

Minimización de funciones booleanas

Tema 7



Contenido

- › Métodos de minimización
- › Mapas de Karnaugh
 - 2 variables
 - 3 variables
 - 4 variables
 - 5 variables
 - 6 variables y más

Métodos de minimización

› Destacan:

- Método algebraico
 - › Es un método teórico poco operativo
- Método de Quine-McCluskey
 - › Método potente de gran valor computacional, pero de laboriosa aplicación a mano
- Método de los mapas de Karnaugh
 - › Método gráfico poco potente, pero de fácil aplicación manual

Karnaugh, Maurice (November 1953) [1953-04-23, 1953-03-17].
["The Map Method for Synthesis of Combinational Logic Circuits"](#) (PDF). [Transactions of the American Institute of Electrical Engineers, Part I: Communication and Electronics](#). 72 (5): 593–599.



Mapas de Karnaugh

› Para 2 variables (a,b):

	ab	m_i	M_i
0	00	$\bar{a}\bar{b}$	$a + b$
1	01	$\bar{a}b$	$a + \bar{b}$
2	10	$a\bar{b}$	$\bar{a} + b$
3	11	ab	$\bar{a} + \bar{b}$



$\begin{matrix} a \\ b \end{matrix}$	0	1
0		
1		



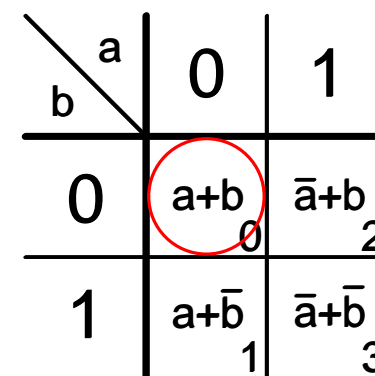
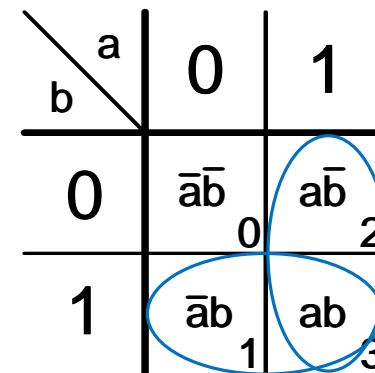
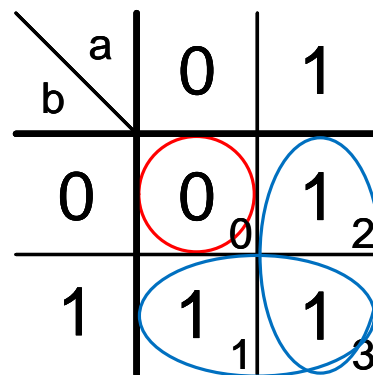
$\begin{matrix} a \\ b \end{matrix}$	0	1
0	$\bar{a}\bar{b}$ 0	$a\bar{b}$ 2
1	$\bar{a}b$ 1	ab 3

$\begin{matrix} a \\ b \end{matrix}$	0	1
0	$a+b$ 0	$\bar{a}+b$ 2
1	$a+\bar{b}$ 1	$\bar{a}+\bar{b}$ 3

Mapas de Karnaugh

› Para 2 variables (a,b): ejemplo 1

	ab	m_i	M_i	F
0	00	$\bar{a}\bar{b}$	$a + b$	0
1	01	$\bar{a}b$	$a + \bar{b}$	1
2	10	$a\bar{b}$	$\bar{a} + b$	1
3	11	ab	$\bar{a} + \bar{b}$	1



$$F(a, b) = a + b$$

$$F(a, b) = a\bar{b} + ab + \bar{a}b + ab = a(\bar{b} + b) + b(\bar{a} + a) = a + b$$

Los mapas de Karnaugh aplican: $XY + X\bar{Y} = X$ $(X + Y)(X + \bar{Y}) = X$
 $X + X = X$ $X \cdot X = X$

Mapas de Karnaugh

› Para 2 variables (a,b): ejemplo 2

	ab	m_i	M_i	G
0	00	$\bar{a}\bar{b}$	$a + b$	1
1	01	$\bar{a}b$	$a + \bar{b}$	1
2	10	$a\bar{b}$	$\bar{a} + b$	0
3	11	ab	$\bar{a} + \bar{b}$	0

$\begin{matrix} a \\ b \end{matrix}$	0	1
0	1 ₀	0 ₂
1	1 ₁	0 ₃

$\begin{matrix} a \\ b \end{matrix}$	0	1
0	$\bar{a}\bar{b}$ ₀	$a\bar{b}$ ₂
1	$\bar{a}b$ ₁	ab ₃

$$G(a, b) = \bar{a}\bar{b} + \bar{a}b = \bar{a}(\bar{b} + b) = \bar{a}$$

Mapas de Karnaugh

› Para 3 variables (a,b,c)

– Las últimas dos filas se permutan para conservar la adyacencia

	abc	m_i	Minitérmino	M_i	Maxitérmino
0	000	m_0	$\bar{a}\bar{b}\bar{c}$	M_0	$a + b + c$
1	001	m_1	$\bar{a}\bar{b}c$	M_1	$a + b + \bar{c}$
2	010	m_2	$\bar{a}b\bar{c}$	M_2	$a + \bar{b} + c$
3	011	m_3	$\bar{a}bc$	M_3	$a + \bar{b} + \bar{c}$
4	100	m_4	$a\bar{b}\bar{c}$	M_4	$\bar{a} + b + c$
5	101	m_5	$a\bar{b}c$	M_5	$\bar{a} + b + \bar{c}$
6	110	m_6	$ab\bar{c}$	M_6	$\bar{a} + \bar{b} + c$
7	111	m_7	abc	M_7	$\bar{a} + \bar{b} + \bar{c}$



a \ bc	0	1
00	0	4
01	1	5
11	3	7
10	2	6

Ejemplo 1

$$f(a, b, c) = \sum m(3, 4, 5, 6, 7) = m_3 + m_4 + m_5 + m_6 + m_7$$

$a \backslash bc$	0	1
00	0	1 ₄
01	1	1 ₅
11	1 ₃	1 ₇
10	2	1 ₆

Diagram illustrating the Karnaugh map for the function $f(a, b, c)$. The map shows the function value (0 or 1) for each combination of inputs a, b, c . The inputs a, b, c are labeled on the axes. The function value is 1 for the combinations (0,1,0), (0,1,1), (1,1,0), (1,1,1), and (1,0,1), which are circled in red. The function value is 0 for the combinations (0,0,0), (0,0,1), (1,0,0), and (1,0,1), which are not circled. A green circle highlights the cells (1,1,0) and (1,1,1), with an arrow pointing to the label bc . A red arrow points from the bottom of the red circle to the label a .

$$f(a, b, c) = a + bc$$

Ejemplo 2

$$f(a, b, c) = m_1 + m_3 + m_5$$

$\begin{array}{c} a \\ \backslash bc \end{array}$	0	1
00	0	4
01	1	5
11	1	7
10	2	6

Diagram illustrating the Karnaugh map for the function $f(a, b, c) = m_1 + m_3 + m_5$. The map shows the values of the function for all combinations of a, b, c . The variables a, b, c are represented by the rows and columns. The map is divided into four groups of 1s, each representing a prime implicant:

- Group 1 (Red circle): $\bar{a}c$ (covers m_1, m_3)
- Group 2 (Green circle): $\bar{b}c$ (covers m_1, m_5)
- Group 3 (Red circle): $\bar{a}b$ (covers m_3, m_7)
- Group 4 (Red circle): $\bar{a}b$ (covers m_3, m_7)

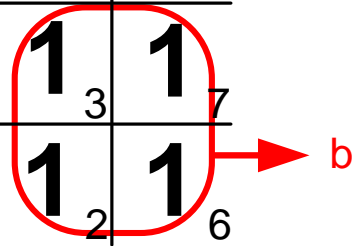
$$f(a, b, c) = \bar{a}c + \bar{b}c$$

Ejemplo 3

$$f(a, b, c) = \sum m(2, 3, 6, 7)$$

$\begin{array}{c} a \\ \backslash bc \end{array}$	0	1
00	0	4
01	1	5
11	1 ₃	1 ₇
10	1 ₂	1 ₆

$$f(a, b, c) = b$$



Ejemplo 4

$$f(a, b, c) = \prod M(0,1,2,3,5,7) = \sum m(4,6)$$

$\begin{array}{c} a \\ \backslash bc \end{array}$	0	1
00		1
01		
11		
10		1

Diagram illustrating the Karnaugh map for the function $f(a, b, c) = \sum m(4,6)$. The map shows two 1s at positions (0,1) and (1,1). A red circle groups the 1s at (0,1) and (1,1), and a red arrow points to the expression $a\bar{c}$.

$$f(a, b, c) = a\bar{c}$$

Ejemplo 5

$$f(a, b, c) = ab\bar{c} + \bar{b}c + \bar{a}$$

$\begin{array}{c} a \\ \backslash bc \end{array}$	0	1
00	1	0
01	1	1
11	1	0
10	1	1

Diagram illustrating the Karnaugh map for the function $f(a, b, c) = ab\bar{c} + \bar{b}c + \bar{a}$. The map shows the function's value (0 or 1) for each combination of inputs a, b, c . The variables a, b, c are represented by the rows and columns. The map is divided into four groups of 1s, each corresponding to a term in the sum of products:

- Group 1 (Red circle): \bar{a} (all 1s in the first column, where $a=0$).
- Group 2 (Red circle): $\bar{b}c$ (all 1s in the second column, where $b=0$).
- Group 3 (Red circle): $b\bar{c}$ (all 1s in the third column, where $c=0$).
- Group 4 (Red circle): $ab\bar{c}$ (all 1s in the fourth column, where $c=0$).

$$f(a, b, c) = \bar{a} + \bar{b}c + b\bar{c}$$

Ejemplo 6: varias formas simples

$$f(a, b, c) = \sum m(0, 1, 2, 5, 6, 7)$$

$\begin{array}{c} a \\ \backslash bc \end{array}$	0	1
00	1 ₀	0 ₄
01	1 ₁	1 ₅
11	0 ₃	1 ₇
10	1 ₂	1 ₆

$\bar{a}\bar{c}$ (from cell 0)
 $\bar{b}c$ (from cells 1, 5)
 ab (from cells 2, 6)

$$f(a, b, c) = \bar{a}\bar{c} + \bar{b}c + ab$$

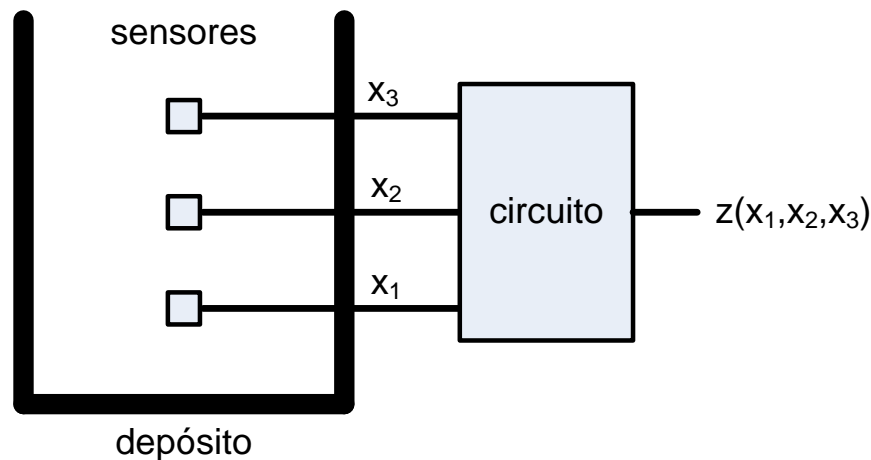
$\begin{array}{c} a \\ \backslash bc \end{array}$	0	1
00	1 ₀	0 ₄
01	1 ₁	1 ₅
11	0 ₃	1 ₇
10	1 ₂	1 ₆

$\bar{a}\bar{b}$ (from cell 0)
 ac (from cells 1, 7)
 $b\bar{c}$ (from cells 2, 6)

$$f(a, b, c) = \bar{a}\bar{b} + ac + b\bar{c}$$

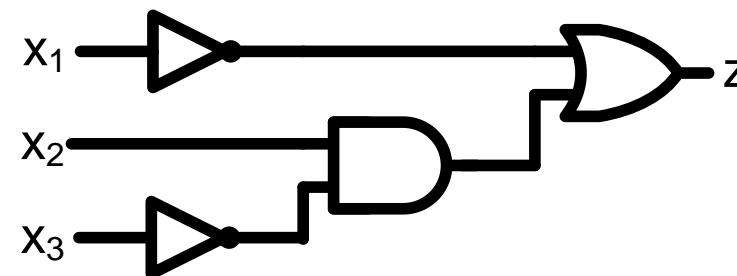
Ejemplo 7:

- › Diseñar un circuito que produce señal 1 cuando el líquido de un depósito está por debajo del nivel x_1 o bien entre x_2 y x_3 :



x_1	x_2	x_3	z
0	0	0	1
0	0	1	X
0	1	0	X
0	1	1	X
1	0	0	0
1	0	1	X
1	1	0	1
1	1	1	0

x_1	0	1
$x_2 x_3$		
00	1 ₀	0 ₄
01	X ₁	X ₅
11	X ₃	0 ₇
10	X ₂	1 ₆



$$z(x_1, x_2, x_3) = \sum m(0,6) + \sum d(1,2,3,5) = x_1 + x_2 \bar{x}_3$$

Mapas de Karnaugh

- › Para 4 variables (a,b,c,d):
 - Se permutan las 2 últimas filas y las 2 últimas columnas para conservar la adyacencia
 - Adyacencias de hasta tercer orden (agrupaciones de 8 casillas para simplificar 3 variables)

$\begin{matrix} & ab \\ cd \end{matrix}$	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Ejemplo 1

$$f(a, b, c, d) = \sum m(0,5,6,7,8,13,14,15)$$

ab \ cd	00	01	11	10
00	1 ₀			1 ₈
01		1 ₅	1 ₁₃	
11		1 ₇	1 ₁₅	
10		1 ₆	1 ₁₄	

$$f(a, b, c, d) = bd + bc + \bar{b}\bar{c}\bar{d}$$

$\bar{b}\bar{c}\bar{d}$

bd

bc

Ejemplo 2

$$f(a, b, c, d) = \sum m(0, 2, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15)$$

ab \ cd	00	01	11	10
00	1 ₀	1 ₄	1 ₁₂	1 ₈
01		1 ₅		
11		1 ₇	1 ₁₅	1 ₁₁
10	1 ₂	1 ₆	1 ₁₄	1 ₁₀

$$f(a, b, c, d) = \bar{d} + \bar{a}b + ac$$

Ejemplo 3

$$f(a, b, c, d) = \sum m(1, 3, 4, 5, 10, 12, 13)$$

ab \ cd	00	01	11	10
00		1 ₄	1 ₁₂	
01	1 ₁	1 ₅	1 ₁₃	
11	1 ₃			
10				1 ₁₀

→ $b\bar{c}$ (red circle around m4, m5, m12, m13)
→ $\bar{a}bd$ (green circle around m1, m3)
→ $a\bar{b}c\bar{d}$ (blue circle around m10)

$$f(a, b, c, d) = b\bar{c} + \bar{a}bd + a\bar{b}c\bar{d}$$

Ejemplo 4

$$f(a, b, c, d) = \sum m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$$

ab \ cd	00	01	11	10
00	1 ₀			1 ₈
01		1 ₅		
11	1 ₃	1 ₇	1 ₁₅	1 ₁₁
10	1 ₂	1 ₆	1 ₁₄	1 ₁₀

$$f(a, b, c, d) = c + \bar{b}\bar{d} + \bar{a}bd$$

Ejemplo 5

$$f(a, b, c, d) = \sum m(1, 3, 5, 7, 9) + \sum d(6, 12, 13)$$

ab \ cd	00	01	11	10
00			X ₁₂	
01	1 ₁	1 ₅	X ₁₃	1 ₉
11	1 ₃	1 ₇		
10		X ₆		

$$f(a, b, c, d) = \bar{a}d + \bar{c}d$$

Ejemplo 6: producto de sumas mínimo

$$f(a, b, c, d) = \sum m(0, 2, 3, 4, 8, 10, 11, 15) = \prod M(1, 5, 6, 7, 9, 12, 13, 14)$$

ab \ cd	00	01	11	10
00	1 ₀	1 ₄	0 ₂	1 ₈
01	0 ₁	0 ₅	0 ₁₃	0 ₉
11	1 ₃	0 ₇	1 ₁₅	1 ₁₁
10	1 ₂	0 ₆	0 ₁₄	1 ₁₀

$$f(a, b, c, d) = (c + \bar{d})(a + \bar{b} + \bar{c})(\bar{a} + \bar{b} + d)$$

Ejemplo 7:

- › Simplificar $f(a, b, c, d) = \sum m(0, 2, 8, 12, 13)$ mediante POS y SOP. ¿Qué realización es más económica?

ab \ cd	00	01	11	10
00	1 ₀		1 ₁₂	1 ₈
01			1 ₁₃	
11				
10	1 ₂			

Diagram illustrating the Karnaugh map for the function $f(a, b, c, d) = \sum m(0, 2, 8, 12, 13)$. The map shows the following groupings:

- Red circle around cells (0,0) and (0,1) labeled $\bar{b}\bar{c}\bar{d}$.
- Blue circle around cells (1,1) and (1,0) labeled $ab\bar{c}$.
- Green circle around cells (0,0) and (1,0) labeled $\bar{a}\bar{b}\bar{d}$.

$$f(a, b, c, d) = ab\bar{c} + \bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{d}$$

3 AND y 1 OR

Ejemplo 7:

- › Simplificar $f(a, b, c, d) = \sum m(0, 2, 8, 12, 13)$ mediante POS y SOP. ¿Qué realización es más económica?

ab \ cd	00	01	11	10
00	0	0	12	8
01	0	0	13	9
11	0	0	15	11
10	2	0	14	10

Diagram illustrating the Karnaugh map for the function $f(a, b, c, d) = \sum m(0, 2, 8, 12, 13)$. The map shows the function value (0 or 1) for each combination of inputs a, b, c, d . The map is grouped into four regions, each representing a product term in the POS expression:

- Red box: $a + \bar{b}$
- Green box: $b + \bar{d}$
- Blue box: $\bar{a} + \bar{c}$
- Blue box: $a + \bar{b}$

$$f(a, b, c, d) = ab\bar{c} + \bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{d}$$

3 AND y 1 OR

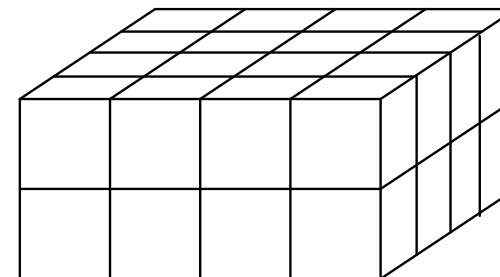
$$f(a, b, c, d) = (b + \bar{d})(\bar{a} + \bar{c})(a + \bar{b})$$

3 OR y 1 AND

La segunda emplea menos transistores al tener las puertas lógicas menor número de entradas

Mapas de Karnaugh

- › Para 5 variables (a,b,c,d,e):
 - 2 mapas de 4 variables
 - Adyacencias de hasta cuarto orden



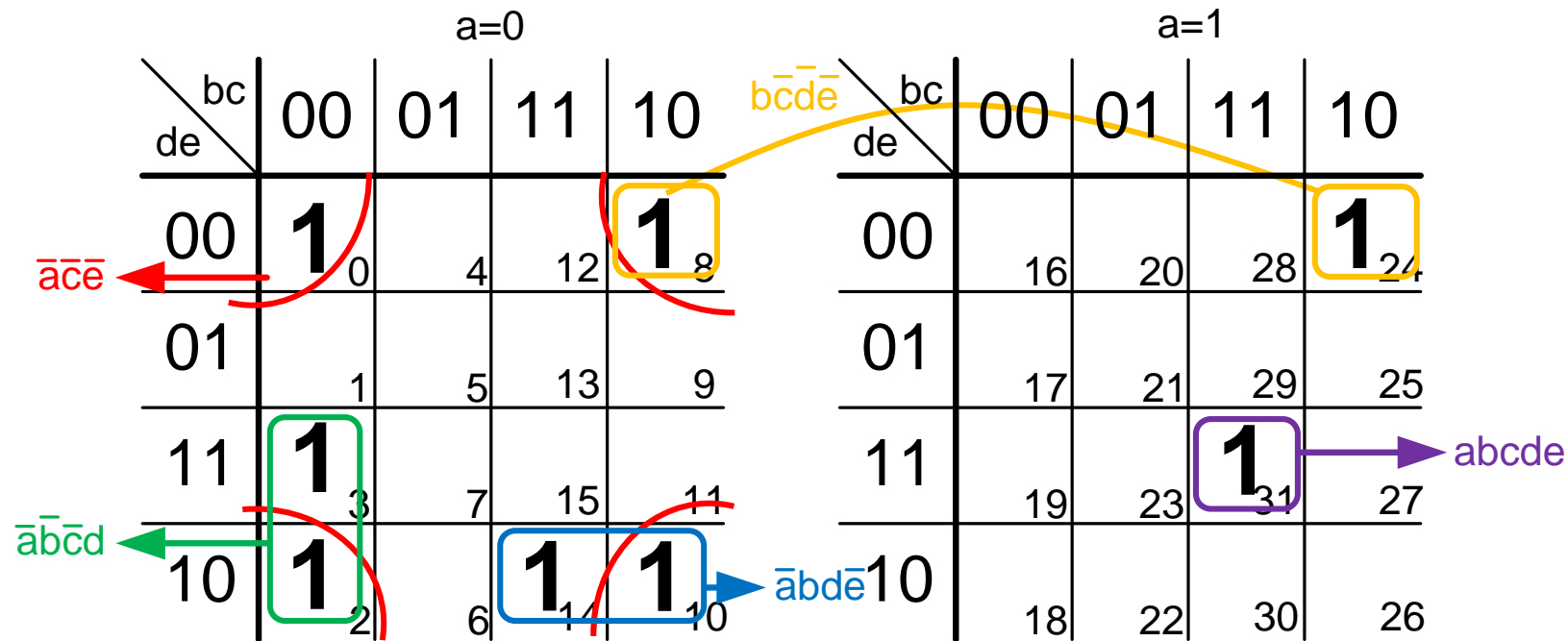
a=0				
bc \ de	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

a=1				
bc \ de	00	01	11	10
00	16	20	28	24
01	17	21	29	25
11	19	23	31	27
10	18	22	30	26

Ejemplo 1

$$f(a, b, c, d, e) = \sum m(0, 2, 3, 8, 10, 14, 24, 31)$$

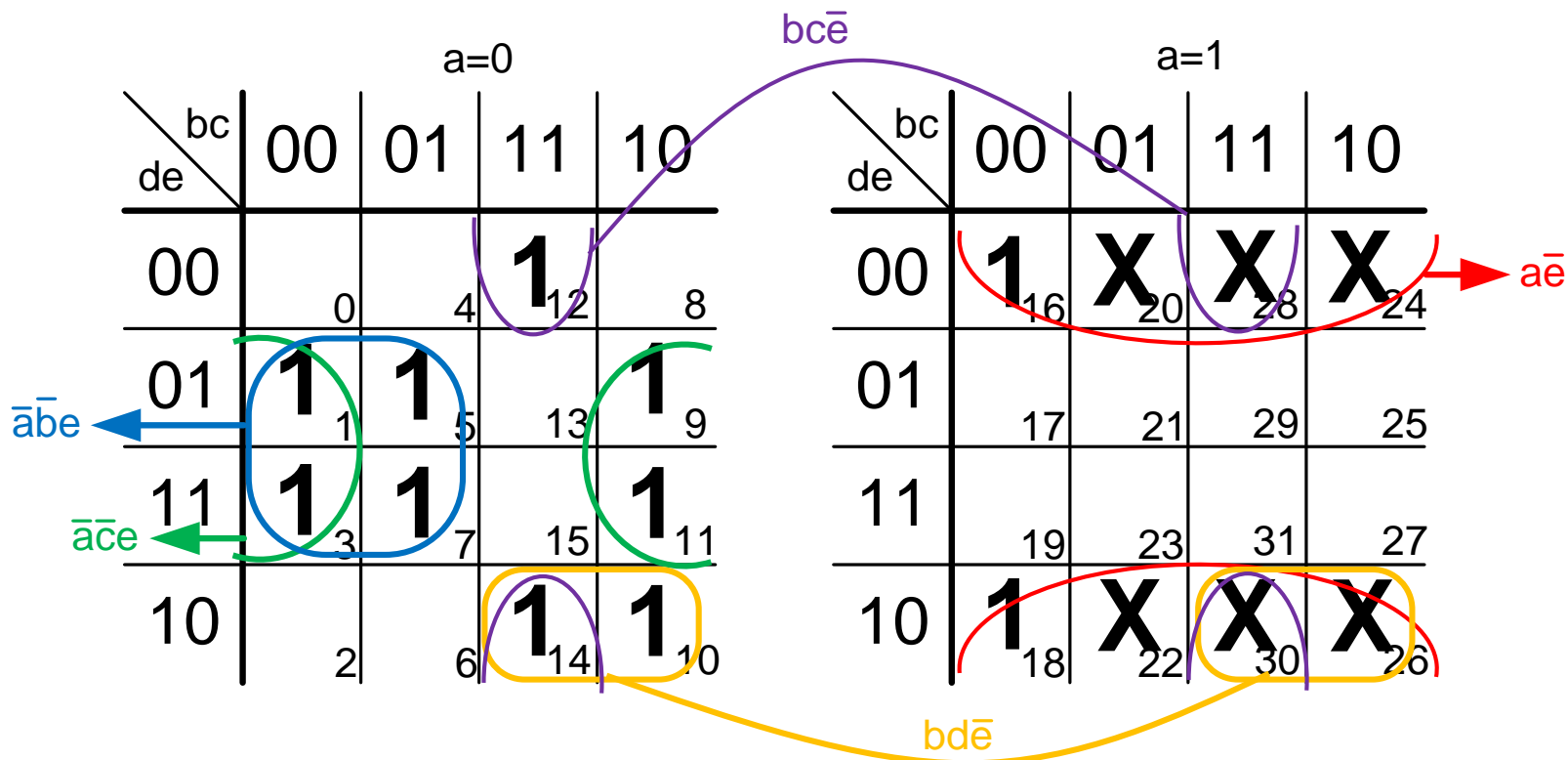
$$f(a, b, c, d, e) = \bar{a}\bar{c}\bar{e} + \bar{a}\bar{b}\bar{c}d + \bar{a}bd\bar{e} + b\bar{c}\bar{d}\bar{e} + abcde$$



Ejemplo 2

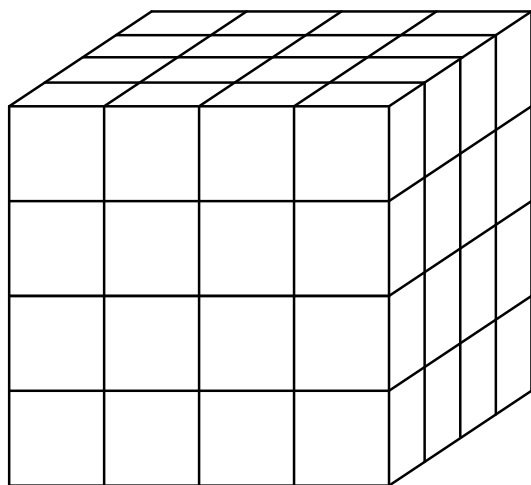
$$f(a, b, c, d, e) = \sum m(1,3,5,7,9,10,11,12,14,16,18) + \sum d(20,22,24,26,28,30)$$

$$f(a, b, c, d, e) = a\bar{e} + \bar{a}\bar{b}e + \bar{a}\bar{c}e + bc\bar{e} + bd\bar{e}$$



Mapas de Karnaugh

- › Para 6 variables (a,b,c,d,e,f):
 - 4 mapas de 4 variables
 - Adyacencias de hasta quinto orden



		ab=00				
		cd	00	01	11	10
ef	00		0	4	12	8
	01		1	5	13	9
	11		3	7	15	11
	10		2	6	14	10

		ab=01				
		cd	00	01	11	10
ef	00		16	20	28	24
	01		17	21	29	25
	11		19	23	31	27
	10		18	22	30	26

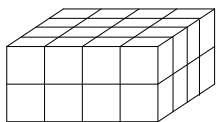
		ab=10			
		cd	00	01	11
ef	00				
	01				
	11				
	10				

		ab=11				
		cd	00	01	11	10
ef	00		48	52	60	56
	01		49	53	61	57
	11		51	55	63	59
	10		50	54	62	58

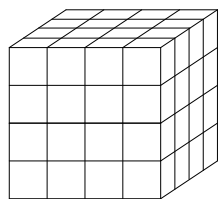
Mapas de Karnaugh

› Y más...

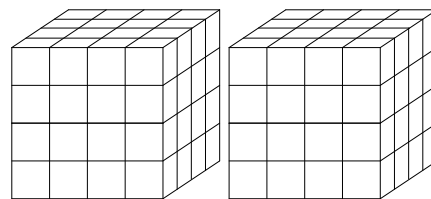
5 variables



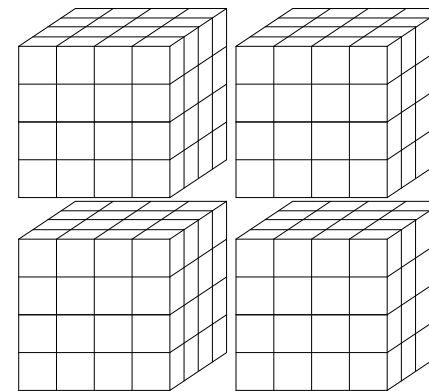
6 variables



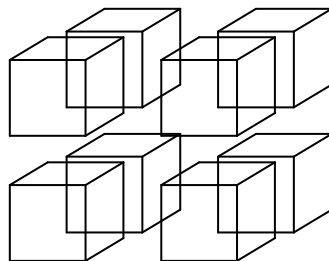
7 variables



8 variables



9 variables



10 variables...

