Homework 11: Hybrid Systems Simulation

24-760 Robot Dynamics & Analysis
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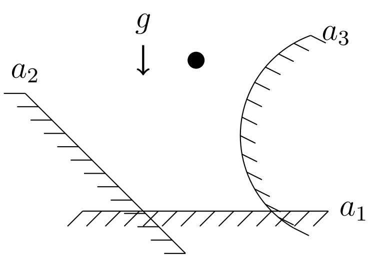
Problem 1) Falling Ball



Consider a point particle that can make plastic frictionless impact with several constraints. Assume the particle is mass 1 and gravity is 9.8. Let the constraints be $a_1(x, y) = y$, $a_2(x, y) = x + y + 1$, and $a_3(x, y) = (x - 2)^2 + (y - 1)^2 - 2$.

1.1) What is the hybrid dynamical system for this problem? That is, what are all of the components of $\mathcal{H} = (\mathcal{J}, \mathit{\Gamma}, \mathcal{D}, \mathcal{F}, \mathcal{G}, \mathcal{R})$? Consider both impact (IV complementarity) and liftoff (FA complementarity) transitions. You may limit the hybrid system to only the feasible transitions ($\tilde{\mathit{\Gamma}}$ instead of $\mathit{\Gamma}$). For simplicity, assume the particle does not impact multiple constraints at once from the unconstrained mode.

1.2) Simulate the system in Matlab using `ode45` and an event function. The `odefun` should capture the continuous dynamics $\mathcal{F}$, while the event function detects the guard conditions $\mathcal{G}$. Apply the reset function outside of the `ode45` execution. You may want to make separate Matlab functions to calculate $a, A, \dot{A}, \mathcal{F}, \mathcal{R}$, the block matrix inverse, etc. To solve the complementarity problems you do not need to use a computationally efficient algorithm, simply check the complementarity conditions for all possible modes (modes in the local scope, $\mathcal{I}$) and return the (hopefully unique) mode that satisfies the constraints. Here are two pages documenting these Matlab features:
https://www.mathworks.com/help/matlab/ref/ode45.html
https://www.mathworks.com/help/matlab/math/ode-event-location.html
Hint: Start with just a single constraint a_1, and then add in a_2 and a_3. If your simulation is missing events, you may want to try using the `MaxStep` option.

1.3) Run four simulations starting at $(0, 5)$, $(-1.5, 5)$, $(1.5, 5)$, and $(1, 5)$ with zero velocity. Run each simulation for 5 seconds. What contact mode transitions occur and at what times?

ANSWERS:-

1.1

$\mu = 1$

$g = 9.81$

$a_1(x, y) = y$

$a_2(x, y) = x + y + 1$

$a_3(x, y) = (x-2)^2 + (y-1)^2 - 2$

Contact modes;

$$J = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}$$

Feasible Transitions;

$$F = \{(\{\}, \{1\}), (\{\}, \{2\}), (\{\}, \{3\}), (\{3\}, \{\}), (\{1\}, \{2\}), (\{2\}, \{1\}), (\{1\}, \{1,3\})\}$$

Now the domain of our hybrid system is,

$$D = \begin{cases} D_{\emptyset} = \{ q, \dot{q} : a_1(q) \geq 0, a_2(q) \geq 0, a_3(q) \geq 0 \} \\ D_1 = \{ q, \dot{q} : a_1(q) = 0, A_1 \dot{q} = 0, a_2(q) \geq 0, a_3(q) \geq 0 \} \\ D_2 = \{ q, \dot{q} : a_1(q) \geq 0, a_2(q) = 0, A_2 \dot{q} = 0, a_3(q) \geq 0 \} \\ D_3 = \{ q, \dot{q} : a_1(q) \geq 0, a_2(q) \geq 0, a_3(q) = 0, A_3 \dot{q} = 0 \} \\ D_{12} = \{ q, \dot{q} : a_1(q) = 0, A_1 \dot{q} = 0, a_2(q) = 0, A_2 \dot{q} = 0, a_3(q) \geq 0 \} \\ D_{13} = \{ q, \dot{q} : a_1(q) = 0, A_1 \dot{q} = 0, a_2(q) \geq 0, a_3(q) = 0, A_3 \dot{q} = 0 \} \end{cases}$$

Now,

$$\left.\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 \end{bmatrix} \\ A_2 &= \begin{bmatrix} 1 & 1 \end{bmatrix} \\ A_3 &= \begin{bmatrix} 2x-4 & 2y-2 \end{bmatrix} \end{aligned}\right\} A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2x-4 & 2y-2 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \bar{N} = \begin{bmatrix} 0 \\ 9.81 \end{bmatrix}$$

$$\Upsilon = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathcal{F} = \{ F_1 : \bar{M} \ddot{q} + \bar{N}(q, \dot{q}) + A_1^T \lambda = 0 \}$$

$$F_{\{1\}} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$F_{\{2\}} = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$F_{\{3\}} = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} + \begin{bmatrix} 2x-4 \\ 2y-2 \end{bmatrix}\lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$F_{\{1,3\}} = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} + \begin{bmatrix} 0 & 2x-4 \\ 1 & 2y-2 \end{bmatrix}\lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$F_{\{\}} = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Guard sets;

$$G = \left\{ \begin{array}{l}
G_{\{\},1} = q, \dot{q} \in D_{\{\}} : a_1(q) = 0,\ A_1\dot{q} < 0 \\
G_{\{\},2} = q, \dot{q} \in D_{\{\}} : a_2(q) = 0,\ A_2\dot{q} < 0 \\
G_{\{\},3} = q, \dot{q} \in D_{\{\}} : a_3(q) = 0,\ A_3\dot{q} < 0 \\
G_{1,2} = q, \dot{q} \in D_1 : a_2(q) = 0,\ A_2\dot{q} < 0 \\
G_{2,1} = q, \dot{q} \in D_2 : a_1(q) = 0,\ A_1\dot{q} < 0 \\
G_{\{1\},\{1,3\}} = q, \dot{q} \in D_1 : a_3(q) = 0,\ A_3\dot{q} < 0 \\
G_{3\{\}} = q, \dot{q} \in D_3 : M(\lambda_3) < 0
\end{array} \right\}$$

Reset map;

$$R = \begin{cases} R_{\{3\},1}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_1 \\ R_{\{3\},2}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_2 \\ R_{\{3\},3}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_3 \\ R_{1,2}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_2 \\ R_{2,1}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_1 \\ R_{\{1\},\{1,3\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_1 \\ R_{3,\{2\}}(q, \dot{q}^-) = (q, \dot{q}^-) \in D_{\{2\}} \end{cases},$$

$$\dot{q}^+ = \dot{q}^- - A_J^{\#T}(q) A_J(q) \dot{q}^-$$

1.2 MATLAB CODE

1.3 → For $[0, 5]$,

contact mode $\{1\}$ at $t = 1.0102\,s$

→ For $[-1.5, 5]$,

contact mode $\{2\}$ at $t = 0.95831\,s$

contact mode {1} at t = 1.0595 s

contact mode {1 3} at t = 1.4447 s

→ For [1.5, 5],

contact mode {3} at t = 0.73916 s

contact mode { } at t = 0.89201 s

contact mode {1} at t = 1.3706 s

contact mode {2} at t = 1.7155 s

contact mode {1} at t = 2.2402 s

contact mode {1 3} at t = 3.7959 s

Intial condition: [1 5],

contact mode {3} at t = 0.78246 s

contact mode { } at t = 0.78246 s

contact mode {1} at t = 1.1404 s

contact mode {2} at t = 1.3041 s

contact mode $\{1\}$ at $t = 2.0866\,s$

contact mode $\{1, 3\}$ at $t = 3.1298\,s$