Non-classical Behaviour and Pattern Formation in DOPO

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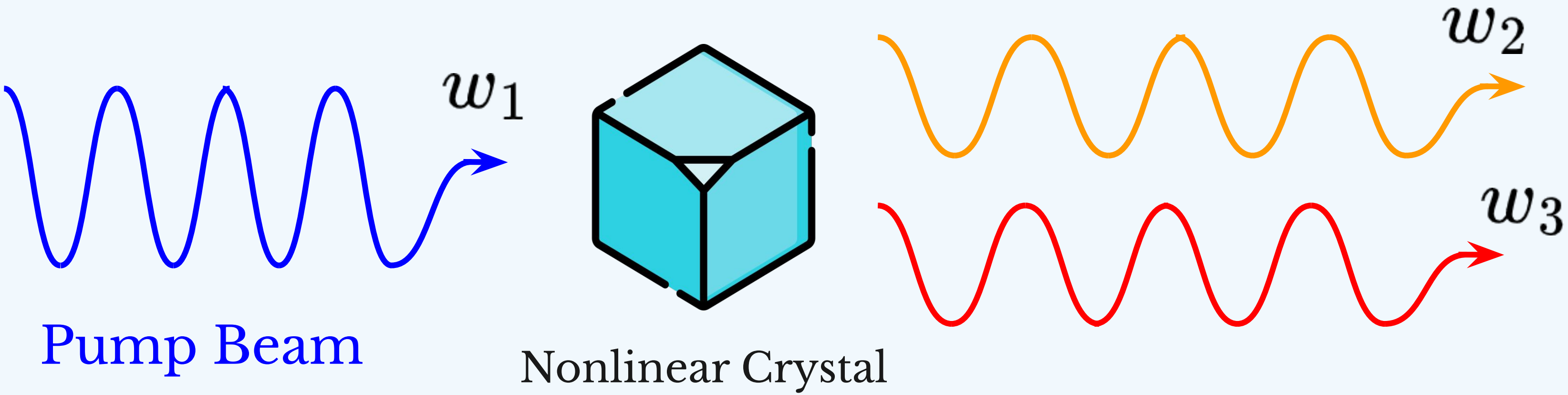
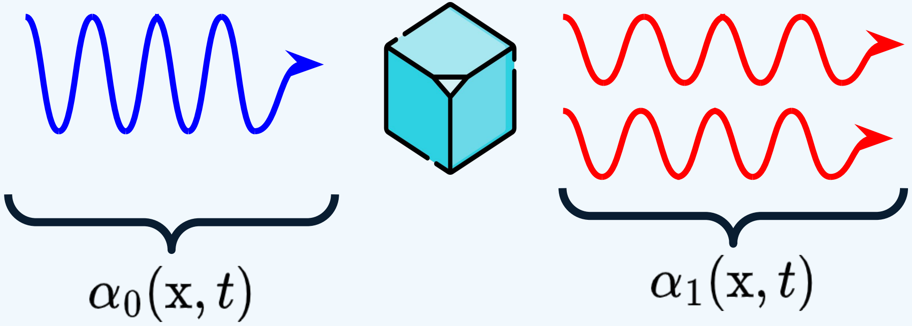
Introduction

The **Optical Parametric Oscillator** (OPO) employs a nonlinear crystal in a cavity to convert a high intensity **pump beam**, in two coherent beams at lower energies.



Degenerated Optical Parametric Oscillator (DOPO): $w_2 = w_3$

DOPO: Equations in Q-representation



Objectives

- Study pattern formation in the system.
- Understand how does the pattern change with the system size.

$$\begin{cases} \partial_t \alpha_0(\mathrm{x}, t) = -\left[(1 + \mathrm{i}\Delta_0) - i\partial_x^2\right]\alpha_0(\mathrm{x}, t) + E - \dfrac{1}{2}\alpha_1^2(\mathrm{x}, t) + \sqrt{\dfrac{2}{d}\dfrac{g}{\gamma}}\xi_0(\mathrm{x}, t), \\ \partial_t \alpha_1(\mathrm{x}, t) = -\left[(1 + \mathrm{i}\Delta_1) - 2i\partial_x^2\right]\alpha_1(\mathrm{x}, t) + \alpha_0(\mathrm{x}, t)\alpha_1^*(\mathrm{x}, t) + \sqrt{\dfrac{2}{d}\dfrac{g}{\gamma}}\xi_1(\mathrm{x}, t) \end{cases}$$

$$\xi_1(x,t) = \left[\frac{-\alpha_{0\mathrm{I}}(x,t)}{2\sqrt{2+\alpha_{0\mathrm{R}}(x,t)}} + \frac{\mathrm{i}}{2}\sqrt{2+\alpha_{0\mathrm{R}}(x,t)}\right]\phi(x,t) + \sqrt{\frac{1-\frac{|\alpha_0(x,t)|^2}{4}}{2+\alpha_{0\mathrm{R}}(x,t)}}\psi(x,t)$$

$\xi_0(x,t),\ \psi(x,t),\ \phi(x,t)$ are uncorrelated gaussian white noises in space and time.

<u>The set of equations can be concisely written as:</u>

$$\partial_t \alpha_k(\mathrm{x},t) = -\left[z_k - a_k i\partial_x^2\right]\alpha_k(\mathrm{x},t) + f_k(\alpha_0(\mathrm{x},t),\alpha_1(\mathrm{x},t),E) + b_k\xi_k(\mathrm{x},t)$$

$$z_k = 1 + \mathrm{i}\Delta_k,\quad a_0 = 1,\quad a_1 = 2,\quad b_k = \sqrt{\frac{2}{d}\frac{g}{\gamma}},\quad k = 0,1$$

$$f_0(\alpha_1(\mathrm{x},t),E) = E - \frac{1}{2}\alpha_1^2(\mathrm{x},t),\quad f_1(\alpha_0(\mathrm{x},t),\alpha_1(\mathrm{x},t)) = \alpha_0(\mathrm{x},t)\alpha_1^*(\mathrm{x},t).$$

Pattern Formation: Linear Stability

Classical equations (zero noise)

$$\begin{cases} \partial_t \alpha_0(\mathrm{x},t) = -\left[(1+\mathrm{i}\Delta_0) - i\partial_x^2\right]\alpha_0(\mathrm{x},t) + E - \dfrac{1}{2}\alpha_1^2(\mathrm{x},t) \\ \partial_t \alpha_1(\mathrm{x},t) = -\left[(1+\mathrm{i}\Delta_1) - 2i\partial_x^2\right]\alpha_1(\mathrm{x},t) + \alpha_0(\mathrm{x},t)\alpha_1^*(\mathrm{x},t) \end{cases}$$

Steady and homogeneous solution

$$\alpha_0^{st} = \frac{E}{1+i\Delta_0}$$

$$\alpha_1^{st} = 0$$

Dispersion Relation

$$\lambda_\pm(k) = -1 \pm \sqrt{|\alpha_0^{st}| - (\Delta_1 + 2k^2)^2}$$

Critical values

$$k_c = \sqrt{-\Delta_1/2} \qquad E_c = \sqrt{1+\Delta_0^2}$$

Numerical Solution

- Using the centered-space Heun method, the numerical integration is given by:

$$
\begin{aligned}
A^{(1)}_{k,n}(t_j) =& A_{k,n}(t_j) + \Delta t\left[-z_k A_{k,n}(t_j) + D_k \delta A_{k,n}(t_j) + f_k\right] + b_k\sqrt{\frac{\Delta t}{\Delta x}}w_{k,n}(t_j) \\
A_{k,n}(t_{j+1}) =& A_{k,n}(t_j) - \frac{\Delta t z_k}{2}\left(A_{k,n}(t_j) + A^{(1)}_{k,n}(t_j)\right) + \frac{\Delta t D_k}{2}\left(\delta A_{k,n}(t_j) + \delta A^{(1)}_{k,n}(t_j)\right) \\
&+ \frac{\Delta t}{2}\left(f_k(A_{0,n}(t_j), E) + f_k(A^{(1)}_{0,n}(t_j), E)\right) + \frac{1}{2}b_k\sqrt{\frac{\Delta t}{\Delta x}}(w_{k,n}(t_j) + w^{(1)}_{k,n}(t_j))
\end{aligned}
$$

$$
D_k \equiv i\frac{a_k}{(\Delta \mathrm{x})^2}, \quad w_{0,n}(t_j) = w^{(1)}_{0,n}(t_j) \equiv u_n(t_j), \quad w_{1,n}(t_j) \equiv \sum_{l=1}^{2} u_{l,n}(t_j) g_l(A_{0,n}(t_j)),
$$

$$
w^{(1)}_{1,n}(t_j) \equiv \sum_{l=1}^{2} u_{l,n}(t_j) g_l(A^{(1)}_{0,n}(t_j)), \quad \delta A_{k,n}(t_j) \equiv A_{k,n+1}(t_j) - 2A_{k,n}(t_j) + A_{k,n-1}(t_j)
$$

Parameters

$$\Delta_0 = 0,\ \ \Delta_1 = -0.18,\ \ \frac{g}{\sqrt{d}\gamma} = 10^{-4}$$

$$E_c = 1.0,\ \ k_c = 0.3,\ \ \lambda_c = \frac{2\pi}{k_c}$$

Initial Conditions

$$A_{0,n}(0) = E\ \ \forall n,\ \ A_{1,n}(0) = 10^{-5}(\epsilon(x_n) + 10\sin(k_c x_n))$$

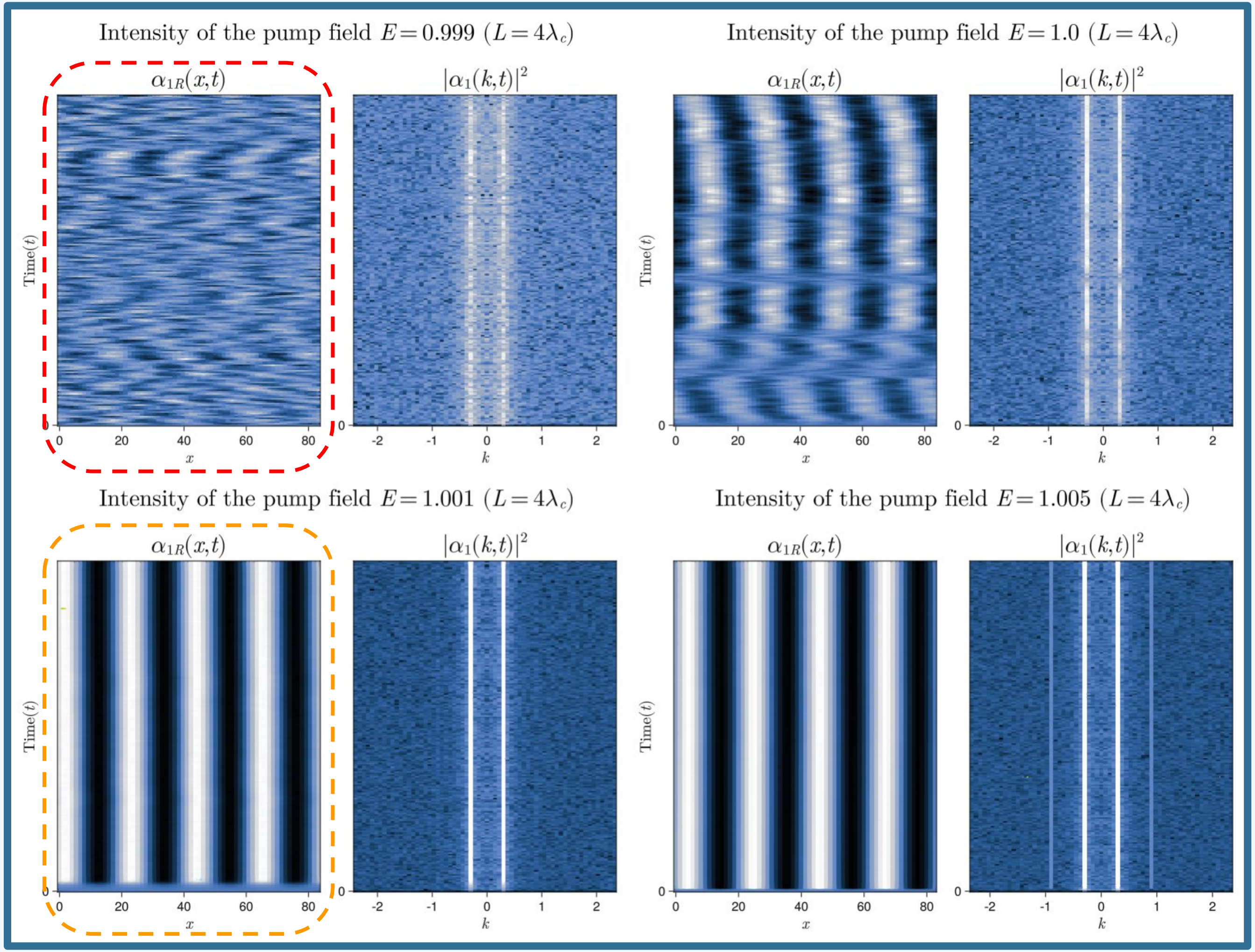
Discretization

$$\Delta t = 0.01,\ \ \Delta x = \frac{L}{n-1},\ n = 64$$

Results for $L = 4\lambda_c$

Noisy precursor produced by quantum fluctuations

By slightly increasing the intensity of the pump field above threshold, the pattern becomes fully formed

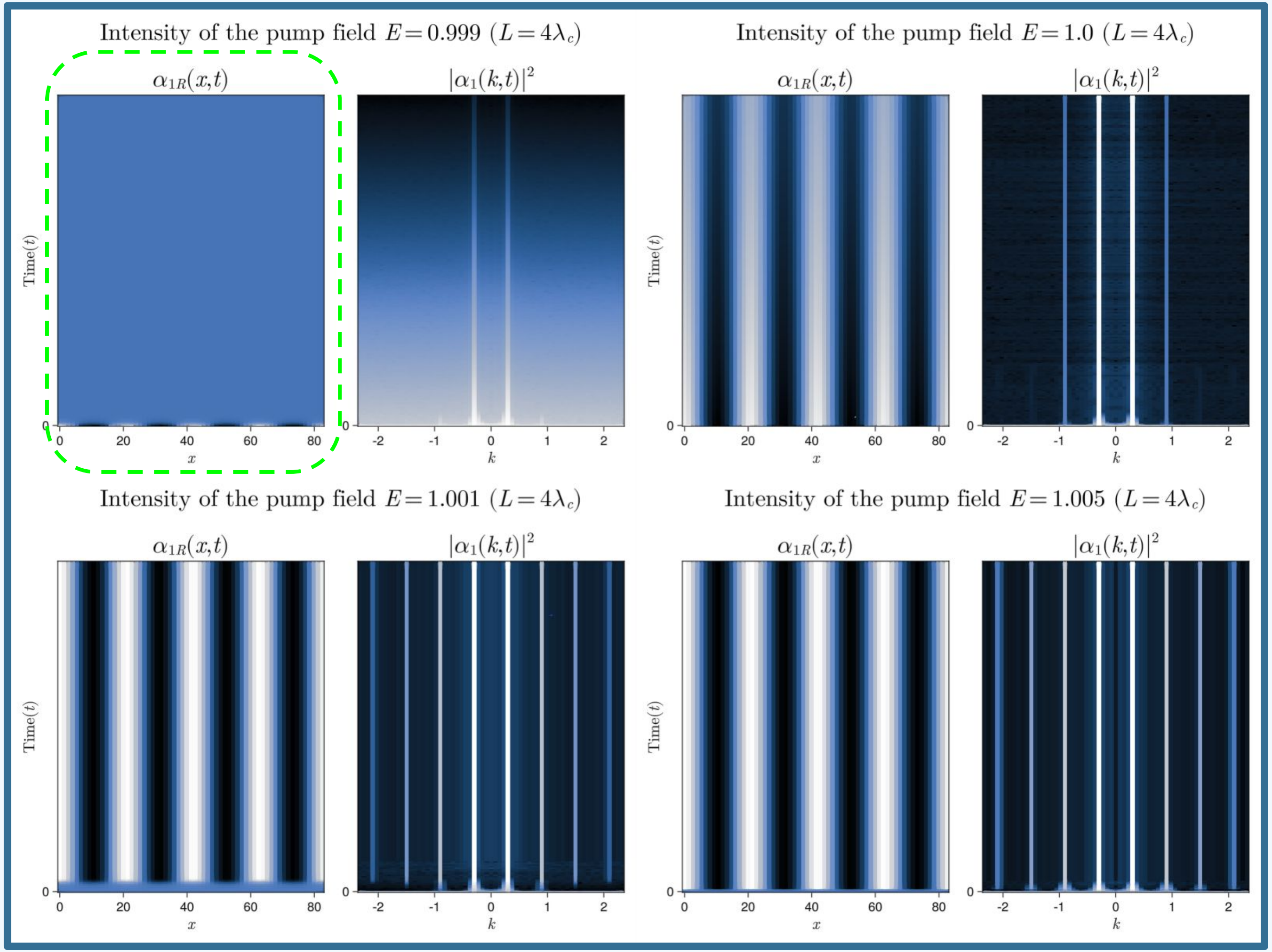


Results for $L = 4\lambda_c$

Zero noise

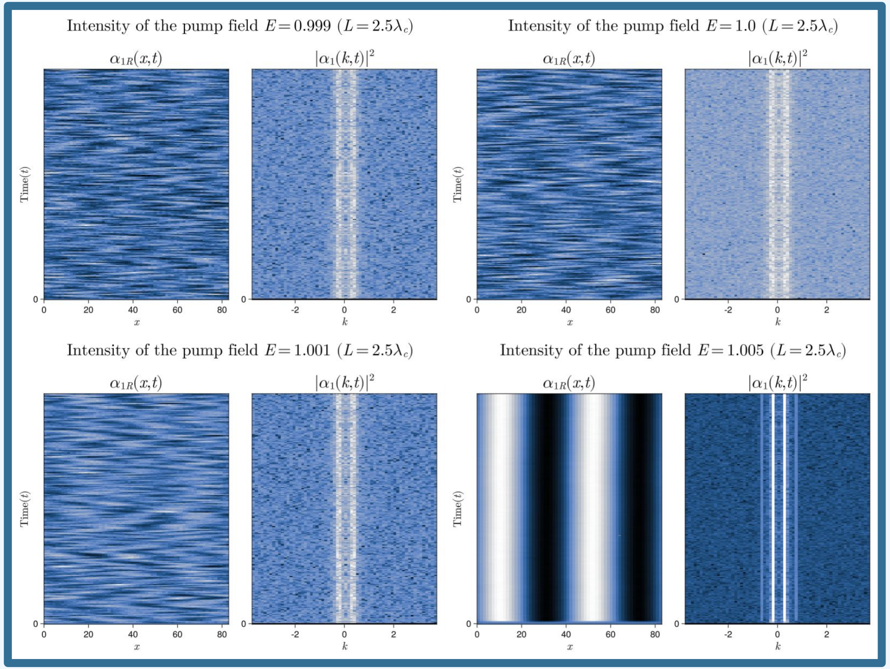
$$b_k = 0$$

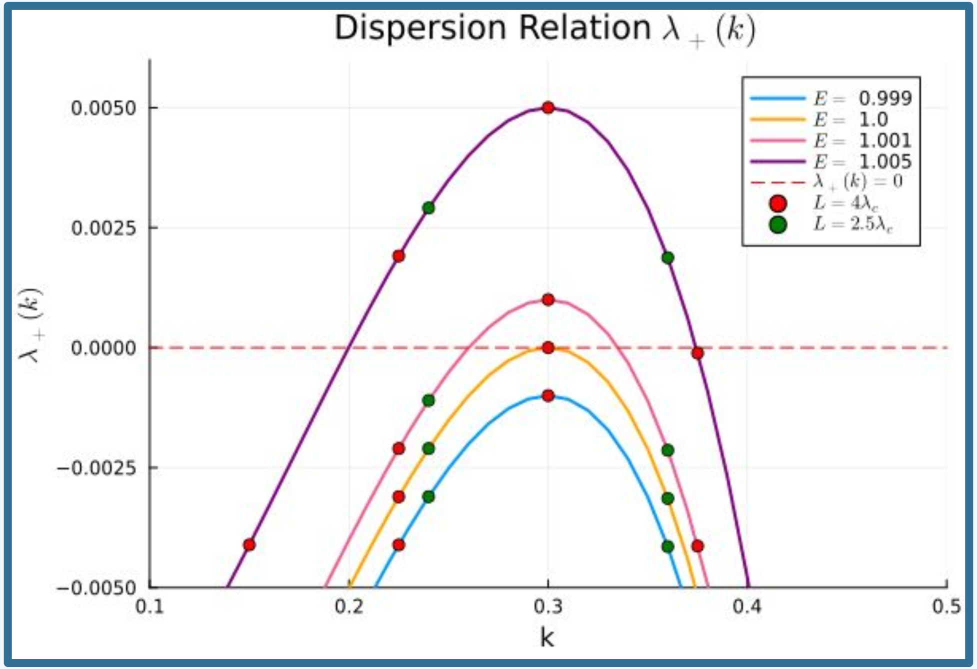
Turning off the noise, we obtain an homogeneous field



Results for $L = 2.5\lambda_c$

The threshold can change due to the finite size of the system.





Conclusions

- The noisy precursor produced by quantum fluctuations has been obtained as well as the striped pattern that appears above threshold.
- We have shown that the threshold can change due to the finite size effects.

Thanks for your attention

Reference

- Roberta Zambrini et al. “Non-classical behavior in multimode and disordered transverse structures in OPO: Use of the Q-representation”. In:Springer 22 (2003), pp. 461–471. DOI: `https://doi.org/10.1140/epjd/e2003-00019-7`

Appendix: Generalized Stochastic Heun Method

$$\frac{d\mathrm{x}}{dt} = q\big(\mathrm{x}(t), t\big) + \sum_{j=1}^{d} g_j\big(\mathrm{x}(t), t\big)\xi_j(t)$$

$$\mathrm{x}^{(1)}(t_i) = \mathrm{x}(t_i) + \Delta t q\big(\mathrm{x}(t_i), t_i\big) + \Delta t \sum_{j=1}^{d} u_j(t_i) g\big(\mathrm{x}(t_i), t_i\big)$$

$$\begin{aligned}\mathrm{x}(t_{i+1}) = & \mathrm{x}(t_i) + \frac{\Delta t}{2}\Big[q\big(\mathrm{x}(t_i), t_i\big) + q\big(\mathrm{x}^{(1)}(t_i), t_{i+1}\big)\Big] \\ & + \frac{\sqrt{\Delta t}}{2}\sum_{j=1}^{d} u_j(t_i)\Big[g\big(\mathrm{x}(t_i), t_i\big) + g\big(\mathrm{x}^{(1)}(t_i), t_{i+1}\big)\Big]\end{aligned}$$

Appendix: Finite differences, Centered-Space Method

$$A_{k,n}(t) = \frac{1}{\sqrt{\Delta \mathrm{x}}} \int_{\mathrm{x}_n}^{\mathrm{x}_{n+1}} \alpha_k(\mathrm{x}, t) d\mathrm{x}, \quad \xi^{cg}_{k,n}(t) = \frac{1}{\sqrt{\Delta \mathrm{x}}} \int_{\mathrm{x}_n}^{\mathrm{x}_{n+1}} \xi_k(\mathrm{x}, t) d\mathrm{x}$$

$$\frac{dA_{k,n}(t)}{dt} = - z_k A_{k,n}(t) + a_k i \left[\frac{A_{k,n+1}(t_i) - 2A_{k,n}(t_i) + A_{k,n-1}(t_i)}{(\Delta \mathrm{x})^2} \right] + f_k(A_{k,0}(t), A_{k,1}(t), t), E) + \frac{b_k}{\sqrt{\Delta x}} \xi_{k,n}(t)$$

Appendix: Code Implementation

$$
\begin{aligned}
\lambda_{k,n}(t_j) =& \Delta t \left[-z_k A_{k,n}(t_j) + i \frac{a_k}{(\Delta \mathrm{x})^2} \delta A_{k,n}(t_j) + f_k(A_{0,n}(t_j), A_{1,n}(t_j), E) \right] + b_k \sqrt{\frac{\Delta t}{\Delta x}} w_{k,n}, \\
A_{k,n}^{(1)}(t_j) =& A_{k,n}(t_j) + \lambda_{k,n}(t_j), \\
A_{k,n}(t_{j+1}) =& A_{k,n}(t_j) + \frac{1}{2}\lambda_k + \frac{\Delta t}{2}\left(-z_k A_{1,n}^{(1)}(t_j) + D_k \delta A_{1,n}^{(1)}(t_j) + f_k(A_{0,n}^{(1)}(t_i), A_{1,n}^{(1)}(t_i), E) \right) \\
& + \frac{1}{2} b_k \sqrt{\frac{\Delta t}{\Delta x}} w_{k,n}^{(1)}
\end{aligned}
$$