Assignment 4 (DB) - Answers

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Question

Solve the following system of linear equations using Gauss elimination method:

1.

$$\begin{aligned} 4y + 2z &= 1 \\ 2x + 3y + 5z &= 0 \\ 3x + y + z &= 11 \end{aligned}$$

2.

$$\begin{aligned} 3x_1 + 6x_2 - 9x_3 &= 15 \\ 2x_1 + 4x_2 - 6x_3 &= 10 \\ -2x_1 - 3x_2 + 4x_3 &= -6 \end{aligned}$$

Answer

1.

The given system of linear equations has the following augmented matrix: $A = \left(\begin{array}{ccc|c} 0 & 4 & 2 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 1 & 1 & 11 \end{array}\right)$.

$$A = \left(\begin{array}{ccc|c} 0 & 4 & 2 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 1 & 1 & 11 \end{array}\right) \xrightarrow{R_{12}} \left(\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & 4 & 2 & 1 \\ 3 & 1 & 1 & 11 \end{array}\right) \xrightarrow{R_1 = \frac{1}{2} \times R_1} \left(\begin{array}{ccc|c} 1 & 1.5 & 2.5 & 0 \\ 0 & 4 & 2 & 1 \\ 3 & 1 & 1 & 11 \end{array}\right) \xrightarrow{R_3 = R_3 - 3 \times R_1}$$

$$\left(\begin{array}{ccc|c} 1 & 1.5 & 2.5 & 0 \\ 0 & 4 & 2 & 1 \\ 0 & -3.5 & -6.5 & 11 \end{array}\right) \xrightarrow{R_2 = \frac{R_2}{4}} \left(\begin{array}{ccc|c} 1 & 1.5 & 2.5 & 0 \\ 0 & 1 & 0.5 & 0.25 \\ 0 & -3.5 & -6.5 & 11 \end{array}\right) \xrightarrow[R_3 = R_3 + 3.5 \times R_2]{R_1 = R_1 - 1.5 \times R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1.75 & -0.375 \\ 0 & 1 & 0.5 & 0.25 \\ 0 & 0 & -4.75 & 11.875 \end{array}\right)$$

$$\xrightarrow{R_3=\frac{R_3}{-4.75}} \left(\begin{array}{ccc|c} 1 & 0 & 1.75 & -0.375 \\ 0 & 1 & 0.5 & 0.25 \\ 0 & 0 & 1 & -2.5 \end{array}\right) \xrightarrow[R_2=R_2-0.5\times R_3]{R_1=R_1-1.75\times R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & -2.5 \end{array}\right).$$

Hence, the solution to the given system of linear equations is: $\begin{cases} x = 4 \\ y = 1.5 \\ z = -2.5 \end{cases}$.

2.

The given system of linear equation has the following augmented matrix: $B = \left(\begin{array}{ccc|c} 3 & 6 & -9 & 15 \\ 2 & 4 & -6 & 10 \\ -2 & -3 & 4 & -6 \end{array}\right)$.

$$B = \left(\begin{array}{ccc|c} 3 & 6 & -9 & 15 \\ 2 & 4 & -6 & 10 \\ -2 & -3 & 4 & -6 \end{array}\right) \xrightarrow{R_1=\frac{R_1}{3}} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 2 & 4 & -6 & 10 \\ -2 & -3 & 4 & -6 \end{array}\right) \xrightarrow[R_3=R_3+2\times R_1]{R_2=R_2-2\times R_1} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 4 \end{array}\right) \xrightarrow{R_{23}}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1=R_1-2\times R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

Hence, there are infinitely many solutions for the given system of linear equations, subject to the constraints: $\begin{cases} x_1 + x_3 = -3 \\ x_2 - 2x_3 = 4 \end{cases}$; e.g. $(0, -2, -3)$ is a solution.