HOMEWORK-1: FUNDAMENTALS
ROBOT DYNAMICS & ANALYSIS

Problem (1) - Functions:-

(i) Tan:

→ Domain:- All real numbers except those in form of $\pi(n+\frac{1}{2})$

→ Image: $\mathbb{R}$

→ Differentiability class: Not C^∞, because it's not defined at $\pi(n+\frac{1}{2})$

→ It's onto, but it can also be one-to-one when the domain restricted within π

(ii) arctan:

→ Domain: $\mathbb{R}$

→ Image: $(-\frac{\pi}{2}, \frac{\pi}{2})$

→ Differentiability class: C^∞ on its domain

→ ~~one to one:~~

→ It is one to one

(iii) abs:

→ Domain: $\mathbb{R}$

→ Image: $[0, \infty)$

→ Differentiability class: It is not C^∞; abs $\in C^\infty$ in $\mathbb{R}-\{0\}$ → $|x|$ ↻ $x>0$ or $x<0$

→ It is neither one-to-one nor onto.

(iv) ln :

→ Domain : $(0, \infty)$

→ Image : $\mathbb{R}$

→ Differentiability class : C^∞

→ It is both onto and one to one

(V) floor :

→ Domain : $\mathbb{R}$

→ Image : $\mathbb{Z}$, all Integers

→ Differentiability class :- not C^∞ , when Domain $= \mathbb{R} - \mathbb{Z}$.

→ It is onto

(vi) $f(x) = \begin{cases} x^2 , & x \geq 0 \\ -x^2 , & x < 0 \end{cases}$

→ Domain → $\mathbb{R}$

→ Image → $\mathbb{R}$

→ differentiability class → not C^∞, because can't differentiate in 0. $[C^1]$

→ It is both onto & one to one

(vii) $g(x, y) = x^2 + xy$

→ Domain : $\mathbb{R}^2$

→ Image : $\mathbb{R}$

→ Differentiability class : C^∞

→ It is onto

(viii) $f(g(x,y)) =$

$\Downarrow$

$f \circ g$

→ Domain : $\mathbb{R}^2$

→ Image : $\mathbb{R}$

→ Differentiability class :

→ It is onto

Differentiability class:- If f & g are C^∞, then its C^∞

→ For one to one:- If $f(g(x_1, y_1))$ is different from $f(g(x_2, y_2))$
Then its one to one

→ For onto:- If ~~Real nos.~~ $(x, y) \in$ Real nos
$f(g(x,y)) \in$ Real nos, then its onto

Problem (2)

2.1 Given contraints. $C(x(t), y(t)) := x^2+y^2-r^2 = 0$ & $r=1$,
~~Derivat~~ $D_y C(x,y) = 2y$
which is, $\Rightarrow \pm 2\sqrt{1-x^2}$
~~This D_yC~~ $D_yC(x,y)$ is invertible for all x values except when $y=0$, Thus. $x \in (-1, 1)$ or $x = \pm 1$

2.2 eqn $\Rightarrow x^2+y^2=1$, can be expressed in 2 ways.

$\sqrt{1-x^2}$ $(g_1(x))$ $\qquad$ $-\sqrt{1-x^2}$ $(g_2(x))$

For a point x_0 is $g_1(x_0)$ & $g_2(x_0)$

$$g_1(x_0) \Downarrow \sqrt{1-x_0^2} \qquad g_2(x_0) \Downarrow -\sqrt{1-x_0^2}$$

So if x_0 is 0 the point is $[0,1]$.
The largest open set V_i for which $C(x, g_2(x)) = 0$ is
also $-1 \leq x \leq 1$

2.3 This is similar to 2.2, we now express x in 'y' terms.

$$x^2+y^2=1$$

$$C(g_1(y), y)=0$$

$$\Rightarrow g_1^2(y)+y^2-1=0$$

$$\Rightarrow g_1(y) = \pm\sqrt{1-y^2}.$$

$$\left(g_3(y)\right) = \sqrt{1-y^2} \qquad \left(g_4(y)\right) = -\sqrt{1-y^2}$$

Considering initial point $y_0=0$, the point is $[1,0]$

The largest open set V_3 or V_4 for both fns

$$-1 \leq y \leq 1$$

2.4 $x^2+y^2-1=0$

differenting wrt 't',

$$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow 2x\,dx + 2y\,dy = 0$$

$$\Rightarrow x\,dx + y\,dy = 0$$

$$\Rightarrow \underline{x\dot{x}+y\dot{y}=0} \Leftarrow$$ This is constraint on $\dot{x}$ & $\dot{y}$

2.5 To give relationship b/w velocities for each co-ords:

$$\dot{y} = \frac{dy}{dx}\dot{x}$$

For $g_1(x) \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$ & $\dot{y} = \frac{-x\dot{x}}{\sqrt{1-x^2}}$

$g_2(x) \Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{1-x^2}} \Rightarrow \frac{x\dot{x}}{\sqrt{1-x^2}}$

$$g_3(y) \Rightarrow \frac{dx}{dy} = \frac{-y}{\sqrt{1-y^2}} \Rightarrow \dot{x} = \frac{-y\dot{y}}{\sqrt{1-y^2}}$$

$$g_4(y) \Rightarrow \frac{dy}{dx} = \frac{y}{\sqrt{1-y^2}} \Rightarrow \dot{x} = \frac{y\dot{y}}{\sqrt{1-y^2}}$$

(3) Problem

(3.1) Integration of eqn of motion :-

given
$$\begin{cases} x(0)=0 \\ y(0)=1 \\ \dot{x}(0)=v\cos\phi \\ \dot{y}(0)=v\sin\phi \\ \ddot{x}(t)=0 \\ \ddot{y}(t)=-g \end{cases}$$

For along x-axis;

$\ddot{x}(t)=0$

$\dot{x}(0)=v\cos\phi$

$\Rightarrow \dot{x}(t) = v\cos(\phi) + \cancel{c_1} \to 0$

$\dot{x}(t) = v\cos(\phi)$

Now, $x(t) = v\cos(\phi)\, t$

For along y axis;

$\ddot{y}(t) = -g$

$\dot{y}(0) = v\sin(\phi)$

$\dot{y}(t) = -gt + v\sin(\phi) + \cancel{c_3} \to 0$

$y(t) = \frac{1}{2}gt^2 + v\sin(\phi)t + c_4$ → This is not 0 ($y(0)=1$, $c_4=1$)

Now, $y(t) = -\frac{1}{2}gt^2 + v\sin(\phi)t + 1$

$$\boxed{\begin{array}{l} v = u + at \\ S = ut + \frac{1}{2}at^2 \\ 2as = v^2 - u^2 \end{array}}$$

when the cannon ball hits groud the y should be 0. ($y(t)=0$)

$$0 = \frac{-gt^2}{2} + v\sin\phi\, t + H$$

$$\frac{gt^2}{2} - v\sin\phi\, t \; H = 0$$

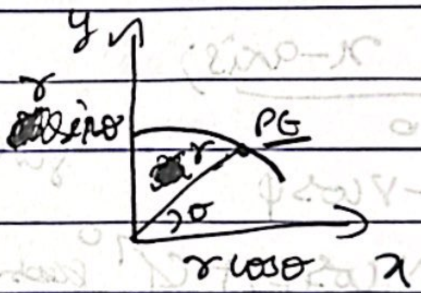
using quadratic eqns, $t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$t \Rightarrow \frac{v\sin(\phi) \pm \sqrt{(v\sin\phi)^2 \cdot 2g}}{g}$$

3.2 Polar coordinates:-

Gn; $x = r\cos\theta$

$y = r\sin\theta$



From Phy theorm,

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

Hence, $r(t) = \sqrt{(v\cos(\phi)t)^2 + \left(-\frac{gt^2}{2} + v\sin(\phi)t + 1\right)^2}$

$$\theta = \tan^{-1}(y/x)$$

$$\theta = \tan^{-1}\left(\frac{-\frac{gt^2}{2} + v\sin(\phi)t + H}{v\cos(\phi)t}\right)$$

(3.3) velocity in Polar coordinates :-

using change of coordinates a direct differential

$$r^2 = (vt)^2 + \left(1 - \frac{1}{2}gt^2\right)^2$$

differential,

$$2r\frac{dr}{dt} = 2(v)(vt) + 2\left(1 - \tfrac{1}{2}gt^2\right)(-gt)$$

$$\dot{r} = \frac{2v^2t + (2-gt^2)(-gt)}{2r}$$

$$\dot{r} = \frac{2v^2t + (2-gt^2)(-gt)}{2\sqrt{(vt)^2 + (1-\frac{1}{2}gt^2)^2}}$$

now for $\dot{\theta} = \dfrac{1}{1+\left(\frac{1-gt^2}{vt}\right)^2} \times \dfrac{(vt)(-gt) - (v)\left(1-\frac{gt^2}{2}\right)}{(vt)^2}$

$$= \frac{-gt(vt) - v\left(1-\frac{gt^2}{2}\right)}{\sqrt{\left(vt^2 + 1 - \frac{gt^2}{2}\right)}}$$

using change of coordinates;

v of $\dot{x} = \dot{r}\cos(\theta) - r\dot{\theta}\sin(\theta)$ $\times \cos\theta$

$-gt$ of $\dot{y} = \dot{r}\sin(\theta) + r\dot{\theta}\cos(\theta)$ $\times \sin\theta$

solving eqns;

$\Rightarrow v\cos\theta = \dot{r}\cos^2\theta - r\dot{\theta}\cos\theta\sin\theta$

$-gt\sin\theta = \dot{r}\sin^2\theta + r\dot{\theta}\cos\theta\sin\theta$

$v\cos\theta - gt\sin\theta = \dot{r}(1)$

putting r, we get

$$\dot{\theta} = \cancel{\ } -\frac{(v\sin\theta + gt\cos\theta)}{r}$$

~~Now, $\dot{\theta} = \frac{gt(vt}{r}$~~

Now, $$\dot{\theta} = \frac{gt\,vt - v(1-gt^2/2)}{r}$$

$$\dot{\theta} = \frac{gt(vt) - v(1-gt^2/2)}{\sqrt{(vt)^2 + (1-\frac{gt^2}{2})}}$$

$\hat{\theta}$ Sinew

Now, take $r = v\cos\theta - gt\sin\theta$

$$\dot{r} = v\left(\frac{vt}{r}\right) - gt\left(\frac{1-gt^2/2}{r}\right)$$

$$\cancel{vt - gt(1-gt^2/2)}$$

$$\dot{r} \Rightarrow \frac{v^2t - gt + g^2t^3}{2\sqrt{(vt)^2 + (1-\frac{gt^2}{2})^2}}$$

Therefore, they ~~[illegible]~~ are same as the differential ones