Vector Algebra

1 12^{th} Maths - Chapter 10

This is Problem-8 from Miscellaneous Exercise

1. Show that the points A$(1, -2, -8)$, B$(5, 0, -2)$ and C$(11, 3, 7)$ are collinear, and find the ratio in which B divides AC.
   Solution: We know that points $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are collinear, if

$$\text{rank}\begin{pmatrix}\mathbf{AB}\\ \mathbf{AC}\end{pmatrix} = 1 \tag{1}$$

$$\begin{pmatrix}\mathbf{AB}\\ \mathbf{AC}\end{pmatrix} = \begin{pmatrix}4 & 2 & 6\\ 10 & 5 & 15\end{pmatrix} \tag{2}$$

Performing a sequence of row reduction operations

$$\xleftrightarrow[R_1 \to \frac{10}{4}R_1]{R_2 \to R_2 - R_1} \begin{pmatrix}10 & 5 & 15\\ 0 & 0 & 0\end{pmatrix} \tag{3}$$

$$\xleftrightarrow{R_1 \to \frac{4}{10}R_1} \begin{pmatrix}4 & 2 & 6\\ 0 & 0 & 0\end{pmatrix} \tag{4}$$

Therefore, the rank of the matrix is 1. Hence, referring to equation 1, the points are collinear as the rank of the matrix is equal to 1.

Let us assume B divides AC in k:1 ratio. Therefore, we get:

$$\implies B = \frac{kC + A}{k+1} \tag{5}$$

$$= \frac{(11k, 3k, 7k) + (1, -2, -8)}{k+1} \tag{6}$$

Substituting B $= (5, 0, 2)$

$$\implies \frac{(11k, 3k, 7k) + (1, -2, -8)}{k+1} = (5, 0, -2) \tag{7}$$

$$\implies (11k + 1, 3k - 2, 7k - 8) = (5k + 5, 0, -2k - 2) \tag{8}$$

$$\implies k = \frac{2}{3} \tag{9}$$

B divides AC in 2:3 ratio.