Vector Dot Product

1 12^{th} Maths - Chapter 10

This is Problem-6 from Exercise 10.3

1. Find $\|\mathbf{a}\|$ and $\|\mathbf{b}\|$ if $(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b}) = 8$ and $\|\mathbf{a}\| = 8\|\mathbf{b}\|$.
   Solution:

   Given

$$(\mathbf{a}+\mathbf{b})^\top (\mathbf{a}-\mathbf{b}) = 8 \tag{1}$$
$$\mathbf{a}^\top\mathbf{a} + \mathbf{b}^\top\mathbf{a} - \mathbf{a}^\top\mathbf{b} - \mathbf{b}^\top\mathbf{b} = 8 \tag{2}$$
$$\|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 = 8 \tag{3}$$

   Substituting $\|\mathbf{a}\| = 8\|\mathbf{b}\|$

$$64\|\mathbf{b}\|^2 - \|\mathbf{b}\|^2 = 8 \tag{4}$$
$$63\|\mathbf{b}\|^2 = 8 \tag{5}$$
$$\|\mathbf{b}\|^2 = \frac{8}{63} \tag{6}$$
$$\|\mathbf{b}\| = \frac{2\sqrt{2}}{3\sqrt{7}} \tag{7}$$
$$\|\mathbf{a}\| = \frac{16\sqrt{2}}{3\sqrt{7}} \tag{8}$$