Proposal:
Tools for Quantum State Distinguishability and Exclusion

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Overview

This is a project proposal for Google Summer of Code 2025, enhancing the |toqito> Python library that provides various tools for quantum information theory. Currently, it possesses modules dedicated to specific aspects of quantum state distinguishability (QSD) (ppt_distinguishability.py, state_distinguishability.py, is_distinguishable.py) and quantum state exclusion (QSE) (is_antidistinguishable.py, state_exclusion.py).

A key aspect is determining whether a quantum state is separable (non-entangled) or entangled, the Positive Partial Transpose (PPT) criterion plays a crucial role in various checks in is_separable.py module. A part of this project involves using polynomial-based checks for separability in cases where the density matrix ρ satisfies $r(\rho)+r(\rho^{TA})\leq 2MN-M-N+2$, which is computationally challenging for higher ordered systems.

Background and Skills

I have bachelor's degree in Electronics and Communication and have knowledge in Quantum Technologies. I have knowledge of Quantum Information Theory (Basics) and linear algebra. I worked as an Embedded developer so I have experience in software development in Python and C languages.

Proposals

1. Enhance Separability Checks for Separable Measurements
2. Incorporate Practical Polynomial-Based Check
3. Develop a General Ensemble Module to return figure-of-merit.

Approach and Plan

1. Enhance Separability Checks for Separable Measurements: The existing is_separable.py module lacks certain checks (present in established tools like QETLAB) so extend the current module to incorporate additional tests based on modern criteria (referencing QETLAB).

2. This involves setting up and solving the system of polynomial equations for separability in systems where $r(\rho)+r(\rho^{T_A})\leq 2MN-M-N+2$ holds for small systems (e.g., 3×3). Use external libraries such as Sympy (with Gröbner basis computation) or other external packages to symbolically reduce systems and solve the system equations.
3. Develop a General Ensemble Analysis Module: Create a new module that accepts a quantum ensemble (states rho_i, probabilities p_i) and calculates key figures-of-merit for both QSD (e.g., minimum-error probability, unambiguous discrimination probability) and QSE using SDP solvers (cvxpy, picos). This module should also aim to provide constructive decompositions for states identified as separable, potentially drawing on methods outlined for specific cases.

Plan

1. Phase1: (3-4weeks)
   Conduct literature review and understand existing project architecture and codebase and follow the established conventions.
2. Phase2: (6-7 weeks)
   Implementation of Core Functionality of all the approaches stated above.
3. Phase3: (2-3 weeks)
   Comprehensive Documentation of the functionalities.

Conclusion

While the full general solution for systems of coupled high-order polynomial equations is computationally challenging, this project proposal focuses on practical implementable methods for low-dimensional cases (such as 3×3 systems).