

12.10.3.6

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CLASS 12, CHAPTER 10, EXERCISE 3.6

Find $\|\mathbf{a}\|$ and $\|\mathbf{b}\|$ if $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = 8$ and $\mathbf{a} = 8\|\mathbf{b}\|$.

SOLUTION(3.6)

$$\begin{aligned}(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) &= (\mathbf{a} + \mathbf{b})^T (\mathbf{a} - \mathbf{b}) \\(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) &= (\mathbf{a}^T + \mathbf{b}^T)(\mathbf{a} - \mathbf{b}) \\(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) &= \mathbf{a}^T \mathbf{a} + \mathbf{b}^T \mathbf{a} - \mathbf{a}^T \mathbf{b} - \mathbf{b}^T \mathbf{b} \\(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) &= \|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 \\\|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 &= 8\end{aligned}$$

We know that

$$\|\mathbf{a}\|^2 = 64\|\mathbf{b}\|^2$$

Therefore, substituting :

$$\begin{aligned}63\|\mathbf{b}\|^2 &= 8 \\\|\mathbf{b}\| &= \frac{2\sqrt{2}}{3\sqrt{7}} \\\text{and} \\\|\mathbf{a}\| &= \frac{16\sqrt{2}}{3\sqrt{7}}\end{aligned}$$