Homework 5: Contact Kinematics

24-760 Robot Dynamics & Analysis
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Problem 1) Grasp Properties

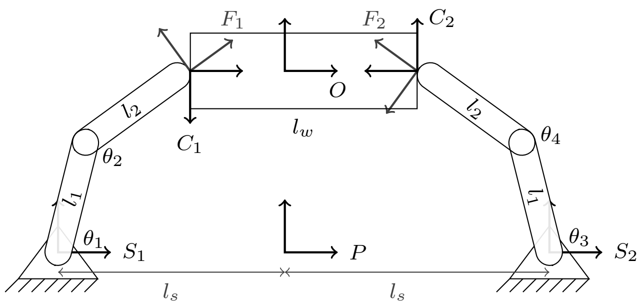


Figure 1: Two fingered robot. For each frame, the label indicates the x-axis.

Consider a planar two finger hand with two links per finger, as shown in Figure 1. The base of each finger is located at $\pm l_s$ along the x-axis, with link lengths of l_1 and l_2. The fingers are holding an object of width l_w and height l_h, with an object frame O at its center. For this planar problem, define the contact frames C_1 and C_2 with the y-axis pointing into the object (normally the C frame is defined with the z-axis pointing inward). For each problem you may solve the problem in SE(3) if you would like and then drop the z direction and rotations out of the plane (which should all be zero). See notes posted under readings on planar kinematics if you have questions.

The object location and orientation relative to the palm is $o = [x_o, y_o, \phi_o]^T$ (*note that the book usually uses the notation x_o instead of o for the whole vector*), and the joint variables are $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T$. Assume frictional contact with coefficient μ, and that the contact points are along the object's x-axis (but note that the object frame can otherwise move or rotate from the configuration as drawn). You are encouraged to use Matlab to help with the calculations. Please make sure your answer is consistent in dimensions (planar or spatial).

1.1) What is the grasp map, G? What is the combined friction cone, FC?

1.2) Is this a force-closure grasp?

1.3) In a static scene, if the object mass is m what is the object wrench, F_e due to gravity and what is a feasible vector of contact forces, f_c, that resist this wrench? Write your solution in terms of the object configuration o.

1.4) What are the possible internal forces?

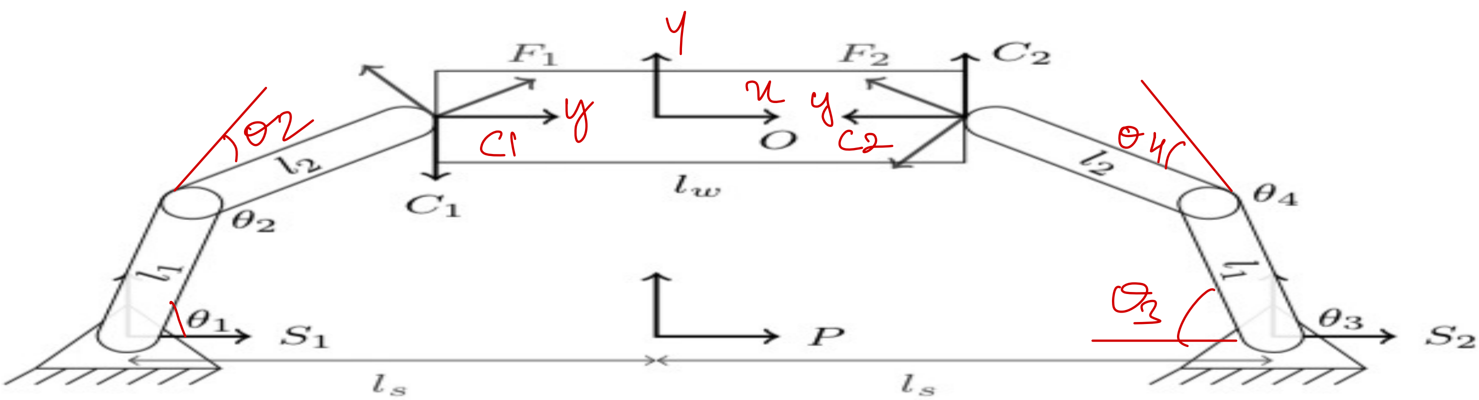
1.5) What if any is the minimal internal force to hold the object with respect to gravity?

1.6) What is the hand Jacobian, J_h?

1.7) What are the possible internal motions?

1.8) The robot has 4 degrees of freedom (DOF), two per finger, meaning that in nonsingular configurations there is a 4 dimensional space of motions it can produce. The object in the plane is only 3 DOF. What does the fourth DOF correspond to?

ANSWERS:



$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} \quad \& \quad O = \begin{bmatrix} x_0 \\ y_0 \\ \phi_0 \end{bmatrix}$$

PROBLEM 1.1:-

To calculate Grasp map with the equation

$$G = \left[Ad^T_{g_{oc_1}} B_{c_1} \quad Ad^T_{g^{-1}_{oc_2}} B_{c_2} \right]$$

Now,

$$g_{oc_1} [3D] = \begin{bmatrix} 0 & 1 & 0 & -\frac{l\omega}{2} \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad g_{oc_2} [3D] = \begin{bmatrix} 0 & -1 & 0 & l\omega/2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{oc_1} [2D] = \begin{bmatrix} 0 & 1 & -l\omega/2 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad g_{oc_2} [2D] = \begin{bmatrix} 0 & -1 & \frac{l\omega}{2} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, the frictional contacts are;

$$B_{c_1} [3D] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = B_{c_2} [3D] ; \quad B_{c_1} [2D] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = B_{c_2} [2D]$$

Now, for G matrix,

$$G [3D] = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{l\omega}{2} & 0 & \frac{l\omega}{2} & 0 \end{bmatrix} ; \quad G [2D] = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{l\omega}{2} & 0 & \frac{l\omega}{2} & 0 \end{bmatrix}$$

For Friction Cone,

$$FC = \mu(fc_1) = \begin{bmatrix} fc_{1y} \\ \mu fc_{1y} - \sqrt{fc_{1x}^2 + fc_{1z}^2} \end{bmatrix} \geq 0$$

$$= \mu(fc_2) = \begin{bmatrix} fc_{2y} \\ \mu fc_{2y} - \sqrt{fc_{2x}^2 + fc_{2z}^2} \end{bmatrix} \geq 0$$

$$\boxed{FC = FC_1 \times FC_2}$$

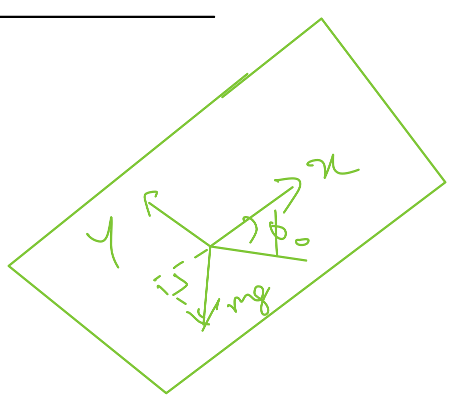
PROBLEM 1.2 :-

In this particular Grasp scenario, the rank of the grasp map (G) = 3, which says the columns can span the entire 3D-Planar space and the 2 columns are linearly dependent.

→ can have wrench on the object.

And Friction cone donot have any limitations on this span. Thus it is a **force-closure grasp**

PROBLEM 1.3 :-



$$f_{c\,[2D]} = \begin{bmatrix} mg\sin(\phi_0) \\ mg\cos(\phi_0) \\ 0 \end{bmatrix}$$

$$F_e = -G f_c \quad \text{--- (1)}$$

$$\sum F_y = 0$$

$$F_{e\,[3D]} = \begin{bmatrix} -mg\sin\phi_0 \\ -mg\cos\phi_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad G_{[3D]} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix}$$

$$G_{(2D)} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix}$$

From eqn 1, $\quad Gf_c = -F_e$,

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix} \begin{bmatrix} f_{c_1x} \\ f_{c_1y} \\ f_{c_2x} \\ f_{c_2y} \end{bmatrix} = \begin{bmatrix} mg\sin\phi_0 \\ mg\cos\phi_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad - [3D] \rightarrow 1$$

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix} \begin{bmatrix} f_{c_1x} \\ f_{c_1y} \\ f_{c_2x} \\ f_{c_2y} \end{bmatrix} = \begin{bmatrix} mg\sin\phi_0 \\ mg\cos\phi_0 \\ 0 \end{bmatrix} \quad - [2D] \rightarrow 2$$

From eqns ① & ②, we can get

$$f_{c_1y} - f_{c_2y} = mg\sin\phi_0 \quad — ③$$

$$f_{c_2x} - f_{c_1x} = mg\cos\phi_0 \quad — ④$$

$$\frac{l\omega}{2} f_{c_1x} + \frac{l\omega}{2} f_{c_2x} = 0 \quad — ⑤$$

Solving ④ & ⑤,

$$f_{c_2x} - \cancel{f_{c_1x}} = mg\cos\phi_0$$

$$\cancel{f_{c_1x}} + f_{c_2x} = 0$$

$$\boxed{f_{c_2x} = \frac{mg\cos\phi_0}{2}}$$

So, $\boxed{f_{c_1x} = -\frac{mg\cos\phi_0}{2}}$

$$\mu f_{c_1y} - \sqrt{f_{c_1x}^2} \geq 0$$

$$\mu f_{c_1y} - \sqrt{\left(\frac{mg\cos\phi_0}{2}\right)^2} \geq 0$$

$$\boxed{f_{c_1y} = mg + \left|\frac{mg\cos\phi_0}{2\mu}\right|}$$

and, by substituting

we get $\boxed{f_{c2y} = mg\left(1-\sin\phi_0\right) + \left|\frac{mg\cos\phi_0}{2\mu}\right|}$

PROBLEM 1.4 :-

For Internal forces;

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix} \begin{bmatrix} f_{c1x} \\ f_{c1y} \\ f_{c2x} \\ f_{c2y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$f_{c1y} - f_{c2y} = 0 \quad \text{——— } 1$$

$$f_{c2x} - f_{c2x} = 0 \quad \text{——— } 2$$

$$\frac{lw}{2} f_{c1x} + \frac{lw}{2} f_{c2x} = 0 \quad \text{——— } 3$$

where, $\boxed{f_{c1y} = f_{c2y}}$

$$f_{c1x} = f_{2x}$$

$$f_{c1x} = -f_{c2x}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \lambda = f$$

Thus,
f_{c1y} & f_{c2y} are the Internal forces

PROBLEM 1.5 :-

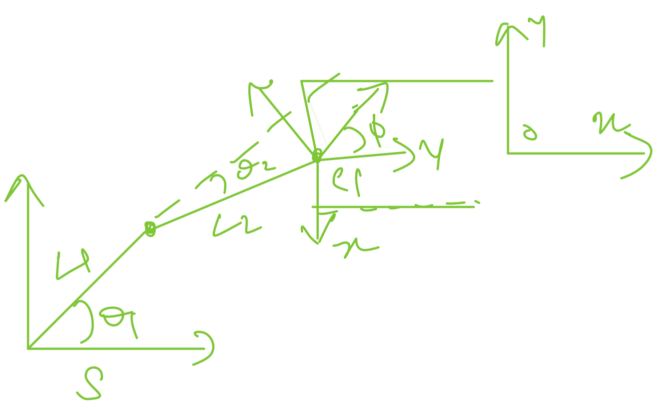
To hold an object we need the internal forces to generate some friction forces.

we know, $G f_c = -F_e$

And, the friction cone for this system should be $f_{c_1 x}, f_{c_2 x} \le \mu (f_{c_1 y}, f_{c_2 y})$

$$\boxed{\text{So, } f = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \lambda \;\hookrightarrow\; \geq \left| \frac{mg\cos\phi_0}{2\mu} \right|}$$

PROBLEM 1.6 :-



$$G = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix}$$

$$B_{c_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_h = \begin{bmatrix} B_{c_1}^T Ad^{-1}_{g_{s_1 c_1}} J^S_{s_1 f_1} \theta_{f_1} & 0 \\ 0 & B_{c_2}^T Ad^{-1}_{g_{s_2 c_2}} J^S_{s_2 f_2} \theta_{f_2} \end{bmatrix}$$

Now,

$$J^S_{s_1 f_1} = \begin{bmatrix} 0 & l_1 \sin(\theta_1) \\ 0 & -l_1 \cos(\theta_1) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \quad \& \quad J^S_{s_2 f_2} = \begin{bmatrix} 0 & l_1 \sin\theta_3 \\ 0 & -l_1 \cos\theta \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

By utilizing Finger Frame,

$$g_{s_1 c_1} = \begin{bmatrix} \sin\phi_0 & \cos\phi_0 & 0 & l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1 \\ -\cos\phi_0 & \sin\phi_1 & 0 & l_2\sin(\theta_1+\theta_2) + l_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{s_2 c_2} = \begin{bmatrix} -\sin\phi_0 & -\cos\phi_0 & 0 & l_2\cos(\theta_3+\theta_4) + l_1\cos\theta_3 \\ \cos\phi_0 & -\sin\phi_0 & 0 & l_2\sin(\theta_3+\theta_4) + l_1\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By combining,

$$J_h = \begin{bmatrix} -l_2\cos(\theta_1-\phi_0+\theta_2) - l_1\cos(\phi_0-\theta_1) & -l_2\cos(\theta_1-\phi_0+\theta_2) \\ l_1\sin(\phi_0-\theta_1) - l_2\sin(\theta_1-\phi_0+\theta_2) & -l_2\sin(\theta_1-\phi_0+\theta_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \cdots + l_1\cos(\phi_0-\theta_3) & l_2\cos(\theta_3-\phi_0+\theta_4) \\ \cdots - l_1\sin(\phi_0-\theta_3) & l_2\sin(\theta_3-\phi_0+\theta_4) \end{bmatrix}$$

hand Jacobian

PROBLEM 1.7 :-

The internal motions refer to the joint velocity changes $\dot{\theta}$ that result in $\boxed{\text{no movement}}$ of the contact point when viewed from the perspective of the contact wrench bases

$$\boxed{\dot{\theta} = \begin{bmatrix} \frac{-l_2}{l_1 + l_2} \\ 1 \\ 0 \\ 0 \end{bmatrix} \omega}$$

PROBLEM 1.8 :-

Here while there are no internal movements in typical configurations, there can be degree of freedom associated with the internal forces. This means the robot has the capability to maneuver the object within a plane.