Applications of Derivatives: JEE Maths

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1. The largest of $\cos(ln\theta)$ and $\ln(\cos\theta)$ if $e^{\frac{\pi}{2}} < \theta < \frac{\pi}{2}$ is ....
2. The function
$$y = 2x^2 - ln|x|$$
is monotonically increasing for values of $x(\neq 0)$ satisfying the inequalities ...... and monotonically decreasing for values of x satisfying the inequalities .....
3. The set of all x for which $\ln(1 + x) \leq x$ is equal to ......
4. Let P be a variable point on the ellipse
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{4.1}$$
with the foci F_1 and F_2. If A is the area of the triangle PF_1 F_2 then the maximum value of A is......
5. Let C be the curve
$$y^2 - 3xy + 2 = 0 \tag{5.1}$$
If H is the set of points on the curve C where the tangent is horizontal and V is the set of the point on the curve C where the tangent is vertical then H = ... and V = ...

True/False:

6. If x - r is the factor of the polynomial
$$f(x) = a_n x^4 + ... + a_0,$$
repeated m times $(1 < m < n)$, then r is a root of $f'(x) = 0$ repeated m times.
7. For $0 < a < x$, the minimum value of the function $log_a x + log_x$ a is 2.

MCQs with One Correct Answer:

8. If a + b + c = 0,Then the quadratic equation
$$3ax^2 + 2bx + c = 0 \tag{8.1}$$
has
   a) at least one root in [0 , 1]
   b) one root in [2,3] and the other in [-2,-1]
   c) imaginary roots
   d) none of these
9. AB is a diameter of a circle and C is any point on the circumference of the circle.Then
   a) The area of ΔABC is maximum when it is isosceles
   b) The area of ΔABC is maximum when it is isosceles
   c) The perimeter of ΔABC is maximum when it is isosceles
   d) none of these
10. The normal to the curve
$$x = a(\cos\theta + \theta\sin\theta)$$
$$y = a(\sin\theta - \theta\cos\theta)$$
at any point $'\theta'$ is such that.
   a) it makes a constant angle with x-axis
   b) it passes through the origin
   c) it is at a constant distance from the origin
   d) none of these
11. If
$$y = alnx + bx^2 + x \tag{11.1}$$
has its extremum values at x = -1 and x = 2 then
   a) a = 2, b = -1
   b) a = 2, b = $\frac{-1}{2}$
   c) a = -2, b = $\frac{1}{2}$
   d) none of these
12. Which one of the following curves cut the parabola
$$y^2 = 4ax \tag{12.1}$$
at right angles?
   a) $x^2 + y^2 = a^2$
   b) $y = e^{\frac{x}{2a}}$
   c) y = ax
   d) $x^2 = 4ay$

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13. The function defined by
$$f(x) = (x+2)e^{-x}$$
is
a) decreasing for all x
b) decreasing in ($-\infty$, -1) and increasing in (-1, ∞)
c) increasing for all x
d) decreasing in (-1, ∞) and increasing in ($-\infty$, -1)

14. The function
$$f(x) = \frac{ln(\pi + x)}{ln(e + x)}$$
a) increasing on $(0, \infty,)$
b) decreasing on $(0, \infty)$
c) increasing on $(0, \frac{\pi}{e})$, decreasing on $(\frac{\pi}{e}, \infty)$
d) decreasing on $(0, \frac{\pi}{e})$, increasing on $(\frac{\pi}{e}, \infty)$

15. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at the point
a) 0
b) $\frac{1}{4}$
c) $\frac{1}{2}$
d) $\frac{1}{3}$

16. The slope of the tangent to a curve $y = f(x)$ at $[x, f(x)]$ is 2x + 1. If the curve passes through the point (1, 2), then the area bounded by the curve, the x-axis and line x = 1 is
a) $\frac{5}{6}$
b) $\frac{6}{5}$
c) $\frac{1}{6}$
d) 6

17. If
$$f(x) = \frac{x}{\sin x} and$$
$$g(x) = \frac{x}{\tan x}$$
where $0 < x \leq 1$, then in this interval
a) both $f(x)$ and $g(x)$ are increasing functions
b) both $f(x)$ and $g(x)$ are decreasing functions
c) $f(x)$ is an increasing function
d) $g(x)$ is an increasing function

18. The function
$$f(x) = sin^4 x + cos^4 x$$
increasing if
a) $0 < x < \frac{\pi}{8}$
b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$
d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

19. Consider the following statements in S and R
S: Both sin x and cos x are decreasing functions in the interval $(\frac{\pi}{2}, \pi)$
R: The differentiable function decrease in an interval (a, b), then its derivative also decreases in (a,b).
Which of the following is true
a) Both S and R are wrong
b) Both S and R are correct but R is not the correct explanation for S
c) S is correct and R is correct explanation for S
d) S is Correct and R is wrong

20. Let
$$f(x) = \int e^x(x-1)(x-2)dx$$
Then f decrease in the interval
a) $(-\infty, -2)$
b) $(-2, -1)$
c) (1, 2)
d) $(2, \infty)$

21. If the normal ti the curve $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3) =$
a) -1
b) $\frac{-3}{4}$
c) $\frac{4}{3}$
d) 1

22. Let
$$f(x) = \left\{\begin{cases} |x| & 0 < |x| \leq 2 \\ 1 & x = 0 \end{cases}\right\}$$
then at x = 0, f has
a) a local maximum
b) no local maximum
c) a local minimum
d) no extremum

23. For all $x \in (0, 1)$
a) $e^x < 1 + x$
b) $log_e(1+x) < x$
c) $\sin x > x$
d) $log_e x > x$

24. If $f(x) = xe^{x(1-x)}$ then f(x) is
a) increasing on $[\frac{-1}{2}, 1]$

b) decreasing on R
c) increasing on R
d) decreasing on $[\frac{-1}{2}, 1]$

25. the triangle formed by the tangents of the curve

$$f(x) = x^2 + bx - b$$

at the point (1, 1) and the coordinate axes, lies in the first quadrant. If its area is 2, Then the value of b is

a) -1
b) 3
c) -3
d) 1

26. Let

$$f(x) = (1 + b^2)x^2 + 2bx + 1 \tag{26.1}$$

and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is

a) $[0, 1]$
b) $(0, \frac{1}{2}]$
c) $[\frac{1}{2}, 1]$
d) $(0, 1]$

27. The length of the longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing is

a) $\frac{\pi}{3}$
b) $\frac{\pi}{2}$
c) $\frac{3\pi}{2}$
d) π

28. The points on the curve

$$y^3 + 3x^2 = 12y \tag{28.1}$$

where the tangent is vertical, is

a) $(\pm\frac{4}{\sqrt{3}}, -2)$
b) $(\pm\sqrt{\frac{11}{3}}, 1)$
c) $(0, 0)$
d) $(\pm\frac{4}{\sqrt{3}}, 2)$

29. In [0, 1] Lagranges Mean Value theorem is NOT applicable to

a) f(x) =
$$\begin{cases} \frac{1}{2} - x & x < \frac{1}{2} \\ (\frac{1}{2} - x)^2 & x \geq \frac{1}{2} \end{cases}$$
b) f(x) =
$$\begin{cases} \frac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$
c) $f(x) = x|x|$
d) $f(x) = |x|$

30. Tangent is drawn to ellipse

$$\frac{x^2}{27} + y^2 = 1at(3\sqrt{3}\cos\theta, \sin\theta)(where\theta \in (0, \frac{\pi}{2}))$$

Then the value of θ such that sum of intercepts on axes made by this tangent is minimum is

a) $\frac{\pi}{3}$
b) $\frac{\pi}{6}$
c) $\frac{\pi}{8}$
d) $\frac{\pi}{4}$

31. If

$$f(x) = x^3 + bx^2 + cx + d$$

and $0 < b^2 < c$, then in $(-\infty, \infty)$

a) $f(x)$ is strictly increasing function
b) $f(x)$ has local maxima
c) $f(x)$ is a strictly decreasing function
d) $f(x)$ is bounded

32. If

$$f(x) = x^\alpha logx$$

and $f(0) = 0$ then the value of α for which Rolle's theorem can be applied in [0, 1] is

a) -2
b) -1
c) 0
d) $\frac{1}{2}$

33. If $P(x)$ is a polynomial of degree less than or equal to 2 then S is the set of all such polynomials so that p(0) = 0, P(1) = 1 and $P'(x) > 0\forall x \in [0, 1]$ then

a) S = Φ
b) $S = ax + (1 - a)x^2 \forall a \in (0, 2)$
c) $S = ax + (1 - a)x^2 \forall a \in (0, \infty)$
d) $S = ax + (1 - a)x^2 \forall a \in (0, 1)$

34. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$

a) on the left of x = c
b) on the right of x = c
c) at no point
d) at all points

35. Consider the two curves

$$C_1 : y^2 = 4x$$

$$C_2 : x^2 + y^2 - 6x + 1 = 0$$

then,

a) C_1 and C_2 touch only each other at one point
b) C_1 and C_2 touch each other exactly at two point
c) C_1 and C_2 intersect at exactly two points
d) C_1 and C_2 neither intersect nor touch each other

36. The total number of local minima and local maxima of the function f(x)=

$$\begin{cases} (2+x)^3 & -3 < x \leq -1 \\ x^{\frac{2}{3}} & -1 < x < 2 \end{cases}$$

is

a) 0
b) 1
c) 2
d) 3

37. Let the function g: $(-\infty, \infty) \to (\frac{-\pi}{2}, \frac{\pi}{2})$ be given by

$$g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$$

Then g is

a) even and it is strictly increasing in (0,∞)
b) odd and is strictly decreasing in $(-\infty, \infty)$
c) odd and is strictly increasing in $(-\infty, \infty)$
d) neither even nor odd but it is strictly increasing in $(-\infty, \infty)$

38. The least value of $a \in R$ for which $4\alpha x^2 + \frac{1}{x} \leq 1$, for all $x > 0$ is

a) $\frac{1}{64}$
b) $\frac{1}{32}$
c) $\frac{1}{27}$
d) $\frac{1}{25}$

39. If $f : R \to R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$, and $f(\frac{1}{2}) = \frac{1}{2}$, $f(1) = 1$, then

a) $f'(1) \leq 0$
b) $0 < f'(1) \leq \frac{1}{2}$
c) $\frac{1}{2} < f'(1) \leq 1$
d) $f'(1) > 1$

MCQ's with One or More than One Correct Answer:

40. Let

$$P(x) = a_0 + a_1x^2 + a_2x^4.....a_nx^{2n}$$

be a polynomial equation in real variable x with $0 < a_0 < a_1 < a_2 < ..... < a_n$.The function $P(x)$ has

a) neither a maximum nor a minimum
b) only one maximum
c) only one minimum
d) only one minimum and one maximum
e) none of these

41. If the line ax + by + c = 0 is a normal to the curve xy = 1 then

a) $a > 0, b > 0$
b) $a > 0, b < 0$
c) $a < 0, b > 0$
d) $a < 0, b < 0$
e) none of these

42. The smallest positive root of the equation, $\tan x - x = 0$ lies in

a) $(0, \frac{\pi}{2})$
b) $(\frac{\pi}{2}, \pi)$
c) $(\pi, \frac{3\pi}{2})$
d) $(\frac{3\pi}{2}, 2\pi)$

43. Let f and g be the increasing and decreasing functions respectively from $[0, \infty)$ to $[0, \infty)$. Let $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is

a) always zero
b) always negative
c) always positive
d) strictly increasing
e) None of these

44. If

$$f(x) = \left\{ \begin{array}{ll} 3x^2 + 12x - 1 & -1 \leq x \leq 2 \\ 37 - x & 2 < x \leq 3 \end{array} \right\}$$

then:

a) $f(x)$ is increasing on [-1, 2]
b) $f(x)$ is continues on [-1, 3]
c) $f(2)$ does not exist
d) $f(x)$ has the maximum value at x = 2

45. If

$$h(x) = f(x) - (f(x))^2 + (f(x))^3$$

for every real number x. Then

a) h is increasing whenever f is increasing
b) h is increasing whenever f is decreasing
c) h is decreasing whenever f is decreasing
d) nothing can be said in general

46. If
$$f(x) = \frac{x^2-1}{x^2+1}$$
for every real number then x then the minimum value of f
   a) does not exist because f is unbounded
   b) is not attained even though f is bounded
   c) is equal to 1
   d) is equal to -1
47. The number of values of x where function
$$f(x) = \cos x + \cos(\sqrt{2}x)$$
attains its maximum is
   a) 0
   b) 1
   c) 2
   d) infinite
48. The function
$$f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5 dt$$
has a local minimum at x =
   a) 0
   b) 1
   c) 2
   d) 3
49. $f(x)$ is a cubic polynomial with $f(2) = 18$ and $f(1) = -1$. Also $f(x)$ has local maxima at x = -1 and $f'(x)$ has local minima at x = 0, then
   a) the distance between (-1, 2) and $(af(a))$, where x = a is the point of local minima is $2\sqrt{2}$
   b) $f(x)$ is increasing for $x \in [1, 2\sqrt{5}]$
   c) $f(x)$ has local minima at x = 1
   d) the value of $f(0) = 15$
50. Let f(x)=
$$\begin{cases} e^x & 1 < x \leq 1 \\ 2 - e^{x-1} & 1 < x \leq 2 \\ x - e & 2 < x \leq 3 \end{cases}$$
and $g(x) = \int_0^x f(t)dt,\ x \in [1, 3]$ then $g(x)$ has
   a) local maxima at x = 1 + ln2 and local minima at x = e
   b) local maxima at x = 1 and local minima at x = 2
   c) no local maxima
   d) no local minima
51. For the function
$$f(x) = x\cos\frac{1}{x}, x \geq 1,$$
   a) for atleast one x in the interval
$$[1, \infty), f(x+2) - f(x) < 2$$
   b) $\lim_{x\to\infty} f'(x) = 1$
   c) for all x in the interval
$$[1, \infty), f(x+2) - f(x) > 2$$
   d) $f'(x)$ is strictly decreasing for the interval $[1, \infty)$
52. If
$$f(x) = \int_0^x e^{x^2}(t-2)(t-3)dt$$
for all $x \in (0, \infty)$, then
   a) f has local maxima at x = 2
   b) f is decreasing on (2, 3)
   c) there exist some $c \in (0, \infty)$, such that $f'(c) = 0$
   d) f has a local minimum at x = 3
53. A rectangular sheet of fixed perimeter with sides having length in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are
   a) 24
   b) 32
   c) 45
   d) 60
54. Let $f : (0, \infty) \to R$ be given by
$$f(x) \int_{\frac{1}{x}}^{x} e^{-(t+\frac{1}{t})\frac{dt}{t}}$$
then
   a) $f(x)$ is monotonically increasing on $[1, \infty)$
   b) $f(x)$ is monotonically decreasing on(0, 1)
   c) $f(x) + f(\frac{1}{x}) = 0$ for all $x \in (0, \infty)$
   d) $f(2^x)$ is an odd function of x on R
55. Let $f, g : [-1, 2] \to R$ be continuous functions which are twice differentiable on the interval (-1, 2). Let the values of f and g at the point -1, 0 and 2 be as given in the following table

	x = -1	x = 0	x = 2
f(x)	3	6	0
g(x)	0	1	-1

in each of the intervals (-1, 0) and (0, 2) the function $(f-3g)''$ never vanishes.Then the correct statement (s) is (are)

a) $f'(x)-3g'(x) = 0$ has exactly three solutions in (-1, 0) ∪ (0, 2)
b) $f'(x) - 3g'(x) = 0$ has exactly one solutions in (-1, 0)
c) $f'(x) - 3g'(x) = 0$ has exactly one solutions in (0, 2)
d) $f'(x) - 3g'(x) = 0$ has exactly two solutions in (-1 ,0), exactly two solutions in (0, 2)

56. Let $f : R \to R$ is a differentiable functions such that $f'(x) > 2f(x)$ for all $x \in R$ and $f(0) = 1$, then

a) $f(x)$ is increasing in $(0, \infty)$
b) $f(x)$ is decreasing in $(0, \infty)$
c) $f(x) > e^{2x}$ in $(0, \infty)$
d) $f'(x) > e^{2x}$ in $(0, \infty)$

57. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$ Then

a) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$
b) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$
c) $f(x)$ attains its maximum at x = 0
d) $f(x)$ attains its minimum at x = 0

58. Defines collections $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles as follows:

$$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1 \tag{58.1}$$

R_1: Rectangle of largest area, with parallel sides to the axes inscribed in E_1

$$E_n : Ellipse \frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1 \tag{58.2}$$

of largest area inscribed in R_{n-1}, $n > 1$; R_n: Rectangle of largest area with sides parallel to the axes inscribed in E_n, $n > 1$.
Then which of the following options are correct?

a) The eccentricities of E_18 and E_19 are NOT equal
b) The length of latus rectum of E_9 is $\frac{1}{6}$
c) $\sum_{n=1}^{N}$ (area of R_n) < 24 for each positive integer N
d) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

59. Let $f : R \to R$ be given by

$$f(x) = (x-1)(x-2)(x-5)$$

Define

$$F(x) = \int_0^x f(t)dt, x > 0$$

Then when of the following options is/are correct?

a) F has a local maximum at x = 2
b) F has a local minimum at x = 1
c) F has two local maximum and one local minimum $(0, \infty)$
d) F(x) 0 for all x ∈ (0, 5)

60. Let

$$f(x) = \frac{\sin \pi x}{x^2}, x > 0$$

Let $x_1 < x_2 < x_3...... < x_n <....$ be all the points of local maximum of f and $y_1 < y_2 < y_3 <....< y_n <....$be all the points of local minimum of f. Then which of the following options is/are correct?

a) $x_{n+1} - x_n > 2$
b) $x_n \in (2n, 2n + \frac{1}{2})$ for every n
c) $|x_n - y_n| > 1$ for every n
d) $x_1 < y_1$

E.Subjective Problems

61. Prove that minimum value of

$$\frac{(a+x)(b+x)}{(c+x)}, a, b > c, x > -c$$

is

$$(\sqrt{a-c} + \sqrt{b-c})^2$$

62. Let x and y be two real variables such that $x > 0$ and xy = 1. Find minimum value of x + y.
63. For all x in [0, 1], Let the second derivative $f''(x)$ of a function $f(x)$ exist and satisfy $|f''(x)| < 1$. If f(0) = f(1) Then show that $|f'(x)| < 0$ for all x in [0, 1].
64. Use the function $f(x) = x^{\frac{1}{x}}$, $x > 0$ to determine the biggest of the two numbers e^{π} and π^{e}.
65. If $f(x)$ and $g(x)$ are differentiable function for $0 \leq x \leq 1$ such that $f(0) = 2$, $g(0) = 0$, $f(1) = 6$, $g(1) = 2$, then show that there exist

c satisfying $0 < c < 1$ and $f'(c) = 2g'(c)$.

66. Find the shortest distance of the points(0, c) from the parabola $y = x^2$ where $0 \le c \le 5$.
67. If $ax^2 + \frac{b}{x} \ge c$ for all positive x where $a > 0$ and $b > 0$ show that $27ab^2 \ge 4c^3$.
68. Show that $1 + xln(x + \sqrt{x^2+1}) \ge \sqrt{1+x^2}$ for all $x \ge 0$.
69. Find the coordinates of the points on the curve $y = \frac{x}{1+x^2}$ where the tangent to the curve has the greatest slope.
70. Find all the tangents to the curve $y = \cos(x+y)$, $-2\pi \le x \le 2\pi$ that are parallel to the line x + 2y = 0.
71. Let $f(x) = \sin^3 x + \lambda \sin^2 x$, $\frac{-\pi}{2} < x < \frac{\pi}{2}$ find the intervals in which λ should lie in order that $f(x)$ has exactly one minumum and one maximum.
72. Find the point on the curve
$$4x^2 + a^2y^2 = 4a^2, 4 < a^2 < 8$$
that is farthest from the point (0, -2).
73. Investigate maxima and minima the function
$$f(x) = \int_1^x [2(t-1)(t-2)^2 + 3(t-1)^2(t-2)^2]dt$$
74. Find all maxima and minima of the function $y = x(x-1)^2, 0 \le x \le 2$ also determine the area bounded by the curve $y = x(x-1)^2$ the y-axis and the line y = 2.
75. Show that $2\sin x + \tan x \ge 3x$ where $0 \le x < \frac{\pi}{2}$.
76. A point P is given on the circumference of a circle of radius r. Chord QR is parallel to the tangent at P. Determine the maximum possible area of the triangle PQR.
77. A window of perimeter P is in the form of rectangle surrounded by a semicircle. The semicicular portion is fitted with coloured glass while the rectangular portion is fitted with the clear glass transmits three times as much light per square meter as the colour glass does. What is the ratio for the sides of rectangle so that the window transmits the maximum light?
78. A cubic $f(x)$ vanishes at x = 2 and has relative minimum and maximum at x = -1 and $x = \frac{1}{3}$ if
$$\int_{-1}^{1} f dx = \frac{14}{3}$$
find the cubic f(x).
79. What normal to the curve $y = x^2$ forms the shortest chord?
80. Find the equation of normal to the curve
$$y = (1+x)^y + \sin^{-1}(\sin^2 x)$$
at x=0.
81. Let f(x) =
$$\begin{cases} -x^3 + \frac{(b^3-b^2+b-1)}{(b^2+3b+2)} & 0 \le x < 1 \\ 2x-3 & 1 \le x \le 3 \end{cases}$$
Find all possible real values of b such that $f(x)$ has the smallest value at x = 1.
82. The curve
$$y = ax^3 + bx^2 + cx + 5 \tag{82.1}$$
touches the x-axis at P(-2, 0) and cuts the y axis at a point Q where its gradient is 3 . Find a, b, c
83. The circle
$$x^2 + y^2 = 1 \tag{83.1}$$
cuts the x-axis at Pand Q another circle with centre at Q and variable radius intersects the first circle at R above the x-axis and the line segment PQ at S Find the maximum area of the triangle QSR.
84. Let (h, k) be a fixed point where $h > 0, k > 0$. A stright line passing through this point cuts the positive direction of the coordinate axis at points P and Q. Find the minimum area of triangle OPQ, O being the origin.
85. A curve $y = f(x)$ passes through the point p(1, 1). The normal to the curve at P is $a(y-1)+(x-1) = 0$. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point, determine th equation of the curve also obtain the area bounded by the y-axis the curve and the normal to the curve at P.
86. Determine the points of maxima and minima of the function
$$f(x) = \frac{1}{8}lnx - bx + x^2$$
x > 0 where $b \ge 0$ is a constant.
87. Let f(x)=
$$\begin{cases} xe^{ax} & x \le 0 \\ x + ax^2 - x^3 & x > 0 \end{cases}$$
where a is positive constant. Find the interval in which $f'(x)$ is increasing.

Match the Following Questions:

88. In this questions there are entries in column I and column II. Each entry in column I ia related to exactly one entry in column II.Write the correct letter from column II againest the entry number in column I in your answer book.Let the functions defined in column I have domain $(-\frac{\pi}{2}, \frac{\pi}{2})$

Column I	**Column II**
(A) X + $\sin X$	(p) increasing
(B) $\sec x$	(q) decreasing
	(r) neither increasing nor decreasing

By appropriately matching the matching the information given in the three columns of the following table. Let f(x)=x+$log_e x - xlog_e x$ $x \in (0, \infty)$

a) Column 1 contains information about zeros of f(x),f'(x) and f"(x).
b) Column 2 contains information about the limiting behaviour of f(x),f'(x) and f"(x) at infity
c) Column 3 contains information about the increasing/decreasing nature of f(x) and f'(x).

Column-1	Column-2	Column-3
(I) f(x)=0 for some $x \in (1, e^2)$	(i) $lim_{x\to\infty} f(x) = 0$	(P) f is increasing on (0, 1)
(II)f'(x) for $x \in (1, e)$	(ii)$lim_{x\to\infty} f(x) = -\infty$	(Q) f is increasing in (e, e^2)
(III)f'(x) for $x \in (0, 1)$	(iii)$lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0, 1)
(IV)f"(x) for $x \in (1, e)$	(iv)$lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

89. Which of the following option is the only correct combination
a) (I)(i)(P)
b) (II)(ii)(Q)
c) (III)(iii)(R)
d) (IV)(iv)(S)

90. Which of the following option is the only correct combination
a) (I)(ii)(R)
b) (II)(iii)(S)
c) (III)(iv)(P)
d) (IV)(i)(S)

91. Which of the following option is the only incorrect combination
a) (I)(iii)(P)
b) (II)(iv)(Q)
c) (III)(i)(R)
d) (II)(iii)(P)

Comprehension Based Questions:

PASSAGE-1

If a continuous function f defined on the real line R,assume positive and negative values in R then the equation f(x)=0 has a root in R.For example ,if it is known that a continuous function f on R is positive at some point and its minimum value is negative then the equation f(x)=0 has a root in R.consider f(x)=$ke^x - x$ for all real x where k is a real constant.

92. The line y = x meets $y = ke^x$ for $k \leq 0$ at
   a) no point
   b) one point
   c) two points
   d) more than two points
93. The positive value of k for $ke^x - x = 0$ has only one root is
   a) $\frac{1}{e}$
   b) 1
   c) e
   d) $log_e 2$
94. for $k > 0$ the set of all values of k for which $ke^x - x = 0$ has two distinct roots
   a) (0, $\frac{1}{e}$)
   b) ($\frac{1}{e}$, 1)
   c) ($\frac{1}{e}$, ∞)
   d) (0, 1)

PASSAGE-2

Let f(x) = $(1 + x)^2 \sin^2 x + x^2$ for all $x\ inIR$ and let g(x) = $\int_1^x (\frac{2(t-1)}{t+1} - lnt)$ f(t)dt for all $x \in (1, \infty)$

95. Consider the following statements:
   P: There exist some x ∈ R such that f(x) + 2x = 2(1 + x^2)
   Q: There exist some $x \in R$ such that 2f(x) + 1 = 2x(1 + x) Then
   a) Both P and Q are true
   b) P is true and Q is false
   c) P is false and Q is true
   d) Both P and Q are false
96. Which of the following is true?
   a) g is increasing on (0, ∞)
   b) g is decreasing on (1, ∞)
   c) g is increasing on (1, 2) and decreasing on(2, ∞)
   d) g is decreasing on (1, 2) and increasing on (2, ∞)

PASSAGE-3

Let f:[0, 1] → R be a function suppose the function f is twice differenciable, f(0) = f(1) = 0 and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, x ∈ [0, 1].

97. Which of the following is true for $0 < x < 1$?
   a) $0 < f(x) < \infty$
   b) $-\frac{1}{2} < f(x) < \frac{1}{2}$
   c) $-\frac{1}{4} < f(x) < 1$
   d) $-\infty < f(x) < 0$
98. If function e^{-x} f(x) assumes its minimum in the interval [0, 1] at x = $\frac{1}{4}$ which of the following is true?
   a) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$
   b) $f'(x) > f(x), 0 < x < \frac{1}{4}$
   c) $f'(x) < f(x), 0 < x < \frac{1}{4}$
   d) $f'(x) < f(x), \frac{3}{4} < x < 1$

Integer Value Correct Type:

99. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set A = $\{|x|x^2 + 20 \leq 9x\}$
100. Let p(x) be the polynomial of degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}(1 + \frac{p(x)}{x^2}) = 2$. then the value of p(2) is
101. Let f be a real valued differential function on R such that f(1) = 1. If the y-intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P then find the value of f(-3).
102. Let f be a function defined on R such that $f'(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$ forall $x \in R$. If g is a function defined on R with values in the interval (0, ∞) such that f(x) = ln(g(x)),for all $x \in R$. Then the number of points at which g has a local maximum is
103. Let $f : IR \to IR$ be defined as f(x) = $|x| + |x^2 - 1|$. The total number of points at which f attains either local maximum or local minimum is
104. Let p(x) be a real polynomials of least degree which has a local maximum at x = 1 and local minimum at x = 3. If p(1) = 6 and p(3) = 2, Then $p'(0)$ is
105. A vertical line passing through the point(h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangent to the ellipse at P and Q meet at the point R. If Δ(h) = area of the triangle PQR,$\Delta_1 = \max_{\frac{1}{2}\leq h\leq 1}$ Δ(h) and $\Delta_2 = \min_{\frac{1}{2}\leq h\leq 1}$ Δ(h), then $\frac{8}{\sqrt{5}}\Delta_1$ - $8\Delta_2$ =
106. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1, 3) is
107. A cylindrical container is to be made from certain solid material with the following con-

straints. It has a fixed inner volume of V mm^3 has a 2mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2mm and is of radius equal to the outer radius of the container. If the volume of the material used to make the container is minimum when the inner radius of the container is 10mm, then the values of $\frac{v}{250\pi}$ is

108. Let $-1 \le p \le 1$. Show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval $[\frac{1}{2}, 1]$ and identify it.
109. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line x + y = 7, is minimum.
110. Using the relation $2(1 - \cos x) < x^2$, $x \neq 0$ or otherwise prove that $\sin(\tan x) \geq x \forall x \in [0, \frac{\pi}{4}]$.
111. If the function $f : [0, 4] \to R$ is differentiable then show that
    a) for $a, b \in (0, 4)$, $(f(4))^2 - (f(0))^2 = 8f'(a)f(b)$
    b) $\int_0^4 f(t)dt = 2[\alpha f(\alpha^2) + \beta f(\beta^2)] \forall 0 < \alpha, \beta < 2$
112. If $p(1) = 0$ and $\frac{dP(x)}{dx} > P(x)$ for all x ≥ 1 then prove that $P(x) > 0$, for all x > 1.
113. Using Rolle's theorem prove that there is at least one root for $(45^{\frac{1}{100}}, 46)$ of polynomial
$$P(x) = 51x^{101} - 2323(x)^{100} - 45x + 1035$$
114. Prove that for $x \in [0, \frac{\pi}{2}]$, $\sin x + 2x \geq \frac{3x(x+1)}{\pi}$ explain the identity if any used in the proof.
115. $|f(x_1) - f(x_2)| < (x_1 - x_2)^2$ for $x_1, x_2 \in R$. Find the equation of the tangent to the curve $y = f(x)$ at the point (1, 2).
116. If p(x) be the polynomial of degree 3 satisfying p(-1) = 10, p(1) = -6 and p(x) has maxima at x = -1 and $p'(x)$ has minima at x = 1. Find the distance between the local maxima and local minima of the curve.
117. For a twice differenciable function f(x), g(x) is defined as g(x) = $(f'(x)^2 + f''(x))$ f(x) on [a, e] If for $a < b < c < d < e$, f(a) = 0, f(b) = 2, f(c) = -1, f(d) = 2, f(e) = 0 then find the minimum number of zeros of g(x).
118. Let a + b = 4, where a < 2 and let $g(x)$ be a differentiable function. If $\frac{dg}{dx} > 0$ for all x Prove that $\int_0^a g(x)dx + \int_0^b$ g(x) dx increases as (b - a) increasing.
119. Suppose $f(x)$ is a function statisfying th following conditions
    a) f(0) = 2, f(1) = 1
    b) f has a minimum value at x = $\frac{5}{2}$ and
    c) for all x
$$f'(X) = \begin{vmatrix} 2ax & 2ax - 1 & 2ax + b + 1 \\ b & b + 1 & -1 \\ 2(ax + b) & 2ax + 2b + 1 & 2ax + b \end{vmatrix}$$
where a, b be are some constants. Determine the constants a, b and the function f(x).
120. A curve C has the property that if the tangent drawn at any point P on C the co-ordinate axes at A and B then P is the mid point of AB. The curve passes through the point (1, 1). Determine the equation of the curve.
121. Suppose
$$p(x) = a_0 + a_1x + ... + a_nx^n$$
If $|p(x)| \leq |e^{x-1} - 1|$ for all x ≥ 0. Prove that
$$|a_1 + 2a_2 + ... + na_n| \leq 1.$$

Section-B:

122. The maximum distance from origin of a point on the curve
$$x = a \sin t - b \sin \frac{at}{b}$$
$$y = a \cos t - b \cos \frac{at}{b}, a, b > 0$$
    a) a - b
    b) a + b
    c) $\sqrt{a^2 + b^2}$
    d) $\sqrt{a^2 - b^2}$
123. If 2a + 3b + 6c = 0,$(a, b, c \in R)$ then the quadratic equation $ax^2 + bx + c = 0$ has
    a) at least one root in [0, 1]
    b) at least one root in [2, 3]
    c) at least one root in [4, 5]
    d) none of these
124. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2 + 1$, where $a > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$ then a equals
    a) $\frac{1}{2}$
    b) 3
    c) 1
    d) 2
125. A point on the parabola $y^2 = 18x$ at which the ordinate increase at twice the rate of the abscissa is
    a) $(\frac{9}{8}, \frac{9}{2})$

b) (2, -4)
c) $(-\frac{9}{8}, \frac{9}{2})$
d) (2, 4)

126. A function y = f(x) has a second order derivative $f''(x) = 6(1-x)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is y = 3x - 5 then the function is
a) $(x+1)^2$
b) $(x-1)^3$
c) $(x+1)^3$
d) $(x-1)^2$

127. The normal to the curve x = $(1 + \cos\theta)$, y = $a\sin\theta$ at θ always passes through the fixed point
a) (a, a)
b) (0, a)
c) (0, 0)
d) (a, 0)

128. If 2a + 3b + 6c = 0 then at least one root of the equation $ax^2+bx+c = 0$ lies in the interval
a) (1, 3)
b) (1, 2)
c) (2, 3)
d) (0, 1)

129. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
a) 2ab
b) ab
c) $\sqrt{ab}$
d) $\frac{a}{b}$

130. The normal to the curve x = a$(\cos\theta + \theta\sin\theta)$, y = a$(\sin\theta - \theta\cos\theta)$ at any point $'\theta'$ is such that
a) it passes through the origin
b) it makes an angle $\frac{\pi}{2} + \theta$ with the x-axis
c) it passes through (a$\frac{\pi}{2}$, -a)
d) it is at constant distance from the origin

131. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness of ice is 5cm then the rate at which the thickness of ice decrease is
a) $\frac{1}{36\pi} cm/min$
b) $\frac{1}{18\pi} cm/min$
c) $\frac{1}{54\pi} cm/min$
d) $\frac{5}{6\pi} cm/min$

132. if the equation $a_n x^n + a_{n-1}x^{n-1}$....... + $a_1 x = 0$, $a_1 \neq 0, n \geq 2$, has a positive root x = α then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1$ = 0 has a positive root which is
a) greater than α
b) smaller than α
c) greater than or equal to α
d) equal to α

133. The function f(x) = $\frac{x}{2} + \frac{2}{x}$ has a local minimum at
a) x = 2
b) x = -2
c) x = 0
d) x = 1

134. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank.The two sides having fence are of same length x,the maximum area enclosed by the park is
a) $\frac{3}{2}x^2$
b) $\sqrt{\frac{x^3}{8}}$
c) $\frac{1}{2}$
d) πx^2

135. A value of C for which conclusion of Mean Value Theorem holds for the function f(x) = $\log_e^x$ on the interval [1, 3] is
a) $\log_3 e$
b) $\log_e 3$
c) $2\log_3 e$
d) $\frac{1}{2}\log_3 e$

136. The function f(x) = $\tan^{-1}(\sin x + \cos x)$ is an increasing function in
a) $(0, \frac{\pi}{2})$
b) $(-\frac{\pi}{2}, \frac{\pi}{2})$
c) $(\frac{\pi}{4}, \frac{\pi}{2})$
d) $(-\frac{\pi}{2}, \frac{\pi}{4})$

137. If p and q are positive real numbers such that $p^2 + q^2 = 1$ then the maximum value of (p + q) is
a) $\frac{1}{2}$
b) $\frac{1}{\sqrt{2}}$
c) $\sqrt{2}$
d) 2

138. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$ Then which one of the following holds?
a) the cubic has minimum at $\sqrt{\frac{p}{3}}$ and maximum at $-\sqrt{\frac{p}{3}}$

b) the cubic has minimum at $-\sqrt{\frac{p}{3}}$ and maximum at $\sqrt{\frac{p}{3}}$
c) the cubic has minimum at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
d) the cubic has maximum at $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

139. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?
a) 7
b) 1
c) 3
d) 5

140. Let f(x) = x$|x|$ and g(x) = sin x.
a) Statement 1: gof is differentiable at x = 0 and its derivative is continuous at that point.
b) Statement 2: gof is twice differentiable at x = 0.
a) statement-1 is true,statement-2 is true statement-2 is not correct explination for statement-1
b) statement-1 is true,statement-2 is false
c) statement-1 is false and statement-2 is true
d) statement-1 is true,statement-2 is true statement-2 is correct explination of statement-1

141. Given P(x) = $x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of $P'(x) = 0$. If P(-1) < P(1), Then in the interval [-1, 1]:
a) P(-1) is not a minimum but P(1) is the maximum of P
b) P(-1) is the minimum but P(1) is not the maximum of P
c) Neither P(-1) is a minimum nor P(1) is the maximum of P
d) P(-1) is a minimum but P(1) is the maximum of P

142. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$ that is parallel to the x-axis is
a) y = 1
b) y = 2
c) y = 3
d) y = 0

143. Let $f : R \to R$ be defined by f(x) =

$$\begin{cases} k - 2x & if x \le -1 \\ 2x + 3 & if x > -1 \end{cases}$$

if f has a local minimum at x = -1 then a possible value of k is
a) 0
b) $\frac{-1}{2}$
c) -1
d) 1

144. Let $f : R \to R$ be a continuous function defined by f(x) = $\frac{1}{e^x + 2e^{-x}}$
a) Statement-1: f(c) = $\frac{1}{3}$ for some $c \in R$
b) Statement -2: $0 < f(x) \le \frac{1}{2\sqrt{2}}$ for all $x \in R$
a) statement-1 is true,statement-2 is true statement-2 is not correct explination for statement-1
b) statement-1 is true,statement-2 is false
c) statement-1 is false and statement-2 is true
d) statement-1 is true,statement-2 is true statement-2 is correct explination of statement-1

145. The shortest distance between line y - x = 1 and curve x = y^2 is
a) $\frac{3\sqrt{2}}{8}$
b) $\frac{8}{3\sqrt{2}}$
c) $\frac{4}{\sqrt{3}}$
d) $\frac{\sqrt{3}}{4}$

146. For $x \in (0, \frac{5\pi}{2})$ define f(x) = $\int_0^x \sqrt{t} \sin t$dt then f has
a) local minimum at $\pi and 2\pi$
b) local minimum at π and local maximum at 2π
c) local maximum at π and local minimum at 2π
d) local maximum at π and 2π

147. A spherical balloon filled with 4500π cubic meters of helium gas. If a leak in balloon causes the gas to escape at the rate of 72π cubic meters per minute then then rate at which the radius of balloon decreases 49 minutes after the leakage began is:
a) $\frac{9}{7}$
b) $\frac{7}{9}$
c) $\frac{2}{9}$
d) $\frac{9}{2}$

148. Let a, b ∈ R be such that the function f given by f(x) = ln$|x|$ + bx^2 + ax, $x \neq 0$ has extreme values at x = -1 and at x = 2.
a) Statement-1: f has local maximum at x = -1 and at x = 2
b) Statement-2: a = $\frac{1}{2}$ and $b = -\frac{1}{4}$
a) Statement-1 is false,Statement-2 is true
b) Statement-1 is true,statement-2 is true statement-2 is a correct explanation of statement -1
c) Statement-1 is true,statement-2 is true statement-2 is not a correct explanation of statement -1
d) Statement-1 is true and statement-2 is false

149. A line is drawn through the point [1, 2] to meet

the coordinates axes at P and Q such that it forms a triangle OPQ where O is the origin. If the area of the triangle OPQ is least then the slope of the line PQ is:
a) $\frac{-1}{4}$
b) -4
c) -2
d) $\frac{-1}{2}$

150. The intercepts on the axis made by tangents to the curve y = $\int_0^x |t|$ dt, $x \in R$ which are parallel to the line y = 2x are equal to
a) ±1
b) ±2
c) ±3
d) ±4

151. If f and g are differentiable functions in [0, 1] satisfying f(0) = 2 = g(1), g(0) = 0 and f(1) = 6, Then for some c ∈ [0, 1]
a) $f'(c) = g'(c)$
b) $f'(c) = 2g'(c)$
c) $2f'(c) = g'(c)$
d) $2f'(c) = 3g'(c)$

152. Let f(x) be the polynomial of degree four having extreme values at x = 1 and x = 2. If $\lim_{x\to 0}$ [1 + $\frac{f(x)}{x^2}$] = 3, then f(2) is equal to:
a) 0
b) 4
c) -8
d) -4

153. Consider:

$$f(x) = \tan^{-1}(\sqrt{\frac{1+\sin x}{1-\sin x}})x \in (0, \frac{\pi}{2})$$

A normal to y = f(x) at x = $\frac{p}{6}$ also passes through the point:
a) ($\frac{\pi}{6}$, 0)
b) ($\frac{\pi}{4}$, 0)
c) (0, 0)
d) (0, $\frac{2\pi}{3}$)

154. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle os radius = r units. If sum of the areas of the squares and the circle so formed is minimum then,
a) x = 2r
b) 2x = r
c) 2x = $(\pi + 4)$r
d) $(4 - \pi)$x = πr

155. The function $f : R \to [\frac{-1}{2}, \frac{1}{2}]$ defined as f(x) = $\frac{x}{1+x^2}$ is
a) neither injective nor surjective
b) invertible
c) injective but not surjective
d) surjective but not injective

156. The Normal to the curve y(x - 2)(x - 3) = x + 6 at the point where the curve intersects the y-axis passes through the point:
a) $(\frac{1}{2}, \frac{1}{3})$
b) $(-\frac{1}{2}, -\frac{1}{2})$
c) $(\frac{1}{2}, \frac{1}{2})$
d) $(\frac{1}{2}, -\frac{1}{3})$

157. Twenty meter of wire is available for fencing off a flower bed in the form of circular sector Then the maximum area of flower bed is:
a) 30
b) 12.5
c) 10
d) 25

158. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$ If one of its directices is x = -4 then the equation of normal to it at (1, $\frac{3}{2}$) is ;
a) x + 2y = 4
b) 2y - x = 2
c) 4x - 2y = 1
d) 4x + 2y = 7

159. Let f(x) = $x^2 + \frac{1}{x^2}$ and g(x) = x - $\frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If h(x) = $\frac{f(x)}{g(x)}$ then local minimum value of h(x) is:
a) -3
b) -2$\sqrt{2}$
c) 2$\sqrt{2}$
d) 3

160. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersects each other at right angles then the value of b is:
a) $\frac{1}{2}$
b) 4
c) $\frac{9}{2}$
d) 6

161. The maximum volume of the right circular cone having slant height 3m is
a) 6π
b) $3\sqrt{3}\pi$
c) $\frac{4}{3}\pi$

d) $2\sqrt{3}\pi$

162. If q denotes the acute angle between the curves, y = $10 - x^2$ and y = 2 + x^2 at a point of their intersection then $|\tan\theta|$ is equal to
   a) $\frac{4}{9}$
   b) $\frac{8}{15}$
   c) $\frac{7}{17}$
   d) $\frac{8}{17}$
163. If f(x) is a non-zero polynomial of degree four having local extreme points at x = -1, 0, 1 then the set S = {x R: f(x) = f(0)} contains exactly
   a) four irrational numbers.
   b) four rational numbers.
   c) two irrational and two rational number.
   d) two irrational and one rational number.
164. If the tangent to the curve y = x3 + ax - b at the point (1, -5) is perpendicular to the line -x + y + 4 = 0, then which one of the following points lie on the curve?
   a) (-2, 1)
   b) (-2, 2)
   c) (2, -1)
   d) (2, -2)
165. Let S be the set of all values of x for which the tangent to the curves y = f(x) = $x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(a)) and (-1, f(-1)) then S is equal to:
   a) {$\frac{1}{3}$, 1}
   b) {$-\frac{1}{3}$, -1}
   c) {$\frac{1}{3}$, -1}
   d) {$-\frac{1}{3}$, 1}