# ProgrammierParadigmen

Übung - Gruppe 1 & 2

Tobias Kahlert

# Typsystem

$$\textsc{Const}: \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \textsc{Var}: \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

$$\textsc{Abs}: \frac{\Gamma, x : \tau_1 \vdash t : \tau_2}{\Gamma \vdash \lambda x.\, t : \tau_1 \to \tau_2} \qquad \textsc{App}: \frac{\Gamma \vdash t_1 : \tau_2 \to \tau \qquad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1\ t_2 : \tau}$$

# Typinferenz

$$\lambda \texttt{f}.\ \lambda \texttt{x}.\ \texttt{f}\ \texttt{x}$$

$$C = \{ \qquad \}$$

$$\textsc{Const}: \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \textsc{Var}: \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

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# Typinferenz

$$\vdash \texttt{\lambda f. \lambda x. f x} : \alpha_1$$

$$C = \{ \qquad \}$$

$$\textsc{Const}: \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \textsc{Var}: \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

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# Typinferenz

$$\text{ABS: } \frac{\Gamma, x : \tau_1 \vdash t : \tau_2}{\Gamma \vdash \lambda x.\ t : \tau_1 \to \tau_2}$$

$$\vdash \lambda \mathtt{f}.\ \lambda \mathtt{x}.\ \mathtt{f}\ \mathtt{x} : \alpha_1$$

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$$\text{CONST: } \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \text{VAR: } \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

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# Typinferenz

$$\text{Abs: } \frac{\Gamma, x : \tau_1 \vdash t : \tau_2}{\Gamma \vdash \lambda x.\ t : \tau_1 \to \tau_2}$$

$$\textit{Abs}\ \frac{\mathtt{f} : \alpha_2 \vdash \lambda\mathtt{x.\ f\ x} : \alpha_3}{\vdash \lambda\mathtt{f}.\ \lambda\mathtt{x.\ f\ x} : \alpha_1}$$

$$C = \{\alpha_1 = \alpha_2 \to \alpha_3 \qquad \}$$

$$\text{Const: } \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \text{Var: } \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

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# Typinferenz

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$$C = \{\alpha_1 = \alpha_2 \to \alpha_3 \qquad \}$$

$$\textsc{Const}: \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \textsc{Var}: \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

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$$C = \{\alpha_1 = \alpha_2 \to \alpha_3 \qquad \}$$

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# Typinferenz

$$\text{Abs: } \frac{\Gamma, x : \tau_1 \vdash t : \tau_2}{\Gamma \vdash \lambda x.\ t : \tau_1 \to \tau_2}$$

$$\dfrac{\dfrac{\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{f\ x} : \alpha_5}{\mathtt{f} : \alpha_2 \vdash \lambda \mathtt{x}.\ \mathtt{f\ x} : \alpha_3}\,\mathit{Abs}}{\vdash \lambda \mathtt{f}.\ \lambda \mathtt{x}.\ \mathtt{f\ x} : \alpha_1}\,\mathit{Abs}$$

$$C = \{\alpha_1 = \alpha_2 \to \alpha_3, \alpha_3 = \alpha_4 \to \alpha_5 \qquad \}$$

$$\text{Const: } \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \text{Var: } \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

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# Typinferenz

$$\dfrac{\dfrac{\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{f\ x} : \alpha_5}{\mathtt{f} : \alpha_2 \vdash \lambda\mathtt{x}.\ \mathtt{f\ x} : \alpha_3}\ \mathit{Abs}}{\vdash \lambda\mathtt{f}.\ \lambda\mathtt{x}.\ \mathtt{f\ x} : \alpha_1}\ \mathit{Abs}$$

$$C = \{\alpha_1 = \alpha_2 \to \alpha_3, \alpha_3 = \alpha_4 \to \alpha_5 \qquad \}$$

$$\textsc{Const}: \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \textsc{Var}: \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

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# Typinferenz

$$\text{APP: } \frac{\Gamma \vdash \boxed{t_1} : \tau_2 \to \boxed{\tau} \qquad \Gamma \vdash \boxed{t_2} : \tau_2}{\Gamma \vdash \boxed{t_1}\ \boxed{t_2} : \boxed{\tau}}$$

$$\textit{Abs}\ \dfrac{\textit{Abs}\ \dfrac{\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \boxed{\mathtt{f}}\ \boxed{\mathtt{x}} : \boxed{\alpha_5}}{\mathtt{f} : \alpha_2 \vdash \lambda \mathtt{x}.\ \mathtt{f}\ \mathtt{x} : \alpha_3}}{\vdash \lambda \mathtt{f}.\ \lambda \mathtt{x}.\ \mathtt{f}\ \mathtt{x} : \alpha_1}$$

$$C = \{\alpha_1 = \alpha_2 \to \alpha_3, \alpha_3 = \alpha_4 \to \alpha_5 \qquad\qquad \}$$

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# Typinferenz

$$\textsc{App}: \frac{\Gamma \vdash t_1 : \tau_2 \to \tau \qquad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1\ t_2 : \tau}$$

$$
\dfrac{\dfrac{\dfrac{\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{f} : \alpha_6 \qquad \mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{x} : \alpha_7}{\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{f}\ \mathtt{x} : \alpha_5}\,\mathit{App}}{\mathtt{f} : \alpha_2 \vdash \lambda \mathtt{x}.\ \mathtt{f}\ \mathtt{x} : \alpha_3}\,\mathit{Abs}}{\vdash \lambda \mathtt{f}.\ \lambda \mathtt{x}.\ \mathtt{f}\ \mathtt{x} : \alpha_1}\,\mathit{Abs}
$$

$$C = \{\alpha_1 = \alpha_2 \to \alpha_3, \alpha_3 = \alpha_4 \to \alpha_5, \alpha_6 = \alpha_7 \to \alpha_5 \qquad \}$$

$$\textsc{Const}: \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \textsc{Var}: \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

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# Typinferenz

$$\dfrac{\dfrac{\dfrac{\texttt{f} : \alpha_2, \texttt{x} : \alpha_4 \vdash \texttt{f} : \alpha_6 \qquad \texttt{f} : \alpha_2, \texttt{x} : \alpha_4 \vdash \texttt{x} : \alpha_7}{\texttt{f} : \alpha_2, \texttt{x} : \alpha_4 \vdash \texttt{f x} : \alpha_5}\,\mathit{App}}{\texttt{f} : \alpha_2 \vdash \lambda\texttt{x.\ f x} : \alpha_3}\,\mathit{Abs}}{\vdash \lambda\texttt{f.\ }\lambda\texttt{x.\ f x} : \alpha_1}\,\mathit{Abs}$$

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# Typinferenz

$$\text{VAR: } \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{f} : \alpha_6 \qquad \mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{x} : \alpha_7}{\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{f\ x} : \alpha_5}\,\mathit{App}}{\mathtt{f} : \alpha_2 \vdash \lambda\mathtt{x}.\ \mathtt{f\ x} : \alpha_3}\,\mathit{Abs}}{\vdash \lambda\mathtt{f}.\ \lambda\mathtt{x}.\ \mathtt{f\ x} : \alpha_1}\,\mathit{Abs}$$

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# Typinferenz

$$\text{VAR: } \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

$$\textit{Abs}\;\dfrac{\textit{Abs}\;\dfrac{\textit{App}\;\dfrac{\textit{Var}\;\dfrac{(\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4)\,(\mathtt{f}) = \alpha_6}{\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{f} : \alpha_6} \qquad \mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{x} : \alpha_7}{\mathtt{f} : \alpha_2, \mathtt{x} : \alpha_4 \vdash \mathtt{f}\ \mathtt{x} : \alpha_5}}{\mathtt{f} : \alpha_2 \vdash \lambda \mathtt{x}.\ \mathtt{f}\ \mathtt{x} : \alpha_3}}{\vdash \lambda \mathtt{f}.\ \lambda \mathtt{x}.\ \mathtt{f}\ \mathtt{x} : \alpha_1}$$

$$C = \{\alpha_1 = \alpha_2 \to \alpha_3, \alpha_3 = \alpha_4 \to \alpha_5, \alpha_6 = \alpha_7 \to \alpha_5, \alpha_2 = \alpha_6 \qquad \}$$

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# Typinferenz

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\dfrac{\dfrac{\dfrac{(\mathtt{f}:\alpha_2,\mathtt{x}:\alpha_4)\,(\mathtt{f}) = \alpha_6}{\mathtt{f}:\alpha_2,\mathtt{x}:\alpha_4 \vdash \mathtt{f}:\alpha_6}\,\textit{Var} \qquad \mathtt{f}:\alpha_2,\mathtt{x}:\alpha_4 \vdash \mathtt{x}:\alpha_7}{\mathtt{f}:\alpha_2,\mathtt{x}:\alpha_4 \vdash \mathtt{f}\ \mathtt{x}:\alpha_5}\,\textit{App}}{\dfrac{\mathtt{f}:\alpha_2 \vdash \lambda\mathtt{x}.\ \mathtt{f}\ \mathtt{x}:\alpha_3}{\vdash \lambda\mathtt{f}.\ \lambda\mathtt{x}.\ \mathtt{f}\ \mathtt{x}:\alpha_1}\,\textit{Abs}}\,\textit{Abs}
$$

$$C = \{\alpha_1 = \alpha_2 \to \alpha_3, \alpha_3 = \alpha_4 \to \alpha_5, \alpha_6 = \alpha_7 \to \alpha_5, \alpha_2 = \alpha_6 \qquad\qquad \}$$

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$$\dfrac{\dfrac{\dfrac{(\texttt{f}:\alpha_2, \texttt{x}:\alpha_4)(\texttt{f}) = \alpha_6}{\texttt{f}:\alpha_2, \texttt{x}:\alpha_4 \vdash \texttt{f}:\alpha_6}\,\textit{Var} \quad \dfrac{(\texttt{f}:\alpha_2, \texttt{x}:\alpha_4)(\texttt{x}) = \alpha_7}{\texttt{f}:\alpha_2, \texttt{x}:\alpha_4 \vdash \texttt{x}:\alpha_7}\,\textit{Var}}{\texttt{f}:\alpha_2, \texttt{x}:\alpha_4 \vdash \texttt{f x}:\alpha_5}\,\textit{App}}{\dfrac{\texttt{f}:\alpha_2 \vdash \lambda\texttt{x}.\ \texttt{f x}:\alpha_3}{\vdash \lambda\texttt{f}.\ \lambda\texttt{x}.\ \texttt{f x}:\alpha_1}\,\textit{Abs}}\,\textit{Abs}$$

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$$\sigma_C = [\qquad\qquad\qquad\qquad]$$

$$\sigma_C(\alpha_1) =$$

$$\textsc{Const}: \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \textsc{Var}: \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

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# Typinferenz

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\dfrac{\dfrac{\dfrac{\dfrac{(\mathtt{f}:\alpha_2,\mathtt{x}:\alpha_4)(\mathtt{f})=\alpha_6}{\mathtt{f}:\alpha_2,\mathtt{x}:\alpha_4\vdash\mathtt{f}:\alpha_6}\,\textit{Var}\qquad\dfrac{(\mathtt{f}:\alpha_2,\mathtt{x}:\alpha_4)(\mathtt{x})=\alpha_7}{\mathtt{f}:\alpha_2,\mathtt{x}:\alpha_4\vdash\mathtt{x}:\alpha_7}\,\textit{Var}}{\mathtt{f}:\alpha_2,\mathtt{x}:\alpha_4\vdash\mathtt{f}\ \mathtt{x}:\alpha_5}\,\textit{App}}{\mathtt{f}:\alpha_2\vdash\lambda\mathtt{x}.\ \mathtt{f}\ \mathtt{x}:\alpha_3}\,\textit{Abs}}{\vdash\lambda\mathtt{f}.\ \lambda\mathtt{x}.\ \mathtt{f}\ \mathtt{x}:\alpha_1}\,\textit{Abs}
$$

$$C=\{\alpha_1=\alpha_2\to\alpha_3,\alpha_3=\alpha_4\to\alpha_5,\alpha_6=\alpha_7\to\alpha_5,\alpha_2=\alpha_6,\alpha_4=\alpha_7\}$$

$$\sigma_C=[\alpha_1 \Rightarrow (\alpha_7\to\alpha_5)\to\alpha_7\to\alpha_5,\alpha_2\Rightarrow\alpha_7\to\alpha_5,\alpha_3\Rightarrow\alpha_7\to\alpha_5,\alpha_4\Rightarrow\alpha_7,\alpha_6\Rightarrow\alpha_7\to\alpha_5]$$

$$\sigma_C(\alpha_1)=$$

$$\textsc{Const}:\ \frac{c\in\mathit{Const}}{\Gamma\vdash c:\tau_c}\qquad\textsc{Var}:\ \frac{\Gamma(x)=\tau}{\Gamma\vdash x:\tau}$$

$$\textsc{Abs}:\ \frac{\Gamma,x:\tau_1\vdash t:\tau_2}{\Gamma\vdash\lambda x.\ t:\tau_1\to\tau_2}\qquad\textsc{App}:\ \frac{\Gamma\vdash t_1:\tau_2\to\tau\qquad\Gamma\vdash t_2:\tau_2}{\Gamma\vdash t_1\ t_2:\tau}$$

# Typinferenz

$$
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  \dfrac{
    \dfrac{(\texttt{f} : \alpha_2, \texttt{x} : \alpha_4)(\texttt{f}) = \alpha_6}{\texttt{f} : \alpha_2, \texttt{x} : \alpha_4 \vdash \texttt{f} : \alpha_6}\,\textit{Var}
    \qquad
    \dfrac{(\texttt{f} : \alpha_2, \texttt{x} : \alpha_4)(\texttt{x}) = \alpha_7}{\texttt{f} : \alpha_2, \texttt{x} : \alpha_4 \vdash \texttt{x} : \alpha_7}\,\textit{Var}
  }{\texttt{f} : \alpha_2, \texttt{x} : \alpha_4 \vdash \texttt{f x} : \alpha_5}\,\textit{App}
}{
  \dfrac{\texttt{f} : \alpha_2 \vdash \lambda \texttt{x}.\ \texttt{f x} : \alpha_3}{\vdash \lambda \texttt{f}.\ \lambda \texttt{x}.\ \texttt{f x} : \alpha_1}\,\textit{Abs}
}\,\textit{Abs}
$$

$$C = \{\alpha_1 = \alpha_2 \to \alpha_3, \alpha_3 = \alpha_4 \to \alpha_5, \alpha_6 = \alpha_7 \to \alpha_5, \alpha_2 = \alpha_6, \alpha_4 = \alpha_7\}$$

$$\sigma_C = [\alpha_1 \Rightarrow (\alpha_7 \to \alpha_5) \to \alpha_7 \to \alpha_5, \alpha_2 \Rightarrow \alpha_7 \to \alpha_5, \alpha_3 \Rightarrow \alpha_7 \to \alpha_5, \alpha_4 \Rightarrow \alpha_7, \alpha_6 \Rightarrow \alpha_7 \to \alpha_5]$$

$$\sigma_C(\alpha_1) = (\alpha_7 \to \alpha_5) \to \alpha_7 \to \alpha_5$$

$$\textsc{Const}: \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \textsc{Var}: \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

$$\textsc{Abs}: \frac{\Gamma, x : \tau_1 \vdash t : \tau_2}{\Gamma \vdash \lambda x.\ t : \tau_1 \to \tau_2} \qquad \textsc{App}: \frac{\Gamma \vdash t_1 : \tau_2 \to \tau \qquad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1\ t_2 : \tau}$$

# Blatt 9

- λ-Terme und die Herleitung ihrer allgemeinsten Typen
- Typabstraktion
- Typinferenz, let-Polymorphismus

# Typregeln

$$\textsc{Const}: \frac{c \in \mathit{Const}}{\Gamma \vdash c : \tau_c} \qquad \textsc{Var}: \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

$$\textsc{Abs}: \frac{\Gamma, x : \tau_1 \vdash t : \tau_2}{\Gamma \vdash \lambda x.\, t : \tau_1 \to \tau_2} \qquad \textsc{App}: \frac{\Gamma \vdash t_1 : \tau_2 \to \tau \qquad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1\ t_2 : \tau}$$

---

$$\textsc{Var}: \frac{\Gamma(x) = \tau' \qquad \tau' \succeq \tau}{\Gamma \vdash x : \tau}$$

$$\textsc{Abs}: \frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \qquad \tau_1 \text{ kein Typschema}}{\Gamma \vdash \lambda x.\, t : \tau_1 \to \tau_2}$$

$$\textsc{Let}: \frac{\Gamma \vdash t_1 : \tau_1 \qquad \Gamma, x : ta(\tau_1, \Gamma) \vdash t_2 : \tau_2}{\Gamma \vdash \mathtt{let}\ x = t_1\ \mathtt{in}\ t_2 : \tau_2}$$

# Unifikation

```
if C == ∅ then []
else let {τ1 = τ2} ∪ C' = C in
  if τ1 == τ2 then unify(C')
  else if τ1 == α and α ∉ FV(τ2) then unify([α ⇨ τ2] C') ∘ [α ⇨ τ2]
  else if τ2 == α and α ∉ FV(τ1) then unify([α ⇨ τ1] C') ∘ [α ⇨ τ1]
  else if τ1 == (τ1' → τ1'') and τ2 == (τ2' → τ2'')
       then unify(C' ∪ {τ1' = τ2', τ1'' = τ2''})
  else fail
```