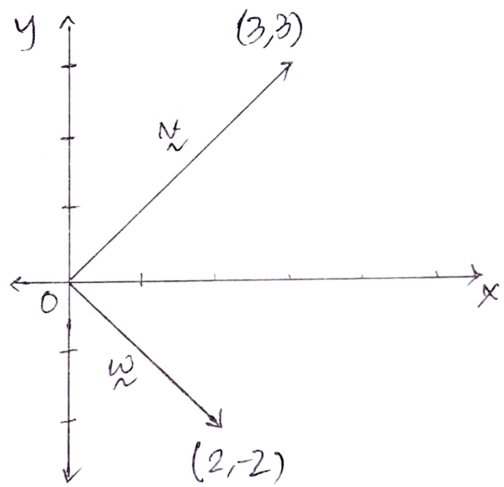
Answer 1)

$\underset{\sim}{v} + \underset{\sim}{w} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$; $\underset{\sim}{v} - \underset{\sim}{w} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$\therefore \underset{\sim}{v} = \frac{1}{2}\begin{pmatrix} 5+1 \\ 1+5 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$; $\underset{\sim}{w} = \frac{1}{2}\begin{pmatrix} 5-1 \\ 1-5 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$



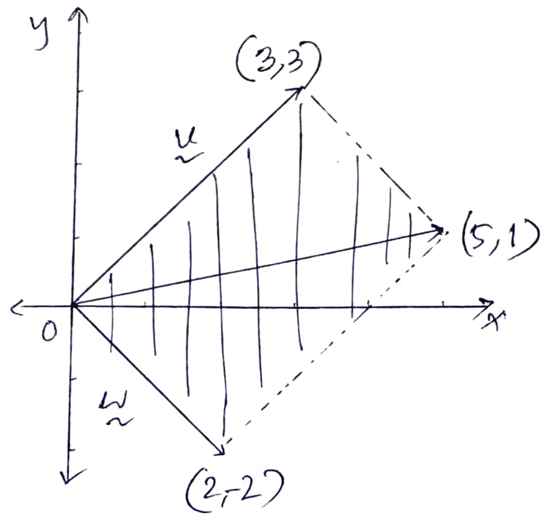
Answer 2)

Shade of the region bounded by

$[c\underset{\sim}{v} + d\underset{\sim}{w}]$;

where $c, d \in [0,1]$:→

Answer) The rectangle formed by the points $(0,0), (3,3), (5,1), (2,-2)$ on the x-y plane.



Answer 3)

• A Unit vector $\underset{\sim}{u}$ in the direction of $\underset{\sim}{v} = (3,7)$

$$= \frac{1}{|\underset{\sim}{v}|} \cdot \underset{\sim}{v} = \frac{1}{\sqrt{9+49}}(3,7) = \left(\frac{3}{\sqrt{58}}, \frac{7}{\sqrt{58}}\right)$$

• A Unit vector $\underline{e}$ that is perpendicular to $\underline{u} = \left(\frac{3}{\sqrt{58}}, \frac{7}{\sqrt{58}}\right)$

would be $\left(\frac{7}{\sqrt{58}}, \frac{-3}{\sqrt{58}}\right)$.

• There are two possibilities for $\underline{e}$, the other one being

$$\left(\frac{-7}{\sqrt{58}}, \frac{3}{\sqrt{58}}\right)$$

Answer 4)

(i) Angle b/w $\underset{\sim}{u} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$= \cos^{-1}\left(\frac{1\cdot 1 + \sqrt{3}\cdot 0}{\sqrt{1+3} \cdot \sqrt{1+0}}\right) = \cos^{-1}\frac{1}{2} = 60^\circ$$

(ii) Angle b/w $\underset{\sim}{u} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$:

$$= \cos^{-1}\left(\frac{2\cdot 2 + 2\cdot(-1) + (-1)\cdot 2}{\sqrt{4+4+1} \cdot \sqrt{4+1+4}}\right) = \cos^{-1} 0 = 90^\circ$$

Answer 5 >

The plane perpendicular to $(1,1,0)$ consists of all vectors of the form $(c,-c,d)$, $[c,d \in \mathbb{R}]$.

In this plane, $(1,-1,0)$ and $(0,0,1)$ are perpendicular to each-other.

Hence, the two examples of vectors that are perpendicular to $(1,1,0)$ and to each-other would be: $(1,-1,0)$ and $(0,0,1)$