Control Systems - Problem 07

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Problem Statement

A system is described by the following differential equation:

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x \tag{2.1}$$

Find the expression for the transfer function of the system G(s)= $\frac{Y(s)}{X(s)}$.

Finding the Laplace transform of derivatives

Given the equation is

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x \tag{3.1}$$

Finding Laplace transform of $\frac{df(t)}{dt}$

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt \tag{3.2}$$

$$\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st} f'(t)dt \tag{3.3}$$

$$= \left[e^{-st} f(t)\right]_0^\infty - \int\limits_0^\infty f(t)(-se^{-st})\, dt \tag{3.4}$$

$$= -f(0) + s\mathcal{L}\{f(t)\} \tag{3.5}$$

Hence Laplace transform of first derivative is

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0) \tag{3.6}$$

similarly Laplace transform of second derivative is

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0) \tag{3.7}$$

Laplace transform of third derivative is

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0) \tag{3.8}$$

Laplace transform of nth derivative is

$$\mathcal{L}\{f^n(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0)....... - f^{n-1}(0) \tag{3.9}$$

Finding the Transfer Function

Taking the Laplace transform on both sides of the given equation

$$s^3Y(s) + 3s^2Y(s) + 5sY(s) + Y(s) + \tag{3.10}$$

$$\text{initial condition terms involving y(t)}$$

$$= s^3X(s) + 4s^2X(s) + 6sX(s) + 8X(s) + \tag{3.11}$$

$$\text{initial condition terms involving x(t)}$$

Let's assume that all initial conditions to be zero then the equation reduces to

$$(s^3 + 3s^2 + 5s + 1)Y(s) = (s^3 + 4s^2 + 6s + 8)X(s) \tag{3.12}$$

Hence the Transfer Function G(s) is

$$G(s) = \frac{Y(s)}{X(s)} = \frac{(s^3 + 4s^2 + 6s + 8)}{(s^3 + 3s^2 + 5s + 1)} \tag{3.13}$$