

# Vector Dot Product

## 1 12<sup>th</sup> Maths - Chapter 10

This is Problem-6 from Exercise 10.3

1. Find  $\|\mathbf{a}\|$  and  $\|\mathbf{b}\|$  if  $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = 8$  and  $\|\mathbf{a}\| = 8 \|\mathbf{b}\|$ .

**Solution:**

$$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = (\mathbf{a} + \mathbf{b})^\top (\mathbf{a} - \mathbf{b}) \quad (1)$$

$$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = \mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{a} - \mathbf{a}^\top \mathbf{b} - \mathbf{b}^\top \mathbf{b} \quad (2)$$

$$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = \|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 \quad (3)$$

$$\implies \|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (4)$$

$$(5)$$

Substituting  $\|\mathbf{a}\| = 8 \|\mathbf{b}\|$ .

$$64 \|\mathbf{b}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (6)$$

$$63 \|\mathbf{b}\|^2 = 8 \quad (7)$$

$$\|\mathbf{b}\|^2 = \frac{8}{63} \quad (8)$$

$$\|\mathbf{b}\| = \frac{2\sqrt{2}}{3\sqrt{7}} \quad (9)$$

$$\|\mathbf{a}\| = \frac{16\sqrt{2}}{3\sqrt{7}} \quad (10)$$