Assignment No.5

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Download latex-tikz codes from

https://github.com/sravani706/Assignment5/main/main.tex

Download python codes from

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Question taken from

Quadratic_forms, exercise 2

1 Question No 2.30

Find the equation of the hyperbola with vertices $\begin{pmatrix} 0 \\ \pm \frac{\sqrt{11}}{2} \end{pmatrix}$ and foci are $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$

2 Solution

We have been provided with values for vertices and foci

The given vertices are- $\binom{0}{\pm \frac{\sqrt{11}}{2}}$

The given vertices are in the form of $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$ Here, The major axis is along X axis

The equation of conic is given as

$$\mathbf{u}^{T}(\mathbf{t}^{T} - \mathbf{n}\mathbf{n}^{T})\mathbf{u} + 2(\mathbf{c}\mathbf{n} - \mathbf{t}\mathbf{f}^{T}\mathbf{u} + \mathbf{t} ||F||^{2} - \mathbf{c}^{2} = 0$$
(2.0.1)

Thus,

$$\mathbf{F} = \begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{c} = 0, \mathbf{t} = \frac{1}{9}$$
 (2.0.2)

$$\mathbf{n}^T \left(\frac{1}{9} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} \right) \mathbf{u} + \qquad (2.0.3)$$

$$2\left(0 - \frac{1}{9} \begin{pmatrix} 0\\ \pm 3 \end{pmatrix}\right)^{T} \mathbf{u} + \frac{1}{9} \left\| \begin{pmatrix} 0\\ \pm 3 \end{pmatrix} \right\|^{2} - 0 = 0 \quad (2.0.4)$$

$$\mathbf{u}^T \left(\begin{pmatrix} \frac{-8}{9} & 0\\ 0 & \frac{-8}{9} \end{pmatrix} \right) \mathbf{u} + 2 \left(0 & \frac{1}{3} \right) \mathbf{u} + 1 = 0$$
 (2.0.5)

Replacing u by $\binom{u}{v}$ in (2.0.3) $\frac{\mathbf{u}^2}{\frac{9}{9}} + \frac{\mathbf{v}^2}{\frac{9}{9}} + \frac{2}{3}\mathbf{v} = 1$ (2.0.6)

$$\frac{8}{9}\mathbf{u}^2 + \frac{8}{9}\mathbf{v}^2 + \frac{2}{3}\mathbf{v} = 1 \tag{2.0.7}$$

$$8\mathbf{u}^2 + 8\mathbf{v}^2 + 6\mathbf{v} = 9 \tag{2.0.8}$$

(2.0.9)

: this is the equation of hyperbola

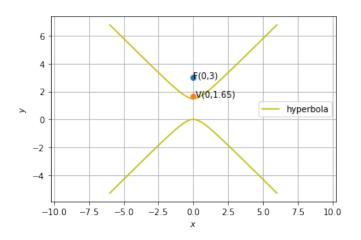


Fig. 0: Hyperbola