SC-MD_Nov-21_1

Srijan Kundu

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```r
set.seed(1)
```

Question 1:

Let X be a random variable such that $X \sim Bin(10, 0.3)$. Then,

Question 1(a):

Find: - $P(X = 5)$

```r
dbinom(5, 10, 0.3)
```

```
## [1] 0.1029193
```

- $P(X \leq 1)$

```r
pbinom(1, 10, 0.3)
```

```
## [1] 0.1493083
```

- $P(X > 4)$

```r
pbinom(4, 10, 0.3, lower.tail = FALSE)
```

```
## [1] 0.1502683
```

- The median of X

```r
qbinom(0.5, 10, 0.3)
```

```
## [1] 3
```

Question 1(b)

Draw a random sample of size 100 from the distribution of X and obtain the proportion of observations which are ≤ 3.

```
data = rbinom(100, 10, 0.3)
length(data[data <= 3])/100
```

```
## [1] 0.61
```

Question 2:

A car hire company has two cars which it hires out a day. The number of demands for a car on each day has a Poisson distribution with mean 1.5. Find the proportion of days on which:

Question 2(a):

Neither car is used:

```
dpois(0, 1.5)
```

```
## [1] 0.2231302
```

Question 2(b):

One car remains idle:

```
dpois(1, 1.5)
```

```
## [1] 0.3346952
```

Question 2(c):

Some demands are refused:

```
ppois(2, 1.5, lower.tail = FALSE)
```

```
## [1] 0.1911532
```

Question 3:

A batch of parts contains 100 parts from a local supplier of tubing and 200 parts from a supplier of the neighboring state. If four parts are selected randomly and without replacement

Question 3(a):

What is the probability that they are all from the local supplier?

```
dhyper(4, 100, 200, 4)
```

```
## [1] 0.01185408
```

Question 3(b):

What is the probability that two or more parts in the sample are from the local supplier?

```
phyper(1, 100, 200, 4, lower.tail = FALSE)
```

```
## [1] 0.4074057
```

Question 3(c):

What is the probability that at least one part in the sample is from the local supplier?

```
1 - phyper(0, 100, 200, 4)
```

```
## [1] 0.8044538
```

Question 4:

Suppose a fair coin is tossed until the first head appears. Find the probability that:

Question 4(i):

There will be exactly five tails before the first head appears.

```
dgeom(5, 0.5)
```

```
## [1] 0.015625
```

Question 4(ii):

There will be at most 2 tails before the first head.

```
pgeom(2, 0.5)
```

```
## [1] 0.875
```

Question 4(iii):

There will be at least 5 tails before the first head appears.

```
pgeom(4, 0.5, lower.tail = FALSE)
```

```
## [1] 0.03125
```

Question 4(iv):

The median number of tails before the first head.

```r
qgeom(0.5, 0.5)
```

```
## [1] 0
```

Question 5:

When Stephane plays chess against his favorite computer program, he wins with probability 0.60. Assuming independence find the probability that Stephane's fifth win happens when he plays his eighth game.

```r
dnbinom(3, 5, 0.6)
```

```
## [1] 0.1741824
```

Here, $x \sim NB(5, 0.6)$*,where* x *is the number of failures preceding* r^{th} *success.*

Question 6:

The height of students in a large college is found to have a normal distribution with mean 162.50 cm and standard deviation 6 cm. Find the probability that a student selected at random will have:

Question 6(i):

Height greater than 168 cm:

```r
pnorm(168, 162.5, 6, lower.tail = FALSE)
```

```
## [1] 0.1796587
```

Question 6(ii):

Height less than or equal to 150 cm:

```r
pnorm(150, 162.5, 6, lower.tail = TRUE)
```

```
## [1] 0.01861043
```

Question 6(iii):

Height between 150 and 168 cm

```r
pnorm(168, 162.5, 6) - pnorm(150, 162.5, 6)
```

```
## [1] 0.8017309
```

Question 7:

The marks obtained by candidates in Mathematics (full marks 100) in a certain examination are found to be normally distributed with a certain mean and standard deviation. If 10% of the candidates obtain 60% or more marks, 40% failed to pass (minimum marks for pass is 30), find the mean and standard deviation of the distribution of marks.

```
qnorm(0.4, 0, 1)
```

```
## [1] -0.2533471
```

```
qnorm(0.1, 0, 1, lower.tail = FALSE)
```

```
## [1] 1.281552
```

Here, we need to solve for μ & σ where

$$\frac{30-\mu}{\sigma} = -0.25, \text{ and}$$
$$\frac{60-\mu}{\sigma} = 1.28$$

Question 8:

It is known that the lifetime t of electron tubes is distributed as exponential with mean m. Find the probability that an electron tube chosen at random survives for more than 400 hours when m=200.

```
pexp(400, 1/200, lower.tail = FALSE)
```

```
## [1] 0.1353353
```

For what value of m is this probability 0.5?

```
qexp(0.5, 1/400)
```

```
## [1] 277.2589
```

Question 9:

A daily consumption of milk in a city, in excess of 20000 liters, is approximately distributed as a Gamma with shape parameter 0.0001 and scale parameter 2. The city has a daily stock of 30000 liters. What is the probability that the stock is insufficient on a particular day?

Question 10:

The percentage of impurities per batch in a certain chemical product is a random variable X following a beta(3,2) distribution. A batch with more than 40% impurities cannot be sold. What is the probability that a randomly selected batch cannot be sold because of excessive impurities?

Question 11:

Suppose the lifetime of a motor has a lognormal distribution with shape parameter 11 hours and scale parameter 1.3 hours. What is the probability that the lifetime exceeds 12,000 hours?