AI1103-Assignment-3

Kodavanti Rama Sravanth,CS20BTECH11027

Download all python codes from

```
https://github.com/Sravanth−k27/AI1103/tree/main/
    Assignment−3/codes
```

Download latex-tikz codes from

```
https://github.com/Sravanth−k27/AI1103/tree/
    main/Assignment−3/Assignment−3.tex
```

Question:Gate 2012 (MA) ,Q.29

If a random variable X assumes only positive integral values ,with the probability

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, ..., \tag{0.0.1}$$

then $E(X)$ is

(A) $\frac{2}{9}$
(B) $\frac{2}{3}$
(C) 1
(D) $\frac{3}{2}$

Solution:Gate 2012 (MA) ,Q.29

Given that random variable X assumes only positive integral values and its probability is:

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1} \tag{0.0.2}$$

The expectation value E(X) is given by

$$E(X) = \sum_{i=1}^{\infty} i \times P(X = i) \tag{0.0.3}$$

Let $E(X) = S$
so,

$$S = \sum_{i=1}^{\infty} i \times P(X = i) \tag{0.0.4}$$

$$\implies S = \sum_{i=1}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \tag{0.0.5}$$

$$\implies S = \frac{2}{3} + \sum_{i=2}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \tag{0.0.6}$$

As

$$\sum_{i=2}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} = \sum_{i=1}^{\infty}(i+1) \times \frac{2}{3}\left(\frac{1}{3}\right)^{i} \tag{0.0.7}$$

Now substituting (0.0.7) in (0.0.6)

$$\implies S = \frac{2}{3} + \sum_{i=1}^{\infty}(i+1) \times \frac{2}{3}\left(\frac{1}{3}\right)^{i} \tag{0.0.8}$$

$$\implies S = \frac{2}{3} + \sum_{i=1}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i} + \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \tag{0.0.9}$$

Dividing with 3 on both sides in (0.0.5) gives

$$\frac{S}{3} = \sum_{i=1}^{\infty} i \times \frac{2}{3}\left(\frac{1}{3}\right)^{i} \tag{0.0.10}$$

Now substituting (0.0.10) in (0.0.9) gives

$$\implies S = \frac{2}{3} + \frac{S}{3} + \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \tag{0.0.11}$$

$$\implies \frac{2S}{3} = \frac{2}{3} + \frac{2}{3}\sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{i} \tag{0.0.12}$$

$$\implies \frac{2S}{3} = \frac{2}{3}\left(1 + \sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{i}\right) \tag{0.0.13}$$

$$\implies S = 1 + \sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{i} \tag{0.0.14}$$

$$\implies S = 1 + \frac{\frac{1}{3}}{1 - \frac{1}{3}} \tag{0.0.15}$$

$$\implies S = 1 + \frac{1}{2} = \frac{3}{2} \tag{0.0.16}$$

$$\implies E(X) = S = \frac{3}{2} \tag{0.0.17}$$

$\therefore$ **Option D is correct**

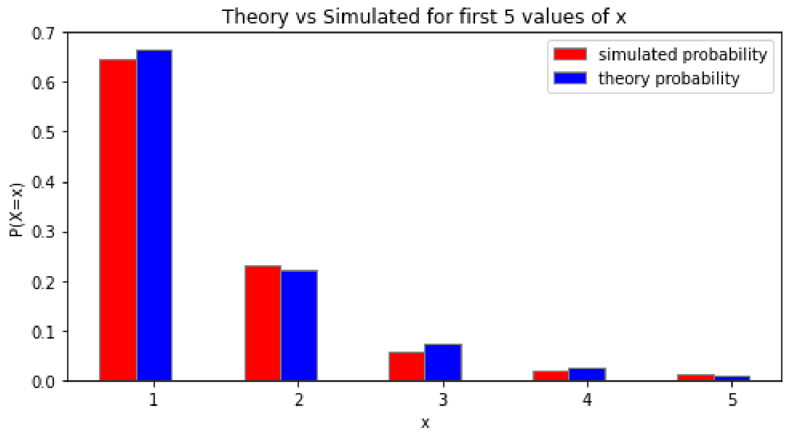

Fig. 4: Theory vs simulated of P(X=x)