1. Determine all finite sets S of at least three points in the plane which satisfy the following condition:
   for any two distinct points A and B in S, the perpendicular bisector of the line segment AB is an axis of symmetry for S.

2. Let n be a fixed integer, with $n \geq 2$.
   (a) Determine the least constant C such that the inequality

$$\sum_{1\leq i<j\leq n} x_i x_j \left(x_i^2 + x_j^2\right) \leq C \left(\sum_{1\leq i\leq n} x_i \right)^4$$

   holds for all real numbers $x_1, ..., x_n \geq 0$.
   (b) For this constant C, determine when equality holds.

3. Consider an $n \times n$ square board, where n is a fixed even positive integer. The board is divided into n^2 unit squares. We say that two different squares on the board are adjacent if they have a common side.
   N unit squares on the board are marked in such a way that every square (*markedorunmarked*) on the board is adjacent to at least one marked square.
   Determine the smallest possible value of N.

4. Determine all pairs (n, p) of positive integers such that
   p is a prime,
   n not exceeded $2p$,and
   $(p-1)^n + 1$ is divisible by n^{p-1}.

5. Two circles G_1 and G_2 are contained inside the circle G, and are tangent to G at the distinct points M and N, respectively. G_1 passes through the center of G_2. The line passing through the two points of intersection of G_1 and G_2 meets G at A and B. The lines MA and MB meet G_1 at C and D, respectively.
   Prove that CD is tangent to G_2.

6. Determine all functions $f : \mathbf{R} \to \mathbf{R}$ such that
   $f\left(x - f(y)\right) = f\left(f(y)\right) + xf(y) + f(x) - 1$