SECTION-A

1. Write the coordinates of the point which is the reflection of the point (α, β, γ) in the XZ-plane.

2. Find the position vector of the point which divides the join of points with position vectors $\vec{a} + 3\vec{b}$ and $\vec{a} - \vec{b}$ internally in the ratio 1 : 3.

3. If $\left|\vec{a}\right| = 4, \left|\vec{b}\right| = 3$,and $\vec{a} \cdot \vec{b} = 6\sqrt{3}$,then find the value of $\left|\vec{a} \times \vec{b}\right|$.

4. Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

5. If $\mathbf{A} = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ and $BA=\left(b_{ij}\right)$,find $b_{21} + b_{32}$.

6. Write the number of all possible matrices of order 2×3 with each entry 1 or 2.

SECTION-B

7. Find the equation of the tangent line to the curve $y = \sqrt{5x-3} - 5$, which is parallel to the line $4x - 2y + 5 = 0$.

8. Solve the differential equation:
$\left(x^2 + 3xy + y^2\right) dx - x^2 dy = 0$ given that $y = 0$, when $x = 1$.

9. On her birthday Seema decided to donate some money to the children of an orphanage home. If there were 8 children less, every one would have got ₹10 more. However, if there were 16 children more, every one would have got ₹10 less. Using the matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision?

10. Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$ intersect.
Find their point of intersection.

11. Show that the function f given by:
f(x)=$\begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, if x \neq 0 \\ -1, if x = 0 \end{cases}$
is discontinuous at $x = 0$.

12. Find:$\int \frac{2x+1}{(x^2+1)(x^2+4)}$dx

13. If $x = e^{\cos 2t} and\ y = e^{\sin 2t}$,prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

14. Verify Mean Value theorem for the function $f(x) = 2\sin x + \sin 2x$ on $\left[0, \pi\right]$.

15. Solve for x: $\tan^{-1}\left(\frac{2-x}{2+x}\right) = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$,$x > 0$.

16. Prove that $2sin^{-1}\left(\frac{3}{5}\right) - tan^{-1}\left(\frac{17}{31}\right)$=$\frac{\pi}{4}$.

17. Evaluate: $\int_1^5 \{\left|x-1\right| + \left|x-2\right| + \left|x-3\right|\}$dx

18. Evaluate:$\int_0^{\pi} \frac{xsinx}{1+3cos^2x}$dx

19. $x\frac{dy}{dx} + y - x + xycotx = 0;\ x \neq 0$.

20. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

21. Find the angle between the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ if $\vec{a}$=$2\hat{i} - \hat{j} + 3\hat{k}$, and $\vec{b}$=$3\hat{i} + \hat{j} - 2\hat{k}$ hence find a vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$

22. Find: $\int (3x+5)\ \sqrt{5+4x-2x^2}$ dx

SECTION-C

23. Using integration, find the area of the triangle formed by negative x-axis and tangent and normal to the circle $x^2 + y^2 = 9$ at $\left(-1, 2\sqrt{2}\right)$.

24. A company manufactures two types of cardigans: type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cardigan. The company can make at most 300 cardigans and spend at most ₹72,000 a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than 200. The company makes a profit of ₹100 for each cardigan of type A and ₹50 for every cardigan of type B.
Formulate this problem as a linear programming problem to maximise the profit to the company. Solve it graphically and find maximum profit.

25. Find the coordinates of the foot of perpendicular and perpendicular distance from the point $P(4, 3, 2)$ to the plane $x + 2y + 3z = 2$ Also find the image of P in the plane.

26. Solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix}$=0,using properties of determinants.

27. Using elementary row operation find the inverse of matrix X $A=\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ and hence solve the following system of equations $3x - 3y + 4z = 21, 2x - 3y + 4z = 20, -y + z = 5$.

28. A, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first.

29. Show that the relation R defined by (a, b) R $(c, d) \Rightarrow a + d = b + c$ on the $A \times A$, where $A = \{1, 2, 3........, 10\}$ is an equivalence relation.Hence write the equivalence class $\left[(3, 4)\right]$; $a, b, c, d \epsilon A$.

30. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi-vertical angle α is one-third that of the cone and the greatest volume of the cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

31. Find the intervals in which the function f(x)=$\frac{4sinx}{2+cosx}$ - x;$0 \leq x \leq 2\pi$ is strictly incresing or strictly decreasing