Problem Set 1

Sravan Ramawamy

https://github.com/SravanjR/bc-micro-methods/blob/main/psets/pset_auctions/Problem-Set-1.pdf

```
#Load Libraries
library(MASS)
library(ggplot2)
library(evd)
```

```
#Load Bid Data
DataDir = paste(getwd(), "/bids1.csv",sep = "")
Data = read.csv(DataDir, header = FALSE)
bids = Data[,1]
```

Question 1: Estimate Density of Bids

Normal Distribution

```
Normpdf = dnorm(bids, mean = mean(bids), sd = sd(bids))
Normpdf
```

```
##   [1] 0.31329357 0.11180397 0.08426095 0.28693490 0.48806370 0.25910471
##   [7] 0.45086042 0.13677822 0.42098683 0.37770831 0.19492840 0.34610848
##  [13] 0.45554863 0.53176934 0.52915949 0.28700977 0.50672589 0.52346217
##  [19] 0.20643271 0.53234470 0.48334108 0.44850495 0.44307080 0.33864327
##  [25] 0.17433787 0.27377211 0.48249626 0.42014156 0.53487959 0.53700344
##  [31] 0.52209656 0.53181485 0.11097815 0.28728169 0.47775445 0.36566785
##  [37] 0.45247802 0.47949723 0.27945773 0.41902632 0.35732042 0.27141677
##  [43] 0.14967786 0.53174021 0.35243700 0.51244202 0.34430065 0.41863817
##  [49] 0.53653701 0.46250681 0.53620091 0.31019068 0.08302195 0.39632456
##  [55] 0.33839189 0.51497230 0.06269825 0.25938864 0.36123394 0.48704605
##  [61] 0.13650387 0.24307104 0.23351374 0.49373643 0.18129943 0.53174021
##  [67] 0.09646250 0.18097531 0.49869230 0.49103565 0.17149436 0.22805234
##  [73] 0.18335949 0.31940978 0.48779494 0.47740111 0.39321799 0.12201192
##  [79] 0.45447580 0.51449529 0.06710415 0.52923588 0.51260731 0.09417020
##  [85] 0.35807228 0.19826340 0.22368206 0.46885772 0.45807978 0.30111563
##  [91] 0.32616604 0.50729450 0.52675598 0.08049987 0.51271981 0.26350484
##  [97] 0.38215831 0.11760435 0.30576472 0.51564074 0.44756656 0.47560164
## [103] 0.22890939 0.47390944 0.29161157 0.49694678 0.53037374 0.43411984
## [109] 0.36257948 0.35676619 0.25208868 0.38689984 0.50285893 0.23510120
## [115] 0.08615729 0.51961899 0.17125937 0.51564074 0.33839189 0.33192363
## [121] 0.19522792 0.46242904 0.44977604 0.32994606 0.32754064 0.52231316
## [127] 0.33442792 0.37620961 0.53683646 0.32855159 0.48324261 0.32617802
## [133] 0.51040738 0.52308843 0.48249626 0.53554448 0.23701993 0.45715953
## [139] 0.52973321 0.46625899 0.14856001 0.53769128 0.29009055 0.32587033
## [145] 0.51621502 0.52712715 0.24533419 0.14991260 0.39507159 0.45659720
## [151] 0.46103016 0.51358657 0.11132322 0.30041649 0.40451731 0.36631687
```

```
## [157] 0.11974248 0.51999673 0.38001571 0.52616882 0.44731065 0.53201512
## [163] 0.53745158 0.47307268 0.52484983 0.42185913 0.13183382 0.10970769
## [169] 0.42759880 0.45954278 0.51301737 0.21934819 0.29684903 0.27360063
## [175] 0.38151959 0.32207836 0.52057734 0.35179388 0.21088257 0.49743140
## [181] 0.52082152 0.14292658 0.48313038 0.53279882 0.53519345 0.40825825
## [187] 0.53764259 0.52921971 0.23218492 0.41766808 0.32516704 0.48949448
## [193] 0.21758062 0.47194832 0.06126004 0.52353192 0.14585507 0.36137605
## [199] 0.49951601 0.53373137 0.36578588 0.36432548 0.30776774 0.37570691
## [205] 0.34727853 0.31804785 0.51696908 0.47105995 0.33602106 0.28321249
## [211] 0.27502785 0.31070499 0.50738139 0.53769258 0.51688553 0.14403391
## [217] 0.51497230 0.45536214 0.46680422 0.53044459 0.45247802 0.19394637
## [223] 0.46453986 0.46586507 0.51224301 0.49766197 0.14781375 0.22729894
## [229] 0.48764977 0.53760327 0.43633222 0.51250041 0.33162802 0.47731549
## [235] 0.52740844 0.43354519 0.09566950 0.32634186 0.53112335 0.47621245
## [241] 0.05643744 0.31497362 0.27130012 0.52845332 0.32688131 0.16323353
## [247] 0.29191595 0.23940728 0.15566159 0.49335092 0.43988468 0.33127646
## [253] 0.51876719 0.53742531 0.39163430 0.43652222 0.43784897 0.52659313
## [259] 0.25809257 0.07489733 0.27703537 0.33697512 0.40613026 0.07823853
## [265] 0.13547057 0.08217881 0.18459424 0.36145105 0.36955660 0.41121928
## [271] 0.52202352 0.48164606 0.46823718 0.46226716 0.44359050 0.29684903
## [277] 0.09857543 0.17714595 0.51379012 0.18039986 0.53760630 0.50633125
## [283] 0.24274086 0.36432548 0.35806042 0.48036074 0.17301019 0.05125580
## [289] 0.47122317 0.26740409 0.05452151 0.52128419 0.17557462 0.50724147
## [295] 0.04640083 0.49504710 0.33082502 0.18426696 0.48227594 0.52914325
```

Gaussian Kernel

```
n = length(bids)
iq = IQR(bids)
iq
```

```
## [1] 1.144875
```

```
bw_plugin = .9*min(sd(bids),iq/1.34)*n^(-1/5)
bw_plugin
```

```
## [1] 0.2133987
```

```
gk  <- density(bids,bw=bw_plugin, kernel = c("gaussian"))
gk
```

```
##
## Call:
##  density.default(x = bids, bw = bw_plugin, kernel = c("gaussian"))
##
## Data: bids (300 obs.);   Bandwidth 'bw' = 0.2134
##
##        x                  y
##  Min.   :-0.03085   Min.   :0.0001336
##  1st Qu.: 1.09669   1st Qu.:0.0398139
##  Median : 2.22423   Median :0.2095719
##  Mean   : 2.22423   Mean   :0.2215020
##  3rd Qu.: 3.35176   3rd Qu.:0.4070823
##  Max.   : 4.47930   Max.   :0.4472902
```

```r
BAR1 <- with(gk, approxfun(x = x, y = y))
gDensity <- BAR1(bids)
gDensity
```

```
##   [1] 0.33394041 0.15645653 0.09859234 0.35796713 0.44574617 0.33593433
##   [7] 0.42448506 0.17385091 0.42053385 0.38262771 0.26773508 0.39109706
##  [13] 0.43491892 0.43106419 0.43819312 0.30963501 0.43887180 0.43503677
##  [19] 0.28175184 0.43545979 0.44463812 0.42337688 0.42972320 0.35485820
##  [25] 0.20044212 0.29653878 0.44442017 0.40841670 0.43285268 0.42835236
##  [31] 0.44269049 0.43595165 0.15582166 0.35821083 0.44307011 0.39915490
##  [37] 0.42523716 0.44359794 0.35249141 0.40778637 0.36873773 0.29415848
##  [43] 0.18248282 0.43602014 0.36522021 0.44616899 0.35919931 0.41958205
##  [49] 0.43080918 0.43768794 0.42865942 0.37300321 0.12913445 0.41084494
##  [55] 0.38764270 0.44550592 0.09989901 0.28185995 0.37149427 0.44554045
##  [61] 0.17366891 0.32117078 0.25510420 0.44671022 0.25011355 0.43602014
##  [67] 0.14345377 0.24968148 0.44719698 0.44630026 0.19822843 0.24954545
##  [73] 0.25284847 0.37817107 0.43754499 0.43495505 0.39248714 0.16388368
##  [79] 0.42615172 0.43776528 0.10712698 0.43813333 0.43813166 0.11507197
##  [85] 0.39612408 0.22070953 0.24513075 0.43209865 0.43594202 0.32300071
##  [91] 0.34487648 0.44701280 0.43994050 0.09237388 0.44610316 0.33972118
##  [97] 0.40547984 0.15405576 0.37036195 0.44530807 0.43161032 0.43440907
## [103] 0.30686971 0.43386071 0.31406464 0.43881387 0.43181609 0.41607639
## [109] 0.37243536 0.39559456 0.32966206 0.38853811 0.43902227 0.25674095
## [115] 0.13277893 0.44384890 0.23642062 0.44530807 0.38764270 0.34956414
## [121] 0.26811004 0.42959523 0.42397416 0.34796632 0.38241535 0.43547802
## [127] 0.35155115 0.38164186 0.42840318 0.38292472 0.43655357 0.38172881
## [133] 0.43847574 0.44216913 0.44442017 0.43207425 0.25871399 0.42735089
## [139] 0.43215207 0.43111487 0.20330714 0.42866178 0.31261016 0.38157378
## [145] 0.43736503 0.43968633 0.26730906 0.20536006 0.41036768 0.42710007
## [151] 0.43711162 0.44589553 0.14367313 0.32235517 0.39934811 0.39941056
## [157] 0.15757345 0.43628887 0.38412628 0.43388922 0.43150362 0.43576767
## [163] 0.42828889 0.43359171 0.43446786 0.42088886 0.17056079 0.15483231
## [169] 0.42324535 0.43652503 0.44603226 0.24079267 0.36474265 0.29636567
## [175] 0.40523755 0.34146969 0.44342488 0.39354034 0.28695561 0.44711992
## [181] 0.43601229 0.19465678 0.44458363 0.43049446 0.42917411 0.41545262
## [187] 0.42889516 0.43241832 0.31028017 0.40701594 0.38121944 0.43786776
## [193] 0.29458337 0.44116916 0.06124475 0.44193033 0.19917284 0.39745319
## [199] 0.43897333 0.42997157 0.37464567 0.39862652 0.32903994 0.40302949
## [205] 0.36143021 0.37743316 0.44486995 0.43289448 0.38654002 0.35528946
## [211] 0.34909514 0.33165677 0.43883121 0.42854123 0.43719700 0.17867888
## [217] 0.44550592 0.42655140 0.43933002 0.43177806 0.42523716 0.21683380
## [223] 0.43044518 0.43897534 0.43819395 0.43886703 0.18122164 0.30517229
## [229] 0.43751631 0.42835808 0.41724444 0.43814998 0.38443353 0.44293451
## [235] 0.43949081 0.41577016 0.14269555 0.34502195 0.43141639 0.44259268
## [241] 0.08874345 0.37573372 0.34615561 0.43280409 0.38208315 0.22504672
## [247] 0.31435554 0.31758195 0.21397717 0.44665589 0.41908119 0.38426143
## [253] 0.44420785 0.42941457 0.40906214 0.41734327 0.41803509 0.43369525
## [259] 0.33504716 0.11867316 0.29980769 0.35355535 0.41462198 0.12315533
## [265] 0.17298136 0.09515029 0.25447318 0.37164606 0.40066926 0.40330766
## [271] 0.43558625 0.44419814 0.43986177 0.43759577 0.42994135 0.36474265
## [277] 0.14541959 0.24452156 0.43791166 0.24891336 0.42900414 0.44711256
## [283] 0.32085176 0.39862652 0.36926604 0.44383802 0.23885074 0.07875544
## [289] 0.43295124 0.29008164 0.05074005 0.44308723 0.20141671 0.43883991
## [295] 0.06884396 0.44688009 0.34867773 0.25404311 0.43631550 0.43245752
```

Epanechikov Kernel

```
ek <- density(bids,bw=bw_plugin, kernel = c("epanechnikov"))
ek
```

```
##
## Call:
##  density.default(x = bids, bw = bw_plugin, kernel = c("epanechnikov"))
##
## Data: bids (300 obs.);   Bandwidth 'bw' = 0.2134
##
##        x                  y
##  Min.   :-0.03085   Min.   :0.00000
##  1st Qu.: 1.09669   1st Qu.:0.04091
##  Median : 2.22423   Median :0.21610
##  Mean   : 2.22423   Mean   :0.22151
##  3rd Qu.: 3.35176   3rd Qu.:0.41447
##  Max.   : 4.47930   Max.   :0.44543
```

```
BAR2 <- with(ek, approxfun(x = x, y = y))
eDensity1 <- BAR2(bids)
eDensity1
```

```
##   [1] 0.32702930 0.15001105 0.10182838 0.34964181 0.43368818 0.33086993
##   [7] 0.42916422 0.17263889 0.43326573 0.38400005 0.27099674 0.38832431
##  [13] 0.43926817 0.43760351 0.43927657 0.30247298 0.42471109 0.43140084
##  [19] 0.28548972 0.44052618 0.43441560 0.42855024 0.43803160 0.34849729
##  [25] 0.21020410 0.28972352 0.43454943 0.41715346 0.44211503 0.44458931
##  [31] 0.43563426 0.44031648 0.14928057 0.34988841 0.43558580 0.39979887
##  [37] 0.42950879 0.43510275 0.34480097 0.41661654 0.36615449 0.28761421
##  [43] 0.18474428 0.44028999 0.36167108 0.43173969 0.35375672 0.43238139
##  [49] 0.44311523 0.43906050 0.44316846 0.36814235 0.12279264 0.41954091
##  [55] 0.38456764 0.43288840 0.09925181 0.27677793 0.36958239 0.43386493
##  [61] 0.17240533 0.31951708 0.25785400 0.43249617 0.25179540 0.44028999
##  [67] 0.13661034 0.25131406 0.43123641 0.43311593 0.20716309 0.25449878
##  [73] 0.25483774 0.37487756 0.42647553 0.42789398 0.39777420 0.15929983
##  [79] 0.42982237 0.42821321 0.10533456 0.43931360 0.42729711 0.11840327
##  [85] 0.39530698 0.23330658 0.25171381 0.42928295 0.43926660 0.31622584
##  [91] 0.33775501 0.43001114 0.43805292 0.09545167 0.43186216 0.33387542
##  [97] 0.40943566 0.15511395 0.36466222 0.43319853 0.43869723 0.42817240
## [103] 0.30807720 0.42845166 0.30709736 0.42521912 0.43581224 0.42299080
## [109] 0.37074846 0.39453135 0.32613920 0.39209467 0.42459512 0.25895987
## [115] 0.12626299 0.43468583 0.23633011 0.43319853 0.38456764 0.34259903
## [121] 0.27140143 0.43005782 0.42888338 0.34094113 0.37957916 0.43086906
## [127] 0.34476590 0.38266132 0.44428759 0.38007321 0.42706680 0.37891326
## [133] 0.42617257 0.43612773 0.43454943 0.44259987 0.26029430 0.43006760
## [139] 0.43509440 0.42965788 0.20102241 0.44543146 0.30559439 0.37876289
## [145] 0.42894694 0.43823906 0.26623054 0.20327860 0.41860238 0.43002300
## [151] 0.43916446 0.43224841 0.14544175 0.31556314 0.40741898 0.40018095
## [157] 0.15832022 0.43009885 0.38604318 0.43284572 0.43866670 0.44039288
## [163] 0.44526121 0.42858865 0.43212910 0.43358731 0.16832823 0.14813179
## [169] 0.43507812 0.43922941 0.43199405 0.24878358 0.35732203 0.28957011
## [175] 0.40903194 0.33428872 0.43499803 0.39158172 0.29050260 0.43153764
## [181] 0.43033562 0.19170002 0.43444906 0.43898050 0.44175765 0.42739214
## [187] 0.44533762 0.43465878 0.31087678 0.41593975 0.37841920 0.42627887
```

```
## [193] 0.29745037 0.43728989 0.06030700 0.43635411 0.19650350 0.39726591
## [199] 0.42492832 0.44012681 0.37351473 0.39900925 0.32235572 0.40556688
## [205] 0.35670445 0.37396454 0.43376377 0.42892791 0.38352252 0.34717182
## [211] 0.34191534 0.32487307 0.42479310 0.44540725 0.42922110 0.17912633
## [217] 0.43288840 0.42992544 0.43851537 0.43589351 0.42950879 0.22957978
## [223] 0.42986806 0.43866586 0.42710745 0.42513646 0.18286414 0.30666563
## [229] 0.42649328 0.44534948 0.42383256 0.42724133 0.38151871 0.43571818
## [235] 0.43838224 0.42277397 0.13591120 0.33790476 0.43673918 0.43604925
## [241] 0.08961276 0.37177855 0.33935739 0.43408152 0.37925695 0.22407139
## [247] 0.30739795 0.31669878 0.21256152 0.43258940 0.42522084 0.38135399
## [253] 0.43440828 0.44470251 0.41612131 0.42390552 0.42441618 0.43304908
## [259] 0.33019420 0.11436338 0.29268492 0.34700743 0.42615987 0.11779284
## [265] 0.17152297 0.09830028 0.25662525 0.36977046 0.40208456 0.41227832
## [271] 0.43074054 0.43468908 0.43824300 0.43908367 0.43811573 0.35732203
## [277] 0.13843554 0.24550820 0.42788625 0.25045834 0.44524869 0.42989108
## [283] 0.31926658 0.39900925 0.36681533 0.43493729 0.23907712 0.08073262
## [289] 0.42890043 0.28401662 0.04816781 0.43529055 0.21148255 0.42477553
## [295] 0.07165066 0.43216525 0.34167568 0.25615209 0.42719752 0.43459944
```

Question 2: Least-Squares cross-validation for epanechikov kernel

```
set.seed(1)
ek2 <- density(bids,bw="ucv", kernel = c("epanechnikov"))
```

```
## Warning in bw.ucv(x): minimum occurred at one end of the range
```

```
ek2
```

```
##
## Call:
##  density.default(x = bids, bw = "ucv", kernel = c("epanechnikov"))
##
## Data: bids (300 obs.);   Bandwidth 'bw' = 0.2702
##
##        x                 y
##  Min.   :-0.2012   Min.   :0.00000
##  1st Qu.: 1.0115   1st Qu.:0.02723
##  Median : 2.2242   Median :0.19023
##  Mean   : 2.2242   Mean   :0.20593
##  3rd Qu.: 3.4369   3rd Qu.:0.39399
##  Max.   : 4.6496   Max.   :0.44387
```

```
BAR3 <- with(ek2, approxfun(x = x, y = y))
eDensity2 <- BAR3(bids)
eDensity2
```

```
##   [1] 0.32419581 0.15079846 0.10986063 0.34195506 0.43199227 0.31970370
##   [7] 0.40980161 0.18085332 0.41707965 0.37264178 0.26180927 0.38813828
##  [13] 0.42432813 0.43972185 0.43558285 0.30420364 0.42624012 0.43444721
##  [19] 0.27190026 0.43813496 0.43085754 0.40898847 0.42092539 0.34410800
##  [25] 0.21678333 0.29472766 0.43064054 0.39790976 0.44034008 0.44334982
##  [31] 0.43425249 0.43759499 0.14977597 0.34223144 0.42943795 0.40005293
##  [37] 0.41033305 0.42987381 0.33654760 0.39744186 0.35904076 0.29309782
##  [43] 0.19506296 0.43752131 0.35536849 0.43381648 0.34884342 0.41680914
##  [49] 0.44191430 0.42610395 0.44260738 0.36121943 0.11780857 0.41133114
##  [55] 0.38274741 0.43391791 0.09508884 0.28432920 0.36182725 0.43176474
```

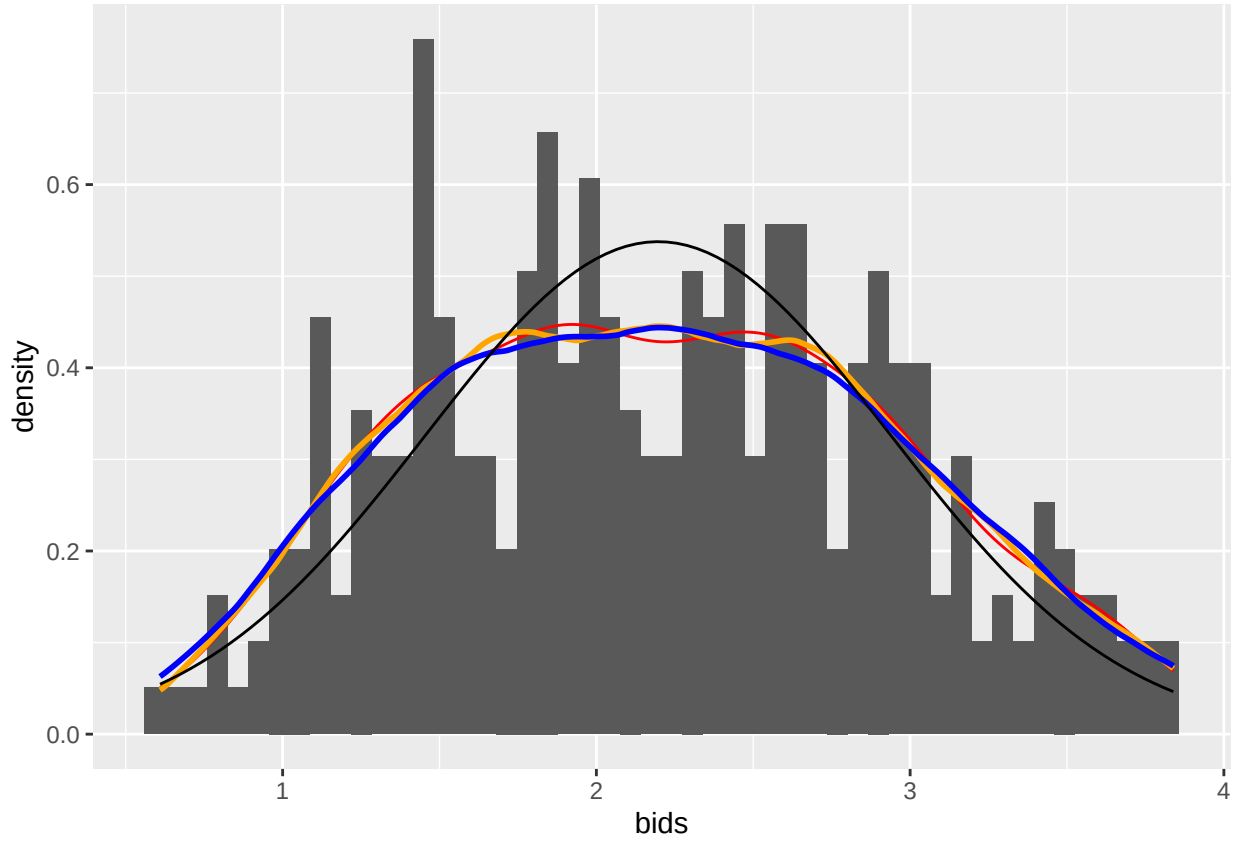
```
##  [61] 0.18053435 0.30437081 0.26326133 0.43304741 0.24871134 0.43752131
##  [67] 0.13311825 0.24837011 0.43363879 0.43259569 0.21450110 0.25850062
##  [73] 0.25087245 0.36876494 0.42253640 0.41864984 0.38286547 0.16311223
##  [79] 0.41098908 0.42942187 0.10038207 0.43560956 0.42860149 0.12459180
##  [85] 0.39579678 0.23428119 0.25472436 0.41532417 0.42496476 0.31511535
##  [91] 0.33419996 0.43398234 0.43486627 0.10410196 0.43381484 0.32372613
##  [97] 0.40674737 0.16093304 0.35751408 0.43397761 0.42219172 0.41790666
## [103] 0.29128389 0.41716463 0.30772251 0.42453652 0.43877095 0.40361846
## [109] 0.36277208 0.39500931 0.31306562 0.37869930 0.42545976 0.26464164
## [115] 0.12133071 0.43421429 0.23750662 0.43397761 0.38274741 0.33864885
## [121] 0.26208452 0.41343496 0.40942654 0.33708940 0.37497282 0.43368156
## [127] 0.34064850 0.37166938 0.44319024 0.37573006 0.42093006 0.37394802
## [133] 0.42764644 0.43428029 0.43064054 0.44091076 0.26626723 0.41182605
## [139] 0.43833495 0.41453105 0.20846487 0.44381305 0.30654633 0.37371661
## [145] 0.43017809 0.43496675 0.27316719 0.21036048 0.41093041 0.41164984
## [151] 0.42572001 0.43380964 0.15069528 0.31459517 0.39013541 0.40037017
## [157] 0.16429550 0.43220112 0.37416519 0.43616640 0.42212117 0.43779615
## [163] 0.44363358 0.41682769 0.43534719 0.41717418 0.17507851 0.14819275
## [169] 0.41776551 0.42533723 0.43381306 0.25114365 0.35005678 0.29460913
## [175] 0.40653056 0.33104356 0.43423896 0.39190105 0.27566492 0.43352307
## [181] 0.43270937 0.20037223 0.43080329 0.44039412 0.44192044 0.41478530
## [187] 0.44369446 0.43799255 0.29412951 0.39685025 0.37318768 0.42299314
## [193] 0.28146848 0.42808571 0.07379131 0.43430994 0.20464027 0.39771558
## [199] 0.42497968 0.44097473 0.36494367 0.39932491 0.32004920 0.40445915
## [205] 0.35128704 0.36768407 0.43407144 0.41607229 0.38108056 0.33929223
## [211] 0.33323503 0.32224191 0.42642317 0.44385666 0.43050807 0.18909435
## [217] 0.43391791 0.41126440 0.42704033 0.43882014 0.41033305 0.23124335
## [223] 0.41405805 0.42684689 0.42844534 0.42466273 0.19314187 0.28989276
## [229] 0.42249154 0.44376894 0.40451038 0.42855556 0.37795342 0.42932940
## [235] 0.43504361 0.40338779 0.13220503 0.33433351 0.43929190 0.42905802
## [241] 0.08726839 0.36520786 0.33034805 0.43749993 0.37447695 0.22798527
## [247] 0.30795808 0.30081653 0.21823686 0.43299113 0.40587783 0.37769932
## [253] 0.43417671 0.44333065 0.40983306 0.40458727 0.40509865 0.43639911
## [259] 0.31875930 0.10875995 0.29703057 0.34272444 0.41427861 0.11247227
## [265] 0.17935003 0.10666494 0.25210810 0.36197981 0.40191358 0.39381247
## [271] 0.43348675 0.43042292 0.42732896 0.42604145 0.42107339 0.35005678
## [277] 0.13551811 0.24421608 0.42911587 0.24776349 0.44363010 0.43398524
## [283] 0.30404611 0.39932491 0.35958673 0.43009346 0.23952328 0.08102193
## [289] 0.41613321 0.29026199 0.06287010 0.43425152 0.21774704 0.42638395
## [295] 0.07451343 0.43324038 0.33778249 0.25178306 0.42053960 0.43794244
```

Question 3: Plot of Estimated Density Functions over Bid Histogram

```
data <- as.data.frame(bids)

ggplot(data,aes(bids))  + geom_histogram(aes(y = stat(density)), bins = 50) +
  geom_line(stat="density",bw=bw_plugin, col = 'red') +
  geom_line(stat="density", bw=bw_plugin,lwd = 1, col = 'Orange', kernel = c("epanechnikov")) +
  geom_line(stat="density", bw="ucv",lwd = 1, col = 'blue', kernel = c("epanechnikov")) +
    stat_function(fun = dnorm, args = list(mean = mean(bids), sd = sd(bids)))
```

```
## Warning in bw.ucv(x): minimum occurred at one end of the range
```



While the tails are somewhat similar, the the density function consisting of the epanechnikov and the plug-in estimate appear to fit the data best. This line is given by the orange curve above.

Question 4: GPV and the cross-validated Epanechnikov kernel to recover the valuation implied for each bid

```
BAR <- with(ek2, approxfun(x = x, y = y))
eDensity <- BAR(bids)

#Calculate CDF Values
f<- approxfun(ek2$x, ek2$y, yleft=0, yright=0)

cdf <- vector()
for(i in 1:length(bids)){
    temp <- integrate(f, lower = -Inf, bids[i])
    cdf[i] <- temp$value
}

value <- bids + cdf/(2*eDensity)
value
```

```
##   [1]  4.2328571  6.6598867  0.8813189  1.6012402  2.2921587  1.5190113
##   [7]  3.4867781  6.0094500  2.0168524  3.8516244  1.3183005  1.7719425
##  [13]  2.1482991  2.9394734  2.5790589  4.4072870  3.1782955  3.0394468
##  [19]  1.3568490  2.6177162  2.2693031  3.4982228  2.0996953  4.0739512
##  [25]  5.4143352  4.4979285  2.2653109  3.6340083  2.6583979  2.8252310
```

```
##  [31]  2.5096626  2.6107106  6.6860325  1.6022743  2.2433580  1.8308837
##  [37]  3.4789348  2.2513445  1.5790372  3.6392598  3.9619339  4.5141797
##  [43]  5.7602238  2.6097487  3.9900721  2.4349923  4.0388406  2.0082242
##  [49]  2.6974134  2.1767044  2.8527828  1.6679131  7.7282318  1.9306001
##  [55]  1.7493219  2.4530321  8.8693111  4.6015719  3.9400206  2.2871753
##  [61]  6.0153653  1.4719491  4.8171952  2.3213175  1.2715514  2.6097487
##  [67]  7.1705433  1.2704336  2.3484668  2.3072087  5.4489023  4.8681104
##  [73]  1.2786365  1.6941995  3.2926816  3.3514954  3.7694110  6.3672600
##  [79]  3.4691817  3.1201858  8.5621365  2.5799277  3.1350366  0.9319245
##  [85]  1.8076528  5.1601750  4.9097023  3.3976514  2.1585354  4.3112080
##  [91]  4.1512584  2.4003148  2.5533534  0.8610903  2.4369447  1.5318602
##  [97]  1.8836341  1.0363534  1.6553186  2.4579159  2.1168921  3.3614613
## [103]  1.4289630  3.3708330  4.3756473  3.2397468  2.9596047  3.5674756
## [109]  3.9325655  1.8037141  1.4985126  3.8027951  3.2036321  4.8026816
## [115]  7.5879480  2.4887055  1.2367303  2.4579159  1.7493219  4.1157240
## [121]  1.3193127  3.4300356  3.4920571  4.1279849  1.7176596  3.0509858
## [127]  4.1001927  3.8595568  2.8318877  1.7205825  3.3187272  1.7137187
## [133]  3.1517701  2.5184794  2.2653109  2.6721006  4.7855491  3.4560750
## [139]  2.9681940  3.4109090  1.1553856  2.7635743  4.3861070  1.7128223
## [145]  3.1061985  2.5571352  4.7139776  1.1603366  1.9264030  3.4588323
## [151]  2.1706041  2.4431009  1.0102839  4.3157852  3.7103653  1.8329117
## [157]  1.0451232  3.0731219  3.8393120  3.0107184  2.1159040  2.6133215
## [163]  2.8019082  3.3753816  3.0250136  2.0200808  6.1194408  6.7271187
## [169]  2.0414904  2.1645013  2.4390485  4.9507452  1.6298670  4.4990804
## [175]  1.8815440  4.1767935  2.4966280  1.7888019  1.3715081  2.3413854
## [181]  3.0654051  1.1345868  2.2683042  2.9233622  2.8781756  1.9712826
## [187]  2.7526247  2.9746910  1.4391404  3.6457562  1.7107977  3.2830012
## [193]  1.3931541  2.2173312  0.7455485  2.5224951  1.1454177  1.8176986
## [199]  3.2242682  2.9073607  3.9150815  1.8267521  4.2681882  1.8626604
## [205]  4.0207227  1.6902947  2.4678628  3.3861410  1.7423672  1.5901805
## [211]  1.5658586  4.2493847  3.1737849  2.7719180  3.1005665  5.8619234
## [217]  2.4530321  3.4648699  2.1949028  2.9586319  3.4789348  5.2026277
## [223]  3.4194927  2.1908763  3.1378404  3.2354741  5.7925717  1.4239263
## [229]  3.2935131  2.7888230  3.5568255  3.1358579  1.7295395  2.2413638
## [235]  2.5600420  3.5702294  7.2003554  4.1501743  2.9489441  2.2363690
## [241]  9.3860690  1.6815056  1.5547836  2.9842712  1.7157541  1.2083118
## [247]  4.3735619  1.4610556  1.1811860  2.3192744  3.5397693  1.7285161
## [253]  2.4818454  2.7334348  1.9149328  3.5559083  3.5495471  3.0060171
## [259]  1.5160579  8.1286996  4.4753832  4.0843544  1.9638783  7.9568122
## [265]  6.0375688  0.8702193  1.2828949  3.9388204  1.8430489  3.6769054
## [271]  3.0538376  2.2613199  2.2010847  2.1757140  2.1016667  1.6298670
## [277]  7.0938003  1.2572033  3.1257834  1.2684476  2.7480570  2.3941630
## [283]  1.4709734  1.8267521  3.9577239  2.2553329  1.2428546  9.8694352
## [289]  3.3852728  4.5424771  0.6989396  2.5026113  5.3997791  3.1747502
## [295] 10.4500563  2.3283180  4.1225291  1.2817722  3.3242535  2.9756644
```

Question 5: Estimate the Distribution of Value using another Epanechikov Kernel using the Plug-in Bandwidth

```
n = length(value)
iq = IQR(value)
bw_plugin = .9*min(sd(value),iq/1.34)*n^(-1/5)
valDist  <- density(value,bw=bw_plugin, kernel = c("epanechnikov"))
```

```
valDist
```

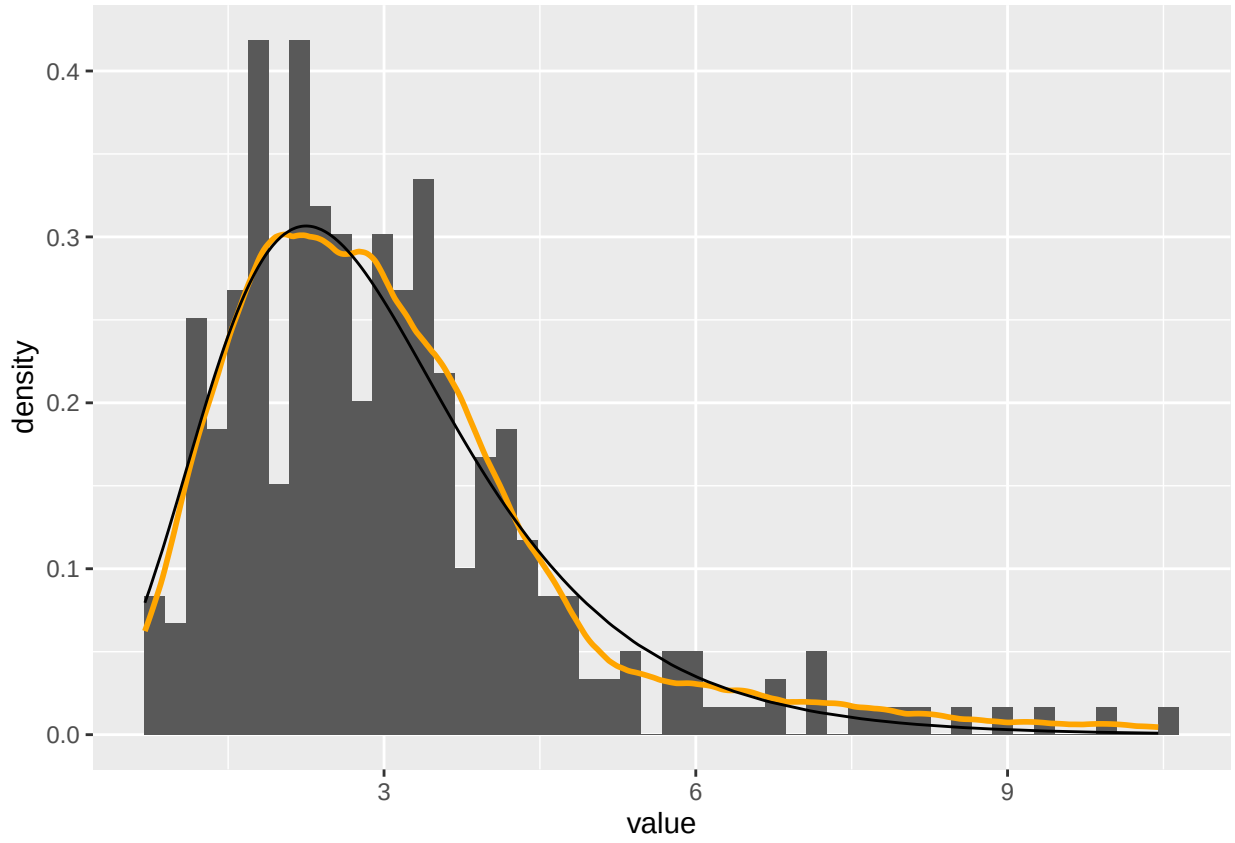
```
##
## Call:
##  density.default(x = value, bw = bw_plugin, kernel = c("epanechnikov"))
##
## Data: value (300 obs.);  Bandwidth 'bw' = 0.38
##
##        x                y
##  Min.   :-0.4411   Min.   :0.000000
##  1st Qu.: 2.5667   1st Qu.:0.007458
##  Median : 5.5745   Median :0.026354
##  Mean   : 5.5745   Mean   :0.083032
##  3rd Qu.: 8.5823   3rd Qu.:0.150940
##  Max.   :11.5901   Max.   :0.301433
```

Question 6: Guess the Distribution that the Valuations were generated with

```
m1 <- dgev(value, 2.25, 1.2, 0)

data2 <- as.data.frame(value)

ggplot(data,aes(value))  + geom_histogram(aes(y = stat(density)), bins = 50) +
geom_line(stat="density", bw=bw_plugin,lwd = 1, col = 'Orange', kernel = c("epanechnikov")) +
geom_line(aes(y = m1))
```



The Valuations seem to have been generated from a Generalized Extreme Value distribution. The Location parameter is around 2.25, the scale parameter is around 1.2 and the shape parameter is about 0 which suggests that the distribution of bidding valuations is specifically a Gumbel/Type 1 Extreme value Distribution.