Course Project Report

Optimizing Inventory for Zoyo Neighborhood Yogurt

A Linear Programming Model

SCM 518 : Analytical Decision Modeling

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Background

Zoyo Neighborhood Yogurt offers a wide variety of frozen yogurt flavors and toppings. Its main ingredients include **milk**, **cream**, and **yogurt cultures**, which are purchased from dairy suppliers. The shop also serves **smoothies** and **parfaits**, which require additional ingredients like **fruit** and **granola**.

During the winter season, customer visits tend to decline, reducing the demand for these products.

Ordering the same quantities as in peak seasons can lead to inventory surplus and financial losses. To avoid wastage and minimize costs, Zoyo needs a plan to optimize the procurement of dairy products and other key ingredients.

Problem Statement

The goal is to determine the optimal quantities of **milk**, **cream**, and **yogurt cultures** to order, while meeting the reduced winter demand for **frozen yogurt**, **smoothies**, and **parfaits**, staying within storage and budget constraints, and minimizing costs.

Objective

Minimize the total cost of purchasing the required ingredients while:

1. Ensuring adequate stock to meet customer demand for all menu items.
2. Not exceeding storage space or budget constraints.

Data we collected

1. Menu Items and Ingredient Requirements

For each menu item, the quantity of key ingredients required (milk, cream, yogurt cultures, fruit, and granola) per unit needs to be determined.

Menu/Ingredients	Milk(L/unit)	Cream(L/unit)	Yogurt cultures(kg/unit)	Dry Fruit(kg/unit)	Granola(kg/unit)	Demand (units)
Vanilla Yogurt	0.8	0	0.1	0	0	50
Chocolate Yogurt	0.6	0.2	0.15	0	0	40
Strawberry Yogurt	0.7	0	0.1	0.2	0	35
Smoothie	0.4	0	0.05	0.3	0	30
Parfait	0	0	0.2	0.15	0.1	20

2. Ingredient Costs, Storage Requirements & available budget and capacity

Ingredient	Cost per unit	Storage per unit
Milk(L/unit)	$2 per liter	0.5 sq.ft.per.liter
Cream(L/unit)	$3 per liter	0.7 sq.ft.per.liter
Yogurt cultures(kg/unit)	$5 per kg	1.0 sq.ft.per.kg
dry Fruit(kg/unit)	$4 per kg	0.8 sq.ft.per.kg
Granola(kg/unit)	$3 per kg	0.6 sq.ft.per.kg

Total storage capacity (sq.ft)	180
total budget	$ 450.00

Mathematical model

Inputs:

- $i \in \{1,2,3,4,5\}$ - Index to represent Ingredients (1 = Milk, 2 = Cream, 3 = Yogurt Cultures, 4 = Fruit, 5 = Granola);
- $j \in \{1,2,3,4,5\}$ - Index to represent Menu items (1 = Vanilla Yogurt, 2 = Chocolate Yogurt, 3 = Strawberry Yogurt, 4 = Smoothie, 5 = Parfait);

- C_i: Cost per unit of ingredient i($), ($C_1$=2$ per liter);
- S_i: Storage requirement per unit of ingredient i (sq. ft.), (S_1=0.5 sq.ft.per.liter);
- A_{ij}: Amount of ingredient i required to produce one unit of menu item j,(A_{14}=0.4 - 0.4 liters of milk needed for 1 unit of Smoothie);
- D_j: Projected demand for menu item j (units), (D_1=50);
- B: Total budget available ($),(B=450);
- SC: Total storage capacity available (sq. ft.), (SC=180);

Decision Variables:

X_i: Quantity of ingredient i to order, (X_1: Quantity of milk to order (liters));

Objective:

Minimize the total cost of ingredients:

$$\text{Minimize: } \sum_{i \in \{1,2,3,4,5\}} C_i * X_i$$

Where C_i is the cost per unit of ingredient i, and X_i is the amount of ingredient i ordered.

Constraints:

i) $X_i >= \sum_{i \in \{1,2,3,4,5\}} \sum_{j \in \{1,2,3,4,5\}} A_{ij} * D_j$, (Demand Constraints - The ordered quantity of each ingredient i must be sufficient to meet the demand for all menu items);

ii) $\sum_{i \in \{1,2,3,4,5\}} S_i * X_i <= SC$, (Storage Constraint - The total storage required for the ordered ingredients must not exceed the available storage space);

iii) $\sum_{i \in \{1,2,3,4,5\}} C_i * X_i <= B$, (Budget Constraint - The total cost of the ordered ingredients must not exceed the available budget);

iv) $\sum_{i \in \{1,2,3,4,5\}} X_i >= 0$, (Non-Negativity Constraints - The quantities ordered for each ingredient must be non-negative);

Excel - Solution

The below figure illustrates the linear programming model developed to optimize inventory procurement for Zoyo Neighborhood Yogurt. The model integrates key data points such as menu item ingredient requirements, costs, and storage needs, along with constraints like demand, budget, and storage capacity.

Key Highlights

1. *Objective Function:* Minimize total procurement costs while satisfying the constraints.

2. *Decision Variables:* Quantities of ingredients (milk, cream, yogurt cultures, fruit, granola) to order based on calculated demand.

3. Constraints:

- Demand constraints ensure sufficient quantities of each ingredient to meet product requirements.
- Storage constraints limit total storage use to 92.25 sq. ft., well below the 180 sq. ft. capacity.

- Budget constraints ensure total cost ($407) is within the allocated budget of $450.

Menu/Ingredients	Milk(L/unit)	Cream(L/unit)	Yogurt cultures(kg/unit)	dry Fruit(kg/unit)	Granola(kg/unit)	Demand (units)
Vanilla Yogurt	0.8	0	0.1	0	0	50
Chocolate Yogurt	0.6	0.2	0.15	0	0	40
Strawberry Yogurt	0.7	0	0.1	0.2	0	35
Smoothie	0.4	0	0.05	0.3	0	30
Parfait	0	0	0.2	0.15	0.1	20

Ingredient	Cost per unit	Storage per unit
Milk(L/unit)	$2	0.5
Cream(L/unit)	$3	0.7
Yogurt cultures(kg/unit)	$5	1
dry Fruit(kg/unit)	$4	0.8
Granola(kg/unit)	$3	0.6

Total storage capacity (sq.ft)	180
total budget	$ 450.00

Decision Variables

Ingredient	Quantity to be ordered
Milk(L/unit)	101
Cream(L/unit)	8
Yogurt cultures(kg/unit)	20
dry Fruit(kg/unit)	19
Granola(kg/unit)	2

Constraints

Ingredient	demand constraints
Milk(L/unit)	100.5
Cream(L/unit)	8
Yogurt cultures(kg/unit)	20
dry Fruit(kg/unit)	19
Granola(kg/unit)	2

		storage constraints		
		92.25	<=	180
		Budget Constraint		
		$407	<=	$ 450.00

Objective	
Minimize	407

Figure: Inventory Optimization Model for Zoyo Neighborhood Yogurt

This structured approach successfully balances cost efficiency, storage optimization, and demand satisfaction, showcasing practical real-world applicability for Zoyo's seasonal inventory management needs.

Excel - Solver Visual

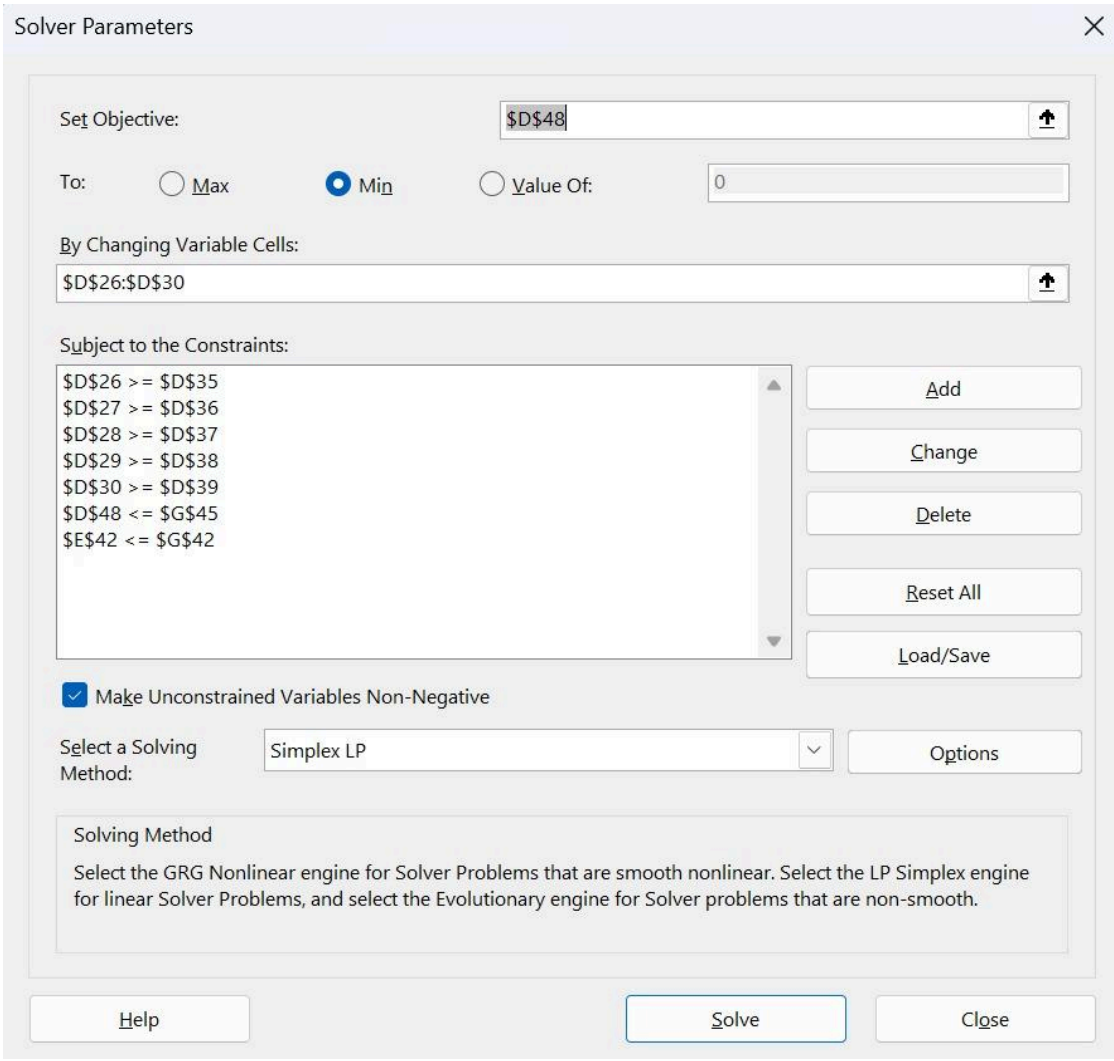


Figure: Excel Solver Parameters for Inventory Optimization

The above figure demonstrates the use of Excel's Solver tool to optimize the inventory procurement plan for Zoyo Neighborhood Yogurt. Solver is configured to minimize the total cost of ingredient procurement while satisfying all constraints as outlined below:

1. *Objective Function:* The cell $D$48 represents the total cost, which is set to be minimized.

2. *Decision Variables:* Cells $D$26:$D$30 correspond to the quantities of ingredients (milk, cream, yogurt cultures, fruit, granola) to be ordered.
3. *Constraints:*

- Demand constraints ensure the ordered quantities meet the required minimum levels for each ingredient ($D$26 >= $D$35, etc.).
- Budget constraint ensures the total cost does not exceed the allocated budget ($D$48 <= $G$45).
- Storage constraint ensures the total storage requirement remains within the available capacity ($E$42 <= $G$42).

4. *Non-Negativity:* The "Make Unconstrained Variables Non-Negative" option ensures all decision variables are non-negative.

5. *Solving Method:* The **Simplex LP** method is selected for solving this linear programming problem efficiently.

Python - Solution

```
Gurobi Optimizer version 12.0.0 build v12.0.0rc1 (linux64 - "Ubuntu 22.04.3 LTS")

CPU model: Intel(R) Xeon(R) CPU @ 2.20GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 1 physical cores, 2 logical processors, using up to 2 threads

Optimize a model with 7 rows, 5 columns and 15 nonzeros
Model fingerprint: 0x14e6b4a9
Coefficient statistics:
  Matrix range     [5e-01, 5e+00]
  Objective range  [2e+00, 5e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [2e+00, 4e+02]
Presolve removed 7 rows and 5 columns
Presolve time: 0.01s
Presolve: All rows and columns removed
Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.0700000e+02   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.01 seconds (0.00 work units)
Optimal objective  4.070000000e+02
Optimal solution found:
Milk: 100.50 units
Cream: 8.00 units
Yogurt Cultures: 20.00 units
Dry Fruit: 19.00 units
Granola: 2.00 units
Total Cost: $407.00
```

Figure: Gurobi Optimizer Results for Inventory Optimization

The above figure displays the output from the Gurobi Optimizer used to solve the inventory optimization problem for Zoyo Neighborhood Yogurt. The linear programming model was solved efficiently, yielding the following insights:

1. *Optimal Objective:* The total cost of $407 was minimized while satisfying all constraints.

2. *Optimal Solution:* The quantities of ingredients to order were determined as:

- Milk: 100.50 units
- Cream: 8.00 units
- Yogurt Cultures: 20.00 units
- Dry Fruit: 19.00 units
- Granola: 2.00 units

3. Performance:

- The model was solved in 0 iterations, taking only 0.01 seconds, showcasing the efficiency of Gurobi's presolve phase.
- All constraints, including demand, storage, and budget, were satisfied.

This output validates the effectiveness of the linear programming model in optimizing inventory procurement while adhering to operational constraints and cost objectives.

Conclusion

Summary of Optimal Quantities and Cost Validation

Ingredient	Optimal Quantity (units)	Cost ($)
Milk	101 L	202
Cream	8 L	24
Yogurt Cultures	20 kg	100
Fruit	19 kg	76
Granola	2 kg	6

Constraint Validation

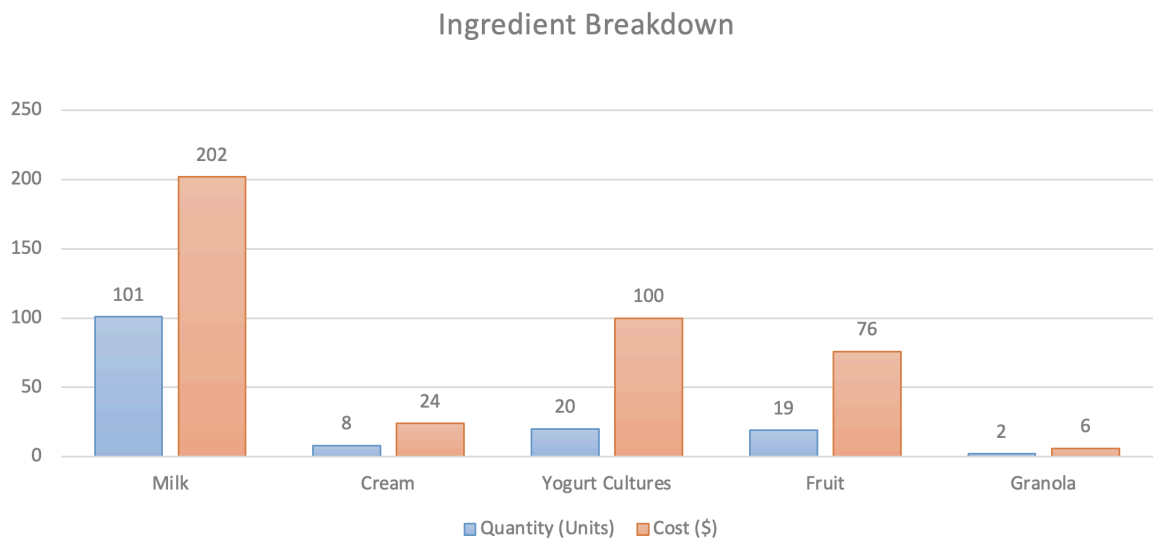
- **Storage:** 92.25 sq.ft. <= 180 sq.ft.
- **Budget:** $407 <= $450

This table summarizes the optimal ingredient quantities and their associated costs, with storage and budget constraints successfully validated.

Visualizations

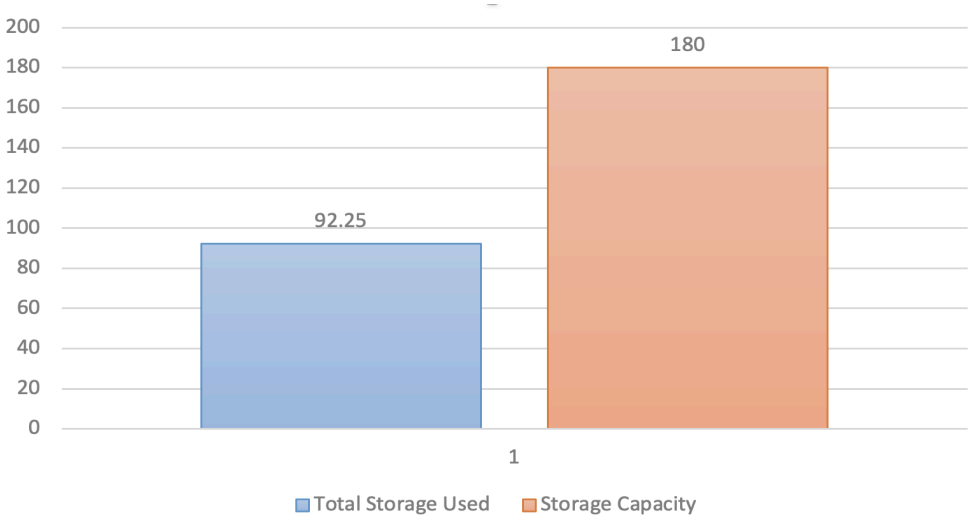
Ingredient Breakdown Chart:

This chart displays the quantities and costs of ingredients, with milk being the highest contributor in both aspects.



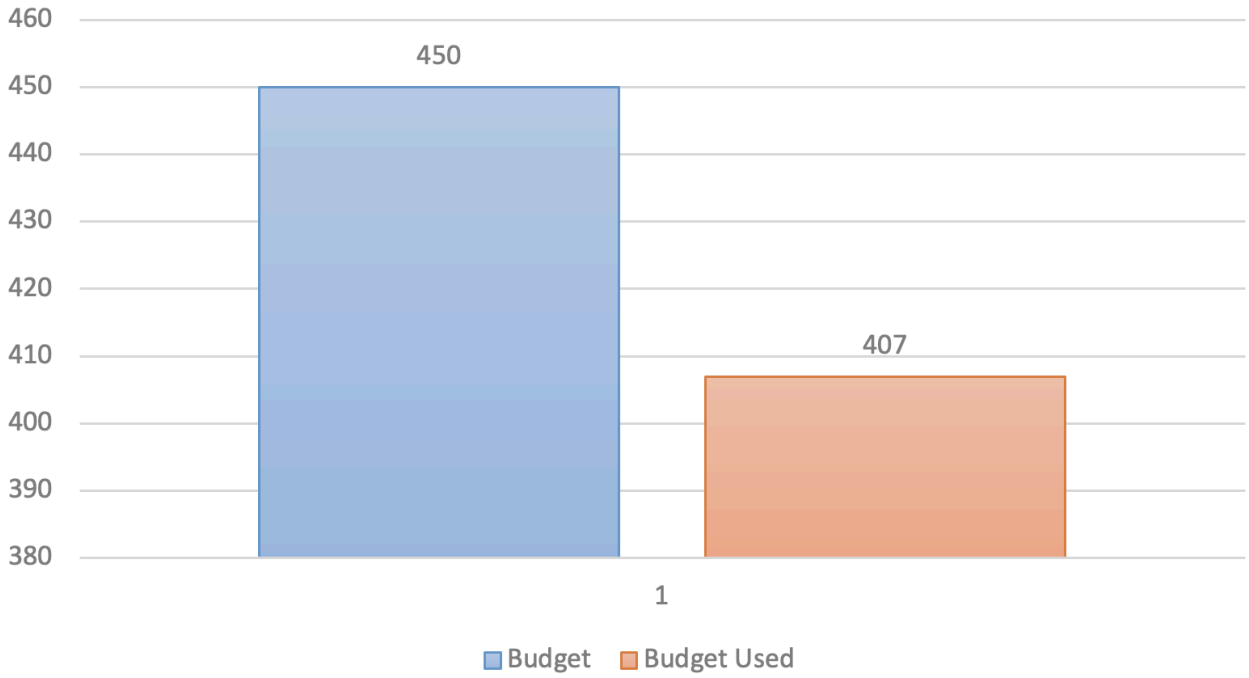
Storage Utilization Chart:

This chart shows that only 92.25 sq. ft. of the 180 sq. ft. storage capacity was utilized, ensuring efficient use of space.



Budget Utilization Chart:

This chart highlights that $407 of the $450 budget was used, leaving a margin for flexibility.



To summarise, in this project we successfully developed and implemented a linear programming model to optimize inventory procurement for Zoyo Neighborhood Yogurt. By balancing costs, storage constraints, and demand requirements, the model achieved a cost-efficient solution, saving $43 under the allocated budget and utilizing only 51.25% of the available storage space. This approach demonstrates the practicality and scalability of optimization techniques in addressing real-world business challenges, paving the way for more informed and efficient decision-making in resource management.