

# Vector Dot Product

## 1 12<sup>th</sup> Maths - Chapter 10

This is Problem-6 from Exercise 10.3

1. Find  $\|\mathbf{a}\|$  and  $\|\mathbf{b}\|$  if  $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = 8$  and  $\|\mathbf{a}\| = 8\|\mathbf{b}\|$ .

**Solution:**

Given

$$(\mathbf{a} + \mathbf{b})^\top (\mathbf{a} - \mathbf{b}) = 8 \quad (1)$$

$$\implies \mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{a} - \mathbf{a}^\top \mathbf{b} - \mathbf{b}^\top \mathbf{b} = 8 \quad (2)$$

$$\implies \|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (3)$$

Substituting  $\|\mathbf{a}\| = 8\|\mathbf{b}\|$  in (3)

$$\implies 64\|\mathbf{b}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (4)$$

$$\implies 63\|\mathbf{b}\|^2 = 8 \quad (5)$$

$$\implies \|\mathbf{b}\|^2 = \frac{8}{63} \quad (6)$$

$$\implies \|\mathbf{b}\| = \frac{2\sqrt{2}}{3\sqrt{7}} \quad (7)$$

$$\implies \|\mathbf{a}\| = \frac{16\sqrt{2}}{3\sqrt{7}} \quad (8)$$