

AI1103-Assignment 1

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Download all python codes from

<https://github.com/Sravanth-k27/AI1103-Assignment-1/Codes>

and latex-tikz codes from

<https://github.com/Sravanth-k27/AI1103-Assignment-1/Assignment-1.tex>

QUESTION(2.13):

A die is thrown three times. Events A and B are defined as below:

- 1) A : 4 on the third throw.
- 2) B : 6 on the first and 5 on the second throw.

Find the probability of A given that B has already occurred?

SOLUTION(2.13):

Let $X_i \in \{1, 2, 3, 4, 5, 6\}$ where $i = 1, 2, 3$ be the random variables representing the outcomes of throwing a die three times.

- 1) Probability of event A happening=Probability of $X_3 = 4$

$$\Pr(A) = \Pr(X_3 = 4) \quad (0.0.1)$$

Since all the outcomes are equally likely their probabilities are same
so

$$\Pr(A) = \Pr(X_3 = 4) = \frac{1}{6} \quad (0.0.2)$$

- 2) Probability of event B happening=Probability of $X_1 = 6, X_2 = 5$.
so

$$\Pr(B) = \Pr(X_1 = 6, X_2 = 5) \quad (0.0.3)$$

Random variable X_1 depends on first throw of die and random variable X_2 depends on second

throw of die so X_1 and X_2 are independent.
so

$$\begin{aligned} \Pr(X_1 = 6, X_2 = 5) &= \Pr(X_1 = 6) \Pr(X_2 = 5) \\ &= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \end{aligned}$$

$$\Pr(B) = \Pr(X_1 = 6, X_2 = 5) = \frac{1}{36} \quad (0.0.4)$$

Also A,B are also independent events
therefore from 0.0.2 and 0.0.4

$$\Pr(AB) = \Pr(A) \Pr(B)$$

$$\frac{1}{6} \times \frac{1}{36}$$

$$\Pr(AB) = \frac{1}{216} \quad (0.0.5)$$

Since we have to find probability of A given that B has already happened.
so $\Pr(A|B)$

- 3) By formula of conditional probability

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \quad (0.0.6)$$

From 0.0.4 and 0.0.5

$$\Rightarrow \Pr(A|B) = \left(\frac{\frac{1}{216}}{\frac{1}{36}} \right) \quad (0.0.7)$$

$$\Rightarrow \Pr(A|B) = \frac{1}{6} \quad (0.0.8)$$

So the probability of A given that B has already happened = $\Pr(A|B) = \frac{1}{6}$