AI1103-Assignment-4

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Download all python codes from

https://github.com/Sravanth-k27/AI1103/tree/main/Assignment-4/codes

Download latex-tikz codes from

https://github.com/Sravanth-k27/AI1103/tree/main/Assignment-4/Assignment-4.tex

Question Gate 2021 (EC) Q.27 (EC engg section):

A box contains following three coins.

I. A coin with head on one face and tail on other face.
II. A coin with heads on both the faces.
III. A coin with tails on both the faces.

A coin is picked randomly from the box and tossed .Out of the two remaining coins in the box ,one coin is then picked randomly and tossed.If the first toss results in a head,Then the probability of getting head in second toss is :

(A) $\frac{2}{5}$ (C) $\frac{1}{2}$

(B) $\frac{1}{3}$ (D) $\frac{2}{3}$

Solution Gate 2021 (EC) Q.27 (EC engg section):

Let $X \in \{1,2\}$ be a random variable representing the trail happening.$X = 1, X = 2$ for the trail 1,trail 2 respectively.

Let $Y \in \{1,2,3\}$ be a random variable representing the coin tossed .$Y = 1, Y = 2, Y = 3$ for the coin 1,coin 2 ,coin 3 respectively.
Since all the coins are equally likely

$$\Pr(Y = 1) = \Pr(Y = 2) = \Pr(Y = 3) = \frac{1}{3} \quad (0.0.1)$$

Let $Z \in \{0,1\}$ be a random variable representing the outcomes of the coin.$Z = 0, Z = 1$ for tail,head respectively.

Now we need to find
$\Pr(X = 2, Z = 1 | X = 1, Z = 1)$= a (let)

From conditional probability we have

$$a = \frac{\Pr(X = 1, X = 2, Z = 1)}{\Pr(X = 1, Z = 1)} \quad (0.0.2)$$

Event	Definition
$Y = 1$	selecting coin 1 for a trail
$Y = 2$	selecting coin 2 for a trail
$Y = 3$	selecting coin 3 for a trail
$X = 1, Z = 1 \mid Y = 1$	getting head on first trail by tossing coin 1
$X = 1, Z = 1 \mid Y = 2$	getting head on first trail by tossing coin2
$X = 1, Z = 1 \mid Y = 3$	getting head on first trail by tossing coin3
$X = 1, Z = 1$	getting head on first trail
$X = 1, X = 2, Z = 1$	getting head on both first and second trails

TABLE 4: TABLE-1.

$$\Pr(X = 1, Z = 1) = \sum_{i=1}^{3} \Pr(X = 1, Z = 1 | Y = i) \times \Pr(Y = i) \quad (0.0.3)$$

Probability	Value
$\Pr(Y=1)$	$\frac{1}{3}$
$\Pr(Y=2)$	$\frac{1}{3}$
$\Pr(Y=3)$	$\frac{1}{3}$
$\Pr(X=1,Z=1\|Y=1)$	$\frac{1}{2}$
$\Pr(X=1,Z=1\|Y=2)$	1
$\Pr(X=1,Z=1\|Y=3)$	0
$\Pr(X=1,Z=1)$	$=\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1+\frac{1}{3}\times 0$ $=\frac{1}{2}$
$\Pr(X=1,X=2,Z=1)$	$=\frac{1}{3}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{3}\times 0$ $=\frac{1}{6}$

TABLE 4: Table-2.

from 4

$$\Pr(X=2,Z=1|X=2,Z=1)=\frac{\frac{1}{6}}{\frac{1}{2}} \qquad (0.0.4)$$

$$\Pr(X=2,Z=1|X=2,Z=1)=\frac{1}{3} \qquad (0.0.5)$$

Hence the required probability is $\frac{1}{3}$

$\therefore$ **Option B is correct**