AI1103-Assignment-3

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QUESTION:GATE 2012 (MA) ,Q.29

If a random variable X assumes only positive integral values ,with the probability

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, ..., \quad (0.0.1)$$

then $E(X)$ is

(A) $\frac{2}{9}$ 　　　　　　　 (C) 1

(B) $\frac{2}{3}$ 　　　　　　　 (D) $\frac{3}{2}$

SOLUTION:GATE 2012 (MA) ,Q.29

Given that random variable X assumes only positive integral values and its probability is:

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1} \quad (0.0.2)$$

The expectation value E(X) is given by

$$E(X) = \sum_{i=1}^{\infty} i \times P(X = i) \quad (0.0.3)$$

Let $E(X) = S$
so,

$$S = \sum_{i=1}^{\infty} i \times P(X = i) \quad (0.0.4)$$

$$\implies S = \sum_{i=1}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \quad (0.0.5)$$

$$\implies S = \frac{2}{3} + \sum_{i=2}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \quad (0.0.6)$$

As

$$\sum_{i=2}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} = \sum_{i=1}^{\infty}(i + 1) \times \frac{2}{3}\left(\frac{1}{3}\right)^{i} \quad (0.0.7)$$

Now substituting (0.0.7) in (0.0.6)

$$\implies S = \frac{2}{3} + \sum_{i=1}^{\infty}(i + 1) \times \frac{2}{3}\left(\frac{1}{3}\right)^{i} \quad (0.0.8)$$

$$\implies S = \frac{2}{3} + \sum_{i=1}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i} + \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \quad (0.0.9)$$

Dividing with 3 on both sides in (0.0.5) gives

$$\frac{S}{3} = \sum_{i=1}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i} \quad (0.0.10)$$

Now substituting (0.0.10) in (0.0.9) gives

$$\implies S = \frac{2}{3} + \frac{S}{3} + \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \quad (0.0.11)$$

$$\implies \frac{2S}{3} = \frac{2}{3} + \frac{2}{3}\sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{i} \quad (0.0.12)$$

$$\implies \frac{2S}{3} = \frac{2}{3}\left(1 + \sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{i}\right) \quad (0.0.13)$$

$$\implies S = 1 + \sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{i} \quad (0.0.14)$$

$$\implies S = 1 + \frac{\frac{1}{3}}{1 - \frac{1}{3}} \quad (0.0.15)$$

$$\implies S = 1 + \frac{1}{2} = \frac{3}{2} \quad (0.0.16)$$

$$\implies E(X) = S = \frac{3}{2} \quad (0.0.17)$$

∴ **Option D is correct**