

# Formas canónicas de una función booleana

Tema 5

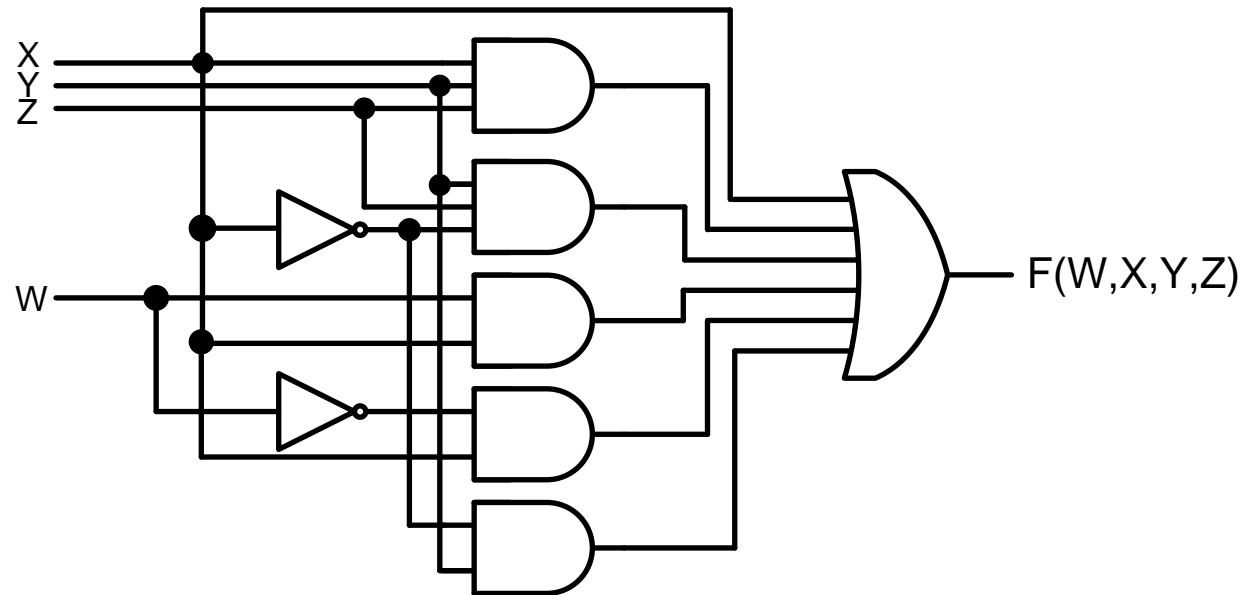
# Contenido

- › Forma normal disyuntiva. Suma de minitérminos
- › Forma normal conjuntiva. Producto de maxitérminos
- › Relación entre formas normales

# Ejercicio 1: Simplificar

$$F(W,X,Y,Z) = X + XYZ + YZ\bar{X} + WX + \bar{W}X + \bar{X}Y$$

W	X	Y	Z	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



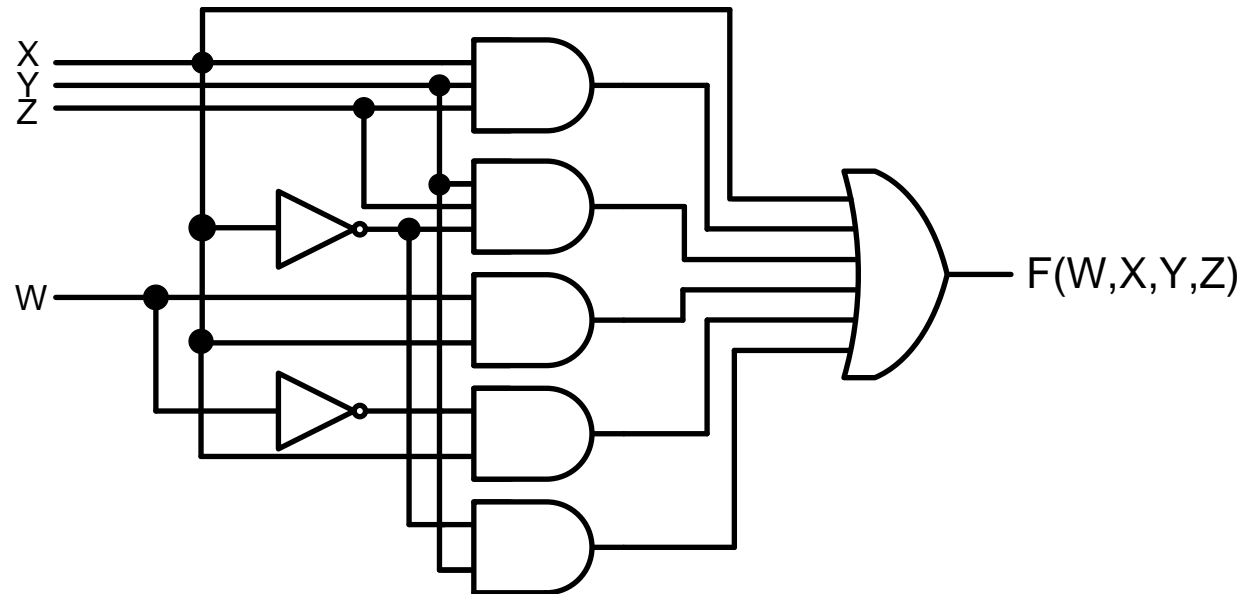
# Ejercicio 1: Simplificar

$$\begin{aligned} F(W, X, Y, Z) &= X + XYZ + YZ\bar{X} + WX + \bar{W}X + \bar{X}Y \\ &= X + YZ(X + \bar{X}) + X(W + \bar{W}) + \bar{X}Y \\ &= X + YZ + X + \bar{X}Y \\ &= YZ + X + X + \bar{X}Y \\ &= YZ + X \cdot 1 + \bar{X}Y \\ &= YZ + X(Y + \bar{Y}) + \bar{X}Y \\ &= YZ + XY + X\bar{Y} + \bar{X}Y \\ &= YZ + XY + XY + X\bar{Y} + \bar{X}Y \\ &= YZ + XY + X\bar{Y} + XY + \bar{X}Y \\ &= YZ + X(Y + \bar{Y}) + Y(X + \bar{X}) \\ &= YZ + X + Y = X + Y + YZ \\ &= X + Y(1 + Z) \\ &= X + Y \end{aligned}$$

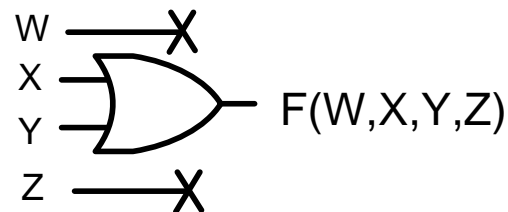
# Ejercicio 1: Simplificar

$$F(W,X,Y,Z) = X + XYZ + YZ\bar{X} + WX + \bar{W}X + \bar{X}Y$$

W	X	Y	Z	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



$$F(W,X,Y,Z) = X + Y$$



## Ejercicio 2: Simplificar

$$\begin{aligned}F(X, Y, Z) &= Y\bar{Z}(X\bar{Z} + \bar{Z}) + (\bar{X} + \bar{Z})(\bar{X}Y + Z\bar{X}) \\&= Y\bar{Z} \cdot \bar{Z}(X + 1) + (\bar{X} + \bar{Z})(\bar{X}Y + Z\bar{X}) \\&= Y\bar{Z} \cdot \bar{Z} + (\bar{X} + \bar{Z})(\bar{X}Y + Z\bar{X}) \\&= Y\bar{Z} + (\bar{X} + \bar{Z})(\bar{X}Y + Z\bar{X}) \\&= Y\bar{Z} + \bar{X} \cdot \bar{X}Y + \bar{X} \cdot Z\bar{X} + \bar{X}Y \cdot \bar{Z} + Z\bar{X} \cdot \bar{Z} \\&= Y\bar{Z} + \bar{X}Y + Z\bar{X} + \bar{X}Y\bar{Z} + \bar{X}Z\bar{Z} \\&= Y\bar{Z} + \bar{X}Y + Z\bar{X} + \bar{X}Y\bar{Z} \\&= Y\bar{Z} + \bar{X}Y\bar{Z} + \bar{X}Y + Z\bar{X} \\&= Y\bar{Z}(1 + \bar{X}) + \bar{X}Y + Z\bar{X} \\&= Y\bar{Z} + \bar{X}Y + Z\bar{X} \\&= Z\bar{X} + Y\bar{Z} + \bar{X}Y \\&= \bar{X}Z + Y\bar{Z} + \bar{X}Y \\&= \bar{X}Z + Y\bar{Z}\end{aligned}$$

# Forma normal disyuntiva. Suma de minitérminos

- › Un minitérmino es un producto lógico que contiene todas las variables, negadas o no.
- › Para 3 variables: a, b, c

	abc	Minitérmino	$m_i$
0	000	$\bar{a}\bar{b}\bar{c}$	$m_0$
1	001	$\bar{a}\bar{b}c$	$m_1$
2	010	$\bar{a}b\bar{c}$	$m_2$
3	011	$\bar{a}bc$	$m_3$
4	100	$a\bar{b}\bar{c}$	$m_4$
5	101	$a\bar{b}c$	$m_5$
6	110	$ab\bar{c}$	$m_6$
7	111	$abc$	$m_7$

# Forma normal disyuntiva. Suma de minitérminos

- › Obtención de la forma canónica a partir de la Tabla de verdad. Ejemplo:

$m_i$	abc	f
$m_0$	000	0
$m_1$	001	1
$m_2$	010	0
$m_3$	011	1
$m_4$	100	0
$m_5$	101	0
$m_6$	110	0
$m_7$	111	1

$$f(a, b, c) = \sum m(1,3,7) = m_1 + m_3 + m_7 = \bar{a}\bar{b}c + \bar{a}bc + abc$$

$$f(0,0,0) = 0 + 0 + 0 = 0$$

$$f(0,0,1) = 1 + 0 + 0 = 1$$

$$f(0,1,0) = 0 + 0 + 0 = 0$$

$$f(0,1,1) = 0 + 1 + 0 = 1$$

$$f(1,0,0) = 0 + 0 + 0 = 0$$

$$f(1,0,1) = 0 + 0 + 0 = 0$$

$$f(1,1,0) = 0 + 0 + 0 = 0$$

$$f(1,1,1) = 0 + 0 + 1 = 1$$



# Forma normal conjuntiva. Producto de maxitérminos

- › Un maxitérmino es una suma lógica que contiene todas las variables, negadas o no.
- › Para 3 variables: a, b, c

	abc	Maxitérmino	$M_i$
0	000	$a + b + c$	$M_0$
1	001	$a + b + \bar{c}$	$M_1$
2	010	$a + \bar{b} + c$	$M_2$
3	011	$a + \bar{b} + \bar{c}$	$M_3$
4	100	$\bar{a} + b + c$	$M_4$
5	101	$\bar{a} + b + \bar{c}$	$M_5$
6	110	$\bar{a} + \bar{b} + c$	$M_6$
7	111	$\bar{a} + \bar{b} + \bar{c}$	$M_7$

# Forma normal disyuntiva. Suma de maxitérminos

- › Obtención de la forma canónica a partir de la Tabla de verdad. Ejemplo:

$M_i$	abc	g
$M_0$	000	0
$M_1$	001	1
$M_2$	010	0
$M_3$	011	1
$M_4$	100	1
$M_5$	101	1
$M_6$	110	0
$M_7$	111	1

$$g(a, b, c) = \prod M(0,2,6) = M_0 \cdot M_2 \cdot M_6$$

$$= (a + b + c)(a + \bar{b} + c)(\bar{a} + \bar{b} + c)$$

$$g(0,0,0) = 0 \cdot 1 \cdot 1 = 0$$

$$g(0,0,1) = 1 \cdot 1 \cdot 1 = 1$$

$$g(0,1,0) = 1 \cdot 0 \cdot 1 = 0$$

$$g(0,1,1) = 1 \cdot 1 \cdot 1 = 1$$

$$g(1,0,0) = 1 \cdot 1 \cdot 1 = 1$$

$$g(1,0,1) = 1 \cdot 1 \cdot 1 = 1$$

$$g(1,1,0) = 1 \cdot 1 \cdot 0 = 0$$

$$g(1,1,1) = 1 \cdot 1 \cdot 1 = 1$$

# Relación entre las formas normales

› Maxitérmino y minitérmino son opuestos:

$$- \overline{m_i} = M_i$$

$$- \overline{M_i} = m_i$$

# Relación entre las formas normales

- › Ejemplo: obtener la forma normal disyuntiva a partir de la forma conjuntiva

$$h(a, b) = m_3 = ab$$

ab	h	$M_i$	$m_i$
00	0	$M_0 = a + b$	$m_0 = \bar{a}\bar{b}$
01	0	$M_1 = a + \bar{b}$	$m_1 = \bar{a}b$
10	0	$M_2 = \bar{a} + b$	$m_2 = a\bar{b}$
11	1	$M_3 = \bar{a} + \bar{b}$	$m_3 = ab$

$$h(a, b) = \prod M(0,1,2) = M_0 \cdot M_1 \cdot M_2$$

$$= (a + b)(a + \bar{b})(\bar{a} + b)$$

$$= (aa + a\bar{b} + ba + b\bar{b})(\bar{a} + b)$$

$$= (a + a\bar{b} + ba)(\bar{a} + b)$$

$$= a\bar{a} + ab + a\bar{b}\bar{a} + a\bar{b}b + ba\bar{a} + bab$$

$$= ab + ab$$

$$= ab$$