Assignment No.5

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Download latex-tikz codes from

https://github.com/sravani706/Assignment5/main/main.tex

Download python codes from

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Question taken from

Quadratic_forms, exercise 2

1 Question No 2.30

Find the equation of the hyperbola with vertices $\begin{pmatrix} 0 \\ \pm\frac{\sqrt{11}}{2} \end{pmatrix}$ and foci are $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$

2 Solution

We have been provided with values for vertices and foci

The given vertices are- $\begin{pmatrix} 0 \\ \pm\frac{\sqrt{11}}{2} \end{pmatrix}$

The given vertices are in the form of $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$ Here,

The major axis is along X axis

The equation of conic is given as

$$\mathbf{u}^T(\mathbf{t}^T - \mathbf{n}\mathbf{n}^T)\mathbf{u} + 2(\mathbf{c}\mathbf{n} - \mathbf{t}\mathbf{f}^T\mathbf{u} + \mathbf{t}\,\|F\|^2 - \mathbf{c}^2 = 0 \tag{2.0.1}$$

Thus,

$$\mathbf{F} = \begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{c} = 0, \mathbf{t} = \frac{1}{9} \tag{2.0.2}$$

$$\mathbf{n}^T\left(\frac{1}{9}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix}\right)\mathbf{u}+ \tag{2.0.3}$$

$$2\left(0 - \frac{1}{9}\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}\right)^T \mathbf{u} + \frac{1}{9}\left\|\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}\right\|^2 - 0 = 0 \tag{2.0.4}$$

$$\mathbf{u}^T\left(\begin{pmatrix} \frac{-8}{9} & 0 \\ 0 & \frac{-8}{9} \end{pmatrix}\right)\mathbf{u} + 2\begin{pmatrix} 0 & \frac{1}{3} \end{pmatrix}\mathbf{u} + 1 = 0 \tag{2.0.5}$$

$$\mathbf{u}\ \mathbf{v}\begin{pmatrix} \frac{-8}{9} & 0 \\ 0 & \frac{-8}{9} \end{pmatrix}\mathbf{u} + 2\begin{pmatrix} 0 & \frac{1}{3} \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} + 1 = 0 \tag{2.0.6}$$

$$\begin{pmatrix} \frac{-8}{9}\mathbf{u} & \frac{-8}{9}\mathbf{v} \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} + 6\mathbf{v} + 1 = 0 \tag{2.0.7}$$

$$8\mathbf{u}^2 = 8\mathbf{v}^2 + 6\mathbf{v} = 9 \tag{2.0.8}$$

$\therefore$ this is the equation of hyperbola

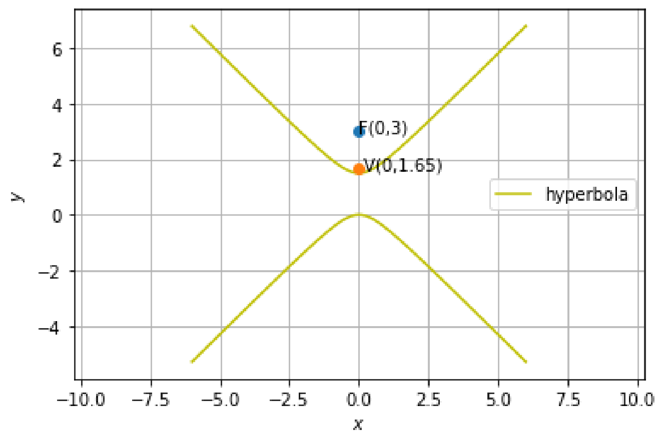

Fig. 0: Hyperbola