Vector Algebra

1 12^{th} Maths - Chapter 10

This is Problem-8 from Miscellaneous Exercise

1. Show that the points A(1, –2, –8), B(5, 0, –2) and C(11, 3, 7) are collinear, and find the ratio in which B divides AC.
Solution: We know that points $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are collinear, if

$$\operatorname{rank}\begin{pmatrix}\mathbf{A}-\mathbf{B}\\ \mathbf{A}-\mathbf{C}\end{pmatrix} = 1 \tag{1}$$

$$\begin{pmatrix}\mathbf{A}-\mathbf{B}\\ \mathbf{A}-\mathbf{C}\end{pmatrix} = \begin{pmatrix}4 & 2 & 6\\ 10 & 5 & 15\end{pmatrix} \tag{2}$$

Performing a sequence of row reduction operations

$$\xleftrightarrow[R_1\to\frac{10}{4}R_1]{R_2\to R_2-R_1} \begin{pmatrix}10 & 5 & 15\\ 0 & 0 & 0\end{pmatrix} \tag{3}$$

$$\xleftrightarrow{R_1\to\frac{4}{10}R_1} \begin{pmatrix}4 & 2 & 6\\ 0 & 0 & 0\end{pmatrix} \tag{4}$$

Therefore, the rank of the matrix is 1. Hence, referring to equation 1, the points are collinear as the rank of the matrix is equal to 1.

Let us assume B divides AC in $k : 1$ ratio. Therefore, we get:

$$\implies \mathbf{B} = \frac{k\mathbf{C}+\mathbf{A}}{k+1} \tag{5}$$

$$= \frac{\begin{pmatrix}11k\\ 3k\\ 7k\end{pmatrix} + \begin{pmatrix}1\\ -2\\ -8\end{pmatrix}}{k+1} \tag{6}$$

Substituting $\mathbf{B} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$

$$\implies \frac{\begin{pmatrix} 11k \\ 3k \\ 7k \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}}{k+1} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} \tag{7}$$

$$\implies \begin{pmatrix} 11k+1 \\ 3k-2 \\ 7k-8 \end{pmatrix} = \begin{pmatrix} 5k+5 \\ 0 \\ -2k-2 \end{pmatrix} \tag{8}$$

$$\implies k = \frac{2}{3} \tag{9}$$

B divides AC in 2:3 ratio.