Assignment-07

Sravani sandhya
MD/2020/706

Download latex-tikz codes from

https://github.com/sravani706/Assignment7.git

Download python codes from

https://github.com/sravani706/Assignment7.git

Question taken from

Optimization , exercises question 2.17

1 Question No 2.17

A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

2 Solution

Type	No.of Bags	Element A	Element B	Element C	Cost per bag
P	X	3 units	2.5 units	2 units	Rs 250
Q	y	1.5 units	11.25 units	3 units	Rs 200
Total		4.5 units	13.5 units	5 units	$250X + 200Y$

Let the Brand P of cattle feed is X and The Brand Q of cattle feed is Y number of such that : From the given information,

$$x \geq 0 \tag{2.0.1}$$

$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$3x + 1.5y \geq 18 \implies x + 0.5y \geq 6 \tag{2.0.3}$$

$$\tag{2.0.4}$$

and,

$$2.5x + 11.25y \geq 45 \implies 0.3x + 2.25y \geq 9 \tag{2.0.5}$$

$$\tag{2.0.6}$$

$$2x + 2.25y \geq 24 \tag{2.0.7}$$

$\therefore$ Our problem is

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 250 & 200 \end{pmatrix} \mathbf{x} \tag{2.0.8}$$

$$s.t. \quad \begin{pmatrix} 1 & 0.5 \\ 0.3 & 2.25 \\ 2 & 3 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 6 \\ 9 \\ 24 \end{pmatrix} \tag{2.0.9}$$

Lagrangian function is given by

$$\begin{aligned} & L(\mathbf{x}, \boldsymbol{\lambda}) \\ & = \begin{pmatrix} 250 & 2000 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 1 & 0.5 \end{pmatrix} \mathbf{x} + 6 \right] \right. \\ & + \left[\begin{pmatrix} 0.5 & 2.25 \end{pmatrix} \mathbf{x} + 9 \right] \\ & + \left[\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} + 24 \right] + \left[\begin{pmatrix} -100 \end{pmatrix} \mathbf{x} \right] \\ & + \left[\begin{pmatrix} 0 & 0 & -1 \end{pmatrix} x \right] \boldsymbol{\lambda} \end{aligned} \tag{2.0.10}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \end{pmatrix} \tag{2.0.11}$$

Now,

$$\nabla L(\mathbf{x}, \lambda) = \begin{pmatrix} 250 + \begin{pmatrix} 1 & 0.5 & -1 & 0 & 0 \end{pmatrix} \lambda \\ 200 + \begin{pmatrix} 0.3 & 2.25 & 0 & -1 & 0 \end{pmatrix} \lambda \\ \begin{pmatrix} 1 & 0.5 \end{pmatrix} \mathbf{x} + 6 \\ \begin{pmatrix} 0.3 & 2.25 \end{pmatrix} \mathbf{x} + 9 \\ \begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} + 24 \\ \begin{pmatrix} -1 & 0 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.12}$$

$\therefore$ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0.5 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 2.25 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 & 3 & 0 & 0 & -1 \\ 1 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.3 & 2.25 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -250 \\ -200 \\ 50 \\ 6 \\ 9 \\ 24 \end{pmatrix} \tag{2.0.13}$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0.3 & 2.25 \\ 0 & 0 & 2 & 3 \\ 1 & 0.5 & 0 & 0 \\ 0.3 & 2.25 & 0 & 0 \\ 2 & 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -250 \\ -200 \\ 50 \\ 6 \\ 9 \\ 24 \end{pmatrix} \tag{2.0.14}$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0.5 & 0 \\ 0 & 0 & 0.3 & 2.25 & 0 \\ 0 & 0 & 2 & 3 & 0 \\ 1 & 0.5 & 0 & 0 & 0 \\ 0.3 & 2.25 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -250 \\ -200 \\ 50 \\ 6 \\ 9 \\ 24 \end{pmatrix} \tag{2.0.15}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{-6}{7} & \frac{9}{14} \\ 0 & 0 & 0 & 0 & \frac{4}{7} & \frac{-2}{21} \\ 0 & 0 & 0 & 1 & \frac{4}{7} & \frac{-25}{42} \\ 0 & \frac{-6}{7} & \frac{9}{14} & 0 & 0 & 0 \\ 0 & \frac{4}{7} & \frac{-2}{21} & 0 & 0 & 0 \\ 1 & \frac{4}{7} & \frac{-25}{42} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -250 \\ -200 \\ 50 \\ 6 \\ 9 \\ 24 \end{pmatrix} \tag{2.0.16}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} \frac{-54}{7} \\ \frac{60}{21} \\ \frac{285}{14} \\ \frac{2300}{21} \\ \frac{4175}{21} \end{pmatrix} \tag{2.0.17}$$

$\therefore$ Optimal solution is given by

$$\mathbf{x} \tag{2.0.18}$$

$$y = \begin{pmatrix} \frac{-54}{7} \\ \frac{60}{21} \end{pmatrix} \tag{2.0.19}$$

$$Z = \begin{pmatrix} 250 & 200 \end{pmatrix} \mathbf{x} \tag{2.0.20}$$

$$= \begin{pmatrix} 250 & 200 \end{pmatrix} \begin{pmatrix} \frac{-54}{7} \\ \frac{60}{21} \end{pmatrix} \tag{2.0.21}$$

$$= 1350 \tag{2.0.22}$$

Hence the Minimum cost of mixture per bag $\boxed{Z = 1350}$.
This is verified in Fig. 0.

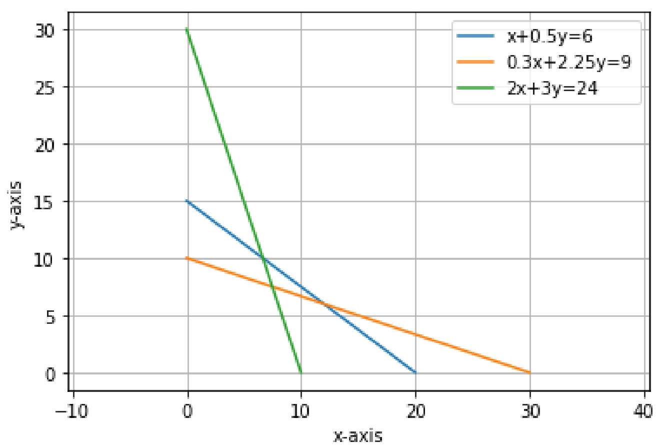

Fig. 0: graphical solution