Vector Algebra

1 12^{th} Maths - Chapter 10

This is Problem-8 from Miscellaneous Exercise

1. Show that the points A$(1, -2, -8)$, B$(5, 0, -2)$ and C$(11, 3, 7)$ are collinear, and find the ratio in which B divides AC.
   Solution: We know that points $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are collinear, if

$$\text{rank}\begin{pmatrix} \mathbf{A}-\mathbf{B} \\ \mathbf{A}-\mathbf{C} \end{pmatrix} = 1 \tag{1}$$

$$\begin{pmatrix} \mathbf{A}-\mathbf{B} \\ \mathbf{A}-\mathbf{C} \end{pmatrix} = \begin{pmatrix} 4 & 2 & 6 \\ 10 & 5 & 15 \end{pmatrix} \tag{2}$$

Performing a sequence of row reduction operations

$$\xleftrightarrow[R_1 \to \frac{10}{4}R_1]{R_2 \to R_2 - R_1} \begin{pmatrix} 10 & 5 & 15 \\ 0 & 0 & 0 \end{pmatrix} \tag{3}$$

$$\xleftrightarrow{R_1 \to \frac{4}{10}R_1} \begin{pmatrix} 4 & 2 & 6 \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

Therefore, the rank of the matrix is 1. Hence, referring to equation 1, the points are collinear as the rank of the matrix is equal to 1.

Let us assume B divides AC in k:1 ratio. Therefore, we get:

$$\implies \mathbf{B} = \frac{k\mathbf{C}+\mathbf{A}}{k+1} \tag{5}$$

$$= \frac{\begin{pmatrix} 11k \\ 3k \\ 7k \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}}{k+1} \tag{6}$$

Substituting $\mathbf{B} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$

$$\implies \frac{\begin{pmatrix} 11k \\ 3k \\ 7k \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}}{k+1} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} \tag{7}$$

$$\implies \begin{pmatrix} 11k+1 \\ 3k-2 \\ 7k-8 \end{pmatrix} = \begin{pmatrix} 5k+5 \\ 0 \\ -2k-2 \end{pmatrix} \tag{8}$$

$$\implies k = \frac{2}{3} \tag{9}$$

B divides AC in 2:3 ratio.