AI1103-Assignment-4

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QUESTION GATE 2021 (EC) Q.27 (EC ENGG SECTION):

A box contains following three coins.

   I. A coin with head on one face and tail on other face.
  II. A coin with heads on both the faces.
 III. A coin with tails on both the faces.

A coin is picked randomly from the box and tossed .Out of the two remaining coins in the box ,one coin is then picked randomly and tossed.If the first toss results in a head,Then the probability of getting head in second toss is :

(A) $\frac{2}{5}$ (C) $\frac{1}{2}$

(B) $\frac{1}{3}$ (D) $\frac{2}{3}$

SOLUTION ate 2021 (EC) Q.27 (EC ENGG SECTION):

Let $X_1 \in \{0, 1, 2\}$ be a random variable representing the coin tossed in first toss.
Here $X_1 = 0, X_1 = 1, X_1 = 2$ represents coin 1 ,coin 2,coin 3 respectively
Since all the coins are equally likely

$$\Pr(X_1 = 0) = \Pr(X_1 = 1) = \Pr(X_1 = 2) = \frac{1}{3}$$
(0.0.1)

Let $X_2 \in \{0, 1, 2\}$ be a random variable representing the coin tossed in second toss.

Here $X_2 = 0, X_2 = 1, X_2 = 2$ represents coin 1 ,coin 2,coin 3 respectively

$$\text{if } X_1 = 0 : \Pr(X_2 = 1) = \Pr(X_2 = 2) = \frac{1}{2} \quad (0.0.2)$$

$$\text{if } X_1 = 1 : \Pr(X_2 = 0) = \Pr(X_2 = 2) = \frac{1}{2} \quad (0.0.3)$$

$$\text{if } X_1 = 2 : \Pr(X_2 = 0) = \Pr(X_2 = 1) = \frac{1}{2} \quad (0.0.4)$$

Let $Y, Z \in \{0, 1\}$ be the random variables which represents outcomes of first and second toss respectively.Here 0 represents tail and 1 represents head respectively.
Now we have to find $\Pr(Z = 1|Y = 1)$

$$\Pr(Z = 1|Y = 1) = \frac{\Pr(Z = 1, Y = 1)}{\Pr(Y = 1)} \quad (0.0.5)$$

As,

$$\Pr(Y = 1) = \sum_{i=0}^{2} \Pr(Y = 1|X_1 = i) \times \Pr(X_1 = i)$$
(0.0.6)

Probability	definition
$\Pr(Y = 1|X_1 = 0) = \frac{1}{2}$	probability of getting head on 1st throw by throwing coin 1
$\Pr(Y = 1|X_1 = 1) = 1$	probability of getting head on 1st throw by throwing coin 2
$\Pr(Y = 1|X_1 = 2) = 0$	probability of getting head on 1st throw by throwing coin 3

TABLE 4: Table 1.

from the 4 and (0.0.6)

$$\Pr(Y = 1) = \frac{1}{2} \times \frac{1}{3} + 1 \times \frac{1}{3} + 0 \times \frac{1}{3} \quad (0.0.7)$$

$$\Pr(Y = 1) = \frac{1}{2} \quad (0.0.8)$$

As,

$$\Pr(Z = 1, Y = 1) = \sum_{i=0}^{2} \Pr(Z = 1|X_1 = i) \times$$
$$\Pr(Y - 1|X_1 = i) \times \Pr(X_1 = i) \quad (0.0.9)$$

now substituting (0.0.19) and (0.0.8) in (0.0.5) gives

$$\Pr(Z = 1|Y = 1) = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3} \quad (0.0.20)$$

$\therefore$ **Option B is correct**

Probability	definition
$\Pr(Z = 1|X_2 = 0) = \frac{1}{2} = a(\text{let})$	probability of getting head on 2nd throw by throwing coin 1
$\Pr(Z = 1|X_2 = 1) = 1 = b(\text{let})$	probability of getting head on 2nd throw by throwing coin 2
$\Pr(Z = 1|X_2 = 2) = 0 = c(\text{let})$	probability of getting head on 2nd throw by throwing coin 3

TABLE 4: Table 2.

from (0.0.2) and (0.0.3) and (0.0.4)

$$\Pr(Z = 1|X_1 = 0) = b \times \Pr(X_2 = 1) + c \times \Pr(X_2 = 2) \quad (0.0.10)$$

$$\Pr(Z = 1|X_1 = 0) = 1 \times \frac{1}{2} + 0 \times \frac{1}{2} \quad (0.0.11)$$

$$\Pr(Z = 1|X_1 = 0) = \frac{1}{2} \quad (0.0.12)$$

$$\Pr(Z = 1|X_1 = 1) = a \times \Pr(X_2 = 0) + c \times \Pr(X_2 = 2) \quad (0.0.13)$$

$$\Pr(Z = 1|X_1 = 1) = \frac{1}{2} \times \frac{1}{2} + 0 \times \frac{1}{2} \quad (0.0.14)$$

$$\Pr(Z = 1|X_1 = 1) = \frac{1}{4} \quad (0.0.15)$$

$$\Pr(Z = 1|X_1 = 2) = a \times \Pr(X_2 = 0) + b \times \Pr(X_2 = 1) \quad (0.0.16)$$

$$\Pr(Z = 1|X_1 = 2) = \frac{1}{2} \times \frac{1}{2} + 1 \times \frac{1}{2} \quad (0.0.17)$$

$$\Pr(Z = 1|X_1 = 2) = \frac{3}{4} \quad (0.0.18)$$

substituting (0.0.12) and (0.0.15) and (0.0.18) in (0.0.9) gives

$$\Pr(Z = 1, Y = 1) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} + \frac{1}{4} \times 1 \times \frac{1}{3} + \frac{3}{4} \times 0 \times \frac{1}{3}$$

$$\Pr(Z = 1, Y = 1) = \frac{1}{6}$$

$$(0.0.19)$$