Vector Algebra

1 12^{th} Maths - Chapter 10

This is Problem-8 from Miscellaneous Exercise

1. Show that the points A $\begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$, B $\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$ and C $\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix}$ are collinear, and find the ratio in which B divides AC.

   Solution: To find collinearity of the given points, we use rank method. If the rank is less than 3, then the points are said to be collinear.

$$\begin{pmatrix} 1 & 5 & 11 \\ -2 & 0 & 3 \\ -8 & -2 & 7 \\ 1 & 1 & 1 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 + 2R_1} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ -8 & -2 & 7 \\ 1 & 1 & 1 \end{pmatrix} \tag{1}$$

$$\xleftarrow{R_3 \to 8R_1 + R_3} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ 0 & 38 & 95 \\ 1 & 1 & 1 \end{pmatrix} \tag{2}$$

$$\xleftarrow{R_4 \to -R_1 + R_4} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ 0 & 38 & 95 \\ 0 & -4 & -10 \end{pmatrix} \tag{3}$$

$$\xleftarrow{R_3 \to -\frac{-38R_2}{10} + R_3} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ 0 & 0 & 0 \\ 0 & -4 & -10 \end{pmatrix} \tag{4}$$

$$\xleftarrow{R_4 \to \frac{4R_2}{10} + R_4} \begin{pmatrix} 1 & 5 & 11 \\ 0 & 10 & 25 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5}$$

Therefore, the rank of the matrix is 2. Hence, the points are collinear as the rank of the matrix is less than 3.

Let us assume B divides AC in $k:1$ ratio. Therefore, using section formula we get:

$$\implies \mathbf{B} = \frac{k\mathbf{C} + \mathbf{A}}{k+1} \tag{6}$$

Substituting the values of $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ in (5)

$$\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} = \frac{\begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix} k + \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}}{k+1} \tag{7}$$

$$k \begin{pmatrix} -6 \\ -3 \\ -9 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ -6 \end{pmatrix} \tag{8}$$

Performing dot product on both sides by $\begin{pmatrix} -4 \\ -2 \\ -6 \end{pmatrix}^\top$ yields,

$$k \begin{pmatrix} -6 \\ -3 \\ -9 \end{pmatrix} \begin{pmatrix} -4 & -2 & -6 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ -6 \end{pmatrix} \begin{pmatrix} -4 & -2 & -6 \end{pmatrix} \tag{9}$$

$$\implies k\,(84) = (56) \tag{10}$$

$$\implies k = \frac{2}{3} \tag{11}$$

Hence the desired ratio is $\dfrac{2}{3}$.