

1.  $AB$  is tangent to the circles  $CAMN$  and  $NMBD$ .  $M$  lies between  $C$  and  $D$  on the line  $CD$ , and  $CD$  is parallel to  $AB$ . The chords  $NA$  and  $CM$  meet at  $P$ ; the chords  $NB$  and  $MD$  meet at  $Q$ . The rays  $CA$  and  $DB$  meet at  $E$ . Prove that  $PE = QE$ .
2.  $A, B, C$  are positive reals with product 1. Prove that  $\left(A - 1 + \frac{1}{B}\right)\left(B - 1 + \frac{1}{C}\right)\left(C - 1 + \frac{1}{A}\right) \leq 1$ .
3.  $k$  is a positive real.  $N$  is an integer greater than 1.  $N$  points are placed on a line, not all coincident. A *move* is carried out as follows. Pick any two points  $A$  and  $B$  which are not coincident. Suppose that  $A$  lies to the right of  $B$ . Replace  $B$  by another point  $B'$  to the right of  $A$  such that  $AB' = kBA$ . For what values of  $k$  can we move the points arbitrarily far to the right by repeated moves?
4. 100 cards are numbered 1 to 100 (*each card different*) and placed in 3 boxes (*at least one card in each box*). How many ways can this be done so that if two boxes are selected and a card is taken from each, then the knowledge of their sum alone is always sufficient to identify the third box?
5. Can we find  $N$  divisible by just 2000 different primes, so that  $N$  divides  $2^N + 1$ ? [ $N$  may be divisible by a prime power.]
6.  $A_1A_2A_3$  is an acute-angled triangle. The foot of the altitude from  $A_i$  is  $K_i$  and the incircle touches the side opposite  $A_i$  at  $L_i$ . The line  $K_1K_2$  is reflected in the line  $L_1L_2$ . Similarly, the line  $K_2K_3$  is reflected in  $L_2L_3$  and  $K_3K_1$  is reflected in  $L_3L_1$ . Show that the three new lines form a triangle with vertices on the incircle.