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Saving Federal Travel Dollars

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Source: *Interfaces*, Vol. 31, No. 5 (Sep. - Oct., 2001), pp. 13-23

Published by: INFORMS

Stable URL: <https://www.jstor.org/stable/25062735>

Accessed: 21-10-2018 01:22 UTC

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# Saving Federal Travel Dollars

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Federal employees frequently travel to professional-development conferences and training events. Conference planners often decide on event locations arbitrarily with little regard for travel cost. We developed a method for evaluating travel costs and determining the least expensive conference site from over 261 US cities with government-contracted airfares. It involves using the relaxation method for solving the noncapacitated shortest-path-network problem and computing interairport distances using latitude and longitude data. A decision support tool called OffSite allows conference planners to assess collective travel costs for attendees arriving from geographically dispersed locations. Event planners input a list of origin cities for conference participants, and OffSite then determines the lowest-cost location. This tool can restrict the search for meeting venues to the origin locations of potential hosts for the conference. In addition, OffSite is flexible enough to allow planners to choose a preferred destination.

**W**e developed and delivered this decision support tool in less than two months. The compressed development schedule was possible only because

we leveraged the Internet and other technological resources. At the time of the project, we were physically located on opposite coasts. Randy Zimmerman was in

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0092-2102/01/3105/0013/\$05.00  
1526-551X electronic ISSN  
This paper was refereed.

TRAVEL—MODEL-ROUTE CHOICE  
GOVERNMENT—AGENCIES

Richmond, Virginia, and Jeff Huisingh and Harold Yamauchi were in Monterey, California. The Internet, e-mail, high-speed data processors, and long-distance telephone calls enabled us to collaborate in ways that were previously impossible. For example, we frequently e-mailed spreadsheets and databases to each other and reviewed the data over the telephone. The Internet proved to be an invaluable tool for finding sources of data, telephone numbers of offices and individuals, and the code to solve the network problem. The network optimization code, RELAX IV, was posted on the Internet by Dimitri Bertsekas from MIT. He posted his optimization code on the Internet for other researchers to use in solving network problems.

### **The Problem**

The Defense Contract Management Command (DCMC), headquartered at Fort Belvoir, Virginia, has 15,000 people stationed in offices dispersed throughout the United States with recurring professional-development training requirements. The travel costs associated with this training constitute a substantial portion of the DCMC's budget. In fiscal year 1998, DCMC managers faced a 25 percent budget reduction while maintaining the same training requirement as in previous years.

Mission requirements often require government employees to travel to diverse destinations on temporary duty (TDY) assignments. Such considerations as physical security, specialized equipment, scheduling conflicts, or availability of travel dates for key personnel may limit travel choices to a specific site. However, in the case of DCMC, training planners told us that the

event locations were selected arbitrarily. In other words, a decision maker had discretion in deciding where to host the event. Since travel costs vary with the location, different conference or training sites can produce radical differences in total TDY costs for a training event. The problem, therefore, is to select the most cost-effective training location among many possible choices so that the DCMC minimizes travel expenses and remains within annual travel-budget constraints.

### **The Development Process**

In an effort to solve the problem, the DCMC contacted the Defense Logistics Agency Office of Operations Research and Resource Analysis (DORRA) in February 1998. DORRA conducted an initial literature

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## **Different sites can produce radical differences in TDY costs.**

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review and determined that no previous work had been published that documented solving this specific problem. During the course of the literature review, DORRA partnered with the United States Army Training and Doctrine Command Analysis Center-Monterey (TRAC-Monterey) to solve the problem. At the time, Jeff Huisingh was an operations analyst at TRAC-Monterey and Randy Zimmerman was an operations research analyst at DORRA. The partnership was possible because they were students together at the Colorado School of Mines in the operations research program taught by Gene Woolsey and had collaborated on previous work.

At the School of Mines, Woolsey teaches

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his students to work within the system that has the problem in order to understand the real problem, to break problems down into their component parts, to identify the political constraints on the problem, and to solve problems with the appropriate tools. The first step in the process was to identify all of the costs associated with a DCMC training event. We identified the applicable expenses as airfare, lodging, meals and incidental expenses, rental cars, meeting-facility fees, and equipment-rental fees. We determined that the variability by location of meeting-facility fees, equipment-rental fees, and rental cars precluded their being effectively modeled. However, the *Joint Travel Regulations* (JTR) [Per Diem Committee 1998] that govern all federal employee travel stipulate the maximum allowable amounts that travelers can be reimbursed for official travel to all areas of the United States.

The next step of the process was to identify and map where all of the DCMC offices were located. We quickly realized that the offices were scattered around the United States and located in many towns without major airports. We then contacted the General Services Administration (GSA) city-pair contract office for the list of cities for which the GSA had contracted airfares. The GSA provided an Excel spreadsheet with more than 5,000 city-pair combinations. The list was sorted and filtered to establish a list of 261 airports in the contiguous United States with GSA airfares. Favoring the low-tech approach of a box of pushpins, a \$10 sheet of Styrofoam, and a *National Geographic* map of the United States, Huisingh then plotted each airport

to establish the span of our transportation network.

With the list of GSA contract airfares in hand, the next step was to contact the Per Diem Travel and Transportation Allowance Committee in the Office of the Secretary of Defense for the per-diem rates for the 261 cities and to map that information with the cities. Again, we were given an Excel spreadsheet with all of the established rates for the cities. Mapping the per-diem rates to the city-pair cities was a challenging endeavor. We found numerous spelling discrepancies between the

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For producing a comparative picture of aggregate travel costs for a conference, OffSite is unprecedented.

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lists, for example, cities on the city-pair list that were not on the per-diem list and different abbreviations for cities. When no specified per-diem rate is available, the JTR establishes a flat rate of \$85 per day for lodging, meals, and incidental expenses. These differences prevented us from using an automated mapping for the rates, and each rate was manually verified.

By working within the travel system, we identified constraints on the problem and made assumptions to deal with them. For example, travelers who have to travel less than 100 miles usually drive to the meeting or training event the morning of the event and then return home when it ends. Also, people who fly to a meeting will generally depart the day prior to the event and return home the day it ends. Given that a traveler's itinerary and mode of travel are dependent on distance to the

meeting, we elected to use proximity between airports to qualify the drive-versus-fly decision. To determine the distances between airports, we obtained the geographic coordinates for each airport. No automated mapping of the coordinates was available; thus, we had to manually verify each latitudinal and longitudinal coordinate. In all, we confirmed by hand over 40,000 different numeric values.

Having identified the customer's needs and assembled and verified the data, we still had to solve a substantial networking problem and deliver a product that the DCMC travel planners could use immediately. At this point, Huisingh solicited programming support from Yamauchi to develop a Windows-based program as the deliverable product for the customer.

### The Solution

The JTR stipulates how the government compensates travelers for TDY expenses incurred in performance of duties, which include the costs of transportation, meals, lodging, and incidentals. The least-cost training or conference location minimizes total TDY costs and is represented as follows:

$$\text{Minimize } \sum_j^N X_j * \left( \sum_i^N M_i * (T_{ij} + D * P_j) \right)$$

$$\text{Subject to } \sum_j^N X_j \geq 1$$

where

$i, j$  = cities,

$D$  = duration of event (days),

$M_i$  = number of participants from city  $i$ ,

$P_j$  = per-diem rate at city  $j$ ,

$T_{ij}$  = the cost of travel to city  $j$  from city  $i$ ,

$X_j$  = 1 if conference in city  $j$ , 0 otherwise.

### Transportation Costs

We assumed that travelers would not fly if the distance to the destination city were less than 100 miles. The city-to-city travel cost,  $T_{ij}$ , where city  $j$  is less than or equal to 100 statute miles from city  $i$  is the distance times the standard driving rate authorized for personally owned vehicles (POVs).

We calculated driving rates using a standard \$.325-cents-per-mile rate authorized by the JTR for POV travel. We determined the distance between airports using a great-circle distance calculator [Shufeldt 1980] using latitudinal and longitudinal coordinates.

$$C = ((\sin L1) (\sin L2)) + ((\cos L1) (\cos L2) * (\cos (N2 - N1))),$$

$$A = \text{Radians} (\arccos C),$$

$$D = A * R,$$

where  $L1$  and  $L2$  represent latitudes and  $N1$  and  $N2$  longitudes of the two surveyed airports and  $R$  equals the radius of the earth.

Rental car costs are not considered in this model for a variety of reasons. Currently, the GSA does not negotiate standardized rates with rental car agencies. Hence, it is impractical to model the variability of rates by city and rental agency. Also, training-location planners can choose to use airport hotels for their training events. Frequently airport hotels provide no-cost shuttle transportation from the hotel to the airport, avoiding the cost of rental cars altogether.

The GSA annually contracts with several US air carriers for routes throughout the world. Each route is composed of two cities known as a city pair. The GSA-negotiated fare for city-pair travel is based on one-

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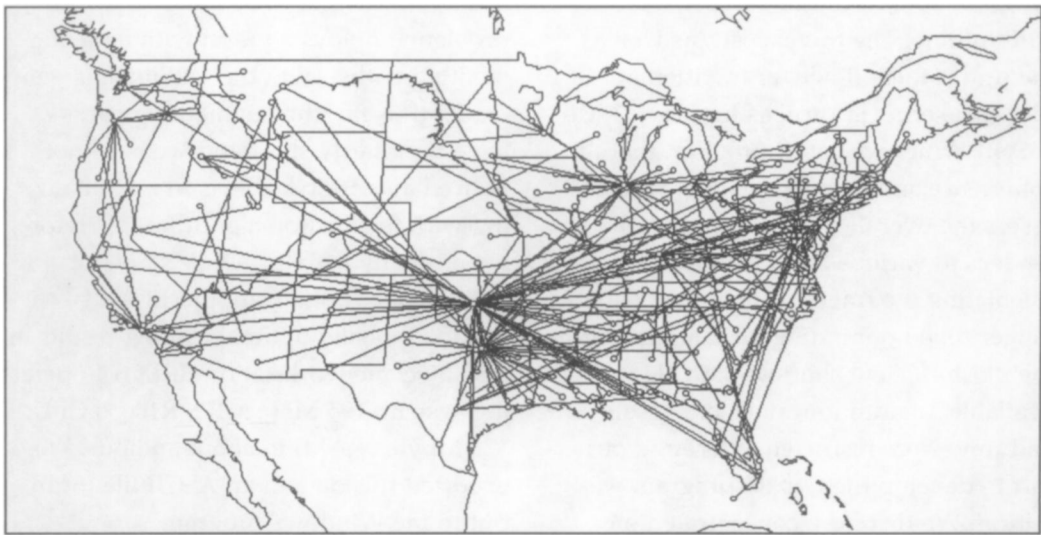
way travel and is valid bidirectionally, for example, the Chicago-Atlanta fare equals the Atlanta-Chicago fare. Although this pricing method was incongruent with our personal experiences (it is usually cheaper to book a round-trip ticket than two one-way fares), government travel agents use the GSA city-pair rates, which are static for the duration of the contract and updated yearly. Although the DCMC had employees overseas, management wished to limit potential conference sites to locations within the coterminous 48 states. This request bounded the problem to 4,640 GSA city pairs connecting 261 US cities.

Since no airport services every other airport, a trip is often composed of multiple legs; for example, a trip between Monterey and Seattle would require a transfer in either Los Angeles or San Francisco. Assuming the least-expensive routing is through San Francisco, the government traveler is charged the sum of the fares between

Monterey and San Francisco and San Francisco and Seattle. Finding the lowest-cost routing for all city pairs is not a trivial matter and requires referencing fares from the 4,600-plus contracted city pairs as well as optimally computing the remaining 29,000 city-pair combinations.

So for distances greater than 100 miles requiring commercial flights, we determined the city-to-city travel costs,  $T_{ij}$ , by solving the following shortest-path subproblem:

Suppose we have a directed graph with a set of nodes  $N$  and a set of arcs  $A$  with nodes numbered from 1 to  $N$  representing the cities and arcs  $(i, j)$  where a commercial flight connects city  $i$  with city  $j$  (Figure 1). Let  $c_{ij}$  represent the cost of flying from city  $i$  to city  $j$ . Let a path in the directed graph be represented by  $(n_1, n_2, \dots, n_k)$  and be composed exclusively of forward arcs. In terms of a minimum-cost-flow problem, we want to find the shortest path from  $n_1$  to  $n_k$  by solving the following:



**Figure 1:** This map depicts the lowest-cost routing of all 261 cities in the model to Oklahoma City.

Minimize  $\sum_{(i,j) \in A} c_{ij}x_{ij}$

Subject to

$$\sum_{\{j \mid (i,j) \in A\}} x_{ij} - \sum_{\{j \mid (j,i) \in A\}} x_{ji} = s_i, \forall i \in N \quad (1)$$

$$x_{ji} \geq 0, \forall (i, j) \in A. \quad (2)$$

Constraint (1) is the conservation-of-flow constraint where  $s_i = 1$  if  $i = n_1$ ,  $s_i = -1$  if  $i = n_k$ , and  $s_i = 0$  for all  $i \neq n_1$  or  $n_k$ . Constraint (2) is the capacity constraint.

The shortest-path problem is well studied. Dijkstra's algorithm is the best-known algorithm for the problem and is discussed in many books [Evans and Minieka 1992]. The algorithm has been implemented with various heap (or priority queue) data structures, for example, the Fibonacci heap [Fredman and Tarjan 1987], to enhance run-time performance. Our approach to developing the OffSite decision support system, however, did not rely on using the theoretically most-efficient algorithm. By design, the OffSite program obtains the travel costs between cities offline. The travel costs, as well as the routes to be flown, are written out to files that serve, in turn, as inputs to a Microsoft Windows-based program that the conference and training planner runs to assess the overall costs of bringing the attendees to various alternate locations. By calculating the travel costs offline, we no longer relied on finding and implementing the most-efficient shortest-path algorithm available. In addition, since our resources and time were restricted, allocating our effort to develop the OffSite program was critical. We therefore concentrated on building the database and coding the Windows program and relied on finding either

a commercial or a public-domain software optimization package capable of solving the shortest-path problem. We found a program in the public domain that implements the relaxation method to solve the minimum-cost-flow problem [Bertsekas 1985, 1991; Bertsekas and Tseng 1988]. The relaxation method is a dual-ascent method that generally appears to outperform classic primal-dual and primal-simplex methods [Bertsekas 1985; Bertsekas and Tseng 1988, 1994]. We obtained FORTRAN code

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**It has been well received throughout the federal government.**

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that implements the relaxation method, called RELAX IV, with the permission of Bertsekas. This code also implements an optional procedure that provides good initial prices and flows for difficult problems using the auction algorithm [Bertsekas 1991; Bertsekas and Tseng 1994]. The relaxation method can be quite slow if the problem employs a graph with long augmenting paths. With the OffSite data generated thus far, the augmenting paths have been fairly short and we have not noticed any appreciable gain by initializing with the auction algorithm. Therefore, we normally use RELAX IV's default (nonauction) initialization. RELAX IV yielded globally optimal routing results on a SGI Octane workstation (IRIX 6.4 operating system, 195 MHz MIPS R10000 CPU, 128 Mbytes RAM) in about an hour. We exported this data in an ASCII file for input to the Windows program.

When a destination city contains multiple major airports, for example, New York

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and Chicago, we tried to calculate the cost of bringing all travelers to a midpoint between the airports. For example, a traveler flying to New York may fly into JFK, LaGuardia, or Newark. We applied a cab fare to the air-travel fare in these situations to transport all passengers to a central point, which is located roughly in south Manhattan. This maintains standardization with other cities where all travelers are brought to one location, the local airport. We obtained the cab-fare rates used in the model by calling actual cab companies serving the areas, requesting fee schedules and any applicable airport departure or arrival fees, zone fees, and so forth. Since all cities served by several airports regulate cab fares, fares remain constant throughout each city.

By constraining the model to select only one arrival airport in a multiple-airport city, the OffSite program initially produced artificially high cost estimates. To

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### Our satisfaction comes from using existing tools for this application.

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eliminate this problem, we relaxed the constraint to terminate travel at one airport to allow passengers to arrive at the lowest-cost airport serving such a city.

#### **Per-Diem Costs**

Per-diem rates are published annually as established by the Per Diem Committee, a federal interagency group. Per-diem costs consist of lodging, meals, and incidental expenses (M and IE). M and IE are geographically dependent and are paid at a flat rate for each full day on TDY. M and IE include \$2 per day for incidental ex-

penses. In accordance with the JTR, the OffSite model allows only 75 percent of the M and IE allowance for the first and last day's travel.

The committee also sets the rates for lodging, which are both temporally and geographically dependent; for example, reimbursable lodging rates for Vail, Colorado are \$226 per night from November through March and \$99 per night from April through October. In cases where seasonal per-diem rates are available, the high-season rate is used regardless of the time of year the conference takes place.

#### **Assumptions**

As a decision support tool, OffSite was never intended to handle every travel situation that could arise in government service. However, for rapidly producing a comparative picture of aggregate travel costs for a specific conference, OffSite is unprecedented. Inherent in this model are several assumptions:

- (1) All travel is done individually, that is, with no ride or room sharing.
- (2) Travelers will use commercial air or POV.
- (3) No government meals or lodging are available.
- (4) Travelers with permanent duty stations (PDSs) greater than 100 statute miles from the site will fly to the training event site; otherwise, they will drive.
- (5) Attendees who fly to the training event will arrive the day before the event begins and will return to their PDSs on the last day.
- (6) Attendees who drive to the training event will depart from their PDSs the day the training event begins and return to their PDSs on the last day.



(7) The GSA-contracted airfare is available for all flights.

If mission requirements demand modification of these assumptions, the user has a limited capability to modify the rate structure to accommodate special situations. For example, the planner could modify the lodging-rate table to allow for government lodging, if available.

Example Solution

The solution methodology used to optimize the results for this problem involves simple arithmetic. Given travelers from different cities that must come together for a meeting, the total travel cost is the sum of the products of the number of travelers from each city and the round-trip-transportation cost from that city to a candidate city. The additional cost of lodging and M and IE is the product of the number of people traveling to the training event site and the duration of the meeting in days. We can best illustrate the methodology by example.

Consider the situation in which 11 people originating from four cities must attend a four-day meeting at one of these origin cities. The expense data varies by city (Table 1). To compute the total cost for holding the meeting in St. Louis, we calculate transportation costs from all points of origin:

$(\# \text{ ATL passengers}) \times (\text{ATL-STL airfare}) = (2 \text{ people}) \times (\$204/\text{person}) = \$408$   
 $(\# \text{ ORD passengers}) \times (\text{ORD-STL airfare}) = (3 \text{ people}) \times (\$70/\text{person}) = \$210$   
 $(\# \text{ SPI}) \times (\text{SPI-STL mileage allowance}) = (2 \text{ people}) \times (168 \text{ miles/person} \times (\$ .31/\text{mile})) = \$104$

The total transportation costs are \$722.

We compute meals and incidental expenses (M and IE) as per *The Joint Travel Regulations*, allowing 75 percent for the first and last days traveled with 100 percent given for full travel days. We can calculate the St. Louis M and IE as follows:

$(\text{Total number of passengers flying to St. Louis}) \times (\text{St. Louis M and IE}) \times (\# \text{ of days}-0.5$

Origin City	Number of Travelers	Meals and Incidental Expenses/Day	Lodging/Day
Atlanta, Georgia	2	\$38	\$97
Chicago, Illinois	3	\$42	\$120
Springfield, Illinois	2	\$30	\$55
St. Louis, Missouri	4	\$42	\$75

Route	Round Trip Travel Cost/Person	Cab Fare to Central Airport Point
Atlanta(ATL)—Chicago(ORD)	\$148	\$8
Atlanta(ATL)—St. Louis(STL)	\$204	N/A
Atlanta(ATL)—Springfield(SPI)	\$254	N/A
Chicago(ORD)—Springfield(SPI)	\$106	\$8
Chicago(ORD)—St. Louis(STL)	\$70	\$8
Springfield(SPI)—St. Louis(STL)	\$52 (Driving, 84 miles)	N/A

Table 1: We used the above travel costs for Atlanta, Chicago, Springfield, and St. Louis in the initial model. The Defense Logistics Agency now updates this information yearly.

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(assuming the meeting is more than one day)) = (5 people)\*(\$42/day per person)\*(4.5 days) = \$945

(Total number of people driving to St. Louis)\*(St. Louis M and IE)\*(# of days-0.5 (assuming the meeting is more than one day)) = (2 people)\*(\$42/day per person)\*(3.5 days) = \$294

The total M and IE costs are \$1,239.

We calculate lodging costs for people that fly, assuming they arrive the day before the meeting and depart the day the meeting ends.

(Total number of people who fly to STL)\*(STL maximum lodging rate)\*(# of days for the training event) = (5 people)\*(\$75/day per person)\*(4 days) = \$1,500.

We calculate lodging costs for people that drive, assuming they arrive the day of the meeting and depart the day the meeting ends.

(Total number of people who drive to STL)\*(STL maximum lodging rate)\*(# of days for the training event - 1) = (2 people)\*(\$75/day per person)\*(3 days) = \$450

The total lodging costs are \$1,950.

The total cost to host the training event in St. Louis is thus \$3,911.

We compute the costs for Atlanta, Chicago, and Springfield using the same

City	Travel	M and IE	Lodging	Total
Atlanta, Georgia	\$1,768	\$1,539	\$3,492	\$6,799
Chicago, Illinois	\$788	\$1,512	\$3,840	\$6,140
St. Louis, Missouri	\$722	\$1,239	\$1,950	\$3,911
Springfield, Illinois	\$1,034	\$1,095	\$1,760	<b>\$3,889</b>

**Table 2: Aggregating the costs of travel, meals and incidental expenses, and lodging for all possible locations yields Springfield as the lowest-cost meeting site.**

methodology (Table 2). The true strength of this methodology is that it may, as this example shows, provide a counterintuitive answer to the problem. In this case, the intuitive answer is to send everyone to St. Louis since it is the city with the largest number of resident attendees. However, the reduced M and IE and lodging costs for Springfield outweigh the transportation benefits provided by staying in St. Louis.

## Application

It quickly becomes obvious that while manually calculating all of the costs is a straight-forward, albeit time-consuming task, a spreadsheet would be useful for performing the calculations. However, obtaining the data required makes developing a spreadsheet-based solution to this problem difficult.

We tested OffSite using data from a two-day training event the DCMC sponsored in Monterey, California. Representatives from every DCMC office attended the training event. OffSite revealed that temporary duty expenses incurred during the Monterey event cost the taxpayer \$37,582; over \$10,000 more than the least-cost alternative, which was Tulsa, Oklahoma. This represents a 39 percent savings on this training event alone. If OffSite were constrained to choose one of the DCMC office locations, then St. Louis, Missouri would have been chosen at \$29,896. St. Louis would have been 26 percent less expensive than Monterey. The 10 least-expensive places in the United States for this example would have been

Tulsa, Oklahoma	\$27,060,
Oklahoma City, Oklahoma	\$27,180,

Little Rock, Arkansas	\$27,622,
Moline, Illinois	\$28,114,
Louisville, Kentucky	\$28,857,
Reno, Nevada	\$29,148,
Ontario, California	\$29,314,
Albuquerque, New Mexico	\$29,742,
Lawton, Oklahoma	\$29,792, and
St. Louis, Missouri	\$29,896.

If an origin city hosted the conference, the least-expensive cities would have been

St. Louis, Missouri	\$29,896,
Indianapolis, Indiana	\$30,110,
Dayton, Ohio	\$30,129,
Orlando, Florida	\$30,466,
Hartford, Connecticut	\$30,513,
Cleveland, Ohio	\$30,624,
Birmingham, Alabama	\$30,658,
Albany, New York	\$31,081,
Wichita, Kansas	\$31,157, and,
Jacksonville, Florida	\$31,381.

If DCMC could save 39 percent on every training event, it would realize a \$3.1 million savings based upon the FY97 training budget. Even with a more conservative estimate of five percent annual savings, DCMC could save \$400,000 a year on travel related to training.

The inherent beauty in the OffSite model is that its power is disguised in a very intuitive Windows-based presentation. Most new users can open the program, input data, and obtain results in less than 10 minutes. Since OffSite's introduction in 1998, it has been well received throughout the federal government. Because the program can now be downloaded from the Web and forwarded as an e-mail attachment, tracking the extent of savings is impossible. However, the executive director for plans and operations of the Defense Logistics Agency, Christine

Gallo, substantiates cost-avoidance savings of well over a million dollars for her agency. As a statement to OffSite's broad applicability, Major General Craig Bambrough, deputy commanding general of the United States Army Reserve, mandated the use of OffSite for travel cost-benefit analysis for the entire army reserve of 1,064,000 soldiers.

Our satisfaction from completing this project comes not from delivering a product to meet a customer's needs on time and under budget but from using existing tools for this application. From Bertsekas and Tseng's RELAX IV program to the bitmap of the United States (Figure 1) recycled from a combat simulation, the OffSite program is a model of the efficient use of resources.

#### Acknowledgments

We are indebted to Dimitri Bertsekas for allowing us to incorporate his code into the OffSite program pro bono. The use of the RELAX IV program saved us a tremendous amount of time and money that we could have spent trying to invent or to purchase another solver at a time when we had a very limited budget to support the project.

#### APPENDIX

##### The Relaxation Method

For the shortest-path problem, suppose we have a directed graph with a set of nodes  $N$  (numbered from 1 to  $N$ ) and a set of arcs  $A$ . Each arc  $(i, j)$  has a cost  $c_{ij}$  associated with it. Let a path  $P$  in the directed graph be represented by  $(n_1, n_2, \dots, n_k)$  and be composed exclusively of forward arcs. In terms of a minimum-cost-flow problem, we want to find the shortest path from  $n_1$  to  $n_k$  by solving the following:

$$\text{Minimize } \sum_{(i,j) \in A} c_{ij} x_{ij}$$

Subject to

$$\sum_{(j| (i,j) \in A)} x_{ij} - \sum_{(j| (j,i) \in A)} x_{ji} = s_i, \forall i \in N \quad (1)$$

$$0 \leq x_{ij}, \forall (i, j) \in A. \quad (2)$$

Constraint (1) is the conservation-of-flow constraint where  $s_i = 1$  if  $i = n_1$ ,  $s_i = -1$  if  $i = n_k$ , and  $s_i = 0$  for all  $i \neq n_1$  or  $n_k$ . Constraint (2) is the capacity constraint. It is assumed that  $c_{ij}$ ,  $x_{ij}$ , and  $s_i$  are all integer. If the flow  $\bar{x}$  through  $P$  is an optimal solution to the minimum-cost-flow problem,  $\bar{x}$  will be defined by

$$x_{ij} = \begin{cases} 1 & \text{if } (i, j) \in P, \\ 0 & \text{otherwise.} \end{cases}$$

If the above problem is called the primal problem, its dual can be formulated by associating a price  $p_i$  with each node  $i \in N$ . The dual problem is

Maximize  $p_{n_1} - p_{n_k}$

Subject to  $p_i - p_j \leq c_{ij}, \forall (i, j) \in A$

$p_i \geq 0, \forall i \in N$ .

From duality theory, given  $\bar{x}$  is a feasible flow vector for the primal problem and  $\bar{p}$  is the price vector for the dual problem;  $\bar{x}$  and  $\bar{p}$  make up an optimal primal-dual solution pair if and only if the following complementary slackness conditions are met:

$$p_i - p_j \leq c_{ij}, \quad \forall (i, j) \in A.$$

$$p_i - p_j = c_{ij}, \quad \forall (i, j) \in A \text{ where } x_{ij} = 1.$$

Primal-dual methods start with a price vector and iteratively try to obtain new price vectors that increase the dual functional while also maintaining the flow vector such that it satisfies complementary slackness with the price vector. The iterations involve a change of the price vector along a direction that provides the maximal rate of improvement of the dual functional per-unit change of the price vector. During the course of the iteration, a connected subset of nodes  $S$  is built until either the nodes in  $S$  form an augmenting

path or  $S$  defines the steepest dual ascent direction. If the former, there is a path originating from the first node added to  $S$ ,  $n_1$ , to the last node added,  $n_k$ , whose arcs have room for a flow increase in the direction  $n_1$  to  $n_k$ . If the latter, the prices of the nodes in  $S$  are changed by an equal amount.

The relaxation method is similar to the primal-dual method. The major difference between the two is that the relaxation method performs a price change as soon as  $S$  defines an ascent direction. This often occurs when the first node enters  $S$ , that is, the ascent direction is along a single coordinate. This is a key factor to the algorithm's efficiency.

## References

- The Per Diem Travel and Transportation Allowance Committee 1998, *The Joint Travel Regulations*, Volume 1, Uniformed Service Members, Department of Defense, Washington.
- Bertsekas, D. P. 1985, "A unified framework for minimum cost network flow problems," *Mathematical Programming*, Vol. 32, No. 2, pp. 125-145.
- Bertsekas, D. P. 1991, *Linear Network Optimization, Algorithms and Codes*, MIT Press, Cambridge, Massachusetts.
- Bertsekas, D. P. and Tseng, P. 1988, "Relaxation methods for minimum cost ordinary and generalized network flow problems," *Operations Research*, Vol. 36, No. 1, pp. 93-114.
- Bertsekas, D. P. and Tseng, P. 1994, "RELAX IV: A faster version of the RELAX code for solving minimum cost flow problems," Report No. LIDS-P-2276, Massachusetts Institute of Technology.
- Evans, J. R. and Minieka, E. 1992, *Optimization Algorithms for Networks and Graphs*, Marcel Dekker, New York.
- Fredman, M. L. and Tarjan, R. E. 1987, "Fibonacci heaps and their uses in improved network optimization algorithms," *Journal of the Association for Computing Machinery*, Vol. 34, No. 3, pp. 596-615.
- Shufeldt, H. H. 1980, *The Calculator Afloat: Mariner's Guide to the Electronic Calculator*, Naval Press Institute, Annapolis, Maryland.