

ASSIGNMENT 5

Sravani sandhya

Download all python codes from

<https://github.com/Sravani706/Assignment-5>

and latex-tikz codes from

<https://github.com/Sravani706/Assignment-5>

1 QUADRATIC FORMS-2.30

Find the equation of the hyperbola with vertices $\begin{pmatrix} 0 \\ \pm \frac{\sqrt{11}}{2} \end{pmatrix}$, foci $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$

2 SOLUTION

The standard equation of hyperbola is

$$\frac{\mathbf{x}^T \mathbf{V} \mathbf{x}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1$$

Since focus $\mathbf{F} = \begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$, the vertices lie in the y-axis. Thus,

$$\mathbf{c} = -\mathbf{V}^{-1} \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2)$$

$$\mathbf{u} = \mathbf{0} \quad (3)$$

$$f = 1 \quad (4)$$

$$\mathbf{V} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad (5)$$

where $\lambda_1 > 0$ and $\lambda_2 < 0$. Vertices is given by,

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = \frac{\sqrt{11}}{2} \quad (6)$$

$$\sqrt{\frac{-1}{\lambda_2}} = \frac{\sqrt{11}}{2} \quad (7)$$

$$\lambda_2 = -\frac{4}{11} \quad (8)$$

$$\mathbf{F} = \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_1 - \lambda_2)}{\lambda_1 \lambda_2}} \right) \mathbf{p}_2 \quad (9)$$

where $\mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\pm \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \pm \left(\sqrt{\frac{(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (10)$$

$$3 = \sqrt{\frac{(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \quad (11)$$

$$3 = \sqrt{\frac{(-4 - 11\lambda_1)}{-4\lambda_1}} \quad (12)$$

$$\lambda_1 = \frac{4}{25} \quad (13)$$

The equation of the given hyperbola is

$$\mathbf{x}^T \begin{pmatrix} \frac{4}{25} & 0 \\ 0 & -\frac{4}{11} \end{pmatrix} \mathbf{x} = -1 \quad (14)$$

(1) The plot of the hyperbola is given in the fig 2.1

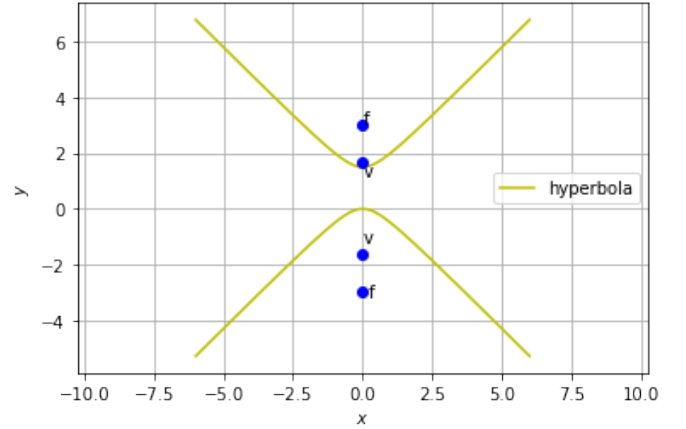


Fig. 2.1. Hyperbola